Econ720 - TA Session 12

Yanran Guo

UNC-Chapel Hill

2020. 11. 06

PS6 - Stochastic Patent Duration

Questions:

1. Solve the problem of the final goods producer.

Answer key:

$$\max_{L_t,\{x_{jt}\}_{j=0}^{N_t}} (1-\beta)^{-1} L_t^{\beta} \int_0^{N_t} x_{jt}^{1-\beta} dj - w_t L_t - \int_0^{N_t} p_{jt} x_{jt} dj$$

$$[L_t]: \quad \frac{\beta}{1-\beta} \frac{Y_t}{L_t} = w_t$$

$$[x_{jt}]: \quad L_t^{\beta} x_{jt}^{-\beta} = p_{jt}$$

Hence the demand function for monopolistic inputs is $x_{jt}^m = L_t (p_{jt}^m)^{-\frac{1}{\beta}}$

The demand function for competitive inputs is $x_{jt}^c = L_t (p_{jt}^c)^{-\frac{1}{\beta}}$

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Discussion:

- While prices differ across intermediate inputs, the final good producer takes these prices as given.
- Hence there is no need to explicitly separate monopolistic and competitive intermediates as

$$\max_{L_t, \{x_{jt}\}_{j=0}^{N_t}} (1-\beta)^{-1} L_t^{\beta} \int_0^{N_t} x_{jt}^{1-\beta} dj - w_t L_t$$
$$- \int_0^{N_{1t}} p_{jt}^m x_{jt}^m dj - \int_0^{N_{2t}} p_{jt}^c x_{jt}^c dj$$

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2. Solve the problem of a monopolist intermediate input producer.

Answer key:

Monopolist intermediates producer maximize her profit by choosing price, given demand function (derived from the final good producer's problem).

$$\max_{p_{jt}^m} \pi_{jt}^m \;\rightarrow\; \max_{p_{jt}^m} p_{jt}^m x_{jt}^m - \psi x_{jt}^m$$

$$\max_{p_{jt}^m} p_{jt}^m L_t (p_{jt}^m)^{-\frac{1}{\beta}} - \psi L_t (p_{jt}^m)^{-\frac{1}{\beta}}$$

From the FOC, $p_{jt}^m = \frac{\psi}{1-\beta}$.

Hence $x_{jt}^m = L_t (\frac{\psi}{1-\beta})^{-\frac{1}{\beta}}$ and $\pi_{jt}^m = \beta L_t (\frac{\psi}{1-\beta})^{1-\frac{1}{\beta}}$

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3. Solve for equilibrium interest rate.

Discussion:

- Equilibrium interest rate is derived from free entry condition.
- Free entry condition only means: *cost = benefit*
  e.g. FE doesn't imply $\eta V = 1$. This equation is NOT universally true!
- In this model, it costs η units of final good to create a new type of intermediate good.
- Hence the cost of creating a new type of intermediate is

$$\text{Cost} = \eta \cdot price_{final\ goods} = \eta$$

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Notes on Poisson Process

- In a discrete time model, in each period, with probability p a monopolistic firm loses its monopoly power. Then the present value of the stream of profits is

$$
\begin{aligned}
V &= \pi_s + \frac{1}{1+r}(1-p)\pi_{s+1} + \left(\frac{1}{1+r}\right)^2(1-p)^2\pi_{s+2} + \dots \\
&= \sum_{t=s}^{\infty}\left(\frac{1}{1+r}\right)^{t-s}(1-p)^{t-s}\pi_t
\end{aligned}
$$

- Under continuous time setup, the probability of no event over a period of length τ is $e^{-\delta\tau}$, where δ is the Poisson arrival rate.

Hence, $e^{-\delta(t-s)}$ is the continuous time analogous of $(1-p)^{t-s}$

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Answer Key:

- Cost of creating a new type of intermediate is η
- Benefit of creating a new type of intermediate is

$$
\begin{aligned}
V &= \int_s^\infty e^{-r(t-s)} e^{-\delta(t-s)} \pi_t^m dt \\
&= \int_s^\infty e^{-(r+\delta)(t-s)} \beta L (\frac{\psi}{1-\beta})^{1-\frac{1}{\beta}} dt \\
&= \beta L (\frac{\psi}{1-\beta})^{1-\frac{1}{\beta}} \frac{1}{r+\delta}
\end{aligned}
$$

- Free entry condition: $\eta = V$

$$
r = \beta L (\frac{\psi}{1-\beta})^{1-\frac{1}{\beta}} / \eta - \delta
$$

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4. Derive equilibrium growth rate. Which patent duration δ maximizes growth? Does this also maximize welfare?

Answer key:

EE can be obtained by solving HH problem.

$$g = \frac{\dot{C}_t}{C_t} = \frac{r-\rho}{\theta} = \frac{\beta L(\frac{\psi}{1-\beta})^{1-\frac{1}{\beta}}/\eta - \delta - \rho}{\theta}$$

Since $\delta \geq 0$, $\delta = 0$ will maximize growth rate. And it doesn't maximize welfare.

