A survey of IND-CCA constructions

Ganyu (Bruce) Xu (g66xu)

CO 789, Winter 2024

1 Introduction

Indistinguishability under (adaptive) chosen-ciphertext attack (IND-CCA2) is widely recognized as the desirable security notion for public-key cryptography. However, directly achieving IND-CCA2 security is difficult. Previous attempts at deploying public-key cryptography in production, such as the usage of RSA PKCS1 v1.5 in early versions of SSl/TLS, were found to be vulnerable to adaptive chosen-ciphertext attacks.

Instead of directly constructing IND-CCA2 secure cryptosystem from NP-hard problems, recent works approached this problem by proposing generic transformation that take cryptographic primitives of lesser strengths (e.g. OW-CPA, IND-CPA) and produce encryption schemes that lose only a negligible amount of security. One such transformation was proposed by Fujisaki and Okamoto in 1999 and later improved by Hofheinz, Hovelmann, and Kiltz in 2017. With simple construction and robust security reduction, the FO transformation is adopted by submissions to NIST's post-quantum cryptography competition (notably by Kyber, which has been standardized in FIPS 203 in 2024).

In this paper, we will review the constructions of the Fujisaki-Okamoto transformation and its variations. We will also review their security results, including the techniques used in the security reduction. Finally, we will discuss open problems and propose some optimization.

2 Preliminaries

2.1 Spread and correctness

The spread of measures the randomness of ciphertext in a probabilistic encryption scheme, where the randomness of the encryption routine is determined by a random coin $r \xleftarrow{\$}$ Coin from the coin space.

Definition 2.1 (γ-spread)**.** *The spread γ of an asymmetric encryption scheme* (KeyGen, E, D) *for some fixed public key pk and plaintext m is defined by*

$$\gamma(pk, m) = -\log \|E_{pk}(m)\|_\infty$$

Where $\|E_{pk}(m)\|_\infty$ is the min-entropy of the ciphertext with respect to the coin:

$$\|E_{pk}(m)\|_\infty = \max_{c \in \mathcal{C}} P[E_{pk}(m) = c]$$

Definition 2.2 (correctness)**.** *A public-key encrpytion scheme* (KeyGen, E, D) *is δ-correct if across all possible keypairs and messages, the probability of decryption error is at most δ:*

$$\max_{m \in \mathcal{M}, (pk,sk) \xleftarrow{\$} \text{KeyGen}()} P[D_{sk}(E_{pk}(m)) \neq m] \leq \delta$$

2.2 Difference lemma

The difference lemma is used extensively in the sequence of games to estimate the loss of security when one game is transformed into another

Lemma 1 (Difference lemma)**.** *Let A, B, F be events defined on the same probability space. If $P[A \cap \neg F] = P[B \cap \neg F]$, then $P[A] - P[B] \leq P[F]$*

Proof.

$$\begin{aligned} P[A] - P[B] &= (P[A \cap F] + P[A \cap \neg F]) - (P[B \cap F] + P[B \cap \neg F]) \\ &= (P[A \cap F] - P[B \cap F]) + (P[A \cap \neg F] - P[B \cap \neg F]) \\ &= P[A \cap F] - P[B \cap F] \\ &\leq P[A \cap F] \\ &\leq P[F] \end{aligned}$$

□

In the context of security reduction using sequence of game, the difference lemma is invoked with event A and B set to "adversary winning one game" and "adversary winning another game", and event F set to the event that would cause the two games two diverge. If the diverging event F does not happen, then the two games are identical to each other, so the advantage of the adversary remains the same across the two games; if the diverging event F happens, then the advantage of the adversary does not change by more than the probability of the diverging event F.

