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Applications of Quantitative Methods in Financial Market

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Biography

- MSc in Financial Mathematics from HKUST
- MSc in Information and Computational Science from Sun Yat-sen University
- CFA
- FRM
- SFC representative licenses of type 1, 4 & 9
- Senior Quantitative Researcher
    - Alpha model development
    - Risk management
    - Portfolio construction
    - Portfolio implementation
    - Performance attribution

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Company introduction

- Magnum Research Limited - AQUMON
- Founded in 2016
- SFC License of Type 1, 4 & 9
- Robo-advisor & asset management firm
- Automation of KYC, advisory, execution, portfolio management
- Business:
  - Advisory
  - Wealth management
  - Asset management

Modern Portfolio Theory

- Development of Modern Portfolio Theory (MPT)
  - Risk-return tradeoff
  - Capital Market Pricing Model (CAPM)
  - Arbitrage Pricing Theory (APT)
  - Market anomalies
  - Behavioral finance

Modern Portfolio Theory

1. Asset allocation
   a. Mean-variance optimization
2. Asset pricing and factor investment
   a. Capital Asset Pricing Model (CAPM)
   b. Arbitrage Pricing Theory (APT)
   c. Fama-French three factor model
   d. Factor zoo
   e. Non-linear model
   f. Multi-factor investing

Asset Allocation

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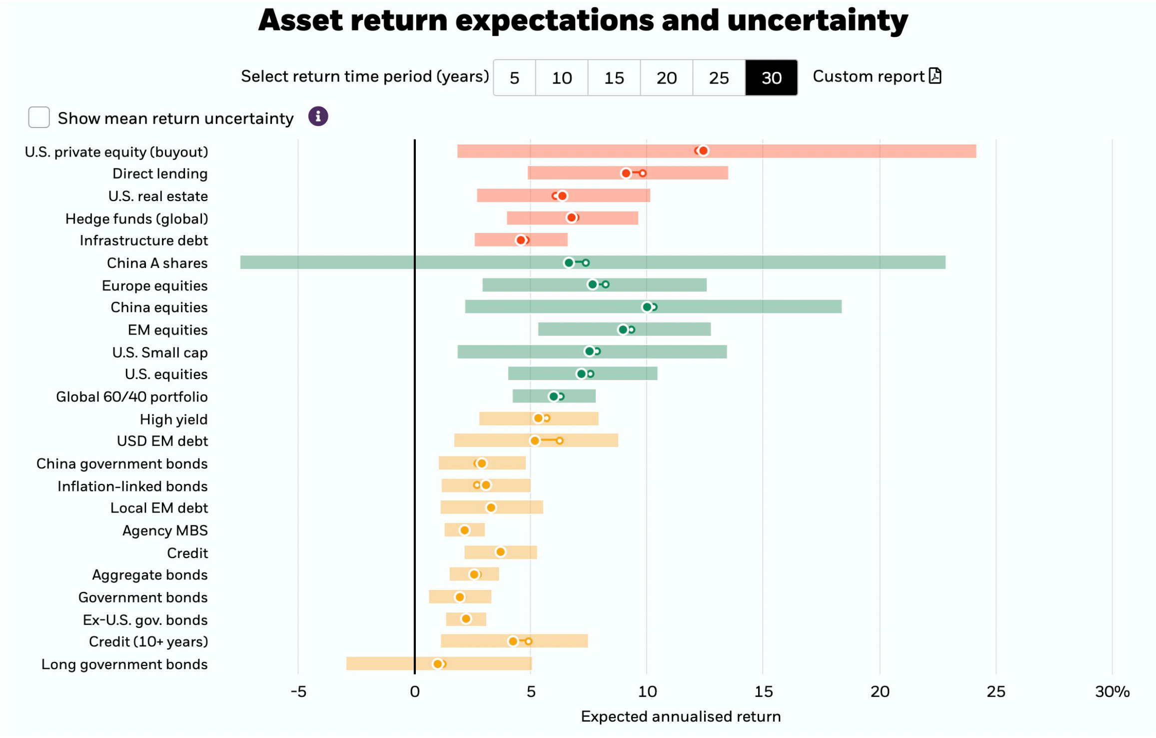
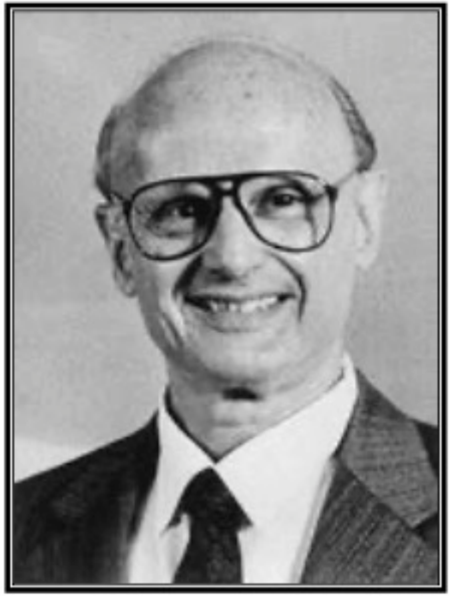
Asset Performance

- Asset classes
  - US Bonds
  - Int'l Bonds
  - US Growth Equity
  - US Value Equity
  - Developed Market Equity
  - Emerging Market Equity
- Historical asset returns
- All-in to US private Equity?



Return vs. Risk

- "... If the investor were only interested in expected values of securities, he or she would only be interested in the expected value of the portfolio, and to maximize the expected value of a portfolio one need invest only in a single security..."
- "... This, I knew, was not the way investors did or should act. Investors diversify because they are concerned with risk as well as return. Variance came to mind as a measure of risk..."
- "... The fact that portfolio variance depended on security covariances added to the plausibility of the approach. Since there were two criteria, risk and return, it was natural to assume that investors selected from the set of ... optimal risk-return combinations."



