Homework 6

WU Jiamin 20659352

3. (a) $X_1 : n \times k$ $B: n \times (N-n)$ $C: (N-n) \times (N-n)$

$A = X_1 X_1^T$ $B = X_1 X_2^T$

$A = U \Lambda U^T \Rightarrow X_1 = U_k \Lambda_k^{\frac{1}{2}}$

$X_2 = B^T X_1 = B^T U_k \Lambda_k^{\frac{1}{2}}$

(b) $x_{ij} = \begin{cases} \sqrt{\lambda_j} U_{ij} & \text{if } j \le n \\ \sum_p B_{pi} U_{pj} / \sqrt{\lambda_j} & \text{o.w.} \end{cases}$

$A = X_1 X_1^T$

$= (U_k \Lambda_k^{\frac{1}{2}})(U_k \Lambda_k^{\frac{1}{2}})^T$

$= U \Lambda U^T$

$B = X_1 X_2^T$

$= (U_k \Lambda_k^{\frac{1}{2}})(B^T U_k \Lambda_k^{\frac{1}{2}})^T$

when k is of rank m or Less

$\hat{k} = \begin{bmatrix} A & B \\ B^T & B^T A^{-1} B \end{bmatrix}$

(d) A is invertible

$\det(K) = \det \begin{pmatrix} A & B \\ B^T & C \end{pmatrix}$

$= \det(A) \det(C - B^T A^{-1} B)$

where $C - B^T A^{-1} B = K/A$ is the Schur Complement of A in K.

$\det(K) = \det(A) \cdot \det(K/A)$

(e) Similarly, $\text{rank}(K) = \text{rank}(A) + \text{rank}(K/A)$.