Compressed Sensing for Structure Health Monitoring Data: Case in Tunnel Monitoring

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Abstract

In structural health monitoring (SHM) of civil structures, the large volumes of sensor data generated from the monitoring system has challenged the data transfer and storage. The Compressed Sensing (CS) can compress and reconstruct signal lower than Nyquist rate, which shows promising potential in SHM data compression. To investigate the application of CS in SHM, this project used a tunnel SHM dataset containing 10000 data sample as case study, then compressively sampled the monitoring data at different sampling ratio, subsequently reconstructed the signal with orthogonal matching pursuit (OMP) and basis pursuit (BP) algorithms and analyses the reconstruction error. The results show that (1) the tunnel SHM date can use CS to efficiently compress and successfully reconstruct; (2) BP has higher reconstruction accuracy on tunnel SHM date than that of OMP, while BP has worse performance on computation time and consumption than OMP; (3) increasing sampling rate can reduce reconstruction error and computation time for both BP and OMP. CS on SHM data of operating tunnel can reduce the transmission and storage pressure on the original monitoring data

1 Introduction

Structure Health Monitoring (SHM) engages multiply sensors to continuously or periodically measure various structural indicators (e.g., structure displacement, structure vibration) to obtain the real-time structural health status and evaluate safety risk in abnormal situation (Bao et al., 2011). The SHM system needs to transmit and maintain large amounts of data due to the large scale of structure and high sampling rate, which requires high demand on processing power and memory space (Jayawardhana et al., 2017). For example, a tunnel (5 Km) SHM system consists about 800 sensors and operates nearly 24 hours every day with the 30-100Hz monitoring frequency, which will generate millions of data samples in a day (Liu et al., 2022). However, the widely-used sensor node (e.g., Imot2) only has about 64-128 MB combined memory (Kang et al., 2023), so storing and processing such a large amount of data could be a challenge for the SHM.

To mitigate the problem caused by big data volume, compression techniques like wavelet-based methods, are developed to reduce data volume without losing key information. Typical data compression schemes usually involve regular sampling on the already sampled signal and then compressing the data size utilizing different algorithms (Li et al., 2019). But, the compression efficiency and modeling errors of those method are limited because the key information in most of SHM data is carried only by a few samples compared to the total number of samples. Recently, the Compressed Sensing (CS) which can reconstruct data with smaller number of acquired samples (lower sampling rate) than that defined by Nyquist's theorem, has attracted considerable attention in various fields like computer vision, remote sensing, medical, etc (Rani et al., 2018).

CS relies on the signal's sparsity and can randomly sample the signals with compressed modality, then reconstruct the original signals from the linear measurements. The random matrix and linear projection of CS simplifies the data sampling procedure and downsizes the sample volume (Candès et al., 2006). Ambient vibration responses of structures are generally considered to be sparse (Kang et al., 2023) because in most of time the structure is in healthy condition only few data samples are abnormal and carry the key information for SHM, which satisfy the sparsity condition for utilizing CS. Thus, CS have potential to alleviate the problem in transmitting and maintaining large amounts of SHM data by compressively sampling the structural responses measured by sensors.

Therefore, the research questions of this project could be summarized as *how can we apply Compressed Sensing on Structure Health Monitoring data to reduce its data volume and recover data accurately*? Addressing such problem, this project engaged a dataset of tunnel SHM as case study, then compressively sampled the monitoring data at different sampling ratio, subsequently reconstructed the signal with difficult algorithms (i.e., orthogonal matching pursuit, basis pursuit) and compared their reconstruction accuracy.

The remainder of this report is organized as follows. Section 2 Methodology briefly reviews the theory and algorithms of CS. Section 3 Case Study introduces the background of the case and analyzes the results of CS in such case. Section 4 summarizes report and draw the conclusions.

