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TOWARD A BETTER UNDERSTANDING OF THE INTERVENIN(OPPORTUNITIES MODEL†

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1. INTRODUCTION

THE INTERVENING opportunities model is one of a number of mathematical models c trip distribution process. The purpose of a trip distribution model is to predict the de: tions of trips within an area, given: (1) their origins, (2) a measure of the separation bet origins and possible destinations and (3) specific model parameters. A general mathem: representation of trip distribution models might be the following function f:

$$T = f(\mathbf{O}, S, \mathbf{P})$$

where:

T is a matrix containing the number of trips from each origin zone to each de: tion zone.

$\mathbf{O}$ is a vector containing the number of trips beginning in each origin zone.

S is a matrix containing a measure of the separation from each origin zone to destination zone.

$\mathbf{P}$ is a vector of model parameters.

The intervening opportunities model, or, more briefly, the opportunity model, wa: applied to the trip distribution process in an urban area by Schneider (CATS, 1960). distinguishing feature of the model is its unique independent variable, intervening o] tunities. Although new to the urban transportation field when proposed, the variabl been a feature of earlier models of human behavior in the fields of population migratio inter-city travel. Some of these earlier intervening opportunities models have summarized by Leonard and Delano (1965).

The first application of the opportunity model was to the Chicago metropolitan r by the Chicago Area Transportation Study (CATS) in the late fifties. A version o model having only two parameters was found to duplicate adequately existing t patterns and to predict satisfactorily future patterns. Subsequent attempts to appl original form of the model to small areas within the urban region were not successful. I necessary to go back to the theoretical bases of the model to determine how to include parameters in the model and how to specify these parameters for particular applica of the model.

This paper will discuss the improved understanding of the opportunity model whic been gained since the model has been applied to small areas. The derivation of the n

† The research leading to the findings presented in this paper was conducted while the author w the staff of the Chicago Area Transportation Study.

and a description of initial applications are repeated in Section 2 before discussing interpretations of the model and its parameters in Section 3. Methods of specifying parameters for a particular run—calibrating the model—are presented in Section 4. A discussion of possible extensions of the model is included in Section 5, followed by conclusions in Section 6.

2. THE OPPORTUNITY MODEL

2.1. *Its derivation from hypotheses of human behavior*

The hypotheses upon which the opportunity model is based are the following (CATS, 1960):

1. Total travel time from a point is minimized, subject to the condition that every destination point has a stated probability of being accepted if it is considered.
2. The probability of a destination being accepted, if it is considered, is a constant, independent of the order in which destinations are considered.

These hypotheses lead to the following mathematical formulation, in terms of limitingly small quantities:

$$dP = L[1-P(V)]\,dV \tag{1}$$

where:

dP = the probability that a trip will terminate when considering dV possible destinations:

$P(V)$ = the total probability that a trip will terminate by the time V possible destinations are considered:

V = possible destinations already considered, or subtended volume:

L = the constant probability of a possible destination being accepted if it is considered.

The solution of the differential equation (1) which satisfies $P(0) = 0$ is:

$$P(V) = 1-\exp(-LV) \tag{2}$$

The expected interchange (T_{ij}) from zone i to zone j is the volume of trip origins O_i at zone i multiplied by the probability of a trip from i terminating in j:

$$T_{ij} = O_i[P(V_{j+1})-P(V_j)] \tag{3}$$

or

$$T_{ij} = O_i[\exp(-LV_j)-\exp(-LV_{j+1})] \tag{4}$$

The subtended volumes (V's) are the sums of the possible destinations considered before reaching a given zone. When a 24-hr time period is considered, it can be assumed that a zone's trip origins equal its trip destinations. Therefore, V_j can be defined in terms of the trip origins reached before reaching zone j:

$$V_j = \sum_{k=1}^{j-1} O_k \tag{5}$$

where the O_k's are arranged in order of increasing travel time from zone i.

Although (5) could be substituted into (4) to express T_{ij} completely in terms of trip origins and the L value, it is more convenient to leave the equation as given in (4).

