Question Bank - Permutations and combinations

LEVEL–I

1. Three numbers are chosen from 1, 2, 3 . . . , n . In how many ways can the numbers be chosen such that either maximum of these numbers is s or minimum of these numbers is r ($r < s$)?

2. Six candidates are called for interview to fill four posts in an office. Assuming that each candidate is fit for each post, determine the number of ways in which

(i) First and second posts can be filled

(ii) First three posts can be filled.

3. In how many ways 4 identical white balls and 6 identical black balls be arranged in a row so that no two white balls are together?

4. Find the sum of the digits in the unit place of all the number formed with the help of 3, 4, 5, 6 taken all at a time.

5. There are 21 balls, which are either white or black. Balls of the same colour are alike. Find the number of white balls, so that the number of arrangements of these balls in a row be maximum.

6. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.

7. How many different words can be formed with the letters of the word ORDINATE so that

(i) the vowels occupy odd places.

(ii) the word begin with O.

(iii) the word begin with O and end with E.

8. The result of 11 chess matches (as win, loose or draw) are to be forecast. Out of all possible forecasts, find how many will have 8 correct and 3 incorrect results.

9. 5 boys and 4 girls sit in a straight line. Find the number of ways in which they can be seat if 2 girls are together and the other 2 are also together but separate from the first 2.

10. How many 7 digit numbers are there the sum of whose digits is even ?

LEVEL–II

1. Prove that the number of words that can be formed out of the letter a, b, c, d, e and f taken three together, each word containing at least one vowel atleast is 96.

2. A candidate is required to answer 7 out of 15 questions which are divided into three groups A, B and C each containing 4, 5 and 6 questions respectively. He is required to select at least 2 questions from each group. In how many ways can he make up his choice?

3. Four digit numbers are to be formed by using the digits 0, 1, 2, 3, 4, 5. What is the number of such numbers if

(i) repetition is not allowed **(ii)** repetition is allowed,

(iii) at least one digit is repeated?

4. There are 12 balls of which 4 are red, 3 black and 5 white. In how many ways can the balls be arranged in a line so that no two white balls occupy consecutive positions, if balls of the same colour are

(i) identical. **(ii)** different.

5. A tea party is arranged for 2m people about two sides of a long table with m-chairs on each side r-men wish to sit on one particular side and s on the other. In how many ways they can be seated. (r, s $\leq$ m).

6. Find the number of subsets containing 2 elements of the set $\{1, 2, 3, \ldots, 100\}$, sum of whose elements is divisible by 3.

7. In a chess tournament, where the participants are to play one game with one another two players fell ill, having played only 3 games each . If the total number of games played in the tournament is equal to 84 then find the number of participants in the beginning.

8. In a certain examination of 6 papers each paper has 100 marks as maximum marks. Show that the number of ways in which a candidate can secure 40% marks in the whole examination is

$$\frac{1}{5!}\left\{\frac{245!}{240!} - 6.\frac{144!}{139!} + 15.\frac{43!}{38!}\right\}.$$

9. In an examination the maximum marks for each of the three papers are n and for the fourth paper is 2n. Prove that the number of ways in which a candidate can get 3n marks is $\frac{1}{6}(n+1)(5n^2+10n+6)$.

10. Find the number of ways in which p identical white balls, q identical black balls & r identical red balls can be put in n different bags, if one or more of the bags remain empty.

IIT JEE PROBLEMS (OBJECTIVE)

A. Fill in the blanks

1. In a certain test, a_i students gave wrong answers to at least i question, where i = 1, 2, . . . k. No student gave more than k wrong answers. The total number of wrong answers given is .... **[IIT - 82]**

2. The side AB, BC and CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is .... **[IIT - 84]**

3. Total number of ways in which six '+' and four '–' signs occur together is .... **[IIT - 88]**

4. There are four balls of different colours and four boxes same as these of the balls. The number of ways in which the balls, one in each box could be placed such that a ball does not go in a box of its own colour is.... **[IIT - 92]**

5. Let n and k be positive integers such that $n \geq \frac{k(k+1)}{2}$. The number of solutions **[IIT - 96]**

$(x_1, x_2 \ldots x_k)$, $x_1 \geq 1,\ x_2 \geq 2, \ldots x_k \geq k$,all integers, satisfying $x_1 + x_2 + \ldots + x_k = n$, is....