2 Methodology

2.1 Fundamental of Compressed Sensing

CS mainly contains three parts, (i) sparse representation of the collected signal, (ii) design of measurement matrix and (iii) signal reconstruction (Donoho, 2006). The concept of CS can be represented as a linear equation Eq (1).

$$y = \Phi x \tag{1}$$

Where $x \in \mathbb{R}^{N\times 1}$ is the original signal with sparsity, the $\Phi \in \mathbb{R}^{M\times N}$ is the measurement matrix which is usually designed as a random matrix to satisfy the RIP (Restricted Isometry Property) requirement, the $y \in \mathbb{R}^{M\times 1}$ is the compressed measurement which is obtained by linear transformation of original signal. The data size of compressed measurement is significantly less than that of original signal because $M \ll N$ and can overcome the Nyquist rate.

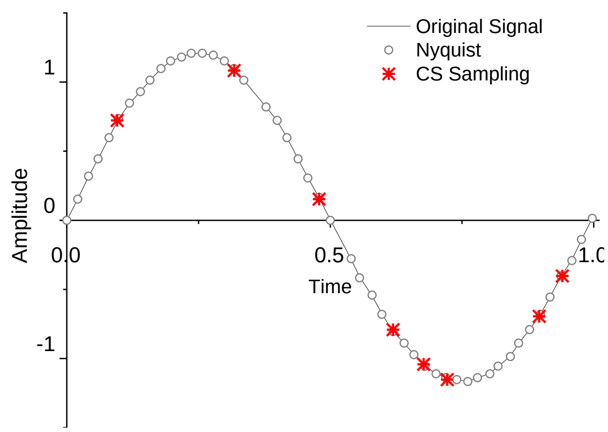

Figure 1. Compressively sampling Process

The objective of CS is to reconstruct original signal x from compressed measurement y, which need to solve the Eq (1). The x is high dimensional but spare, the x can be represented as $x = \theta s$ by assuming it has k-sparse in domain $\theta \in \mathbb{R}^{N\times M}$ (k is the number of non-zero values in basis coefficient $s \in \mathbb{R}^{M\times 1}$, $k \ll N$). The solution of the equation can be solved via the minimum the l_0-norm optimization as Eq. (2) shows, which is a NP-hard combinatorial optimization problem.

$$s = \arg\ \min\|s\|_0 \\ s.t.\ y = \Phi\theta s \tag{2}$$

2.2 Reconstruction Algorithm

(1) Convex optimization: Basis Pursuit

Solving the NP-hard problem is computational expensive as it requires to try all possible combination for Eq (2). Thus, alternative solutions with convert relaxation have been proposed to obtain a solution similar to the l_0-norm. A typical method is the basis pursuit (BP) proposed by Chen et al (1998), which solve the Eq (2) by minimum the l_1-norm. The BP can be represented as the optimization problem in Eq (3), which is a tractable linear programming problem and can be effectively solved by simplex algorithm or interior-point algorithm.

$$s = \arg\ \min\|s\|_1 \\ s.t.\ y = \Phi\theta s \tag{3}$$

(2) Greedy Algorithm: Orthogonal Matching Pursuit

Another type of reconstruction method is greedy algorithms. For example, the Orthogonal Matching Pursuit (OMP) utilize the combination of local optimization to explore the nonzero coefficients to reconstruct the original data (Tropp, 2004). The OMP starts the search by finding a column of Θ (Θ is called sensing matrix, $\Theta = \Phi\theta$) with maximum correlation with measurements y at the first step and thereafter at each iteration it searches for the column of Θ with maximum correlation with the current residual. In each iteration, the estimation of the signal vector is updated by the highly correlated column of Θ. The OMP can be performed as the following pseudocode.

