

# CMSC 3540I: The Interplay of Learning and Game Theory (Autumn 2022)

## Introduction to Game Theory (I)

Instructor: Haifeng Xu



# Outline

- Games and its Basic Representation
- Nash Equilibrium and its Computation
- Other (More General) Classes of Games

# (Recall) Example 1: Prisoner's Dilemma

- Two members A,B of a criminal gang are arrested
- They are questioned in two separate rooms
  - ❖ No communications between them

	B	B stays silent	B betrays
A		-1	0
A stays silent	-1		-3
A betrays	0	-3	-2

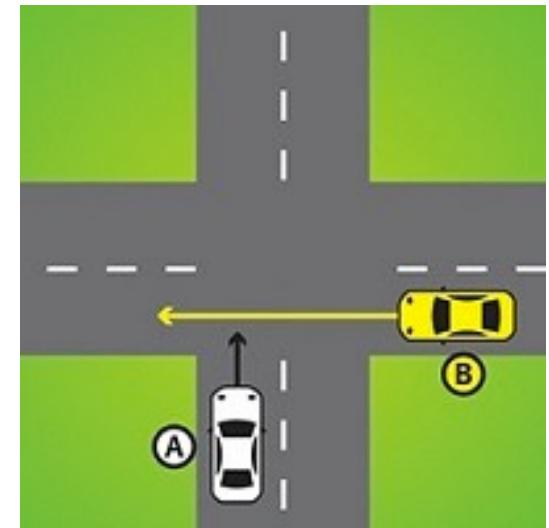
Q: How should each prisoner act?

- Both of them betray, though (-1,-1) is better for both

## Example 2: Traffic Light Game

- Two cars heading to orthogonal directions

		B
		STOP
A		GO
STOP	STOP	(-3, -2)
	GO	(0, -2)
GO	STOP	(-3, 0)
GO	GO	(-100, -100)



Q: what are the equilibrium statuses?

Answer: (STOP, GO) and (GO, STOP)

# Example 3: Rock-Paper-Scissor

		Player 2		
		Rock	Paper	Scissor
Player 1	Rock	(0, 0)	(-1, 1)	(1, -1)
	Paper	(1, -1)	(0, 0)	(-1, 1)
	Scissor	(-1, 1)	(1, -1)	(0, 0)

Q: what is an equilibrium?

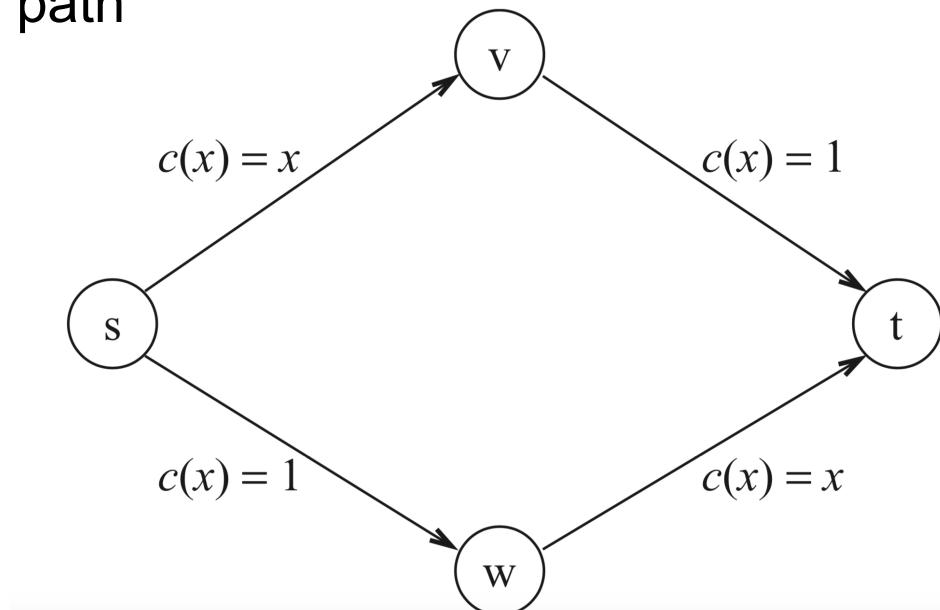
- Need to randomize – any deterministic action pair cannot make both players happy
- Common sense suggests  $(1/3, 1/3, 1/3)$

## Example 4: Selfish Routing

- One unit flow from  $s$  to  $t$  which consists of (infinite) individuals, each controlling an infinitesimal small amount of flow
- Each individual wants to minimize his own travel time

Q: What is the equilibrium status?

