CMSC 35401: The Interplay of Learning and Game Theory (Autumn 2022)

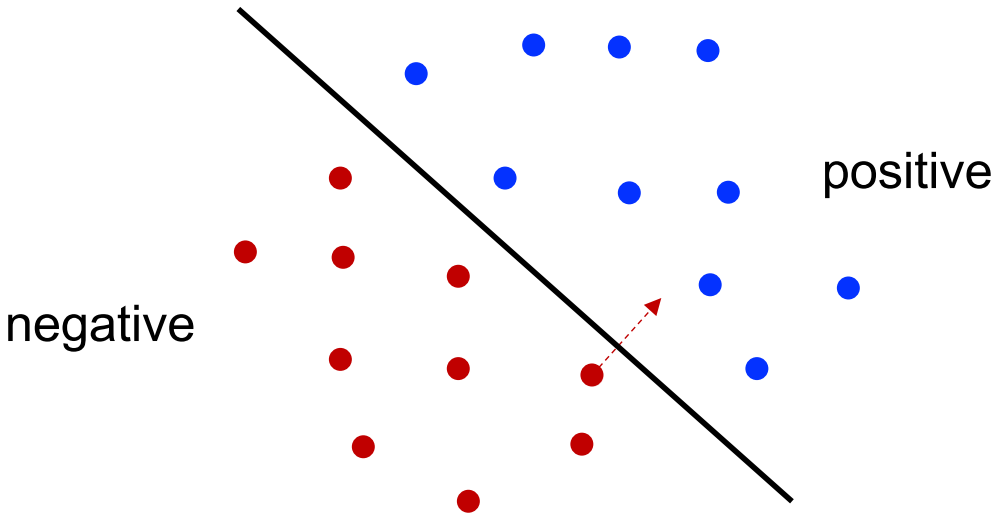
Learning From Strategic Data Sources

Instructor: Haifeng Xu

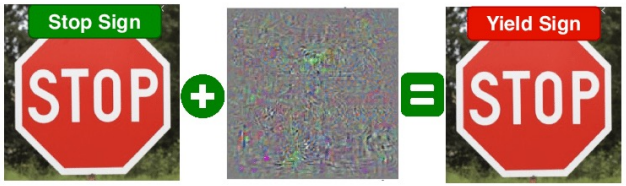
Outline

- Introduction to Strategic Classification
- Learnability and Computability of Strategic Classifiers
- Beyond Classification

Classification

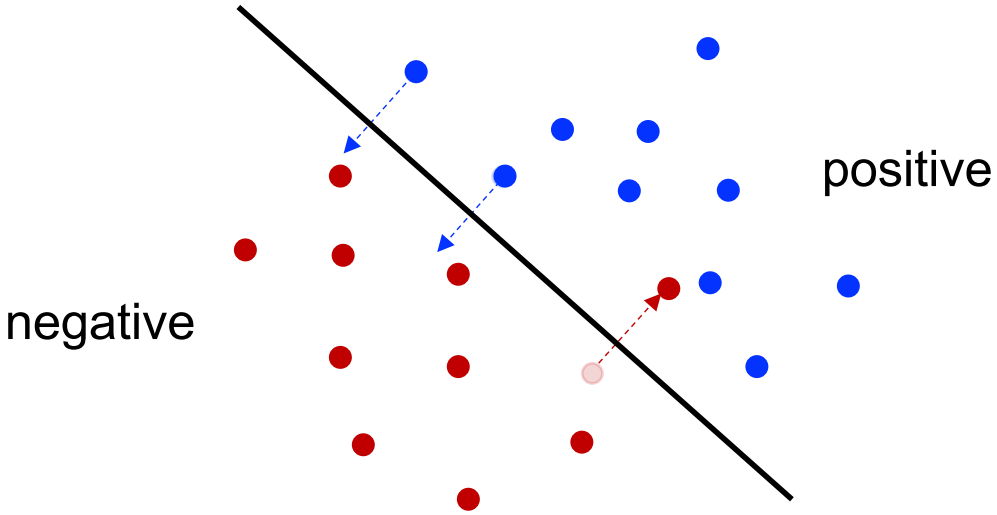


Data points' features may be manipulated



Adversarial attack

[Goodfellow et al.'15]
[Eykholt et al.'18]
[Cullina et al.'18]
……



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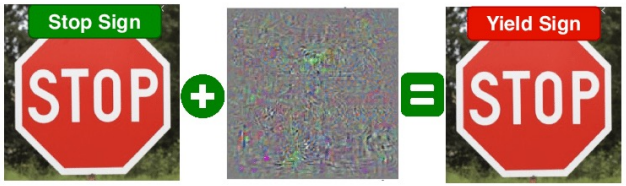
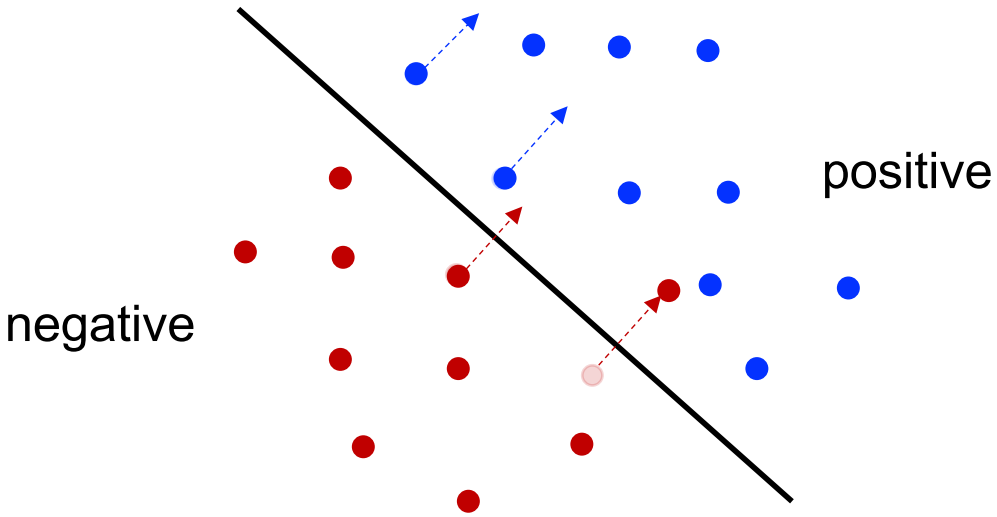