Input Θ, y
Output x
Initialization: $r_0 = y,\ x_0 = 0,\ S_0 = \emptyset$
Repeat if $\|r_t\|_2 > \epsilon$

1. $\mathrm{j_t} = \mathrm{argmax}_{1\le \mathrm{j} \le \mathrm{N}} \left|\langle \Theta_j, r_{t-1}\rangle\right|$
2. $\mathrm{S_t} = S_{t-1} \cup J_t$
3. $\mathrm{x_t} = \mathrm{argmin}_{x\in\mathbb{R}^{N\times 1}} \left\|y - A_{S_t}x\right\|$
4. $r_t = b - Ax_t$

Return x_t

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3 Case study

3.1 Case Description

The data in this case is obtained from the Wuhan Metro tunnel SHM system. The tunnel in the monitoring interval contains a two-lane tunnel with an inner diameter of 6.2 m. The line spacing between the left and right lines is 14-19 m, and the tunnel burial depth is 15-42 m. The acceleration sensors were installed as Figure 2 shows to monitoring the vibration of the tunnel structure and evaluate its healthy performance.

In this report, six groups of acceleration sensor data (from DK9+696.728 to DK9+736.728) with close geometrical locations, numbered from 1 to 6 in order, are selected as the data source for the analysis of CS technique. Figure 3 presents 10000 data sample of different sensors (monitoring frequency is 50 Hz) from the SHM system. From Figure 3, the data measured from six acceleration sensors are relatively smooth and their amplitude fluctuations are relatively similar, which indicates the data has good quality. However, the original data is not sparse, so the Fourier transform is utilized to obtain the frequency domain of the monitoring data, as show in Figure 4. The frequency domain shows sparsity, thus it is feasible to use CS to compress data and reconstruct signal.

