

9/11/2020 Recitation

Big Picture:

Sector problem

Equilibrium

What to know where the economy is heading to
and will end in

State variable

k_{t+1} law of motion

Steady state
↑

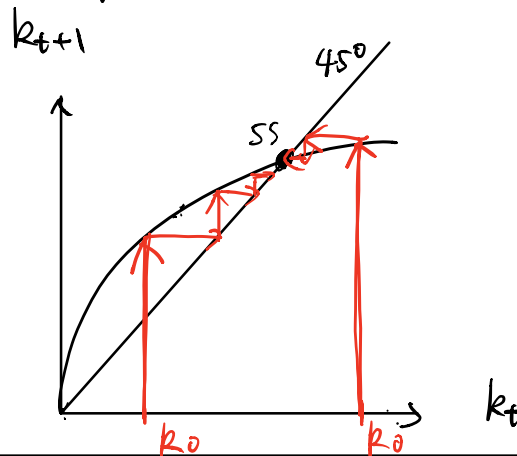
Money in OLG

m state variable.

→ law of motion for m

In standard model, state variable is k .

Law of motion for k is $k_{t+1} = \frac{f}{1+\beta} \frac{1-\theta}{1+n} k_t^\theta$



stable

In OLG with money.

$$m_t = s((1+n)m_{t+1}/m_t) \quad ??$$

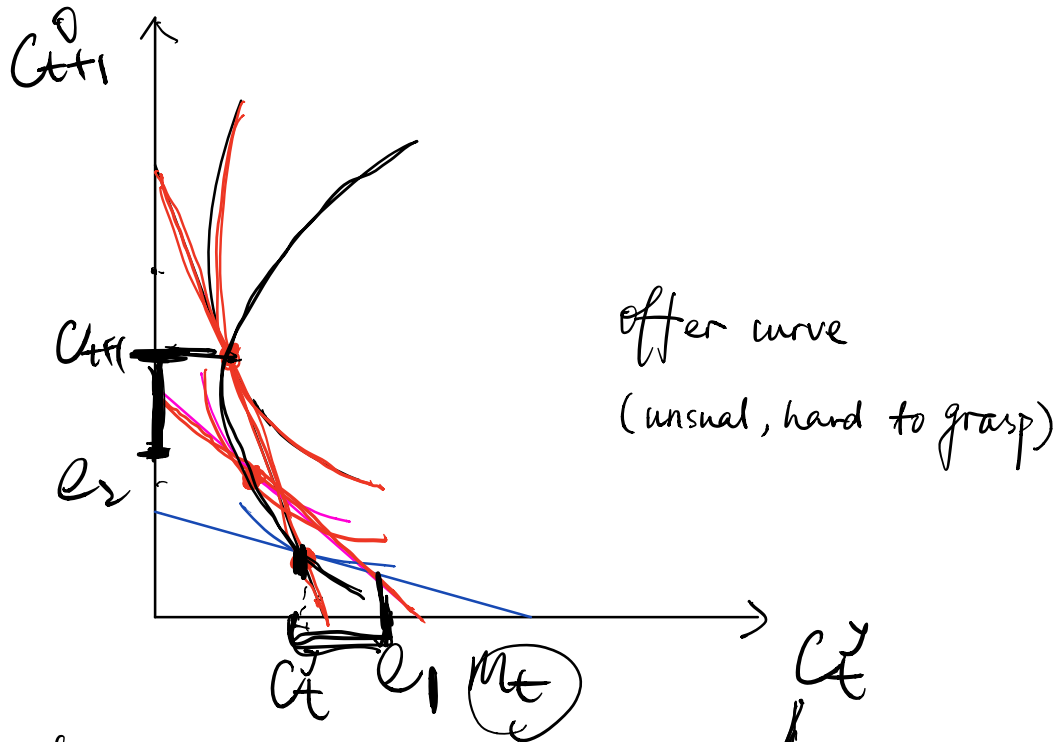
Want to map m_t into m_{t+1} . How?