2.2. *Initial applications*

Early experiments with the opportunity model showed that it would be necessary to specify more than one value for L, because of the differing probabilities of acceptance associated with different types of trips. For example, people are more selective in choosing places to work than they are in choosing places to shop for groceries. It was found that three trip sub-populations with two L values satisfactorily represented empirical trip data for large regions.

These three trip sub-populations are defined approximately as follows:

1. Long residential trips—all trips from work to home and from the Central Business District (CBD). Also, all external trips to residential land.
2. Long non-residential trips—all trips from home to work and to the CBD. Also, all external trips to non-residential land.
3. Short trips—all other trips.

The two L values are termed "long" and "short" corresponding to the trip groups with which they are used. The subtended volume which attracts long residential trips is made up of long non-residential trips, and vice versa. Short trips serve as their own attractors.

A mathematical statement of the opportunity model, as used by CATS is, therefore:

$$T_{ij} = \sum_{k=1}^{3} O_{ik}[\exp(-L_k V_{jk}) - \exp(-L_k V_{j+1,k})] \tag{6}$$

where k ranges over the three trip sub-populations. No provision was made for the variation of the two L values from origin zone to origin zone.

The discussion of interpretations of the model and the L value which follows will be based on just one of the trip sub-populations of (6). It will be assumed that the model holds in the simple form of (3).

3. INTERPRETATIONS OF THE MODEL

The opportunity model can be considered in its broadest sense as an explanation of human behavior, as stated in the two hypotheses given above. The fact that the model has proved to be satisfactory for metropolitan regions indicates that, when averaged over a large area and when considered in just three sub-populations, people do behave as hypothesized in the model.

In the following paragraphs, interpretations of the mathematics of the model, characteristics of the model's parameter and relationships between the L value and trip parameters are discussed.

3.1. *Interpretation of the mathematics of the model*

The mathematics of the model can be interpreted by considering (2) as an equation which is to be fitted to empirical data by adjusting the parameter L. Such empirical data fitting can be facilitated by linearizing (2) to the following form:

$$-LV = \ln[1-P(V)] \tag{7}$$

A typical plot of empirical values of V and $1-P(V)$ for a given origin zone is shown in Fig. 1. Theoretically, this semi-log plot will be a straight line for all trip types and all origin zones. The parameter, or L value, can be found using curve-fitting techniques. Values of V and $1-P(V)$ can be determined from survey data, once actual trip interchanges from a given zone are arranged in travel time order. In a more limited sense, therefore, the opportunity model can be interpreted as a statement that a semi-log relationship tends to exist between V and $1-P(V)$.

3.2. *Characteristics of the L value*

The curve-fitting approach discussed above leads to a graphic interpretation of the L value. This parameter can be viewed simply as the slope of the straight line which best fits a set of empirical V and $\ln[1-P(V)]$ data.

Mathematically, the L value can be expressed:

$$L = \frac{-\ln[1-P(V)]}{V} \tag{8}$$

Two characteristics of the L value can be seen by investigating (8). The sign of L will always be positive. The units of L are (1/opportunities), or (1/trip ends). Experience with empirical data indicates that L is always very small—usually of the order of 10^{-5} and always much less than one.

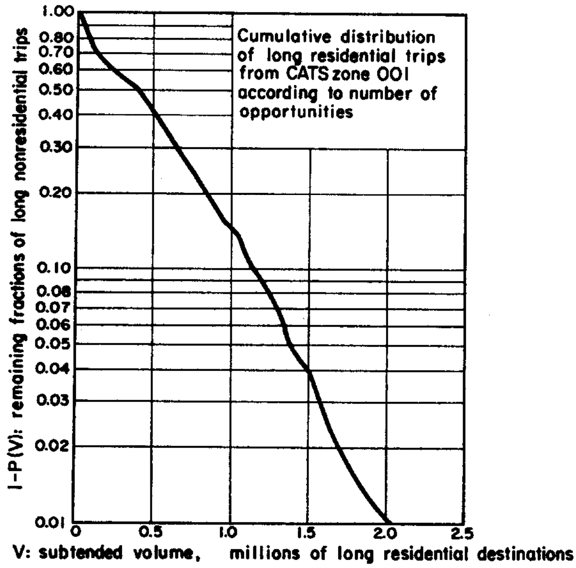

FIG. 1.

