Robust PCA and Its Video Applications

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Introduction

In many contemporary' industry fields (bioinformation, network traffic, image processing & computer vision), the obtained data to be analyzed often has high dimensionality and multivariate, which bring great challenges to data analysis, such as computational cost and computer memory requirements. Principal Component Analysis (PCA) or its robust extensions is one of the most popular low-rank matrix recovery tools for high-dimensional data analysis [1].

PCA was introduced by Karl Pearson in 1901 and first widely used in statistics, but this general method has many drawbacks. In order to address outlier corrupted data problem, recent years, people have been multiple attempts called robust PCA (RPCA), which replace the standard estimation of the covariance matrix with a robust estimator or by using projection pursuit techniques. Furthermore, Candes addressed the robustness by decomposing data matrix ***D*** into the sum of low-rank matrix ***L*** and a sparse matrix ***S*** (outlier matrix). Then, in order to avoid large subspace dimension, a better model is assumed that the tracking data lying in slowly changing subspace called Dynamic-RPCA or recursive-RPCA [2].

In current age, RPCA algorithm was applied with a great success in the image processing and video surveillance fields. The image processing applications include Low-level imaging (denoising, colorization, multi-focus image & face recognition), Medical imaging (reconstruction, data acquisition & real-time dynamic imaging for organs) and 3D motion recovery requiring camera position and manual alignment. For the video processing, RPCA problem formulations be mostly used in background/foreground separation, background initialization, moving target detection, denoising and video object segmentation [3].

In this project, we utilized Accelerated Alternating Projections PCA (AccAltProj) algorithm [4] & DECOLOR algorithm [5] to automatically analyze one given video, compared the results of these two methods, and shows the great Image recovery performance of RPCA algorithm.

Dataset

In this project, we used the video data from the website: https://drive.google.com/file/d/10-wwUl10fzzgvVF_YX0E1bEuU2Q9hGNG/view?usp=sharing.

RPCA Model

Part1. RPCA – (AccAltProj)

Compared with the traditional PCA, Robust PCA (RPCA), which introduces a sparse part, works well with recovering a subspace or low-rank matrix robustly when existing outliers. RPCA looks for the decomposition ***D*** = ***L*** + ***S***, where ***L*** is a low rank matrix, ***S*** is a sparse matrix. Then RPCA can be modeled as seeking ***L'*** and ***S'*** such that their sum fits ***D*** as close as possible:

$$\begin{aligned}&\min_{\boldsymbol{L}',\boldsymbol{S}'\in\mathbb{R}^{m\times n}} \left\|\boldsymbol{D}-\boldsymbol{L}'-\boldsymbol{S}'\right\|_F \\ &\text{subject to } \operatorname{rank}\left(\boldsymbol{L}'\right)\le r \text{ and } \left\|\boldsymbol{S}'\right\|_0 \le |\Omega|\end{aligned} \tag{1}$$

where $\|\cdot\|_F$ denotes the Frobenius Norm of matrices, $\|\cdot\|_0$ counts the number of non-zero entries in matrices, and Ω denotes the support set of the underlying sparse matrix (HQ Cai, 2019).
Accelerated Alternating Projections (AccAltProj), proposed by Cai (2019), focuses on the non-convex opti- mization for RPCA in (1)

Part 2. DECOLOR

There exist many formulations of RPCA, when modeling the outlier support explicitly, Zhou (2012) proposed a framework called DEtecting Contiguous Outliers in the LOw-rank Representation (DECOLOR). (It can be viewed as a ℓ_0-penalty regularized RPCA.)
Zhou proposed to minimize the following energy:

$$\min_{\boldsymbol{L},S_{ij}\in\{0,1\}} \frac{1}{2}\|\mathcal{P}_{\boldsymbol{S}^\perp}(\boldsymbol{D}-\boldsymbol{L})\|_F^2+\alpha\|\boldsymbol{L}\|_*+\beta\|\boldsymbol{S}\|_1+\gamma\|\boldsymbol{A}\,\mathrm{vec}(\boldsymbol{S})\|_1 \tag{2}$$

Here, $\boldsymbol{L} \in \mathbb{R}^{m\times n}$ denotes the underlying background images. $\boldsymbol{S} \in \{0, 1\}^{m\times n}$ denotes the foreground support:

$$S_{ij}=\begin{cases}0, & \text{if } ij \text{ is background}\\ 1, & \text{if } ij \text{ is foreground}\end{cases}$$

