

Announcements

- HW 2 is due next Tuesday
 - No class on next Tuesday, but TAs will be here to collect HW

CS6501:Topics in Learning and Game Theory (Fall 2019)

Prediction Markets (as a Forecasting Tool)



Instructor: Haifeng Xu

JOURNAL ARTICLE

Orange Juice and Weather

Richard Roll



The American Economic Review
Vol. 74, No. 5 (Dec., 1984),
pp. 861-880 (20 pages)

Futures of orange juice can be used to predict weather



SAT	SUN	MON
Mostly Sunny High: 76° Low: 52°	T-storms High: 72° Low: 37°	Sunny High: 55° Low: 32°

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Events of Interest for Prediction

- Will there be a HW4 for this course?
- Will UVA win NCAA championship in 2020?
- Will bit coin price exceed \$9K tomorrow?
- Will Tesla's stock exceed \$300 by the end of this year?
- Will the number of iPhones sold in 2019 exceed 150 million?
- Will Trump win the election in 2020
- Will there be a cure for cancer by 2025?
- Will the world be peaceful in 2050?
- . . .

The Prediction Problem

- An uncertain event to be predicted
 - Will Tesla stock exceed \$300 by Dec 2019?
- Dispersed information/evidence
 - Tesla employees, Tesla drivers, other EV company employees, government policy makers, etc.
- Goal: generate a prediction that is based on information from all sources
 - ML can also do prediction, but will see why markets have advantages

Bet \approx Credible Opinion

Q: will P vs NP problem be solved by the end of 20'th century?

P vs NP would be solved by the end of the 20th century, if not sooner. The terms: **one ounce of pure gold**



Michael Sipser

- Other examples: stock trading, gambling, . . .
- Betting intermediaries: Wall Street, Las Vegas, InTrade, . . .

Prediction Markets

A prediction market is a **financial market** that is designed for **event prediction** via information aggregation

- Payoffs of the traded **contract** are determined by outcomes of future events

\$1 if UVA wins NCAA
\$0 otherwise

A contract

Price of a contract? $\$1 \times$ percentage of shares that bet on UVA winning?

This is what we will be designing!

Prediction Markets: Examples

Screenshot of the PredictIt website showing the 2020 U.S. presidential election prediction market.

The page title is "Who will win the 2020 U.S. presidential election?"

Contract	Latest Yes Price	Best Offer	Buy Yes	Buy No	Best Offer
Donald Trump	41¢ NC	41¢	Buy Yes	Buy No	60¢
Elizabeth Warren	31¢ NC	32¢	Buy Yes	Buy No	69¢
Joe Biden	13¢ NC	13¢	Buy Yes	Buy No	88¢
Bernie Sanders	11¢ 2¢↑	11¢	Buy Yes	Buy No	91¢
Andrew Yang	8¢ NC	9¢	Buy Yes	Buy No	92¢
Pete Buttigieg	7¢ NC	7¢	Buy Yes	Buy No	94¢
Mike Pence	3¢ NC	3¢	Buy Yes	Buy No	98¢

Prediction Markets: Examples

Will Dwayne "the Rock" Johnson run for president in 2020?

The screenshot shows a prediction market interface for Dwayne 'the Rock' Johnson running for president in 2020. At the top, there's a header with 'Latest Yes Price' (4¢), 'Best Offer' (5¢), and another 'Best Offer' (97¢). Below this is a photo of Dwayne Johnson and a 'Latest Price' section. To the right are 'Buy Yes' and 'Buy No' buttons. A 'The Rules' section explains the requirements for candidacy. A candlestick chart shows price movement over time. On the right, there are 'Related Markets' for the 2020 Democratic nominee, Iowa Dem caucus winner, and New Hampshire Dem primary winner, each with their own charts and data.

Latest Yes Price Best Offer Best Offer

Latest Price 4¢ NC 5¢ Buy Yes Buy No 97¢

The Rules

Actor and professional wrestler Dwayne Johnson shall become a candidate for president of the United States in the 2020 general election, by filing a Statement of Candidacy and/or a Statement of Organization of an exploratory committee for the office of president with the Federal Election Commission.

