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An Agent-Based Model of Wealth Distribution*

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Abstract

This model belongs to the large group of essentially neo-classical models that neglect work, production, and productive relations, but rather focuses on distributive interactions in a hunter-gatherer society. We set up an agent-based model where each agent has internal states and behavioral rules. Some states are fixed for the agent's life, while others change through interaction with other agents or with the external environment. In our model, the agent's metabolic rate and vision are fixed while wealth can change as agents move around and interact with the environment and indirectly with their co-agents as well. Life in an artificial society takes place in an environment governed by its own rules to provide a space of renewable resources that agents can get, metabolize and accumulate. Finally, there are rules of behavior for agents and for sites of the environment. In our model a movement rule for agent is to look around as far as one can, find the richest site, go there, get the resource. The rule for the environment sites could state: each site refreshes its initial resource capacity at the end of each period. Using this simple microstructure the model tries to "grow" a collective structure such as the wealth distribution. In the first palce, we find that agents' skill (vision) heterogeneity increases wealth inequality, measured by the Gini coefficient. Secondly, we discover that, switching to a different measure of inequality, the gap between the richest and the poorest agents, does not allow us to obtain a clear understanding of the effects of skill's heterogeneity. But, we do find that the increase in average vision increases the average wealth. Moreover, we find that inheritance of wealth, neighborhood at birth, metabolism etc. reduces the Gini, while vision inheritance increases it. Finally, our model reproduces a wealth distribution much more skewed than the Pareto's law.

JEL Classification: B41, C65, D31.

Keywords: Agent-Based Models, Computational Economics, Income Distribution, Economic Methodology.

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1 Methodological introduction

In this paper we apply agent-based computer modeling technique to study the dynamics of wealth distribution. The basic feature of this model is that "fundamental social structures and aggregate behaviors emerge from the interaction of individual agent operating on artificial environments under rules that place only bounded demands on each agent's information and computation capacity" (Epstein and Axtell (1996), henceforth E&A 96, p.6). Before going deeply into the characteristics of this technique we will discuss some methodological issues related to this method of inquiring into economic theory, in order to understand the peculiarity of this modern approach. To this end, we will examine the methodological features of two different conceptions of economic theory following one upon the other as dominant theory in the life of the discipline: the British classical political economy and the contemporary neoclassical economic theory.

In the first place, the two theories can be distinguished by the choice of the point of view taken to observe economic phenomena from: the neo-classical economists opt for an individualistic perspective and as a result analysis focuses on the simple elements of these phenomena, or in other words on the single parts constituting the social whole. It is an approach that takes society as the sum of separate individuals: social phenomena are assumed to be attributable to the actions of individuals as involuntary products. In this approach - termed methodological individualism in the social sciences - when individuals inter-relate forming a social whole they continue to behave on the basis of their own intrinsic characteristics and the fact that they belong to a collective body does not modify their behavior. Thus society is the mere sum of isolated individuals and the plurality of people does not affect the constitution of human subjectivity. As Menger put it, the original factors of economic theory consist of the elements of the "single economies", namely needs, the resources nature has to offer and the aspiration to satisfy these needs. Political economy shows how the more complex phenomena of "social economy" develop from these factors (see Menger 1985 p.45) The laws of "social economy" are the theoretical representation of the ratios between economic magnitudes (prices, quantities of goods, income, wages, profits, rents), which are thus deduced from the relations between individuals, needs and the natural environment, without considering those relations between individuals that take on concrete form in entities external to the individuals, consisting in institutions, moral rules, social conventions and, more generally speaking, the culture of a people. By contrast, the classical economists favor a systemic or - as the social sciences have it - "holistic" approach, which consists in taking a more general position to observe reality, focusing more on the product of interaction between subjects in society than on the actions of the individuals. Here the individual is seen as a social being, an entity that generates society and is generated by it; in social interaction the individual is not taken as an isolated subject acting independently of the others, but rather as occupying roles and positions within society, which in turn influences his behavior. Classical economics takes these collective roles into consideration as products of the historical interaction between individuals, and goes on to analyze the functioning of the social system consisting of groups, classes, hierarchies, institutions social conventions and moral rules. With this more general perspective, classical theory is able to bring into its analysis of economic phenomena those social and historical factors that define the economic problem in all its complexity.

The second central aspect we focus on is closely bound up with the first, being a matter of the different interpretations of the nature of economic generalizations in terms of the two approaches considered. Neo-classical analysis based on the model of the natural sciences and the positivist conception of science makes of economic theory "an extension of the field of application of exact logic" (Schumpeter p..). These economists display great confidence in the possibility of the field of application of the quantitative relations with general properties sufficiently well defined in form. Together with a rigorously individualistic approach, this leads them to exclude from their analysis forces determining the fundamental economic relations - i.e. historical and social factors - confining any possible effect of the latter to the exogenous elements of their theory (taste, labor endowment, technological conditions). By contrast, classical analysis takes a more cautious approach to the use of mathematical tools, dividing the structure of analysis into two separate fields (see Garegnani 1990). The first field consists of a theoretical "core", which is identified with the determination of relative prices and the rate of profits, where economic phenomena can be represented with general quantitative relations of a definite form. Economic relations are more complex in the second field where we can find the analysis of the forces determining capitalistic accumulation and real wages; these forces cannot be represented with a system of rigorous quantitative relations, but analysis must be conducted with a more inductive method of inquiry involving specific experience of real events. By limiting the sphere of application of axiomatic procedures the classical economists succeed in taking account of historical and social factors in explaining economic phenomena, thus affording the study of such factors an integral part in economic theory.

Now that we have presented two important methodological benchmarks we can use them to discuss the basic features of the agent-based models. The agent-based (AB) modeling proposes a different approach that tries to deal with the potential complexity of human interaction using computer simulations and experiments. The agent-based modeling methodology has a long history beginning with Von Neumann's work on self-reproducing automata (1966). Since then it has had a lot of developments and applications in various fields. The efforts of the pioneers of this methodology were constrained by the limited computational power available in the seventies and eighties. It is only in the nineties that advances in computer science have made agent-based modeling powerful and practical. The recent efforts to use agent-based modeling in social science include the works of Albin and Foley (1990), Arthur et al. (1996, 1997), Marimon, McGrattan and Sargent (1990) .

The basic idea of this approach is to "grow" social structure of interest in the computer in order to discover the micro mechanisms that are sufficient to reproduce them. The main aim is, of course, to be able to control the micro mechanism in such a way that the macro structure we grow is as close as possible to the real phenomena. In a further step the conditions of divergent results are assessed. We can now go deeper in analyzing some important characteristic. In macroeconomics the use of representative agents eliminates heterogeneity of empirical agents. In the agent-based models the population is highly heterogeneous and consists of agents of distinct resources and genetically and culturally transmitted traits. Individual traits can change - adapt - in the course of life as a result of interaction with other agents and with the environment.

Action in these models takes place locally and no agent has global information. When trade is modeled, price formation is a process of completely decentralized bilateral trade between neighbors. There is neither any computation by any agent of the market-clearing price, nor any super-agent like Walrasian auctioneer doing that job for all the others. This is in stark contrast with neoclassical theory, which relies on aggregate excess demand functions - or some other for of global information - for the existence and convergence to equilibrium. Furthermore the equilibrium prices that emerge are different from the general equilibrium prices of neoclassical theory, it is a statistical equilibrium. If we try to predict the price of a transaction we should take into account that each agent's utility depends not only on her own internal valuation (marginal rate of substitution) but also on that of her trading partner; obviously, this calculation can only be made probabilistically. One important consequence of this is that agents with identical endowments and preferences can end up in very different welfare states through the decentralized trade: they encounter different people, bargain to different prices, and trade different quantities, producing initially small differences in their respective welfare states, which can be amplified with time. This result is related to the fact that with decentralized bilateral trade the economy is perpetually out of equilibrium (in the neoclassical sense) and the story of the efficient market allocation told by the First Welfare Theorem does not hold .

Another important characteristic of the agent-based models is that the point of departure is the individual: the economist gives agents rules of behavior then a simulation is run and a macroscopic social structure emerges. This approach contrasts with what we called the "holistic approach" and with the highly aggregate perspective of macroeconomics, in which social aggregates like classes, states and institutions constitute the starting points of the analysis. Thus, to that extent agent-based models can be characterized as methodological individualists. However, as E&A (96) point out, agent-based models belong to that version of methodological individualism that believe that the collective structures or "institutions" that emerge can have feedback effects in the agent population, altering the behavior of individuals (see p.16-17). This allows studying the interactions between individuals and institutions, actions and structure, and invention and nature.

Finally, we want to tell something about the idea of explanation that is behind the agent-based models. E&A observe that this kind of models is going to change the idea of explanation in social science. They suggest that in order to ask ourselves if we can explain a phenomenon, we should ask if we can "grow" it (p.29). The central aim of economics and of all social science should be to look for the microspecifications that allow us to generate in silico the macrophenomena we are interested in. Such experiments lead to hypotheses of social concern that may be tested statistically against data.

Now we will try to address the issue of how to place the agent based modeling technique in the epistemological debate between classical and neoclassical approach. In the first place, we already observe that AB models share with neoclassical theory the methodological individualism. However, the idea of the feedback effects from the social structures to the single agent recall the classical concern for social and historical factors in explaining economic phenomena. Furthermore, the local dimension of individual interactions allows the AB models to analyze the inherent complexity of human relations in the economic sphere. As Foley observes, "the neoclassical rational economic agent needs not to consider the reactions of other actors in their potential complexity, since the equilibrium market prices convey enough summary information to allow her to make a rational plan" (1998 p.54-55). Thus, the reductionism of neoclassical theory lies in the faith that the complexity level of the economy as a whole is lower than that of the agents who constitute it.

