Question 4

With a ternary Merkle tree (3MT for short), each piece of data being committed will have 2 sibling nodes at each layer in the 3MT. If the 3MT has 3^t leaf nodes, then the authentication paths will contain $2t$ hashes (compared to t hashes in regular MT).

In the SPX hypertree, each layer produces one WOTS signature and one Merkle tree authentication path. If we use 3MT for SPX then:

$$|\sigma^{\text{SPX}}| = d(|\sigma^{\text{WOTS}}| + 2t|H|) + |\sigma^{\text{FORS}}|$$

where H is the hash function used in the Merkle tree.

Note that if we use 3MT, then each XMSS will have 3^t leaf nodes, and the entire hypertree will be able to sign 3^{dt} distinct messages. With a fixed security level λ and t, the value of d will decreased, meaning that we will need fewer layers in the hypertree (so the signature size is not strictly increasing).