The Surplus Theory of Social Stratification and the Size Distribution of Personal Wealth*

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Abstract

The Surplus Theory of Social Stratification explains inequality of wealth in terms of (1) the fugitivity of wealth not needed to sustain the production of more wealth, (2) the tendency of wealth to flow into the hands of those who are already disproportionately wealthy, and (3) the ability of workers in an industrial society to retain a greater share of the wealth they produce than workers in societies with more primitive technologies. Size distributions of wealth from societies at different levels of technology can be fitted by a family of gamma distributions, whose shape parameter is related to a society's level of technology. The Surplus Theory implies a stochastic process that generates gamma-like distributions. Analysis of this process, the Inequality Process, explains many facts about size distributions of personal wealth.

This paper presents a general theory of the size distribution of personal wealth. The paper shows that the Surplus Theory of Social Stratification implies a stochastic process, called here the "inequality process," that reproduces the major features of size distributions of personal wealth in societies at various technological levels. This paper brings together two literatures without cross-citations. These are (1) the literature on the fitting of probability density functions to size distributions of personal wealth, and (2) the literature in anthropology, social archeology, and sociology on the emergence of inequality as populations of hunter/gatherers developed agriculture, the literature of the Surplus Theory of Social Stratification.

This paper assumes that the concept of wealth has very nearly the same meaning in societies as different as those of hunter/gatherers on the one hand and industrial societies on the other. However, no comparisons

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between amounts of wealth in such different societies are made, only comparisons of the shape of the size distribution of personal wealth, which means that wealth is defined in terms that make sense in each type of society. This intuitive approach is taken to avoid difficult questions. This paper assumes, in the interest of simplicity, that income and wealth are indicators of each other. In this view an income stream is evidence of wealth. Thus, if abilities and skills generate income, they are wealth. This form of wealth is called human capital, but the concept of wealth indicated by income streams is also extended to include pension rights, social insurance, and membership in organizations that confer benefits on their members, such as the Communist party in Communist states. A lack of interest in the distinction between income and wealth stems from the kind of evidence at hand. The evidence that gave rise to the Surplus Theory, distributions of grave wealth (the artifacts interred with the dead), gives no information about income. On the other hand, the most readily available information on inequality in industrial societies is income data since some industrial forms of wealth, such as skills and rights under redistributive schemes, are difficult to measure in any way other than by the income they generate.

Fitting Density Functions to the Size Distribution of Personal Wealth

The size distribution of personal wealth is a frequency distribution: the number of people falling into ranked categories of wealth, that is, the number of people having between x and y units of wealth. A probability density function can be fitted to a frequency distribution and provides a convenient summary of the information in the frequency distribution (Cowell 1977). Since a density function may have a known relationship to a stochastic process, fitting a density function can also possibly give clues to the nature of the social processes responsible for generating inequality of wealth.

As an example of a density function that fits a size distribution of wealth, Salem and Mount (1974) fitted a two-parameter gamma density function to size distributions of household income in the U.S. in the 1960s. They achieve close fits. Equation 1 gives the two-parameter gamma (Johnson and Kotz 1970, p. 166):

$$P_x(x) = (\beta^{\alpha}\Gamma(\alpha))^{-1} x^{\alpha-1} e^{-x/\beta} \qquad (1)$$

where,

$\alpha>0$
$\beta>0$
$x>0$.

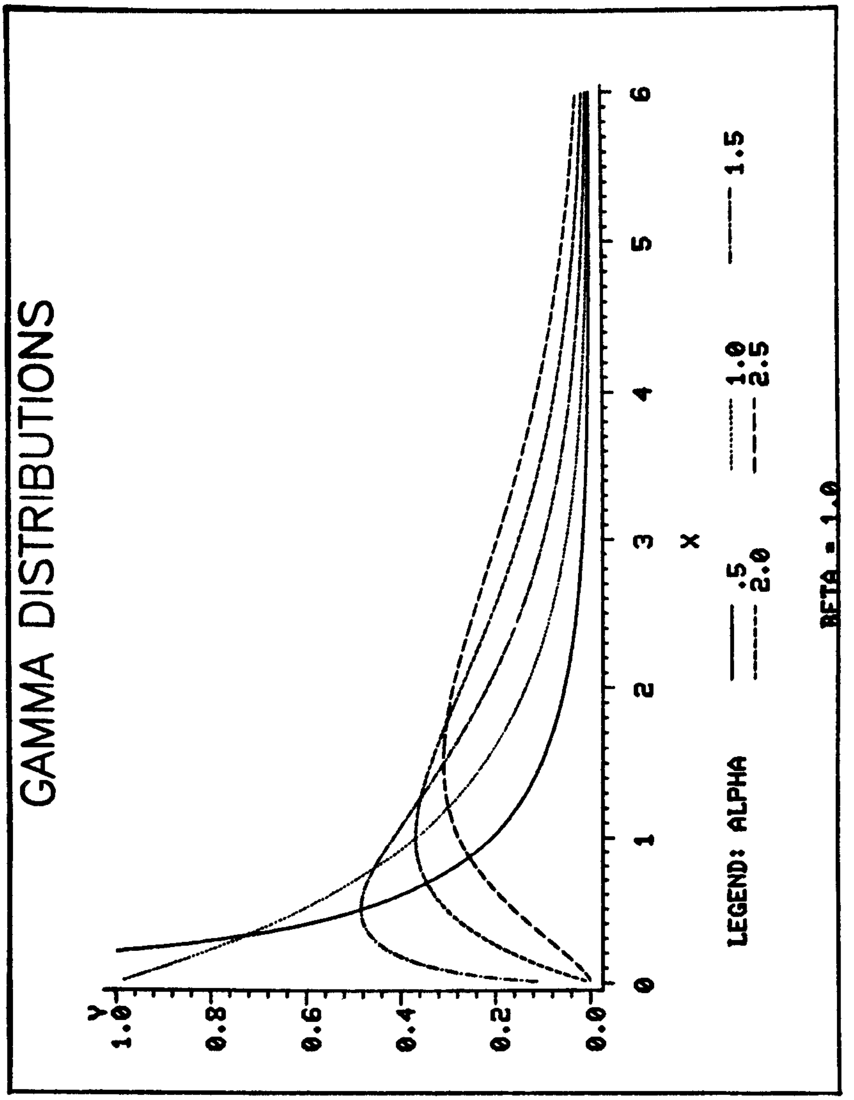

Figure 1.

Figure 1 displays the family of two-parameter gamma density functions. These have a fixed mean. Alpha is the shape parameter of the gamma distribution. A gamma distribution with a shape parameter in the range of 2.0 to 2.5 approximates the shape of the size distribution of household incomes examined by Salem and Mount. The scale parameter, beta, depends on the units in which x is measured. If $\beta = 1$ when x is measured in dollars, then $\beta = 2$ when x is in half dollars. Alpha, beta and the mean of the distribution, $\bar{x}$, are related by the formula, $\alpha\beta = \bar{x}$.

Perhaps the most remarkable fact about size distributions of personal wealth is their positive skew, expressed in long, gently tapering right tails, the distribution of people over great wealth. Pareto (1897) was so struck by the fact that size distributions of personal wealth from societies far removed in time, space, or culture all exhibit this characteristic tail that he inferred that a common cause, or in more modern terms, a common process, underlay the distribution of wealth to persons in all societies. This conclusion is the most general form of what is known as the Pareto Law. There is a narrower version, given by Davis (1941a):

> In all places and times the distribution of income in a stable economy, when the origin of measurement is at a sufficiently high income level [incomes of poorer people are discarded] will be given approximately by the empirical formula $Y = aX^{-v}$, where Y is the number of people having income X or greater and v is approximately 1.5 (p. 23).

Davis (1941b) suggested that the value of 1.5 for v is an equilibrium value and that if a society's size distribution of wealth could be fitted by a Pareto function with a substantially different parameter, revolution might ensue. Lydall (1968) notes that few economists put much credence in the narrow form of Pareto's Law. In fact, v has a considerable range when the Pareto function is fitted to Pareto's own data (cf. Creedy 1977). Lydall also observes that the Pareto function is really only a good fit to the top 20 percent of incomes in a society. The Pareto density function, the negative derivative with respect to x of the Pareto function, is widely recognized as a mediocre fit to the central region of the distribution of wealth or income, and a bad fit to the left tail, the distribution of people over small amounts of wealth (Cowell 1977).[1]

What density function can be fitted to a whole size distribution of wealth? The lognormal was until recently the most widely recognized candidate. The lognormal provides only an approximate fit to size distributions of personal income (Metcalf 1972), one that can be improved on by fitting a gamma distribution (Salem and Mount 1974), or a beta distribution (McDonald and Ransom 1979). A point in favor of the lognormal distribution, though, as a candidate for the "right" function to fit is that it can be generated by a stochastic process that seems plausible and is certainly parsimonious, the "law of proportional effect" (cf. Aitchison and Brown 1957; Gibrat 1931). This "law" is that you might expect larger incomes or wealth to vary, over time, as much proportionately as smaller incomes or wealth, which means that, in absolute terms, the larger incomes or wealth will vary more. A Markov process such as $Y_t = Y_{t-1} + e_t$ where e_t is independent of Y_{t-1}, the expectation of e_t is zero, and its variance is constant, will asymptotically generate a normal distribution (Lydall 1968). The Markov process $X_t = X_{t-1} + e_t$ where $X_{t-1} = ln\, Y_{t-1}$, will generate a lognormal distribution asymptotically and is the mathematical statement of the law of proportional effect. The law of proportional effect, although parsimonious and able to generate positively skewed distributions, cannot be, without modification, *the* Inequality Process (the process underlying the generation of size distributions of personal wealth) because it generates distributions whose variances increase with time without bound (Kalecki 1945).

