Faster generic IND-CCA secure KEM using encrypt-then-MAC

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Abstract. ML-KEM is a lattice-based IND-CCA secure key encapsulation mechanism (KEM) standardized by NIST in FIPS 203. ML-KEM achieves chosen-ciphertext security by applying the Fujisaki-Okamoto transformation to an IND-CPA secure public-key encryption (PKE) scheme. The Fujisaki-Okamoto transformation uses de-randomization and re-encryption to ensure ciphertext non-malleability, but because ML-KEM's underlying PKE encryption is significantly slower than the underlying PKE decryption, ML-KEM's decapsulation performance is dominated by re-encryption. In this paper, we propose an alternative generic IND-CCA secure KEM transformation that achieves chosen-ciphertext security by applying the encryptthen-MAC mechanism to an OW-PCA secure PKE and an existentially unforgeable MAC. Compared to the Fujisaki-Okamoto transformation, our encrypt-then-MAC transformation replaces de-randomization and re-encryption with computing a MAC tag. We instantiate our proposed KEM with the PKE sub-routines of ML-KEM and call it ML-KEM⁺. We then implement ML-KEM⁺ with a wide selection of MACs including Poly1305, GMAC, CMAC, and KMAC. At the cost of minimal increase in encapsulation CPU cycles (+1.8%) and ciphertext size (+2.1%), ML-KEM⁺ achieves a massive reduction of decapsulation CPU cycles (-72.2%) compared to ML-KEM. Furthermore, we implement key exchange protocols and measure realistic network round trip times (RTT), where ML-KEM⁺ reduces RTT by 23.9%-39.5% compared to ML-KEM.

Keywords: Key encapsulation mechanism, Message authentication code, Post-quantum cryptography, Lattice cryptography, Fujisaki-Okamoto transformation

1 Introduction

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A key encapsulation mechanism (KEM) [Sho01] is a cryptographic primitive that allows two parties to establish a shared secret over an insecure channel. The desired security standard for a KEM is called indistinguishability under chosen-ciphertext attack (IND-CCA). Intuitively speaking, IND-CCA security requires that no efficient adversary can distinguish a pseudorandom shared secret from a uniformly random bit string of equal 31 length, even with access to a decapsulation oracle throughout the attack. However, building a provably IND-CCA secure KEM is tremendously difficult. Early attempts without formal proofs, such as RSA encryption defined in PKCS#1 v1.5 [Kal98], were later shown to be vulnerable to practical chosen-ciphertext attacks [Ble98]. In recent decades, the most viable approach has been to start with cryptographic primitives possessing weaker security properties, such as a public-key encryption (PKE) scheme with one-way security under chosen-plaintext attack (OW-CPA), then add steps to achieve ciphertext non-malleability [BN00]. Some of the earliest proposals for generic IND-CCA secure constructions include OAEP [BR94], Fujisaki-Okamoto transformation [FO99] [FO13], REACT [OP01b], and GEM [CHJ⁺02]. Some other notable examples for constructing IND-CCA secure public-key cryptosystems in the standard model include the constructions of Naor and Yung [NY90], Cramer and Shoup [CS98] and Canetti, Halevi and Katz [CHK04]. The construction of Naor and Yung uses an IND-CPA secure PKE and a non-interactive zero-knowledge

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(NIZK) proof system. Canetti, Halevi and Katz [CHK04] proposed a construction to build an IND-CCA secure PKE scheme from an IND-CPA secure IBE scheme and a one-time signature scheme.

On the other hand, chosen-ciphertext security is a solved problem in symmetric cryptography. It is well understood that, by combining an IND-CPA secure symmetric encryption scheme with an existentially unforgeable message authentication code (MAC) in a pattern called encrypt-then-MAC [Kra01], one can build an authenticated encryption scheme [BN00] that achieves IND-CCA security. The encrypt-then-MAC (EtM) mechanism was standardized in ISO 19772 [fS09]. While this technique cannot be directly applied in the context of public-key cryptography due to the lack of a shared symmetric key between the two communicating parties, the concept of authenticating ciphertext using a MAC still has strong merits. Abdalla, Rogaway, and Bellare proposed DHIES (also known as "Hashed ElGamal")[ABR99][ABR01], a hybrid public-key encryption (HPKE) scheme whose IND-CCA security reduces to the Gap Diffie-Hellman assumption [OP01a] under the random oracle model. The technique behind DHIES is to derive both the shared secret and a symmetric MAC key by hashing a random PKE plaintext, encrypting the PKE plaintext, then authenticating the PKE ciphertext using the previously derived MAC key. If the Gap Diffie-Hellman assumption holds and the MAC is existentially unforgeable, no efficient adversary can recover the decryption of an unknown ciphertext even with access to a decryption oracle because it cannot produce a valid tag for such unknown ciphertext.

5 1.1 Contributions

Our main contributions are threefold:

- New IND-CCA secure KEM construction using encrypt-then-MAC. We propose a generic IND-CCA secure KEM transformation called the encrypt-then-MAC transformation and prove that the IND-CCA security of the encrypt-then-MAC transformation reduces tightly to the OW-PCA security of the underlying PKE and the existential unforgeability of the underlying MAC. In addition, we argue that the encrypt-then-MAC transformation can be instantiated with one-time MAC such as Poly1305 for further performance improvements.
- ML-KEM⁺: Efficient CCA-secure KEM based on ML-KEM. We present ML-KEM⁺, an IND-CCA secure KEM constructed by applying the encrypt-then-MAC transformation to ML-KEM's underlying PKE sub-routines. Compared to ML-KEM, ML-KEM⁺ adds a small amount of performance penalty to the encapsulation routine and a small increase in ciphertext size, but replaces the expensive re-encryption step in decapsulation with computing and verifying a MAC tag, which yields substantial performance improvements.
- Performance evaluation and comparisons. We implemented ML-KEM⁺ in C by adapting ML-KEM's reference implementation [BDK⁺24] and compared its performance with ML-KEM in a variety of scenarios. Compared to ML-KEM, ML-KEM⁺ achieves 72.2%-74.7% reduction of CPU cycle count for decapsulation while only increasing encapsulation's CPU cycle count by 1.8%-13.8% and ciphertext size by 2.0% (see Table 1). We also implemented the Kyber key exchange protocols [BDK⁺18] with ML-KEM⁺ and measured the round trip time (RTT) in realistic network settings. Compared to ML-KEM, ML-KEM⁺ achieves 23.7%-25.4% reduction of round trip time in unauthenticated key exchange (KE), 28.4%-32.3% in unilaterally authenticated key exchange (UAKE), and 30.6%-44.9% reduction in mutually authenticated key exchange (AKE). See Table 2 for a summary of round trip times.

imminal penalty in encapsulation performance and explicitly size.			
	ML-KEM ⁺ -512	ML-KEM ⁺ -768	ML-KEM ⁺ -1024
	Poly1305	KMAC 192-bit tag	KMAC 256-bit tag
Encap	93157	155219	228357
(ccl/tick)	(+1.8%)	(+13.8%)	(+8.8%)
Decap	33733	52415	62269
(ccl/tick)	(-72.2%)	(-71.9%)	(-74.7%)
CT size	784	1112	1600
(bytes)	(+2.1%)	(+2.2%)	(+2.0%)
	Encap (ccl/tick) Decap (ccl/tick) CT size	ML-KEM ⁺ -512 Poly1305 Encap 93157 (ccl/tick) (+1.8%) Decap 33733 (ccl/tick) (-72.2%) CT size 784	ML-KEM ⁺ -512 ML-KEM ⁺ -768 Poly1305 KMAC 192-bit tag

Table 1: ML-KEM⁺ achieves impressive reduction of decapsulation CPU cycles (ccl) while incurring minimal penalty in encapsulation performance and ciphertext size.

Table 2: ML-KEM⁺ achieves substantial RTT savings in key exchange protocols despite increased encapsulation cost and ciphertext size. RTT is measured in microseconds μs .

	ML-KEM ⁺ -512	ML-KEM ⁺ -768	ML-KEM ⁺ -1024
	Poly1305	KMAC 192-bit tag	KMAC 256-bit tag
KE RTT	70	103	144
(μs)	(-23.9%)	(-23.7%)	(-25.4%)
UAKE RTT	103	154	213
(μs)	(-29.0%)	(-28.4%)	(-32.3%)
AKE RTT	133	204	282
(μs)	(-39.5%)	(-30.6%)	(-44.9%)

1.2 Related works

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Optimal Asymmetric Encryption Padding (OAEP) [BR94][BDPR98] is a generic chosen-ciphertext secure PKE. Under the random oracle model, the chosen-ciphertext security of the OAEP encryption scheme reduces to the one-wayness of the input trapdoor permutation. Although it was discovered that there exist trapdoor permutations with which the OAEP encryption scheme does not achieve full IND-CCA security [Sho02], Fujisaki et al. later proved that OAEP is IND-CCA secure when combined with the RSA trapdoor permutation [FOPS01][RSA78]. RSA-OAEP was standardized in PKCS#1 v2 [MKJR16] and is currently the most recommended of all RSA-based encryption schemes. Unfortunately, OAEP's requirement for a trapdoor permutation is immensely difficult to satisfy, and no other practical instantiation saw widespread adoption to this day.

The Fujisaki-Okamoto transformation [FO99, FO13] is another generic IND-CCA secure transformation. In the original publication, Fujisaki and Okamoto proposed a hybrid public-key encryption scheme whose IND-CCA security reduces non-tightly to the OW-CPA security of the underlying PKE and the IND-CPA security of the symmetric encryption scheme. Later works [Den03, HHK17, DNR04, HHM22, BP18] tightened the security reduction, accounted for imperfect correctness, adapted the original proposal to build a KEM, and proved its security in the quantum random oracle model (QROM).

