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CIA Model.

(1). H.H. BC.

$$P_t C_t + P_t k_{t+1} + q_t h_{t+1} + M_{t+1}^d = P_t R k_t + q_t h_t + r_t h_t + M_t$$

$$\Rightarrow P_t C_t + P_t k_{t+1} + q_t h_{t+1} + M_{t+1} - P_t T_t = P_t R k_t + q_t h_t + r_t h_t + M_t$$

$$\Rightarrow C_t + k_{t+1} + \frac{q_t}{P_t} h_{t+1} + \frac{M_{t+1}}{P_t} - T_t = R k_t + \frac{q_t}{P_t} h_t + \frac{r_t}{P_t} h_t + \frac{M_t}{P_t}$$

(2). Bellman Equation.

$$V(M, k, h) = \max_{c, M', k', h'} u(c, h) + \beta V(M', k', h') + \lambda \underbrace{\left(Rk + \frac{q+r}{P} h + \frac{M}{P} - c - k' - \frac{q}{P} h' - \frac{M'}{P} + \tau\right)}_{\text{B.C.}} + \gamma \left(\underbrace{\frac{M}{P} - c}_{\text{CIA constraint}}\right)$$

Notice that I use M_{t+1}^d to denote my money demand in B.C.

But then I substitute M_{t+1}^d out by using $M_{t+1}^d + P_t T_t = M_{t+1}$, where M_{t+1} represents money holding. It is because, from value function, $V(M, k, h)$, it is clear that M, money holding, is the state variable. Hence on the RHS of Bellman Equation, we should use M', instead of $M^{d\prime}$.

A simple example is $\underset{\text{today's capital}}{V(k)} = \max u(f(k) + (1-\delta)k - k') + \beta V(k')$ — tomorrow's capital

$\Rightarrow$ The only difference between k and k' is time.

But M is today's money holding, $M^{d\prime}$ is money demand tomorrow.

The difference between M and $M^{d\prime}$ is more than just time.

(3).

FOC:

$$[c] \quad u_c(c,h) = \lambda + \gamma$$

$$[M'] \quad \beta V_M(M',k',h') = \lambda \frac{1}{P}$$

$$[k'] \quad \beta V_k(M',k',h') = \lambda$$

$$[h'] \quad \beta V_h(M',k',h') = \lambda \frac{q}{P}$$

EC:

$$[M] \quad V_M(M,k,h) = \frac{1}{P}(\lambda+\gamma)$$

$$[k] \quad V_k(M,k,h) = R\lambda$$

$$[h] \quad V_h(M,k,h) = u_h(c,h) + \frac{r+q}{P}\lambda$$

(4).

$$[c] + [M'] + [M] \Rightarrow \beta u_c(c',h')\frac{P}{P'} = u_c(c,h) - \gamma$$

$$[c] + [k'] + [k] \Rightarrow \beta R'\left(u_c(c',h') - \gamma'\right) = u_c(c,h) - \gamma$$

$$[c] + [h'] + [h] \Rightarrow \beta\left[\frac{P}{q} u_h(c',h') + \frac{P}{q}\frac{r'+q'}{P'}\left(u_c(c',h') - \gamma'\right)\right] = u_c(c,h) - \gamma$$

(5). is on page 4.

(6). When CIA condition does not bind, $\gamma = 0$.

Hence, the EEs become

$$U_c(c,h) = \beta U_c(c',h') \frac{p}{p'}$$ return of money

$$U_c(c,h) = \beta U_c(c',h') R$$ return of capital

$$U_c(c,h) = \beta U_c(c',h') \frac{p}{q}\frac{r'+q'}{p'} + \beta \frac{p}{q} U_h(c',h')$$

return of land.

$\therefore \frac{p}{p'} = R$, indicating that money and capital have the same return.

$\because U_h > 0$ and $\beta U_c(c',h')\frac{p}{q}\frac{r'+q'}{p'} + \beta\frac{p}{q}U_h(c',h') = \beta U_c(c',h')R$

$$\therefore \beta U_c(c',h')\frac{p}{q}\frac{r'+q'}{p'} < \beta U_c(c',h')R$$

$\therefore \frac{p}{q}\frac{r'+q'}{p'} < R$ since $U_c > 0$, $0<\beta<1$

$\therefore$ Return of land, which is $\frac{p}{q}\frac{r'+q'}{p'}$, is less than returns of money and capital.

(7). Define $\pi_{t+1} = \frac{P_{t+1}}{P_t}$

$\therefore \beta U_c(c',h')\frac{p}{p'} = U_c(c,h) - \gamma \Rightarrow \beta U_c(c',h')\frac{1}{\pi'} = U_c(c,h) - \gamma$ ①.

