

# A survey of generic IND-CCA2 transformations

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## 1 Preliminaries

### 1.1 Public-key encryption schemes

A public key encryption scheme  $\text{PKE} = (\text{KeyGen}, \text{E}, \text{D})$  is a collection of three routines.  $\text{KeyGen}(1^\lambda)$  takes the security parameter as input and returns a keypair  $(\text{pk}, \text{sk})$ .  $\text{E}(\text{pk}, m)$  takes some public key and some plaintext message  $m \in \mathcal{M}_{\text{PKE}}$  and output a ciphertext  $c \in \mathcal{C}_{\text{PKE}}$ . Where the encryption routine is probabilistic, we model the randomness using a coin  $r \in \mathcal{R}$  such that  $E(\text{pk}, m; r)$  is deterministic with an explicit  $r$ . Finally,  $\text{D}(\text{sk}, c)$  uses the secret key to decrypt the ciphertext.

#### 1.1.1 Correctness

Conventionally we require a PKE to be perfectly correct. This means that for all possible key pairs  $(\text{pk}, \text{sk})$  and plaintexts  $m$ , the decryption routine always correctly inverts the encryption routine:  $\text{D}(\text{sk}, \text{E}(\text{pk}, m)) = m$ .

Where perfect correctness is not achieved, such as with most lattice-based encryption schemes, we need to account for the possibility that decryption can fail. If under some keypair  $(\text{pk}, \text{sk})$ , a plaintext-ciphertext pair  $(m, c)$  is such that  $c \stackrel{\$}{\leftarrow} \text{E}(\text{pk}, m)$  is obtained from encrypting  $m$  (probabilistically) but  $m \neq \text{D}(\text{sk}, c)$ , we call it a decryption failure. For probabilistic encryption routines where the coin is uniformly sampled from the coin space, we can quantify the probability that  $m$  triggers a decryption failure. From here, we can take the distribution of all keypairs and quantify the “correctness” of a (possibly imperfectly correct) encryption scheme. The following definition of  $\delta$ -correctness is directly taken from [BDK<sup>+</sup>18].

**Definition 1.1** ( $\delta$ -correctness). *A probabilistic public-key encryption scheme  $\text{PKE} = (\text{KeyGen}, \text{E}, \text{D})$  is  $\delta$ -correct if the expected maximal probability of decryption failure taken across the distribution of keypairs is at most  $\delta$ :*

$$E \left[ \max_{m \in \mathcal{M}} P[D(\text{sk}, E(\text{pk}, m))] \right] \leq \delta$$

where the expectation is taken over the distribution of keypairs and the probability is taken over the distribution of coins.

[HHK17] also defined an adversarial game in which the adversary’s goal is to find some plaintext message to trigger a decryption failure. This adversarial game meaningfully models the real-world scenario in which decryption failure can reveal information about the secret key. Notice that when evaluating the win condition, the coin is uniformly random instead of being chosen by the adversary.

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**Algorithm 1** CORs

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- 1:  $(\text{pk}, \text{sk}) \stackrel{\$}{\leftarrow} \text{KeyGen}(1^\lambda)$
  - 2:  $m \stackrel{\$}{\leftarrow} A_{\text{CORs}}(1^\lambda, \text{pk}, \text{sk})$
  - 3: **return**  $\llbracket \text{D}(\text{sk}, \text{E}(\text{pk}, m)) \neq m \rrbracket$
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Figure 1: The correctness game CORs

The definition of  $\delta$ -correctness sets an explicit upper bound on the probability that any plaintext triggers decryption failure. This means that even if the **CORS** adversary actually finds the message  $m$  that is the most likely to trigger decryption failure, the probability of winning the **CORS** game is still upper-bounded by  $\delta$ .

**Lemma 1.0.1.** *If PKE is  $\delta$ -correct, then for all CORS adversaries  $A$ , even computationally unbounded ones, the probability of winning the CORS game is at most  $\delta$*

The values of  $\delta$  for Kyber are taken directly from [ABD<sup>+</sup>19]

security level	$\delta$
Kyber512	$2^{-139}$
Kyber768	$2^{-164}$
Kyber1024	$2^{-174}$

Table 1: Concrete  $\delta$  for Kyber

### 1.1.2 Security

An one-way adversary  $A = (A_1, A_2)$  consists of two sub-routines  $A_1$  and  $A_2$ .  $s \xleftarrow{\$} A_1^{\mathcal{O}_1}(1^\lambda, \text{pk})$  takes the security parameter, some public-key, access to some oracle(s)  $\mathcal{O}_1$ , and outputs some intermediate state  $s$ .  $\hat{m} \leftarrow A_2^{\mathcal{O}_2}(1^\lambda, \text{pk}, c^*)$  resumes from the output of  $A_1$  and takes some challenge ciphertext, then tries to guess the corresponding decryption.

