Econ720 - TA Session 11

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Questions:

1. Solve the problem of the final goods producer.

Answer key:

- Final goods sector: perfect competition
- Goods producers are price takers

Normalize final goods price to be 1

$$\max_{L_t,\{x_{jt}\}_{j=0}^{N_t}} (1-\beta)^{-1} L_t^{\beta} \int_0^{N_t} x_{jt}^{1-\beta} dj - w_t L_t - \int_0^{N_t} p_{jt} x_{jt} dj$$

$$[L_t]: \quad \beta \frac{Y_t}{L_t} = w_t$$

$$[x_{jt}]: \quad L_t^{\beta} x_{jt}^{-\beta} = p_{jt}$$

Hence the demand function for monopolistic inputs is $x_{jt}^m = L_t (p_{jt}^m)^{-\frac{1}{\beta}}$

The demand function for competitive inputs is $x_{jt}^c = L_t (p_{jt}^c)^{-\frac{1}{\beta}}$

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2. Solve the problem of a monopolist intermediate input producer.

Answer key:

Monopolist intermediates producer j maximizes the present value of her profit stream

$$V_j = \int_s^\infty e^{-r(t-s)} e^{-\delta(t-s)} \pi_{j,t}^m dt$$

However, since this is a static problem, it is equivalent to maximizing the instantaneous profit.

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Monopolist intermediates producer j maximizes her profit by choosing price, given demand function (derived from the final good producer's problem).

$$\max_{p_{jt}^m} \pi_{jt}^m \rightarrow \max_{p_{jt}^m} p_{jt}^m x_{jt}^m - \psi x_{jt}^m$$

$$\max_{p_{jt}^m} p_{jt}^m L_t (p_{jt}^m)^{-\frac{1}{\beta}} - \psi L_t (p_{jt}^m)^{-\frac{1}{\beta}}$$

From the FOC, $p_{jt}^m = \frac{\psi}{1-\beta}$.

Hence $x_{jt}^m = L_t (\frac{\psi}{1-\beta})^{-\frac{1}{\beta}}$ and $\pi_{jt}^m = \beta L_t (\frac{\psi}{1-\beta})^{1-\frac{1}{\beta}}$

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Discussion:

There are two types of intermediate producers!!!

- Monopoly

$$p_{jt}^{m} = \frac{\psi}{1-\beta} \quad \rightarrow \quad x_{jt}^{m} = L_t\left(\frac{\psi}{1-\beta}\right)^{-\frac{1}{\beta}}$$

- Perfect competition
  price=MC, which means $p_{jt}^{c} = \psi$
  Plug price into demand function, $x_{jt}^{c} = L_t \psi^{-\frac{1}{\beta}}$

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3. Solve for equilibrium interest rate.

Discussion:

- Equilibrium interest rate is derived from free entry condition.
- Free entry condition means: *cost = benefit*
- The cost of creating a new type of intermediate is $1/\eta$ units of final goods

$$\text{Cost} = 1/\eta \cdot price_{final\ goods} = 1/\eta$$

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Answer Key:

- Benefit of creating a new type of intermediate is

$$
\begin{aligned}
V &= \int_s^\infty e^{-r(t-s)} e^{-\delta(t-s)} \pi_t^m dt \\
&= \int_s^\infty e^{-(r+\delta)(t-s)} \beta L\left(\frac{\psi}{1-\beta}\right)^{1-\frac{1}{\beta}} dt \\
&= \beta L\left(\frac{\psi}{1-\beta}\right)^{1-\frac{1}{\beta}} \frac{1}{r+\delta}
\end{aligned}
$$

- Free entry condition: $1/\eta = V$

$$
r = \beta L\left(\frac{\psi}{1-\beta}\right)^{1-\frac{1}{\beta}} \eta - \delta
$$

Note that we are applying the equilibrium labor market clearing condition $L_t = 1$ to get rid of the time subscript of L.

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4. Derive equilibrium growth rate. Which patent duration δ maximizes growth? Does this also maximize welfare?

Answer key:

EE can be obtained by solving HH problem.

$$g = \frac{\dot{C}_t}{C_t} = \frac{r-\rho}{\theta} = \frac{\beta L(\frac{\psi}{1-\beta})^{1-\frac{1}{\beta}}\eta - \delta - \rho}{\theta}$$

Since $\delta \geq 0$, $\delta = 0$ will maximize growth rate. And it doesn't maximize welfare.

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Reason:

- When $\delta = 0$, intermediate goods market is monopolistic. The monopolists will set higher price to get more profit, which leads to inefficiency. This can be formally proved using a planner problem.

