Programming Exercise 5 - Regularized Linear Regression and Bias v.s. Variance

March 9, 2017

0.1 Programming Exercise 5 - Regularized Linear Regression and Bias v.s. Variance

```
In [1]: # %load ../../../standard_import.txt
        import pandas as pd
        import numpy as np
        import matplotlib as mpl
        import matplotlib.pyplot as plt

        from scipy.io import loadmat
        from scipy.optimize import minimize

        from sklearn.linear_model import LinearRegression, Ridge
        from sklearn.preprocessing import PolynomialFeatures

        pd.set_option('display.notebook_repr_html', False)
        pd.set_option('display.max_columns', None)
        pd.set_option('display.max_rows', 150)
        pd.set_option('display.max_seq_items', None)

        #%config InlineBackend.figure_formats = {'pdf',}
        %matplotlib inline

        import seaborn as sns
        sns.set_context('notebook')
        sns.set_style('white')

In [2]: data = loadmat('data/ex5data1.mat')
        data.keys()

Out[2]: dict_keys(['__header__', '__version__', '__globals__', 'X', 'y', 'Xtest', '

In [3]: y_train = data['y']
        X_train = np.c_[np.ones_like(data['X']), data['X']]

        yval = data['yval']
        Xval = np.c_[np.ones_like(data['Xval']), data['Xval']]
```

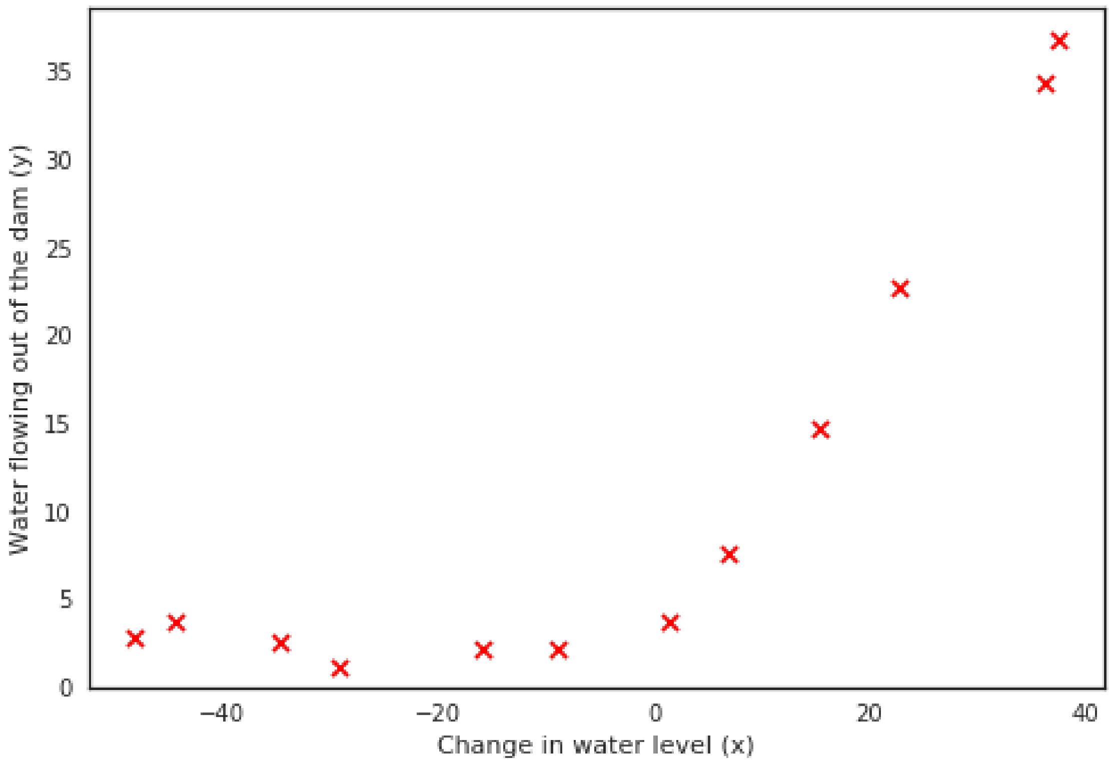
```
print('X_train:', X_train.shape)
print('y_train:', y_train.shape)
print('Xval:', Xval.shape)
print('yval:', yval.shape)
```

```
X_train: (12, 2)
y_train: (12, 1)
Xval: (21, 2)
yval: (21, 1)
```

