

```
In[6]:= Clear["Global`*"];
```

## 1. Expressions in different Tables

Table 8

```
In[7]:= A22 = 4 * r2^2 + r^2 * (1 - 3 * Cos[2 * θ2]) - 2 * r * r2 * Cos[θ2];  
In[8]:= A12 = 4 * r1^2 + r^2 * (1 - 3 * Cos[2 * θ2]) - 2 * r * r1 * Cos[θ2];  
In[9]:= A21 = 4 * r2^2 + r^2 * (1 - 3 * Cos[2 * θ1]) - 2 * r * r2 * Cos[θ1];  
In[10]:= A11 = 4 * r1^2 + r^2 * (1 - 3 * Cos[2 * θ1]) - 2 * r * r1 * Cos[θ1];  
In[11]:= B2 = r^3 * (Sin[θ2])^2 * Cos[θ2];  
In[12]:= B1 = r^3 * (Sin[θ1])^2 * Cos[θ1];  
In[13]:= C22 = r2 - r * Cos[θ2];  
In[14]:= C12 = r1 - r * Cos[θ2];  
In[15]:= C21 = r2 - r * Cos[θ1];  
In[16]:= C11 = r1 - r * Cos[θ1];  
In[17]:= l22 = Sqrt[r^2 + r2^2 - 2 * r * r2 * Cos[θ2]];  
In[18]:= l12 = Sqrt[r^2 + r1^2 - 2 * r * r1 * Cos[θ2]];  
In[19]:= l21 = Sqrt[r^2 + r2^2 - 2 * r * r2 * Cos[θ1]];  
In[20]:= l11 = Sqrt[r^2 + r1^2 - 2 * r * r1 * Cos[θ1]];
```

Table 9

```
In[21]:= D22 = r2^2 + r^2 * (3 * (Cos[θ2])^2 - 2) + r * r2 * Cos[θ2];  
In[22]:= D12 = r1^2 + r^2 * (3 * (Cos[θ2])^2 - 2) + r * r1 * Cos[θ2];  
In[23]:= D21 = r2^2 + r^2 * (3 * (Cos[θ1])^2 - 2) + r * r2 * Cos[θ1];  
In[24]:= D11 = r1^2 + r^2 * (3 * (Cos[θ1])^2 - 2) + r * r1 * Cos[θ1];
```

Table 10

```
In[25]:= E22 = r^4 + r^2 * r2^2 + 4 * r2^4 - r * r2 * (r^2 + 4 * r2^2) * Cos[θ2] -  
r^2 * (3 * r^2 + r2^2) * Cos[2 * θ2] + 3 * r^3 * r2 * Cos[3 * θ2];  
In[26]:= E12 = r^4 + r^2 * r1^2 + 4 * r1^4 - r * r1 * (r^2 + 4 * r1^2) * Cos[θ2] -  
r^2 * (3 * r^2 + r1^2) * Cos[2 * θ2] + 3 * r^3 * r1 * Cos[3 * θ2];  
In[27]:= E21 = r^4 + r^2 * r2^2 + 4 * r2^4 - r * r2 * (r^2 + 4 * r2^2) * Cos[θ1] -  
r^2 * (3 * r^2 + r2^2) * Cos[2 * θ1] + 3 * r^3 * r2 * Cos[3 * θ1];
```

```
In[8]:= E11 = r^4 + r^2 * r1^2 + 4 * r1^4 - r * r1 * (r^2 + 4 * r1^2) * Cos[θ1] -
r^2 * (3 * r^2 + r1^2) * Cos[2 * θ1] + 3 * r^3 * r1 * Cos[3 * θ1];
```

**Table 11**

```
In[9]:= F2 = -3 * Cos[θ2] *
(8 * (λ2 - λ1) * (Sin[θ2])^2 + 4 * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1] * (5 - Cos[2 * θ2]));
```

```
In[10]:= F1 = -3 * Cos[θ1] *
(8 * (λ2 - λ1) * (Sin[θ1])^2 + 4 * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1] * (5 - Cos[2 * θ1]));
```

```
In[11]:= H2 = -4 * (r^4 + r^2 * r2^2 + 4 * r2^4) * (λ2 - λ1);
```

```
In[12]:= H1 = -4 * (r^4 + r^2 * r1^2 + 4 * r1^4) * (λ2 - λ1);
```

```
In[13]:= I22 = 2 * r^2 * ((3 * r^2 + r2^2) * Cos[2 * θ2] - 3 * r * r2 * Cos[3 * θ2]) *
(2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]);
```

```
In[14]:= I12 = 2 * r^2 * ((3 * r^2 + r1^2) * Cos[2 * θ2] - 3 * r * r1 * Cos[3 * θ2]) *
(2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]);
```

```
In[15]:= I21 = 2 * r^2 * ((3 * r^2 + r2^2) * Cos[2 * θ1] - 3 * r * r2 * Cos[3 * θ1]) *
(2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]);
```

```
In[16]:= I11 = 2 * r^2 * ((3 * r^2 + r1^2) * Cos[2 * θ1] - 3 * r * r1 * Cos[3 * θ1]) *
(2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]);
```

```
In[17]:= J2 = -4 * (13 * r^4 + 13 * r^2 * r2^2 + 4 * r2^4) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1];
```

```
In[18]:= J1 = -4 * (13 * r^4 + 13 * r^2 * r1^2 + 4 * r1^4) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1];
```

```
In[19]:= K22 = 4 * r * r2 * Cos[θ2] *
((r^2 + 4 * r2^2) * (λ2 - λ1) + (25 * r^2 + 4 * r2^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);
```

```
In[20]:= K12 = 4 * r * r1 * Cos[θ2] *
((r^2 + 4 * r1^2) * (λ2 - λ1) + (25 * r^2 + 4 * r1^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);
```

```
In[21]:= K21 = 4 * r * r2 * Cos[θ1] *
((r^2 + 4 * r2^2) * (λ2 - λ1) + (25 * r^2 + 4 * r2^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);
```

```
In[22]:= K11 = 4 * r * r1 * Cos[θ1] *
((r^2 + 4 * r1^2) * (λ2 - λ1) + (25 * r^2 + 4 * r1^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);
```

**Table 12**

```
In[23]:= L2 = -3 * Cos[θ2] *
(8 * (λ2 - λ1) * (Sin[θ2])^2 - 4 * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1] * (5 - Cos[2 * θ2]));
```

```
In[24]:= L1 = -3 * Cos[θ1] *
(8 * (λ2 - λ1) * (Sin[θ1])^2 - 4 * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1] * (5 - Cos[2 * θ1]));
```

```
In[25]:= M22 = 2 * r^2 * ((3 * r^2 + r2^2) * Cos[2 * θ2] - 3 * r * r2 * Cos[3 * θ2]) *
(2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]);
```

```

In[6]:= M12 = 2 * r^2 * ((3 * r^2 + r1^2) * Cos[2 * θ2] - 3 * r * r1 * Cos[3 * θ2]) *
          (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]) ;

In[7]:= M21 = 2 * r^2 * ((3 * r^2 + r2^2) * Cos[2 * θ1] - 3 * r * r2 * Cos[3 * θ1]) *
          (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]) ;

In[8]:= M11 = 2 * r^2 * ((3 * r^2 + r1^2) * Cos[2 * θ1] - 3 * r * r1 * Cos[3 * θ1]) *
          (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]) ;

In[9]:= N22 = 4 * r * r2 * Cos[θ2] *
          ((r^2 + 4 * r2^2) * (λ2 - λ1) - (25 * r^2 + 4 * r2^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]) ;

In[10]:= N12 = 4 * r * r1 * Cos[θ2] *
           ((r^2 + 4 * r1^2) * (λ2 - λ1) - (25 * r^2 + 4 * r1^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]) ;

In[11]:= N21 = 4 * r * r2 * Cos[θ1] *
           ((r^2 + 4 * r2^2) * (λ2 - λ1) - (25 * r^2 + 4 * r2^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]) ;

In[12]:= N11 = 4 * r * r1 * Cos[θ1] *
           ((r^2 + 4 * r1^2) * (λ2 - λ1) - (25 * r^2 + 4 * r1^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]) ;

