ECE271A HW2 Report

Problem6

a. Last week what I did is calculating prior probabilities based on the number of samples from grass and cheetah in training sets:

P_Y(cheetah) = 250/(1053+250) = 0.1919
P_Y(grass) = 1053/(1053+250) = 0.8081

This week the problem 2 shows that the optimal solution is c/n, where c is the number of times one class is observed in all observations and n is total number of observation. So the result of prior probabilities remains the same with last week's answer.

b. The Maximum Likelihood Estimate for Gaussian Classifier is based on:

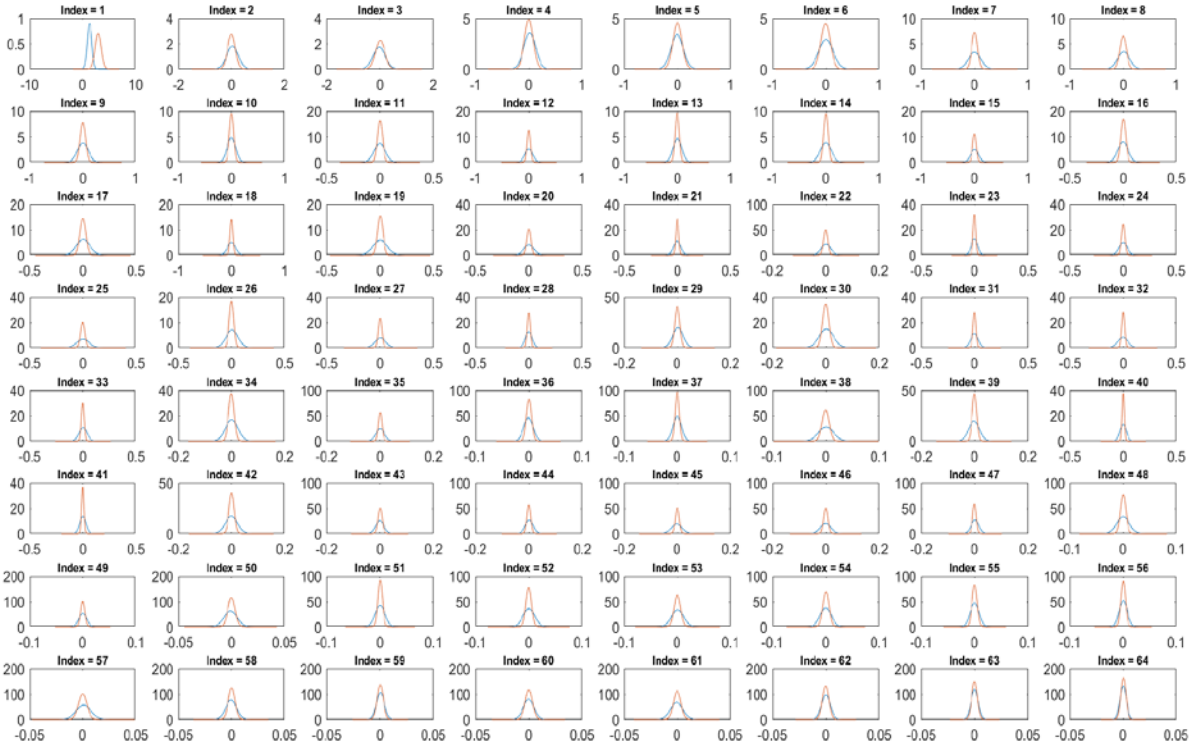
$$f(T) = \frac{1}{(\sigma_T\sqrt{2\pi})^N} e^{-\frac{1}{2}(\frac{T-\bar{T}}{\sigma_T})^2}$$

$$L(T_1, T_2, \cdots, T_N|\bar{T}, \sigma_T) = L = \prod_{i=1}^{N}[\frac{1}{\sigma_T\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{T-\bar{T}}{\sigma_T})^2}]$$