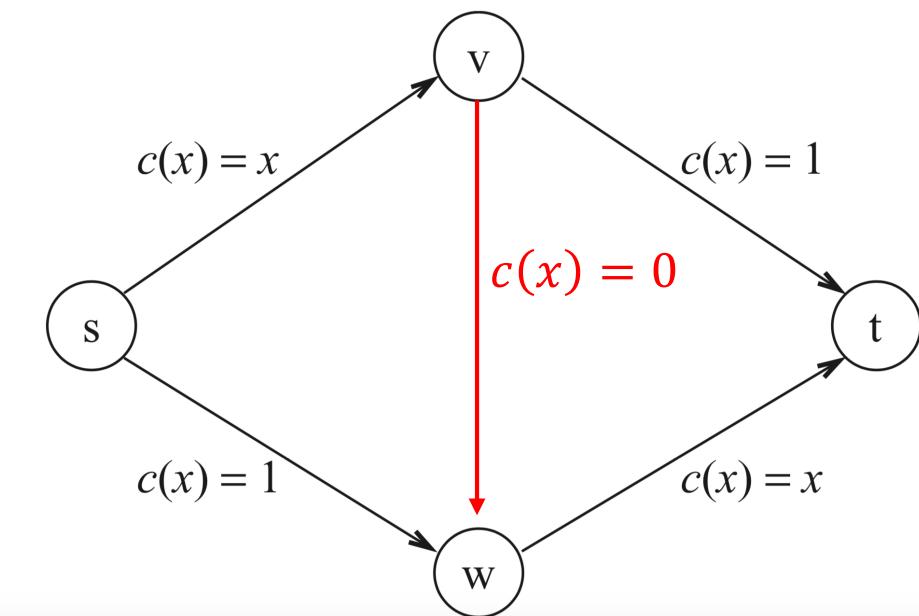
- Half unit flow through each path
- Social cost =  $3/2$



## Example 4: Selfish Routing

- One unit flow from  $s$  to  $t$  which consists of (infinite) individuals, each controlling an infinitesimal small amount of flow
- Each individual wants to minimize his own travel time

Q: What is the equilibrium status after adding a superior high way with 0 traveling cost?

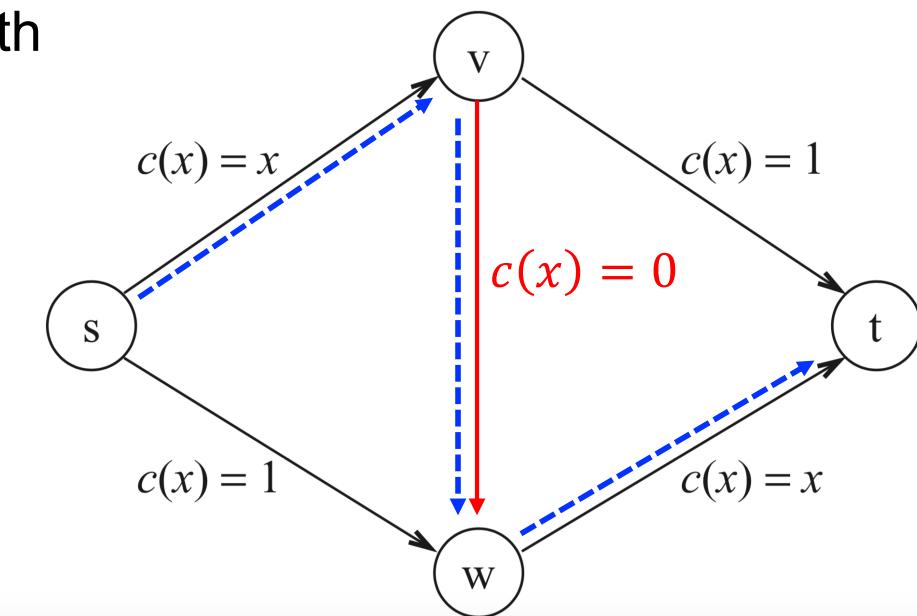


## Example 4: Selfish Routing

- One unit flow from  $s$  to  $t$  which consists of (infinite) individuals, each controlling an infinitesimal small amount of flow
- Each individual wants to minimize his own travel time

Q: What is the equilibrium status after adding a superior high way with 0 traveling cost?