Adversarial attack

Strategic manipulation

[Hardt et al.'16]
[Hu et al.'19]
[Ghalme et al.'21]
……



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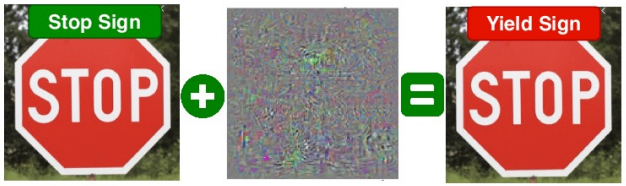
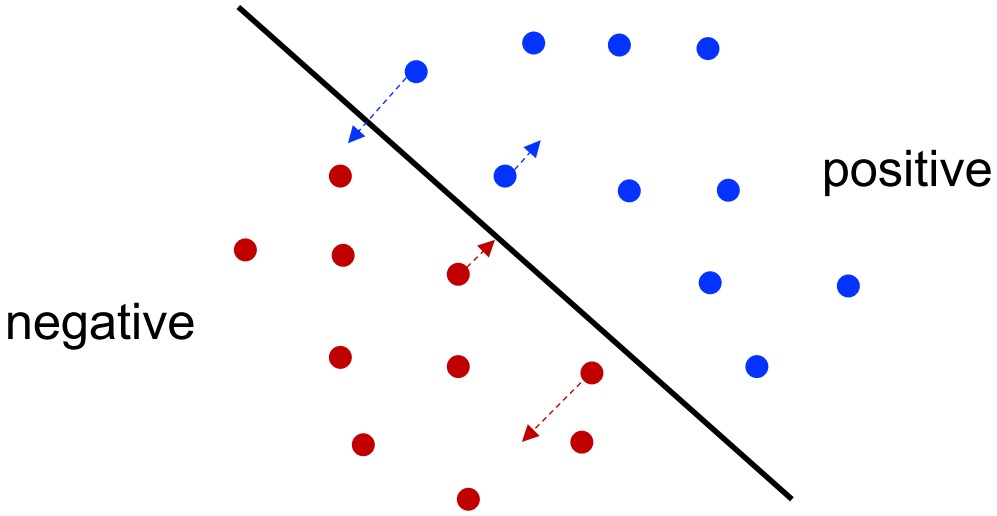


Adversarial attack

[SVXY'21]

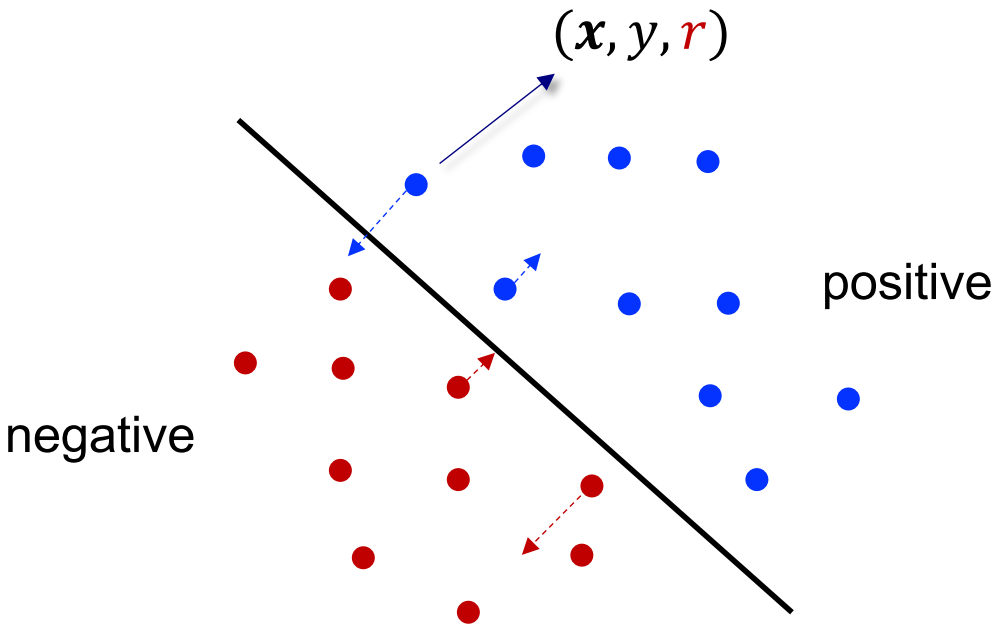
Strategic manipulation



Data points' features may be manipulated

A Unified Model of Strategic Classification

- Each data point is an individual agent, represented by $(\boldsymbol{x}, y, r)$
  - $r \in \mathbb{R}$ capture the point's incentive of being classified as positive



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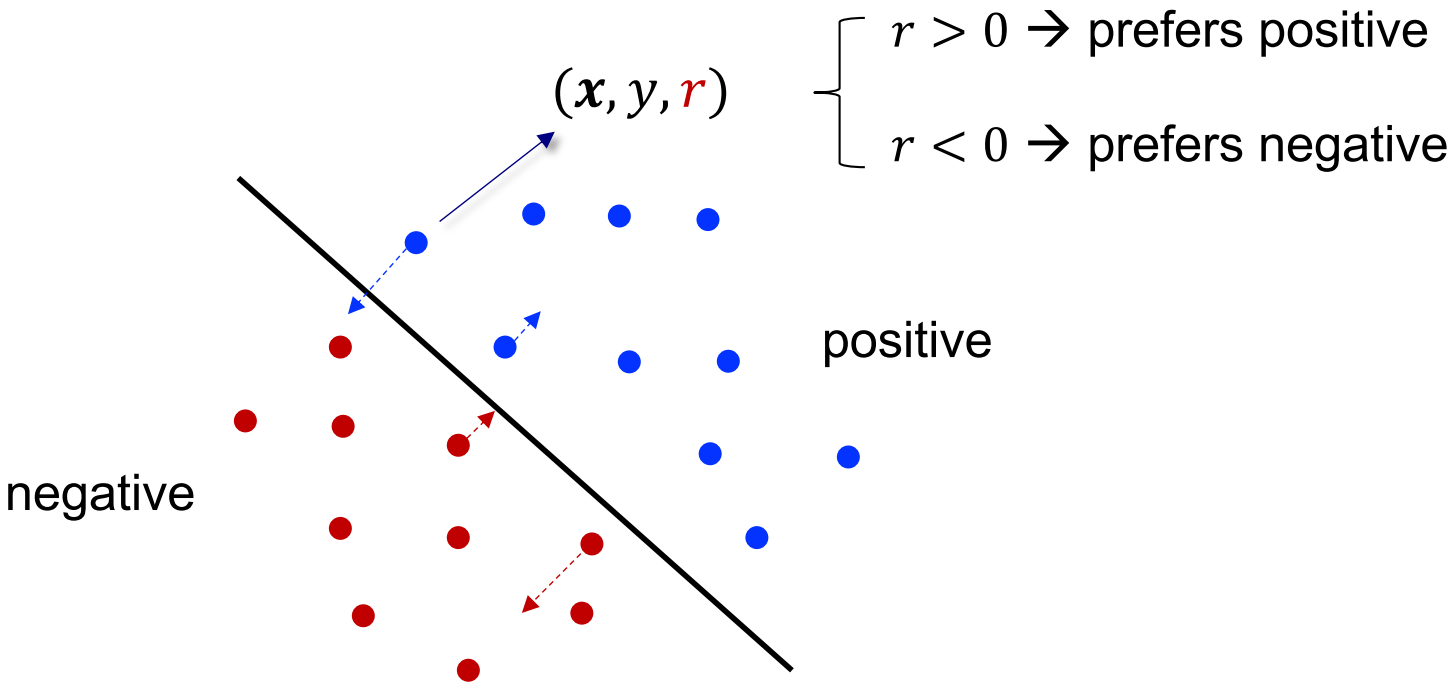