These three characteristics all support the interpretation of the L value as a modified probability quantity. It lies between zero and one, but is not unitless. It can be interpreted as the probability, per individual opportunity or trip end, of destination acceptance. This interpretation of the L value is in agreement with the model's theoretical basis.

An observation worth mentioning at this point is that the interpretation of the L value places no restrictions on the variation of L values from origin zone to origin zone. In fact, the interpretations which have been given here are based on one origin zone and would be seriously limited if L values could not vary from zone to zone.

The L value can be further characterized by determining the mean of the opportunity model probability distribution (2):

$$\bar{V} = \int_0^\infty V \mathrm{d}P(V)$$

or

$$\bar{V} = \int_0^\infty LV \exp(-LV) \mathrm{d}V = \frac{1}{L} \tag{9}$$

This relationship would be a useful method of specifying L values if average subtended volumes could be easily observed and predicted. Since this is not the case, L values must be related to trip parameters in order to obtain useful methods of specifying their values. This is done in the next section.

3.3. *Relationships of L values and trip parameters*

The L value has been interpreted in terms of subtended volume and fraction o unsatisfied, and in terms of average subtended volumes. For the extremely sim situation in which trip end density is assumed to be constant, the L value can be exp in terms of average trip length and average trip end density. Since trip length and tri densities are more common trip parameters than subtended volume, the expr obtained, although a simplification, provides insights into the nature of the L value.

In addition to the assumption of uniform trip end density, it is necessary to assum the time ranking of possible destinations can be replaced by a distance ranking witho of accuracy. This assumption would be true if the speed in all parts of the transpor system were constant, or nearly so.

The average trip length ($\bar{r}$) may be found by performing the following integration t the expected value of the distance variable when weighted by the opportunity distribution function:

$$\bar{r} = E(d) = \int_0^\infty f(V)\,\mathrm{d}P(V)$$

where:

d = distance variable (miles);

$E(d)$ = expected value of the distance variable (miles);

$f(V)$ = the expression of distance in terms of the variable V (subtended volume

$\mathrm{d}P(V)$ = the density function of the variable V, obtained from (2).

Because of the assumptions of constant density and distance ranking, subtended v can be expressed in terms of distance as follows:

$$V = \rho\pi d^2$$

where ρ = average trip end density (trip ends/mile^2). Equation (11) can be solved fc obtain $f(V)$:

$$d = f(V) = \left[\frac{V}{\rho\pi}\right]^{1/2}$$

Substituting into (9):

$$\bar{r} = \int_0^\infty \left[\frac{V}{\rho\pi}\right]^{1/2} L\exp(-LV)\,\mathrm{d}V$$

Carrying out the integration and simplifying, the following expression for $\bar{r}$ is obt

$$\bar{r} = \frac{1}{2}\left[\frac{1}{\rho L}\right]^{1/2}$$

or solving for L,

$$L = \frac{1}{4\rho\bar{r}^2}$$

Although it must be remembered that (15) is a gross approximation, it does in that L tends to be inversely proportional to trip end density and to the square of a trip length. Experience with the model has confirmed these tendencies.

A dimensional analysis of (15) indicates that, since ρ is expressed in trip ends per s mile and $\bar{r}$ is expressed in miles, the L value has the units (1/trip ends). Thus, (15) dimensionally with the theoretical basis of the opportunity model.

4. CALIBRATION OF THE OPPORTUNITY MODEL

A major benefit of the improved understanding of the opportunity model has been the enhanced ability to determine the parameters for—to calibrate—individual applications of the model. Calibration techniques are desired so that empirical data or predictions, such as total vehicle mileage and screenline traffic counts, can be duplicated by the traffic assignment process, which uses the output of the trip distribution process.

4.1. *The calibration criterion*

The matching of actual or predicted average trip lengths by the predicted trip interchanges has been the criterion which has led to the most useful calibration techniques. Since the number of trips in an area must be specified before the model can be applied, and since total vehicle miles of travel is the product of total trips and their average trip length, the matching of average trip lengths means that actual or predicted vehicle miles of travel will be matched by the model.

