HW1

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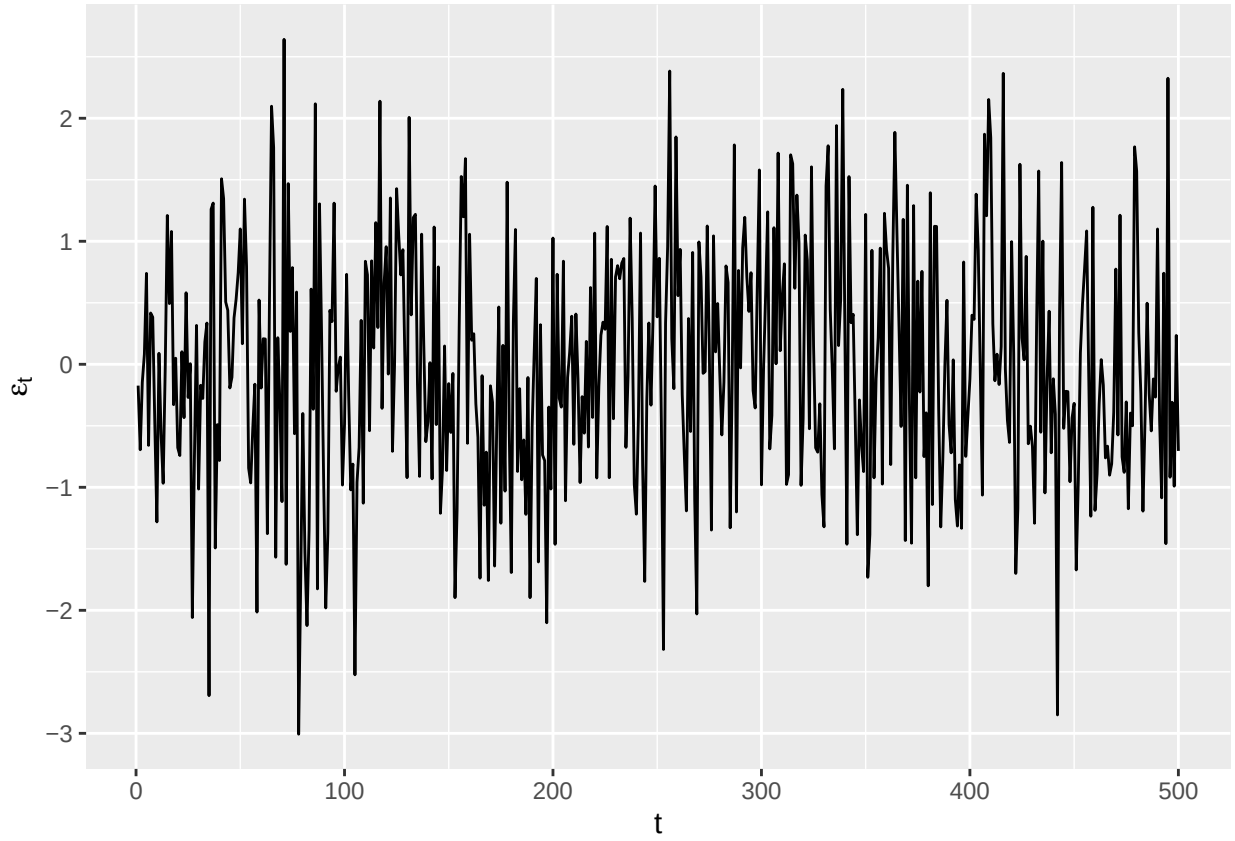
Contents

1)Generate ϵ_t~N(0,1) ; IID samples, $1 \leq t \leq 500$

a)

ϵ_t(vs)t

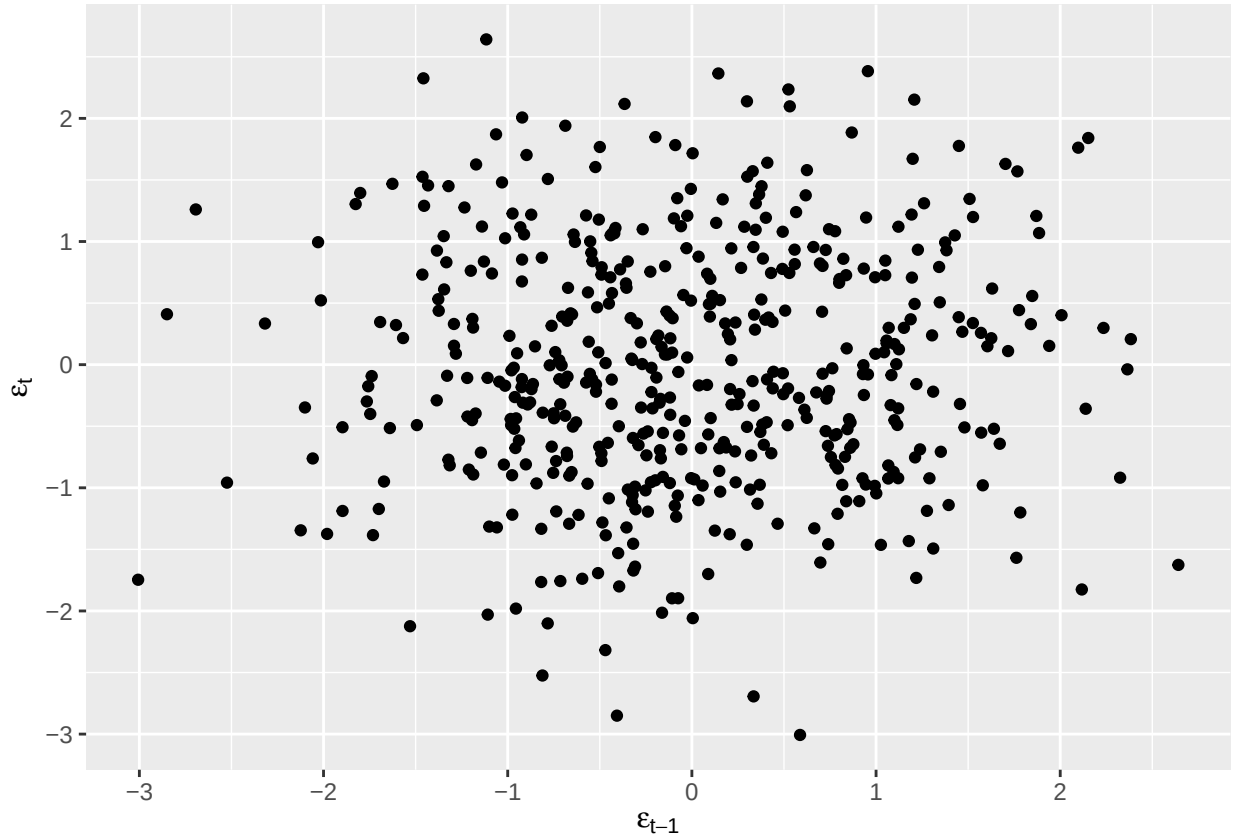
```
library(ggplot2)
x<-1:500
set.seed(6666)
y<-rnorm(500)
df<-data.frame(x=x,y=y)
p<-ggplot(df,aes(x=x,y=y))+
  geom_line()+
  xlab("t")+
  ylab(expression(epsilon[t]))
p
```



