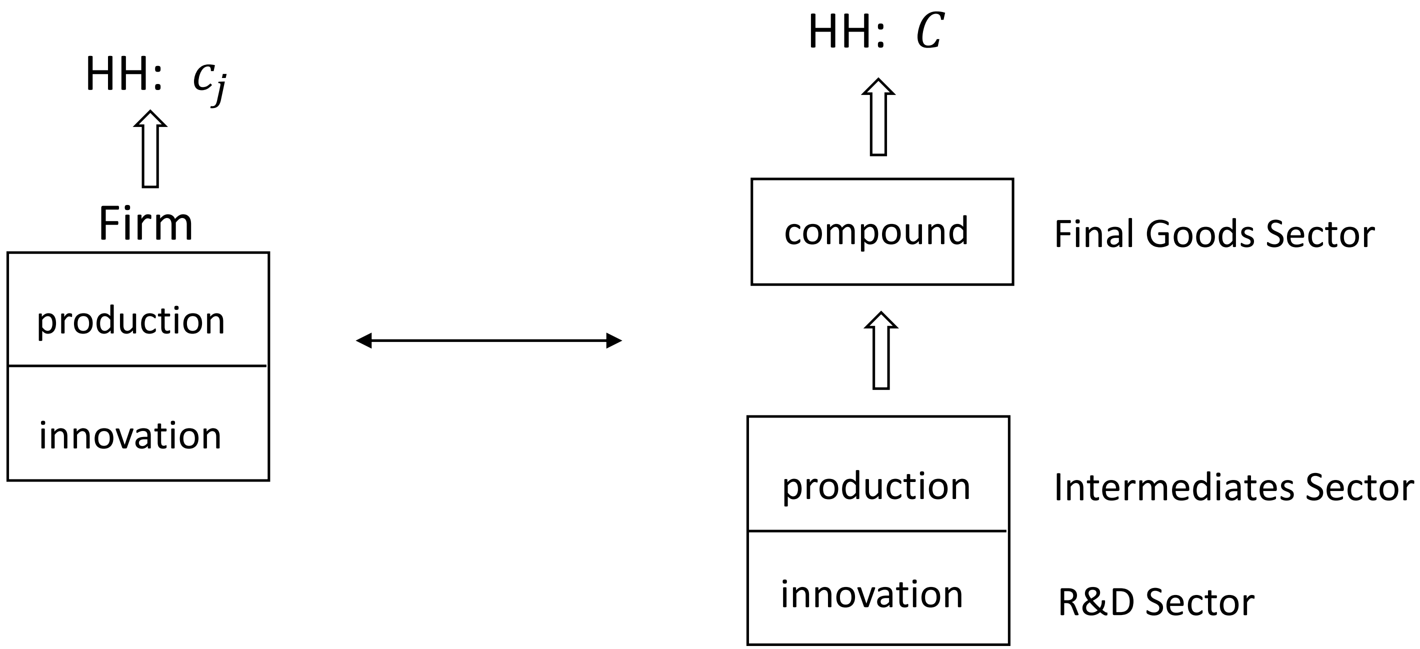
Econ 720 Recitation #11

Date: 11/12/2021

TA: Yanran Guo

## Endogenous Growth Model



## A Brief Summary of Endogenous Growth Model

| Problem of Each Sector | Variety Expansion Model | Quality Ladder Model |
|---|---|---|
| ① Household Sector<br>⇒ Get EE which shows consumption growth rate | $\max \int_0^\infty e^{-\rho t}u(c_t)dt$<br>Complicated BC or a reduced form $\dot{a}_t = r_t a_t + w_t - c_t + \Pi_t$<br>$g(c_t) = \frac{r_t - \rho}{\sigma(c_t)}$ | |
| ② Final Goods Sector (**Perfect Competition**)<br>Profit maximization given factor prices<br>⇒ Find the optimal demand for $x_{jt}$ and $L_t$<br><br>! Notice that final goods price is normalized to 1 | Production function: $Y_t = (1-\beta)^{-1}(\int_0^{N_t} x_{jt}{}^{1-\beta}\,dj)L_t^\beta$<br>$\max\ Y_t - \int_0^{N_t} p_{jt}x_{jt}\,dj - w_tL_t$<br>$\overset{FOC}{\Longrightarrow}$ Optimal demand for intermediates and labor input<br>$[x_{jt}]$: $x_{jt}^{-\beta}L_t^\beta = p_{jt}$<br>$[L_t]$: $\frac{\beta}{1-\beta}\left(\int_0^{N_t} x_{jt}{}^{1-\beta}\,dj\right)L_t^{\beta-1} = w_t$ | $Y_t = (1-\beta)^{-1}(\int_0^1 q_{jt}\cdot x_{jt}{}^{1-\beta}\,dj)L_t^\beta$<br>$\max\ Y_t - \int_0^1 p_{jt}x_{jt}\,dj - w_tL_t$<br>$\overset{FOC}{\Longrightarrow}$ Demand for intermediates and labor<br>$[x_{jt}]$: $x_{jt} = (q_{jt}/p_{jt})^{1/\beta}L_t$<br>$[L_t]$: $\beta Y_t/L_t = w_t$ |