In the second place, the interpretation of the nature of economic generalizations in the AB models is a little bit controversial. As we said these models propose a new idea of explication in social science and probably this rules out the possibility of using old categories like the causal relations. The connection between the emerged macrostructure and the micro mechanism, which can grow it, is certainly not experimental, in the sense that our results are the product of experiments and not analytic-logical deductions from some basic postulates, rather the nexus is stochastic. Features from the inductive approach come to the fore insofar as the agents are modeled in accordance to empirical human beings.

But are we sure that in economic analysis explanation doesn't matter at all? Somebody could observe that this kind of argument is close to Friedman's idea of economic theory as a machine to create good predictions. No matter how we construct the model, the validation of a theory is determined by its capacity of generating good predictions (see Friedman 1953). The critics to this approach are centered on two basic arguments: on one hand, theories focusing on forecasting at the expense of explanation do not contribute to our understanding of economics if by this understanding we mean progressive steps towards the discovery of the truths in causal relations between economic magnitudes. Moreover, the forecast criterion brings research activity up against a blind wall; two theories with the same forecasting capacity are considered equally good, and there is no way of judging the specific validity and related areas of application. E&A do not seem to be worried about this criticism when they observe "the mapping from micro-rules to macrostructure could be many to one. In the social sciences, that could be an embarrassment of riches; in many areas, any to one would be an advance" . On the other hand, causal relation could serve as a good manual in the case that our theoretic machine breaks down and we need to look under the hood to fix the problem. In the next paragraphs, we will present a model for "growing" the relevant phenomena of wealth inequality and we try to evaluate the consequences of the methodological features discussed directly in the field of the application.

2 The Basic Model

While the agent-based modelling approach provides models for phenomena of many scientific areas, we pick one characteristic feature of capitalist societies. The model simulates the distribution of wealth using the Net Logo Beta 8.0 program. Our model is grounded on the wealth distribution model of netlogo, which is mainly based on E&A's Sugarscape Model (1996 ch. 2). It belongs to the large group of essentially neo-classical models that neglect work, production, and productive relations, but rather focuses on distributive interactions in a hunter-gatherer tribe. Basically, such a model has three ingredients: agents, an environment or space, and rules. Where agents and rules of decision are familiar from neo-classical models, an objective and autonomous environment unimaginable in neo-classical theory. Our set-up further deviates insofar as initially not only the agents, but the environment is endowed as well. The decision taking aims to maximize, but the constraints are neither global nor obvious.

Agents are the people of the artificial society we construct. Each agent has internal states and behavioral rules. Some states are fixed for the agent's life, while others change through interaction with other agents or with the external environment. In our model, the agent's metabolic rate and vision are fixed while wealth can change as agents move around and interact with the environment and indirectly with their co-agents as well. Life in an artificial society takes place in an environment governed by its own rules to provide a space of renewable resources that agents can get, metabolize and accumulate.

Finally, there are rules of behavior for agents and for sites of the environment. In our model a movement rule for agent is to look around as far as one can, find the richest site, go there, get the resource. The rule for the environment sites could state: each site refreshes its initial resource capacity at the end of each period. Using this simple microstructure the model tries to "grow" a collective structure such as the wealth distribution. In the E&A model wealth is represented by an amount of sugar, while we call it grain. In the following subsection we introduce the main features of our setting.

2.1 Description of the basic setting

The model can be specified describing its three main component: agents' characteristics, the nature of the environment, and the rules of the welath accumulation mechanism.

1. The Agents

For each agent a location on the grain space is specified. In the model, agents are randomly distributed in the grain space. Some agents are born at or in vicinity of rich patches, others at poor ones. Two and more agents are allowed to occupy the same patch. Each agent has a "genetic endowment" consisting of the grain metabolism, a level of vision, and the agent's initial positioning that can be described as its "environmental endowment". As the agents have different values for these endowment parameters, they are heterogeneous agents.

Agent's metabolism is simply the constant grain units she burns per time step. It is specified for each individual agent at the beginning, as an uniform distribution out of a narrow parameter space. Each period the agents can accumulate the difference of what they get from their search and their metabolism. If an agent's metabolism rate is higher than what she has gathered in a period, she consumes out of her stock.

The concept of vision is fourfold restricted. Three restrictions establish general bounds on the whole population conceptualizing bounded rationality. The agents are first prevented from global vision over all patches, second from unlimited Moore vision over all patches in diagonal and orthogonal adjacent directions, and from unlimited von-Neumann-vision over all patches in orthogonal adjacent directions. Finally on the individual level, the agent is restricted to a certain limited scope of vision within the specified parameter space. So, our agents live in a von Neumann neighborhood, where the individual agent with vision x can see x patches in the four lattice directions: north, south, east, west. The individual values are uniformly distributed within the specified parameter space. The restrictions on the vision are a form of imperfect information that bound agent's rationality.

The agent's initial endowments of grain are randomly distributed over the parameter space. It is calculated as sum of the individual metabolism rate per period and random value out of the parameter space. In contrast to the patches maximum bearing capacity, the turtle's accumulation of the potential gathering surplus over its metabolism is unrestricted. This supports the intellectual interpretation of the lattice insofar as the agent's "mobility" is not affected by the size of its stock.

To study the distribution of wealth we need to impose another restriction on the model. If we let agents lived forever, the only cause of death could be for starvation - when the agent's grain stock is smaller then his metabolism rate. Even though a stationary wealth distribution can be achieved with eternally living agents (ref. 3.2.3), in accordance with our realism principle we model finitely living agents by setting a life span for the agents. The distribution is again uniform.

2. The Environment

The environmental space is a lattice where a grain stock indicating the current amount of grain and a grain capacity being the maximum of the grain the patch can carry is individually assigned to each patch. The capacity is set at the level of its initial endowment. During the initialization process each patch depends on their neighbors' capacity. On the lattice areas of higher (hills) and of lower concentration of grain (valleys) are built. This structured altering could neither be reached through a randomized process nor through equally distribution of grain capacity units over all the patches. Thus, for the patches there is a spatial dimension of the lattice. Our scenario imposes highly unequal conditions in term of grain endowment of the neighboring patches on the initial positioning of the agents. From the agent's point of view the space is purely intellectual, since moving on it does not cause any effort. However, insofar as the grain on a patch is divided among the agents on that patch, if there is more than one turtle on the same patch, a spatial dimension not between, but on the same patch reappears.

3. The Rules

There are different rules applying to the entities, agents and patches. The agent's movement rule tells to look around you as far as the respective individual vision permits; identify the site having the greatest amount of grain. If several best patches appear, choose the one that minimizes your turn to the right. Get the grain and take this position as initial position for the next period.

The patches' rule regulates the self-reproduction of the resources (grain) in the model. At each patch the grain grows by 9 units per period up to the capacity of that patch.Given that we have chosen finitely living agents we need to implement a rule for the replacement of the agents. To abstract from population development as it is familiar to the classical approach that implements effective demand instead. The replacement rule generates a constant population by this mechanism: When an agent dies she is replaced by an agent of age 0, random genetic endowments, random position on the grain space, random initial endowment of grain, and a life span randomly chosen in the specified range.

As we have mentioned above, the structure of the basic model is close to E&A's sugarscape model (op. cit. ch.2). They take the Gini coefficient as the measure of inequality and find that their model can actually "grow" wealth inequality. Their results show an increasing Gini coefficient as the agents interact in the artificial world. Their stationary value seems to be around 0.5. They observe that this value is much lower that what we can see looking at the statistics of the industrialized societies. For they suggest that adding agent's rules for inheritance or trade could help the simulation results matching closer with reality. Finally they observe that the skewed wealth distribution that emerge form the simulation seems to be a " characteristic of heterogeneous agents extracting resources from a landscape of fixed capacity" (op. cit. p.33). Their epistemological goal for this kind of economic research is to reproduce macrostructures and not to explain causal relations, they do not attempt to offer any explanation, neither of theoretical by deciphering the very micro-mechanics behind their models nor plausible reasoning by providing an economic reasonable explanation, of why such a micro specification unequal wealth distribution. Only in a note they timidly observe that, "Agents having wealth above the mean have both high vision and low metabolism. In order to become one of the very wealthiest agents one might be born high on the Sugarscape and live a long life" (op. cit. note 20, p.33). As E&A's computer program is not obtainable, their data are not tractable, but the story sounds like the worldly wisdom of supportive forces: parental home, best education, thrifty life-style and ripe old age.

The goal of our work is on the one hand, to test the robustness of the model for various parameter specifications and to check the validity of E&A's story. Out of the four reasons they mention we concentrate on vision. Furthermore we try to enrich the model by adding new behavioral rules as inheritance and discuss the Gini coefficient as a valid summary statistic to measure inequality. On the other hand, we try to link the macrostructure we obtain to the micro world we construct. Likely, that this is not a smart attempt, as these models are not invented to give explanations. But we regard it interesting whether our knowledge of economic theory would allow us to reconcile the algorithmic man with some kind of smart and updated version of the homo oeconomicus. Furthermore, we believe that a good result in modeling complex dynamic systems involve, at least, the capacity of knowing the effects of the control variables on the state variables; E&A don't seem to be worried about this requirement

2.2 Parametrization and exploration of the grainscape model

2.2.1 Parameterization of the environment and agents

Since we are not dealing with an analytically tractable model, it is worth using the Monte Carlo method to explore the model's characteristics. The model has seven parameters to be set in the experiment, each representing one dimension. A parameter space is recommended for every parameter: number of turtle [1, 1000], vision [1, 50], metabolism [1, 25], growth rate [1, 10], growth quantity [1, 10], percentage share of best land [1, 25], and the minimum and maximum ages [1, 100].

