



北京国际数学研究中心  
BEIJING INTERNATIONAL CENTER FOR  
MATHEMATICAL RESEARCH

Brainiac 博伴智界

# AI-Powered Math Education Practice at Peking University

Leheng Chen Zihao Liu  
2025.9



# About Us



## Leheng Chen

PhD Candidate in BICMR (PKU)  
& Zhongguancun Academy  
Gold Medalist, China Mathematical Olympiad  
Focus: AI Engineering for Education Equity



## Zihao Liu

PhD Candidate in BICMR (PKU)  
& Zhongguancun Academy  
Initiator & early developer of the project  
Focus: AI + Education Integration

**Brainiac** is an **EdTech company** originates from Peking University's School of Mathematic science, **leveraging AI and large models** to enhance knowledge understanding, improve teaching processes, and reshape the education ecosystem.

# Project Milestones

September 2023–May 2024 : First-generation product Brainiac Buddy launched —  
China's earliest “AI + Education”(AITutor) applications.

 AI Tutor
Course Study
Knowledge
Logout

绪论

[绪论.pdf](#)

**Knowledges:**

[Mitchell, 1997] 给出了一个更形式化的定义：假设有  $P$  来评估计算机程序在某任务类  $T$  上的性能，若一个程序通过利用经验  $E$  在  $T$  中任务上获得了性能改善，则我们就说关于  $T$  和  $P$ ，该程序对  $E$  进行了学习。

## 第1章 绪 论

### 1.1 引言

傍晚小街面上沁出微雨后的湿润，和煦的细风吹来，抬头看看天边的晚霞，嗯，明天又是一个好天气。走到水果摊旁，挑了个根蒂蜷缩、敲起来声音浊响的青绿西瓜，一边满心期待着皮薄肉厚瓢甜的爽落感，一边愉快地想着，这学期狠下了工夫，基础概念弄得清清楚楚，算法作业也是信手拈来，这门课成绩一定差不了！

希望各位在学期结束时有这样的感觉。作为开场，我们先大致了解一下什么是“机器学习”(machine learning)。

回头看第一段话，我们会发现这里涉及很多基于经验做出的预判。例如，为什么看到微湿路面、感到和风、看到晚霞，就认为明天是好天呢？这是因为在我们的生活经验中已经遇见过很多类似情况，头一天观察到上述特征后，第二天天气通常会很好。为什么色泽青绿、根蒂蜷缩、敲声浊响，就能判断出是正熟的好瓜？因为我们吃过、看过很多西瓜，所以基于色泽、根蒂、敲声这几个特征我们就可以做出相当好的判断。类似的，我们从以往的学习经验知道，下足了工夫、弄清了概念、做好了作业，自然会取得好成绩。可以看出，我们能做出有效的预判，是因为我们已经积累了许多经验，而通过对经验的利用，就能对新情况做出有效的决策。

上面对经验的利用是靠我们人类自身完成的。计算机能帮忙吗？

机器学习正是这样一门学科，它致力于研究如何通过计算的手段，利用经验来改善系统自身的性能。在计算机系统中，“经验”通常以“数据”形式存在，因此，机器学习所研究的主要内容，是关于在计算机上从数据中产生“模型”(model)的算法，即“学习算法”(learning algorithm)。有了学习算法，我们把经验数据提供给它，它就能基于这些数据产生模型；在面对新的情况时(例如看到一个没剖开的西瓜)，模型会给我们提供相应的判断(例如好瓜)。如果说计算机科学是研究关于“算法”的学问，那么类似的，可以说机器学习是研究关于“学习算法”的学问。

learning, suggests that simpler models are preferred over more complex ones if they can achieve the same level of accuracy.

