

Fast Fujisaki-Okamoto transformation using encrypt-then-mac and applications to Kyber

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Abstract. The modular Fujisaki-Okamoto (FO) transformation takes public-key encryption with weaker security and constructs a key encapsulation mechanism (KEM) with indistinguishability under adaptive chosen ciphertext attacks. While the modular FO transform enjoys tight security bound and quantum resistance, it also suffers from computational inefficiency due to using de-randomization and re-encryption for providing ciphertext integrity. In this work, we propose an alternative modular FO transformation that replaces re-encryption with a message authentication code (MAC) and prove the security bound of our construction. We then instantiate a concrete instance with ML-KEM and show that when re-encryption incurs significant computational cost, our construction provides substantial runtime speedup and reduced memory footprint.

Keywords: Key encapsulation mechanism, post-quantum cryptography, lattice cryptography, Fujisaki-Okamoto transformation

1 Introduction

The Fujisaki-Okamoto transformation [FO99] is a generic construction that takes cryptographic primitives of lesser security and constructs a public-key encryption scheme with indistinguishability under adaptive chosen ciphertext attacks. Later works extended the original transformation to the construction of key encapsulation mechanism, which has been adopted by many post-quantum schemes such as Kyber [BDK⁺18] (standardized by NIST into ML-KEM [KE23]).

The current state of the FO transformation enjoys tight security bound and quantum resistance [HHK17], but also leaves many open questions. One such problem is the use of re-encryption for providing ciphertext integrity [BP18], which requires the decryption/decapsulation to run the encryption routine as a subroutine. In many post-quantum schemes, such as Kyber, the encryption routine is substantially computationally more expensive than the decryption routine.

The problem of ciphertext integrity was solved in symmetric cryptography. Given a semantically secure symmetric cipher and an existentially unforgeable message authentication code, combining them using “encrypt-then-mac” provides authenticated encryption [BN00]. We took inspiration from this strategy and applied a similar technique to provide ciphertext integrity for a public-key encryption scheme, which then translates to an IND-CCA secure KEM. Using a message authentication code for ciphertext integrity replaces the re-encryption step in decryption with the computation of a tag, which should offer significant performance improvements while maintaining comparable level of security.

The main challenge in applying “encrypt-then-mac” to public-key cryptography is the lack of a pre-shared MAC key. We proposed to derive the shared MAC key by hashing the plaintext message. We will prove in section 3 that, under the random oracle model, the MAC key is securely hidden behind the hash function, and producing a valid pair of ciphertext and tag without full knowledge of the plaintext constitutes a forgery attack on the message authentication code. Thanks to the modular construction in [HHK17],

providing ciphertext integrity in the underlying encryption scheme gives us an IND-CCA secure KEM for free.

In section 4.2, we instantiate concrete instances of our proposed transformation by modifying ML-KEM. We will demonstrate that, at the cost of small increase in encryption runtime and ciphertext size, our construction reduces both the runtime and memory footprint of the decryption routine.

2 Preliminaries and previous results

2.1 Public-key encryption scheme

We define a public key encryption scheme PKE to be a collection of three routines ($\text{Gen}, \text{Enc}, \text{Dec}$) defined over a finite message space \mathcal{M} and some ciphertext space \mathcal{C} . Many encryption routines are probabilistic, and we define their source of randomness to come from some coin space \mathcal{R} .

The encryption routine $\text{Enc}(\text{pk}, m)$ takes a public key, a plaintext message, and outputs a ciphertext $c \in \mathcal{C}$. Where the encryption routine is probabilistic, specifying a pseudorandom seed $r \in \mathcal{R}$ will make the encryption routine behave deterministically. The decryption routine $\text{Dec}(\text{sk}, c)$ takes a secret key, a ciphertext, and outputs the decryption \hat{m} if the ciphertext is valid under the given secret key, or the rejection symbol \perp if the ciphertext is invalid.

