FINAL EXAM

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Contents

1.Consider the ARMA(1, 2) model

$$Z_t = 0.5 + 0.8Z_{t-1} + \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2}$$

a. Compute the theoretical ACF and plot it

definition

$$Z_t = c + \phi_1 Z_{t-1} + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}$$

theoretical Autocovariance

$$Z_t - \mu = \phi_1(Z_{t-1} - \mu) + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}$$

$$\begin{aligned}\gamma_0 &= E[Z_t - \mu]^2 \\ &= E[\phi_1(Z_{t-1} - \mu) + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}]^2 \\ &= \phi_1^2\gamma_0 + \sigma^2 + \theta_1^2\sigma^2 + \theta_2^2\sigma^2 + 2\phi_1\theta_1\sigma^2 + 2\phi_1\theta_2(\phi_1 + \theta_1)\sigma^2\end{aligned}$$

$$\gamma_0 = \frac{\sigma^2[1 + 2\phi_1\theta_1 + 2\phi_1\theta_2(\phi_1 + \theta_1) + \theta_1^2 + \theta_2^2]}{1 - \phi_1^2}$$

$$\begin{aligned}\gamma_1 &= E[(Z_t - \mu)(Z_{t-1} - \mu)] \\ &= E\{[\phi_1(Z_{t-1} - \mu) + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}][Z_{t-1} - \mu]\} \\ &= \phi_1\gamma_0 + \theta_1\sigma^2 + \theta_2(\phi_1 + \theta_1)\sigma^2\end{aligned}$$

$$\begin{aligned}\gamma_2 &= E[(Z_t - \mu)(Z_{t-2} - \mu)] \\ &= E\{[\phi_1(Z_{t-1} - \mu) + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}][Z_{t-2} - \mu]\} \\ &= \phi_1\gamma_1 + \theta_2\sigma^2\end{aligned}$$

$for\ j \geqslant 3$

$$\begin{aligned}
\gamma_j &= E[(Z_t - \mu)(Z_{t-j} - \mu)] \\
&= E\{[\phi_1(Z_{t-1} - \mu) + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2}][Z_{t-j} - \mu]\} \\
&= \phi_1 E[(Z_{t-1} - \mu)(Z_{t-j} - \mu)] \\
&= \phi_1\gamma_{j-1}
\end{aligned}$$

theoretical Autocorrelation

$$\begin{aligned}
\gamma_0 &= \frac{\sigma^2[1 + 2\phi_1\theta_1 + 2\phi_1\theta_2(\phi_1 + \theta_1) + \theta_1^2 + \theta_2^2]}{1 - \phi_1^2} \\
\gamma_1 &== \phi_1\gamma_0 + \theta_1\sigma^2 + \theta_2(\phi_1 + \theta_1)\sigma^2 \\
\gamma_2 &== \phi_1\gamma_1 + \theta_2\sigma^2 \\
\gamma_j &= \phi_1\gamma_{j-1} \quad for\ j \geqslant 3 \\
\rho_1 &= \frac{\gamma_1}{\gamma_0} \\
\rho_2 &= \frac{\gamma_2}{\gamma_0} \\
\rho_j &= \frac{\gamma_j}{\gamma_0}
\end{aligned}$$

plot

$$x_t = \phi$$

$$x_t = \phi$$

$$x_t = \phi$$

$$x_t = \phi$$