# Package 'tensorMam'

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Type Package

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<b>Description</b> A tensor Estimation approach to multivariate additive models. The B-splines are used to approximate unknown function. The number of predictors can be diverged as sample size increases, in which the penalty LASSO, MCP or SCAD can be used.	
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tensorMam-package

A tensor estimation approach to multivariate additive models

#### **Description**

For a high-dimensional multivariate additive model (MAM) using B-splines, with or without aparsity assumptions, treating the coefficients as a third-order tensor and borrowing Tucker decomposition to reduce the number of parameters. The multivariate sparse group lasso (mcp or scad) and the coordinate descent algorithm are used to estimate functions for sparsity situation.

#### **Details**

This section should provide a more detailed overview of how to use the package, including the most important functions.

#### Author(s)

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#### References

A tensor estimation approach to multivariate additive models.

breastData

Breast cancer gene expression and DNA copy number dataset

#### **Description**

The breast cancer dataset includes gene expressions and comparative genomic hybridization measurements for 89 subjects, which is from Chin et al. (2006). This dataset has been considered by Witten et al. (2009) and Chen et al. (2013). In our paper, we selected chromosome 21, including q=44 variables for copy-number variations and p=227 variables for gene expression. As in Chen et al. (2013), we consider copy-number variations as the responses and gene expressions as the predictors.

# Usage

data(breastData)

#### **Details**

The "breastData" is formated as a list with elements:

dna: the CGH spots, a matrix with size  $2149 \times 89$  and the smaple size 89

rna: genes, a matrix with size  $19672 \times 89$  and the smaple size 89 chrom: chromosomal location of each CGH spot, a 2149-vector nuc: nucleotide position of each CGH spot, a 2149-vector gene: an accession number for each gene, a 19672-vector

genenames: gene name, a 19672-vector

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genechr: chromosomal location of each gene, a 19672-vector

genedesc: description of each gene, a 19672-vector

genepos: nucleotide position of each gene, a 19672-vector

#### References

Chin, K., DeVries, S., Fridlyand, J., Spellman, P., Roydasgupta, R., Kuo, W.-L., Lapuk, A., Neve, R., Qian, Z., Ryder, T., Chen, F., Feiler, H., Tokuyasu, T., Kingsley, C., Dairkee, S., Meng, Z., Chew, K., Pinkel, D., Jain, A., Ljung, B., Esserman, L., Albertson, D., Waldman, F. & Gray, J. (2006). Genomic and transcriptional aberrations linked to breast cancer pathophysiologies. Cancer cell **10** (6), 429-541.

Witten, D. M., Tibshirani, R. and Hastie, T. (2009). A penalized matrix decomposition, with applications to sparse principal components and canonical correlation analysis. Biostatistics **10** (3), 515-534.

Chen, K., Dong, H., and Chan, K. S. (2013). Reduced rank regression via adaptive nuclear norm penalization. Biometrika, **100** (**4**), 901-920.

# **Examples**

```
data(breastData)
attach(breastData)
Y = t(dna[chrom==21,])
Xt = t(rna[which(genechr==21),])
n = nrow(Y)

minX = apply(Xt,2,min)
maxX = apply(Xt,2,max)
X = (Xt - matrix(rep(minX,each = n),n))/matrix(rep(maxX-minX,each = n),n)
Y = scale(Y)
fit <- mam_sparse_dr(Y[,1:5], X[,1:10])
D3hat <- fit$Dnew
opt <- fit$rk_opt
detach(breastData)</pre>
```

generateData

Generate data from MAM model.

# **Description**

Generate data for a high-dimensional multivariate additive model, with or without aparsity assumptions.

# Usage

```
generateData(n, q, p, s, D2, sigma2=NULL, indexF=NULL, seed_id=NULL)
```

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# **Arguments**

n	Sample size.
q	The number of responses, $q \ge 1$ .
p	The number of covariates, $p \ge 1$ .
S	The true covariates associating to response, $s \ge 1$ .
D2	The mode of unfolding $D_{(2)}$ .
sigma2	err variance. Default is 0.1.
indexF	A $q \times s$ matrix. The index of significant predictors corresponding to response $y_l$ . Default is the matrix with each row being $(1, 2, \dots, s)$ .
seed_id	A positive integer, the seed for generating the random numbers.

#### **Details**

This function gives pq functional coefficients' estimators of MAM. The singular value matrix of tensor is a  $r_1 \times r_2 \times r_3$ -tensor. We choose  $r_1$ ,  $r_2$  and  $r_3$  by BIC, AIC, EBIC, CV, or GCV.