2.3 IND-CPA to OW-CPA

It is trivially true that if a public-key encryption scheme is not one-way secure, then it is not indistinguishable. The converse if also almost true, with the caveat that the message space must be sufficiently large:

Lemma 2. *Let* (KeyGen, E, D) *be a public-key encryption scheme with a finite message space $\mathcal{M}$. For every IND-CPA adversary A, there exists an OW-CPA adversary B such that*

$$\epsilon_{\mathit{OW\text{-}CPA}} = \epsilon_{\mathit{IND\text{-}CPA}} + \frac{1}{|\mathcal{M}|}$$

3 Related works

Key encapsulation mechanisms (KEM) usually have weaker security requirements than public-key encryption schemes (PKE) thanks to the fact that properly designed KEMs obfuscate the plaintext through a key-derivation function (which is usually a hash function and thus a random oracle under the RO assumption) in the decryption routine. As a result, we can implement a simple IND-CCA2 KEM using an IND-CCA2 PKE:

Algorithm 1: KeyGen

$(\text{pk}, \text{sk}) \xleftarrow{\$} \text{KeyGen}()$;
return (pk, sk)

Algorithm 2: Encap

Input: pk
$m \xleftarrow{\$} \mathcal{M}$;
$c \xleftarrow{\$} E(\text{pk}, m)$;
$K \leftarrow H(m, c)$;
return (c, K);

Algorithm 3: Decap

Input: $\text{sk}, c \in \mathcal{C}$
$\hat{m} \leftarrow D(\text{sk}, c)$;
$\hat{K} \leftarrow H(\hat{m}, c)$;
return $\hat{K}$;

It is straightforward to simulate the KEM game using a PKE adversary. The PKE adversary samples two random messages $(m_0, m_1) \xleftarrow{\$} \mathcal{M}$ and obtains the challenge encryption for one of them $c^* \leftarrow E(\text{pk}, m_b)$. The PKE adversary then computes $K^* \leftarrow H(c^*, m_0)$ and passes (c^*, K^*) as the challenge encapsulation to the KEM adversary. If c^* is an encryption of m_0, then K^* is pseudorandom, otherwise under the random oralce model, K^* is truly random from the perspective of the KEM adversary. Therefore, $\epsilon_{\text{PKE}} = \epsilon_{\text{KEM}}$.

Using IND-CCA2 PKE as an IND-CCA2 KEM has been in production for a long time. For example, TLS 1.2 [2] offers this implementation for key exchange using RSA-OAEP[1] as the underlying IND-CCA2 PKE. The key exchange protocol proceeds as follows:

1. Server sends its public key through the X.509 certificate
2. Client samples a master secret, encrypts the secret using server's public key, then sends the ciphertext to the server
3. Server decrypts the ciphertext to obtain the master secret

This approach to key exchange has since been deprecated, and RSA is no longer supported for key exchange starting with TLS 1.3. In addition to the difficulty of implementing RSA correctly and securely (RSA is notoriously hard to implement without side-channel vulnerabilities), using the server's long-term keypair for establishing common secrets fails to provide forward secrecy, meaning that if the long-term key is compromised, all prior communications can be trivially decrypted.

While the modern approach is to use ephemeral Diffie-Hellman (with either prime field or elliptic curve), it is difficult to adapt this approach to the post-quantum setting. Instead, post-quantum key exchange usually builds an IND-CCA2 KEM, then perform the key exchange using ephemeral keypairs. At each session's handshake, the client generates the a new keypair and sends the public key to the server. The server uses the public key to run the encapsulation routine and sends back the ciphertext. Finally, the client runs the decapsulation routine on the ciphertext to recover the shared secret.

4 Fujisaki-Okamoto transformation and variations

In 1999, Fujisaki and Okamoto [3] proposed a generic transformation that outputs an IND-CCA2-secure public-key encryption scheme using an IND-CPA symmetric cipher, an OW-CPA asymmetric cipher, and two hash functions. Later on in 2017, Hofheinz, Hovelmann, and Kiltz [4] improved on the security of the Fujisaki-Transformation and adapted the original approach to constructing IND-CCA2 KEMs. In this section, we will give a quick overview of the individual transformations and their security results. We will then discuss the techniques used in the security reduction. Finally, we will provide a sketch of the actual proof.

4.1 An overview of transformed routines and security results

The original 1999 Fujisaki-Okamoto transformation takes as input a symmetric cipher $(E^{\text{sym}}, D^{\text{sym}})$, an asymmetric cipher $(\text{KeyGen}, E^{\text{asym}}, D^{\text{asym}})$, a key derivation function (aka a hash function) $G : \mathcal{M}^{\text{asym}} \to \mathcal{K}^{\text{sym}}$, and a hash functino $H : \{0,1\}^* \to \text{Coin}^{\text{asym}}$.