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Reason:

- $\delta = 0$ means once innovator succeeds, she can maintain monopoly power forever. It increases the benefit of doing R&D. Hence more potential entrants will participate in doing innovation. The variety of intermediate inputs increases quickly. That increases the production of final goods, hence the consumption.
- When $\delta = 0$, intermediate goods market is monopolistic. The monopolists will set higher price to get more profit, which leads to inefficiency. This can be formally proved using a planner problem.

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5. Consider the balanced growth path. Show that $\frac{N_1}{N_2} = \frac{g}{\delta}$

Idea:

- Step-1. Prove $g_{N_2} = g_N$ using the law of motion for N_2
- Step-2. Prove $g_N = g_Y$ using production function
- Step-3. Prove $g_N = g_C$ using resource constraint

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Answer Key:

- Step-1. Prove $g_{N_2} = g_N$ using the law of motion for N_2

$$\dot{N}_{2t} = \delta N_{1t} = \delta(N_t - N_{2t})$$
$$g_{N_2} = \frac{\dot{N}_{2t}}{N_{2t}} = \delta(\frac{N_t}{N_{2t}} - 1)$$

On BGP, since g_{N_2} is constant, $\frac{N_t}{N_{2t}}$ must be constant.
Hence, on BGP, $g_{N_2} = g_N$

Answer Key:

- Step-2. Step-2. Prove $g_N = g_Y$ using production function

$$
\begin{aligned}
Y_t &= (1-\beta)^{-1} L^{\beta} \int_0^{N_t} x_{jt}^{1-\beta} dj \\
&= (1-\beta)^{-1} L^{\beta} \left\{ \int_0^{N_{1t}} (x_{jt}^{m})^{1-\beta} dj + \int_0^{N_{2t}} (x_{jt}^{c})^{1-\beta} dj \right\} \\
&= (1-\beta)^{-1} L^{\beta} \left\{ N_{1t} (L(\frac{\psi}{1-\beta})^{-\frac{1}{\beta}})^{1-\beta} + N_{2t} (L\psi^{-\frac{1}{\beta}})^{1-\beta} \right\} \\
&= \underbrace{\Omega_1}_{\text{constant}} N_{1t} + \underbrace{\Omega_2}_{\text{constant}} N_{2t} \\
&= \Omega_1 (N_t - N_{2t}) + \Omega_1 N_{2t}
\end{aligned}
$$

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$$\frac{Y_t}{N_t} = \Omega_1 + (\Omega_2 - \Omega_1)\frac{N_{2t}}{N_t}$$

On BGP, since $\frac{N_{2t}}{N_t}$ is constant, $\frac{Y_t}{N_t}$ is constant.

Hence, on BGP, $g_N = g_Y$

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Answer Key:

- Step-3. Prove $g_N = g_C$ using resource constraint

$$
\begin{aligned}
Y_t &= C_t + X_t + Z_t \\
&= C_t + \left\{ \int_0^{N_{1t}} \psi x_{jt}^m dj + \int_0^{N_{2t}} \psi x_{jt}^c dj \right\} + \frac{1}{\eta} \dot{N}_t \\
&= C_t + \left\{ \underbrace{\Omega_3}_{\text{constant}} N_{1t} + \underbrace{\Omega_4}_{\text{constant}} N_{2t} \right\} + \frac{1}{\eta} \dot{N}_t \\
&= C_t + \Omega_3 N_t + (\Omega_4 - \Omega_3) N_{2t} + \frac{1}{\eta} \dot{N}_t
\end{aligned}
$$

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$$\frac{Y_t}{N_t} = \frac{C_t}{N_t} + \Omega_3 + (\Omega_4 - \Omega_3)\frac{N_{2t}}{N_t} + \frac{1}{\eta}\frac{\dot{N}_t}{N_t}$$

On BGP, $\frac{\dot{N}_t}{N_t}$, $\frac{N_{2t}}{N_t}$, $\frac{Y_t}{N_t}$ are constant.

Hence $\frac{C_t}{N_t}$ is constant, which means $g_N = g_C = g$

Hence on BGP, $g_{N_2} = g_N = g$

Due to law of motion of N_2, $\dot{N}_{2t} = \delta N_{1t}$

$$g = \frac{\dot{N}_{2t}}{N_{2t}} = \delta \frac{N_{1t}}{N_{2t}} \quad \rightarrow \quad \frac{N_{1t}}{N_{2t}} = \frac{g}{\delta}$$

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6. Define a competitive equilibrium.

Answer key:

$\{C_t, L_t, A_t, Y_t, x_{jt}^m, x_{jt}^c, \pi_{jt}^m, V_j, N_{1t}, N_{2t}, Z_t\}$ and $\{p_{jt}^m, p_{jt}^c, w_t, r_t\}$

- Household: EE, BC
- Final good producer: FOC for labor, FOC for competitive intermediates, FOC for monopolistic intermediates, definition for Y_t
- Monopoly intermediate producer: p_{jt}^m, x_{jt}^m, π_{jt}^m
- Competitive intermediate producer: x_{jt}^c
- R&D sector: free entry condition
- Market clearing: RC, intermediates, labor, asset
- Law of motion for N_{2t}