- Everyone takes the blue path
- Social cost = 2



# Key Characteristics of These Games

- Each agent wants to maximize her own payoff
- An agent's payoff depends on other agents' actions
- The interaction stabilizes at a state where no agent can increase his payoff via **unilateral deviation**

# Strategic Games Are Ubiquitous

## ➤ Pricing

<input type="checkbox"/> Spirit Airlines (2) \$438	6:30am - 8:15am United Very Good Flight (8.1/10) <a href="#">Details &amp; baggage fees</a>	2h 45m (Nonstop) BOS - ORD	5 left at <b>\$236</b> roundtrip	<b>Select</b>
<input type="checkbox"/> Morning (5:00am - 11:59am)				
<input type="checkbox"/> Afternoon (12:00pm - 5:59pm)				
<input type="checkbox"/> Evening (6:00pm - 11:59pm)				
<b>Departure time - Boston</b>				
<input type="checkbox"/> Early Morning (12:00am - 4:59am)	9:23am - 11:27am American Airlines Very Good Flight (8.3/10) <a href="#">Details &amp; baggage fees</a>	3h 4m (Nonstop) BOS - ORD	<b>\$236</b> roundtrip	<b>Select</b>
<input type="checkbox"/> Morning (5:00am - 11:59am)				
<input type="checkbox"/> Afternoon (12:00pm - 5:59pm)				
<input type="checkbox"/> Evening (6:00pm - 11:59pm)				
<b>Arrival time - Chicago</b>				
<input type="checkbox"/> Early Morning (12:00am - 4:59am)	7:01am - 9:10am American Airlines Very Good Flight (8.3/10) <a href="#">Details &amp; baggage fees</a>	3h 9m (Nonstop) BOS - ORD	<b>\$236</b> roundtrip	<b>Select</b>
<input type="checkbox"/> Morning (5:00am - 11:59am)				
<input type="checkbox"/> Afternoon (12:00pm - 5:59pm)				
<input type="checkbox"/> Evening (6:00pm - 11:59pm)				
<input type="checkbox"/> Run this search to ad	5:30am - 8:50am Delta Satisfactory Flight (6.4/10) <a href="#">Details &amp; baggage fees</a>	4h 20m (1 stop) BOS - 42m in DTW - ORD	1 left at <b>\$246</b> roundtrip	<b>Select</b>

# Strategic Games Are Ubiquitous

- Pricing
- Sponsored search
  - Drives 90%+ of Google's revenue

The screenshot shows a Google search results page with the query "where to buy cruise vacation". The results are as follows:

- Carnival Cruise Line Ad**: \$1.03. The ad features a large price of \$1.03 and text about 2-5 day and 6-9 day cruises.
- Expedia Cruises Ad**: \$1.02. The ad features a large price of \$1.02 and text about finding the perfect cruise at the best price.
- VacationsToGo.com Ad**: \$0.60. The ad features a large price of \$0.60 and text about 2019 cruise deals.
- KAYAK.com Ad**: \$0.21. The ad features a large price of \$0.21 and text about finding the cheapest cruise deals.
- See cruise vac...** (Sponsored)
- 3-D Cruise Ship Centerpiece Zoom Party**: \$0.65. An image of a 3D cruise ship model.

# Strategic Games Are Ubiquitous

- Pricing
- Sponsored search
  - Drives 90%+ of Google's revenue
- FCC's Allocation of spectrum to radio frequency users

The screenshot shows the top navigation bar of the FCC website. It features the FCC logo and name on the left, followed by two search/filter options: "Browse by CATEGORY" and "Browse by BUREAUS & OFFICES". To the right is a search bar with a magnifying glass icon. Below the header is a blue navigation bar with links: "About the FCC", "Proceedings & Actions", "Licensing & Databases", "Reports & Research", "News & Events", and "For Consumers".