Specifically, the asymmetric cipher is assumed to be probabilistic, but with pseudorandomness seeded on some coin $r \in \text{Coin}^{\text{asym}}$. When E^{asym} is called without specifying a value for coin, it is assumed that a coin is uniformly sampled from the coin space and passed in as the pseudorandom seed:

$$c \xleftarrow{\$} E^{\text{asym}}(\text{pk}, m) \Leftrightarrow c \leftarrow E^{\text{asym}}(\text{pk}, m, r \xleftarrow{\$} \text{Coin})$$

The output of the transformation is a public-key encryption scheme $(\text{KeyGen}, E^{\text{hy}}, D^{\text{hy}})$. Note that the key generation routine is exactly the same as the input asymmetric cipher's routine, so no distinction of notation is needed. The encryption and decryption routines are as follows:

Algorithm 4: Hybrid encryption E^{hy}

Input: $\text{pk}, m \in \mathcal{M}^{\text{sym}}$

$\sigma \xleftarrow{\$} \mathcal{M}^{\text{asym}}$;
$a \leftarrow G(\sigma)$; $c \leftarrow E_a^{\text{sym}}(m)$;
$r \leftarrow H(\sigma, c)$; $e \leftarrow E^{\text{asym}}(\text{pk}, m, r)$;
return (c, e);

Algorithm 5: Hybrid decryption D^{hy}

Input: $\text{sk}, c \in \mathcal{C}^{\text{sym}}, e \in \mathcal{C}^{\text{asym}}$

$\hat{\sigma} \leftarrow D^{\text{asym}}(\text{sk}, e)$;
$\hat{r} \leftarrow H(\hat{\sigma}, c)$;
if $E^{asym}(pk, \hat{\sigma}, \hat{r}) \neq e$ **then**
  return $\perp$; `// the "re-encryption"`
end
$\hat{a} \leftarrow G(\hat{\sigma})$;
$\hat{m} \leftarrow D_{\hat{a}}^{\text{sym}}(c)$;
return $\hat{m}$;

Theorem 4.1 (Security result for $(E^{\text{hy}}, D^{\text{hy}})$)**.** *If the input asymmetric cipher has γ spread, then for every IND-CPA adversary $\mathcal{A}^{sym}_{IND\text{-}CPA}$ against the underlying symmetric cipher with advantage ϵ^{sym} and OW-CPA adversary $\mathcal{A}^{asym}_{OW\text{-}CPA}$ against the input asymmetric cipher with advantage ϵ^{asym}, there exists an IND-CCA2 adversary with against the hybrid scheme making q_H hash queries and q_D decryption queries such that:*

$$\epsilon^{hy} \leq q_D 2^{-\gamma} + \epsilon^{sym} + q_H \epsilon^{asym}$$

Hofheinz, Hovenmann, and Kiltz pointed out in their 2017 paper a number of drawbacks of the original FO transformation:

1. Decryption error, which is especially possible with lattice-based schemes, is not accounted for
2. the $q_H \epsilon^{\text{asym}}$ term causes security to be non-tight, which have implications when choosing the security parameters

In their proposals, IND-CCA2 security is achieved through a two-step transformation. First, an OW-CPA asymmetric scheme is transformed into an OW-PCVA asymmetric scheme. The OW-PCVA scheme is then transformed into an IND-CCA2 KEM. Here PCVA refers to the adversary's ability to access some plaintext-checking oracle and some ciphertext validation oracle. Although PCVA is a non-standard security definition, it is entirely contained within the two-step transformation and serves only as an intermediary step.

