

- (a) $T(n) = O(n \log n)$
- (b) $T(n) = O(n^{\log_4 3})$
- (c) $T(n) = O(n^2)$
- (d) The recurrence relation would be: $T(n) = c * T(n/c) + \log n$
Solving this relation would give us:
 $T(n) = O(n \log n)$
- (e) The running time ceases to be constant. It would now change based on the function n .

(a) Parent(i):

return $((i - 1) / d) + 1$

(b) Child(i, k):

return $(i * d) + k + 1$

(c) The height of the d -array heap would be $O(\log \text{ base } d \text{ of } n)$

(d) The asymptotic running time of Heapify is $O(d * \log \text{ base } d \text{ of } n)$ since for every level it does d comparisons and it is of height $\log \text{ base } d \text{ of } n$.

while the running time of Increase-key will be $O(\log \text{ base } d \text{ of } n)$ since it does one comparison per each of the $\log \text{ base } d \text{ of } n$ levels in the heap.

(e) Since the height is $O(\log \text{ base } d \text{ of } n)$:

If $d = O(1)$, the resulting running time is $O(\log n)$

If $d = O(\log n)$, the resulting running time becomes $O(\log \text{ base } \log n \text{ of } n)$ which simplifies to $O(\log n / (\log (\log n)))$

If $d = O(n)$, the resulting running time becomes $O(\log \text{ base } n \text{ of } n)$ which simplifies to $O(1)$

(f) The running times for Heapify:

a) when $d = O(1)$, would be $O(\log n)$

b) when $d = O(\log n)$, would be $O(\log n * \log n / (\log (\log n)))$

c) when $d = O(n)$, would be $O(n)$

The running times for Increase-key:

a) when $d = O(1)$, would be $O(\log n)$

b) when $d = O(\log n)$, would be $O(\log n / (\log (\log n)))$

c) when $d = O(n)$, would be $O(1)$

When d is increased the running times increase for Heapify but decrease for Increase-Key