```

**Table 13**

```

In[13]:= O22 = r2^3 * (9 * r^2 * r2 + 4 * r2^3 - 4 * r * (r^2 + 3 * r2^2) * Cos[θ2] + 3 * r^2 * r2 * Cos[2 * θ2]) ;

In[14]:= O12 = r1^3 * (9 * r^2 * r1 + 4 * r1^3 - 4 * r * (r^2 + 3 * r1^2) * Cos[θ2] + 3 * r^2 * r1 * Cos[2 * θ2]) ;

In[15]:= O21 = r2^3 * (9 * r^2 * r2 + 4 * r2^3 - 4 * r * (r^2 + 3 * r2^2) * Cos[θ1] + 3 * r^2 * r2 * Cos[2 * θ1]) ;

In[16]:= O11 = r1^3 * (9 * r^2 * r1 + 4 * r1^3 - 4 * r * (r^2 + 3 * r1^2) * Cos[θ1] + 3 * r^2 * r1 * Cos[2 * θ1]) ;

```

**Table 14**

```

In[17]:= P22 = 6 * r^2 * r2^4 * Cos[2 * θ2] * (2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]) ;

In[18]:= P12 = 6 * r^2 * r1^4 * Cos[2 * θ2] * (2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]) ;

In[19]:= P21 = 6 * r^2 * r2^4 * Cos[2 * θ1] * (2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]) ;

In[20]:= P11 = 6 * r^2 * r1^4 * Cos[2 * θ1] * (2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]) ;

In[21]:= U22 = Sqrt[1 - (2 * r * r2 * Cos[θ2]) / (r^2 + r2^2)] ;

In[22]:= U12 = Sqrt[1 - (2 * r * r1 * Cos[θ2]) / (r^2 + r1^2)] ;

In[23]:= U21 = Sqrt[1 - (2 * r * r2 * Cos[θ1]) / (r^2 + r2^2)] ;

In[24]:= U11 = Sqrt[1 - (2 * r * r1 * Cos[θ1]) / (r^2 + r1^2)] ;

```

```

In[8]:= Q22 = -Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1] *
(5*r^6*U22 + r2^6*(5*U22 - 16) + r^2*r2^4*(15*U22 - 52) + r^4*r2^2*(15*U22 - 16));
In[9]:= Q12 = -Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1] *
(5*r^6*U12 + r1^6*(5*U12 - 16) + r^2*r1^4*(15*U12 - 52) + r^4*r1^2*(15*U12 - 16));
In[10]:= Q21 = -Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1] *
(5*r^6*U21 + r2^6*(5*U21 - 16) + r^2*r2^4*(15*U21 - 52) + r^4*r2^2*(15*U21 - 16));
In[11]:= Q11 = -Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1] *
(5*r^6*U11 + r1^6*(5*U11 - 16) + r^2*r1^4*(15*U11 - 52) + r^4*r1^2*(15*U11 - 16));
In[12]:= R22 = 2*r*r2*Cos[θ2] *
((λ2 - λ1)*(5*r^4*U22 + r2^4*(5*U22 - 24) + 2*r^2*r2^2*(5*U22 - 4)) +
(r^2 + r2^2)*(5*r^2*U22 + r2^2*(5*U22 - 24))*Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1]);
In[13]:= R12 = 2*r*r1*Cos[θ2] *
((λ2 - λ1)*(5*r^4*U12 + r1^4*(5*U12 - 24) + 2*r^2*r1^2*(5*U12 - 4)) +
(r^2 + r1^2)*(5*r^2*U12 + r1^2*(5*U12 - 24))*Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1]);
In[14]:= R21 = 2*r*r2*Cos[θ1] *
((λ2 - λ1)*(5*r^4*U21 + r2^4*(5*U21 - 24) + 2*r^2*r2^2*(5*U21 - 4)) +
(r^2 + r2^2)*(5*r^2*U21 + r2^2*(5*U21 - 24))*Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1]);
In[15]:= R11 = 2*r*r1*Cos[θ1] *
((λ2 - λ1)*(5*r^4*U11 + r1^4*(5*U11 - 24) + 2*r^2*r1^2*(5*U11 - 4)) +
(r^2 + r1^2)*(5*r^2*U11 + r1^2*(5*U11 - 24))*Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1]);
In[16]:= S22 = -5*(r^2 + r2^2)^3*(λ2 - λ1)*U22;
In[17]:= S12 = -5*(r^2 + r1^2)^3*(λ2 - λ1)*U12;
In[18]:= S21 = -5*(r^2 + r2^2)^3*(λ2 - λ1)*U21;
In[19]:= S11 = -5*(r^2 + r1^2)^3*(λ2 - λ1)*U11;
In[20]:= T2 = 4*r2^4*(9*r^2 + 4*r2^2)*(λ2 - λ1);
In[21]:= T1 = 4*r1^4*(9*r^2 + 4*r1^2)*(λ2 - λ1);

```

## Table 15

```

In[1]:= V22 = 6*r^2*r2^4*Cos[2*θ2]*(2*λ2 - 2*λ1 - Sin[2*(λ - λ1)] + Sin[2*(λ - λ2)]);
In[2]:= V12 = 6*r^2*r1^4*Cos[2*θ2]*(2*λ2 - 2*λ1 - Sin[2*(λ - λ1)] + Sin[2*(λ - λ2)]);
In[3]:= V21 = 6*r^2*r2^4*Cos[2*θ1]*(2*λ2 - 2*λ1 - Sin[2*(λ - λ1)] + Sin[2*(λ - λ2)]);
In[4]:= V11 = 6*r^2*r1^4*Cos[2*θ1]*(2*λ2 - 2*λ1 - Sin[2*(λ - λ1)] + Sin[2*(λ - λ2)]);
In[5]:= W22 = 2*r*r2*Cos[θ2] *
((λ2 - λ1)*(5*r^4*U22 + r2^4*(5*U22 - 24) + 2*r^2*r2^2*(5*U22 - 4)) -
(r^2 + r2^2)*(5*r^2*U22 + r2^2*(5*U22 - 24))*Cos[2*λ - λ1 - λ2]*Sin[λ2 - λ1]);

```

```
In[8]:= W12 = 2 * r * r1 * Cos[\theta2] *
  ((\lambda2 - \lambda1) * (5 * r^4 * U12 + r1^4 * (5 * U12 - 24) + 2 * r^2 * r1^2 * (5 * U12 - 4)) -
   (r^2 + r1^2) * (5 * r^2 * U12 + r1^2 * (5 * U12 - 24)) * Cos[2 * \lambda - \lambda1 - \lambda2] * Sin[\lambda2 - \lambda1]);
In[9]:= W21 = 2 * r * r2 * Cos[\theta1] *
  ((\lambda2 - \lambda1) * (5 * r^4 * U21 + r2^4 * (5 * U21 - 24) + 2 * r^2 * r2^2 * (5 * U21 - 4)) -
   (r^2 + r2^2) * (5 * r^2 * U21 + r2^2 * (5 * U21 - 24)) * Cos[2 * \lambda - \lambda1 - \lambda2] * Sin[\lambda2 - \lambda1]);
In[10]:= W11 = 2 * r * r1 * Cos[\theta1] *
  ((\lambda2 - \lambda1) * (5 * r^4 * U11 + r1^4 * (5 * U11 - 24) + 2 * r^2 * r1^2 * (5 * U11 - 4)) -
   (r^2 + r1^2) * (5 * r^2 * U11 + r1^2 * (5 * U11 - 24)) * Cos[2 * \lambda - \lambda1 - \lambda2] * Sin[\lambda2 - \lambda1]);
```

