In[ ]:=
```
Clear["Global`*"];
```

0. Readme

1. The below derivations of the Vx and Vy of a tesseroid are based on Eqs. (A.2) and (A.3) of Deng and Shen (2019) SGG, where λ3, θ3, and r3 represent λ', θ', and r' of the integration point.
2. VxSphericalZonalBand and VySphericalZonalBand are the Vx and Vy of a spherical zonal band, which are equal to zero.
3. VxSphericalShell and VySphericalShell are the Vx and Vy of a spherical shell, which are equal to zero.

In[ ]:=
```
φ = Pi / 2;(*φ=Pi/2-θ, θ=0*)
```

In[ ]:=
```
φ3 = Pi / 2 - θ3;
```

In[ ]:=
```
cosPhi = Sin[φ] * Sin[φ3] + Cos[φ] * Cos[φ3] * Cos[λ3 - λ]
```

Out[ ]=
```
Cos[θ3]
```

In[ ]:=
```
Δx = r3 * (Cos[φ] * Sin[φ3] - Sin[φ] * Cos[φ3] * Cos[λ3 - λ])
```

Out[ ]=
```
-r3 Cos[λ - λ3] Sin[θ3]
```

In[ ]:=
```
Δy = r3 * Cos[φ3] * Sin[λ3 - λ]
```

Out[ ]=
```
-r3 Sin[θ3] Sin[λ - λ3]
```

In[ ]:=
```
Δz = r3 * cosPhi - r
```

Out[ ]=
```
-r + r3 Cos[θ3]
```

1. Vx

In[ ]:=
```
FullSimplify[∫ r3^2 Sin[θ3] (-r3 Cos[λ - λ3] Sin[θ3] / (Sqrt[r^2 + r3^2 - 2 * r * r3 * Cos[θ3]])^3) dλ3]
```

Out[ ]=
$$\frac{r3^3 \sin[\theta 3]^2 \sin[\lambda - \lambda 3]}{\left(r^2 + r3^2 - 2\, r\, r3 \cos[\theta 3]\right)^{3/2}}$$

In[ ]:=
```
Vxλ3[λ3_] := r3^3 Sin[θ3]^2 Sin[λ - λ3] / (r^2 + r3^2 - 2 r r3 Cos[θ3])^(3/2)
```

In[ ]:=
```
FullSimplify[Vxλ3[λ2] - Vxλ3[λ1]]
```

Out[ ]=
$$\frac{r3^3 \sin[\theta 3]^2 \left(-\sin[\lambda - \lambda 1] + \sin[\lambda - \lambda 2]\right)}{\left(r^2 + r3^2 - 2\, r\, r3 \cos[\theta 3]\right)^{3/2}}$$

In[ ]:=
```
FullSimplify[∫(r3^3 Sin[θ3]^2 (-Sin[λ-λ1]+Sin[λ-λ2]))/(r^2+r3^2-2 r r3 Cos[θ3])^(3/2) dr3]
```

Out[ ]=

$$\left(3\,r\,\text{ArcTanh}\left[\frac{r3-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r3^2-2\,r\,r3\,\text{Cos}[\theta 3]}}\right]\text{Cos}[\theta 3]+\left(\left(r^2+r3^2-r\,r3\,\text{Cos}[\theta 3]-\left(3\,r^2+r3^2\right)\text{Cos}[2\,\theta 3]+3\,r\,r3\,\text{Cos}[3\,\theta 3]\right)\text{Csc}[\theta 3]^2\right)\Big/\left(2\sqrt{r^2+r3^2-2\,r\,r3\,\text{Cos}[\theta 3]}\right)\right)\text{Sin}[\theta 3]^2\,(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2])$$