Combining equation ① and $\beta R'(U_c(c',h') - \gamma') = U_c(c,h) - \gamma$, we get

$$\beta R'(U_c(c',h') - \gamma') = \beta U_c(c',h')\frac{1}{\pi'}$$

Date back by one period, $\beta R(\underbrace{U_c(c,h) - \gamma}) = \beta U_c(c,h)\frac{1}{\pi}$

$$= \beta U_c(c',h')\frac{1}{\pi'} \text{ by equation ①}$$

$$\therefore \beta R U_c(c',h')\frac{1}{\pi'} = U_c(c,h)\frac{1}{\pi}$$

$$\therefore U_c(c,h) = \beta R U_c(c',h')\frac{\pi}{\pi'}$$

$\because$ β is discount factor

$\therefore$ $0 < \beta < 1$

$\because$ It is given that $R > \frac{1}{\beta}$

$\therefore$ $R\beta > 1$

When inflation rate is rising over time, it means $\frac{\pi}{\pi'} < 1$

$$\therefore \quad \frac{U_c(c,h)}{U_c(c',h')} = \beta R \frac{\pi}{\pi'} < \beta R.$$

βR is the ratio of $U_c(c,h)$ and $U_c(c',h')$ when there is no inflation.

$\therefore$ $\frac{U_c(c,h)}{U_c(c',h')}$ is lower in the economy where there is inflation.

It indicates that, compared to an economy without inflation, in the economy where there is inflation, HHs tend to consume more today relative to tomorrow. That is because inflation drives down the value of their saving. So it is more optimal for households to consume more today and save less.

(5).

- $U_c(c,h) = \lambda + \gamma$.

LHS is the cost I have when decreasing my consumption by 1 unit.
RHS is the benefit I have when decreasing my consumption by 1 unit.
The benefit comes from two parts:

① Since I decrease my consumption, it means my B.C. is relaxed.
λ is the value of relaxing my B.C.

②. Decreasing my consumption also relaxes my CIA constraint.
γ is the value of relaxing my CIA constraint.

- EE for M: $u_c(c,h) = \gamma + \beta u_c(c',h')\frac{p}{p'}$

LHS is the cost I have when decreasing my consumption by 1 unit today.
RHS is the benefit I have when decreasing my consumption by 1 unit today.
It comes from two parts:

① Decreasing my consumption today relaxes my today's CIA constraint. γ is the value of relaxing my CIA constraint today.

② Since I decrease my consumption by 1 unit today, it means that I have one unit of extra good, which is p units of money. Then I take this p units of money to next period. In next period, this p amount of money can buy me $\frac{p}{p'}$ units of good. So I get benefit by eating this extra $\frac{p}{p'}$ units of good. Measure the benefit in terms of utility by multiplying $u_c(c',h')$ and discount it back to today's value by multiplying β.

- EE for K: $u_c(c,h) = \gamma + \beta R'\left(u_c(c',h') - \gamma'\right)$

LHS: same as above

RHS: same as above

The benefit comes from two parts:

① γ: same as above.

②. I decrease my consumption by 1 unit today. I invest this extra unit of good in K. So it gives me R more units of good next period. So I get benefit by eating this extra R units of good. Normally we use $u_c(c',h')$ to measure this benefit. But notice that, because I consume more in period t+1, this makes my CIA constraint tighter. So my benefit from consuming more goods is mitigated by a tighter CIA constraint. That's why we use $u_c(c',h')-\gamma'$ to multiply R', instead of just $u_c(c',l')$. $-\gamma'$ captures the cost of tightening my CIA constraint in next period. Again, since it's the benefit I get tomorrow, I need to multiply it by β to discount it back to today's value.

- EE for h: $\underbrace{u_c(c,h)}_{\text{cost}} = \underbrace{\gamma}_{\substack{\text{benefit}\\\text{from relaxing}\\\text{CIA today}}} + \beta\Big[\underbrace{\frac{p}{q} u_h(c',h')}_{\substack{\text{benefit from}\\\text{holding more}\\\text{land tomorrow}}} + \underbrace{\frac{p}{q}\frac{r'+q'}{p'}\left(u_c(c',h')-\gamma'\right)}_{\substack{\text{benefit from}\\\text{consuming more}\\\text{tomorrow.}}}\Big]$

And the extra goods I can eat tomorrow is brought by holding more land

benefit from consuming more is mitigated by a tight CIA constraint.

Exhausted...

please write the interpretation for EE of h yourself.