Home / Economics and Analytics /

## Auctions

### Proceedings & Actions

Proceedings and Actions Overview

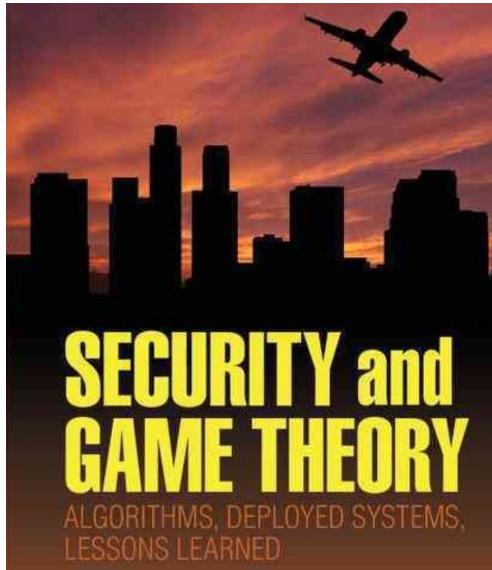
Since 1994, the Federal Communications Commission (FCC) has conducted auctions of licenses for electromagnetic spectrum. These auctions are open to any eligible company or individual that submits an application and upfront payment, and is found to be a qualified bidder by the Commission ([More About Auctions...](#))

#### Go to an Auction

Select an Au...

# Strategic Games Are Ubiquitous

- Pricing
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- FCC's Allocation of spectrum to radio frequency users
- National security, border patrolling, counter-terrorism



Optimize resource allocation against  
attackers/adversaries

# Strategic Games Are Ubiquitous

- Pricing
- Sponsored search
  - Drives 90%+ of Google's revenue
- FCC's Allocation of spectrum to radio frequency users
- National security, border patrolling, counter-terrorism
- Kidney exchange – decides who gets which kidney at when

The screenshot shows the UNOS website with a blue header bar containing the UNOS logo and navigation links: Transplant, Solutions, Technology, Data, Policy, Community, Resources, News, and a magnifying glass icon for search. Below the header, a breadcrumb trail reads "Home > Transplant > Kidney paired donation". The main content area has a blue background and features the text "Kidney paired donation" in white. At the bottom left of this area is a button labeled "Download PDF".

Kidney paired donation (KPD) is a transplant option for candidates who have a living donor who is medically able, but cannot donate a kidney to their intended candidate because they are incompatible (i.e., poorly matched).

[Download PDF](#)

[Learn about kidney paired donation](#)

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# Strategic Games Are Ubiquitous

- Pricing
- Sponsored search
  - Drives 90%+ of Google's
- FCC's Allocation of spectrum
- National security, border control
- Kidney exchange – decisions
- Entertainment games: poker, blackjack, Go, chess . . .
- Social choice problems such as voting, fair division, etc.



# Strategic Games Are Ubiquitous

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- FCC's Allocation of spectrum to radio frequency users
- National security, border patrolling, counter-terrorism
- Kidney exchange – decides who gets which kidney at when
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- Social choice problems such as voting, fair division, etc.

These are just a few example domains *where computer science has made significant impacts*; There are many others.

# Main Components of a Game

- **Players**: participants of the game, each may be an individual, organization, a machine or an algorithm, etc.
- **Strategies**: actions available to each player
- **Outcome**: the profile of player strategies
- **Payoffs**: a function mapping an outcome to a utility for each player

# Normal-Form Representation

- $n$  players, denoted by set  $[n] = \{1, \dots, n\}$
- Player  $i$  takes action  $a_i \in A_i$
- An outcome is the **action profile**  $a = (a_1, \dots, a_n)$ 
  - As a convention,  $a_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$  denotes all actions excluding  $a_i$
- Player  $i$  receives payoff  $u_i(a)$  for any outcome  $a \in \prod_{i=1}^n A_i$ 
  - $u_i(a) = u_i(a_i, a_{-i})$  depends on other players' actions
- $\{A_i, u_i\}_{i \in [n]}$  are public knowledge

This is the most basic game model

- There are game models with richer and more intricate structures

# Illustration: Prisoner's Dilemma

- 2 players: 1 and 2
- $A_i = \{\text{silent, betray}\}$  for  $i = 1, 2$
- An outcome can be, e.g.,  $a = (\text{silent, silent})$
- $u_1(a), u_2(a)$  are pre-defined, e.g.,  $u_1(\text{silent, silent}) = -1$
- The whole game is public knowledge; players take actions simultaneously
  - Equivalently, take actions without knowing the others' actions

# Dominant Strategy

An action  $a_i$  is a **dominant strategy** for player  $i$  if  $a_i$  is better than any other action  $a'_i \in A_i$ , **regardless** what actions other players take.