Harry Markowitz,
Nobel Prize 1990

Benefit of Asset Allocation

The following example illustrates that the portfolio of assets may achieve higher risk adjusted return compared to the component assets. Example of two assets:

Asset	Expected Return	Volatility
A	10%	15%
B	10%	20%

Equally weighted portfolio, i.e. 50% of asset A and 50% of asset B, but with different correlation:

Correlation	Expected Return	Volatility
1	10%	17.5%
0.5	10%	15.2%
0	10%	12.5%
-0.5	10%	9.0%
-1	10%	2.5%

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Asset Allocation

- Modern Portfolio Theory (Markowitz, 1952)
    - Reward: Mean return
    - Risk: Variance

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Simple Case

Consider a portfolio of two assets. Let

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

be their mean and variances of returns.

Consider a portfolio with weights $\boldsymbol{w} = (w, 1 - w)^T$ which places weight w on asset 1 and $1 - w$ on asset 2. Then the mean and variance of this portfolio are

$$\mu_P = w\mu_1 + (1 - w)\mu_2 = \boldsymbol{w}^T \mu,$$

$$\sigma_P^2 = w^2\sigma_{11} + 2w(1 - w)\sigma_{12} + (1 - w)^2\sigma_{22} = \boldsymbol{w}^T \Sigma \boldsymbol{w}.$$

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General Case

Consider a portfolio of p assets. Let

$$\mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_p \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1p} \\ \vdots & \ddots & \vdots \\ \sigma_{p1} & \cdots & \sigma_{pp} \end{pmatrix}$$

be their mean and variances of returns.

Consider a portfolio with weights $\boldsymbol{w} = \left(w_1, \cdots, w_p\right)^T$ where w_i is the weight on asset i. Then the mean and variance of this portfolio are

$$\mu_P = \boldsymbol{w}^T \mu,$$
$$\sigma_P^2 = \boldsymbol{w}^T \Sigma \boldsymbol{w}.$$

Mean-variance Optimization

Suppose we set a target of mean return μ_0 and wish to minimize the risk. It is an optimization problem:

$$\text{argmin}_{\boldsymbol{w}}\ \boldsymbol{w}^T \Sigma \boldsymbol{w} \quad s.t.\ \boldsymbol{w}^T \mu = \mu_0.$$

A different but equivalent formulation is, we set a target risk of σ_0^2 and wish to maximize the mean return:

$$\text{argmax}_{\boldsymbol{w}}\ \boldsymbol{w}^T \mu \quad s.t.\ \boldsymbol{w}^T \Sigma \boldsymbol{w} = \sigma_0^2.$$

By introducing λ as risk aversion parameter, another equivalent formulation is

$$\text{argmax}_{\boldsymbol{w}}\ \boldsymbol{w}^T \mu - \frac{1}{2} \lambda \boldsymbol{w}^T \Sigma \boldsymbol{w}.$$

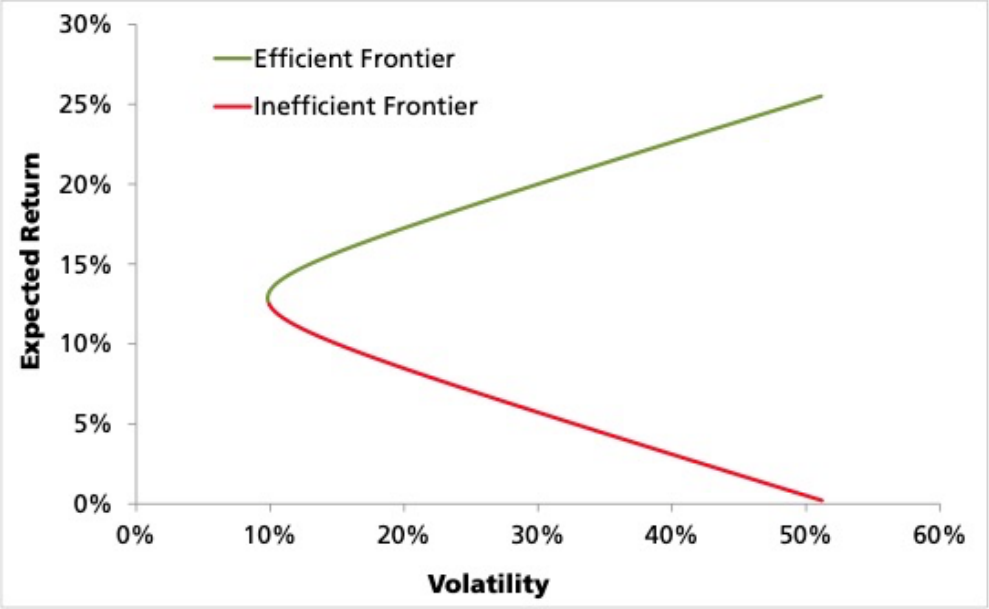
Analytical Solution

With the first order condition, the analytical solution is

$$\boldsymbol{w} = \frac{1}{\lambda}\Sigma^{-1}\mu.$$

Efficient Frontier

Efficient portfolio: the one showing the highest expected return given the variance.
Efficient frontier: the set of all the efficient portfolio.



MVO with Constraints

The above general case of Markowitz's mean-variance portfolio allows short sell. In other words, the weights can be negative. In reality, many assets are traded under constraints, typically the no-short-sell constraint.