ϵ_t fluctuates around 0

ϵ_t (vs) ϵ_{t-1}

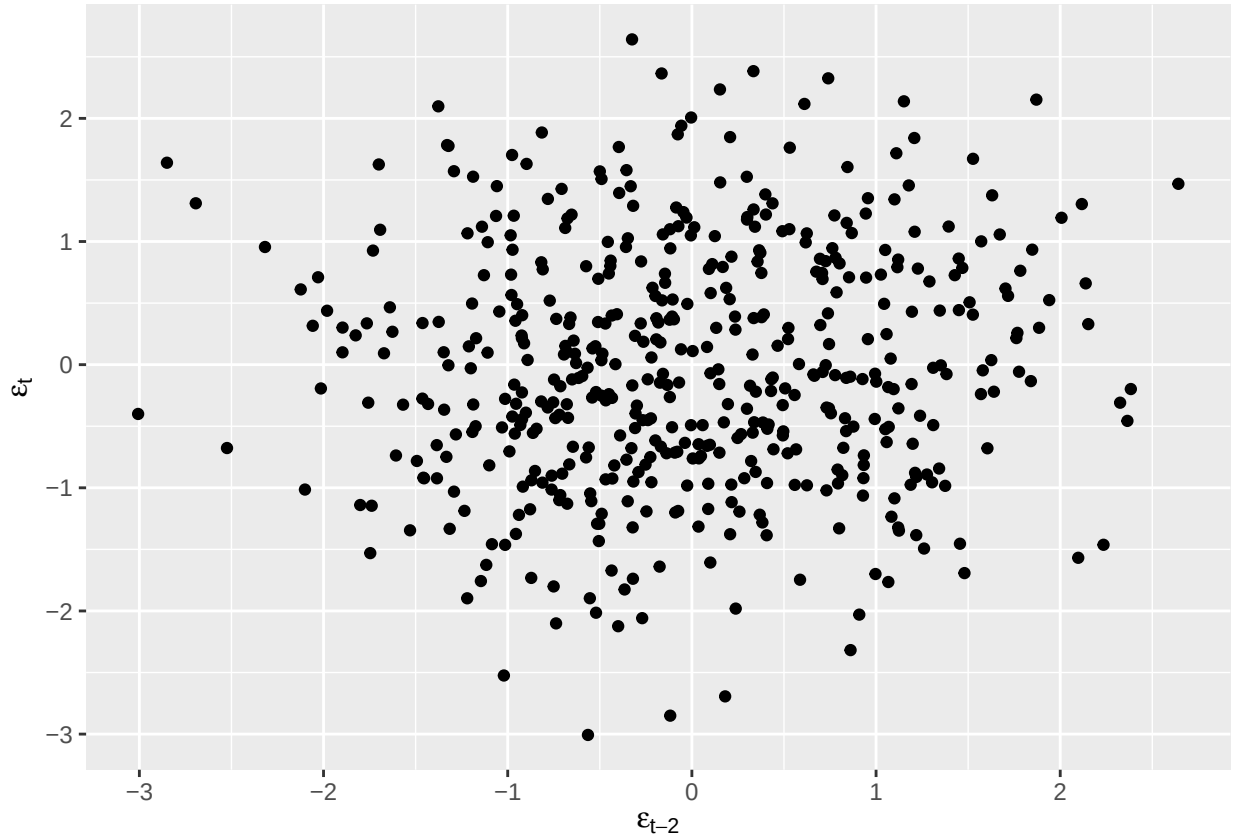
```
set.seed(6666)
y<-rnorm(501)
t<-501
df<-data.frame(x=y[1:c(t-1)],y=y[2:t])
p<-ggplot(df,aes(x=x,y=y))+
  geom_point()+
  xlab(expression(epsilon[t-1]))+
  ylab(expression(epsilon[t]))
p
```



it seems no correlation between ϵ_t and ϵ_{t-1}

ϵ_t (vs) ϵ_{t-2}

```
set.seed(6666)
y<-rnorm(502)
t<-502
df<-data.frame(x=y[1:(t-2)],y=y[3:t])
p<-ggplot(df,aes(x=x,y=y))+
  geom_point()+
  xlab(expression(epsilon[t-2]))+
  ylab(expression(epsilon[t]))
p
```



it seems no correlation between ϵ_t and ϵ_{t-2}

b)

mean

```
set.seed(6666)
y<-rnorm(500)
mean(y)
```

```
## [1] -0.02309015
```

variance

```
var(y)
```

```
## [1] 0.9683645
```

covariance γ_k

```
set.seed(6666)
y<-rnorm(1000)
getCov<-function(k){
  x1<-y[1:500]
  x2<-y[(k+1):(500+k)]
  cov(x1,x2)
}
k<-0:500
cov1<-sapply(k,getCov)
dfCov<-data.frame(k=k,cov=cov1)
head(dfCov)
```

```
##   k        cov
## 1 0 0.96836447
## 2 1 0.04149649
## 3 2 0.05859499
## 4 3 0.04959531
## 5 4 0.05376657
## 6 5 0.01807119
```

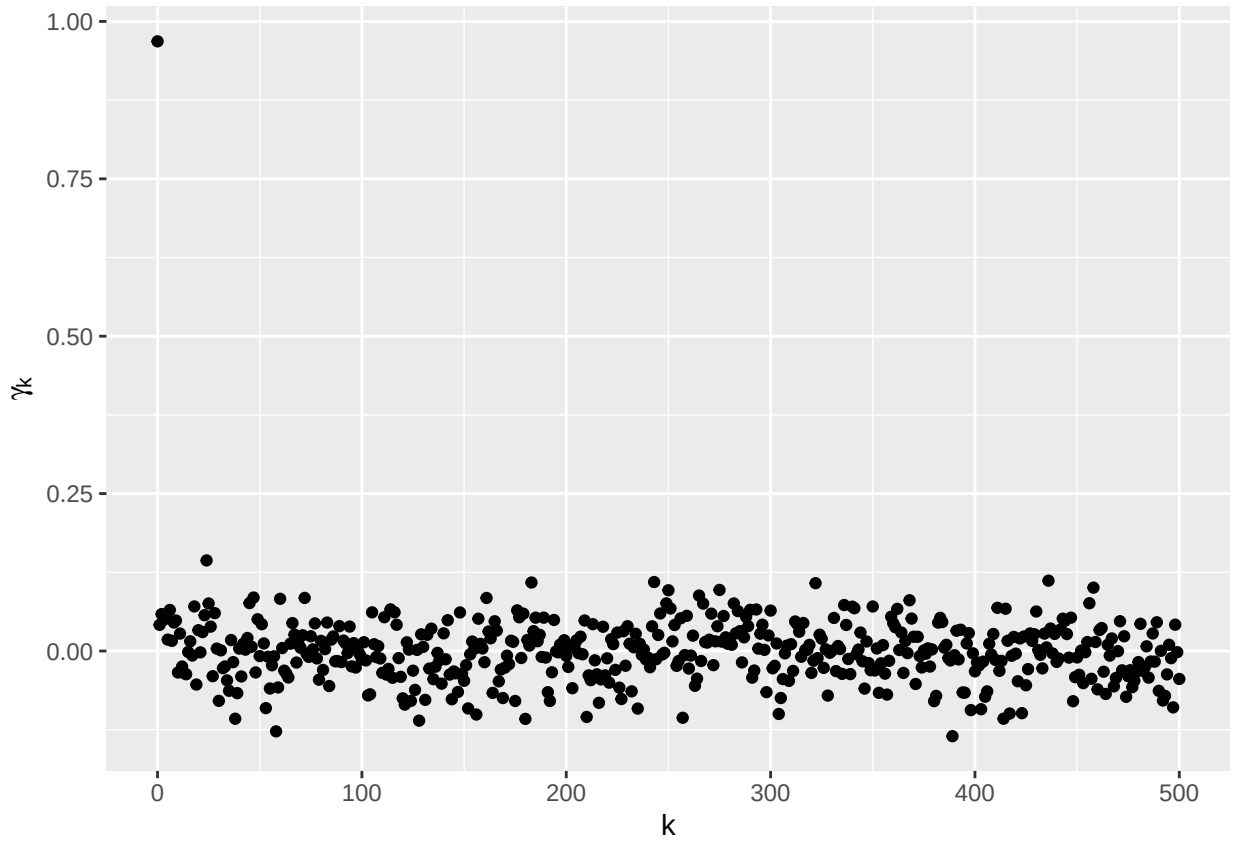
ρ_k

```
p<-cov1/cov1[1]
dfP<-data.frame(k=k,p=p)
head(dfP)
```

```
##   k          p
## 1 0 1.00000000
## 2 1 0.04285214
## 3 2 0.06050923
## 4 3 0.05121554
## 5 4 0.05552307
## 6 5 0.01866156
```

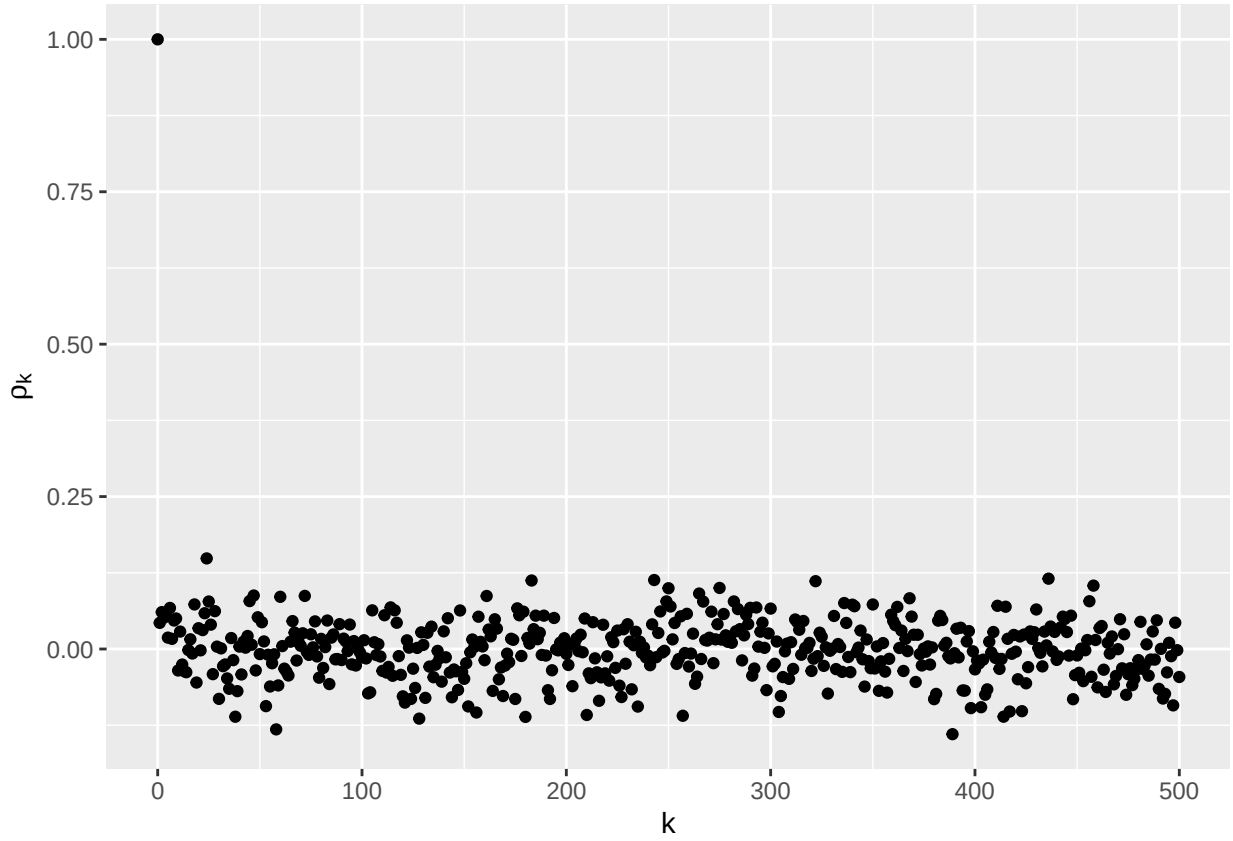
γ_k **(vs)** k

```
p<-ggplot(dfCov,aes(x=k,y=cov))+
  geom_point()+
  xlab(expression(k))+
  ylab(expression(gamma[k]))
p
```