Neither the OW-PCVA nor the IND-CCA transformation makes use of any symmetric cipher, so we ditch the distinction. The OW-PCVA transformation takes as input an asymmetric cipher (E, D) and a hash function $G : \mathcal{M} \to \text{Coin}$. The transformed encryption and decryption routines (E^T, D^T) are as follows:

Algorithm 6: OW-PCVA E^T

Input: $\text{pk}, m \in \mathcal{M}$
$r \leftarrow G(m)$;
$c \leftarrow E(\text{pk}, m, r)$;
return c;

Algorithm 7: OW-PCVA D^T

Input: $\text{sk}, c \in \mathcal{C}$
$\hat{m} \leftarrow D(\text{sk}, c)$;
$\hat{r} \leftarrow G(\hat{m})$;
if $E(pk, \hat{m}, \hat{r}) \neq c$ **then**
  return $\perp$;
end
return $\hat{m}$;

Depending on whether the implementation chooses implicit or explicit rejection of invalid ciphertext for the final KEM, the second transformation has slightly different key generation and decryption routine. The explicit rejection KEM is denoted by $U^{\perp}$, and its key generation is identical to the input asymmetric cipher.

Algorithm 8: $U^{\perp}$ Encap

Input: pk
$m \xleftarrow{\$} \mathcal{M}$;
$c \leftarrow E^T(\text{pk}, m)$;
$K \leftarrow H(m, c)$ `// H is a hash function`;
return (c, K)

Algorithm 9: $U^{\perp}$ Decap

Input: $c \in \mathcal{C}, \text{sk}$
$\hat{m} \leftarrow D^T(\text{sk}, c)$;
if $m = \perp$ **then**
  return $\perp$;
end
return $H(\hat{m}, c)$

On the other hand, the implicit rejection KEM, denoted by $U^{\not\perp}$, generate a random message $m' \xleftarrow{\$} \mathcal{M}$ as the fake input to H when the ciphertext is invalid

Algorithm 10: KeyGen

$\text{pk}, \text{sk} \xleftarrow{\$} \text{KeyGen}()$;
$m' \xleftarrow{\$} \mathcal{M}$;
return $(pk, (sk, m'))$;

Algorithm 11: Encap

$m \xleftarrow{\$} \mathcal{M}$;
$c \leftarrow E^T(\text{pk}, m)$;
$K \leftarrow H(m, c)$;
return (c, K)

Algorithm 12: Decap

$\hat{m} \leftarrow D^T(\text{sk}, c)$;
if $m = \perp$ **then**
  return $H(m', c)$;
end
return $H(\hat{m}, c)$

The security result is similarly expressed in two theorems:

Theorem 4.2. *If the input asymmetric scheme is δ-correct and γ-spread, then for every OW-PCVA adversary that issues at most q_G hash queries and q_V ciphertext validation queries with advantage ϵ^T, there exists an IND-CPA adversary against the underlying asymmetric scheme with advantage ϵ such that*

$$\epsilon^T \leq q_G \cdot \delta + q_V \cdot 2^{-\gamma} + \frac{2q_G + 1}{|\mathcal{M}|} + 3\epsilon$$

Theorem 4.3. *For every IND-CCA2 adversary against the $U^{\perp}$ KEM with advantage $\epsilon^{U^{\perp}}$, there exists an OW-PCVA adversary against the input scheme (E^T, D^T) with advantage ϵ^T such that*

$$\epsilon^{U^{\perp}} \leq \epsilon^T$$

Theorem 4.4. *For every IND-CCA2 adversary against the $U^{\not\perp}$ KEM that issues q_H hash queries with advantage $\epsilon^{U^{\not\perp}}$, there exists an OW-PCVA adversary against the input scheme (E^T, D^T) with advantage ϵ^T such that*

$$\epsilon^{U^{\not\perp}} \leq \epsilon^T + \frac{q_H}{|\mathcal{M}|}$$

4.2 A discussion of proof techniques

The 1999 Fujisaki-Okamoto transformation and its 2017 variations deployed similar strategies for their respective security reductions: start with the appropriate standard IND-CCA game, then gradually transform the game into something that can be simulated by some adversary playing another standard OW-CPA or IND-CPA game (or in the case of the IND-CCA KEM, some adversary playing the OW-PCVA game).