Calculating the total number of different possibilities for the model we use the formula:

$$N_r = N_i \prod_{i=1}^{7} g_i \tag{1}$$

Where N_r is the number of runs, N_i is the number of reiterations and g_i is the incremental step per parameter. Thereby we obtain $N_r = 1,057,343,750,000,000$ possible parameterizations of the model. Since each run is not deductively derived from these initial settings, but influenced by many stochastic processes each parameterization should be run more than once. If we ran each parameterization 20 times in order to approximate the normal distribution and each run takes about 30 minutes, exploring the model in this diligent manner would take us about 1,206,187,257,586 years; this is more than 100 times the estimated age of the universe. Viewed from this approach, the task of analyzing the model looks overwhelming.

This hurdle can be cleared by two different approaches. The first synthetic approach allows just playing around with all the parameters in an arbitrary way to get a certain result, e.g. wealth equality. From this state one could investigate the sensitiveness on the parameterization. We try to model an equal distribution following that track and find that inequality is caused by the unequal initial endowment, the positioning on an unequal landscape, the unequal rate of metabolism, and the unequal degree of vision. But even the equalization of all these parameters cannot give rise to equality as long as the turtles have different ages. Older turtles just had more time for the gathering of grain than the younger ones. By simultanizing the age over all the turtles we achieve equality but with a significant lack of reality.

The second realistic approach finds an escape in reality. All the parameters are to be specified at a level that is adequate by analogy with a realistic assessment of the state of the social world. We study some causal mechanisms by varying only the vision parameter and observing the development over time. The benchmarks of that realism are the total amount of grain available, the fading out of special generation effects, initial wealth, and divergence in vision. We have chosen the parameters as following: best land = 23, growth rate = 1, growth quantity = 9, number of turtles = 217, vision [1, 50], age minimum = 45, age maximum = 83, metabolism = [1, 2].

With the setting of the age parameters (min = 45, max = 83) the theoretical average life expectancy, irrespectable of starvations, is 64. This age structure deviates from social demography, in that first the arithmetic mean in reality is quite close to the mode, while in the model all values have the same theoretical frequency, and that second the model lacks death besides starvation before 45. The setting of the vision, metabolism, and the technical parameters will be discussed in detail.

Since robustness of the model in a strict sense is not achievable in the time we had, we have chosen a practical rule: We run the model with a certain parameter a few times and take the mean value of those runs. The variance is with <2.5% over all runs and time segments negligible and outliers appear only in the early periods. If the results appeared to be ambiguous we ran the parameterization more often. Following this rule, our dataset is mostly based on less than twenty reiterations of the same setting.

2.2.2 Exploration

The Notion of vision The notion of vision could somehow be misleading. There is nothing like foresight in the agent's behavior. No agent can look beyond the current period. Even more, the behavior of the others during the same period is not at all taken into account. This could come up as a kind of malicious penalty for the agents with high vision, as we have mentioned already. In this aspect, the model we discuss deviates from the E&A model, as their model prevents any patch from being occupied by more than one turtle bringing with it the violation of a strict simultaneousness of the turtles actions within a period, when turtles act one after another. E&A try to correct this trough employing a stochastic order of taking action within each period.

The meaning of endowing an agent with vision from 1 to 50 remains to be explored. The landscape is 41 patches square. The respective turtle standing on one patch sees all patches that are principally visible from this angle, if its vision is 20. Further increasing the maximum vision therefore does not make the most skilled turtle seeing more, but still elevates the average level of vision among the population, since more will be endowed with vision 20 or higher. This effect is demonstrated in the graphs on the richest and the average turtle's wealth when we research on inequality in absolute terms.

Initial wealth and metabolism Dealing with the initial wealth we face the following problems. Wealth is what a turtle accumulates over its life span without metabolizing it. Metabolism is an exogenous restriction not interacting with the turtle's activity, skills or income. The higher the metabolism parameter the more difficulties the turtle have to survive until its predefined date of death. Since the turtle's behavior and genetics are not affected by the kind of death of its parent, we try to reduce the influence of this exogenous parameter to explore the endogenous mechanisms. In our approach the metabolism is only slightly influenced by the initial wealth. Especially with a low rate of metabolism, the initial wealth has to be low in relation to the wealth acquired during the life span for the dynamics to make sense. If the turtle's initial endowment would be too high the wealth of the individual turtle, and its ranking in the social distribution, would mostly be independent of the other parameters (vision, initial positioning). We show this result in run vis07_01. Therefore we have rather distributed the grain on the environment ("fruitful patches"), rather than allocated to the turtles (low initial endowment), enabling an easy survival by gathering.

Production parameter fixed The technical situation affects the turtles' behavior. If the "mobility" of the agents is high (high vision parameter), then the more dire their situation is, i.e. the slower the patches regain their capacity level of grain, the more they apply their ability to "move". So the turtle's behavior depends on the technical structure of the landscape. In this perspective the best land parameter makes the next best piece of land more distant on the average and the "growth rate" makes the agents leaving their fertile (in terms of potentially, but not actually rich since they have already "grazed" it) area to search for another accumulation of resources while the patches they are on are just growing again.

The technical parameters (best land =23, growth rate =1, growth quantity =9, number of turtles=217) are set in order to obtain a sufficient product. With this setting, 38.7% of the hypothetical amount of grain that is available if the best land parameter was 100%, is distributed over the patches. The model is constructed and modified for analyzing wealth distribution. Our idea is to abstract from varying the technical parameters in order to endow the landscape with a given total amount of grain. This abstraction leaves our model suited for a classical analysis even though the abstraction might be too clear-cut in a way since the amount of resources strongly influences the development. The effect of these technical parameters ruling the amount of resources on inequality is outlined below.

Lacking the exclusiveness of access to the patches by only one turtle, the question is whether the greater vision could generate a kind of rational clustering, rational insofar as the turtles are applying their rule correctly. But could that lack of exclusiveness bring forth turtles' concentration on a few very best patches? Figure 1 shows that there is never an absolute scarcity of grain on the patches in the setting we employ. Out of the total amount of this maximum capacity, at least 80% are available on the landscape at the end of the 1148th period after the harvest and before growing again in the next period has passed by. Since this initial assignment is generated by random mechanism and changing with a variance of 2% over the runs, the lower line ("real of potential") expresses the relation to a total hypothetical capacity if every patch had the maximum grain capacity of 50 units. The initial assignment left the patches on average with 38.7% of the potential capacity of the landscape.

Spatial concentration To assess the sufficiency of the size of the landscape in relation to the number of turtles we have to analyze the relation between the available and the used space. Keeping in mind that 217 turtles are moving around and 1681 patches are accessible, of which 387, i.e. 23%, have maximum capacity, can we see the turtles using the space they are offered efficiently? By this, we also study the effect of the rule that is offering multiple accesses to the patches, since the exclusivity rule prevents turtles from clustering. If there is only a small amount of wealth, increased vision could become a burden when each turtle cannot predict the behavior of the others and anticipate in their decision rule to get 50 units of grain, but get only one if 50 turtles would have made the same calculation. To contain the influence of this effect, the concept of bounded rationality as incorporated in the von-Neumann-neighborhood is sufficient.

In figure 2 we see that the clustering of turtles on patches decreases with a higher degree of vision. At vision 1 almost no turtle is alone on its patch and about 77% are together with more than 5 others on their patch. So the turtles are by no mean, spread out over the huge landscape. This concentration on only a few patches decreases a lot as vision is increased, but it does not disappear. This phenomenon is confirmed in figure 3 from the perspective of the populated patches indicating how many turtles are on the average on the populated patches. From the perspective of all patches the percentage of the populated patches increases from 2% at vision 2 to more than 10% at vision 50. It requires further research to clarify why and how the clustering of the turtles increases with lower vision. With many starvations the initial positioning that works randomly becomes more influential. This should counteract the forces driving the turtles together.

Fading of the generation effect All turtles start at the beginning of an experiment with the initial age of zero. Besides starving the turtles do not die before 45 and not later than 82. Thus there is a huge wave of turtle at the same age. This is only blurred over time due to starving and divergent stochastic resetting. When is this generation effect artificially induced by equal initial age of all turtles leveled out? For answering this question we have to determine the minimum duration of a run.

We have picked a special age segment for the whole. Encompassing the shortest range while the overall population remains constant, the age segment below twenty years is especially volatile in comparison to the age segments below 40 or below 60. The simulation showed in figure 4 suggests that the initial generation wave does not fade out before 300 periods have passed by. Even then we cannot claim that the variance has disappeared. So, the age structure does not become stable within the time range analyzed here. One special run has shown that this does not change over the first 10000 runs also. With 1148 periods the duration of our experiments can be regarded as sufficient from this point of view.

We have faced the problem that reproduction in this model in no way depends on the previous generations. Far from Malthus' demographic assumptions, the population and the wealth are not connected at all. If the agents are very rich they do not produce a larger offspring. The same is true for dire conditions of life. Since the agent is always automatically reproduced and always receives an initial endowment, it is independent of the wealth of their one "parent" (homozygous). But this one parent only initiates the birth of the offspring as it dies by age or starvation, the offspring's geno- and phenotype are totally independent of it. Analyzing distribution in a stable environment for our parameterization to make sense, we have to assure that there is no a hidden stabilizing force ruling the roost. As this reproduction rule is pretty strong we have introduced starvation indicators to demonstrate the reliance on this strong exogenous stabilizer. We employed this indicator as a threshold to the interpretability of the experiments.

