2:

2. There exists negative value

(c): the distance is not totally equal

(d): The distribution is similar to real world distribution. dim-2 (top 2) coordinate is same to dim-3's (top 3) first 2-dim

3:

3.

a: if all eigenvalue are nonnegative

$\forall \vec{x}$ and $||\vec{x}|| \geq 0$

$\vec{x}^T K x = \vec{q}\,\vec{v} \geq 0$

$\vec{q}$ is eigenvalues ≥ 0

$\vec{v}$ is corresponding

b: $d_{ij} = K_{ii} + K_{ij} - 2K_{ij}$

$= \vec{u_i}^T K \vec{u_i} + \vec{u_j}^T K \vec{u_j} - 2(\vec{u_i}^T K \vec{u_j})$

$= (\vec{u_i} - \vec{u_j})^T K (\vec{u_i} - \vec{u_j})$

K is symmetric and positive semi-definite

dij is squared Euclidean distance

d:

4:

4.

(a): No

$d(x_1, x_3) = 4$. $d(x_1, x_2) = 4$

$d(x_2, x_3) = 7$

$4+4 > 7$ satisfies ~~...~~ triangle inequality.

$16+16 < 49$ so d^2 is not distance function

(b): Yes

1. $d(x,y) \geq 0 \rightarrow \sqrt{d(x,y)} \geq 0$

2. ~~...~~ $\forall d(x,y) = 0 = \sqrt{d(x,y)}$, $x = y$

3. $d(x,y) = d(y,x)$

$\sqrt{d(x,y)} = \sqrt{d(y,x)}$

4. ~~...~~ Given arbitrary $d_1 \leq d_2 + d_3$

$\sqrt{d_1} \leq \sqrt{d_2} + \sqrt{d_3}$

$\downarrow$

$d_1 \leq d_2 + d_3 + 2\sqrt{d_2 d_3}$