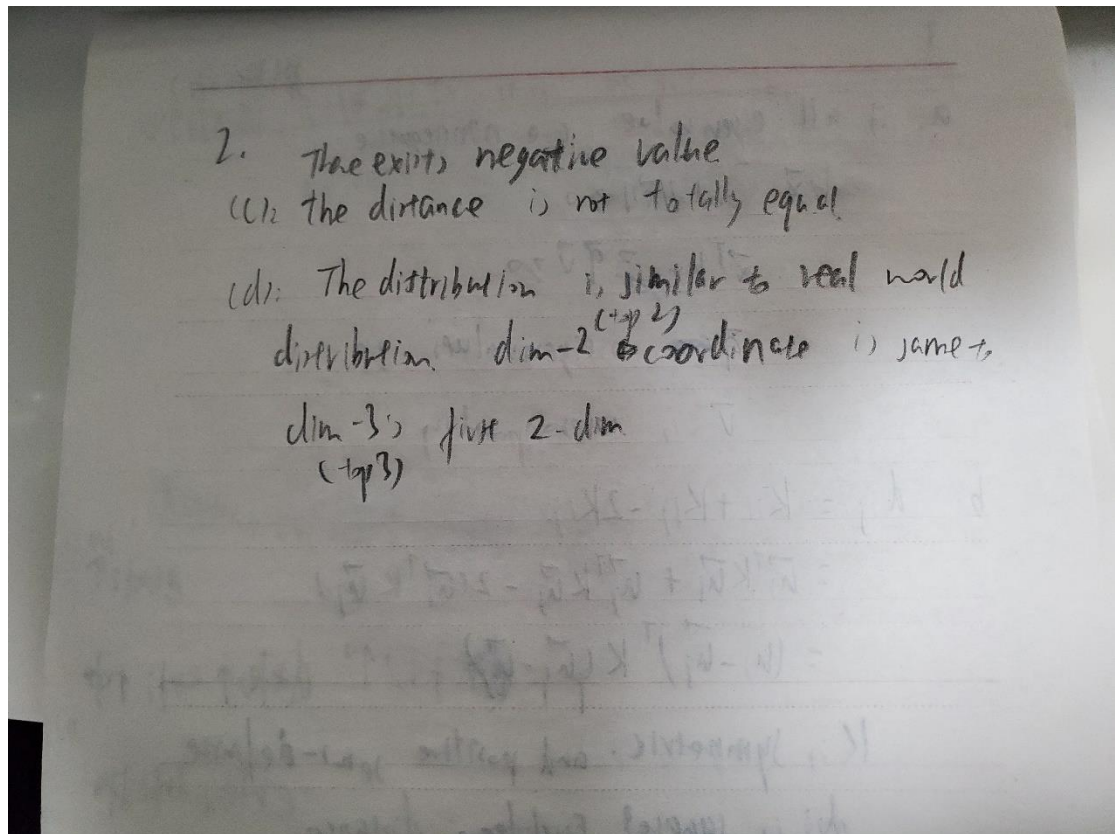


2:



3:

3.

a: if all eigenvalue are nonnegative

$$\|\vec{x}\| \text{ and } \|\vec{x}'\| \geq 0$$

$$\vec{x}'^T K \vec{x} = \vec{q}^T \vec{V} \geq 0$$

\vec{q} are eigenvalues ≥ 0

\vec{V} is corresponding

b: $d_{ij} = K_{ii} + K_{jj} - 2K_{ij}$

$$= \vec{u}_i^T K \vec{u}_i + \vec{u}_j^T K \vec{u}_j - 2(\vec{u}_i^T K \vec{u}_j)$$

$$= (\vec{u}_i - \vec{u}_j)^T K (\vec{u}_i - \vec{u}_j)$$

K is symmetric and positive semi-definite

d_{ij} is squared Euclidean distance

d_i

4:

4.

(a): No

$$d(x_1, x_2) = 4 \quad d(x_1, x_3) = 4$$

$$d(x_2, x_3) = 7$$

$4+4 > 7$ satisfies triangle inequality.

$16+16 < 49$ so d^2 is not distance function

(b): Yes

1. $d(x, y) \geq 0 \rightarrow \sqrt{d(x, y)} \geq 0$

2. ~~\forall~~ $\forall d(x, y) = 0 \Rightarrow \sqrt{d(x, y)} = 0, x = y$

3. $d(x, y) = d(y, x)$
 $\sqrt{d(x, y)} = \sqrt{d(y, x)}$

4. Given arbitrary $d_1 \leq d_2 + d_3$

$$\sqrt{d_1} \leq \sqrt{d_2} + \sqrt{d_3}$$

$$\downarrow$$
$$d_1 \leq d_2 + d_3 + 2\sqrt{d_2 d_3}$$