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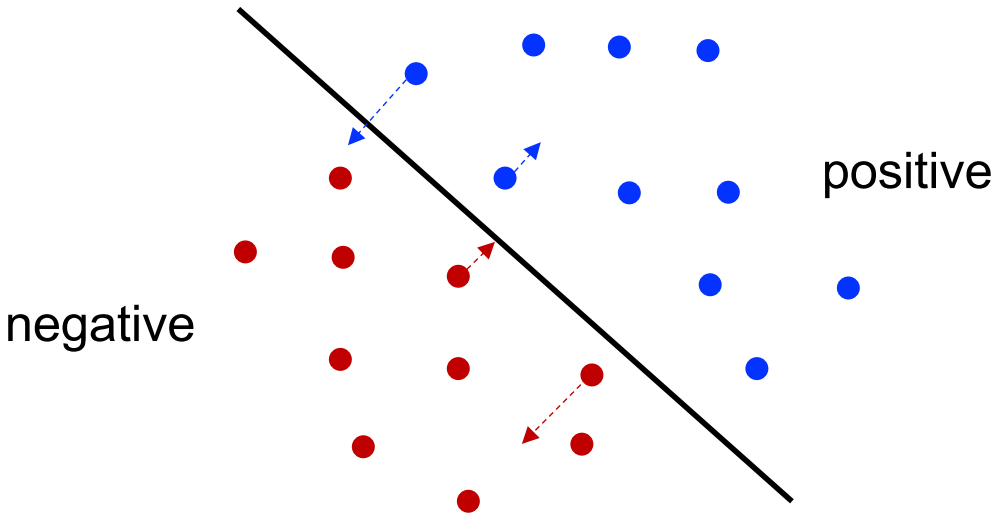
  - c is an arbitrary semi-norm



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- Given classifier $f: X \to \{0, 1\}$, data point $(\boldsymbol{x}, y, r)$ will manipulate its feature to $\boldsymbol{z}$ that maximizes utility

$$\underbrace{r \cdot \mathbb{I}(f(\boldsymbol{z}) = 1)}_{\text{reward from classification outcome}} - \underbrace{c(\boldsymbol{x} - \boldsymbol{z})}_{\text{Manipulation cost}}$$

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$$\boldsymbol{z}^*(\boldsymbol{x}, r; f) = \arg\max_{\boldsymbol{z} \in X} \left[r \cdot \mathbb{I}(f(\boldsymbol{z}) = 1) \; - \; c(\boldsymbol{x} - \boldsymbol{z}) \right]$$

This is a game now!

A Unified Model of Strategic Classification

The Strategic Classification Problem

Input: n uncontaminated training data $(\boldsymbol{x}_1, y_1, r_1), \cdots, (\boldsymbol{x}_n, y_n, r_n) \sim \mathcal{D}$

Learning goal: compute a classifier f that predicts well based only on the manipulated feature $\boldsymbol{z}^*(\boldsymbol{x}, r; f)$

