# Order the Face Images by Manifold Learning

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# Introduction

- Ordering orientation of unstructured images
  - Helps estimate the right motion of object or camera
  - Applications:
    object tracking, multi-view reconstruction, etc.

- Similarity Measurement and Dimensinality Reduction

# Objective

- Order the 33 face images using different manifold learning methods
- Compare performances of these methods
- Conduct parameter test for some methods

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# Dataset

Figure 1: The ground truth ordered face images

Table 1: Labelled rank number of ground truth face images

| Image | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rank | 9 | 13 | 19 | 32 | 6 | 18 | 28 | 7 | 17 | 1 | 5 | 16 | 12 | 10 | 4 | 21 | 22 |

| Image | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rank | 26 | 33 | 11 | 2 | 24 | 3 | 27 | 29 | 23 | 14 | 30 | 31 | 20 | 15 | 25 | 8 |

The dataset contains 33 face images ($X \in \mathbb{R}^{112 \times 92 \times 33}$).

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# Methodology

- 6 manifold learning methods
  - Diffusion Map (DM)
  - Multi-Dimensional Scaling (MDS)
  - Isometric Mapping (ISOMAP)
  - Locally Linear Embedding (LLE)
  - Local Tangent Space Alignment (LTSA)
  - t-distributed Stochastic Neighbor Embedding (t-SNE)
- Compare methods
  - Convert the ground truth label and 6 predicted rank labels into hot matrices.
  - Calculate the Frobenius norm of the distance between predicted and ground truth hot matrices.
- Test parameters
  - Number of neighbors to consider for each point: 2, 4, 5, 8, 16, 32.
  - Test 3 methods: ISOMAP, LLE, and LTSA.

# 6 Methods I

- DM
  - Dataset, $X \in \mathbb{R}^{n \times p}$ $(\mathbb{R}^{33 \times 10304})$
    Gaussian kernel, $k(x, y) = \exp\left(-\frac{||x-y||^2}{\alpha}\right)$
    Diffusion matrix, $P = D^{-1}K$
  - Approximate diffusion distance
- MDS
  - Dissimilarity matrix of $X$, $D_{ij} = ||x_i - x_j||_2$
  - Optimize

$$\underset{p_1,\ldots,p_n}{\operatorname{argmin}} \sum_{i<j} \left(\|p_i - p_j\|_2 - D_{ij}\right)^2$$

# 6 Methods II

- ISOMAP
  - Construct a neighboring graph $G$
  - Compute the shortest paths distance $d_G(i,j)$
  - Construct a lower dimensional embedding. Apply the classical MDS to $D_G = \{d_G(i,j)\}$.
- LLE
  - Local Fit

$$\min \left\| x_i - \sum_{j \in N_i} w_{ij} x_j \right\|^2 \quad \text{s.t.} \quad \sum_{j \in N_i} w_{ij} = 1$$

  - Global Alignment

$$\min_Y \sum_i \left\| y_i - \sum_{j=1}^n W_{ij} y_j \right\|^2$$

# 6 Methods III

- LTSA
  - Given $x_1, x_2, ..., x_n \in \mathbb{R}^p$,
    Compute the k-nearest neighbours,
    Compute local SVD on neighbourhood
  - Search the nearest neighbors
  - Align the tangent space
  - Do eigenvalue decomposition
- t-SNE
  - Compute pairwise affinities with perplexity
  - Sample intitial solution
  - Compute low-dimensional affinities and gradient

