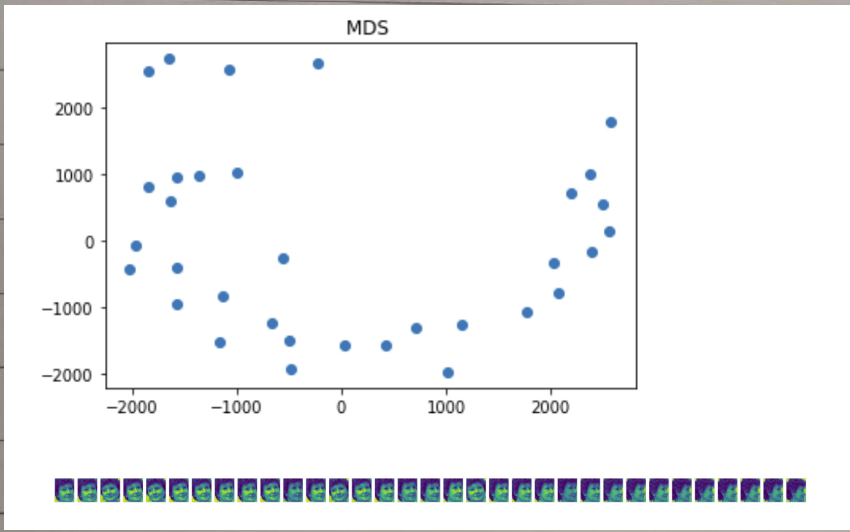
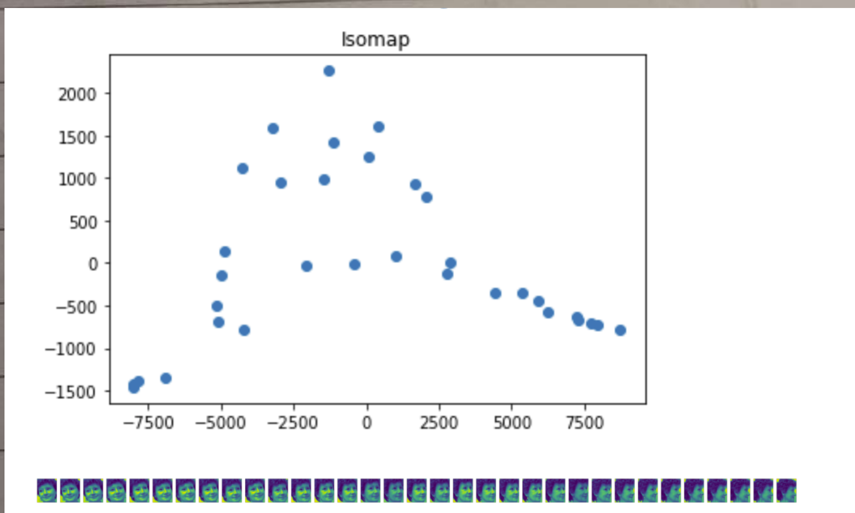
CSIC 5011 HW 6

DATE: / /   MO TU WE TH FR SA SU

Q-1) a) Using MDS for ordering the face dataset:

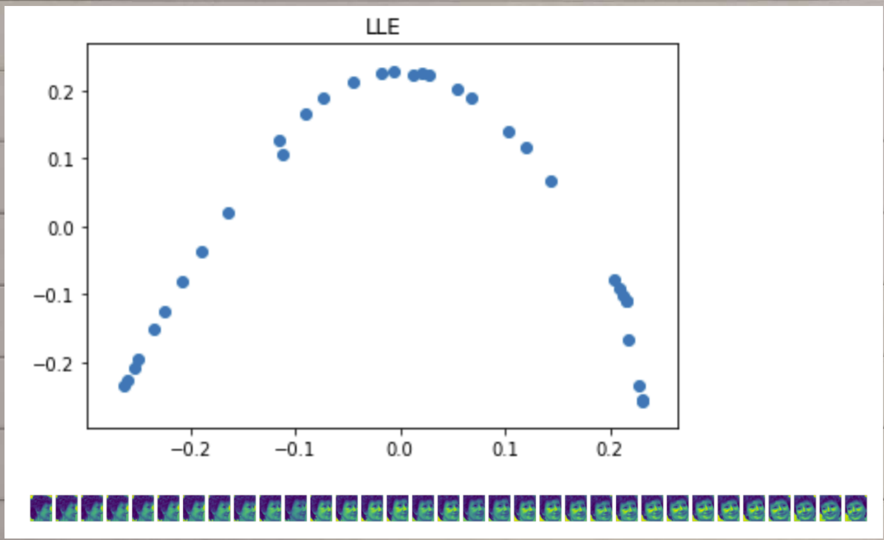


b) Using ISOMAP for ordering the faces:



