Question Bank - Matrices and Determinants

LEVEL–I

1. If a, b and c are three distinct non zero real numbers, then show that the system of equations $x+y+z=0$, $ax+by+cz=0$, $\frac{1}{a}x+\frac{1}{b}y+\frac{1}{c}z=0$, cannot have infinity many solutions.

2. Let $A=\begin{bmatrix} 2x^{\log_4 3} & 0 & 15 \\ 10 & 16 & 3^{\log_4 x} \\ 1 & 0 & 1 \end{bmatrix}$. If trace $A=\sum_{i\neq j} a_{ij}$, find x.

3. Show if A is idempotent, then $(I+A)^n = I+(2^n-1)A$.

4. Explain why the notation A/B is ambiguous when A and B are matrices, even if det $B \neq 0$.

5. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix . Then for $n \geq 2$, show that $A^n = nA - (n-1)I$.

6. Prove the followings :
(i) Adjoint of a symmetric matrix is a symmetric matrix .
(ii) