Linear Programs

$$\begin{aligned} \min_{\mathbf{x}}\ & \mathbf{f}^T\mathbf{x} \\ \text{subject to: }\ & \mathbf{A}\mathbf{x} \le \mathbf{b} \\ & \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\ & \mathbf{l}_{\text{b}} \le \mathbf{x} \le \mathbf{u}_{\text{b}} \end{aligned}$$

Ex1 LP

$$\begin{aligned} \min_{\mathbf{x}}\ & -6x_1 - 5x_2 \\ \text{subject to: }\ & x_1 + 4x_2 \le 16 \\ & 6x_1 + 4x_2 \le 28 \\ & 2x_1 - 5x_2 \le 6 \\ & 0 \le \mathbf{x} \le 10 \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{x}}\ & \begin{bmatrix} -6 \\ -5 \end{bmatrix}^T \mathbf{x} \\ \text{subject to: }\ & \begin{bmatrix} 1 & 4 \\ 6 & 4 \\ 2 & -5 \end{bmatrix} \mathbf{x} \le \begin{bmatrix} 16 \\ 28 \\ 6 \end{bmatrix} \\ & 0 \le \mathbf{x} \le 10 \end{aligned}$$

Ex2 LP

$$\begin{aligned} \min_{\mathbf{x}}\ & -x_1 - 2x_2 - 3x_3 \\ \text{subject to: }\ & -x_1 + x_2 + x_3 \le 20 \\ & x_1 - 3x_2 + x_3 \le 30 \\ & x_1 + x_2 + x_3 = 40 \\ & 0 \le x_1 \le 40 \\ & 0 \le x_2 \\ & 0 \le x_3 \end{aligned}$$

MI Linear Programs

$$
\begin{aligned}
\min_{\mathbf{x}}\ & \mathbf{f}^T\mathbf{x} \\
\text{subject to: }\ & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\
& \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\
& \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \\
& x_i \in \mathbb{Z} \\
& x_j \in \{0, 1\}
\end{aligned}
$$

Ex1 MILP

$$
\begin{aligned}
\min_{\mathbf{x}}\ & -6x_1 - 5x_2 \\
\text{subject to: }\ & x_1 + 4x_2 \leq 16 \\
& 6x_1 + 4x_2 \leq 28 \\
& 2x_1 - 5x_2 \leq 6 \\
& 0 \leq \mathbf{x} \leq 10 \\
& \mathbf{x} \in \mathbb{Z}
\end{aligned}
$$

Ex2 MILP

$$
\begin{aligned}
\min_{\mathbf{x}}\ & -x_1 - x_2 - 3x_3 - 2x_2 - 2x_5 \\
\text{subject to: }\ & -x_1 - x_2 + x_3 + x_4 \leq 30 \\
& x_1 + x_3 - 3x_4 \leq 30 \\
& 0 \leq x_1 \leq 40 \\
& 0 \leq x_2 \leq 1 \\
& 0 \leq x_3 \\
& 0 \leq x_4 \\
& 0 \leq x_5 \leq 1
\end{aligned}
$$

Quadratic Programs

$$
\begin{aligned}
\min_{\mathbf{x}}\ & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{f}^T\mathbf{x} \\
\text{subject to: }\ & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\
& \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\
& \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}}
\end{aligned}
$$

$$
\begin{aligned}
\min_{\mathbf{x}}\ & 0.5x_1^2 + x_2^2 - x_1x_2 - 2x_1 - 6x_2 \\
\text{subject to:}\quad & x_1 + x_2 \le 2 \\
& -x_1 + 2x_2 \le 2 \\
& 2x_1 + x_2 \le 3 \\
& \mathbf{0} \le \mathbf{x}
\end{aligned}
$$

$$
\begin{aligned}
\min_{\mathbf{x}}\ & -2x_1x_2 \\
\text{subject to:}\ & -\mathbf{0.5} \le \mathbf{x} \le \mathbf{1}
\end{aligned}
$$