The Fujisaki-Okamoto transformation is remarkably successful because of the simplicity of its construction, the tightness of the security bound, and the proven (though non-tight) security against quantum adversaries. It was adopted by many submissions to NIST's post-quantum cryptography competition, including Kyber [BDK+18], Saber [DKRV18], FrodoKEM [BCD+16], and classic McEliece [ABC+20] among others. The $\mathtt{KEM}_m^{\not\perp}$ variant of the Fujisaki-Okamoto transformation (see Figure 1) is adopted in ML-KEM [oST24].

However, the Fujisaki-Okamoto transformation is not perfect. Because it uses reencryption for achieving ciphertext non-malleability, the Fujisaki-Okamoto transformation suffers from the following two problems:

- Computational inefficiency. The decapsulation routine needs to re-encrypt the decryption to ensure ciphertext has not been tempered with. For input PKE whose encryption routine carries substantial computational cost, such as most lattice-based cryptosystems, re-encryption slows down decapsulation significantly.
- Side-channel vulnerability. Re-encryption also introduces side-channels that

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$\texttt{KEM}_m^{\not\perp}.\texttt{KeyGen}()$	$\overline{\texttt{KEM}_m^{\not\perp}.\texttt{Encap}(\texttt{pk})}$	$\overline{\mathtt{KEM}_m^{\not\perp}.\mathtt{Decap}(\mathtt{sk},c)}$
1: $(pk, sk') \stackrel{\$}{\leftarrow} PKE.KeyGen()$ 2: $z \stackrel{\$}{\leftarrow} \mathcal{M}$ 3: $sk \leftarrow (sk', pk, z)$ 4: $\mathbf{return} \ (pk, sk)$	1: $m \stackrel{\$}{\leftarrow} \mathcal{M}$ 2: $r \leftarrow G(m)$ 3: $c \leftarrow \texttt{PKE.Enc}(\texttt{pk}, m, r)$ 4: $K \leftarrow H(m)$ 5: $\mathbf{return}\ (c, K)$	1: $\hat{m} \leftarrow \texttt{PKE.Dec}(\texttt{sk}',c)$ 2: $\hat{r} \leftarrow G(m)$ 3: $\hat{c} \leftarrow \texttt{PKE.Enc}(\texttt{pk},\hat{m},\hat{r})$ 4: if $\hat{c} = c$ then 5: $K \leftarrow H(\hat{m})$ 6: else 7: $K \leftarrow H(z,c)$ 8: end if 9: return K

Figure 1: The $\mathtt{KEM}^{\not\perp}_m$ variant of Fujisaki-Okamoto transformation is used in ML-KEM

can leak information about the decrypted PKE plaintext. As demonstrated in [UXT⁺22, RRCB19, TUX⁺23], these side-channels can be converted into efficient plaintext-checking attacks that can fully recover the secret key.

The rest of the paper is organized as follows. In Section 2, we define and discuss the preliminary concepts and theorems. In Section 3, we present our encrypt-then-MAC transformation in details, prove its security results, and discusses its relationship to the ElGamal cryptosystem. In Section 4, we present ML-KEM⁺, an instantiation of the encrypt-then-MAC transformation using ML-KEM's PKE sub-routines, then discuss some implementation rationales. In Section 5, we benchmark and compare the performance of individual KEM routines and key exchange round trip times between ML-KEM⁺ and ML-KEM.

2 Preliminaries

2.1 Public-key encryption scheme

Syntax. A public-key encryption scheme PKE(KeyGen, Enc, Dec) is a collection of three routines defined over some plaintext space \mathcal{M} and some ciphertext space \mathcal{C} . Key generation $(\mathtt{pk},\mathtt{sk}) \overset{\$}{\leftarrow} \mathtt{KeyGen}()$ is a randomized routine that returns a keypair. The encryption routine $\mathtt{Enc}: (\mathtt{pk},m) \mapsto c$ encrypts the input plaintext m under the input public key \mathtt{pk} and produces a ciphertext c. The decryption routine $\mathtt{Dec}: (\mathtt{sk},c) \mapsto m$ decrypts the input ciphertext c under the input secret key \mathtt{sk} and produces a plaintext m. Where the encryption routine is randomized, we denote the randomness by a coin $r \in \mathcal{R}$, where \mathcal{R} is called the coin space. The decryption routine is assumed to always be deterministic.

Correctness. Following the definition in [DNR04], a PKE is δ -correct if:

$$E\left[\max_{m \in \mathcal{M}} P\left[\mathtt{Dec}(\mathtt{sk}, c) \neq m \mid c \xleftarrow{\$} \mathtt{Enc}(\mathtt{pk}, m)\right]\right] \leq \delta.$$

Where the expectation is taken with respect to the probability distribution of all possible keypairs $(pk, sk) \stackrel{\$}{\leftarrow} PKE.KeyGen()$. For many lattice-based cryptosystems, including ML-KEM, decryption failures could leak information about the secret key, although the probability of a decryption failure is low enough that classical adversaries cannot exploit decryption failure more than they can defeat the underlying lattice problem.

Security. The security of public-key encryption is conventionally discussed within the context of adversarial games played between a challenger and an adversary [GM82]. There are two main types of games: i) in the one-wayness (OW-ATK) game, the adversary

is given a random encryption, then asked to produce the correct decryption; ii) in the indistinguishability (IND-ATK) game, the adversary is given the encryption of one of two adversary-chosen plaintexts, then asked to decide which of the plaintexts corresponds with the given encryption. Depending on the attack model, the adversary may have access to various oracles. Within the context of public-key cryptography, adversaries are always assumed to have the public key with which they can mount chosen-plaintext attack (CPA). If the adversary has access to a plaintext-checking oracle (PCO) [OP01b] then it can mount plaintext-checking attack (PCA). Where the adversary has access to a decryption oracle, it can mount chosen-ciphertext attacks (CCA).

OW-ATK Game	IND-ATK Game	$\overline{\mathcal{O}_{\texttt{PCO}}(m,c)}$
$\mathbf{1:}\ (\mathtt{pk},\mathtt{sk}) \xleftarrow{\$} \mathtt{KeyGen}(1^{\lambda})$	$1 \colon \left(\mathtt{pk}, \mathtt{sk} \right) \overset{\$}{\leftarrow} \mathtt{KeyGen}(1^{\lambda})$	1: $\mathbf{return} \ \llbracket m = Dec(sk, c) \rrbracket$
2: $m^* \stackrel{\$}{\leftarrow} \mathcal{M}$ 3: $c^* \stackrel{\$}{\leftarrow} \operatorname{Enc}(\operatorname{pk}, m^*)$	2: $(m_0, m_1) \stackrel{\$}{\leftarrow} A^{\mathcal{O}_{\mathtt{ATK}}}(1^{\lambda}, \mathtt{pk})$	
	$b \stackrel{\$}{\leftarrow} \{0,1\}$	
4: $\hat{m} \overset{\$}{\leftarrow} A^{\mathcal{O}_{\mathtt{ATK}}}(1^{\lambda}, \mathtt{pk}, c^{*})$	$c^* \stackrel{\$}{\leftarrow} \operatorname{Enc}(\operatorname{pk}, m_b)$	$\underline{\mathcal{O}_{\mathtt{Dec}}(c)}$
5: return $[\hat{m} = m^*]$	5: $\hat{b} \overset{\$}{\leftarrow} A^{\mathcal{O}_{\mathtt{ATK}}}(1^{\lambda}, \mathtt{pk}, c^{*})$	1: $\mathbf{return}\ \mathtt{Dec}(\mathtt{sk},c)$
	6: return $[\hat{b} = b]$	

Figure 2: The one-way game, indistinguishability game, plaintext-checking oracle (PCO), and decryption oracle. $ATK \in \{CPA, PCA, CCA\}$

The advantage of an adversary in the OW-ATK game is the probability that it outputs the correct decryption. The advantage of an adversary in the IND-ATK game is defined below. A PKE is OW-ATK/IND-ATK secure if no efficient adversary has non-negligible advantage in the corresponding security game.

$$\mathtt{Adv}_{\mathtt{IND-ATK}}(A) = \left| P\left[A^{\mathcal{O}_{\mathtt{ATK}}}(1^{\lambda}, \mathtt{pk}, c^*) = b \right] - \frac{1}{2} \right|.$$

2.2 Key encapsulation mechanism (KEM)

Syntax. A key encapsulation mechanism KEM(KeyGen, Encap, Decap) is a collection of three routines defined over some ciphertext space $\mathcal C$ and some key space $\mathcal K$. The key generation routine takes the security parameter 1^λ and outputs a keypair $(\mathtt{pk},\mathtt{sk}) \overset{\$}{\leftarrow} \mathtt{KeyGen}(1^\lambda)$. Encap(pk) is a probabilistic routine that takes a public key pk and outputs a pair of values (c,K) where $c \in \mathcal C$ is the ciphertext (also called encapsulation) and $K \in \mathcal K$ is the shared secret (also called session key). Decap(sk, c) is a deterministic routine that takes the secret key sk and the encapsulation c and returns the shared secret K if the ciphertext is valid. Some KEM constructions use explicit rejection, where if c is invalid then Decap will return a rejection symbol \bot ; other KEM constructions use implicit rejection, where if c is invalid then Decap will return a fake session key that depends on the ciphertext and some other secret values.