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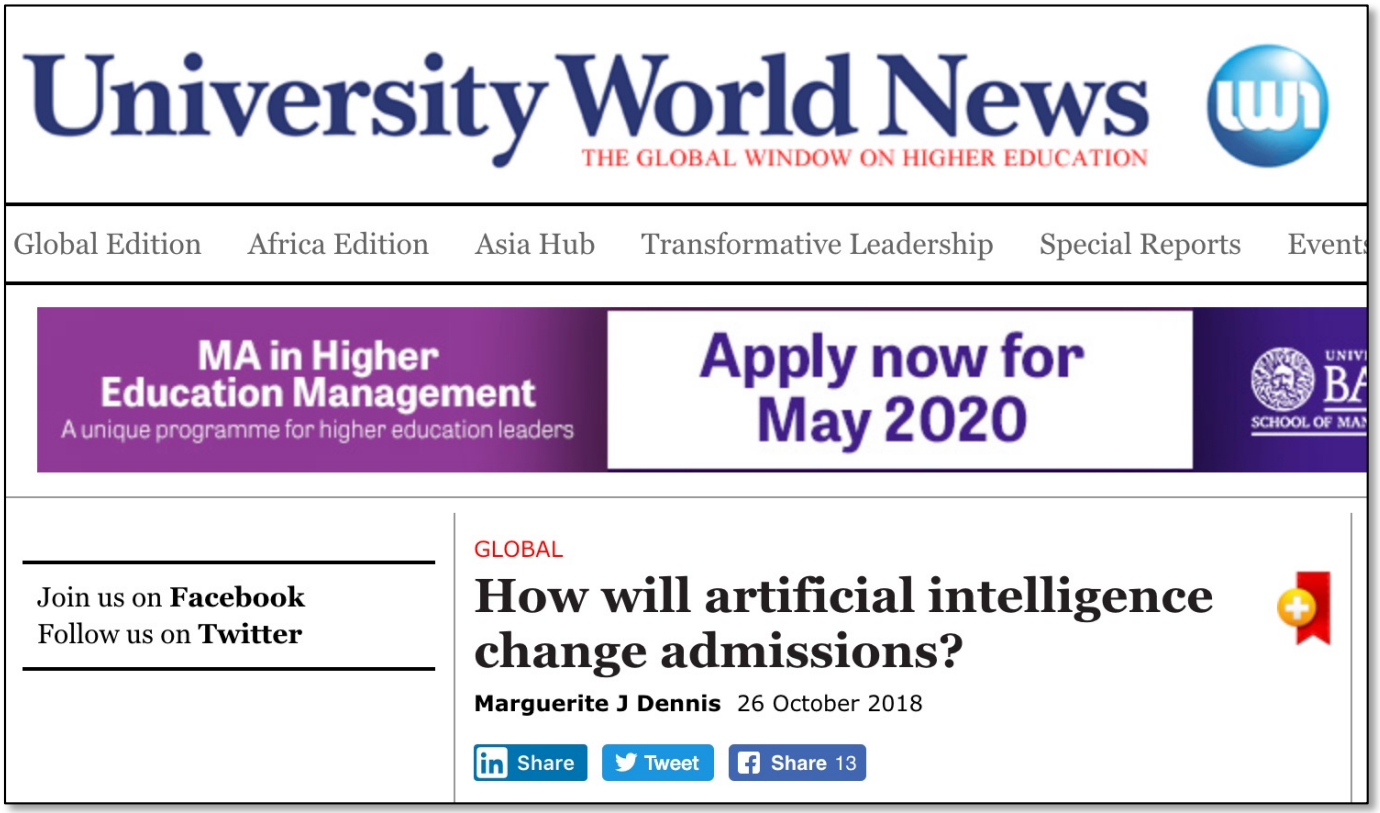
Some notably special cases

- ✓ $r \equiv 0$ → classic classification
- ✓ $r \equiv 1$ → strategic classification (cf. [Hardt et al.'16])
- ✓ $r = -y$ → adversarial classification (cf. [Cullina et al.'18])
- ✓ $\mathrm{sgn}(r) = -y$ → generalized adversarial classification

Remark: manipulation here does not change true label

Strategic Classifications are Everywhere

- University admissions
  - Students academic records are selectively revealed
  - Heterogeneous preferences: not all students prefer the same school



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- Classify loan lending decisions
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  - Heterogeneous preferences: not all borrowers prefer the same loan

30-Year Fixed *V.S* 15-Year Fixed

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- We decide which restaurants to go based on Yelp rating
  - Yelp may selectively show you the ratings
- Hiring job candidates in various scenarios

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- Hiring job candidates in various scenarios
- Note: this problem deserves study even you do classification manually instead of using an automated classifier

Slides adapted from Xintong Wang

Manipulation in Stock Trading

Spoofing is the practice of submitting large **spurious** buy (sell) orders to create artificial demand (supply) and mislead other traders.

UBS, Deutsche Bank and HSBC to pay millions in spoofing settlement, CFTC says

- Deutsche Bank will pay $30 million, UBS $15 million and HSBC $1.6 million to settle civil charges that some of their traders engaged in spoofing in the precious metals market.
- The CFTC charged six individuals, and the Department of Justice charged eight with crimes related to deceptive trading in a wide-ranging investigation.

Liz Moyer
Published 2:29 PM ET Mon, 29 Jan 2018 | Updated 8:32 AM ET Wed, 31 Jan 2018
CNBC

Luke MacGregor | Reuters

Flash Crash Trader E-Mails Show Spoofing Strategy, U.S. Says

by Tom Schoenberg Tschoenberg22 Suzi Ring journosooz Janan Hanna

September 3, 2015 – 4:03 PM EDT *Updated on* September 4, 2015 – 9:32 AM EDT

Navinder Singh Sarao leaves Westminster Magistrates' Court in London, on Friday, Aug. 28, 2015. Photographer: Chris Ratcliffe/Bloomberg Photographer: Chris Ratcliffe/Bloomberg

- Failed orders are `costing me,' Sarao said to tell programmer
- Indictment's new details seen bolstering U.S. extradition case

US seals first prosecution against stock market trader for 'spoofing'

A jury convicts Michael Coscia on six charges of commodities fraud and six charges of spoofing, all of the charges he faced

9 0 4 13 Email

Prosecutors said Michael Coscia wanted to lure other traders to markets by creating an illusion of demand so that he could make money on smaller trades Photo: AP

