

```
In[*]:= Clear["Global`*"];
```

1. Expressions in different Tables

Table 8

```
In[*]:= A22 = 4 * r2^2 + r^2 * (1 - 3 * Cos[2 * θ2]) - 2 * r * r2 * Cos[θ2];
```

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In[*]:= A12 = 4 * r1^2 + r^2 * (1 - 3 * Cos[2 * θ2]) - 2 * r * r1 * Cos[θ2];
```

```
In[*]:= A21 = 4 * r2^2 + r^2 * (1 - 3 * Cos[2 * θ1]) - 2 * r * r2 * Cos[θ1];
```

```
In[*]:= A11 = 4 * r1^2 + r^2 * (1 - 3 * Cos[2 * θ1]) - 2 * r * r1 * Cos[θ1];
```

```
In[*]:= B2 = r^3 * (Sin[θ2])^2 * Cos[θ2];
```

```
In[*]:= B1 = r^3 * (Sin[θ1])^2 * Cos[θ1];
```

```
In[*]:= C22 = r2 - r * Cos[θ2];
```

```
In[*]:= C12 = r1 - r * Cos[θ2];
```

```
In[*]:= C21 = r2 - r * Cos[θ1];
```

```
In[*]:= C11 = r1 - r * Cos[θ1];
```

```
In[*]:= l22 = Sqrt[r^2 + r2^2 - 2 * r * r2 * Cos[θ2]] ;
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```
In[*]:= l12 = Sqrt[r^2 + r1^2 - 2 * r * r1 * Cos[θ2]] ;
```

```
In[*]:= l21 = Sqrt[r^2 + r2^2 - 2 * r * r2 * Cos[θ1]] ;
```

```
In[*]:= l11 = Sqrt[r^2 + r1^2 - 2 * r * r1 * Cos[θ1]] ;
```

Table 9

```
In[*]:= D22 = r2^2 + r^2 * (3 * (Cos[θ2])^2 - 2) + r * r2 * Cos[θ2];
```

```
In[*]:= D12 = r1^2 + r^2 * (3 * (Cos[θ2])^2 - 2) + r * r1 * Cos[θ2];
```

```
In[*]:= D21 = r2^2 + r^2 * (3 * (Cos[θ1])^2 - 2) + r * r2 * Cos[θ1];
```

```
In[*]:= D11 = r1^2 + r^2 * (3 * (Cos[θ1])^2 - 2) + r * r1 * Cos[θ1];
```

Table 10

```
In[*]:= E22 = r^4 + r^2 * r2^2 + 4 * r2^4 - r * r2 * (r^2 + 4 * r2^2) * Cos[θ2] -  
r^2 * (3 * r^2 + r2^2) * Cos[2 * θ2] + 3 * r^3 * r2 * Cos[3 * θ2];
```

```
In[*]:= E12 = r^4 + r^2 * r1^2 + 4 * r1^4 - r * r1 * (r^2 + 4 * r1^2) * Cos[θ2] -  
r^2 * (3 * r^2 + r1^2) * Cos[2 * θ2] + 3 * r^3 * r1 * Cos[3 * θ2];
```

```
In[*]:= E21 = r^4 + r^2 * r2^2 + 4 * r2^4 - r * r2 * (r^2 + 4 * r2^2) * Cos[θ1] -  
r^2 * (3 * r^2 + r2^2) * Cos[2 * θ1] + 3 * r^3 * r2 * Cos[3 * θ1];
```

$$\text{In[*]:= E11} = r^4 + r^2 * r1^2 + 4 * r1^4 - r * r1 * (r^2 + 4 * r1^2) * \text{Cos}[\theta1] - \\ r^2 * (3 * r^2 + r1^2) * \text{Cos}[2 * \theta1] + 3 * r^3 * r1 * \text{Cos}[3 * \theta1];$$

Table 11

$$\text{In[*]:= F2} = -3 * \text{Cos}[\theta2] * \\ (8 * (\lambda2 - \lambda1) * (\text{Sin}[\theta2])^2 + 4 * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1] * (5 - \text{Cos}[2 * \theta2]));$$

$$\text{In[*]:= F1} = -3 * \text{Cos}[\theta1] * \\ (8 * (\lambda2 - \lambda1) * (\text{Sin}[\theta1])^2 + 4 * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1] * (5 - \text{Cos}[2 * \theta1]));$$

$$\text{In[*]:= H2} = -4 * (r^4 + r^2 * r2^2 + 4 * r2^4) * (\lambda2 - \lambda1);$$

$$\text{In[*]:= H1} = -4 * (r^4 + r^2 * r1^2 + 4 * r1^4) * (\lambda2 - \lambda1);$$

$$\text{In[*]:= I22} = 2 * r^2 * ((3 * r^2 + r2^2) * \text{Cos}[2 * \theta2] - 3 * r * r2 * \text{Cos}[3 * \theta2]) * \\ (2 * \lambda2 - 2 * \lambda1 + \text{Sin}[2 * (\lambda - \lambda1)] - \text{Sin}[2 * (\lambda - \lambda2)]);$$

$$\text{In[*]:= I12} = 2 * r^2 * ((3 * r^2 + r1^2) * \text{Cos}[2 * \theta2] - 3 * r * r1 * \text{Cos}[3 * \theta2]) * \\ (2 * \lambda2 - 2 * \lambda1 + \text{Sin}[2 * (\lambda - \lambda1)] - \text{Sin}[2 * (\lambda - \lambda2)]);$$

$$\text{In[*]:= I21} = 2 * r^2 * ((3 * r^2 + r2^2) * \text{Cos}[2 * \theta1] - 3 * r * r2 * \text{Cos}[3 * \theta1]) * \\ (2 * \lambda2 - 2 * \lambda1 + \text{Sin}[2 * (\lambda - \lambda1)] - \text{Sin}[2 * (\lambda - \lambda2)]);$$

$$\text{In[*]:= I11} = 2 * r^2 * ((3 * r^2 + r1^2) * \text{Cos}[2 * \theta1] - 3 * r * r1 * \text{Cos}[3 * \theta1]) * \\ (2 * \lambda2 - 2 * \lambda1 + \text{Sin}[2 * (\lambda - \lambda1)] - \text{Sin}[2 * (\lambda - \lambda2)]);$$

$$\text{In[*]:= J2} = -4 * (13 * r^4 + 13 * r^2 * r2^2 + 4 * r2^4) * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1];$$

$$\text{In[*]:= J1} = -4 * (13 * r^4 + 13 * r^2 * r1^2 + 4 * r1^4) * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1];$$

$$\text{In[*]:= K22} = 4 * r * r2 * \text{Cos}[\theta2] * \\ ((r^2 + 4 * r2^2) * (\lambda2 - \lambda1) + (25 * r^2 + 4 * r2^2) * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1]);$$

$$\text{In[*]:= K12} = 4 * r * r1 * \text{Cos}[\theta2] * \\ ((r^2 + 4 * r1^2) * (\lambda2 - \lambda1) + (25 * r^2 + 4 * r1^2) * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1]);$$

$$\text{In[*]:= K21} = 4 * r * r2 * \text{Cos}[\theta1] * \\ ((r^2 + 4 * r2^2) * (\lambda2 - \lambda1) + (25 * r^2 + 4 * r2^2) * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1]);$$

$$\text{In[*]:= K11} = 4 * r * r1 * \text{Cos}[\theta1] * \\ ((r^2 + 4 * r1^2) * (\lambda2 - \lambda1) + (25 * r^2 + 4 * r1^2) * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1]);$$

Table 12

$$\text{In[*]:= L2} = -3 * \text{Cos}[\theta2] * \\ (8 * (\lambda2 - \lambda1) * (\text{Sin}[\theta2])^2 - 4 * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1] * (5 - \text{Cos}[2 * \theta2]));$$

$$\text{In[*]:= L1} = -3 * \text{Cos}[\theta1] * \\ (8 * (\lambda2 - \lambda1) * (\text{Sin}[\theta1])^2 - 4 * \text{Cos}[2 * \lambda - \lambda1 - \lambda2] * \text{Sin}[\lambda2 - \lambda1] * (5 - \text{Cos}[2 * \theta1]));$$

$$\text{In[*]:= M22} = 2 * r^2 * ((3 * r^2 + r2^2) * \text{Cos}[2 * \theta2] - 3 * r * r2 * \text{Cos}[3 * \theta2]) * \\ (2 * \lambda2 - 2 * \lambda1 - \text{Sin}[2 * (\lambda - \lambda1)] + \text{Sin}[2 * (\lambda - \lambda2)]);$$

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In[*]:= M12 = 2 * r^2 * ((3 * r^2 + r1^2) * Cos[2 * θ2] - 3 * r * r1 * Cos[3 * θ2]) *
          (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]);

In[*]:= M21 = 2 * r^2 * ((3 * r^2 + r2^2) * Cos[2 * θ1] - 3 * r * r2 * Cos[3 * θ1]) *
          (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]);

In[*]:= M11 = 2 * r^2 * ((3 * r^2 + r1^2) * Cos[2 * θ1] - 3 * r * r1 * Cos[3 * θ1]) *
          (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]);

In[*]:= N22 = 4 * r * r2 * Cos[θ2] *
          ((r^2 + 4 * r2^2) * (λ2 - λ1) - (25 * r^2 + 4 * r2^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

In[*]:= N12 = 4 * r * r1 * Cos[θ2] *
          ((r^2 + 4 * r1^2) * (λ2 - λ1) - (25 * r^2 + 4 * r1^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

In[*]:= N21 = 4 * r * r2 * Cos[θ1] *
          ((r^2 + 4 * r2^2) * (λ2 - λ1) - (25 * r^2 + 4 * r2^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

In[*]:= N11 = 4 * r * r1 * Cos[θ1] *
          ((r^2 + 4 * r1^2) * (λ2 - λ1) - (25 * r^2 + 4 * r1^2) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

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Table 13

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In[*]:= O22 = r2^3 * (9 * r^2 * r2 + 4 * r2^3 - 4 * r * (r^2 + 3 * r2^2) * Cos[θ2] + 3 * r^2 * r2 * Cos[2 * θ2]);

In[*]:= O12 = r1^3 * (9 * r^2 * r1 + 4 * r1^3 - 4 * r * (r^2 + 3 * r1^2) * Cos[θ2] + 3 * r^2 * r1 * Cos[2 * θ2]);

In[*]:= O21 = r2^3 * (9 * r^2 * r2 + 4 * r2^3 - 4 * r * (r^2 + 3 * r2^2) * Cos[θ1] + 3 * r^2 * r2 * Cos[2 * θ1]);

In[*]:= O11 = r1^3 * (9 * r^2 * r1 + 4 * r1^3 - 4 * r * (r^2 + 3 * r1^2) * Cos[θ1] + 3 * r^2 * r1 * Cos[2 * θ1]);

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Table 14

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In[*]:= P22 = 6 * r^2 * r2^4 * Cos[2 * θ2] * (2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]);

In[*]:= P12 = 6 * r^2 * r1^4 * Cos[2 * θ2] * (2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]);

In[*]:= P21 = 6 * r^2 * r2^4 * Cos[2 * θ1] * (2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]);

In[*]:= P11 = 6 * r^2 * r1^4 * Cos[2 * θ1] * (2 * λ2 - 2 * λ1 + Sin[2 * (λ - λ1)] - Sin[2 * (λ - λ2)]);

In[*]:= U22 = Sqrt[1 - (2 * r * r2 * Cos[θ2]) / (r^2 + r2^2)];

In[*]:= U12 = Sqrt[1 - (2 * r * r1 * Cos[θ2]) / (r^2 + r1^2)];

In[*]:= U21 = Sqrt[1 - (2 * r * r2 * Cos[θ1]) / (r^2 + r2^2)];

In[*]:= U11 = Sqrt[1 - (2 * r * r1 * Cos[θ1]) / (r^2 + r1^2)];

