Review of One Period Model

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Introduction

What are economic models

- A description or representation of the world.
- Formally articulate **assumptions** and tease out relationships behind assumptions.

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Purpose of economic models

- simulation

Given what we know about the behavioural workings of the economy, and taking these mostly as given, how might the economy respond to, say, an elimination of colleges?

- forecast

Forecast future with current information. The further out the forecast, the larger the structural uncertainties making model projections at best illustrative.

7 key properties of good models

1. **Parsimony**: the simplest solution tends to be the best one
   parsimony
2. **Tractability**: easy to analyze (by pen and paper or by computer)
3. **Conceptually insightfulness**: reveal fundamental properties of economic behavior or economic system
4. **Generalizability**: applicable to wide range of situations
5. **Falsifiability**: predictions that can be empirically falsified
6. **Empirical consistency**: consistent with available data
   consistency
7. **Predictive precision**: make "strong" predictions, not "precise"

Basic model components

Subject of study

Are you examining questions related to household? firm? or both?

Household's problem

Households maximize utility, subject to budget constraint.

Firm's problem

Firms maximize profit, given input costs.

Study both households and firms

- standard friction-less economy market clearing conditions connect HHs and firms
- frictional economy uses matching function and wage setting rule to connect HHs and firms

Categories of variables

Exogenous variables

Apparatus of a lab. Given to you as the environment. Setting the operation boundary. Often called parameters than variables. Potential for counterfactual/policy experiments.

Endogenous variables

Within the boundary of exogenous variables, the model generates endogenous variables, as its results.

- **State variables**: variables carry history of information, linking the model from period to period. Determines where the economy is at the current *STATE*.
- **Float variables**: value generates within the period, hence "float". Given parameters, and the state variables, you get the float variables. They are memory-less.

HH model framework - in a one-period model

$$
\begin{aligned}
V &= \max_{c,l}\{u(c,l)\} \\
&\text{s.t.} \\
c &= wn + \pi - T \\
l + n &= 1
\end{aligned} \tag{1}
$$

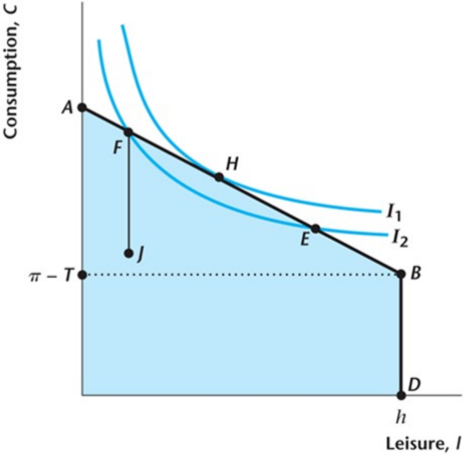
Give profit income π, lump-sum tax T, and wage rate w, the HH maximizes her value function V (utility of this period $u(\cdot)$), by choosing consumption c and leisure l.
The HH allocates l share of time to leisure and $n = 1 - l$ share of time to work.

HH optimization

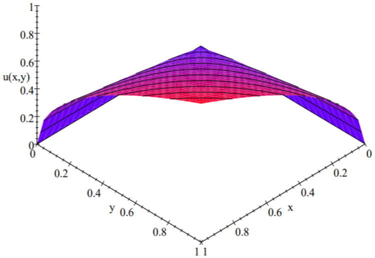
Decision rules to satisfy optimal labor-leisure tradeoff:

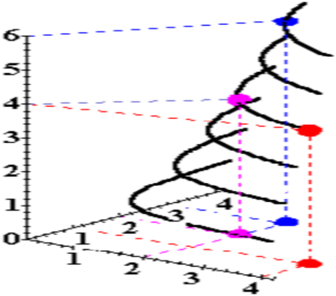
$$MRS_{l,c} = w$$

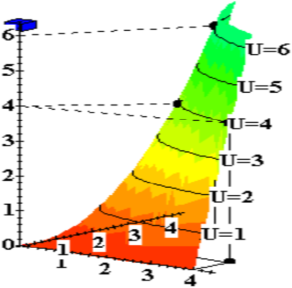
Or graphically:

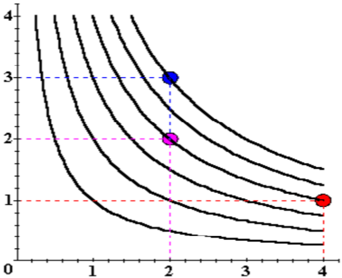


Utility function vs indifference curve









Utility functions commonly used in macro

CRRA utility

$$u(c) = \frac{c^{1-\rho}}{1-\rho}$$

where ρ represents coefficient of risk aversion, and $1/\rho$ represents elasticity of inter-temporal substitution (EIS)

Logarithmic utility

When $\rho \to 1$, CRRA utility function becomes:

$$u(c) = logC$$

Income effect and substitution effect cancels out in log utility.