0.1.1 Regularized Linear Regression

```
In [4]: plt.scatter(X_train[:,1], y_train, s=50, c='r', marker='x', linewidths=1)
        plt.xlabel('Change in water level (x)')
        plt.ylabel('Water flowing out of the dam (y)')
        plt.ylim(ymin=0);
```

```
/home/ubuntu/anaconda3/lib/python3.6/site-packages/matplotlib/font_manager.py:1297:
  (prop.get_family(), self.defaultFamily[fontext]))
```



Regularized Cost function

```
In [5]: def linearRegCostFunction(theta, X, y, reg):
            m = y.size

            h = X.dot(theta)

            J = (1/(2*m))*np.sum(np.square(h-y)) + (reg/(2*m))*np.sum(np.square(the

            return(J)
```

Gradient

```
In [6]: def lrgradientReg(theta, X, y, reg):
            m = y.size

            h = X.dot(theta.reshape(-1,1))

            grad = (1/m)*(X.T.dot(h-y))+ (reg/m)*np.r_[[[0]],theta[1:].reshape(-1,1

            return(grad.flatten())
```

```
In [7]: initial_theta = np.ones((X_train.shape[1],1))
        cost = linearRegCostFunction(initial_theta, X_train, y_train, 0)
        gradient = lrgradientReg(initial_theta, X_train, y_train, 0)
        print(cost)
        print(gradient)
```

```
303.951525554
[ -15.30301567  598.16741084]
```

```
In [8]: def trainLinearReg(X, y, reg):
            #initial_theta = np.zeros((X.shape[1],1))
            initial_theta = np.array([[15],[15]])
            # For some reason the minimize() function does not converge when using
            # zeros as initial theta.

            res = minimize(linearRegCostFunction, initial_theta, args=(X,y,reg), me
                           options={'maxiter':5000})

            return(res)
```

```
In [9]: fit = trainLinearReg(X_train, y_train, 0)
        fit
```

```
Out[9]:       fun: 1604.4002999186634
         hess_inv: array([[ 1.03142187,  0.00617881],
                [ 0.00617881,  0.001215  ]])
```

```
     jac: array([  3.42437190e-12,  -5.70371898e-10])
 message: 'Optimization terminated successfully.'
    nfev: 6
     nit: 4
    njev: 6
  status: 0
 success: True
       x: array([ 13.08790351,   0.36777923])
```