Security. The security of a KEM is similarly discussed in adversarial games (Figure 3), although the win conditions differ slightly from the win conditions of a PKE indistinguishability game. In a KEM's indistinguishability game, an adversary is given the public key and a challenge ciphertext, then asked to distinguish a pseudorandom shared secret K_0 associated with the challenge ciphertext from a truly random bit string of equal length.

The decapsulation oracle $\mathcal{O}^{\mathtt{Decap}}$ takes a ciphertext c and returns the output of the Decap routine using the secret key. The advantage of an IND-CCA adversary $\mathcal{A}_{\mathtt{IND-CCA}}$ is

IND-ATK game	$\mathcal{O}_{ exttt{Decap}}(c)$
1: $(\mathtt{pk},\mathtt{sk}) \xleftarrow{\$} \mathtt{KeyGen}(1^{\lambda})$	1: $\mathbf{return} \; \mathtt{Decap}(\mathtt{sk}, c)$
$2: (c^*, K_0) \stackrel{\$}{\leftarrow} \mathtt{Encap}(\mathtt{pk})$	
$K_1 \stackrel{\$}{\leftarrow} \mathcal{K}$	
$4: b \stackrel{\$}{\leftarrow} \{0,1\}$	
5: $\hat{b} \overset{\$}{\leftarrow} A^{\mathcal{O}_{\mathtt{ATK}}}(1^{\lambda}, \mathtt{pk}, c^*, K_b)$	
6: $\mathbf{return} \ \llbracket \hat{b} = b \rrbracket$	

Figure 3: IND-ATK game for KEM and decapsulation oracle $\mathcal{O}_{\mathtt{Decap}}$

defined by the adversary's ability to correctly distinguish the two cases beyond a blind guess:

$$\mathtt{Adv}_{\mathtt{IND-CCA}}(A) = \left| P[A^{\mathcal{O}_{\mathtt{Decap}}}(a^{\lambda}, \mathtt{pk}, c^*, K_b) = b] - \frac{1}{2} \right|.$$

A KEM is IND-ATK secure if no efficient adversary has non-negligible advantage in the corresponding security game.

2.3 Message authentication code (MAC)

Syntax. A message authentication code MAC(KeyGen, Sign, Verify) is a collection of routines defined over some key space \mathcal{K} , some message space \mathcal{M} , and some tag space \mathcal{T} . The signing routine $\operatorname{Sign}(k,m)$ authenticates the message m under the secret key k by producing a tag t (also called digest) (we define the process that generates an authentication tag t over message m a signing routine in this paper). The verification routine $\operatorname{Verify}(k,m,t)$ takes the triplet of secret key k, message m, and tag t, and outputs 1 if the message-tag pair is valid under the secret key, or 0 otherwise. Many MAC constructions are deterministic. For these constructions it is simpler to denote the signing routine by $t \leftarrow \operatorname{MAC}(k,m)$ and perform verification using a simple comparison.

Security. The security of a MAC is defined in an adversarial game in which an adversary, with access to a MAC oracle that can answer signing queries $MAC(k, m) \leftarrow \mathcal{O}_{MAC}(m)$, tries to forge a new valid message-tag pair that has never been queried before. The ability to access a MAC oracle is called *chosen-message attack (CMA)*. The ability to produce a valid tag on some arbitrary message is called *existential forgery*. The existential unforgeability under chosen message attack (EUF-CMA) game is shown below:

Figure 4: The existential forgery game and the MAC oracle

The advantage Adv_{EUF-CMA} of the existential forgery adversary is the probability that it wins the EUF-CMA game. Some MACs are one-time existentially unforgeable, meaning that each secret key can be used to authenticate only a single message. The corresponding security game is modified such that the MAC oracle will only answer a single signing query.

3 The encrypt-then-MAC transformation

Our technique. We introduce our encrypt-then-MAC transformation that transforms a OW-PCA secure PKE and an one-time existentially unforgeable MAC into an IND-CCA secure KEM. Our scheme mainly differs from DHIES in its versatility and input requirement. Whereas the IND-CCA security of DHIES reduces specifically to the Gap Diffie-Hellman assumption, the chosen-ciphertext security of the encrypt-then-MAC KEM reduces more generally to the OW-PCA security [OP01b] of the input scheme. In addition, we propose that because each call to encapsulation samples a fresh PKE plaintext, the encrypt-then-MAC KEM can be instantiated with one-time secure MAC such as Poly1305 for further performance improvements (Abdalla, Rogaway, and Bellare originally proposed to use HMAC and CBC-MAC, which are many-time secure MAC but less efficient than one-time MAC, see Section 4.2). The encapsulation data flow is illustrated in Figure 5.

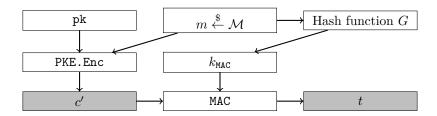


Figure 5: Combining PKE with MAC using encrypt-then-MAC to ensure ciphertext integrity

In Section 3.2 we reduce the IND-CCA security of the KEM tightly to the OW-PCA security of the underlying PKE, and non-tightly to the unforgeability of the MAC. In Section 3.3, we show that DHIES is a special case of the encrypt-then-MAC transformation by reducing the OW-PCA security of the ElGamal cryptosystem to the Gap Diffie-Hellman assumption.

3.1 The generic KEM construction

Let \mathcal{B}^* denote the set of finite bit strings. Let PKE(KeyGen, Enc, Dec) be a public-key encryption scheme defined over message space \mathcal{M} and ciphertext space \mathcal{C} . Let MAC: $\mathcal{K}_{\text{MAC}} \times \mathcal{B}^* \to \mathcal{T}$ be a deterministic message authentication code that takes a key $k \in \mathcal{K}_{\text{MAC}}$, some message $m \in \mathcal{B}^*$, and outputs a tag $t \in \mathcal{T}$. Let $G: \mathcal{M} \to \mathcal{K}_{\text{MAC}}$ be a hash function that maps from PKE's plaintext space to MAC's key space. Let $H: \mathcal{B}^* \to \mathcal{K}_{\text{KEM}}$ be a hash function that maps bit strings into the set of possible shared secrets. The encrypt-then-MAC transformation EtM[PKE, MAC, G, H] constructs a key encapsulation mechanism KEM_{EtM}(KeyGen, Encap, Decap), whose routines are described in Figure 6.

Since the encrypt-then-MAC transformation removes re-encryption in decapsulation, there is no longer the need for fixing the pseudorandom coin r in the PKE's encryption routine. If the input PKE is already rigid, then the shared secret may be derived from hashing the PKE plaintext alone. However, if the input PKE is not rigid, then the shared secret must be derived from hashing both the PKE plaintext and the PKE ciphertext.

Security analysis. The chosen-ciphertext security of the encrypt-then-MAC scheme can be intuitively argued through an adversary's inability to learn additional information from the decapsulation oracle. For an adversary A to produce a valid tag t for some unauthenticated ciphertext c' under the symmetric key $k \leftarrow G(\text{Dec}(sk',c'))$ implies that A must either know the symmetric key k or produce a forgery. Under the random oracle model, A also cannot know k without knowing its pre-image Dec(sk',c'), so A must either

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$\mathtt{KEM}_{\mathtt{EtM}}.\mathtt{KeyGen}()$	$\underbrace{\texttt{KEM}_{\texttt{EtM}}.\texttt{Decap}(\texttt{sk},c)}$
1: $(pk, sk') \stackrel{\$}{\leftarrow} PKE.KeyGen()$ 2: $z \stackrel{\$}{\leftarrow} \mathcal{M}$ 3: $sk \leftarrow (sk', z)$ 4: $\mathbf{return} \ (pk, sk)$	1: $(c',t) \leftarrow c$ 2: $(\mathtt{sk}',z) \leftarrow \mathtt{sk}$ 3: $\hat{m} \leftarrow \mathtt{PKE.Dec}(\mathtt{sk}',c')$ 4: $\hat{k} \leftarrow G(\hat{m})$ 5: if $\mathtt{MAC}(\hat{k},c')=t$ then 6: $K \leftarrow H(\hat{m},c)$ 7: else
$ \begin{split} \text{KEM}_{\texttt{EtM}}. & \texttt{Encap}(\texttt{pk}) \\ 1: \ m \overset{\$}{\leftarrow} \mathcal{M} \\ 2: \ k \leftarrow G(m) \\ 3: \ c' \overset{\$}{\leftarrow} \ \texttt{PKE}. \ \texttt{Enc}(\texttt{pk}, m) \\ 4: \ t \leftarrow \texttt{MAC}(k, c') \\ 5: \ c \leftarrow (c', t) \\ 6: \ K \leftarrow H(m, c) \\ 7: \ \textbf{return} \ (c, K) \end{split} $	8: $K \leftarrow H(z,c)$ 9: end if 10: return K

Figure 6: The encrypt-then-MAC transformation builds a KEM, denoted by KEM_{EtM} , using a PKE(KeyGen, Enc, Dec), a MAC, and two hash functions G, H

have produced c' honestly, or have broken the one-way security of PKE. This means that the decapsulation oracle will not give out information on decryption that the adversary does not already know.

However, a decapsulation oracle can still give out some information: for a known plaintext m, all possible encryptions $c' \stackrel{\$}{\leftarrow} \mathtt{Enc}(\mathtt{pk},m)$ can be correctly signed, while ciphertexts that don't decrypt back to m cannot be correctly signed. This means that a decapsulation oracle can be converted into a plaintext-checking oracle, so every chosen-ciphertext attack against the KEM can be converted into a plaintext-checking attack against the underlying PKE.