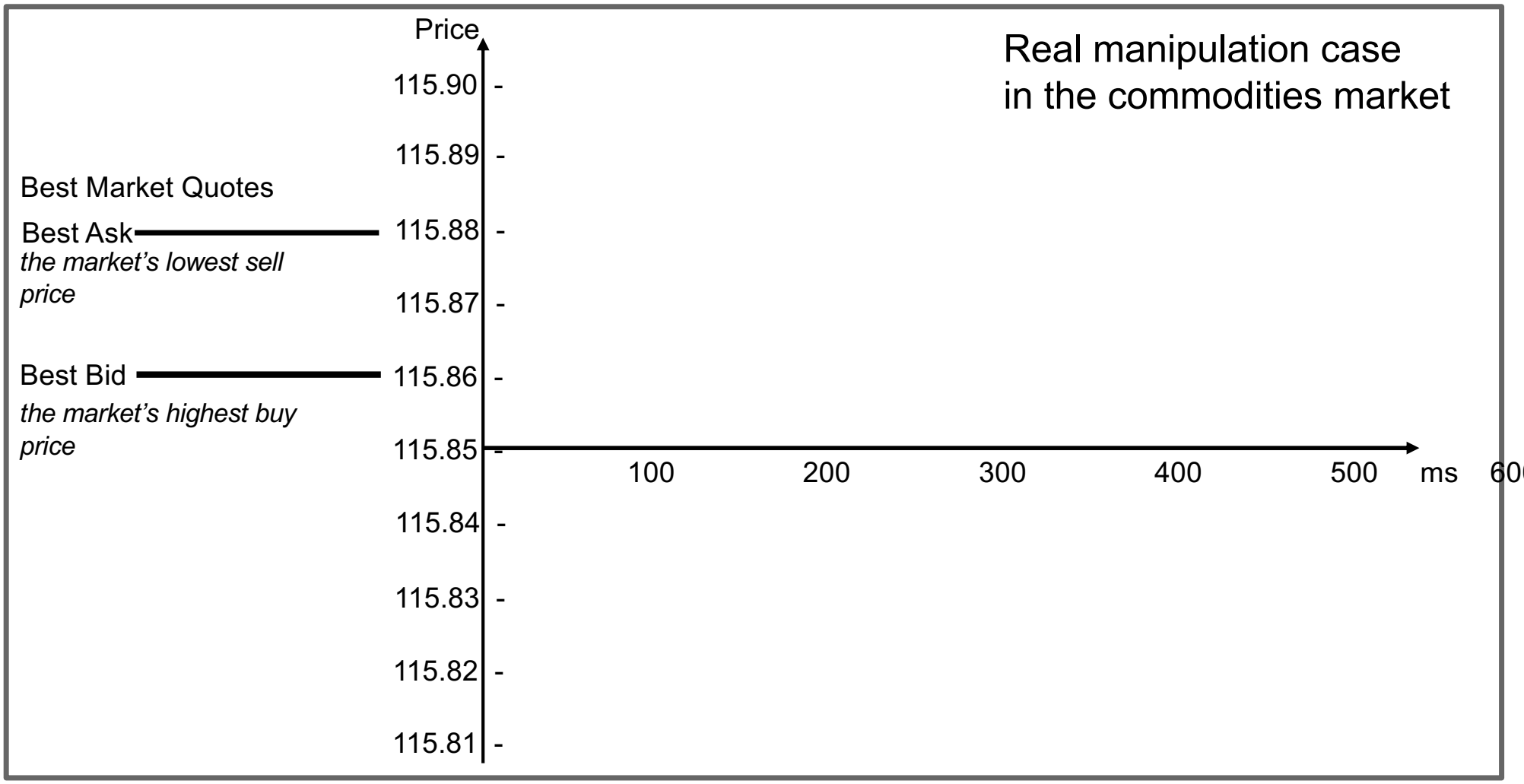
By Reuters
11:48PM GMT 03 Nov 2015

A US jury has found high-frequency trader Michael Coscia guilty of commodities fraud and "spoofing" in the US government's first criminal

Useful sources
https://www.fca.org.uk/publication/final-notices/coscia.pdf
https://www.fca.org.uk/publication/final-notices/coscia-appendix-1a.pdf

Slides adapted from Xintong Wang

Manipulation in Stock Trading



Source: Financial Conduct Authority, Animated Example of Mr. Coscia's Trading

Slides adapted from Xintong Wang

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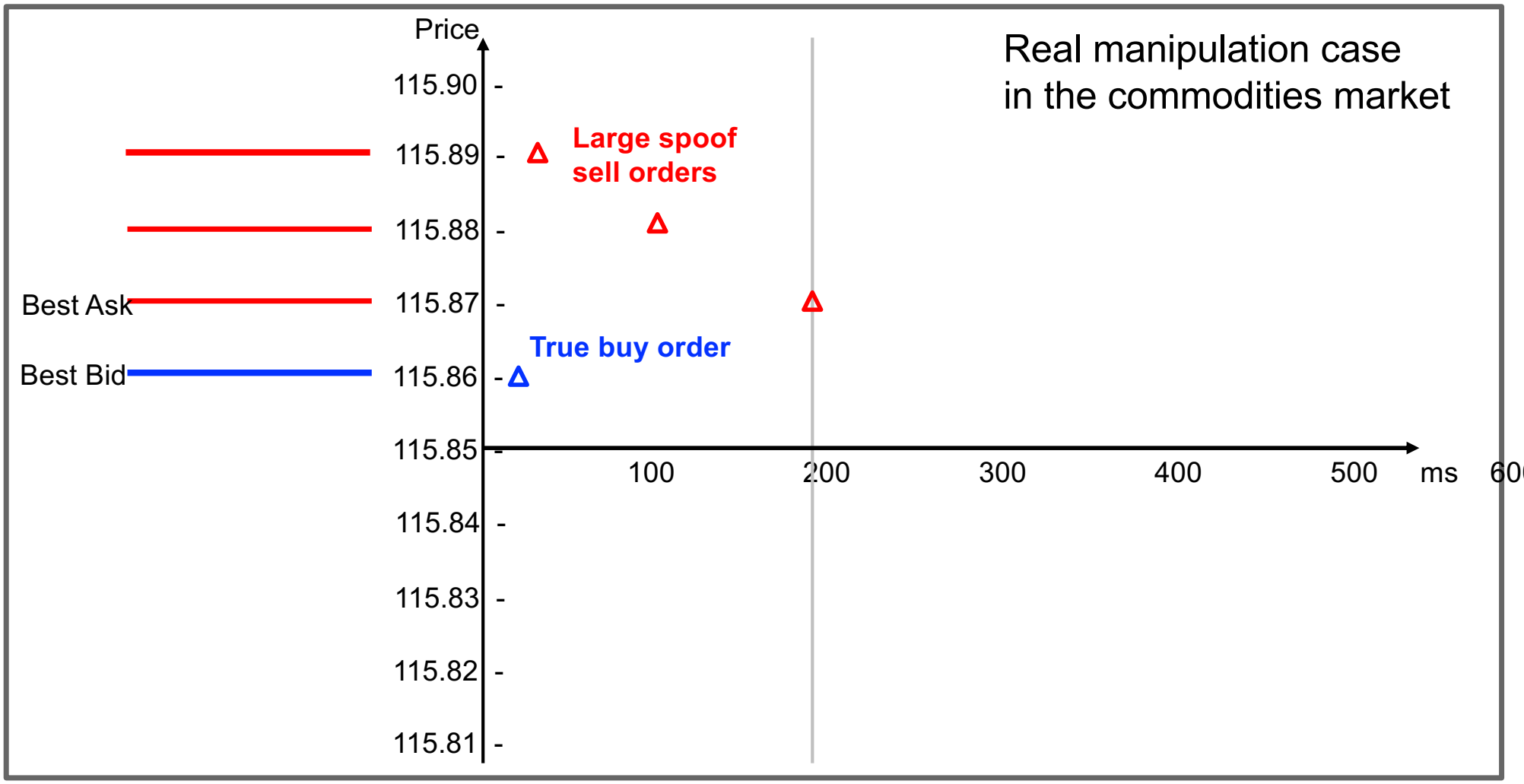

