Econ720 - TA Session 5

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1. Growth Model

Infinite horizon

When defining the solution, always remember to include boundary conditions (e.g. k_0 is given; TVC).

Discrete time

- Sequence language /sequential solution - set up Lagrangean to find sequences of real variables.
- Recursive formulation - use Dynamic Programming (DP) to find a sequence of value functions and policy functions

2. Dynamic Programming

- Define **state variables**
  Variables carried over into the current period from the last period
  Variables that are predetermind in the current period
- Define control variables
- Value function: V(state variables)
- Utility + continuation value

$$\Rightarrow \begin{cases} \text{Bellman equation} \\ \text{FOC} \\ \text{Envelope condition} \\ \text{Euler equation} \\ \text{Law of motion} \end{cases}$$

2. Dynamic Programming - Example

$$
\begin{aligned}
& max \sum_{t=0}^{\infty} \beta^t u(c_t) \\
& s.t. \quad k_{t+1} = f(k_t) - c_t
\end{aligned}
$$

What are the state variables?

What are the control variables?

How to write the Bellman equation?

2. Dynamic Programming - Example

Bellman equ.

$$V(k) = max\ u(c) + \beta V(k') + \lambda(f(k) - c - k') \quad \text{(state } k\text{, control } c, k'\text{)}$$

or

$$V(k) = max\ u(f(k) - k') + \beta V(k') \quad \text{(state } k\text{, control } k'\text{)}$$

or

$$V(k) = max\ u(c) + \beta V(f(k) - c) \quad \text{(state } k\text{, control } c\text{)}$$

Notice:

- It is conventional to drop the time subscript and use a prime to denote next period variables.
- Remember to write 「max」! The value function tells us the maximum utility obtainable from tomorrow onwards for any value of the state variables.
- Remember to write β in front of next period's V

2. Dynamic Programming - Solution

The solution of a DP problem is

- Policy function: each control variable has a policy function, which is a function of state variables.

  e.g. $c = \phi(k)$, notice that it should be a function of k, not k'!
- Value function: $V(\cdot)$

A DP problem can be solved by

- FOC w.r.t control variables, Envelope condition w.r.t state variables $\Rightarrow$ EE
- Guess & Verify (only for special cases!!)

  Solve explicitly for the value function and the policy function in a DP problem.

  Guess the value function and verify that guess by plugging it into the RHS of Bellman equ. Then solve the maximization problem, and verify that the solution, the value function on the LHS, has the conjecture form.

2. Dynamic Programming - Some Tips

1. Time matters because of an aggregate state variable
   e.g. stochastic productivity following AR(1) process: $f(k_t, A_t)$, $A_{t+1} = G(A_t)$
   $\rightarrow$ Add A_t as a state variable to the value function
2. Finite horizon
   $\rightarrow$ Add t as a state variable to the value function. This indicates value function changes over time
3. $u(c_t, c_{t-1})$
   $\rightarrow$ Define a new state variable $s_t = c_{t-1}$ and add it to the value function. Remember to define the law of motion for s!

3. Recursive Competitive Equilibrium

Recall that the time-t prices faced by agents depend on the equilibrium quantity of capital (assume inelastic labor supply)

$$q_t = F_K(K_t, 1) \quad w_t = F_L(K_t, 1)$$

This suggests that in a recursive setup, for the households to take future prices into account in their decisions they need to know how

- prices depend on the equilibrium (aggregate) capital stock
- and the capital stock evolves over time

So in a dateless and recursive formulation, we write all prices as functions of the aggregate state variable, K:

$$q(K) \quad w(K)$$

We endow the household with knowledge of the law of motion for aggregate capital

$$K' = \varphi(K)$$

3. Recursive Competitive Equilibrium

The household must think of itself as atomistic

- we must formulate the problem so that the household does not believe that its choices affect prices
- but the household has to believe that its choices affect its own outcomes

For this reason we introduce a distinction between the household's own capital stock, k, and the economy wide capital stock, K

$\Rightarrow$ RCE is often used in heterogeneous model and model with uncertainty.

3. Recursive Competitive Equilibrium

When define a RCE

- everything is written as functions of state variables
- don't forget to include law of motion for aggregate state variables in objects
- don't forget the consistency condition

The consistency condition is the distinctive feature of the recursive formulation of competitive equilibrium. The requirement is that, whenever the individual consumer is endowed with a level of capital equal to the aggregate level (for example, only one single agent in the economy owns all the capital), his own individual behavior will exactly mimic the aggregate behavior. The term consistency points out the fact that the aggregate law of motion perceived by the agent must be consistent with the actual behavior of individuals.
– *Real Macroeconomic Theory, Per Krusell, 2014, P83*

4. Exercise

Midterm Exam. Econ720. Fall 2015

Capital Depreciates in 2 Periods

- Consider a standard growth model in discrete time where capital fully depreciates in 2 periods.
- Demographics: There is a representative agent of mass 1 who lives forever.
- Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$
- Endowments: k_0 at $t = 0$
- Technology: $f(k_t) = c_t + i_t$ with $k_{t+1} = i_t + i_{t-1}$

Questions:

(1). Write down the planner's Bellman equation.
(2). Define a solution to the planner's problems.

4. Exercise

Capital depreciates fully in two periods, and does not depreciate at all before that:

$$k_t = i_{t-1} + i_{t-2}$$

Define two auxiliary state variables: $z_t = i_{t-1}$ and$x_t = i_{t-2}$.
Then $k_t = z_t + x_t$
The laws of motion are $z_{t+1} = i_t = f(z_t + x_t) - c_t$, $x_{t+1} = i_{t-1} = z_t$
Bellman equ.

$$V(z,x) = max\ u(f(x+z) - i) + \beta V(z', x')$$

4. Exercise

Notice that there are two state variables in this problem. That is unavoidable here; there is no way of summarizing what one needs to know at a point in time with only one variable. For example, the total capital stock in the current period is not informative enough, because in order to know the capital stock next period we need to know how much of the current stock will disappear between this period and the next. Both i_{-1} and i_{-2} are natural state variables: they are predetermined, they affect outcomes and utility, and neither is redundant: the information they contain cannot be summarized in a simpler way.