Report: Portfolio Comparison with PCA

Qing Du, Jiahao He

Abstract

When we apply principal component analysis on daily stock returns, each component corresponds to a weight vector over the stocks and can be regarded as a portfolio. Moreover, the value of the principal portfolios are uncorrelated. Combining uncorrelated assets helps increase the diversification and thus reduce risk exposure. We consider 3 portfolios. The first one consists of stocks, where the long/short direction is in favor of the holder and the holding volume is proportional to the trading volume of the stocks. The rest of them are built on principal portfolios along with traded volume information. In this report, we compare the performance of these portfolios. Reproducible codes of this report are available at https://github.com/qdddddd/csic5011_project1.

1 Problem Definition

In this project, we analyse SNP'500 data. Let $X = (X_1, X_2, \dots, X_T) \in \mathbb{R}^{n\times T}$ where X_{it} represents the log return of stock i on day t. If we apply PCA on X, $X = U\Sigma V$, component u_i is a weight over the set of stocks and can be regarded as a portfolio. Moreover, the returns of the principal components are uncorrelated. Let $V = (V_1, \dots, V_n)^T \in \mathbb{R}^n$ where V_i is the total traded volume of stock i in the total T trading days, we construct a number of simple assets as follows.

1. $\mathbf{a} = (a_1, a_2, \dots, a_n)^T$ where a_i represents the normalized position of stock i and is given by
$$a_i = \operatorname{sign}\left(\sum_{t=1}^{T} X_{it}\right) \frac{V_i}{\sum_{j=1}^{n} V_j},$$
i.e., we long(short) a stock whose cumulative return is positive(negative) to a position that is proportional to its traded volume. We refer to this asset as the volume-weighted stock portfolio.
2. $\mathbf{b} = \frac{u_1}{\|u_1\|_1}$, i.e., hold the first principal portfolio.
3. Pick the first m principal portfolios $u_1, u_2, \dots, u_m$ and hold
$$\operatorname{sign}\left(\sum_{t=1}^{T} u_k^T X_t\right) \frac{u_k^T V}{\sum_{j=1}^{n} u_j^T V}$$
proportion of u_k, i.e., the normalized position of stock i is given by $c_i := \sum_{k=i}^{n} u_{ki} \operatorname{sign}\left(\sum_{t=1}^{T} u_k^T X_t\right) \frac{u_k^T V}{\sum_{j=1}^{n} u_k^T V}$. We refer to this asset as the volume-weighted principal portfolio. Note that when $m = n$, $\mathbf{c}$ reduces to $\mathbf{a}$.

2 Principal Portfolios Visualization

We first report the explained variance ratio of the principal components, based on which we select an appropriate m, the number of principal portfolios we used to construct our volume-weighted principal portfolio. We then visualize the first few principal components as weights in different stocks.

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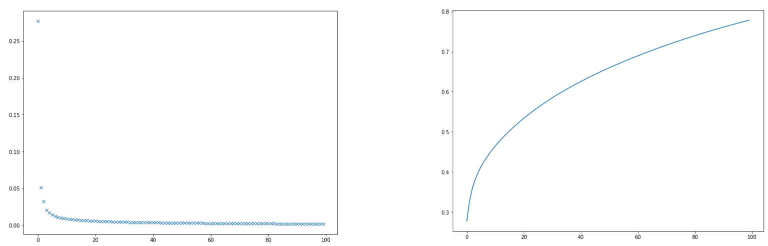
2.1 Explained Variance Ratio

Figure 1 reports the explained variance ratio of the first 100 components. We also observe that the first component explains 26.94% of the total variance, the first 10 components explain 44.59% of it, and the first 112 components explain 80% of it. We will choose $m = 1$ and 10 in our volume-weighted principal portfolio.



(a) Explained Variance Ratio (b) Cumulative Explained Variance Ratio

Figure 1: Explained Variance

2.2 Largest Components and Low Dimensional Representation of Daily Return

Figures 2–4 report the first two components. We observe that all weights in the first component are positive and stocks with high weights in the first few components also have high volatility. [h!]