The Inequality Process, the true process generating the distribution of wealth, unlike the law of proportional effect, converges quickly to its stationary (asymptotic) distribution. Some have observed that even the efforts of the most serious and bloodthirsty revolutionaries to abolish differences of wealth have come to naught as size distributions of wealth quickly reemerge in postrevolutionary societies, if indeed they were ever gone (Bernadelli 1944; Kelley and Klein 1977; Lenski 1978). Studies in societies where the size distribution of personal wealth or income can be measured back centuries show change, but slow, gradual change, in response to industrial development (Lydall 1976; Smith and Franklin 1974; Soltow 1968, 1971).

There are a great variety of density functions that have been fitted to size distributions of personal wealth or income (Blinder 1974; Cowell 1977; Lydall 1968; McDonald and Ransom 1979; Metcalf 1972; Nygard and Sandström 1981; Ord 1975; Sahota 1978). McDonald (1984) has shown, however, that for fits to the size distribution of family income in the U.S. in recent years, the generalized beta and gamma are best. Naturally, distributions with a larger number of parameters can achieve closer fits than distributions with a smaller number. Among two- and three-parameter distributions, the standard beta and gamma achieve the closest fits (McDonald and Ransom 1979).

The Surplus Theory of Social Stratification

The Surplus Theory of Social Stratification, as it is known to anthropologists, archeologists, and sociologists, is perhaps the best known and most widely accepted theory of inequality of wealth. Harris (1959) observes that the "surplus theory is so widely accepted among anthropologists that many regard it as an innocuous truism" (p. 185). The essentials of the theory can be traced to Adam Smith (Orans 1966), Marx and Engels (Dalton 1960, 1963), and to nineteenth-century writers on social evolution, such as Lewis Henry Morgan, on whose work Engels ([1884] 1972) commented. In the twentieth century, archeologists such as V. Gordon Childe (1944, 1951) have developed the theory to explain the dramatic and startlingly universal fact that ". . . the equal distribution of wealth in societies with surpluses is so rare as to be almost non-existent" (Herskovits 1940, p. 371). In archeological excavations, evidence of substantial inequality of wealth is usually first found in the same strata as the first evidence of agriculture and concomitant food abundance.

The theory has two core definitions and two core propositions—

Definition 1: Subsistence is wealth necessary to keep producers alive and cover the long-term costs of production, including investments, which include keeping the families of producers alive.

Definition 2: Surplus is the difference between subsistence and total production of wealth; societal net product.

Both concepts can be defined at the level of the individual person as well as society, as they are here.

Proposition 1: Where people are able to produce a surplus, some of the surplus will be fugitive and leave the possession of the people who produce it.

Proposition 2: Wealth confers on those who possess it the ability to extract wealth from others. So netting out each person's ability to do this in a general competition for surplus wealth, the rich tend to take surplus away from the poor.

There is a narrow and a wide version of the Surplus Theory. The above statement is the wide version, the way the theory is usually stated. However, the narrow version is the way it is often used and is the original version in anthropology and archeology, what anthropologists and archeologists really have in mind when they think of the Surplus Theory. In the narrow version, "surplus" means excess capacity to produce food or more food than the people who produce it will need to survive and maintain production. While the wide version of the theory can apply to all societies, the scope of the narrow version is just the transition from hunter/gatherers to the next higher technocultural type, the "ranked" society (Fried 1967, p. 109) also called the "chiefdom" (Service 1962, p. 143). Service also distinguishes a transitional societal type between "band" hunter/gatherers and the chiefdom, the "tribe." The narrow Surplus Theory explains why hunter/gatherers, who usually eke out a subsistence by collecting or hunting wild foods, are usually described as egalitarian (Fried 1967) and become inegalitarian (Renfrew 1974) as well as richer when they learn to domesticate plants and animals and thereby produce larger, more dependable food supplies. The narrow Surplus Theory presupposes peasant farmers who produce and possess their crops and have some of the crops taken from them by (1) theft, (2) extortion, (3) taxation, (4) exchange coerced by unequal power between the participants, (5) genuinely voluntary exchange, or (6) gift. The concept of transfer or exchange as the basis of the economics of primitive peoples is well established (Earle and Ericson 1977; Firth 1939; Naroll 1973; Pryor 1977; Renfrew and Shennan 1982;

Sahlins 1972). Writers in the tradition of the Surplus Theory have long viewed unequal exchange as the mechanism by which surpluses are concentrated in fewer hands (Gilman 1981).

The Surplus Theory, both narrow and wide, assumes that producers of wealth are more likely to retain subsistence wealth than surplus wealth because (1) they will put up more resistance as more and more of their surplus is taken, reducing them to Pareto's "wolf-point," the point at which less means death, and (2) extractors of surplus will act in their own long-term interest and will show husbandry toward the extractees, that is, leave them enough to live on and keep producing, perhaps leaving even a little of the surplus as motivation. Causal direction in both wide and narrow versions of the Surplus Theory goes from surplus to inequality because, as Herskovits (1940) notes, inequality appears wherever there is a surplus. There is no reason to think the emergence of inequality contributed to the invention of agriculture. Inequality emerges in hunter/gatherer groups in especially favored ecological niches who enjoy surplus without agriculture (Fried 1967). It is surplus, not agriculture, that causes inequality. The subsistence/surplus distinction is not as important in the wider version of the theory as in the narrow since in an industrial society, subsistence is a small fraction of total wealth.

WHAT DOES "SURPLUS" MEAN?

Most of the anthropological literature on the Surplus Theory is an attempt to answer the question, "What does 'surplus' mean?" Pearson (1957) suggested that the Surplus Theory was meaningless since the concept of surplus was hard to measure. There was a flurry of defenses and clarifications (Dalton 1960, 1963; Harris 1959; Orans 1966). Still, most anthropological writings after all these clarifications still use the definition of surplus that annoyed Pearson—all production above minimum nutritional requirements—and the casual assumption that all surplus is available for redistribution. The essential issue in the definition of surplus is availability for redistribution. When one measures total production by the dichotomy, subsistence/surplus, one divides production into something essential for life and further production, which, if taken away, would disrupt further production, and production that can be taken without harm. The simplicity of dichotomous measurement hides the possibility that there are degrees of availability of nonsubsistence wealth.

HOW DO TRANSFERS OF SURPLUS WEALTH OCCUR?

Proposition #2 of the Surplus Theory explains how transfers of wealth occur. The proposition asserts that wealth itself confers on its possessor some ability to take wealth from others. In a general competition for wealth a richer person encountering a poorer person would have an advantage over the poorer person and be able to take surplus wealth from the poorer person. Most people know this principle as "the rich get richer, the poor get poorer." Naroll (1980) has called this process the "snowball." But the snowball has a problem as an explanation of transfers. If a richer person always has an absolute advantage over a poorer person, wealth quickly becomes concentrated in the hands of one person, the richest person; everyone else would be stripped of surplus wealth and become a population of slaves. No society has had quite this situation; approximations to it are rare; an unmodified snowball process is not plausible.

What determines success in encounters in which surplus wealth changes hands? Personal characteristics surely. Some people are more eager for wealth than others; some are bright, others not; some are genial, others boorish; some able-bodied, others lame; in a word, some are lucky, others not. From the point of view of a system of transfers of surplus wealth, individual characteristics are just noise, a lottery, an irrelevant stochastic process. If the outcome of the transfer is not a chance thing, then ability to influence the outcome can be modelled as a chance event. In modelling encounters in which surplus wealth changes hands, some simplifying assumptions have to be made. It is assumed here that there is a regression toward the mean over generations, so even if personal characteristics that affect the accumulation of wealth were culturally or genetically inherited, the descendants of gifted individuals would lose the trait and become more like the whole population. Human natural selection and the distribution of wealth is too complex a topic to investigate here, and, if the Inequality Process is as fast converging as it appears to be, one that need not be considered.

It is plausible, as well as logically necessary, then to model success in encounters in which surplus changes hands as the outcome of both wealth differences (according to Proposition #2) and personal characteristics, modelled as white noise. Whether greater wealth trumps luck is unknown, but it is certain that it does not trump every time since that generates wealth distributions that are not observed.

