

Announcements

- HW 2 is due next Tuesday
 - No class on next Tuesday, but TAs will be here to collect HW

CS6501:Topics in Learning and Game Theory (Fall 2019)

Prediction Markets (as a Forecasting Tool)



Instructor: Haifeng Xu

JOURNAL ARTICLE
Orange Juice and Weather

Richard Roll



The American Economic Review
Vol. 74, No. 5 (Dec., 1984),
pp. 861-880 (20 pages)

Futures of orange juice can be used to predict weather



SAT	SUN	MON
Mostly Sunny	T-storms	Sunny
High: 76° Low: 52°	High: 72° Low: 37°	High: 55° Low: 32°

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Events of Interest for Prediction

- Will there be a HW4 for this course?
- Will UVA win NCAA championship in 2020?
- Will bit coin price exceed \$9K tomorrow?
- Will Tesla's stock exceed \$300 by the end of this year?
- Will the number of iPhones sold in 2019 exceed 150 million?
- Will Trump win the election in 2020
- Will there be a cure for cancer by 2025?
- Will the world be peaceful in 2050?
- . . .

The Prediction Problem

- An uncertain event to be predicted
 - Will Tesla stock exceed \$300 by Dec 2019?
- Dispersed information/evidence
 - Tesla employees, Tesla drivers, other EV company employees, government policy makers, etc.
- Goal: generate a prediction that is based on information from all sources
 - ML can also do prediction, but will see why markets have advantages

Bet \approx Credible Opinion

Q: will P vs NP problem by solved by the end of 20'th century?

P vs NP would be solved by the end of the 20th century, if not sooner. The terms: **one ounce of pure gold**



Michael Sipser

- Other examples: stock trading, gambling, . . .
- Betting intermediaries: Wall Street, Las Vegas, InTrade, . . .

Prediction Markets

A prediction market is a **financial market** that is designed for **event prediction** via information aggregation

- Payoffs of the traded **contract** are determined by outcomes of future events

\$1 if UVA wins NCAA
\$0 otherwise

Price of a contract? $\$1 \times$ percentage of shares that bet on UVA winning?

This is what we will be designing!

A contract

Prediction Markets: Examples

Who will win the 2020 U.S. presidential election?

Contract	Latest Yes Price	Best Offer	Best Offer
 Donald Trump	41¢ NC	41¢	Buy Yes Buy No 60¢
 Elizabeth Warren	31¢ NC	32¢	Buy Yes Buy No 69¢
 Joe Biden	13¢ NC	13¢	Buy Yes Buy No 88¢
 Bernie Sanders	11¢ 2¢▲	11¢	Buy Yes Buy No 91¢
 Andrew Yang	8¢ NC	9¢	Buy Yes Buy No 92¢
 Pete Buttigieg	7¢ NC	7¢	Buy Yes Buy No 94¢
 Mike Pence	3¢ NC	3¢	Buy Yes Buy No 98¢

Prediction Markets: Examples

Will Dwayne "the Rock" Johnson run for president in 2020?

The screenshot shows a prediction market interface for Dwayne 'the Rock' Johnson running for president in 2020. At the top, there's a photo of Dwayne Johnson, a "Latest Price" section showing 4¢ NC, and a trading area with "Buy Yes" (green) and "Buy No" (gray) buttons. The "Buy Yes" button is highlighted. The price is 5¢ and the "Best Offer" is 97¢. Below this, there's a "The Rules" section with a link to "Read the Full Rules". A large candlestick chart shows the price movement over time, with a significant jump from around 18¢ to 30¢. To the right, there are "Related Markets" sections for the 2020 Democratic nominee, Iowa Dem caucus winner, and NH Dem primary winner, each featuring a logo and a list of candidates with their current prices.

Latest Yes Price Best Offer Best Offer

Latest Price 4¢ NC 5¢ Buy Yes Buy No 97¢

The Rules

Actor and professional wrestler Dwayne Johnson shall become a candidate for president of the United States in the 2020 general election, by filing a Statement of Candidacy and/or a Statement of Organization of an exploratory committee for the office of president with the Federal Election Commission.