**机器学习如何在大选中发挥作用？**

**什么是版本空间？如何计算版本空间的数量？**

**How can machine learning algorithms learn from data?**

**在有噪声的数据中，如何选择一致性假设？**

**What is the definition of machine learning?**

**+ New Chat** **Formula Insert**

Input question, mathematical formulas start and end with '\$'...(Enter: Alt + Enter)

# Project Milestones



October–December 2024: Co-developed PKU Quest with Center for Faculty Development, selected by China Education Ministry as a **model case in “AI + Higher Education” initiatives.**

The screenshot shows a web-based AI teaching platform. At the top, there's a navigation bar with tabs like '课程' (Courses), '衔接课' (Bridge Courses), '工具箱' (Toolbox), and '文档' (Documents). A user profile '刘梓豪' is visible. Below the navigation, a document titled '从零开始学Python数据可视化.pdf' is open. The main content area displays a Python code snippet for creating a line plot:

```
import matplotlib.pyplot as plt

# 数据
x = [1, 2, 3, 4, 5]
y = [10, 14, 12, 23, 18]

# 创建折线图
plt.plot(x, y, marker='o', linestyle='-', color='b', label='Sample Data')

# 设置标题和标签
plt.title('Simple Line Plot Example')
plt.xlabel('X Axis')
plt.ylabel('Y Axis')

# 显示图表
plt.legend()

# 显示图表
plt.show()
```

On the right side of the interface, there's an AI interaction window with a message from AI: "AI已经能帮我们非常轻松地写程序做数据分析和可视化了。现在，你可以直接点击右边内置的提问，或是在对话框提问，尝试用自然语言指导AI编程一段数据可视化的程序！". Below this, there are two input fields: "写一段用numpy和matplotlib可视化数据的完整例子" and "什么是数据可视化？". At the bottom, there are buttons for '+ 新对话' (New Conversation), '历史' (History), '公式' (Formula), '提问模板' (Question Template), '笔记' (Notes), and '已连接' (Connected). The footer includes page navigation buttons: '上一页', '1 / 5', '页码', '查询', and '下一页'.

## 软件产品服务合同

甲方：北京大学教师教学发展中心  
地址：北京市海淀区颐和园路5号

乙方：博伴智界（北京）科技有限公司  
地址：北京市海淀区中关村大街32号5层A505

甲方向乙方（博伴智界（北京）科技有限公司）购买“北大问学”——基于生成式语言模型的AI教学平台产品服务。本着双方诚信互利的原则，依据《合同法》等相关法律，经双方协商，制订条款如下：

## 第二批“人工智能+高等教育”应用场景典型案例名单

序号	高校	案例名称
1	北京大学	北大问学——智能教学平台

# Project Milestones

March 2025: PKU Quest launched math&teaching plugin and was selected for the Ministry of Education's National Smart Platform AI Model Pilot.



**AI试验场**

欢迎回来，一起试试我们的新能力！

**学生学习**  
Student learning



北京大学  
中国移动提供算力支持

**数学解题助手**  
Step-by-step problem solving, targeted explanation, multi-path exploration...



北京大学  
中国移动提供算力支持

**代码纠错助手**  
Locate code problems, guide solutions, provide...



北京大学  
中国移动提供算力支持

**代码解答助手**  
Explain code snippets, analyze key points, detailed notes...



北京大学  
中国移动提供算力支持

**教师教学**  
Teacher teaching



清华大学  
中国移动提供算力支持

**AI课堂**  
Students interact with multiple AI characters in real-time, assisting...



北京大学  
中国移动提供算力支持

**AI助教**  
Integrate intelligent teaching assistants and AI tutoring, supporting...



上海交通大学  
中国移动提供算力支持

**智能出题助手**  
Smart batch question generation, turning "questioning" into "selecting".



北京大学  
中国移动提供算力支持



APP下载



# Project Milestones

May 2025 – Present: Partnered with **leading universities** (PKU, Fuyao University of Science and Technology, and Zhongguancun Academy), joined the **Ministry of Education's AI Q&A project** and introduced the comprehensive **AI Knowledge Intelligence Solution** (AKIS)



北京大学  
PEKING UNIVERSITY



福耀科技大学  
FUYAO UNIVERSITY OF SCIENCE AND TECHNOLOGY

北京中关村学院  
Zhongguancun Academy

关于国家中小学智慧教育平台的数学 AI 解题研究与  
应用项目协议书

甲方：教育部教育技术与资源发展中心（中央电化教育馆）

地址：北京市西城区复兴门内大街 160 号，邮编：100031

电话：010-66490201

乙方：博伴智界（北京）科技有限公司

地址：北京市海淀区燕园街道颐和园路 5 号，邮编：100871

电话：



AKIS  
AI Knowledge  
Intelligence Solution

# Early Practice of PKUQuest

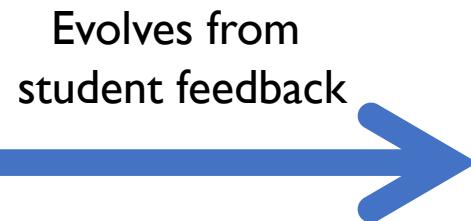


- ✓ For teachers: AI teaching tools for syllabus design and question generation.
- ✓ For students: Knowledge-base Q&A based on lecture notes