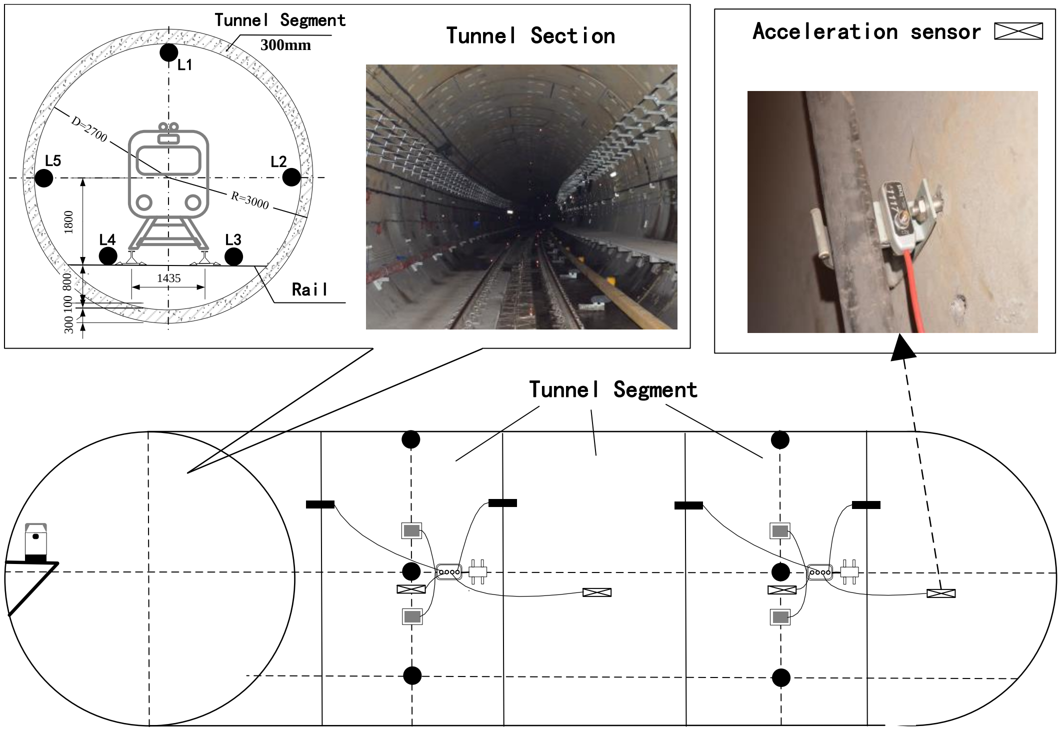

Figure 2. Sensors of SHM system

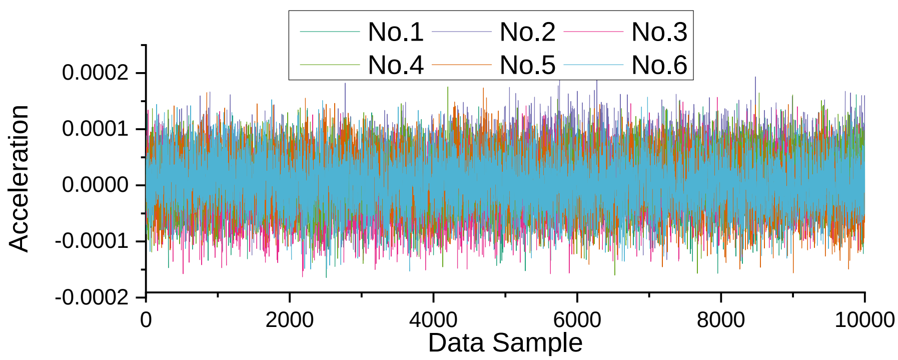

Figure 3. Monitoring data of accelerometers (No.1 Sensor – No.6 Sensor)

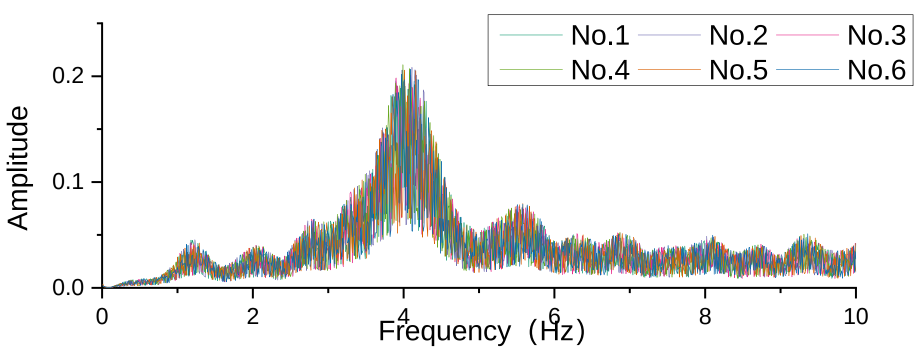

Figure 4. Frequency domain of monitoring data (No.1 Sensor – No.6 Sensor)

3.2 Compression of the collected signal

The original data is compressively sampled to reduce its volume, where several data samples under original sampling frequency is randomly selected using random matrix. As an example, the 10000 data samples obtained from sensor No. 1 was compressed and sampled at 10%, 15%, 20% and 30% sampling rates, and the results are shown in Fig. 5.

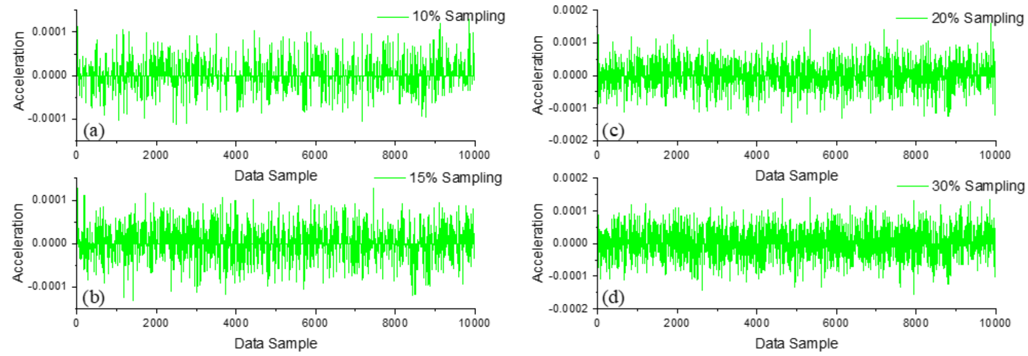

Figure 5. Compressed data: (a) 10% sampling rates, (b) 15% sampling rates, (c) 20% sampling rates, (d) 30% sampling rates

3.3 Signal Reconstruction

As Figure 5 shows, the data volume is significantly reduced after compressing. Then, the BP and OMP algorithms are engaged to reconstruct the signal from the compressed data. The data of sensor No. 1 were utilized as a example to present the reconstruction result, which is shown in Figure 6. From Figure 6, both BP can OMP can successfully reconstruct the signal from the compressed date, even for low sampling rate (i.e., 10%).

