CSIC 5011 HW7

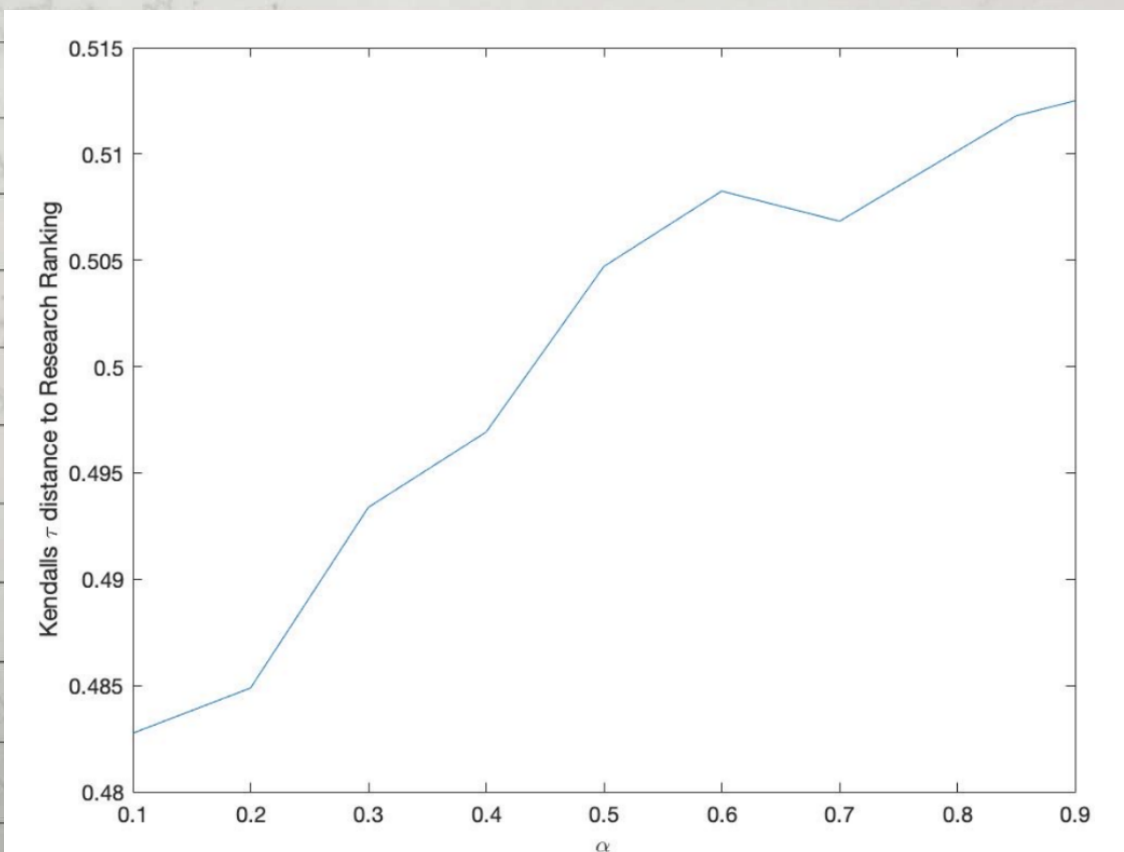
DATE: / / MO TU WE TH FR SA SU

Q) 1) a), b) see the Matlab code

Q) 1) (c) Kendall's τ distance of Google Page rank, HITS Hub & Authority ranking to Research ranking

$$\rho_{HITS\_Authority} > \rho_{Google\text{-}PR} > \rho_{HITS\_HUB}$$
$$(0.5741) > (0.5118) > (0.3885)$$

Q) 1) d) plot of Kendall's τ distance to research ranking vs α



It is observed that as α increases the Kendall's τ distance increases.

DATE: / / MO TU WE TH FR SA SU

Q) 2) $A > 0$ $\max \delta$

s.t $Ax \geq \delta x$

$x \geq 0$

$x \neq 0$

a) proof by contradiction

let us assume $Av^* \neq \lambda^* v^* \Rightarrow$ for some i,

$[Av^*]_i > \lambda^* v_i^*$ — ①

let $\tilde{v} = v^* + \epsilon e_i$, $\epsilon > 0$ & e_i is standard bases vector with 1 at ith entry.

