Econ720 - TA Session 1

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0. Macro is saying hi to you!

Office Hour:

Thursday, 2:30 - 3:30, Zoom

- Link: `https://unc.zoom.us/j/92880780119`
- Appointment by email: `yanran@ad.unc.edu`

How to learn Macro?

- Go through the slides and think about the model. Think hard!
- Solve the model without looking at the slides
- Review Question

0. Macro is saying hi to you!

Today's Task:

- How to define competitive equilibrium (On the Next: a detailed OLG example)
- Walras Law
- Arrow-Debreu and Sequential Trading

1. How to set up a competitive equilibrium?

1. Describe the economy (**Find how many sectors there are**)
2. Solve each sector's problem (e.g. Household, Firm, etc.)
   endogenous/ choice variables
   price variables

   Example

$$max\ u(c_1, c_2)$$
$$s.t.\ p_1 c_1 + p_2 c_2 = p_1 e_1 + p_2 e_2$$

Choice variables?
Written in real terms: $c_1 + pc_2 = e_1 + pe_2$, where $p = \frac{p_2}{p_1}$

1. How to set up a competitive equilibrium?

2. Solve each sector's problem (e.g. Household)

$$\max \; ln(c_1) + \beta ln(c_2)$$
$$s.t. \; c_1 + pc_2 = e_1 + pe_2$$

- Set up Lagrangean
- Get FOCs by taking derivative with respect to all choice variables in the Lagrangean.

1. How to set up a competitive equilibrium?

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- Set up Lagrangean
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$$\mathscr{L} = ln(c_1) + \beta \, ln(c_2) + \lambda(e_1 + pe_2 - c_1 - pc_2)$$

1. How to set up a competitive equilibrium?

2. Solve each sector's problem (e.g. Household)

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1. How to set up a competitive equilibrium?

3. State the market clearing condition
   - How to find markets?
     $\rightarrow$ Start with choice variables
   - How to write market clearing conditions?
     $\rightarrow$ Aggregate supply $=$ Aggregate demand
4. Define the equilibrium
   Allocations { ... } and prices { ... } that satisfy

$$\text{Optimality conditions for each sector} \begin{cases} \text{Household problem} \\ \text{Firm problem} \\ \ldots \end{cases}$$

Market clearing condition
Accounting identity

N objects, N+1 equations (Walras' Law)

2. Walras' Law

Define the equilibrium

Allocations $\{c_1, c_2\}$ and prices $\{p\}$ that satisfy

- Optimality conditions for household
  1. FOC: $\beta \frac{c_1}{c_2} = p$
  2. BC: $c_1 + pc_2 = e_1 + pe_2$
- Market clearing condition
  3. $c_1 = e_1$
  4. $c_2 = e_2$

3 objects, 4 equations (Walras' Law)

Linear combination exists in these 4 equations: equation 2 is a linear combination of equation 3 and 4

3. Arrow-Debreu vs. Sequential Trading

Two-period Example

Demographics: N identical household live for 2 periods, $t = 1,\ 2$

Commodities: $c_1,\ c_2$

Preference: $u(c_1,\ c_2)$

Endowments: $e_1,\ e_2$

3. Arrow-Debreu vs. Sequential Trading

Two-period $t = 1,\ 2$

- c_1: consumption in period 1
- c_2: consumption in period 2

Goods in different periods can be considered different goods

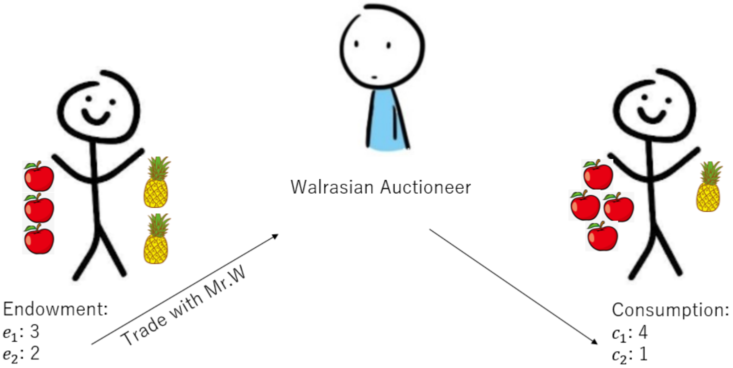
- c_1: apple
- c_2: pineapple

For each different goods there is a corresponding market.