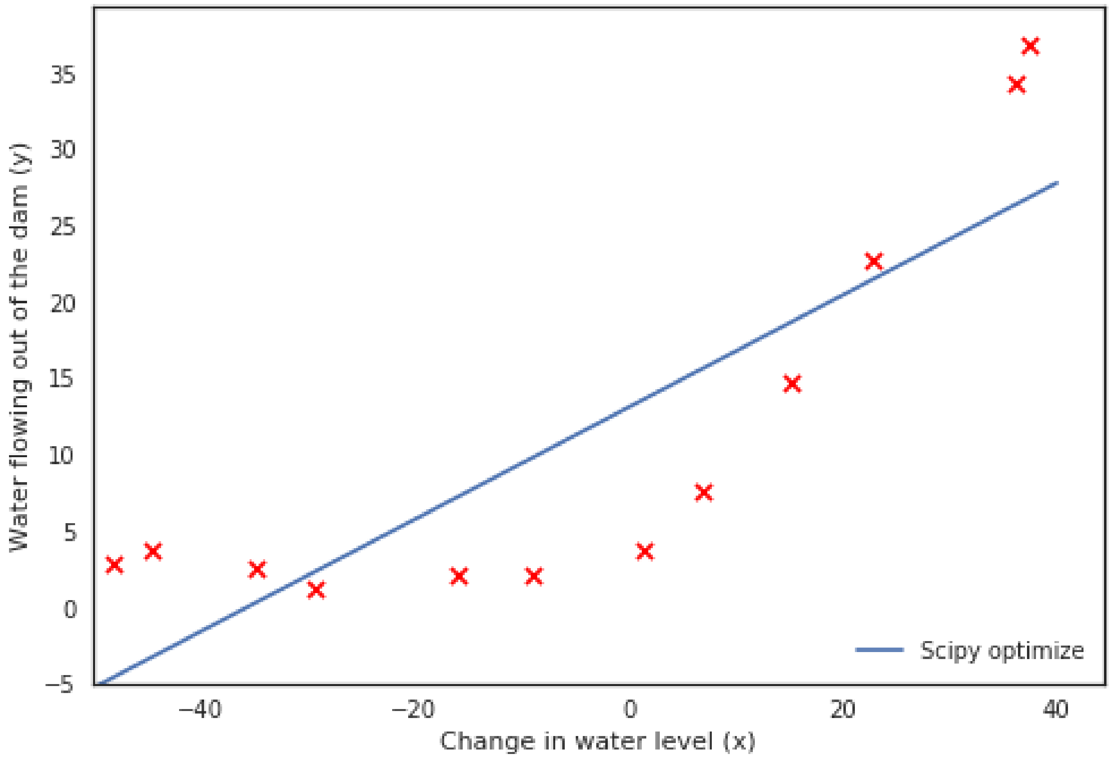
Comparison: coefficients and cost obtained with LinearRegression in Scikit-learn

```
In [10]: regr = LinearRegression(fit_intercept=False)
         regr.fit(X_train, y_train.ravel())
         print(regr.coef_)
         print(linearRegCostFunction(regr.coef_, X_train, y_train, 0))
```