In[ ]:=
```
Vxr3[r3_] := (3 r ArcTanh[(r3 - r Cos[θ3])/Sqrt[r^2 + r3^2 - 2 r r3 Cos[θ3]]] Cos[θ3] +
   ((r^2 + r3^2 - r r3 Cos[θ3] - (3 r^2 + r3^2) Cos[2 θ3] + 3 r r3 Cos[3 θ3]) Csc[θ3]^2)/
    (2 Sqrt[r^2 + r3^2 - 2 r r3 Cos[θ3]])) Sin[θ3]^2 (-Sin[λ - λ1] + Sin[λ - λ2])
```

In[ ]:=
```
Vxr3[r2] - Vxr3[r1]
```

Out[ ]=

$$-\left(\left(3\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\right]\text{Cos}[\theta 3]+\frac{\left(r^2+r1^2-r\,r1\,\text{Cos}[\theta 3]-\left(3\,r^2+r1^2\right)\text{Cos}[2\,\theta 3]+3\,r\,r1\,\text{Cos}[3\,\theta 3]\right)\text{Csc}[\theta 3]^2}{2\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\right)\text{Sin}[\theta 3]^2\,(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2])\right)+$$

$$\left(3\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\right]\text{Cos}[\theta 3]+\frac{\left(r^2+r2^2-r\,r2\,\text{Cos}[\theta 3]-\left(3\,r^2+r2^2\right)\text{Cos}[2\,\theta 3]+3\,r\,r2\,\text{Cos}[3\,\theta 3]\right)\text{Csc}[\theta 3]^2}{2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\right)\text{Sin}[\theta 3]^2\,(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2])$$

In[ ]:=
$$\int\Bigg(-\Bigg(\Bigg(3\,r\,\text{ArcTanh}\Big[\frac{r1-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\Big]\,\text{Cos}[\theta 3]+\frac{\left(r^2+r1^2-r\,r1\,\text{Cos}[\theta 3]-\left(3\,r^2+r1^2\right)\text{Cos}[2\,\theta 3]+3\,r\,r1\,\text{Cos}[3\,\theta 3]\right)\text{Csc}[\theta 3]^2}{2\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\Bigg)\,\text{Sin}[\theta 3]^2\,(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2])\Bigg)+\Bigg(3\,r\,\text{ArcTanh}\Big[\frac{r2-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\Big]\,\text{Cos}[\theta 3]+\frac{\left(r^2+r2^2-r\,r2\,\text{Cos}[\theta 3]-\left(3\,r^2+r2^2\right)\text{Cos}[2\,\theta 3]+3\,r\,r2\,\text{Cos}[3\,\theta 3]\right)\text{Csc}[\theta 3]^2}{2\,\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\Bigg)\,\text{Sin}[\theta 3]^2\,(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2])\Bigg)\,\text{d}\theta 3$$

Out[ ]=

$$\frac{1}{6}\left(-\frac{4\,r1^2\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}{(r-r1)^2}}\;\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}-\right.$$

$$\left(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}{(r-r1)^2}}\right.$$

$$\left.\left.\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\right)\Big/$$

$$\left(r^2\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}\right)+\frac{4\,r2^2\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}{(r-r2)^2}}\;\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}+$$

$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}{(r-r2)^2}}\right.\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\right.$$

$$\left.\left.\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 3])\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}\;\text{Sin}[\theta 3]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 3])\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}\;\text{Sin}[\theta 3]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\right]\text{Sin}[\theta 3]^3+$$

$$\left.6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\right]\text{Sin}[\theta 3]^3\right)$$

$$(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2])$$

In[•]:=
$$\text{Vx}\theta 3[\theta 3\_] := \frac{1}{6}\left(-\frac{4\,r1^2\,\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}{(r-r1)^2}}\;\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}-\right.$$
$$\left(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}{(r-r1)^2}}\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\right.\right.$$
$$\left.\left.\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\right)\Big/\left(r^2\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}\right)+$$
$$\frac{4\,r2^2\,\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}{(r-r2)^2}}\;\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}+$$
$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}{(r-r2)^2}}\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\right.\right.$$
$$\left.\left.\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\,\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}\right)+$$
$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 3])\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}\;\text{Sin}[\theta 3]}{r}-$$
$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 3])\,\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}\;\text{Sin}[\theta 3]}{r}-$$
$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\right]\text{Sin}[\theta 3]^3+$$
$$\left.6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\right]\text{Sin}[\theta 3]^3\right)(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2])$$

