Econ720 - TA Session 9

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Phase Diagram

Ramsey Model

Social Planner Problem

$$max \int_0^\infty e^{-(\rho-n)t}\frac{c_t^{1-\sigma}}{1-\sigma}dt$$
$$\dot{k}_t = f(k_t) - (n+\delta)k_t - c_t$$

Phase Diagram

Current Value Hamiltonian:

$$H = \frac{c_t^{1-\sigma}}{1-\sigma} + \mu_t(f(k_t) - (n+\delta)k_t - c_t)$$

FOCs

$$\begin{aligned} [c_t]: &\quad c_t^{-\sigma} = \mu_t \\ [k_t]: &\quad \mu_t[f'(k_t) - (n+\delta)] = -\dot{\mu}_t + (\rho - n)\mu_t \end{aligned}$$

Phase Diagram

$$c_t^{-\sigma} = \mu_t \quad \rightarrow \quad -\sigma c_t^{-\sigma-1} \dot{c}_t = \dot{\mu}_t$$

Plug into

$$\mu_t[f'(k_t) - (n+\delta)] = -\dot{\mu}_t + (\rho - n)\mu_t$$

Then we have

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - (\delta + \rho)}{\sigma}$$

Phase Diagram

How to draw phase diagram?

1. Find the dynamics for control and state
   Differential equation for control and for state
   Tips:
   - Usually differential equation for control can be derived by substituting out co-state.
   - Differential equation for state variable is usually given.
2. Think about the steady state $\rightarrow$ "dot" equals zero
3. Plot the two steady state equations separately
4. Decide the movement of control and state respectively.

Phase Diagram

Step-1: Find differential equation for control and for state

Control variable c_t

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - (\delta + \rho)}{\sigma}$$

State variable k_t

$$\dot{k}_t = f(k_t) - (n + \delta)k_t - c_t$$

Phase Diagram

Step-2: How do these two equations look like in SS

$$\frac{\dot{c}_t}{c_t} = 0 \quad \Rightarrow \quad f'(k^*) = \delta + \rho$$

Hence k^* is a constant

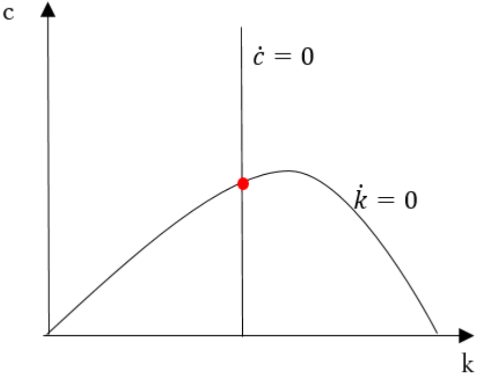
$$\dot{k}_t = 0 \quad \Rightarrow \quad c^* = f(k^*) - (n+\delta)k^*$$

Hence c^* is a function of k^*

Phase Diagram

Step-3: Plot the two SS equations

$$\dot{c} = 0: \quad f'(k^*) = \delta + \rho$$
$$\dot{k} = 0: \quad c^* = f(k^*) - (n+\delta)k^*$$



Phase Diagram

Step-4: Decide the movement of these two variables separately

- c (control) is the vertical axis $\rightarrow$ c moves up and down
- k (state) is the horizontal axis $\rightarrow$ k moves left and right

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - (\delta + \rho)}{\sigma}$$

$$\dot{k}_t = f(k_t) - (n + \delta)k_t - c_t$$

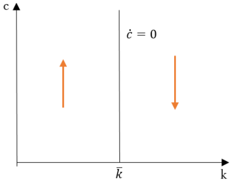
1. Use $\dot{c}$ equation and change the value of k to study the movement of c
2. Use $\dot{k}$ equation and change the value of c to study the movement of k
3. Put them together

Phase Diagram

Use $\dot{c}$ equation and change the value of k to study the movement of c

$$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - (\delta + \rho)}{\sigma}$$

- Take one k on $\dot{c} = 0$ curve, $k = \overline{k}$, $\frac{\dot{c}}{c} = \frac{f'(\overline{k}) - (\delta+\rho)}{\sigma} = 0$
- For each k to the left of $\overline{k}$, $k < \overline{k}$, $\frac{\dot{c}}{c} = \frac{f'(k) - (\delta+\rho)}{\sigma} > \frac{f'(\overline{k}) - (\delta+\rho)}{\sigma} = 0$ c increases
- For each k to the right of $\overline{k}$, $k > \overline{k}$, $\frac{\dot{c}}{c} = \frac{f'(k) - (\delta+\rho)}{\sigma} < \frac{f'(\overline{k}) - (\delta+\rho)}{\sigma} = 0$ c decreases

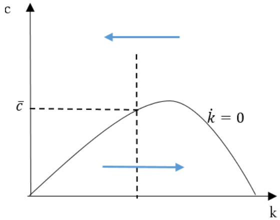


Phase Diagram

Use $\dot{k}$ equation and change the value of c to study the movement of k

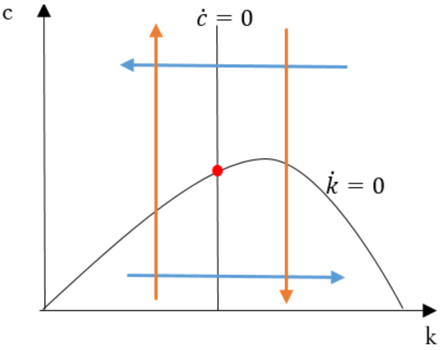
$$\dot{k}_t = f(k_t) - (n+\delta)k_t - c_t$$

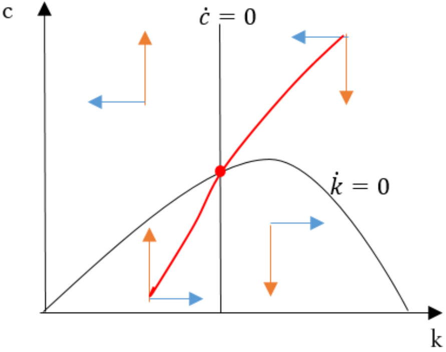
- Take any c on $\dot{k} = 0$ curve, $c = \bar{c}$, $\dot{k} = f(k) - (n+\delta)k - \bar{c} = 0$
- For each c above $\bar{c}$, $c > \bar{c}$,
  $\dot{k} = f(k) - (n+\delta)k - c < f(k) - (n+\delta)k - \bar{c}$ k decreases
- For each c under $\bar{c}$, $c < \bar{c}$,
  $\dot{k} = f(k) - (n+\delta)k - c > f(k) - (n+\delta)k - \bar{c}$ k increases



Phase Diagram

Put them together





Detrending

- Balanced growth path: real variables keep growing at constant growth rates
- Want SS
- Detrend

Fall 2017 Final

Question-1. Continuous time growth model