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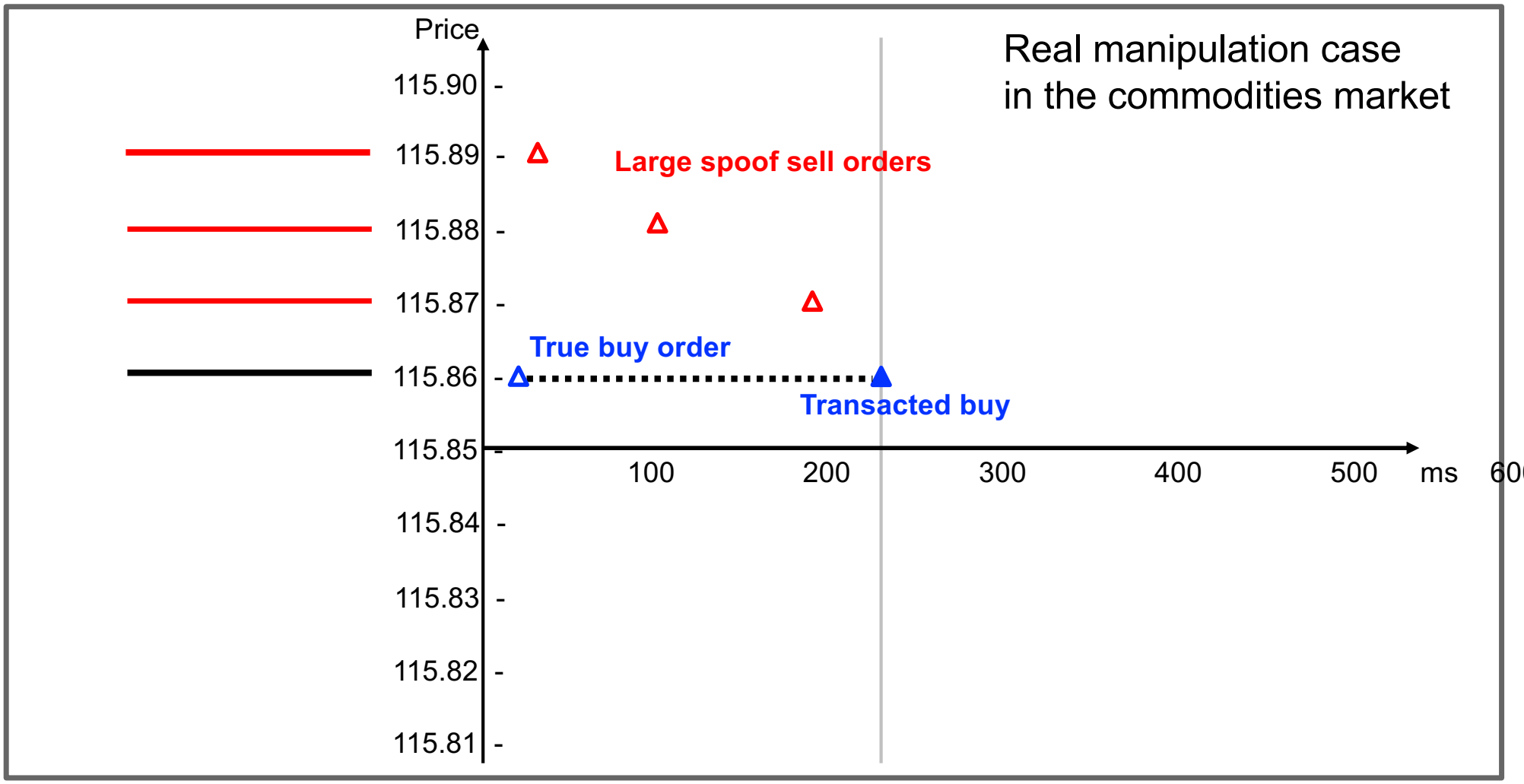

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Recall…

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Learning goal: compute a classifier f that predicts well based only on the manipulated feature $\boldsymbol{z}^*(\boldsymbol{x}, r; f)$

But will this general problem still be learnable? 

In classic ML setup

- ✓ Learnability (sample complexity) of a hypothesis class is governed by its VC-dimension
- ✓ The learning algorithm is the empirical risk minimization (ERM)

The Learnability of Strategic Classifiers

… is governed by a variant, coined strategic VC-dimension (SVC)

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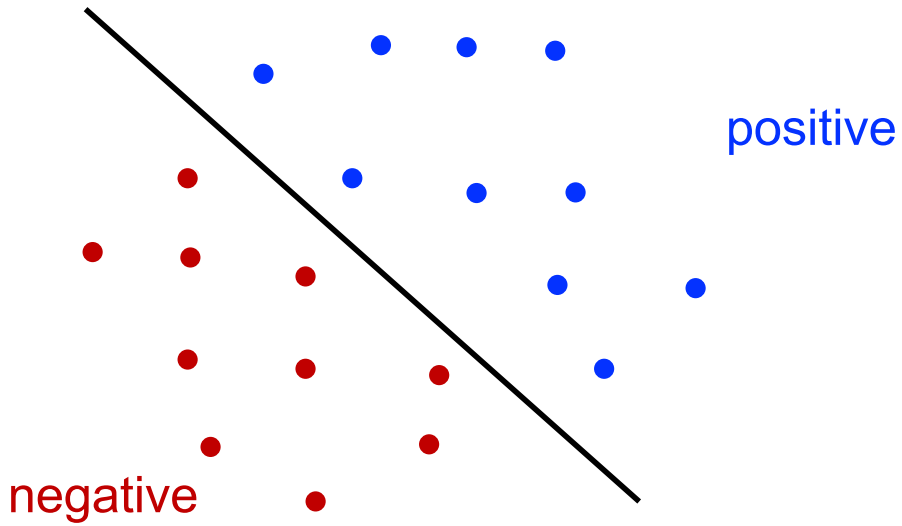
- Defined over the equilibrium of the classification outcome



Challenge is to characterize the classification outcomes under strategic manipulation

The Learnability of Strategic Classifiers

… is governed by a variant, coined strategic VC-dimension (SVC)

Theorem. Any strategic classification instance is (PAC) learnable via a strategic variant of ERM, with sample complexity

$$n(\epsilon, \delta) = \Theta\left(\frac{SVC + \log(1/\delta)}{\epsilon^2}\right)$$

where ϵ is accuracy loss and δ is the failure probability.