In[•]:=
$$\text{G}*\text{rho}*(\text{Vx}\theta 3[\theta 2]-\text{Vx}\theta 3[\theta 1])$$

Out[•]=
$$\text{G rho}\left(-\frac{1}{6}\left(-\frac{4\,r1^2\,\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}{(r-r1)^2}}\;\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}}-\right.\right.$$
$$\left(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}{(r-r1)^2}}\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 1}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\right.\right.$$
$$\left.\left.\left.\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\right)\Big/\left(r^2\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}\right)+$$

$$\frac{4\,r2^2\,\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}{(r-r2)^2}}\;\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}}+$$

$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}{(r-r2)^2}}\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 1])\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}\;\text{Sin}[\theta 1]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 1])\,\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}\;\text{Sin}[\theta 1]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 1]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}}\right]\text{Sin}[\theta 1]^3+$$

$$6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 1]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}}\right]\text{Sin}[\theta 1]^3\Bigg)\,(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2])+$$

$$\frac{1}{6}\Bigg(-\frac{4\,r1^2\,\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}{(r-r1)^2}}\;\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}}-$$

$$\left(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}{(r-r1)^2}}\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}\right)+$$

$$\frac{4\,r2^2\,\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}{(r-r2)^2}}\;\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}}+$$

$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}{(r-r2)^2}}\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 2])\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}\;\text{Sin}[\theta 2]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 2])\,\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}\;\text{Sin}[\theta 2]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 2]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}}\right]\text{Sin}[\theta 2]^3+$$

$$6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 2]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}}\right]\text{Sin}[\theta 2]^3\Bigg)\Bigg)\left(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2]\right)$$

In[•]:=
```
Vx[λ1_, λ2_, θ1_, θ2_] :=
 G rho (-1/6 (-(4 r1^2 Sqrt[(r^2+r1^2-2 r r1 Cos[θ1])/(r-r1)^2] EllipticF[θ1/2, -(4 r r1)/(r-r1)^2])/Sqrt[r^2+r1^2-2 r r1 Cos[θ1]] -
    (2 (2 r^2-r1^2) Sqrt[(r^2+r1^2-2 r r1 Cos[θ1])/(r-r1)^2] ((r-r1)^2 EllipticE[θ1/2, -(4 r r1)/(r-r1)^2] -
       (r^2+r1^2) EllipticF[θ1/2, -(4 r r1)/(r-r1)^2]))/(r^2 Sqrt[r^2+r1^2-2 r r1 Cos[θ1]]) +
    (4 r2^2 Sqrt[(r^2+r2^2-2 r r2 Cos[θ1])/(r-r2)^2] EllipticF[θ1/2, -(4 r r2)/(r-r2)^2])/Sqrt[r^2+r2^2-2 r r2 Cos[θ1]] +
    (2 (2 r^2-r2^2) Sqrt[(r^2+r2^2-2 r r2 Cos[θ1])/(r-r2)^2] ((r-r2)^2 EllipticE[θ1/2, -(4 r r2)/(r-r2)^2] -
       (r^2+r2^2) EllipticF[θ1/2, -(4 r r2)/(r-r2)^2]))/(r^2 Sqrt[r^2+r2^2-2 r r2 Cos[θ1]]) +
    (2 (r1+3 r Cos[θ1]) Sqrt[r^2+r1^2-2 r r1 Cos[θ1]] Sin[θ1])/r -
    (2 (r2+3 r Cos[θ1]) Sqrt[r^2+r2^2-2 r r2 Cos[θ1]] Sin[θ1])/r -
    6 r ArcTanh[(r1-r Cos[θ1])/Sqrt[r^2+r1^2-2 r r1 Cos[θ1]]] Sin[θ1]^3 +
    6 r ArcTanh[(r2-r Cos[θ1])/Sqrt[r^2+r2^2-2 r r2 Cos[θ1]]] Sin[θ1]^3) (-Sin[λ-λ1]+Sin[λ-λ2]) +
 1/6 (-(4 r1^2 Sqrt[(r^2+r1^2-2 r r1 Cos[θ2])/(r-r1)^2] EllipticF[θ2/2, -(4 r r1)/(r-r1)^2])/Sqrt[r^2+r1^2-2 r r1 Cos[θ2]] -
```