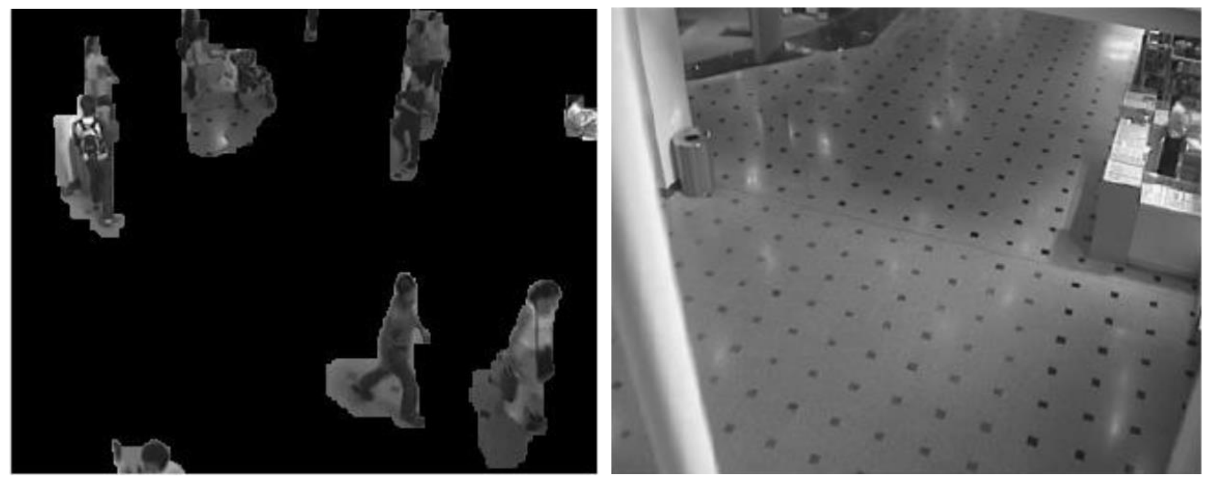
$\mathcal{P}_{\boldsymbol{S}}(\boldsymbol{X})$ represents the orthogonal projection of $\boldsymbol{X}$ onto the linear space of matrices supported by $\boldsymbol{S}$:

$$\mathcal{P}_{\boldsymbol{S}}(\boldsymbol{X})(i,j)=\begin{cases}0, & \text{if } S_{ij}=0\\ X_{ij}, & \text{if } S_{ij}=1\end{cases}$$

and $\mathcal{P}_S(\boldsymbol{X}) + \mathcal{P}_{S^\perp}(\boldsymbol{X}) = \boldsymbol{X}$.
$\|\cdot\|_*$ denotes the nuclear norm of matrices, $\|\cdot\|_1$ denotes the ℓ_1-norm of matrices, $\boldsymbol{A}$ is the node-edge incidence matrix of graph $\mathcal{G}$, $\alpha > 0$ is a parameter, $\beta, \gamma > 0$ are constants.

Results

Image recovery performance of DECOLOR (Input with missing pixels)



Foreground/background separation with DECOLOR

Results

Foreground/background separation with AccAltProj

Conclusion

In this project, we applied two RPCA algorithms, DECOLOR and AccAltProj, to address the video processing problem. It shows that the DECOLOR enables to achieve the image recovery perfectly (with 50% missing pixels input). And moreover, we find that both these two algorithms can separate the foreground component (sparse components) and background component (low-rank components). Compare with each other, DECOLOR performs better than AccAltProj method, which can subtract background image entirely. AccAltProj may be unable to achieve video foreground/background separation thoroughly, which still lefts some components in background image.

Discussion

Although the RPCA formulation has been greatly successful in many computer vision applications due to its robustness to outliers and flexibility to different types of outliers, there are still many significant issues need to be solved in the future work and applications. First concerns the guarantee for dynamic RPCA under even weaker assumption and need to pay more attention to unsampled dynamic RPCA and tensor RPCA method. Furthermore, we need to make RPCA achievement in more general computer vision problems, such as subspace clustering (or scalar subspace clustering) and phaseless RPCA & subspace tracking.

References

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Contribution

AccAltProj Model → Xueyang QUAN, Haoyi CHENG
DECOLOR Model → Zhiyuan YU