[Read the Full Rules](#)

Stats 24hr 7 Day 30 Day 90 Day Candle Line

PredictIt

Volume 0.0

30
24
18
12

Price

6¢
5¢
4¢
3¢

2020 Democratic nominee?

Elizabeth Warren 45¢ 2¢ ↑
Joe Biden 20¢ 1¢ ↓
67.6M Shares Traded

2020 Iowa Dem caucus winner?

Elizabeth Warren 51¢ 3¢ ↓
Bernie Sanders 17¢ 1¢ ↑
5.3M Shares Traded

2020 NH Dem primary winner?

Elizabeth Warren 52¢ 1¢ ↓

Prediction Markets: Examples

HI HAIFENG XU ▾ CLAIMS ▾ ABOUT BLOG Search SEARCH OUR TEAM RULES SCHEDULE

* Demographics and Interests (Required)
 Forecasting Replications Quiz
 Statistics Background & Quiz

SURVEYS

FORECAST TOPICS

All Claims	AJ PS American Journal of Political Science
Economics	Exp E Experimental Economics
JE SP Journal of Experimental Social Psychology	Jo LE Journal of Labor Economics
Jo M Journal of Marketing	Jo OB Journal of Organizational Behavior
JP SP Journal of Personality and Social Psychology	JA MS Journal of the Academy of Marketing Science
Management & Marketing & others	Political Science
Psychological Science	Psychology

LEADERBOARDS Expected Points

1	sattuma	1,020
2	adamlgreen	1,009
3	BradleyJBaker	949
4	Luthor113	929
5	ncerutti	920
6	ulrich	871
7	stanlaurel	817
8	unipedal	803
9	ted	803
10	Achim	791
11	cteno4	782

RECENT ACTIVITY Participants

NO RESULTS FOUND.

OUR TEAM

	Christian Pfeiffer, Harvard University		Anna Dreber, Stockholm School of Economics		Magnus Johannesson, Stockholm School of Economics
Incentives, Scoring, IRB					
	Yiling Chen, Harvard University		Prof. Yang Liu, University of California Santa Cruz		Prof. Haifeng Xu, University of Virginia

Replication Market

Prediction Markets: Examples

The screenshot shows a prediction market interface for the Augur platform. The main title of the market is "Will Snap Q3 2019 Earnings per share (EPS) exceed USD \$ -0.05?". The resolution source is listed as "General knowledge". The market is currently open, with 0 ETH in volume and an estimated fee of 1.0100%. The reporting start date is set for October 11, 2019, at 6:52 AM (UTC -4), and the type is Yes/No. The reporting starts on October 23, 2019, at 8:00 AM (UTC 0). The market author is identified by the address 0x0960da039bb8151cacfef620476e8baf34bd9565.

RESOLUTION SOURCE
General knowledge

ADDITIONAL DETAILS

Snap Inc. are expected to release their next earnings report before October 23rd 2019 8am UTC. These results will be widely available from financial media.

Firms are required to round to the nearest cent when the calculated EPS is not an integer in cents. In this case USD \$ -0.05 or lower resolves as no, USD \$ -0.04 and higher resolves as yes.

If the reporting start time is delayed beyond the reporting start time of this market, the market shall resolve as invalid.
[read more](#)

YES PRICE (ETH)

Volume	Open	High	Low	Close	Period
1.0000					PAST WEEK, DAILY
0.7500					
0.5000					

No Completed Trades

MARKET DEPTH

Qty	Price	Depth
1.5750		
1.4175		
1.2600	Mid Price 0.7000 ETH	
1.1025		
0.9450		
0.7875		
0.6300		

Augur: the first decentralized prediction markets

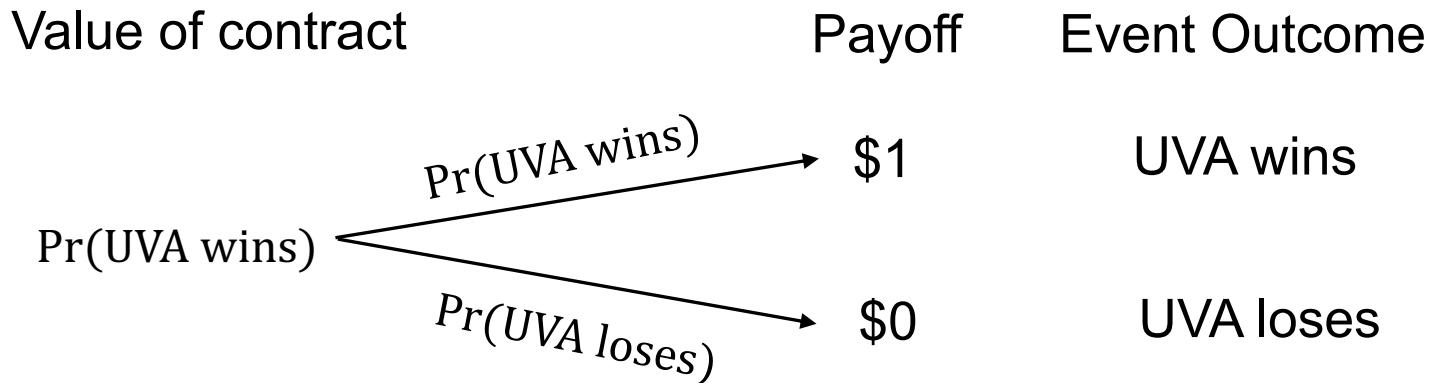
Does It Work?