RESISTANCE TO EXTRACTION OF SURPLUS

The surplus/subsistence dichotomy is too simplistic. Degrees of availability of wealth within surplus ought to be recognized. "Top" layers in the surplus should be recognized as more likely to change hands than "bottom" layers, what is left if the person is reduced almost to subsistence. The Surplus Theory asserts that subsistence wealth will be retained, whether by extractee resistance or resource-management considerations on the part of the extractors, but has little to say about how or why. There is one discussion of how much surplus producers keep. Lenski (1966) suggests that while workers in industrial economies produce a bigger surplus than workers with more primitive technologies (here "surplus" means net societal product, not just excess food-producing capacity), they manage to keep a larger proportion of it. This is a remarkable assertion since workers in industrial societies rarely possess what they produce in the same exclusive way a peasant farmer possesses crops. Lenski's explanation for the declining inequality of wealth with industrialization is that, on average, workers in industrial societies are more skilled than workers with more primitive technologies and have more bargaining power vis-à-vis employers, the rich, as a result. Lenski hypothesizes that rich people choose absolute gains in wealth (by employing more productive, skilled workers who nevertheless get to keep more of what they produce) over relative gains (which can be had by employing less skilled workers from whom a relatively larger share of product can be extracted). Lenski's hypothesis is supported by the "bases of compliance" literature in organizations theory (Etzioni 1975; Galbraith 1967), which notes that industrialization shifted the basis of compliance of workers with employers away from compulsion toward incentive because it is harder to compel the more productive work of skilled workers than it is to reward it.

Two more propositions can be added to the list of central propositions of the Surplus Theory—

Proposition 3: As surplus wealth is transferred away from the person who produced it, less of what surplus is left is available for transfer.

Proposition 4: A smaller proportion of surplus wealth is extracted from producers of wealth in industrial societies than in societies with more primitive technologies.

Criteria for a Theory of Inequality

There are a number of facts about size distributions of personal wealth that a theory of wealth distribution should be able to explain. These are:

1. THE GENERAL PARETO LAW

The uniformity of the shape of the right tails of size distributions of personal wealth and the universality of the effect of food surpluses on hunting and gathering societies suggest that a single process of distribution operates in all societies.

2. THE NARROW PARETO LAW

The Narrow Pareto Law is that the right tails of size distributions of personal wealth should be fitted by a Pareto density function (or, equivalently, the complementary cumulative distribution should be fitted by the Pareto Function). The narrowest form of the Pareto Law is the assertion that the parameter of the distribution, v, has an "equilibrium" value of 1.5 and that if the estimated parameter for a society is much different a revolution may be expected to ensue.

3. STABILITY

An adequate theory of the size distribution of personal wealth should imply a process that generates a stable distribution. Stability, in this context, means that the distribution generated by a model of the Inequality Process should converge quickly to one that fits empirical size distributions of personal wealth. Stability in the shape of the generated distribution is what is of interest. Shape can be measured by the variance and higher moments or by the value of the shape parameter of a fitted function. Stability in shape means that there is no trend in the estimates of these statistics and their standard deviations are small as the process goes on.

4. EVOLUTION OF INEQUALITY WITH TECHNOLOGY

There is often some lack of specificity in the anthropological and sociological literature as to what is meant by "inequality." The concept is usually operationalized by the Gini Concentration Ratio in social science although there are many other measures (Alker 1965, p. 42; Nygard and Sandström 1981, pp. 240–41; Shryock and Siegel 1973, p. 179). The Gini ratio is a measure of the concentration of wealth, a generalization of cruder measures of concentration such as the percentage of the society's wealth the richest 1, 5, 10, or 20 percent of the population has. Often in discussing the origin of inequality, though, anthropologists and archeologists use evidence of differences of wealth, such as a measure akin to the range, the difference in wealth between the richest and the average person. The range is a measure of dispersion. The dispersion of a size distribution of personal wealth affects the likelihood that a person will encounter a person with a different amount of wealth and hence experience inequality, whether as the richer or poorer party. Thus in a ranked society where commoners have just about a subsistence and constitute the great majority of the population there is a kind of egalitarianism even though there may be concentration of great wealth in a small group of people.

The concentration of wealth has a clear evolutionary pattern as

Lenski (1966) observes. Wealth concentration is at a minimum in hunting and gathering societies without long-term food surpluses (Binford 1971; Fried 1967; Service 1962; Werner 1981). Chiefdoms, which emerge out of hunting and gathering societies, often through an intermediate "bigmen" phase (Orme 1981, p. 139), clearly have some concentration of wealth in chiefs and their entourages. The impact of technological advance on hunting and gathering societies is an increase in the concentration of wealth. The application of industrial technology to production has reduced the concentration of wealth, that is, as an agrarian society industrializes the concentration of wealth, over a time span measured in decades and centuries, decreases (Kuznets 1963; Lenski 1966; Paukert 1973; Soltow 1968). Rapid spurts of economic growth, measured in terms of years or a few decades, may be accompanied by temporary increases in concentration of wealth. Kuznets (1955) demonstrates by simulation how an increase in mean wealth (although accompanied by a decrease in concentration) in a small industrial sector of the economy, when considered in the aggregate of a larger, poorer rural sector, may make industrialization appear to cause greater concentration in the country as a whole. Paukert reviews both cross-sectional studies, comparing currently industrialized countries with currently less developed countries, and longitudinal studies, comparing currently industrialized countries with their less developed past. Both kinds of studies point toward the same conclusion: industrialization lessens concentration of wealth.[2]

5. EVOLUTION IN THE SHAPE OF THE SIZE DISTRIBUTION OF PERSONAL WEALTH

Certain things are known about what the shape of the size distribution of personal surplus wealth in hunter/gatherer societies is like. First, there is a left limit, subsistence. The limit may fluctuate, but can be statistically summarized as a point. Second, there is a limit on the right side, the rich side. While some hunter/gatherers may have more than others in their band, they do not have very many multiples of what the others have. When anthropologists discuss the egalitarianism of hunter/gatherer society, they refer to the fact that all appear to have about the same quantity of goods. The fact that, to people perched just above subsistence, small differences in wealth may have enormous utility does not enter this view. The archeological and even anthropological approach to wealth does not emphasize the subjective utility of wealth. Neither does the view of wealth in this paper. The size distribution of personal wealth then among hunter/gatherers resembles a gamma distribution with a shape parameter of about .5 (see Figure 1). Most people are close to the left limit of subsistence, zero surplus, while the right tail, people with more than just subsistence, is short, that is, no one has many multiples of mean wealth.

An examination of grave wealth in ranked societies shows that these societies' size distribution of personal wealth has a substantially different shape from that of hunter/gatherer societies (Lederman 1980; Randsborg 1973, 1974, 1980; Shennan 1975). The two distributions must resemble each other on the left side since it is known that most people in ranked societies had barely more than subsistence too. The difference is in the right tail, the side of the personal wealth distribution that grave wealth documents. In ranked societies the richest individuals had many multiples of what the average individual had. The right tails of grave wealth distributions of ranked societies are long and tapering. Ranked societies then have a size distribution of personal wealth at least approximately shaped like a gamma distribution with a shape parameter in the vicinity of 1.0. Most of the probability density (people) is near the left limit, as in hunter/gatherer society, but the right tail is longer and fatter than that of a gamma with a shape parameter of .5. Salem and Mount (1974) fitted gammas with shape parameters between 1.94 and 2.51 to the size distribution of household income in the U.S. between 1960 and 1969. Such a gamma distribution is roughly characteristic of the size distribution of personal income in industrialized societies.

The Surplus Theory and the Inequality Process

The Surplus Theory has within itself an implied process of transfers of surplus wealth. From here on, this process implied by the Surplus Theory is called the "inequality process" with lower-case letters, while the process that generates real size distributions of personal wealth is called "Inequality Process" with capital letters. To find out what the implications of the inequality process are for the size distribution of personal wealth the propositions of the Surplus Theory have to be translated into algebra and equations. This section of the paper does that, proposition by proposition.

Proposition #1, the fugitivity of surplus wealth principle, is the most basic proposition. It implies encounters in which surplus wealth changes hands fairly readily. These encounters can be modelled by two equations, one for each party to the encounter. For simplicity only pairwise encounters are modelled. The Surplus Theory is not concerned with the creation or destruction of wealth, only its distribution, so these encounters are "zero-sum," that is, what one party gains, the other loses. Who wins the encounter? Proposition #2, the snowball, has something to say about that. How much does the loser lose? Proposition #3, the principle of greater resistance to a proportionately larger loss of surplus, has something to say about that. But Equations 2a and 2b just model Proposition #1 so the chance of either party winning is a uniform .5 probability (i.e., either could win; nothing is known; ignorance is uniform) and the amount lost by the loser is a uniform random proportion (again because ignorance is uniform). "Winning" consists of taking some of the other party's surplus while not giving up your own. Equations 2a,b describe what Proposition #1 implies.

$$X_{mt} = X_{m(t-1)} + d\,U\,X_{n(t-1)} - (1-d)\,U\,X_{m(t-1)} \qquad \text{(2a)}$$

$$X_{nt} = X_{n(t-1)} + (1-d)\,U\,X_{m(t-1)} - d\,U\,X_{n(t-1)} \qquad \text{(2b)}$$

where,

X_{mt} = m's surplus wealth after an encounter with n
$X_{n(t-1)}$ = n's surplus wealth before an encounter with m

$$d = \begin{cases} 1 \text{ with probability } .5 \\ 0 \text{ with probability } .5 \end{cases}$$

and

U = a 0,1 continuous uniform random variate.

