Econ720 - TA Session 6

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1. Midterm

- Technical tools
  - How to set up the model $\rightarrow$ AD vs. ST
  - How to solve the model $\rightarrow$ sequential language vs. DP
  - How to present the results $\rightarrow$ Define CE vs. RCE
  - How to analyze equilibrium $\rightarrow$ Steady state
- Model: OLG
- Interpretation

2. Dynamic Programming

Two ways to solve the model:

- Sequence language /sequential solution - set up Lagrangean to find sequences of real variables.
- Recursive formulation - use Dynamic Programming (DP) and set up Bellman equation to find a sequence of value functions and policy functions

2. Dynamic Programming

- Define **state variables**
  Variables carried over into the current period from the last period
  Variables that are predetermind in the current period
- Define control variables
- Value function: V(state variables)
- Utility + continuation value

$$\Rightarrow \begin{cases} \text{Bellman equation} \\ \text{FOC} \\ \text{Envelope condition} \\ \text{Euler equation} \\ \text{Law of motion} \end{cases}$$

2. Dynamic Programming - Example

$$max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t. \quad k_{t+1} = f(k_t) - c_t$$

What are the state variables?

What are the control variables?

How to write the Bellman equation?

2. Dynamic Programming - Example

Bellman equ.

$$V(k) = max\ u(c) + \beta V(k') + \lambda(f(k) - c - k') \quad \text{(state } k\text{, control } c, k'\text{)}$$

or

$$V(k) = max\ u(f(k) - k') + \beta V(k') \quad \text{(state } k\text{, control } k'\text{)}$$

Notice:

- Remember to write 「*max*」! The value function tells us the maximum utility obtainable from tomorrow onwards for any value of the state variables.
- Remember to write β in front of next period's V

2. Dynamic Programming - Some Tips

1. Finite horizon

   Time consistency and stationarity $\rightarrow$ one Bellman equation. However, stationary doesn't hold with finite horizon. Value function changes over time.
   Write Bellman equation for each period (DP slides P14-20), or add t as a state variable (DP slides P29-30)

2. Lagged variable in utility or BC, e.g. $u(c_t, c_{t-1})$

   Define a new state variable $s_t = c_{t-1}$ and add it to the value function. Remember to define the law of motion for s! (DP slides P33-41)

2. Dynamic Programming

(CE slides P5) How to define solution when using

- sequence language
- DP/ recursive formulation

3. Recursive Competitive Equilibrium

*The Growth Model: Discrete Time Competitive Equilibrium

CE vs. Recursive CE

	CE	Recursive CE
HH	DP or Lagrangean	DP (P15)
Firm	Same as before (P6)	agg. state var. (P16)
Equilibrium	Same as before (P8)	RCE (P17)

Recursive Competitive Equilibrium

Key feature: aggregate state

- HH optimal decision depends on private state and aggregate state

$$k' = h(k, K)$$

- Firm optimal input depends on price which depends on aggregate state

$$q(K), \quad w(K)$$

3. Recursive Competitive Equilibrium

Recall that the time-t prices faced by agents depend on the equilibrium quantity of capital (assume inelastic labor supply)

$$q_t = F_K(K_t, 1) \quad w_t = F_L(K_t, 1)$$

This suggests that in a recursive setup, for the households to take future prices into account in their decisions they need to know how

- prices depend on the equilibrium (aggregate) capital stock
- and the capital stock evolves over time

So in a dateless and recursive formulation, we write all prices as functions of the aggregate state variable, K:

$$q(K) \quad w(K)$$

We endow the household with knowledge of the law of motion for aggregate capital

$$K' = \varphi(K)$$

3. Recursive Competitive Equilibrium

The household must think of itself as atomistic

- we must formulate the problem so that the household does not believe that its choices affect prices
- but the household has to believe that its choices affect its own outcomes

For this reason we introduce a distinction between the household's own capital stock, k, and the economy wide capital stock, K

$\Rightarrow$ RCE is often used in heterogeneous model and model with uncertainty.

3. Recursive Competitive Equilibrium

When define a RCE

- everything is written as functions of state variables
- don't forget to include law of motion for aggregate state variables in objects
- don't forget the consistency condition

The consistency condition is the distinctive feature of the recursive formulation of competitive equilibrium. The requirement is that, whenever the individual consumer is endowed with a level of capital equal to the aggregate level (for example, only one single agent in the economy owns all the capital), his own individual behavior will exactly mimic the aggregate behavior. The term consistency points out the fact that the aggregate law of motion perceived by the agent must be consistent with the actual behavior of individuals.
– *Real Macroeconomic Theory, Per Krusell, 2014, P83*