On the other hand, if the underlying PKE is OW-PCA secure and the underlying MAC is one-time existentially unforgeable, then the encrypt-then-MAC KEM is IND-CCA secure:

Theorem 1. For every IND-CCA adversary A against KEM_{EtM} that makes q decapsulation queries, there exists an OW-PCA adversary B who makes at least q plaintext-checking queries against the underlying PKE, and an one-time existential forgery adversary C against the underlying MAC such that

$$Adv_{IND-CCA}(A) \leq q \cdot Adv_{OT-MAC}(C) + 2 \cdot Adv_{OW-PCA}(B).$$

3.2 Proof of Theorem 1

We will prove Theorem 1 using a sequence of game. A summary of the the sequence of games can be found in Figure 7 and 8. From a high level we made three incremental modifications to the IND-CCA game for KEM_{ELM} :

- 1. Replace the true decapsulation oracle with a simulated decapsulation oracle
- 2. Replace the pseudorandom MAC key $k^* \leftarrow G(m^*)$ with a truly random $k^* \stackrel{\$}{\leftarrow} \mathcal{K}_{\text{MAC}}$

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3. Replace the pseudorandom shared secret $K_0 \leftarrow H(m^*, c)$ with a truly random shared secret $K_0 \stackrel{\$}{\leftarrow} \mathcal{K}_{\texttt{KEM}}$

A OW-PCA adversary can then simulate the modified IND-CCA game for the KEM adversary, and the advantage of the OW-PCA adversary is associated with the probability of certain behaviors of the KEM adversary.

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IND-CCA game for KEM _{EtM}	Decap oracle $\mathcal{O}^{\mathtt{Decap}}(c)$
1: $(pk, sk) \stackrel{\$}{\leftarrow} KEM_{EtM} . KeyGen()$ 2: $m^* \stackrel{\$}{\leftarrow} \mathcal{M}$ 3: $c' \stackrel{\$}{\leftarrow} PKE . Enc(pk, m^*)$ 4: $k^* \leftarrow G(m^*)$ \triangleright Game 0-1 5: $k^* \stackrel{\$}{\leftarrow} \mathcal{K}_{MAC}$ \triangleright Game 2-3 6: $t \leftarrow MAC(k^*, c')$ 7: $c^* \leftarrow (c', t)$ 8: $K_0 \leftarrow H(m^*, c^*)$ \triangleright Game 0-2 9: $K_0 \stackrel{\$}{\leftarrow} \mathcal{K}_{KEM}$ \triangleright Game 3	1: $(c',t) \leftarrow c$ 2: $\hat{m} = \text{Dec}(sk',c')$ 3: $\hat{k} \leftarrow G(\hat{m})$ 4: if $\text{MAC}(\hat{k},c') = t$ then 5: $K \leftarrow H(\hat{m},c)$ 6: else 7: $K \leftarrow H(z,c)$ 8: end if 9: return K
10: $K_1 \stackrel{\$}{\leftarrow} \mathcal{K}_{\text{KEM}}$ 11: $b \stackrel{\$}{\leftarrow} \{0, 1\}$ 12: $\hat{b} \leftarrow A^{\mathcal{O}^{\text{Decap}}}(\text{pk}, c^*, K_b)$ \triangleright Game 0 13: $\hat{b} \leftarrow A^{\mathcal{O}^{\text{Decap}}}(\text{pk}, c^*, K_b)$ \triangleright Game 1-3 14: return $[\![\hat{b} = b]\!]$	$\begin{array}{c} \frac{\mathcal{O}_{1}^{Decap}(c)}{1:\;(c',t) \leftarrow c} \\ 0 & 2: \; \mathbf{if} \; \exists (\tilde{m},\tilde{k}) \in \mathcal{L}^{G} : \tilde{m} \; = \; Dec(\mathtt{sk'},c') \land \\ 0 & MAC(\tilde{k},c') = t \; \mathbf{then} \\ 0 & 3: K \leftarrow H(\tilde{m},c) \\ 0 & 4: \; \mathbf{else} \end{array}$
Hash oracle $\mathcal{O}^{G}(m)$ 1: if $\exists (\tilde{m}, \tilde{k}) \in \mathcal{L}^{G} : \tilde{m} = m$ then 2: return \tilde{k}	5: $K \leftarrow H(z,c)$ 6: end if 7: return K
2: return k 3: end if 4: $k \stackrel{\$}{\leftarrow} \mathcal{K}_{\texttt{MAC}}$ 5: $\mathcal{L}^G \leftarrow \mathcal{L}^G \cup \{(m,k)\}$ 6: return k	$\begin{array}{l} \mathcal{O}^{H}(m,c) \\ \hline 1: \ \mathbf{if} \ \exists (\tilde{m},\tilde{c},\tilde{K}) \in \mathcal{L}^{H} : \tilde{m} = m \wedge \tilde{c} = c \\ \mathbf{then} \\ 2: \ \mathbf{return} \ \tilde{K} \\ \hline 3: \ \mathbf{end} \ \mathbf{if} \\ 4: \ K \xleftarrow{\$} \mathcal{K}_{\texttt{KEM}} \\ 5: \ \mathcal{L}^{H} \leftarrow \mathcal{L}^{H} \cup \{(m,c,K)\} \\ 6: \ \mathbf{return} \ K \end{array}$

Figure 7: Sequence of games in the proof of Theorem 1

Proof. Game 0 is the standard KEM IND-CCA game. The decapsulation oracle $\mathcal{O}^{\mathsf{Decap}}$ executes the decapsulation routine using the challenge keypair and return the results faithfully. The queries made to the hash oracles $\mathcal{O}^G, \mathcal{O}^H$ are recorded to their respective tapes $\mathcal{L}^G, \mathcal{L}^H$.

Game 1 is identical to game 0 except that the true decapsulation oracle $\mathcal{O}^{\mathsf{Decap}}$ is replaced with a simulated oracle $\mathcal{O}^{\mathsf{Decap}}_1$. Instead of directly decrypting c' as in the decapsulation routine, the simulated oracle searches through the tape \mathcal{L}^G to find a matching query (\tilde{m}, \tilde{k}) such that \tilde{m} is the decryption of c'. The simulated oracle then uses \tilde{k} to validate the tag t against c'.

If the simulated oracle accepts the queried ciphertext as valid, then there is a matching query that also validates the tag, which means that the queried ciphertext is honestly