- $m_t = s(R_{t+1})$
- m_{t+1} can be linked to $s(R_{t+1})$

\Rightarrow Young B.C and old B.C are linked together.
affected by R_{t+1}

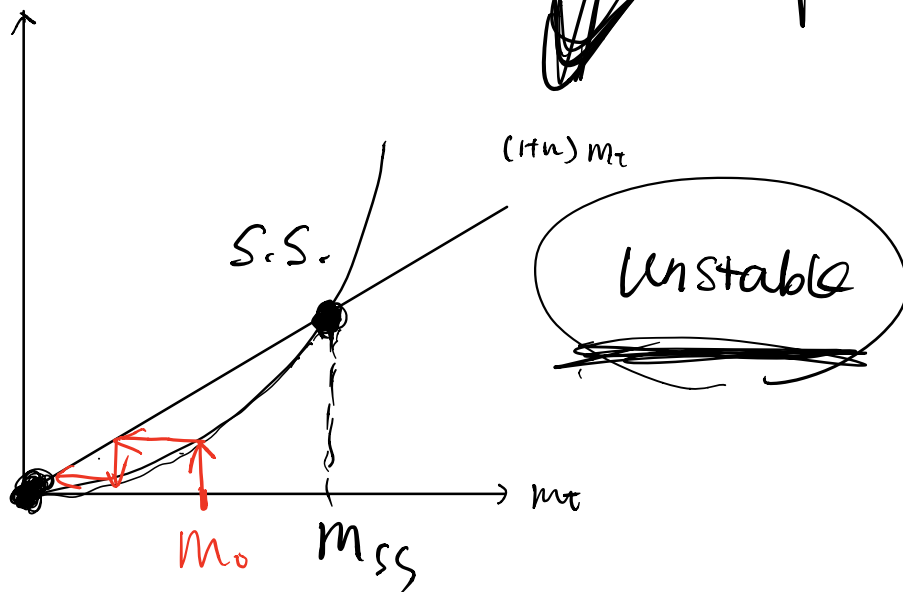
\Rightarrow Lifetime BC

$$G_{t+1}^0 = -R_{t+1} G_t^y + R_{t+1} (e_1 + e_2) \quad \text{constant}$$



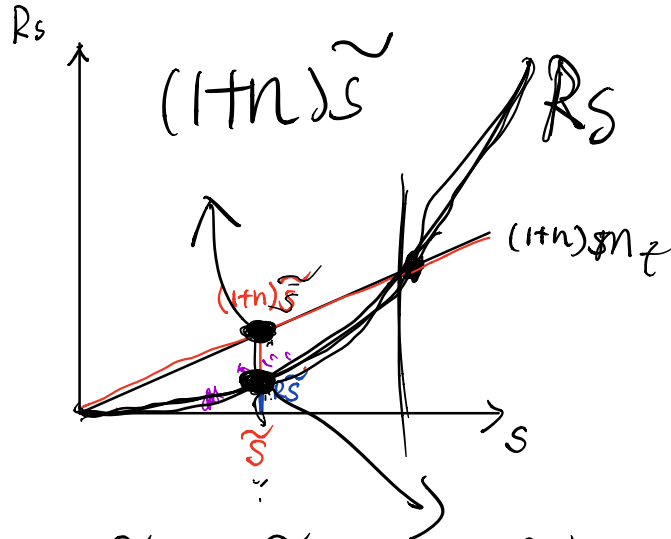
Law of motion, steady state

Flip



$$s = m$$

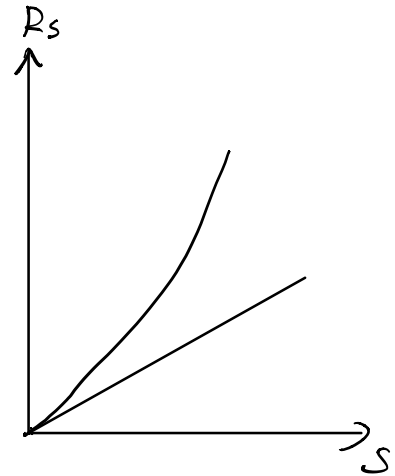
Samuelson (P30)



$$(1+n)\tilde{s} > R\tilde{s}$$

$$\therefore (1+n) > R$$

classical



$$R > (1+n)$$

Dynamic Efficiency

$$f'(k_{GR}) = n + \delta$$

$$f'(k) < n \Rightarrow \text{dynamic inefficiency.}$$

$$(1+r) < 1+n$$

$$R < 1+n$$

Fiscal Theory of the Price Level

- Gov:
$$\frac{M_{t+1} - M_t}{N_{t+1} P_{t+1}} = P_{t+1} G_{t+1}$$

Gov:

$$M_{t+1} = M_t (1 + \theta)$$

- HH:
$$\max u(C_t^y) + \beta u(C_{t+1}^o)$$

$$\text{s.t. } P_t C_t^y + P_t \lambda_t = P_t e_1$$

$$P_{t+1} C_{t+1}^o = P_{t+1} e_2 + P_t \lambda_t$$

$$\mathcal{L} = u(C_t^y) + \beta u(C_{t+1}^o) + \lambda_t \left[e_1 - C_t^y + \frac{P_{t+1}}{P_t} e_2 - \frac{P_{t+1}}{P_t} C_{t+1}^o \right]$$

$$\text{F.O.C. } u'(C_t^y) = \lambda_t$$

$$\beta u'(C_{t+1}^o) = \lambda_t \frac{P_{t+1}}{P_t} \Rightarrow \text{E.E. } u'(C_t^y) = \frac{P_t}{P_{t+1}} u'(C_{t+1}^o)$$

• CE:

Allocations $\{C_t^y, C_{t+1}^o, \lambda_t, M_t\}$ price $\{P_t\}$, s.t.