The poster class conditional probability could be calculated by:

$$P_{X|Y}(x|i) = \frac{1}{\sqrt{(2\pi)^d|\sum_i|}}\exp\{-\frac{1}{2}(x-\mu_i)^T\sum_i^{-1}(x-\mu_i)\}$$

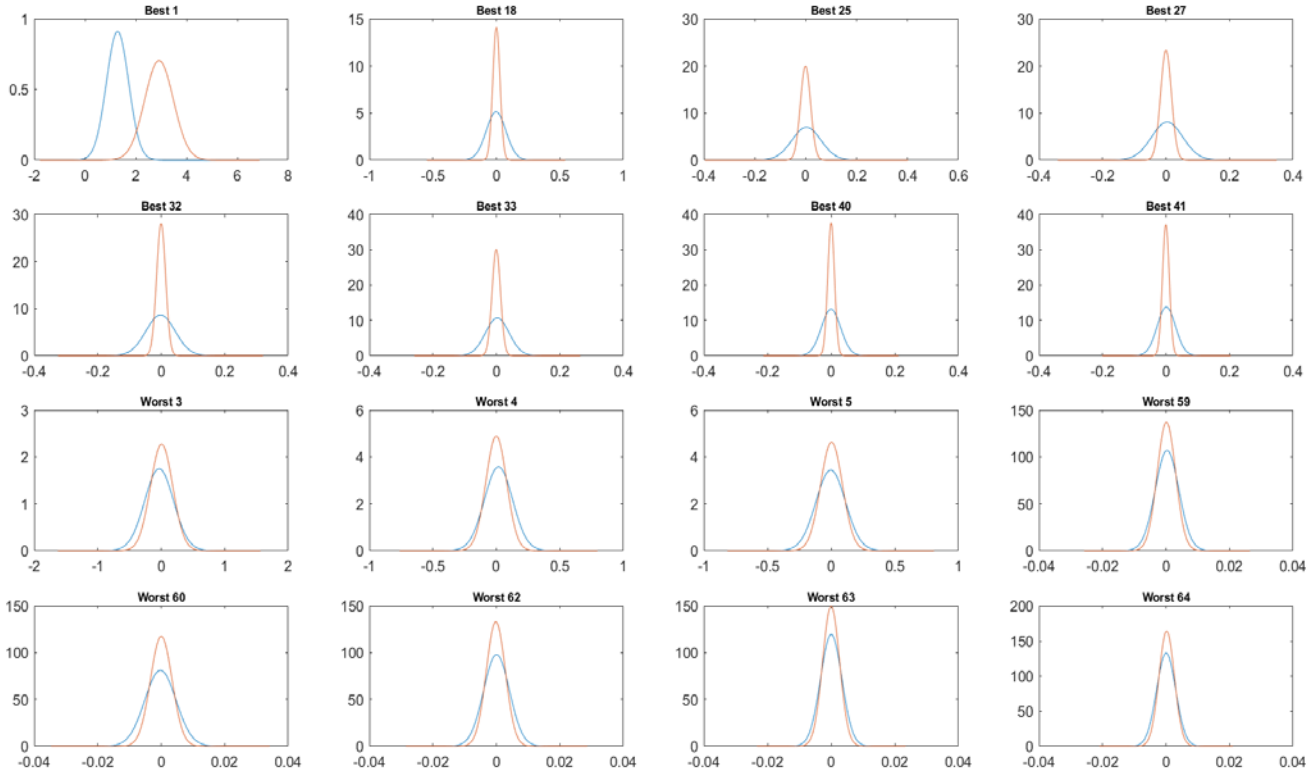
This process is included in my code. As for the marginal distribution plots, they are here:



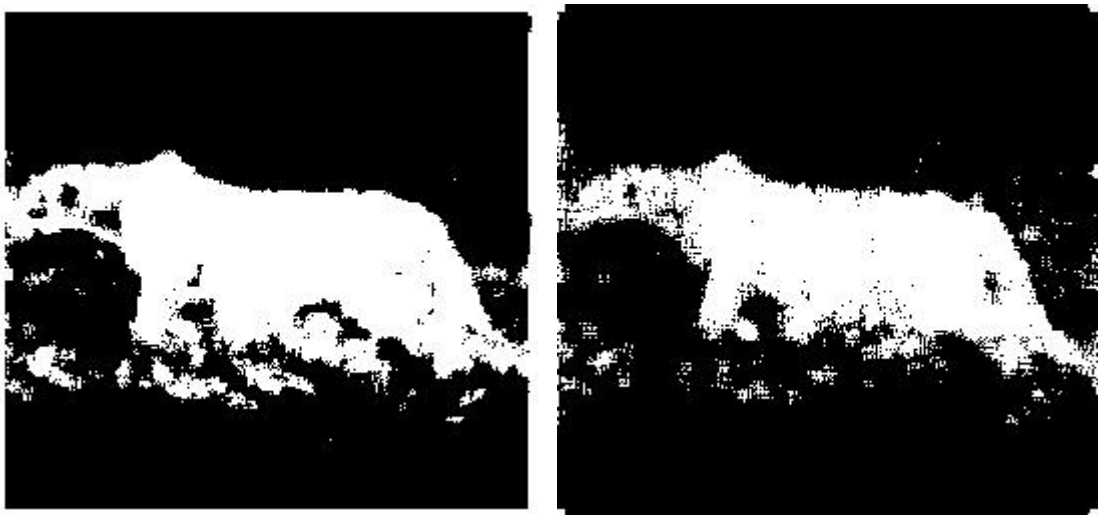
Then I used Kullback-Leibler distance (KL distance) to measure the similarity of the 2-class marginal distribution. The bigger difference indicates that the feature in this dimension has higher value for classification.

The 8 best index is [1, 18, 25, 27, 32, 33, 40, 41].

The 8 worst index is [3, 4, 5, 59, 60, 62, 63, 64].



c. The Bayesian Decision Rule is applied to given image to give prediction based on training set.



(i) 64 dimension feature prediction

(ii) 8 best feature prediction

Probability of error is calculated based on:

$$\text{error(FG)} = \frac{\#\text{FG pixels misclassified as BG}}{\#\text{FG pixels in ground truth of test set}} \times \text{prior probability of FG}$$

$$\text{error(BG)} = \frac{\#\text{BG pixels misclassified as FG}}{\#\text{BG pixels in ground truth of test set}} \times \text{prior probability of BG}$$

$$\text{error} = \text{error(FG)} + \text{error(BG)}$$

The result of 64 dimension feature is error(FG) = 0.0107, error(BG)=0.1250, **error = 0.1357**

The result of 8 dimension feature is error(FG) = 0.0143, error(BG)=0.0334, **error = 0.0477**

The result shows that the performance of 8 best features we chose is better than all 64 dimension features, which means that the feature selection is significant for classification task and the best feature should be the one whose distribution on different classes are not similar.

Code:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% problem a
n = 1053+250;
c_cheetah = 250;
c_grass = 1053;
prior_cheetah = c_cheetah/n;
prior_grass = c_grass/n;

% From Problem 2 we got p* = c/n
% where c is the number of times that a certain class samples are observed.
% n is the number of times for observation totally.
% Compared to the result of last week, the prior probability is identical.
% Last week, the result of prior probability is obtained from number of observation directly.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% problem b
train_set = load('TrainingSamplesDCT_8_new.mat');
FGmat = train_set.TrainsampleDCT_FG;
BGmat = train_set.TrainsampleDCT_BG;

