Homework 7

WU Jiamin 20659352

2. (a). $A > 0$

Assume $Av^* \neq \lambda^* v^* \Rightarrow Av^* > \lambda^* v^*$

then $Av^* = Av^* - \lambda^* v^* + \lambda^* v^*$

$\Rightarrow A(Av^*) = A(Av^* - \lambda^* v^*) + A\lambda^* v^*$

We have $A > 0$ $\lambda^* > 0$ $v^* > 0$ $Av^* - \lambda^* v^* > 0$

So $A(Av^*) > \lambda^* Av^* + Av^* - \lambda^* v^*$

Conflict So $Av^* = \lambda^* v^*$

(b) $v^* = \dfrac{Av^*}{\lambda^*}$

$1^T v^* = 1 \Rightarrow v^* \neq 0$

$v^* \geq 0$ $\Rightarrow v^* > 0$

3. (a) $N(i,i) = \mathbb{E}\left(\sum_{m=0}^{\infty} 1\{\text{jumps } m \text{ times from } i \text{ to } i\}\right)$

$= \sum_{m=0}^{\infty} \mathbb{P}\{\text{jumps } m \text{ times from } i \text{ to } i\}$

$= 1 + \sum_{k=1}^{\infty} N(i,k)\,Q(k,i)$

(b) $N(i,j) = \mathbb{E}\left(\sum_{m=0}^{\infty} 1\{\text{jumps } m \text{ times from } i \text{ to } j\}\right)$

$= \sum_{m=0}^{\infty} \mathbb{P}\{\text{jumps } m \text{ times from } i \text{ to } j\}$

$= \sum_{k=1}^{\infty} N(i,k)\,Q(k,j)$

(c) $N(i,i) = \sum_{k} N(i,k)\,Q(k,i) + 1$

$\Rightarrow N - NQ = I$

(d) $B(i) = \mathbb{E}\left(\sum_{m=0}^{\infty} 1\{\text{jumps } m \text{ times from } i \text{ to } n+j\}\right)$

$= \sum_{m=0}^{\infty} \sum_{k} \mathbb{P}\{\text{jumps } (m-1) \text{ times from } i \text{ to } k\} R(k,j)$

$= \sum_{k=1}^{\infty} N(i,k) R(k)$

$\Rightarrow B = NR$