DATE: / / MO TU WE TH FR SA SU

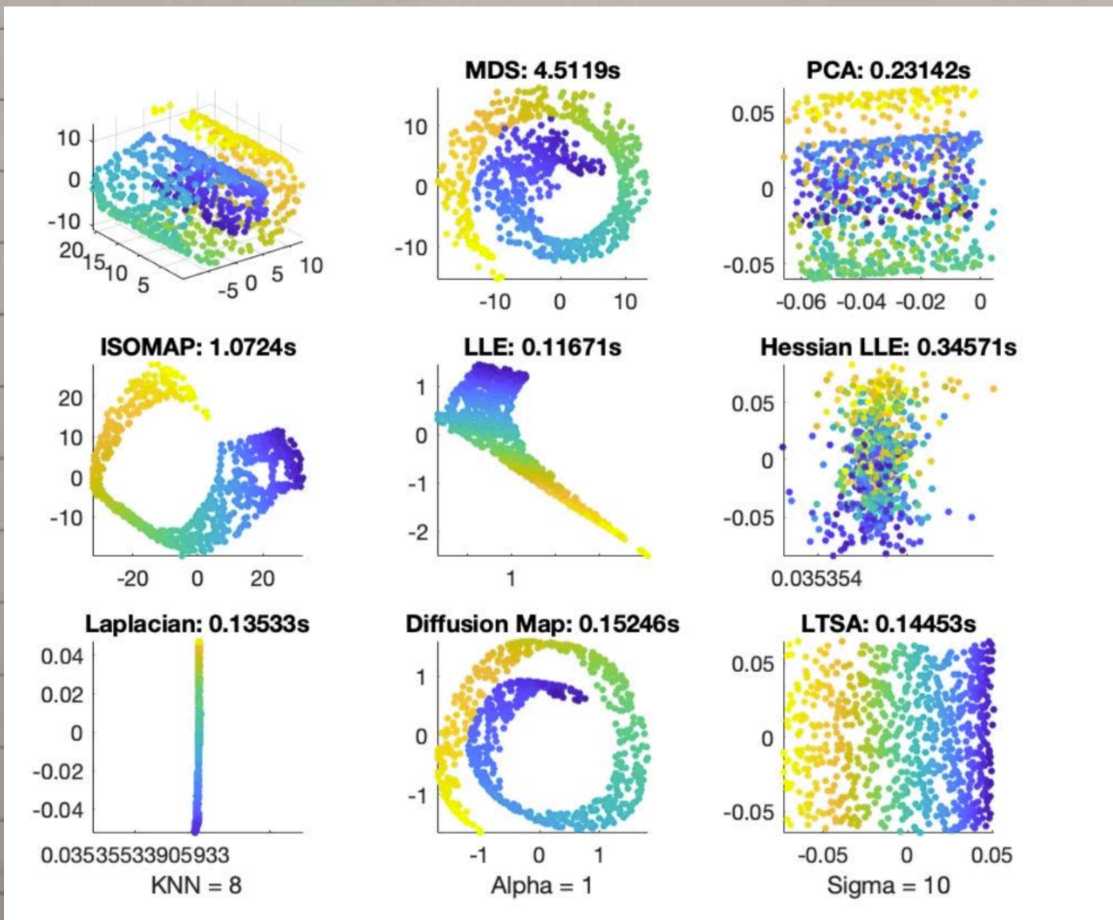
c) Using LLE for ordering the faces:



- It is observed that ISOMAP & LLE are able to order the faces well by preserving the distances between different face angles.
- ISOMAP orders the faces as the view changes from left to right (according to observer's view). whereas LLE orders the faces as the view changes from right to left.
- MDS is not able to order the faces according to the view well. The reason is that MDS works on euclidean distances which might not be accurate on the face manifold.