# Value

Υ	Response, a $n \times q$ -matrix.
Χ	Design matrix, a $n \times p$ -matrix
fO	True functions

# References

A tensor estimation approach to multivariate additive models.

#### See Also

```
mam_sparse
```

```
# Example 1
D2 <- matrix(runif(30, 0.7, 1), 2, 15)
mydata <- generateData(200, 3, 5, 5, D2)</pre>
Y <- mydata$Y
X \leftarrow mydata$X
# Example 2
n <- 500
p <- 10
q <- 10
s <- 10
K <- 6
s0 <- s
r10=r20=r30=2
S3 \leftarrow matrix(runif(r10*r20*r30,3,7),nrow = r30)
T1 <- matrix(rnorm(s0*r10), nrow = s0)
U1 <- qr.Q(qr(T1))
```

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```
T1 <- matrix(rnorm(K*r20),nrow = K)
U2 <- qr.Q(qr(T1))
T1 <- matrix(rnorm(q*r30),nrow = q)
U3 <- qr.Q(qr(T1))
D3 <- U3%*%S3%*%t(kronecker(U2,U1))
D2 <- TransferModalUnfoldings(D3,3,2,s0,K,q)
mydata <- generateData(n,q,p,s0,D2)
```

mam

Fit MAM without sparsity assumption and with fixed ranks.

# **Description**

Fit a low-dimensional multivariate additive model using B-splines, without aparsity assumptions, and given ranks  $r_1, r_2, r_3$ .

# Usage

```
mam(Y, X, K = 6, r1 = NULL, r2 = NULL, r3 = NULL, SABC = NULL,

intercept = TRUE, degr = 3, eps = 1e-4, max\_step = 20)
```

# **Arguments**

Υ	A $n \times q$ numeric matrix of responses.
Χ	A $n \times p$ numeric design matrix for the model.
K	The number of B-spline base function, that is the plus of both degrees of base function and the number of knots. Default is 6.
degr	the number of knots of B-spline base function. Default is 3.
r1	The first dimension of single value matrix of the tensor. Default is 2.
r2	The second dimension of single value matrix of the tensor. Default is 2.
r3	The third dimension of single value matrix of the tensor. Default is 2.
SABC	A user-specified list of initial coefficient matrix of $S, A, B, C$ . By default, initial matrices are provided by random.
intercept	Should intercept(s) be fitted (default=TRUE) or set to zero (FALSE)?
eps	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-4.
max_step	Maximum number of iterations. Default is 20.

# **Details**

This function gives pq functional coefficients' estimators of MAM. The singular value matrix of tensor is a  $r_1 \times r_2 \times r_3$ -tensor. We choose  $r_1$ ,  $r_2$  and  $r_3$  by BIC or CV.

# Value

Dnew

	(3).
mu	Estimator of intercept $\mu$ .
rss	Residual sum of squares (RSS).
Υ	Response $Y$ .
Χ	Design matrix $X$ .
Z	Design matrix of Bspline approximation.

Estimator of  $D_{(3)}$ .

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#### References

A tensor estimation approach to multivariate additive models.

#### See Also

mam\_sparse

#### **Examples**