$\rho_{\mathbf{k}}$ **(vs)** $\mathbf{k}$

```
p<-ggplot(dfP,aes(x=k,y=p))+
  geom_point()+
  xlab(expression(k))+
  ylab(expression(rho[k]))
p
```



c)

min

```
set.seed(6666)
y<-rnorm(500)
min(y)
```

```
## [1] -3.006701
```

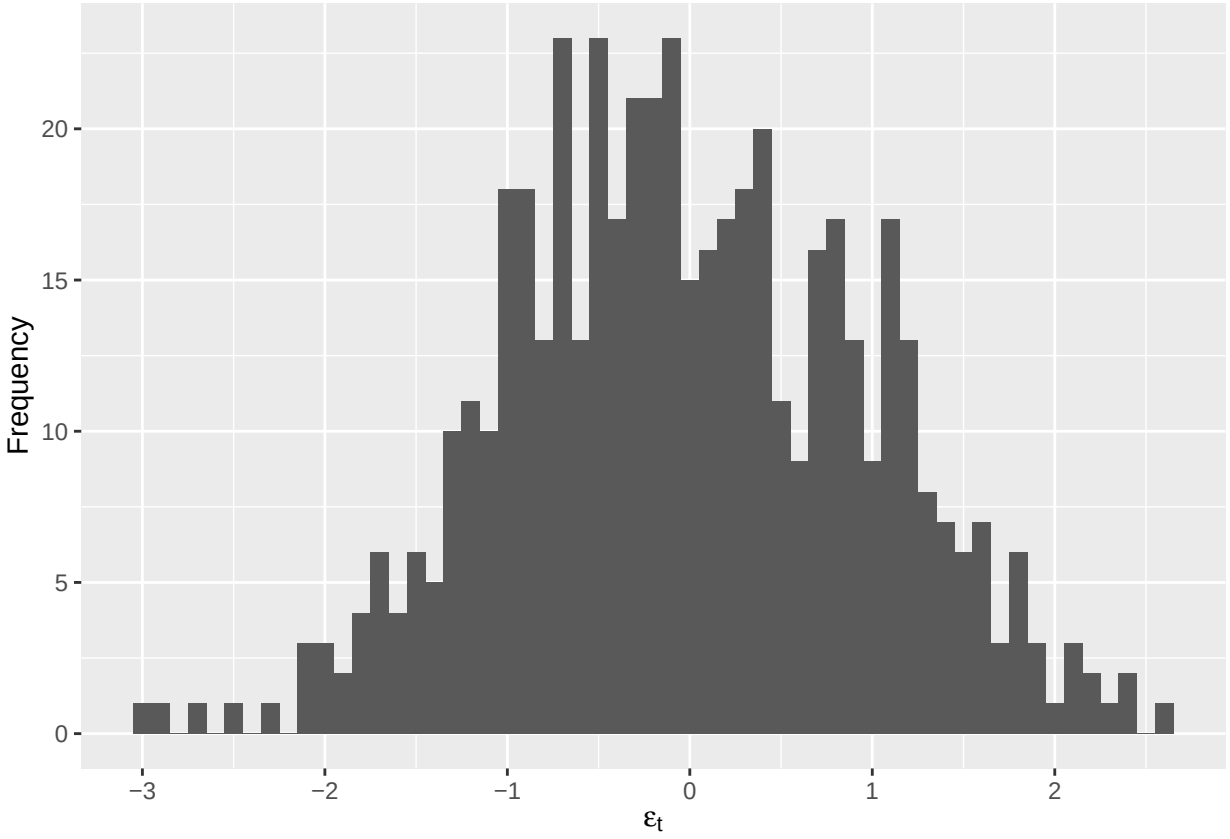
max

```
max(y)
```

```
## [1] 2.641487
```

histogram

```
qplot(y,  binwidth=0.1, geom = "histogram", xlab=expression(epsilon[t]), ylab="Frequency")
```

