

HW2_Yi_Xiong

Yi Xiong

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Contents

1) Generate one set of 100 samples of $x_t \sim \text{iid } N(0,1)$, and compute its mean \bar{x}, and variance $\hat{\sigma}^2$	1
a) Repeat this experiment 100 times and obtain 100 estimates of \bar{x} , and $\hat{\sigma}^2$	1
b) Compute the mean, variance of \bar{x}_i , for $1 \leq i \leq 100$	2
c) Compute the mean, variance of $\hat{\sigma}_i^2$, for $1 \leq i \leq 100$	2
2) Generate $\varepsilon_t \sim \text{iid } N(0,1)$ for $1 \leq t \leq 500$	2
a) Define $x_t = \varepsilon_t + 0.5\varepsilon_{t-1}$ called MA(1) process	3
b) Plot x_t (vs) x_{t-1} , x_t (vs) x_{t-2} , x_t (vs) t	3
c) Compute the mean, variance and autocorrelation of x_t and plot	5
3) Plot $N(0, \sigma^2)$ for $\sigma^2 = 0.1, 0.5, 1.0, 2.0, 5.0$ on the same plot and comment	7

1) Generate one set of 100 samples of $x_t \sim \text{iid } N(0,1)$, and compute its mean \bar{x} , and variance $\hat{\sigma}^2$

```
set.seed(6666)
x<-rnorm(100)
# mean
(x_mean<-mean(x))
```

```
## [1] -0.1192927
```

```
# variance
(x_var<-var(x))
```

```
## [1] 1.124203
```

a) Repeat this experiment 100 times and obtain 100 estimates of \bar{x} , and $\hat{\sigma}^2$

```

xt <- data.frame(T = 1:100)
for (i in 1:100) {
  xt <- cbind(xt, rnorm(100))
}
colnames(xt)[-1] <- paste0("S", 1:100)
# mean
head(xt_mean <- colMeans(xt[, -1]))

```

```

##           S1           S2           S3           S4           S5           S6
## -0.13158945  0.09087463  0.08560193 -0.04104514  0.14779573  0.12226931

```

```

# variance
head(xt_var <- apply(xt[, -1], 2, var))

```

```

##           S1           S2           S3           S4           S5           S6
## 0.9685090 0.7678836 1.0244348 0.9493001 1.0950379 1.0968314

```

b) Compute the mean, variance of \bar{x}_i , for $1 \leq i \leq 100$

```

# mean
(xt_mean_mean <- mean(xt_mean))

```

```

## [1] 0.008002791

```

```

# variance
(xt_mean_var <- var(xt_mean))

```

```

## [1] 0.00922678

```

c) Compute the mean, variance of $\widehat{\sigma}_i^2$, for $1 \leq i \leq 100$

```

# mean
(xt_var_mean <- mean(xt_var))

```

```

## [1] 0.9880721

```

```

# variance
(xt_var_var <- var(xt_var))