# From Knowledge-base Chat to Subject-specific AI Agents



LLM-based AI teaching platform  
(knowledge-base chat)



Heuristic, interactive tool  
for textbook exercises  
(Socratic, step-guided)

# Math Tutor



A screenshot of the Peking University 'Ask PKU' website interface. The top navigation bar includes links for '课程' (Courses), '衔接课' (Bridge Courses), '工具箱' (Toolbox) which is highlighted with a red underline, '文档' (Documents), and a dropdown menu for '北大问学-学生'. Below the navigation is a grid of icons representing various tools: a calculator icon labeled 'Math Copilot', a book with a pi symbol icon labeled '数学习题课' (Math Problem Class), a stack of books icon labeled '更多工具' (More Tools), a magnifying glass over a code editor icon labeled '代码抓虫' (Code Bug Catcher), a code editor icon labeled '代码精讲' (Code精讲), and a 'Math Tutor' icon circled in red. At the bottom of the page is the URL <https://ask.pku.edu.cn>.

Math Tutor is a heuristic interactive AI assistant

# Math Tutor: Hallucination-Free Heuristic Problem-Solving Assistance



"How would you use derivatives to determine the monotonicity of a function on a given interval?"

题目：求函数的单调区间与极值点:  $y = x^2 + \frac{1}{x^2}$ .

1 首先, 求函数的一阶导数:

$$y' = \frac{d}{dx} \left( x^2 + \frac{1}{x^2} \right) = 2x - \frac{2}{x^3}$$

2 令一阶导数等于0, 解方程求出可能的极值点:

$$2x - \frac{2}{x^3} = 0$$

化简得:

$$2x^4 - 2 = 0 \Rightarrow x^4 = 1 \Rightarrow x = \pm 1$$

因此, 可能的极值点为  $x = 1$  和  $x = -1$ 。

非常好! 你正确地找到了  $x = 1$  和  $x = -1$  这两个点。现在, 我们需要确定这些点是否是极值点, 以及函数在这些点附近的单调性。你能告诉我, 你会如何利用导数来判断函数在某个区间内的单调性吗?

提示一下

法... (按Enter发送, Shift+Enter换行)

公式 提示 语音输入 发送

- Embedding Human-Annotated Correct Answers— Ensuring Answer Accuracy
- Heuristic Guidance— Enabling Students to Solve Independently