Utility functions commonly used in macro

Power (dis)utility

$$u(l) = \psi \frac{(1-l)^{1+1/\sigma}}{1+1/\sigma}$$

where σ represents Frisch Elasticity, or intertemporal elasticity of labor supply, measuring how hours respond to wage changes abstracting from its effect on wealth.

Linear disutility (lottery)

$$u(l) = \psi(1-n)$$

Individuals have some disutility towards working. Everyone draw a lottery, whoever wins gets to enjoy leisure. But everyone in a HH shares resources.

Commonly used separable $U(c, l)$ in macro

Power separable

$$U(c, l) = \frac{c^{1-\rho}}{1-\rho} - \psi\frac{(1-l)^{1+1/\sigma}}{1+1/\sigma}$$

Indivisible labor

$$U(c, l) = logc + \psi l$$

Cobb-Douglas utility

$$U(c, l) = \frac{(c^{\alpha} l^{\sigma})^{1-\delta}}{1-\delta}$$

Or just omit leisure (HH doesn't value leisure)

$$U(c, l) = U(c)$$

Other common formats of $U(c, l)$ in macro

GHH utility

$$U(c,l) = (\frac{c - \psi(1-l)^{1+\gamma}}{1+\gamma})^{1-\delta}$$

No wealth/income effect on labor supply. Wage increases; labor supply increases.

Epstine-Zin utility

$$U_t(c_t, l_t) = [(c_t^v(1-l_t)^{1-v})^{(1-\gamma)/\theta} + \beta(E_t U_{t+1}^{1-v})^{1/\theta}]^{\frac{\theta}{1-\gamma}}$$

Separating risk aversion and EIS, and help generate risk premium.

Many others..

Subsistence consumption utility, habit formation utility, etc...

Firm

$$\pi = \max_{N} \{F(K, N) - wN\} \tag{2}$$

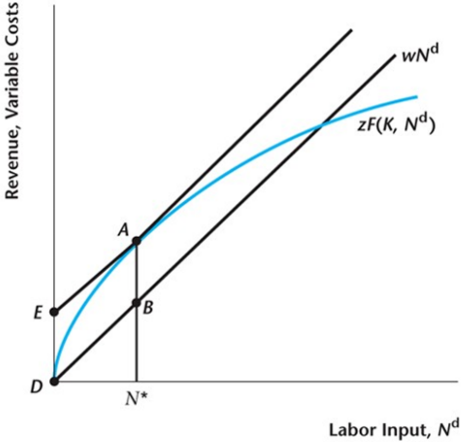
In a one-period framework, firms with production function $F(\cdot)$ are given capital endowment K, face competitive market wage rate w, and make labor demand decisions N, in order to maximizes profit π.

Firm optimization

Desicion rules to maximize profit in a competitive labor market:

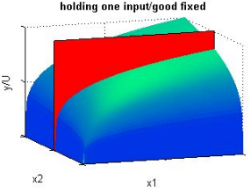
$$MRT_{l,c} = w$$

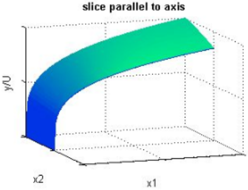
Or graphically:

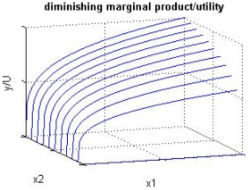


Production function

Relationship between the two inputs and output:

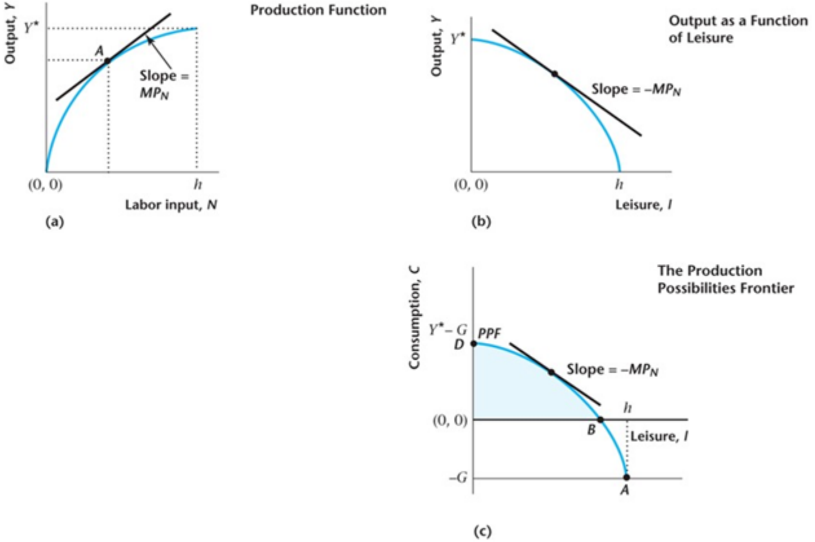






Production function vs PPF

Switch the horizontal axis from N to l, we flipped the production function into PPF (production possibility frontier) for GE purpose:



Production function commonly used in macro

Cobb-Douglas Production

$$Y = AK^{\alpha}N^{\beta}$$

If $\alpha + \beta > 1$, we have increasing returns to scale (IRS);
If $\alpha + \beta = 1$, we have constant returns to scale (CRS);
If $\alpha + \beta < 1$, we have decreasing returns to scale (DRS);

CES Production

$$Y = A(aK^{\gamma} + (1-a)N^{\gamma})^{1/\gamma}$$

where γ represents elasticity of substitution between K and N, and a represents relative importance of K and N in the production.