```

```

## [1] 0.02034064

```

2) Generate $\varepsilon_t \sim \text{iid } N(0,1)$ for $1 \leq t \leq 500$

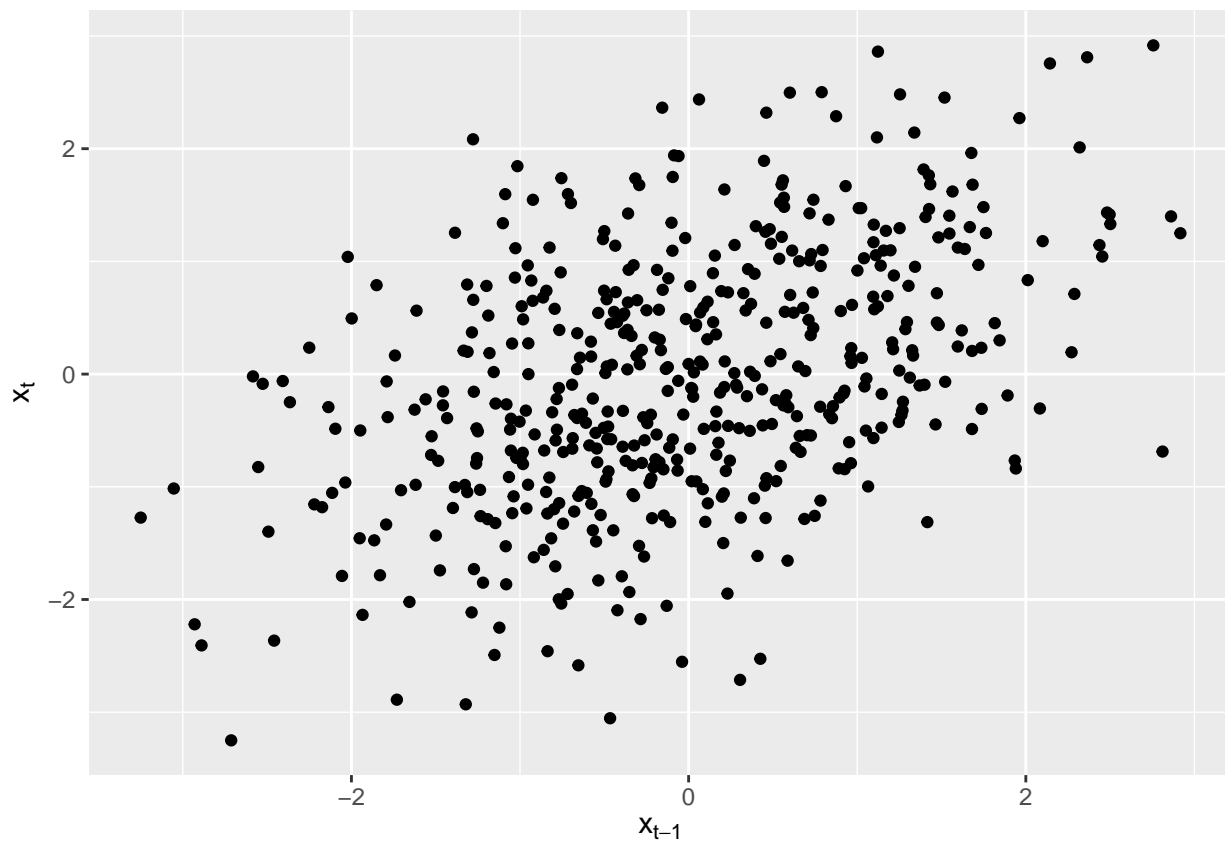
```
t<-500
set.seed(6666)
x<-rnorm(500)
```

a) Define $x_t = \varepsilon_t + 0.5\varepsilon_{t-1}$ called MA(1) process

```
xt<-c(x[1])
for (t in 2:500){
  xt[t]<-x[t]+0.5*x[t-1]
}
```