With the no-short-sell constraint, the weights must be non-negative. Another common constraint is the no-leverage constraint. To make sure no leverage, the sum of weights must be 1. Thus, the efficient portfolio is the optimization of

$$\operatorname{argmax}_{\boldsymbol{w}} \boldsymbol{w}^T \mu - \frac{1}{2} \lambda \boldsymbol{w}^T \Sigma \boldsymbol{w},$$
$$s.t.\ \boldsymbol{w}^T \mathbf{1} = 1,$$
$$\boldsymbol{w} \geq 0.$$

MVO with Constraints

Many funds restrict the holding of one single asset in their portfolios to be lower than a given threshold, say 5%. Then, the efficient portfolio is the optimization of

$$\text{argmax}_{\boldsymbol{w}}\ \boldsymbol{w}^T\mu - \frac{1}{2}\lambda\boldsymbol{w}^T\Sigma\boldsymbol{w},$$
$$s.t.\ \boldsymbol{w}^T\boldsymbol{1} = 1,$$
$$\boldsymbol{0} \leq \boldsymbol{w} \leq 5\%.$$

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Considerations

- Risks are only represented by the portfolio variance.
- The solution of MVO is sensitive to the input parameter.
- Estimation difficulties for the input data (expected return and covariance)
- Transaction costs, tax and many other cost cannot be easily incorporated.

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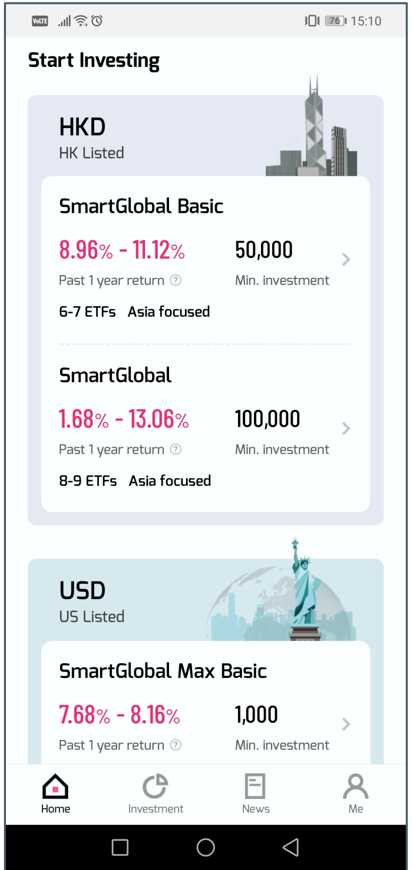
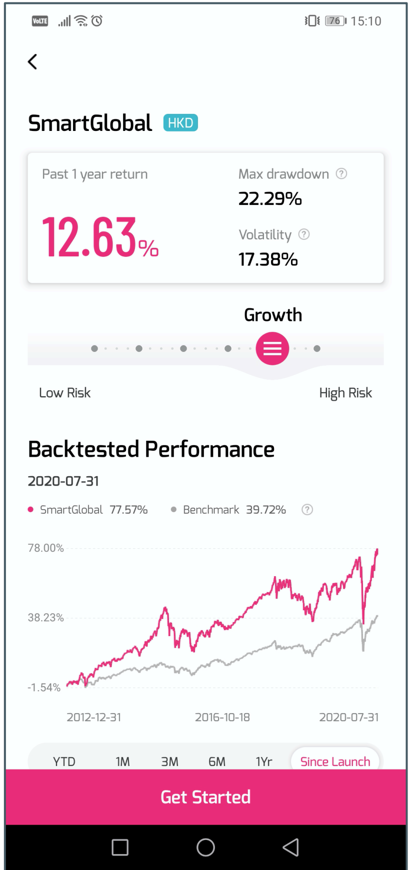
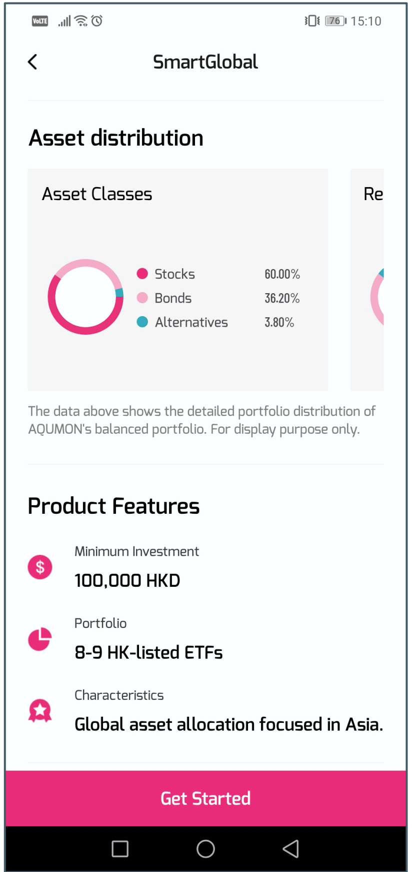
Application - Robo-advisory







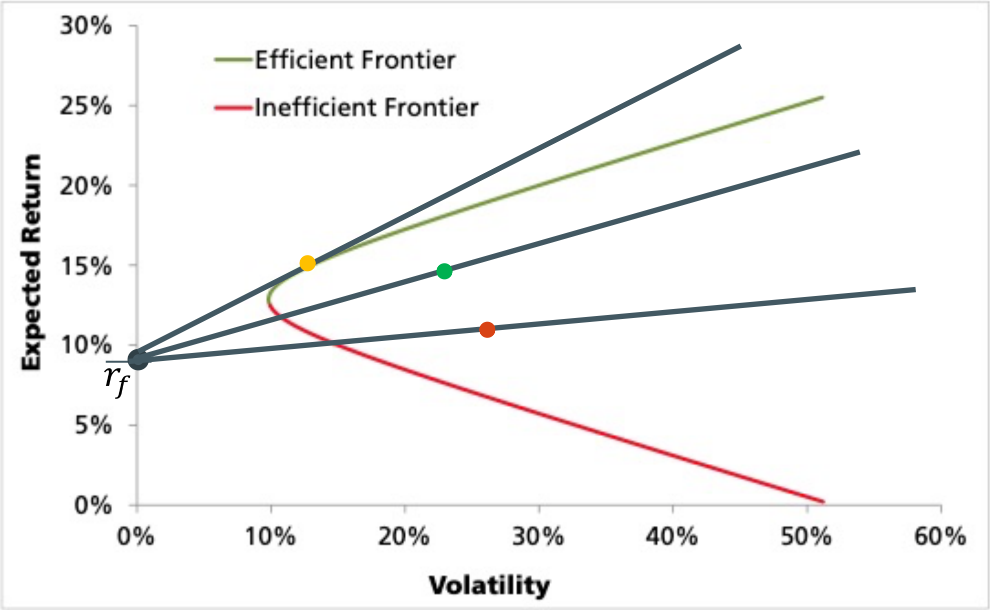
Asset Pricing and Factor Investment

Captial Asset Pricing Model (CAPM)

- What is the relation between the expected return and risk of an efficient portfolio
- What is the proper risk measure for an efficient portfolio
- What is the relation between the expected return and risk of a single security (or inefficient portfolio)
- What is the proper risk measure for a single securities (or inefficient portfolio)

Efficient Frontier

If introduce risk-free asset, how would a rational investor allocation between risk-free asset and risky asset?