```

```

In[*]:= Q22 = -Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1] *
  (5 * r6 * U22 + r2 * (5 * U22 - 16) + r2 * r2 * (15 * U22 - 52) + r4 * r2 * (15 * U22 - 16));

In[*]:= Q12 = -Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1] *
  (5 * r6 * U12 + r2 * (5 * U12 - 16) + r2 * r2 * (15 * U12 - 52) + r4 * r2 * (15 * U12 - 16));

In[*]:= Q21 = -Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1] *
  (5 * r6 * U21 + r2 * (5 * U21 - 16) + r2 * r2 * (15 * U21 - 52) + r4 * r2 * (15 * U21 - 16));

In[*]:= Q11 = -Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1] *
  (5 * r6 * U11 + r2 * (5 * U11 - 16) + r2 * r2 * (15 * U11 - 52) + r4 * r2 * (15 * U11 - 16));

In[*]:= R22 = 2 * r * r2 * Cos[θ2] *
  ((λ2 - λ1) * (5 * r4 * U22 + r2 * (5 * U22 - 24) + 2 * r2 * r2 * (5 * U22 - 4)) +
  (r2 + r2) * (5 * r2 * U22 + r2 * (5 * U22 - 24)) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

In[*]:= R12 = 2 * r * r1 * Cos[θ2] *
  ((λ2 - λ1) * (5 * r4 * U12 + r2 * (5 * U12 - 24) + 2 * r2 * r2 * (5 * U12 - 4)) +
  (r2 + r2) * (5 * r2 * U12 + r2 * (5 * U12 - 24)) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

In[*]:= R21 = 2 * r * r2 * Cos[θ1] *
  ((λ2 - λ1) * (5 * r4 * U21 + r2 * (5 * U21 - 24) + 2 * r2 * r2 * (5 * U21 - 4)) +
  (r2 + r2) * (5 * r2 * U21 + r2 * (5 * U21 - 24)) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

In[*]:= R11 = 2 * r * r1 * Cos[θ1] *
  ((λ2 - λ1) * (5 * r4 * U11 + r2 * (5 * U11 - 24) + 2 * r2 * r2 * (5 * U11 - 4)) +
  (r2 + r2) * (5 * r2 * U11 + r2 * (5 * U11 - 24)) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

In[*]:= S22 = -5 * (r2 + r2)3 * (λ2 - λ1) * U22;

In[*]:= S12 = -5 * (r2 + r2)3 * (λ2 - λ1) * U12;

In[*]:= S21 = -5 * (r2 + r2)3 * (λ2 - λ1) * U21;

In[*]:= S11 = -5 * (r2 + r2)3 * (λ2 - λ1) * U11;

In[*]:= T2 = 4 * r2 * (9 * r2 + 4 * r2) * (λ2 - λ1);

In[*]:= T1 = 4 * r2 * (9 * r2 + 4 * r2) * (λ2 - λ1);

```

Table 15

```

In[*]:= V22 = 6 * r2 * r2 * Cos[2 * θ2] * (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]);

In[*]:= V12 = 6 * r2 * r2 * Cos[2 * θ2] * (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]);

In[*]:= V21 = 6 * r2 * r2 * Cos[2 * θ1] * (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]);

In[*]:= V11 = 6 * r2 * r2 * Cos[2 * θ1] * (2 * λ2 - 2 * λ1 - Sin[2 * (λ - λ1)] + Sin[2 * (λ - λ2)]);

In[*]:= W22 = 2 * r * r2 * Cos[θ2] *
  ((λ2 - λ1) * (5 * r4 * U22 + r2 * (5 * U22 - 24) + 2 * r2 * r2 * (5 * U22 - 4)) -
  (r2 + r2) * (5 * r2 * U22 + r2 * (5 * U22 - 24)) * Cos[2 * λ - λ1 - λ2] * Sin[λ2 - λ1]);

```

$$\begin{aligned} \text{In[*]}:= \text{W12} &= 2 * r * r1 * \text{Cos}[\theta 2] * \\ &\left((\lambda 2 - \lambda 1) * (5 * r^4 * U12 + r1^4 * (5 * U12 - 24) + 2 * r^2 * r1^2 * (5 * U12 - 4)) - \right. \\ &\quad \left. (r^2 + r1^2) * (5 * r^2 * U12 + r1^2 * (5 * U12 - 24)) * \text{Cos}[2 * \lambda - \lambda 1 - \lambda 2] * \text{Sin}[\lambda 2 - \lambda 1] \right); \end{aligned}$$

$$\begin{aligned} \text{In[*]}:= \text{W21} &= 2 * r * r2 * \text{Cos}[\theta 1] * \\ &\left((\lambda 2 - \lambda 1) * (5 * r^4 * U21 + r2^4 * (5 * U21 - 24) + 2 * r^2 * r2^2 * (5 * U21 - 4)) - \right. \\ &\quad \left. (r^2 + r2^2) * (5 * r^2 * U21 + r2^2 * (5 * U21 - 24)) * \text{Cos}[2 * \lambda - \lambda 1 - \lambda 2] * \text{Sin}[\lambda 2 - \lambda 1] \right); \end{aligned}$$

$$\begin{aligned} \text{In[*]}:= \text{W11} &= 2 * r * r1 * \text{Cos}[\theta 1] * \\ &\left((\lambda 2 - \lambda 1) * (5 * r^4 * U11 + r1^4 * (5 * U11 - 24) + 2 * r^2 * r1^2 * (5 * U11 - 4)) - \right. \\ &\quad \left. (r^2 + r1^2) * (5 * r^2 * U11 + r1^2 * (5 * U11 - 24)) * \text{Cos}[2 * \lambda - \lambda 1 - \lambda 2] * \text{Sin}[\lambda 2 - \lambda 1] \right); \end{aligned}$$