$$\left(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}{(r-r1)^2}}\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}\right)+$$

$$\frac{4\,r2^2\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}{(r-r2)^2}}\,\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}}+$$

$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}{(r-r2)^2}}\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 2])\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}\,\text{Sin}[\theta 2]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 2])\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}\,\text{Sin}[\theta 2]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 2]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}}\right]\text{Sin}[\theta 2]^3+$$

$$6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 2]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}}\right]\text{Sin}[\theta 2]^3\Bigg)\left(-\text{Sin}[\lambda-\lambda 1]+\text{Sin}[\lambda-\lambda 2]\right)\Bigg)$$

In[•]:= `VxSphericalZonalBand = Vx[0, 2 * Pi, θ1, θ2]`

Out[•]=

$$0$$

In[•]:= `VxSphericalShell = Vx[0, 2 * Pi, 0, Pi]`

Out[•]=

$$0$$

2. Vy

In[•]:= `FullSimplify[∫ r3^2 Sin[θ3] (-r3 Sin[θ3] Sin[λ - λ3] / (√(r^2 + r3^2 - 2 * r * r3 * Cos[θ3]))^3) dλ3]`

Out[•]=

$$-\frac{r3^3\,\text{Cos}[\lambda-\lambda 3]\,\text{Sin}[\theta 3]^2}{\left(r^2+r3^2-2\,r\,r3\,\text{Cos}[\theta 3]\right)^{3/2}}$$

In[•]:= `Vyλ3[λ3_] := -r3^3 Cos[λ - λ3] Sin[θ3]^2 / (r^2 + r3^2 - 2 r r3 Cos[θ3])^(3/2)`

In[•]:=
```
FullSimplify[Vyλ3[λ2] - Vyλ3[λ1]]
```

Out[•]=

$$\frac{r3^3\ (\text{Cos}[\lambda-\lambda1]-\text{Cos}[\lambda-\lambda2])\ \text{Sin}[\theta3]^2}{\left(r^2+r3^2-2\ r\ r3\ \text{Cos}[\theta3]\right)^{3/2}}$$

In[•]:=
```
FullSimplify[∫(r3^3 (Cos[λ - λ1] - Cos[λ - λ2]) Sin[θ3]^2)/(r^2 + r3^2 - 2 r r3 Cos[θ3])^(3/2) dr3]
```

Out[•]=

$$(\text{Cos}[\lambda-\lambda1]-\text{Cos}[\lambda-\lambda2])\left(3\ r\ \text{ArcTanh}\left[\frac{r3-r\ \text{Cos}[\theta3]}{\sqrt{r^2+r3^2-2\ r\ r3\ \text{Cos}[\theta3]}}\right]\text{Cos}[\theta3]+\frac{\left(r^2+r3^2-r\ r3\ \text{Cos}[\theta3]-\left(3\ r^2+r3^2\right)\text{Cos}[2\ \theta3]+3\ r\ r3\ \text{Cos}[3\ \theta3]\right)\text{Csc}[\theta3]^2}{2\ \sqrt{r^2+r3^2-2\ r\ r3\ \text{Cos}[\theta3]}}\right)\text{Sin}[\theta3]^2$$

