

Example: OLG with Capital, Land and Bonds

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Consider a standard two-period overlapping generations model with the following characteristics:

Demographics

- Each period a cohort of size $N_t = 1$ are born. Each cohort lives for two periods.
- All cohorts are identical and behave competitively.

Endowments and Preferences

- Each young cohort is endowed with 1 unit of labor which they supply inelastically.
- At $t = 0$, the old cohort is endowed with k_0 units of capital and x_0 units of land.
- Each cohort born in generic period t maximizes the following utility function:

$$U = u(c_t^y) + \beta u(c_{t+1}^o)$$

where c_t^y and c_{t+1}^o represent consumption when young and old respectively and the utility function $u(\cdot)$ satisfies the usual conditions.

Technology

- Capital k_t , land x_t , and bonds b_t can be traded among households in spot markets. Bonds can be stored intertemporally costlessly.
- Capital and consumption goods can be freely transformed one to another (one-to-one).
- Land is available in fixed supply. (Additional land above x_0 cannot be accumulated)
- Firms are identical and perfectly competitive.
- Firms rent capital and land from old households and labor (L_t) from young households to produce a final good with the following production function:

$$y_t = f(K_t, X_t, L_t)$$

where $f(\cdot)$ satisfies the usual Inada conditions and y_t is in units of consumption.

- Capital depreciates after use at rate $0 \leq \delta \leq 1$. Land does not depreciate (Land is a durable good).

Markets

- Bonds are issued by households with interest rate R_{t+1} (in units of account) and have a one-period maturity.
- Capital may be traded at price P_t^k and rented to firms at rate R_t^k (in units of account).
- Land may be traded at price P_t^x and rented to firms at rate R_t^x (in units of account).
- Consumption goods may be traded at price P_t^c .
- Goods market must hold for consumption and capital.

Questions

1. What are the representative household's budget constraint in each period?
2. Have we defined a numeraire yet? If not, let's do so.
3. What is the representative household's lifetime budget constraint?
4. Write down and solve the representative household's problem.
5. Write down and solve the firm's problem.
6. Define a competitive equilibrium.