E&A distinguish two steps of experiment. In the first step they determine the carrying capacity of the landscape. How many turtles can reach their predetermined age of death? They set the initial population at this level and proceed to the second step of exploring the influence of the parameters. Figure 5 displays that starving occurs only with a low vision parameter. Then the turtle is mostly dependent on the fertility of the patch where it has been set initially by random. They cannot escape from a dire area since they do not "see" richer ones. Remember that the grain is not totally randomly distributed but clustered over the landscape. By increasing the vision parameter (i.e. the average and the maximum) starvations vanish almost completely from the vision level of 7 upwards. At this level only 0.07% of approximately 3900 deaths during one run are caused by starving. At the level of 1 and 2 the dynamical process relies pretty strong on the reproduction after starving with 41% and 34% of all deaths.

This exogenous force substitutes turtles with low wealth on stony patches by young ones with low wealth at patches of different quality. At first this is causing a concentration through selection by the landscape on better patches, since turtles on these patches live longer. Secondly as the real life expectancy decreases, being the most important parameter for the accumulation of wealth, the average wealth per turtle should decrease. Finally the results at high starving rates are less interesting since the relative influence of all the other parameters has been reduced by the overpowering influence of this exogenous force.

3 Simulation Results I: Emerging Inequality

Conceptualizing inequality we have employed the Gini coefficient as a measurement of relative inequality. The Gini coefficient has been criticized for not being equally sensitive to redistributions at different scales. But transfers from a richer to a poorer person always reduce it. Describing the relative inequality in a country, we would have preferred the Lorenz curve. The Lorenz curve is more accurate in the description of inequality as it does not imply assumptions on the interpersonal comparison. Two Lorenz curves can intersect when the lower class as well as the middle class is larger in one country, but the Gini coefficient could still be the same regardless this difference. But we have used the Gini since the comparison of just one index number for all the different societies and class structures is possible along one yardstick within this concept. Since the calculation of the Gini coefficient implies a complex computational procedure, we have calculated it only for every fifth period. Therefore concerning the Gini we are not comparing averaged data from all the periods but just a sample of it.

1. **Levels and dynamics of the Gini coefficient**

E&A's emerging inequality (see above) can be explained quite easily in our model. A very successful way of creating an equal distribution is to model a homogenous age structure over all the turtles, since the time turtles have for gathering is what makes them rich. So, if all the turtles are young at the same time, they are all poor, but relatively equally poor. As they become older they get richer, again equally. This mechanism is showed in figure 6 analyzing the age structure in relation to vision and through time. Each color indicates a certain period of time within the 1148 periods, e.g. the pink square the first 48 periods. Three lines for the early periods indicate this special effect at the beginning. With increasing vision avoiding more and more starvations the share of turtles that are older than 20 decreases. This effect diminishes as the starvation diminishes (see diagram on starvations). The age structure does not converge to a specific share, but oscillates within limits.

From this explanation it should be possible to link at least the inequality of the first 100 periods to the age structure. From this it becomes clear how important avoiding systematic influences from the age structure was. All generation should be more or less equally sized.

To get a quantitative measure of the level of inequality that the model produces we can look at the range of the Gini coefficients when we control for the age structure and we allow the vision to vary in a reasonable parameter space (1, 20). The experiment provides us with Gini values from 30% to 33% for wealth. Comparing these data with the real world Gini coefficients for the distribution of income from the World Bank's statistics about world development indicators, sets our model into a cluster of many countries: Canada 31.5, France 32.7, Germany 30.0, Greece 32.7, Indonesia 31.7, Rep. Korea 31.6, Latvia 32.4, Lithuania 32.4, Netherlands 32.6, Pakistan 31.2, Poland 31.6, Spain 32.5.

2. **Resources & inequality**

Total resources and inequality are inversely correlated at least for poor landscapes. It can be shown that with such small amounts of grain available on the patches, the relative inequality in terms of the Gini coefficient is very small. But then most turtles die by starving and the relative equality is reached by preventing turtles from accumulation and just comparing their initial endowments, the rule of reproduction is stabilizing the distribution.

3. **Vision & inequality**

The comparative-static analysis of the influence of the vision parameter is shown in figure 8. The overall trend is that the Gini decreases in the dymension of the vision space, the only exception beeing at the first 3 values of the vision space. With vision 1 all turtles have the same vision, i.e. just deciding on the four neighboring patches from where to grasp the most grain. At this stage there is no difference in vision among the agents and so inequality levels are low. At vision level 2 wealth inequality reaches its top value because with a binary vision space no agent has the average vision, but all are situated either at the upper or at the lower end (the vision space is discrete). As the absolute range is increased, the average level of vision will be increased and this produce a reduction of wealth inequality. Moreover, as the vision increases above 20, more and more turtles have the highest vision and the Gini returns close to the value it had when all turtles were equally endowed with vision 1. Therefore, the proxy for agents' skill heterogeneity has an important role in shaping the dynamics of wealth distribution.

4. **Scenarios of inheritance & inequality**

The effects of introducing inheritance into the model depend on the notion of inheritance. The evolutionary biologist would claim that what matters is the inheritance of vision. For an economist the inheritance of wealth is at the focus. Therefore we have distinguished between different kinds of inheritance. We have carried out different scenarios of inheritance and compared them always to the standard case that is described above. Due to a lack of time all these scenarios could not be explored to the degree of thoroughness reached in the standard scenario. We have simply made numerical comparisons of several samples from each setting, to claim whether the result has changed in comparison to the standard scenario and, if it has changed, in which direction.

First, we observe a reduction in the Gini when we let the child be born where its parent has died -inheritance of the neighborhood. If agents inherited all states from their parent the Gini is a little bit lower, which mainly should be assigned to the generational learning effect since besides that the individual agents are virtually immortal as they inherit every state from their parents. Thereby, the initial random endowment and setting stays with each individual. Finally, setting all parameters as inherited besides the vision parameter causes the Gini to be mostly smaller. This suggests that vision is a key parameter in the ineaulity generating mechanism. The intution for the role of vision in the setting with inheritance is that a random vision at the birth makes wealth dynamics converge to lower level of inequality with respect to a fixed, inherited, distribution of vision.

5. **Dynamics of the tails of wealth distribution**

In this simulation we study the outcomes of the model in terms of a different measure of inequality, the gap between the richest and the poorest agent, that allows us to study the links between skills heterogeneity (vision's dispersion) and the dynamics of specific parts of the wealth distribution. Figure 9 and 10 plots the simulated relation between the richest and poorest turtles' wealth and their degree of vision. First it can be stated again that the wealth depends on the duration of the process. For the first fifty periods it seems that the gap between richest and poorest agents shrinks when vision increases. In fact, we can observe a pretty stable path of the relationship between vision and wealth for the richest and an increasing path for the poorest. With more than 50 runs the gap between the two seems to make a discrete jump uo (in runs 50 to 100) and than stays substantially constant.

Finally, figure 10 shows the relationship between the average agent's wealth and the degree of vision when population is assumed constant. The figure shows the pattern of wealth of the average agent is positively influenced by a better average vision. Streaching a parallel with economic theory this could be interpreted as an increase in the general skill level of the population.

4 Simulation Results II: Pareto's law

More in line with the synthetic approach we try to model a Pareto in our model. Pareto originally has identified this relationship with the logarithm of the households on the ordinate and the logarithms of the income on the abscissa. He has found that this relationship can be described by

$$\ln x = A - \alpha \ln y \tag{2}$$

where x is the number of households with income less than y, A is a parameter and α is the slope. Using data for England, Germany, a number of Italian cities, Paris and Peru, Pareto found that α was clustered around 1.5 (Persky 1992 p.183)[1]. While the value of has changed over time and between countries in a range from 1 to 3, empirical distributions often conform with this law. According to Steindl in the New Palgrave, the range is larger for wealth than for income.

We have integrated a schedule drawing the logarithmic relation of agents and wealth our algorithm in order to identify whether we can generate a Pareto law. The basic requirements to draw a distribution of wealth as a Pareto law are a strongly unequal distribution with high values for the upper end not breaking down. We present four different plots of distribution in order to identify a power law. Figures 11 to 14 are generated respectively with vision space 1, (1,2), (1,4) and (1,50). As in Pareto's original analysis we have found that a skewed distribution of wealth. As we have already seen, the heterogeneity in vision reduces inequality, thus when we increase the vision space from 1 to 2, 4 and 50 we notice a more skewed distribution. The slope for the first two cases is about -1, which would indicate a Zipf law indicating a distribution even more unequal than Pareto's original.

5 Conclusion

This paper has analyzed some causal links between the parameters of our very stylized economy and the emergence of wealth inequality. We find that skill (vision) heterogeneity increases wealth inequality measured by the Gini coefficient. Using the gap between the richest and the poorest agents as an index of inequality does not allow us to obtain a clear understanding of the effects of skill's heterogeneity. But, we do find that the increaase in average vision increases the average wealth. Moreover, we find that inheritance of wealth, neighborhood at birth, methabolism etc. reduce the Gini, while vision inheritance increses it. Finally, our model reproduces a wealth distribution much more skewed than the Pareto's law.

We conclude with a methodological remark. As the reader can easily verify, our experiment suggests that agent-based modeling does not allow the economist to build immediate bridges between the model and the reality. Many times, as showed above, we obtain results that are hard to interpret with the lenses of economic theory and/or with what we observe in real life. Most of this problems are related to the very stylized economy of our model; to bring more realist feature into it we should leave our primitive hunter-gather society and introduce, at least, trade between agents, a production process and some form of expectations. But, some of the lack in the explanatory power is feature that this new modeling technique shares with the neoclassical theory and it is basically related to the choice of microfunding economic phenomena.