Table 16

$$\text{In[*]}:= \text{X22} = 4 * r2^4 * (9 * r^2 + 4 * r2^2 + 3 * r^2 * \text{Cos}[2 * \theta 2]);$$

$$\text{In[*]}:= \text{X12} = 4 * r1^4 * (9 * r^2 + 4 * r1^2 + 3 * r^2 * \text{Cos}[2 * \theta 2]);$$

$$\text{In[*]}:= \text{X21} = 4 * r2^4 * (9 * r^2 + 4 * r2^2 + 3 * r^2 * \text{Cos}[2 * \theta 1]);$$

$$\text{In[*]}:= \text{X11} = 4 * r1^4 * (9 * r^2 + 4 * r1^2 + 3 * r^2 * \text{Cos}[2 * \theta 1]);$$

$$\text{In[*]}:= \text{Y22} = 2 * r * r2 * \text{Cos}[\theta 2] * (5 * r^4 * U22 + r2^4 * (5 * U22 - 24) + 2 * r^2 * r2^2 * (5 * U22 - 4));$$

$$\text{In[*]}:= \text{Y12} = 2 * r * r1 * \text{Cos}[\theta 2] * (5 * r^4 * U12 + r1^4 * (5 * U12 - 24) + 2 * r^2 * r1^2 * (5 * U12 - 4));$$

$$\text{In[*]}:= \text{Y21} = 2 * r * r2 * \text{Cos}[\theta 1] * (5 * r^4 * U21 + r2^4 * (5 * U21 - 24) + 2 * r^2 * r2^2 * (5 * U21 - 4));$$

$$\text{In[*]}:= \text{Y11} = 2 * r * r1 * \text{Cos}[\theta 1] * (5 * r^4 * U11 + r1^4 * (5 * U11 - 24) + 2 * r^2 * r1^2 * (5 * U11 - 4));$$

$$\text{In[*]}:= \text{Z22} = -5 * (r^2 + r2^2)^3 * U22;$$

$$\text{In[*]}:= \text{Z12} = -5 * (r^2 + r1^2)^3 * U12;$$

$$\text{In[*]}:= \text{Z21} = -5 * (r^2 + r2^2)^3 * U21;$$

$$\text{In[*]}:= \text{Z11} = -5 * (r^2 + r1^2)^3 * U11;$$

2. Expressions for the Vzz, Vxx, Vyy, Vzzz, Vxxz, Vyyz of a tesseroïd and ZonalBandVzz, ZonalBandVxx, ZonalBandVyy, ZonalBandVzzz, ZonalBandVxxz, ZonalBandVyyz of a spherical zonal band

$$\begin{aligned}
 In[*] := & \text{Vzz} = \frac{G * \rho * (\lambda_2 - \lambda_1)}{6 * r^3} * \\
 & \left(\frac{E_{22}}{l_{22}} - \frac{E_{12}}{l_{12}} - \frac{E_{21}}{l_{21}} + \frac{E_{11}}{l_{11}} + 6 * B_2 * \text{Log}\left[\frac{C_{22} + l_{22}}{C_{12} + l_{12}}\right] + 6 * B_1 * \text{Log}\left[\frac{C_{11} + l_{11}}{C_{21} + l_{21}}\right] \right) (*\text{Eq. (9)} *) \\
 Out[*] = & \frac{1}{6 r^3} \\
 & G (-\lambda_1 + \lambda_2) \rho \left((r^4 + r^2 r_1^2 + 4 r_1^4 - r r_1 (r^2 + 4 r_1^2) \cos[\theta_1] - r^2 (3 r^2 + r_1^2) \cos[2 \theta_1] + \right. \\
 & \quad 3 r^3 r_1 \cos[3 \theta_1]) / (\sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_1]}) - \\
 & \quad (r^4 + r^2 r_2^2 + 4 r_2^4 - r r_2 (r^2 + 4 r_2^2) \cos[\theta_1] - r^2 (3 r^2 + r_2^2) \cos[2 \theta_1] + \\
 & \quad 3 r^3 r_2 \cos[3 \theta_1]) / (\sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_1]}) - \\
 & \quad (r^4 + r^2 r_1^2 + 4 r_1^4 - r r_1 (r^2 + 4 r_1^2) \cos[\theta_2] - r^2 (3 r^2 + r_1^2) \cos[2 \theta_2] + \\
 & \quad 3 r^3 r_1 \cos[3 \theta_2]) / (\sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_2]}) + \\
 & \quad (r^4 + r^2 r_2^2 + 4 r_2^4 - r r_2 (r^2 + 4 r_2^2) \cos[\theta_2] - r^2 (3 r^2 + r_2^2) \cos[2 \theta_2] + \\
 & \quad 3 r^3 r_2 \cos[3 \theta_2]) / (\sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_2]}) + \\
 & \quad 6 r^3 \cos[\theta_1] \text{Log}\left[\frac{r_1 - r \cos[\theta_1] + \sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_1]}}{r_2 - r \cos[\theta_1] + \sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_1]}}\right] \sin[\theta_1]^2 + \\
 & \quad \left. 6 r^3 \cos[\theta_2] \text{Log}\left[\frac{r_2 - r \cos[\theta_2] + \sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_2]}}{r_1 - r \cos[\theta_2] + \sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_2]}}\right] \sin[\theta_2]^2 \right)
 \end{aligned}$$

$$In[*]:= V_{xx} = \frac{G * \rho}{48} * \left(F2 * \text{Log}\left[\frac{C22 + l22}{C12 + l12}\right] + F1 * \text{Log}\left[\frac{C11 + l11}{C21 + l21}\right] + \right. \\ \left. \frac{1}{r^3 * l22} * (H2 + I22 + J2 + K22) - \frac{1}{r^3 * l12} * (H1 + I12 + J1 + K12) - \right. \\ \left. \frac{1}{r^3 * l21} * (H2 + I21 + J2 + K21) + \frac{1}{r^3 * l11} * (H1 + I11 + J1 + K11) \right) (*Eq. (14) *)$$