b) Plot x_t (vs) x_{t-1} , x_t (vs) x_{t-2} , x_t (vs) t

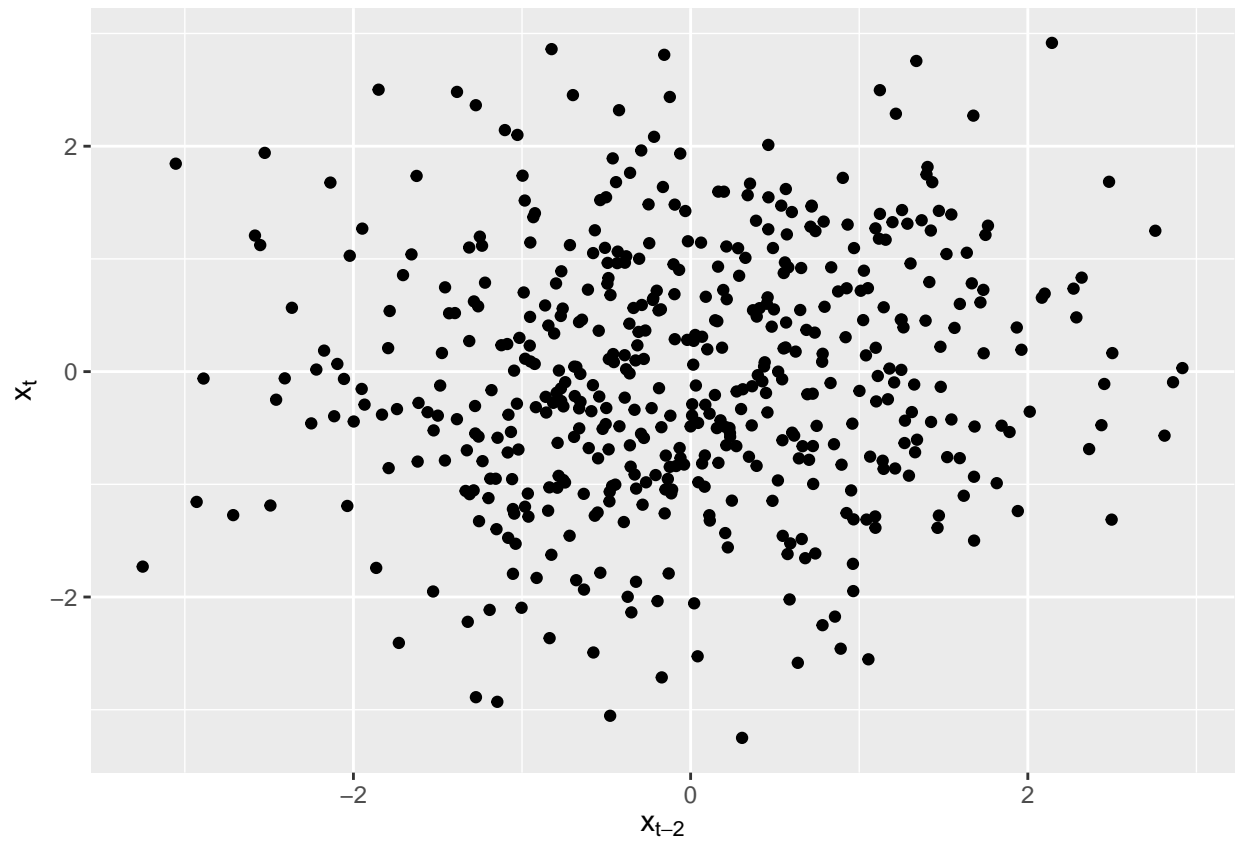
```
library(ggplot2)
# t vs t-1
df<-data.frame(x=xt[1:(t-1)],y=xt[2:t])
p<-ggplot(df,aes(x=x,y=y))+
  geom_point()+
  xlab(expression(x[t-1]))+
  ylab(expression(x[t]))
p
```



```

# t vs t-2
df<-data.frame(x=xt[1:(t-2)],y=xt[3:t])
