Econ.720 Recitation-6

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1 Define CE

Model

↙ ↘

Solve the model using sequential language
=Lagrangian

$L = \sum_{t=0}^{\infty} \{\beta^t u(c_t) + \lambda_t (BC)\}$
FOC, EE
...

Solve the model using recursive language
=Dynamic Programming (DP)

$V(k) = \max_{c,k'} u(c) + \beta V(k') + \lambda(BC)$
FOC, Envelope Condition (EC), EE
...

↖ ↗

Define CE
as before

2 Define Recursive CE

Model

↓

Solve the model using DP with aggregate state variable

↓

Define recursive CE

3 Example

Model setup:

- Household

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$s.t.\ c_t + k_{t+1} = (1-\delta+q_t)k_t + w_t$$

  No population growth, k_0 is given

- Firm

$$\max\ F(K_t, L_t) - w_t L_t - q_t K_t$$

(1). Define CE

- Household

  Sequential language:

  $$L = \sum_{t=0}^{\infty}\{\beta^t u(c_t) + \lambda_t[(1-\delta+q_t)k_t + w_t - c_t - k_{t+1}]\}$$

  FOC

  $[c_t]: \quad \beta^t u'(c_t) = \lambda_t$

  $[k_{t+1}]: \quad \lambda_{t+1}(1-\delta+q_{t+1}) = \lambda_t$

  EE

  $u'(c_t) = \beta u'(c_{t+1})(1-\delta+q_{t+1})$

  The solution to the HH problem is a sequence $\{c_t, k_{t+1}\}$ that satisfy

  - BC
  - EE
  - k_0 is given
  - TVC

  Recursive language:

  $$V(k) = \max u(c) + \beta V(k')$$
  $$s.t.\ c + k' = (1-\delta+q)k + w$$

  Bellman equation

  $$V(k) = \max u((1-\delta+q)k + w - k') + \beta V(k')$$

  FOC $[k']: \quad \beta V'(k') = u'(c)$

  EC $[k]: \quad V'(k) = u'(c)(1-\delta+q)$

  EE $\beta u'(c')(1-\delta+q') = u'(c)$

  The solution to the HH problem is

  policy functions: $c = \phi(k)$, $k' = h(k)$

  value function: V

  s.t.

  - The policy functions solve the "max" part of Bellman equation, given V
  - The value function is a fixed point of Bellman equation, given c and k'

- Firm

$$\max \ F(K_t, L_t) - w_t L_t - q_t K_t$$

$$\text{FOC}$$

$$[K_t]: \quad F_K(K_t, L_t) = q_t$$

$$[L_t]: \quad F_L(K_t, L_t) = w_t$$

- CE

  Allocations $\{c_t, k_t, K_t, L_t\}$ and prices $\{w_t, q_t\}$, s.t.

  Household: BC, EE

  Firm: FOCs

  Market Clearing condition

  - Good market: $F(K_t, L_t) = c_t + K_{t+1} - (1-\delta)K_t$
  - Capital market: $K_t = k_t$
  - Labor market: $L_t = 1$

(2). Define Recursive CE

- Household
  - Cannot use sequential language any more!
  - DP + key feature (aggregate state variable)
  - Aggregate state variable: the joint distribution of the households over their states.

    Representative Agent: $k \to \kappa$

    Heterogeneous Agent: joint distribution (9/23 slides P19 example)
  - Aggregate state variables enter value function $V(k, \kappa)$.

    Prices are functions of state variables $q(\kappa)$ and $w(\kappa)$
  - Households take aggregate state variables as exogenously given

Bellman equation:

$$V(k, \kappa) = \max \; u((1 - \delta + q(\kappa))k + w(\kappa) - k') + \beta V(k', \kappa')$$

FOC $[k']$: $\quad \beta V_1(k', \kappa') = u'(c)$

EC $[k]$: $\quad V_1(k, \kappa) = u'(c)(1 - \delta + q(\kappa))$

EE: $\quad \beta u'(c')(1 - \delta + q(\kappa')) = u'(c)$

The solution to the HH problem is

policy functions: $c = \phi(k, \kappa)$, $k' = h(k, \kappa)$

value function: V

s.t.

- The policy functions solve the "max" part of Bellman equation, given V
- The value function is a fixed point of Bellman equation, given c and k'

- Firm

$$\max \; F(K,L) - w(\kappa)L - q(\kappa)K$$

FOC

$$[K]: \quad F_K(K,L) = q(\kappa)$$

$$[L]: \quad F_L(K,L) = w(\kappa)$$

Solution: $K(\kappa)$ and $L(\kappa)$

- Recursive CE

  Objects

  - Household: policy functions $c = \phi(k,\kappa)$, $k' = h(k,\kappa)$ and value function V
  - Firm: $K(\kappa)$ and $L(\kappa)$
  - Price functions: $w(\kappa)$ and $q(\kappa)$
  - Law of motion for aggregate state variable: $\kappa' = G(\kappa)$

  that satisfy

  - Solution to HH problem
  - Firm FOC
  - Market Clearing condition
    * Good market: $F(K(\kappa), L(\kappa)) = \phi(k,\kappa) + h(k,\kappa) - (1-\delta)K(\kappa)$
    * Capital market: $K(\kappa) = k$
    * Labor market: $L(\kappa) = 1$
  - Consistency: $G(\kappa) = h(\kappa,\kappa)$

1. Please solve the household problem

   $\rightarrow$ Be careful with how to define the solution

2. Please solve the household problem using sequential language

   $\rightarrow$ Lagrangian

3. Please solve the household problem using recursive/ functional language

   $\rightarrow$ DP, Bellman equation

4. State the household's dynamic program $\rightarrow$ DP, Bellman equation

5. Write down the Bellman equation for the household

6. Define a competitive equilibrium

7. Define a recursive competitive equilibrium