| ③ Intermediates Sector (**Monopolistic Competition**)<br>Profit maximization by choosing price<br>⇒ Find the optimal price $p_{jt}$ that maximizes profit<br><br>What's the demand function? What's the cost? | **1 units of good $j$ is produced by $\varphi$ units of final goods**<br>$\max\ \pi_{jt} \Leftrightarrow \max\ p_{jt}x_{jt} - \varphi x_{jt} \Leftrightarrow \max\ (p_{jt} - \varphi)\, L_t p_{jt}^{-1/\beta}$<br>$\overset{FOC}{\Longrightarrow}$ Optimal price $p_{jt} = \frac{\varphi}{1-\beta}$ for $\forall t,\ \forall j$<br><br>Hence, $x_{jt} = L_t(\frac{\varphi}{1-\beta})^{-1/\beta}$ and $\pi_{jt} = L_t(\frac{\varphi}{1-\beta})^{-1/\beta}\frac{\beta\varphi}{1-\beta}$ | **Marginal cost is $\varphi q_{jt}$ units of final goods**<br>$\max\ \pi_{jt} \Leftrightarrow \max\ p_{jt}x_{jt} - \varphi q_{jt}x_{jt}$<br>$\overset{FOC}{\Longrightarrow}$ Optimal price $p_{jt} = \frac{\varphi q_{jt}}{1-\beta}$<br><br>So, $x_{jt} = L_t(\frac{\varphi}{1-\beta})^{-1/\beta}$, $\pi_{jt} = L_t(\frac{\varphi}{1-\beta})^{-1/\beta}\frac{\beta\varphi q_{jt}}{1-\beta}$ |
| ④ R&D Sector<br>We only think about equilibrium<br>In equilibrium, **Free Entry Condition** holds<br>⇒ Get the present value of profit $V_j$ and $r$ | $\dot{N}_t = \eta Z_t$<br>For simplicity, set $\varphi = 1 - \beta$<br>Cost of creating a new variety $= 1/\eta$<br>Benefit of creating a new variety is $V_j = \int_s^\infty e^{-r(t-s)}\pi_{jt}\ dt$<br>Free Entry: cost=benefit ⇒ $\frac{1}{\eta} = V_j$,<br><br>hence $\frac{1}{\eta} = \frac{\beta L}{r}$, therefore $r = \beta L\eta$ | $n_{jt}\Delta t = (\eta/q_{jt})Z_{jt}\Delta t$<br>Innovation takes quality from $q_{jt}$ to $\lambda q_{jt}$<br>Suppose current quality is $q_{jt}/\lambda$<br><br>Free Entry: $q_{jt}/\lambda\eta = V(j,t\|q_{jt})$<br><br>What's the value of $V(j,t\|q_{jt})$? Asset pricing! |