In the standard games, the decryption oracle, the decapsulation oracle, the plaintext checking oracle, and the ciphertext validation oracles all behave as if they possess the true secret key and use the secret key to execute the decryption routine on their query inputs. However, thanks to the design of the transformed schemes, there are ways to simulate the behaviors of those oracles under the random oracle assumption by modeling the hash functions as probabilistic turing machines whose state is taped. The simulated oracles will behave differently from the true oracles in certain circumstances, so the transformed game will diverge from the original game, but because of the difference lemma, we can bound the loss of security by the probability of the diverging event.

4.2.1 Simulate decryption oracle with random oracle

The decryption oracle can be simulated without secret key by taking advantage of the fact that in an honest execution of the encryption routine, a hash of the message is used to generate the pseudorandom coin. Assuming that the random oracle records its queries, if the ciphertext is honestly generated, then the random oracle should contain exactly one record whose input contains the correct message.

Here is an example of comparing the standard decryption oracle with the simulated decryption oracle in the 1999 Fujisaki-Okamoto transformation, where the hash function $H : \{0,1\}^* \to \text{Coin}$ is modeled as a probabilistic turing machine and used to recover the plaintext

Algorithm 13: Standard decryption oracle

Input: $\tilde{c} \in \mathcal{C}^{\text{sym}}, \tilde{e} \in \mathcal{C}^{\text{asym}}$

`// Check algorithm 5 for` D^{hy}

$\hat{m} \leftarrow D^{\text{hy}}(\text{sk}, (\tilde{c}, \tilde{e}))$;

return $\hat{m}$

Algorithm 14: Simulated decryption oracle

Input: $\tilde{c} \in \mathcal{C}^{\text{sym}}, \tilde{e} \in \mathcal{C}^{\text{asym}}$

if $\exists(\sigma, c, r) \in \mathcal{O}^H$ *such that* $c = \tilde{c}$ *and* $E^{asym}(pk, \sigma, r) = \tilde{e}$ **then**

  `// We have found a decryption`

  $\hat{a} \leftarrow G(\sigma)$;

  $\hat{m} \leftarrow D^{\text{sym}}_{\hat{a}}(c)$;

  return $\hat{m}$

end

return $\perp$;

The two oracles will behave differently if and only if the queried ciphertext is generated without querying the hash oracle, but if the hash oracle is not consulted, then the coin is indistinguishable from truly random. Therefore, the simulated oracle's behavior diverges from the standard oracle if and only if the adversary correctly guesses the ciphertext generated from a truly random coin. The probaility of such an event is bounded by the spread of the underlying encryption scheme.

4.2.2 Random until queried

In the 1999 transformation, both the symmetric key $a \leftarrow G(\sigma)$ and the asymmetric coin $r \leftarrow H(\sigma, c)$ are pseudorandom. In the 2017 transformation, the asymmetric coin $r \leftarrow H(m)$ is also pseudorandom. On the other hand, in the standard security game against any symmetric cipher, the secret key is truly random, and in the standard security game against an asymmetric cipher, the encryption coin is also truly random. This difference prevents an adversary playing the standard game to correctly simulate the challenge encryption in the IND-CCA2 game.

Under the random oracle model, this difference can be resolved by observing that there is no distinction between querying a random oracle once and randomly sampling from the specified probability space. In other words, unless the IND-CCA2 adversary makes a query to the random oracle with the specific input, from the IND-CCA2 adversary's perspective there is no difference between pseudorandom and truly random values, so the sequence of games can safely replace pseudorandom values with truly random values, and the difference in advantage will be bounded by the probability that the IND-CCA2 adversary indeed makes some very specific query. Finally, this probability can be bounded by a standard game adversary who can take advantage of the specific value being queried.