```
p <- 5 q <- 5 D2 <- matrix(runif(2*p*q, 0.7, 1), 2, p*q) # tensor with size 5*2*5 mydata <- generateData(200, q, p, p, D2) fit <- mam(mydata\$Y, mydata\$X) K <- fit\$K D3hat <- fit\$Dnew # A q*(Kp) matrix with (p,K,q)=(5,6,5) D2hat <- TransferModalUnfoldings(D3hat,3,2,p,K,q)
```

mam\_dr

Fit MAM without sparsity assumption, and with ranks selected by BIC, AIC, EBIC, CV,  $or\ GCV$ .

# Description

Fit a low-dimensional multivariate additive model using B-splines, without aparsity assumptions, and with ranks  $r_1, r_2, r_3$  selected by BIC, AIC, EBIC, CV, or GCV.

# Usage

# **Arguments**

Υ	A $n \times q$ numeric matrix of responses.
Χ	A $n \times p$ numeric design matrix for the model.
method	The method to be applied to select parameters. Either BIC (the default), AIC, EBIC, ${\sf CV}$ , or ${\sf GCV}$ .
ncv	The number of cross-validation folds. Default is 10. If method is not "CV", ncv is useless.
K_index	A user-specified sequence of K values, where K is he number of B-spline base function. Default is k_index=6.
r1_index	A user-specified sequence of $r_1$ values, where $r_1$ is the first dimension of single value matrix of the tensor. Default is r1_index= $1, \cdots, \min(\log(n)\rceil, p)$ .
r2_index	A user-specified sequence of $r_2$ values, where $r_2$ is the second dimension of single value matrix of the tensor. Default is $r2\_index = 1, \cdots, max\{K\_index\}$ .
r3_index	A user-specified sequence of $r_3$ values, where $r_3$ is the third dimension of single value matrix of the tensor. Default is $r3\_index = 1, \dots, \min(\log(n)], q)$ .

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SABC	A user-specified list of initial coefficient matrix of $S$ , $A$ , $B$ , $C$ , which is a list with values $S$ , $A$ , $B$ , $C$ . By default, initial matrices are provided by random.
intercept	Should intercept(s) be fitted (default=TRUE) or set to zero (FALSE)?
degr	the number of knots of B-spline base function. Default is 3.
eps	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-4.
max_step	Maximum number of iterations. Default is 20.

# **Details**

This function gives pq functional coefficients' estimators of MAM. The singular value matrix of tensor is a  $r_1 \times r_2 \times r_3$ -tensor. We choose  $r_1$ ,  $r_2$  and  $r_3$  by BIC or CV.

# Value

Dnew	Estimator of $D_{(3)}$ .
mu	Estimator of intercept $\mu$ .
rss	Residual sum of squares (RSS).
rk_opt	The optimal parametres that slected by BIC, AIC, EBIC, CV, or GCV. It is a vector with length 4, which are selected $r_1,r_2,r_3$ , and $K$ .
selected	Which $\lambda$ is selection.
Υ	Response $Y$ .
Χ	Design matrix $X$ .
Z	Design matrix of Bspline approximation.

# References

A tensor estimation approach to multivariate additive models.

# See Also

```
mam, mam_sparse_dr
```

```
p <- 5
q <- 5
D2 <- matrix(runif(2*p*q, 0.7, 1), 2, p*q) # tensor with size 5*2*5
mydata <- generateData(200, q, p, p, D2)

fit <- mam_dr(mydata$Y, mydata$X)
K <- fit$K
D3hat <- fit$Dnew # A q*(Kp) matrix with (p,K,q)=(5,6,5)
D2hat <- TransferModalUnfoldings(D3hat,3,2,p,K,q)
opt <- fit$rk_opt</pre>
```

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mam_sparse	Fit MAM with sparsity assumption and fixed ranks.	

# Description

Fit a high-dimensional multivariate additive model using B-splines, with or without aparsity assumptions, and given ranks  $r_1, r_2, r_3$ . The multivariate sparse group LASSO, MCP or SCAD) and the coordinate descent algorithm are used to estimate functions for sparsity situation.

# Usage