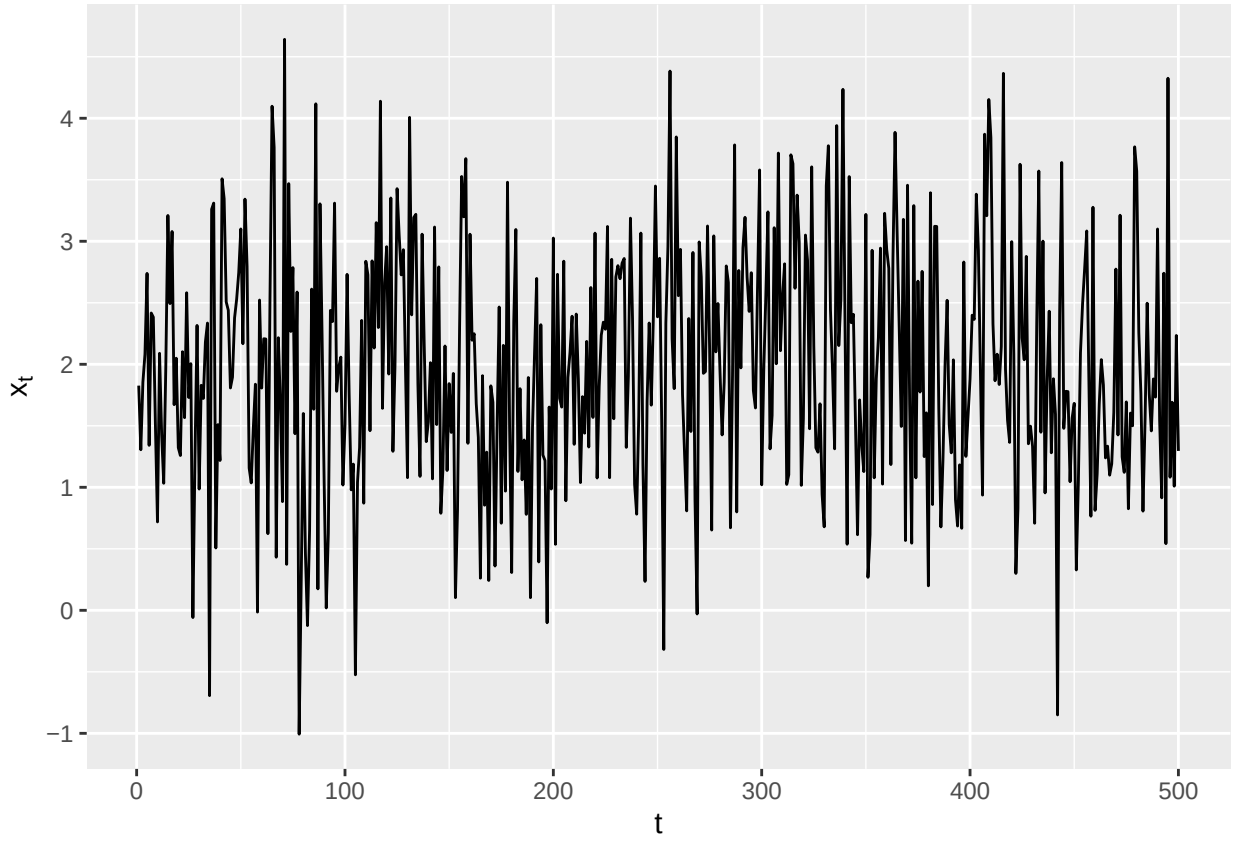


2) Repeat (Q1) on x_t when x_t is given by

a) x_t=2.0+ϵ_t, x_0=0

x_t(vs)t

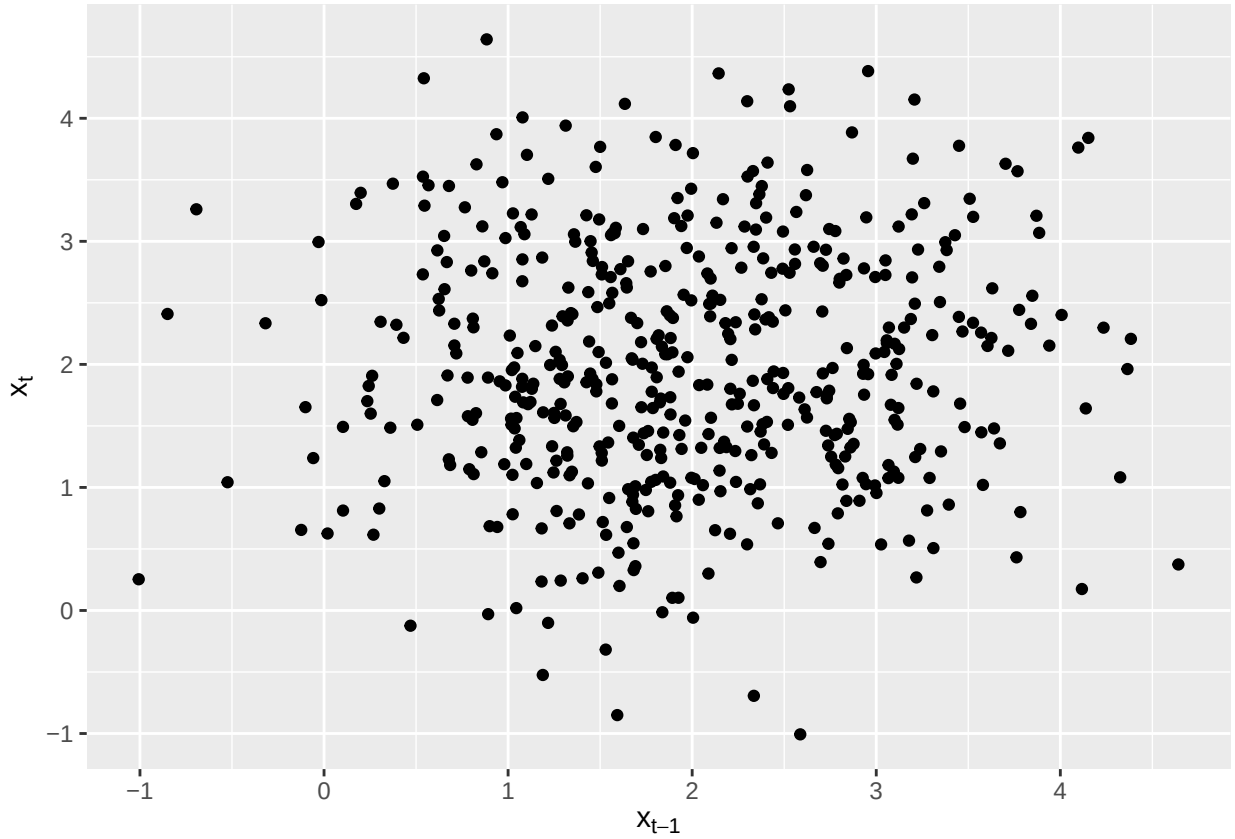
```
t<-1:500
set.seed(6666)
y<-rnorm(500)+2.0
df<-data.frame(x=t,y=y)
p<-ggplot(df,aes(x=x,y=y))+
  geom_line()+
  xlab("t")+
  ylab(expression(x[t]))
p
```



x_t fluctuates around 2

x_t (vs) x_{t-1}

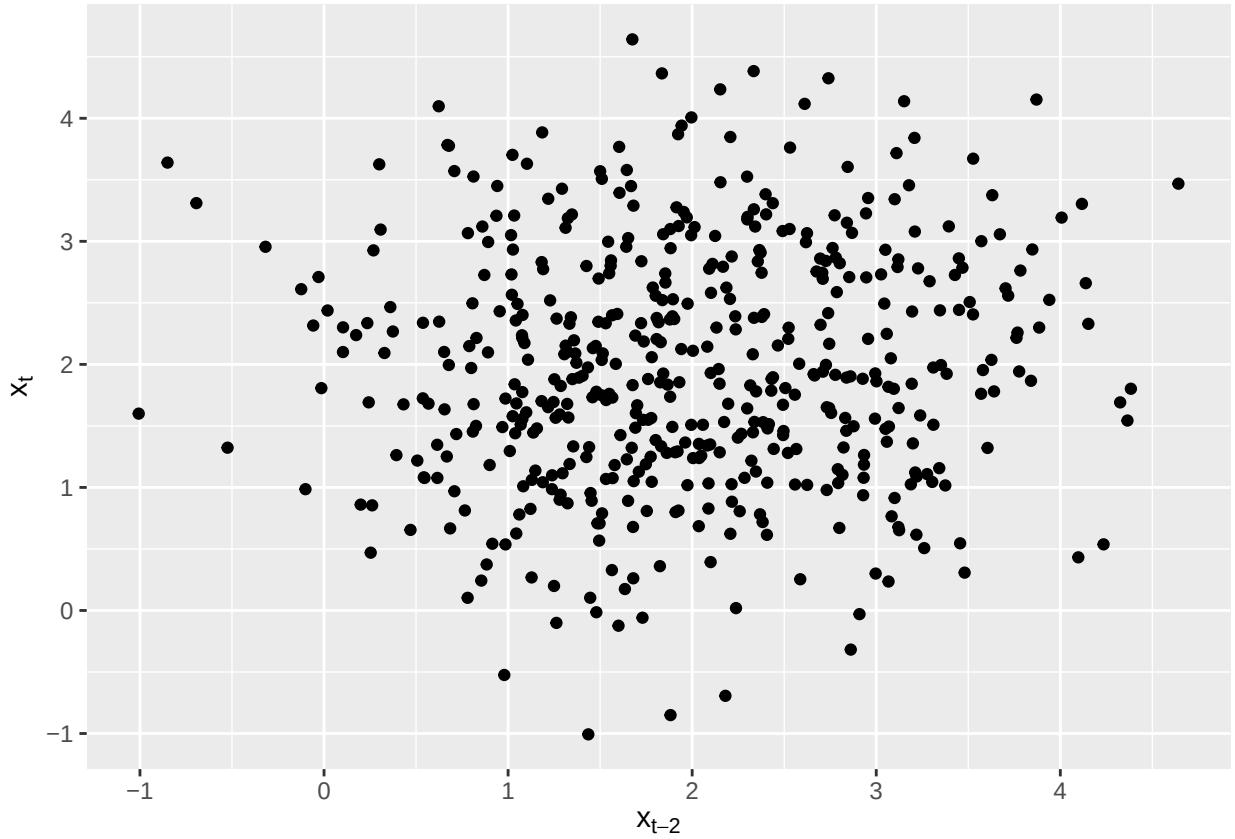
```
set.seed(6666)
y<-rnorm(501)+2.0
t<-501
df<-data.frame(x=y[1:c(t-1)],y=y[2:t])
p<-ggplot(df,aes(x=x,y=y))+
  geom_point()+
  xlab(expression(x[t-1]))+
  ylab(expression(x[t]))
p
```



it seems no correlation between x_t and x_{t-1}

x_t (vs) x_{t-2}

```
set.seed(6666)
y<-rnorm(502)+2.0
t<-502
df<-data.frame(x=y[1:(t-2)],y=y[3:t])
p<-ggplot(df,aes(x=x,y=y))+
  geom_point()+
  xlab(expression(x[t-2]))+
  ylab(expression(x[t]))
p
```



it seems no correlation between x_t and x_{t-2}

mean

```
set.seed(6666)
y<-rnorm(501)+2.0
mean(y)
```

```
## [1] 1.977736
```

variance

```
var(y)
```

```
## [1] 0.9667695
```

covariance γ_k

```
set.seed(6666)
y<-rnorm(1000)+2.0
```

```
getCov<-function(k){
  x1<-y[1:500]
  x2<-y[(k+1):(500+k)]
  cov(x1,x2)
}
k<-0:500
cov1<-sapply(k,getCov)
dfCov<-data.frame(k=k,cov=cov1)
head(dfCov)
```

```
##   k        cov
## 1 0 0.96836447
## 2 1 0.04149649
## 3 2 0.05859499
## 4 3 0.04959531
## 5 4 0.05376657
## 6 5 0.01807119
```