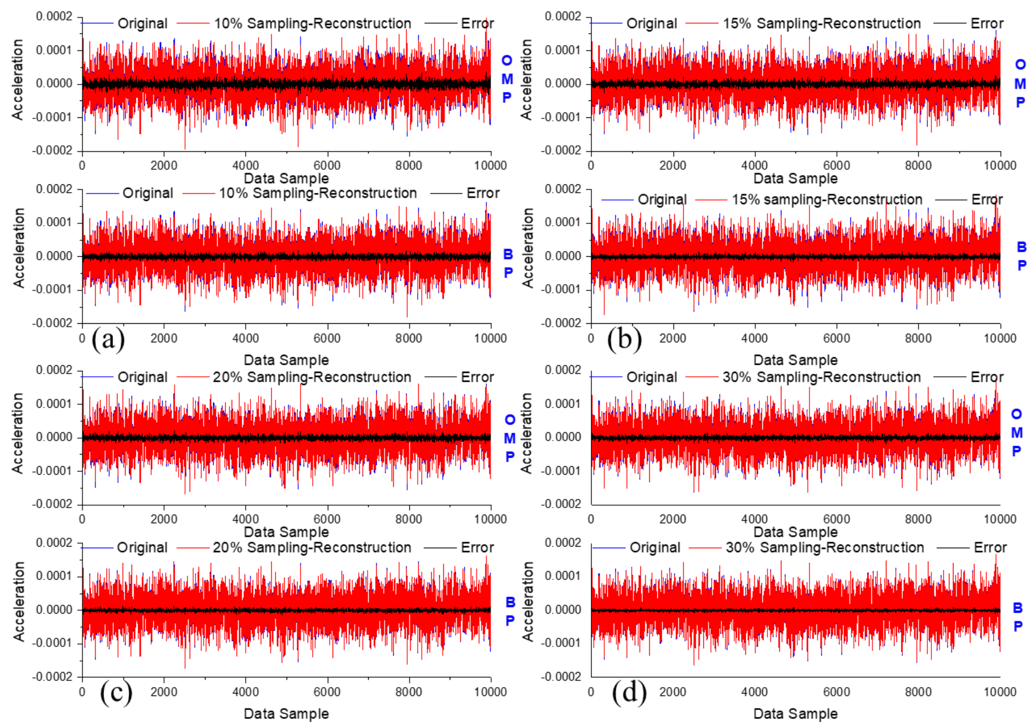

Figure 6. Reconstructed data: (a) reconstructing from10% sampling, (b) reconstructing from 15% sampling, (c) reconstructing from 20% sampling, (d) reconstructing from 30% sampling

3.4 Signal Reconstruction Accuracy Analysis

The errors generated in the compressing and reconstructing process demonstrate the applicability and reliability of the CS algorithm. From a qualitative standpoint, it can be inferred from Figure 6 that the reconstruction accuracy of the BP algorithm surpasses that of the OMP algorithm in a given sampling rate. Moreover, the reconstruction precision is positively correlated with the data sampling rate, indicating that a higher data sampling rate leads to a higher reconstruction precision.

To quantitatively analyze the differences between different CS algorithms, Eq (4) are employed to calculate the error between the original data and the reconstructed data.

$$\xi = \frac{\|\bar{u}-u\|}{\|u\|_2} \tag{4}$$

Where $\bar{u}$ is the reconstructed data, u is the original data.

Figure 7 illustrates the relationship between the reconstruction error and the sampling rate at six different measurement points using both BP and OMP algorithms. Additionally, to analyze the computational efficiency of the two algorithms, the required computation time (on MATLB 2022b with AMD R7-5800H CPU) for each algorithm is also presented in Figure 7.