```
mam_sparse(Y, X, method="BIC",ncv=10, K = 6, r1 = NULL, r2 = NULL, r3 = NULL,
           penalty="LASSO", isPenColumn=TRUE, lambda = NULL, SABC = NULL, intercept = TRUE,
           initMethod="LASSO", degr = 3, nlam = 20, lam_min = 1e-3, eps1 = 1e-4, maxstep1 = 20,
           eps2 = 1e-4, maxstep2 = 20, gamma = 2, dfmax = NULL, alpha = 1)
```

#### Arg

nlam

guments	guments		
Υ	A $n \times q$ numeric matrix of responses.		
Χ	A $n \times q$ numeric design matrix for the model.		
method	The method to be applied to select parameters. Either BIC (the default), AIC, EBIC, CV, or GCV.		
ncv	The number of cross-validation folds. Default is 10. If method is not CV, ncv is useless.		
penalty	The penalty to be applied to the model. Either LASSO (the default), MCP or SCAD.		
isPenColumn	A logical value indicating whether the coefficients associating with $X_j$ that affects whole response $y$ is penalized. Default is TRUE. If isPenColumn is TRUE, the coefficients associating with $X_j$ that affects simultaneously whole response $y$ is penalized for each $j \in \{1, \cdots, p\}$ . If isPenColumn is FALSE, the coefficients associating with $X_j$ that affects single response $y_l$ is penalized for each $j \in \{1, \cdots, p\}$ , where $l \in \{1, \cdots, q\}$ .		
K	The number of B-spline base function, that is the plus of both degrees of base function and the number of knots. Default is 6.		
degr	The number of knots of B-spline base function. Default is 3.		
r1	The first dimension of single value matrix of the tensor. Default is 2.		
r2	The second dimension of single value matrix of the tensor. Default is 2.		
r3	The third dimension of single value matrix of the tensor. Default is 2.		
lambda	A user-specified sequence of lambda values. By default, a sequence of values of length nlam is computed, equally spaced on the log scale.		
SABC	A user-specified list of initial coefficient matrix of $S, A, B, C$ . By default, initial matrices are provided by random.		
intercept	Should intercept(s) be fitted (default=TRUE) or set to zero (FALSE)?		
initMethod	One can estimate the initial tensor $D_{(3)}$ as a metrix by choosing a penalty to penalize group-column wise. initMethod can be LASSO, MCP or SCAD. The default		

is LASSO

The number of lambda values. Default is 20.

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lam\_min The smallest value for lambda, as a fraction of lambda.max. Default is 1e-3.

eps1 Convergence threshhold. The algorithm iterates until the relative change in any

coefficient is less than eps1. Default is 1e-4.

maxstep1 Maximum number of iterations. Default is 20.

eps2 Convergence threshhold. The Coordinate descent method algorithm iterates un-

til the relative change in any coefficient is less than eps2. Default is 1e-4.

maxstep2 The maximum iterates number of coordinate descent method. Default is 20.

gamma The tuning parameter of the MCP/SCAD penalty (see details).

dfmax Upper bound for the number of nonzero coefficients. Default is no upper bound.

However, for large data sets, computational burden may be heavy for models

with a large number of nonzero coefficients.

alpha Tuning parameter for the Mnet estimator which controls the relative contri-

butions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha

may be arbitrarily small, but not exactly 0.

#### **Details**

This function gives pq functional coefficients' estimators of MAM. The singular value matrix of tensor is a  $r_1 \times r_2 \times r_3$ -tensor.  $r_1$ ,  $r_2$  and  $r_3$  are fixed.

#### Value

betapath Solution path of  $\beta$ .

rss Residual sum of squares (RSS).

df Degrees of freedom.

lambda The sequence of regularization parameter values in the path.

lambda\_opt The value of lambda with the minimum BIC value. selectedID The index of lambda corresponding to lambda\_opt. activeX The active set of X. It is a p dimensional vector. activeF The active set of functions. It is a  $q \times p$  matrix.

Dnew Estimator of  $D_{(3)}$ .

mu Estimator of intercept  $\mu$ .

Y Response Y.

X Design matrix X.

Z Design matrix of Bspline approximation  $\lambda$ .

#### References

A tensor estimation approach to multivariate additive models.

# See Also

mam, mam\_sparse\_dr

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#### **Examples**

```
p <- 10
q <- 5
s <- 5
D2 <- matrix(runif(2*s*q, 0.7, 1), 2, s*q) # tensor with size 5*2*5
mydata <- generateData(200, q, p, s, D2)

fit <- mam_sparse(mydata$Y, mydata$X)
K <- fit$K
D3hat <- fit$Dnew # A q*(Kp) matrix with (p,K,q)=(10,6,5)
D2hat <- TransferModalUnfoldings(D3hat,3,2,p,K,q)
D1hat <- TransferModalUnfoldings(D3hat,3,1,p,K,q)
which(rowSums(D1hat^2)>0)
fit$activeX
```

mam\_sparse\_dr

Fit MAM with sparsity assumption and ranks selected by BIC, AIC, EBIC, CV, or GCV.

#### **Description**

Fit a high-dimensional multivariate additive model using B-splines, with or with aparsity assumptions and ranks selected by BIC, AIC, EBIC, CV, or GCV. The multivariate sparse group LASSO, MCP or SCAD) and the coordinate descent algorithm are used to estimate functions for sparsity situation. The tuning parameter is selected by BIC, AIC, EBIC, CV, or GCV, which matchs the method of rank selection.

# Usage

