Econ720 - TA Session 2

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1. Recap: how to set up a competitive equilibrium?

1. Read the question carefully and find how many sectors there are
2. Solve each sector's problem (e.g. Household, Firm)
   - Choice variables
   - Prices
   - Rewrite HH BC in real terms if it is in unit of accounts
3. State the market clearing condition
4. Define the equilibrium

   Allocations { ... } and prices { ... } that satisfy

$$\text{Optimality conditions for each sector} \begin{cases} \text{Household problem} \\ \text{Firm problem} \\ \dots \end{cases}$$

   Market clearing condition
   Accounting identity

N objects, N+1 equations (Walras' Law)

2. Example: OLG with Capital, Land and Bonds

Model

Consider a standard two-period overlapping generations model with the following characteristics:

Demographics

- Each period a cohort of size $N_t = 1$ are born. Each cohort lives for two periods.
  $\rightarrow$ 2 periods: c_t^y, c_{t+1}^o; No population growth
- All cohorts are identical and behave competitively.
  $\rightarrow$ WLOG, we may consider representative households.

2. Example: OLG with Capital, Land and Bonds

Endowments and Preferences

- Each young cohort is endowed with 1 unit of labor
- At $t = 0$, the old cohort is endowed with k_0 units of capital and x_0 units of land.
- Each cohort born in generic period t maximizes the following utility function:

$$U = u(c_t^y) + \beta u(c_{t+1}^o)$$

where c_t^y and c_{t+1}^o represent consumption when young and old respectively and the utility function $u(\cdot)$ satisfies the usual conditions.

$\rightarrow$ Utility only comes from consumption. So household supplies all their labor endowment, so that they can have more income, thus to support more consumption.

$\rightarrow L_t = N_t = 1$

2. Example: OLG with Capital, Land and Bonds

Technology

- Capital k_t, land x_t, and bonds b_t can be traded among households in spot markets. Bonds can be stored intertemporally costlessly.
  No depreciation on bond.
- Capital and consumption goods can be freely transformed one to another (one-to-one)
  $\rightarrow$ k_t and c_t have the same price. Hence if the price of c_t is normalized to 1, the price of capital is also 1.
- Land is available in fixed apply. (Additional land above x_0 cannot be accumulated)
  $\rightarrow$ The total amount of land is always x_0.
- Firms are identical and perfectly competitive.
  $\rightarrow$ Firms are price taker.

2. Example: OLG with Capital, Land and Bonds

- Firms **rent capital and land from old households and labor (L_t) from young households** to produce a final good with the following production function:

$$y_t = f(K_t,\ X_t,\ L_t)$$

  where $f(\cdot)$ satisfies the usual Inada conditions and y_t is in units of consumption.
- Capital depreciates after use at rate $0 \leq \delta \leq 1$. Land does not depreciate (Land is a durable good.)

2. Example: OLG with Capital, Land and Bonds

Markets

- Bonds are issued by households with interest rate R_{t+1} (in units of account) and have a one-period maturity.
  $\rightarrow$ A nominal monetary unit of measure
  $\rightarrow$ Buy bonds in current period, get returns in the next period.
- Capital may be traded at price P_t^k and rented to firms at rate R_t^k (in units of account)
- Land may be traded at price P_t^x and rented to firms at rate R_t^x (in units of account)
- Consumption goods may be traded at price P_t^c
- Goods market must hold for consumption and capital
  $\rightarrow$ Goods produced in each period is used for consumption and capital accumulation.

  $\rightarrow$ Goods market clearing condition:

$$y_t = C_t + I_t = C_t + K_{t+1} - (1-\delta)K_t$$

2. Example: OLG with Capital, Land and Bonds

Questions

1. What are the representative household's budget constraint in each period?
2. Have we defined a numeraire yet? If not, let's do so.
3. What is the representative household's lifetime budget constraint?
4. Write down and solve the representative household's problem
5. Write down and solve the firm's problem
6. Define a competitive equilibrium

2. Example: OLG with Capital, Land and Bonds

1. What are the representative household's budget constraint in each period?

   $(c_t^y,\ c_{t+1}^o,\ k_t,\ x_t,\ b_t)$
   Key idea: Income = Consumption + Saving

Young:

$$W_t = P_t^c c_t^y + P_t^k k_t + P_t^x x_t + P_t^c b_t$$

Old:

$$R_{t+1}^k k_t + P_{t+1}^k (1-\delta) k_t + R_{t+1}^x x_t + P_{t+1}^x x_t + R_{t+1} b_t + P_{t+1}^c b_t = P_{t+1}^c c_{t+1}^o$$

2. Example: OLG with Capital, Land and Bonds

2. Have we defined a numeraire yet? If not, let's do so.

The budget constraints are in nominal units. But for most cases, it is more convenient to deal with real units.