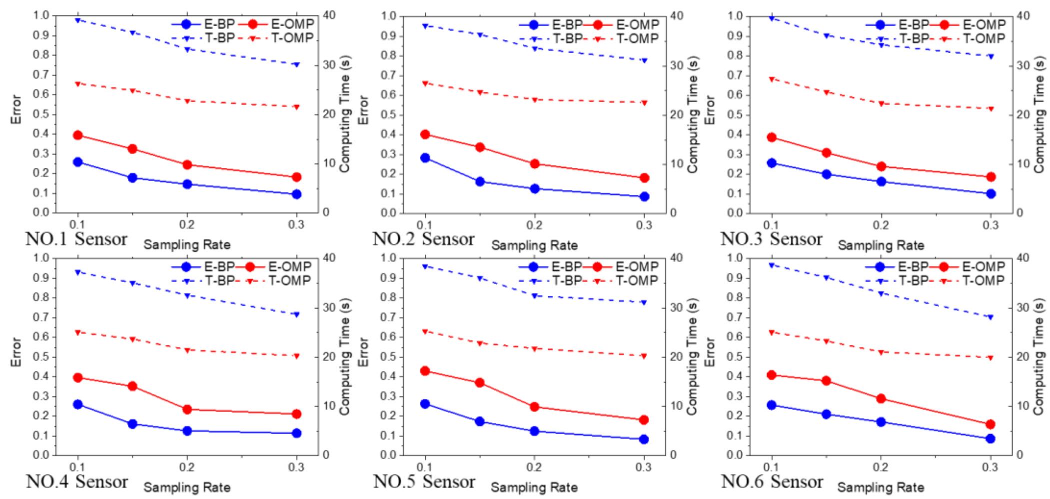

Figure 7. Reconstruction Accuracy Analysis

From Figure 7, the data from difficult sensor shows similar trends in reconstruction accuracy and computation time. The results of ξ in different sensors indicate that the BP algorithm has better accuracy on CS of SHM data than OMP algorithm, the ξ of BP is about 30% lower than using OMP to reconstruct. However, the computational efficiency has opposite relationship, the OMP algorithm requires significantly less time (spend about 10 seconds less) to reconstruct the signal from compressed data comparing with BP algorithm.

The BP and OMP have different properties due to their different theoretical base. The BP convert the l0-norm optimization into l1-norm optimization and use linear programming to solve the optimization problem, which can find the solution close to the original solution but requires high computational consumption. The OMP calculate iterations to achieve the combination of local optimization, which has less computational consumption and fast convergence. The OMP can conveniently implement on the outdated monitoring system with limited processing capability.

In addition, the sampling rate can influence the reconstruction accuracy and computation time, specifically, increasing sampling rate can reduce reconstruction error and computation time. If the sampling rate is 30%, the CS algorithms are capable to reconstruct the signal with less than 20% relative error, which can guarantee both the monitoring efficiency and precision in SHM system.

4 Conclusion

Addressing the problem in transmitting and maintaining large amounts of SHM data, this project used a tunnel SHM dataset containing 10000 data sample as case study, then compressively sampled the monitoring data at different sampling ratio, subsequently reconstructed the signal with BP and OMP algorithms and analyses the reconstruction error. The results show that (1) the tunnel SHM date can use CS to efficiently compress and successfully reconstruct; (2) BP has higher reconstruction accuracy on tunnel SHM date than that of OMP, while BP has worse performance on computation time and consumption than OMP; (3) increasing sampling rate can reduce reconstruction error and computation time for both BP and OMP, 30% sampling rate can guarantee both the monitoring efficiency and precision in SHM system.

Compressing and reconstructing the structural health monitoring data of operating tunnel can reduce the transmission and storage pressure on the original monitoring data, providing a data basis for later tunnel health status evaluation, which provide decision support for the tunnel safety maintenance measures.

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