```
mam_sparse_dr(Y, X, method = "BIC", ncv = 10, penalty = "LASSO", isPenColumn=TRUE,
    K_index = NULL, r1_index = NULL, r2_index = NULL, r3_index = NULL,
    lambda = NULL, SABC = NULL, intercept = TRUE, initMethod="LASSO",
    nlam = 50, degr = 3, lam_min = 0.01, eps1 = 1e-4, maxstep1 = 20,
    eps2 = 1e-4, maxstep2 = 20, gamma = 2, dfmax = NULL, alpha = 1)
```

#### **Arguments**

Y A  $n \times q$  numeric matrix of responses.

X A  $n \times q$  numeric design matrix for the model.

method The method to be applied to select parameters. Either BIC (the default), AIC,

EBIC, CV, or GCV.

ncv The number of cross-validation folds. Default is 10. If method is not CV, ncv is

useless.

penalty The penalty to be applied to the model. Either LASSO (the default), MCP or SCAD.

is PenColumn A logical value indicating whether the coefficients associating with  $X_j$  that af-

fects whole response y is penalized. Default is TRUE. If isPenColumn is TRUE, the coefficients associating with  $X_j$  that affects simultaneously whole response y is penalized for each  $j \in \{1, \cdots, p\}$ . If isPenColumn is FALSE, the coefficients associating with  $X_j$  that affects single response  $y_l$  is penalized for each

 $j \in \{1, \dots, p\}$ , where  $l \in \{1, \dots, q\}$ .

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K_index	A user-specified sequence of K values, where K is he number of B-spline base function. Default is k_index=6.
r1_index	A user-specified sequence of $r_1$ values, where $r_1$ is the first dimension of single value matrix of the tensor. Default is $r1\_index = 1, \cdots, min(\lceil log(n) \rceil, p)$ .
r2_index	A user-specified sequence of $r_2$ values, where $r_2$ is the second dimension of single value matrix of the tensor. Default is $r2\_index = 1, \cdots, max\{K\_index\}$ .
r3_index	A user-specified sequence of $r_3$ values, where $r_3$ is the third dimension of single value matrix of the tensor. Default is $r3\_index = 1, \cdots, min(\lceil log(n) \rceil, q)$ .
lambda	A user-specified sequence of lambda values. By default, a sequence of values of length nlam is computed, equally spaced on the log scale.
SABC	A user-specified list of initial coefficient matrix of $S, A, B, C$ . By default, initial matrices are provided by random.
intercept	Should intercept(s) be fitted (default=TRUE) or set to zero (FALSE)?
initMethod	One can estimate the initial tensor $D_{(3)}$ as a metrix by choosing a penalty to penalize group-column wise. initMethod can be LASSO, MCP or SCAD. The default is LASSO
nlam	The number of lambda values. Default is 50.
degr	The number of knots of B-spline base function. Default is degr = 3.
lam_min	The smallest value for lambda, as a fraction of lambda.max. Default is 1e-2.
eps1	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps1. Default is 1e-4.
maxstep1	Maximum number of iterations. Default is 20.
eps2	Convergence threshhold. The Coordinate descent method algorithm iterates until the relative change in any coefficient is less than eps2. Default is 1e-4.
maxstep2	The maximum iterates number of coordinate descent method. Default is 20.
gamma	The tuning parameter of the MCP/SCAD penalty (see details).
dfmax	Upper bound for the number of nonzero coefficients. Default is no upper bound. However, for large data sets, computational burden may be heavy for models with a large number of nonzero coefficients.
alpha	Tuning parameter for the Mnet estimator which controls the relative contributions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha may be arbitrarily small, but not exactly 0.

# Details

This function gives pq functional coefficients' estimators of MAM. The singular value matrix of tensor is a  $r_1 \times r_2 \times r_3$ -tensor. We choose  $r_1$ ,  $r_2$  and  $r_3$  by BIC or CV.

# Value

Dnew	Estimator of $D_{(3)}$ .
mu	Estimator of intercept $\mu$ .
rss	Residual sum of squares (RSS).
df	Degrees of freedom.

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The active set of X. It is a p dimensional vector. activeX The active set of functions. It is a  $q \times p$  matrix. activeF lambda The sequence of regularization parameter values in the path. selectedID The index of lambda corresponding to lambda\_opt. The value of lambda with the minimum BIC, AIC, EBIC, CV, or GCV value. lambda\_opt The values of BIC or CV, which is a vector. **RSS** rk\_opt The optimal parametres that slected by BIC, AIC, EBIC, CV, or GCV. It is a vector with length 4, which are selected  $r_1$ ,  $r_2$ ,  $r_3$ , and K. Υ Response Y. Χ Design matrix X. Ζ Design matrix of Bspline approximation.

#### References

A tensor estimation approach to multivariate additive models.

#### See Also