Planner's Problem

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5. Consider the balanced growth path. Show that $\frac{N_1}{N_2} = \frac{g}{\delta}$

Idea:

- Step-1. Prove $g_{N_2} = g_N$ using the law of motion for N_2
- Step-2. Prove $g_N = g_Y$ using production function
- Step-3. Prove $g_N = g_C$ using resource constraint

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Answer Key:

- Step-1. Prove $g_{N_2} = g_N$ using the law of motion for N_2

$$\dot{N}_{2t} = \delta N_{1t} = \delta(N_t - N_{2t})$$
$$g_{N_2} = \frac{\dot{N}_{2t}}{N_{2t}} = \delta(\frac{N_t}{N_{2t}} - 1)$$

On BGP, since g_{N_2} is constant, $\frac{N_t}{N_{2t}}$ must be constant.
Hence, on BGP, $g_{N_2} = g_N$

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Answer Key:

- Step-2. Prove $g_N = g_Y$ using production function

$$
\begin{aligned}
Y_t &= (1-\beta)^{-1} L^{\beta} \int_0^{N_t} x_{jt}^{1-\beta} dj \\
&= (1-\beta)^{-1} L^{\beta} \left\{ \int_0^{N_{1t}} (x_{jt}^{m})^{1-\beta} dj + \int_0^{N_{2t}} (x_{jt}^{c})^{1-\beta} dj \right\} \\
&= (1-\beta)^{-1} L^{\beta} \left\{ N_{1t} (L(\frac{\psi}{1-\beta})^{-\frac{1}{\beta}})^{1-\beta} + N_{2t} (L\psi^{-\frac{1}{\beta}})^{1-\beta} \right\} \\
&= \underbrace{\frac{L^{\beta}}{1-\beta} (L(\frac{\psi}{1-\beta})^{-\frac{1}{\beta}})^{1-\beta}}_{\text{constant, } \Omega_1} N_{1t} + \underbrace{\frac{L^{\beta}}{1-\beta} (L\psi^{-\frac{1}{\beta}})^{1-\beta}}_{\text{constant, } \Omega_2} N_{2t} \\
&= \Omega_1 (N_t - N_{2t}) + \Omega_1 N_{2t}
\end{aligned}
$$

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$$\frac{Y_t}{N_t} = \Omega_1 + (\Omega_2 - \Omega_1)\frac{N_{2t}}{N_t}$$

On BGP, since $\frac{N_{2t}}{N_t}$ is constant, $\frac{Y_t}{N_t}$ is constant.

Hence, on BGP, $g_N = g_Y$

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Answer Key:

- Step-3. Prove $g_N = g_C$ using resource constraint

$$
\begin{aligned}
Y_t &= C_t + X_t + Z_t \\
&= C_t + \left\{ \int_0^{N_{1t}} \psi x_{jt}^m dj + \int_0^{N_{2t}} \psi x_{jt}^c dj \right\} + \eta \dot{N}_t \\
&= C_t + \left\{ \underbrace{\Omega_3}_{\text{constant}} N_{1t} + \underbrace{\Omega_4}_{\text{constant}} N_{2t} \right\} + \eta \dot{N}_t \\
&= C_t + \Omega_3 N_t + (\Omega_4 - \Omega_3) N_{2t} + \eta \dot{N}_t
\end{aligned}
$$

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$$\frac{Y_t}{N_t} = \frac{C_t}{N_t} + \Omega_3 + (\Omega_4 - \Omega_3)\frac{N_{2t}}{N_t} + \eta\frac{\dot{N}_t}{N_t}$$

On BGP, $\frac{\dot{N}_t}{N_t}$, $\frac{N_{2t}}{N_t}$, $\frac{Y_t}{N_t}$ are constant.

Hence $\frac{C_t}{N_t}$ is constant, which means $g_N = g_C = g$

Hence on BGP, $g_{N_2} = g_N = g$

Due to law of motion of N_2, $\dot{N}_{2t} = \delta N_{1t}$

$$g = \frac{\dot{N}_{2t}}{N_{2t}} = \delta\frac{N_{1t}}{N_{2t}} \quad \rightarrow \quad \frac{N_{1t}}{N_{2t}} = \frac{g}{\delta}$$

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6. Define a competitive equilibrium.