2.1.1 Correctness

It is common to require a PKE to be perfectly correct, meaning that for all possible keypairs (pk, sk) and plaintext messages $m \in \mathcal{M}$, $\text{Dec}(\text{sk}, \text{Enc}(\text{pk}, m)) = m$ at all times. However, some encryption schemes, including many popular lattice-based schemes, admit a non-zero probability of decryption failure: $\text{Dec}(\text{sk}, \text{Enc}(\text{pk}, m)) \neq m$. Furthermore, [HHK17] and [ABD⁺19] explained how decryption failure played a role in an adversary's advantage. In this paper, we inherit the definition for correctness from [HHK17]:

Definition 1 (δ -correctness). A public key encryption scheme PKE is δ -correct if

$$\mathbf{E}[\max_{m \in \mathcal{M}} P[\text{Dec}(\text{sk}, c) \neq m \mid c \xleftarrow{\$} \text{Enc}(\text{pk}, m)]] \leq \delta$$

Where the expectation is taken over the probability distribution of keypairs $(\text{pk}, \text{sk}) \xleftarrow{\$} \text{Gen}()$

2.1.2 Security

We discuss the security of a PKE using the sequence of games described in [Sho04]. Specifically, we first define the OW-ATK and the IND-CPA game as they pertain to a public key encryption scheme. In later section we will define the IND-CCA game as it pertains to a key encapsulation mechanism.

In the OW-ATK game, an adversary's goal is to recover the decryption of a randomly generated ciphertext.

The adversary \mathcal{A} with access to oracle(s) \mathcal{O}_{ATK} wins the game if its guess \hat{m} is equal to the challenge plaintext m^* . The *advantage* $\epsilon_{\text{OW-ATK}}$ of an adversary in this game is the probability that it wins the game.

The choice of oracle(s) \mathcal{O}_{ATK} depends on the choice of ATK. Specifically:

Algorithm 1 The OW-ATK game

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1:  $(\mathbf{pk}, \mathbf{sk}) \xleftarrow{\$} \text{Gen}(1^\lambda)$ 
2:  $m^* \xleftarrow{\$} \mathcal{M}$ 
3:  $c^* \xleftarrow{\$} \text{Enc}(\mathbf{pk}, m)^*$ 
4:  $\hat{m} \xleftarrow{\$} \mathcal{A}^{\mathcal{O}_{\text{ATK}}}(1^\lambda, \mathbf{pk}, c^*)$ 
5: return  $\llbracket m^* = \hat{m} \rrbracket$ 

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Figure 1: The OW-ATK game**Algorithm 2** PCO($m \in \mathcal{M}, c \in \mathcal{C}$)

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1: return  $\llbracket \text{Dec}(\mathbf{sk}, c) = m \rrbracket$ 

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Algorithm 3 CVO($c \in \mathcal{C}$)

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1: return  $\llbracket \text{Dec}(\mathbf{sk}, c) \in \mathcal{M} \rrbracket$ 

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Figure 2: The Plaintext-Checking Oracle PCO**Figure 3:** the Ciphertext-Validation Oracle CVO

$$\mathcal{O}_{\text{ATK}} = \begin{cases} - & \text{ATK} = \text{CPA} \\ \text{PCO} & \text{ATK} = \text{PCA} \\ \text{CVO} & \text{ATK} = \text{VA} \\ \text{PCO}, \text{CVO} & \text{ATK} = \text{PCVA} \end{cases}$$

83 Where the definitions of plaintext-checking oracle PCO and the ciphertext-validation
84 oracle CVO are inherited from [HHK17]

85 In the IND-CPA game, an adversary's goal is to distinguish the encryption of one message
86 from the encryption of another message. Given the public key, the adversary outputs two
87 adversarially chosen messages and obtains the encryption of a random choice between
88 these two messages. The adversary wins the IND-CPA game if it correctly identifies which
89 message the encryption is obtained from.