Formally,

$$u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i}), \quad \forall a'_i \neq a_i \text{ and } \forall a_{-i}$$

Note: “strategy” is just another term for “action”

	B	B stays silent	B betrays
A			
A stays silent	-1	0	
A betrays	-1	-3	

Prisoner's Dilemma

- Betray is a dominant strategy for both
- Dominant strategies do not always exist
  - For example, the traffic light game

	STOP	GO
STOP	(-3, -2)	(-3, 0)
GO	(0, -2)	(-100, -100)

# Equilibrium

- An outcome  $a^*$  is an equilibrium if no player has incentive to deviate **unilaterally**. More formally,

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*), \quad \forall a_i \in A_i$$

- A special case of Nash Equilibrium, a.k.a., *pure strategy NE*
- If each player has a dominant strategy, they form an equilibrium

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A stays silent	-1	-1	0
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Prisoner's Dilemma

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- If each player has a dominant strategy, they form an equilibrium
- But, an equilibrium does not need to consist of dominant strategies

Quiz: find equilibrium

		B	
		STOP	GO
		STOP	(-3, -2)
A	STOP	(0, -2)	(-100, -100)
	GO	(-3, 0)	

Traffic Light Game

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Traffic Light Game

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Quiz: find equilibrium

	L	M	R
U	4,3	5,1	6,2
M	2,1	8,4	3,6
D	3,0	9,6	2,5

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Quiz: find equilibrium

	Rock	Paper	Scissor
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissor	(-1, 1)	(1, -1)	(0, 0)

Pure strategy NE does not always exist...

# Outline

- Games and its Basic Representation
- Nash Equilibrium and its Computation
- Other (More General) Classes of Games

# Pure vs Mixed Strategy

- Pure strategy: take an action deterministically
- Mixed strategy: can randomize over actions
  - Described by a distribution  $x_i$  where  $x_i(a_i) = \text{prob. of taking action } a_i$
  - $|A_i|$ -dimensional simplex  $\Delta_{A_i} := \{x_i: \sum_{a_i \in A_i} x_i(a_i) = 1, x_i(a_i) \geq 0\}$  contains all possible mixed strategies for player  $i$
  - Players draw their own actions *independently*
- Given **strategy profile**  $x = (x_1, \dots, x_n)$ , expected utility of  $i$  is
$$\sum_{a \in A} u_i(a) \cdot \prod_{i \in [n]} x_i(a_i)$$
  - Often denoted as  $u(x)$  or  $u(x_i, x_{-i})$  or  $u(x_1, \dots, x_n)$
  - When  $x_i$  corresponds to some pure strategy  $a_i$ , we also write  $u(a_i, x_{-i})$
  - Fix  $x_{-i}$ ,  $u(x_i, x_{-i})$  is **linear** in  $x_i$

# Best Responses

Fix any  $x_{-i}$ ,  $x_i^*$  is called a best response to  $x_{-i}$  if

$$u_i(x_i^*, x_{-i}) \geq u_i(x_i, x_{-i}), \quad \forall x_i \in \Delta_{A_i}.$$

Claim. There always exists a pure best response

Proof: linear program “ $\max u_i(x_i, x_{-i})$  subject to  $x_i \in \Delta_{A_i}$ ” has a vertex optimal solution

Remark: If  $x_i^*$  is a best response to  $x_{-i}$ , then any  $a_i$  in the support of  $x_i^*$  (i.e.,  $x_i^*(a_i) > 0$ ) must be equally good and are all “pure” best responses

# Nash Equilibrium (NE)

A mixed strategy profile  $x^* = (x_1^*, \dots, x_n^*)$  is a **Nash equilibrium** if

$$u_i(x_i^*, x_{-i}^*) \geq u_i(x_i, x_{-i}^*), \quad \forall x_i \in \Delta_{A_i}, \forall i \in [n].$$

That is, for any  $i$ ,  $x_i^*$  is a best response to  $x_{-i}^*$ .