- Yes, evidence from real markets, lab experiments, and theory
 - I.E.M. beat political polls 451/596 [Forsythe 1992, 1999][Oliven 1995][Rietz 1998][Berg 2001][Pennock 2002]
 - HP market beats sales forecast 6/8 [Plott 2000]
 - Sports betting markets provide accurate forecasts of game outcomes [Gandar 1998][Thaler 1988][Debnath EC'03][Schmidt 2002]
 - Laboratory experiments confirm information aggregation [Plott 1982;1988;1997][Forsythe 1990][Chen, EC'01]
 - Theory: “rational expectations” [Grossman 1981][Lucas 1972]
 - More ...

Why Can Markets Aggregate Information?

- Price $\approx \text{Prob}(\text{event} | \text{all information})$

\$1 if UVA wins NCAA title, \$0 otherwise



Value of contract $\approx P(\text{ UVA wins }) \approx \text{Equilibrium price}$

Market Efficiency (a design goal)

Markets vs Other Prediction Approaches

Opinion Poll

- Sampling
- No incentive to be truthful
- Equally weighted information
- Hard to be real-time

Ask Experts

- Identifying experts can be hard
- Combining opinions is difficult

Prediction Markets

- Self-selection
- Monetary incentive and more
- Money-weighted information
- Real-time
- Self-organizing

Other Prediction Approaches vs Markets

Machine Learning

- Historical data
- Assume past and future are related
- Hard to incorporate recent new information

Prediction Markets

- No need for data
- No assumption on past and future
- Immediately incorporate new information

Caveat: markets have their own problems too – manipulations, irrational traders, etc.

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets (PMs)
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Some Design Objectives of PMs

Liquidity: people can find counterparties to trade whenever they want

Bounded loss: total loss of the market institution is bounded

Market efficiency: Price reflects predicted probabilities.

Computational efficiency: The process of operating the market should be computationally manageable.

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.12

\$0.09

\$0.15

➤ Seller orders

\$0.30

\$0.17

\$0.13

Price = \$0.14

What's Wrong with CDA?

➤ Thin market problem

- When there are not enough traders, trade may not happen.

➤ No trade theorem [Milgrom & Stokey 1982]

- Why trade? These markets are zero-sum games (negative sum w/ transaction fees)
- For all money earned, there is an equal (greater) amount lost; am I smarter than average?
- Rational risk-neutral traders will *never* trade
- But trade happens ...

An Alternative: Market Maker (MM)



- A market maker is the market institution who sets the prices and is willing to accept orders (buy or sell) at the price specified.
- Why? **Liquidity!**
- Market makers bear risk. Thus, we desire mechanisms that can **bound the loss of market makers**.

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- An (automated) market marker (MM)
- Sell or buy back contracts $\boxed{\$1 \text{ iff } e_1}$ \cdots $\boxed{\$1 \text{ iff } e_n}$
- Value function ($q = (q_1, \dots, q_n)$ is current sales quantity)

$$V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$$

Parameter b
adjusts liquidity

- Price function

$$p_i(q) = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}} = \frac{\partial V(q)}{\partial q_i}$$

- To buy $x \in \mathbb{R}^n$ amount, a buyer pays: $V(q + x) - V(q)$
 - Negative x_i 's mean selling contracts to MM
 - Negative payment means market maker pays the buyer
 - Market starts with $V(0) = b \log n$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

- Say, you buy $x \in \mathbb{R}^n$ amount
- You pay $V(q + x) - V(q)$; Your expected return is $\sum_{j \in [n]} \lambda_j \cdot x_j$
- Expected utility is

$$U(x) = \sum_{j \in [n]} \lambda_j \cdot x_j - b \log \sum_{j \in [n]} e^{(q_j+x_j)/b} + V(q)$$

- Which x maximizes your utility?

$$\frac{\partial U(x)}{\partial x_i} = \lambda_i - \frac{e^{(q_i+x_i)/b}}{\sum_{j \in [n]} e^{(q_j+x_j)/b}} = 0$$

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The market price of contract i
after your purchase

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ . Your expected utility of purchasing this amount is always non-negative.