[Read the Full Rules](#)

Stats 24hr 7 Day 30 Day 90 Day Candle Line

PredictIt

Volume

30
24
18
12

4¢
5¢
6¢

2020 Democratic nominee?

Candidate	Price	Change
Elizabeth Warren	45¢	2¢ ↑
Joe Biden	20¢	1¢ ↓

67.6M Shares Traded

2020 Iowa Dem caucus winner?

Candidate	Price	Change
Elizabeth Warren	51¢	3¢ ↓
Bernie Sanders	17¢	1¢ ↑

5.3M Shares Traded

IOWA

2020 NH Dem primary winner?

Candidate	Price	Change
Elizabeth Warren	52¢	1¢ ↓

New Hampshire

Prediction Markets: Examples

HI HAIFENG XU ▾ CLAIMS ▾ ABOUT BLOG

SURVEYS

* Demographics and Interests (Required)
Forecasting Replications Quiz
Statistics Background & Quiz

FORECAST TOPICS

 All Claims	 American Journal of Political Science
 Economics	 Experimental Economics
 Journal of Experimental Social Psychology	 Journal of Labor Economics
 Journal of Marketing	 Journal of Organizational Behavior
 Journal of Personality and Social Psychology	 Journal of the Academy of Marketing Science
 Management & Marketing & others	 Political Science
 Psychological Science	 Psychology

LEADERBOARDS

1 sattuma	1,020
2 adamlgreen	1,009
3 BradleyJBaker	949
4 Luthor113	929
5 ncerutti	920
6 ulrich	871
7 stanlaurel	817
8 unipedal	803
9 ted	803
10 Achim	791
11 cteno4	782

RECENT ACTIVITY

NO RESULTS FOUND.

OUR TEAM


Christian Pfeiffer,
Princeton University


Anna Dreber,
Stockholm School of Economics


Magnus Johannesson,
Stockholm School of Economics

Incentives, Scoring, IRB


Jiliing Chen,
Harvard University


Prof. Yang Liu,
University of California Santa Cruz


Prof. Haifeng Xu,
University of Virginia

Replication Market

Prediction Markets: Examples

The screenshot shows a prediction market on the Augur platform. The market title is "Will Snap Q3 2019 Earnings per share (EPS) exceed USD \$ -0.05?". The resolution source is "General knowledge". Additional details mention that Snap Inc. are expected to release their next earnings report before October 23rd 2019 8am UTC. Firms are required to round to the nearest cent when the calculated EPS is not an integer in cents. The market is currently open, with 0 ETH in volume and an estimated fee of 1.0100%. The reporting start time is set for October 23, 2019 at 8:00 AM (UTC 0). The market author is 0x0960da039bb8151cacfef620476e8baf34bd9565. The market depth chart shows the current price of 0.7000 ETH.

RESOLUTION SOURCE
General knowledge

ADDITIONAL DETAILS
Snap Inc. are expected to release their next earnings report before October 23rd 2019 8am UTC. These results will be widely available from financial media.
Firms are required to round to the nearest cent when the calculated EPS is not an integer in cents. In this case USD \$ -0.05 or lower resolves as no, USD \$ -0.04 and higher resolves as yes.
[read more](#)

VOLUME 0 ETH EST. FEE 1.0100% PHASE Open

Always make sure that the **title, details, reporting start outcomes** are not in direct conflict with each other [1](#)

CREATED October 11, 2019 6:52 AM (UTC -4) TYPE Yes/No

REPORTING STARTS October 23, 2019 8:00 AM (UTC 0)
October 23, 2019 4:00 AM (EDT) (Your Timezone)

MARKET AUTHOR 0x0960da039bb8151cacfef620476e8baf34bd9565

YES PRICE (ETH)

Volume	Open	High	Low	Close	Period
1.0000					PAST WEEK, DAILY

Market Depth

Qty	Price	Depth
1.5750		
1.4175		
1.2600		
1.1025		
0.9450		
0.8785		
0.6300		

No Completed Trades

Mid Price 0.7000 ETH

Augur: the first decentralized prediction markets

Does It Work?

- Yes, evidence from real markets, lab experiments, and theory
 - I.E.M. beat political polls 451/596 [Forsythe 1992, 1999][Oliven 1995][Rietz 1998][Berg 2001][Pennock 2002]
 - HP market beats sales forecast 6/8 [Plott 2000]
 - Sports betting markets provide accurate forecasts of game outcomes [Gandar 1998][Thaler 1988][Debnath EC'03][Schmidt 2002]
 - Laboratory experiments confirm information aggregation [Plott 1982;1988;1997][Forsythe 1990][Chen, EC'01]
 - Theory: “rational expectations” [Grossman 1981][Lucas 1972]
 - More ...

Why Can Markets Aggregate Information?