Algorithm 4 The IND-CPA game

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1:  $(\mathbf{pk}, \mathbf{sk}) \xleftarrow{\$} \text{Gen}(1^\lambda)$ 
2:  $(m_0, m_1) \xleftarrow{\$} \mathcal{A}(a^\lambda, \mathbf{pk})$ 
3:  $b \xleftarrow{\$} \{0, 1\}$ 
4:  $c^* \xleftarrow{\$} \text{Enc}(\mathbf{pk}, m_b)$ 
5:  $\hat{b} \xleftarrow{\$} \mathcal{A}(1^\lambda, \mathbf{pk}, c^*)$ 
6: return  $\llbracket b = \hat{b} \rrbracket$ 

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90 The *advantage* $\epsilon_{\text{IND-CPA}}$ of an IND-CPA adversary \mathcal{A} is defined by

$$\epsilon_{\text{IND-CPA}} = \left| P[\hat{b} = b] - \frac{1}{2} \right|$$

2.2 Key encapsulation mechanism

A key encapsulation mechanism KEM is a collection of three routines ($\text{Gen}, \text{Encap}, \text{Decap}$) defined over some ciphertext space \mathcal{C} and some key space \mathcal{K} . The key generation routine takes the security parameter 1^λ and outputs a keypair $(\text{pk}, \text{sk}) \xleftarrow{\$} \text{Gen}(1^\lambda)$. $\text{Encap}(\text{pk})$ is a probabilistic routine that takes a public key pk and outputs a pair of values (c, K) where $c \in \mathcal{C}$ is the encapsulation (or ciphertext) of the shared secret $k \in \mathcal{K}$. $\text{Decap}(\text{sk}, c)$ is a deterministic routine that takes the secret key sk and the encapsulation c and returns the shared secret k if the ciphertext is valid, or the rejection symbol \perp if the ciphertext is invalid.

The IND-CCA security of a KEM is defined by an adversarial game in which an adversary's goal is to distinguish pseudorandom shared secret (generated by running the Encap routine) and a truly random value.

Algorithm 5 IND-CCA game for KEM

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1:  $(\text{pk}, \text{sk}) \xleftarrow{\$} \text{Gen}(1^\lambda)$ 
2:  $(c^*, k_0) \xleftarrow{\$} \text{Encap}(\text{pk})$ 
3:  $k_1 \xleftarrow{\$} \mathcal{K}$ 
4:  $b \xleftarrow{\$} \{0, 1\}$ 
5:  $\hat{b} \xleftarrow{\$} \mathcal{A}_{\text{IND-CCA}}^{\mathcal{O}^{\text{Decap}}}(1^\lambda, \text{pk}, c^*, k_b)$ 
6: return  $\llbracket \hat{b} = b \rrbracket$ 

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The decapsulation oracle $\mathcal{O}^{\text{Decap}}$ takes a ciphertext c and returns the output of the Decap routine using the secret key. The advantage $\epsilon_{\text{IND-CCA}}$ of an IND-CCA adversary $\mathcal{A}_{\text{IND-CCA}}$ is defined by

$$\epsilon_{\text{IND-CCA}} = \left| P[\hat{b} = b] - \frac{1}{2} \right|$$

2.3 Message authentication code

A message authentication code MAC is a collection of routines ($\text{MAC}, \text{MAC.Verify}$) defined over some key space \mathcal{K} , some message space \mathcal{M} , and some tag space \mathcal{T} . The signing routine $\text{MAC}(k, m)$ takes the secret key $k \in \mathcal{K}$ and some message, and outputs a tag t . The verification routine $\text{MAC.Verify}(k, m, t)$ takes the triplet of secret key, message, and tag, and outputs 1 if the message-tag pair is valid under the secret key, or 0 otherwise.

The security of a MAC is defined in an adversarial game in which an adversary, with access to some signing oracle $\mathcal{O}_{\text{MAC}}(m)$, tries to forge a new valid message-tag pair that has never been queried before. The existential unforgeability under chosen message attack (EUF-CMA) game is shown below:

Algorithm 6 The EUF-CMA game

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1:  $k^* \xleftarrow{\$} \mathcal{K}$ 
2:  $(\hat{m}, \hat{t}) \xleftarrow{\$} \mathcal{A}^{\mathcal{O}_{\text{MAC}}}()$ 
3: return  $\llbracket \text{MAC.Verify}(k^*, \hat{m}, \hat{t}) \text{ and } (\hat{m}, \hat{t}) \notin \mathcal{O}_{\text{MAC}} \rrbracket$ 

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The advantage $\epsilon_{\text{EUF-CMA}}$ of the existential forgery adversary is the probability that it wins the EUF-CMA game.