The advantage  $\text{Adv}_{\text{OW-ATK}}(A)$  of an OW-ATK adversary is the probability that its guess is correct.

**Definition 1.2.** *A PKE is OW-ATK secure if for all efficient adversaries  $A$ , the advantage in the OW-ATK game is negligible with respect to the security parameter:*

$$\text{Adv}_{\text{OW-ATK}}(A) \leq \text{negl}(\lambda)$$

An **indistinguishability** adversary  $A = (A_1, A_2)$  similarly consists of two sub-routines. The first sub-routine adversarially chooses two distinct plaintext messages, and the second sub-routine tries to distinguish which of the two plaintext messages is the decryption of the challenge encryption. The advantage of an indistinguishability adversary is defined by  $\text{Adv}_{\text{IND-ATK}}(A) = P[\hat{b} = b] - \frac{1}{2}$

**Definition 1.3.** *A PKE is IND-ATK secure if for all efficient adversaries  $A$ , the advantage in the indistinguishability game is negligible with respect to the security parameter*

$$\text{Adv}_{\text{IND-ATK}}(A) \leq \text{negl}(\lambda)$$

The security games are described in details in figure 2

Algorithm 2 OW-ATK game	Algorithm 3 IND-ATK game
1: $(\text{pk}, \text{sk}) \xleftarrow{\$} \text{KeyGen}(1^\lambda)$	1: $(\text{pk}, \text{sk}) \xleftarrow{\$} \text{KeyGen}(1^\lambda)$
2: $s \xleftarrow{\$} A_1^{\mathcal{O}_1}(1^\lambda, \text{pk})$	2: $(m_0, m_1) \xleftarrow{\$} A_1^{\mathcal{O}_1}(1^\lambda, \text{pk})$
3: $m^* \xleftarrow{\$} \mathcal{M}$	3: $b \xleftarrow{\$} \{0, 1\}$
4: $c^* \xleftarrow{\$} \text{E}(\text{pk}, m^*)$	4: $c^* \xleftarrow{\$} \text{E}(\text{pk}, m_b)$
5: $\hat{m} \leftarrow A_2^{\mathcal{O}_2}(1^\lambda, \text{pk}, s, c^*)$	5: $\hat{b} \leftarrow A_2^{\mathcal{O}_2}(1^\lambda, \text{pk}, s, c^*)$
6: <b>return</b> $[\hat{m} = m^*]$	6: <b>return</b> $[\hat{b} = b]$

Figure 2: The indistinguishability security game

The capabilities of the adversaries are modeled using different collections of oracles. In standard security requirements, adversaries with access to no additional oracles can only mount chosen plaintext attacks (CPA),

adversaries with access to decryption oracle  $\mathcal{O}^D$  only before the receiving challenge ciphertext can mount non-adaptive chosen ciphertext attacks (CCA1), adversaries with access to decryption oracle both before and after receiving the challenge ciphertext can mount adaptive chosen ciphertext attacks (CCA2).

[HHK17] also defined two non-standard oracles and the corresponding attacks. The plaintext checking oracle  $\text{PCO}(m, c)$  returns 1 if  $m$  is a decryption of  $c$  and 0 otherwise. The ciphertext validation oracle  $\text{CVO}(c)$  returns 1 if  $c$  is a valid ciphertext and 0 otherwise.

<b>Algorithm 4</b> $\text{PCO}(m, c)$	<b>Algorithm 5</b> $\text{CVO}(c)$
<b>return</b> $\llbracket \text{D}(\text{sk}, c) = m \rrbracket$	1: <b>return</b> $\llbracket \text{D}(\text{sk}, c) \in \mathcal{M} \rrbracket$

Figure 3: PCO and CVO

Here is an overview of the various kinds of attacks and their associated oracles

ATK	$\mathcal{O}_1$	$\mathcal{O}_2$
CPA	—	
CCA1	$\mathcal{O}^D$	-
CCA2	$\mathcal{O}^D$	
PCVA	PCO, CVO	
PCA	PCO	
VA	CVO	

Table 2: Attacks and associated oracle access

## 2 Modular Fujisaki-Okamoto transformation

The Fujisaki-Okamoto transformation [FO99] and its KEM variations [HHK17].

## References

- [ABD<sup>+</sup>19] Roberto Avanzi, Joppe Bos, Léo Ducas, Eike Kiltz, Tancrede Lepoint, Vadim Lyubashevsky, John M Schanck, Peter Schwabe, Gregor Seiler, and Damien Stehlé. Crystals-kyber algorithm specifications and supporting documentation. *NIST PQC Round*, 2(4):1–43, 2019.
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