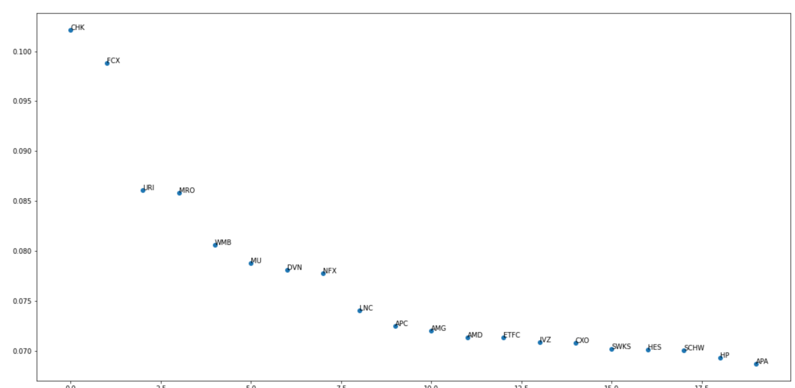


Figure 2: Stocks with the Largest Weights in Component 1

3 Comparison Among the Nominated Portfolios

We now compare the value of our nominated portfolios. As we anticipate the sign of total of the total return when constructing our portfolio, it is not surprising that all portfolios will have positive total return. We report the profit and loss curve in Figures 5 and 6 and their Sharpe ratio in Table 1.

Asset	$\mathbf{a}$	$\mathbf{b}$	$\mathbf{c}, m = 1$	$\mathbf{c}, m = 10$
Sharpe ratio	1.07	1.05	1.027	0.69

Table 1: Sharpe Ratio

4 Conclusion

From the experiments, it shows that the best performance is given by constructing the portfolio directly by the weights given by the volumes. The Sharpe Ratio is higher when constructing the

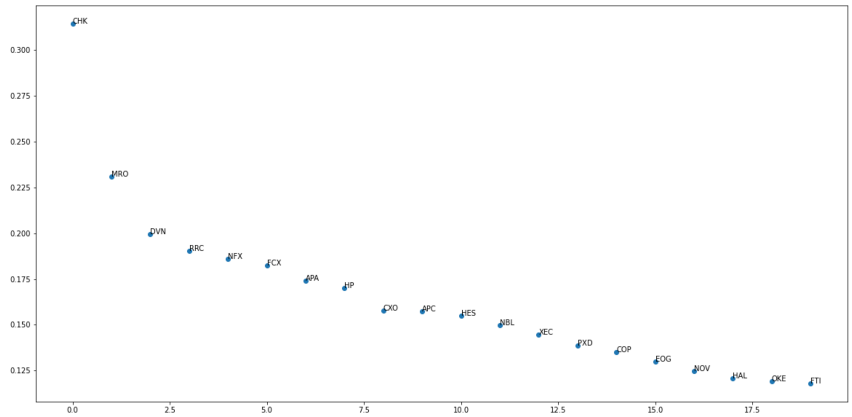


Figure 3: Stocks with the Largest Positive Weights in Component 2

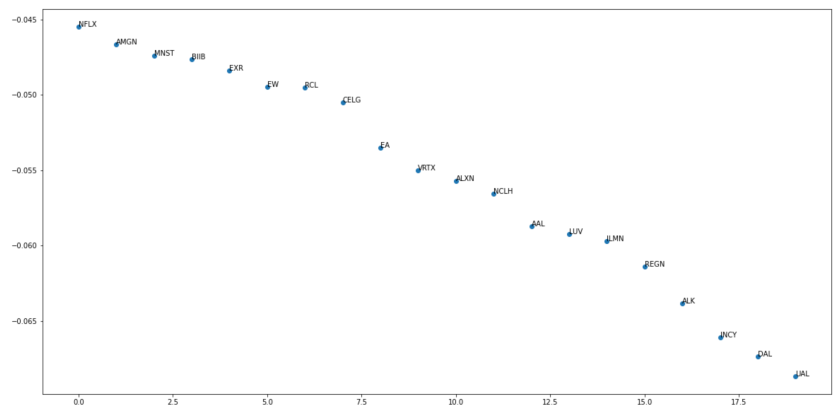
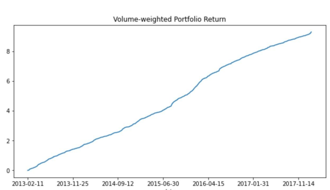
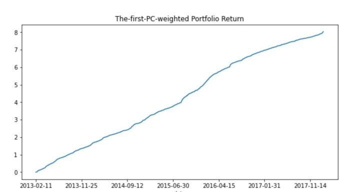


Figure 4: Stocks with the Largest Negative Weights in Component 2



(a) Volume-weighted Stock Portfolio



(b) First Principal Portfolio

Figure 5: Profit and Loss for $\mathbf{a}$ and $\mathbf{b}$

portfolio according to the weights given by the principle components, since it weighs more on the stocks that has higher return variance, and thus is more risky.

Remarks In this project, we both contribute to the idea of the research topic. Jiahao He wrote most of the report and also the coding. Qing Du was responsible for the calculation and the plots of the experiment results.

(a) Volume-weighted Principal Portfolio, $m = 1$

(b) Volume-weighted Principal Portfolio, $m = 10$

Figure 6: Profit and Loss for **c** with $m = 1, 10$