Econ.720 Recitation#8

Example 1. RQ#6 2.3 Capital Adjustment Costs

Consider a version of the standard one sector, neoclassical growth model with no technological progress, inelastic labor supply and zero population growth. We will examine the planner's problem in an economy in which capital is costly to adjust.

The planner maximizes the lifetime utility of the representative household:

$$max \int_0^\infty e^{-\rho t} u(c_t) dt$$

There is one final good which is produced using labor and capital using a production technology which can be written in intensive form as $f(k_t)$, where $f'(\cdot) > 0$, $f''(\cdot) < 0$, and the usual Inada conditions apply.

Capital is costly to adjust however, so the resource constraint is given by

$$f(k_t) = c_t + i_t(1 + T(\frac{i_t}{k_t}))$$

where i_t is investment, and $T(\cdot)$ represents additional costs incurred whenever capital is adjusted. We assume that $T(0) = 0$ and $T'(\cdot) > 0$. Capital depreciates at rate δ, implying that

$$\dot{k}_t = i_t - \delta k_t$$

The initial capital stock, k_0, is given.

Solve the social planner's problem

Example 2. Macroeconomics Qualifying Examination January 2012. An Investment Problem

Consider the problem of an infinitely lived firm that invests in capital K_t subjects to an adjustment cost. Time is continuous. The profit stream is given by

$$\pi_t = f(k_t) - I_t - \phi(I_t)$$

where f obeys Inada conditions and the adjustment cost is convex: $\phi' > 0$ and $\phi'' > 0$. $\phi(0) = 0$.

The firm maximizes the discounted present value of profits

$$\max_{I_t, K_t; t \geq 0} \int_0^\infty e^{-rt} \pi_t dt$$

subject to the law of motion

$$\dot{k_t} = I_t - \delta k_t$$

1. Derive the necessary conditions for the firm's optimal investment plan, including the TVC.

2. From the necessary conditions, derive the differential equation for I_t.

3. Draw a phase diagram in (k_t, I_t) space. For simplicity, assume that the $\dot{I} = 0$ locus is downward sloping.

4. Discuss the stability properties of the steady state.