DATE: / / MO TU WE TH FR SA SU

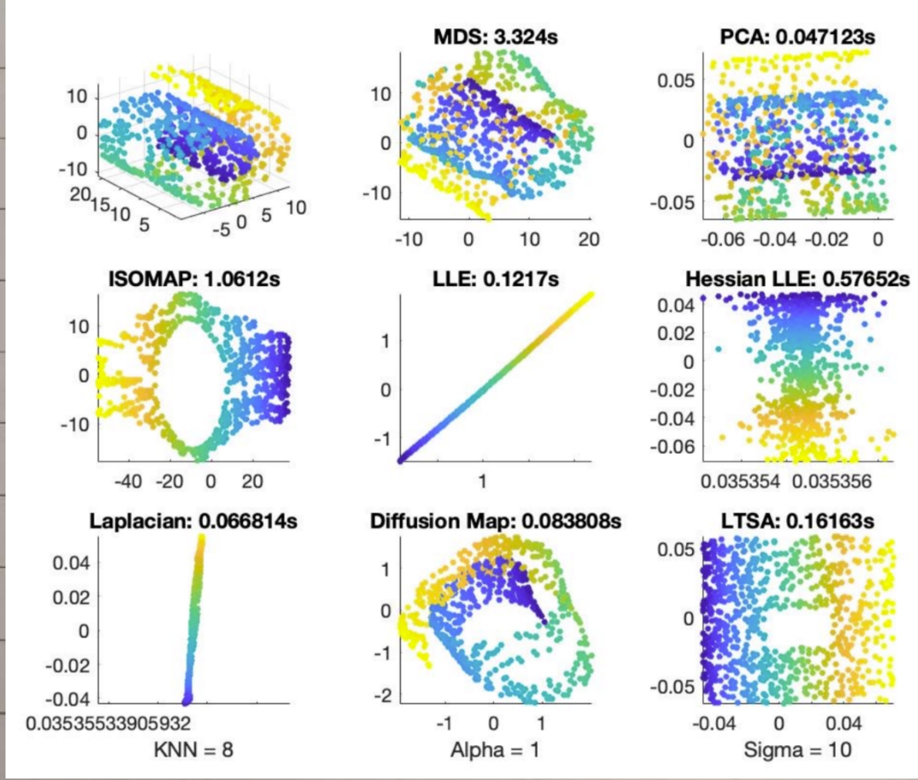
Q)2) Manifold Learning Algorithms on Swiss Roll Dataset



- It is observed that MDS & PCA cannot preserve the manifold structure in low dimension embedding.
- LTSA performs the best in order to preserve the manifold structure in low ~~structure~~ dimension embedding. LLE, Hessian LLE & Laplacian eigen map also do a good job, Hessian LLE being the best.
- ISOMAP & Diffusion Map are not so good on the swiss roll Dataset.
- LTSA is the fastest algorithm & also preserves the structure of manifold.

DATE: / / MO TU WE TH FR SA SU

(b) Swiss Hole Dataset

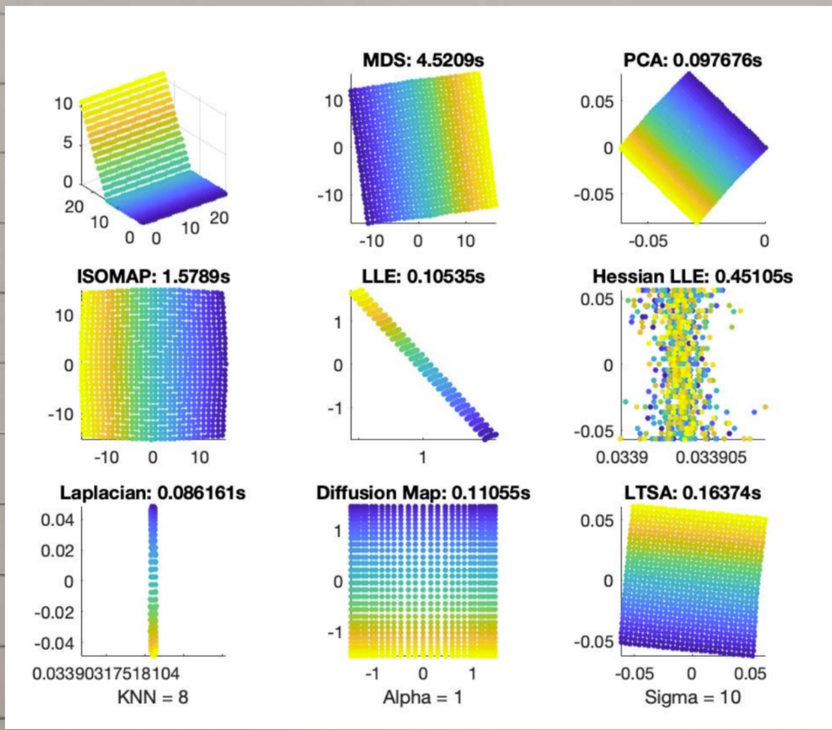


- As before PCA & MDS are not good in this Dataset.
- ISOMAP is also not able to preserve the high dimensional manifold structure due to non convexity in the dataset.
- LLE, Hessian LLE, LTSA & laplacian are able to work well with LTSA having the best performance in low dimension embedding.
- Diffusion map does not work well in this dataset to separate the points as per their position in the manifold.

