MID-TERM EXAM - 2

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Contents

1.(20+20 pts) Apply the method of module 7.2 for estimating the parameters a and b in the model

from a set of m samples $(y_i, x_i),\ 1 \leqslant i \leqslant m$.

A) $y = a + e^{-bx}$

define a linear relation

$$y_i = a + bx_i$$

by the Least Squares method we estimate the parameters a and b

$$\begin{aligned}\hat{b} &= \frac{\bar{xy} - \bar{x}\bar{y}}{\bar{x^2} - (\bar{x})^2} \\ &= \frac{m\sum_{i=1}^{m} x_i y_i - (\sum_{i=1}^{m} x_i)(\sum_{i=1}^{m} y_i)}{m(\sum_{i=1}^{m} x_i^2) - (\sum_{i=1}^{m} x_i)^2}\end{aligned}$$

$$\begin{aligned}\hat{a} &= \frac{1}{m}\left[\sum_{i=1}^{m} y_i - \hat{b}\sum_{i=1}^{m} x_i\right] \\ &= \bar{y} - \hat{b}\bar{x}\end{aligned}$$

convert

$$y = a + e^{-bx}$$
$$\ln(y) = \ln(a) - bx$$

let ${y_i}' = \ln(y_i)\ a' = \ln(a)\ b' = -b$

$$y_i{}' = a' + b'x_i$$

$$\begin{aligned}
\hat{b'} &= \frac{x\,\bar{\ln}(y) - \bar{x}\bar{ln}(y)}{\bar{x^2} - (\bar{x})^2} \\
&= \frac{m\sum_{i=1}^{m} x_i \ln(y_i) - (\sum_{i=1}^{m} x_i)(\sum_{i=1}^{m} \ln(y_i))}{m(\sum_{i=1}^{m} x_i^2) - (\sum_{i=1}^{m} x_i)^2} \\
\hat{b} &= -\hat{b'}
\end{aligned}$$

$$\begin{aligned}
\hat{a'} &= \frac{1}{m}\left[\sum_{i=1}^{m} \ln(y_i) - \hat{b'}\sum_{i=1}^{m} x_i\right] \\
&= \bar{\ln}(y) - \hat{b'}\bar{x} \\
a &= e^{a'} \\
a &= \bar{y}e^{\hat{b}\bar{x}}
\end{aligned}$$

B) $y = c + mx$

let $a' = c$ $b' = m$

$$y_i = a' + b'x_i$$

convert

$$\begin{aligned}
\hat{m} &= \frac{\bar{xy} - \bar{x}\bar{y}}{\bar{x^2} - (\bar{x})^2} \\
&= \frac{m\sum_{i=1}^{m} x_i y_i - (\sum_{i=1}^{m} x_i)(\sum_{i=1}^{m} y_i)}{m(\sum_{i=1}^{m} x_i^2) - (\sum_{i=1}^{m} x_i)^2}
\end{aligned}$$

$$\begin{aligned}
\hat{c} &= \frac{1}{m}\left[\sum_{i=1}^{m} y_i - \hat{m}\sum_{i=1}^{m} x_i\right] \\
&= \bar{y} - \hat{m}\bar{x}
\end{aligned}$$