Unifies learnability of all previous special cases

- Generalizes the fundamental theorem of classic PAC learning ($r = 0$)
- Recovers the main sample complexity result of [Hardt et al.'16] with $r = 1$, for which we show their $SVC = 3$
- Generalizes learnability of adversarial classification [Cullina et al.'18] with $r = -y$

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Implies learnability of many new setups with heterogeneous data preferences: loan approval, student admission, classifying job candidates,…

Strategic Empirical Risk Minimization

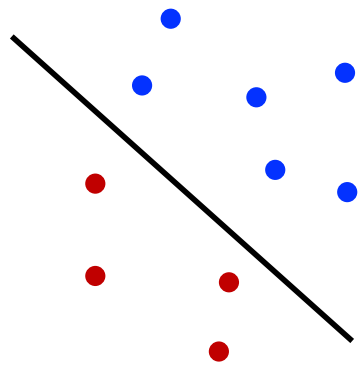
Input: n uncontaminated training data $(\boldsymbol{x}_1, y_1, r_1), \cdots, (\boldsymbol{x}_n, y_n, r_n) \sim \mathcal{D}$

Output: a classifier $h(\boldsymbol{z})$ that minimizes "strategic risk"

SERM: $\min_h \sum_{i=1}^{n} \mathbb{I}[h(\boldsymbol{z}_i) = y_i]$

$$\text{s.t.} \quad \boldsymbol{z}_i = \arg\max_{\boldsymbol{z} \in X} \left[r_i \cdot \mathbb{I}(h(\boldsymbol{z}) = 1) - c(\boldsymbol{x}_i - \boldsymbol{z}_i) \right], \forall i$$

- Strategic ERM minimizes empirical risk by accounting for manipulation



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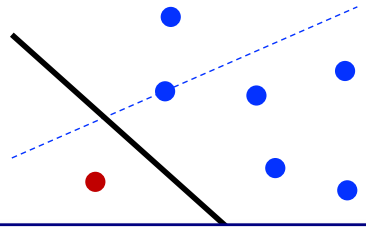
- Strategic ERM minimizes empirical risk by accounting for manipulation
- This is a bi-level optimization problem (a Stackelberg game with n followers)
  - Difficult to solve due to non-smooth objective functions

Instantiation to Linear Classification

Theorem. The SVC of d-dimensional linear classifiers is at most $d + 1$.

- $d + 1$ is the VC of linear classifiers in classic setup
- Learning strategic linear classifiers is no harder statistically
- Why can SVC be smaller than VC dimension?



Lessons Learned

Flexibility of manipulating features reduces the "richness" of possible classification outcomes, and may make it easier to learn

[SVXY, ICML'21]

Computing Strategic Linear Classifier

Unfortunately, not all news is good…

Theorem. Strategic empirical risk minimization (ERM) is NP-hard for linear classification.

[SVXY, ICML'21]

Computing Strategic Linear Classifier

Unfortunately, not all news is good…

Theorem. Strategic empirical risk minimization (ERM) is NP-hard for linear classification. But, strategic ERM can be solved in polynomial time when the instance is *essentially adversarial*.

$$min^- = \min\{r : (\boldsymbol{x}, y, r) \textit{ with } y = -1\} \textit{ and}$$
$$max^+ = \max\{r : (\boldsymbol{x}, y, r) \textit{ with } y = +1\}$$

Essentially adversarial if $min^- \geq max^+$

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Vignette 1: Manipulation in Multi-Armed Bandits

$\text{Reward}_1 \sim$ 

$\text{Reward}_2 \sim$ 

$\text{Reward}_3 \sim$ 

$\text{Reward}_4 \sim$ 

Paper: The Intrinsic Robustness of Stochastic Bandits to Strategic Manipulation

Vignette 1: Manipulation in Multi-Armed Bandits

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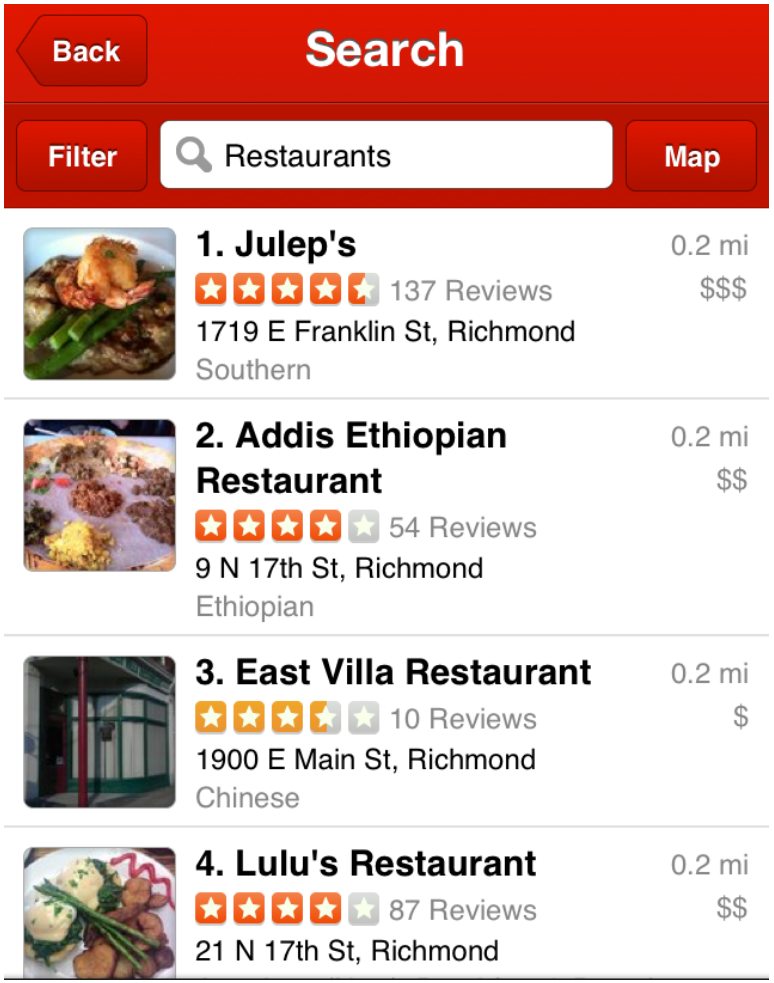


