Search and Match Model

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Introduction

Shortcomings of a "standard" model derived:
competitive equilibrium (as our first set of lecture notes)
planner's economy (as the previous optimal growth models)

- cannot describe the labor market dynamics, in terms of **unemployment, vacancy posting, and labor market tightness**
- cannot describe the relationship/dynamics between an individual worker-job pair

DMP Search and Match model:

Peter **Diamond**, Dale **Mortensen** and Christopher **Pissarides** were awarded the 2010 Nobel Prize in Economics for their fundamental contribution to matching theories.

Environment of the Model

- unemployed workers seek jobs and firms with unfilled positions seek unemployed workers
- matching function transforms measures of unemployed workers and unfilled jobs into matches
- paired worker and firm bargain over wage, determining how 'match surplus' (value lost if match breaks down) is split
- search externality: agents do not consider the implications of their actions for 'labor market tightness' and thus transition rates of other unmatched agents

preferences and technologies

- unit measure of identical, infinitely-lived, risk-neutral workers
  - employed period return: $u(c) = w$ (real wage)
  - unemployed period return: $u(c) = b$ (unemp. benefit/leisure)
  - discount future utility by factor β
- continuum of risk-neutral, profit-maximizing managers (firm), with same β as workers (stationary equilibrium)
  - production technology $y = zh$, where $h \in \{0, 1\}$
  - unmatched manager must pay κ to post (advertise) vacancy
  - free entry by managers (we will describe later)
- matches and separations
  - $u = 1 - n$, where n is aggregate employment
  - $M(u, v)$, matching function
  - exogenous job destruction at rate δ

More on matching technology

$$M(u, v) = Au^{\alpha} v^{1-\alpha}$$

- α: elasticity of matching w.r.t. u
- $\theta = \frac{v}{u}$ market tightness
- probability of worker finding a job: $p_1 = \frac{M(u,v)}{u} = A\theta^{1-\alpha}$
- probability of job filling a vacancy: $p_2 = \frac{M(u,v)}{u} = A\theta^{-\alpha}$

Decentralized equilibrium and necessary conditions

- value of unemployed worker:

$$V^u(z_i) = b + \beta(p_1(z_i)\sum_{j=1}^{N_z}\pi_{ij}V^e(z_j) + (1-p_1(z_i))\sum_{j=1}^{N_z}\pi_{ij}V^u(z_j)) \tag{1}$$

- value of employed worker:

$$V^e(z_i) = w(z_i) + \beta((1-\delta)\sum_{j=1}^{N_z}\pi_{ij}V^e(z_j) + \delta\sum_{j=1}^{N_z}\pi_{ij}V^u(z_j)) \tag{2}$$

- value of unmatched job:

$$J^u(z_i) = -\kappa + \beta(p_2(z_i)\sum_{j=1}^{N_z}\pi_{ij}J^e(z_j) + (1-p_2(z_i))\sum_{j=1}^{N_z}\pi_{ij}J^u(z_j)) \tag{3}$$

- value of matched job:

$$J^e(z_i) = z - w + \beta((1-\delta)\sum_{j=1}^{N_z}\pi_{ij}J^e(z_j) + \delta\sum_{j=1}^{N_z}\pi_{ij}J^u(z_j)) \tag{4}$$

Free entry condition

- If $J^u < 0$, no job wants to hire
- If $J^u > 0$, everyone wants to hire
- If $J^u = 0$, just enough firms hiring that drives equilibrium

$$J^u(z_i) = -\kappa + \beta(p_2(z_i)\sum_{j=1}^{N_z}\pi_{ij}J^e(z_j) + (1 - p_2(z_i))\sum_{j=1}^{N_z}\pi_{ij}J^u(z_j)) \tag{5}$$

$$0 = -\kappa + \beta(p_2(z_i)\sum_{j=1}^{N_z}\pi_{ij}J^e(z_j) + 0 \tag{6}$$

$$\kappa = \beta(p_2(z_i)\sum_{j=1}^{N_z}\pi_{ij}J^e(z_j) \tag{7}$$

Re-cap of essential conditions

- value of unemployed worker:

$$V^u(z_i) = b + \beta(p_1(z_i)E_z V^e(z_j) + (1 - p_1(z_i))E_z V^u(z_j)) \quad (8)$$

- value of employed worker:

$$V^e(z_i) = w(z_i) + \beta((1 - \delta)E_z V^e(z_j) + \delta E_z V^u(z_j)) \quad (9)$$

- value of unmatched job:

$$J^u(z_i) = 0 \quad (10)$$

- value of matched job:

$$J^e(z_i) = z - w + \beta(1 - \delta)E_z J^e(z_j) \quad (11)$$

- Evolution of labor market:

$$u' = \delta(1 - u) + (1 - p_1)u \quad (12)$$

Nash Bargaining to set wage

- Worker's matched surplus:

$$V^e(w;z) - V^u(z))$$

- Firm/manager's matched surplus is $J^e(w;z) - J^u(z)$, but really:

$$J^e(w;z)$$

Manager and worker matched and bargain for wage to maximize their own surplus following the rule:

$$w(z) = arg \max_w \{(V^e(w;z) - V^u(z))^\phi J^e(w;z)^{1-\phi}\} \tag{13}$$

Here, ϕ is an arbitrary bargaining power of the worker. If ϕ is large, the worker has more bargaining power, hence can bargain for a higher wage.