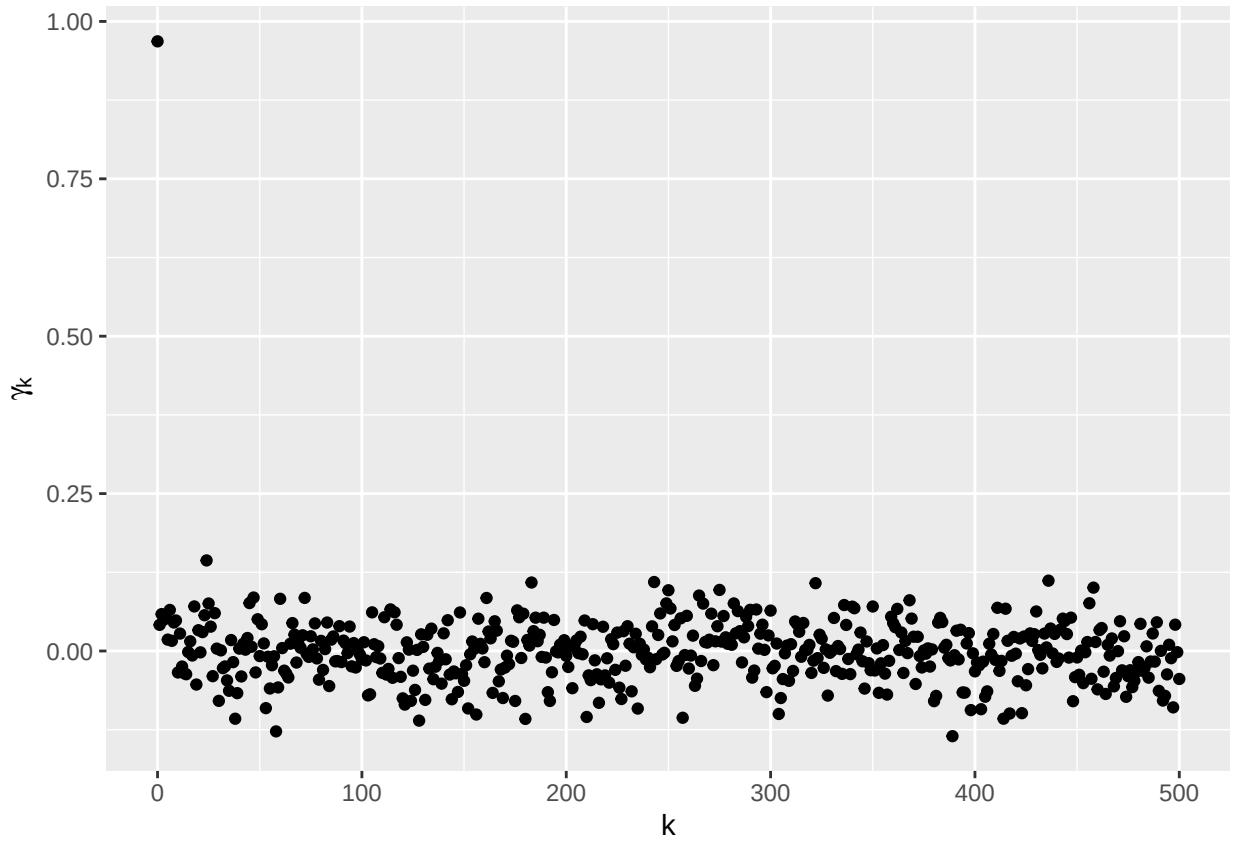
$\rho_{\mathbf{k}}$

```
p<-cov1/cov1[1]
dfP<-data.frame(k=k,p=p)
head(dfP)
```

```
##   k          p
## 1 0 1.00000000
## 2 1 0.04285214
## 3 2 0.06050923
## 4 3 0.05121554
## 5 4 0.05552307
## 6 5 0.01866156
```

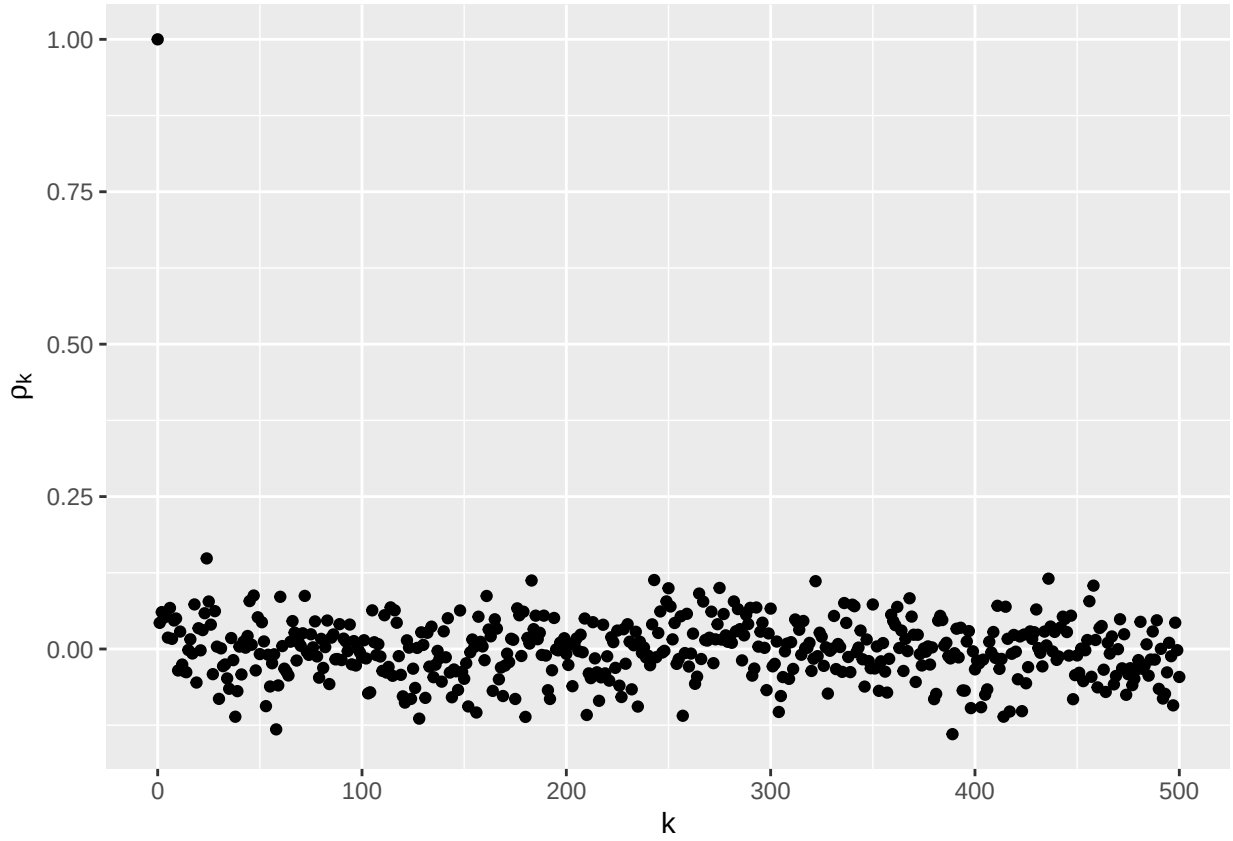
$\gamma_{\mathbf{k}}$ (vs) k

```
p<-ggplot(dfCov,aes(x=k,y=cov))+
  geom_point()+
  xlab(expression(k))+
  ylab(expression(gamma[k]))
p
```



$\rho_{\mathbf{k}}$ **(vs)** $\mathbf{k}$

```
p<-ggplot(dfP,aes(x=k,y=p))+
  geom_point()+
  xlab(expression(k))+
  ylab(expression(rho[k]))
p
```