p<-ggplot(df,aes(x=x,y=y))+
  geom_point()+
  xlab(expression(x[t-2]))+
  ylab(expression(x[t]))
p

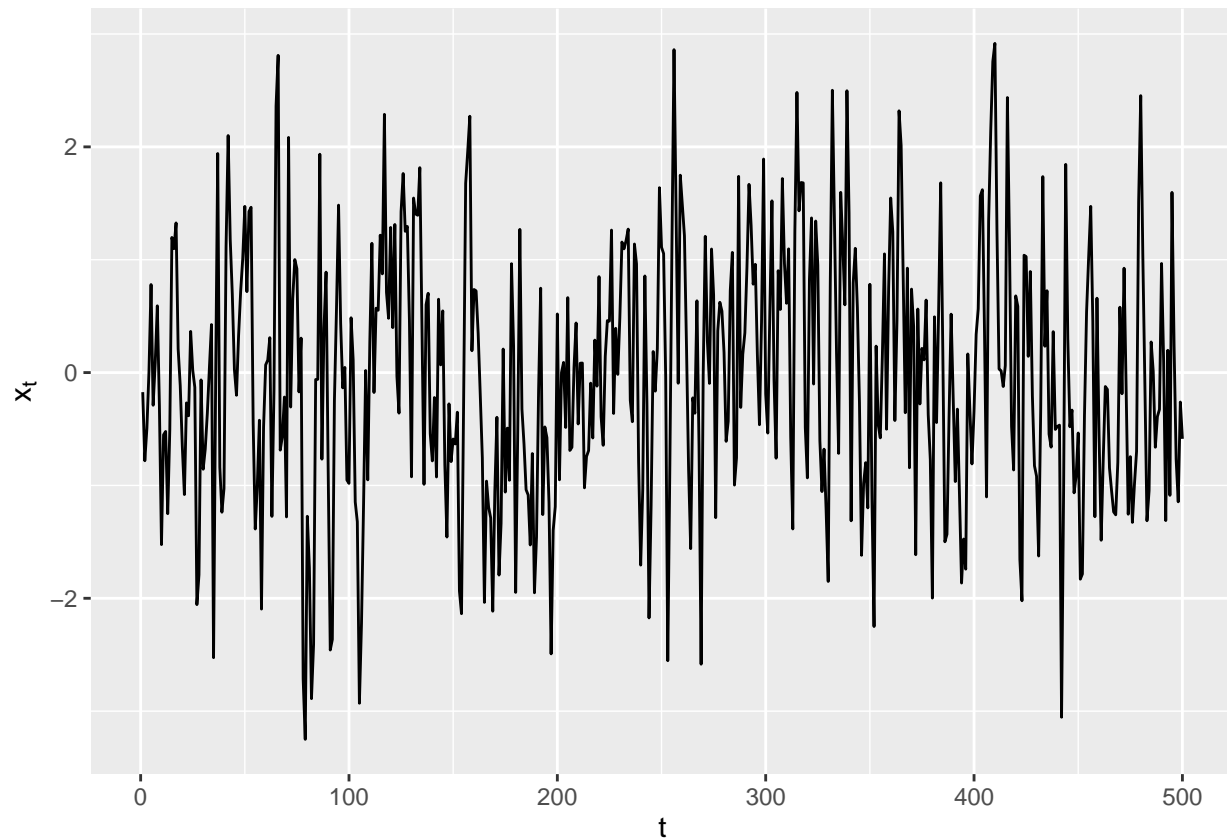
```



```

# xt vs t
df<-data.frame(x=1:t,y=xt)
p<-ggplot(df,aes(x=x,y=y))+
  geom_line()+
  xlab(expression(t))+
  ylab(expression(x[t]))
p

