Programming Exercise 2 - Logistic Regression

February 21, 2017

0.1 Programming Exercise 2 - Logistic Regression

- Logistic regression
- Regularized logistic regression

In [1]:
```
# %load ../../standard_import.txt
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

from scipy.optimize import minimize

from sklearn.preprocessing import PolynomialFeatures

pd.set_option('display.notebook_repr_html', False)
pd.set_option('display.max_columns', None)
pd.set_option('display.max_rows', 150)
pd.set_option('display.max_seq_items', None)

#%config InlineBackend.figure_formats = {'pdf',}
%matplotlib inline

import seaborn as sns
sns.set_context('notebook')
sns.set_style('white')
```

In [2]:
```
def loaddata(file, delimeter):
    data = np.loadtxt(file, delimiter=delimeter)
    print('Dimensions: ',data.shape)
    print(data[1:6,:])
    return(data)
```

In [3]:
```
def plotData(data, label_x, label_y, label_pos, label_neg, axes=None):
    # Get indexes for class 0 and class 1
    neg = data[:,2] == 0
    pos = data[:,2] == 1
```

```
    # If no specific axes object has been passed, get the current axes.
    if axes == None:
        axes = plt.gca()
    axes.scatter(data[pos][:,0], data[pos][:,1], marker='+', c='k', s=60, l
    axes.scatter(data[neg][:,0], data[neg][:,1], c='y', s=60, label=label_n
    axes.set_xlabel(label_x)
    axes.set_ylabel(label_y)
    axes.legend(frameon= True, fancybox = True);
```

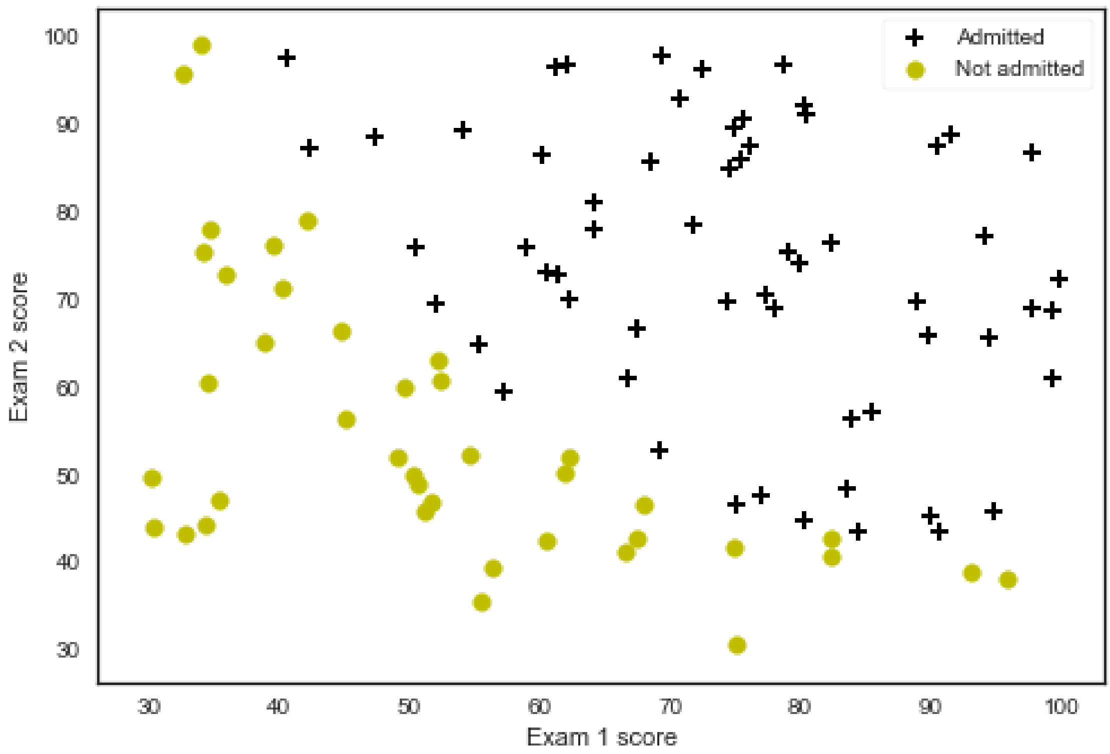
0.1.1 Logistic regression

```
In [4]: data = loaddata('data/ex2data1.txt', ',')
```

```
Dimensions:  (100, 3)
[[ 30.28671077  43.89499752   0.        ]
 [ 35.84740877  72.90219803   0.        ]
 [ 60.18259939  86.3085521    1.        ]
 [ 79.03273605  75.34437644   1.        ]
 [ 45.08327748  56.31637178   0.        ]]
```

```
In [5]: X = np.c_[np.ones((data.shape[0],1)), data[:,0:2]]
        y = np.c_[data[:,2]]
```

```
In [6]: plotData(data, 'Exam 1 score', 'Exam 2 score', 'Admitted', 'Not admitted')
```