- HH: 2 BC, 1 EE

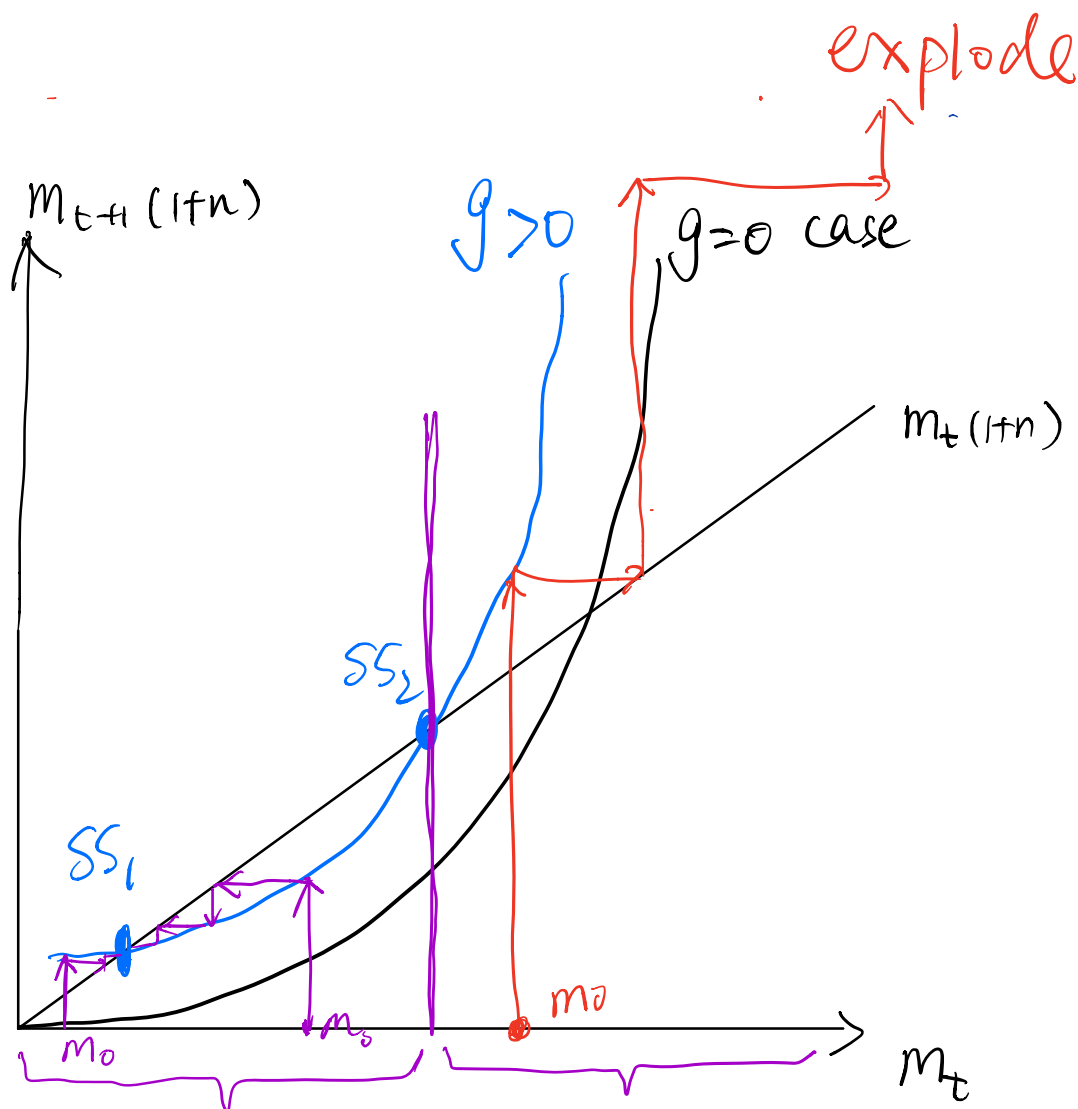
- Gov: BC

- Goods Mkt

- Money Mkt

Characterize equilibrium

$$m_{t+1} = s \left((1+n) \frac{m_{t+1} - g_{t+1}}{m_t} \right)$$



Any starting point m_0 will converge to SS_1

if economy starts in this region, it explodes