Out[*]=

$$\frac{1}{48} G \rho \left(-3 \cos[\theta 1] \text{Log}\left[\frac{r1 - r \cos[\theta 1] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 1]}}{r2 - r \cos[\theta 1] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 1]}}\right] \right. \\ \left(8 (-\lambda 1 + \lambda 2) \sin[\theta 1]^2 - 4 (5 - \cos[2 \theta 1]) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) - \\ 3 \cos[\theta 2] \text{Log}\left[\frac{r2 - r \cos[\theta 2] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 2]}}{r1 - r \cos[\theta 2] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 2]}}\right] \\ \left(8 (-\lambda 1 + \lambda 2) \sin[\theta 2]^2 - 4 (5 - \cos[2 \theta 2]) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) + \\ \frac{1}{r^3 \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 1]}} \\ \left(-4 (r^4 + r^2 r1^2 + 4 r1^4) (-\lambda 1 + \lambda 2) + 2 r^2 \left((3 r^2 + r1^2) \cos[2 \theta 1] - 3 r r1 \cos[3 \theta 1] \right) \right. \\ \left(-2 \lambda 1 + 2 \lambda 2 + \sin[2 (\lambda - \lambda 1)] - \sin[2 (\lambda - \lambda 2)] \right) + \\ 4 (13 r^4 + 13 r^2 r1^2 + 4 r1^4) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 4 r r1 \cos[\theta 1] \\ \left((r^2 + 4 r1^2) (-\lambda 1 + \lambda 2) - (25 r^2 + 4 r1^2) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \left. \right) - \\ \frac{1}{r^3 \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 2]}} \\ \left(-4 (r^4 + r^2 r1^2 + 4 r1^4) (-\lambda 1 + \lambda 2) + \right. \\ 2 r^2 \left((3 r^2 + r1^2) \cos[2 \theta 2] - 3 r r1 \cos[3 \theta 2] \right) \\ \left(-2 \lambda 1 + 2 \lambda 2 + \sin[2 (\lambda - \lambda 1)] - \sin[2 (\lambda - \lambda 2)] \right) + \\ 4 (13 r^4 + 13 r^2 r1^2 + 4 r1^4) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 4 r r1 \cos[\theta 2] \\ \left((r^2 + 4 r1^2) (-\lambda 1 + \lambda 2) - (25 r^2 + 4 r1^2) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \left. \right) - \\ \frac{1}{r^3 \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 1]}} \\ \left(-4 (r^4 + r^2 r2^2 + 4 r2^4) (-\lambda 1 + \lambda 2) + \right. \\ 2 r^2 \left((3 r^2 + r2^2) \cos[2 \theta 1] - 3 r r2 \cos[3 \theta 1] \right) \\ \left(-2 \lambda 1 + 2 \lambda 2 + \sin[2 (\lambda - \lambda 1)] - \sin[2 (\lambda - \lambda 2)] \right) + \\ 4 (13 r^4 + 13 r^2 r2^2 + 4 r2^4) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 4 r r2 \cos[\theta 1] \\ \left((r^2 + 4 r2^2) (-\lambda 1 + \lambda 2) - (25 r^2 + 4 r2^2) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \left. \right) + \\ \frac{1}{r^3 \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 2]}} \\ \left(-4 (r^4 + r^2 r2^2 + 4 r2^4) (-\lambda 1 + \lambda 2) + \right. \\ 2 r^2 \left((3 r^2 + r2^2) \cos[2 \theta 2] - 3 r r2 \cos[3 \theta 2] \right) \\ \left(-2 \lambda 1 + 2 \lambda 2 + \sin[2 (\lambda - \lambda 1)] - \sin[2 (\lambda - \lambda 2)] \right) + \\ 4 (13 r^4 + 13 r^2 r2^2 + 4 r2^4) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 4 r r2 \cos[\theta 2] \\ \left. \left((r^2 + 4 r2^2) (-\lambda 1 + \lambda 2) - (25 r^2 + 4 r2^2) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \right) \right)$$

$$In[*]:= \text{Vyy} = \frac{G * \rho}{48} * \left(L2 * \text{Log}\left[\frac{C22 + l22}{C12 + l12}\right] + L1 * \text{Log}\left[\frac{C11 + l11}{C21 + l21}\right] + \right. \\ \left. \frac{1}{r^3 * l22} * (H2 + M22 - J2 + N22) - \frac{1}{r^3 * l12} * (H1 + M12 - J1 + N12) - \right. \\ \left. \frac{1}{r^3 * l21} * (H2 + M21 - J2 + N21) + \frac{1}{r^3 * l11} * (H1 + M11 - J1 + N11) \right) (*Eq. (15) *)$$

Out[*]=

$$\frac{1}{48} G \rho \left(-3 \cos[\theta 1] \text{Log}\left[\frac{r1 - r \cos[\theta 1] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 1]}}{r2 - r \cos[\theta 1] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 1]}}\right] \right. \\ \left(8 (-\lambda 1 + \lambda 2) \sin[\theta 1]^2 + 4 (5 - \cos[2 \theta 1]) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) - \\ 3 \cos[\theta 2] \text{Log}\left[\frac{r2 - r \cos[\theta 2] + \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 2]}}{r1 - r \cos[\theta 2] + \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 2]}}\right] \\ \left(8 (-\lambda 1 + \lambda 2) \sin[\theta 2]^2 + 4 (5 - \cos[2 \theta 2]) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) + \\ \frac{1}{r^3 \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 1]}} \\ \left(-4 (r^4 + r^2 r1^2 + 4 r1^4) (-\lambda 1 + \lambda 2) + 2 r^2 \left((3 r^2 + r1^2) \cos[2 \theta 1] - 3 r r1 \cos[3 \theta 1] \right) \right. \\ \left. (-2 \lambda 1 + 2 \lambda 2 - \sin[2 (\lambda - \lambda 1)] + \sin[2 (\lambda - \lambda 2)]) - \right. \\ \left. 4 (13 r^4 + 13 r^2 r1^2 + 4 r1^4) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 4 r r1 \cos[\theta 1] \right. \\ \left. \left((r^2 + 4 r1^2) (-\lambda 1 + \lambda 2) + (25 r^2 + 4 r1^2) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \right) - \\ \frac{1}{r^3 \sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta 2]}} \\ \left(-4 (r^4 + r^2 r1^2 + 4 r1^4) (-\lambda 1 + \lambda 2) + \right. \\ \left. 2 r^2 \left((3 r^2 + r1^2) \cos[2 \theta 2] - 3 r r1 \cos[3 \theta 2] \right) \right. \\ \left. (-2 \lambda 1 + 2 \lambda 2 - \sin[2 (\lambda - \lambda 1)] + \sin[2 (\lambda - \lambda 2)]) - \right. \\ \left. 4 (13 r^4 + 13 r^2 r1^2 + 4 r1^4) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 4 r r1 \cos[\theta 2] \right. \\ \left. \left((r^2 + 4 r1^2) (-\lambda 1 + \lambda 2) + (25 r^2 + 4 r1^2) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \right) - \\ \frac{1}{r^3 \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 1]}} \\ \left(-4 (r^4 + r^2 r2^2 + 4 r2^4) (-\lambda 1 + \lambda 2) + \right. \\ \left. 2 r^2 \left((3 r^2 + r2^2) \cos[2 \theta 1] - 3 r r2 \cos[3 \theta 1] \right) \right. \\ \left. (-2 \lambda 1 + 2 \lambda 2 - \sin[2 (\lambda - \lambda 1)] + \sin[2 (\lambda - \lambda 2)]) - \right. \\ \left. 4 (13 r^4 + 13 r^2 r2^2 + 4 r2^4) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 4 r r2 \cos[\theta 1] \right. \\ \left. \left((r^2 + 4 r2^2) (-\lambda 1 + \lambda 2) + (25 r^2 + 4 r2^2) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \right) + \\ \frac{1}{r^3 \sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta 2]}} \\ \left(-4 (r^4 + r^2 r2^2 + 4 r2^4) (-\lambda 1 + \lambda 2) + \right. \\ \left. 2 r^2 \left((3 r^2 + r2^2) \cos[2 \theta 2] - 3 r r2 \cos[3 \theta 2] \right) \right. \\ \left. (-2 \lambda 1 + 2 \lambda 2 - \sin[2 (\lambda - \lambda 1)] + \sin[2 (\lambda - \lambda 2)]) - \right. \\ \left. 4 (13 r^4 + 13 r^2 r2^2 + 4 r2^4) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] + 4 r r2 \cos[\theta 2] \right. \\ \left. \left((r^2 + 4 r2^2) (-\lambda 1 + \lambda 2) + (25 r^2 + 4 r2^2) \cos[2 \lambda - \lambda 1 - \lambda 2] \sin[\lambda 1 - \lambda 2] \right) \right) \left. \right)$$

$$In[*]:= V_{zzz} = -\frac{G * \rho * (\lambda_2 - \lambda_1)}{2 * r^4} * \left(\frac{022}{l_{22}^3} - \frac{012}{l_{12}^3} - \frac{021}{l_{21}^3} + \frac{011}{l_{11}^3} \right) (*Eq. (17)*)$$