## Remarks

- An equivalent condition:  $u_i(x_i^*, x_{-i}^*) \geq u_i(a_i, x_{-i}^*), \forall a_i \in A_i, \forall i \in [n]$ 
  - Since there always exists a pure best response
- It is not clear yet that such a mixed strategy profile would exist
  - Recall that pure strategy Nash equilibrium may not exist

# Nash Equilibrium (NE)

**Theorem (Nash, 1951):** Every finite game (i.e., finite players and actions) admits at least one mixed strategy Nash equilibrium.

- A foundational result in game-theory
- Example: rock-paper-scissor – what is a mixed strategy NE?

- $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$  is a best response to  $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

		1/3	1/3	1/3
ExpU = 0	Rock	(0, 0)	(-1, 1)	(1, -1)
ExpU = 0	Paper	(1, -1)	(0, 0)	(-1, 1)
ExpU = 0	Scissor	(-1, 1)	(1, -1)	(0, 0)

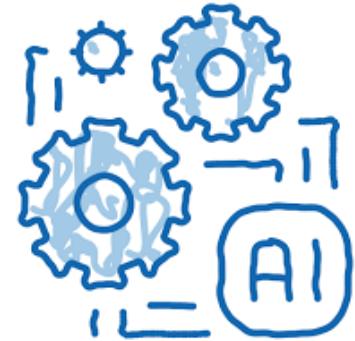
# Nash Equilibrium (NE)

**Theorem (Nash, 1951):** Every finite game (i.e., finite players and actions) admits at least one mixed strategy Nash equilibrium.

- An equilibrium outcome is not necessarily the best for players
  - Equilibrium only describes where the game stabilizes at
  - Many researches on understanding how self-interested behaviors reduces overall social welfare (recall the selfish routing game)
- A game may have many, even infinitely many, NEs
  - Which equilibrium you think it will stabilize at? → **the issue of equilibrium selection**

	B	B stays silent	B betrays
A			
A stays silent	-1	-1	0
A betrays	0	-3	-2

# Computing a NE



Why we want to compute?

- Reason 1: want to predict what would happen when a multi-agent system stabilizes
  - E.g., if each self-driving car optimizes the time for its driver, would the road be efficient overall?
  - If each seller on Amazon tries to optimize their own revenue, what would be the ultimate price?
- Reason 2: want to figure out best action to take
  - Just like why we want to solve single-agent optimization problem
  - E.g., want to figure out best GO/Poker agent strategy

# Intractability of Finding a NE

**Theorem:** Computing a Nash equilibrium for any two-player normal-form game is PPAD-hard.

Note: PPAD-hard problems are believed to not admit poly time algorithm

- A two player game can be described by  $2mn$  numbers –  $u_1(i, j)$  and  $u_2(i, j)$  where  $i \in [m]$  is player 1's action and  $j \in [n]$  is player 2's.
- Theorem implies no  $\text{poly}(mn)$  time algorithm to compute an NE for any input game
- Ok, so what can we hope?
  - If the game has good structures, maybe we can find an NE efficiently
  - For example, zero-sum ( $u_1(i, j) + u_2(i, j) = 0$  for all  $i, j$ ), some resource allocation games

# An Exponential-Time Alg for Two-Player Nash

- What if we know the support of the NE:  $S_1, S_2$  for player 1 and 2?
- The NE can be formulated by a **linear feasibility** problem with variables  $x_1^*, x_2^*, U_1, U_2$

$$\forall j \in S_2: \quad \sum_{i \in S_1} u_2(i, j) x_1^*(i) = U_2$$

$$\forall j \notin S_2: \quad \sum_{i \in S_1} u_2(i, j) x_1^*(i) \leq U_2$$

# An Exponential-Time Alg for Two-Player Nash

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$$\forall j \notin S_2: \quad \sum_{i \in S_1} u_2(i, j) x_1^*(i) \leq U_2$$

$$\sum_{i \in [m]} x_1^*(i) = 1$$

$$\forall i \notin S_1: \quad x_1^*(i) = 0$$

$$\forall i \in [m]: \quad x_1^*(i) \geq 0$$

# An Exponential-Time Alg for Two-Player Nash

- What if we know the support of the NE:  $S_1, S_2$  for player 1 and 2?
- The NE can be formulated by a **linear feasibility** problem with variables  $x_1^*, x_2^*, U_1, U_2$