MI Quadratic Programs

$$
\begin{aligned}
\min_{\mathbf{x}}\ & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{f}^T\mathbf{x} \\
\text{subject to:}\ & \mathbf{A}\mathbf{x} \le \mathbf{b} \\
& \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\
& \mathbf{l}_{\text{b}} \le \mathbf{x} \le \mathbf{u}_{\text{b}} \\
& x_i \in \mathbb{Z} \\
& x_j \in \{0, 1\}
\end{aligned}
$$

$$
\begin{aligned}
\min_{\mathbf{x}}\ & 0.5x_1^2 + x_2^2 - x_1x_2 - 2x_1 - 6x_2 \\
\text{subject to:}\quad & x_1 + x_2 \le 2 \\
& -x_1 + 2x_2 \le 2 \\
& 2x_1 + x_2 \le 3 \\
& \mathbf{0} \le \mathbf{x} \\
& x_1 \in \mathbb{Z}
\end{aligned}
$$

QC Quadratic Programs

$$
\begin{aligned}
\min_{\mathbf{x}}\ & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{f}^T\mathbf{x} \\
\text{subject to:}\ & \mathbf{A}\mathbf{x} \le \mathbf{b} \\
& \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\
& \mathbf{l}_{\text{b}} \le \mathbf{x} \le \mathbf{u}_{\text{b}} \\
& \mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{l}^T\mathbf{x} \le \mathbf{r}
\end{aligned}
$$

QC Row

$$\mathbf{q}_{\mathrm{rl}} \le \mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{l}^T\mathbf{x} \le \mathbf{q}_{\mathrm{ru}}$$

$$\begin{aligned} \min_{\mathbf{x}}\ & 0.5x_1^2 + 0.5x_2^2 - 2x_1 - 2x_2 \\ \text{subject to:}\quad & -x_1 + x_2 \le 2 \\ & x_1 + 3x_2 \le 5 \\ & x_1^2 + x_2^2 - 2x_2 \le 1 \\ & \mathbf{0} \le \mathbf{x} \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{x}}\ & 0.5x_1^2 + 0.5x_2^2 - 2x_1 - 2x_2 \\ \text{subject to:}\quad & -x_1 + x_2 \le 2 \\ & x_1 + 3x_2 \le 5 \\ & x_1^2 + x_2^2 - 2x_2 \le 1 \\ & x_1^2 + x_2^2 - x_1 + 2x_2 \le 1.2 \\ & \mathbf{0} \le \mathbf{x} \end{aligned}$$

MIQC Quadratic Programs

$$\begin{aligned} \min_{\mathbf{x}}\ & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{f}^T\mathbf{x} \\ \text{subject to:}\ & \mathbf{A}\mathbf{x} \le \mathbf{b} \\ & \mathbf{A}_{\mathrm{eq}}\mathbf{x} = \mathbf{b}_{\mathrm{eq}} \\ & \mathbf{l}_{\mathrm{b}} \le \mathbf{x} \le \mathbf{u}_{\mathrm{b}} \\ & \mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{l}^T\mathbf{x} \le \mathbf{r} \\ & x_i \in \mathbb{Z} \\ & x_j \in \{0, 1\} \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{x}}\ & 0.5x_1^2 + 0.5x_2^2 - 2x_1 - 2x_2 \\ \text{subject to:}\quad & -x_1 + x_2 \le 2 \\ & x_1 + 3x_2 \le 5 \\ & x_1^2 + x_2^2 - 2x_2 \le 1 \\ & \mathbf{0} \le \mathbf{x} \\ & x_1 \in \mathbb{Z} \end{aligned}$$

SDP

$$
\begin{aligned}
\min_{\mathbf{x}} \ & \mathbf{f}^T\mathbf{x} \\
\text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\
& \mathbf{l}_\mathrm{b} \leq \mathbf{x} \leq \mathbf{u}_\mathrm{b} \\
& \mathbf{X} = \sum_{i=1}^{n} x_i \mathbf{F}_i - \mathbf{F}_0 \\
& \mathbf{X} \succeq \mathbf{0} \text{ [Positive Semidefinite]}
\end{aligned}
$$

