Question 6

We will prove equivalence by showing that each statement implies the other.

If (1) is true for all $\mathbf{c}_1 \neq \mathbf{c}_2$, then we can fix $\mathbf{c}_2 = \mathbf{0}$, and statement (1) becomes

$$\|\mathbf{c}_1 - \mathbf{0}\|_H = \|\mathbf{c}_1\|_H \geq d$$

for all non-zero $\mathbf{c}_1$, thus statement (2) is true.

Now suppose (2) is true. For all $\mathbf{c}_2 \neq \mathbf{c}_1$, $\mathbf{c}_2 - \mathbf{c}_1$ is a non-zero code in C because C is linear. Therefore:

$$\|\mathbf{c}_2 - \mathbf{c}_1\|_H \geq d$$

thus statement (1) is true.