Logistic regression hypothesis

$$h_\theta(x) = g(\theta^T x)$$

$$g(z) = \frac{1}{1+e^{-z}}$$

```
In [7]: def sigmoid(z):
            return(1 / (1 + np.exp(-z)))
```

Scipy actually has a convenience function which does exactly the same: http://docs.scipy.org/doc/scipy/reference/generated/scipy.special.expit.html#scipy.special.expit

Cost Function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\big[-y^{(i)}\, log\,(h_\theta\,(x^{(i)})) - (1-y^{(i)})\, log\,(1-h_\theta(x^{(i)}))\big]$$

Vectorized Cost Function

$$J(\theta) = \frac{1}{m}\big((\,log\,(g(X\theta))^T y + (\,log\,(1-g(X\theta))^T(1-y)\big)$$

```
In [8]: def costFunction(theta, X, y):
            m = y.size
            h = sigmoid(X.dot(theta))

            J = -1*(1/m)*(np.log(h).T.dot(y)+np.log(1-h).T.dot(1-y))

            if np.isnan(J[0]):
                return(np.inf)
            return(J[0])
```

Partial derivative

$$\frac{\delta J(\theta)}{\delta\theta_j} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$$

Vectorized

$$\frac{\delta J(\theta)}{\delta \theta_j} = \frac{1}{m} X^T (g(X\theta) - y)$$

```
In [9]: def gradient(theta, X, y):
            m = y.size
            h = sigmoid(X.dot(theta.reshape(-1,1)))

            grad =(1/m)*X.T.dot(h-y)

            return(grad.flatten())
```

```
In [10]: initial_theta = np.zeros(X.shape[1])
         cost = costFunction(initial_theta, X, y)
         grad = gradient(initial_theta, X, y)
         print('Cost: \n', cost)
         print('Grad: \n', grad)
```

```
Cost:
 0.69314718056
Grad:
 [ -0.1        -12.00921659 -11.26284221]
```

Optimize cost function

```
In [11]: res = minimize(costFunction, initial_theta, args=(X,y), method=None, jac=g
         res
```

```
C:\Users\Lenovo\Anaconda3\lib\site-packages\ipykernel\__main__.py:5: RuntimeWarning
C:\Users\Lenovo\Anaconda3\lib\site-packages\ipykernel\__main__.py:5: RuntimeWarning
```

```
Out[11]:       fun: 0.20349770158950992
          hess_inv: array([[  2.85339493e+03,  -2.32908823e+01,  -2.27416470e+01],
                 [ -2.32908823e+01,   2.04489131e-01,   1.72969525e-01],
                 [ -2.27416470e+01,   1.72969525e-01,   1.96170322e-01]])
               jac: array([ -2.68557621e-09,   4.36433486e-07,  -1.39671757e-06])
           message: 'Optimization terminated successfully.'
              nfev: 34
               nit: 25
              njev: 30
            status: 0
           success: True
                 x: array([-25.16131634,   0.2062316 ,   0.20147143])
```