Ex1 SDP

$$
\begin{aligned}
\min_{\mathrm{x}} \ & x \\
\text{subject to: } & \begin{bmatrix} x & \sqrt{2} \\ \sqrt{2} & x \end{bmatrix} \succeq \mathbf{0}
\end{aligned}
$$

Ex1 SDP Equation

$$
x \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{F}_1} - \underbrace{\begin{bmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & 0 \end{bmatrix}}_{\mathbf{F}_0} \succeq \mathbf{0}
$$

Ex2 SDP

$$
\begin{aligned}
\min_{\mathbf{x}} \ & x_1 + x_2 \\
\text{subject to: } & \begin{bmatrix} x_1 & 2 \\ 2 & x_2 \end{bmatrix} \succeq \mathbf{0} \\
& \mathbf{0} \leq \mathbf{x} \leq \mathbf{10}
\end{aligned}
$$

Ex2 SDP Equation

$$
x_1 \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{A}_1} + x_2 \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{A}_2} - \underbrace{\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}}_{\mathbf{C}} \succeq \mathbf{0}
$$

Ex3 SDP

$$\begin{aligned} &\min_{\mathbf{x}}\ x_1 \\ \text{subject to: } &\begin{bmatrix} x_2 & x_3 \\ x_3 & x_4 \end{bmatrix} \preceq \begin{bmatrix} x_1 & 0 \\ 0 & x_1 \end{bmatrix} \\ &\begin{bmatrix} x_2 & x_3 \\ x_3 & x_4 \end{bmatrix} \succeq \begin{bmatrix} 1 & 0.2 \\ 0.2 & 1 \end{bmatrix} \end{aligned}$$

Nonlinear Least Squares

$$\begin{aligned} \min_{\mathbf{x}}\ & \|\mathbf{F}\left(\mathbf{x}\right)\|_2^2 \\ \text{subject to: } & \mathbf{A}\mathbf{x} \le \mathbf{b} \\ & \mathbf{A}_{\mathrm{eq}}\mathbf{x} = \mathbf{b}_{\mathrm{eq}} \\ & \mathbf{l}_{\mathrm{b}} \le \mathbf{x} \le \mathbf{u}_{\mathrm{b}} \end{aligned}$$

$$\min_{\mathbf{x}}\ \sum_i \left(\mathrm{F}_i\left(\mathbf{x}, \mathbf{xdata}\right) - ydata_i\right)^2$$

Ex1

$$\min_{\mathbf{x}}\ \left\|\begin{matrix} 100\left(x_2 - x_1^2\right) \\ 1 - x_1 \end{matrix}\right\|_2^2$$

Ex2

$$\begin{aligned} \min_{\mathbf{x}}\ & \left\|\begin{matrix} 100\left(x_2 - x_1^2\right) \\ 1 - x_1 \end{matrix}\right\|_2^2 \\ \text{subject to: } & -2 \le \mathbf{x} \le 0.5 \end{aligned}$$

Ex3

$$\mathbf{F}\left(\mathbf{x}, \mathbf{xdata}\right) = x_1 e^{x_2 \mathbf{xdata}}$$

System of Nonlinear Equations

$$\mathbf{F}\left(\mathbf{x}\right) = \mathbf{0}$$

$$
\begin{aligned}
2x_1 - x_2 - e^{-x_1} &= 0 \\
-x_1 + 2x_2 - e^{-x_2} &= 0
\end{aligned}
$$

Nonlinear Programs

$$
\begin{aligned}
\min_{\mathbf{x}} \;& f\left(\mathbf{x}\right) \\
\text{subject to: } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\
& \mathbf{A}_{\text{eq}}\mathbf{x} = \mathbf{b}_{\text{eq}} \\
& \mathbf{l}_{\text{b}} \leq \mathbf{x} \leq \mathbf{u}_{\text{b}} \\
& \mathbf{c}\left(\mathbf{x}\right) \leq \mathbf{d} \\
& \mathbf{c}_{\text{eq}}\left(\mathbf{x}\right) = \mathbf{d}_{\text{eq}}
\end{aligned}
$$

Ex2 NLP

$$
\begin{aligned}
\min_{\mathbf{x}} \;& 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \\
\text{subject to: } & -5 \leq \mathbf{x} \leq 5
\end{aligned}
$$

