Question 1

We will prove $|c_1 - (c_2 + e)| \geq |c_1 - c_2| - |e|$ by proving a more general result:

Theorem 1 (variation of triangle inequality)**.** *let $|\cdot|$ be a metric, then for a, b in the vector space:*

$$|a - b| \geq |a| - |b|$$

The desired result is a special case of the theorem above where $a = c_1 - c_2$ and $b = e$

Proof. By the triangle inequality we know that $|a + b| \leq |a| + |b|$. Let $c = a + b$, then $a = c - b$, and the triangle inequality becomes

$$|c| \leq |c - b| + |b|$$

Re-arranging the equation above gives us the following:

$$|c - b| \geq |c| - |b|$$

We made no assumption of a, b, or c, so the inequality above holds for all vectors b, c in the vector space □