Review of Real Inter-temporal Model with Investment

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Introduction

- This simple framework combines both one period model and simple two-period endowment model.
- and I promise this is the last one we see before we start Matlab.

Plan

Start with

We will start with a real inter-temporal model with investment in a competitive economy

End with

We will end this session with a social planner format of the model. i.e. **One-sector Optimal Growth Model**

Model structure

- A representative HH works and earn income every period, and decides how much to spend and save.
- A representative firm hires worker every period to maximize profit.
- The firm also decides how much investment to put into the second period (from profit of first period).

HH model framework - in a one-period model

$$
\begin{aligned}
V = \max_{c,l,c',l',s} &\{u(c,l) + \beta u(c', l')\} \\
\text{s.t.} & \\
c + s &= wn + \pi \\
l + n &= 1 \\
c' &= w'n' + (1+r)s + \pi' \\
l' + n' &= 1
\end{aligned} \tag{1}
$$

HH maximizes lifetime utility by choose c, c', l, l', given wage rate and none wage income at each period.
β is the discount rate, how much one discount future.

Q: What's the state variable here for HH?

HH optimization

Decision rules:

- optimal labor-leisure tradeoff at $t = 1$:

$$MRS_{l,c} = w$$

- optimal labor-leisure tradeoff at $t = 2$:

$$MRS_{l',c'} = w'$$

- optimal consumption-savings tradeoff at $t = 1$:

$$MRS_{c,c'} = 1 + r$$

Firm

$$
\begin{aligned}
\Omega &= \max_{N,I,N'} \{\pi + \frac{\pi}{1+r}\} \\
\text{where} & \\
\pi &= F(K,N) - wN - I \\
\pi' &= F(K',N') - w'N' + (1-d)K'
\end{aligned} \tag{2}
$$

Given wage rate at both periods, the firm maximizes lifetime profit Ω, by choosing employment level today and tomorrow (N, N'), and investment I.

Q: What's the state variable here?

Firm

Investment is the "saving" strategy/technology of the firm. It allows firm to "save" some of the product from profit today and use it for tomorrow's production/profit.

$$K' = (1-d)K + I \tag{3}$$

Tomorrow's production capital K' comes from un-depreciated capital from today. (The things that didn't get used up and didn't depreciate can still be used tomorrow.) AND new things added to it, as investment I.

Firm's optimization

- employment today: $MRT_{l,c} = w$
- employment tomorrow: $MRT_{l',c'} = w'$
- **investment today:** $MPK' - d = r$

Competitive general equilibrium concept

- Representative consumer optimizes given market prices.
- Representative firm optimizes given market prices.
- Labor market, goods market, and **CREDIT** market clears.
- The government budget constraint is satisfied.

Competitive general equilibrium definition

A competitive equilibrium is a set of functions

$$\{V, \Omega, c, c', N^s, N^d, N'^s, N'^d, s, I, K'\} \tag{4}$$

and prices $\{w, w', r\}$, such that:

1. given w, w', π, π', N^s, N'^s, c, C', and s solves HH's problem in (1)
2. given w, w', K, N^d, N'^d, and I (hence K') solves firm's problem in (2)
3. policy function g evolves as $g(K) = (1-d)K + I$
4. price is competitively determined: $w = D_2F(K, N)$
5. markets clear: $S = I$, $N^d = N^s$ and $Y = C + I$

Benevolent social planner

Another way of describing the previous setting of the economy, is we can use a benevolent planner scenario. We know that a competitive market will without frictions or externalities will generate optimal allocation of resources (i.e. both HH and firm are achieving their optimization. Everyone is happy).

Assuming that there is a great benevolent planner who knows what everyone wants, and observes everyone's utility function, as well as knowing the production function of the firm perfectly, it can create a planned economy, by arbitrarily assign the number of people working and the amount of consumption everyone gets from the output for the purpose of achieving maximized utility in the society.

Optimal Growth Model

We write the optimal growth model using a planned economy format to describe the results generated by the previously presented competitive market economy.

$$
\begin{aligned}
V = \max_{c,L,c',L',K'} & \{u(c,l) + \beta u(c',l')\} \\
\text{s.t.} & \\
c + K' &= F(K,N) + (1-d)K \\
L + N &= 1 \text{ and } L' + N' = 1 \\
c' &= F(K',N') + (1-d)K' \\
\text{K given} &
\end{aligned}
\tag{5}
$$

Notice here, the budget constrain is the allocation of the resources (RHS) into consumption and investment.
Remember the aggregate accounting? Components of GDP?
$Y = C + I + G + NX$ Without NX and assuming no tax or government spending, $Y = C + I$ in period 1; and $Y = C$ in period 2. (*hint:* $I = K' - (1-d)K$)