Ex2 NLP

$$
\begin{aligned}
\min_{\mathbf{x}} \;& \log(1 + x_1^2) - x_2 \\
\text{subject to: } & \left(1 + x_1^2\right)^2 + x_2^2 = 4
\end{aligned}
$$

Ex2 NLP

$$
\begin{aligned}
\min_{\mathbf{x}} \;& x_1 x_4 \left(x_1 + x_2 + x_3\right) + x_3 \\
\text{subject to: } & x_1 x_2 x_3 x_4 \geq 25 \\
& x_1^2 + x_2^2 + x_3^2 + x_4^2 = 40 \\
& 1 \leq \mathbf{x} \leq 5
\end{aligned}
$$

Ex2 LinearCon NLP

$$
\begin{aligned}
\min_{\mathbf{x}} \;& (x_1 - x_2)^2 + (x_2 - x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2 \\
\text{subject to: } & x_1 + 3x_3 = 4 \\
& x_3 + x_4 - 2x_5 = 0 \\
& x_2 - x_5 = 0
\end{aligned}
$$

Ex 3

$$\min_{\mathbf{x}} \ 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

MI Nonlinear Programs

$$\begin{aligned} \min_{\mathbf{x}} \ & f(\mathbf{x}) \\ \text{subject to: } & \mathbf{A}\mathbf{x} \le \mathbf{b} \\ & \mathbf{A}_{\mathrm{eq}}\mathbf{x} = \mathbf{b}_{\mathrm{eq}} \\ & \mathbf{l}_{\mathrm{b}} \le \mathbf{x} \le \mathbf{u}_{\mathrm{b}} \\ & \mathbf{c}(\mathbf{x}) \le \mathbf{d} \\ & \mathbf{c}_{\mathrm{eq}}(\mathbf{x}) = \mathbf{d}_{\mathrm{eq}} \\ & x_i \in \mathbb{Z} \\ & x_j \in \{0, 1\} \end{aligned}$$

EX1 MINLP

$$\begin{aligned} \min_{\mathbf{x}} \ & -x_1 - x_2 - x_3 \\ \text{subject to: } & (x_2 - 0.5)^2 + (x_3 - 0.5)^2 \le 0.25 \\ & x_1 - x_2 \le 0 \\ & x_1 + x_3 + x_4 \le 2 \\ & x_1 \le 1 \\ & x_4 \le 5 \\ & \mathbf{0} \le \mathbf{x} \\ & x_1 \in \{0, 1\} \\ & x_4 \in \mathbb{Z} \end{aligned}$$

EX2 MINLP

$$\begin{aligned} \min_{\mathbf{x}} \ & 20 + x_1^2 + x_2^2 - 10(\cos 2\pi x_1 + \cos 2\pi x_2) \\ \text{subject to: } & 5\pi \le x_1 \le 20\pi \\ & -20\pi \le x_2 \le -4\pi \\ & x_1 \in \mathbb{Z} \end{aligned}$$

Constraint Stuff

$$\mathbf{r_l} \le \mathbf{A}\mathbf{x} \le \mathbf{r_u}$$

1st Derivatives

Gradient

$$\nabla f = \frac{\partial f}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} & \cdots & \frac{\partial f}{\partial x_n} \end{bmatrix}$$

Jacobian

$$\nabla \mathbf{F} = \frac{\partial \mathbf{F}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Ex1

$$f(\mathbf{x}) = x_1 x_4 (x_1 + x_2 + x_3) + x_3$$

2nd Derivatives

$$\nabla^2 f = \frac{\partial^2 f}{\partial \mathbf{x}^2} = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

Hess Lagrangian

$$\nabla^2 \mathbf{L} = \sigma \nabla^2 f + \sum_i \lambda_i \nabla^2 c_i$$

Adv Options
Tolr

$$\frac{|f_i - f_{i-1}|}{|f_i|} \leq \text{tol}_{rel}$$

Tola

$$|f_i - f_{i-1}| \leq \text{tol}_{abs}$$

Toli

$$|x_r - x_z| \leq \text{tol}_{int}$$