[1] A value of $\alpha = 1.5$ means that the top 10 percent of the income recipients receives 48 of the the total income.

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Figure 1. Efficiency of harvesting and vision

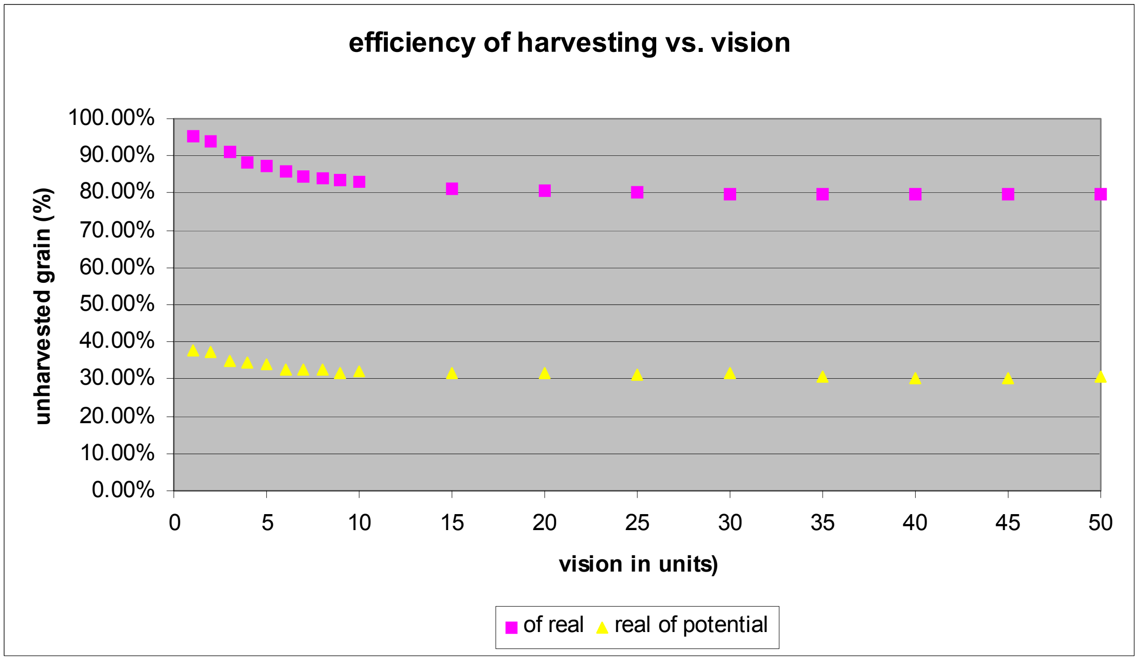


Figure 2. Concentration on patches and vision

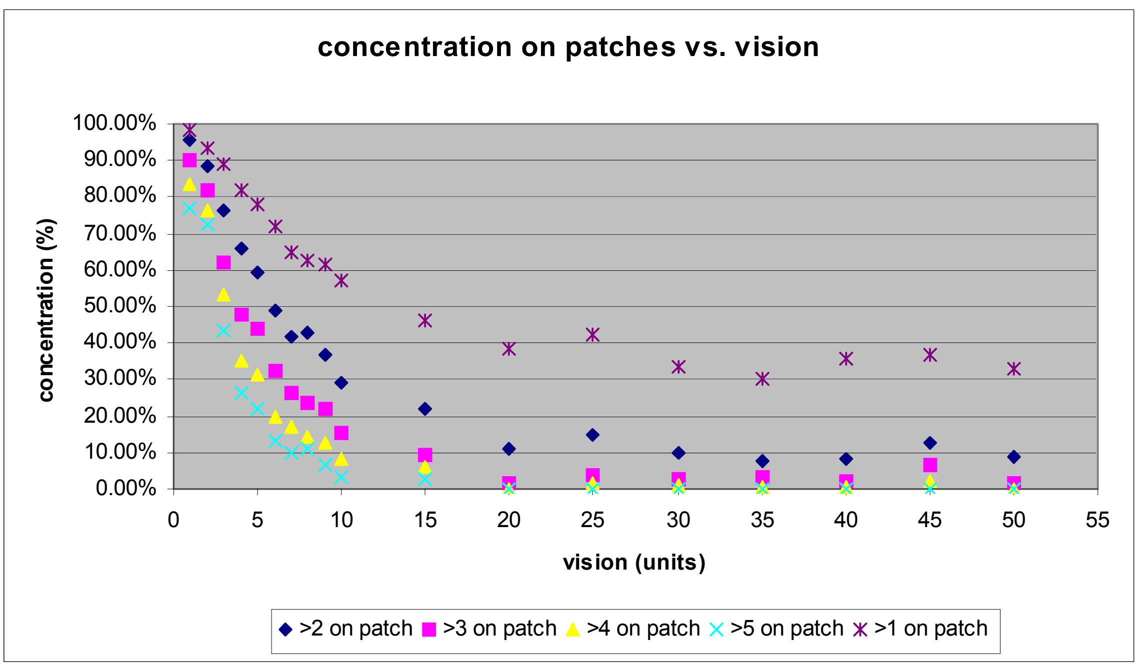
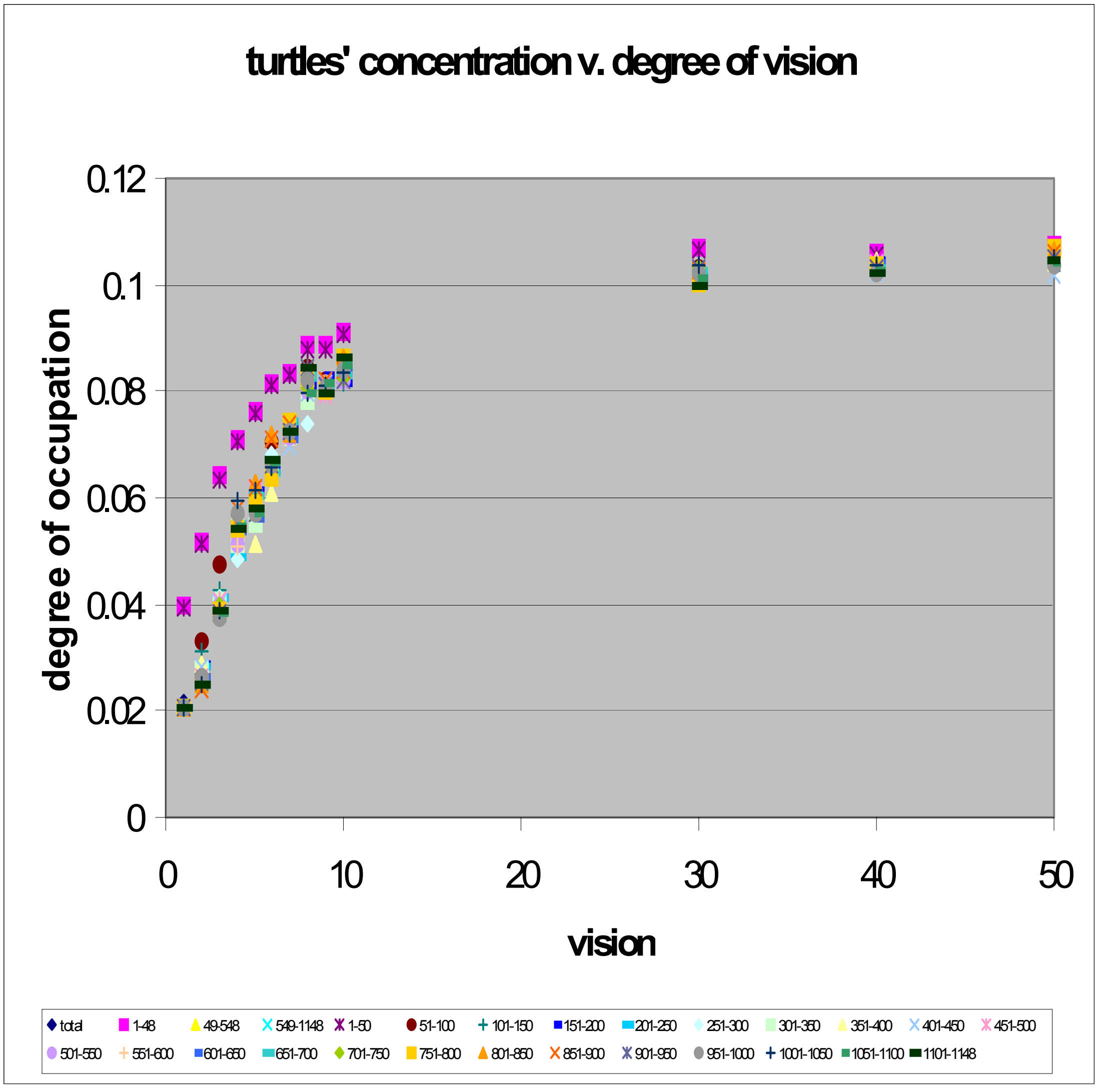
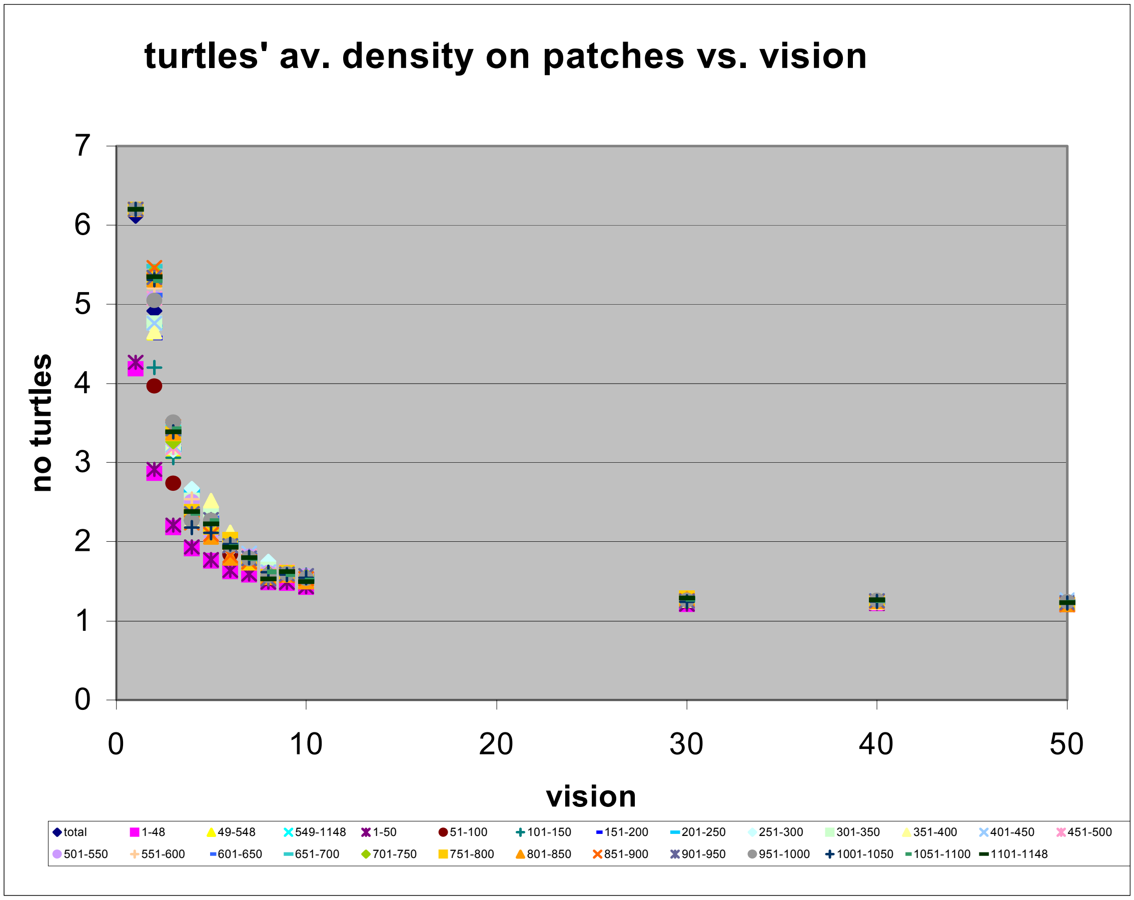


