# Homework–6

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## Contents



## 1.Derive the expression for the power spectrum of MA(2). Plot this power spectrum for $0 \leqslant \omega \leqslant \pi$

### MA(2) definition

$$y_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}$$

### Autocovariance of MA(2)

$$\begin{aligned}\gamma_0 &= (1 + \theta_1^2 + \theta_2^2)\sigma^2 \\ \gamma_1 &= (\theta_1 + \theta_2\theta_1)\sigma^2 \\ \gamma_2 &= \theta_2\sigma^2 \\ \gamma_j &= 0,\ j \geqslant 3\end{aligned}$$

### Generating functions for ACF of MA(2)

$$\begin{aligned}g_y(z) &= \sum_{j=-\infty}^{\infty} \gamma_j z^j, (\gamma_{-j} = \gamma_j) \\ &= \gamma_2 z^{-2} + \gamma_1 z^{-1} + \gamma_0 + \gamma_1 z^1 + \gamma_2 z^{-2} \\ &= \sigma^2 \left[\theta_2 z^{-2} + (\theta_1 + \theta_2\theta_1)z^{-1} + (1 + \theta_1^2 + \theta_2^2) + (\theta_1 + \theta_2\theta_1)z + \theta_2 z^2\right] \\ &= \sigma^2 \left[\theta_2(z^{-2} + z^2) + (\theta_1 + \theta_2\theta_1)(z^{-1} + z) + (1 + \theta_1^2 + \theta_2^2)\right]\end{aligned}$$

### Setting $z = e^{-iw}$ and simplifying

$$\begin{aligned} S_y(\omega) &= \frac{1}{2\pi} g_y(e^{-i\omega}) \\ &= \frac{1}{2\pi}\sigma^2[2\theta_2 cos(2\omega) + 2(\theta_1 + \theta_2\theta_1)cos(\omega) + (1 + \theta_1^2 + \theta_2^2)] \\ &= \frac{1}{2\pi}[\gamma_0 + 2\gamma_1 cos(\omega) + 2\gamma_2 cos(2\omega)] \end{aligned}$$

### Plot

```
library(ggplot2)
theta1<-0.5
theta2<-0.5
sigmaSQ<-1
omega<-seq(0,pi,0.001)
Sy<-1/2*pi*sigmaSQ*(2*theta2*cos(2*omega)
                    +2*(theta1+theta2*theta1)*cos(omega)+1+theta1**2+theta2**2)
dfP <- data.frame(x = omega, y = Sy)
p <- ggplot(dfP, aes(x = x, y = y)) +
  geom_line() +
  xlab(expression(omega)) +
  ylab(expression(Sy[(omega)]))
p
```