A case could conceivably be made for using the criterion of matching actual or predicted trip travel times which are more directly related to the opportunity model. The main reason for rejecting this criterion is that experience indicates travel time data obtained in travel surveys are much less reliable than travel distance data. Travel time data must be estimated by the trip maker, and this estimation is often quite gross. Travel distances, on the other hand, can be calculated from origin and destination information, which typically is much more accurately reported in travel surveys.

A second reason for using average trip lengths is that the travel distance for any given zonal interchange is invariant from start of opportunity model calculation through traffic assignment. This is not true of travel times, which are changed by the capacity restraint portion of the assignment method. Travel distances are therefore useful since they can be determined prior to calculation of trip distribution and traffic assignment data.

The prediction of future average trip lengths must, of course, be accomplished when the model is to be used to predict a future situation. The problem of predicting future average trip lengths is being studied. A National Cooperative Highway Research Project has as its goal the determination of trends in average trip lengths which can be observed over time on an area-wide basis (Voorhees *et al.*, 1966).

An investigation of zone by zone variations in average trip lengths, as they existed at the time of CATS 1966 travel surveys, has been conducted. One result of the investigation has been the discovery that an important source of zone-to-zone variation in average trip lengths is zonal trip end density. A hand-fitted plot of zonal average trip length versus zonal trip end density is shown in Fig. 2. The fact that zones with few trips tend to have a long average trip length is apparent. This tendency is an affirmation of the hypotheses of the opportunity model, which states that the satisfaction of trips, and, therefore, average trip length, is affected by the number of available destinations.

As more sources of variation in zonal average trip lengths which can be applied to a future situation are discovered, it will be possible to predict zonal average trip lengths, once area-wide trends have been determined. For example, if it is assumed that the curve of Fig. 2 will remain constant over time, future average trip lengths can be determined easily by reading values from the curve once future trips have been generated for each zone.

4.2. *Single L value calibration method*

The first attempt to develop a calibration method was the application of (14) to the case in which only one L value is desired per trip population. This method has been attributed to Morton Schneider (Muranyi, 1963). Since (14) is approximate, and it has been found that the "constant" term is not exactly 0·25 in each case, the equation has been modified to a ratio form so that the model results using one parameter can be used as added information when revising model parameters. The ratio form of (14) is:

$$\frac{\bar{r}_1}{\bar{r}_2} = \frac{\sqrt{(L_2 \rho_2)}}{\sqrt{(L_1 \rho_1)}} \tag{16}$$

where the average trip lengths ($\bar{r}_1$ and $\bar{r}_2$) and average trip end densities (ρ_1 and ρ_2) are obtained by considering the entire assignment area. The subscripts 1 and 2 refer to two particular times, places or assignment runs.

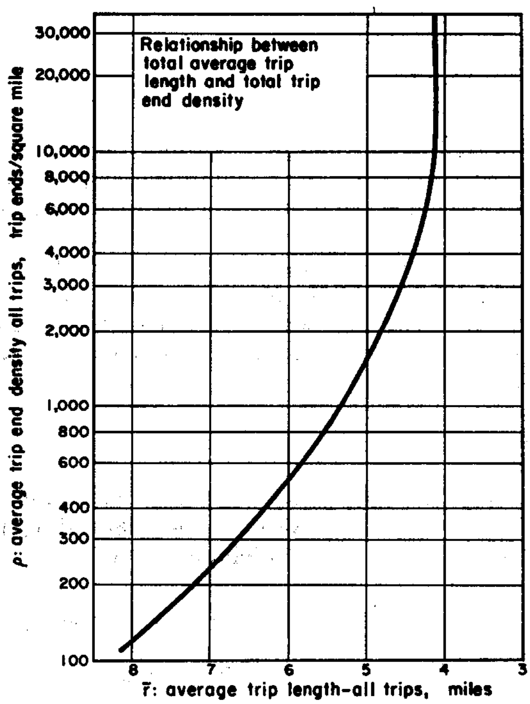

FIG. 2.

The L value obtained using (16) can be only regarded as a first approximation. The results of the assignment run using this first L value must be investigated to determine the direction in which the second L value must differ from the first. This trial-and-error process must continue until the desired accuracy is achieved.