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B(pk, c'^*)
  1: z \stackrel{\$}{\leftarrow} \mathcal{M}
2: k \stackrel{\$}{\leftarrow} \mathcal{K}_{\text{MAC}}
                                                                                                            1: (c',t) \leftarrow c
                                                                                                            2: if \exists (\tilde{m}, \tilde{k}) \in \mathcal{L}^G : PCO(\tilde{m}, c') = 1 \land
                                                                                                                  MAC(\tilde{k}, c') = t then
  3: t \leftarrow \text{MAC}(k, c'^*)
  4: c^* \leftarrow (c'^*, t)
                                                                                                                          K \leftarrow H(\tilde{m}, c)
                                                                                                            4:
                                                                                                                  else
  5: K \overset{\$}{\leftarrow} \mathcal{K}_{\mathtt{KEM}}
6: \hat{b} \leftarrow A^{\mathcal{O}_{B}^{\mathtt{Decap}}, \mathcal{O}_{B}^{G}, \mathcal{O}_{B}^{H}}(\mathtt{pk}, c^{*}, K)
                                                                                                                          K \leftarrow H(z, c)
                                                                                                            6: end if
  7: if ABORT(m) then
                                                                                                            7: return K
                return m
  9: end if
                                                                                                         \mathcal{O}_B^G(m)
\mathcal{O}_B^H(m,c)
                                                                                                            1: if PCO(m, c'^*) = 1 then
     if PCO(m, c'^*) = 1 then
                                                                                                                          ABORT(m)
                                                                                                            2:
             ABORT(m)
                                                                                                            3: end if
                                                                                                            4: if \exists (\tilde{m}, \tilde{k}) \in \mathcal{L}^G : \tilde{m} = m then
     if \exists (\tilde{m}, \tilde{c}, \tilde{K}) \in \mathcal{L}^H : \tilde{m} = m \land \tilde{c} = c then
                                                                                                                          return \tilde{k}
             return K
                                                                                                            6: end if
     end if
                                                                                                           7: k \overset{\$}{\leftarrow} \mathcal{K}_{\texttt{MAC}}
8: \mathcal{L}^G \leftarrow \mathcal{L}^G \cup \{(m,k)\}
    \begin{split} K &\overset{\$}{\leftarrow} \mathcal{K}_{\texttt{KEM}} \\ \mathcal{L}^H &\leftarrow \mathcal{L}^H \cup \{(m,c,K)\} \end{split}
                                                                                                            9: \mathbf{return} \ k
```

Figure 8: OW-PCA adversary B simulates game 3 for IND-CCA adversary A in the proof for Theorem 1

generated. Therefore, the true oracle must also accept the queried ciphertext. On the other hand, if the true oracle rejects the queried ciphertext, then the tag is simply invalid under the MAC key k = G(Dec(sk',c')). Therefore, there could not have been a matching query that also validates the tag, and the simulated oracle must also rejects the queried ciphertext.

This means that from the adversary A's perspective, game 1 and game 0 differ only when the true oracle accepts while the simulated oracle rejects, which means that t is a valid tag for c' under k = G(Dec(sk',c')), but k has never been queried. Under the random oracle model, such k is a uniformly random sample of \mathcal{K}_{MAC} that the adversary does not know, so for A to produce a valid tag is to produce a forgery against the MAC under an unknown and uniformly random key. Furthermore, the security game does not include a MAC oracle, so this is a zero-time forgery. While zero-time forgery is not a standard security definition for a MAC, we can bound it by the advantage of a one-time forgery adversary C:

$$P\left[\mathcal{O}^{\mathtt{Decap}}(c) \neq \mathcal{O}^{\mathtt{Decap}}_1(c)\right] \leq \mathtt{Adv}_{\mathtt{OT-MAC}}(C).$$

Across all q decapsulation queries, the probability that at least one query is a forgery is thus at most $q \cdot P \left[\mathcal{O}^{\text{Decap}}(c) \neq \mathcal{O}^{\text{Decap}}_1(c) \right]$. By the difference lemma:

$$Adv_{G_0}(A) - Adv_{G_1}(A) \leq q \cdot Adv_{OT-MAC}(C)$$
.

Game 2 is identical to game 1, except that the challenger samples a uniformly random MAC key $k^* \stackrel{\$}{\leftarrow} \mathcal{K}_{\texttt{MAC}}$ instead of deriving it from m^* . From A's perspective the two games

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are indistinguishable, unless A queries G with the value of m^* . Denote the probability that A queries G with m^* by P[QUERY G], then:

$$Adv_{G_1}(A) - Adv_{G_2}(A) \leq P [QUERY G].$$

Game 3 is identical to game 2, except that the challenger samples a uniformly random shared secret $K_0 \stackrel{\$}{\leftarrow} \mathcal{K}_{\texttt{KEM}}$ instead of deriving it from m^* and t. From A's perspective the two games are indistinguishable, unless A queries H with (m^*, \cdot) . Denote the probability that A queries H with (m^*, \cdot) by P[QUERY H], then:

$$\operatorname{Adv}_{G_2}(A) - \operatorname{Adv}_{G_3}(A) \leq P \left[\operatorname{QUERY} \ \operatorname{H} \right].$$

Since in game 3, both K_0 and K_1 are uniformly random and independent of all other variables, no adversary can have any advantage: $Adv_{G_3}(A) = 0$.

We will bound $P[\mathtt{QUERY}\ \mathtt{G}]$ and $P[\mathtt{QUERY}\ \mathtt{H}]$ by constructing a OW-PCA adversary B against the underlying PKE that uses A as a sub-routine. B's behaviors are summarized in Figure 8.

B simulates game 3 for A: receiving the public key pk and challenge encryption c'^* , B samples random MAC key and session key to produce the challenge encapsulation, then feeds it to A. When simulating the decapsulation oracle, B uses the plaintext-checking oracle to look for matching queries in \mathcal{L}^G . When simulating the hash oracles, B uses the plaintext-checking oracle to detect when $m^* = \text{Dec}(sk', c'^*)$ has been queried. When m^* is queried, B terminates A and returns m^* to win the OW-PCA game. In other words:

$$P[\text{QUERY G}] \leq \text{Adv}_{\text{OW-PCA}}(B),$$

 $P[\text{QUERY H}] \leq \text{Adv}_{\text{OW-PCA}}(B).$

Combining all equations above produce the desired security bound.

3.3 ElGamal is OW-PCA secure

We show that the DHAES/DHIES hybrid encryption scheme is a special case of the encrypt-then-MAC transformation. Specifically, we will sketch a proof of the following lemma:

Lemma 1. For every OW-PCA adversary A against the ElGamal cryptosystem, there exists a Gap Diffie-Hellman problem solver B such that:

$$Adv_{GapDH}(B) = Adv_{OW-PCA}(A).$$

In other words, ElGamal is OW-PCA secure under the Gap Diffie-Hellman assumption.

Each ElGamal cryptosystem [Gam85] is parameterized by a cyclic group $G = \langle g \rangle$ of prime order q > 2. A summary of the routine is shown in Figure 9:

KeyGen()	$\overline{{\tt Enc}({\tt pk}=g^x, m\in G)}$	$\overline{ \operatorname{Dec}(\operatorname{sk} = x, c = (w, v) \in G^2) }$
1: $x \stackrel{\$}{\leftarrow} \mathbb{Z}_q$	Require: $m \in G$	$\frac{1}{1} \cdot \hat{m} \leftarrow (w^x)^{-1} \cdot v$
$2: \mathbf{sk} \leftarrow \overset{\mathbf{q}}{x}$	1: $y \stackrel{\$}{\leftarrow} \mathbb{Z}_q$	2: return \hat{m}
$3: \ \mathtt{pk} \leftarrow g^x$	2: $w \leftarrow g^y$	
4: return (pk, sk)	3: $v \leftarrow m \cdot (g^x)^y$	
	4: return $c = (w, v)$	

Figure 9: ElGamal cryptosystem

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The security of ElGamal cryptosystem reduces to the conjectured intractability of the computational Diffie-Hellman problem and the decisional Diffie-Hellman problem:

Definition 1 (computational Diffie-Hellman problem (CDH)). Let $x, y \stackrel{\$}{\leftarrow} \mathbb{Z}_q$ be uniformly random samples. Given (g, g^x, g^y) , compute g^{xy} .

Definition 2 (decisional Diffie-Hellman problem (DDH)). Let $x, y, z \stackrel{\$}{\leftarrow} \mathbb{Z}_q$ be uniformly random samples. Let $h \stackrel{\$}{\leftarrow} \{g^z, g^{xy}\}$ be randomly chosen between g^z and g^{xy} . Given (g, g^x, g^y, h) , determine whether h is g^{xy} or g^z

