Econ 720 Recitation #11 11/15/2019

A Brief Summary of Endogenous Growth Model

Problem of Each Sector	Variety Expansion Model	Quality Ladder Model
① Household Sector ⇒ Get EE which shows consumption growth rate	$\max \int_0^\infty e^{-\rho t}u(c_t)dt$ Complicated BC or a reduced form $\dot{a}_t = r_t a_t + w_t - c_t$ $g(c_t) = \frac{r_t - \rho}{\sigma(c_t)}$	
② Final Goods Sector (**Perfect Competition**) Profit maximization given factor prices ⇒ Find the optimal demand for x_{jt} and L_t ! Notice that final goods price is normalized to 1	Production function: $Y_t = (1-\beta)^{-1}(\int_0^{N_t} x_{jt}{}^{1-\beta}\, dj)L_t^\beta$ $\max\ Y_t - \int_0^{N_t} p_{jt}x_{jt}\, dj - w_t L_t$ $\overset{FOC}{\Longrightarrow}$ Optimal demand for intermediates and labor input $[x_{jt}]$: $x_{jt}^{-\beta}L_t^\beta = p_{jt}$ $[L_t\ ]$: $\frac{\beta}{1-\beta}\left(\int_0^{N_t} x_{jt}{}^{1-\beta}\, dj\right)L_t^{\beta-1} = w_t$	. $Y_t = (1-\beta)^{-1}(\int_0^1 q_{jt}\cdot x_{jt}{}^{1-\beta}\, dj)L_t^\beta$ $\max\ Y_t - \int_0^1 p_{jt}x_{jt}\, dj - w_t L_t$ $\overset{FOC}{\Longrightarrow}$ Demand for intermediates and labor $[x_{jt}]$: $x_{jt} = (q_{jt}/p_{jt})^{1/\beta}L_t$ $[L_t\ ]$: $\beta Y_t/L_t = w_t$
③ Intermediates Sector (**Monopolistic Competition**) Profit maximization by choosing price ⇒ Find the optimal price p_{jt} that maximizes profit What's the demand function? What's the cost?	**1 units of x_{jt} is produced by φ units of final goods** $\max\ \pi_{jt} \Leftrightarrow \max\ p_{jt}x_{jt} - \varphi x_{jt} \Leftrightarrow \max\ (p_{jt}-\varphi)\, L_t p_{jt}^{-1/\beta}$ $\overset{FOC}{\Longrightarrow}$ Optimal price $p_{jt} = \frac{\varphi}{1-\beta}$ for $\forall t,\ \forall j$ Hence, $x_{jt} = L_t(\frac{\varphi}{1-\beta})^{-1/\beta}$ and $\pi_{jt} = L_t(\frac{\varphi}{1-\beta})^{-1/\beta}\frac{\beta\varphi}{1-\beta}$	**The marginal cost is φq_{jt} units of final goods** $\max\ \pi_{jt} \Leftrightarrow \max\ p_{jt}x_{jt} - \varphi q_{jt}x_{jt}$ $\overset{FOC}{\Longrightarrow}$ Optimal price $p_{jt} = \frac{\varphi q_{jt}}{1-\beta}$ So, $x_{jt} = L_t(\frac{\varphi}{1-\beta})^{-1/\beta}$, $\pi_{jt} = L_t(\frac{\varphi}{1-\beta})^{-1/\beta}\frac{\beta\varphi q_{jt}}{1-\beta}$
④ R&D Sector We only think about equilibrium In equilibrium, **Free Entry Condition** holds ⇒ Get the present value of profit V_j and r For simplicity, only focus on BGP, so r is constant	$\boldsymbol{\dot{N}_t = \eta Z_t}$ For simplicity, set $\varphi = 1-\beta$ Cost of creating a new variety $= 1/\eta$ Benefit of creating a new variety is $V_j = \int_s^\infty e^{-r(t-s)}\pi_{jt}\ dj$ Free Entry: $\frac{1}{\eta} = V_j$, hence $\frac{1}{\eta} = \frac{\beta L}{r}$, therefore $r = \beta L\eta$	$\boldsymbol{n_{jt}\Delta t = (\eta/q_{jt})Z_{jt}\Delta t}$ Innovation takes quality from q_{jt} to λq_{jt} Suppose current quality is q_{jt}/λ Free Entry: $1=(\lambda\eta/q_{jt})V(j,t\vert q_{jt})$ What's the value of $V(j,t\vert q_{jt})$? Asset pricing!