GRAVITY MODELS AND TRIP DISTRIBUTION THEORY

by Morton Schneider

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THIS IS PRIMARILY A PRESENTATION IN THEORY, treating the special problem of trip distribution in a trip generating complex. Although some practical procedures will be sketched and a few real numbers may turn up in the text, the main concern of this paper is the theoretical approach it advances -- a concern that defers to a critical prologue.

It is not trivial to ask if interchanges -- trip movements between zones -- can be or ought to be predicted. Interchange volumes are of no great worth in themselves; they are important only as intermediates in the solution of traffic flow. Perhaps it would be better all around to let them lie implicit in some more forceful attack on the basic problems of trip behavior. Perhaps it simply is not possible to compute them as independent quantities. These points can be argued with considerable cogency but not much knowledge. Custom, and the absence of anything else to do, has committed practice to the explicit use of interchanges, and used they will be. The "can" and the "ought" of the matter rest in an understanding that does not now obtain; there they will stay for the remainder of this paper.

Interchanges are commonly predicted by an iterative "growth factor" method or by a "gravity" model. Growth factor methods will not be discussed here -- they are more or less arbitrary algorithms of small theoretical interest -- but something should be said of the gravity concept. The appellation "gravity" applied to the computation of interchanges is, or should be a misnomer. There is no real kinship between a gravitational field and a trip generating system. Newtonian gravity is an energy-force field characterizing the motions of particles, not their intentions. It does not deal in statistical partitions: a cluster of mass points under the influence of circumambient masses does not break into flights aimed at the separate attractive poles -- each member of the cluster is content to be governed by the vector resultant obtaining at its point in space, and this vector varies from point to point only in a mathematically continuous fashion. "Lines of force" is a reasonable visualization, but this is not at all the same as lines of movement. Moreover, the computation of a gravitational force is uncompromising and unexceptionable; the elements of the computation have stable dimension, meaning, and measure. To make the point absurdly complete, relativistic gravity -- a subject well outside the domain of this paper (and of this author, it has been imputed) -- postulates a quite different "geodesic" field which, again, has none of the properties of trip distribution.

The theoretical supports on which the gravity method rests appear to be these: an interchange between two regions is clearly a descending function of the distance between them, and inverse proportion descends with engaging convenience. These are by no means contemptible grounds, but neither are they entirely satisfactory. The cardinal failure of the gravity model is that it is not explanatory and does not really try to be. If it were an adequate predictor of interchange volumes, certainly a contestable point, it would still be only a specific answer to a very special problem. Regression analysis, even log log regression, is no more than a species, and a risky one, of curve-fitting; it merely summarizes data. Many classic problems had been empirically solved with great exactness before Newton. Galileo could calculate the speeds and distances traveled of falling bodies; there isn't much Kepler didn't know about planetary motion. The stature of Newton does not stem from his many excellent computations or the numerous problems he personally solved. His transcending contribution was his synthesis of countless notions and data of physics; his recognition that the problems of physics need not be treated as particular, isolate perplexities. The image of spherically radiating lines of force, the corollary inverse-square law, the three laws of motion were his fundamental implementations of this recognition. It helps, of course, to have that kind of insight.

At this writing, an empirical study of the trip distribution theory presented below is in progress at the Chicago Area Transportation Study. Certain preliminary results seem promising, but the bulk of the work remains to be done. The dispersions of data, due to the manner of collecting and manipulating them in an origin-destination survey, and to simple statistics, make it difficult to construct critical tests for most hypotheses, or to adduce conclusive evidence. In any case, this attempt doesn't really expect to go too far. Its highest hope is to do as well as other "ideal" theories, which is rather badly indeed. It's the discrepancies that count.

The assumptions (to be distinguished from simplifications) made here are these: that the probability of a trip finding a terminal in any element of a region is proportional to the number of terminal opportunities contained in the element; that a trip prefers to be as short as possible, lengthening only as it fails to find a terminal. The simplifications are abundant and conspicuous. Most notably, the trip receiving region is regarded as an unbounded plane surface over which trip terminal opportunities are evenly distributed. It should be mentioned that the formulation is amenable to generalization on this score, and others; but it seems best to start with the simplest situation.