# AI-Driven Digital Textbooks

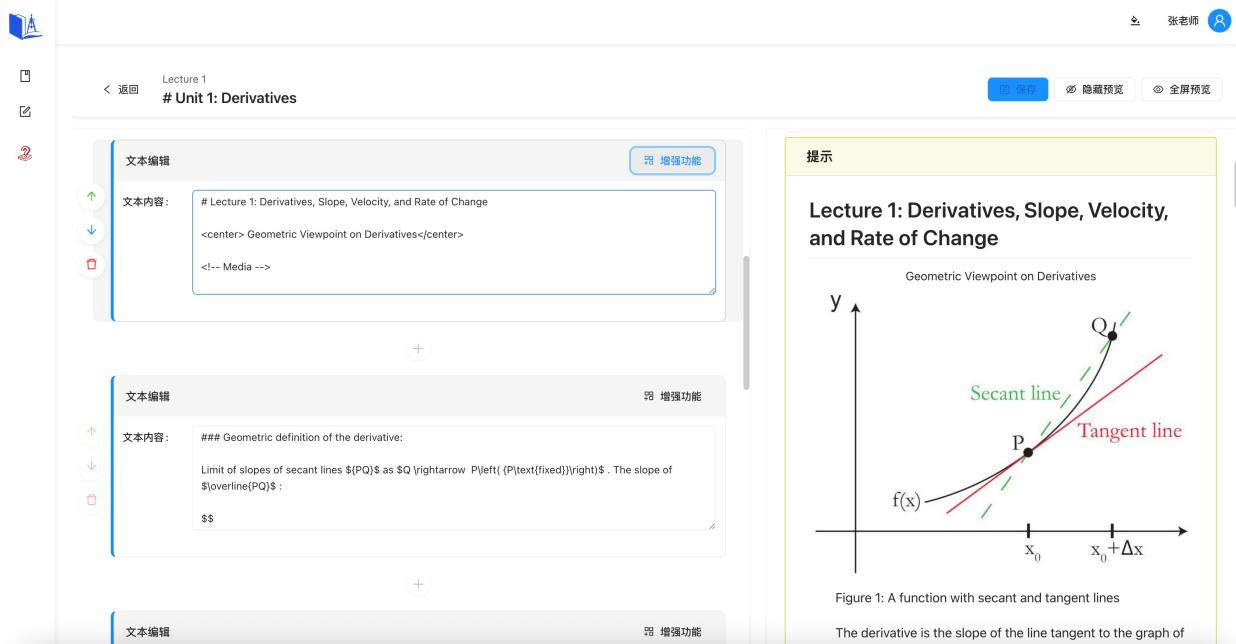


A screenshot of a digital textbook interface. At the top right, there is a user profile for "张老师" (Teacher Zhang) with a blue circular icon. Below the title, there is a back arrow and the text "返回教材列表". The main content area shows a list of chapters for "Calculus-1" (共 5 个章节). Each chapter entry includes the chapter number, unit name, and three action buttons: 阅读 (Read), 编辑 (Edit), and 删除 (Delete). The chapters listed are:

- 第 1 章 # Unit 1: Derivatives
- 第 2 章 # Unit 2: Applications of Differentiation
- 第 3 章 # Unit 3: The Integral
- 第 4 章 # Unit 4: Techniques of Integration
- 第 5 章 # Unit 5: Infinite Series

At the bottom center of the interface, the text "welcome to AI Book" is displayed in a stylized font.

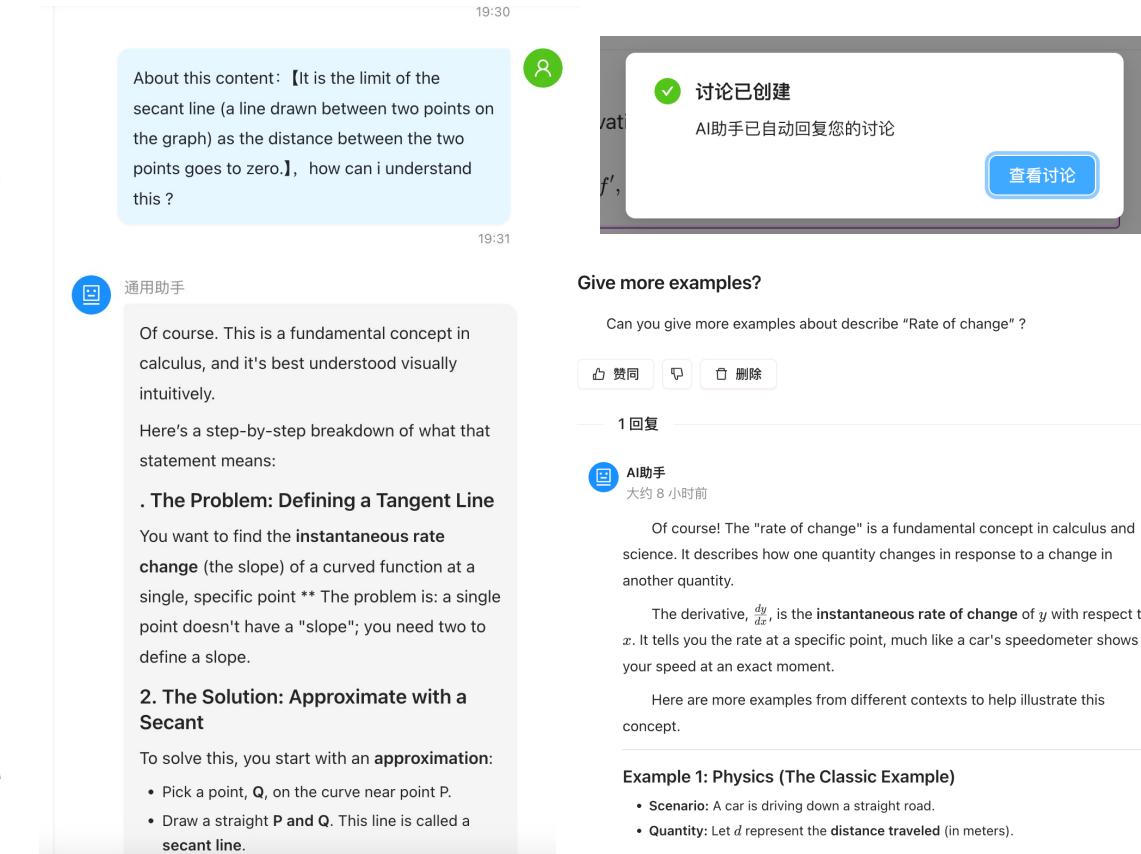
# AIBOOKS: A living course material that evolves continuously