- Price $\approx \text{Prob}(\text{event} | \text{all information})$

\$1 if UVA wins NCAA title, \$0 otherwise

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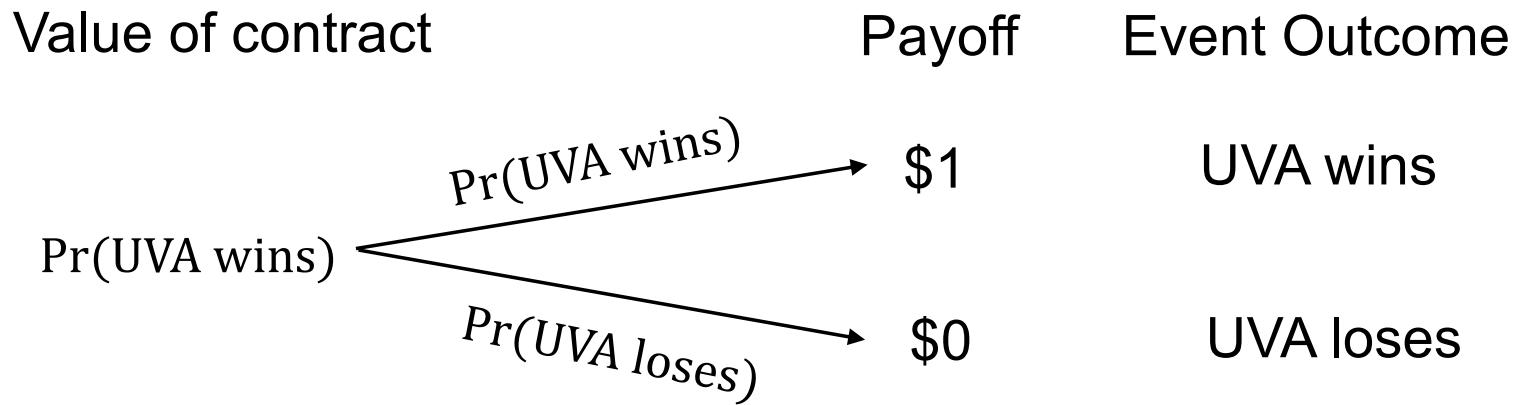
\$1 if UVA wins NCAA title, \$0 otherwise

Value of contract	Payoff	Event Outcome
?	\$1	UVA wins
	\$0	UVA loses

Why Can Markets Aggregate Information?

- Price $\approx \text{Prob}(\text{event} | \text{all information})$

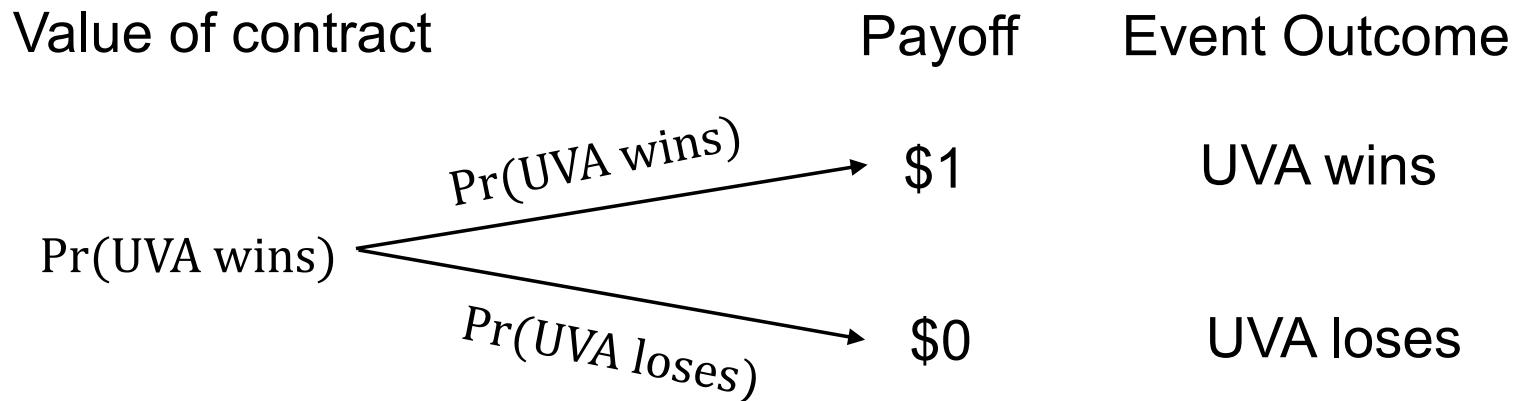
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Why Can Markets Aggregate Information?

