Econ720 - TA Session 9

Yanran Guo

UNC-Chapel Hill

2019. 11. 1

Phase Diagram

Ramsey Model

Social Planner Problem

$$max \ \int_0^\infty e^{-(\rho - n)t} u(c_t) dt$$
$$\dot{k}_t = f(k_t) - (n + \delta) k_t - c_t$$

Phase Diagram

Current Value Hamiltonian:

$$H = u(c_t) + \mu_t(f(k_t) - (n+\delta)k_t - c_t)$$

FOCs

$$\begin{aligned} [c_t]: \quad & u'(c_t) = \mu_t \\ [k_t]: \quad & \mu_t[f'(k_t) - (n+\delta)] = -\dot{\mu}_t + (\rho - n)\mu_t \end{aligned}$$

Hence

$$\frac{u''(c_t)\dot{c}_t}{u'(c_t)} = (\delta + \rho) - f'(k_t)$$

Phase Diagram

For simplicity, we use CRRA utility

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

Hence,

$$\frac{u''(c_t)}{u'(c_t)} = -\frac{\sigma}{c_t}$$

Then the equation $\frac{u''(c_t)\dot{c}_t}{u'(c_t)} = (\delta+\rho) - f'(k_t)$ becomes

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - (\delta+\rho)}{\sigma}$$

Phase Diagram

How to draw phase diagram?

1. Find the dynamics for control and state
   Differential equation for control and for state
2. Think about the steady state
3. Plot the two steady state equations separately
4. Decide the movement of control and state respectively.

Phase Diagram

Step-1: Find differential equation for control and for state

Control variable c_t

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - (\delta + \rho)}{\sigma}$$

State variable k_t

$$\dot{k}_t = f(k_t) - (n + \delta)k_t - c_t$$

Phase Diagram

Step-2: How do these two equations look like in SS

$$\frac{\dot{c}_t}{c_t} = 0 \quad \Rightarrow \quad f'(k^*) = \delta + \rho$$

Hence k^* is a constant

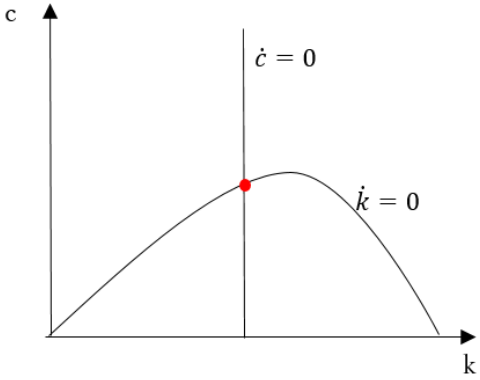
$$\dot{k}_t = 0 \quad \Rightarrow \quad c^* = f(k^*) - (n + \delta)k^*$$

Hence c^* is a function of k^*

Phase Diagram

Step-3: Plot the two SS equations

$$\dot{c} = 0: \quad f'(k^*) = \delta + \rho$$
$$\dot{k} = 0: \quad c^* = f(k^*) - (n+\delta)k^*$$



Phase Diagram

Step-4: Decide the movement of these two variables separately

- c is the vertical axis $\rightarrow$ c moves up and down
- k is the horizontal axis $\rightarrow$ k moves left and right

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - (\delta + \rho)}{\sigma}$$
$$\dot{k}_t = f(k_t) - (n + \delta)k_t - c_t$$

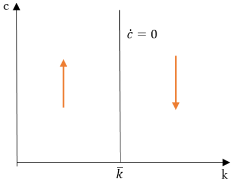
1. Use $\dot{c}$ equation and change the value of k to study the movement of c
2. Use $\dot{k}$ equation and change the value of c to study the movement of k
3. Put them together

Phase Diagram

Use $\dot{c}$ equation and change the value of k to study the movement of c

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - (\delta + \rho)}{\sigma}$$

- Take one k on $\dot{c} = 0$ curve, $k = \bar{k}$, $\frac{\dot{c}}{c} = \frac{f'(\bar{k}) - (\delta+\rho)}{\sigma} = 0$
- For each k to the left of $\bar{k}$, $k < \bar{k}$, $\frac{\dot{c}}{c} = \frac{f'(k) - (\delta+\rho)}{\sigma} > \frac{f'(\bar{k}) - (\delta+\rho)}{\sigma} = 0$ c increases
- For each k to the right of $\bar{k}$, $k > \bar{k}$, $\frac{\dot{c}}{c} = \frac{f'(k) - (\delta+\rho)}{\sigma} < \frac{f'(\bar{k}) - (\delta+\rho)}{\sigma} = 0$ c decreases

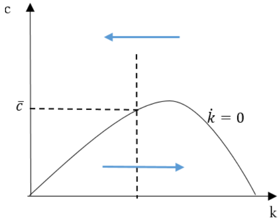


Phase Diagram

Use $\dot{k}$ equation and change the value of c to study the movement of k

$$\dot{k}_t = f(k_t) - (n+\delta)k_t - c_t$$

- Take one c on $\dot{k}=0$ curve, $c=\bar{c}$, $\dot{k} = f(k) - (n+\delta)k - \bar{c} = 0$
- For each c above $\bar{c}$, $c > \bar{c}$,
  $\dot{k} = f(k) - (n+\delta)k - c < f(k) - (n+\delta)k - \bar{c}$ k decreases
- For each c under $\bar{c}$, $c < \bar{c}$,
  $\dot{k} = f(k) - (n+\delta)k - c > f(k) - (n+\delta)k - \bar{c}$ k increases



Phase Diagram

Put them together