Out[*]=

$$-\frac{1}{2 r^4} G (-\lambda_1 + \lambda_2) \rho \left(\frac{r_1^3 (9 r^2 r_1 + 4 r_1^3 - 4 r (r^2 + 3 r_1^2) \cos[\theta_1] + 3 r^2 r_1 \cos[2 \theta_1])}{(r^2 + r_1^2 - 2 r r_1 \cos[\theta_1])^{3/2}} - \frac{r_2^3 (9 r^2 r_2 + 4 r_2^3 - 4 r (r^2 + 3 r_2^2) \cos[\theta_1] + 3 r^2 r_2 \cos[2 \theta_1])}{(r^2 + r_2^2 - 2 r r_2 \cos[\theta_1])^{3/2}} - \frac{r_1^3 (9 r^2 r_1 + 4 r_1^3 - 4 r (r^2 + 3 r_1^2) \cos[\theta_2] + 3 r^2 r_1 \cos[2 \theta_2])}{(r^2 + r_1^2 - 2 r r_1 \cos[\theta_2])^{3/2}} + \frac{r_2^3 (9 r^2 r_2 + 4 r_2^3 - 4 r (r^2 + 3 r_2^2) \cos[\theta_2] + 3 r^2 r_2 \cos[2 \theta_2])}{(r^2 + r_2^2 - 2 r r_2 \cos[\theta_2])^{3/2}} \right)$$

$$In[*]:= V_{xxz} = \frac{G * \rho}{16 * r^4} * \left(\frac{P_{22} + Q_{22} + R_{22} + S_{22} + T_2}{l_{22}^3} - \frac{P_{12} + Q_{12} + R_{12} + S_{12} + T_1}{l_{12}^3} - \frac{P_{21} + Q_{21} + R_{21} + S_{21} + T_2}{l_{21}^3} + \frac{P_{11} + Q_{11} + R_{11} + S_{11} + T_1}{l_{11}^3} \right) (*Eq. (18)*)$$

Out[*]=

$$\frac{1}{16 r^4} G \rho \left(\frac{1}{(r^2 + r_1^2 - 2 r r_1 \cos[\theta_1])^{3/2}} \left(4 r_1^4 (9 r^2 + 4 r_1^2) (-\lambda_1 + \lambda_2) - 5 (r^2 + r_1^2)^3 (-\lambda_1 + \lambda_2) \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} + 6 r^2 r_1^4 \cos[2 \theta_1] \right. \right. \\ \left. \left. (-2 \lambda_1 + 2 \lambda_2 + \sin[2 (\lambda - \lambda_1)] - \sin[2 (\lambda - \lambda_2)]) + \left(5 r^6 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} + r_1^6 \left(-16 + 5 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + r^2 r_1^4 \left(-52 + 15 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + \right. \right. \right. \\ \left. \left. r^4 r_1^2 \left(-16 + 15 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) \cos[2 \lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] + \right. \\ \left. 2 r r_1 \cos[\theta_1] \left((-\lambda_1 + \lambda_2) \left(5 r^4 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} + r_1^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + 2 r^2 r_1^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) \right. \right. \\ \left. \left. (r^2 + r_1^2) \left(5 r^2 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} + r_1^2 \left(-24 + 5 \sqrt{1 - \frac{2 r r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) \cos[2 \lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \right) \right) - \\ \frac{1}{(r^2 + r_2^2 - 2 r r_2 \cos[\theta_1])^{3/2}} \left(4 r_2^4 (9 r^2 + 4 r_2^2) (-\lambda_1 + \lambda_2) - \right.$$

$$\begin{aligned}
& 5 (r^2 + r2^2)^3 (-\lambda1 + \lambda2) \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + \\
& 6 r^2 r2^4 \cos[2 \theta1] (-2 \lambda1 + 2 \lambda2 + \sin[2 (\lambda - \lambda1)] - \sin[2 (\lambda - \lambda2)]) + \\
& \left(5 r^6 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + r2^6 \left(-16 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) + r^2 r2^4 \right. \\
& \quad \left. \left(-52 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) + r^4 r2^2 \left(-16 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) \right) \\
& \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 2 r r2 \cos[\theta1] \\
& \left((-\lambda1 + \lambda2) \left(5 r^4 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + r2^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) + 2 r^2 \right. \right. \\
& \quad \left. r2^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) \right) - (r^2 + r2^2) \left(5 r^2 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} + \right. \\
& \quad \left. r2^2 \left(-24 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \right) \right) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] \Bigg) - \\
& \frac{1}{(r^2 + r1^2 - 2 r r1 \cos[\theta2])^{3/2}} \left(4 r1^4 (9 r^2 + 4 r1^2) (-\lambda1 + \lambda2) - \right. \\
& 5 (r^2 + r1^2)^3 (-\lambda1 + \lambda2) \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + \\
& 6 r^2 r1^4 \cos[2 \theta2] (-2 \lambda1 + 2 \lambda2 + \sin[2 (\lambda - \lambda1)] - \sin[2 (\lambda - \lambda2)]) + \\
& \left(5 r^6 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + r1^6 \left(-16 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) + r^2 r1^4 \right. \\
& \quad \left. \left(-52 + 15 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) + r^4 r1^2 \left(-16 + 15 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) \right) \\
& \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] + 2 r r1 \cos[\theta2] \\
& \left((-\lambda1 + \lambda2) \left(5 r^4 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + r1^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) + 2 r^2 \right. \right. \\
& \quad \left. r1^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) \right) - (r^2 + r1^2) \left(5 r^2 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} + \right. \\
& \quad \left. r1^2 \left(-24 + 5 \sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \right) \right) \cos[2 \lambda - \lambda1 - \lambda2] \sin[\lambda1 - \lambda2] \Bigg) + \\
& \frac{1}{(r^2 + r2^2 - 2 r r2 \cos[\theta2])^{3/2}} \left(4 r2^4 (9 r^2 + 4 r2^2) (-\lambda1 + \lambda2) - \right. \\
& 5 (r^2 + r2^2)^3 (-\lambda1 + \lambda2) \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + \\
& 6 r^2 r2^4 \cos[2 \theta2] (-2 \lambda1 + 2 \lambda2 + \sin[2 (\lambda - \lambda1)] - \sin[2 (\lambda - \lambda2)]) + \\
& \left(5 r^6 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} + r2^6 \left(-16 + 5 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) + r^2 r2^4 \right. \\
& \quad \left. \left(-52 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) + r^4 r2^2 \left(-16 + 15 \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \right) \right)
\end{aligned}$$

$$\cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] + 2r_1 r_2 \cos[\theta_2]$$

$$\left((-\lambda_1 + \lambda_2) \left(5r^4 \sqrt{1 - \frac{2r_1 r_2 \cos[\theta_2]}{r^2 + r_2^2}} + r_2^4 \left(-24 + 5 \sqrt{1 - \frac{2r_1 r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) + 2r^2 \right. \right. \\ \left. \left. r_2^2 \left(-4 + 5 \sqrt{1 - \frac{2r_1 r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) \right) - (r^2 + r_2^2) \left(5r^2 \sqrt{1 - \frac{2r_1 r_2 \cos[\theta_2]}{r^2 + r_2^2}} + \right. \right. \\ \left. \left. r_2^2 \left(-24 + 5 \sqrt{1 - \frac{2r_1 r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) \right) \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \right) \right)$$

$$In[*]:= V_{yyz} = \frac{G * \rho}{16 * r^4} * \left(\frac{V_{22} - Q_{22} + W_{22} + S_{22} + T_2}{l_{22}^3} - \frac{V_{12} - Q_{12} + W_{12} + S_{12} + T_1}{l_{12}^3} - \right. \\ \left. \frac{V_{21} - Q_{21} + W_{21} + S_{21} + T_2}{l_{21}^3} + \frac{V_{11} - Q_{11} + W_{11} + S_{11} + T_1}{l_{11}^3} \right) (*Eq. (19)*)$$

