Review of Two Period Model

Dr. Guanyi Yang

St. Lawrence University

January 22, 2019

Introduction

- Two-period model is the foundation of DSGE (dynamic stochastic general equilibrium) framework (without stochastic in our version)
- It extends from one period model by introducing inter-temporal dynamics
- In this simple framework, we only examine the inter-temporal decisions of HH; hence we abstract away from firms
- It gives us insight into how consumption-savings motive changes when interest rate and/or income change
- Another founding block to infinite horizon dynamic models

Model structure

- There is a representative HH and a government.
- HH receive some endowment income every period (no need to work), and decides how much to consume VS to save in period one.
- The saved income is lent to the government (as a Treasury Bond). Government needs to pay the principal back in $t = 2$, with interest.

HH model framework - in a two-period model

$$
\begin{aligned}
V &= \max_{c,c'} \{u(c, c')\} \\
&\text{s.t.} \\
c + s &= y - t \\
c' &= (1 + r)s + y' - t'
\end{aligned}
\tag{1}
$$

Model description

HH receives endowment income in period one y and in period two as y', pays tax in period one t and period two t'. HH optimize the utility by choosing how much to consume today c and how much to consume the next day c'.

Lifetime budget constraint

Because we have this amazing savings technology, both periods are linked together!!

$$c + s + \frac{c'}{1+r} = y - t + \frac{y' - t'}{1+r} + \frac{(1+r)s}{1+r}$$

or just (2)

$$c + \frac{c'}{1+r} = y - t + \frac{y' - t'}{1+r}$$

Inter-temporal linkage

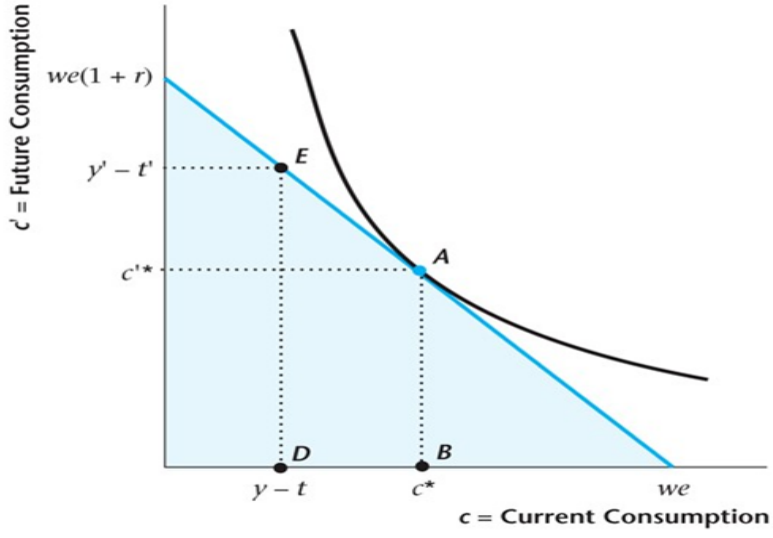
HH can inter-temporally allocate resources (income) through savings technology s (they can save to defer current income to future, or borrow from future income and use it now.)
This decision is affected by income in both periods ($y - t$ and $y' - t'$) AND interest rate r.

HH optimization

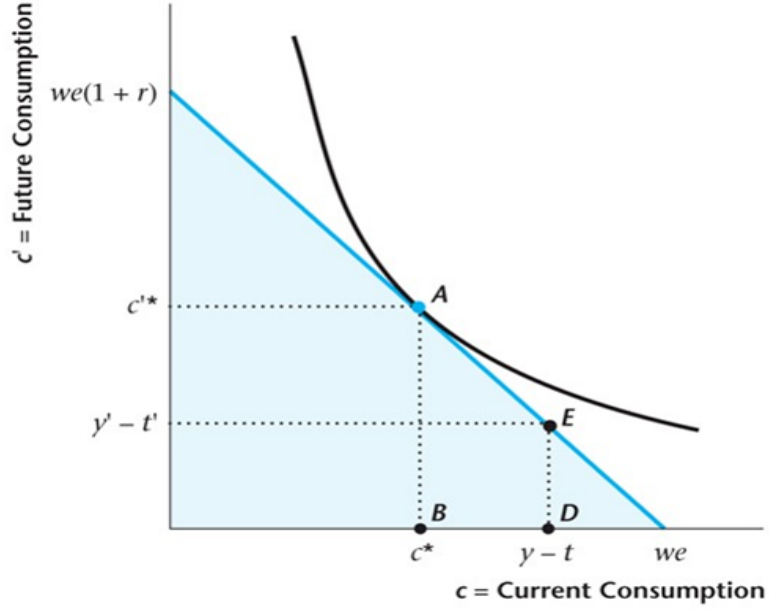
Decision rules to satisfy optimal consumption-savings tradeoff:

$$MRS_{c',c} = 1 + r$$

Or graphically:



borrower



lender/saver

Government

$$
\begin{aligned}
G &= T + B \\
G' &= T' - (1 + r)B
\end{aligned}
\tag{3}
$$

- In $t = 1$, given government spending G, it decides how much to levy as tax T, and how much as bond borrowing from HH B.
- In $t = 2$, given government spending G', it decides how much to levy as tax T', and NEED to pay back the borrowing from $t = 1$.

Government

Borrowing technology links government decisions across periods:

$$G + \frac{G'}{1+r} = T + B + \frac{T'}{1+r} - \frac{(1+r)B}{1+r}$$

or simply as (4)

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

- The point of this inter-temporal linkage through savings/borrowing, is that it doesn't mess with how much resource you really have. It only allocates it across time! What you get is still how much you have.

General equilibrium concept

- Representative consumer optimizes given market prices.
- The government budget constraint is satisfied.
- Credit market clears.

GE definition

A competitive equilibrium is a set of functions

$$\{V, c, c', s, B, T\} \tag{5}$$

and prices $\{r\}$, such that:

1. given r, T, T', y, y', c and s (and hence c') solves HH's problem in (1)
2. given r, G, G', government balances budget by setting T, T' and B
3. price r adjusts to clear credit market: $S = B$