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$$\forall j \notin S_2: \quad \sum_{i \in S_1} u_2(i, j) x_1^*(i) \leq U_2$$

$$\sum_{i \in [m]} x_1^*(i) = 1$$

$$\forall i \notin S_1: \quad x_1^*(i) = 0$$

$$\forall i \in [m]: \quad x_1^*(i) \geq 0$$

Symmetric constraints for player 2

- The challenge of computing a NE is to find the correct supports
  - No general tricks, typically just try all possibilities
  - Some pre-processing may help, e.g., eliminating dominated actions
- This approach does not work for  $> 2$  players games (why?)

# An Example

	A	B	C
D	0, 0	-1, 1	1, -1
E	1, -1	0, 0	-1, -2
F	-1, 1	1, -1	0, -2

Step 1: pre-processing

- Column player never wants to play C

# An Example

	A	B	C
D	0, 0	-1, 1	1, -1
E	1, -1	0, 0	-1, -2
F	-1, 1	1, -1	0, -2

Step 1: pre-processing

- Column player never wants to play C

# An Example

		$q_1$	$q_2$	0
		A	B	C
0	D	0, 0	-1, 1	1, -1
$p_2$	E	1, -1	0, 0	-1, -2
$p_3$	F	-1, 1	1, -1	0, -2

Step 2: Guess support and parameterize the equilibrium

$$\begin{aligned} u_1(E, A) \times q_1 + u_1(E, B) \times q_2 &= u \\ u_1(F, A) \times q_1 + u_1(F, B) \times q_2 &= u \end{aligned} \quad \left. \begin{array}{l} \text{Row player indifferent} \\ \text{between } \{E, F\} \end{array} \right\}$$

$$u_1(D, A) \times q_1 + u_1(D, B) \times q_2 \leq u \quad \rightarrow \quad \begin{array}{l} \text{Row player prefers } \{E, \\ F\} \text{ over } \{D\} \end{array}$$

... same for column player

Solve LP for  $p_2, p_3, q_1, q_2, u, v$

# An Example

		1/3	2/3	0
		A	B	C
0	D	0, 0	-1, 1	1, -1
2/3	E	1, -1	0, 0	-1, -2
1/3	F	-1, 1	1, -1	0, -2

Turns out our guess of support is correct

- If not, LP will be infeasible;
- In general, try all possibilities of support → Nash's theorem guarantees that one of LP systems must be feasible

# Intractability of Finding “Best” NE

**Theorem:** It is NP-hard to compute the NE that maximizes the sum of players' utilities or any single player's utility even in two-player games.

- Proofs of these results for NEs are beyond the scope of this course

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- Other (More General) Classes of Games

# Bayesian Games

- Previously, assumed players have complete knowledge of the game
- What if players are uncertain about the game?
- Can be modeled as a Bayesian belief about the state of the game
  - This is typical in Bayesian decision making, but not the only way

	B	B stays silent	B betrays
A			
A stays silent		$\theta - 1$ $+ \theta$	0 $+ \theta$
A betrays		$\theta - 3$	-2