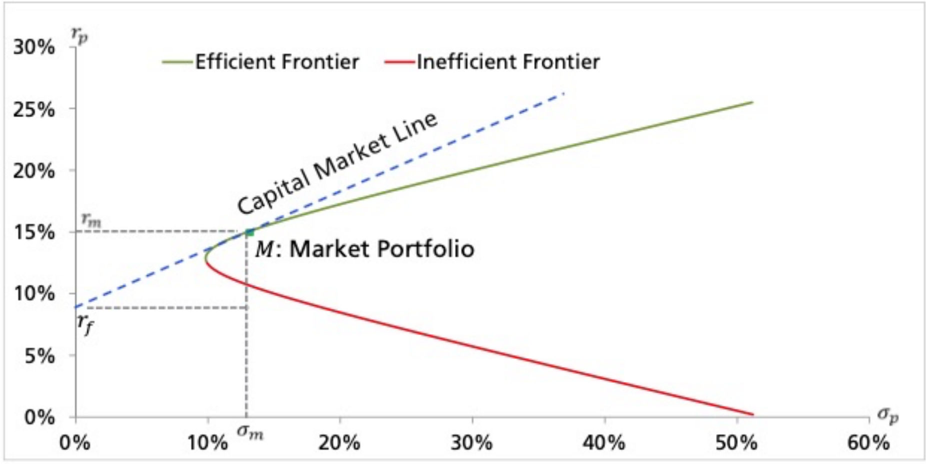


Capital Market Line (CML)

Capital Market Line (CML):

$$r_P = r_f + \frac{r_m - r_f}{\sigma_m}\sigma_P$$

where r_f is the risk-free rate; r_m and σ_m are the expected return and volatility of market portfolio; r_P and σ_P are the expected return and volatility of portfolio on the CML.



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Security Market Line (SML)

Proposition (Security Market Line): For any asset i, CAPM implies:

$$r_i - r_f = \beta_i(r_m - r_f)$$

And

$$R_i - r_f = \beta_i(R_m - r_f) + \epsilon_i$$

where $\beta_i = \mathrm{Cov}\,(R_m, R_i)/\sigma_m^2$ and $\mathrm{E}(\epsilon_i) = 0, \mathrm{Cov}(R_m, R_i)$. ϵ_i is the unsystematic/idiosyncratic risk.

Why the expected return model of inefficient portfolio is different CML?

- Unsystematic risk can be diversified away, but systematic risk cannot. Thus, only systematic risk have compensation.

CAPM Assumptions

1. Investors are rational (using expected return and volatility to evaluate portfolio) and risk averse
2. Efficient market: it implies that all investors have access to all the information
3. Assets are divisible; investors are allowed to buy any amount of assets
4. There is risk-free assets. Investors can borrow or lend at risk-free rate
5. No tax and transaction cost
6. Homogeneous expectations; all investors will have the same expectations (asset returns and covariances) and make the same choices given a particular set of circumstances
7. The economy is closed
8. Market is equilibrium

Considerations

- All investors have access to the same information and agree about the risk and expected return of all assets
- Variance in the only risk factor
- Missing risk factors: inflation, liquidity, non-financial asset like Human Capital, etc.

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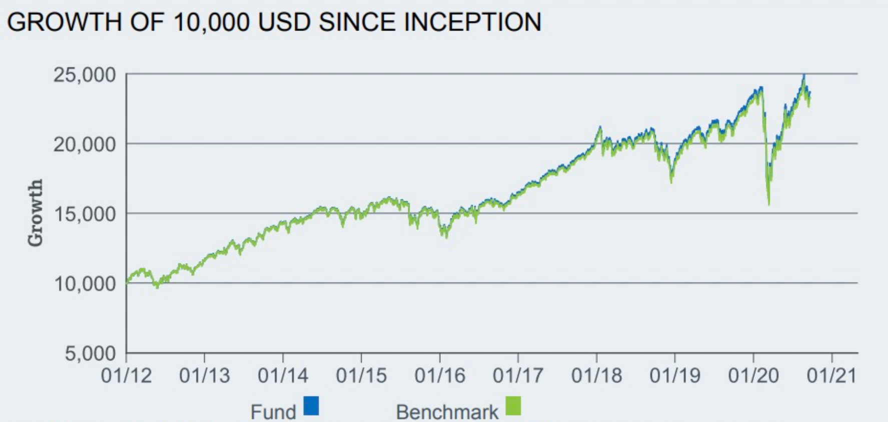
Application – Index Fund

- iShares MSCI World ETF (URTH)
- Fidelity Advantage Portfolio Fund - World Equity Index Fund
- Vanguard Global Stock Index Fund

Features:

- The target is to track index
- Management fee is much lower



The Growth of $10,000 chart reflects a hypothetical $10,000 investment and assumes reinvestment of dividends and capital gains. Fund expenses, including management fees and other expenses were deducted.