B. True /False

6. The product of any r consecutive natural numbers is always divisible by r!. **[IIT - 85]**

C. Multiple choice questions with one or more than one correct answer

7. An n - digit number is a positive number with exactly 'n' digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is **[IIT - 98]**

(A) 6 (B) 7 (C) 8 (D) 9

D. Multiple choice questions with one correct answer

8. The number of ways in which five identical balls can distributed among ten identical boxes such that no box contains more than one ball, is **[IIT - 73]**

(A) 10! (B) $\frac{10!}{5!}$ (C) $\frac{10!}{(5!)^2}$ (D) none of these

9. The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth player just one card, is **[IIT - 79]**

(A) $\frac{52!}{17!}$ (B) 52! (C) $\frac{52!}{(17!)^3}$ (D) none of these

10. Ten different letters of an alphabet are given. Words with five letters are formed from the given letters, then the number of words which have atleast one letter repeated is **[IIT - 80]**

(A) 69760 (B) 30240 (C) 99748 (D) none of these

11. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst chairs marked 1 to 4, and then men select the chairs from amongst the remaining. The number of possible arrangements is **[IIT - 80]**

(A) ${}^6C_3 \times {}^4C_2$ (B) ${}^4P_3 \times {}^4P_2$ (C) ${}^4C_3 + {}^4C_3$ (D) none of these

12. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subsets of A consisting of all determinants with value – 1. Then **[IIT - 81]**

(A) C is empty (B) B has as many elements as C

(C) $A = B \cup C$ (D) B has twice as many elements as many elements as C

13. The value of ${}^nP_r$ is equal to

(A) ${}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$ (B) $n\,.\,{}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$

(C) $n({}^{n-1}P_r + r\,{}^{n-1}P_{r-1})$ (D) ${}^{n-1}P_{r-1} + {}^{n-1}P_r$ **[IIT -83]**

14. The sides AB, BC, CA of a triangle ABC have respectively 3, 4 and 5 points lying on them. The number of triangles that can be constructed using these points as vertices is **[IIT - 84]**

(A) 205 (B) 220 (C) 210 (D) none of these

15. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw **[IIT - 86]**

(A) 64 (B) 45 (C) 46 (D) none of these

16. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4, 5 without repetition. The total number of ways this can be done is **[IIT - 89]**

(A) 216 (B) 600 (C) 240 (D) 3125

17. A student is allowed to select at most, n books from a collection of (2n + 1) books. If the total number of ways in which he can select a book is 63, then n is equal **[IIT - 96]**

(A) 3 (B) 4 (C) 5 (D) 6

18. Number of divisors of the form $4n + 2(n \geq 0)$ of the integer 240 is **[IIT - 98]**

(A) 4 (B) 8 (C) 10 (D) 3

19. In a college of 300 students, every student reads 5 news papers & every paper is read by 60 students. The number of news papers is **[IIT - 98]**

(A) atleast 30 (B) almost 20 (C) exactly 25 (D) none of these

20. The number of ways in which 5 male and 2 female members of committee can be seated around a table so that two female members are not seated together is **[REE - 99]**

(A) 480 (B) 600 (C) 720 (D) 840

21. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that odd digits occupy even positions ? **[IIT - 2000]**

(A) 16 (B) 36 (C) 60 (D) 180

22. The maximum number of triangles that can be formed by choosing the vertices from a set of 12 points, 7 of which are in a straight line are **[REE - 2000]**

(A) 50 (B) 105 (C) 175 (D) 185

23. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of 'n' sides. If $T_{n+1} - T_n = 21$, then 'n' equals **[IIT - 2001]**

(A) 5 (B) 7 (C) 6 (D) 4

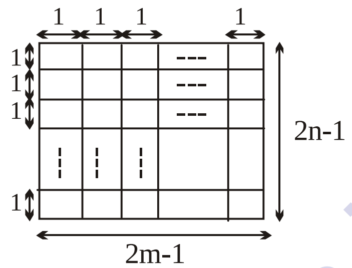
24. Let E = {1, 2, 3, 4} & F = {1, 2}. Then the number of onto functions from E to F is

(A) 14 (B) 16 (C) 12 (D) 8 **[IIT - 2001]**

25. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is **[IIT - 2002]**

(A) 40 (B) 60 (C) 80 (D)100

26. A rectangle with sides $2m - 1$ and $2n - 1$ is divided into square of unit length by drawing parallel lines as shown in diagram, then the number of rectangles possible with odd side length is



(A) $(m + n - 1)^2$ (B) 4^{m+n-1}

(C) m^2n^2 (D) $m(m + 1) n(n + 1)$ **[IIT - 2005]**

27. If r, s, t are prime numbers and p, q are the positive integers such that LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is **[IIT - 2006]**