Predict

```
In [12]: def predict(theta, X, threshold=0.5):
             p = sigmoid(X.dot(theta.T)) >= threshold
             return(p.astype('int'))
```

```
In [13]: # Student with Exam 1 score 45 and Exam 2 score 85
         # Predict using the optimized Theta values from above (res.x)
         sigmoid(np.array([1, 45, 85]).dot(res.x.T))
```

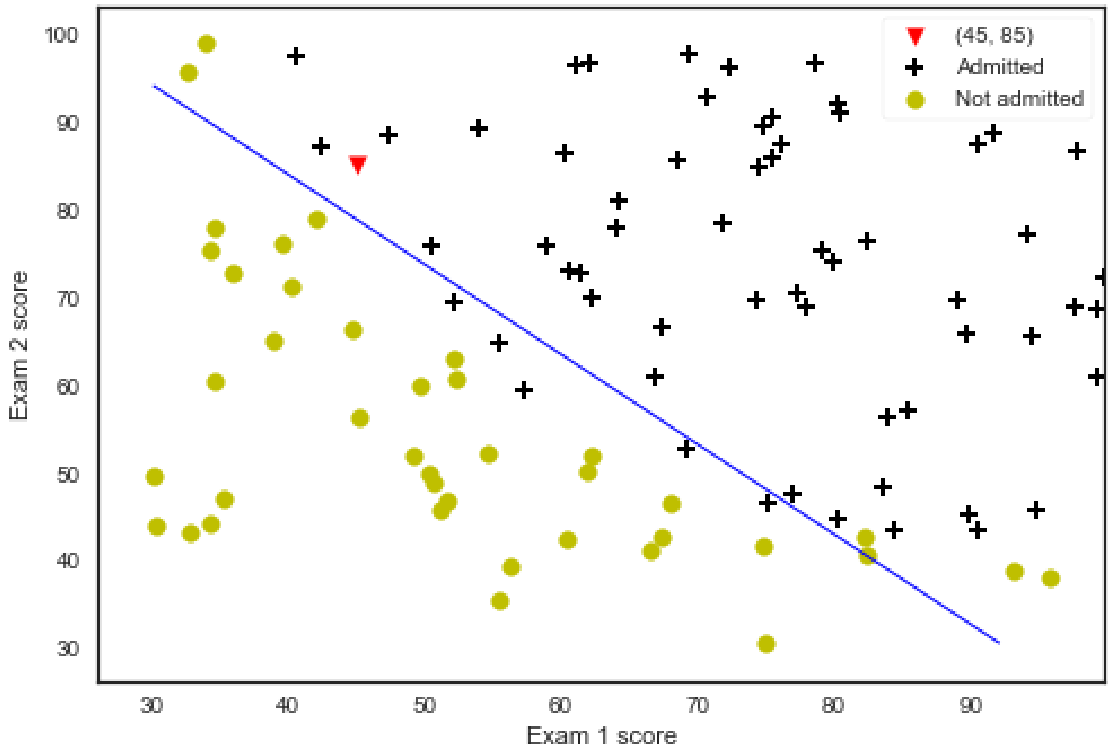
```
Out[13]: 0.776290324933310213
```

```
In [14]: p = predict(res.x, X)
         print('Train accuracy {}%'.format(100*sum(p == y.ravel())/p.size))
```

```
Train accuracy 89.0%
```

Decision boundary

```
In [15]: plt.scatter(45, 85, s=60, c='r', marker='v', label='(45, 85)')
         plotData(data, 'Exam 1 score', 'Exam 2 score', 'Admitted', 'Not admitted')
         x1_min, x1_max = X[:,1].min(), X[:,1].max(),
         x2_min, x2_max = X[:,2].min(), X[:,2].max(),
         xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2
         h = sigmoid(np.c_[np.ones((xx1.ravel().shape[0],1)), xx1.ravel(), xx2.rave
         h = h.reshape(xx1.shape)
         plt.contour(xx1, xx2, h, [0.5], linewidths=1, colors='b');
```