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# Results

Table 2: The rank results of various manifold learning methods

| Rank | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ground Truth | 10 | 21 | 23 | 15 | 11 | 5 | 8 | 33 | 1 | 14 | 20 | 13 | 2 | 27 | 31 | 12 | 9 |
| DM | 10 | 21 | 23 | 15 | 11 | 5 | 8 | 1 | 33 | 14 | 20 | 13 | 2 | 27 | 31 | 12 | 9 |
| MDS | 10 | 21 | 23 | 19 | 14 | 20 | 5 | 11 | 33 | 8 | 4 | 13 | 29 | 1 | 28 | 15 | 25 |
| ISOMAP | 10 | 21 | 23 | 15 | 5 | 11 | 1 | 33 | 14 | 8 | 20 | 13 | 2 | 27 | 31 | 12 | 9 |
| LLE | 10 | 21 | 23 | 15 | 5 | 11 | 8 | 1 | 33 | 14 | 20 | 13 | 2 | 27 | 31 | 12 | 9 |
| LTSA | 10 | 21 | 23 | 15 | 11 | 5 | 33 | 8 | 1 | 14 | 20 | 13 | 2 | 31 | 27 | 12 | 9 |
| t-SNE | 21 | 10 | 3 | 31 | 26 | 23 | 15 | 30 | 6 | 27 | 22 | 13 | 17 | 9 | 1 | 14 | 16 |

| Rank | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ground Truth | 6 | 3 | 30 | 16 | 17 | 26 | 22 | 32 | 18 | 24 | 7 | 25 | 28 | 29 | 4 | 19 |
| DM | 6 | 3 | 30 | 16 | 17 | 22 | 26 | 32 | 18 | 24 | 7 | 25 | 29 | 28 | 4 | 19 |
| MDS | 27 | 2 | 7 | 24 | 9 | 18 | 31 | 12 | 32 | 6 | 17 | 26 | 30 | 16 | 3 | 22 |
| ISOMAP | 6 | 3 | 30 | 16 | 17 | 26 | 22 | 32 | 18 | 24 | 7 | 25 | 28 | 29 | 4 | 19 |
| LLE | 6 | 3 | 30 | 16 | 17 | 22 | 26 | 32 | 18 | 24 | 7 | 25 | 28 | 29 | 4 | 19 |
| LTSA | 6 | 3 | 30 | 16 | 17 | 26 | 22 | 32 | 18 | 24 | 7 | 25 | 25 | 29 | 4 | 19 |
| t-SNE | 18 | 32 | 5 | 12 | 33 | 28 | 2 | 25 | 7 | 8 | 11 | 29 | 24 | 20 | 19 | 4 |

Table 3: One-hot error of different algorithms

| Algorithm | Diffusion Map | MDS | ISOMAP | LLE | LTSA | t-SNE |
|---|---|---|---|---|---|---|
| Error | 3.46410 | 7.74560 | 3.16228 | 3.46410 | 2.82843 | 7.87401 |