DATE: / /

MO TU WE TH FR SA SU

(c) Corner planes:



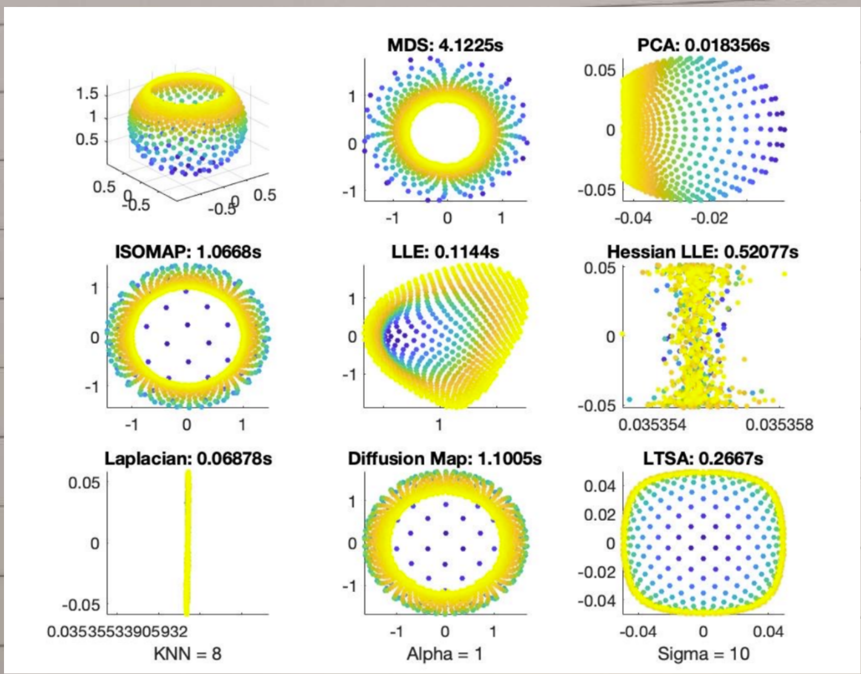
- In this case PCA & MDS are able to preserve the manifold structure in lower dimension since the euclidean distance is approximately equal to the Geodesic distance.

- ISOMAP, LLE, Laplacian, Diffusion Map & LTSA also work very well in the low dimension embedding.

- The Hessian LLE does not work very well, possibly because the manifold doesnot have high curvature in this case.

DATE: / / MO TU WE TH FR SA SU

(d) Punchned Sphere:



- PCA & MDS perform somewhat good in low dimension embedding possibly due to equivalence in euclidean & geodesic distances.

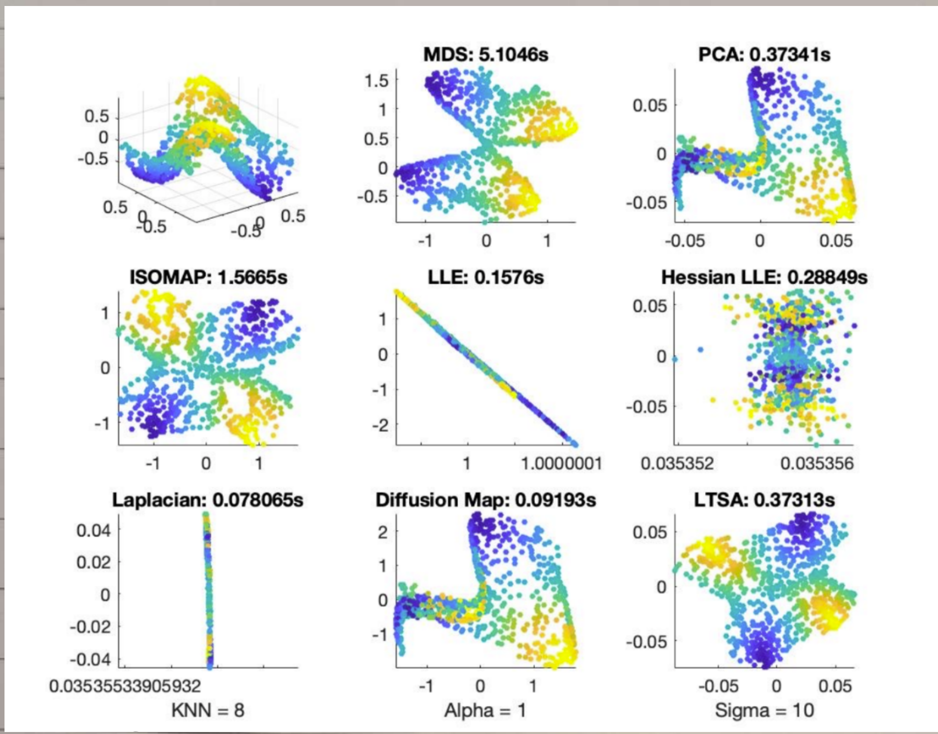
- ISOMAP & Diffusion map don't work well possibly due to non convexity in the dataset.

