

# Announcements

- HW 3 due next Tuesday
- No HW 4

# CS6501:Topics in Learning and Game Theory (Fall 2019)

## Crowdsourcing Information and Peer Prediction

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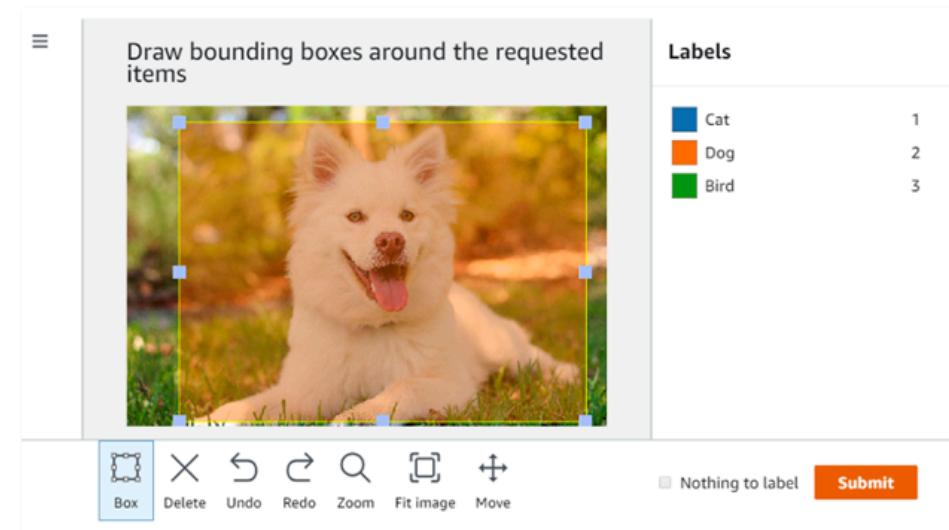
Instructor: Haifeng Xu

# Outline

- Eliciting Information without Verification
- Equilibrium Concept and Peer Prediction Mechanism
- Bayesian Truth Serum

# Crowdsourcing Information

- Recruit AMT workers to label images
  - Cannot check ground truth (too costly)



# Crowdsourcing Information

- Recruit AMT workers to label images
  - Cannot check ground truth (too costly)
- Peer grading (of, e.g., essays) on MOOC
  - Don't know true scores

The screenshot shows the Coursera website interface. At the top, there is a navigation bar with the Coursera logo, search links for 'Explore Catalog', 'Degrees', 'Certificates', and 'For Enterprise', and a search bar with a magnifying glass icon. A promotional banner offers 'Get 50% off' when referring a friend, with a link to 'Learn More' and a 'See terms and conditions' link. Below the banner, the course title 'Getting Started with Essay Writing' is displayed, along with its rating of '4.7' based on '1,622 ratings' and '446 reviews'. The course is part of the 'Academic English: Writing Specialization' under the 'Arts and Humanities > Music and Art' category. It is offered by 'UCI Extension Continuing Education'. A yellow button encourages users to 'Enroll for Free' starting on 'Oct 30'. The text 'Financial aid available' is also present. A large statistic at the bottom indicates that '109,835' people have already enrolled in the course.

# Crowdsourcing Information

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- Peer grading (of, e.g., essays) on MOOC
  - Don't know true scores
- Elicit ratings for various entities (e.g., on Yelp or Google)
  - We never find out the true quality/rating

The image consists of two side-by-side screenshots of web pages.

**Left Screenshot (Yelp):** A screenshot of the Yelp website for Orlando, FL. The search bar shows "hospitals". The results list three hospitals: Winnie Palmer Hospital for Women & Babies, Arnold Palmer Hospital For Children, and Florida Hospital. Each listing includes a small photo, the hospital's name, address, phone number, and a short review from a user. Below the list is a map of Orlando with red dots indicating the locations of the hospitals.