## Table 16

```
In[1]:= X22 = 4 * r2^4 * (9 * r^2 + 4 * r2^2 + 3 * r^2 * Cos[2 * \theta2]);
In[2]:= X12 = 4 * r1^4 * (9 * r^2 + 4 * r1^2 + 3 * r^2 * Cos[2 * \theta2]);
In[3]:= X21 = 4 * r2^4 * (9 * r^2 + 4 * r2^2 + 3 * r^2 * Cos[2 * \theta1]);
In[4]:= X11 = 4 * r1^4 * (9 * r^2 + 4 * r1^2 + 3 * r^2 * Cos[2 * \theta1]);
In[5]:= Y22 = 2 * r * r2 * Cos[\theta2] * (5 * r^4 * U22 + r2^4 * (5 * U22 - 24) + 2 * r^2 * r2^2 * (5 * U22 - 4));
In[6]:= Y12 = 2 * r * r1 * Cos[\theta2] * (5 * r^4 * U12 + r1^4 * (5 * U12 - 24) + 2 * r^2 * r1^2 * (5 * U12 - 4));
In[7]:= Y21 = 2 * r * r2 * Cos[\theta1] * (5 * r^4 * U21 + r2^4 * (5 * U21 - 24) + 2 * r^2 * r2^2 * (5 * U21 - 4));
In[8]:= Y11 = 2 * r * r1 * Cos[\theta1] * (5 * r^4 * U11 + r1^4 * (5 * U11 - 24) + 2 * r^2 * r1^2 * (5 * U11 - 4));
In[9]:= Z22 = -5 * (r^2 + r2^2)^3 * U22;
In[10]:= Z12 = -5 * (r^2 + r1^2)^3 * U12;
In[11]:= Z21 = -5 * (r^2 + r2^2)^3 * U21;
In[12]:= Z11 = -5 * (r^2 + r1^2)^3 * U11;
```

## 2. Expressions for the Vzz, Vxx, Vyy, Vzzz, Vxxz, Vyyz of a tesseroid and ZonalBandVzz, ZonalBandVxx, ZonalBandVyy, ZonalBandVzzz, ZonalBandVxxz, ZonalBandVyyz of a spherical zonal band

$$\text{In}[\circ]:= \mathbf{Vzz} = \frac{\mathbf{G} * \rho * (\lambda 2 - \lambda 1)}{6 * r^3} * \left( \frac{\mathbf{E22}}{\mathbf{l22}} - \frac{\mathbf{E12}}{\mathbf{l12}} - \frac{\mathbf{E21}}{\mathbf{l21}} + \frac{\mathbf{E11}}{\mathbf{l11}} + 6 * \mathbf{B2} * \text{Log}\left[\frac{\mathbf{C22} + \mathbf{l22}}{\mathbf{C12} + \mathbf{l12}}\right] + 6 * \mathbf{B1} * \text{Log}\left[\frac{\mathbf{C11} + \mathbf{l11}}{\mathbf{C21} + \mathbf{l21}}\right] \right) (*\text{Eq. (9)*})$$

Out[ $\circ$ ] =

$$\frac{1}{6 r^3}$$

$$\begin{aligned} & \mathbf{G} (-\lambda 1 + \lambda 2) \rho \left( \left( r^4 + r^2 r1^2 + 4 r1^4 - r r1 (r^2 + 4 r1^2) \cos[\theta 1] - r^2 (3 r^2 + r1^2) \cos[2 \theta 1] + \right. \right. \\ & \quad \left. \left. 3 r^3 r1 \cos[3 \theta 1] \right) / \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 1]} \right) - \\ & \quad (r^4 + r^2 r2^2 + 4 r2^4 - r r2 (r^2 + 4 r2^2) \cos[\theta 1] - r^2 (3 r^2 + r2^2) \cos[2 \theta 1] + \\ & \quad 3 r^3 r2 \cos[3 \theta 1]) / \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 1]} - \\ & \quad (r^4 + r^2 r1^2 + 4 r1^4 - r r1 (r^2 + 4 r1^2) \cos[\theta 2] - r^2 (3 r^2 + r1^2) \cos[2 \theta 2] + \\ & \quad 3 r^3 r1 \cos[3 \theta 2]) / \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 2]} + \\ & \quad (r^4 + r^2 r2^2 + 4 r2^4 - r r2 (r^2 + 4 r2^2) \cos[\theta 2] - r^2 (3 r^2 + r2^2) \cos[2 \theta 2] + \\ & \quad 3 r^3 r2 \cos[3 \theta 2]) / \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 2]} + \\ & \quad \left. 6 r^3 \cos[\theta 1] \text{Log}\left[\frac{r1 - r \cos[\theta 1] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 1]}}{r2 - r \cos[\theta 1] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 1]}}\right] \sin[\theta 1]^2 + \right. \\ & \quad \left. 6 r^3 \cos[\theta 2] \text{Log}\left[\frac{r2 - r \cos[\theta 2] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 2]}}{r1 - r \cos[\theta 2] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 2]}}\right] \sin[\theta 2]^2 \right) \end{aligned}$$

$$\begin{aligned}
In[6]:= \mathbf{Vxx} = & \frac{G * \rho}{48} * \left( F2 * \text{Log}\left[\frac{C22 + l22}{C12 + l12}\right] + F1 * \text{Log}\left[\frac{C11 + l11}{C21 + l21}\right] + \right. \\
& \frac{1}{r^3 * l22} * (H2 + I22 + J2 + K22) - \frac{1}{r^3 * l12} * (H1 + I12 + J1 + K12) - \\
& \left. \frac{1}{r^3 * l21} * (H2 + I21 + J2 + K21) + \frac{1}{r^3 * l11} * (H1 + I11 + J1 + K11) \right) (*\text{Eq. } (14)*)
\end{aligned}$$