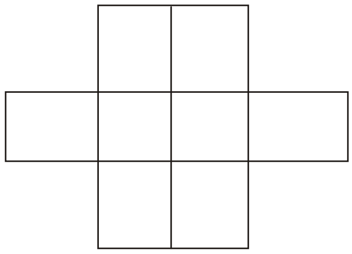
(A) 252 (B) 254 (C) 225 (D) 224

28. The letters of the **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word **COCHIN** is

(A) 360 (B) 192 (C) 96 (D) 48 **[IIT - 2007]**

IIT JEE PROBLEMS (SUBJECTIVE)

1. Six X have to be placed in the squares of Figure below in such a way that each row contains at least one X. In how many different ways can this be done ? **[IIT - 78]**



2. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Find the number of words which have at least one letter repeated (although not necessarily consecutively). **[IIT - 80]**

3. Five balls of different colours are to be placed in three boxes of different size. Each box can hod all five. In how many different ways can we placed the balls so that no box remains empty ? **[IIT -81]**

4. Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Prove that there is a bijective mapping from A to B. **[IIT - 81]**

5. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated is $\frac{m!(m+1)!}{(m-n+1)!}$. **[IIT - 83]**

6. Show that the number of ways in which 3 numbers in arithmetical progression can be selected from 1, 2, 3, ...n is $\frac{1}{4}(n-1)^2$ or $\frac{1}{4}n(n-2)$, according as 'n' is odd or even. **[REE- 84]**

7. A man has 7 relatives, 4 of them are ladies and 3 gentlemen; his wife also has 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can they a dinner party of 3 ladies and 3 gentlemen so that there are 3 of the man's relatives and 3 of wife's relatives ? **[IIT - 85]**

8. A Palindrome is a word which reads the same backward as forward (i.e. madam or anna). Find how many Palindrome of length n can be formed from an alphabet of k . **[REE-85]**

9. Let A be a set of n distinct elements. Find the total number of distinct functions from A to A. Also find how many of these functions are onto. **[IIT - 85]**

10. There are 'n' straight lines in a plane, no 2 of which parallel, and no 3 pass through the same point. Their point of intersection are joined. Show that the number of fresh lines thus introduced are $\frac{n(n-1)(n-2)(n-3)}{8}$. **[REE- 86]**

11. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw. **[IIT - 86]**

12. A student is allowed to select almost n books from a collection of $(2n + 1)$ books. If the total number of ways in which he can select at least one book is 63, find the value of n. **[IIT - 87]**

13. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side. Determine the number of ways in which the seating arrangements can be made. **[IIT - 91]**

14. In a football championship, there were played 153 matches. Every two teams played one match with each other. Find the number of teams, participating in the championship. **[IIT - 92]**

15. There are four balls of different colors and four boxes of colors, same as those of the balls. Find the number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own color. **[IIT - 92]**

16. A committee of 12 is to be formed from 9 women and 8 men. In how many ways can this be done if at least five women have to be included in a committee ? In how many of these committees :
(i) the women are in majority ? **(ii)** then men are in majority ? **[IIT - 94]**

17. In how many ways can three girls and nine boys be selected in two vans, each having numbered seats, 3 in the front and 4 in the back ? How many seating arrangements are possible if the three girls should sit together in a back row on adjacent seats ? Now, if all the seating arrangements are equally, what is the probability of 3 girls sitting together in a back row on adjacent seats ? **[IIT - 96]**

18. Find the total number of ways of selecting five letters from the letters of the word INDEPENDENT . **[IIT - 96]**

19. Find the number of divisors of the form $4n + 2$ $(n \geq 0)$ of the integer 240? **[IIT – 1998]**

20. How many different 9 digit numbers can be formed from the number 223355888 by rearranging its digits, so that the odd digits occupy even position? **[IIT – 2000]**

21. Using permutation or otherwise, prove that $\dfrac{(n^2)!}{(n!)^n}$ is an integer, where n is a positive integer. **[IIT – 2004]**

22. Prove that $\dfrac{(n^2)!}{(n!)^n}$ is an integer, using permutations or otherwise. **[IIT – 2004]**

23. If total number of runs scored in n matches is $\left(\dfrac{n+1}{4}\right)(2^{n+1} - n - 2)$, where $n > 1$, and the runs scored in the k^{th} match are given by $k \,.\, 2^{n+1-k}$, where $1 \leq k \leq n$. Find n. **[IIT – 2005]**

Permutations and combinations

SET–I

1. In a college examination, a candidate is required to answer 6 out of 10 questions which are divided into two sections each containing 5 questions. Further the candidate is not permitted to attempt more than 4 questions from either of the section. The number of ways in which he can make up a choice of 6 questions, is
(A) 200 (B) 150 (C) 100 (D) 50

2. The number of way of seven digit number of the form $a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7$, $(a_i \neq 0\ \forall i = 1, 2, ..., 7)$ be present in decimal system, such that $a_1 < a_2 < a_3 < a_4 > a_5 > a_6 > a_7$, is
(A) 820 (B) 720 (C) 620 (D) 4878