- Price $\approx \text{Prob}(\text{event} | \text{all information})$

\$1 if UVA wins NCAA title, \$0 otherwise



Value of contract $\approx P(\text{ UVA wins }) \approx$ Equilibrium price

Market Efficiency (a design goal)

Markets vs Other Prediction Approaches

Opinion Poll

- Sampling
- No incentive to be truthful
- Equally weighted information
- Hard to be real-time

Ask Experts

- Identifying experts can be hard
- Combining opinions is difficult

Prediction Markets

- Self-selection
- Monetary incentive and more
- Money-weighted information
- Real-time
- Self-organizing

Other Prediction Approaches vs Markets

Machine Learning

- Historical data
- Assume past and future are related
- Hard to incorporate recent new information

Prediction Markets

- No need for data
- No assumption on past and future
- Immediately incorporate new information

Caveat: markets have their own problems too – manipulations, irrational traders, etc.

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets (PMs)
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Some Design Objectives of PMs

Liquidity: people can find counterparties to trade whenever they want

Bounded loss: total loss of the market institution is bounded

Market efficiency: Price reflects predicted probabilities.

Computational efficiency: The process of operating the market should be computationally manageable.

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

➤ Seller orders

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.12

➤ Seller orders

\$0.30

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.12

\$0.09

➤ Seller orders

\$0.30

\$0.17

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.12

\$0.09

➤ Seller orders

\$0.17

\$0.30

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.12

\$0.09

\$0.15

➤ Seller orders

\$0.17

\$0.30

\$0.13

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.15

\$0.12

\$0.09

➤ Seller orders

\$0.13

\$0.17

\$0.30

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.15

\$0.12

\$0.09

Price = \$0.14

➤ Seller orders

\$0.13

\$0.17

\$0.30

What's Wrong with CDA?

- Thin market problem
 - When there are not enough traders, trade may not happen.
- No trade theorem [Milgrom & Stokey 1982]
 - Why trade? These markets are zero-sum games (negative sum w/ transaction fees)
 - For all money earned, there is an equal (greater) amount lost; am I smarter than average?
 - Rational risk-neutral traders will *never* trade
 - But trade still happens ...

An Alternative: Market Maker (MM)



- A market maker is the market institution who sets the prices and is willing to accept orders (buy or sell) at the price specified.
- Why? **Liquidity!**
- Market makers bear risk. Thus, we desire mechanisms that can **bound the loss of market makers**.

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- An (automated) market marker (MM)
- Sell or buy back contracts \$1 iff e_1 · · · \$1 iff e_n
- Value function ($q = (q_1, \dots, q_n)$ is current sales quantity)

$$V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$$

Parameter b
adjusts liquidity

- Price function

$$p_i(q) = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}} = \frac{\partial V(q)}{\partial q_i}$$

- To buy $x \in \mathbb{R}^n$ amount, a buyer pays: $V(q + x) - V(q)$
 - Negative x_i 's mean selling contracts to MM
 - Negative payment means market maker pays the buyer
 - Market starts with $V(0) = b \log n$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

- Say, you buy $x \in \mathbb{R}^n$ amount
- You pay $V(q + x) - V(q)$; Your expected return is $\sum_{j \in [n]} \lambda_j \cdot x_j$
- Expected utility is

$$U(x) = \sum_{j \in [n]} \lambda_j \cdot x_j - b \log \sum_{j \in [n]} e^{(q_j+x_j)/b} + V(q)$$

- Which x maximizes your utility?

$$\frac{\partial U(x)}{\partial x_i} = \lambda_i - \frac{e^{(q_i+x_i)/b}}{\sum_{j \in [n]} e^{(q_j+x_j)/b}} = 0$$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

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The market price of contract i
after your purchase

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ . Your expected utility of purchasing this amount is always non-negative.

- Why non-negative?
 - Buy 0 amount leads to 0, so optimal amount is at least as good

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ . Your expected utility of purchasing this amount is always non-negative.

- This is the expected utility you believe, but **may be incorrect since your λ may be inaccurate!**
- So, buy only when your prediction is really more accurate than the current market prediction
 - **Achieves market efficiency:** price = current best market prediction

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q2: If market ends up with $q = (q_1, \dots, q_n)$ shares for each contract, how much money did the MM collect?