Consider a field characterized by a probability-density point function, G. By probability-density is meant probability per unit of area, so that GdA, where dA is an element of area, is defined to give the probability dP, that a trip originating at some point, O, will terminate within a region of area dA. Clearly, dP may be regarded as the product of two probabilities: the probability, p, that a trip will get to dA at all and the probability, r, that it will terminate there if it does. Thus,

$$dP = GdA = pr \tag{1}$$

Since the entire region is considered to have constant trip terminal density, the first assumption requires that r be proportional to dA. Now the second assumption requires that p be simply the probability that the trip has not found a terminal closer to O. Therefore

$$dP = GdA = psdA, \text{ and } G = sp \tag{2}$$

But p is 1 minus the probability that the trip has found a terminal, so that

$$G = s\left(1 - \int_0^P dP\right) \quad , \tag{3}$$

or

$$G = s\left(1 - \int_0^A GdA\right) \quad . \tag{4}$$

Equation (4) is easily solved by differentiating with respect to A,

$$\frac{dG}{dA} = -sG \quad , \tag{5}$$

and solving equation (5):

$$G = Ke^{-sA} \tag{6}$$

K is an integration constant that can be identified by inserting equation (6) into equation (4) or by remembering that

$$\int_0^\infty GdA = 1 \quad . \tag{6a}$$

Finally, K turns out to be s, and

$$G = se^{-sA} \quad . \tag{7}$$

In a plane surface, $A = \pi t^2$. So

$$G = se^{-s\pi t^2} \tag{8}$$

t is the radial distance from the origin. This need not be ordinary geographical distance; it could be any cost metric -- the Chicago Study is working with minimum standard travel time(obtaining these times, by the way, is a major project of some interest that will be reported elsewhere). However, the region will not, in general, remain planar under a change of dimension.

The important parameter, s, can be calculated in several ways. The most convenient is through the median trip length, which can be roughly estimated. The median trip length corresponds to a median area, A_m. It follows from the definition of median that

$$\int_0^{A_m} se^{-sA} dA = .5 \tag{9}$$

This becomes

$$-e^{-sA_m} + 1 = .5 \tag{10}$$

or,

$$e^{-sA_m} = .5 \quad . \tag{11}$$

Thus,

$$s = -\frac{1}{A_m} \ln (.5) \quad . \tag{12}$$

Computed in this manner for the Chicago area, s is about .01 miles $^{-2}$, which seems to be the right order of magnitude judging by practical trials.

Actual calculations may be formulated as follows: consider a destination zone to be small enough so G is constant everywhere within it. Then

$$P_j = GA_j = se^{-s\pi t^2} A_j \tag{13}$$

where P_j is the probability of a trip terminating in the given zone and A_j is the area of that zone. If the number of trip terminal opportunities at a zone is regarded as equal to its trip generation, V_j, equation (13) can be represented by

$$P_j = s\frac{V_j}{\rho} e^{-s\pi t^2} \tag{14}$$

where ρ is the trip generation density of the region, i.e.,

$$\rho = \frac{V}{A} \tag{15}$$

Using (15), equation (14) becomes

$$P_j = sA\frac{V_j}{V} e^{-s\pi t^2} \quad \text{or} \quad R\frac{V_j}{V} e^{-s\pi t^2} \tag{16}$$

In a true unbounded, uniform density plane, total V and A would be infinite, but any arbitrary region would yield the correct ratio. In a real case, one must simply choose a large region and hope that it is representative. If the trip generation volume at the point of origin is V_i, then the expected interchange between that origin and the considered destination zone is

$$V_{ij} = V_i P_j \tag{17}$$

The probability of any given deviation from this expected value can be computed from the Binomial distribution. The only rigorous test of the method would be to see if deviations occur with their proper theoretical frequencies. This is a difficult test to make for many reasons, not the least of which is the doubtful statistical properties of data in the field.

A number of avenues toward the generalization of this formulation suggest themselves. The three most salient, and important, should be indicated.

The density might be generalized, that is a variable density postulated as a continuous point function. Loosely, this would involve recasting equation (4) into the form

$$G = s\,(1 - \iint G\rho J\,du\,dv) \tag{18}$$

(u and v are generalized coordinates. J is the transformation to a real measure.)

a far more difficult mathematical construct to work with. An interesting conjecture is that some metric exists with respect to which trip generation density *is* constant.

The area might be generalized -- computed by the appropriate function for its proper geometry, whether bounded plane or curved surface.

Trip behavior might be generalized. Perhaps all trips do not observe the postulates of this theory, perhaps there are different modes within those postulates. To some extent this is being explored at the Chicago Study.

Aside from these major points, there is a plentiful supply of improvements to be made in realism, elegance, and rigor. Apply to the author for an indexed catalogue.