*Out[6]=*

$$\begin{aligned}
& \frac{1}{48} G \rho \left( -3 \cos[\theta1] \text{Log}\left[\frac{r1 - r \cos[\theta1] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}}{r2 - r \cos[\theta1] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}}\right] \right. \\
& (8 (-\lambda1 + \lambda2) \sin[\theta1]^2 - 4 (5 - \cos[2 \theta1]) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2]) - \\
& 3 \cos[\theta2] \text{Log}\left[\frac{r2 - r \cos[\theta2] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}}{r1 - r \cos[\theta2] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}}\right] \\
& (8 (-\lambda1 + \lambda2) \sin[\theta2]^2 - 4 (5 - \cos[2 \theta2]) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2]) + \\
& \frac{1}{r^3 \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}} \\
& (-4 (r^4 + r^2 r1^2 + 4 r1^4) (-\lambda1 + \lambda2) + 2 r^2 ((3 r^2 + r1^2) \cos[2 \theta1] - 3 r r1 \cos[3 \theta1])) \\
& (-2 \lambda1 + 2 \lambda2 + \sin[2 (\lambda - \lambda1)] - \sin[2 (\lambda - \lambda2)]) + \\
& 4 (13 r^4 + 13 r^2 r1^2 + 4 r1^4) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 4 r r1 \cos[\theta1] \\
& ((r^2 + 4 r1^2) (-\lambda1 + \lambda2) - (25 r^2 + 4 r1^2) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2])) - \\
& \frac{1}{r^3 \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}} (-4 (r^4 + r^2 r1^2 + 4 r1^4) (-\lambda1 + \lambda2) + \\
& 2 r^2 ((3 r^2 + r1^2) \cos[2 \theta2] - 3 r r1 \cos[3 \theta2])) \\
& (-2 \lambda1 + 2 \lambda2 + \sin[2 (\lambda - \lambda1)] - \sin[2 (\lambda - \lambda2)]) + \\
& 4 (13 r^4 + 13 r^2 r1^2 + 4 r1^4) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 4 r r1 \cos[\theta2] \\
& ((r^2 + 4 r1^2) (-\lambda1 + \lambda2) - (25 r^2 + 4 r1^2) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2])) - \\
& \frac{1}{r^3 \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}} (-4 (r^4 + r^2 r2^2 + 4 r2^4) (-\lambda1 + \lambda2) + \\
& 2 r^2 ((3 r^2 + r2^2) \cos[2 \theta1] - 3 r r2 \cos[3 \theta1])) \\
& (-2 \lambda1 + 2 \lambda2 + \sin[2 (\lambda - \lambda1)] - \sin[2 (\lambda - \lambda2)]) + \\
& 4 (13 r^4 + 13 r^2 r2^2 + 4 r2^4) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 4 r r2 \cos[\theta1] \\
& ((r^2 + 4 r2^2) (-\lambda1 + \lambda2) - (25 r^2 + 4 r2^2) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2])) + \\
& \frac{1}{r^3 \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}} (-4 (r^4 + r^2 r2^2 + 4 r2^4) (-\lambda1 + \lambda2) + \\
& 2 r^2 ((3 r^2 + r2^2) \cos[2 \theta2] - 3 r r2 \cos[3 \theta2])) \\
& (-2 \lambda1 + 2 \lambda2 + \sin[2 (\lambda - \lambda1)] - \sin[2 (\lambda - \lambda2)]) + \\
& 4 (13 r^4 + 13 r^2 r2^2 + 4 r2^4) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 4 r r2 \cos[\theta2] \\
& ((r^2 + 4 r2^2) (-\lambda1 + \lambda2) - (25 r^2 + 4 r2^2) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2])) \Big)
\end{aligned}$$

$$\begin{aligned}
In[8]:= \text{Vyy} = & \frac{G * \rho}{48} * \left( L2 * \text{Log}\left[\frac{C22 + l22}{C12 + l12}\right] + L1 * \text{Log}\left[\frac{C11 + l11}{C21 + l21}\right] + \right. \\
& \frac{1}{r^3 * l22} * (H2 + M22 - J2 + N22) - \frac{1}{r^3 * l12} * (H1 + M12 - J1 + N12) - \\
& \left. \frac{1}{r^3 * l21} * (H2 + M21 - J2 + N21) + \frac{1}{r^3 * l11} * (H1 + M11 - J1 + N11) \right) (*\text{Eq. } (15)*)
\end{aligned}$$

*Out[8]=*

$$\begin{aligned}
& \frac{1}{48} G \rho \left( -3 \cos[\theta1] \text{Log}\left[\frac{r1 - r \cos[\theta1] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}}{r2 - r \cos[\theta1] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}}\right] \right. \\
& (8 (-\lambda1 + \lambda2) \sin[\theta1]^2 + 4 (5 - \cos[2 \theta1]) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2]) - \\
& 3 \cos[\theta2] \text{Log}\left[\frac{r2 - r \cos[\theta2] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}}{r1 - r \cos[\theta2] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}}\right] \\
& (8 (-\lambda1 + \lambda2) \sin[\theta2]^2 + 4 (5 - \cos[2 \theta2]) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2]) + \\
& \frac{1}{r^3 \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}} \\
& (-4 (r^4 + r^2 r1^2 + 4 r1^4) (-\lambda1 + \lambda2) + 2 r^2 ((3 r^2 + r1^2) \cos[2 \theta1] - 3 r r1 \cos[3 \theta1]) \\
& (-2 \lambda1 + 2 \lambda2 - \sin[2 (\lambda - \lambda1)] + \sin[2 (\lambda - \lambda2)]) - \\
& 4 (13 r^4 + 13 r^2 r1^2 + 4 r1^4) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 4 r r1 \cos[\theta1] \\
& ((r^2 + 4 r1^2) (-\lambda1 + \lambda2) + (25 r^2 + 4 r1^2) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2])) - \\
& \frac{1}{r^3 \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}} (-4 (r^4 + r^2 r1^2 + 4 r1^4) (-\lambda1 + \lambda2) + \\
& 2 r^2 ((3 r^2 + r1^2) \cos[2 \theta2] - 3 r r1 \cos[3 \theta2]) \\
& (-2 \lambda1 + 2 \lambda2 - \sin[2 (\lambda - \lambda1)] + \sin[2 (\lambda - \lambda2)]) - \\
& 4 (13 r^4 + 13 r^2 r1^2 + 4 r1^4) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 4 r r1 \cos[\theta2] \\
& ((r^2 + 4 r1^2) (-\lambda1 + \lambda2) + (25 r^2 + 4 r1^2) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2])) - \\
& \frac{1}{r^3 \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}} (-4 (r^4 + r^2 r2^2 + 4 r2^4) (-\lambda1 + \lambda2) + \\
& 2 r^2 ((3 r^2 + r2^2) \cos[2 \theta1] - 3 r r2 \cos[3 \theta1]) \\
& (-2 \lambda1 + 2 \lambda2 - \sin[2 (\lambda - \lambda1)] + \sin[2 (\lambda - \lambda2)]) - \\
& 4 (13 r^4 + 13 r^2 r2^2 + 4 r2^4) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 4 r r2 \cos[\theta1] \\
& ((r^2 + 4 r2^2) (-\lambda1 + \lambda2) + (25 r^2 + 4 r2^2) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2])) + \\
& \frac{1}{r^3 \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}} (-4 (r^4 + r^2 r2^2 + 4 r2^4) (-\lambda1 + \lambda2) + \\
& 2 r^2 ((3 r^2 + r2^2) \cos[2 \theta2] - 3 r r2 \cos[3 \theta2]) \\
& (-2 \lambda1 + 2 \lambda2 - \sin[2 (\lambda - \lambda1)] + \sin[2 (\lambda - \lambda2)]) - \\
& 4 (13 r^4 + 13 r^2 r2^2 + 4 r2^4) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 4 r r2 \cos[\theta2] \\
& ((r^2 + 4 r2^2) (-\lambda1 + \lambda2) + (25 r^2 + 4 r2^2) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2])) \Big)
\end{aligned}$$

$$\text{In}[\circ]:= \mathbf{V}_{\mathbf{ZZZ}} = -\frac{G * \rho * (\lambda 2 - \lambda 1)}{2 * r^4} * \left( \frac{022}{l_{22}^3} - \frac{012}{l_{12}^3} - \frac{021}{l_{21}^3} + \frac{011}{l_{11}^3} \right) (*\text{Eq. } (17)*)$$