```



c) Compute the mean, variance and autocorrelation of x_t and plot

```
# mean
(xt_mean<-mean(xt))
```

```
## [1] -0.03393109
```

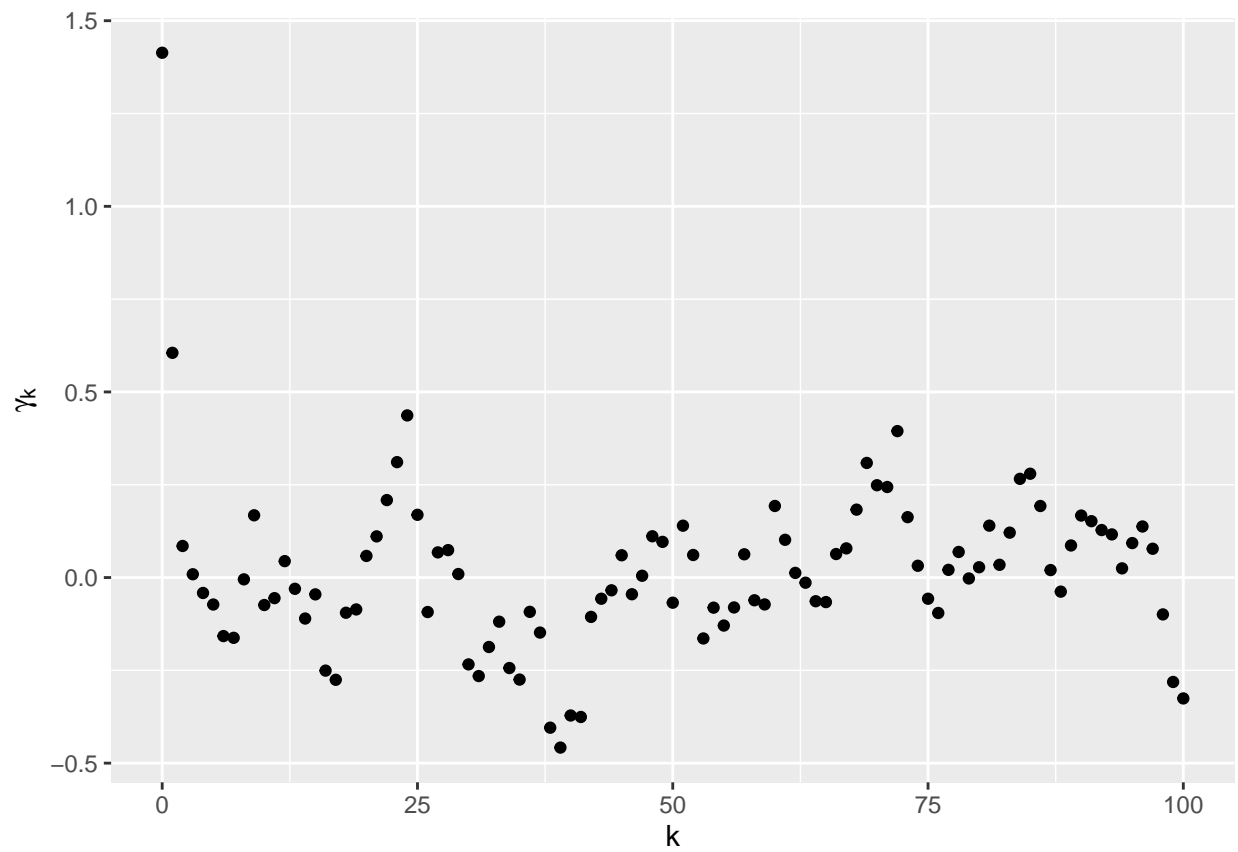
```
# var
(xt_var<-var(xt))
```

```
## [1] 1.252277
```

```
# autocorrelation
getCov<-function(k){
  x1<-xt[1:100]
  x2<-xt[(k+1):(100+k)]
  cov(x1,x2)
}
k<-0:100
cov1<-sapply(k,getCov)
dfCov<-data.frame(k=k,cov=cov1)
p<-ggplot(dfCov,aes(x=k,y=cov))+
```