It is also conjectured in [ABR01] (and later extensively studied in [OP01a]) that for certain choice of cyclic group G, the computational Diffie-Hellman problem remains intractable even if the adversary as access to a restricted decisional Diffie-Hellman oracle. This assumption is captured in the Gap Diffie-Hellman problem:

Definition 3 (Gap Diffie-Hellman problem). Let $G = \langle g \rangle$ be a cyclic group of prime order q > 2. Let $x, y \stackrel{\$}{\leftarrow} \mathbb{Z}_q$ be uniformly random samples. Given (g, g^x, g^y) and a restricted DDH oracle $\mathcal{O}^{\text{DDH}} : (u, v) \mapsto \llbracket u^x = v \rrbracket$, compute g^{xy} .

We now present the proof for Lemma 1.

Proof. We will prove by a sequence of games. A summary can be found in Figure 10

$G_0 - G_2$		$\overline{\text{PCO}(m,c=(w,v))}$
1: $x \stackrel{\$}{\leftarrow} \mathbb{Z}_q$		1: return $[m = (w^x)^{-1} \cdot v]$
2: $m^* \stackrel{\$}{\leftarrow} G$ 3: $y \stackrel{\$}{\leftarrow} \mathbb{Z}_q, w \leftarrow g^y$		
3: $y \stackrel{\mathfrak{d}}{\leftarrow} \mathbb{Z}_q, w \leftarrow g^y$		
$4: v \leftarrow m^* \cdot (g^x)^y$	$\triangleright G_0$ - G_1	$PCO_1(m,c=(w,v))$
5: $v \stackrel{\$}{\leftarrow} G$	$\triangleright G_2$	1: return $[(w^x) = m^{-1} \cdot v]$
$6: c^* \leftarrow (w, v)$		
7: $\hat{m} \stackrel{\$}{\leftarrow} A^{\texttt{PCO}}(g^x, c^*)$	$\triangleright G_0$	
8: $\hat{m} \stackrel{\$}{\leftarrow} A^{\texttt{PCO}_1}(g^x, c^*)$	$\triangleright G_1$ - G_2	
9: return $[\hat{m} = m^*]$	$\triangleright G_0$ - G_1	
10: return $[\hat{m} = w^{-x}]$	v $v $ v	

Figure 10: The sequence of games in proving Lemma 1

Game θ is the OW-PCA game. Adversary A has access to the plaintext-checking oracle PCO and wins the game if it can correctly recover the challenge plaintext m^* .

Game 1 is identical to game 0, except that the formulation of the PCO is changed. When servicing the plaintext-checking query (m, c = (w, v)), PCO₁ checks whether w^x is equal to $m^{-1} \cdot v$. Observe that in the cyclic group G, the algebraic expressions in PCO and PCO₁ are equivalent, which means that PCO₁ behaves identically to PCO.

Game 2 is identical to game 1 except for two modifications: first, when computing the challenge ciphertext, v is no longer computed from m^* but is randomly sampled; second, the win condition changed from $\hat{m} = m^*$ to $\hat{m} = w^{-x} \cdot v$. It is easy to verify that Game 0 through Game 2 are algebraically equivalent:

$$Adv_0(A) = Adv_1(A) = Adv_2(A).$$

The Gap Diffie-Hellman adversary B can perfectly simulate game 2 for A (see Figure 11): B receives as the Gap Diffie-Hellman problem inputs g^x and g^y . g^x simulates an

ElGamal public key, where as q^y simulates the first component of the challenge ciphertext. As in game 2, the second component of the challenge ciphertext can be randomly sampled. Finally, the PCO₁ from game 2 can be perfectly simulated using the restricted DDH oracle

$B^{\mathcal{O}^{\text{DDH}}}(g, g^x, g^y)$	$\mathcal{O}^{\mathrm{DDH}}(u,v)$
1: $w \leftarrow g^y$	1: return $\llbracket u^x = v \rrbracket$
$2: v \stackrel{\$}{\leftarrow} G$	
$3: c^* \leftarrow (w, v)$	$\overline{\mathtt{PCO}_2(m,c=(w,v))}$
4: $\hat{m} \stackrel{\$}{\leftarrow} A^{\text{PCO}_2}(g^x, c^*)$	1: return $\mathcal{O}^{\mathrm{DDH}}(w, m^{-1} \cdot v)$
5: return $\hat{m}^{-1} \cdot v$	1. Tetain C (w, mt v)

Figure 11: Gap Diffie-Hellman adversary B simulates game 2 for A

If A wins game 2, then its output is $\hat{m} = w^{-x} \cdot v = g^{-xy} \cdot v$, so $m^{-1} \cdot v$ is g^{xy} , the correct 363 answer to the Gap Diffie-Hellman problem. In other words, B solves its Gap Diffie-Hellman problem if and only if A wins the simulated game 2: $Adv_2(A) = Adv_{GapDH}(B)$.

4 Practical instantiation with ML-KEM

ML-KEM is an IND-CCA secure key encapsulation mechanism standardized by NIST 367 in FIPS 203 [oST24]. The chosen-ciphertext security of ML-KEM is achieved in two 368 steps. First, ML-KEM constructs a PKE (called K-PKE, see Figure 16 in Appendix) 369 whose IND-CPA security reduces to the conjectured intractability of the decisional Module 370 Learning with Error (MLWE) problem [Reg05, LPR10, Reg09, LPR10, Pei09]. Then, the 371 $\mathtt{KEM}_{m}^{\underline{f}}$ variant (see Figure 1) of the Fujisaki-Okamoto transformation is applied to convert 372 the IND-CPA secure PKE into an IND-CCA secure KEM (See Figure 17 in Appendix). 373

ML-KEM⁺ 4.1

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We apply the encrypt-then-MAC transformation to the K-PKE routines of ML-KEM. The resulting KEM is called ML-KEM⁺ (See Figure 12). The key generation routines are identical between ML-KEM⁺ and ML-KEM. The ML-KEM⁺ encapsulation and decapsulation algorithms make use of two additional algorithms, namely a MAC and a KDF.

For performance and practical security, ML-KEM⁺ makes two minor modifications to the encrypt-then-MAC transformation:

• Hashing pk into MAC key and shared secret. The public key goes into deriving both the MAC key and the shared secret for the same two reasons why Kyber hashes its public key into the pseudorandom coin and the shared secret [BDK+18]. The first reason is to make the KEM contributory. In a realistic key exchange, the public key and the random PKE plaintext are generated by opposite parties. Hashing both the public key and the PKE plaintext ensures that both parties have inputs into the shared secret. The second reason is multi-user security: if the MAC key is derived from the PKE plaintext alone, then an adversary can pre-compute a large dictionary mapping MAC keys to the pre-image PKE plaintext. Upon receiving some ciphertext, an adversary can search through this key-plaintext dictionary and recover the decryption. Hashing the public key into the MAC key prevents an adversary from computing one large lookup table and use it against multiple keypairs.

ML-KEM ⁺ .KeyGen()	ML-KEM ⁺ .Decap(sk, c)
1: $z \stackrel{\$}{\leftarrow} \{0,1\}^{256}$ 2: $(pk, sk') \stackrel{\$}{\leftarrow} K\text{-PKE.KeyGen}()$ 3: $h \leftarrow H(pk)$ 4: $sk \leftarrow (sk'\ pk\ h\ z)$ 5: $\mathbf{return}\ (pk, sk)$	Require: Secret key $\mathtt{sk} = (\mathtt{sk'} \ \mathtt{pk} \ h \ z)$ Require: Ciphertext $c = (c' \ t)$ 1: $(\mathtt{sk'}, \mathtt{pk}, h, z) \leftarrow \mathtt{sk}$ 2: $(c', t) \leftarrow c$ 3: $\hat{m} \leftarrow \mathtt{K-PKE.Dec}(\mathtt{sk'}, c')$ 4: $(\overline{K}, \hat{k}) \leftarrow G(\hat{m} \ h)$ 5: $\hat{t} \leftarrow \mathtt{MAC}(\hat{k}, c')$
ML-KEM ⁺ .Encap(pk)	6: if $\hat{t} = t$ then
1: $m \stackrel{\$}{\leftarrow} \{0,1\}^{256}$ 2: $(\overline{K}, k) \leftarrow G(m H(\mathtt{pk}))$ 3: $c' \stackrel{\$}{\leftarrow} \mathtt{K-PKE.Enc}(\mathtt{pk}, m)$ 4: $t \leftarrow \mathtt{MAC}(k, c')$ 5: $K \leftarrow \mathtt{KDF}(\overline{K} t)$ 6: $c \leftarrow (c', t)$ 7: $\mathbf{return}(c, K)$	7: $K \leftarrow \texttt{KDF}(\overline{K} \ t)$ 8: else 9: $K \leftarrow \texttt{KDF}(z \ c)$ 10: end if 11: return K

Figure 12: ML-KEM⁺ applies our encrypt-then-MAC transformation to the K-PKE subroutines of ML-KEM. G is SHA3-512, H is SHA3-256, KDF is Shake256

• Hashing the MAC tag instead of the entire ciphertext. Because we allow the PKE to be randomized, the shared secret must have inputs from both the plaintext and the ciphertext. However, we find it unnecessary to hash the entire ciphertext. Instead, since the MAC tag functions as a keyed hash of the ciphertext, we can use the much shorter MAC tag as a substitute ciphertext hash, which yields meaningful performance improvements. On the other hand, not hashing the entire ciphertext will not give an adversary additional advantage because assuming that the adversary does not already know the underlying decryption, it cannot tamper with any part of the challenge ciphertext without making the ciphertext-tag pair invalid, so the decapsulation oracle will always return the implicit rejection.

Compared to ML-KEM, ML-KEM⁺ makes the following performance trade-offs:

- The ML-KEM⁺ encapsulation contains two additional steps: computing the MAC tag and hashing the MAC tag into the shared secret.
- An ML-KEM⁺ ciphertext size increases by the size of the MAC tag.
- The ML-KEM⁺ decapsulation replaces re-execution K-PKE.Enc with computing the MAC tag.

4.2 Choosing a message authentication code (MAC)

In Figure 12, while H, G and KDF are instantiated by SHA3-256, SHA3-512 and Shake256, respectively, there could be various choices of standardized and well-analyze MAC algorithms. For implementation, we instantiated the MAC with a selection that covered a wide range of MAC designs, including Poly1305 [Ber05], GMAC [MV04], CMAC [IK03][BR05], and KMAC [Gro13]. All MACs are parameterized with a 256-bit key to ensure that MAC keys are at least as hard to guess as the underlying PKE plaintext. On the other hand, all MACs use a 128-bit tag except for KMAC, which can have a variable tag length. We suspect that a 128-bit tag might be insufficient for the 192-bit and 256-bit security levels,