Out[*]=

$$\frac{1}{16 r^4}$$

$$G \rho \left(\frac{1}{(r^2 + r_1^2 - 2r_1 r_1 \cos[\theta_1])^{3/2}} \left(4r_1^4 (9r^2 + 4r_1^2) (-\lambda_1 + \lambda_2) - 5(r^2 + r_1^2)^3 (-\lambda_1 + \lambda_2) \right. \right. \\ \left. \left. \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} + 6r^2 r_1^4 \cos[2\theta_1] \right. \right. \\ \left. \left. (-2\lambda_1 + 2\lambda_2 - \sin[2(\lambda - \lambda_1)] + \sin[2(\lambda - \lambda_2)]) - \left(5r^6 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} + \right. \right. \right. \\ \left. \left. r_1^6 \left(-16 + 5 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + r^2 r_1^4 \left(-52 + 15 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + \right. \right. \\ \left. \left. r^4 r_1^2 \left(-16 + 15 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] + \right. \\ \left. 2r_1 r_1 \cos[\theta_1] \left((-\lambda_1 + \lambda_2) \left(5r^4 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} + r_1^4 \right. \right. \right. \\ \left. \left. \left(-24 + 5 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) + 2r^2 r_1^2 \left(-4 + 5 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) \right) + \\ \left. (r^2 + r_1^2) \left(5r^2 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} + r_1^2 \left(-24 + 5 \sqrt{1 - \frac{2r_1 r_1 \cos[\theta_1]}{r^2 + r_1^2}} \right) \right) \right) \\ \left. \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \right) \right) - \\ \frac{1}{(r^2 + r_2^2 - 2r_1 r_2 \cos[\theta_1])^{3/2}} \left(4r_2^4 (9r^2 + 4r_2^2) (-\lambda_1 + \lambda_2) - \right. \\ \left. 5(r^2 + r_2^2)^3 (-\lambda_1 + \lambda_2) \sqrt{1 - \frac{2r_1 r_2 \cos[\theta_1]}{r^2 + r_2^2}} + \right. \\ \left. 6r^2 r_2^4 \cos[2\theta_1] (-2\lambda_1 + 2\lambda_2 - \sin[2(\lambda - \lambda_1)] + \sin[2(\lambda - \lambda_2)]) - \right. \\ \left. \left(5r^6 \sqrt{1 - \frac{2r_1 r_2 \cos[\theta_1]}{r^2 + r_2^2}} + r_2^6 \left(-16 + 5 \sqrt{1 - \frac{2r_1 r_2 \cos[\theta_1]}{r^2 + r_2^2}} \right) + r^2 r_2^4 \right. \right.$$

$$\begin{aligned}
& \left(-52 + 15 \sqrt{1 - \frac{2 r r_2 \cos[\theta_1]}{r^2 + r_2^2}} \right) + r^4 r_2^2 \left(-16 + 15 \sqrt{1 - \frac{2 r r_2 \cos[\theta_1]}{r^2 + r_2^2}} \right) \Bigg) \\
& \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] + 2 r r_2 \cos[\theta_1] \\
& \left((-\lambda_1 + \lambda_2) \left(5 r^4 \sqrt{1 - \frac{2 r r_2 \cos[\theta_1]}{r^2 + r_2^2}} + r_2^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r_2 \cos[\theta_1]}{r^2 + r_2^2}} \right) + 2 r^2 \right. \right. \\
& \quad \left. \left. r_2^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r_2 \cos[\theta_1]}{r^2 + r_2^2}} \right) \right) \right) + (r^2 + r_2^2) \left(5 r^2 \sqrt{1 - \frac{2 r r_2 \cos[\theta_1]}{r^2 + r_2^2}} + \right. \\
& \quad \left. r_2^2 \left(-24 + 5 \sqrt{1 - \frac{2 r r_2 \cos[\theta_1]}{r^2 + r_2^2}} \right) \right) \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \Bigg) - \\
& \frac{1}{(r^2 + r_1^2 - 2 r r_1 \cos[\theta_2])^{3/2}} \left(4 r_1^4 (9 r^2 + 4 r_1^2) (-\lambda_1 + \lambda_2) - \right. \\
& \quad 5 (r^2 + r_1^2)^3 (-\lambda_1 + \lambda_2) \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} + \\
& \quad 6 r^2 r_1^4 \cos[2\theta_2] (-2\lambda_1 + 2\lambda_2 - \sin[2(\lambda - \lambda_1)] + \sin[2(\lambda - \lambda_2)]) - \\
& \quad \left(5 r^6 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} + r_1^6 \left(-16 + 5 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} \right) + r^2 r_1^4 \right. \\
& \quad \left. \left(-52 + 15 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} \right) + r^4 r_1^2 \left(-16 + 15 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} \right) \right) \Bigg) \\
& \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] + 2 r r_1 \cos[\theta_2] \\
& \left((-\lambda_1 + \lambda_2) \left(5 r^4 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} + r_1^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} \right) + 2 r^2 \right. \right. \\
& \quad \left. \left. r_1^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} \right) \right) \right) + (r^2 + r_1^2) \left(5 r^2 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} + \right. \\
& \quad \left. r_1^2 \left(-24 + 5 \sqrt{1 - \frac{2 r r_1 \cos[\theta_2]}{r^2 + r_1^2}} \right) \right) \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \Bigg) + \\
& \frac{1}{(r^2 + r_2^2 - 2 r r_2 \cos[\theta_2])^{3/2}} \left(4 r_2^4 (9 r^2 + 4 r_2^2) (-\lambda_1 + \lambda_2) - \right. \\
& \quad 5 (r^2 + r_2^2)^3 (-\lambda_1 + \lambda_2) \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} + \\
& \quad 6 r^2 r_2^4 \cos[2\theta_2] (-2\lambda_1 + 2\lambda_2 - \sin[2(\lambda - \lambda_1)] + \sin[2(\lambda - \lambda_2)]) - \\
& \quad \left(5 r^6 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} + r_2^6 \left(-16 + 5 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) + r^2 r_2^4 \right. \\
& \quad \left. \left(-52 + 15 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) + r^4 r_2^2 \left(-16 + 15 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) \right) \Bigg) \\
& \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] + 2 r r_2 \cos[\theta_2] \\
& \left((-\lambda_1 + \lambda_2) \left(5 r^4 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} + r_2^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) + 2 r^2 \right. \right. \\
& \quad \left. \left. r_2^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) \right) \right) + (r^2 + r_2^2) \left(5 r^2 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} + \right. \\
& \quad \left. r_2^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) \right) \cos[2\lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \Bigg) +
\end{aligned}$$

$$r^2 \left(-24 + 5 \sqrt{1 - \frac{2 r r_2 \cos[\theta_2]}{r^2 + r_2^2}} \right) \cos[2 \lambda - \lambda_1 - \lambda_2] \sin[\lambda_1 - \lambda_2] \Bigg) \Bigg)$$

In[*]:= ZonalBandVzz =

$$\frac{\text{Pi} * \text{G} * \rho}{3 * r^3} * \left(\frac{\text{E22}}{\text{l22}} - \frac{\text{E12}}{\text{l12}} - \frac{\text{E21}}{\text{l21}} + \frac{\text{E11}}{\text{l11}} + 6 * \text{B2} * \text{Log}\left[\frac{\text{C22} + \text{l22}}{\text{C12} + \text{l12}}\right] + 6 * \text{B1} * \text{Log}\left[\frac{\text{C11} + \text{l11}}{\text{C21} + \text{l21}}\right] \right)$$

(*Eq. (24)*)

Out[*]=

$$\begin{aligned} & \frac{1}{3 r^3} \text{G} \pi \rho \left(\left(r^4 + r^2 r_1^2 + 4 r_1^4 - r r_1 (r^2 + 4 r_1^2) \cos[\theta_1] - \right. \right. \\ & \quad \left. r^2 (3 r^2 + r_1^2) \cos[2 \theta_1] + 3 r^3 r_1 \cos[3 \theta_1] \right) / \left(\sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_1]} \right) - \\ & \quad \left(r^4 + r^2 r_2^2 + 4 r_2^4 - r r_2 (r^2 + 4 r_2^2) \cos[\theta_1] - r^2 (3 r^2 + r_2^2) \cos[2 \theta_1] + \right. \\ & \quad \left. 3 r^3 r_2 \cos[3 \theta_1] \right) / \left(\sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_1]} \right) - \\ & \quad \left(r^4 + r^2 r_1^2 + 4 r_1^4 - r r_1 (r^2 + 4 r_1^2) \cos[\theta_2] - r^2 (3 r^2 + r_1^2) \cos[2 \theta_2] + \right. \\ & \quad \left. 3 r^3 r_1 \cos[3 \theta_2] \right) / \left(\sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_2]} \right) + \\ & \quad \left(r^4 + r^2 r_2^2 + 4 r_2^4 - r r_2 (r^2 + 4 r_2^2) \cos[\theta_2] - r^2 (3 r^2 + r_2^2) \cos[2 \theta_2] + \right. \\ & \quad \left. 3 r^3 r_2 \cos[3 \theta_2] \right) / \left(\sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_2]} \right) + \\ & \quad 6 r^3 \cos[\theta_1] \text{Log}\left[\frac{r_1 - r \cos[\theta_1] + \sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_1]}}{r_2 - r \cos[\theta_1] + \sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_1]}}\right] \sin[\theta_1]^2 + \\ & \quad \left. 6 r^3 \cos[\theta_2] \text{Log}\left[\frac{r_2 - r \cos[\theta_2] + \sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_2]}}{r_1 - r \cos[\theta_2] + \sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_2]}}\right] \sin[\theta_2]^2 \right) \end{aligned}$$