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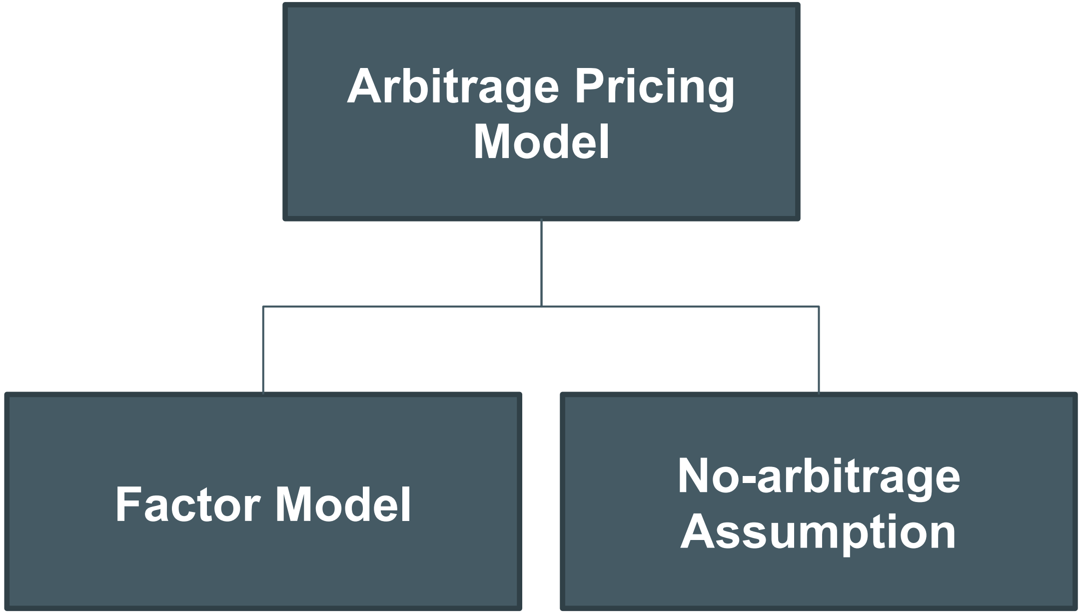
Arbitrage Pricing Theory (APT)



Factor Model

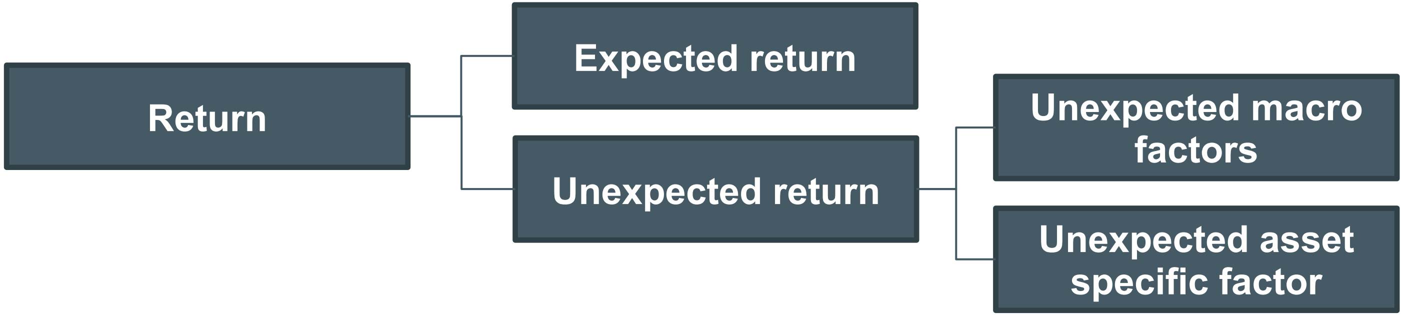
Asset's sensitivity to the factor

$$R_i = a_i + \sum_{k=1}^{m} b_{ik} f_k + \epsilon_i \,, i = 1, \ldots, n$$

Expected return of the asset

Unexpected macro events that have broad impact on asset returns

Unexpected asset specific events that have impact on the asset return



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Arbitrage

Arbitrage: is the practice to taking positive expected return from overvalued or undervalued securities in the inefficient market without any incremental risk and zero additional investments.

No-arbitrage principle: if there is no-arbitrage opportunities, the return for portfolio without any incremental risk and any additional investments will be zero.

Arbitrage activities → Price adjustment → Same price for same type of assets → No arbitrage opportunities

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APT Assumptions

- Perfectly competitive and frictionless capital market
- There exists large amount of assets so that investors can build a portfolio of assets where idiosyncratic risk is eliminated through diversification
- No arbitrage opportunity exists among well-diversified portfolios. If any arbitrage opportunities do exist, they will be exploited away by investors.

APT

The APT states that if asset returns follow a factor structure then the following relation exists between expected return and the factor sensitivities:

$$\mu_j = r_f + \lambda_{j1} RP_1 + \cdots + \lambda_{jn} RP_n,$$

where RP_k is the risk premium of the factor k, and r_f is the risk-free rate.

APT vs. CAPM

One factor vs. multi-factors

- In CAPM, risk on asset return is only related to market risk; and it doesn't explain where does the market risk comes from.
- APT admits that there are multiple risk factors which would potentially effect the asset return.

=> It provides a framework for portfolio risk analysis

Fama-French Three Factor Model

Fama-French Three Factors Model:

$$R_j - r_f = \beta_{j1}(R_M - r_f) + \beta_{j2}\mathrm{SMB} + \beta_{j3}\mathrm{HML} + \epsilon_j$$

where R_j is the return of security j, r_f is the risk-free return, R_M is the return on the value-weight market portfolio, SMB (standing for small minus big) is the return on a diversified portfolio of small stocks minus the return on a diversified portfolios of big stocks, HML (standing for high minus low) is the difference between the returns on diversified portfolio of high and low book-to-market stocks, and ϵ_j is a zero-mean error term.