**Right Screenshot (Google):** A screenshot of a Google search results page for "atlantic ocean". The top result is a link to Wikipedia with the heading "Atlantic Ocean". To the right of the link are the standard Google snippet details: a 3.9 rating from 13,797 reviews, a "Write a review" button, and sorting options. Below this are several user reviews. One review by "Necro Null" is highlighted, showing a 5-star rating and a comment: "Much better than the Pacific Ocean and I had a great view as I was sinking from the ship we were on when we struck some stray iceberg, but it was chilly and a couple people drowned sadly. Still think it's worth the trip though I'm pretty sure the Titanic was decommissioned." Another review by "Dominick Raffaini" is also partially visible.

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- Elicit ratings for various entities (e.g., on Yelp or Google)
  - We never find out the true quality/rating
- And many other applications...

# Common Features in These Applications

- We (the designer) elicit information from population
- Cannot or too costly to know ground truth
  - The reason of using crowdsourcing info elicitation
  - Key difference from prediction markets
- Agents/experts may misreport

**Challenge:** cannot verify the report/prediction

**Solution:** let multiple agents compete for the same task, and score them against each other (thus the name “**peer prediction**”)

Where else did we see a similar idea?

# A Simple and Concrete Example

- Elicit Alice's and Bob's truthful rating  $A, B$  about UVA dinning
  - $A, B \in \{High, Low\}$
  - There is a common joint belief:  $P([A, B] = [H, H]) = 0.5$ ;  $P([A, B] = [H, L]) = 0.24$ ;  $P([A, B] = [L, H]) = 0.24$ ;  $P([A, B] = [L, L]) = 0.02$

Let's try to understand this distribution ...

- It is symmetric among Alice and Bob
- $P(A = H) = 0.5 + 0.24 = 0.74$ 
  - Each expert very likely rates  $H$
- $P(A = H | B = H) = \frac{P(A=H, B=H)}{P(B=H)} = \frac{0.5}{0.74} = \frac{25}{37}$ 
  - Given that one rates  $H$ , the other very likely rates  $H$  as well
- $P(A = H | B = L) = \frac{P(A=H, B=L)}{P(B=L)} = \frac{0.24}{0.26} = \frac{12}{13}$ 
  - Given that one rates  $L$ , the other still very likely rates  $H$

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  - $P(A = H) = 0.74$ ;  $P(A = H | B = H) = \frac{25}{37}$ ;  $P(A = H | B = L) = \frac{12}{13}$

**Q:** What are some natural peer comparison and rewarding mechanisms?

- One simple idea is to **reward agreement**
  - Ask Alice and Bob to report their signals  $\bar{A}, \bar{B}$  (may misreport)
  - Award 1 to both if  $\bar{A} = \bar{B}$ , otherwise reward 0

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- Does this work?
  - If  $A = H$ , what should Alice report?
  - If  $A = L$ , what should Alice report?

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Truthful report is not an equilibrium!

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**Q:** What are some natural peer comparison and rewarding mechanisms?

- Both players always report  $H$  (i.e.,  $\bar{A} = \bar{B} = H$ ) is a Nash Equ.
- Why?
  - Well, under “rewarding agreement”, they both get 1, the maximum possible
  - In fact, both always reporting  $L$  is also a NE

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# The Model of Peer Prediction

- Two experts Alice and Bob, each holding a signal  $A \in \{A_1, \dots, A_n\}$  and  $B \in \{B_1, \dots, B_m\}$  respectively
  - A joint distribution  $p$  of  $(A, B)$  is publicly known
  - Everything we describe generalize to  $n$  experts
- We would like to elicit Alice's and Bob's true signals
  - We never know what signals they truly have

A seemingly richer but equivalent model

- We want to estimate distribution of random var  $E$
- Joint prior distribution  $p$  of  $(A, B, E)$  is publicly known
  - E.g.,  $E$  is true quality of our dinning, which we never observe
- Goal: elicit  $A, B$  to refine our estimation of  $E$

# A Subtle Issue

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## Eliciting **signals** vs distributions

- In prediction markets, we asked experts to report distributions
- Here, could have done the same thing
  - Alice could report  $p(E|A)$ , the dist. of  $E$  conditioned on her signal  $A$