3. Ramesh has 6 friends. In how many ways can be invite one or more of them at a dinner ?
(A) 61 (B) 62 (C) 63 (D) 64

4. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is
(A) 11 (B) 12 (C) 27 (D) 63

5. The straight lines I_1, I_2, I_3 are parallel and lie in the same plane. A total number of m points are taken on I_1, n points on I_2, k points on I_3. The maximum number of triangles formed with vertices at these points are
(A) ${}^{m+n+k}C_3$ (B) ${}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$
(C) ${}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3$ (D) none of these

6. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is
(A) 10^2 (B) 1023 (C) 2^{10} (D) 10 !

7. How many numbers between 5000 and 10000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number ?
(A) $5 \times {}^8P_3$ (B) $5 \times {}^8C_8$ (C) $5! \times {}^8P_3$ (D) $5 \times {}^8C_3$

8. The total number of permutations of 4 letters that can be made out of the letters of the word EXAMINATION is
(A) 2454 (B) 2436 (C) 2545 (D) none of these

9. The total number of selections (taking at least one) of fruit which can be made from 3 bananas, 4 apples and 2 oranges is
(A) 59 (B) 315 (C) 512 (D) none of these

10. A lady gives a dinner party to 5 guests to be selected from nine friends. The number of ways of forming the party of 5, given that two of the friends will not attend the party together is
(A) 56 (B) 126 (C) 91 (D) none of these

11. The greatest possible number of points of intersection of 8 straight lines and 4 circles is
(A) 32 (B) 64 (C) 76 (D) 104

12. The number of all the possible selections which a student can make for answering one or more questions out of eight given questions in a paper, when each question has an alternative is
(A) 256 (B) 6560 (C) 6561 (D) none of these

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¨À¯ ¯ã ¨à¯ $\frac{6^n - 6}{n} + 6$ ¨Å¯ $^{n+5}C_5$ ¨ã¯ ãÐãÛ Ðû õìÛÕÛ

14. A father with 8 children takes them 3 at a time to the Zoological Gardens, as often as he can without taking the same 3 children together more than once. The number of times each child (and father)will go to the garden respectively

(A) 56, 21 (B) 21, 56 (C) 112, 336 (D) none of these

15. The number of ways in which four letters can be selected from the word DEGREE is

(A) 7 (B) 6 (C) $\frac{6!}{3!}$ (D) none of these

16. Number of ways in which Rs. 18 can be distributed amongst four persons such that no body receives less than Rs. 4 is

(A) 4^2 (B) 2^4 (C) 4! (D) none of these

17. There are five different green dyes, four different blue dyes and three different red dyes. The total number of combinations of dyes that can be chosen taking at least one green and one blue dye is

(A) 3255 (B) 2^{12} (C) 3720 (D) none of these

18. There are n straight lines in a plane, no two of which are parallel, and no three pass through the same point. Their points of intersection are joined. Then the number of fresh lines thus obtained is

(A) $\frac{n(n-1)(n-2)}{8}$ (B) $\frac{n(n-1)(n-2)(n-3)}{6}$

(C) $\frac{n(n-1)(n-2)(n-3)}{8}$ (D) none of these

19. All possible two-factor products are formed from the numbers 1, 2, ..., 100. The number of factors out of the total obtained which are multiple of 3 is

(A) 2211 (B) 4950 (C) 2739 (D) none of these

20. The number of ways of choosing a committee of 4 women and 5 men from 10 women and 9 men, if Mr. A refuses to serve on the committee if Ms. B is member of the committee, cannot exceed

(A) 20580 (B) 21000 (C) 21580 (D) 22000

SET–II

1. The number of signals that can be generated by using 6 different coloured flags, when any number of them may be hoisted at a time is

(A) 1956 (B) 1957 (C) 1958 (D) 1959

2. The number of ways of distributing 5 prizes to 4 boys is (when each boy is eligible for any number of prizes)

(A) 56 (B) 625 (C) 600 (D) 1024

3. Total number of regions in which 'n' coplanar circles can divide the plane, it is known that each pair of circles intersect in two different points and no three of them have common point of intersection, is equal to

(A) $\frac{1}{2}(n^2+n+2)$ (B) $\frac{1}{2}(n+3n^2)$ (C) $\frac{1}{2}(3n+n^2)$

(D) $(n^2 - n + 2)$

4. Number of sub parts into which 'n' straight lines (no two are parallel and no three are concurrent) in a plane can divide it is :

(A) $\frac{n^2+n+2}{2}$ (B) $\frac{n^2+n+4}{2}$ (C) $\frac{n^2+n+6}{2}$

(D) none of these

5. The total number of arrangements which can be made out of the letters of the word **ALGEBRA** without altering the relative position of vowels and consonants is