In[•]:=
```
Vyr3[r3_] :=
 (Cos[λ - λ1] - Cos[λ - λ2]) (3 r ArcTanh[(r3 - r Cos[θ3])/Sqrt[r^2 + r3^2 - 2 r r3 Cos[θ3]]] Cos[θ3] +
   ((r^2 + r3^2 - r r3 Cos[θ3] - (3 r^2 + r3^2) Cos[2 θ3] + 3 r r3 Cos[3 θ3]) Csc[θ3]^2)/(2 Sqrt[r^2 + r3^2 - 2 r r3 Cos[θ3]])) Sin[θ3]^2
```

In[•]:=
```
Vyr3[r2] - Vyr3[r1]
```

Out[•]=

$$-\left((\text{Cos}[\lambda-\lambda1]-\text{Cos}[\lambda-\lambda2])\left(3\ r\ \text{ArcTanh}\left[\frac{r1-r\ \text{Cos}[\theta3]}{\sqrt{r^2+r1^2-2\ r\ r1\ \text{Cos}[\theta3]}}\right]\text{Cos}[\theta3]+\frac{\left(r^2+r1^2-r\ r1\ \text{Cos}[\theta3]-\left(3\ r^2+r1^2\right)\text{Cos}[2\ \theta3]+3\ r\ r1\ \text{Cos}[3\ \theta3]\right)\text{Csc}[\theta3]^2}{2\ \sqrt{r^2+r1^2-2\ r\ r1\ \text{Cos}[\theta3]}}\right)\text{Sin}[\theta3]^2\right)+(\text{Cos}[\lambda-\lambda1]-\text{Cos}[\lambda-\lambda2])\left(3\ r\ \text{ArcTanh}\left[\frac{r2-r\ \text{Cos}[\theta3]}{\sqrt{r^2+r2^2-2\ r\ r2\ \text{Cos}[\theta3]}}\right]\text{Cos}[\theta3]+\frac{\left(r^2+r2^2-r\ r2\ \text{Cos}[\theta3]-\left(3\ r^2+r2^2\right)\text{Cos}[2\ \theta3]+3\ r\ r2\ \text{Cos}[3\ \theta3]\right)\text{Csc}[\theta3]^2}{2\ \sqrt{r^2+r2^2-2\ r\ r2\ \text{Cos}[\theta3]}}\right)\text{Sin}[\theta3]^2$$

In[•]:=
$$\int\left(-\left((\text{Cos}[\lambda-\lambda 1]-\text{Cos}[\lambda-\lambda 2])\left(3\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\right]\text{Cos}[\theta 3]+\frac{\left(r^2+r1^2-r\,r1\,\text{Cos}[\theta 3]-\left(3\,r^2+r1^2\right)\text{Cos}[2\,\theta 3]+3\,r\,r1\,\text{Cos}[3\,\theta 3]\right)\text{Csc}[\theta 3]^2}{2\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\right)\text{Sin}[\theta 3]^2\right)+(\text{Cos}[\lambda-\lambda 1]-\text{Cos}[\lambda-\lambda 2])\left(3\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\right]\text{Cos}[\theta 3]+\frac{\left(r^2+r2^2-r\,r2\,\text{Cos}[\theta 3]-\left(3\,r^2+r2^2\right)\text{Cos}[2\,\theta 3]+3\,r\,r2\,\text{Cos}[3\,\theta 3]\right)\text{Csc}[\theta 3]^2}{2\,\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\right)\text{Sin}[\theta 3]^2\right)\text{d}\theta 3$$

Out[ ]=

$$\frac{1}{6}\left(\text{Cos}[\lambda-\lambda 1]-\text{Cos}[\lambda-\lambda 2]\right)\left(-\frac{4\,r1^2\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}{(r-r1)^2}}\;\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}-\right.$$

$$\left(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}{(r-r1)^2}}\right.$$

$$\left.\left.\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\right)\Big/$$

$$\left(r^2\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}\right)+\frac{4\,r2^2\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}{(r-r2)^2}}\;\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}+$$