Then for $j \neq i$,

$(A\tilde{v})_j = (Av^*)_j + \epsilon (Ae_i)_j = \lambda^* v_j^* + \epsilon A_{ji} > \lambda^* v_j^* = \lambda^* \tilde{v}_j$

since $A > 0$

for $j = i$:

$(A\tilde{v})_i = (Av^*)_i + \epsilon (Ae_i)_i > \lambda^* v_i^* + \epsilon A_{ii}$ → from ①

since, $\lambda^* \tilde{v}_i = \lambda^* v_i^* + \epsilon \lambda^*$ subtracting.

DATE: / / MO TU WE TH FR SA SU

$\Rightarrow (A\tilde{v})_i - (\lambda^* \tilde{v})_i + \epsilon (A_{ii} - \lambda^*) = (Av^*)_i - (\lambda^* v_i^*) - \epsilon(\lambda^* - A_{ii}) > 0$

$\Rightarrow$ Holds for small $\epsilon > 0 \Rightarrow$ for small $\epsilon > 0$, $(A\tilde{v}) > \lambda^* \tilde{v}$

Thus, λ^* is not optimal $\Rightarrow$ contradicts the assumption.

Hence, $\boxed{Av^* = \lambda v^*}$

b) To show $v^* > 0$ proof by contradiction:

let us assume for some k, $v_k^* = 0$, then $(Av^*)_k = \lambda^* v_k^* = 0$

But $A > 0$, $v^* \geq 0$ & $v^* \neq 0$, $\Rightarrow \exists i$, $v_i^* > 0$, which implies $Av^* > 0 \Rightarrow$ This contradicts previous conclusion

$\Rightarrow v^* > 0$ (Since $\lambda^* > 0$)

c) Show that for every $v \geq 0$, $Av = \mu v \Rightarrow \mu = \lambda^*$

Hence, A must have a left Perron vector $w^* > 0$, s.t $A^T w^* = \lambda^* w^*$

Then, $\lambda^* (w^{*T} v) = w^{*T} A v = \mu (w^{*T} v)$

Since $w^{*T} v > 0$ $(w^* > 0, v \geq 0)$ there must be $\lambda^* = \mu$, ie. λ^* is unique & v^* is unique

DATE: / / MO TU WE TH FR SA SU

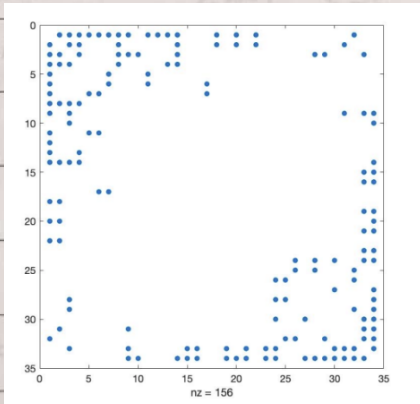
(d) for any other eigenvalue $Az = \lambda z$, $A|z| \geq |Az| = |\lambda||z|$ so $|\lambda| \leq \lambda^*$ Then using lecture notes, $\Rightarrow |\lambda| < \lambda^*$

Q)4) Karate Club Network: See the Matlab Code attached for implementation details

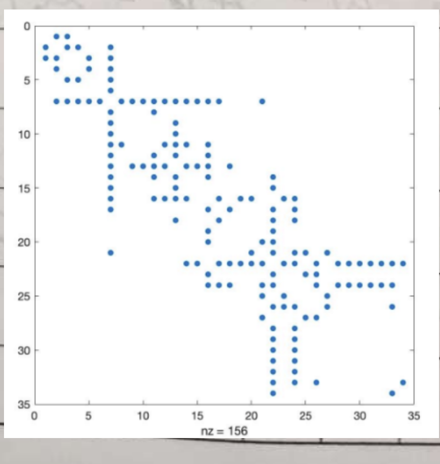
(a) $\lambda_2 = 0.4685$

(b) see the code

Adjacency Matrix: A →



As Sorted Adjacency Matrix →.



DATE: / / MO TU WE TH FR SA SU

(c) $\alpha_f = 0.1467$

$S^* = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 17, 18, 20, 22\}$

(d) yes, $\lambda_2 > \alpha_f$

(e) $h_{S^+} = 0.1515$

$S^+ = \{1, 2, 4, 5, 6, 7, 8, 11, 12, 13, 14, 17, 18, 20, 22\}$

It can be seen that the suboptimal cut S^+ differs from the optimal cut S^* in terms of 2 nodes which are missing in $S^+ \Rightarrow \{3, 9\}$

(f) see Matlab code attached.