- LLE & ~~LES~~ LTSA perform well in preserving the manifold structure in lower dimension.

- Hessian LLE & laplacian don't work well on the dataset.

DATE: / / MO TU WE TH FR SA SU

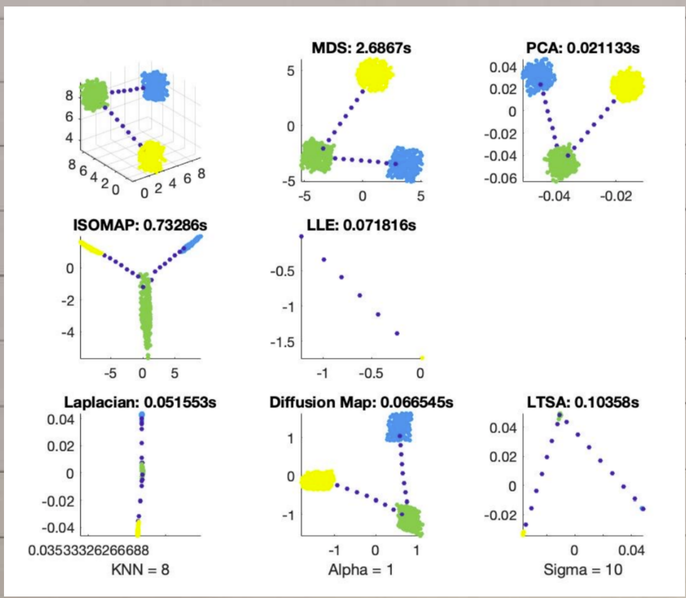
(e) Twin peaks:



- MDS & PCA don't work well on this dataset.
- ISOMAP & LTSA work well in preserving the manifold structure in lower dimension.
- LLE & laplacian give a straight line in 2D which does not tell much about the manifold structure.
- Hessian LLE & Diffusion map are not so good in this case.

DATE: / / MO TU WE TH FR SA SU

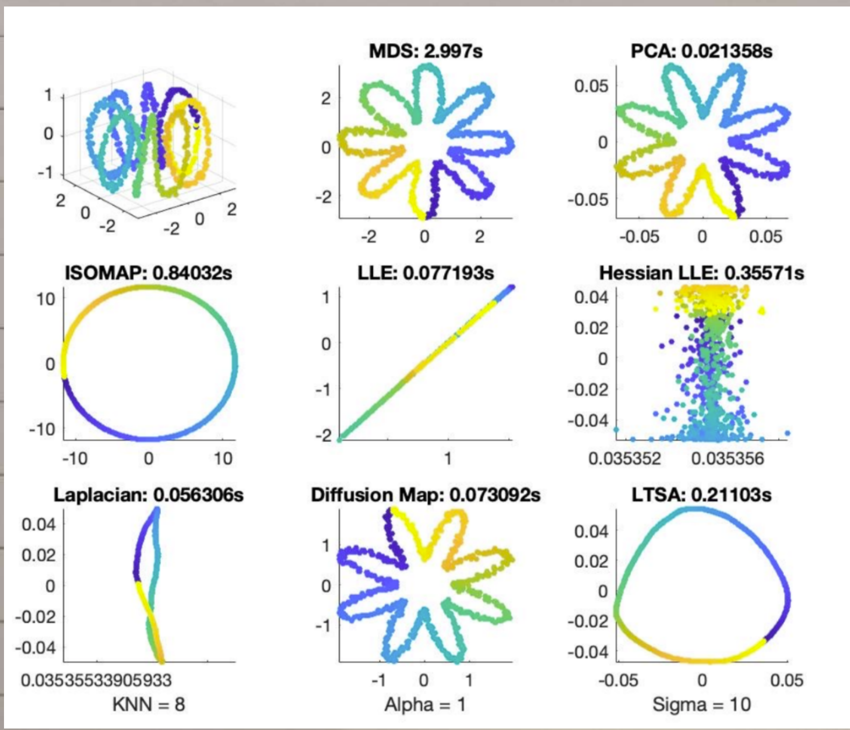
(f) 3D clusters:



- Here, MDS & PCA work well since the euclidean distances & geodesic distances are almost equal.
- Among other manifold algorithms, Diffusion map & LTSA work well in preserving the manifold structure in 2D.
- LLE does not work well. Hessian LLE has no output possibly since the curvature is not well defined on their dataset.