In[*]:= ZonalBandVxx =

$$-\frac{\text{Pi} * \text{G} * \rho}{6 * r^3} * \left(\frac{\text{E22}}{\text{l22}} - \frac{\text{E12}}{\text{l12}} - \frac{\text{E21}}{\text{l21}} + \frac{\text{E11}}{\text{l11}} + 6 * \text{B2} * \text{Log}\left[\frac{\text{C22} + \text{l22}}{\text{C12} + \text{l12}}\right] + 6 * \text{B1} * \text{Log}\left[\frac{\text{C11} + \text{l11}}{\text{C21} + \text{l21}}\right] \right)$$

(*Eq. (25)*)

Out[*]=

$$\begin{aligned} & -\frac{1}{6 r^3} \text{G} \pi \rho \left(\left(r^4 + r^2 r_1^2 + 4 r_1^4 - r r_1 (r^2 + 4 r_1^2) \cos[\theta_1] - \right. \right. \\ & \quad \left. r^2 (3 r^2 + r_1^2) \cos[2 \theta_1] + 3 r^3 r_1 \cos[3 \theta_1] \right) / \left(\sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_1]} \right) - \\ & \quad \left(r^4 + r^2 r_2^2 + 4 r_2^4 - r r_2 (r^2 + 4 r_2^2) \cos[\theta_1] - r^2 (3 r^2 + r_2^2) \cos[2 \theta_1] + \right. \\ & \quad \left. 3 r^3 r_2 \cos[3 \theta_1] \right) / \left(\sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_1]} \right) - \\ & \quad \left(r^4 + r^2 r_1^2 + 4 r_1^4 - r r_1 (r^2 + 4 r_1^2) \cos[\theta_2] - r^2 (3 r^2 + r_1^2) \cos[2 \theta_2] + \right. \\ & \quad \left. 3 r^3 r_1 \cos[3 \theta_2] \right) / \left(\sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_2]} \right) + \\ & \quad \left(r^4 + r^2 r_2^2 + 4 r_2^4 - r r_2 (r^2 + 4 r_2^2) \cos[\theta_2] - r^2 (3 r^2 + r_2^2) \cos[2 \theta_2] + \right. \\ & \quad \left. 3 r^3 r_2 \cos[3 \theta_2] \right) / \left(\sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_2]} \right) + \\ & \quad 6 r^3 \cos[\theta_1] \text{Log}\left[\frac{r_1 - r \cos[\theta_1] + \sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_1]}}{r_2 - r \cos[\theta_1] + \sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_1]}}\right] \sin[\theta_1]^2 + \\ & \quad \left. 6 r^3 \cos[\theta_2] \text{Log}\left[\frac{r_2 - r \cos[\theta_2] + \sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_2]}}{r_1 - r \cos[\theta_2] + \sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_2]}}\right] \sin[\theta_2]^2 \right) \end{aligned}$$

In[*]:= ZonalBandVyy =

$$-\frac{\text{Pi} * \text{G} * \rho}{6 * r^3} * \left(\frac{\text{E22}}{\text{l22}} - \frac{\text{E12}}{\text{l12}} - \frac{\text{E21}}{\text{l21}} + \frac{\text{E11}}{\text{l11}} + 6 * \text{B2} * \text{Log}\left[\frac{\text{C22} + \text{l22}}{\text{C12} + \text{l12}}\right] + 6 * \text{B1} * \text{Log}\left[\frac{\text{C11} + \text{l11}}{\text{C21} + \text{l21}}\right] \right)$$

(*Eq. (25)*)

Out[*]=

$$\begin{aligned} & -\frac{1}{6 r^3} \text{G} \pi \rho \left(\left(r^4 + r^2 r_1^2 + 4 r_1^4 - r r_1 (r^2 + 4 r_1^2) \cos[\theta_1] - \right. \right. \\ & \quad \left. r^2 (3 r^2 + r_1^2) \cos[2 \theta_1] + 3 r^3 r_1 \cos[3 \theta_1] \right) / \left(\sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_1]} \right) - \\ & \quad \left(r^4 + r^2 r_2^2 + 4 r_2^4 - r r_2 (r^2 + 4 r_2^2) \cos[\theta_1] - r^2 (3 r^2 + r_2^2) \cos[2 \theta_1] + \right. \\ & \quad \left. 3 r^3 r_2 \cos[3 \theta_1] \right) / \left(\sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_1]} \right) - \\ & \quad \left(r^4 + r^2 r_1^2 + 4 r_1^4 - r r_1 (r^2 + 4 r_1^2) \cos[\theta_2] - r^2 (3 r^2 + r_1^2) \cos[2 \theta_2] + \right. \\ & \quad \left. 3 r^3 r_1 \cos[3 \theta_2] \right) / \left(\sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_2]} \right) + \\ & \quad \left(r^4 + r^2 r_2^2 + 4 r_2^4 - r r_2 (r^2 + 4 r_2^2) \cos[\theta_2] - r^2 (3 r^2 + r_2^2) \cos[2 \theta_2] + \right. \\ & \quad \left. 3 r^3 r_2 \cos[3 \theta_2] \right) / \left(\sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_2]} \right) + \\ & \quad 6 r^3 \cos[\theta_1] \text{Log}\left[\frac{r_1 - r \cos[\theta_1] + \sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_1]}}{r_2 - r \cos[\theta_1] + \sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_1]}}\right] \sin[\theta_1]^2 + \\ & \quad \left. 6 r^3 \cos[\theta_2] \text{Log}\left[\frac{r_2 - r \cos[\theta_2] + \sqrt{r^2 + r_2^2 - 2 r r_2 \cos[\theta_2]}}{r_1 - r \cos[\theta_2] + \sqrt{r^2 + r_1^2 - 2 r r_1 \cos[\theta_2]}}\right] \sin[\theta_2]^2 \right) \end{aligned}$$

$$\text{In}[*]:= \text{ZonalBandVzzz} = -\frac{\text{Pi} * \text{G} * \rho}{r^4} * \left(\frac{022}{l22^3} - \frac{012}{l12^3} - \frac{021}{l21^3} + \frac{011}{l11^3} \right) (*\text{Eq. (26)} *)$$

Out[*]=

$$-\frac{1}{r^4} \text{G} \pi \rho \left(\frac{r1^3 (9 r^2 r1 + 4 r1^3 - 4 r (r^2 + 3 r1^2) \cos[\theta1] + 3 r^2 r1 \cos[2 \theta1])}{(r^2 + r1^2 - 2 r r1 \cos[\theta1])^{3/2}} - \frac{r2^3 (9 r^2 r2 + 4 r2^3 - 4 r (r^2 + 3 r2^2) \cos[\theta1] + 3 r^2 r2 \cos[2 \theta1])}{(r^2 + r2^2 - 2 r r2 \cos[\theta1])^{3/2}} - \frac{r1^3 (9 r^2 r1 + 4 r1^3 - 4 r (r^2 + 3 r1^2) \cos[\theta2] + 3 r^2 r1 \cos[2 \theta2])}{(r^2 + r1^2 - 2 r r1 \cos[\theta2])^{3/2}} + \frac{r2^3 (9 r^2 r2 + 4 r2^3 - 4 r (r^2 + 3 r2^2) \cos[\theta2] + 3 r^2 r2 \cos[2 \theta2])}{(r^2 + r2^2 - 2 r r2 \cos[\theta2])^{3/2}} \right)$$