(A) 4! . 3! (B) $\frac{4!3!}{2}$ (C) 2(4! . 3!) (D) none of these

6. m parallel lines in a plane are intersected by a family of n parallel lines. The total number of parallelograms so formed is

(A) $\frac{(m-1)(n-1)}{4}$ (B) $\frac{mn}{4}$

(C) $\frac{nm(m-1)(n-1)}{2}$ (D) $\frac{nm(m-1)(n-1)}{4}$

7. There are 20 persons among whom two are brothers. The number of ways in which we can arrange them around a circle so that there is exactly one person between the two brothers, is

(A) 2(18!) (B) 18! (C) (18! × 18!) (D) none of these

8. The number of five digit numbers in which digits decrease from left to right, is

(A) ${}^{9}C_5$ (B) ${}^{10}C_5$ (C) $2 \times {}^{10}C_5$ (D) ${}^{10}P_5$

9. The number of ways in which one can post 5 letters in 7 letter boxes is

(A) 5^7 (B) ${}^{7}P_5$ (C) 7^5 (D) 35

10. The number of permutations of all the letters of the word 'MISSISSIPPI' is

(A) 46504 (B) 34650 (C) 77880 (D) none of these

11. A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be selected so that there are at least two balls of each colour

(A) 425 (B) 426 (C) 452 (D) none of these

12. The number of ways of arranging m positive and n (< m + 1) negative signs in a row so that no two negative signs are together, is

(A) ${}^{m+1}P_n$ (B) ${}^{n+1}P_m$ (C) ${}^{m+1}C_n$ (D) ${}^{n+1}C_m$

13. The total number of ways of selecting six coins out of 20 one rupee coins, 10 fifty paise coins and 7 twenty five paise coins, is

(A) 26 (B) 28 (C) 56 (D) none of these

14. There are m copies of each of n different books in a university library. The number of ways in which one or more than one book can be selected is

(A) $m^n + 1$ (B) $(m + 1)^n - 1$ (C) $(n + 1)^n - m$ (D) m

15. The number of 5 - digit numbers in which no two consecutive digits are identical is

(A) $9^2 \times 8^3$ (B) 9×8^4 (C) 9^5 (D) none of these

16. There are 12 balls numbered from 1 to 12. The number of ways in which they can be used to fill 8 places in a row so that the balls are with numbers in ascending or descending order, is equal to

(A) ${}^{12}C_8$ (B) ${}^{12}P_8$ (C) $2 \times {}^{12}P_8$ (D) $2 \times {}^{12}C_8$

17. 'n' is selected from the set {1, 2, 3,.........100} and the number $2^n + 3^n + 5^n$ is formed. Total number of ways of selecting 'n' so that the formed number is divisible by 4, is equal to

(A) 50 (B) 49 (C) 48 (D) none of these

18. For a game in which two partners oppose two other partners, 6 men are available. If every possible pair must play every other pair, the number of games played is

(A) 15 (B) 45 (C) 90 (D) 105

19. Five persons including one lady are to deliver lectures to an audience. The organizer can arrange the presentation of their lectures, so that the lady is always in the middle, is

(A) ${}^5P_5$ ways (B) $4. {}^4P_4$ ways (C) 4! ways (D) ${}^5C_4$ ways

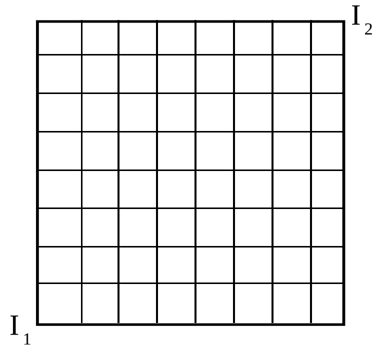
20. Let m denote the number of ways in which 4 different books are distributed among 10 persons, each receiving none or one only and let n denote the number of ways of distribution if the books are all alike. Then

(A) m = 4n (B) n = 4m (C) m = 24n (D) none of these

SET–III

More than one correct

1. On the normal chess board as shown, I_1 and I_2 are two insects which starts moving towards each other. Each insect moving with the same constant speed. Insect I_1 can move only to the right or upward along the lines while the insect I_2 can move only to the left or downward along the lines of the chess board. The total number of ways the two insects can meet at same point during their trip is



(A) $\binom{9}{8}\binom{10}{7}\binom{11}{6}\binom{12}{5}\binom{13}{4}\binom{14}{3}\binom{15}{2}\binom{16}{1}$