min

```
set.seed(6666)
y<-rnorm(500)+2.0
min(y)
```

```
## [1] -1.006701
```

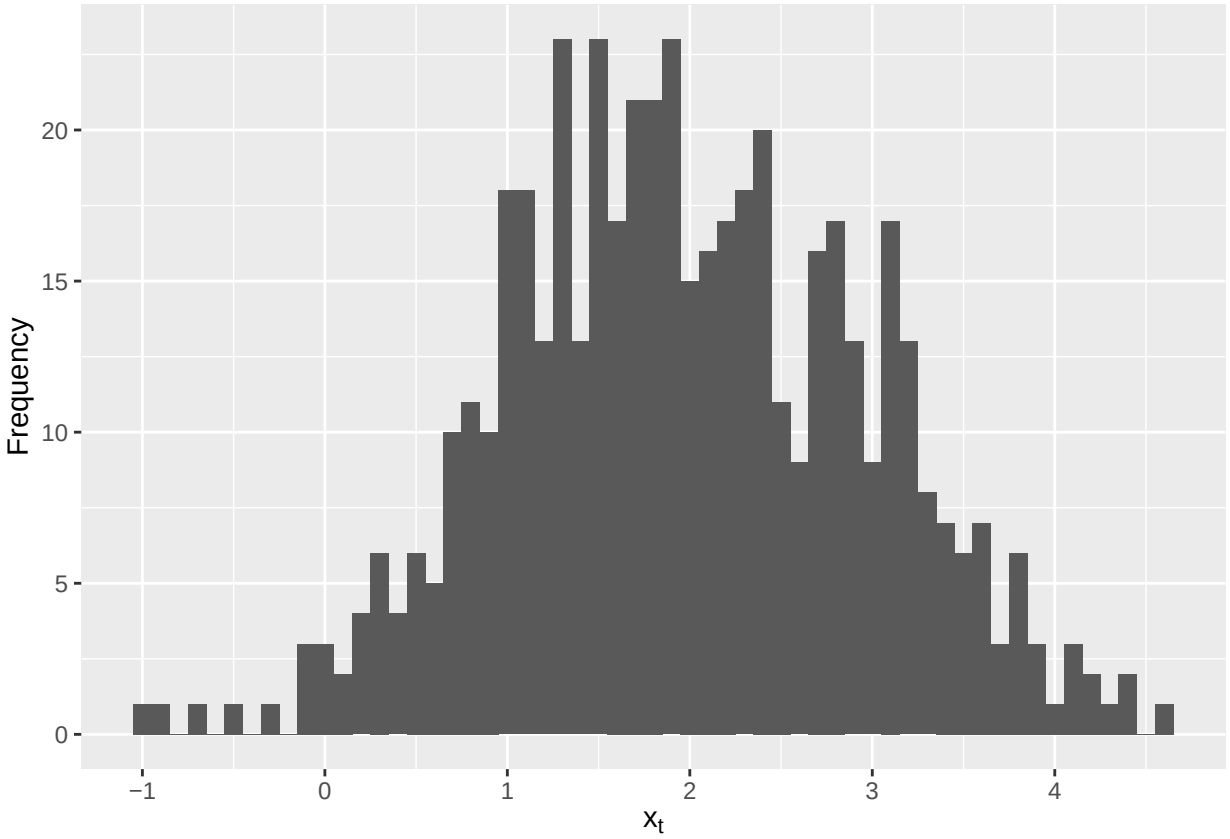
max

```
max(y)
```

```
## [1] 4.641487
```

histogram

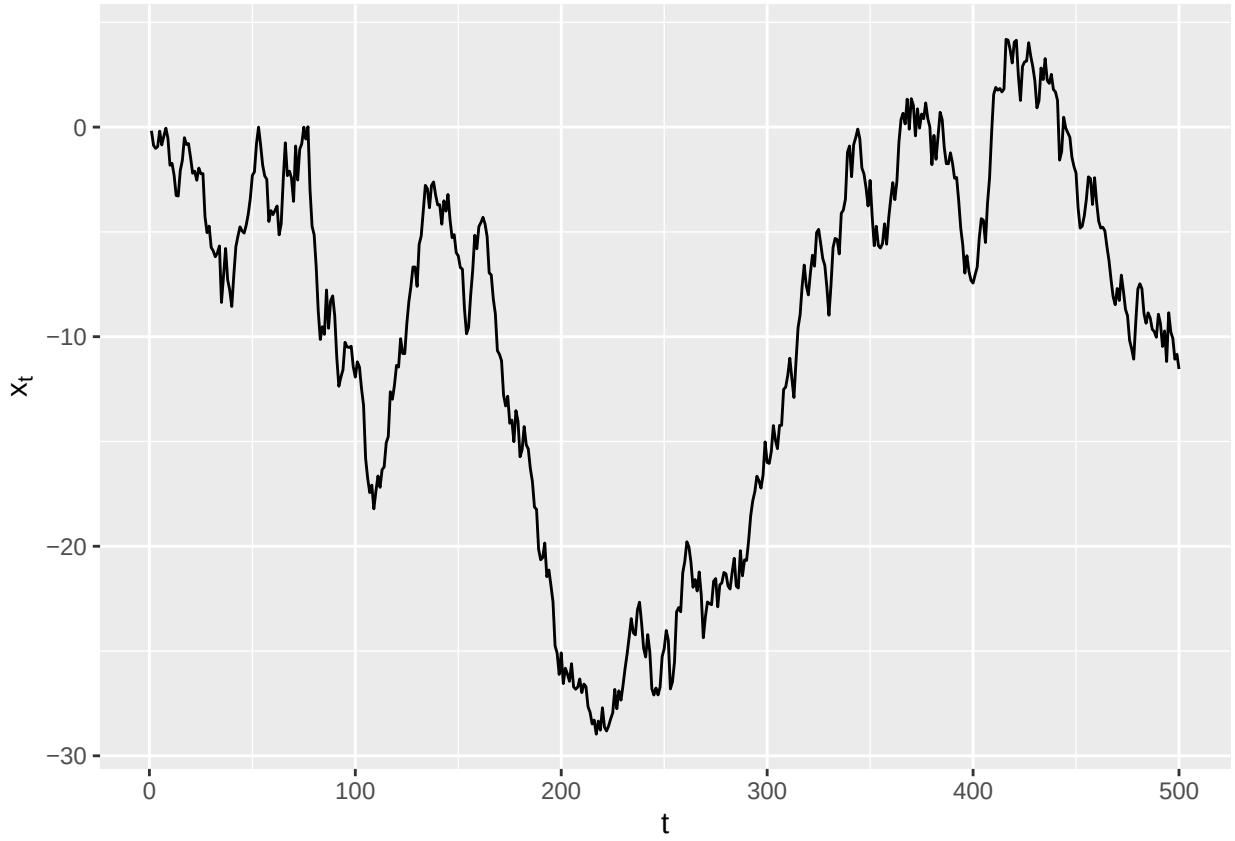
```
qplot(y,  binwidth=0.1, geom = "histogram", xlab=expression(x[t]), ylab="Frequency")
```



b) $x_t = x_{t-1} + \epsilon_t$, $x_0 = 0$

x_t(vs)t

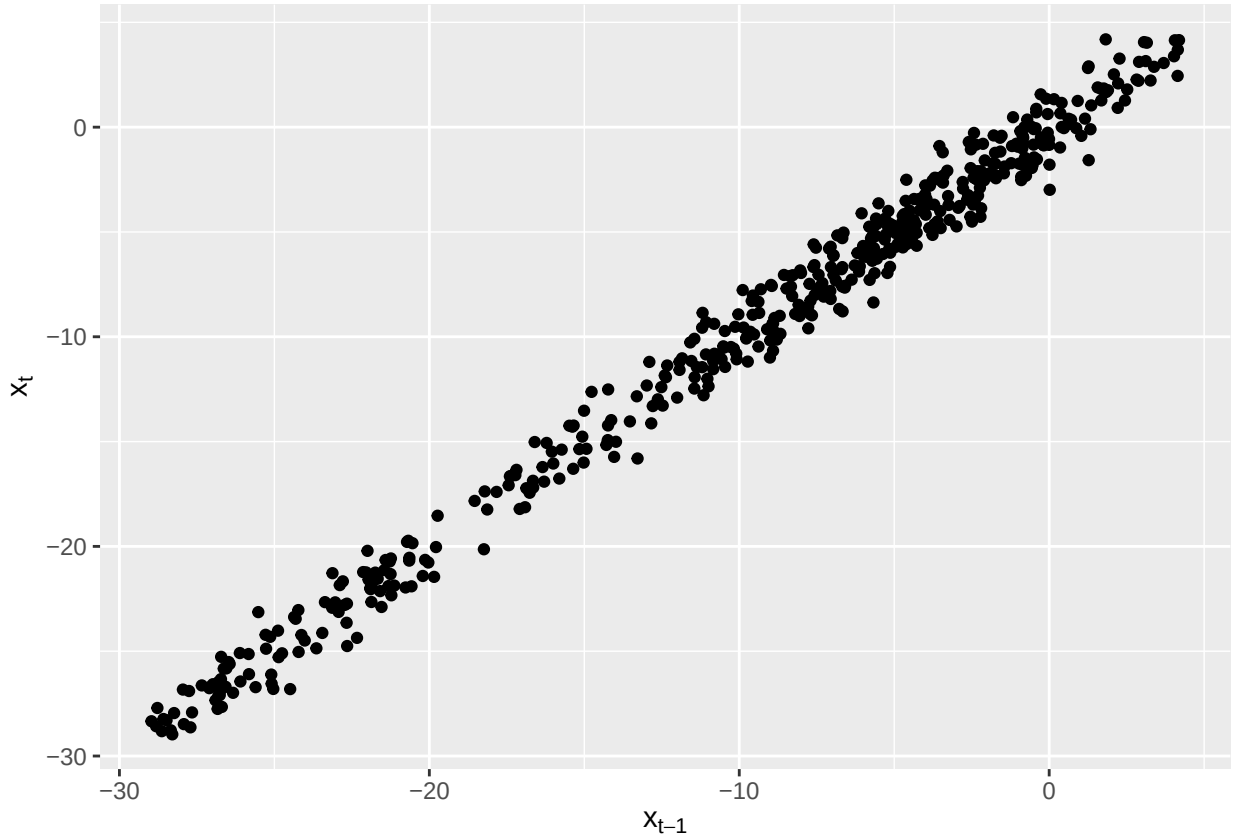
```
t<-1:500
set.seed(6666)
y<-rnorm(500)
xt<-function(t){
  if (t==0) return(0)
  else return(xt(t-1)+y[t])
}
y<-sapply(1:500,xt)
df<-data.frame(x=t,y=y)
p<-ggplot(df,aes(x=x,y=y))+
  geom_line()+
  xlab("t")+
  ylab(expression(x[t]))
p
```