```
mam_dr, mam_sparse
```

```
#Example 1
p <- 10
q <- 5
s <- 5
D2 <- matrix(runif(2*s*q, 0.7, 1), 2, s*q) # tensor with size 5*2*5
mydata <- generateData(200, q, p, s, D2)</pre>
fit <- mam_sparse_dr(mydata$Y, mydata$X)</pre>
K <- fit$K
D3hat <- fit$Dnew # A q*(Kp) matrix with (p,K,q)=(5,6,5)
D2hat <- TransferModalUnfoldings(D3hat,3,2,p,K,q)
D1hat <- TransferModalUnfoldings(D3hat,3,1,p,K,q)
opt <- fit$rk_opt
which(rowSums(D1hat^2)>0)
fit$activeX
#Example 2
data(breastData)
attach(breastData)
Y = t(dna[chrom==21,])
Xt = t(rna[which(genechr==21),])
n = nrow(Y)
minX = apply(Xt, 2, min)
maxX = apply(Xt, 2, max)
X = (Xt - matrix(rep(minX, each = n), n))/matrix(rep(maxX-minX, each = n), n)
Y = scale(Y)
fit <- mam_sparse_dr(Y[,1:5], X[,1:10])</pre>
K <- fit$K
```

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```
D3hat <- fit$Dnew # A q*(Kp) matrix with (p,K,q)=(10,6,5)
D1hat <- TransferModalUnfoldings(D3hat,3,1,10,K,5)
opt <- fit$rk_opt
which(rowSums(D1hat^2)>0)
fit$activeX
detach(breastData)
```

mvrblockwise

Estimate coefficients of high-dimensional multivariate regression for the grouped-column-wise

# Description

This function provides the coefficient matrix estimator of high-dimensional multivariate regression (MVR) with penalty LASSO, MCP or SCAD). The tuning parameter is selected by BIC (the default), AIC, EBIC, CV, or GCV.

# Usage

#### **Arguments**

Υ	The response, a vector of size $n$ or a matrix of size $n \times q$ .
X	The covariates to be penalized, a matrix with dimension $n \times p$ .
Z	The covariates without penalization, a matrix with dimension $n\times d$ . The default is NULL.
method	The method to be applied to select parameters. Either BIC (the default), AIC, EBIC, CV, or GCV.
ncv	The number of cross-validation folds. Default is 10. If method is not CV, ncv is useless.
penalty	The penalty to be applied to the model. Either LASSO (the default), MCP or SCAD.
isPenColumn	A logical value indicating whether the coefficients associating with $X_j$ that affects whole response $Y$ is penalized. Default is TRUE. If isPenColumn is TRUE, the coefficients associating with $X_j$ that affects simultaneously whole response $y$ is penalized for each $j \in \{1, \cdots, p\}$ . If isPenColumn is FALSE, the coefficients associating with $X_j$ that affects single response $Y_l$ is penalized for each $j \in \{1, \cdots, p\}$ , where $l \in \{1, \cdots, q\}$ .
group	A integer vector describing the grouping of the coefficients. For example, we can preset group = $rep(1:G,each=K)$ . If no grouping, group = $rep(1:ncol(X))$ . The default is group = $rep(1:ncol(X))$ .
lambda	A user-specified sequence of lambda values. By default, a sequence of values of length nlam is computed, equally spaced on the log scale.
nlam	The number of lambda values. Default is 50.
intercept	Should intercept(s) be fitted (default=TRUE) or set to zero (FALSE)?
lam_min	The smallest value for lambda, as a fraction of lambda.max. Default is 1e-3.

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eps Convergence threshhold. The algorithm iterates until the relative change in any

coefficient is less than eps1. Default is 1e-4.

max\_step Maximum number of iterations. Default is 50.

gamma\_pen The tuning parameter of the MCP/SCAD penalty (see details).

dfmax Upper bound for the number of nonzero coefficients. Default is no upper bound.

However, for large data sets, computational burden may be heavy for models

with a large number of nonzero coefficients.

alpha Tuning parameter for the Mnet estimator which controls the relative contri-

butions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha

may be arbitrarily small, but not exactly 0.

#### Value

Bhat Estimator of coefficients of X.

Residual sum of squares (RSS).

active X The active set of X. It is a p dimensional vector.

lambda The sequence of regularization parameter values in the path.

selectedID The index of lambda corresponding to lambda\_opt. lambda\_opt The value of lambda with the minimum BIC value.

bic BIC value used to select variables.

muhat Estimator of intercept  $\mu$ . It is NULL if intercept is FALSE. Chat Estimator of coefficients of Z. Chat is NULL if Z is NULL.

group The input group. Y Response Y. X Design matrix X.

#### References

A tensor estimation approach to multivariate additive models. Manuscript.