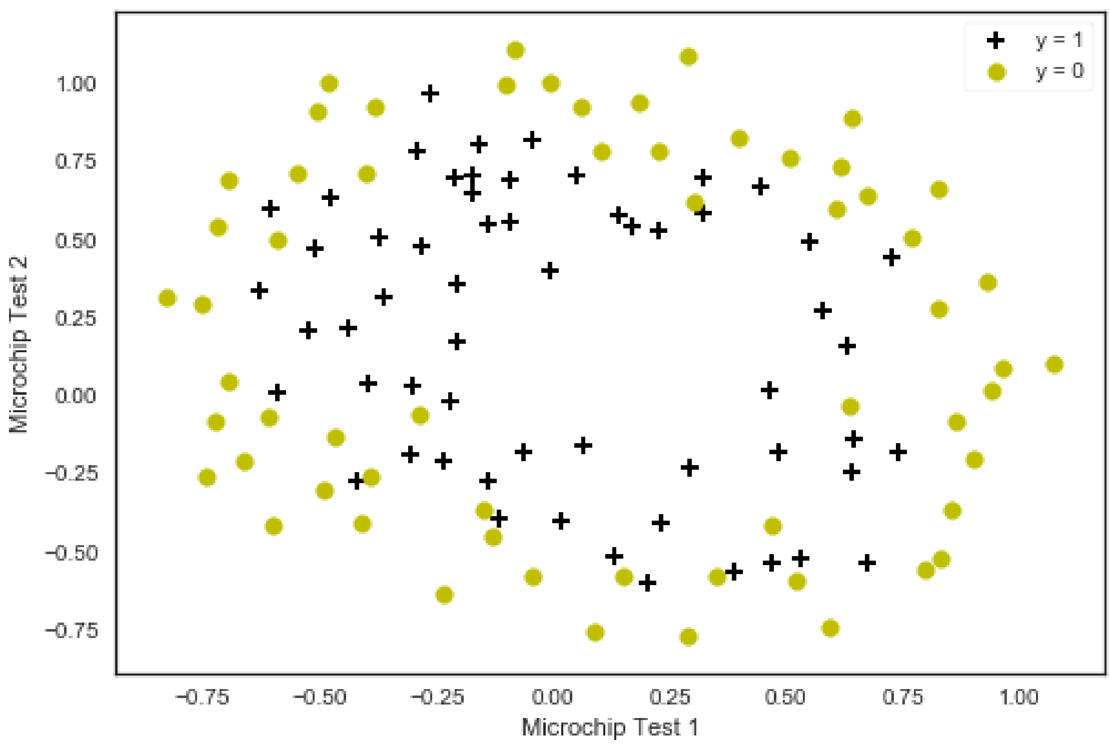
0.1.2 Regularized logistic regression

```
In [16]: data2 = loaddata('data/ex2data2.txt', ',')
```

```
Dimensions:  (118, 3)
[[-0.092742  0.68494   1.        ]
 [-0.21371   0.69225   1.        ]
 [-0.375     0.50219   1.        ]
 [-0.51325   0.46564   1.        ]
 [-0.52477   0.2098    1.        ]]
```

```
In [17]: y = np.c_[data2[:,2]]
         X = data2[:,0:2]
```

```
In [18]: plotData(data2, 'Microchip Test 1', 'Microchip Test 2', 'y = 1', 'y = 0')
```



Polynomials

```
In [19]: poly = PolynomialFeatures(6)
         XX = poly.fit_transform(data2[:,0:2])
         XX.shape
```

Out[19]: (118, 28)

Regularized Cost Function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\big[-y^{(i)}\, log\,(h_\theta\,(x^{(i)})) - (1-y^{(i)})\, log\,(1-h_\theta(x^{(i)}))\big] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

Vectorized Cost Function

$$J(\theta) = \frac{1}{m}\big((\,log\,(g(X\theta))^T y + (\,log\,(1-g(X\theta))^T(1-y)\big) + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

```
In [20]: def costFunctionReg(theta, reg, *args):
             m = y.size
             h = sigmoid(XX.dot(theta))

             J = -1*(1/m)*(np.log(h).T.dot(y)+np.log(1-h).T.dot(1-y)) + (reg/(2*m))

             if np.isnan(J[0]):
                 return(np.inf)
             return(J[0])
```