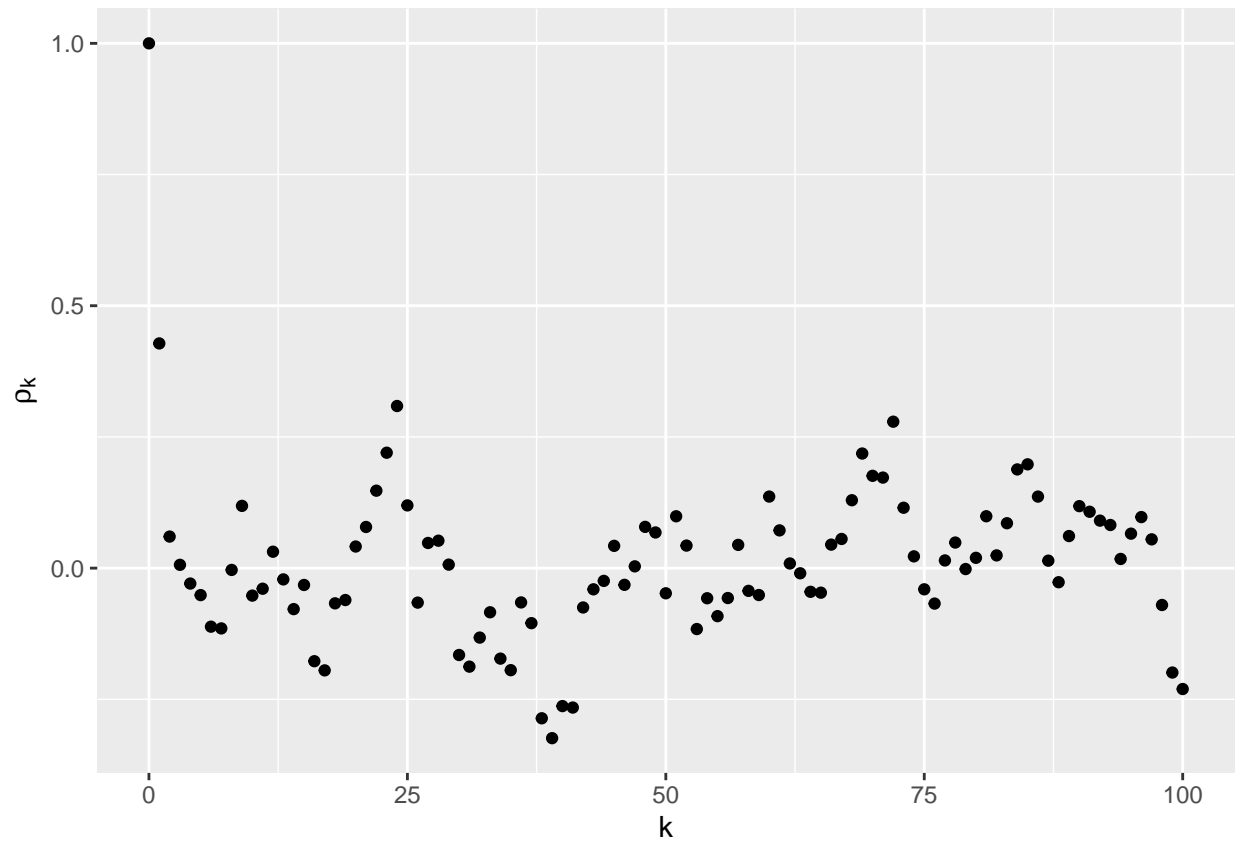
```
geom_point()+
xlab(expression(k))+
ylab(expression(gamma[k]))
```

p



```
p<-cov1/cov1[1]
dfP<-data.frame(k=k,p=p)
p<-ggplot(dfP,aes(x=k,y=p))+
  geom_point()+
  xlab(expression(k))+
  ylab(expression(rho[k]))
```

p



3) Plot $N(0, \sigma^2)$ for $\sigma^2 = 0.1, 0.5, 1.0, 2.0, 5.0$ on the same plot and comment

