Unitary Group Approach

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1 Gel'fand-Tsetlin basis

Define the composite creation operators $X_k^\dagger(n_k)$,

$$X_k^\dagger(n_k) = \begin{cases} I, & \text{if } n_k = 0 \\ a_{k\mu}^\dagger, & \text{if } n_k = 1 \\ a_{k\uparrow}^\dagger a_{k\downarrow}^\dagger, & \text{if } n_k = 2. \end{cases} \tag{1}$$

The electronic Gel'fand-Tsetlin state is obtained by sequentially coupling the orbital level 0, 1 or 2-particle states,

$$|\psi\rangle = \sum_{\{M_k\}} \sum_{\{\mu_k\}} \prod_k^n \langle S_{k-1} M_{k-1} s_k \mu_k | S_k M_k \rangle \, X_k^\dagger(n_k) \, |0\rangle \, , \tag{2}$$

where $|0\rangle$ is the physical vacuum state, $\langle S'M'S''M''|SM\rangle$ are the Clebsch-Gordan coefficient, S_k are the intermediate spin quantum numbers, and M_k the corresponding magnetic quantum numbers (s_z), and $s_k = \mu_k = 0$ if $n_k = 0, 2$, while $s_k = 1/2, \mu_k = \pm 1/2$ if $n_k = 1$. The summation extends over all μ_k $(k = 1, 2, \ldots, n)$ and M_k $(k = 1, \ldots, n-1)$ while $S_0 = M_0 = 0$, $S_n = S$, $M_n = M$.