Equation 2a says that m's current wealth is m's wealth before the encounter with a gain from n or a loss to n. If m wins from n (when $d=1$), m receives $d\,U\,X_{n(t-1)}$ from n, that is, the product of d (when $d=1$), U (a uniform 0,1 random variable) and $X_{n(t-1)}$, what n had going into the encounter.

Proposition #2 of the Surplus Theory asserts that the richer party to an encounter has an advantage in the encounter. The extent of this advantage is determined by the parameter, δ, *delta*, in Equations 3a and 3b.

$$X_{mt} = X_{m(t-1)} + d\,U\,X_{n(t-1)} - (1-d)\,U\,X_{m(t-1)} \qquad \text{(3a)}$$

$$X_{nt} = X_{n(t-1)} + (1-d)\,U\,X_{m(t-1)} - d\,U\,X_{n(t-1)} \qquad \text{(3b)}$$

where,

$$d = \begin{cases} 1 \text{ with probability } \delta \text{ if } X_{m(t-1)} \text{ greater than } X_{n(t-1)} \text{ and} \\ \quad \text{probability } 1-\delta \text{ if } X_{n(t-1)} \text{ greater than } X_{m(t-1)} \\ 0 \text{ otherwise} \end{cases}$$

and the other variables are defined as in Equations 2a and 2b. It is expected under Proposition #2 that *delta* is greater than .5, in other words, that the richer party is more likely to win an encounter than the poorer party.

Proposition #3 of the Surplus Theory asserts that surplus should be viewed as being made up of layers and that the top layers are more fugitive, more easily lost than the bottom layers, those close to the level of subsistence. Let L be the number of layers distinguished in surplus wealth, then a simple way to model the reduction in the likelihood of loss in each succeedingly lower layer of surplus is to make loss the average of a power series of L terms in U, a random 0,1 uniform variate. A loser loses a U_1 proportion of the top layer, a U_2^2 proportion of the next lower layer (where U_2^2 is the square of U_2 and U_2 is independent of U_1), and so on down to the lowest layer of surplus, in which the proportion U_L^L is lost by the loser to the encounter. Thus, the expected loss in each layer by a loser gets smaller and smaller in succeedingly lower layers. L can be arbitrarily set at any integer greater than or equal to one. Clearly, the larger L is, the smaller the expected loss of surplus by a loser. A loser will lose a

$$\sum_{i=1}^{L} U_i^i / L = Z$$

proportion of surplus wealth. So incorporating Proposition #3 into the equations of the inequality process yields:

$$X_{mt} = X_{m(t-1)} + dZ\, X_{n(t-1)} - (1-d)Z\, X_{m(t-1)} \quad (4a)$$

$$X_{nt} = X_{n(t-1)} + (1-d)Z\, X_{m(t-1)} - dZ\, X_{n(t-1)} \quad (4b)$$

Proposition #4, the Lenski hypothesis of increased resistance to extraction at higher levels of technology, can be modelled by increasing L.

SIMULATION OF THE INEQUALITY PROCESS

Equations 4a and 4b describe a process of encounters between parties in which wealth can change hands as the Surplus Theory implies. This is the inequality process. The simulation of the inequality process in a population of cases endowed with wealth can find what size distribution of wealth results from the inequality process, that is, what size distribution of wealth is implied by the Surplus Theory. A FORTRAN program was written to simulate the inequality process. The program generates cases, endows them with wealth, arranges encounters and transfers of wealth according to the inequality process, and periodically samples the values of cases and records them. The transfers of wealth are consistent with equations 4a and 4b but are not simple evaluations of the algebra in the two statements. The Appendix contains the FORTRAN code for the transfers of wealth. The FORTRAN program that simulates the inequality process calls subroutines of the International Mathematical and Statistical Library, the IMSL (1982), wherever appropriate. For example, IMSL routines generate 0,1 uniform random variates and sort vectors. Another FORTRAN program analyzes the distribution generated by the inequality process. This program calls an IMSL routine to perform the one-sample Kolmogorov–Smirnov Test.

The number of cases generated by the simulation program and the number of iterations of the inequality process permitted determine the N of the generated distribution. There are two constraints in the choice of N:

(1) the N should be large enough so that the shape of the generated distribution can be recognized and tested, and (2) cost minimization. Two hundred cases are generated and endowed with exactly the same amount of wealth, 4.0 units (4.0 is an arbitrary number but there is no loss of generality). Mean wealth has to be greater than zero and the shape of the distributions resulting from a given L and delta, the parameters of the inequality process, is independent of the mean. Each of the 200 cases has one encounter with each of the other 199 during an iteration of the process. The Surplus Theory gives no guidance as to who interacts with whom or how many are party to an encounter, so, for simplicity, everybody has one pairwise encounter with everybody else once during an iteration. There are $(200 \times 199)/2 = 19{,}900$ unique encounters during one iteration. Two full iterations are run before case values are sampled to remove the "edge effect," the effect of the initial distribution. Then another iteration is run. After this iteration, case values are recorded. Thus each case has had $(199 \times 3) = 597$ encounters in which surplus wealth can change hands before it is first sampled. The case is then sampled again after each iteration, that is, after 199 encounters, another 9 times. The sample of observations on the inequality process contains 2,000 observations, 200 cases sampled 10 times. Observations on the same case are independent since serial correlations approximate zero after just a few exchanges of surplus wealth, as will be shown. Fixing the mean makes one case dependent on the other 199 but this dependence is negligible.

Findings

The inequality process was simulated a number of times with different values of its parameters L and delta. L is the degree of resistance to loss of surplus wealth and delta, δ, is the degree of advantage of the richer party to an encounter. Although the Surplus Theory suggests *delta* should be greater than .5, giving advantage to the richer party, some simulations were run with delta less than .5. Which parameter values were selected? Particular values for the parameters are not hypothesized. Rather, particular values are experimented with to see if they can reproduce the shapes of empirical size distributions of surplus wealth. A mnemonic for the combinations of the parameters is as follows: "*P*" indicates advantage to the poorer party ($\delta = .4$), that is, the richer party has a less than even chance of coming away the winner to the encounter; "*R*" indicates the richer party has a better than even chance of winning ($\delta = .6$); "*F*" indicates a fair encounter ($\delta = .5$). "1*L*" indicates one layer of resistance to extraction of surplus; "2*L*" two layers, and so on. So "*F*3*L*" means a simulation of an inequality process in which $\delta = .5$ and $L = 3$.

Can size distributions generated by the inequality process be fitted by gamma probability density functions? Yes. The technique of fitting is to first estimate the parameters of the gamma distribution from the observations, making the assumption that they are gamma-distributed. Knowledge of the arithmetic and geometric means is sufficient to calculate the maximum likelihood estimate of the shape parameter of the two-parameter gamma distribution. Here the method of Greenwood and Durand (1960) is followed. Since the mean of the observations is fixed, the scale parameter of the two-parameter gamma distribution, β, can be computed using the relationship, $\alpha\beta = \bar{x}$. Once the parameters of the two-parameter gamma distribution are estimated, a "theoretical" gamma distribution with the same parameters can be compared with the observed distribution. The Kolmogorov–Smirnov (K–S) Test compares the observed cumulative distribution with the theoretical distribution function. A one-sample, two-sided K–S Test is performed using the IMSL routine NKS1. The IMSL routine MDGAM is used to generate values from the theoretical gamma distribution function.

The results of the K–S Test are displayed in Table 1. These are p-values, the probabilities, under the null hypothesis, that one would be in error if one rejected the null hypothesis that the distributions are the same. A number close to 1.0 indicates near-indistinguishability between the distributions. There are several excellent fits—p-values close to 1.0—in Table 1. Only for two simulations, P1L and F1L, do the p-values of the K–S Test indicate rejection of the null hypothesis. In both cases if the method of moments estimate of α, the shape parameter, is used to generate the theoretical gamma distribution for the test, there is a fit. Gamma distributions with shape parameters smaller than .5, what P1L and F1L generate, are intrinsically more difficult to estimate than those with larger shape parameters. The other sixteen simulations generate distributions that can be well fitted by a gamma.