2.4 Modular Fujisaki-Okamoto transformation

The Fujisaki-Okamoto transformation (FOT) [FO99] is a generic transformation that takes a PKE with weaker security (such as OW-CPA or IND-CPA) and outputs a PKE with stronger security. A later variation [HHK17] improved the original construction in [FO99] by accounting for decryption failures, tightening security bounds, and providing a modular construction that first transforms OW-CPA/IND-CPA PKE into OW-PCVA PKE by providing ciphertext integrity through re-encryption (the T transformation), then transforming the OW-PCVA PKE into an IND-CCA KEM (the U transformation).

Particularly relevant to our results are two variations of the U transformation: U^\perp (KEM with explicit rejection) and U^χ (KEM with implicit rejection). If PKE is OW-PCVA secure, then U^\perp transforms PKE into an IND-CCA secure KEM^\perp :

Theorem 1. *For any IND-CCA adversary \mathcal{A}_{KEM} against KEM^\perp with advantage ϵ_{KEM} issuing at most q_D decapsulation queries and at most q_H hash queries, there exists an OW-PCVA adversary \mathcal{A}_{PKE} against the underlying PKE with advantage ϵ_{PKE} that makes at most q_H queries to PCO and CVO such that*

$$\epsilon_{KEM} \leq \epsilon_{PKE}$$

Similarly, if PKE is OW-PCA secure, then U^χ transforms PKE into an IND-CCA secure KEM^χ

Theorem 2. *For any IND-CCA adversary \mathcal{A}_{KEM} against KEM^χ with advantage ϵ_{KEM} issuing at most q_D decapsulation queries and at most q_H hash queries, there exists an OW-CPA adversary \mathcal{A}_{PKE} against the underlying PKE with advantage ϵ_{PKE} issuing at most q_H queries to PCO such that:*

$$\epsilon_{KEM} \leq \frac{q_H}{|\mathcal{M}_{PKE}|} + \epsilon_{PKE}$$

The modularity of the T and U transformation allows us to tweak only the T transformation (see section 3), obtain OW-PCVA security, then automatically get IND-CCA security for free. This means that we can directly apply our contribution to existing KEM's already using this modular transformation, such as ML-KEM [KE23], and obtain performance improvements while maintaining comparable levels of security (see section 4.2).

3 The “encrypt-then-MAC” transformation

Let $PKE(\text{Gen}, \text{Enc}, \text{Dec})$ be a public-key encryption scheme. Let MAC be a deterministic message authentication code. Let $G : \mathcal{M}_{PKE} \rightarrow \mathcal{K}_{MAC}$ and $H : \{0, 1\}^* \rightarrow \mathcal{K}_{KEM}$ be hash functions, where \mathcal{K}_{KEM} denote the set of all possible session keys. The EtM transformation outputs a key encapsulation mechanism $KEM_{\text{EtM}}(\text{Gen}_{\text{EtM}}, \text{Encap}_{\text{EtM}}, \text{Decap}_{\text{EtM}})$. The three routines are described in figure 4.

Algorithm 7 Gen_{EtM}	Algorithm 8 $\text{Encap}_{\text{EtM}}(\text{pk})$	Algorithm 9 $\text{Decap}_{\text{EtM}}(\text{sk}, (c, t))$
1: $(\text{pk}, \text{sk}_{\text{PKE}}) \xleftarrow{\$} \text{Gen}(1^\lambda)$	1: $m \xleftarrow{\$} \mathcal{M}_{\text{PKE}}$	1: $(\text{sk}_{\text{PKE}}, z) \leftarrow \text{sk}$
2: $z \xleftarrow{\$} \mathcal{M}_{\text{PKE}}$	2: $k \leftarrow G(m)$	2: $\hat{m} \leftarrow \text{Dec}(\text{sk}_{\text{PKE}}, c)$
3: $\text{sk} \leftarrow (\text{sk}_{\text{PKE}}, z)$	3: $c \xleftarrow{\$} \text{Enc}(\text{pk}, m)$	3: $\hat{k} \leftarrow G(\hat{m})$
4: return (pk, sk)	4: $t \leftarrow \text{MAC}(k, c)$	4: if $\text{MAC}(\hat{k}, c) \neq t$ then
	5: $K \leftarrow H(m, c)$	5: return $H(z, c)$
	6: return $((c, t), K)$	6: end if
		7: return $H(\hat{m}, c)$