(B) $2^8\binom{1}{1}\binom{3}{2}\binom{5}{3}\binom{7}{4}\binom{9}{5}\binom{11}{6}\binom{13}{7}\binom{15}{8}$

(C) $\binom{2}{1}\binom{6}{2}\binom{10}{3}\binom{14}{4}\binom{18}{5}\binom{22}{6}\binom{26}{7}\binom{30}{8}$

(D) C (16, 8)

2. Number of quadrilaterals which can be constructed by joining the vertices of a convex polygon of 20 sides if none of the side of the polygon is also the side of the quadrilateral is

(A) ${}^{17}C_4 - {}^{15}C_2$ (B) $\frac{{}^{15}C_3 . 20}{4}$ (C) 2275 (D) 2125

3. Consider the expansion, $(a_1 + a_2 + a_3 + ... + a_p)^n$ where $n \in N$ and $n \le p$. The correct statement(s) is/are

(A) Number of different terms in the expansion is, ${}^{n+p-1}C_n$

(B) Co-efficient of any term in which none of the variables $a_1, a_2, ..., a_p$ occur more than once is 'n'

(C) Co-efficient of any term in which none of the variables $a_1, a_2, ..., a_p$ occur more than once is $n!$

(D) Number of terms in which none of the variables $a_1, a_2, ..., a_p$ occur more than once is $\binom{p}{n}$

4. In an examination of 9 papers, a candidate has to pass in more papers than the number of papers in which he fails, in order to be successful. The number of ways in which he can be unsuccessful is

(A) 255 (B) 256 (C) 193 (D) 319

5. Identify the correct statement(s).

(A) Number of naughts standing at the end of $\lfloor 125$ is 30

(B) A telegraph has 10 arms and each arm is capable of 9 distinct positions excluding the position of rest. The number of signals that can be transmitted is $10^{10} - 1$

(C) Number of numbers greater than 4 lacs which can be formed by using only the digits 0, 2, 2, 4, 4 and 5 is 90

(D) In a table tennis tournament, every player plays with every other player. If the number of games played is 5050 then the number of players in the tournament is 100

W I Let us examine the operation of addition and substraction on the set of natural numbers. If we add two natural numbers, we use to get a natural number but in case of substraction of one natural number with other output need not to be a natural number. Hence operation '+' is a binary operation on the set of natural numbers (it is commutative also), but operation '–' is not a binary operation on the set of natural number. Note that operation '–' is a binary on the set of integers.

In general let an operation $*$ on a non-empty set S is defined. If $a_i * a_j \in S \ \forall \ a_i, \ a_j \in S$, then it is said to be a binary operation and it is said to be commutative if $a_i * a_j = a_j * a_i \ \forall \ a_i, \ a_j \in S$.

Let $S = \{a_1, a_2, ..., a_n\}$

6. Total number of binary operation on S will be

(A) n^2 (B) n^n (C) n^{n^2} (D) none of these

7. Total number of binary operation on S which are commutative is

(A) $\frac{n^2+n}{2}$ (B) $\frac{n^2-n}{2}$ (C) $n^{\frac{n^2+n}{2}}$ (D) $n^{\frac{n^2}{2}}$

8. Total number of binary operation on S such that $a_i * a_j \neq a_i * a_k, \ \forall \ j \neq k$, is

(A) $n!\,n$ (B) $(n!)^n$ (C) $n^{n!}$ (D) $(n + 1)!$

9. Let $a_1, a_2, a_3, ...a_n$ be the distinct real numbers, then total number of binary operation on S such that $a_i * a_j \le a_i * a_{j+1} \ \forall \ i, j$, is

(A) $(n-1)^{\frac{n^2-n}{2}}$ (B) $(n)^{\frac{n^2-n}{2}}$ (C) $n^{\frac{n}{2}}$ (D) none of these

W II. The number of ways in which certain distinct object can be divided into some non-numbered groups

$$= \frac{\textit{Factorial of total number of object}}{\begin{array}{c}\textit{Product of factorial number of elements in each group} \times \textit{product of factorial}\\ \textit{of number of groups having same number of elements (if any)}\end{array}}$$

And number of ways in which certain distinct object can be divided into some numbered groups (this is also known as distribution)

$$= \frac{\begin{array}{c}\textit{Number of grouping (as in the previous)} \times \textit{factorial of number of persons in which object was}\\ \textit{suppose to be distributed}\end{array}}{\textit{factorial of number of person who got nothing (if any)}}$$

Example :

The number of ways in which 20 distinct object can be divided in 6 groups 3 having two elements each, two having three elements each and one having eight elements, is

$$\frac{20!}{2!\ 2!\ 2!\ 3!\ 3!\ 8!\ 3!\ 2!}$$

Example :