%%% maximum likelihood estimates
%%% foreground
meanFG = mean(FGmat);
covFG = cov(FGmat);
varFG = var(FGmat);
mleFG = zeros(c_cheetah,1);
for i=1:c_cheetah
    tmp = 0;
    for j=1:64
        tmp = tmp + ((FGmat(i,j)-meanFG(j))/sqrt(varFG(j)))^2;
    end
    mleFG(i,1) = exp(-32*log(2*pi)-64*log(sqrt(varFG(j)))-0.5*tmp);
end
%%% background
meanBG = mean(BGmat);
covBG = cov(BGmat);
varBG = var(BGmat);
mleBG = zeros(c_grass,1);
for i=1:c_grass
    ave = mean(BGmat(i,:));
    sigma = std(BGmat(i,:));
    tmp = 0;
    for j=1:64
        tmp = tmp + ((BGmat(i,j)-meanBG(j))/sqrt(varBG(j)))^2;
    end
    mleBG(i,1) = exp(-32*log(2*pi)-64*log(sqrt(varBG(j)))-0.5*tmp);
end

%%% marginal plot
kl = [];
for i=1:64
    subplot(8,8,i)
    set(gcf,'Position',get(0,'ScreenSize'))
    ave_FG = mean(FGmat(:,i));
    variance_FG = var(FGmat(:,i));
    sigma_FG = sqrt(variance_FG);
    ave_BG = mean(BGmat(:,i));
    variance_BG = var(BGmat(:,i));
    sigma_BG = sqrt(variance_BG);
    xFG = ave_FG-7*sigma_FG:sigma_FG/50:ave_FG+7*sigma_FG;
    xBG = ave_BG-7*sigma_BG:sigma_BG/50:ave_BG+7*sigma_BG;
    x = sort([xFG xBG]);
    y_cheetah = normpdf(x,ave_FG,sigma_FG);
    y_grass = normpdf(x,ave_BG,sigma_BG);

    plot(x,y_cheetah,x,y_grass)
```

```
    title(['Index = ' num2str(i)],'FontSize',8);

    % calculate KL distance
    kl_tmp = KLdiv(y_cheetah,y_grass);
    kl = [kl kl_tmp];
end
print('64_plots','-dpng');


%%% Find Best and Worst based on KL distance
[KLSorted, KLIdx] = sort(kl);
worstidx = sort(KLIdx(1:8));
bestidx = sort(KLIdx(57:64));

figure();
%%% Make output for best and worst cases
for i=1:8
    idx_w = worstidx(i);
    idx_b = bestidx(i);
    % Worst
    subplot(4,4,i+8)
    ave_FG = mean(FGmat(:,idx_w));
    variance_FG = var(FGmat(:,idx_w));
    sigma_FG = sqrt(variance_FG);
    ave_BG = mean(BGmat(:,idx_w));
    variance_BG = var(BGmat(:,idx_w));
    sigma_BG = sqrt(variance_BG);
    xFG = ave_FG-7*sigma_FG:sigma_FG/50:ave_FG+7*sigma_FG;
    xBG = ave_BG-7*sigma_BG:sigma_BG/50:ave_BG+7*sigma_BG;
    x = sort([xFG xBG]);
    y_cheetah = normpdf(x,ave_FG,sigma_FG);
    y_grass = normpdf(x,ave_BG,sigma_BG);
    plot(x,y_cheetah,x,y_grass)
    title(['Worst ' num2str(idx_w)],'FontSize',8);
    % Best
    subplot(4,4,i)
    ave_FG = mean(FGmat(:,idx_b));
    variance_FG = var(FGmat(:,idx_b));
    sigma_FG = sqrt(variance_FG);
    ave_BG = mean(BGmat(:,idx_b));
    variance_BG = var(BGmat(:,idx_b));
    sigma_BG = sqrt(variance_BG);
    xFG = ave_FG-7*sigma_FG:sigma_FG/50:ave_FG+7*sigma_FG;
    xBG = ave_BG-7*sigma_BG:sigma_BG/50:ave_BG+7*sigma_BG;
    x = sort([xFG xBG]);
    y_cheetah = normpdf(x,ave_FG,sigma_FG);
    y_grass = normpdf(x,ave_BG,sigma_BG);
    plot(x,y_cheetah,x,y_grass)
    title(['Best ' num2str(idx_b)],'FontSize',8);
end
set(gcf,'Position',get(0,'ScreenSize'))
print('BestandWorst_plots','-dpng');