The p-values of the K–S Test are based on an N of about 2,000. How good is the assumption of independence when observations are made on 200 cases and then remade 9 times on the same cases? The answer is "excellent" since the serial correlation between the value of a case at one time and its value later decays quickly to zero. The serial correlation is almost zero after only 5 encounters. Cases are observed 199 encounters apart. The rate of decay of the serial correlation depends slightly on the parameters δ and L but for the range of parameter values used here, the variation is slight relative to 5 encounters, let alone 199 (see Table 2). Because mean wealth is fixed, there are only 199 degrees of freedom in the 200 cases of the simulation. The exact N of the K–S Test is 1,990.

The inequality process as formulated in Equations 4a, 4b generates distributions that when ranked and aggregated into a frequency distribution can be fitted by gamma distributions with shape parameters ranging from .38 to 3.13. This is a range of shapes that, at least approximately,

Table 1. *P*-VALUES OF TWO-SIDED, ONE-SAMPLE KOLMOGOROV-SMIRNOV TESTS OF FIT OF OBSERVATIONS OF VARIOUS PROCESSES TO GAMMA DISTRIBUTIONS*

	Parameter=		
L Parameter (layers)=	.4 P**	.5 F	.6 R
1	.0848***	.0420***	.9270
2	.2289	.5486	.9451
3	.6409	.8798	.8877
4	.9722	.4475	.6565
5	.5480	.8373	.6410
6	.9530	.6554	.6071

*N=2,000. Shape parameter of the theoretical gamma distribution fitted to the observations of each process defined by a δ, L pair is estimated from the observations of the process. These estimates are maximum likelihood estimates. The statistic lnA - lnG, where A is the arithmetic mean of the observations and G is the geometric mean, is computed. The method of Greenwood and Durand (1960) is followed in estimating α, the shape parameter, from lnA - lnG. The Kolmogorov-Smirnov test is performed by an IMSL (1982) routine, NKS1. The theoretical gamma distribution functions required for the test are generated with the estimated parameters and IMSL routine MDGAM. Since the mean of the observations is fixed, one of the two gamma parameters can be calculated from the other since $\alpha\beta=\bar{x}$, where α is the shape parameter and β is the scale parameter.

**The mnemonic for the process whose δ=.5 and whose L=3 is F3L, and so on for other combinations of the parameters.

***These p-values are below two conventional decision points for rejecting the null hypothesis. .0848 is less than .1 and .0420 is less than both .1 and .05. Rejecting the null hypothesis means that the distribution generated by the inequality process does not fit the gamma theoretical distribution with a shape parameter estimated from it. However, when method of moment (MME) estimates are used to generate the gamma theoretical distribution instead of maximum likelihood estimates (MLE), it is possible to fit the empirical distributions with a theoretical gamma. The p-value of the test when the MME estimate is used is .3385 for P1L and .4917 for F1L. Neither can justify rejection of the null hypothesis. MME estimates are computed from the mean and variance of the empirical distribution. The MME estimate of alpha for P1L is .6904, while the MLE estimate is .6686. The MME estimate of alpha for F1L is .5369, while the MLE estimate is .5079.

Table 2. AVERAGE SERIAL CORRELATIONS BETWEEN VALUES OF 200 CASES AFTER 398 ENCOUNTERS AND THEIR VALUES AFTER THE NEXT 5 ENCOUNTERS

	Encounter					
Process	0	1	2	3	4	5
P6L	1.0	.5963	.3569	.1900	.0807	.0143
P5L	1.0	.6097	.3836	.2121	.0874	.0012
P4L	1.0	.5585	.3295	.1673	.0536	-.0214
P3L	1.0	.5669	.3234	.1454	.0424	-.0228
P2L	1.0	.5280	.2589	.0918	-.0163	-.0707
P1L	1.0	.4759	.2244	.0810	.0074	-.0300
F6L	1.0	.6525	.4267	.2588	.1264	.0401
F5L	1.0	.6382	.4038	.2496	.1457	.0642
F4L	1.0	.6414	.4094	.2429	.1326	.0425
F3L	1.0	.6120	.3687	.2111	.1109	.0439
F2L	1.0	.5868	.3348	.1671	.0678	.0023
F1L	1.0	.5291	.2885	.1426	.0562	-.0105
R6L	1.0	.6748	.4611	.2964	.1772	.0886
R5L	1.0	.6625	.4504	.2779	.1579	.0737
R4L	1.0	.6571	.4414	.2889	.1617	.0530
R3L	1.0	.6439	.4116	.2383	.1146	.0281
R2L	1.0	.5993	.3636	.2060	.1028	.0154
R1L	1.0	.5486	.3073	.1648	.0746	.0046

The serial correlation for each case is defined following Granger and Newbold (1977, p. 73) as:

$$r = [\sum_{t=\tau+1}^{n} (x_t-\bar{x})(x_{t-\tau}-\bar{x})]/[\sum_{t=1}^{n} (x_t-\bar{x})^2], \ \tau = 0,1,2,3,4,5$$

where,

$n = 20$
τ = the τth encounter.

The numbers appearing in the table are the average of the r's for each of the 200 cases.

brackets the range of shapes that size distributions of surplus wealth have assumed over the evolution of technology. The gamma distribution with $\alpha=.38$ is generated by R1L, that is, by the inequality process with $\delta=.6$ and $L=1$; the gamma distribution with $\alpha=3.13$ is generated by the inequality process with $\delta=.4$ and $L=6$ (see Table 3).

The larger L is (an increase in L means that it is less likely a loser

Table 3. MAXIMUM LIKELIHOOD ESTIMATES OF ALPHA, α, THE SHAPE PARAMETER OF THE GAMMA DISTRIBUTION.*

L Parameter (layers)=	Parameter= .4 P**	.5 F	.6 R
1	.6686	.5079	.3797
2	1.0331	.9111	.6665
3	1.9029	1.3390	.9286
4	2.3592	1.5809	1.1128
5	2.5798	1.9306	1.2379
6	3.1307	2.2048	1.4461

*N=2,000. α for each set of 2,000 observations generated by a process defined by a δ,L pair is estimated from the statistic lnA - lnG where A is the arithmetic mean of the observations and G is their geometric mean, using Table 1 in Greenwood and Durand (1960, pp. 57, 58). These are maximum likelihood estimates.

**Mnemonic for process whose δ=.6 and whose L=4 is R4L, and so on for other combinations of the parameters.

will lose as large a proportion of wealth), the larger the shape parameter of the gamma is. A glance at Figure 1 illustrates what that means in terms of shape, a shift toward a more modern shape in the size distribution of personal wealth. Proposition #4 of the Surplus Theory, from Lenski (1966), asserts that increasing resistance to extraction will account for the decrease in the degree of concentration of wealth with industrialization. Salem and Mount (1974) and McDonald and Jensen (1979) have shown that if wealth is gamma-distributed then the Gini Ratio is a sole function of the shape parameter. So if the size distribution of surplus wealth is well approximated by a gamma distribution, then Lenski's hypothesis is as much about the shape of the size distribution as about the degree of concentration of wealth. McDonald and Jensen tabulate the Gini Ratio for various values of alpha. As alpha increases, the Gini Ratio decreases, as implied—when wealth is gamma distributed—by Proposition #4, the Lenski hypothesis.

Simulations of the inequality process thus far have shown:

1. That Proposition #1 of the Surplus Theory, the fugitivity of wealth principle, can generate a positively skewed, long right-tailed distribution without the assumption of skullduggery practiced by the rich on the poor,

since simulations in which *delta* is .5 or less (i.e., there is no advantage to being the richer party) also generate positively skewed distributions.

2. That the inequality process generates size distributions of personal wealth that can be closely approximated by a family of gamma distributions.

3. Proposition #4, the Lenski hypothesis that technology enables people to put up more resistance to extraction and that greater resistance to extraction shifts the Gini Ratio downward (toward less concentration), has to be modified. In gamma distributions the Gini Ratio is a sole function of the shape parameter. So the new Lenski hypothesis is that technology can change the shape of the size distribution of personal wealth through its effect on people's ability to resist extraction. In fact, simulations of the inequality process with larger *L*'s, that is, greater resistance to extraction, have more modern shapes, that is, gamma distributions with larger shape parameters.

Skullduggery Practiced by Richer on Poorer Persons?

Is there anything to be said for Proposition #2? Clearly if every time a richer person encounters a poorer person, the richer person walks away with all the poorer person's wealth, or even just a share, an impossibly concentrated distribution of wealth will result. But what if the richer person has just a somewhat better chance at winning an encounter with a poorer person than vice versa? Thus modified, Proposition #2 no longer generates absurd distributions of surplus wealth. The *delta* parameter, δ, controls the probability that the richer party to an encounter will come away a winner. *Delta* has the opposite effect on the shape of the resulting distribution from that of *L*. Making *delta* larger, that is, giving richer people a bigger advantage, pushes the shape of the resulting distribution backwards toward the shape that is characteristic of more primitive societies, a gamma with a smaller shape parameter. Giving poorer people a bigger advantage (making *delta* smaller) pushes the shape of the resulting distribution forward toward that of a modern distribution, that is, a gamma distribution with a larger shape parameter.