The number of ways in which 20 distinct objects can be distributed among 7 person so that 4 of them got 5 each and 3 of them got nothing, is

$$\left(\frac{20!}{5!\ 5!\ 5!\ 5!\ 4!}\right) \times \frac{7!}{3!}$$

10. The number of ways in which 10 digits can be divided in 2 groups of 5 each is

(A) $\frac{10!}{5!\ 5!\ 2!}$ (B) $\frac{10!}{5!\ 5!}$ (C) $2\times\frac{10!}{5!\ 5!}$ (D) none of these

11. The number of ways in which 26 alphabets can be distributed among 3 persons so that 2 of them got 9 each

(A) $\left(\frac{26!}{9!\ 8!\ 2!}\right)\times 3!$ (B) $\left(\frac{26!}{9!\ 9!\ 8!\ 2!}\right)\times 3!$ (C) $\left(\frac{26!}{9!\ 8!\ 2!}\right)$ (D) none of these

12. The number of ways in which 'n' distinct object can be distributed among n persons so that exactly one of them got nothing

(A) ${}^{n}C_2\times{}^{n}C_2\times n!$ (B) $n\times n!$ (C) ${}^{n}C_2\times n!$ (D) none of these

13. The number of ways in which 12 balls be divided into groups of 5, 4 and 3 respectively is

(A) $\frac{12!}{5!4!3!}$ (B) $\frac{12!}{3!(5!4!3!)}$ (C) $\frac{12!}{4!4!4!}$ (D) none of these

14. The number of ways in which 12 different balls be divided between 2 boys, one of them receives 5 and the other 7 balls, is

(A) 1560 (B) 1584 (C) 792 (D) none of these

W III. If we have n-balls and r distinct boxes then number of ways of putting or arranging all the balls inside the boxes is given by following table

Balls are distinct or Identical	**Blank box Allowed**	**Order inside the box required**	**Number of ways**
Distinct	No	Yes	${}^{n-1}C_{r-1}\,.\,n!$
Distinct	Yes	Yes	${}^{n+r-1}C_{r-1}\,.\,n!$
Distinct	No	No	$r^n-{}^{r}C_1(r-1)^n+{}^{r}C_2(r-2)^n-{}^{r}C_3(r-3)^n+\ldots$
Distinct	Yes	No	r^n
Identical	No	No	${}^{n-1}C_{r-1}$
Identical	Yes	No	${}^{n+r-1}C_{r-1}$

15. The number of ways in which 9 non-zero digits can be used to form 3 natural number x, y, z so that each digit used exactly once

(A) ${}^{8}C_2\,.\,9!$ (B) ${}^{11}C_2\,.\,9!$ (C) 3^9 (D) none of these

16. In how many ways a person can post 10 letters in 3 letter box so that in each box at least one letter is post
(A) $3^9 - 2^{10} + 3$ (B) $3^{10} - 3.2^{10} + 3$ (C) $3^9 + 2^{10} + 3$ (D) none of these

17. In how many ways 10 person can stand in 4 rows
(A) ${}^{10}C_3 . 10!$ (B) ${}^{9}C_3 . 10!$ (C) ${}^{13}C_3 . 10!$ (D) none of these

18. The number of ways in which 7 identical balls can be put in three boxes x, y, z
(A) ${}^{9}C_2$ (B) ${}^{10}C_2$ (C) ${}^{12}C_2$ (D) none of these

19. **Fill In The Blanks :**

(i) Number of proper divisors of 2520 which are divisible by 10 is ______ and the sum of these divisors is ______ .

(ii) The smallest positive integer n with 24 divisors (including 1 and n) is _____ .

(iii) Team A and B play in a tournament . The first team that wins two games in a row or wins a total o f four games is considered to win the tournament . The number of ways in which tournament can occur is ________ .

(iv) Number of ways 3 tickets can be selected from a set of 100 tickets numbered , 1 , 2 , 3 ...., 100 so that the number on them are in geometric progression is _____ .

(v) Number of different heptagons which can be formed by joining the vertices of a polygon having 16 sides, if none of the sides of the polygon is the side of the heptagon is _______.

(vi) Number of 9 digits numbers divisible by nine using the digits from 0 to 9 if each digit is used atmost once is K . 8 ! , then K has the value equal to ______ .

(vii) In maths paper there is a question on "Match the column" in which column A contains 6 entries and each entry of column A corresponds to exactly one of the 6 entries given in column B written randomly . 2 marks are awarded for each correct matching and 1 mark is deducted from each incorrect matching . A student having no subjective knowledge decides to match all the 6 entries randomly . The number of ways in which he can answer, to get atleast 25 % marks in this question is ______ .