Each arm has incentives to manipulate its rewards to induce more pulls

Paper: [The Intrinsic Robustness of Stochastic Bandits to Strategic Manipulation]

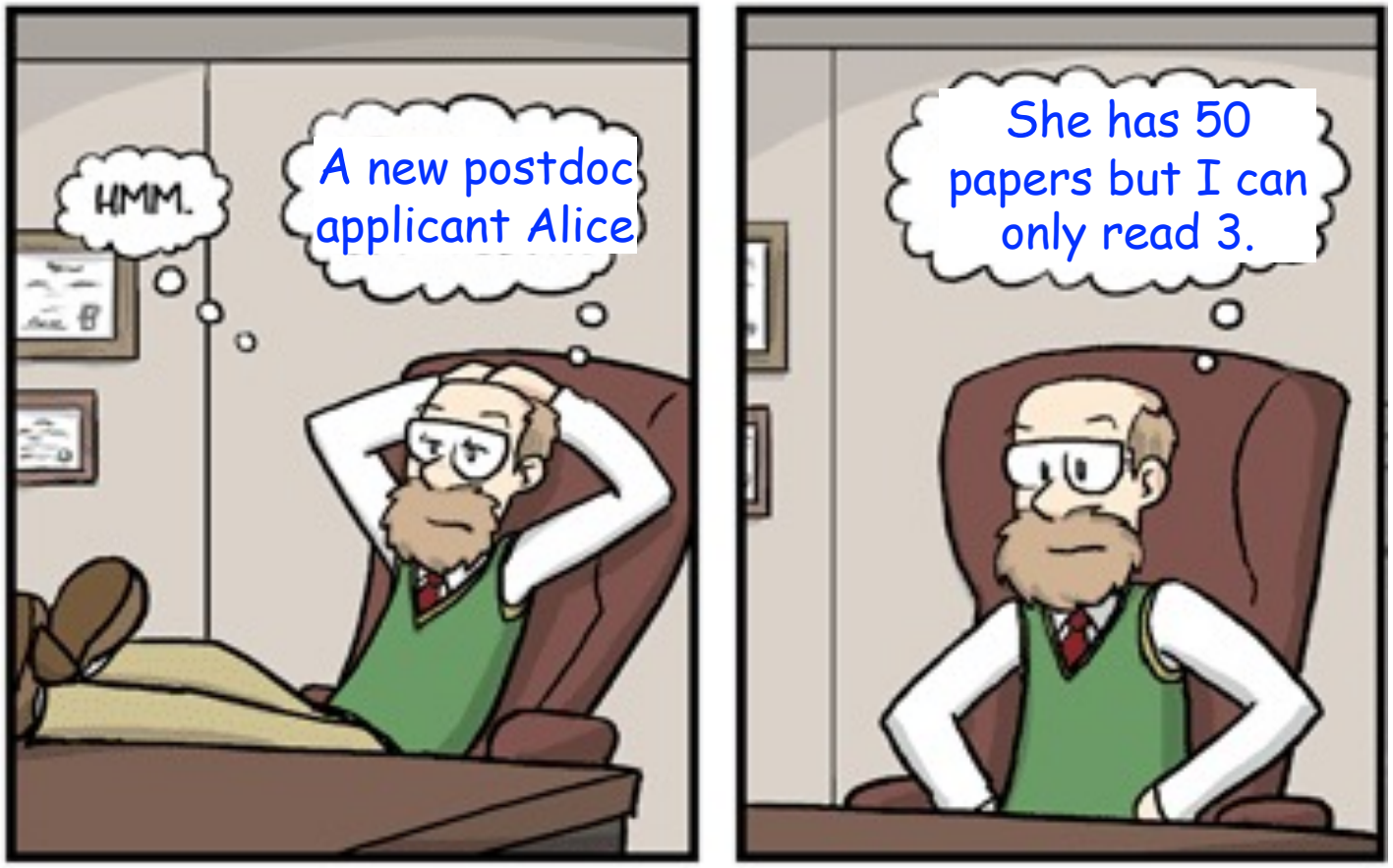
Vignette 1: Manipulation in Multi-Armed Bandits

Theorem. Most standard stochastic bandit algorithms (including UCB, ϵ-Greedy and Thomas Sampling) are all robust to selfish arms' strategic reward manipulation.

- A sharp contrast to adversarial reward attacks, which can ruin all these algorithms easily

Paper: The Intrinsic Robustness of Stochastic Bandits to Strategic Manipulation

Vignette 2: Distinguish Distributions from Strategic Samples



The Trouble of Professor Bob

Paper: When Samples Are Strategically Selected

Vignette 2: Distinguish Distributions from Strategic Samples

Current postdoc Charlie is happy . . .

Paper: When Samples Are Strategically Selected

Vignette 2: Distinguish Distributions from Strategic Samples

They know what each other is thinking…

Paper: When Samples Are Strategically Selected

Vignette 2: Distinguish Distributions from Strategic Samples

- A distribution $l \in \{g, b\}$ arrives, which can be a good distribution (g) or a bad one (b)
- An agent has access to n i.i.d. samples from l, from which he chooses a subset of exactly m samples as his report
  - Agent's goal: persuade a principal to accept l
- Principal observes agent's report, and decides whether to accept
  - Principal's goal: accept when $l = g$ and reject when $l = b$
  - Want to minimize her probability of mistakes

Other applications: e.g., deciding where to hold Olympics based on photographs of different city locations

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Paper: When Samples Are Strategically Selected

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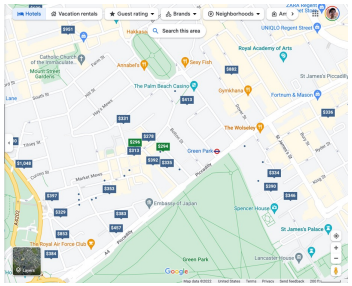

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Paper: When Samples Are Strategically Selected

Concluding Remarks

- Very active research area, with motivations from numerous economic applications
- Strategic studies of classification (online or offline, training time vs testing time), regression, bandits, reinforcement learning…
  - Closely related to adversarial attack and algorithm robustness as well
- Today's lecture – manipulation does not change true nature
  - Next lecture – "strategic improvement"

Thank You

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