# Results - DM & MDS

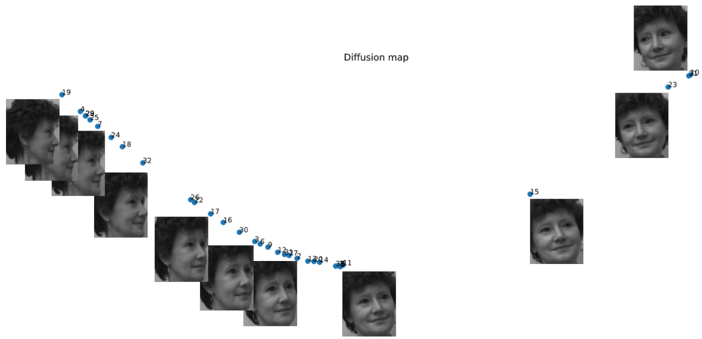


Figure 2: DM Scatter

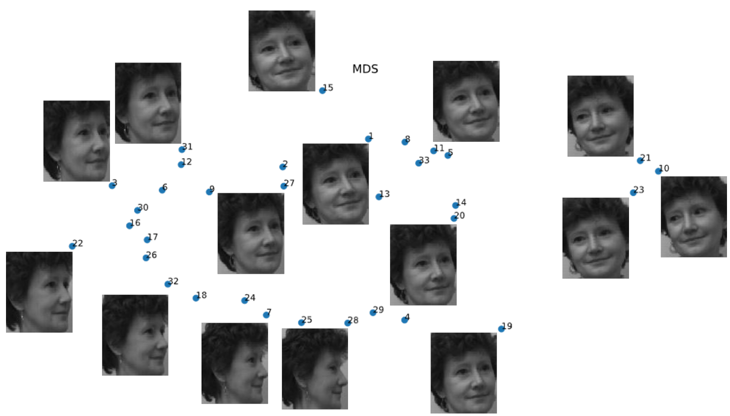


Figure 3: MDS Scatter

# Results - ISOMAP & LLE

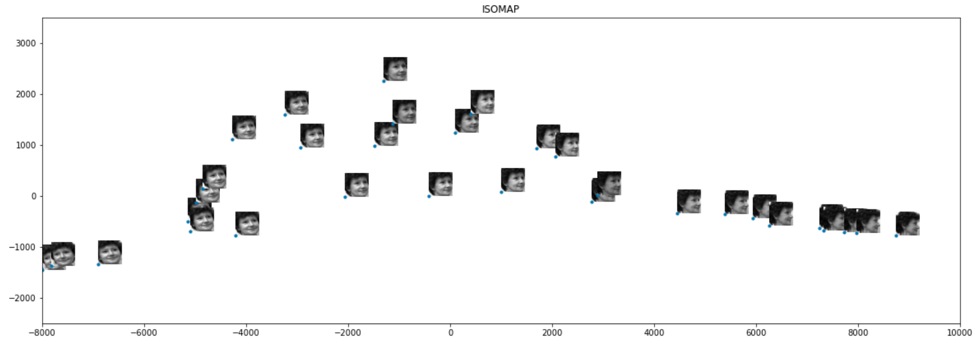


Figure 4: ISOMAP (K=5)

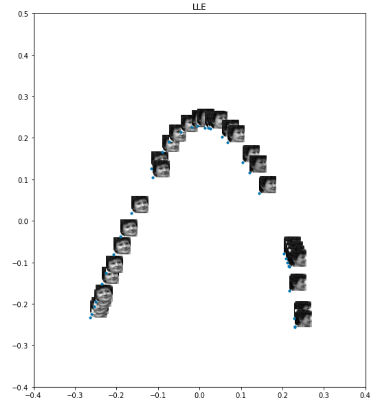


Figure 5: LLE (K=5)

# Results - LTSA & t-SNE

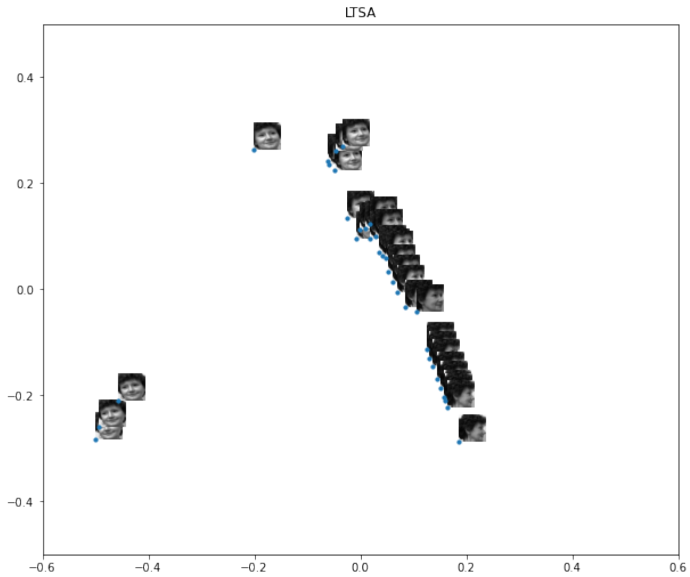


Figure 6: LTSA (K=5)