There is thus a fourth finding:

4. Simulations of the inequality process have shown that varying the degree of advantage or disadvantage of the richer (or poorer) party in the competition for surplus wealth can change the shape of the resulting size distribution of personal wealth.

Conclusions

How well does the inequality process deduced from the Surplus Theory of Social Stratification do in meeting the criteria that a theory of the size distribution of personal wealth ought to satisfy?

CRITERION 1: THE GENERAL PARETO LAW

The inequality process deduced from the Surplus Theory generates a range of distributions that appears to approximate the full range of shapes of size distributions of personal wealth in societies at various levels of technology. A mechanism, suggested by Lenski (1966), by which technology alters the shape of the size distribution of wealth in fact alters the shape of the distribution generated by the inequality process in the predicted direction. This paper has demonstrated that a single process may explain the distribution of wealth to persons in all societies. The General Pareto Law has been explained:

CRITERION 2: THE NARROW PARETO LAW

The Narrow Pareto Law asserts that the size distribution of personal wealth can be fitted by the density function $f(x) = vx_0^v x^{-(v+1)}$ where x is wealth, x_0 is the smallest wealth or income in the data to be fitted, and v is a parameter (Nygard and Sandström 1981). The right tail of the gamma distribution is different from that of the Pareto but there may be difficulty telling them apart, particularly the far right tail, given the way wealth or income data are usually treated: by aggregation into wider and wider categories with greater wealth. In fact, if one generates gamma observations with the IMSL routine, GGAMR, aggregates them into wider and wider categories for larger and larger values, and fits a Pareto Function to the category means, one will have great fits.[3] (See Table 4; explanation in note.) The inequality process explains the Narrow Pareto Law.

CRITERION 3: STABILITY

The mean of the distribution generated by the inequality process is fixed at 4.0 but other statistics on the distribution are free to vary. The variance, third and fourth central moments, median, and estimated alpha parameter of the gamma distribution with the same mean and variance are calculated after each of 100 iterations of the process. There are 100 cases observed in this simulation of the inequality process. Stability in the shape of the distribution means that these statistics are not trended and have little dispersion over the 100 iterations. A glance at the coefficients of variation

Table 4. LEAST-SQUARES FITS OF PARETO FUNCTION TO GAMMA DISTRIBUTED OBSERVATIONS FOR EACH OF FIVE SHAPE PARAMETERS*

Shape Parameter	Largest 20% of Observations		Largest 90% of Observations	
	r^2	-v	r^2	-v
.5	.988	-2.16	.759	-.592
1.0	.976	-2.51	.805	-.926
1.5	.985	-2.96	.824	-1.20
2.0	.967	-3.55	.838	-1.40
2.5	.987	-3.54	.859	-1.64

* Pareto Function fitted to gamma observations is ln y=ln a -v ln x, where y is the number of people having wealth x or greater and v is the Pareto parameter. x is gamma distributed. A set of a thousand observations are generated for each of five gamma shape parameters, 0.5, 1.0, 1.5, 2.0, and 2.5, by IMSL routine, GGAMR. The observations in each set are ranked from low to high. The largest n (n=200 or 900) observations are taken in order. These are grouped into categories of 20 each. The average of each category is taken. Then the natural logarithm of the number of people with wealth as great or greater than that of the mean of the category is taken. The category average is x. The complementary cumulative distribution (number of cases from the highest x value to a particular x value) to the category mean is y. The quantities ln a and -v are estimated by least-squares regression. The BMDP (Dixon and Brown 1979) routine, P1R, is used to estimate ln a and -v.

in Table 5 shows that these are small, indicating little dispersion. A glance at Table 6 shows that there is virtually no linear trend between the estimated shape parameter of the gamma and the number of iterations. There is no obvious nonlinear trend either. The inequality process generates stable distributions.

Table 5. AVERAGE OVER 100 ITERATIONS OF DESCRIPTIVE STATISTICS ON 100 CASES*

Average & Coefficient of Variation for the Following Statistics Over 100 Iterations of Each of the Following Processes:	Processes						
	F2L	P2L	F3L	F4L	R5L	F5L	P5L
Mean (fixed)	4.0	4.0	4.0	4.0	4.0	4.0	4.0
Variance							
Average	16.1	13.0	12.2	9.8	12.7	8.4	5.8
Coefficient of variation	.198	.178	.180	.169	.217	.194	.163
3rd Central Moment							
Average	121.	84.	70.	48.	83.	35.	16.
Coefficient of variation	.597	.527	.606	.547	.711	.707	.574
4th Central Moment							
Average	2075.	1289.	1031.	632.	1317.	444.	163.
Coefficient of variation	.927	.802	.980	.824	1.136	1.145	.862
Median							
Average	2.80	3.04	3.05	3.25	2.99	3.36	3.54
Coefficient of variation	.093	.092	.078	.071	.091	.067	.049
Gini Ratio							
Average	.502	.456	.446	.405	.449	.378	.322
Coefficient of variation	.060	.059	.057	.060	.066	.070	.067
Shape Parameter, Alpha, of Gamma Distribution**							
Average	1.03	1.27	1.36	1.68	1.32	1.98	2.83
Coefficient of Variation	.209	.178	.167	.159	.211	.186	.159

*After each iteration statistics on the distribution generated by the inequality process are calculated. The N of these statistics on each distribution is 100 cases. Each simulation of the inequality process has 100 cases. The simulation is run 100 times producing 100 of these statistics. What appears in the table is the average of the 100 statistics and the coefficient of variation of this average over the 100 iterations.

**The shape parameter of the gamma distribution, α, is estimated by the method of moments from the mean and the variance of the 100 cases in each iteration of the inequality process. Like the other table entries these shape parameters are the average of the estimates over 100 iterations.

CRITERION 4: EVOLUTION OF INEQUALITY WITH TECHNOLOGY

How does the inequality process deduced from the Surplus Theory account for the fact that the concentration of wealth increases with technology in primitive societies but decreases with technological advances in modern industrial societies? The inequality process generates, at least approximately, gamma distributions. In gamma distributions, the Gini Ratio, the most common measure of concentration, is a sole function of the gam-

Table 6. PEARSON CORRELATION COEFFICIENTS BETWEEN ESTIMATED GAMMA SHAPE PARAMETER AND ITERATION NUMBER*

Process	Correlation Coefficient
F2L	.0335
P2L	-.0187
F3L	.0446
F4L	.1408
R5L	-.0269
F5L	-.0153

*The shape parameter of the gamma distribution is estimated by the method of moments from the mean and variance of 100 cases after each iteration of the inequality process. 100 iterations are run. Then the 100 estimates are correlated with the number of iteration, i, from which they are estimated. i=1,100. BMDP (Dixon and Brown 1975) routine P6D is used to estimate the correlation coefficients.

ma's shape parameter, alpha. The relationship between alpha and the Gini Ratio has been tabulated (McDonald and Jensen 1979). Gamma distributions with large alphas have small Gini Ratios, that is, the more modern the shape of a size distribution of personal wealth, the less concentrated wealth is. Conversely, the more primitive the shape of a size distribution of wealth, the more concentrated is wealth.

Doesn't this fact contradict the fact that hunting and gathering societies, the most primitive, are egalitarian? No. The inequality process generates size distributions of surplus wealth, not total wealth, that is, subsistence plus surplus. There is very little surplus wealth in hunter/gatherer societies to be unequally distributed. However, since this small amount has a high utility to those who have it, the inequality process suggests that hunter/gatherer societies may not have been as utopian as they have appeared to anthropologists from industrial societies not especially impressed with the value of the hunter/gatherers' possessions or standard of living. A little surplus wealth may make the difference between life and death in such a society, particularly in a challenging environment, such as a cold climate.

The inequality process explains the rise and then fall in the concentration of wealth as a function of advancing technology in terms of two opposing effects:

1. As technology advances from the hunter/gatherer phase on, subsistence, equally distributed, becomes a smaller fraction of total wealth. Surplus is unequally distributed. An advance in technology at the hunter/gatherer level appears to increase concentration of wealth.

2. The degree of inequality of surplus wealth decreases as a function of technology. Eventually this effect overwhelms the first effect and a decrease in concentration of wealth is noticed, even though the degree of inequality with which surplus wealth is distributed has been declining steadily over the evolution of technology.

CRITERION 5: EVOLUTION IN THE SHAPE OF THE SIZE DISTRIBUTION OF PERSONAL WEALTH

While the inequality process models the distribution of personal surplus wealth, evidence about the shape of the size distribution of personal wealth all pertains to total personal wealth, the sum of subsistence and surplus. Inferences about the shapes of size distributions of surplus wealth can be made from this evidence when one considers that surplus wealth is a small fraction of total wealth in hunter/gatherer societies but a large fraction in industrial societies. The shapes of the size distributions of personal wealth can be approximated by a family of gamma distributions in which the shape parameter, alpha, is linked to technology.

