# Econ.720 Recitation-6

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## 1 Define CE

Model

$\swarrow \quad \searrow$

| Solve the model using sequential language | Solve the model using recursive language |
|---|---|
| =Lagrangian | =Dynamic Programming (DP) |
| $L = \sum_{t=0}^{\infty} \beta^t u(c_t) + \lambda_t(BC)$ | $V(k) = \max_{c,k'} u(c) + \beta V(k') + \lambda(BC)$ |
| FOC, EE | FOC, Envelope Condition (EC), EE |
| ... | ... |

$\nwarrow \quad \nearrow$

Define CE
as before

## 2 Define Recursive CE

Model

$\downarrow$

Solve the model using DP with aggregate state variable

$\downarrow$

Define recursive CE

## 3 Example

Model setup:

- Household

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$s.t.\ c_t + k_{t+1} = (1-\delta+q_t)k_t + w_t$$

No population growth, $k_0$ is given

- Firm

$$\max\ F(K_t, L_t) - w_t L_t - q_t K_t$$

### (1). Define CE

- Household

Sequential language:

$$L = \sum_{t=0}^{\infty}\{\beta^t u(c_t) + \lambda_t[(1-\delta+q_t)k_t + w_t - c_t - k_{t+1}]\}$$

FOC

$[c_t]: \quad \beta^t u'(c_t) = \lambda_t$

$[k_{t+1}]: \quad \lambda_{t+1}(1-\delta+q_{t+1}) = \lambda_t$

EE

$u'(c_t) = \beta u'(c_{t+1})(1-\delta+q_{t+1})$

The solution to the HH problem is a sequence $\{c_t, k_{t+1}\}$ that satisfy

- BC
- EE
- $k_0$ is given
- TVC

Recursive language:

$$V(k) = \max u(c) + \beta V(k')$$
$$s.t.\ c + k' = (1-\delta+q)k + w$$

Bellman equation

$$V(k) = \max u((1-\delta+q)k + w - k') + \beta V(k')$$

FOC $[k']: \quad \beta V'(k') = u'(c)$

EC $[k]: \quad V'(k) = u'(c)(1-\delta+q)$

EE $\beta u'(c')(1-\delta+q') = u'(c)$

The solution to the HH problem is

policy functions: $c = \phi(k)$, $k' = h(k)$

value function: V

s.t.

- The policy functions solve the "max" part of Bellman equation, given V
- The value function is a fixed point of Bellman equation, given $c$ and $k'$

- Firm

$$
\begin{aligned}
&\max\ F(K_t, L_t) - w_t L_t - q_t K_t \\
&\text{FOC} \\
&[K_t]: \quad F_K(K_t, L_t) = q_t \\
&[L_t]: \quad F_L(K_t, L_t) = w_t
\end{aligned}
$$

- CE

  Allocations $\{c_t, k_t, K_t, L_t\}$ and prices $\{w_t, q_t\}$, s.t.

  Household: BC, EE

  Firm: FOCs

  Market Clearing condition

  - Good market: $F(K_t, L_t) = c_t + K_{t+1} - (1-\delta)K_t$
  - Capital market: $K_t = k_t$
  - Labor market: $L_t = 1$

## (2). Define Recursive CE

- Household
  - Cannot use sequential language any more!
  - DP + key feature (aggregate state variable)
  - Aggregate state variables enter value function $V(k, \kappa)$.

    Prices are functions of state variables $q(\kappa)$ and $w(\kappa)$
  - Households take aggregate state variables as exogenously given

  Bellman equation:

$$V(k, \kappa) = \max \; u((1 - \delta + q(\kappa))k + w(\kappa) - k') + \beta V(k', \kappa')$$

  FOC $[k']$ : $\quad \beta V_1(k', \kappa') = u'(c)$

  EC $[k]$ : $\quad V_1(k, \kappa) = u'(c)(1 - \delta + q(\kappa))$

  EE: $\quad \beta u'(c')(1 - \delta + q(\kappa')) = u'(c)$

  The solution to the HH problem is

  policy functions: $c = \phi(k, \kappa)$, $k' = h(k, \kappa)$

  value function: V

  s.t.

  - The policy functions solve the “max” part of Bellman equation, given V
  - The value function is a fixed point of Bellman equation, given $c$ and $k'$

- Firm

$$\begin{aligned}
&\max \ F(K,L) - w(\kappa)L - q(\kappa)K \\
&\text{FOC} \\
&[K]: \quad F_K(K,L) = q(\kappa) \\
&[L]: \quad F_L(K,L) = w(\kappa)
\end{aligned}$$

  Solution: $K(\kappa)$ and $L(\kappa)$

- Recursive CE

  Objects

  - Household: policy functions $c = \phi(k,\kappa)$, $k' = h(k,\kappa)$ and value function V
  - Firm: $K(\kappa)$ and $L(\kappa)$
  - Price functions: $w(\kappa)$ and $q(\kappa)$
  - Law of motion for aggregate state variable: $\kappa' = G(\kappa)$

  that satisfy

  - Solution to HH problem
  - Firm FOC
  - Market Clearing condition
    * Good market: $F(K(\kappa), L(\kappa)) = \phi(k,\kappa) + h(k,\kappa) - (1-\delta)K(\kappa)$
    * Capital market: $K(\kappa) = k$
    * Labor market: $L(\kappa) = 1$
  - Consistency: $G(\kappa) = h(\kappa,\kappa)$

1. Please solve the household problem

   $\rightarrow$ Be careful with how to define the solution

2. Please solve the household problem using sequential language

   $\rightarrow$ Lagrangian

3. Please solve the household problem using recursive/ functional language

   $\rightarrow$ DP, Bellman equation

4. State the household's dynamic program $\rightarrow$ DP, Bellman equation

5. Write down the Bellman equation for the household

6. Define a competitive equilibrium

7. Define a recursive competitive equilibrium