Figure 4: KEM_{EtM} routines

If the input PKE is OW-PCA secure, then KEM_{EtM} is IND-CCA2 secure.

Theorem 3. *For every IND-CCA2 adversary A against KEM_{EtM} that makes at most q_D decapsulation queries, there exists an OW-PCA adversary B against the underlying PKE such that*

$$\text{Adv}_{\text{IND-CCA2}}(A) \leq q_D \cdot \epsilon_{\text{MAC}} + 2 \cdot \text{Adv}_{\text{OW-PCA}}(B)$$

Proof. We will prove using a sequence of games. The complete sequence of games is shown in figure 5

Algorithm 10 Sequence of games $G_0 - G_3$

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1:  $(\text{pk}, \text{sk}) \xleftarrow{\$} \text{Gen}(1^\lambda)$ 
2:  $(m^*, z) \xleftarrow{\$} \mathcal{M}_{\text{PKE}}$ 
3:  $k^* \leftarrow G(m^*)$  ▷ Game 0-1
4:  $k^* \xleftarrow{\$} \mathcal{K}_{\text{MAC}}$  ▷ Game 2-3
5:  $c^* \xleftarrow{\$} \text{Enc}(\text{pk}, m^*)$ 
6:  $t^* \leftarrow \text{MAC}(k^*, c^*)$ 
7:  $K_0 \leftarrow H(m^*, c^*)$  ▷ Game 0-2
8:  $K_0 \xleftarrow{\$} \mathcal{K}_{\text{KEM}}$  ▷ Game 3
9:  $K_1 \xleftarrow{\$} \mathcal{K}_{\text{KEM}}$ 
10:  $b \xleftarrow{\$} \{0, 1\}$ 
11:  $\hat{b} \leftarrow A^{\mathcal{O}^{\text{Decap}}}(1^\lambda, \text{pk}, (c^*, t^*), K_b)$  ▷ Game 0
12:  $\hat{b} \leftarrow A^{\mathcal{O}_1^{\text{Decap}}}(1^\lambda, \text{pk}, (c^*, t^*), K_b)$  ▷ Game 1-3
13: return  $\llbracket \hat{b} = b \rrbracket$ 

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Algorithm 11 $\mathcal{O}^{\text{Decap}}(c, t)$

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1:  $\hat{m} \leftarrow \text{Dec}(\text{sk}_{\text{PKE}}, c)$ 
2:  $\hat{k} \leftarrow G(\hat{m})$ 
3: if  $\text{MAC}(\hat{k}, c) = t$  then
4:   return  $H(\hat{m}, c)$ 
5: end if
6: return  $H(z, c)$ 

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Algorithm 12 $\mathcal{O}_1^{\text{Decap}}(c, t)$

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1: if  $\exists(\tilde{m}, \tilde{k}) \in \mathcal{L}^G :$ 
2:    $\text{Dec}(\text{sk}_{\text{PKE}}, c) = \tilde{m}$ 
3:    $\wedge \text{MAC}(\tilde{k}, c) = t$  then
4:     return  $H(\tilde{m}, c)$ 
5: end if
6: return  $H(z, c)$ 

```

Figure 5: Sequence of games

□

4 Applications to Kyber

4.1 Kyber using FOT+

4.2 Performance comparisons

Comparisons will do for Kyber and Kyber^+ , define it.

4.3 Experimental results

5 Conclusions and future works

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