The screenshot shows the AIBOOKS interface where teachers can contribute to the course material. On the left, there are three text boxes for different sections:

- Lecture 1: Derivatives**  
Text content: # Lecture 1: Derivatives, Slope, Velocity, and Rate of Change  
<center> Geometric Viewpoint on Derivatives</center>  
<!-- Media -->
- 文本编辑**  
Text content: ### Geometric definition of the derivative:  
Limit of slopes of secant lines  $\overline{PQ}$  as  $Q \rightarrow P$ . The slope of  $\overline{PQ}$ :  
$$\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$
- 文本编辑**  
Text content: # Lecture 1: Derivatives, Slope, Velocity, and Rate of Change  
Geometric Viewpoint on Derivatives

On the right, a diagram titled "Geometric Viewpoint on Derivatives" shows a function  $f(x)$  on a coordinate plane. A point  $P$  is on the curve at  $x_0$ . A secant line connects  $P$  to another point  $Q$  on the curve at  $x_0 + \Delta x$ . A tangent line is drawn at point  $P$ , labeled "Tangent line". The text below the diagram states: "Figure 1: A function with secant and tangent lines. The derivative is the slope of the line tangent to the graph of  $f(x)$ ".



The screenshot shows a student interaction with the AI assistant. The student asks a question about the geometric interpretation of the derivative as the limit of a secant line.

**About this content:** It is the limit of the secant line (a line drawn between two points on the graph) as the distance between the two points goes to zero. how can i understand this?

**通用助手**

Of course. This is a fundamental concept in calculus, and it's best understood visually intuitively. Here's a step-by-step breakdown of what that statement means:

- The Problem: Defining a Tangent Line**  
You want to find the instantaneous rate change (the slope) of a curved function at a single, specific point. The problem is: a single point doesn't have a "slope"; you need two to define a slope.
- 2. The Solution: Approximate with a Secant**  
To solve this, you start with an approximation:
  - Pick a point, Q, on the curve near point P.
  - Draw a straight line through P and Q. This line is called a secant line.

**Give more examples?**

Can you give more examples about describe "Rate of change" ?

**AI助手**

Of course! The "rate of change" is a fundamental concept in calculus and science. It describes how one quantity changes in response to a change in another quantity.

The derivative,  $\frac{dy}{dx}$ , is the instantaneous rate of change of  $y$  with respect to  $x$ . It tells you the rate at a specific point, much like a car's speedometer shows your speed at an exact moment.

Here are more examples from different contexts to help illustrate this concept.

**Example 1: Physics (The Classic Example)**

- Scenario: A car is driving down a straight road.
- Quantity: Let  $d$  represent the distance traveled (in meters).

🏆 Instructor Contributions: Teachers can **update and supplement** the latest scientific and technological advances in real time.

🤝 Student–AI Collaboration: Students and the AI assistant co-create a **discussion community**, leaving a **traceable record** of the course.



# AIBOOKS: A living course material that evolves continuously



**注记**

**Notations**

Calculus, rather like English or any other language, was developed by several people. As a result, just as there are many ways to express the same thing, there are many notations for the derivative.