- Why non-negative?
- Buy 0 amount leads to 0, so optimal amount is at least as good

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ . Your expected utility of purchasing this amount is always non-negative.

- This is the expected utility you believe, but **may be incorrect since your λ may be inaccurate!**
- So, buy only when your prediction is really more accurate than the current market prediction
 - **Achieves market efficiency:** price = current best market prediction

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q2: If market ends up with $q = (q_1, \dots, q_n)$ shares for each contract, how much money did the MM collect?

- Answer: $V(q) - V(0) = V(q) - d \log n$
- But after event outcome is realized, MM need to pay based on contracts – what is the worst-case loss of MM?

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

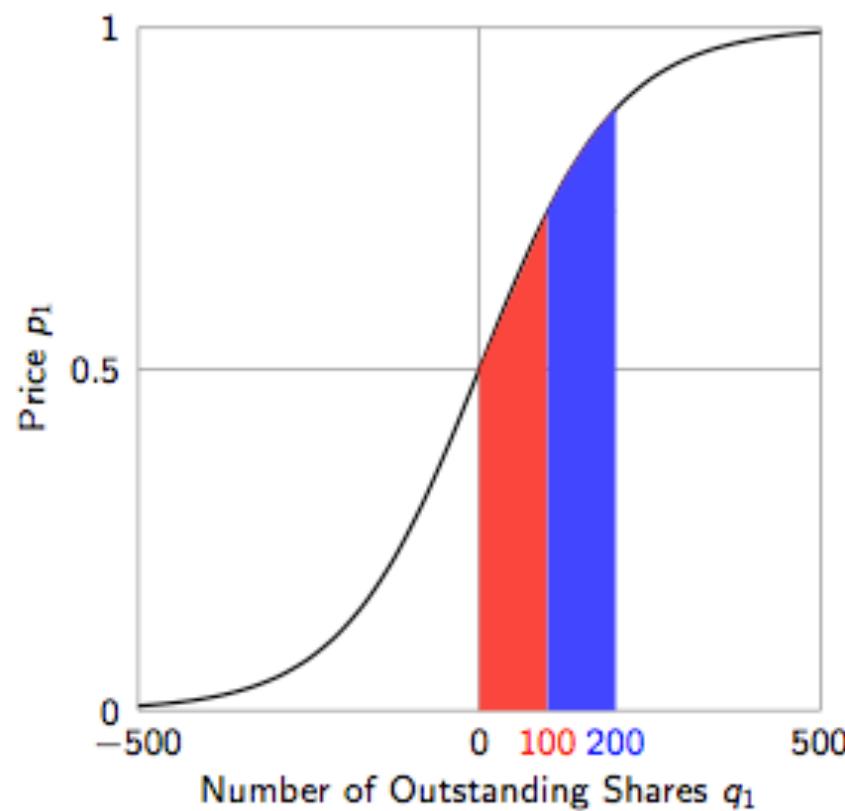
- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Fact. After event outcome realizes and MM pays the contract, worst case MM loses is $d \log n$ (i.e., bounded).

Proof

- Only one event will be realized, say it is event e_i
- MM utility is
$$\begin{aligned} V(q) - d \log n - q_i \\ \geq b \log e^{q_i/b} - d \log n - q_i \\ \geq q_i - d \log n - q_i \\ \geq -d \log n \end{aligned}$$
"=" can be achieved by letting $q_i \rightarrow \infty$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])



Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Has been implemented by several prediction markets
 - E.g., InklingMarkets, Washington Stock Exchange, BizPredict, Net Exchange, and (reportedly) at YooNew.

SELECTED PREDICTION	CURRENT PRICE
Barack Obama	\$57.02

TIP: A price of \$57.02 means there is currently a 57.0% chance this will occur.

Do you think:

- Chances are higher than 57.02% this will occur
- Chances are lower than 57.02% this will occur

TIP: A price of \$57.02 means there is currently a 57.0% chance this will occur.

If you think the current odds of 57% are:

Way too low... Low... Just below... Advanced...