*Out*[\circ]=

$$\begin{aligned} & -\frac{1}{2 r^4} G (-\lambda 1 + \lambda 2) \rho \left( \frac{r 1^3 (9 r^2 r 1 + 4 r 1^3 - 4 r (r^2 + 3 r 1^2) \cos[\theta 1] + 3 r^2 r 1 \cos[2 \theta 1])}{(r^2 + r 1^2 - 2 r r 1 \cos[\theta 1])^{3/2}} - \right. \\ & \frac{r 2^3 (9 r^2 r 2 + 4 r 2^3 - 4 r (r^2 + 3 r 2^2) \cos[\theta 1] + 3 r^2 r 2 \cos[2 \theta 1])}{(r^2 + r 2^2 - 2 r r 2 \cos[\theta 1])^{3/2}} - \\ & \frac{r 1^3 (9 r^2 r 1 + 4 r 1^3 - 4 r (r^2 + 3 r 1^2) \cos[\theta 2] + 3 r^2 r 1 \cos[2 \theta 2])}{(r^2 + r 1^2 - 2 r r 1 \cos[\theta 2])^{3/2}} + \\ & \left. \frac{r 2^3 (9 r^2 r 2 + 4 r 2^3 - 4 r (r^2 + 3 r 2^2) \cos[\theta 2] + 3 r^2 r 2 \cos[2 \theta 2])}{(r^2 + r 2^2 - 2 r r 2 \cos[\theta 2])^{3/2}} \right) \end{aligned}$$

$$\text{In}[\circ]:= \mathbf{V}_{\mathbf{XXX}} = \frac{G * \rho}{16 * r^4} * \left( \frac{P_{22} + Q_{22} + R_{22} + S_{22} + T_2}{l_{22}^3} - \frac{P_{12} + Q_{12} + R_{12} + S_{12} + T_1}{l_{12}^3} - \right. \\ \left. \frac{P_{21} + Q_{21} + R_{21} + S_{21} + T_2}{l_{21}^3} + \frac{P_{11} + Q_{11} + R_{11} + S_{11} + T_1}{l_{11}^3} \right) (*\text{Eq. } (18)*)$$

*Out*[\circ]=

$$\begin{aligned} & \frac{1}{16 r^4} \\ & G \rho \left( \frac{1}{(r^2 + r 1^2 - 2 r r 1 \cos[\theta 1])^{3/2}} \left( 4 r 1^4 (9 r^2 + 4 r 1^2) (-\lambda 1 + \lambda 2) - 5 (r^2 + r 1^2)^3 (-\lambda 1 + \lambda 2) \right. \right. \\ & \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} + 6 r^2 r 1^4 \cos[2 \theta 1] \\ & (-2 \lambda 1 + 2 \lambda 2 + \sin[2 (\lambda - \lambda 1)] - \sin[2 (\lambda - \lambda 2)]) + \left( 5 r^6 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} + \right. \\ & r 1^6 \left( -16 + 5 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} \right) + r^2 r 1^4 \left( -52 + 15 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} \right) + \\ & r^4 r 1^2 \left( -16 + 15 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} \right) \left. \right) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + \\ & 2 r r 1 \cos[\theta 1] \left( (-\lambda 1 + \lambda 2) \left( 5 r^4 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} + r 1^4 \right. \right. \\ & \left. \left. \left( -24 + 5 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} \right) + 2 r^2 r 1^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} \right) \right) - \right. \\ & (r^2 + r 1^2) \left( 5 r^2 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} + r 1^2 \left( -24 + 5 \sqrt{1 - \frac{2 r r 1 \cos[\theta 1]}{r^2 + r 1^2}} \right) \right) \\ & \left. \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \left. \right) - \\ & \frac{1}{(r^2 + r 2^2 - 2 r r 2 \cos[\theta 1])^{3/2}} \left( 4 r 2^4 (9 r^2 + 4 r 2^2) (-\lambda 1 + \lambda 2) - \right. \end{aligned}$$

$$\begin{aligned}
& 5 (r^2 + r2^2)^3 (-\lambda 1 + \lambda 2) \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} + \\
& 6 r^2 r2^4 \cos[2 \theta 1] (-2 \lambda 1 + 2 \lambda 2 + \sin[2 (\lambda - \lambda 1)] - \sin[2 (\lambda - \lambda 2)]) + \\
& \left( 5 r^6 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} + r2^6 \left( -16 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} \right) + r^2 r2^4 \right. \\
& \left. \left( -52 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} \right) + r^4 r2^2 \left( -16 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} \right) \right) \\
& \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 2 r r2 \cos[\theta 1] \\
& \left( (-\lambda 1 + \lambda 2) \left( 5 r^4 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} + r2^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} \right) + 2 r^2 \right. \right. \\
& \left. \left. r2^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} \right) \right) - (r^2 + r2^2) \left( 5 r^2 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} + \right. \\
& \left. \left. r2^2 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta 1]}{r^2 + r2^2}} \right) \right) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) - \\
& \frac{1}{(r^2 + r1^2 - 2 r r1 \cos[\theta 2])^{3/2}} \left( 4 r1^4 (9 r^2 + 4 r1^2) (-\lambda 1 + \lambda 2) - \right. \\
& \left. 5 (r^2 + r1^2)^3 (-\lambda 1 + \lambda 2) \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} + \right. \\
& \left. 6 r^2 r1^4 \cos[2 \theta 2] (-2 \lambda 1 + 2 \lambda 2 + \sin[2 (\lambda - \lambda 1)] - \sin[2 (\lambda - \lambda 2)]) + \right. \\
& \left( 5 r^6 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} + r1^6 \left( -16 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} \right) + r^2 r1^4 \right. \\
& \left. \left( -52 + 15 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} \right) + r^4 r1^2 \left( -16 + 15 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} \right) \right) \\
& \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 2 r r1 \cos[\theta 2] \\
& \left( (-\lambda 1 + \lambda 2) \left( 5 r^4 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} + r1^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} \right) + 2 r^2 \right. \right. \\
& \left. \left. r1^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} \right) \right) - (r^2 + r1^2) \left( 5 r^2 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} + \right. \\
& \left. \left. r1^2 \left( -24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta 2]}{r^2 + r1^2}} \right) \right) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) + \\
& \frac{1}{(r^2 + r2^2 - 2 r r2 \cos[\theta 2])^{3/2}} \left( 4 r2^4 (9 r^2 + 4 r2^2) (-\lambda 1 + \lambda 2) - \right. \\
& \left. 5 (r^2 + r2^2)^3 (-\lambda 1 + \lambda 2) \sqrt{1 - \frac{2 r r2 \cos[\theta 2]}{r^2 + r2^2}} + \right. \\
& \left. 6 r^2 r2^4 \cos[2 \theta 2] (-2 \lambda 1 + 2 \lambda 2 + \sin[2 (\lambda - \lambda 1)] - \sin[2 (\lambda - \lambda 2)]) + \right. \\
& \left( 5 r^6 \sqrt{1 - \frac{2 r r2 \cos[\theta 2]}{r^2 + r2^2}} + r2^6 \left( -16 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta 2]}{r^2 + r2^2}} \right) + r^2 r2^4 \right. \\
& \left. \left( -52 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta 2]}{r^2 + r2^2}} \right) + r^4 r2^2 \left( -16 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta 2]}{r^2 + r2^2}} \right) \right)
\end{aligned}$$

$$\begin{aligned} & \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] + 2r r_2 \cos[\theta_2] \\ & \left( (-\lambda_1 + \lambda_2) \left( 5r^4 \sqrt{1 - \frac{2r r_2 \cos[\theta_2]}{r^2 + r_2^2}} + r_2^4 \left( -24 + 5 \sqrt{1 - \frac{2r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) + 2r^2 \right. \right. \\ & \left. \left. r_2^2 \left( -4 + 5 \sqrt{1 - \frac{2r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) \right) - (r^2 + r_2^2) \left( 5r^2 \sqrt{1 - \frac{2r r_2 \cos[\theta_2]}{r^2 + r_2^2}} + \right. \right. \\ & \left. \left. r_2^2 \left( -24 + 5 \sqrt{1 - \frac{2r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) \right) \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \right) \end{aligned}$$

$$\begin{aligned} \text{In}[^\circ]:= \mathbf{Vyyz} = \frac{G * \rho}{16 * r^4} * \left( \frac{V22 - Q22 + W22 + S22 + T2}{l_{22}^3} - \frac{V12 - Q12 + W12 + S12 + T1}{l_{12}^3} - \right. \\ \left. \frac{V21 - Q21 + W21 + S21 + T2}{l_{21}^3} + \frac{V11 - Q11 + W11 + S11 + T1}{l_{11}^3} \right) (*\text{Eq. } (19)* ) \end{aligned}$$