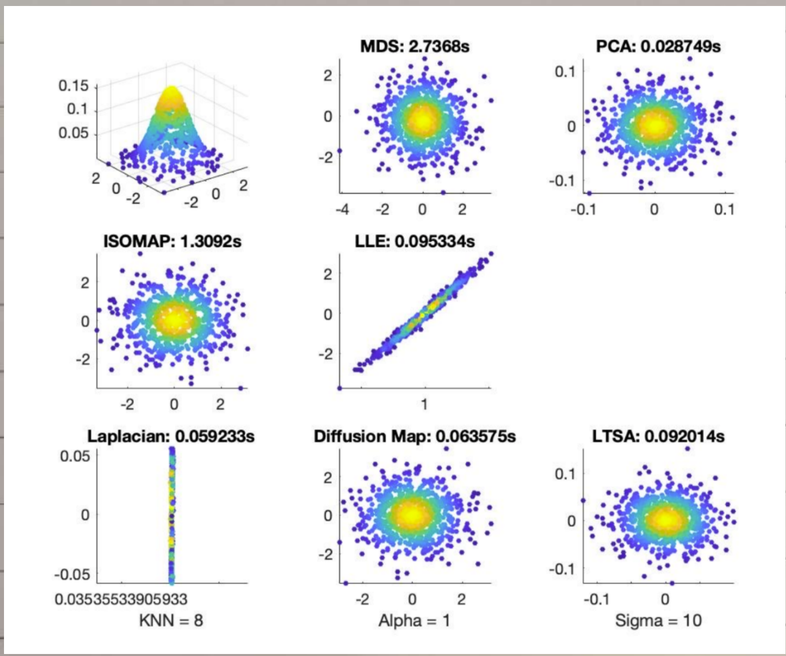
DATE: / / MO TU WE TH FR SA SU

(g) Toroidal helix:



- MDS & PCA preserve the shape in 2D with similar embedding as Diffusion Map.
- ISOMAP & LTSA work well in preserving the distance as per the manifold structure.
- LLE preserves the distance along a line in 2D.
- Hessian LLE & Laplacian give somewhat good result.

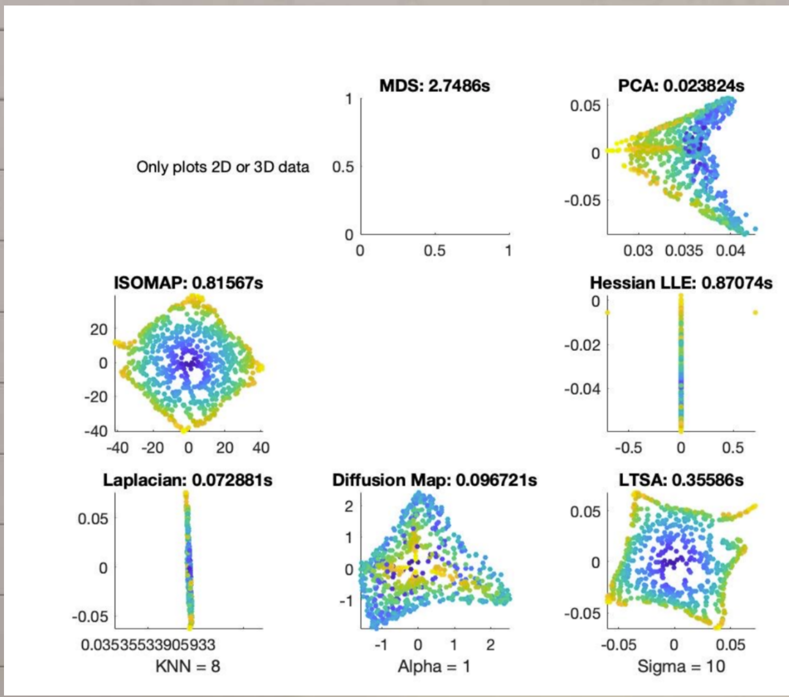
(h) Gaussian:



- In this case MDS, PCA works same as ISOMAP, Diffusion MAP & LTSA.
- LLE & Laplacian squeeze it into a line in 2D embedding.
- Hessian LLE does not give any output.