- Answer: $V(q) - V(0) = V(q) - b \log n$
- But after event outcome is realized, MM need to pay based on contracts – what is the worst-case loss of MM?

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Fact. After event outcome realizes and MM pays the contract, worst case MM loses is $b \log n$ (i.e., bounded).

Proof

- Only one event will be realized, say it is event e_i

- MM utility is
$$V(q) - b \log n - q_i$$

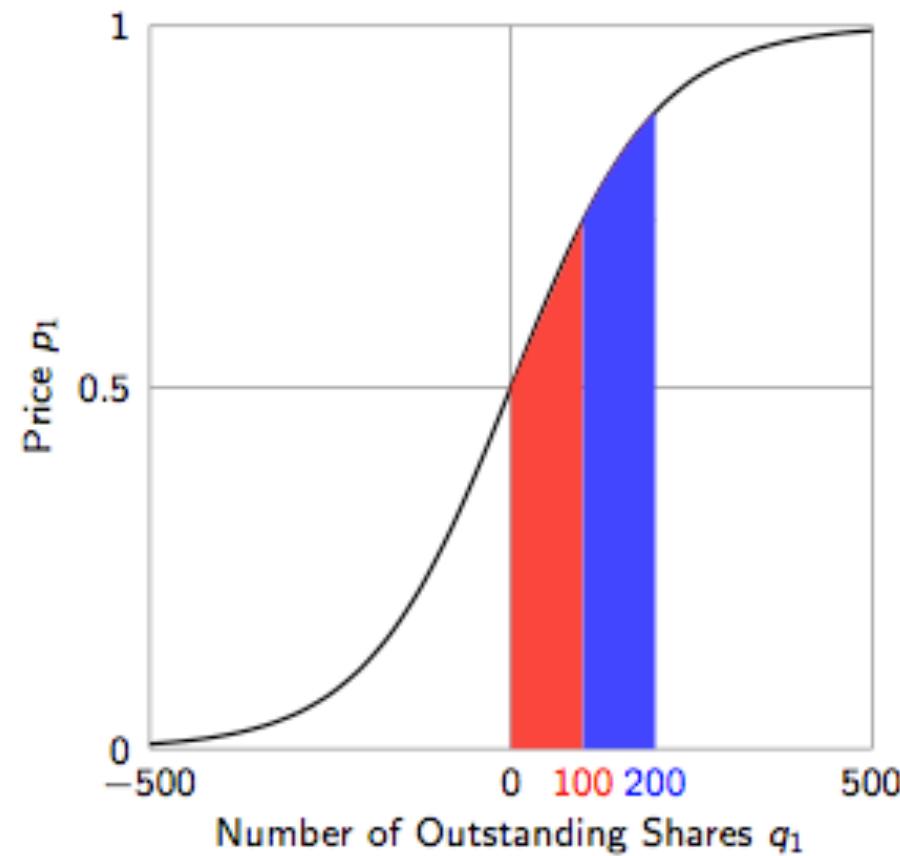
$$\geq b \log e^{q_i/b} - b \log n - q_i$$

$$\geq q_i - b \log n - q_i$$

$$\geq -b \log n$$

“=“ can be achieved by letting $q_i \rightarrow \infty$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])



Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Has been implemented by several prediction markets
 - E.g., InklingMarkets, Washington Stock Exchange, BizPredict, Net Exchange, and (reportedly) at YooNew.

SELECTED PREDICTION	CURRENT PRICE
Barack Obama	\$57.02

TIP: A price of \$57.02 means there is currently a 57.0% chance this will occur.

Do you think:

- Chances are higher than 57.02% this will occur
- Chances are lower than 57.02% this will occur

TIP: A price of \$57.02 means there is currently a 57.0% chance this will occur.

If you think the current odds of 57% are:

Way too low...

Low...

Just below...

Advanced...

Buy 50 shares

your cost
\$2,971.95

estimated new price
\$61.84

Buy 20 shares

your cost
\$1,159.83

estimated new price
\$58.97

Buy 5 shares

your cost
\$286.30

estimated new price
\$57.51

Buy shares

your cost
...

