Notes on Prof. Yinyu Ye's Lecture on 5.8

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Some questions first

- How does the OLP algorithm defend the outlier when the algorithm is processing?
- Why does the second-order condition lead to a better result on the SNL problem?
- What is the algorithm to divide a whole polygon into several polygons to satisfy all the constraints, especially the equal-area constraint?

1 Summary & Further Thinking about the Questions

1.1 OLP

Though offline Linear Programming has achieved extreme success over the past decades, Online Linear Programming is still a meaningful and popular task since this task is more realistic, e.g., online selling or buying systems and advertising. In the online setting, at each time t, the coefficients (r_t, a_t)(reward, constraints) are revealed, and we need to decide the value of x_t instantly. The key to solving the OLP is how to leverage the partial information to construct a policy that is close to the overall best solution. The algorithms and theoretical analysis proposed by Prof. Ye mainly depend on the dual problem which leads to the dual convergence analysis and the action-history-dependent algorithm. Some applications like advertising and distributing in the real world just prove the efficiency of the algorithm.

My question about the algorithm is how the algorithm can defend the outlier to make the decision more reliable. In the original algorithm, there is no specific constraint to detect or remove the outlier. And the convergence analysis is based on the large n and the distribution with some regularity conditions. But in Q&A section, Prof. Ye provided a possible solution that we can formulate the outlier detection as online linear programming and based on the results from the outlier-related OLP we can solve the original task with outliers.

1.2 Second-Order Optimization Method

The second section describes some second-order optimization methods like RDSOM. Compared to the previous second-order method, the algorithm is more efficient that has a smaller number of total computation operations. This method shows significant results in application in sensor network localization.

My question is why a solution with better second-order conditions can give a better result on SNL problems. Following the description in the paper and the talk, I think the better second-order conditions may give a solution lies on a better manifold which can give some optimums more close to the global optimum.

1.3 Decomposition

Another problem is decomposition (divide-and-conquer) which is introduced by solving an equitable convex partition problem. The problem aims at partitioning a polygon with n points inside that the partition should satisfy 1) one point in one region, 2) each region should still be a polygon, and 3) each region has the same area. Some proof has been done to show that such a partition always exists. Prof. Ye, Carlsson, Armbruster propose an algorithm to find such a partition in $O(nN \log N)$ time. After checking the paper, the key idea to developing the algorithm could be divide-and-conquer by using two lemmas recursively (depending on whether n is even or odd). Actually, the proof is so sophisticated for me to understand them in a short time and I think it will be a good further reading to learn such partition tasks.