Answer key:

$\{C_t, L_t, A_t, Y_t, x_{jt}^m, x_{jt}^c, \pi_{jt}^m, V_j, N_{1t}, N_{2t}, Z_t\}$ and $\{p_{jt}^m, p_{jt}^c, w_t, r_t\}$

- Household: EE, BC
- Final good producer: FOC for labor, FOC for competitive intermediates, FOC for monopolistic intermediates, definition for Y_t
- Monopoly intermediate producer: p_{jt}^m, x_{jt}^m, π_{jt}^m
- Competitive intermediate producer: x_{jt}^c
- R&D sector: free entry condition
- Market clearing: RC, intermediates, labor, asset
- Law of motion for N_{2t}

Planner's Problem

Discussion: How to find Pareto efficient allocation? How to solve welfare maximization problem?

$\Rightarrow$ Solve the social planner's problem!

Objective:

- HHs are different (e.g. OLG model): need to think about the weight
- Representative HH, infinite horizon: same as HH's problem

Constraint: RC

Example: simple representative HH, infinite horizon model

$$\max_{C_t, K_{t+1}} \sum_{t=0}^{\infty} u(C_t)$$

$$s.t. Y_t = C_t + \underbrace{K_{t+1} - (1-\delta)K_t}_{I_t}$$

Planner's Problem in VE Model

The planner's problem in variety expansion model is complicated. Why?

$$\text{RC: } Y_t = C_t + X_t + Z_t$$

Deciding investment Z_t doesn't automatically give us consumption. Need to decide X_t first.

- Step 1. Given N, find the optimal X_t, get the net output.
- Step 2. Given the net output ($\tilde{Y}_t = Y_t - X_t$), decide consumption and investment.

Planner's Problem in VE Model

- Step 1. Given N, find the optimal X_t, get the net output.

$$\max_{\{x_{jt}\}_{j=0}^{N}} (1-\beta)^{-1} L^{\beta} \int_0^N x_{jt}^{1-\beta} dj - \int_0^N \psi x_{jt} dj$$

$$[x_{jt}]: \quad L^{\beta} x_{jt}^{-\beta} = \psi$$

$$X_t = \int_0^N \psi x_{jt} dj = \int_0^N \psi \frac{L}{\psi^{1/\beta}} dj = NL\psi^{1-\frac{1}{\beta}}$$

$$Y_t = (1-\beta)^{-1} L^{\beta} \int_0^N x_{jt}^{1-\beta} dj = \frac{1}{1-\beta} LN\psi^{1-\frac{1}{\beta}}$$

$$\tilde{Y}_t = Y_t - X_t = \frac{1}{1-\beta} LN\psi^{1-\frac{1}{\beta}} - NL\psi^{1-\frac{1}{\beta}} = \frac{\beta}{1-\beta} LN\psi^{1-\frac{1}{\beta}}$$

Planner's Problem in VE Model

- Step 2. Given the net output ($\tilde{Y}_t = Y_t - X_t$), decide consumption and investment.

$$\max_{C_t} \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt$$

$$s.t. \quad \dot{N}_t = \eta Z_t$$

$$\tilde{Y}_t = \frac{\beta}{1-\beta} L N_t \psi^{1-\frac{1}{\beta}} = C_t + Z_t$$

Current value Hamiltonian

$$H = \frac{C_t^{1-\theta} - 1}{1-\theta} + \lambda_t [\eta (\frac{\beta}{1-\beta} L N_t \psi^{1-\frac{1}{\beta}} - C_t)]$$

Planner's Problem in VE Model

$$H = \frac{C_t^{1-\theta}-1}{1-\theta} + \lambda_t[\eta(\frac{\beta}{1-\beta}LN_t\psi^{1-\frac{1}{\beta}} - C_t)]$$

$$[C_t]: \quad C_t^{-\theta} = \lambda_t\eta \quad \rightarrow \quad -\theta C_t^{-\theta-1}\dot{C}_t = \dot{\lambda}_t\eta$$

$$[N_t]: \quad \lambda_t\eta\frac{\beta}{1-\beta}L\psi^{1-\frac{1}{\beta}} = -\dot{\lambda}_t + \rho\lambda_t$$

Therefore

$$g(C) = \frac{\dot{C}_t}{C_t} = \frac{\eta\frac{\beta}{1-\beta}L\psi^{1-\frac{1}{\beta}} - \rho}{\theta}$$

which is faster than the maximized consumption growth rate in CE

$$g^{CE}(C) = \frac{\beta L(\frac{\psi}{1-\beta})^{1-\frac{1}{\beta}}\eta - \rho}{\theta}$$

Announcement

Office Hours

- Nov 22, 2-3 pm
  https://unc.zoom.us/j/93649484998
- Nov 29, 11 am - noon
  https://unc.zoom.us/j/99980577757
- Dec 6, 2-5 pm
  https://unc.zoom.us/j/94537204029