Econ720 - TA Session 4

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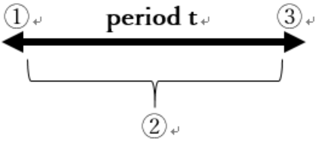
1. Money in OLG Models

Model Setup

Timing:



1. At the beginning of t:
   Each young: goods e_1
   Each old: goods e_2
   money $\frac{M_t}{N_{t-1}}$

1. Money in OLG Models

Why does each old carry $\frac{M_t}{N_{t-1}}$ units of money?

The total amount of money carried by the old when entering period t is M_{t-1}. In period t, government prints additional money in proportion to the current money holdings. Hence the amount of newly printed money in period t is θM_{t-1}. These newly printed money is handed to the old. Hence the total amount of money carried by the old in period t is $M_{t-1} + \theta M_{t-1} = M_t$. Hence each old carries $\frac{M_t}{N_{t-1}}$ units of money.

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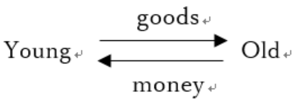
2. During t:



3. At the end of t:
The old give all their money to the young in exchange for goods.

* Notice that in our model setup, we assume that agent's utility only comes from consuming goods, not from holding money (Money In Utility model, PS2-Q1). The old will die at the end of period t. Hence in order to maximize their utility, the old will sell all their money to the young in exchange for goods.

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Translate the setup of timing into equations:

$$\text{Young: } p_t c_t^y + Money_t = p_t e_1$$
$$\text{Old: } p_{t+1} c_{t+1}^0 = p_{t+1} e_2 + Money_t + \theta\, Money_t$$

Rewrite BC in real terms:

$$\text{Young: } c_t^y + \frac{Money_t}{p_t} = e_1 \;\rightarrow\; c_t^y + x_t = e_1$$
$$\text{Old: } c_{t+1}^0 = e_2 + (1+\theta)\frac{Money_t}{p_{t+1}} \;\rightarrow\; c_{t+1}^0 = e_2 + (1+\theta) x_t \frac{p_t}{p_{t+1}}$$

Household problem:

$$\begin{aligned}
&max\ u(c_t^y,\ c_{t+1}^o) \\
&s.t.\ \ c_t^y + x_t = e_1 \\
&\qquad c_{t+1}^0 = e_2 + (1+\theta)x_t\frac{p_t}{p_{t+1}} \\
&\Rightarrow \text{E.E. } u_1(c_t^y,\ c_{t+1}^o) = (1+\theta)\frac{p_t}{p_{t+1}}u_2(c_t^y,\ c_{t+1}^o)
\end{aligned}$$

Define $R_{t+1} \equiv (1+\theta)\frac{p_t}{p_{t+1}}$. R_{t+1} is the real rate of return from holding money.

Tip: real rate of return can be derived by using E.E.

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Competitive equilibrium:

Allocations $\{c_t^y, c_{t+1}^o, x_t, M_t\}$ and prices $\{R_{t+1}, p_t\}$ that satisfy

- Household problem: E.E. (1), BC (2);
- Government: $M_{t+1} = (1+\theta)M_t$;
- Market clearing condition:
  Goods market: $N_t c_t^y + N_{t-1} c_t^o = N_t e_1 + N_{t-1} e_2$
  Money market: $M_t = x_t P_t N_t$
- Accounting identity: $R_{t+1} = (1+\theta)\frac{p_t}{p_{t+1}}$

6 objects, 7 equations

Law of motion of $m_t = \frac{M_t}{P_t N_t}$:

In this model, there are two state variables - M and p. But nominal variables play no role in our economy. Hence we only need to worry about real money balances per capita as the state variable.

$$m_t = s((1+n)\frac{m_{t+1}}{m_t})$$

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$$m_t = s((1+n)\frac{m_{t+1}}{m_t})$$

This is a difference equation that completely determines the path for m (together with a boundary condition). But what does it look like?

$\Rightarrow$ Use offer curve!

Notice that the steady state is unstable.

Dynamics:

- If the offer curve is not backward bending,
  $m_0 > m_{ss}$: m_t will explode;
  $m_0 < m_{ss}$: m_t will converge to 0
  the economy will never reach steady state.
- If the offer curve is backward bending, the economy may reach steady state. The dynamics is complex.

Dynamic efficiency:

Non-monetary interest rate $\geq 1+n$: dynamic efficient

Non-monetary interest rate $< 1+n$: dynamic inefficient

Starting from a dynamically inefficient non-monetary economy (Samuelson case), giving people money results in a dynamically efficient steady state (if money is ever valued and if the steady state is ever reached). But if the economy is dynamically efficient (Classical case), money would never be valued at equilibrium.

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Fiscal theory of the price level:

- Model without government spending: money is valued in equilibrium only in an economy that would be dynamically inefficient without money.
- Model with government spending: there is no non-monetary equilibria

  In this model, we assume that government issues money to cover its spending. It is the government's expenditure plan that gives agents the belief that money has value. Hence, with government spending, money is always valued in equilibrium.