Since  $y = f(x)$ , it's natural to write

$$\Delta y = \Delta f = f(x) - f(x_0) = f(x_0 + \Delta x) - f(x_0)$$

We say "Delta  $y$ " or "Delta  $f$ " or the "change in  $y$ ".

If we divide both sides by  $\Delta x = x - x_0$ , we get two expressions for the difference quotient:

$$\frac{\Delta y}{\Delta x} = \frac{\Delta f}{\Delta x}$$

Taking the limit as  $\Delta x \rightarrow 0$ , we get

$$\frac{\Delta y}{\Delta x} \rightarrow \frac{dy}{dx}$$
 (Leibniz' notation)  
$$\frac{\Delta f}{\Delta x} \rightarrow f'(x_0)$$
 (Newton's notation)

When you use Leibniz' notation, you have to remember where you're evaluating the derivative

- in the example above, at  $x = x_0$ .

Other, equally valid notations for the derivative of a function  $f$  include

$$\frac{df}{dx}, f', \text{ and } Df$$

**示例**

**Example 2.**  $f(x) = x^n$  where  $n = 1, 2, 3 \dots$

What is  $\frac{d}{dx} x^n$ ?

To find it, plug  $y = f(x)$  into the definition of the difference quotient.

$$\frac{\Delta y}{\Delta x} = \frac{(x_0 + \Delta x)^n - x_0^n}{\Delta x} = \frac{(x + \Delta x)^n - x^n}{\Delta x}$$

(From here on, we replace  $x_0$  with  $x$ , so as to have less writing to do.) Since

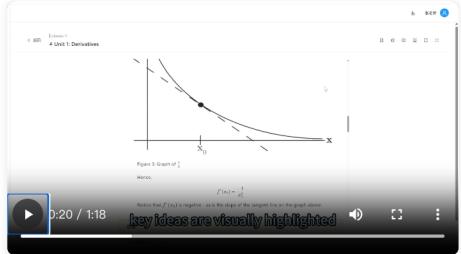
$$(x + \Delta x)^n = (x + \Delta x)(x + \Delta x) \dots (x + \Delta x) \quad n \text{ times}$$

We can rewrite this as

$$x^n + n(\Delta x)x^{n-1} + O((\Delta x)^2)$$

$O(\Delta x)^2$  is shorthand for "all of the terms with  $(\Delta x)^2$ ,  $(\Delta x)^3$ , and so on up to  $(\Delta x)^n$ ".

(This is part of what is known as the binomial theorem; see your textbook for details.)

$$\Delta y = (x + \Delta x)^n - x^n = x^n + n(\Delta x)(x^{n-1}) + O((\Delta x)^2) - x^n = \dots$$


PYTHON

```
1 import marimo as mo
2 app = mo.App(width="medium")
3
4 @app.cell
5 def _():
6     import marimo as mo
7     import numpy as np
8     import matplotlib.pyplot as plt
9
10    # 参数控件
11    a = mo.ui.slider(-5, 5, value=-1, step=0.1, label="参数 a:")
12
13    return mo, np, plt, a
14
15
16
17
18 @app.cell
19 def _(mo, a):
20     mo.md(
21         f"""
22         ## 二次函数图像
23         当前参数:
24         """,
25         style="background-color: #f0f0f0; padding: 10px;
26         font-family: monospace;
27         border: 1px solid #ccc;
28         border-radius: 5px;
29         width: 100px;
30         height: 100px;
31         margin-bottom: 10px;
32     )
33
34     # 画图
35     x = np.linspace(-5, 5, 100)
36     y = a * x ** 2
37     plt.plot(x, y)
38
39     # 显示
40     mo.show()
```