Figure 3. Turtles concentration and degree of vision



note: different colors represent different numbers of runs (i.e. 1-48, the pink points, represent the result after the first 48 runs).

Figure 4. Turtles average density on patches and vision



note: different colors represent different numbers of runs (i.e. 1-48, the pink points, represent the result after the first 48 runs).

Figure 5. Variance of age class below 20 years over time

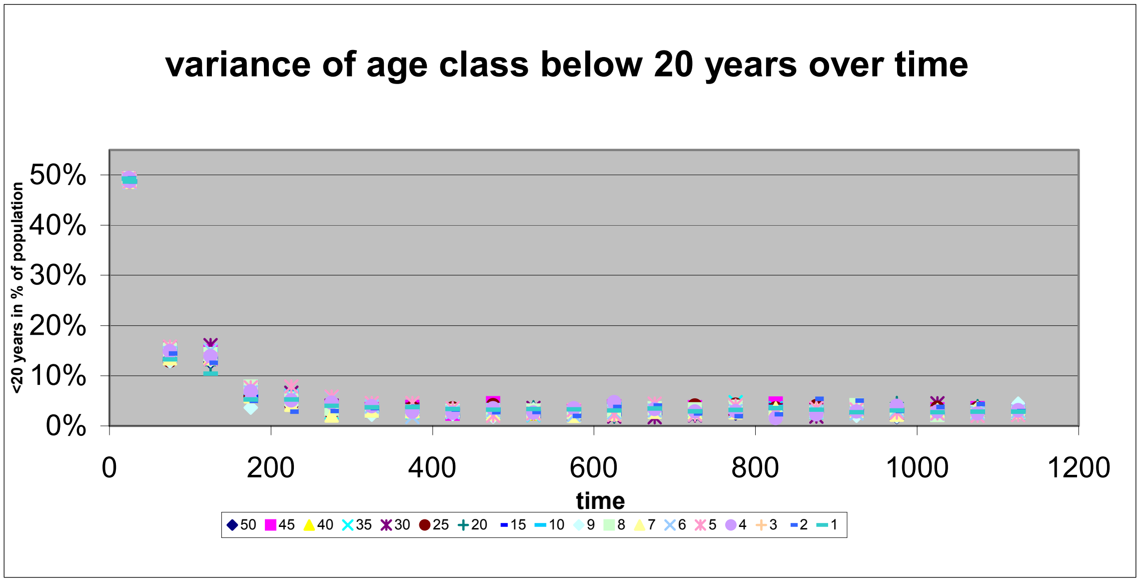


Figure 6. Starving and degree of vision

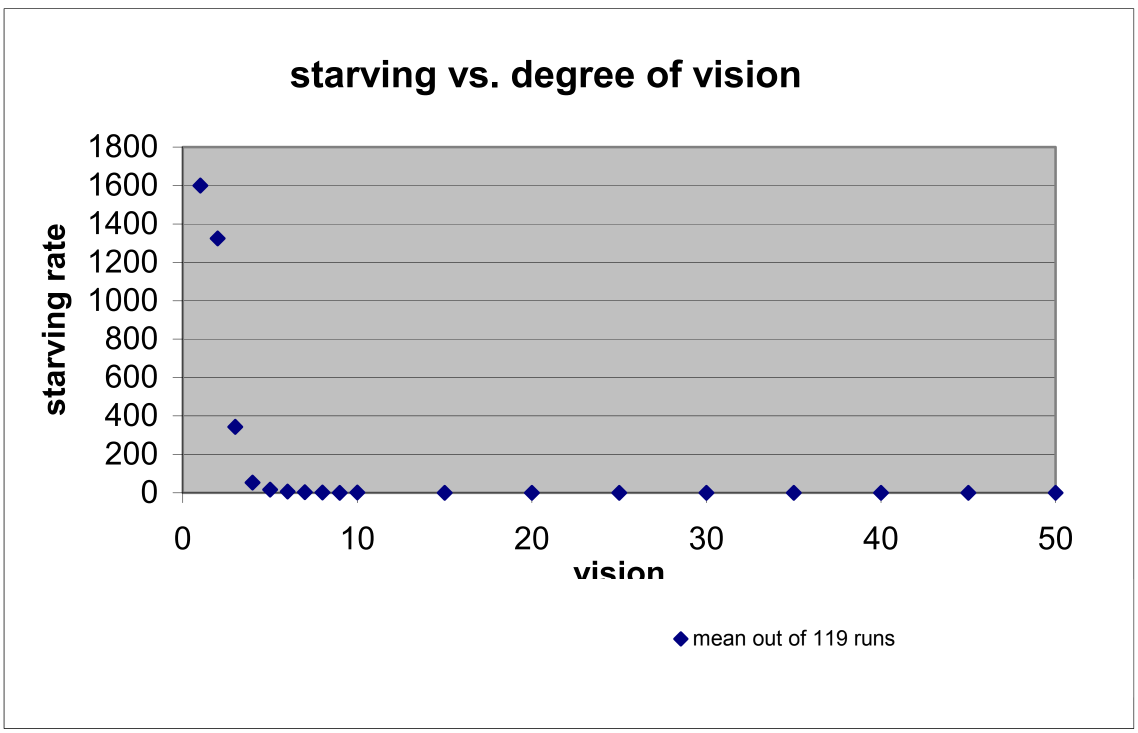
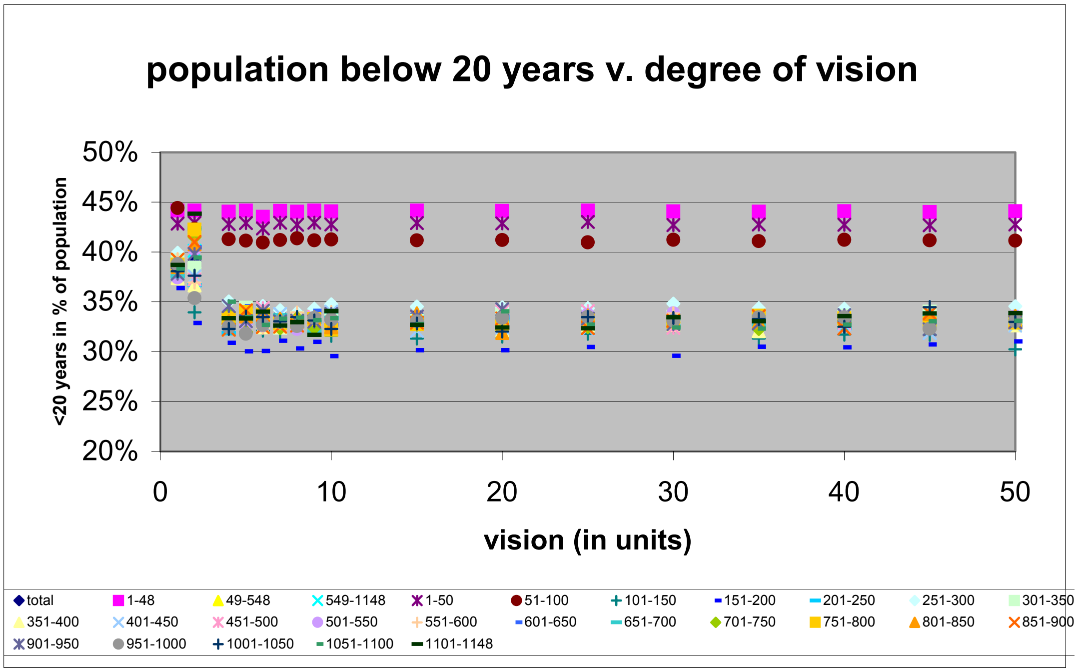
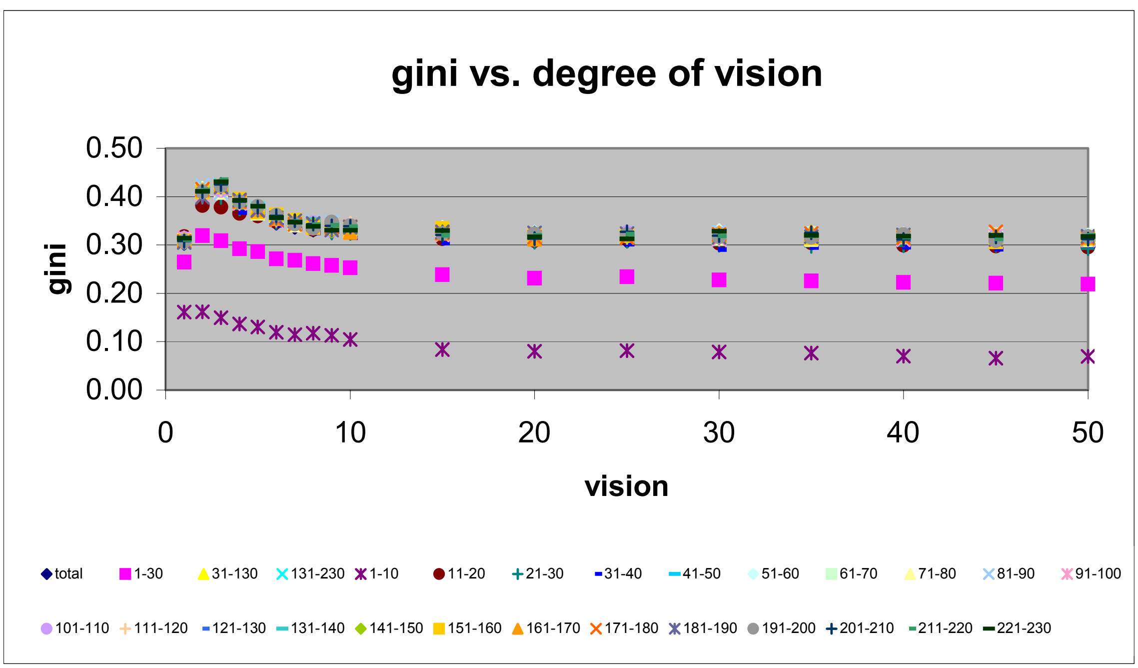