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% problem c

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% (i) Use 64-dimension Gaussion
img = imread('cheetah.bmp');
img = im2double(img);
%%% Add paddle
img = [zeros(size(img,1),4) img];
img = [zeros(4, size(img,2)); img];
img = [img zeros(size(img,1),3)];
img = [img; zeros(3, size(img,2))];

%%% DCT
[m,n] = size(img);
Blocks = ones(m-7,n-7);
```

```
mean_FG = mean(FGmat);
mean_BG = mean(BGmat);
ave_tmp_FG = mean_FG;
ave_tmp_BG = mean_BG;
inv_covFG = inv(covFG);
inv_covBG = inv(covBG);
DcovFG = det(covFG);
DcovBG = det(covBG);

%%% predict
for i=1:m-7
   for j=1:n-7
      DCT = dct2(img(i:i+7,j:j+7));
      zigzag_order = zigzag(DCT);
      feature = zigzag_order;
      g_cheetah = 0;
      g_grass = 0;
      % cheetah
      g_cheetah = g_cheetah + feature*inv_covFG*transpose(feature);
      g_cheetah = g_cheetah - 2*feature*inv_covFG*transpose(ave_tmp_FG);
      g_cheetah = g_cheetah + ave_tmp_FG*inv_covFG*transpose(ave_tmp_FG);
      g_cheetah = g_cheetah + log(DcovFG) - 2*log(prior_cheetah);
      % grass
      g_grass = g_grass + feature*inv_covBG*transpose(feature);
      g_grass = g_grass - 2*feature*inv_covBG*transpose(ave_tmp_BG);
      g_grass = g_grass + ave_tmp_BG*inv_covBG*transpose(ave_tmp_BG);
      g_grass = g_grass + log(DcovBG) - 2*log(prior_grass);
      if g_cheetah >= g_grass
         Blocks(i,j) = 0;
      end
   end
end

%%% save prediction
imwrite(Blocks, 'prediction_64d.jpg');
prediction = mat2gray(Blocks);

ground_truth = imread('cheetah_mask.bmp')/255;
x = size(ground_truth, 1);
y = size(ground_truth, 2);
count1 = 0;
count2 = 0;
count_cheetah_truth = 0;
count_grass_truth = 0;
for i=1:x
   for j=1:y
      if prediction(i,j) > ground_truth(i,j)
         count2 = count2 + 1;
         count_grass_truth = count_grass_truth + 1;
      elseif prediction(i,j) < ground_truth(i,j)
         count1 = count1 + 1;
         count_cheetah_truth = count_cheetah_truth + 1;
      elseif ground_truth(i,j) >0
         count_cheetah_truth = count_cheetah_truth + 1;
      else
         count_grass_truth = count_grass_truth + 1;
      end
   end
end
error1_64 = (count1/count_cheetah_truth) * prior_cheetah;
error2_64 = (count2/count_grass_truth) * prior_grass;
total_error_64 = error1_64 + error2_64;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% (ii) Use 8 best features
img = imread('cheetah.bmp');
img = im2double(img);
%%% Add paddle
img = [zeros(size(img,1),4) img];
img = [zeros(4, size(img,2)); img];
```

```
img = [img zeros(size(img,1),3)];
img = [img; zeros(3, size(img,2))];

%%% DCT
[m,n] = size(img);
Blocks = ones(m-7,n-7);
mean_FG = mean(FGmat(:,bestidx));
mean_BG = mean(BGmat(:,bestidx));
ave_tmp_FG = mean_FG;
ave_tmp_BG = mean_BG;
cov_cheetah = cov(FGmat(:,bestidx));
cov_grass = cov(BGmat(:,bestidx));
inv_covFG = inv(cov_cheetah);
inv_covBG = inv(cov_grass);
DcovFG = det(cov_cheetah);
DcovBG = det(cov_grass);