#### **Examples**

# library(tensorMam)

```
#example 1
n <- 200
q <- 5
s <- 3
p <- 100
B <- matrix(runif(q*s, 2,3), s)
X <- matrix(rnorm(n*p),n,p)
Y <- X[,1:s]%*%B + rnorm(n)
fit <- mvrblockwise(Y,X)
fit$activeX
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$muhat
```

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```
#example 2
K = 5
n <- 200
q <- 5
s <- 4
p <- 100
B1 <- matrix(runif(q*K, 2,3), K)
B2 \leftarrow matrix(0,2*K,q)
B3 <- matrix(runif(q*(s-1)*K, 2,3), (s-1)*K)
B <- rbind(B1,B2,B3)
X <- matrix(rnorm(n*p*K),n)</pre>
Y \leftarrow X[,1:((s+2)*K)]%*B + rnorm(n)
group <- rep(1:p,each=K)</pre>
fit <- mvrblockwise(Y,X,group=group,isPenColumn=TRUE)</pre>
which(fit$activeX==1)
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$muhat
#example 3
K = 5
n <- 200
q <- 5
s <- 4
d <- 3
p <- 100
B1 <- matrix(runif(q*K, 2,3), K)
B2 <- matrix(0,2*K,q)
B3 <- matrix(runif(q*(s-1)*K, 2,3), (s-1)*K)
B <- rbind(B1,B2,B3)
C <- matrix(runif(q*d, 1,2), d)</pre>
X <- matrix(rnorm(n*p*K),n)</pre>
Z <- matrix(rnorm(n*d),n)</pre>
Y \leftarrow X[,1:((s+2)*K)]%*B + Z%*C + rnorm(n)
group <- rep(1:p,each=K)</pre>
fit <- mvrblockwise(Y,X,Z,group=group,isPenColumn=TRUE)</pre>
which(fit$activeX==1)
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$Chat
fit$muhat
```

mvrcolwise

Estimate coefficients of high-dimensional multivariate regression for the column-wise

# **Description**

This function provides the coefficient matrix estimator of high-dimensional multivariate regression (MVR) with penalty LASSO, MCP or SCAD). The tuning parameter is selected by BIC (the default), AIC, EBIC, CV, or GCV.

#### Usage

```
mvrcolwise(Y, X ,Z=NULL, method="BIC", ncv=10, penalty="LASSO",
```

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isPenColumn=TRUE,lambda=NULL, nlam=50, intercept=TRUE,
lam\_min=1e-4,eps=1e-6, max\_step=50, gamma\_pen=2, dfmax=NULL, alpha=1)

#### Arguments

Y The response, a vector of size n or a matrix of size  $n \times q$ .

X The covariates to be penalized, a matrix with dimension  $n \times p$ .

Z The covariates without penalization, a matrix with dimension  $n \times d$ . The default

is NULL.

method The method to be applied to select parameters. Either BIC (the default), AIC,

EBIC, CV, or GCV.

ncv The number of cross-validation folds. Default is 10. If method is not CV, ncv is

useless.

penalty The penalty to be applied to the model. Either LASSO (the default), MCP or SCAD.

is PenColumn A logical value indicating whether the coefficients associating with  $X_i$  that af-

fects whole response Y is penalized. Default is TRUE. If isPenColumn is TRUE, the coefficients associating with  $X_j$  that affects simultaneously whole response y is penalized for each  $j \in \{1, \cdots, p\}$ . If isPenColumn is FALSE, the coefficients associating with  $X_j$  that affects single response  $Y_l$  is penalized for each

 $j \in \{1, \dots, p\}$ , where  $l \in \{1, \dots, q\}$ .

lambda A user-specified sequence of lambda values. By default, a sequence of values of

length nlam is computed, equally spaced on the log scale.

nlam The number of lambda values. Default is 50.

intercept Should intercept(s) be fitted (default=TRUE) or set to zero (FALSE)?

lam\_min The smallest value for lambda, as a fraction of lambda.max. Default is 1e-3.

eps Convergence threshhold. The algorithm iterates until the relative change in any

coefficient is less than eps1. Default is 1e-4.

max\_step Maximum number of iterations. Default is 50.

gamma\_pen The tuning parameter of the MCP/SCAD penalty (see details).

dfmax Upper bound for the number of nonzero coefficients. Default is no upper bound.

However, for large data sets, computational burden may be heavy for models

with a large number of nonzero coefficients.

alpha Tuning parameter for the Mnet estimator which controls the relative contri-

butions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha

may be arbitrarily small, but not exactly 0.

#### Value

Bhat Estimator of coefficients of X.

rss Residual sum of squares (RSS).

active X The active set of X. It is a p dimensional vector.

lambda The sequence of regularization parameter values in the path.

selectedID The index of lambda corresponding to lambda\_opt.

lambda\_opt The value of lambda with the minimum BIC value.

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bic	BIC value used to select variables.
muhat	Estimator of intercept $\mu.$ It is NULL if intercept is FALSE.
Chat	Estimator of coefficients of $Z$ . Chat is NULL if ${\sf Z}$ is NULL.
Υ	Response $Y$ .
Χ	Design matrix $X$ .

#### References

A tensor estimation approach to multivariate additive models. Manuscript.

# **Examples**