x_t is unstable

x_t (vs) x_{t-1}

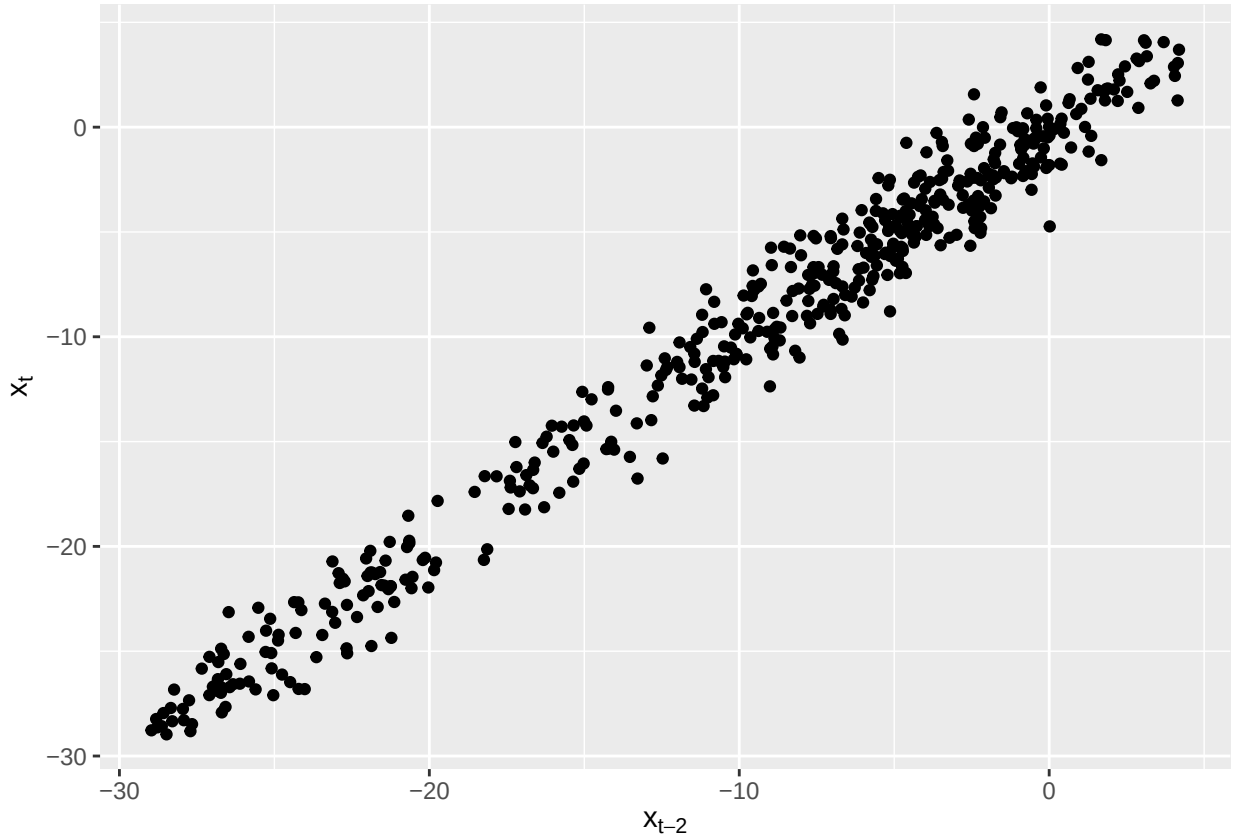
```
t<-501
set.seed(6666)
y<-rnorm(501)
xt<-function(t){
  if (t==0) return(0)
  else return(xt(t-1)+y[t])
}
y<-sapply(1:t,xt)
df<-data.frame(x=y[1:(t-1)],y=y[2:t])
p<-ggplot(df,aes(x=x,y=y))+
  geom_point()+
  xlab(expression(x[t-1]))+
  ylab(expression(x[t]))
p
```



it seems strong correlation between x_t and x_{t-1}

x_t (vs) x_{t-2}

```
t<-502
set.seed(6666)
y<-rnorm(502)
xt<-function(t){
  if (t==0) return(0)
  else return(xt(t-1)+y[t])
}
y<-sapply(1:t,xt)
df<-data.frame(x=y[1:(t-2)],y=y[3:t])
p<-ggplot(df,aes(x=x,y=y))+
  geom_point()+
  xlab(expression(x[t-2]))+
  ylab(expression(x[t]))
p
```



it seems strong correlation between x_t and x_{t-2}

mean

```
set.seed(6666)
y<-rnorm(500)
xt<-function(t){
  if (t==0) return(0)
  else return(xt(t-1)+y[t])
}
y<-sapply(1:500,xt)
mean(y)
```

```
## [1] -9.604342
```

variance

```
var(y)
```

```
## [1] 79.92241
```

covariance γ_k

```
set.seed(6666)
y<-rnorm(1000)
y<-cumsum(y)
getCov<-function(k){
  x1<-y[1:500]
  x2<-y[(k+1):(500+k)]
  cov(x1,x2)
}
k<-0:500
cov1<-sapply(k,getCov)
dfCov<-data.frame(k=k,cov=cov1)
head(dfCov)
```

```
##   k      cov
## 1 0 79.92241
## 2 1 79.35112
## 3 2 78.75464
## 4 3 78.09893
## 5 4 77.39016
## 6 5 76.61015
```

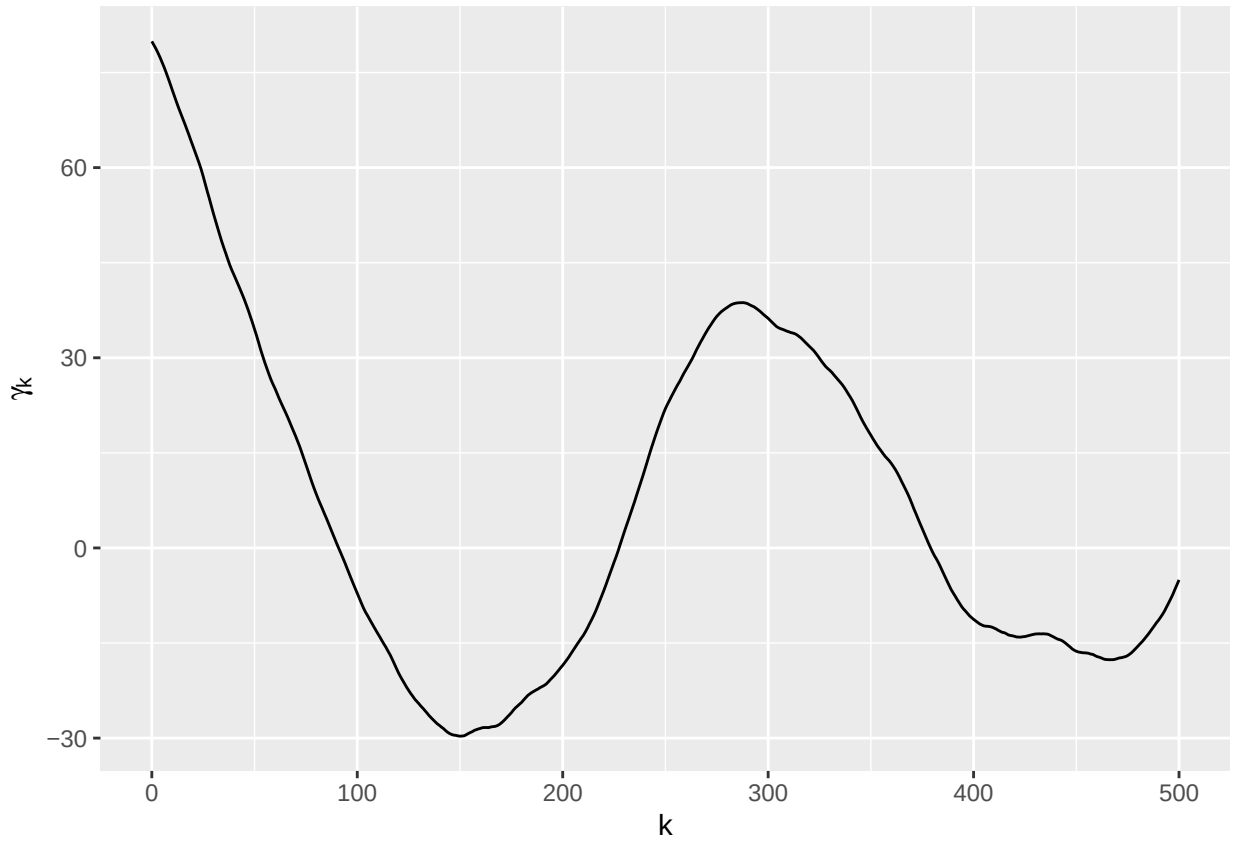
ρ_k

```
p<-cov1/cov1[1]
dfP<-data.frame(k=k,p=p)
head(dfP)
```

```
##   k         p
## 1 0 1.0000000
## 2 1 0.9928520
## 3 2 0.9853887
## 4 3 0.9771844
## 5 4 0.9683162
## 6 5 0.9585566
```