*I will give an additional reward  $\theta$  for whoever staying silent*



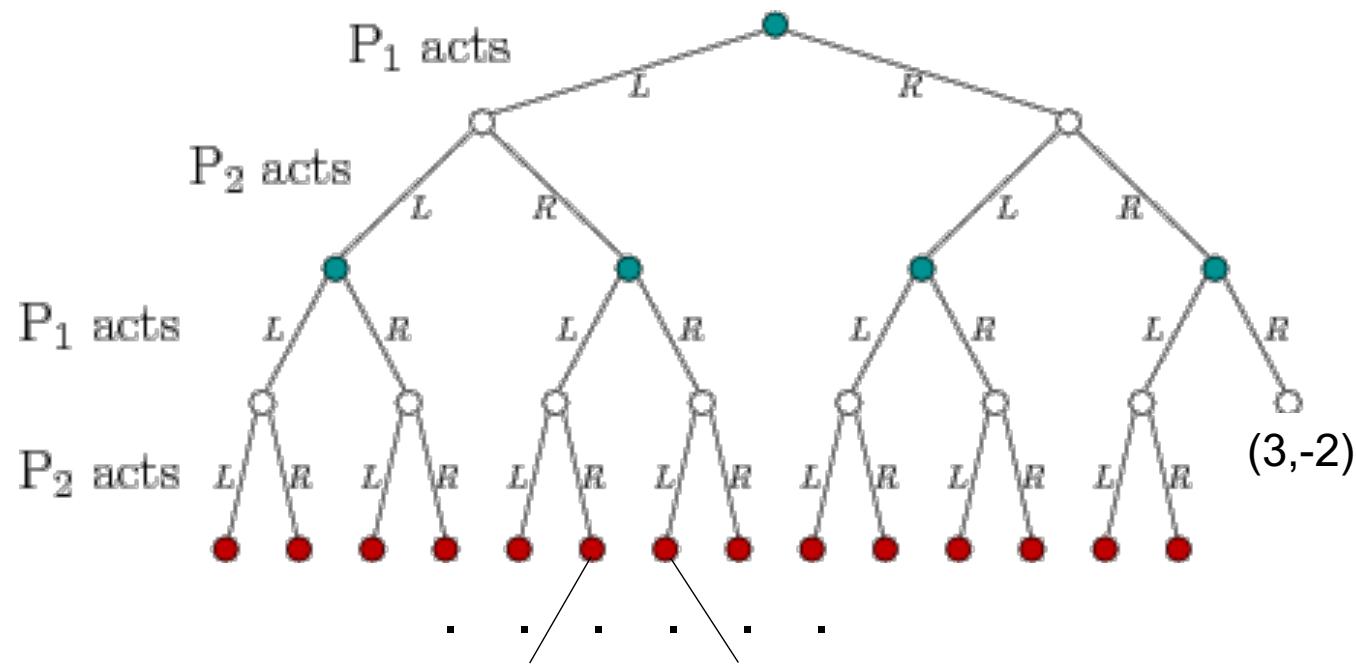
- It is believed that  $\theta \in \{0, 2, 4\}$  uniformly at random
- Or maybe the two players have different beliefs about  $\theta$

# Bayesian Games

- Previously, assumed players have complete knowledge of the game
- What if players are uncertain about the game?
- Can be modeled as a Bayesian belief about the state of the game
  - This is typical in Bayesian decision making, but not the only way
- More generally, can model player  $i$ ' payoffs as  $u_i^\theta$  where  $\theta$  is a **random** state of the game
- Each player obtains a (random) signal  $s_i$  that is correlated with  $\theta$ 
  - A joint prior distribution over  $(\theta, s_1, \dots, s_n)$  is assumed the public knowledge
- Can define a similar notion as Nash equilibrium, but expected utility also incorporates the randomness of the state of the game  $\theta$
- Applications: poker, blackjack, auction design, etc.

# Extensive-Form Games (EFGs)

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- More generally, can move sequentially and for multiple rounds
- Modeled by extensive-form game, described by a **game tree**



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- Previously, assumed players move only once and **simultaneously**
- More generally, can move sequentially and for multiple rounds
- Modeled by extensive-form game, described by a **game tree**
- EFGs are extremely general, can represent almost all kinds of games, but of course very difficult to solve

# A Remark

Sequential move fundamentally differs from simultaneous move

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➤ What is an NE?

- $(a_2, b_2)$  is the unique Nash, resulting in utility pair (1,2)

➤ If A moves first; B sees A's move and then best responds, how should A play?

- Play action  $a_1$  deterministically!

		B	
		$b_1$	$b_2$
		$a_1$	$(2, 1)$
		$a_2$	$(2.01, -2)$
			$(1, 2)$

This sequential game model is called **Stackelberg game**, originally used to model market competition and now adversarial attacks.

# Thank You

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