Saheli Bhattacharya

HW5

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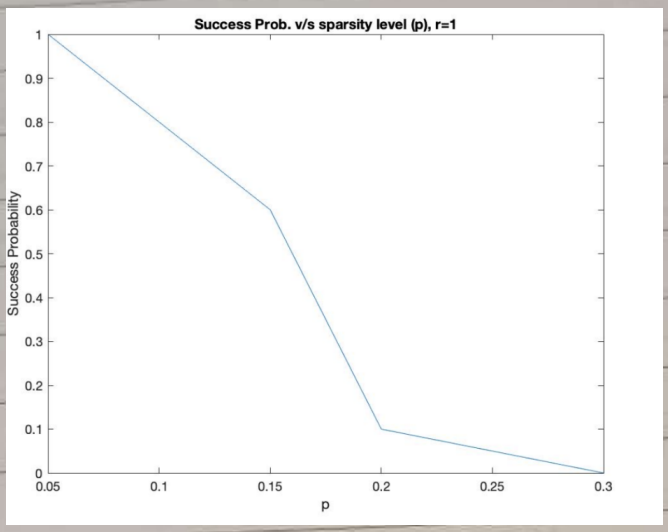
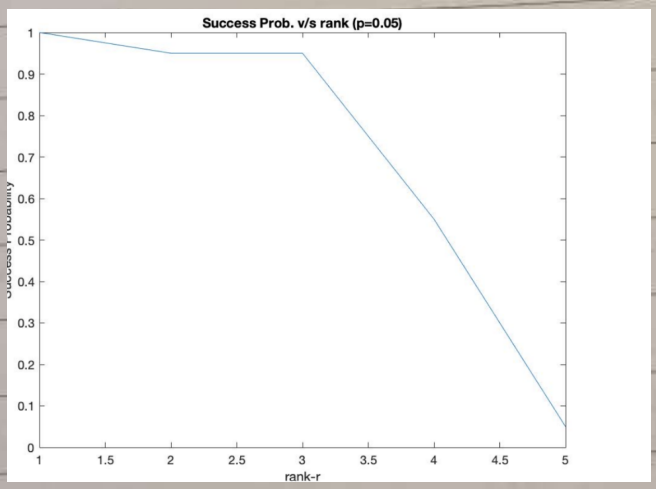
8)1) Success probability decreases as p increases, ie sparsity decreases




Success probability decreases as rank increases.

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8)2) $V_1 \sim N(0, 290)$ $V_2 \sim N(0, 300)$

a) $V_3 = -0.3V_1 + 0.525 V_2 + \epsilon \sim N(0,1)$

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_{10} \end{bmatrix}$$

$x_i = V_1 + \epsilon_i^1 \sim N(0,1)$ , $i = 1, 2, \dots 4$

$x_i = V_2 + \epsilon_i^2 \sim N(0,1)$ , $i = 5, \dots 8$

$x_i = V_3 + \epsilon_i^3 \sim N(0,1)$ , $i = 9, 10$

$\Sigma = E[x x^T]$

$X \rightarrow d \times n$

$H = \frac{I}{n \times n} - \frac{1}{n} \frac{11^T}{n}$

$\Rightarrow E(x_i^2) = 290 + 1 = 291$ ; $i = 1, \dots 4$

$E(x_i^2) = 300 + 1 = 301$ ; $i = 5, \dots 8$

$E(x_i^2) = (0.3)^2 \times 290 + (0.525)^2 \times 300 + 1 + 1$

$= 284.7875$, $i = 9, 10$

$E[x_i x_j] = E[V_1^2] = 290$ ; $i, j = 1, 2, \dots 4$, $i \neq j$

$E[x_i x_j] = 0$ $i = 1, \dots 4$ $j = 5, 6, \dots 8$

$E[x_i x_j] = E[V_2^2] = 300$, $i, j = 5, \dots 8$ $i \neq j$

$E[x_i x_j] = E[V_3^2] = (0.3)^2 + E[V_1^2] + (0.525)^2 E[V_2^2] + 1$

$i, j = 9, 10$ $i \neq j$

$= 26.1 + 256.68 + 1 = 283.78$

$E[x_i x_j] = -0.3 \times E[V_1^2] = -87$ ; $i = 1, \dots 4$, $j = 9, 10$

$E[x_i x_j] = 0.525 \times E[V_2^2] = 277.5$ ; $i = 5, \dots 8$, $j = 9, 10$

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$\Sigma =$

$$\begin{bmatrix} 291 & 290 & 290 & 290 & 0 & 0 & 0 & 0 & -87 & -87 \\ & 291 & 290 & 290 & 0 & 0 & 0 & 0 & -87 & -87 \\ & & 291 & 290 & 0 & 0 & 0 & 0 & -87 & -87 \\ & & & 291 & 0 & 0 & 0 & 0 & -87 & -87 \\ \text{Symmetric} & & & & 301 & 300 & 300 & 300 & 277.5 & 277.5 \\ & & & & & 301 & 300 & 300 & 277.5 & 277.5 \\ & & & & & & 301 & 300 & 277.5 & 277.5 \\ & & & & & & & 284.78 & 283.78 & \\ & & & & & & & & 284.78 & \end{bmatrix}$$

(b) See the Matlab code attached

(c) As $\lambda$ increases the error b/t the cvx soln & normal PCA decreases.

(d) see Matlab code Attached.