$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}{(r-r2)^2}}\right.\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\right.$$

$$\left.\left.\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 3}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 3])\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}\;\text{Sin}[\theta 3]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 3])\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}\;\text{Sin}[\theta 3]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 3]}}\right]\text{Sin}[\theta 3]^3+$$

$$\left.6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 3]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 3]}}\right]\text{Sin}[\theta 3]^3\right)$$

In[•]:=
```
Vyθ3[θ3_] :=
 1/6 (Cos[λ - λ1] - Cos[λ - λ2]) (-(4 r1^2 Sqrt[(r^2 + r1^2 - 2 r r1 Cos[θ3])/(r - r1)^2] EllipticF[θ3/2, -(4 r r1)/(r - r1)^2])/Sqrt[r^2 + r1^2 - 2 r r1 Cos[θ3]] -
 (2 (2 r^2 - r1^2) Sqrt[(r^2 + r1^2 - 2 r r1 Cos[θ3])/(r - r1)^2] ((r - r1)^2 EllipticE[θ3/2, -(4 r r1)/(r - r1)^2] -
 (r^2 + r1^2) EllipticF[θ3/2, -(4 r r1)/(r - r1)^2]))/(r^2 Sqrt[r^2 + r1^2 - 2 r r1 Cos[θ3]]) +
 (4 r2^2 Sqrt[(r^2 + r2^2 - 2 r r2 Cos[θ3])/(r - r2)^2] EllipticF[θ3/2, -(4 r r2)/(r - r2)^2])/Sqrt[r^2 + r2^2 - 2 r r2 Cos[θ3]] +
 (2 (2 r^2 - r2^2) Sqrt[(r^2 + r2^2 - 2 r r2 Cos[θ3])/(r - r2)^2] ((r - r2)^2 EllipticE[θ3/2, -(4 r r2)/(r - r2)^2] -
 (r^2 + r2^2) EllipticF[θ3/2, -(4 r r2)/(r - r2)^2]))/(r^2 Sqrt[r^2 + r2^2 - 2 r r2 Cos[θ3]]) +
 (2 (r1 + 3 r Cos[θ3]) Sqrt[r^2 + r1^2 - 2 r r1 Cos[θ3]] Sin[θ3])/r -
 (2 (r2 + 3 r Cos[θ3]) Sqrt[r^2 + r2^2 - 2 r r2 Cos[θ3]] Sin[θ3])/r -
 6 r ArcTanh[(r1 - r Cos[θ3])/Sqrt[r^2 + r1^2 - 2 r r1 Cos[θ3]]] Sin[θ3]^3 +
 6 r ArcTanh[(r2 - r Cos[θ3])/Sqrt[r^2 + r2^2 - 2 r r2 Cos[θ3]]] Sin[θ3]^3)
```

In[•]:=
```
G * rho * (Vyθ3[θ2] - Vyθ3[θ1])
```

Out[•]=
```
G rho (-1/6 (Cos[λ - λ1] - Cos[λ - λ2]) (-(4 r1^2 Sqrt[(r^2 + r1^2 - 2 r r1 Cos[θ1])/(r - r1)^2] EllipticF[θ1/2, -(4 r r1)/(r - r1)^2])/Sqrt[r^2 + r1^2 - 2 r r1 Cos[θ1]] -
 (2 (2 r^2 - r1^2) Sqrt[(r^2 + r1^2 - 2 r r1 Cos[θ1])/(r - r1)^2] ((r - r1)^2 EllipticE[θ1/2, -(4 r r1)/(r - r1)^2] -
 (r^2 + r1^2) EllipticF[θ1/2, -(4 r r1)/(r - r1)^2]))/(r^2 Sqrt[r^2 + r1^2 - 2 r r1 Cos[θ1]]) +
```

$$\frac{4\,r2^2\,\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}{(r-r2)^2}}\;\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}}+$$