Figure 6. Population below 20 years and degree of vision



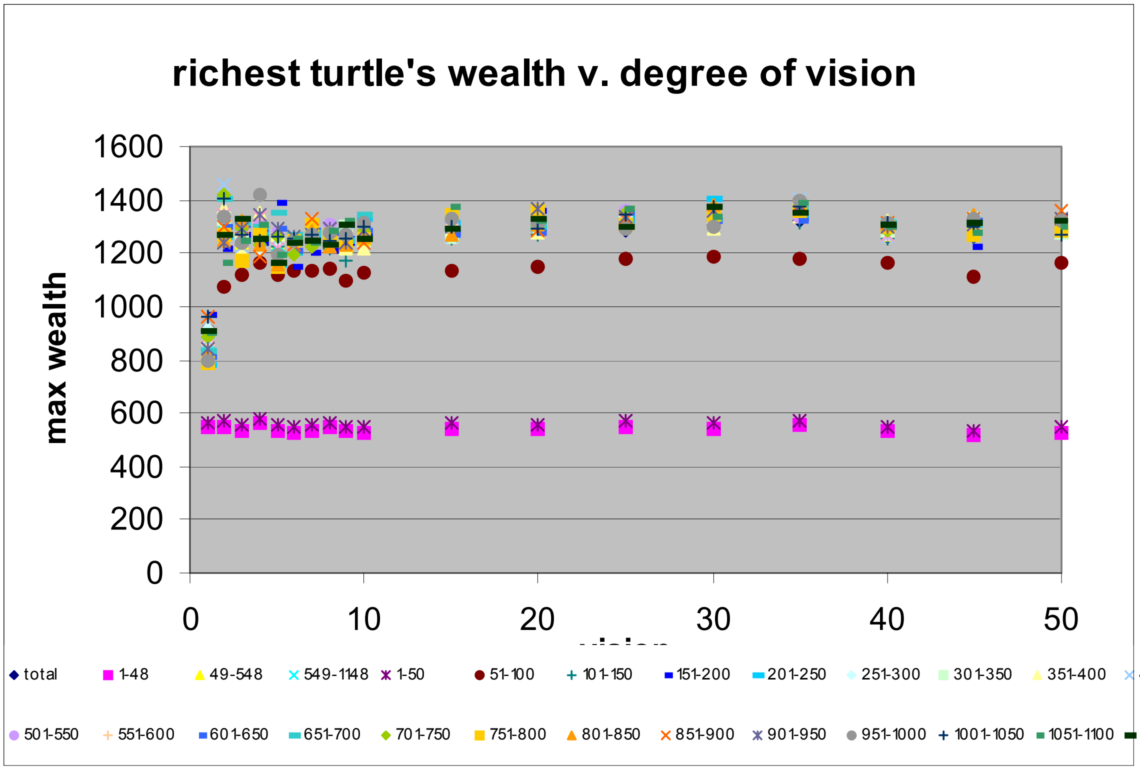
note: different colors represent different numbers of runs (i.e. 1-48, the pink points, represent the result after the first 48 runs).

Figure 7. Gini coefficient and degree of vision



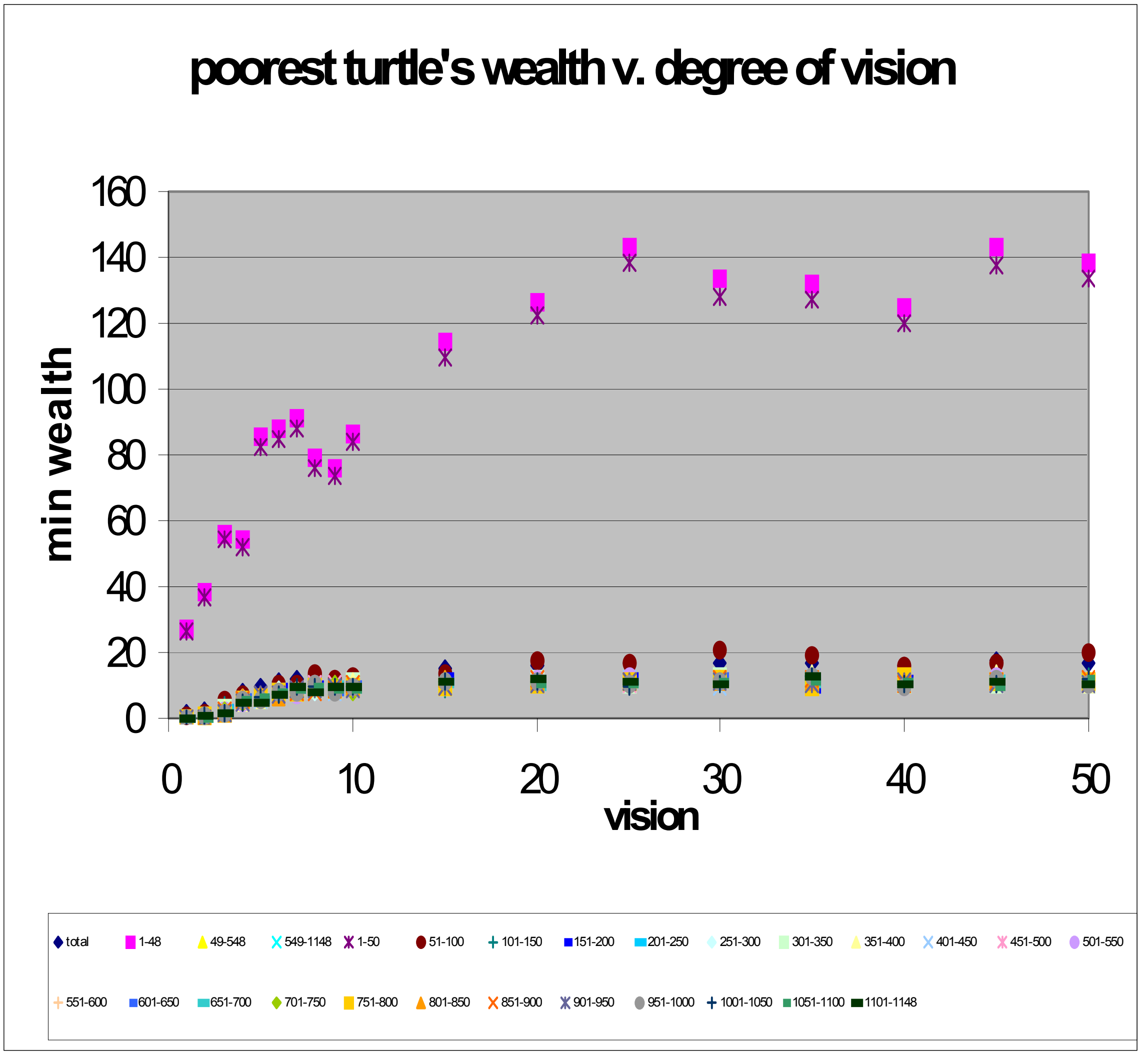
note: different colors represent different numbers of runs (i.e. 1-48, the pink points, represent the result after the first 48 runs).