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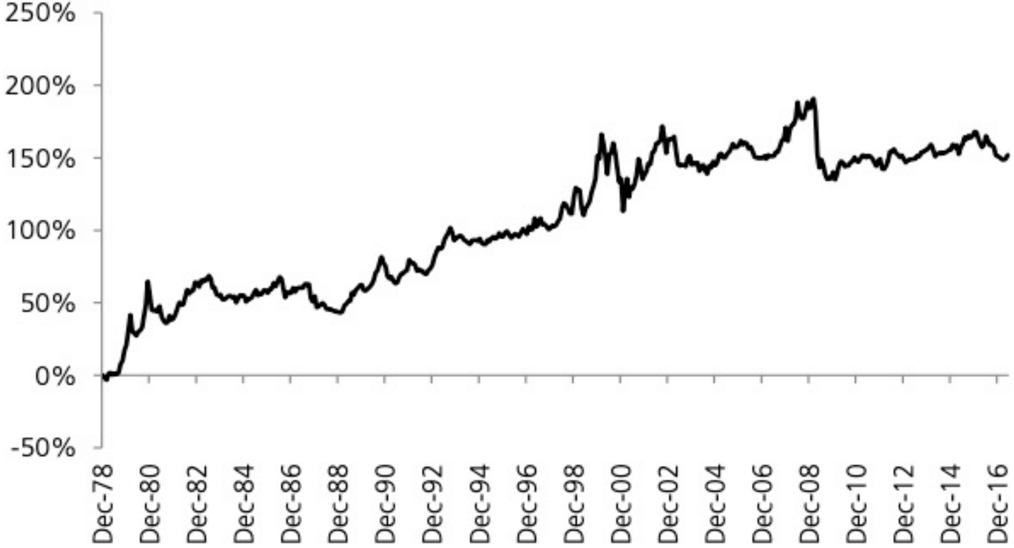
Anomalies and Factor Zoo

Financial Anormalies

- **Jegadeesh and Titman(1993) – 12m Price Momentum**



Note: The chart shows the performance of longing top 1/3 and shorting bottom 1/3 12m price momentum baskets. Back test is run on Russell 3000 index universe. Portfolios are rebalanced on monthly frequency and portfolio returns are weighted by market capitalization.

- Size factor
- Value factor
- Quality
- Momentum
- Volatility
- Dividend
- Illiquidity
- Leverage

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Application: SmartBeta Products

- Market factor
- Size factor
- Value facto
- Dividend factor

aggressiveness

Beta product
e.g. Index fund

SmartBeta product
e.g. SmartBeta fund

Alpha product
e.g. Active fund

Symbol	ETF Name	Asset Class	Total Assets	YTD	Avg Volume	Overall Rating	Factor Style
IWD	iShares Russell 1000 Value ETF	Equity	40,480,200,409.00	12.72%	1,748,637.0	A	Value
IWF	iShares Russell 1000 Growth ETF	Equity	40,089,358,296.00	29.22%	1,367,822.0	A+	Growth
VTV	Vanguard Value ETF	Equity	36,309,751,970.00	16.53%	1,310,537.0	A	Value
VUG	Vanguard Growth ETF	Equity	31,273,637,281.00	27.05%	561,283.0	A-	Growth
VIG	Vanguard Dividend Appreciation ETF	Equity	27,090,583,036.00	20.63%	468,623.0	A	Dividend
VYM	Vanguard High Dividend Yield ETF	Equity	21,182,957,886.00	15.82%	748,762.0	A+	Dividend
IVW	iShares S&P 500 Growth ETF	Equity	19,843,316,508.00	26.86%	644,860.0	A	Growth
DVY	iShares Select Dividend ETF	Equity	17,935,336,513.00	14.10%	502,972.0	B+	Dividend
SDY	SPDR S&P Dividend ETF	Equity	16,590,683,155.00	14.53%	385,134.0	B	Value
IVE	iShares S&P 500 Value ETF	Equity	15,204,987,288.00	14.41%	622,608.0	A	Value
USMV	iShares Edge MSCI Min Vol USA ETF	Equity	15,099,385,814.00	18.68%	1,294,229.0	A	Volatility
RSP	Guggenheim S&P Equal Weight	Equity	14,870,584,664.00	17.48%	447,762.0	B+	Size, Value
VBR	Vanguard Small Cap Value ETF	Equity	12,500,638,434.00	10.54%	315,068.0	A+	Size, Value
IWS	iShares Russell Midcap Value ETF	Equity	11,031,164,156.00	11.92%	441,454.0	A	Size, Value

Non-linear Models

- ordinary least squares (OLS)
- partial least squares (PLS)
- principal components regression (PCR)
- elastic net (ENet)
- generalized linear model with group lasso (GLM)
- random forest (RF)
- gradient boosted regression trees (GBRT)
- neural network (NN)

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Non-linear Models

Empirical Asset Pricing via Machine Learning

Data sources: CRSP for all firms listed in the NYSE, AMEX, NASDAQ

Research Period: 1957 March to 2016 December

Independent variables: 94 characteristics, 74 industry dummies

Other data: 8 macroeconomic variables

Throughout our analysis, we define the baseline set of stock-level covariates $z_{i,t}$ as

$$z_{i,t} = x_t \otimes c_{i,t}, \tag{21}$$

where $c_{i,t}$ is a $P_c \times 1$ matrix of characteristics for each stock i, and x_t is a $P_x \times 1$ vector of macroeconomic predictors (and are thus common to all stocks, including a constant). Thus, $z_{i,t}$ is a $P \times 1$ vector of features for predicting individual stock returns (with $P = P_c P_x$) and includes interactions between stock-level characteristics and macroeconomic state variables. The total number of covariates is $94 \times (8+1) + 74 = 920$.