Gamma distributions with alphas from about 2.0 to 2.5 fit size distributions from an industrial society (McDonald and Ransom 1979; Salem and Mount 1974). These are fits to size distributions of total personal wealth, but in which subsistence is a small part. Ranked societies on the other hand have distributions in which the modal amount is hardly any surplus wealth and the distributions have a large positive skew, much like that of a gamma with an alpha near 1.0. Hunter/gatherer societies have even less surplus and a very short right tail, like that of a gamma with an alpha near .5. The gamma family of distributions, though, can only be an *approximation* to a size distribution of personal wealth since the domain of the gamma density function ranges from zero to positive infinity. The range of personal wealth in all societies has an upper bound. So while the general shape of a gamma with an alpha of .5 approximates the distribution of personal surplus wealth in a hunter/gatherer society, a theoretical gamma with an alpha of .5 might have an observation many times the magnitude of the mean. A real hunter/gatherer society will not likely have such an individual. Notice that the distribution generated by the inequality process, while approximating a gamma, does not share these difficulties of the gamma in fitting empirical size distributions of personal wealth. If adjusted for subsistence, the size distribution of personal wealth generated by the inequality process approximates the observed range of shapes of size distributions of personal wealth in all human societies.

The inequality process meets the criteria that a general theory of the size distribution of personal wealth (and by implication since inequality, however defined, is simply a function of the size distribution of personal wealth, a general theory of inequality, as well) should meet. This very general and powerful theory has been implicit in the Surplus Theory of Social Stratification since the mid-nineteenth century when Lewis Henry Morgan ([1877] 1963) thought it might explain the origins of inequality of wealth and the brutality of many ancient societies. Abstraction and formalization, however, were needed to unlock its implications for the size distribution of personal wealth.[4]

Speculation

The inequality process deduced from the Surplus Theory explains quite a few diverse facts about size distributions of personal wealth. Perhaps it is a good working model of the *the* Inequality Process, the social process that shapes size distributions of wealth. What implications can be drawn from the model of the process for the future of the Inequality Process?

The shape of the size distribution of personal wealth in societies at higher technological levels than now possessed by industrial societies can be represented by gamma distributions with larger shape parameters than 2.5. This is just a simple extrapolation. Future technology may conceivably permit the return of the shape of the size distribution of personal wealth to one like that of a ranked society. Further technological advance may facilitate the extinction of humanity, making the extrapolation irrelevant. Despite these uncertainties, extrapolating a 10,000+ year (since the beginning of agriculture) relationship seems like a good bet. As the shape parameter of the gamma distribution becomes larger, the gamma distribution converges to the normal distribution (Johnson and Kotz 1970). The Gini Ratio, a sole function of the shape parameter of a gamma distribution, decreases as the shape parameter increases but at a slowing rate. A society with a much higher level of technology than our own would not experience the kind of rapid change in the Gini Ratio that an agrarian society experiences in an industrial revolution even though the future pace of technology may increase the shape parameter of the distribution as quickly. The change in the shape of the gamma distribution, and consequently its Gini Ratio, created by a change in alpha from .5 to 1.5 is much greater than that created by a change from 4.5 to 5.5. The propensity of the Gini Ratio in a distribution characteristic of an agrarian society to decrease rapidly as technology forces an increase in the shape parameter of the society's size distribution of surplus wealth may very well explain the vulnerability of this kind of society to political revolution. Power based on relative wealth can shift, with technological and economic development, more rapidly away from the very rich than in any other kind of size distribution of personal wealth.

Is there a hi-tech utopia in the offing, a realization of the faith placed in technology in the eighteenth and nineteenth centuries? Yes, if a society with a small Gini Ratio is your ideal. No, if you prefer a society with less dispersion in the distribution of personal wealth. Here is how the conjecture works. Let x, wealth, be gamma-distributed. The mean of x is $\alpha\beta$, the product of the shape and scale parameters. Mean wealth, $\bar{x} = \alpha\beta$, increases as a function of technology. Alpha also increases with technology, but while surplus wealth increased from near zero in hunter/gatherer societies to many times subsistence in industrial societies, alpha has only increased from somewhere near 0.5 to 2.5. It is clear that β, the scale parameter, given a metric of wealth in which the unit is subsistence or smaller, has increased much faster than α and is much larger than α.

The variance of x is $\alpha\beta^2$. The variance will increase much faster than the mean. So the variance of the size distribution of personal wealth, and in general its dispersion, has increased faster than the mean and may very well continue to do so under the influence of technological advance. Thus, while future size distributions of personal wealth may be normal distributions, they will have large and increasingly larger variances. The difference between this kind of growth in variance and the growth of variance of the distribution generated by the Law of Proportional Effect is that the Law of Proportional Effect itself generates distributions with ever-increasing variance, whereas this inference from the inequality process about future size distributions of wealth is a proposition about change in the parameters of the process, due to an exogeneous effect. Size distributions of personal wealth in the distant future will appear to be a uniform scattering of individuals over the range of possible personal wealth. The probability of a person encountering another person with about the same amount of wealth will approach zero. Of course, except for the poorest people of these more advanced societies of the future, the marginal utility of wealth may have declined so much that differences of wealth may not have the same social meaning as they have today. Perhaps.

Lest this prediction of a size distribution of personal wealth with an ever-increasing dispersion seem farfetched, think of the current worldwide size distribution of personal wealth. Berry, Bourguignon, and Morrisson (1983) have found that most of the world's inequality can be attributed to between-country differences in average wealth rather than to within-country differences. The application of industrial technology has already strung societies out along a continuum of wealth from subsistence (or less) to what seems today to be great wealth. Provided that technology does not carry within itself the seed of the destruction of humanity, there is no reason to expect further technological advance not to continue to increase the dispersion of the size distribution of personal wealth.

THE INEQUALITY PROCESS AS PARADIGM

The inequality process is deduced from the Surplus Theory of Social Stratification and can be viewed as just a formalization of that theory. It may also come to be viewed as a paradigm for social sciences concerned with linking gross structural features of a society to individual actions. The inequality process can be viewed as a mathematical model of individual action that is convenient and compelling. There is much that can be tinkered with in the inequality process. Let us look at several examples of tinkering.

The Surplus Theory is concerned only with how wealth is distributed, not with how it is created. Thus encounters in the inequality process are zero-sum. But why does the Surplus Theory assume that there are encounters? Surely most encounters come from the need to collaborate to create wealth. Thus a generalization of the inequality process (perhaps, the "exchange process") would permit both positive- and negative-sum encounters, which average out to positive-sum encounters, in which wealth is created, and sometimes lost or destroyed, as well as distributed. Wealth might be consumed and waste away in such a model. Perhaps with cases endowed with different abilities, different types of wealth, and different utility functions, one could simulate the development of markets with all their well-known aggregate features.

Another example. Suppose one tinkers with the delta parameter of the inequality process. The delta parameter determines the probability that a party to an encounter will win the encounter on the basis of the characteristics of both parties. In this paper, delta has been set on the basis of who the richer party to the encounter is. But that is simplistic. It would be more sociological to see delta as a function of various characteristics of the parties: their relative abilities, the power of coalitions based on similarity of lifestyle (social classes) or ethnicity, as well as relative wealth. One could simulate the consequences of the inequality process for the size distributions of personal wealth of ethnic groups in the same society. If the results fit real distributions, the delta function would be a working model of how individual actions toward other people result in the gross structural features of wealth distributions in different groups.

Notes

1. The Pareto density function is $f(X) = v\ X_0^v\ X^{-(v+1)}$ where X is wealth, X_0 is the smallest wealth or income in the data to be fitted, and v is a parameter (Nygard and Sandström 1981, p. 30). X_0^v is a constant that makes the function a density function by forcing the integral from X_0 to ∞ to be equal to one. The integral of the Pareto density can be written as $F(X) = aX^{-v}$, where a is a constant. This is the familiar form. This function, usually called the Pareto Function, can be fitted either to the cumulative distribution of income or wealth or to the complementary cumulative distribution. The cumulative distribution begins with the case with the smallest amount of wealth, X_0, counts it, and then ads up cases with successively larger amounts of wealth. The complementary cumulative distribution begins with the case with the greatest wealth and adds up cases with successively smaller amounts of wealth, down to the case with X_0. The complementary cumulative distribution is what the Pareto Function is usually fitted to for two reasons. One is that the top end of the distribution is better measured than the bottom end. X_0 is, in practice, chosen at the point where measurement error becomes uncomfortable. The other reason is that the Pareto Function is a better fit to the right tail of the size distribution of personal wealth than to the central region or left tail.