Out[^\circ]=

$$\begin{aligned} & \frac{1}{16 r^4} \\ & G \rho \left( \frac{1}{(r^2 + r_1^2 - 2r r_1 \cos[\theta_1])^{3/2}} \left( 4r_1^4 (9r^2 + 4r_1^2) (-\lambda_1 + \lambda_2) - 5(r^2 + r_1^2)^3 (-\lambda_1 + \lambda_2) \right. \right. \\ & \left. \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} + 6r^2 r_1^4 \cos[2\theta_1] \right. \\ & \left. (-2\lambda_1 + 2\lambda_2 - \sin[2(\lambda - \lambda_1)] + \sin[2(\lambda - \lambda_2)]) - \left( 5r^6 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} + \right. \right. \\ & \left. r_1^6 \left( -16 + 5 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + r^2 r_1^4 \left( -52 + 15 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + \right. \\ & \left. r^4 r_1^2 \left( -16 + 15 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] + \\ & 2r r_1 \cos[\theta_1] \left( (-\lambda_1 + \lambda_2) \left( 5r^4 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} + r_1^4 \right. \right. \\ & \left. \left( -24 + 5 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + 2r^2 r_1^2 \left( -4 + 5 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) + \\ & (r^2 + r_1^2) \left( 5r^2 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} + r_1^2 \left( -24 + 5 \sqrt{1 - \frac{2r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) \\ & \left. \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \right) - \\ & \frac{1}{(r^2 + r_2^2 - 2r r_2 \cos[\theta_1])^{3/2}} \left( 4r_2^4 (9r^2 + 4r_2^2) (-\lambda_1 + \lambda_2) - \right. \\ & 5(r^2 + r_2^2)^3 (-\lambda_1 + \lambda_2) \sqrt{1 - \frac{2r r_2 \cos[\theta_1]}{r^2 + r_2^2}} + \\ & 6r^2 r_2^4 \cos[2\theta_1] (-2\lambda_1 + 2\lambda_2 - \sin[2(\lambda - \lambda_1)] + \sin[2(\lambda - \lambda_2)]) - \\ & \left. \left( 5r^6 \sqrt{1 - \frac{2r r_2 \cos[\theta_1]}{r^2 + r_2^2}} + r_2^6 \left( -16 + 5 \sqrt{1 - \frac{2r r_2 \cos[\theta_1]}{r^2 + r_2^2}} \right) + r^2 r_2^4 \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left( -52 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) + r^4 r2^2 \left( -16 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) \Bigg) \\
& \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 2 r r2 \cos[\theta1] \\
& \left( (-\lambda1 + \lambda2) \left( 5 r^4 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + r2^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) + 2 r^2 \right. \right. \\
& r2^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) \Big) + (r^2 + r2^2) \left( 5 r^2 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + \right. \\
& \left. \left. r2^2 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) \right) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] \Big) \Big) - \\
& \frac{1}{(r^2 + r1^2 - 2 r r1 \cos[\theta2])^{3/2}} \left( 4 r1^4 (9 r^2 + 4 r1^2) (-\lambda1 + \lambda2) - \right. \\
& 5 (r^2 + r1^2)^3 (-\lambda1 + \lambda2) \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + \\
& 6 r^2 r1^4 \cos[2 \theta2] (-2 \lambda1 + 2 \lambda2 - \sin[2 (\lambda - \lambda1)] + \sin[2 (\lambda - \lambda2)]) - \\
& \left( 5 r^6 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + r1^6 \left( -16 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) + r^2 r1^4 \right. \\
& \left. \left( -52 + 15 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) + r^4 r1^2 \left( -16 + 15 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) \right) \\
& \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 2 r r1 \cos[\theta2] \\
& \left( (-\lambda1 + \lambda2) \left( 5 r^4 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + r1^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) + 2 r^2 \right. \right. \\
& r1^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) \Big) + (r^2 + r1^2) \left( 5 r^2 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + \right. \\
& \left. \left. r1^2 \left( -24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) \right) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] \Big) \Big) + \\
& \frac{1}{(r^2 + r2^2 - 2 r r2 \cos[\theta2])^{3/2}} \left( 4 r2^4 (9 r^2 + 4 r2^2) (-\lambda1 + \lambda2) - \right. \\
& 5 (r^2 + r2^2)^3 (-\lambda1 + \lambda2) \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + \\
& 6 r^2 r2^4 \cos[2 \theta2] (-2 \lambda1 + 2 \lambda2 - \sin[2 (\lambda - \lambda1)] + \sin[2 (\lambda - \lambda2)]) - \\
& \left( 5 r^6 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + r2^6 \left( -16 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) + r^2 r2^4 \right. \\
& \left. \left( -52 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) + r^4 r2^2 \left( -16 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) \right) \\
& \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 2 r r2 \cos[\theta2] \\
& \left( (-\lambda1 + \lambda2) \left( 5 r^4 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + r2^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) + 2 r^2 \right. \right. \\
& r2^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) \Big) + (r^2 + r2^2) \left( 5 r^2 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + \right. \\
& \left. \left. \left. r2^2 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) \right) \right)
\end{aligned}$$

$$r2^2 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] \right) \right)$$

In[8]:= **ZonalBandVzz** =

$$\frac{\pi * G * \rho}{3 * r^3} * \left( \frac{E22}{l22} - \frac{E12}{l12} - \frac{E21}{l21} + \frac{E11}{l11} + 6 * B2 * \log\left[\frac{C22 + l22}{C12 + l12}\right] + 6 * B1 * \log\left[\frac{C11 + l11}{C21 + l21}\right] \right)$$

(\*Eq. (24)\*)

