Econ.720 Recitation#9 11/1/2019

TVC on Page-20 in Perpetual Youth Slide

The standard TVC should be

$$\lim_{t\to\infty} e^{-(\rho+\nu)(t-\tau)}\mu_t a(t|\tau) = 0$$

Due to eq. (11)

$$\frac{\dot{c(t|\tau)}}{c(t|\tau)} = r(t) - \rho$$

Hence

$$c(t|\tau) = c(\tau|\tau)e^{(r(t)-\rho)(t-\tau)}$$

From FOC, we know that

$$\mu_t = \frac{1}{c(t|\tau)} = \frac{1}{c(\tau|\tau)}e^{(\rho-r(t))(t-\tau)}$$

Plug the equation for μ_t back to the TVC

$$\lim_{t\to\infty} e^{-(\rho+\nu)(t-\tau)}\frac{1}{c(\tau|\tau)}e^{(\rho-r(t))(t-\tau)}a(t|\tau) = 0$$

Since $c(\tau|\tau)$ is some constant, so we can take it away. So the TVC becomes

$$\lim_{t\to\infty} e^{-(\rho+\nu)(t-\tau)}e^{(\rho-r(t))(t-\tau)}a(t|\tau) = 0$$

Rearrange the terms

$$\lim_{t\to\infty} e^{[-(\rho+\nu)+(\rho-r(t))](t-\tau)}a(t|\tau) = \lim_{t\to\infty} e^{-(r(t)+\nu)(t-\tau)}a(t|\tau) = 0$$

Notice that in this model, r varies along with time, hence the correct way to writ $r_t(t-\tau)$ should be $\int_\tau^t r_z dz$

Since ν is a parameter, hence $\nu(t-\tau) = \int_\tau^t \nu dz$

$$\lim_{t\to\infty} e^{-(r(t)+\nu)(t-\tau)}a(t|\tau) = \lim_{t\to\infty} e^{-(\int_\tau^t r_z dz + \int_\tau^t \nu dz)}a(t|\tau) = \lim_{t\to\infty} e^{-\int_\tau^t (r_z+\nu)dz}a(t|\tau) = 0$$

Hence the TVC is

$$\lim_{t\to\infty} e^{-\int_\tau^t (r_z+\nu)dz}a(t|\tau) = \lim_{t\to\infty} D_{t,\tau}a(t|\tau) = 0$$

where $D_{t,\tau} = e^{-\int_\tau^t (r_z+\nu)dz}$

Exercise. Macroeconomics Qualifying Examination January 2012. An Investment Problem

Consider the problem of an infinitely lived firm that invests in capital K_t subjects to an adjustment cost. Time is continuous. The profit stream is given by

$$\pi_t = f(k_t) - I_t - \phi(I_t)$$

where f obeys Inada conditions and the adjustment cost is convex: $\phi' > 0$ and $\phi'' > 0$. $\phi(0) = 0$.

The firm maximizes the discounted present value of profits

$$\max_{I_t, K_t; t \geq 0} \int_0^\infty e^{-rt} \pi_t dt$$

subject to the law of motion

$$\dot{k}_t = I_t - \delta k_t$$

1. Derive the necessary conditions for the firm's optimal investment plan, including the TVC.
2. From the necessary conditions, derive the differential equation for I_t.
3. Draw a phase diagram in (k_t, I_t) space. For simplicity, assume that the $\dot{I} = 0$ locus is downward sloping.
4. Discuss the stability properties of the steady state.