

# Block Matrix Multiplication

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The Matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 2 & 2 & 7 \\ 1 & 5 & 6 & 2 \\ 3 & 3 & 4 & 5 \\ 3 & 3 & 6 & 7 \end{bmatrix}$$

can be partitioned into four 2×2 blocks

$$\mathbf{P}_{11} = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}, \quad \mathbf{P}_{12} = \begin{bmatrix} 2 & 7 \\ 6 & 2 \end{bmatrix}, \quad \mathbf{P}_{21} = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}, \quad \mathbf{P}_{22} = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$$

The partitioned matrix can then be written as

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix}$$

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$$A = \left[ \begin{array}{cc|cc} 1 & 2 & 2 & 7 \\ 1 & 5 & 6 & 2 \\ \hline 3 & 3 & 4 & 5 \\ 3 & 3 & 6 & 7 \end{array} \right] \Leftrightarrow \left[ \begin{array}{cc} A_{11} & A_{12} \\ A_{21} & A_{22} \end{array} \right]$$

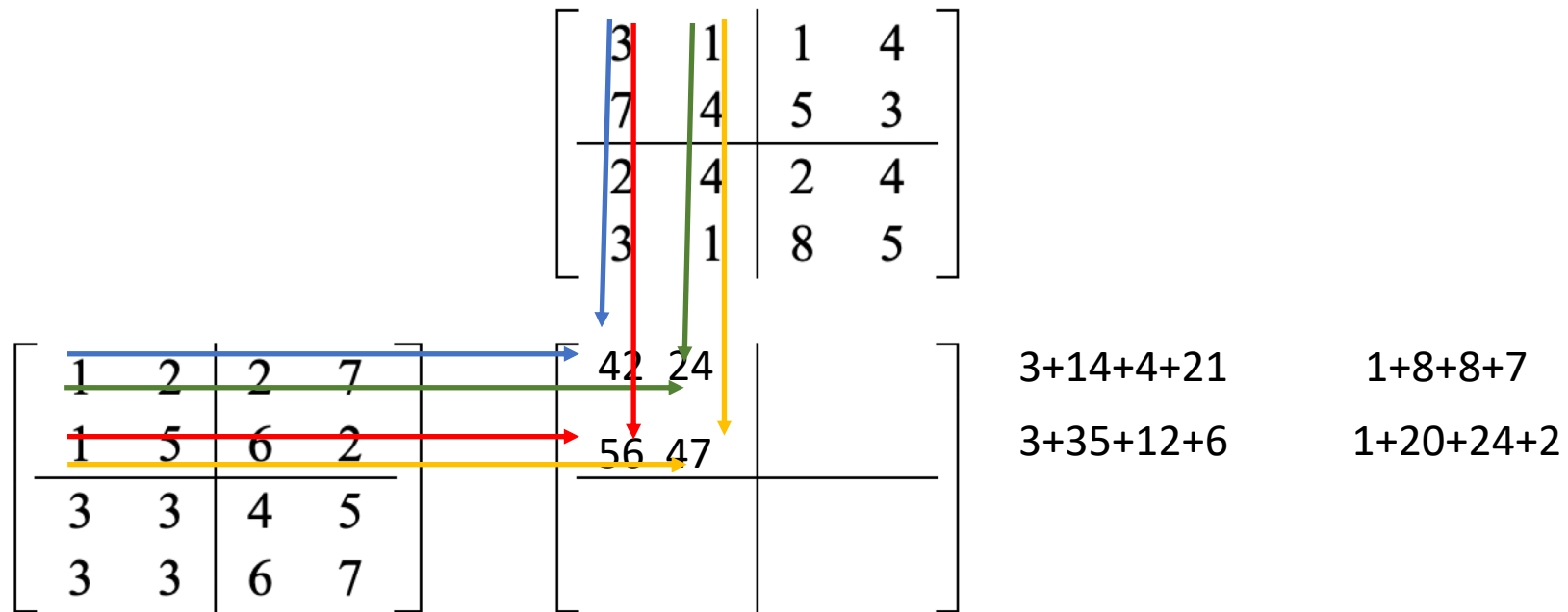
$$B = \left[ \begin{array}{cc|cc} 3 & 1 & 1 & 4 \\ 7 & 4 & 5 & 3 \\ \hline 2 & 4 & 2 & 4 \\ 3 & 1 & 8 & 5 \end{array} \right] \Leftrightarrow \left[ \begin{array}{cc} B_{11} & B_{12} \\ B_{21} & B_{22} \end{array} \right]$$

The block multiplication is performed the same way as before, but now the elements to multiply are matrices:

$$C = A * B = \left[ \begin{array}{cc} C_{11} & C_{12} \\ C_{21} & C_{22} \end{array} \right] = \left[ \begin{array}{cc} A_{11}*B_{11}+ & A_{11}*B_{12}+ \\ & A_{12}*B_{21} & A_{12}*B_{22} \\ A_{21}*B_{11}+ & A_{21}*B_{12}+ \\ & A_{22}*B_{21} & A_{22}*B_{22} \end{array} \right]$$

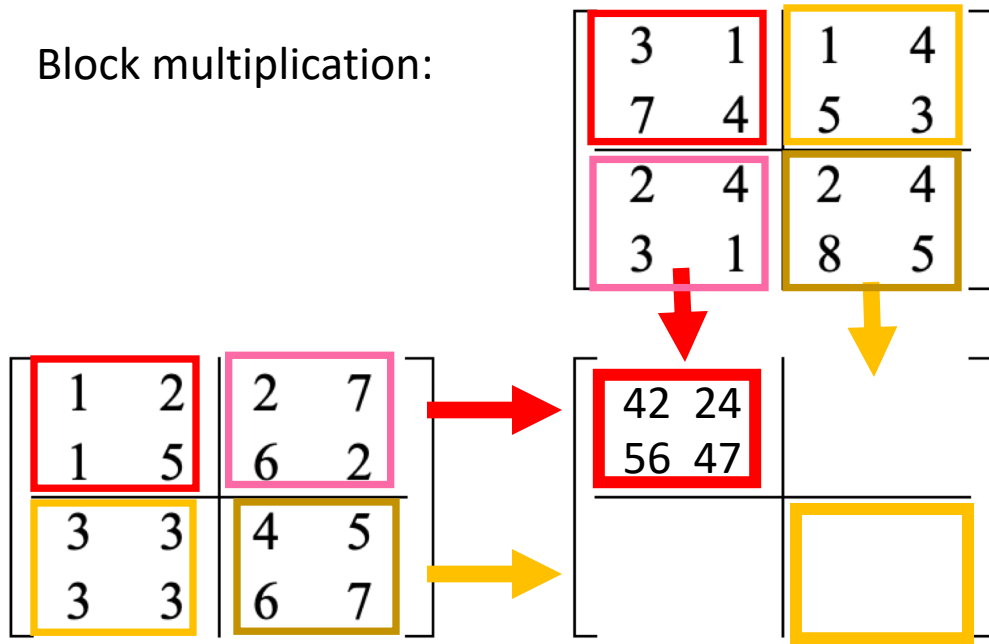
# Block Matrix Multiplication

Single element multiplication:



# Block Matrix Multiplication

Block multiplication:



$$\begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 17 & 9 \\ 38 & 21 \end{bmatrix}$$

+

$$\begin{bmatrix} 2 & 7 \\ 6 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 25 & 15 \\ 18 & 26 \end{bmatrix}$$

=

$$\begin{bmatrix} 42 & 24 \\ 56 & 47 \end{bmatrix}$$