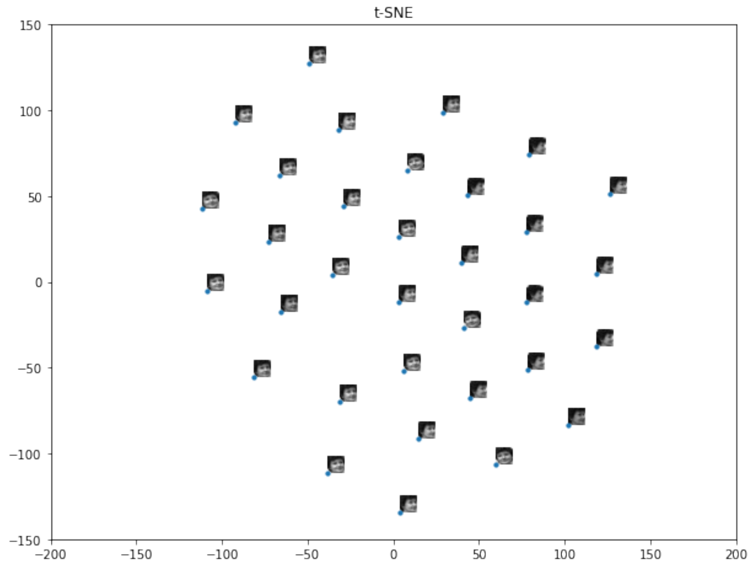


Figure 7: t-SNE

# Results - Test Parameters

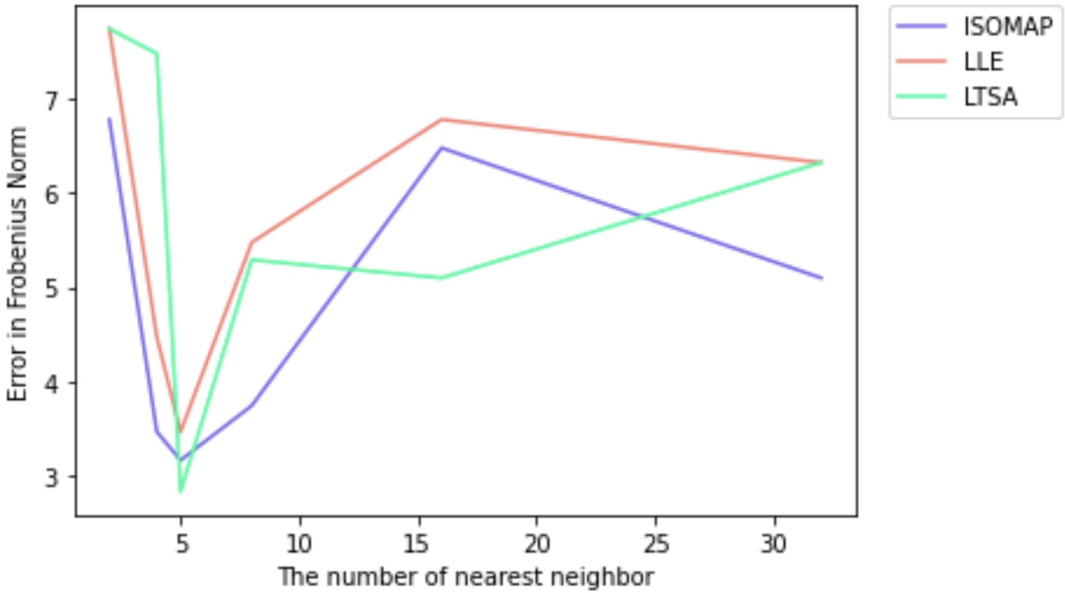


Figure 8: Parameter Test for ISOMAP, LLE, LSTA

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# Conclusion

- Except the t-SNE and MDS methods, other four nonlinear embedding methods obtain the reasonable images order.
- The more detailed the first eigenvector obtain, the better performance the methods show.
- When the number of nearest neighbour closes to 5, the algorithm perform best, which is true for almost all methods.

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# Contribution

- YU Zhiyuan
  - Code in Python for algorithms: DM and MDS, order correct ground truth face images
  - Write report: 1, 2, 3.1 and 3.2
  - Presentation
- CHENG Haoyi
  - Code in Python for algorithms: LTSA and t-SNE
  - Write report: 3.5, 3.6, 4 and 5
  - Presentation
- QUAN Xueyang
  - Code in Python for algorithms: ISOMAP and LLE
  - Write report: 3.3 and 3.4
  - Write PPT
  - Presentation