DATE: / / MO TU WE TH FR SA SU

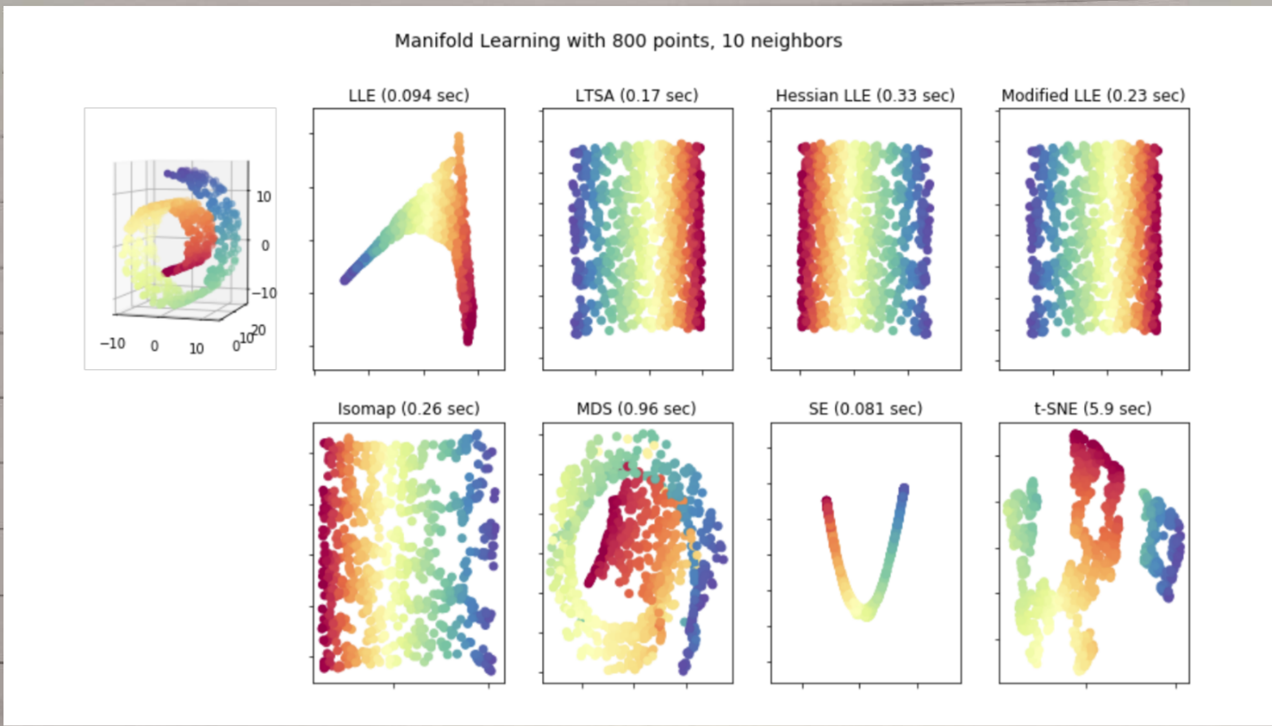
(i) Occluded Disk:



- In this case ISOMAP, DiffusionMap & LTSA work well.
- MDS & LLE donot produce any output.
- Hessian LLE & Laplacian eigen map squeeze the data to a line in 2D embedding.

DATE: / / MO TU WE TH FR SA SU

Result of t-SNE on Swiss Roll Dataset:



Here, we can see that t-SNE does not work very well on the Swiss roll dataset. It tries to separate the points into different clusters. It seems that t-SNE is not good for visualizing continuous data on a manifold. However t-SNE is very good for visualizing categorical datasets like MNIST.

DATE: / / MO TU WE TH FR SA SU

8) 3)

$$K = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

$A \rightarrow n \times n$ $\quad A = U \Lambda U^T$

$\Lambda = diag(\lambda_i), (\lambda_1 \geq \lambda_2 \geq \lambda_3 \cdots \lambda_k \geq \lambda_{k+1} \geq \cdots = 0)$

$U^T U = I$

a) $K = XX^T$, $X = [X_1 ; X_2] \in \mathbb{R}^{N \times R}$;

$X_1 \in \mathbb{R}^{n \times R}$, $X_2 \in \mathbb{R}^{N-n \times R}$.

To show : $X_1 = U_k \Lambda_k^{1/2}$ & $X_2 = B^T U_k \Lambda_k^{-1/2}$

$$K = XX^T = [X_1 ; X_2] \begin{bmatrix} X_1^T \\ X_2^T \end{bmatrix} = \begin{bmatrix} X_1 X_1^T & X_1 X_2^T \\ X_2 X_1^T & X_2 X_2^T \end{bmatrix}$$

$A = X_1 X_1^T$, $B = X_1 X_2^T$

Using EVD of A, & taking top k-eigenvectors X_1 can be approximately.

$$X_1 = U_k \Lambda_k^{1/2}$$

Now, $B = X_1 X_2^T \Rightarrow X_2^T = X_1^{-1} B$

$X_2 = B^T (X_1^{-1})^T$

$X_2 = B^T (\Lambda_k^{-1/2} U_k^{-1})^T$

$X_2 = B^T (\Lambda_k^{-1/2} U_k^T)^T$

$X_2 = B^T U_k \Lambda_k^{-1/2}$

Hence proved.