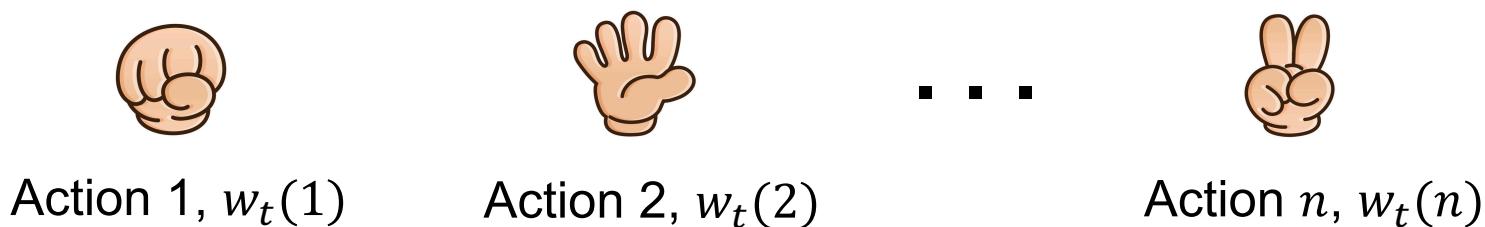
estimated new price
...

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates (EWU)

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \dots, w_t(n)$
- Observe cost vector c_t , and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}, \forall i \in [n]$



$$\begin{aligned}w_{t+1}(i) &= w_t(i) \cdot e^{-\epsilon c_t(i)} \\&= [w_{t-1}(i) \cdot e^{-\epsilon c_{t-1}(i)}] \cdot e^{-\epsilon c_t(i)} \\&= \dots = e^{-\epsilon C_t(i)} \text{ where } C_t(i) = \sum_{\tau \leq t} c_\tau(i)\end{aligned}$$

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \dots, w_t(n)$
- Observe cost vector c_t , and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}, \forall i \in [n]$
- At round $t + 1$, select action i with probability

$$\frac{w_t(i)}{W_t} = \frac{e^{-\epsilon c_t(i)}}{\sum_{j \in [n]} e^{-\epsilon c_t(j)}}$$

where $C_t = \sum_{\tau \leq t} c_\tau$ is the accumulated cost vector

This looks very much like the price function in LMSR (q is the accumulated sales quantity)

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

EWU vs LMSR

➤ Exponential Weight Update

- n actions
- Maintain weight $w_t(i)$
- Total cost $C_T(i) = \sum_{t \leq T} c_t(i)$
- Select i with prob

$$p_i = \frac{e^{-\epsilon C_t(i)}}{\sum_{j \in [n]} e^{-\epsilon C_t(j)}}$$

- Weights reflect how good an action is
- Care about worst case regret

$$C_T(\text{Alg}) - \min_i C_T(i)$$

➤ LMSR

- n contracts (i.e., outcomes)
- Maintain prices $p(i)$
- Total shares sold $q(i)$
- Price of contract i

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

- Prices reflect how probable is an event
- Care about worst case MM loss

$$(\$ \text{ received}) - \min_i q(i)$$

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market markers and no-regret learning algorithms (see [Chen and Vaughan 2010])
- Markets can potentially be a very effective forecasting tool
 - Big on-going project: “replication market” for DARPA SCORE program



DEFENSE ADVANCED
RESEARCH PROJECTS AGENCY

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Defense Advanced Research Projects Agency > Program Information

Systematizing Confidence in Open Research and Evidence (SCORE)

[Dr. Adam Russell](#)

The Department of Defense (DoD) often leverages social and behavioral science (SBS) research to design plans, guide investments, assess outcomes, and build models of human social systems and behaviors as they relate to national security challenges in the human domain. However, a number of recent empirical studies and meta-analyses have revealed that many SBS results vary dramatically in terms of their ability to be independently reproduced or replicated, which could have real-world implications for DoD's plans, decisions, and models. To help address this situation, DARPA's Systematizing Confidence in Open Research and Evidence (SCORE) program aims to develop and deploy automated tools to assign "confidence scores" to different SBS research results and claims. Confidence scores are quantitative measures that should enable a DoD consumer of SBS research to understand the degree to which a particular claim or result is likely to be reproducible or replicable. These tools will assign explainable confidence scores with a reliability that is equal to, or better than, the best current human expert methods. If successful, SCORE will enable DoD personnel to quickly calibrate the level of confidence they should have in the reproducibility and replicability of a given SBS result or claim, and thereby

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market markers and no-regret learning algorithms (see [[Chen and Vaughan 2010](#)])
- Markets can potentially be a very effective forecasting tool
 - Big on-going project: “replication market” for DARPA SCORE program
- Mechanism design for prediction tasks
 - ML is one way but not the only way of making predictions
 - But markets and ML may augment each other

Thank You

Haifeng Xu

University of Virginia

hx4ad@virginia.edu