```
library(tensorMam)
#example 1
n <- 200
q <- 5
s <- 3
p <- 100
B \leftarrow matrix(runif(q*s, 2,3), s)
X \leftarrow matrix(rnorm(n*p),n,p)
Y <- X[,1:s]%*%B + rnorm(n)
fit <- mvrcolwise(Y,X)</pre>
fit$activeX
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$muhat
#example 2
n <- 200
q <- 5
s <- 3
d <- 3
p <- 100
B <- matrix(runif(q*s, 2,3), s)</pre>
C <- matrix(runif(q*d, 1,2), d)</pre>
X <- matrix(rnorm(n*p),n,p)</pre>
Z <- matrix(rnorm(n*d),n)</pre>
Y \leftarrow X[,1:s]%*B + Z%*%C + rnorm(n)
fit <- mvrcolwise(Y,X,Z)</pre>
fit$activeX
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$Chat
fit$muhat
```

plotfuns

Plot the estimated curves from tensorMam.

# **Description**

Plot the curves fitted by mam, mam\_dr, mam\_sparse, and mam\_sparse\_dr

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#### Usage

```
plotfuns(fit,funTrueID,true.curve=FALSE)
```

# Arguments

Object outputting from mam, mam\_dr, mam\_sparse or mam\_sparse\_dr. 
Which function to be plotted. It is a 2-vector. In MAM models, there are  $s_0 \times q$  true functions. Thus, the first argument must be smaller than  $s_0$ , and the second argument must be smaller than q.

true.curve A Logical flag. Plot both true and estimated curves if true.curve=TRUE. Plot estimated curve only if true.curve=FALSE. Default is FALSE.

#### Details

This function gives pq functional coefficients' estimators of MAM. The singular value matrix of tensor is a  $r_1 \times r_2 \times r_3$ -tensor. We choose  $r_1$ ,  $r_2$  and  $r_3$  by BIC or CV.

#### References

A tensor estimation approach to multivariate additive models.

#### See Also

```
mam, mam_dr, mam_sparse, mam_sparse_dr
```

```
n <- 200
p <- 10
q <- 10
s <- 10
K <- 6
s0 <- s
r10=r20=r30=2
S3 \leftarrow matrix(runif(r10*r20*r30,3,7),nrow = r30)
T1 <- matrix(rnorm(s0*r10), nrow = s0)
U1 \leftarrow qr.Q(qr(T1))
T1 <- matrix(rnorm(K*r20),nrow = K)
U2 \leftarrow qr.Q(qr(T1))
T1 <- matrix(rnorm(q*r30), nrow = q)
U3 \leftarrow qr.Q(qr(T1))
D3 <- U3%*%S3%*%t(kronecker(U2,U1))
D2 <- TransferModalUnfoldings(D3,3,2,s0,K,q)
mydata <- generateData(n, q, p, s0, D2)</pre>
fit <- mam(mydata$Y, mydata$X)</pre>
fit$D2 <- D2
fit$s0 <- s0
fit$X0 <- matrix(runif(100*p),100,p)</pre>
plotfuns(fit, c(1,1))
```

# TransferModalUnfoldings

Transfer a tensor's modal unfoldings to another.

# Description

Transfer a tensor's modal unfoldings to another.

# Usage

```
TransferModalUnfoldings(S, d1, d2 , r1, r2, r3)
```

# Arguments

S	A mode-d1-unfolding of a tensor with size $r_1 \times r_2 \times r_3$ , input unfolding
d1	An integer, the mode of unfolding $S_{(d_1)}$
d2	An integer, the mode of output unfolding $S_{\left(d_{2}\right)}$
r1	The fist dimension of tensor
r2	The second dimension of tensor
r3	The third dimension of tensor

# **Details**

This function transfers an input mode-d1-unfolding  $S_{(d_1)}$  to mode-d2-unfolding  $S_{(d_2)}$ 

# Value

```
D the output mode-d2-unfolding, S_{(d_2)}
```

#### References

A tensor estimation approach to multivariate additive models.

```
D1 <- matrix(1:24,nrow = 4) # A tensor unfolding with size 4*6
D2 <- TransferModalUnfoldings(D1,1,2,4,3,2)
```

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