(b) $K = \begin{bmatrix} x_1 x_1^T & x_1 x_2^T \\ x_2 x_1^T & x_2 x_2^T \end{bmatrix} \geq 0.$

let $\hat{K} = \begin{bmatrix} A & B \\ B^T & \hat{C} \end{bmatrix}$ for $\hat{K} \geq 0 \Rightarrow$ (i) $A \geq 0$

we have $A \geq 0$ because $A = x_1 x^T$

and (ii) $\hat{C} - B^T A^{-1} B \geq 0$

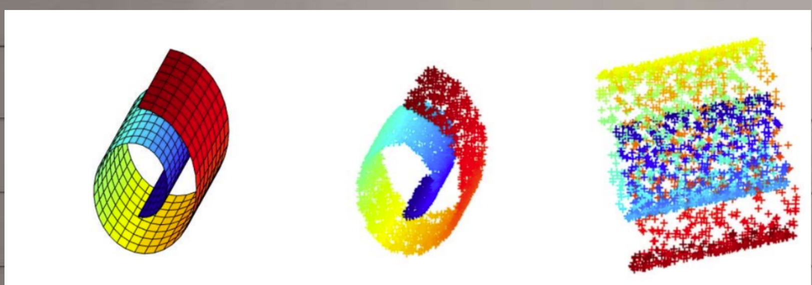
if A is not invertible take the pseudo inverse.

$\Rightarrow \hat{C} - B^T A^{-1} B \geq 0$ choose $\hat{C} = x_2 x_2^T = B^T A^{-1} B$

$\Rightarrow \| \hat{K} - K \|_F^2 = \| C - B^T A^{-1} B \|_F^2$

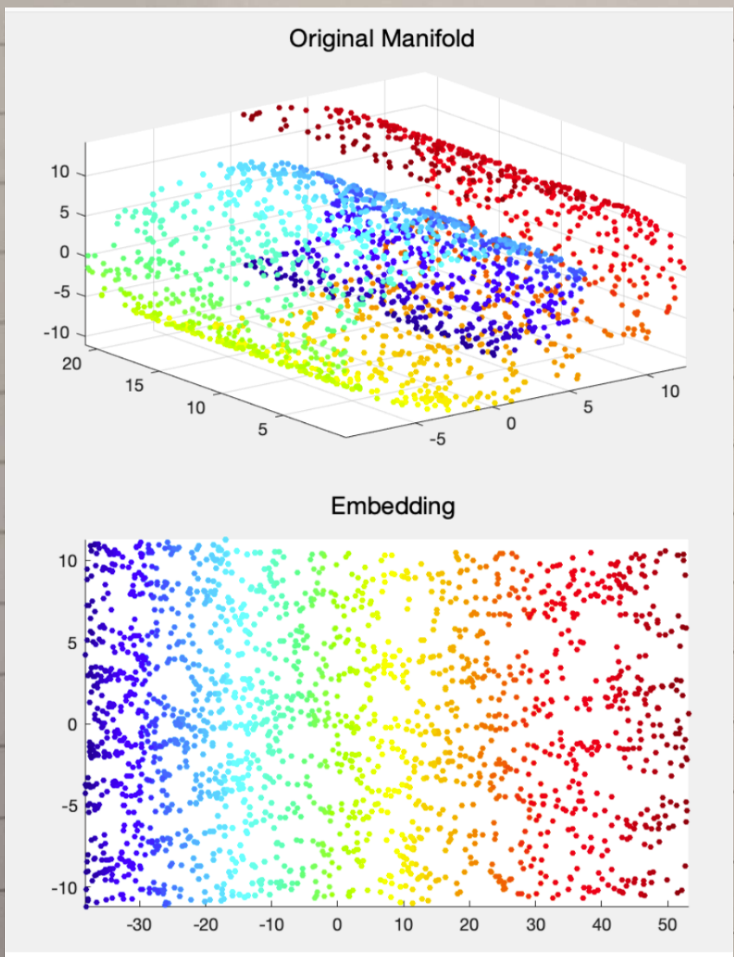
~~Choose~~: $\hat{C} \cancel{=} \cancel{x}$.

(c) Comparing ISOMAP & landmark ISOMAP on swiss Roll Dataset:



DATE: / / MO TU WE TH FR SA SU

(ii) Using random K datapoints as landmark I plot the 2D embedding of Landmark ISOMAP. It is observed that landmark ISOMAP is faster algorithm but it performs poorly when compared to ISOMAP.



DATE: / / MO TU WE TH FR SA SU

(d) To show : $\det(K) = \det(A) \cdot \det(K/A)$

Using the properties of determinant

$\det(CD) = \det(C) \cdot \det(D)$.

& $\det(A) \neq 0$ since $A = x_1 x_1^T \geq 0$

(A is invertible)

$$\det(K/A) = \det(K) - \det(A^{-1}) = \frac{\det(K)}{\det(A)}.$$