- Definition: Intertemporal elasticity of substitution.

$$\sigma(C_t) = -\frac{u''(C_t)\, C_t}{u'(C_t)}$$

- When using CRRA utility function $u(C_t) = \frac{C_t^{1-\theta} - 1}{1-\theta}$, intertemporal elasticity of substitution is constant.

Proof: $u'(C_t) = C_t^{-\theta}$, $u''(C_t) = -\theta\, C_t^{-\theta-1}$

$$\therefore \sigma(C_t) = -\frac{u''(C_t)\, C_t}{u'(C_t)} = \theta$$

- Application.

$$\max \int_0^\infty e^{-\rho t}\, u(C_t)\, dt$$

$$\text{s.t.} \quad \dot{k}_t = f(k_t) - \delta k_t - C_t$$

Current value Hamiltonian:

$$\mathcal{H} = u(C_t) = \lambda_t \left[f(k_t) - \delta k_t - C_t \right]$$

FOC:

$$[C_t] \quad u'(C_t) = \lambda_t \quad \Rightarrow \quad u''(C_t)\, \dot{C}_t = \dot{\lambda}_t$$

$$[k_t] \quad \lambda_t \left[f'(k_t) - \delta \right] = -\dot{\lambda}_t + \rho \lambda_t$$

$$\therefore u'(C_t)\left[f'(k_t) - \delta \right] = -u''(C_t)\, \dot{C}_t + \rho\, u'(C_t)$$

$$\therefore \frac{u''(C_t)}{u'(C_t)}\, \dot{C}_t = \rho + \delta - f'(k_t)$$

$$= -\sigma(C_t)$$

$$\therefore \boxed{\frac{u''(C_t)\, C_t}{u'(C_t)}}\, \frac{\dot{C}_t}{C_t} = \rho + \delta - f'(k_t)$$

$$\therefore \frac{\dot{C}_t}{C_t} = \frac{f'(k_t) - (\rho + \delta)}{\sigma(C_t)}$$