$$\begin{aligned}
In[*]:= & \text{ZonalBandVxxz} = \frac{\text{Pi} * \text{G} * \rho}{8 * r^4} * \\
& \left(\frac{\text{X22} + \text{Y22} + \text{Z22}}{\text{l22}^3} - \frac{\text{X12} + \text{Y12} + \text{Z12}}{\text{l12}^3} - \frac{\text{X21} + \text{Y21} + \text{Z21}}{\text{l21}^3} + \frac{\text{X11} + \text{Y11} + \text{Z11}}{\text{l11}^3} \right) (*\text{Eq. (27)} *) \\
Out[*]= & \frac{1}{8 r^4} \text{G} \pi \rho \\
& \left(\left(-5 (r^2 + r1^2)^3 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta1]}{r^2 + r1^2}} + 2 r r1 \text{Cos}[\theta1] \left(5 r^4 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta1]}{r^2 + r1^2}} + r1^4 \right. \right. \right. \\
& \left. \left. \left(-24 + 5 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta1]}{r^2 + r1^2}} \right) + 2 r^2 r1^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta1]}{r^2 + r1^2}} \right) \right) \right) + \\
& \left. 4 r1^4 (9 r^2 + 4 r1^2 + 3 r^2 \text{Cos}[2 \theta1]) \right) / (r^2 + r1^2 - 2 r r1 \text{Cos}[\theta1])^{3/2} - \\
& \left(-5 (r^2 + r2^2)^3 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta1]}{r^2 + r2^2}} + 2 r r2 \text{Cos}[\theta1] \left(5 r^4 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta1]}{r^2 + r2^2}} + \right. \right. \\
& r2^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta1]}{r^2 + r2^2}} \right) + 2 r^2 r2^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta1]}{r^2 + r2^2}} \right) \right) + \\
& \left. 4 r2^4 (9 r^2 + 4 r2^2 + 3 r^2 \text{Cos}[2 \theta1]) \right) / (r^2 + r2^2 - 2 r r2 \text{Cos}[\theta1])^{3/2} - \\
& \left(-5 (r^2 + r1^2)^3 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta2]}{r^2 + r1^2}} + 2 r r1 \text{Cos}[\theta2] \left(5 r^4 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta2]}{r^2 + r1^2}} + \right. \right. \\
& r1^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta2]}{r^2 + r1^2}} \right) + 2 r^2 r1^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta2]}{r^2 + r1^2}} \right) \right) + \\
& \left. 4 r1^4 (9 r^2 + 4 r1^2 + 3 r^2 \text{Cos}[2 \theta2]) \right) / (r^2 + r1^2 - 2 r r1 \text{Cos}[\theta2])^{3/2} + \\
& \left(-5 (r^2 + r2^2)^3 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta2]}{r^2 + r2^2}} + 2 r r2 \text{Cos}[\theta2] \left(5 r^4 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta2]}{r^2 + r2^2}} + \right. \right. \\
& r2^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta2]}{r^2 + r2^2}} \right) + 2 r^2 r2^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta2]}{r^2 + r2^2}} \right) \right) + \\
& \left. 4 r2^4 (9 r^2 + 4 r2^2 + 3 r^2 \text{Cos}[2 \theta2]) \right) / (r^2 + r2^2 - 2 r r2 \text{Cos}[\theta2])^{3/2} \Big)
\end{aligned}$$

$$\begin{aligned}
\text{In[*]} := \text{ZonalBandVyyz} &= \frac{\text{Pi} * \text{G} * \rho}{8 * r^4} * \\
&\left(\frac{\text{X22} + \text{Y22} + \text{Z22}}{\text{l22}^3} - \frac{\text{X12} + \text{Y12} + \text{Z12}}{\text{l12}^3} - \frac{\text{X21} + \text{Y21} + \text{Z21}}{\text{l21}^3} + \frac{\text{X11} + \text{Y11} + \text{Z11}}{\text{l11}^3} \right) (*\text{Eq. (27)} *) \\
\text{Out[*]} = &\frac{1}{8 r^4} \text{G} \pi \rho \\
&\left(\left(-5 (r^2 + r1^2)^3 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta1]}{r^2 + r1^2}} + 2 r r1 \text{Cos}[\theta1] \left(5 r^4 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta1]}{r^2 + r1^2}} + r1^4 \right. \right. \right. \\
&\quad \left. \left. \left(-24 + 5 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta1]}{r^2 + r1^2}} \right) + 2 r^2 r1^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta1]}{r^2 + r1^2}} \right) \right) \right) + \\
&\quad \left. 4 r1^4 (9 r^2 + 4 r1^2 + 3 r^2 \text{Cos}[2 \theta1]) \right) / (r^2 + r1^2 - 2 r r1 \text{Cos}[\theta1])^{3/2} - \\
&\left(-5 (r^2 + r2^2)^3 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta1]}{r^2 + r2^2}} + 2 r r2 \text{Cos}[\theta1] \left(5 r^4 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta1]}{r^2 + r2^2}} + \right. \right. \\
&\quad \left. r2^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta1]}{r^2 + r2^2}} \right) + 2 r^2 r2^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta1]}{r^2 + r2^2}} \right) \right) \right) + \\
&\quad \left. 4 r2^4 (9 r^2 + 4 r2^2 + 3 r^2 \text{Cos}[2 \theta1]) \right) / (r^2 + r2^2 - 2 r r2 \text{Cos}[\theta1])^{3/2} - \\
&\left(-5 (r^2 + r1^2)^3 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta2]}{r^2 + r1^2}} + 2 r r1 \text{Cos}[\theta2] \left(5 r^4 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta2]}{r^2 + r1^2}} + \right. \right. \\
&\quad \left. r1^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta2]}{r^2 + r1^2}} \right) + 2 r^2 r1^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r1 \text{Cos}[\theta2]}{r^2 + r1^2}} \right) \right) \right) + \\
&\quad \left. 4 r1^4 (9 r^2 + 4 r1^2 + 3 r^2 \text{Cos}[2 \theta2]) \right) / (r^2 + r1^2 - 2 r r1 \text{Cos}[\theta2])^{3/2} + \\
&\left(-5 (r^2 + r2^2)^3 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta2]}{r^2 + r2^2}} + 2 r r2 \text{Cos}[\theta2] \left(5 r^4 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta2]}{r^2 + r2^2}} + \right. \right. \\
&\quad \left. r2^4 \left(-24 + 5 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta2]}{r^2 + r2^2}} \right) + 2 r^2 r2^2 \left(-4 + 5 \sqrt{1 - \frac{2 r r2 \text{Cos}[\theta2]}{r^2 + r2^2}} \right) \right) \right) + \\
&\quad \left. 4 r2^4 (9 r^2 + 4 r2^2 + 3 r^2 \text{Cos}[2 \theta2]) \right) / (r^2 + r2^2 - 2 r r2 \text{Cos}[\theta2])^{3/2}
\end{aligned}$$

3. Test Laplace's equation

$$\begin{aligned}
\text{In[*]} := \text{FullSimplify}[\text{Vxx} + \text{Vyy} + \text{Vzz}] \\
\text{Out[*]} = 0
\end{aligned}$$

```
In[*]:= FullSimplify[
  (Vxxz + Vyyz + Vzzz) /. {
     $\sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \rightarrow \frac{\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}}{\sqrt{r^2 + r2^2}},$ 
     $\sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \rightarrow \frac{\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}}{\sqrt{r^2 + r1^2}},$ 
     $\sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \rightarrow$ 
 $\frac{\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}}{\sqrt{r^2 + r2^2}},$ 
     $\sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} \rightarrow \frac{\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}}{\sqrt{r^2 + r1^2}}$ 
  }]
```

Out[*]=

0

```
In[*]:= FullSimplify[ZonalBandVxx + ZonalBandVyy + ZonalBandVzz]
```

Out[*]=

0

```
In[*]:= FullSimplify[(ZonalBandVxxz + ZonalBandVyyz + ZonalBandVzzz) /.
```

$$\left\{ \sqrt{1 - \frac{2 r r2 \cos[\theta2]}{r^2 + r2^2}} \rightarrow \frac{\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta2]}}{\sqrt{r^2 + r2^2}}, \right.$$

$$\sqrt{1 - \frac{2 r r1 \cos[\theta2]}{r^2 + r1^2}} \rightarrow \frac{\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta2]}}{\sqrt{r^2 + r1^2}}, \sqrt{1 - \frac{2 r r2 \cos[\theta1]}{r^2 + r2^2}} \rightarrow$$

$$\left. \frac{\sqrt{r^2 + r2^2 - 2 r r2 \cos[\theta1]}}{\sqrt{r^2 + r2^2}}, \sqrt{1 - \frac{2 r r1 \cos[\theta1]}{r^2 + r1^2}} \rightarrow \frac{\sqrt{r^2 + r1^2 - 2 r r1 \cos[\theta1]}}{\sqrt{r^2 + r1^2}} \right\}]$$

Out[*]=

0

```
In[*]:= NotebookSave[EvaluationNotebook[]];
```