Figure 8. Richest turtle's wealth and degree of vision



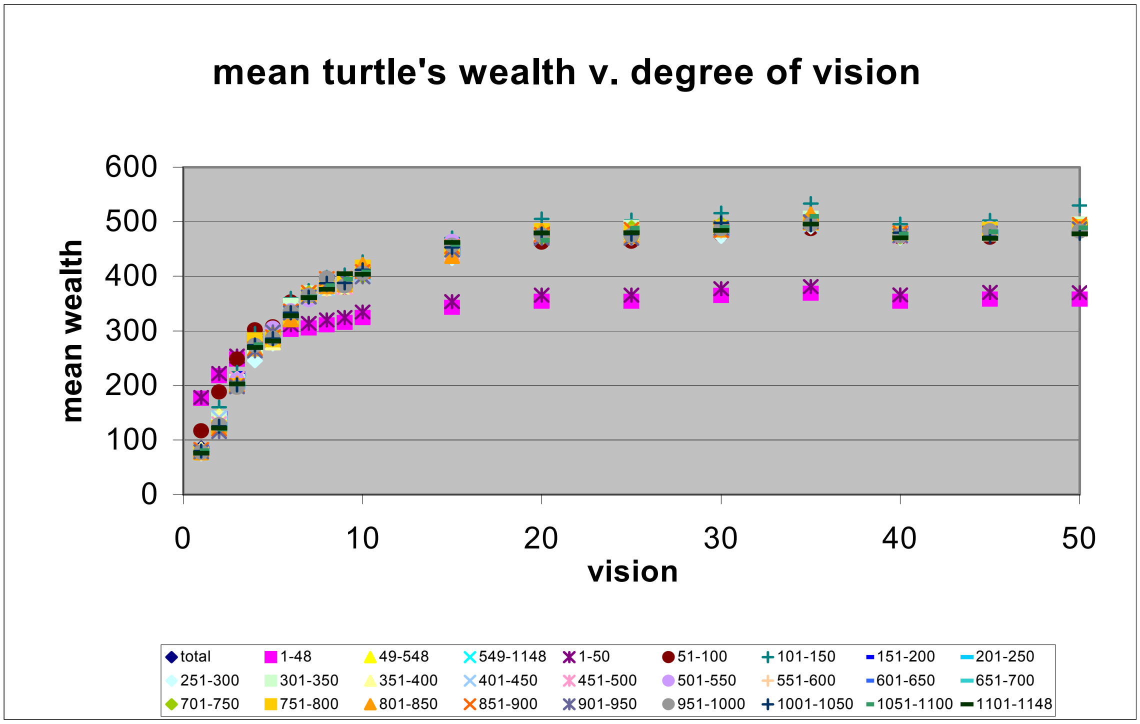
note: different colors represent different numbers of runs (i.e. 1-48, the pink points, represent the result after the first 48 runs).

Figure 9. Poorest turtle's wealth and degree of vision



note: different colors represent different numbers of runs (i.e. 1-48, the pink points, represent the result after the first 48 runs).

Figure 10. Mean turtle's wealth and degree of vision



note: different colors represent different numbers of runs (i.e. 1-48, the pink points, represent the result after the first 48 runs).

Figure 11. Pareto Law (vision 1)

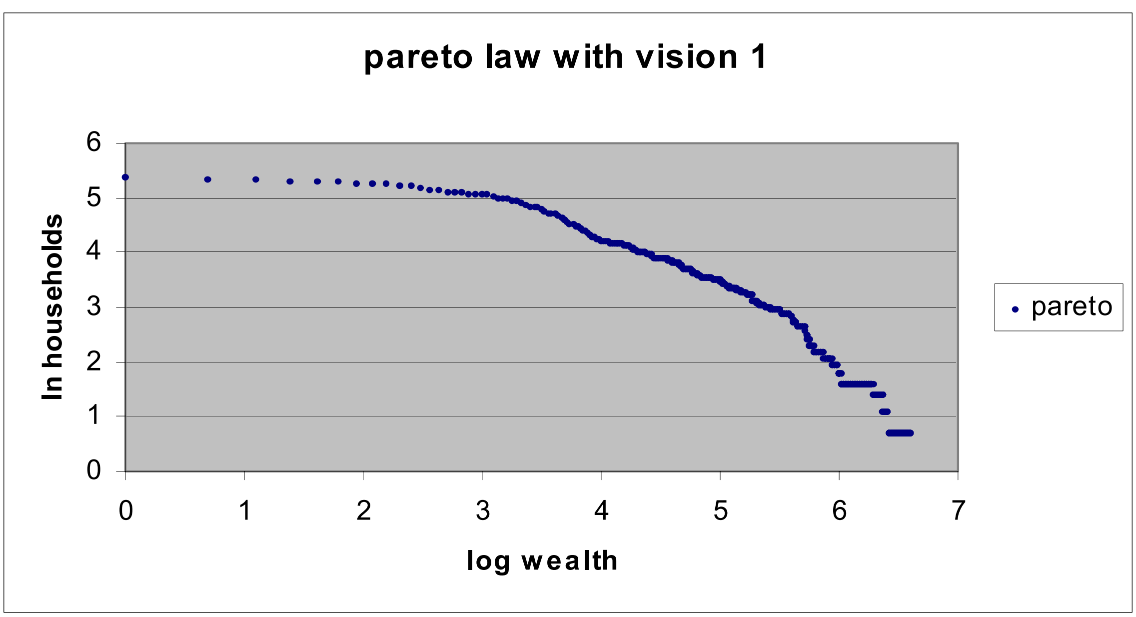


Figure 12. Pareto Law (vision 2)

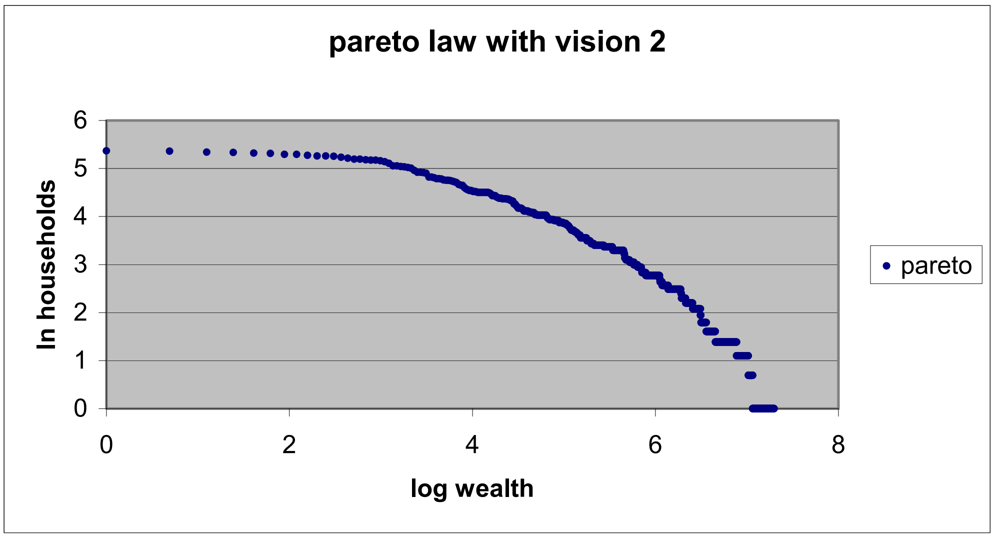


Figure 13. Pareto Law (vision 4)

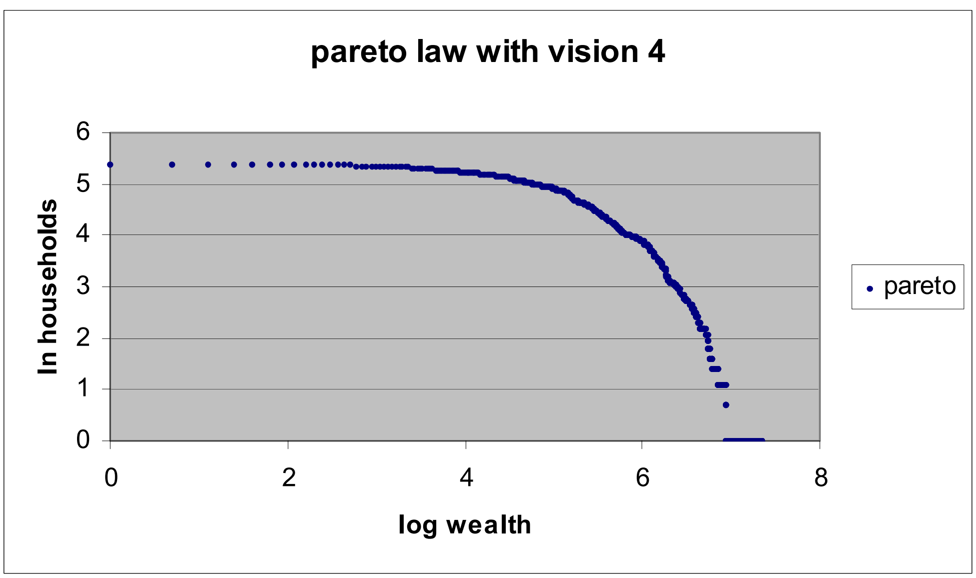


Figure 14. Pareto Law (vision 50)