As an example, in the 1999 transformation's security reduction, consider an IND-CPA adversary against the underlying symmetric cipher $\mathcal{A}^{\text{sym}}_{\text{IND-CPA}}$. $\mathcal{A}^{\text{sym}}_{\text{IND-CPA}}$ tries to simulate the IND-CCA2 game for an adversary against the hybrid scheme $\mathcal{A}^{\text{hy}}_{\text{IND-CCA}}$ in the following steps:

1. $\mathcal{A}^{\text{sym}}_{\text{IND-CPA}}$ runs hybrid KeyGen and passes pk to $\mathcal{A}^{\text{hy}}_{\text{IND-CCA}}$
2. $\mathcal{A}^{\text{sym}}_{\text{IND-CPA}}$ simulates random oracle H, G and the decryption oracle $\mathcal{O}^D$ using method described in section 4.2.1
3. When $\mathcal{A}^{\text{hy}}$ outputs the challenge messages m_0, m_1, $\mathcal{A}^{\text{sym}}$ passes these two messages to the symmetric cipher challenger. The symmetric challenger returns the encryption of one of them $c^* = E^{\text{sym}}_{a^*}(m_b)$ under a **truly random key** a^*
4. $\mathcal{A}^{\text{sym}}$ samples random $\sigma \leftarrow \mathcal{M}^{\text{asym}}$, computes the coin $r \leftarrow H(c^*, \sigma)$ and the asymmetric ciphertext $e^* \leftarrow E^{\text{asym}}(\text{pk}, \sigma, r)$
5. $\mathcal{A}^{\text{sym}}$ passes (c^*, e^*) as the challenge ciphertext to $\mathcal{A}^{\text{hy}}$. When $\mathcal{A}^{\text{hy}}$ outputs the guess $\hat{b}$, $\mathcal{A}^{\text{sym}}$ passes $\hat{b}$ as its own output

Note that in the simulation of the IND-CCA2 game above, the symmetric key for the challenge encryption in the IND-CCA2 game is truly random instead of pseudorandom. Let G_a denote the IND-CCA2 game with truly random symmetric key (aka the simulation above), let G_b denote the IND-CCA2 game with pseudorandom symmetric key, and let QUERY* denote the event that the IND-CCA2 adversary queries G with m_b, which is the diverging event between the two games. By the difference lemma:

$$\epsilon_a - \epsilon_b \leq P[\text{QUERY}^*]$$

On the other hand, if the adversary indeed queries the hash $G(m_b)$, then the tape of the random oracle G should contain an entry that matches one of (m_0, m_1), which can then be used by another IND-CPA adversary:

$$P[\text{QUERY}^*] \leq \epsilon_{\text{IND-CPA}}$$

4.2.3 Plaintext checking, ciphertext validation, and patched decapsulation oracle

The concept of plaintext-checking attack (PCA) was proposed in early 2000s and first used in generic IND-CCA transformation by Coron et al in *GEM: a generic chosen-ciphertext secure encryption method* [5]. Although PCA security is not a standard security definition, it is nonetheless useful as an intermediary step for constructing simple generic IND-CCA secure cryptosystems.

The standard plaintext-checking oracle (PCO) takes as input a plaintext-ciphertext pair (m, c) and uses the secret key to determine whether they correspond to each other. Because lattice-based cryptosystems sometimes suffer from decryption errors, and because the OW-PCVA transformation (E^T, D^T) is a deterministic encryption scheme, the standard PCO checks both encryption and decryption. When trying to simulate PCO without using the secret key, the decryption routine is omitted. Thanks for E^T being deterministic, the behavior of the simulated PCO diverges from the standard PCO if and only if m causes decryption error. The probability of decryption error is bounded by δ-correctness.

Algorithm 15: Standard PCO

Input: $(m \in \mathcal{M}, c \in \mathcal{C})$
if $E^T(pk, m) \neq c$ **then**
  return $\perp$;
end
if $D^T(sk, c) \neq m$ **then**
  return $\perp$;
end

Algorithm 16: Simulated PCO

Input: $(m \in \mathcal{M}, c \in \mathcal{C})$
if $E^T(pk, m) \neq c$ **then**
  return $\perp$;
end

In addition to PCO, Hofheinz [4] also introduced a ciphertext-validation oracle (CVO) that checks whether the queried ciphertext is valid or not. The standard CVO uses the secret key to execute the decryption routine, and the simulated CVO uses the random oracle in a similar trick as described in section 4.2.1 with a similar loss of security.