Solving the model

We solve the model by iterating using the implicit operators defined by the firm, unmatched worker and matched worker functional equations in 11, 8, and 9.

Same as previous algorithm of *Method of Successive Approximation*, we guess the future value of $J^e(z_i), V^u(z_i)$, and $V^e(z_i)$; find the current value $TJ^e(z_i), TV^u(z_i)$, and $TV^e(z_i)$; and use the current value to update future value, until convergence. Each iteration we repeats the steps on the next slide.

Algorithm

We start by defining parameters and shock processes.
Then guess initial $J^e(z_i), V^u(z_i)$, and $V^e(z_i)$. (set them all to zeros)

- At each z_i node, $i = 1, ..., Nz$, compute EJe, EVe, EVu, using the Markov transition probability, Zgrid, and the initial guessed values.
- Use free-entry condition and EJe to find the market tightness θ for each z_i, following Equation 7 and P_2.
- Using θ and EVe, EVu to find TVu, following Equation 8.
- Using EVe, EVu, EJe, TVu to the Nash Bargaining to find optimal wage rate $W(z_i)$, following Equation 13.
- Using $w(z_i)$ to update TVe using Equation 9, and update TJe using Equation 11.

Optimization to solve the Nash Bargaining

You can use previously learned - **grid search** to find the wage rate that optimizes the results. It is inefficient and inaccurate. (set up potential candidates of wage rate as a grid and maximize over to find the largest.)

Golden Section Search

Optimization to solve the Nash Bargaining

Golden Section Search – preparation:

1. Set up initial search boundary *aa* and *bb* (the highest and lowest boundary for wage):
   `aa = b -`
   `beta*(1-delta-A*theta(iz)`$^{(}1 - alpha)) * (eVe - eVu)$;
   `bb = zgrid(iz) + beta*(1-delta)*eJe;`
2. Set up golden ratio: `rr = (3-sqrt(5))/2;`
3. Use Golden ratio to update search using technical parameters:
   `cc = (1-rr)*aa + rr*bb;`
   `dd = rr*aa + (1-rr)*bb;`
4. Assume *cc* is the wage rate, find the value for Nash Bargaining, name the **NEGATIVE** of it as *fc*.
5. And assume *dd* is the wage rate, find the value for Nash Bargaining, name the **NEGATIVE** of it as *fd*.

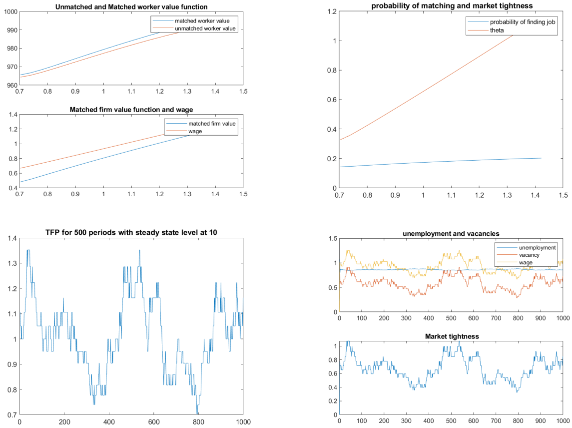
Golden Section Search

Optimization to solve the Nash Bargaining

Golden Section Search – Start:

6. while the distance between cc and dd is larger than the precision, we continue searching.
7. if $fc > fd$, update:
   $aa = cc; cc = dd; fc = fd; dd = rr * aa + (1 - rr) * bb;$, and calculate new fd using the new dd.
8. if $fd > fc$, update:
   $bb = dd; dd = cc; fd = fc; cc = (1 - rr) * aa + rr * bb;$, and calculate new fc using the new cc.
9. search continues until stopping rule is met.

Result



I set parameters with $\kappa = 0.213; \underline{=} 0.4; \phi = 0.72; \alpha = 0.72; A = 0.195; \beta = 0.96^{1/12}; \delta = 1/12; \rho = 0.987; \sigma_e = 0.022; znum = 15;$.