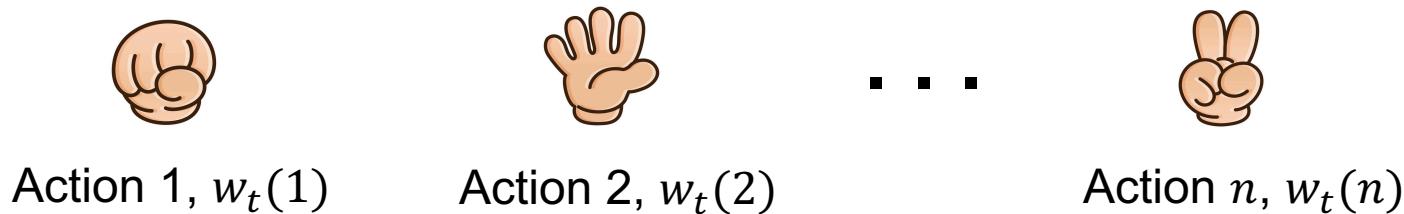
Buy 50 shares your cost \$2,971.95 estimated new price \$61.84	Buy 20 shares your cost \$1,159.83 estimated new price \$58.97	Buy 5 shares your cost \$286.30 estimated new price \$57.51	Buy <input type="text"/> shares your cost ... estimated new price ...
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Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates (EWU)

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \dots, w_t(n)$
- Observe cost vector c_t , and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}, \forall i \in [n]$



$$\begin{aligned}w_{t+1}(i) &= w_t(i) \cdot e^{-\epsilon c_t(i)} \\&= [w_{t-1}(i) \cdot e^{-\epsilon c_{t-1}(i)}] \cdot e^{-\epsilon c_t(i)} \\&= \dots = e^{-\epsilon C_t(i)} \text{ where } C_t(i) = \sum_{\tau \leq t} c_\tau(i)\end{aligned}$$

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \dots, w_t(n)$
- Observe cost vector c_t , and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}, \forall i \in [n]$
- At round $t + 1$, select action i with probability

$$\frac{w_t(i)}{W_t} = \frac{e^{-\epsilon c_t(i)}}{\sum_{j \in [n]} e^{-\epsilon c_t(j)}}$$

where $C_t = \sum_{\tau \leq t} c_\tau$ is the accumulated cost vector

This looks very much like the price function in LMSR (q is the accumulated sales quantity)

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

EWU vs LMSR

➤ Exponential Weight Update

- n actions
- Maintain weight $w_t(i)$
- Total cost $C_T(i) = \sum_{t \leq T} c_t(i)$
- Select i with prob

$$p_i = \frac{e^{-\epsilon C_t(i)}}{\sum_{j \in [n]} e^{-\epsilon C_t(j)}}$$

- Weights reflect how good an action is
- Care about worst case regret

$$C_T(\text{Alg}) - \min_i C_T(i)$$

➤ LMSR

- n contracts (i.e., outcomes)
- Maintain prices $p(i)$
- Total shares sold $q(i)$
- Price of contract i

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

- Prices reflect how probable is an event
- Care about worst case MM loss

$$(\$ \text{ received}) - \min_i q(i)$$

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market markers and no-regret learning algorithms (see [[Chen and Vaughan 2010](#)])
- Markets can potentially be a very effective forecasting tool
 - Big on-going project: “replication market” for DARPA SCORE program



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Systematizing Confidence in Open Research and Evidence (SCORE)

[Dr. Adam Russell](#)

The Department of Defense (DoD) often leverages social and behavioral science (SBS) research to design plans, guide investments, assess outcomes, and build models of human social systems and behaviors as they relate to national security challenges in the human domain. However, a number of recent empirical studies and meta-analyses have revealed that many SBS results vary dramatically in terms of their ability to be independently reproduced or replicated, which could have real-world implications for DoD's plans, decisions, and models. To help address this situation, DARPA's Systematizing Confidence in Open Research and Evidence (SCORE) program aims to develop and deploy automated tools to assign "confidence scores" to different SBS research results and claims. Confidence scores are quantitative measures that should enable a DoD consumer of SBS research to understand the degree to which a particular claim or result is likely to be reproducible or replicable. These tools will assign explainable confidence scores with a reliability that is equal to, or better than, the best current human expert methods. If successful, SCORE will enable DoD personnel to quickly calibrate the level of confidence they should have in the reproducibility and replicability of a given SBS result or claim, and thereby

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market markers and no-regret learning algorithms (see [[Chen and Vaughan 2010](#)])
- Markets can potentially be a very effective forecasting tool
 - Big on-going project: “replication market” for DARPA SCORE program
- Mechanism design for prediction tasks
 - ML is not the only way of making predictions
 - But markets and ML may augment each other...

Thank You

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