Econ720 - TA Session 7

Yanran Guo

UNC-Chapel Hill

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Shopping time

- Demographics: There is a single representative household who lives forever.
- Preferences: The household values consumption (c) and leisure (l) according to $\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$
- Endowments: In each period the household is endowed with 1 unit of time that can be used for leisure (l), work (n), and shopping (s).

$$1 = l_t + n_t + s_t$$

At $t = 0$ the household is endowed with k_0 units of capital and M_0 units of money.

1. Problem Set 4

- Technology: The transactions technology is such that s_t units of time are required to purchase c_t given money balances $m_t = \frac{M_t}{p_t}$: $s_t = g(c_t, m_t)$, where p_t is the price of the good. Obviously, $g_c > 0$ and $g_m < 0$.
  Goods are produced from capital and labor with the production function $f(k_t, n_t)$, which has nice properties. The resource constraint is $f(k, n) + (1 - \delta)k = c + k'$
- Markets: The usual markets for goods, money, capital and labor rental operate. There is no government and the money supply is constant.

1. Problem Set 4

Question:

(1). Define a solution to the H.H. problem.

Answer key:

(1). The H.H. maximizes

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to the BC

$$1 = l_t + n_t + s_t$$
$$s_t = g(c_t, m_t)$$
$$p_t c_t + p_t k_{t+1} + M_{t+1} = W_t n_t + Q_t k_t + (1-\delta) p_t k_t + M_t$$

1. Problem Set 4

$\Rightarrow$

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t - g(c_t, m_t))$$

subject to

$$c_t + k_{t+1} + m_{t+1} \frac{p_{t+1}}{p_t} = w_t n_t + q_t k_t + (1-\delta) k_t + m_t$$

where $m_t = \frac{M_t}{p_t}$, $w_t = \frac{W_t}{p_t}$, $q_t = \frac{Q_t}{p_t}$

What are the state variables?

1. Problem Set 4

Bellman equ.

$$V(k,m) = max\ u(c, 1-n-g(c,m)) + \beta V(k', m') + \lambda(wn + qk + (1-\delta)k + m - c - k' - m'\frac{p'}{p})$$

What are the control variables?

- FOC:

$$\begin{aligned}
[c]: &\quad u_c(c,l) - u_l(c,l)g_c(c,m) = \lambda \\
[n]: &\quad u_l = \lambda w \\
[k']: &\quad \beta V_k(k', m') = \lambda \\
[m']: &\quad \beta V_m(') = \lambda \frac{p'}{p}
\end{aligned}$$

1. Problem Set 4

- Envelope condition:

$$
\begin{aligned}
[k]: &\quad V_k = \lambda(q+1-\delta)\\
[m]: &\quad V_m = -u_l g_m + \lambda
\end{aligned}
$$

- EE:

$$
\begin{aligned}
u_c - u_l g_c &= \beta(q'+1-\delta)[u_c(') - u_l(')g_c(')]\\
u_c - u_l g_c &= \beta\frac{p}{p'}[u_c(') - u_l(')g_c(')] - \beta\frac{p}{p'}u_l(')g_m(')
\end{aligned}
$$

Define $U(c,n,m) = u(c,1-n-g(c,m))$, hence

$$
\begin{aligned}
U_c(c,n,m) &= u_c - u_l g_c\\
U_n(c,n,m) &= -u_l\\
U_m(c,n,m) &= -u_l g_m
\end{aligned}
$$

1. Problem Set 4

Then the first EE can be rewritten as

$$U_c(c,n,m) = \beta(q' + 1 - \delta)U_c(c', n', m')$$

By combining the two EEs,

$$\begin{aligned}
\beta(q' + 1 - \delta)U_c(') &= \beta \frac{p}{p'} U_c(') + \beta \frac{p}{p'} U_m(') \\
(q' + 1 - \delta)\frac{p'}{p} &= \frac{U_c(') + U_m(')}{U_c(')} \\
(q' + 1 - \delta)\frac{p'}{p} - 1 &= \frac{U_m(')}{U_c(')}
\end{aligned}$$

This equation governs the allocation of assets.

1. Problem Set 4

H.H. problem solution:

Policy functions: $c = \Gamma_1(k,m)$, $l = \Gamma_2(k,m)$, $n = \Gamma_3(k,m)$, $s = \Gamma_4(k,m)$, $k' = \Gamma_5(k,m)$, $m' = \Gamma_6(k,m)$

Value function $V(k,m)$, that satisfy

- Given policy functions, value function $V(k,m)$ is a functional fixed point.
- Given functional form of $V(k,m)$, policy functions solve EE and BC.

A solution to the HH problem in sequence form: $\{c_t, m_t, l_t, n_t, s_t, k_t\}$ that satisfy

- HH constraints, EE, static FOC
- k_0 and M_0 are given
- TVC

1. Problem Set 4

(2). Define a competitive equilibrium.

Answer key:

Competitive equilibrium:

Allocations $\{c_t, l_t, n_t, s_t, k_{t+1}, m_{t+1}, k_t^f, n_t^f\}$ and prices $\{p_t, q_t, w_t\}$ that satisfy

- H.H.: EE, BC
- Firm: FOC

  $q_t = f_k(k_t, n_t)$, $w_t = f_n(k_t, n_t)$
- Market clearing:

  Goods market: $f(k_t, n_t) + (1 - \delta)k_t = c_t + k_{t+1}$

  Capital market: $k_t^f = k_t$

  Labor market: $n_t^f = n_t$

  Money market: $p_t m_t = \bar{M}$

(3). Is money neutral in this economy? Prove your answer using the system of equations that define a competitive equilibrium.

Recall:

Money is called neutral if changing the level of M does not affect the real allocation.

It is called super neutral if changing the growth rate of M does not affect the real allocation.

Answer key:

Money is neutral in this economy. A change in the level of the money supply (in all periods) causes a proportional increase in all nominal prices but leaves the equilibrium values of real variables unaffected. We can see this by inspecting the equilibrium conditions and observing that nominal variables always appear in ratios.

1. Problem Set 4

Define the inflation rate as $1+\pi_t = \frac{p_{t+1}}{p_t}$

Define the growth rate of money supply as $1+g_t = \frac{M_{t+1}}{M_t}$

In steady state, $m_{t+1} = m_t = m_{ss} \rightarrow \frac{M_{t+1}}{p_{t+1}} = \frac{M_t}{p_t} \rightarrow g = \pi$

- Due to EE $U_c = \beta(q' + 1 - \delta)U_c(')$, in steady state we have $f'(k_{ss}) = \frac{1}{\beta} + \delta - 1$. k_{ss} does not depend on M.
- Due to goods market clearing condition, in steady state we have $c_{ss} = f(k_{ss}) - \delta k_{ss}$. c_{ss} does not depend on M.

(4). Would money still be neutral if the transactions technology used nominal money balances i.e. $s_t = g(c_t, M_t)$? Explain the intuition. You need not derive your answer.

Answer key:

Money is not neutral. Think about what happens when M and p double in every period. This could not be an equilibrium because the household now needs less time for shopping. Increasing M makes shopping time more 'productive'.