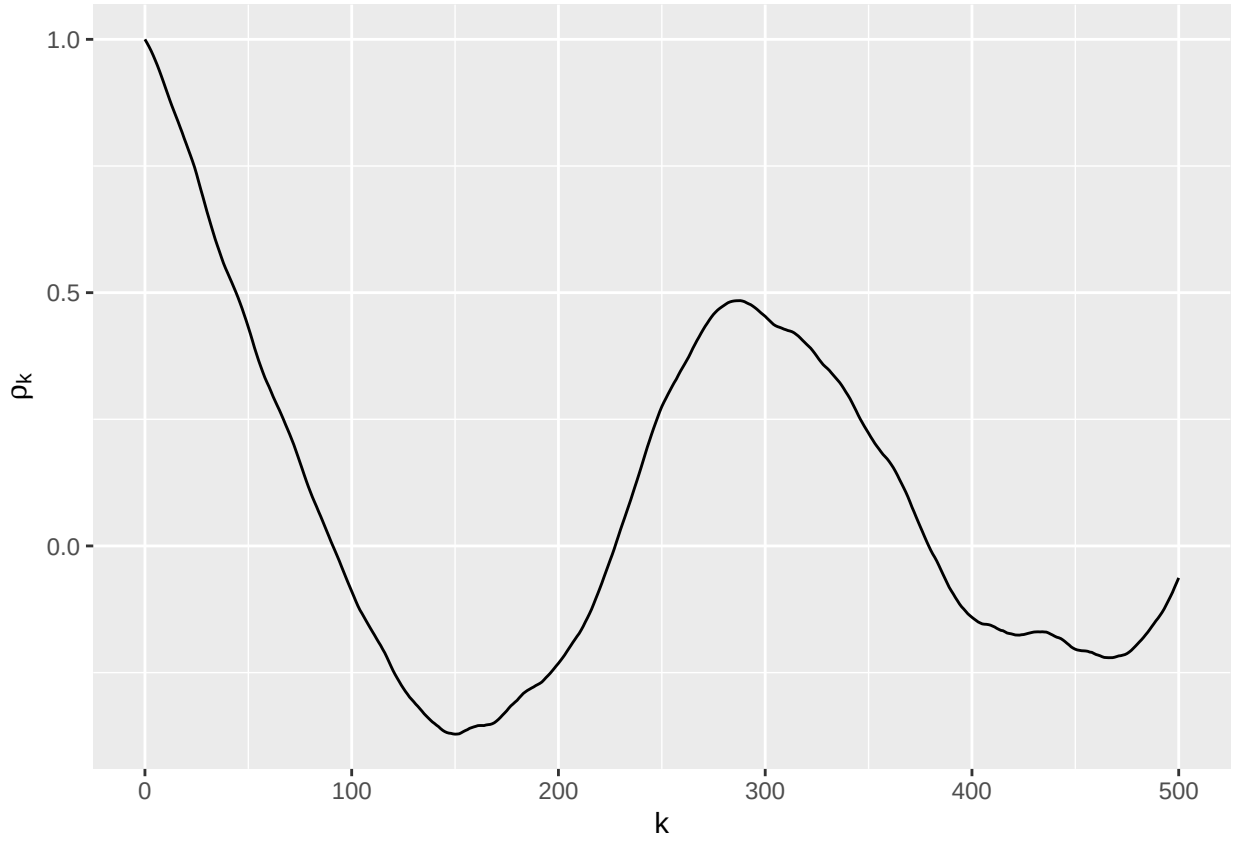
γ_k (vs) k

```
p<-ggplot(dfCov,aes(x=k,y=cov))+
  geom_line()+
  xlab(expression(k))+
  ylab(expression(gamma[k]))
p
```



$\rho_{\mathbf{k}}$ **(vs)** $\mathbf{k}$

```
p<-ggplot(dfP,aes(x=k,y=p))+
  geom_line()+
  xlab(expression(k))+
  ylab(expression(rho[k]))
p
```



min

```
set.seed(6666)
y<-rnorm(500)
y<-cumsum(y)
min(y)
```

```
## [1] -28.96659
```

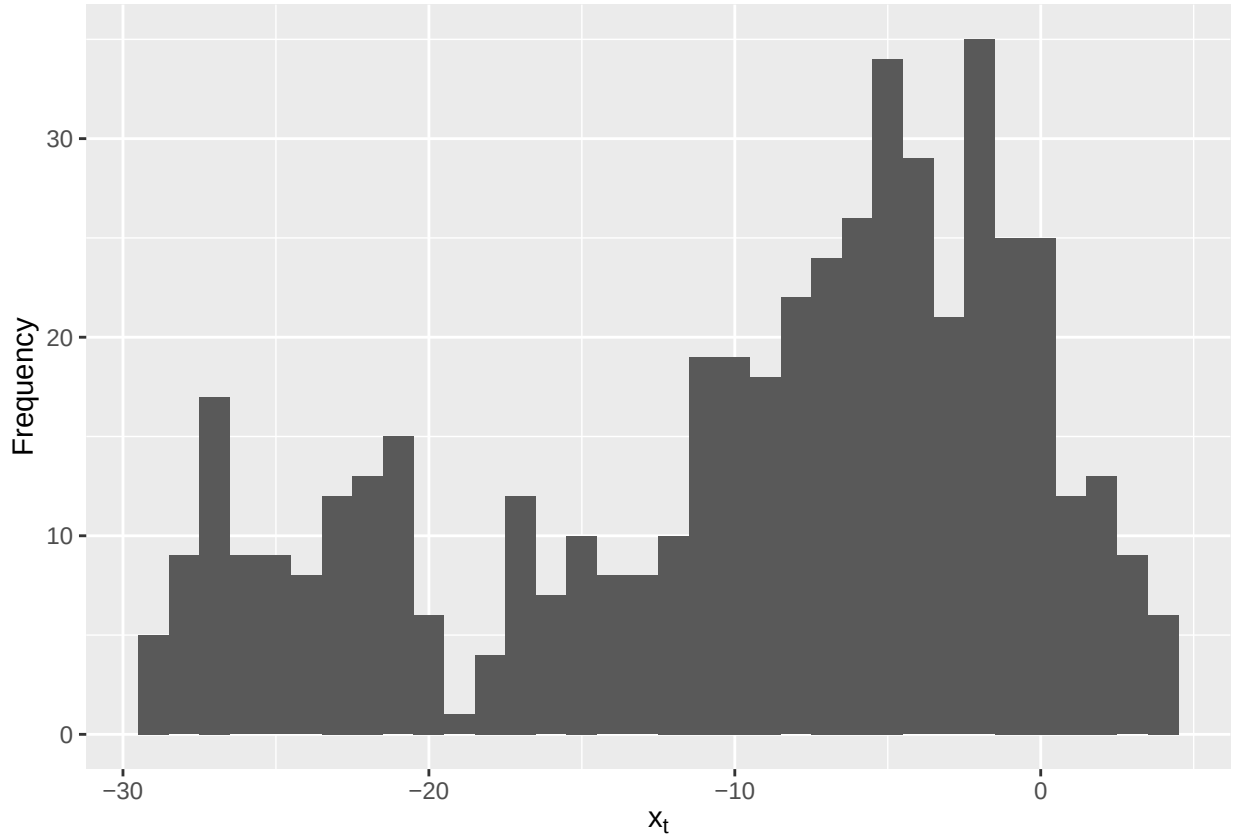
max

```
max(y)
```

```
## [1] 4.188588
```

histogram

```
qplot(y,  binwidth=1, geom = "histogram", xlab=expression(x[t]), ylab="Frequency")
```



3) Verify convariance function $f(i,j)$ is non-negtive definite

```
# define xt~N(0,1)
set.seed(6666)
xt<-rnorm(500)
# each column of xt is x(i) or x(j)
xt<-matrix(xt,ncol = 20)
# verify convariance function f(i,j) is non-negtive definite
totalSum<-0
for (i in 1:20){
  for (j in 1:20){
    totalSum<-totalSum+(t(xt[,i])*cov(xt[,i],xt[,j]))%*%matrix(xt[,j])
  }
}
# TRUE
totalSum>0
```

```
##       [,1]
## [1,] TRUE
```

```
# define xt~N(0,1)
set.seed(6666)
```

```r
xt<-rnorm(500)
# define yt=y(t-1)+xt
yt<-cumsum(xt)
# each column of yt is y(i) or y(j)
yt<-matrix(yt,ncol = 20)
# verify convariance function f(i,j) is non-negtive definite
totalSum<-0
for (i in 1:20){
  for (j in 1:20){
    totalSum<-totalSum+(t(yt[,i])*cov(yt[,i],yt[,j]))%*%matrix(yt[,j])
  }
}
# TRUE
totalSum>0
```

```
##       [,1]
## [1,] TRUE
```