Out[8]=

$$\begin{aligned} & \frac{1}{3 r^3} G \pi \rho \left( (r^4 + r^2 r1^2 + 4 r1^4 - r r1 (r^2 + 4 r1^2) \cos[\theta1] - \right. \\ & \quad \left. r^2 (3 r^2 + r1^2) \cos[2 \theta1] + 3 r^3 r1 \cos[3 \theta1]) / (\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}) - \right. \\ & \quad \left. (r^4 + r^2 r2^2 + 4 r2^4 - r r2 (r^2 + 4 r2^2) \cos[\theta1] - r^2 (3 r^2 + r2^2) \cos[2 \theta1] + \right. \\ & \quad \left. 3 r^3 r2 \cos[3 \theta1]) / (\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}) - \right. \\ & \quad \left. (r^4 + r^2 r1^2 + 4 r1^4 - r r1 (r^2 + 4 r1^2) \cos[\theta2] - r^2 (3 r^2 + r1^2) \cos[2 \theta2] + \right. \\ & \quad \left. 3 r^3 r1 \cos[3 \theta2]) / (\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}) + \right. \\ & \quad \left. (r^4 + r^2 r2^2 + 4 r2^4 - r r2 (r^2 + 4 r2^2) \cos[\theta2] - r^2 (3 r^2 + r2^2) \cos[2 \theta2] + \right. \\ & \quad \left. 3 r^3 r2 \cos[3 \theta2]) / (\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}) + \right. \\ & \quad \left. 6 r^3 \cos[\theta1] \log\left[\frac{r1 - r \cos[\theta1] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}}{r2 - r \cos[\theta1] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}}\right] \sin[\theta1]^2 + \right. \\ & \quad \left. 6 r^3 \cos[\theta2] \log\left[\frac{r2 - r \cos[\theta2] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}}{r1 - r \cos[\theta2] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}}\right] \sin[\theta2]^2 \right) \end{aligned}$$

$$\text{In}[0]:= \text{ZonalBandVxx} = -\frac{\pi * G * \rho}{6 * r^3} * \left( \frac{E22}{l22} - \frac{E12}{l12} - \frac{E21}{l21} + \frac{E11}{l11} + 6 * B2 * \text{Log}\left[\frac{C22 + l22}{C12 + l12}\right] + 6 * B1 * \text{Log}\left[\frac{C11 + l11}{C21 + l21}\right] \right) \\ (*\text{Eq. } (25)*)$$

$$\text{Out}[0]= -\frac{1}{6 r^3} G \pi \rho \left( \left( r^4 + r^2 r1^2 + 4 r1^4 - r r1 (r^2 + 4 r1^2) \cos[\theta1] - \frac{r^2 (3 r^2 + r1^2) \cos[2 \theta1] + 3 r^3 r1 \cos[3 \theta1]}{\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}} \right) - \left( r^4 + r^2 r2^2 + 4 r2^4 - r r2 (r^2 + 4 r2^2) \cos[\theta1] - r^2 (3 r^2 + r2^2) \cos[2 \theta1] + \frac{3 r^3 r2 \cos[3 \theta1]}{\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}} \right) - \left( r^4 + r^2 r1^2 + 4 r1^4 - r r1 (r^2 + 4 r1^2) \cos[\theta2] - r^2 (3 r^2 + r1^2) \cos[2 \theta2] + \frac{3 r^3 r1 \cos[3 \theta2]}{\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}} \right) + \left( r^4 + r^2 r2^2 + 4 r2^4 - r r2 (r^2 + 4 r2^2) \cos[\theta2] - r^2 (3 r^2 + r2^2) \cos[2 \theta2] + \frac{3 r^3 r2 \cos[3 \theta2]}{\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}} \right) + \frac{6 r^3 \cos[\theta1] \text{Log}\left[\frac{r1 - r \cos[\theta1] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}}{r2 - r \cos[\theta1] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}}\right] \sin[\theta1]^2 + 6 r^3 \cos[\theta2] \text{Log}\left[\frac{r2 - r \cos[\theta2] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}}{r1 - r \cos[\theta2] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}}\right] \sin[\theta2]^2 \right)$$

$$\text{In}[0]:= \text{ZonalBandVyy} = -\frac{\pi * G * \rho}{6 * r^3} * \left( \frac{E22}{l22} - \frac{E12}{l12} - \frac{E21}{l21} + \frac{E11}{l11} + 6 * B2 * \text{Log}\left[\frac{C22 + l22}{C12 + l12}\right] + 6 * B1 * \text{Log}\left[\frac{C11 + l11}{C21 + l21}\right] \right) \\ (*\text{Eq. } (25)*)$$

$$\text{Out}[0]= -\frac{1}{6 r^3} G \pi \rho \left( \left( r^4 + r^2 r1^2 + 4 r1^4 - r r1 (r^2 + 4 r1^2) \cos[\theta1] - \frac{r^2 (3 r^2 + r1^2) \cos[2 \theta1] + 3 r^3 r1 \cos[3 \theta1]}{\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}} \right) - \left( r^4 + r^2 r2^2 + 4 r2^4 - r r2 (r^2 + 4 r2^2) \cos[\theta1] - r^2 (3 r^2 + r2^2) \cos[2 \theta1] + \frac{3 r^3 r2 \cos[3 \theta1]}{\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}} \right) - \left( r^4 + r^2 r1^2 + 4 r1^4 - r r1 (r^2 + 4 r1^2) \cos[\theta2] - r^2 (3 r^2 + r1^2) \cos[2 \theta2] + \frac{3 r^3 r1 \cos[3 \theta2]}{\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}} \right) + \left( r^4 + r^2 r2^2 + 4 r2^4 - r r2 (r^2 + 4 r2^2) \cos[\theta2] - r^2 (3 r^2 + r2^2) \cos[2 \theta2] + \frac{3 r^3 r2 \cos[3 \theta2]}{\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}} \right) + \frac{6 r^3 \cos[\theta1] \text{Log}\left[\frac{r1 - r \cos[\theta1] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}}{r2 - r \cos[\theta1] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}}\right] \sin[\theta1]^2 + 6 r^3 \cos[\theta2] \text{Log}\left[\frac{r2 - r \cos[\theta2] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}}{r1 - r \cos[\theta2] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}}\right] \sin[\theta2]^2 \right)$$

$$\begin{aligned}
In[^\circ] := \text{ZonalBandVzzz} = & -\frac{\text{Pi} * \text{G} * \rho}{r^4} * \left( \frac{022}{l_{22}^3} - \frac{012}{l_{12}^3} - \frac{021}{l_{21}^3} + \frac{011}{l_{11}^3} \right) (*\text{Eq. } (26)*)
\\
Out[^\circ] = & -\frac{1}{r^4} G \pi \rho \left( \frac{r1^3 (9 r^2 r1 + 4 r1^3 - 4 r (r^2 + 3 r1^2) \cos[\theta1] + 3 r^2 r1 \cos[2 \theta1])}{(r^2 + r1^2 - 2 r r1 \cos[\theta1])^{3/2}} - \right. \\
& \frac{r2^3 (9 r^2 r2 + 4 r2^3 - 4 r (r^2 + 3 r2^2) \cos[\theta1] + 3 r^2 r2 \cos[2 \theta1])}{(r^2 + r2^2 - 2 r r2 \cos[\theta1])^{3/2}} - \\
& \frac{r1^3 (9 r^2 r1 + 4 r1^3 - 4 r (r^2 + 3 r1^2) \cos[\theta2] + 3 r^2 r1 \cos[2 \theta2])}{(r^2 + r1^2 - 2 r r1 \cos[\theta2])^{3/2}} + \\
& \left. \frac{r2^3 (9 r^2 r2 + 4 r2^3 - 4 r (r^2 + 3 r2^2) \cos[\theta2] + 3 r^2 r2 \cos[2 \theta2])}{(r^2 + r2^2 - 2 r r2 \cos[\theta2])^{3/2}} \right)
\end{aligned}$$

$$\begin{aligned}
In[8]:= \text{ZonalBandVxxz} = & \frac{\text{Pi} * \text{G} * \rho}{8 * r^4} * \\
& \left( \frac{X_{22} + Y_{22} + Z_{22}}{l_{22}^3} - \frac{X_{12} + Y_{12} + Z_{12}}{l_{12}^3} - \frac{X_{21} + Y_{21} + Z_{21}}{l_{21}^3} + \frac{X_{11} + Y_{11} + Z_{11}}{l_{11}^3} \right) (*\text{Eq. } (27)*)
\end{aligned}$$