%%% predict
for i=1:m-7
   for j=1:n-7
      DCT = dct2(img(i:i+7,j:j+7));
      zigzag_order = zigzag(DCT);
      feature = zigzag_order(bestidx);
      g_cheetah = 0;
      g_grass = 0;
      % cheetah
      g_cheetah = g_cheetah + feature*inv_covFG*transpose(feature);
      g_cheetah = g_cheetah - 2*feature*inv_covFG*transpose(ave_tmp_FG);
      g_cheetah = g_cheetah + ave_tmp_FG*inv_covFG*transpose(ave_tmp_FG);
      g_cheetah = g_cheetah + log(DcovFG) - 2*log(prior_cheetah);
      % grass
      g_grass = g_grass + feature*inv_covBG*transpose(feature);
      g_grass = g_grass - 2*feature*inv_covBG*transpose(ave_tmp_BG);
      g_grass = g_grass + ave_tmp_BG*inv_covBG*transpose(ave_tmp_BG);
      g_grass = g_grass + log(DcovBG) - 2*log(prior_grass);
      if g_cheetah >= g_grass
         Blocks(i,j) = 0;
      end
   end
end

%%% save prediction
imwrite(Blocks, 'prediction_8d.jpg');
prediction = mat2gray(Blocks);

ground_truth = imread('cheetah_mask.bmp')/255;
x = size(ground_truth, 1);
y = size(ground_truth, 2);
count1 = 0;
count2 = 0;
count_cheetah_truth = 0;
count_grass_truth = 0;
for i=1:x
   for j=1:y
      if prediction(i,j) > ground_truth(i,j)
         count2 = count2 + 1;
         count_grass_truth = count_grass_truth + 1;
      elseif prediction(i,j) < ground_truth(i,j)
         count1 = count1 + 1;
         count_cheetah_truth = count_cheetah_truth + 1;
      elseif ground_truth(i,j) >0
         count_cheetah_truth = count_cheetah_truth + 1;
      else
         count_grass_truth = count_grass_truth + 1;
      end
   end
end
error1_8 = (count1/count_cheetah_truth) * prior_cheetah;
error2_8 = (count2/count_grass_truth) * prior_grass;
total_error_8 = error1_8 + error2_8;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function output = zigzag(in)
% initializing the variables
%----------------------------------
h = 1;
v = 1;
vmin = 1;
hmin = 1;
vmax = size(in, 1);
hmax = size(in, 2);
i = 1;
output = zeros(1, vmax * hmax);
%----------------------------------
while ((v <= vmax) && (h <= hmax))

   if (mod(h + v, 2) == 0)                % going up
      if (v == vmin)
         output(i) = in(v, h);        % if we got to the first line
         if (h == hmax)
            v = v + 1;
         else
            h = h + 1;
         end
         i = i + 1;
      elseif ((h == hmax) && (v < vmax))   % if we got to the last column
         output(i) = in(v, h);
         v = v + 1;
         i = i + 1;
      elseif ((v > vmin) && (h < hmax))    % all other cases
         output(i) = in(v, h);
         v = v - 1;
         h = h + 1;
         i = i + 1;
   end

   else                                % going down
     if ((v == vmax) && (h <= hmax))       % if we got to the last line
         output(i) = in(v, h);
         h = h + 1;
         i = i + 1;

      elseif (h == hmin)                 % if we got to the first column
         output(i) = in(v, h);
         if (v == vmax)
        h = h + 1;
       else
           v = v + 1;
         end
         i = i + 1;
      elseif ((v < vmax) && (h > hmin))     % all other cases
         output(i) = in(v, h);
         v = v + 1;
         h = h - 1;
         i = i + 1;
      end
   end
   if ((v == vmax) && (h == hmax))          % bottom right element
      output(i) = in(v, h);
      break
   end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function KLD=KLdiv(P,Q)
   P = P/sum(P);
   Q = Q/sum(Q);
   index = P > 0;
   kld = -1 * P(index).* log(Q(index)./P(index));
```

```
    kld(isinf(kld)|isnan(kld)) = 0;
    KLD = sum(kld);
end
```