Algorithm 17: Standard CVO

Input: $c \in \mathcal{C}$
$\hat{m} \leftarrow D^T(\mathrm{sk}, c)$;
if $\hat{m} = \perp$ **then**
  return $\perp$
end
if $E^T(pk, \hat{m}) \neq c$ **then**
  return $\perp$
end

Algorithm 18: Simulated CVO

Input: $c \in \mathcal{C}$
if $\nexists (m, r) \in \mathcal{O}^G$ *such that* $E^T(pk, m) = c$ **then**
  return $\perp$
end

The combination of PCO and CVO allows the decapsulation oracle to be simulated by an OW-PCVA adversary having access to these two oracles. The simulation is achieved by making both the hash oracle H and the simulated decapsulation oracle $\mathcal{O}^D$ stateful, and allowing valid queries to H be added to the state of $\mathcal{O}^D$ so that the output of $\mathcal{O}^D$ stays consistent. In this simulation H **patches** the state of $\mathcal{O}^D$, hence we call this decapsulation "patched".

Algorithm 19: Standard decap oracle

Input: $c \in \mathcal{C}$

$\hat{m} \leftarrow D^T(\mathrm{sk}, c)$;
if $\hat{m} \neq \perp$ **then**
  return $H(\hat{m}, c)$;
end
return $\perp$;

Algorithm 20: Patched hash oracle H

Input: $m \in \mathcal{M}, c \in \mathcal{C}$

if $\exists(\tilde{m}, \tilde{c}, \tilde{K}) \in \mathcal{O}^H$ *such that* $\tilde{m} = m, \tilde{c} = c$ **then**
  return $\tilde{K}$
end
$K \xleftarrow{\$} \mathcal{K}$;
Append (m, c, K) to $\mathcal{O}^H$;
if $\mathrm{PCO}(m, c) \neq$ *bot* **then**
  Append (c, K) to $\mathcal{O}^D$
end
return K

Algorithm 21: Patched decap oracle $\mathcal{O}^D$

Input: $c \in \mathcal{C}$

if $\exists(\tilde{c}, \tilde{K}) \in \mathcal{O}^D$ *such that* $\tilde{c} = c$ **then**
  return $\tilde{K}$
end
if $\mathrm{CVO}(c) \neq \perp$ **then**
  $K \xleftarrow{\$} \mathcal{K}$;
  Append (c, K) to $\mathcal{O}^D$;
  return K
end
return $\perp$; `// Or the implicit rejection`

Under the random oracle model, the combination of patched hash function and patched decapsulation oracle should behave exactly the same as the standard decapsulation oracle.

4.3 A sketch of proof

Here we provide a sketch of the security proof for both the 1999 transformation and the 2017 variations.

4.3.1 Proof of theorem 4.1

Proof. □

4.3.2 Proof of theorem 4.2

4.3.3 Proof of theorem 4.3

5 Future works on generic IND-CCA transformation

References

[1] Mihir Bellare and Phillip Rogaway. Optimal asymmetric encryption. In *Advances in Cryptology—EUROCRYPT'94: Workshop on the Theory and Application of Cryptographic Techniques Perugia, Italy, May 9–12, 1994 Proceedings 13*, pages 92–111. Springer, 1995.

[2] Tim Dierks and Eric Rescorla. Rfc 5246: The transport layer security (tls) protocol version 1.2, 2008.

[3] Eiichiro Fujisaki and Tatsuaki Okamoto. Secure integration of asymmetric and symmetric encryption schemes. In *Annual international cryptology conference*, pages 537–554. Springer, 1999.

[4] Dennis Hofheinz, Kathrin Hövelmanns, and Eike Kiltz. A modular analysis of the fujisaki-okamoto transformation. In *Theory of Cryptography Conference*, pages 341–371. Springer, 2017.

[5] Coron Jean-Sébastien, Helena Handschuh, Marc Joye, Pascal Paillier, David Pointcheval, and Christophe Tymen. Gem: Ag eneric chosen-ciphertext secure e ncryption m ethod. In *Topics in Cryptology—CT-RSA 2002: The Cryptographers' Track at the RSA Conference 2002 San Jose, CA, USA, February 18–22, 2002 Proceedings*, pages 263–276. Springer, 2002.