so for instantiation with the higher security levels, we only use KMAC with 192-bit and 256-bit tag length outputs.

Poly1305 and **GMAC** are both Carter-Wegman style MACs [WC81] that compute the tag using finite field arithmetic. Generically speaking, Carter-Wegman MAC operates by breaking the message into message blocks that can then be parsed into finite field elements. The tag is computed by evaluating a polynomial whose coefficients are the message blocks and whose indeterminate is the secret key (also called a *hash key*). Specifically, Poly1035 operates in the prime field \mathbb{F}_q where $q=2^{130}-5$ whereas GMAC operates in the binary field $\mathbb{F}_{2^{128}}$.

CMAC is based on the CBC-MAC with the block cipher instantiated from AES-256. To compute a CMAC tag, the message is first broke into 128-bit blocks with appropriate padding. Each block is first XORed with the previous block's output, then encrypted under AES using the symmetric key. The final output is XORed with a sub key derived from the symmetric key, before being encrypted for one last time.

KMAC is based on the SHA-3 family of sponge functions [oST15]. We chose KMAC-256, which uses Shake256 as the underlying extendable output functions. KMAC is the only MAC to support variable key and tag length, though we fixed the key length at 256 bits. On the other hand, we chose the appropriate tag length for the corresponding security level: when instantiated with ML-KEM-512, the tag length is 128 bits; with ML-KEM-768, the tag length is 192 bits; with ML-KEM-1024, the tag length is 256 bits.

5 Performance comparison

We measured the real-world performance of the individual routines of our ML-KEM⁺
construction, as well as the round trip time of performing key exchange over a network
in a variety of authentication modes. Our implementation extended from the Kyber
implementation [BDK⁺24]. All C code is compiled with GCC 11.4.1 and OpenSSL 3.0.8.
All binaries are executed on an AWS c7a.medium instance (AMD EPYC 9R14 CPU at 3.7
GHz and 1 GB of RAM) in the us-west-2 region.

5.1 MAC performance

To isolate the performance characteristics of each MAC, we measured the CPU cycles needed for each MAC to compute a tag on random inputs whose sizes correspond to the ciphertext sizes of ML-KEM. The median CPU cycle counts in 10,000 rounds of testing are summarized in Table 3.

Table 3: CPU cycles needed to compute tag on various input sizes

128-bit	security
Tag size	128 bits
CT size	768 bytes
MAC	Median
Poly1305	909
GMAC	3899
CMAC	6291
KMAC	6373

192-bit security		
Tag size	192 bits	
CT size	1088 bytes	
MAC	Median	
KMAC	9697	

256-bit security		
Tag size	256 bits	
CT size	1568 bytes	
MAC	Median	
KMAC	11647	

5.2 Individual routine performance

Compared to the $\mathtt{KEM}_m^{\not\perp}$ variant of Fujisaki-Okamoto transformed used in ML-KEM, the encrypt-then-MAC transformation achieves massive CPU cycle count reduction in decapsulation while incurring only a minimal increase of encapsulation cycle count and

ciphertext size. Since K-PKE.Enc carries significantly more computational complexity than K-PKE.Dec or any MAC we chose, the performance advantage of the encrypt-then-MAC transformation over the $\text{KEM}_m^{\not\perp}$ transformation is dominated by the runtime saving gained from replacing re-encryption with MAC. A comparison between ML-KEM and variations of the ML-KEM⁺ can be found in Table 4.

Remark. We also included the performance of Kyber as it is submitted to the third round of the NIST PQC standardization project, which differs from ML-KEM by deriving the shared secret from hashing the public key, the PKE plaintext, and the PKE ciphertext, while ML-KEM derives its shared secret by hashing only the public key and the PKE plaintext. This results in ML-KEM having meaningful performance improvement in encapsulation over Kyber, though the performance difference in decapsulation is negligible.

Table 4: CPU cycles of each KEM routine

128-bit security	KEM variant	Encap cycles/tick		Decap cycles/tick	
size parameters (byte) KEW Variant	Median	Average	Median	Average
pk size 800	ML-KEM-512	91467	92065	121185	121650
sk size 1632	Kyber512	97811	98090	119937	120299
ct size 768	ML-KEM ⁺ -512 w/ Poly1305	93157	93626	33733	33908
KeyGen cycles/tick	ML-KEM ⁺ -512 w/ GMAC	97369	97766	37725	37831
Median 75945	ML-KEM ⁺ -512 w/ CMAC	99739	99959	40117	39943
Average 76171	ML-KEM ⁺ -512 w/ KMAC	101009	101313	40741	40916

192-bit	security	I.
size param	eters (bytes)	1
pk size	1184	N.
sk size	2400	
ct size	1088	ML-KE
KeyGen (cycles/tick	
Median	129895	
Average	130650	

KEM variant	Encap cycles/tick		Decap cycles/tick	
KEWI Variant	Median	Average	Median	Average
ML-KEM-768	136405	147400	186445	187529
Kyber768	153061	153670	182129	182755
ML-KEM ⁺ -768 w/ KMAC	155219	155848	52415	52611

256-bit security				
size param	eters (bytes)			
pk size	1568			
sk size	3168			
ct size	1568			
KeyGen (cycles/tick			
Median	194921			
Average	195465			

KEM variant	Encap cycles/tick		Decap cycles/tick	
KEW Variant	Median	Average	Median	Average
ML-KEM-1024	199185	199903	246245	247320
Kyber1024	222351	223260	258231	259067
ML-KEM ⁺ -1024 w/ KMAC	216761	217468	62269	62516

5.3 Key exchange protocols

The CRYSTALS-Kyber team [BDK⁺18] proposed three key exchange protocols: Kyber.KE for a passively secure ephemeral key exchange, Kyber.UAKE for unilaterally authenticated key exchange, and Kyber.AKE for mutually authenticated key exchange. First formally proposed in [BCK98] and later analyzed in [CK01], these key exchange protocols all achieve weak forward secrecy, and the authenticated key exchange protocols are resistant to certain categories of active adversaries (e.g. Man-in-the-Middle attacks).

We implemented each of the three Kyber key exchange protocols using ML-KEM⁺ and ML-KEM, then measured the round trip time of each handshake. We denote the party who sends the first message to be the client and the other party to be the server. Round trip time (RTT) is defined to be the time interval between the moment before the client starts generating ephemeral keypairs and the moment after the client derives the final session key. All experiments are run on a pair of AWS c7a.medium instances both located in the us-west-2 region. For each experiment, a total of 10,000 rounds of key exchange are performed, with the median and average round trip time (measured in microsecond μs) recorded.

5.3.1 Unauthenticated key exchange

In unauthenticated key exchange (KE), a single pair of ephemeral keypair $(pk_e, sk_e) \stackrel{\$}{\leftarrow}$ KeyGen() is generated by the client. The client transmits the ephemeral public key pk_e to the server, who runs the encapsulation routine $(c_e, K_e) \stackrel{\$}{\leftarrow} \text{Encap}(pk_e)$ and transmits the ciphertext c_e back to the client. The client finally decapsulates the ciphertext to recover the shared secret $K_e \leftarrow \text{Decap}(sk_e, c_e)$. The key exchange routines are summarized in Figure 13. The key derivation function is instantiated using Shake256, and the final session key is 256 bits in length. ML-KEM⁺-512 with Poly1305 reduces RTT by 23.9% compared to ML-KEM-512; ML-KEM⁺-768 with KMAC reduces RTT by 23.7%; ML-KEM⁺-1024 with KMAC reduces RTT by 25.4% (See Table 5).

KE _C ()	
1: $(pk_e, sk_e) \stackrel{\$}{\leftarrow} KeyGen()$	KE _S ()
1. (pk_e, sk_e) (pk_e) 2: $send(pk_e)$	$1: \ \mathtt{pk}_e \leftarrow \mathtt{read}()$
$c_e \leftarrow \texttt{read}()$	$2:\ (c_e,K_e) \overset{\$}{\leftarrow} \mathtt{Encap}(\mathtt{pk}_e)$
$4: \ K_e \leftarrow \texttt{Decap}(\mathtt{sk}_e, c_e)$	3 : $\mathtt{send}(c_e)$
5: $K \leftarrow \texttt{KDF}(K_e)$	$4: \ K \leftarrow \texttt{KDF}(K_e)$
6: return K	5: return K

Figure 13: Unauthenticated key exchange (KE) routines

Table 9. ILL 1011 comparison				
KEM variant	Client TX bytes	Server TX bytes	RTT (μs)	
KEW Variant	Chefit 1 A bytes	Derver 1 A bytes	Median	Average
ML-KEM-512	800	768	92	97
ML-KEM ⁺ -512 w/ Poly1305	800	784	70	72
ML-KEM ⁺ -512 w/ GMAC	800	784	73	76
ML-KEM ⁺ -512 w/ CMAC	800	784	75	79
ML-KEM ⁺ -512 w/ KMAC	800	784	76	78

Table 5: KE RTT comparison

KEM variant	Client TX bytes	Server TX bytes	RTT (μs)	
TEM variant	Chefit 17 bytes	Derver 17 bytes	Median	Average
ML-KEM-768	1184	1088	135	140
ML-KEM ⁺ -768 w/ KMAC	1184	1120	103	107

KEM variant	Client TX bytes	Server TX bytes	RTT (μs)	
KEW Variant	Chefit 1A bytes	Derver 1 A bytes	Median	Average
ML-KEM-1024	1568	1568	193	199
ML-KEM ⁺ -1024 w/ KMAC	1568	1600	144	149

5.3.2 Unilaterally authenticated key exchange

In unilaterally authenticated key exchange (UAKE), the authenticating party proves its identity to the other party by demonstrating possession of a secret key that corresponds to a published long-term public key. In this implementation, the client possesses the long-term public key pk_S of the server, and the server authenticates itself by correctly decrypting the challenge encryption sent by the client using its long-term secret key sk_S (see Figure 14). ML-KEM⁺-512 with Poly1305 reduces RTT by 29.0% compared to ML-KEM-512; ML-KEM⁺-768 with KMAC reduces RTT by 28.4%; ML-KEM⁺-1024 with KMAC reduces RTT by 31.3% (See Table 6).