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  - Alice could report  $p(E|A)$ , the dist. of  $E$  conditioned on her signal  $A$
  - Let's make a minor assumption:  $p(E|A) \neq p(E|A')$  for any  $A \neq A'$
  - Then, reporting signal  $A$  is equivalent to reporting distribution  $p(E|A)$
  - So, w.l.o.g., eliciting signals is equivalent

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  - So, w.l.o.g., eliciting signals is equivalent
- Drawback: have to assume an accurate and known prior

# Info Elicitation Mechanisms and Equilibrium

- Recall, we elicit info by asking Alice's and Bob's signal  $\bar{A}, \bar{B}$
- As before, will design rewards  $r_A(\bar{A}, \bar{B})$  and  $r_B(\bar{A}, \bar{B})$
- Alice's action is a **report strategy**  $\sigma_A(A) \in \{A_1, \dots, A_n\}$  [Bob similar]
  - This is a pure strategy
  - Will not consider mixed strategy here as we will design  $r_A$  and  $r_B$  so that there is a good pure equilibrium
  - **Truth-telling strategy:**  $\sigma_A(A) = A, \sigma_B(B) = B$
- Then, what outcome is expected to occur? → **equilibrium outcome**
- Generally, it is a Bayesian Nash equilibrium (**BNE**)
  - For simplicity, only define the equilibrium for our particular setting

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**Definition.**  $\sigma_A(A), \sigma_B(B)$  is a Bayesian Nash equilibrium if the following holds

$$\begin{aligned}\mathbb{E}_{B|A} r_A(\sigma_A(A), \sigma_B(B)) &\geq \mathbb{E}_{B|A} r_A(\sigma'_A(A), \sigma_B(B)), & \forall A \\ \mathbb{E}_{A|B} r_B(\sigma_A(A), \sigma_B(B)) &\geq \mathbb{E}_{A|B} r_B(\sigma_A(A), \sigma'_B(B)), & \forall B.\end{aligned}$$

We say it is a **strict BNE** if both “ $\geq$ ” are “ $>$ ”

# Mechanism for Peer Prediction

- Design objective: choose  $r_A, r_B$  so that truth-telling is an Equ.

Any ideas?

- Use proper scoring rules, but don't know signal distributions...
- Alice's signal can be used to estimate a distribution of Bob's signal, and vice versa

# Mechanism for Peer Prediction

## Information Elicitation without Verification

“Parameter”: any **strict proper** scoring rule  $S(i; p)$

1. Elicit Alice’s signal  $\bar{A}$  and Bob’s signal  $\bar{B}$
2. Calculate  $p_{\bar{A}} = \text{dist of } B \text{ conditioned on } \bar{A}$ , and similarly  $p_{\bar{B}}$
3. Award Alice  $r_A(\bar{A}, \bar{B}) = S(\bar{B}; p_{\bar{A}})$  and Bob  $r_B(\bar{A}, \bar{B}) = S(\bar{A}; p_{\bar{B}})$

Note: step 2 relies on the prior distribution  $p$

# Mechanism for Peer Prediction

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**Theorem.** Truth-telling is a strict BNE in the above game

Proof: show  $\sigma_A(A) = A$  is a best response to  $\sigma_B(B) = B$ , and vice versa

- If Bob reports  $B$  truthfully, Alice receives  $S(B; p_{\bar{A}})$  by reporting  $\bar{A}$
- With true signal  $A$ , what is Alice’s best response report  $\bar{A}$ ?
  - By strict properness, Alice wants  $p_{\bar{A}}$  to be exactly her true belief of dist. of  $B$
  - So, Alice should report  $\bar{A} = A$ .