2. Where did the turn in the relationship between technological change and concentration of wealth come? Lenski (1966) first put it just before the onset of industrialization but later put it in the early stages of industrialization (Lenski and Lenski 1974). Henderson (1978) provides evidence that supports the latter conclusion. Historical records indicate that there was considerable concentration of wealth well into the period following the industrial revolution (Gallman 1969; Soltow 1971, 1981a, 1981b) but these are mostly tax records, or other records indicating ownership of substantial real property. Such distributions are typically left-censored (tending to leave out the poor, especially those with nothing to report). Human capital, skills, began to be widely held even among people with few other assets beginning with the industrial revolution. Income distributions measure returns to human capital as well as to real property. Human capital is difficult to measure by any other standard than the income it generates. Not surprisingly, in industrial societies the concentration of income distributions is less than those of asset (excluding human capital) distributions. It is quite possible that the turn in the relationship between technology and concentration of wealth came with the beginning of mass education, somewhat before tax records and other records of real property began showing a decrease in concentration. The exact timing cannot be established here but an adequate theory of the size distribution of personal wealth should explain the reversal of the relationship.

3. To test whether the right tails of the Pareto and gamma distributions can be distinguished if they are treated as size distributions of wealth or income are treated, we need to generate gamma observations and fit a Pareto Function to them in the way that it is usually fitted to income or wealth data. The IMSL (1982) routine, GGAMR, is used to generate gamma distributions with different shape parameters. These are then aggregated into wider and wider categories for larger and larger values, as wealth or income data are.

A thousand observations are generated for each of 5 gamma shape parameters, .5, 1.0, 1.5, 2.0, and 2.5, approximately bracketing the range of shapes of size distributions of personal wealth. The observations are ranked from low to high. The largest 200 are taken in order to be fitted by the Pareto Function, i.e., $x_0 = 801$st observation. These cases are grouped, in order, into 10 categories of 20 each. This treatment is similar to the treatment of income or wealth data in many analyses. In successively more highly ranked categories, the range of the category, the difference between the smallest and largest observations, becomes wider and wider. The average of each category is taken. The x observations to which the Pareto Function, $y = ax^{-v}$, is fitted are the category means; the y's are the values of the complementary cumulative distribution to the category mean, i.e., 190 for the lowest category, 170 for the next lowest, to 10 for the highest category. The natural logarithms of both x's and y's are taken. The equation to be estimated is then $\ln y = \ln a - v \ln x$, a linear equation. The constant term $\ln a$ and the coefficient v can be estimated by linear regression. The P1R regression subroutine of the BMDP package of statistical subroutines (Dixon and Brown 1979) is used to estimate $\ln a$ and v. If the r^2 of the regressions is high, i.e., close to 1.0, then the rightmost 20% of the gamma distribution's right tail is closely fitted by the Pareto density. Table 4 displays the fits for each of the parameters used to generate a gamma distribution. The scale parameter of the gamma has no effect on the fit and is ignored.

The fits are excellent, with r^2's ranging from .967 to .988. If the gamma distributions were data on people's wealth or income, the richest 20 percent of the cases would be readily regarded as well fitted by a Pareto density. So given the way most empirical size distributions of wealth or income are treated, it is quite understandable that even if they were generated by a gamma process, the rightmost part of their tail would be judged Pareto-distributed. The inequality process deduced from the Surplus Theory satisfies the Narrow Pareto Law criterion.

But what about the Pareto parameter, v, and the expectation that it be close to 1.5? The parameter, v, is usually estimated on a much larger proportion of a population than the richest 20 percent. Let us take the richest 90 percent of cases and repeat the fitting procedure, again grouping the cases ranked from low to high into ordered categories of 20 each. The Pareto Function is not expected to fit the richest 90 percent of cases as well as it does the richest 20 percent, and it does not, but also as expected, the range of v's estimated includes the value 1.5. Perhaps unsurprisingly, 1.5 occurs between fits to the two gamma distributions with shape parameters characteristic of industrial societies. When Pareto concluded that 1.5 is an equilibrium value for a society, he was simply concluding that industrial societies are more stable than agrarian societies or societies in the initial phases of industrialization. Estimates of v are very sensitive to censoring of the left tail of a size distribution of personal wealth and should not be viewed as indicative of fundamental processes at work in a society unless censoring is not a problem.

4. It should be pointed out that the inequality process deduced from the Surplus Theory bears a resemblance to other equations known to generate distributions resembling size distributions of wealth or income. For example, Equations 4a, 4b incorporate certain elements of the law of proportional effect. They permit losses to be proportional to the size of wealth possessed. No such principle is permitted for gains, though. Another difference is that Equations 4a, 4b put limits on what ego can gain (everything alter has) and what ego can lose (everything ego has) in any one stochastic event.

Equations 4a, 4b bear some resemblance to a process described by Shorrocks (1975), who was looking for a plausible stochastic process that would converge to a distribution like a size distribution of personal wealth more quickly than some of the classic stochastic models, such as the law of proportional effect. He found that if one has a population in which each person's wealth, j, is subject to a proportional gain or loss in a unit of time, this process leads to a size distribution that is asymptotically negative binomial. The negative binomial can be thought of as a "discretized" gamma distribution. Some of the differences between Equations 4a, 4b and Shorrocks (p. 632) can be verbally described as: (1) Equations 4a, 4b jointly determine wealth exchange between two members of the population; Shorrocks' equation describes events occurring to the wealth of a single individual without reference to other individuals, (2) there are limits to what can be gained or lost in any one stochastic event in the inequality process; there are no limits in Shorrocks' process, (3) encounters in the inequality process are zero-sum so there is no confusion of a purely distributional phenomenon with the creation and destruction of wealth; Shorrocks' process creates and destroys wealth, and (4) Equations 4a, 4b describe a two-stage process in which the winner of the encounter is first determined (and if delta $\neq .5$, information about who has more wealth than the other is used as well) and then the matter of how much the loser gives up to the winner is decided; Shorrocks' process has one stage. The fact that the inequality process and Shorrocks' process, different as they are, lead to about the same family of distributions suggests that the implications of the Surplus Theory for size distributions of personal wealth are robust against tinkering with Equations 4a, 4b and that quite a few different readings of the Surplus Theory would point to the same conclusion.

Appendix. Algorithm for Encounters and Exchanges of Surplus Wealth in the Inequality Process in FORTRAN 77

Code	*Commentary*
`LAST = N - 1`	
`DO 100 I = 1, LAST`	I is the subscript on which case is the "ego" case.
`NI = ISUB(I)`	ISUB( ) is a vector of integers 1,. . . ,N in random order. Order is reshuffled after each iteration.
`NEXT = I +1`	
`DO 100 J = NEXT, N`	J is the subscript on the "alter" cases, the cases "ego" case encounters.
`NJ=ISUB(J)`	
`FRACT = 0`	
`DO 30 K = 1, LAYERS`	LAYERS = L, the number of layers distinguished in a person's surplus. LAYERS is a parameter and remains constant throughout the simulation.
`30 FRACT = FRACT + (RANF( )**K)`	RANF( ) is a large vector of independent 0,1 uniform variates. There are 10,000 such variates in the vector. The subscripting algorithm is omitted for simplicity. It skips through the vector.
`FRACT = FRACT/LAYERS`	FRACT is the fraction of surplus wealth the loser to the encounter will lose.
`IF (SURPLUS(NI) .GT. SURPLUS(NJ)) GO TO 40`	If ego's surplus wealth is greater than alter's surplus wealth, then control is transferred to statement 40.
`IF (RANF( ) .LT. DELTA) GO TO 50`	DELTA is a parameter of the inequality process and remains fixed during a simulation. DELTA is the probability that the richer party will win the encounter.
`GO TO 60`	
`40 IF (RANF( ) .LT. DELTA) GO TO 60`	
`C ALTER WINS SOME OF EGO'S SURPLUS WEALTH`	

```
   50 TRANS = SURPLUS(NI)*FRACT          TRANS is what the loser gives up to the
                                         winner.
      SURPLUS(NI) = SURPLUS(NI) -
      TRANS
      SURPLUS(NJ) = SURPLUS(NJ) +
      TRANS
      GO TO 100
C  EGO WINS SOME OF ALTER'S
   SURPLUS WEALTH
   60 TRANS = SURPLUS(NJ)*FRACT
      SURPLUS(NJ) = SURPLUS(NJ) -
      TRANS
      SURPLUS(NI) = SURPLUS(NI) +
      TRANS
  100 CONTINUE
                                         The sum of surplus wealth of the two par-
                                         ties after the encounter is the same as
                                         before.
```

The way to understand this algorithm is to trace through it in two circumstances. Circumstance #1 is ego is richer. Ego is case I, the location in the vector of cases denoted by subscript NI. Since ego is richer, the condition "If (SURPLUS(NI) .GT. SURPLUS(NJ)) GO TO 40" is met and control is passed to statement 40. There is another test at statement 40. If ego's random number is less than delta, the advantage of the richer party, then control is passed to statement 60 and the statements below it, in which ego takes proportion FRACT out of alter's surplus. If the advantage is to the richer party, that is, delta is greater than .5, statement 40 gives an advantage to ego, the richer party. In fact statement 40 gives the richer party exactly a delta probability of winning. Circumstance #2 in which alter is the richer party is analogous.

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