$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}{(r-r2)^2}}\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 1])\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}\;\text{Sin}[\theta 1]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 1])\,\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}\;\text{Sin}[\theta 1]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 1]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}}\right]\text{Sin}[\theta 1]^3+$$

$$6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 1]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}}\right]\text{Sin}[\theta 1]^3\Bigg)+$$

$$\frac{1}{6}\,(\text{Cos}[\lambda-\lambda 1]-\text{Cos}[\lambda-\lambda 2])\left(-\frac{4\,r1^2\,\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}{(r-r1)^2}}\;\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}}-\right.$$

$$\left(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}{(r-r1)^2}}\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}\right)+$$

$$\frac{4\,r2^2\,\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}{(r-r2)^2}}\;\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}}+$$

$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}{(r-r2)^2}}\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 2])\,\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}\;\text{Sin}[\theta 2]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 2])\,\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}\;\text{Sin}[\theta 2]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 2]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}}\right]\text{Sin}[\theta 2]^3+$$

$$6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 2]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}}\right]\text{Sin}[\theta 2]^3\Bigg)\Bigg)$$

In[•]:= `Vy[λ1_, λ2_, θ1_, θ2_] := G rho`

$$\Bigg(-\frac{1}{6}\left(\text{Cos}[\lambda-\lambda 1]-\text{Cos}[\lambda-\lambda 2]\right)\Bigg(-\frac{4\,r1^2\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}{(r-r1)^2}}\,\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}}-$$

$$\Bigg(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}{(r-r1)^2}}\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 1}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\Bigg)\Big/\left(r^2\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}\right)+$$

$$\frac{4\,r2^2\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}{(r-r2)^2}}\,\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}}+$$

$$\Bigg(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}{(r-r2)^2}}\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 1}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\Bigg)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 1])\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}\,\text{Sin}[\theta 1]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 1])\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}\,\text{Sin}[\theta 1]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 1]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 1]}}\right]\text{Sin}[\theta 1]^3+$$

$$6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 1]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 1]}}\right]\text{Sin}[\theta 1]^3\Bigg)+$$

$$\frac{1}{6}\left(\text{Cos}[\lambda-\lambda 1]-\text{Cos}[\lambda-\lambda 2]\right)\Bigg(-\frac{4\,r1^2\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}{(r-r1)^2}}\,\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}}-$$

$$\left(2\left(2\,r^2-r1^2\right)\sqrt{\frac{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}{(r-r1)^2}}\left((r-r1)^2\,\text{EllipticE}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]-\left(r^2+r1^2\right)\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r1}{(r-r1)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}\right)+$$

$$\frac{4\,r2^2\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}{(r-r2)^2}}\,\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}}+$$

$$\left(2\left(2\,r^2-r2^2\right)\sqrt{\frac{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}{(r-r2)^2}}\left((r-r2)^2\,\text{EllipticE}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]-\left(r^2+r2^2\right)\text{EllipticF}\left[\frac{\theta 2}{2},-\frac{4\,r\,r2}{(r-r2)^2}\right]\right)\right)\Big/\left(r^2\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}\right)+$$

$$\frac{2\,(r1+3\,r\,\text{Cos}[\theta 2])\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}\,\text{Sin}[\theta 2]}{r}-$$

$$\frac{2\,(r2+3\,r\,\text{Cos}[\theta 2])\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}\,\text{Sin}[\theta 2]}{r}-$$

$$6\,r\,\text{ArcTanh}\left[\frac{r1-r\,\text{Cos}[\theta 2]}{\sqrt{r^2+r1^2-2\,r\,r1\,\text{Cos}[\theta 2]}}\right]\text{Sin}[\theta 2]^3+$$

$$6\,r\,\text{ArcTanh}\left[\frac{r2-r\,\text{Cos}[\theta 2]}{\sqrt{r^2+r2^2-2\,r\,r2\,\text{Cos}[\theta 2]}}\right]\text{Sin}[\theta 2]^3\Bigg)\Bigg)$$

In[•]:=
```
VySphericalZonalBand = Vy[0, 2 * Pi, θ1, θ2]
```

Out[•]=

0

In[•]:=
```
VySphericalShell = Vy[0, 2 * Pi, 0, Pi]
```

Out[•]=

0