Econ720 - TA Session 1

Yanran Guo

UNC-Chapel Hill

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0. Macro is saying hi to you!

Office Hour:

Thursday, 1:00 - 2:00, GA 4^{th} Floor, Printer Room

Today's Task:

- How to define competitive equilibrium (On the Next: a detailed OLG example)
- Walras Law
- Arrow-Debreu and Sequential Trading

1. How to set up a competitive equilibrium?

1. Describe the economy (**Find how many sectors there are**)
2. Solve each sector's problem (e.g. Household, Firm)
   endogenous/ choice variables
   price variables

   Example

$$\begin{aligned} &max\ u(c_1, c_2) \\ &s.t.\ p_1 c_1 + p_2 c_2 = p_1 e_1 + p_2 e_2 \end{aligned}$$

   Choice variables?
   Written in real terms: $c_1 + pc_2 = e_1 + pe_2$, where $p = \frac{p_2}{p_1}$

1. How to set up a competitive equilibrium?

2. Solve each sector's problem (e.g. Household, Firm)

$$\max \ ln(c_1) + \beta ln(c_2)$$
$$s.t. \ c_1 + pc_2 = e_1 + pe_2$$

- Set up Lagrangean
- Get FOCs by taking derivative with respect all choice variables in the Lagrangean.

1. How to set up a competitive equilibrium?

3. State the market clearing condition
   - How to find markets?
     $\rightarrow$ Start with choice variables
   - How to write market clearing conditions?
     $\rightarrow$ Aggregate supply $=$ Aggregate demand
4. Define the equilibrium
   Allocations { ... } and prices { ... } that satisfy

   Optimality conditions for each sector $\begin{cases} \text{Household problem} \\ \text{Firm problem} \\ \ldots \end{cases}$

   Market clearing condition
   Accounting identity

   N objects, N+1 equations (Walras' Law)

2. Walras' Law

Define the equilibrium

Allocations $\{c_1, c_2\}$ and prices $\{p\}$ that satisfy

- Optimality conditions for household
  1. FOC: $\beta \frac{c_1}{c_2} = p$
  2. BC: $c_1 + pc_2 = e_1 + pe_2$
- Market clearing condition
  3. $c_1 = e_1$
  4. $c_2 = e_2$

3 objects, 4 equations (Walras' Law)

Linear combination exists in these 4 equations: equation 2 is a linear combination of equation 3 and 4

3. Arrow-Debreu vs. Sequential Trading

Two-period Example

Demographics: N identical household live for 2 periods, $t = 1,\ 2$

Commodities: $c_1,\ c_2$

Preference: $u(c_1,\ c_2)$

Endowments: $e_1,\ e_2$

(1). Arrow-Debreu Trading

All trades take place at $t = 1 \Rightarrow$ Only one BC!!
BC: $p_1 c_1 + p_2 c_2 = p_1 e_1 + p_2 e_2$

$$\begin{aligned} & max \ u(c_1, c_2) \\ & s.t. \ c_1 + pc_2 = e_1 + pe_2 \text{ (BC here is written in real terms.)} \\ & \mathscr{L} = u(c_1, c_2) + \lambda(e_1 + pe_2 - c_1 - pc_2) \\ & \therefore \frac{u_1}{u_2} = \frac{1}{p} \end{aligned}$$

Competitive Equilibrium

Allocations $\{ \ c_1, \ c_2 \ \}$ and price $\{ \ p \ \}$ that satisfy
Household problem solution: F.O.C., BC
Goods market clearing conditions: $c_1 = e_1, \quad c_2 = e_2$

(2). Sequential Trading

Markets open at each date $\Rightarrow$ For each period,there is a BC!!

Budget Constraint:

$$\begin{aligned} p_1 c_1 + (\ ) &= p_1 e_1 \\ p_2 c_2 &= p_2 e_2 + (\ ) \end{aligned}$$

$\Rightarrow$ We need assets to transfer resources between periods!

$$\begin{aligned} p_1 c_1 + Qb &= p_1 e_1 \\ p_2 c_2 &= p_2 e_2 + p_2 b \end{aligned}$$

We can always normalize one price to 1 in each BC.

Normalize the price of good 1 in the first BC: $c_1 + qb = e_1$ where $q = \frac{Q}{p_1}$

Normalize the price of good 2 in the second BC: $c_2 = e_2 + b$

3. Arrow-Debreu vs. Sequential Trading

(2). Sequential Trading

$$
\begin{aligned}
& max\ u(c_1, c_2) \\
& s.t.\ c_1 + qb = e_1 \\
& \qquad c_2 = e_2 + b \\
& \mathscr{L} = u(c_1, c_2) + \lambda(e_1 + qe_2 - c_1 - qc_2) \\
& \therefore \frac{u_1}{u_2} = \frac{1}{q}
\end{aligned}
$$

Competitive Equilibrium

- Allocations $\{\ c_1,\ c_2,\ b\ \}$ and price $\{\ q\ \}$ that satisfy
- Household problem solution: F.O.C., BC
- Market clearing conditions:
  - Goods market: $c_1 = e_1, \quad c_2 = e_2$
  - Bonds market: $b = 0$

4. Why $b_t = 0$?

In equilibrium, the bond market clearing condition is $b_t = 0$

Recall the fundamental rule for market clearing condition:
Aggregate supply = Aggregate demand
$\rightarrow$ Who supplies bonds? Household!
Who demands bonds? Household!
$\rightarrow$ The model setup assumes **Representative Agent**, indicating that we can consider the model as if there was only **A SINGLE HOUSEHOLD** in this economy.

4. Why $b_t = 0$?

Could there be cases where $b_t \neq 0$ in equilibrium?

Yes!

e.g. Government issuing bonds

Heterogeneous agents (See PS1-Question 2)