*Out[8]=*

$$\begin{aligned}
& \frac{1}{8 r^4} \text{G} \pi \rho \\
& \left( \left( -5 (r^2 + r1^2)^3 \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} + 2 r r1 \cos[\theta1] \left( 5 r^4 \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} + r1^4 \right. \right. \right. \\
& \quad \left. \left. \left. \left( -24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} \right) + 2 r^2 r1^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} \right) \right) + \right. \\
& \quad \left. \left. \left. 4 r1^4 (9 r^2 + 4 r1^2 + 3 r^2 \cos[2 \theta1]) \right) \right) / (r^2 + r1^2 - 2 r r1 \cos[\theta1])^{3/2} - \\
& \left( -5 (r^2 + r2^2)^3 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + 2 r r2 \cos[\theta1] \left( 5 r^4 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + \right. \right. \\
& \quad \left. \left. r2^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) + 2 r^2 r2^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) \right) + \right. \\
& \quad \left. \left. \left. 4 r2^4 (9 r^2 + 4 r2^2 + 3 r^2 \cos[2 \theta1]) \right) \right) / (r^2 + r2^2 - 2 r r2 \cos[\theta1])^{3/2} - \\
& \left( -5 (r^2 + r1^2)^3 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + 2 r r1 \cos[\theta2] \left( 5 r^4 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + \right. \right. \\
& \quad \left. \left. r1^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) + 2 r^2 r1^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) \right) + \right. \\
& \quad \left. \left. \left. 4 r1^4 (9 r^2 + 4 r1^2 + 3 r^2 \cos[2 \theta2]) \right) \right) / (r^2 + r1^2 - 2 r r1 \cos[\theta2])^{3/2} + \\
& \left( -5 (r^2 + r2^2)^3 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + 2 r r2 \cos[\theta2] \left( 5 r^4 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + \right. \right. \\
& \quad \left. \left. r2^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) + 2 r^2 r2^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) \right) + \right. \\
& \quad \left. \left. \left. 4 r2^4 (9 r^2 + 4 r2^2 + 3 r^2 \cos[2 \theta2]) \right) \right) / (r^2 + r2^2 - 2 r r2 \cos[\theta2])^{3/2}
\end{aligned}$$

$$\begin{aligned}
In[1]:= \text{ZonalBandVyyz} &= \frac{\text{Pi} * \text{G} * \rho}{8 * r^4} * \\
&\left( \frac{X_{22} + Y_{22} + Z_{22}}{l_{22}^3} - \frac{X_{12} + Y_{12} + Z_{12}}{l_{12}^3} - \frac{X_{21} + Y_{21} + Z_{21}}{l_{21}^3} + \frac{X_{11} + Y_{11} + Z_{11}}{l_{11}^3} \right) (*\text{Eq. } (27)*)
\end{aligned}$$

$$\begin{aligned}
Out[1]:= & \frac{1}{8 r^4} \text{G} \pi \rho \\
& \left( \left( -5 (r^2 + r1^2)^3 \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} + 2 r r1 \cos[\theta1] \left( 5 r^4 \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} + r1^4 \right. \right. \right. \\
& \quad \left. \left. \left. \left( -24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} \right) + 2 r^2 r1^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} \right) \right) + \right. \\
& \quad \left. \left. \left. 4 r1^4 (9 r^2 + 4 r1^2 + 3 r^2 \cos[2 \theta1]) \right) \right) / (r^2 + r1^2 - 2 r r1 \cos[\theta1])^{3/2} - \\
& \left( -5 (r^2 + r2^2)^3 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + 2 r r2 \cos[\theta1] \left( 5 r^4 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + \right. \right. \\
& \quad \left. \left. r2^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) + 2 r^2 r2^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) \right) + \right. \\
& \quad \left. \left. \left. 4 r2^4 (9 r^2 + 4 r2^2 + 3 r^2 \cos[2 \theta1]) \right) \right) / (r^2 + r2^2 - 2 r r2 \cos[\theta1])^{3/2} - \\
& \left( -5 (r^2 + r1^2)^3 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + 2 r r1 \cos[\theta2] \left( 5 r^4 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + \right. \right. \\
& \quad \left. \left. r1^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) + 2 r^2 r1^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) \right) + \right. \\
& \quad \left. \left. \left. 4 r1^4 (9 r^2 + 4 r1^2 + 3 r^2 \cos[2 \theta2]) \right) \right) / (r^2 + r1^2 - 2 r r1 \cos[\theta2])^{3/2} + \\
& \left( -5 (r^2 + r2^2)^3 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + 2 r r2 \cos[\theta2] \left( 5 r^4 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + \right. \right. \\
& \quad \left. \left. r2^4 \left( -24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) + 2 r^2 r2^2 \left( -4 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) \right) + \right. \\
& \quad \left. \left. \left. 4 r2^4 (9 r^2 + 4 r2^2 + 3 r^2 \cos[2 \theta2]) \right) \right) / (r^2 + r2^2 - 2 r r2 \cos[\theta2])^{3/2}
\end{aligned}$$

### 3. Test Laplace's equation

In[2]:= **FullSimplify[Vxx + Vyy + Vzz]**

Out[2]=

0

```
In[8]:= FullSimplify[
  (Vxxz + Vyyz + Vzzz) /. {
    Sqrt[1 - 2 r r2 Cos[\theta2]/(r^2 + r2^2)] -> Sqrt[r^2 + r2^2 - 2 r r2 Cos[\theta2]]/Sqrt[r^2 + r2^2],
    Sqrt[1 - 2 r r1 Cos[\theta2]/(r^2 + r1^2)] -> Sqrt[r^2 + r1^2 - 2 r r1 Cos[\theta2]]/Sqrt[r^2 + r1^2],
    Sqrt[1 - 2 r r2 Cos[\theta1]/(r^2 + r2^2)] -> Sqrt[r^2 + r2^2 - 2 r r2 Cos[\theta1]]/Sqrt[r^2 + r2^2],
    Sqrt[1 - 2 r r1 Cos[\theta1]/(r^2 + r1^2)] -> Sqrt[r^2 + r1^2 - 2 r r1 Cos[\theta1]]/Sqrt[r^2 + r1^2]
  }]
Out[8]= 0
```

```
In[9]:= FullSimplify[ZonalBandVxx + ZonalBandVyy + ZonalBandVzz]
```

```
Out[9]= 0
```

```
In[10]:= FullSimplify[
  (ZonalBandVxxz + ZonalBandVyyz + ZonalBandVzzz) /.
  {
    Sqrt[1 - 2 r r2 Cos[\theta2]/(r^2 + r2^2)] -> Sqrt[r^2 + r2^2 - 2 r r2 Cos[\theta2]]/Sqrt[r^2 + r2^2],
    Sqrt[1 - 2 r r1 Cos[\theta2]/(r^2 + r1^2)] -> Sqrt[r^2 + r1^2 - 2 r r1 Cos[\theta2]]/Sqrt[r^2 + r1^2],
    Sqrt[1 - 2 r r2 Cos[\theta1]/(r^2 + r2^2)] -> Sqrt[r^2 + r2^2 - 2 r r2 Cos[\theta1]]/Sqrt[r^2 + r2^2],
    Sqrt[1 - 2 r r1 Cos[\theta1]/(r^2 + r1^2)] -> Sqrt[r^2 + r1^2 - 2 r r1 Cos[\theta1]]/Sqrt[r^2 + r1^2]
  }]
Out[10]= 0
```

```
In[11]:= NotebookSave[EvaluationNotebook[]];
```