Partial derivative

$$\frac{\delta J(\theta)}{\delta\theta_j} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} + \frac{\lambda}{m}\theta_j$$

Vectorized

$$\frac{\delta J(\theta)}{\delta\theta_j} = \frac{1}{m}X^T(g(X\theta)-y) + \frac{\lambda}{m}\theta_j$$

Note: intercept parameter θ_0 is not to be regularized

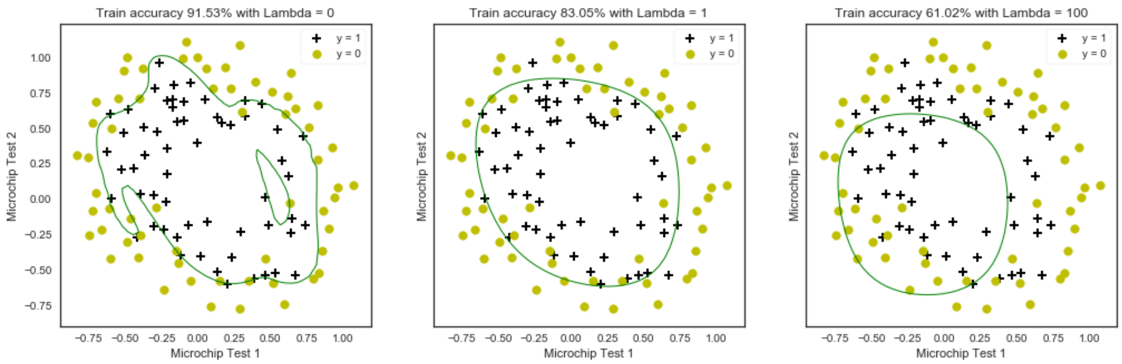
```
In [21]: def gradientReg(theta, reg, *args):
             m = y.size
             h = sigmoid(XX.dot(theta.reshape(-1,1)))

             grad = (1/m)*XX.T.dot(h-y) + (reg/m)*np.r_[[[0]],theta[1:].reshape(-1,

             return(grad.flatten())
```

```
In [22]: initial_theta = np.zeros(XX.shape[1])
         costFunctionReg(initial_theta, 1, XX, y)
```

```
Out[22]: 0.6931471805599454
```

```
In [23]: fig, axes = plt.subplots(1,3, sharey = True, figsize=(17,5))

         # Decision boundaries
         # Lambda = 0 : No regularization --> too flexible, overfitting the trainin
         # Lambda = 1 : Looks about right
         # Lambda = 100 : Too much regularization --> high bias

         for i, C in enumerate([0, 1, 100]):
             # Optimize costFunctionReg
             res2 = minimize(costFunctionReg, initial_theta, args=(C, XX, y), methc

             # Accuracy
             accuracy = 100*sum(predict(res2.x, XX) == y.ravel())/y.size

             # Scatter plot of X,y
             plotData(data2, 'Microchip Test 1', 'Microchip Test 2', 'y = 1', 'y =

             # Plot decisionboundary
             x1_min, x1_max = X[:,0].min(), X[:,0].max(),
             x2_min, x2_max = X[:,1].min(), X[:,1].max(),
             xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_mir
             h = sigmoid(poly.fit_transform(np.c_[xx1.ravel(), xx2.ravel()]).dot(re
             h = h.reshape(xx1.shape)
             axes.flatten()[i].contour(xx1, xx2, h, [0.5], linewidths=1, colors='g'
             axes.flatten()[i].set_title('Train accuracy {}% with Lambda = {}'.forn
```

```
C:\Users\Lenovo\Anaconda3\lib\site-packages\ipykernel\__main__.py:2: RuntimeWarning
  from ipykernel import kernelapp as app
```


Train accuracy 91.53% with Lambda = 0
Train accuracy 83.05% with Lambda = 1
Train accuracy 61.02% with Lambda = 100
y = 1
y = 0
Microchip Test 1
Microchip Test 2