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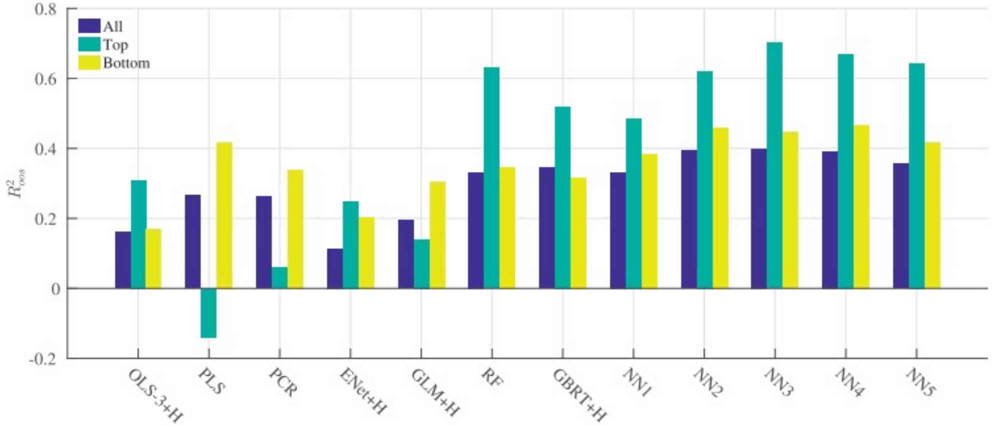
Non-linear Models

Table 1
Monthly out-of-sample stock-level prediction performance (percentage R^2_{oos})

	OLS +H	OLS-3 +H	PLS	PCR	ENet +H	GLM +H	RF	GBRT +H	NN1	NN2	NN3	NN4	NN5
All	–3.46	0.16	0.27	0.26	0.11	0.19	0.33	0.34	0.33	0.39	0.40	0.39	0.36
Top 1,000	–11.28	0.31	–0.14	0.06	0.25	0.14	0.63	0.52	0.49	0.62	0.70	0.67	0.64
Bottom 1,000	–1.30	0.17	0.42	0.34	0.20	0.30	0.35	0.32	0.38	0.46	0.45	0.47	0.42



In this table, we report monthly R^2_{oos} for the entire panel of stocks using OLS with all variables (OLS), OLS using only size, book-to-market, and momentum (OLS-3), PLS, PCR, elastic net (ENet), generalize linear model (GLM), random forest (RF), gradient boosted regression trees (GBRT), and neural networks with 1 to 5 layers (NN1–NN5). "+H" indicates the use of Huber loss instead of the l_2 loss. We also report these R^2_{oos} within subsamples that include only the top-1,000 stocks or bottom-1,000 stocks by market value. The lower panel provides a visual comparison of the R^2_{oos} statistics in the table (omitting OLS because of its large negative values).

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Non-linear Models

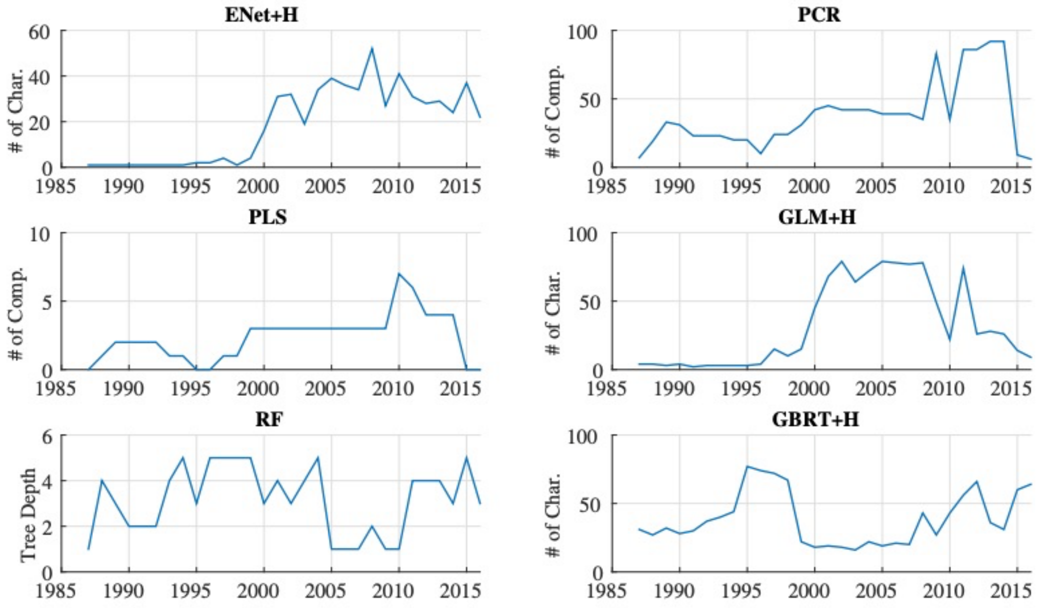

Figure 3
Time-varying model complexity
This figure demonstrates the model's complexity for elastic net (ENet), PCR, PLS, generalized linear model with group lasso (GLM), random forest (RF), and gradient boosted regression trees (GBRT) in each training sample of our 30-year recursive out-of-sample analysis. For ENet and GLM, we report the number of features selected to have nonzero coefficients; for PCR and PLS, we report the number of selected components; for RF, we report the average tree depth; and, for GBRT, we report the number of distinct characteristics entering into the trees.