$\overline{\mathtt{UAKE}_\mathtt{C}(\mathtt{pk}_S)}$	
Require: Server's long-term pk_S	$\overline{\mathtt{UAKE_S}(\mathtt{sk}_S)}$
$1: \; (\mathtt{pk}_e, \mathtt{sk}_e) \xleftarrow{\$} \mathtt{KeyGen}()$	Require: Server's long-term sk_S
$2: (c_S, K_S) \stackrel{\$}{\leftarrow} \mathtt{Encap}(\mathtt{pk}_S)$	1: $(\mathtt{pk}_e, c_S) \leftarrow \mathtt{read}()$
3 : $\mathtt{send}(\mathtt{pk}_e, c_S)$	2: $K_S \leftarrow exttt{Decap}(exttt{sk}_S, c_S)$
$c_e \leftarrow \mathtt{read}()$	$3: \ (c_e, K_e) \overset{\$}{\leftarrow} \mathtt{Encap}(\mathtt{pk}_e)$
5: $K_e \leftarrow \mathtt{Decap}(\mathtt{sk}_e, c_e)$	4 : $\mathtt{send}(c_e)$
6: $K \leftarrow \texttt{KDF}(K_e K_S)$	5: $K \leftarrow \mathtt{KDF}(K_e \ K_S)$
7: $\mathbf{return}\ K$	6: $\mathbf{return}\ K$

Figure 14: Unilaterally authenticated key exchange (UAKE) routines

Tabl	\mathbf{e}	6:	UA	AKE	RTT	comparison

KEM variant	Client TX bytes	Server TX bytes	RTT (μs)	
KEW Variant	Chefit 1A bytes	Server 1 A bytes	Median	Average
ML-KEM-512	1568	768	145	151
ML-KEM ⁺ -512 w/ Poly1305	1584	784	103	106
ML-KEM ⁺ -512 w/ GMAC	1584	784	106	110
ML-KEM ⁺ -512 w/ CMAC	1584	784	108	112
ML-KEM ⁺ -512 w/ KMAC	1584	784	109	113

KEM variant	Client TX bytes	Server TX bytes	RTT (μs)	
TEM variant	Chefit 17 bytes	Derver 17 bytes	Median	Average
ML-KEM-768	2272	1088	215	222
ML-KEM ⁺ -768 w/ KMAC	2288	1104	154	159

KEM variant	Client TX bytes	Server TX bytes	RTT (μs)	
KEW Variant		Server 1 A bytes	Median	Average
ML-KEM-1024	3136	1568	310	318
$\mathrm{ML ext{-}KEM^{+} ext{-}1024}$ w/ KMAC	3152	1584	213	220

5.3.3 Mutually authenticated key exchange (AKE)

Mutually authenticated key exchange is largely identical to unilaterally authenticated key exchange, except for that client authentication is required. This means that client possesses server's long-term public key pk_S and its own long-term secret key sk_C , while the server possesses client's long-term public key pk_C and its own long-term secret key sk_S . The session key is derived by applying KDF onto the concatenation of shared secrets produced under the ephemeral keypair, server's long-term keypair, and client's long-term keypair, in this order. The key exchange routines are described in Figure 15. ML-KEM⁺-512 with Poly1305 reduces RTT by 39.5% compared to ML-KEM-512; ML-KEM⁺-768 with KMAC reduces RTT by 30.6%; ML-KEM⁺-1024 with KMAC reduces RTT by 44.9% (See Table 7).

6 Conclusions and future works

In this paper, we propose a generic KEM transformation based on the encrypt-then-MAC pattern. We reduce the IND-CCA security of the encrypt-then-MAC KEM to the OW-PCA security of the input PKE and the one-time existential unforgeability of the input MAC in the random oracle model. Compared to the Fujisaki-Okamoto transformation, the encrypt-then-MAC transformation replaces the computationally expensive re-encryption with computing a MAC tag. At the cost of minimal increase in encapsulation cost and ciphertext size, the encrypt-then-MAC transformation substantially improves the efficiency

$\overline{\mathtt{AKE}_\mathtt{C}(\mathtt{pk}_S,\mathtt{sk}_C)}$	
Require: Server's long-term pk _S	$\mathtt{AKE_S}(\mathtt{sk}_S,\mathtt{pk}_C)$
Require: Client's long-term sk_C	Require: Server's long-term sk_S
1: $(pk_e, sk_e) \stackrel{\$}{\leftarrow} KeyGen()$	Require: Client's long-term pk_C
$2: \ (c_S,K_S) \xleftarrow{\$} \mathtt{Encap}(\mathtt{pk}_S)$	1: $(\mathtt{pk}_e, c_S) \leftarrow \mathtt{read}()$ 2: $K_S \leftarrow \mathtt{Decap}(\mathtt{sk}_S, c_S)$
3: $\operatorname{send}(\operatorname{pk}_e, c_S)$ 4: $(c_e, c_C) \leftarrow \operatorname{read}()$	$3: \ (c_e, K_e) \overset{\$}{\leftarrow} \mathtt{Encap}(\mathtt{pk}_e)$
5: $K_e \leftarrow \mathtt{Decap}(\mathtt{sk}_e, c_e)$	$4 \colon \left(c_C, K_C\right) \overset{\$}{\leftarrow} \texttt{Encap}(\texttt{pk}_C)$
6: $K_C \leftarrow \mathtt{Decap}(\mathtt{sk}_e, c_C)$	5 : $\mathtt{send}(c_e, c_C)$
7: $K \leftarrow \texttt{KDF}(K_e K_S K_C)$	6: $K \leftarrow \texttt{KDF}(K_e K_S K_C)$
8: return K	7: $\mathbf{return}\ K$

Figure 15: Mutually authenticated key exchange (AKE) routines

Table 7: AKE RTT comparison

		*		
KEM variant	Client TX bytes	Server TX bytes	RTT (μs)	
KEW Variant		Derver 1 A bytes	Median	Average
ML-KEM-512	1568	1536	220	213
ML-KEM ⁺ -512 w/ Poly1305	1584	1568	133	138
ML-KEM ⁺ -512 w/ GMAC	1584	1568	139	143
ML-KEM ⁺ -512 w/ CMAC	1584	1568	143	148
ML-KEM ⁺ -512 w/ KMAC	1584	1568	145	151

KEM variant	Client TX bytes	Server TX bytes	RTT (μs)	
KEW Variant	Chefit 1 A bytes	Server 1 A bytes	Median	Average
ML-KEM-768	2272	2176	294	301
ML-KEM ⁺ -768 w/ KMAC	2288	2208	204	210

KEM variant	Client TX bytes	Sorver TV bytes	RTT (μs)	
TYDIVI Variant	Offent 1A bytes	Derver 17 bytes	Median	Average
ML-KEM-1024	3136	3136	512	511
ML-KEM ⁺ -1024 w/ KMAC	3152	3168	282	288

of the decapsulation routine. Where the input PKE's encryption is slower than decryption, the encrypt-then-MAC transformation achieves meaningful time savings in practical key exchange protocols. We also propose ML-KEM⁺ as an efficient alternative to ML-KEM. Compared to ML-KEM, ML-KEM⁺ achieves great performance improvements in a variety of practical scenarios.

In our future work, we would like to look at applying encryption-then-MAC transformation to other post-quantum cryptosystems for improving their practical implementation efficiency.

Code-based cryptosystems. Because the general problem of decoding a linear code is proven NP hard [BMvT78], code-based cryptosystems does not suffer from the inherent search-decision equivalence of lattice problems and thus be viable candidates with OW-PCA security. Unfortunately, among the code-based submissions to NIST PQC, HQC [MAB⁺18] and BIKE [ABB⁺22] are known to be vulnerable to key-recovery plaintext-checking attacks (KR-PCA) [TUX⁺23]. On the other hand, classic McEliece [ABC⁺20] seems to be PCA secure and thus a viable candidate, although in classic McEliece, the decoding routine is more expensive than the encryption routine, so applying encrypt-then-MAC may yield less impressive performance gains.

lsogeny-based cryptosystems. The intractability assumptions of isogeny-based cryptography resemble the classical Diffie-Hellman assumptions, and it seems possible to formulate a "Gap Diffie-Hellman assumption" in supersingular isogeny [FTTY18]. In fact, SIKE [ACC+17] also uses the Fujisaki-Okamoto transformation, and given the heavy computational cost of isogeny computation, replacing re-encryption with MAC will result in substantial performance improvements. While SIKE and SIDH were found to be insecure [CD23], other isogeny-based cryptosystem such as CSIDH [CLM+18] remains unaffected by the aforementioned attack and might be suitable candidates.

7 Appendix

Below are high-level summaries of the PKE sub-routines and the KEM routines of ML-KEM:

K-PKE.KeyGen()	K-PKE.Enc(pk, m)	K-PKE.Dec(sk, c)
1: $A \stackrel{\$}{\leftarrow} R_q^{k \times k}$ 2: $\mathbf{s} \stackrel{\$}{\leftarrow} \mathcal{X}_{\eta_1}^k$ 3: $\mathbf{e} \stackrel{\$}{\leftarrow} \mathcal{X}_{\eta_1}^k$ 4: $\mathbf{t} \leftarrow A\mathbf{s} + \mathbf{e}$ 5: $pk \leftarrow (A, \mathbf{t})$ 6: $sk \leftarrow \mathbf{s}$ 7: $\mathbf{return} \ (pk, sk)$	Ensure: $\mathtt{pk} = (A, \mathbf{t})$ Ensure: $m \in R_2$ 1: $\mathbf{r} \overset{\$}{\leftarrow} \mathcal{X}^k_{\eta_1}$ 2: $\mathbf{e}_1 \overset{\$}{\leftarrow} \mathcal{X}^k_{\eta_2}$ 3: $e_2 \overset{\$}{\leftarrow} \mathcal{X}_{\eta_2}$ 4: $\mathbf{c}_1 \leftarrow A\mathbf{r} + \mathbf{e}_1$ 5: $c_2 \leftarrow \mathbf{t}^{T}\mathbf{r} + e_2 + m \cdot \lfloor \frac{q}{2} \rfloor$ 6: $\mathbf{return} \ (\mathbf{c}_1, c_2)$	Ensure: $c = (\mathbf{c}_1, c_2)$ Ensure: $\mathbf{sk} = \mathbf{s}$ 1: $\hat{m} \leftarrow c_2 - \mathbf{c}_1^{T} \cdot \mathbf{s}$ 2: $\hat{m} \leftarrow Round(\hat{m})$ 3: $\mathbf{return} \ \hat{m}$

Figure 16: K-PKE is an IND-CPA secure PKE based on the conjectured intractability of the Module Learning with Error (MWLE) problem

ML-KEM.KeyGen()	${\texttt{ML-KEM.Decap}(\texttt{sk},c)}$
1: $(pk, sk') \stackrel{\$}{\leftarrow} K-PKE.KeyGen()$ 2: $h \leftarrow SHA3-256(pk)$ 3: $z \stackrel{\$}{\leftarrow} \mathcal{M}$ 4: $sk \leftarrow (sk', pk, h, z)$ 5: $\mathbf{return} \ (pk, sk)$	1: $\hat{m} \leftarrow \text{K-PKE.Dec}(\text{sk}', c)$ 2: $(\overline{K}, \hat{r}) \leftarrow \text{SHA3-512}(\hat{m}, h)$ 3: $\hat{c} \leftarrow \text{K-PKE.Enc}(\text{pk}, m, \hat{r})$ 4: if $\hat{c} = c$ then 5: $K \leftarrow \overline{K}$ 6: else 7: $K \leftarrow \text{SHA3-256}(c, z)$
ML-KEM.Encap(pk)	8: end if 9: return <i>K</i>
1: $m \stackrel{\$}{\leftarrow} \mathcal{M}$ 2: $h \leftarrow \text{SHA3-256(pk)}$ 3: $(K,r) \leftarrow \text{SHA3-512}(m,h)$ 4: $c \leftarrow \text{K-PKE.Enc(pk}, m, r)$ 5: $\mathbf{return}\ (c,K)$	

Figure 17: ML-KEM uses the $\mathtt{KEM}_m^{\not\perp}$ variant of the modular Fujisaki-Okamoto KEM transformation

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