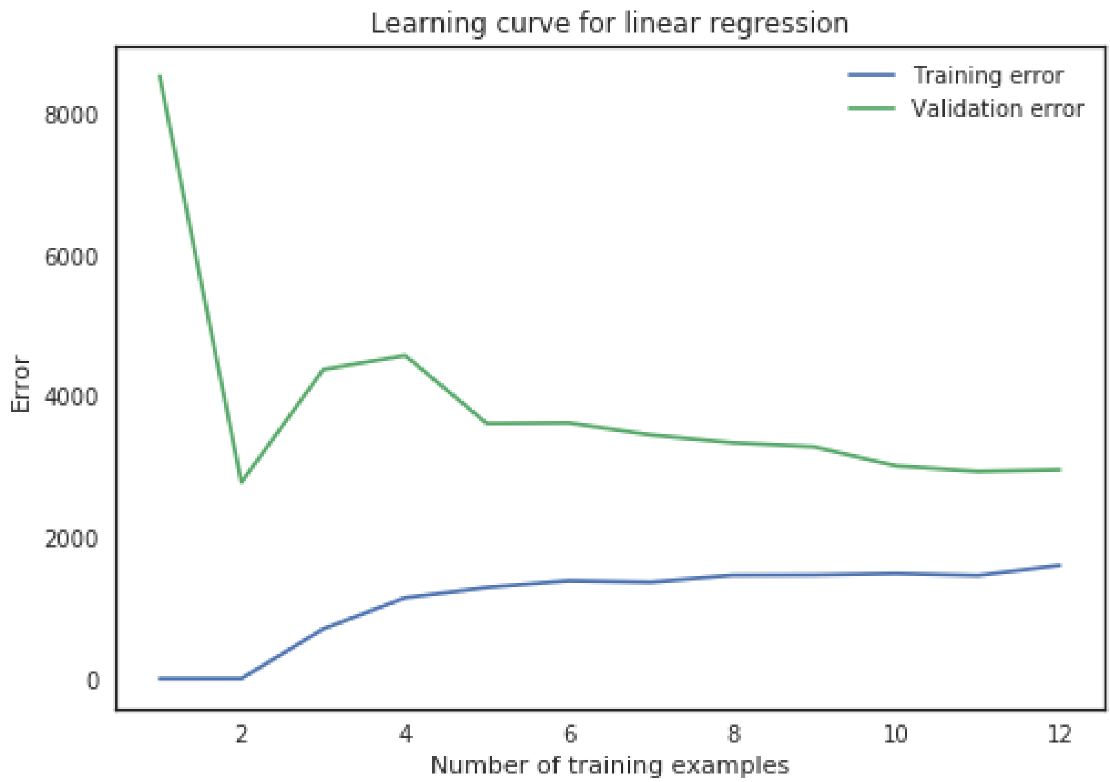
```
[ 13.08790351   0.36777923]
1604.40029992
```

```
In [11]: plt.plot(np.linspace(-50,40), (fit.x[0]+ (fit.x[1]*np.linspace(-50,40))),
         #plt.plot(np.linspace(-50,40), (regr.coef_[0]+ (regr.coef_[1]*np.linspace
         plt.scatter(X_train[:,1], y_train, s=50, c='r', marker='x', linewidths=1)
         plt.xlabel('Change in water level (x)')
         plt.ylabel('Water flowing out of the dam (y)')
         plt.ylim(ymin=-5)
         plt.xlim(xmin=-50)
         plt.legend(loc=4);
```

```
/home/ubuntu/anaconda3/lib/python3.6/site-packages/matplotlib/font_manager.py:1297:
  (prop.get_family(), self.defaultFamily[fontext]))
```




```
In [12]: def learningCurve(X, y, Xval, yval, reg):
             m = y.size

             error_train = np.zeros((m, 1))
             error_val = np.zeros((m, 1))

             for i in np.arange(m):
                 res = trainLinearReg(X[:i+1], y[:i+1], reg)
                 error_train[i] = linearRegCostFunction(res.x, X[:i+1], y[:i+1], re
                 error_val[i] = linearRegCostFunction(res.x, Xval, yval, reg)

             return(error_train, error_val)

In [13]: t_error, v_error = learningCurve(X_train, y_train, Xval, yval, 0)

In [14]: plt.plot(np.arange(1,13), t_error, label='Training error')
         plt.plot(np.arange(1,13), v_error, label='Validation error')
         plt.title('Learning curve for linear regression')
         plt.xlabel('Number of training examples')
         plt.ylabel('Error')
         plt.legend();

/home/ubuntu/anaconda3/lib/python3.6/site-packages/matplotlib/font_manager.py:1297:
  (prop.get_family(), self.defaultFamily[fontext]))
```



0.1.2 Polynomial regression (Scikit-learn)

```
In [15]: poly = PolynomialFeatures(degree=8)
         X_train_poly = poly.fit_transform(X_train[:,1].reshape(-1,1))

         regr2 = LinearRegression()
         regr2.fit(X_train_poly, y_train)

         regr3 = Ridge(alpha=20)
         regr3.fit(X_train_poly, y_train)

         # plot range for x
         plot_x = np.linspace(-60,45)
         # using coefficients to calculate y
         plot_y = regr2.intercept_+ np.sum(regr2.coef_*poly.fit_transform(plot_x.re
         plot_y2 = regr3.intercept_ + np.sum(regr3.coef_*poly.fit_transform(plot_x.

         plt.plot(plot_x, plot_y, label='Scikit-learn LinearRegression')
         plt.plot(plot_x, plot_y2, label='Scikit-learn Ridge (alpha={})'.format(reg
         plt.scatter(X_train[:,1], y_train, s=50, c='r', marker='x', linewidths=1)
         plt.xlabel('Change in water level (x)')
         plt.ylabel('Water flowing out of the dam (y)')
```

```
plt.title('Polynomial regression degree 8')
plt.legend(loc=4);
```

```
/home/ubuntu/anaconda3/lib/python3.6/site-packages/matplotlib/font_manager.py:1297:
  (prop.get_family(), self.defaultFamily[fontext]))
```