The single L value calibration method was used in developing trips for all assignments to the entire CATS area run before August 1965. These assignments have included not only the 1956 "existing" runs and the 1980 "future" runs, but also runs for a number of intervening years. In each case, a combination of trial-and-error methods and the use of the single L value calibration method have resulted, finally, in an acceptable trip distribution and assignment. The number of preliminary runs has varied greatly, and in some cases has been reduced to one. Attempts to apply the single L value calibration method to small areas within the Chicago metropolitan region were not successful. Problems encountered in these small areas led to the adoption of a multiple L value model and multiple L value calibration techniques.

4.3. *Multiple L value calibration method*

Attempts to develop an efficient and accurate multiple L value calibration method have led to an iterative procedure which is easily computerized. The procedure is based on approximately determining the average trip length using the following discrete version of (10) for each origin zone o:

$$\bar{r}_o = \frac{\sum_{j=1}^{n} d_{oj}[\exp(-LV_j) - \exp(-LV_{j+1})]}{1 - \exp(-LV_n)} \tag{17}$$

where:

$\bar{r}_o$ = average trip length for zone o;

d_{oj} = distance from zone o to zone j;

n = total number of zones.

Although zones are ranked by time in the opportunity model, it is necessary to assume for the calibration method that a distance ranking will suffice. Otherwise, it would be necessary to use traffic assignment output in order to determine opportunity model parameters.

The data needed to use (17) to find L values are $\bar{r}_o$'s, d_{oj}'s and V_j's. But the V_j's are just summations of O_i's [see (5)], which are assignment inputs. The $\bar{r}_o$'s must be determined externally as discussed in 4.1. The d_{oj}'s can be calculated if each zone is assigned X and Y grid coordinates. In summary, the data needed to solve (17) can be obtained if the following are specified for each zone:

1. Trip origins;
2. X and Y coordinates;
3. Average trip length.

Solution of (17) is complicated by the fact that the L value cannot be isolated on one side of the equation. However, the following equation, obtained by multiplying both sides of (17) by $L/\bar{r}_o$, can be solved iteratively (Hildebrand, 1956):

$$L = F(L) = \frac{L\sum_{j=1}^{n} d_{oj}[\exp(-LV_j) - \exp(-LV_{j+1})]}{\bar{r}_o[1 - \exp(-LV_n)]} \tag{18}$$

Convergence is guaranteed if

$$\left|\frac{\mathrm{d}F(a)}{\mathrm{d}a}\right| < 1 \tag{19}$$

where

a = true value of L.

Since $F(L)$ depends upon a large number of parameters, it is difficult to check it for convergence in the general case. But it is known that (14) is approximately true, so a test of the $F(L)$ obtained by multiplying both sides of this simpler equation by $L/\bar{r}$ should indicate whether or not the $F(L)$ of equation (18) will converge. Multiplication of (14) by $L/\bar{r}$ results in:

$$L = F(L) = \frac{1}{2\bar{r}}\left[\frac{L}{\rho}\right]^{1/2} \tag{20}$$

Differentiating,

$$\frac{\mathrm{d}F(L)}{\mathrm{d}L} = \frac{1}{4\bar{r}}\left[\frac{1}{\rho L}\right]^{1/2} \tag{21}$$

Once differentiation is complete, $\bar{r}$ can be replaced by its equivalent, as given in (14). The resulting value of the derivative is 0·5, indicating that the condition expressed in (19) is met for the $F(L)$ of (20), and, therefore, should be met for the $F(L)$ of (18). An initial L value can be found using (15) with ρ replaced by O_o/A_o, where A_o is the area of zone o. This necessitates the addition of one more item to the list of zonal data needed, namely, the area of the zone.

A computer program has been written to carry out this iterative multiple L value calibration method for any or all zones within the area of interest. Both long and short L values are calculated.

Tests of the iterative calibration technique indicate, first of all, that average trip lengths resulting using calibrated L values are systematically high. This is caused by the change in ranking procedure from the calibration method (distance) to the model itself (time). A second test result is that the spread of differences between input and output average trip lengths, as measured by the root mean square of the difference, is about 15 per cent of the mean input average trip length. By correcting for the systematic error mentioned above, the root mean square of the difference is reduced to about 7 per cent, an entirely acceptable figure when the accuracy of the input data is considered.

