Inamul haq, q2ad

HW-7

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Q.1 a) Top 10 rank webpages (seed Matlab code)

1- tsinghua.edu.cn
2. pku.edu.cn
3. sjtu.edu.cn
4. nju.edu.cn
5. uestc.edu.cn
6. scut.edu.cn
7. zsu.edu.cn
8. dlut.edu.cn
9. fudan.edu.cn
10. seu.edu.cn

c) Kendall Tau distance τ

$\rho_{\text{HITS authority}} = 0.5741$

$\rho_{\text{google Pr}} = 0.5118$

$\rho_{\text{HITS-hub}} = 0.3885$

d) Plot attached

Q 2 (a) Let us assume $Av^* \neq \lambda^* v^* \Rightarrow$ for some i,

$[Av^*]_i > \lambda^* v_i^*$ — (1)

Let $\tilde{v} = v^* + \epsilon e_i$, $\epsilon > 0$

& e_i standard basis vector with 1 at ith entry

for $j \neq i$,

$(A\tilde{v})_j = (Av^*)_j + \epsilon (Ae_i)_j$

since $A > 0$ $\quad = \lambda^* v_j^* + \epsilon A_{ji} > \lambda^* v_j^* = \lambda^* \tilde{v}_j$

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For $j = i$:

$$(A\tilde{v})_i = (Av^*)_i + \epsilon (Ae_i)_i > \lambda^* v_i^* + \epsilon A_{ii} \quad \text{from (1)}$$

Since $\lambda^* \tilde{v}_i = \lambda^* v_i^* + \epsilon \lambda^*$

$$\Rightarrow (A\tilde{v})_i - (\lambda^* \tilde{v})_i + \epsilon (A_{ii} - \lambda^*)$$

$$= (Av^*)_i - (\lambda^* v_i^*) - \epsilon (\lambda^* - A_{ii}) > 0$$

for small $\epsilon > 0$, $(A\tilde{v}) > \lambda^* \tilde{v}$

Thus λ^* is not optimal $\rightarrow$ contradicts assumption

Hence $\underline{Av^* = \lambda v^*}$

(b) To show ; $v^* > 0$

Assume that for some k, $v_k^* = 0$, then $(Av^*)_k = \lambda^* v_k^* = 0$

But $A > 0$, $v^* \geq 0$ & $v^* \neq 0$, $\Rightarrow \exists i \; v_i^* > 0$

$\Rightarrow Av^* > 0$ $\Rightarrow$ This contradicts previous conclusion.

$v^* > 0$ (Since $\lambda^* > 0$)

c) λ^* is unique & v^* is unique

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$Av = \mu v \Rightarrow \mu = \lambda^*$

Hence, A must have a left Perron vector $w^* > 0$ s.t $A^T w^* = \lambda^* w^*$

Then, $\lambda^* (w^{*T} v) = w^{*T} A v = \mu (w^{*T} v)$

Since $w^{*T} v > 0$ $(w^* > 0, v \geq 0)$, there must be $\lambda^* = \mu$

i.e. λ^* is unique & V^* is unique

d) For any other eigenvalue $Az = \lambda z$,

$A|z| \geq |Az| = |\lambda| |z|$, so $|\lambda| \leq \lambda^*$

Using Lemma 4 of notes, $|\lambda| < \lambda^*$

3 a) $N(i,i)$ - expected no jumps starting from non-absorbing state i & hitting state i before reaching absorbing state $n+1$

1) Jump directly state i to state i in 1 step which contributes 1 to expected no of jumps

2) Jump from another state k to i in 1 step & coming back to state i before absorbing state.

$N(i,i) = 1 + \sum_k N(i,k) Q(k,i)$

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4 a) 2nd smallest generalised eigen vector $\lambda_2 f$

$\lambda_2 = 0.4685$

b) Matlab figures attached

c) $\alpha f = 0.1467$

optimal subset $S^* = \{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 11\ 12\ 13\ 14\ 17\ 18\ 20\ 22\}$

d) Check_lambda

$\lambda_2 > \lambda_s$

e) $h_{S^+} = 0.1515$

$S^+ = \{1, 2, 4, 5, 6, 7, 8, 11, 12, 13, 14, 17, 18, 20, 22\}$

Suboptimal cut S^+ differs for optimal cut S^*
2 nodes missing in S^+ $\{3, 9\}$

f) Matlab code