(viii) There are p intermediate railway stations on a route from one terminus to other . Number of ways in which a train can be stopped at 3 stations if no two stations are consecutive is _______ .

(ix) If $N = 2^{p-1} . (2^p - 1)$, where $2^p - 1$ is a prime, then the sum of the divisors of N expressed in terms of N is equal to ______ .

(x) The number of ways of arranging 2m white and 2n red counters in straight line so that each arrangement is symmetrical with respect to a central mark is ______ .
(assume that all counters are alike except for the colour)

20. Match The Column :

(i) There are 'm' men & 'n' monkeys (n > m) . Then match the entries of column I and II .

	Column I		**Column II**
(a)	Number of ways in which each man may become the owner of one monkey is	(P)	n^m
		(Q)	^nP_m
(b)	Number of ways in which every monkey has a master, if a man may have any number of monkeys is	(R)	mn
		(S)	m^n

(ii) 5 balls are to be placed in 3 boxes . Each box can hold all the 5 balls . Number of ways in which the balls can be placed so that no box remains empty, if

	Column I		**Column II**
(a)	balls are identical but boxes are different	(P)	2
(b)	balls are different but boxes are identical	(Q)	25
(c)	balls as well as boxes are identical	(R)	50
(d)	balls as well as boxes are identical but boxes are kept in a row	(S)	6

LEVEL–I ANSWER

1. ${}^{n-r}C_2 + {}^{s-1}C_2 - (s - r - 1)$

2. **(i)** ${}^{6}P_2$ **(ii)** ${}^{6}P_3$

3. ${}^{7}C_3$

4. 108

5. 10 or 11

6. 1023

7. **(i)** $4! \times 4!$ **(ii)** $7!$ **(iii)** $6!$

8. 1320

9. 43200

10. 4.5×10^6

LEVEL–II

2. 2700

3. **(a)** 300 **(b)** 1080 **(c)** 780

4. **(a)** 1960 **(b)** $\frac{7! \times 8!}{3!}$

5. ${}^{2m-r-s}C_{m-r} \times (m!)^2$

6. ${}^{33}C_2 + 34 \times 33$

7. 15

10. ã ’ d – ˝ Å$_d$ ’ ã ’ Ø – ˝ Å$_Ø$ ’ ã ’ ø – ˝ Å$_ø$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. $a_1 + a_2 + a_3 \ldots + a_k$

2. 205

3. 35

4. 9

5. ${}^{m}C_{k-1}$ where $m = (1/2)(2n - k^2 + k - 2)$

(B)

6. T

(C)

7. B

(D)

8. C	**9.** C	**10.** A	**11.** D	**12.** B	**13.** A	**14.** A
15. A	**16.** A	**17.** A	**18.** A	**19.** C	**20.** A	**21.** C
22. D	**23.** B	**24.** A	**25.** A	**26.** C	**27.** C	**28.** C

IIT JEE PROBLEMS (SUBJECTIVE)

1. 160 **3.** 150 **7.** 485 **11.** 64

12. 3 **13.** $\frac{11!}{5!\times 6!}\times(9!)^2$ **14.** 9

16. **(i)** $^9C_5\,{}^8C_7 + {}^9C_6\,{}^8C_6 + {}^9C_7\,{}^8C_5 + {}^9C_8\,{}^8C_4 + {}^9C_9\,{}^8C_3$ **(ii)** $^9C_5\,{}^8C_7$

18. 72 **19.** 4 **20.** 60 **23.** 7

SET–I

1. A	**2.** D	**3.** C	**4.** D	**5.** B
6. B	**7.** A	**8.** A	**9.** A	**10.** C
11. D	**12.** B	**13.** C	**14.** B	**15.** A
16. D	**17.** C	**18.** C	**19.** C	**20.** A

SET–II

1. A	**2.** D	**3.** D	**4.** A	**5.** B
6. D	**7.** A	**8.** B	**9.** C	**10.** B
11. A	**12.** C	**13.** B	**14.** B	**15.** C
16. D	**17.** B	**18.** B	**19.** C	**20.** C

SET–III

1. ÀàÅå	**2.** ABC	**3.** ACD	**4.** B	**5.** BC
6. A	**7.** B	**8.** C	**9.** C	**10.** B
11. A	**12.** C	**13.** A	**14.** B	**15.** A
16. B	**17.** C	**18.** A		

19.

(i) 17 , 4760 **(ii)** 360 **(iii)** 14 **(iv)** 53 **(v)** 64

(vi) 17 . 8 ! **(vii)** 56 ways **(viii)** $^{p-2}C_3$ **(ix)** 2N **(x)** $\frac{(m+n)!}{n!\ m!}$

20. **(i)** a-Q, b-S **(ii)** a-S, b-Q, c-P, d-S