5. POSSIBLE EXTENSIONS OF THE MODEL

The opportunity model as developed by Schneider (CATS, 1960) can be modified by starting from slightly different basic hypotheses of human behavior. It may be possible, using modified hypotheses, to develop a model in which it would not be necessary to break trips into a number of sub-populations. Two alternative approaches will be mentioned. Although these approaches both show promise, neither has been tested on a large-scale problem.

5.1. *Power function model*

This approach is based on a change of the second hypothesis of the opportunity model to the following statement:

The probability of a destination being accepted, if it is considered, is a function of the number of destinations which already have been considered.

When all trips, rather than just one sub-population of trips, are included in a graph of $\ln[1-P(V)]$ and V, a curve like that in Fig. 3 is obtained. A relationship of the form $L = aV^b$ would provide a better fit to these data than the straight line hypothesized by the opportunity model. Such a relationship would imply the following equation for predicting interzonal trips:

$$T_{ij} = O_i[\exp(-aV_j^{b+1}) - \exp(-aV_{j+1}^{b+1})] \tag{22}$$

5.2. *Time variable model*

Harris (1964) has proposed a number of alternative hypotheses as bases for variants of the opportunity model. One of these is that the L value is proportional to the additional separation, T, needed at any point to reach the "next" opportunity. (Harris uses distance as a measure of separation; time will be used here to remain consistent with the separation measure usually adopted when applying the opportunity model.)

L can then be expressed as

$$L = M\frac{dT}{dV} \tag{23}$$

Substituting (23) into (1),

$$dP = M\frac{dT}{dV}[1-P(V)]\,dV$$

or

$$dP = M[1-P(T)]\,dT \tag{24}$$

This equation leads to the following function for $P(T)$, analogous to (2):

$$P(T) = 1-\exp(-MT) \tag{25}$$

By simply assuming that T is a continuous function of V which has a derivative at all points from $V = 0$ to $V = \infty$, and that $T = 0$ when $V = 0$, the independent variable of the opportunity model becomes T. The exact shape of the V–T function is irrelevant.

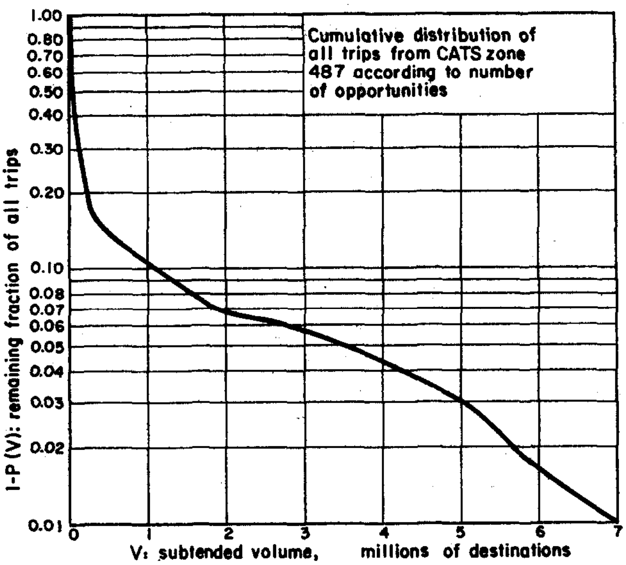

FIG. 3.

6. SUMMARY AND CONCLUSIONS

The opportunity model has been analyzed by interpreting its hypotheses, its mathematical formulation and its parameter, the L value. The L value also has been related to trip parameters. These interpretations have served as the basis of the calibration methods which have been used with the model. Methods of applying the calibrated model to the urban transportation planning process have been described. Possible extensions of the model based on relaxing the assumptions of the model have been presented.

By expanding the opportunity model from single L values per trip sub-population to multiple L values, and by using an efficient calibration technique, the model has been applied successfully to small areas within a large metropolitan region. It is expected that further modifications of the model, based on improved hypotheses of human behavior, and more efficient and accurate calibration procedures, will further enhance the ability of the transportation planner to distribute trips among possible destinations in a wide range of planning problems.

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