Non-linear Models

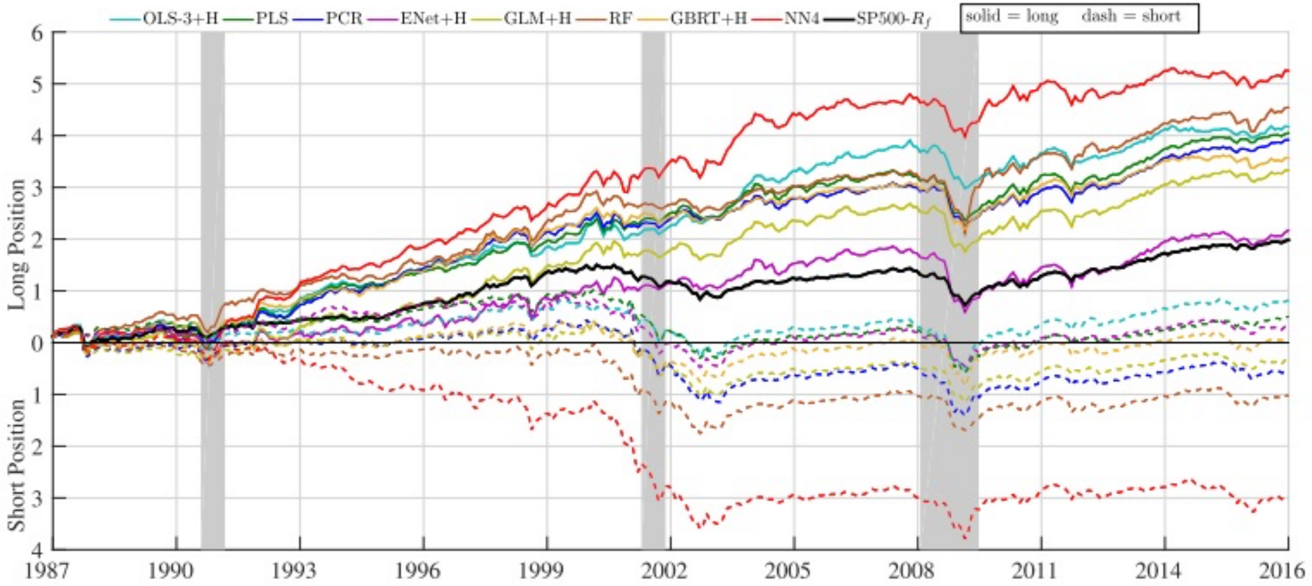

Figure 9
Cumulative return of machine learning portfolios
The figure shows the cumulative log returns of portfolios sorted on out-of-sample machine learning return forecasts. The solid and dashed lines represent long (top decile) and short (bottom decile) positions, respectively. The shaded periods show NBER recession dates. All portfolios are value weighted.

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Data snooping

Picking the factor that worked well in the back testing; but the factor itself doesn’t make any sense.

“If you torture the data long enough, it will confess to anything.”
--- Ronald Coase

Example: Random factor
AMB, AMC, AMD, AME…., BMC, BMD, BME,….,YMZ,
where XMY represents portfolio that long stocks whose name starting with X and short stocks whose name starting with Y.

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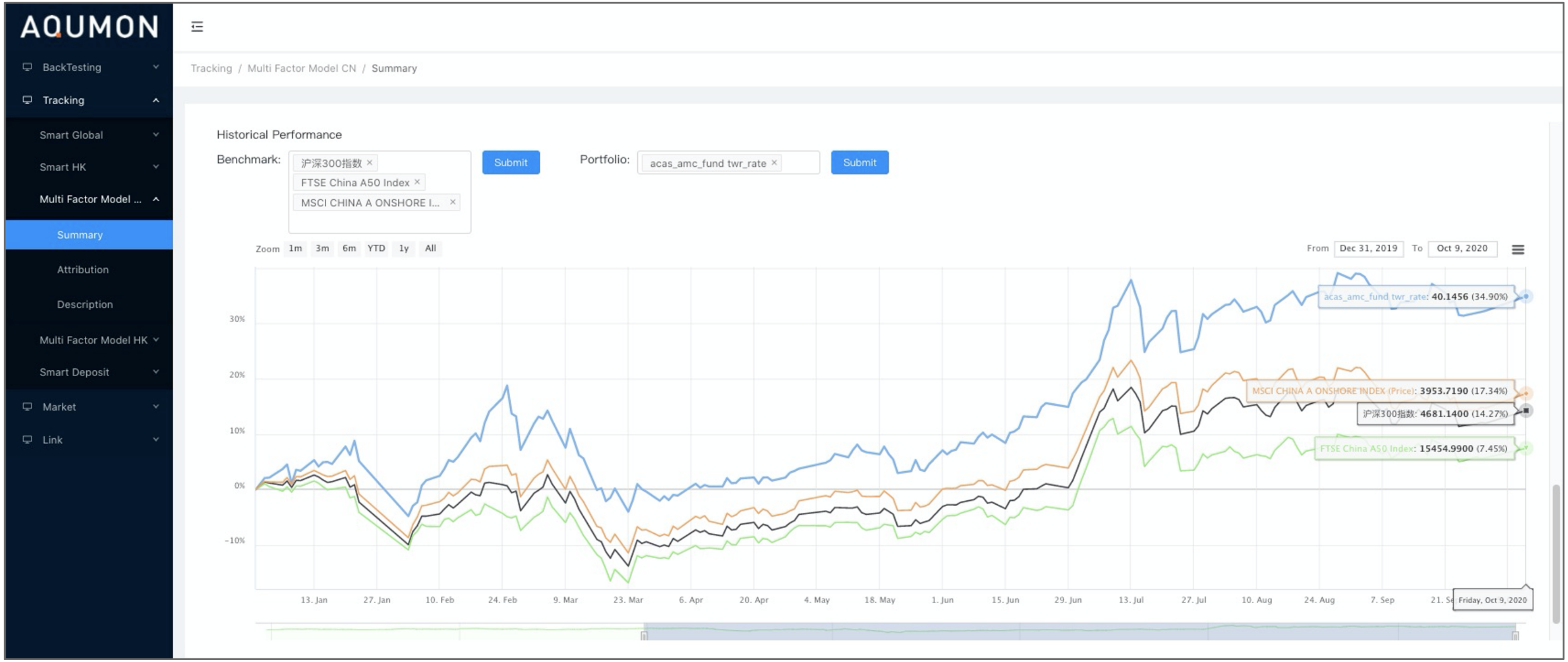
Application in Chinese A-share Market

- Multi-factor model + machine learning



Techniques Have Been Used Above

- Linear regression
- Machine learning models: Random forest, neural network, etc.
- Optimization

Apart from techniques, **background knowledge** is also critical.

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