# Remarks

- Mechanism is only described for two experts, but no difficult to generalize to  $n$  experts
  - Can randomly match each expert to a “peer” as reference
- Serious issues are the following

**Issue 1:** there are many other equilibria in the game

- Dinning rating example with slightly different numbers
  - A common joint belief:  $P([A, B] = [H, H]) = 0.4$ ;  $P([A, B] = [H, L]) = 0.1$ ;  $P([A, B] = [L, H]) = 0.1$ ;  $P([A, B] = [L, L]) = 0.4$
- Both always report  $H$  is also an equilibrium
  - If Bob always say  $H$ , Alice’s reward is always  $S(H; p_{\bar{A}})$  for whatever true  $A$
  - $\bar{A} = H$  makes  $p_{\bar{A}}(H) = P(B = H | \bar{A} = H) = 4/5$
  - $\bar{A} = L$  makes  $p_{\bar{A}}(H) = P(B = H | \bar{A} = L) = 1/5$

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**Issue 1:** there are many other equilibria in the game

- More generally, reporting quantities that are easy to coordinate likely forms an equilibrium
  - E.g., you are asked to grade essays, but you may all report the length of the essay while not its true quality (less effort, more well correlated)
- This is a fundamental issue of peer prediction

**Open question:** how to design mechanisms where truth-telling is unique (or the most plausible) equilibrium

# Remarks

- Mechanism is only described for two experts, but no difficult to generalize to  $n$  experts
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- Serious issues are the following

**Issue 2:** Designer has to know the joint distribution of  $(A, B)$

- Not very realistic, as designer usually has little knowledge
- But, there are remedies for this

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- Bayesian Truth Serum

# Designed for a Special yet Realistic Setting

- We, the designer, want to predict distribution of  $E$
- $n$  experts, each  $i$  has a signal  $S_i \sim p(S|E)$  i.i.d.
  - In this setting, we have to have many experts
  - Assume experts know  $p(S|E)$  but we do not know
- Objective: elicit true signals  $S_1, \dots, S_n$

Key design ideas

# Designed for a Special yet Realistic Setting

- We, the designer, want to predict distribution of  $E$
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## Key design ideas

- Cannot compute posterior distribution conditioned on any expert's signal anymore, but still need it to score him
- So, will elicit both his signal and his posterior belief of others' signals

# Bayesian Truth Serum [Prelec, Science'04]

## The Protocol

1. For each  $i$ , elicit her signal  $\bar{S}_i$  and her prediction  $\bar{p}^i \in \Delta_{|S|}$  of the distribution of any other expert's signal (agents are i.i.d. a-priori)
2. Calculate (geometric) **mean prediction**  $\bar{p}$  where  $\log \bar{p}_S = \frac{1}{n} \sum_i \log \bar{p}_S^i$  for any signal  $S$
3. Compute  $\bar{\lambda}$  to the empirical distribution of reported signals  $\bar{S}_i$ 's.
4. Reward agent  $i$  the following ( $G$  is any proper scoring rule)

$$\log \frac{\bar{\lambda}_{\bar{S}_i}}{\bar{p}_{\bar{S}_i}} + \mathbb{E}_{S \sim \bar{\lambda}} G(S; \bar{p}^i)$$

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Score of  $i$ 's signal report  $S_i$  (good if  $\bar{\lambda}_{\bar{S}_i} \geq \bar{p}_{\bar{S}_i}$ )

➤ That is,  $i$ 's reported type is **surprisingly more common** than predicted probability

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Score of  $i$ 's prediction  $\bar{p}^i$ , against the true signal distribution  $\bar{\lambda}$   
➤ By properness, want  $\bar{p}^i$  to be **close to**  $\bar{\lambda}$

# Bayesian Truth Serum [Prelec, Science'04]

**Theorem.** When  $n \rightarrow \infty$ , truthful report is a Bayesian Nash equilibrium in the previous protocol.

- That is, expert  $i$  should report his true signal  $S_i$  and his true posterior belief of other expert's signals
- $n \rightarrow \infty$  is needed because in that case  $\bar{\lambda} \rightarrow$  the exact signal distribution (under truthful signal report)
  - Several works try to relax this assumption to sufficiently large  $n$
- Proof is a bit intricate (see the Science paper)
- Very insightful, particularly, the design of rewarding “surprisingly common” signals, which is not clear before at all
- The issue of existence of multiple equilibria is still there

# Thank You

Haifeng Xu

University of Virginia

[hx4ad@virginia.edu](mailto:hx4ad@virginia.edu)