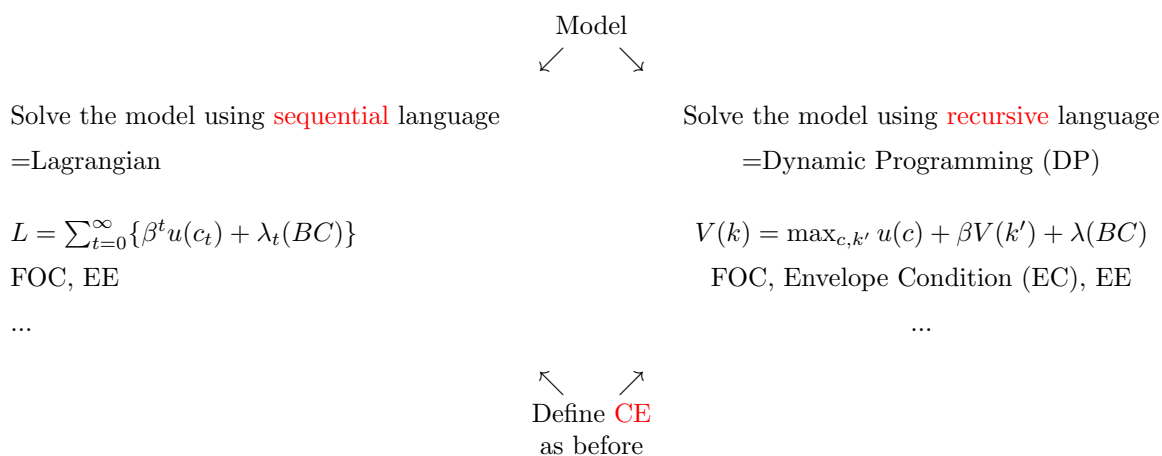


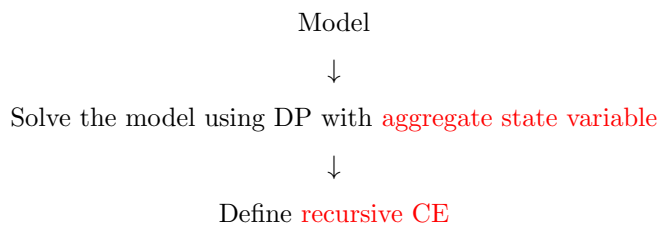
Econ.720 Recitation-6

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1 Define CE



2 Define Recursive CE



3 Example

Model setup:

- Household

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t.} \quad & c_t + k_{t+1} = (1 - \delta + q_t)k_t + w_t \end{aligned}$$

No population growth, k_0 is given

- Firm

$$\max F(K_t, L_t) - w_t L_t - q_t K_t$$

(1). Define CE

- Household

Sequential language:

$$\begin{aligned} L = \sum_{t=0}^{\infty} \{ & \beta^t u(c_t) + \\ & \lambda_t [(1 - \delta + q_t)k_t + w_t - c_t - k_{t+1}] \} \end{aligned}$$

FOC

$$[c_t] : \quad \beta^t u'(c_t) = \lambda_t$$

$$[k_{t+1}] : \quad \lambda_{t+1}(1 - \delta + q_{t+1}) = \lambda_t$$

EE

$$u'(c_t) = \beta u'(c_{t+1})(1 - \delta + q_{t+1})$$

The solution to the HH problem is a sequence $\{c_t, k_{t+1}\}$ that satisfy

– BC

– EE

– k_0 is given

– TVC

Recursive language:

$$\begin{aligned} V(k) = \max & u(c) + \beta V(k') \\ \text{s.t.} \quad & c + k' = (1 - \delta + q)k + w \end{aligned}$$

Bellman equation

$$V(k) = \max u((1 - \delta + q)k + w - k') + \beta V(k')$$

$$\text{FOC } [k'] : \quad \beta V'(k') = u'(c)$$

$$\text{EC } [k] : \quad V'(k) = u'(c)(1 - \delta + q)$$

$$\text{EE } \beta u'(c')(1 - \delta + q') = u'(c)$$

The solution to the HH problem is

policy functions: $c = \phi(k)$, $k' = h(k)$

value function: V

s.t.

– The policy functions solve the “max” part of Bellman equation, given V

– The value function is a fixed point of Bellman equation, given c and k'

- Firm

$$\max F(K_t, L_t) - w_t L_t - q_t K_t$$

FOC

$$[K_t]: F_K(K_t, L_t) = q_t$$

$$[L_t]: F_L(K_t, L_t) = w_t$$

- CE

Allocations $\{c_t, k_t, K_t, L_t\}$ and prices $\{w_t, q_t\}$, s.t.

Household: BC, EE

Firm: FOCs

Market Clearing condition

- Good market: $F(K_t, L_t) = c_t + K_{t+1} - (1 - \delta)K_t$
- Capital market: $K_t = k_t$
- Labor market: $L_t = 1$

(2). Define Recursive CE

- Household
 - Cannot use sequential language any more!
 - DP + key feature (aggregate state variable)
 - Aggregate state variable: the joint distribution of the households over their states.
Representative Agent: $k \rightarrow \kappa$
Heterogeneous Agent: joint distribution (9/23 slides P19 example)
 - Aggregate state variables enter value function $V(k, \kappa)$.
Prices are functions of state variables $q(\kappa)$ and $w(\kappa)$
 - Households take aggregate state variables as exogenously given

Bellman equation:

$$V(k, \kappa) = \max u((1 - \delta + q(\kappa))k + w(\kappa) - k') + \beta V(k', \kappa')$$

$$\text{FOC } [k'] : \quad \beta V_1(k', \kappa') = u'(c)$$

$$\text{EC } [k] : \quad V_1(k, \kappa) = u'(c)(1 - \delta + q(\kappa))$$

$$\text{EE:} \quad \beta u'(c')(1 - \delta + q(\kappa')) = u'(c)$$

The solution to the HH problem is

policy functions: $c = \phi(k, \kappa)$, $k' = h(k, \kappa)$

value function: V

s.t.

- The policy functions solve the “max” part of Bellman equation, given V
- The value function is a fixed point of Bellman equation, given c and k'

- Firm

$$\max F(K, L) - w(\kappa)L - q(\kappa)K$$

FOC

$$[K] : F_K(K, L) = q(\kappa)$$

$$[L] : F_L(K, L) = w(\kappa)$$

Solution: $K(\kappa)$ and $L(\kappa)$

- Recursive CE

Objects

- Household: policy functions $c = \phi(k, \kappa)$, $k' = h(k, \kappa)$ and value function V
- Firm: $K(\kappa)$ and $L(\kappa)$
- Price functions: $w(\kappa)$ and $q(\kappa)$
- Law of motion for aggregate state variable: $\kappa' = G(\kappa)$

that satisfy

- Solution to HH problem
- Firm FOC
- Market Clearing condition
 - * Good market: $F(K(\kappa), L(\kappa)) = \phi(k, \kappa) + h(k, \kappa) - (1 - \delta)K(\kappa)$
 - * Capital market: $K(\kappa) = k$
 - * Labor market: $L(\kappa) = 1$
- Consistency: $G(\kappa) = h(\kappa, \kappa)$

1. Please solve the household problem
→ Be careful with how to define the solution
2. Please solve the household problem using sequential language
→ Lagrangian
3. Please solve the household problem using recursive/ functional language
→ DP, Bellman equation
4. State the household's dynamic program → DP, Bellman equation
5. Write down the Bellman equation for the household
6. Define a competitive equilibrium
7. Define a recursive competitive equilibrium