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Homework 9
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2. BOA=0 => ATW=BW=0 => WERER(AAT+BTB)
                                                                                                                                 > WE ker (d)
                          \langle A x, y \rangle = \langle X, A^T y \rangle
                         <\frac{1}{2}, BT => =<B \frac{1}{2}, 2>
                                  For D= AAT +BTB
                                                 x \in ker(A) \cap ker(B^T) \Rightarrow Ax=B^T x = 0
                                                   => 1 x=0 => 9 (0)
                                                     \Rightarrow ker(\triangle)\geq kenA) \land ker(B)
                                             Suppose Xther (d) => BX=0
                                                              \langle \Delta x, x \rangle = \langle A \circ B^{T} x + B^{T} \circ A x \rangle
                                                                                                     = < BTX, BXX> T<AX, Ax>
                                                                                                    = // BTX112 +1/AX12
                                                      => 1|BTx1|2=1|Ax112=0 => BTx=Ax=0
                                                         =) XE ker (AT) ( ker (B)
                                                            => ker & < ker (A) 1 ker (B)
                                             So ker & = ker (A) () ker(B)
                                         Let Xtker(S) JtIm(A)
                                                 => Ax=BT x=0, H= AZ
                                                  ∠x, y> = ⟨x, A≥> = ⟨B<sup>T</sup>x, ≥> = ⟨0, ₹)=0
                                               Assume y' E Im (B), y' = BT z'

\( \text{X}, \forall ') = \( \text{A} \text{X}, \text{E'} > = \langle 0, \text{Z'} \rangle 0, \text{Z'} \rangle = \langle 0, \text{Z'} \rangle 0, \text{
                                                 ∠8 19'7 = < Az, B7 2'> = <A08 8, 2/> = <0, 2/>=0
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$\Rightarrow im(A) + ker(A) + im(BT) \leq Y \qquad (i)$
$Y = ker(S) + I_m(\Delta)$
for $X=X_1+X_2$, $X_1 \in ker(0)$, $X_2 \in im(0)$
$for y \in Y$
$\chi_2 = \Delta \mathcal{Y} = A(B^T \mathcal{Y}) + (B^T (A \mathcal{Y})) \in 2m(A) + Im(B)$
=) xe ker(0) + In(A) + Im(BT)
=) $Y \subseteq im(A) + ker(A) + im(B) = 0$
$0, 0 \Rightarrow Y = im(A) + ker(D) + im(B).$