Econ720 - TA Session 4

Yanran Guo

UNC-Chapel Hill

2021. 9. 17

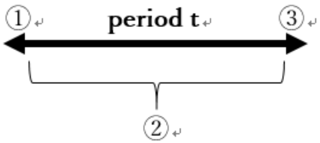
Macro Models

- Short run: equilibrium (solve sector problem and define GE)
- Long run: steady state (convergence, stability)
    - state variable
    - law of motion

We study money in OLG model.

Model Setup

Timing:



1. At the beginning of t:

   Each young: goods e_1

   Each old: goods e_2

   money $\frac{M_t}{N_{t-1}}$

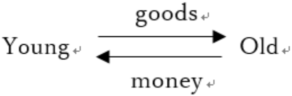
Model Setup

Why does each old carry $\frac{M_t}{N_{t-1}}$ units of money?

The total amount of money carried by the old when entering period t is M_{t-1}. In period t, government prints additional money in proportion to the current money holdings. Hence the amount of newly printed money in period t is θM_{t-1}. These newly printed money is handed to the old. Hence the total amount of money carried by the old in period t is $M_{t-1} + \theta M_{t-1} = M_t$. Hence each old carries $\frac{M_t}{N_{t-1}}$ units of money.

Model Setup

2. During t:



3. At the end of t:

The old give all their money to the young in exchange for goods.

* Notice that in our model setup, we assume that agent's utility only comes from consuming goods, not from holding money (Money In Utility model, PS2-Q2). The old will die at the end of period t. Hence in order to maximize their utility, the old will sell all their money to the young in exchange for goods.

Model Setup

Translate the setup of timing into equations:

$$\text{Young: } p_t c_t^y + Money_t = p_t e_1$$

$$\text{Old: } p_{t+1} c_{t+1}^0 = p_{t+1} e_2 + Money_t + \theta Money_t$$

Rewrite BC in real terms:

$$\text{Young: } c_t^y + \frac{Money_t}{p_t} = e_1 \;\rightarrow\; c_t^y + x_t = e_1$$

$$\text{Old: } c_{t+1}^0 = e_2 + (1+\theta)\frac{Money_t}{p_{t+1}} \;\rightarrow\; c_{t+1}^0 = e_2 + (1+\theta)x_t \frac{p_t}{p_{t+1}}$$

Household Problem

$$max \ u(c_t^y, \ c_{t+1}^o)$$

$$s.t. \ \ c_t^y + x_t = e_1$$

$$c_{t+1}^0 = e_2 + (1+\theta)x_t \frac{p_t}{p_{t+1}}$$

$$\Rightarrow \text{E.E. } u_1(c_t^y, \ c_{t+1}^o) = (1+\theta)\frac{p_t}{p_{t+1}} u_2(c_t^y, \ c_{t+1}^o)$$

Define $R_{t+1} \equiv (1+\theta)\frac{p_t}{p_{t+1}}$. R_{t+1} is the real rate of return from holding money.

Tip: real rate of return can be derived by using E.E.

Competitive Equilibrium

Allocations $\{c_t^y, c_{t+1}^o, x_t, M_t\}$ and prices $\{R_{t+1}, p_t\}$ that satisfy

- Household problem: E.E. (1), BC (2);
- Government: $M_{t+1} = (1+\theta)M_t$;
- Market clearing condition:

  Goods market: $N_t c_t^y + N_{t-1} c_t^o = N_t e_1 + N_{t-1} e_2$

  Money market: $M_t = x_t p_t N_t$
- Accounting identity: $R_{t+1} = (1+\theta)\frac{p_t}{p_{t+1}}$

6 objects, 7 equations

Law of Motion of m_t

Law of motion of $m_t = \frac{M_t}{P_t N_t}$**:**

State variable: real money balances per capita

$$m_t = s((1+n)\frac{m_{t+1}}{m_t})$$

- What's the shape of this law of motion?
- What are the properties of the steady state

Some Side Discussion

9/2 slides P24, OLG with production and log utility

Law of motion of k is

$$k_{t+1} = \frac{1}{1+n}\frac{\beta}{1+\beta}(1-\theta)k_t^{\theta}$$

- What's the shape of this law of motion? Concave
- What are the properties of the steady state? Stable

Law of Motion of m_t

Want to map m_t into m_{t+1}. But how?

$$m_{t+1} = \frac{M_{t+1}}{p_{t+1}N_{t+1}} = \frac{m_t p_t N_t(1+\theta)}{p_{t+1}N_{t+1}} = m_t R_{t+1}\frac{1}{1+n}$$

Hence,

$$(1+n)m_{t+1} = R_{t+1}m_t$$

But this is NOT our final answer. Why?

- $m_t = s(R_{t+1})$
- For each R_{t+1}, there is a corresponding $m_t = s(R_{t+1})$, and there is a corresponding $(1+n)m_{t+1} = R_{t+1}s(R_{t+1})$

Law of Motion of m_t

- $m_t = s(R_{t+1})$
- For each R_{t+1}, there is a corresponding $m_t = s(R_{t+1})$, and there is a corresponding $(1+n)m_{t+1} = R_{t+1}s(R_{t+1})$
- These things can be linked by offer curve.

$$c^o_{t+1} = -R_{t+1}c^y_t + R_{t+1}e_1 + e_2$$

$\rightarrow$ Offer curve (unusual, hard to grasp)

$\rightarrow$ Flip the offer curve, we get the figure for law of motion of m_t.

Dynamics

Properties of steady state: convergence, stability

- If the offer curve is not backward bending,

  $m_0 > m_{ss}$: m_t will explode;

  $m_0 < m_{ss}$: m_t will converge to 0

  the economy will never reach steady state.

- If the offer curve is backward bending, the economy may reach steady state. The dynamics is complex.

Dynamic Efficiency

- Samuelson case: the offer curve at the origin is flatter than $1 + n$
- Classical case: it is steeper than $1 + n$

Fiscal Theory of the Price Level

- Model without government spending: money is valued in equilibrium only in an economy that would be dynamically inefficient without money.
- Model with government spending: there is no non-monetary equilibria

In this model, we assume that government issues money to cover its spending. It is the government's expenditure plan that gives agents the belief that money has value. Hence, with government spending, money is always valued in equilibrium.