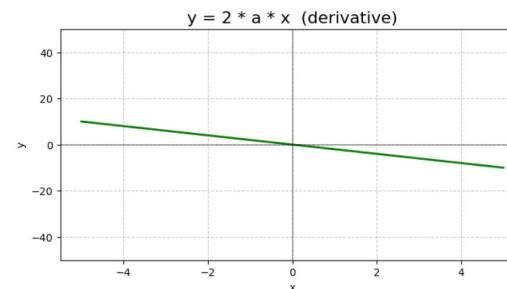
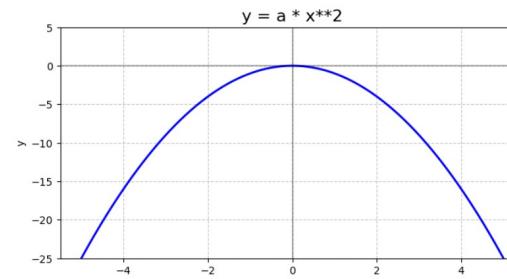
## Quadratic and Its Derivative

Current parameter:  $a = -1.0$

- Top plot: function  $y = ax^2$
- Bottom plot: derivative  $y = 2ax$

Drag the slider to see how both plots change with  $a$ .

Parameter  $a$ :



Multimodal Presentation: Seamless support for **formulas, images, videos, and code**

# EduBridge: Smart Bridging Courses to Bridge High School and University



## 1.1.1 从高中数学到大学数学

北京大学 PEKING UNIVERSITY

第一讲 从高中数学到大学数学

冯宣瑞

北京大学数学科学学院

2025 年 4 月

0:06 / 2:11

北大问学AI助教

高中数学和大学有什么区别?

这是一个很好的问题! 让我们一起来思考几个关键点:

- 首先, 你能回忆一下高中数学主要学习哪些内容吗? 比如代数、几何、概率这些模块?
- 在大学数学中, 你会遇到像微积分、线性代数这样的课程。你觉得这些内容和高中数学有什么联系呢?
- 我注意到你提到了“导数”, 这是个很好的例子。在高中我们学习导数的计算和应用, 但你知道大学会如何更深入地研究这个概念吗?

输入你的问题... (Enter 发送, Shift+Enter 换行)

发送

1,066

freshmen users

13,423

completed bridge lessons

avg. 12.6 per freshmen Within 10 days

## Contents (Selected)

- Advanced Mathematics Bridging Course
- 1. From High School Mathematics to University Mathematics
  - 2. Equations and Inequalities
  - 3. Polar coordinates
  - 4. .... ...

- Widespread student knowledge gaps
- Some students struggle in college due to insufficient high school foundation.



# AI-Powered Visual Explanations

Let  $F(x) = \int_0^x \left(\sin \frac{1}{t}\right)^4 dt$ , then  $\lfloor 100 |F'(0)| \rfloor =$

Recall the identity  $\sin^4 x = \frac{(1 - \cos 2x)^2}{4} = \frac{3}{8} - \frac{1}{2} \cos 2x + \frac{1}{8} \cos 4x$

. Replacing  $x$  by  $\frac{1}{t}$  we obtain  $\sin^4\left(\frac{1}{t}\right) = \frac{3}{8} - \frac{1}{2} \cos \frac{2}{t} + \frac{1}{8} \cos \frac{4}{t}$

By definition  $F'(0) = \lim_{x \rightarrow 0} \frac{F(x) - F(0)}{x} = \lim_{x \rightarrow 0} \frac{1}{x} \int_0^x \left(\sin \frac{1}{t}\right)^4 dt$

## 解题步骤

- (1) Expand sin
- (2) Derivative
- (3) Split Terms
- (4) Key Limit
- (5) Substitute
- (6) Estimate
- (7) Result

# AI Knowledge Intelligent Solution (AKIS)

## — HK Mathematics Proposal



### AI Mathematics Teaching Assistant

Math Copilot & Math Tutor  
for interactive problem solving  
and heuristic guidance



### AIBOOKS Digital Textbooks

Digitized, structured,  
multimodal, and  
collaborative learning hub



### Smart Learning Navigator (Bridging Courses)

Tailored preparatory courses  
to bridge high school—  
university gaps

*“Localized for HK foundational courses (e.g., University Mathematics),  
aligned with teaching style, credit system, and student needs.”*



**Company's Email**  
**business@bbaitutor.com**

**Zihao Liu :**

Tel : (+86) 18811428069

**Leheng Chen :**

Tel : (+86) 13353396997



刘梓豪



Thanks  
for  
Watching



北京大学  
PEKING UNIVERSITY

Brainiac 博伴智界

