1. Deterministic Model vs. Stochastic Model

A simple representative HH, infinite horizon problem

Deterministic

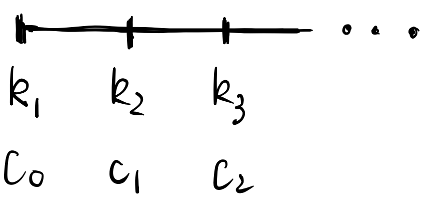
$$\max \sum_{t=0}^{\infty} \beta^t u(C_t)$$

$$\text{s.t. } k_{t+1} + C_t = f(k_t)$$

Solution: sequence $\{C_t, k_{t+1}\}_{t=0}^{\infty}$

that satisfy $\begin{cases} \text{EE} \\ \text{BC} \\ \text{Boundary conditions} \end{cases}$

Given k_0



History is NOT important.

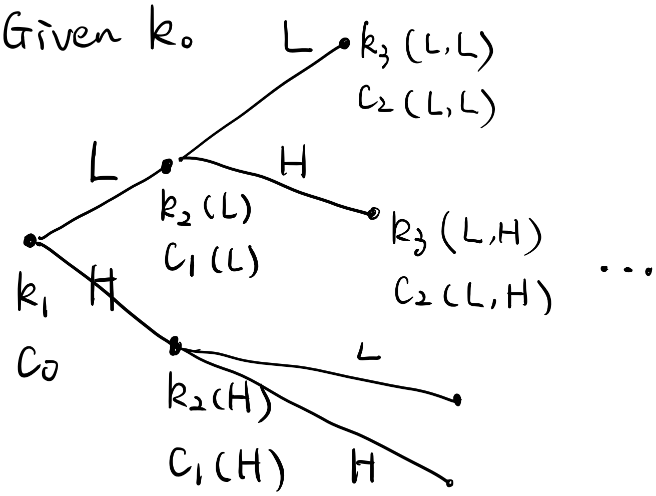
Stochastic

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

$$k_{t+1} + C_t = f(k_t, \theta_t)$$

θ_t: productivity shock.
it takes finite values
e.g. low & high

Given k_0



⇒ We choose sequence $\{c(s^t), k(s^t)\}$ for every possible history path.

History is important!

2. How to Solve Stochastic Model

1). Sequence Approach (simple logic, but messy algebra)

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \Rightarrow \max \sum_{t=0}^{\infty} \sum_{s^t} p(s^t) \beta^t u(c(s^t))$$

$$\text{s.t.} \quad \underbrace{x(s^t)} + c(s^t) = f(k(s^t), \theta(s^t))$$

The capital holding decision I made for tomorrow, given my history path s^t.

$$k(s_{t+1}, s^t) = x(s^t)$$

2) Recursive Approach

Bellman Equation

$$V(\quad) = \max \underline{\qquad} + \beta V(\ '\ )$$

Discussion:

Due to the nature of Bellman equation, we only need to focus on two periods: today and tomorrow. Hence, it simplifies the problem from a whole history path to only two periods, when writing down the problem.

Step 1: find the state variables

- Predetermined variables
- In stochastic growth model, usually the shocks are states.

$\Downarrow$

In order to form expectation for tomorrow, what information do I need?

Example:

- simple Markov process $Pr(\theta' | \theta)$: θ
- $Pr(\theta' | \theta, \theta_{-1})$: θ, θ_{-1}
- $Pr(\theta' | \theta, \theta_{-1}, \theta_{-2}, \cdots \theta_0)$: the whole history.

Step 2: Bellman equation

$$V(k, \theta) = \max u(c) + \beta \mathbb{E}[V(k', \theta') | \theta]$$

s.t. $k' + c = f(k, \theta)$

$$\Rightarrow V(k, \theta) = \max u(f(k, \theta) - k') + \beta \mathbb{E}[V(k', \theta') | \theta]$$

Step 3. Solve the problem

F.O.C

$\Rightarrow$ EE

E.C.

Do NOT take derivative w.r.t. shock.