CES Production with SBTC

$$Y = F(K, H) = K^{\alpha}[\lambda\tilde{A}H^{\eta} + (1-\lambda)L^{\eta}]^{\frac{1-\alpha}{\eta}}$$

where $\tilde{A}$ is some skill biased technology.

General equilibrium concept

- Representative consumer optimizes given market prices.
- Representative firm optimizes given market prices.
- The labor market clears.
- The government budget constraint is satisfied, or $G = T$.

GE definition

A competitive equilibrium is a set of functions

$$\{V, \pi, c, N^s, N^d, T\} \tag{3}$$

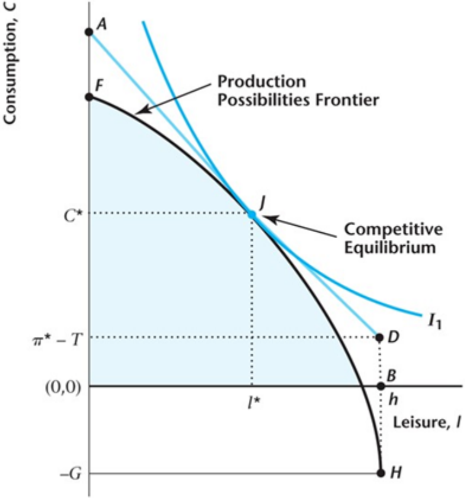
and prices $\{w\}$, such that:

1. given w, π, T, n^s and c solves HH's problem in (1)
2. given w, K, N^d solves firm's problem in (2)
3. given G, government balances budget by setting T
4. price is competitively determined: $w = D_2F(K, N)$
5. markets clear: $N^d = N^s$ and $Y = C + G$

GE graph

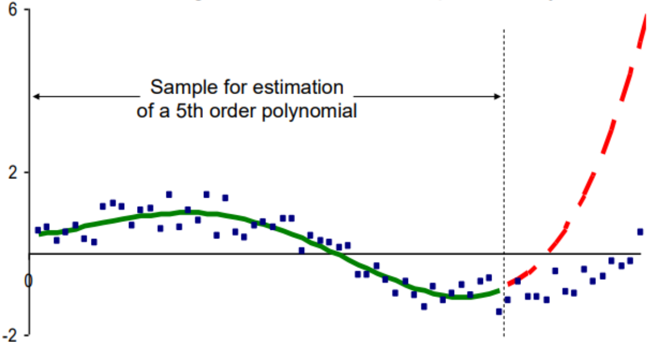
Two conditions determines general equilibrium:

- $MRS_{l,c} = w = MRT_{l,c}$
- $N^d = N^s$



7 key properties of good models

Figure 1: The value of parsimony.

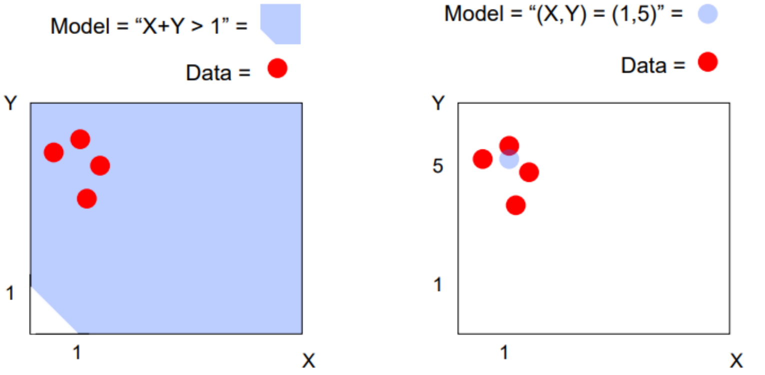


The data (squares) is generated by sin(x/10) + ε, where ε is distributed uniformly between -½ and ½. The sold line fits the first 50 data points to a fifth-order polynomial – a non-parsimonious model. The polynomial has good fit in sample and poor fit out of sample (dashed line).

*Gabaix and Laibson, 2008 7 keys

7 key properties of good models

Figure 2:
Falsifiability, Empirical Consistency, and Predictive Precision



Panel A: Model is falsifiable, empirically consistent, and does not have predictive precision.

Panel B: Model is falsifiable, empirically inconsistent, and has predictive precision.

*Gabaix and Laibson, 2008 7 keys