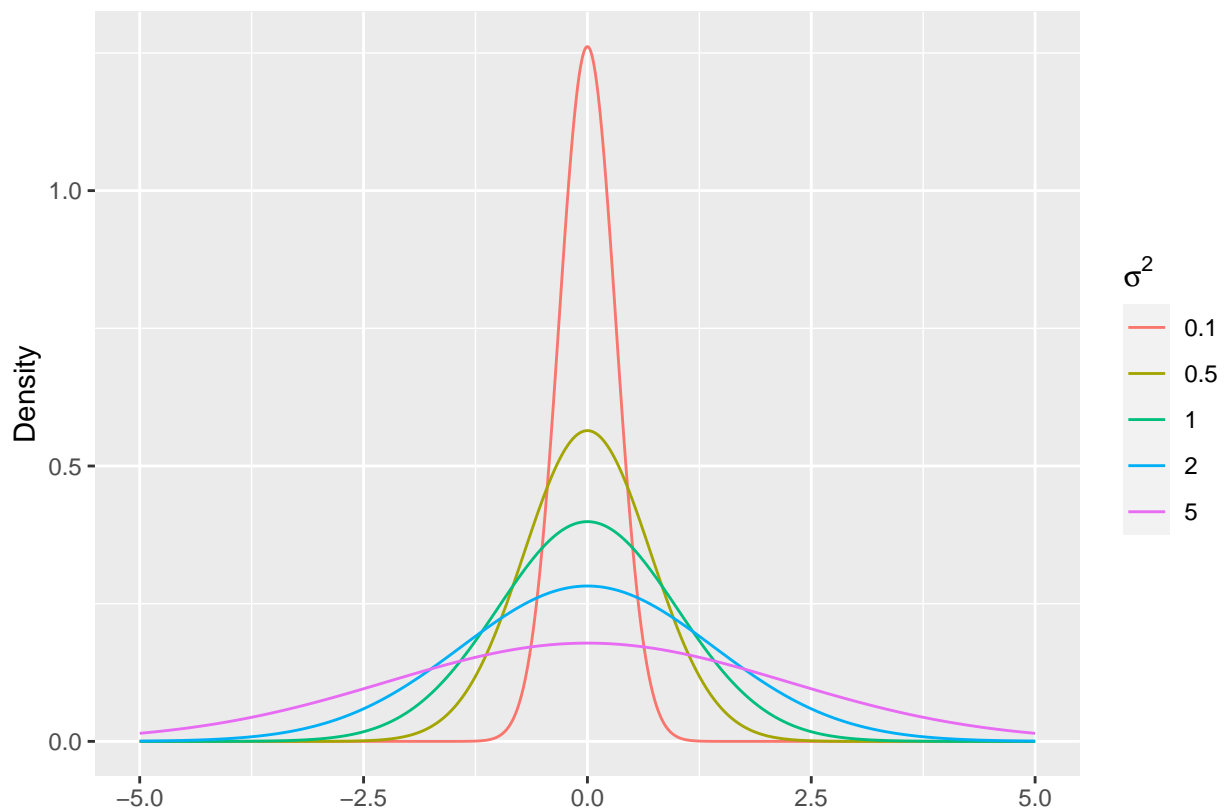
```
# ggplot() + xlim(-5, 5) + geom_function(  
#   fun = function(x)  
#     dnorm(x, sd = sqrt(0.1)),  
#   colour = "red"  
# ) + geom_function(  
#   fun = function(x)  
#     dnorm(x, sd = sqrt(0.5)),  
#   colour = "blue"  
# ) + geom_function(  
#   fun = function(x)  
#     dnorm(x, sd = sqrt(1)),  
#   colour = "green"  
# ) + geom_function(  
#   fun = function(x)  
#     dnorm(x, sd = sqrt(2)),  
#   colour = "yellow"  
# ) + geom_function(  
#   fun = function(x)  
#     dnorm(x, sd = sqrt(5)),  
#   colour = "orange"
```

```

# ) +
#   ylab("Density")+
#   scale_colour_manual(values=cols)

x1<-seq(-5,5,length.out=1001)
d<-sapply(c(0.1,0.5,1,2,5), function(x)dnorm(x1,sd = sqrt(x)))
d<-data.frame(x=rep(x1,times=5),density=c(d),sigma=rep(c(0.1,0.5,1,2,5),each=1001))
d$sigma<-as.factor(d$sigma)
p<-ggplot(d,aes(x=x,y=density,color=sigma))+
  geom_line()+
  xlab("")+
  ylab("Density")+
  scale_color_discrete(name = expression(sigma^2))
p

```



As σ^2 decreases, the tail gets thinner, the overall spread decreases, and the peak at $\mu = 0$ increases,