

Having a Successful Math Conversation

Intuition Behind Analysis

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Outline

- Why Language?

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- Why Language?
- Words and Emotions

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- Conversations and Information

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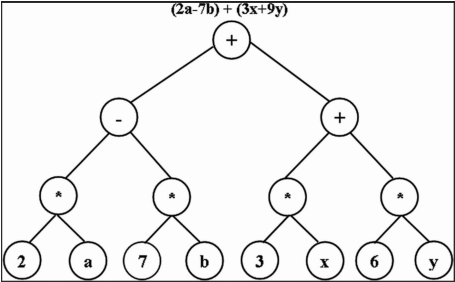
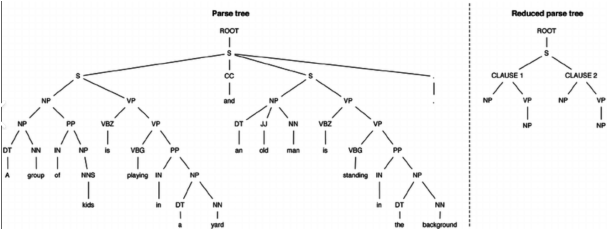
- Why Language?
- Words and Emotions
- Conversations and Information
- Is It Good?

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- Why Language?
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- Conversations and Information
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- Future Work

{Natural, Programming, Math} Language

- Formalism (Syntax)



{Natural, Programming, Math} Language

- Formalism (Syntax)
- Logic

{Natural, Programming, Math} Language

- Formalism (Syntax)
- Logic
- Psychologism

Words and Emotions

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- "Good" proper nouns $\mathbb{N} = \{1, 2, 3, \dots\}$

Conversations

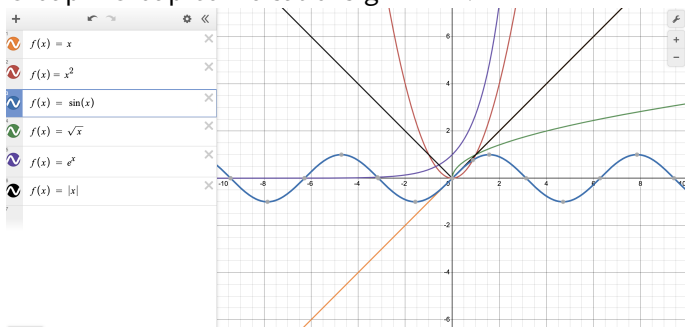
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- Group - Group conversations $g : \mathbb{R}^n \rightarrow \mathbb{R}^n$



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- Examples of conversations: sequences, norms, etc.

Limits and Continuity

- Emotional Distances (Metric)

Limits and Continuity

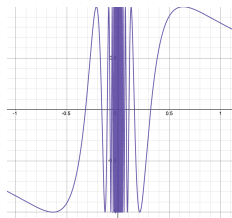
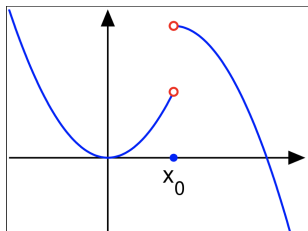
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Limits and Continuity

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- Three Important Theorems
- Uniform Continuity

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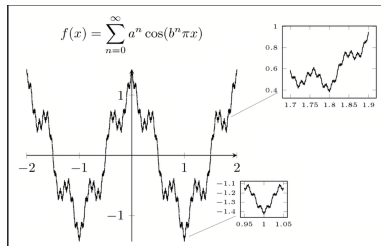
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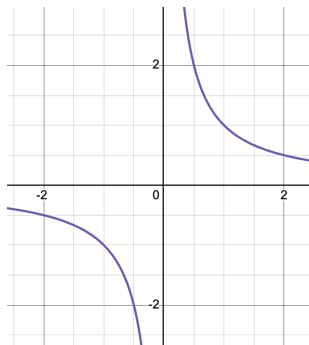


Differentiability

- Increasing/Decreasing, Convex/Concave
- Critical Points
- Mean Value Theorem: $f'(x) = \frac{f(b)-f(a)}{b-a}$
- Differentiable \Rightarrow Continuous

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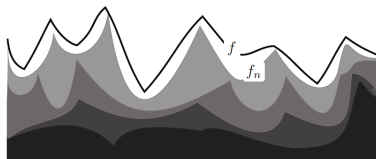
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- Convergence / Improper Integrals

Analyzing Multiple Conversations

- Uniform Convergence

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- Monotone Convergence Theorem



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- Highly Immature
- Infinity is not intuitive enough.
- Difficult to incorporate Topology
- Hard for computational results and theorems.

Future of This Project

- Supplement to other analysis materials?
- Write something?
- xiaohoward.github.io/web