$\Rightarrow$ We numerate the price of consumption!

$\Rightarrow$ Recall that we can define a numeraire in each period's budget constraint.

Young: $(P_t^c = 1)$

$$w_t = c_t^y + k_t + p_t^x x_t + b_t$$

Old: $(P_{t+1}^c = 1)$

$$r_{t+1}^k k_t + (1-\delta)k_t + r_{t+1}^x x_t + p_{t+1}^x x_t + r_{t+1} b_t + b_t = c_{t+1}^o$$

3. What is the representative household's lifetime budget constraint?

Substitute out b_t

$$\begin{aligned} w_t = {} & c_t^y + p_t^x x_t + k_t \\ & + \frac{1}{1+r_{t+1}}[c_{t+1}^o - (p_{t+1}^x + r_{t+1}^x)x_t - (1-\delta+r_{t+1}^k)k_t] \end{aligned}$$

4. Write down and solve the representative household's problem

$$
\begin{aligned}
max \quad & u(c_t^y) + \beta u(c_{t+1}^o) \\
s.t. \quad & w_t = c_t^y + k_t + p_t^x x_t + b_t \\
& r_{t+1}^k k_t + (1-\delta) k_t + r_{t+1}^x x_t + p_{t+1}^x x_t + r_{t+1} b_t + b_t = c_{t+1}^o
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L} = {} & u(c_t^y) + \beta u(c_{t+1}^o) + \lambda \{ w_t - c_t^y - p_t^x x_t - k_t \\
& - \frac{1}{1+r_{t+1}} [c_{t+1}^o - (p_{t+1}^x + r_{t+1}^x) x_t - (1-\delta + r_{t+1}^k) k_t] \}
\end{aligned}
$$

2. Example: OLG with Capital, Land and Bonds

$$[c_t^y] : u'(c_t^y) = \lambda$$

$$[c_{t+1}^o] : \beta u'(c_{t+1}^o) = \frac{\lambda}{1 + r_{t+1}}$$

$$[x_t] : \frac{1}{1 + r_{t+1}}(p_{t+1}^x + r_{t+1}^x) = p_t^x$$

$$[k_t] : \frac{1}{1 + r_{t+1}}(1 - \delta + r_{t+1}^k) = 1$$

$\Rightarrow$

E.E. for bonds: $u'(c_t^y) = \beta u'(c_{t+1}^o)(1 + r_{t+1})$

E.E. for land: $u'(c_t^y) = \beta u'(c_{t+1}^o)\dfrac{p_{t+1}^x + r_{t+1}^x}{p_t^x}$

E.E. for capital: $u'(c_t^y) = \beta u'(c_{t+1}^o)(1 - \delta + r_{t+1}^k)$

INTERPRETATION!!

2. Example: OLG with Capital, Land and Bonds

Solution to household problem is a vector

$$\{c_t^y, c_{t+1}^o, k_t, x_t, b_t\}$$

that satisfies

1. 2 BCs
2. 3 EEs

5. Write down and solve the firm's problem

$$
\begin{aligned}
& max \quad P_t^c y_t - R_t^k K_t - W_t L_t - R_t^x X_t \\
\rightarrow & max \quad f(K_t,\ L_t,\ X_t) - r_t^k K_t - w_t L_t - r_t^x X_t \\
& [K_t] : f_K = r_t^k \\
& [L_t] : f_L = w_t \\
& [X_t] : f_X = r_t^x
\end{aligned}
$$

2. Example: OLG with Capital, Land and Bonds

6. Define a competitive equilibrium

Allocations $\{c_t^y, c_t^o, k_t, x_t, b_t, K_t, L_t, X_t\}$ and prices $\{p_t^x, r_t, r_t^k, r_t^x, w_t\}$ that satisfy

- H.H. Problem: B.C.(2), FOC(3);
- Firm Problem: FOC (3);
- Market Clearing Conditions:
  Goods market:
  $f(K_t,\ L_t,\ X_t) = N_t c_t^y + N_{t-1} c_t^o + K_{t+1} - (1-\delta)K_t$
  Capital rental market: $K_t = N_{t-1}k_{t-1} = k_{t-1}$
  Land rental market: $X_t = N_{t-1}x_{t-1} = x_0$
  Labor rental market: $L_t = N_t = 1$
  Bonds market: $b_t = 0$
- Accounting Identity: $1 + r_{t+1} = 1 - \delta + r_{t+1}^k = \frac{p_{t+1}^x + r_{t+1}^x}{p_t^x}$