3. Arrow-Debreu vs. Sequential Trading

(1). Arrow-Debreu Trading

All trades take place at $t = 1$



(1). Arrow-Debreu Trading

All trades take place at $t = 1 \Rightarrow$ Only one BC!!
BC: $p_1 c_1 + p_2 c_2 = p_1 e_1 + p_2 e_2$

$$max \ u(c_1, c_2)$$
$$s.t. \ c_1 + pc_2 = e_1 + pe_2 \text{ (BC here is written in real terms.)}$$
$$\mathscr{L} = u(c_1, c_2) + \lambda(e_1 + pe_2 - c_1 - pc_2)$$
$$\therefore \frac{u_1}{u_2} = \frac{1}{p}$$

Competitive Equilibrium

Allocations { } and price { } that satisfy
Household problem solution:
Goods market clearing conditions:

(1). Arrow-Debreu Trading

All trades take place at $t = 1 \Rightarrow$ Only one BC!!
BC: $p_1 c_1 + p_2 c_2 = p_1 e_1 + p_2 e_2$

$$max \;\; u(c_1, c_2)$$

$s.t. \;\; c_1 + pc_2 = e_1 + pe_2$ (BC here is written in real terms.)

$$\mathscr{L} = u(c_1, c_2) + \lambda(e_1 + pe_2 - c_1 - pc_2)$$

$$\therefore \frac{u_1}{u_2} = \frac{1}{p}$$

Competitive Equilibrium

Allocations $\{\ c_1,\ c_2\ \}$ and price $\{\ p\ \}$ that satisfy
Household problem solution: F.O.C., BC
Goods market clearing conditions: $c_1 = e_1, \quad c_2 = e_2$

3. Arrow-Debreu vs. Sequential Trading

(2). Sequential Trading

Markets open in each period $\Rightarrow$ For each period,there is a BC!!

Budget Constraint:

$$\begin{aligned} p_1 c_1 + (\;) &= p_1 e_1 \\ p_2 c_2 &= p_2 e_2 + (\;) \end{aligned}$$

$\Rightarrow$ We need assets to transfer resources between periods!

$$\begin{aligned} p_1 c_1 + Qb &= p_1 e_1 \\ p_2 c_2 &= p_2 e_2 + p_2 b \end{aligned}$$

Choose one price to normalize in one BC.

Normalize the price of good 1 in the first BC: $c_1 + qb = e_1$ where $q = \frac{Q}{p_1}$

Normalize the price of good 2 in the second BC: $c_2 = e_2 + b$

(2). Sequential Trading

$$
\begin{aligned}
& max\ \ u(c_1, c_2) \\
& s.t.\ \ c_1 + qb = e_1 \\
& \qquad\quad c_2 = e_2 + b \\
& \mathscr{L} = u(c_1, c_2) + \lambda(e_1 + qe_2 - c_1 - qc_2) \\
& \therefore \frac{u_1}{u_2} = \frac{1}{q}
\end{aligned}
$$

Competitive Equilibrium

Allocations { } and price { } that satisfy

Household problem solution:

Market clearing conditions:

3. Arrow-Debreu vs. Sequential Trading

(2). Sequential Trading

$$
\begin{aligned}
& max \;\; u(c_1, c_2) \\
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& \therefore \frac{u_1}{u_2} = \frac{1}{q}
\end{aligned}
$$

Competitive Equilibrium

Allocations $\{ \; c_1, \; c_2, \; b \; \}$ and price $\{ \; q \; \}$ that satisfy

Household problem solution: F.O.C., BC

Market clearing conditions:

Goods market: $c_1 = e_1, \quad c_2 = e_2$

Bonds market: $b = 0$

4. Why $b_t = 0$?

In equilibrium, the bond market clearing condition is $b_t = 0$

Recall the fundamental rule for market clearing condition:
Aggregate supply = Aggregate demand

$\rightarrow$ Who supplies bonds? Household!
Who demands bonds? Household!

$\rightarrow$ The model setup assumes **Representative Agent**, indicating that we can consider the model as if there was only **A SINGLE HOUSEHOLD** in this economy.

4. Why $b_t = 0$?

Could there be cases where $b_t \neq 0$ in equilibrium?

Yes!

e.g. Government issuing bonds

Heterogeneous agents (See PS1-Question 2)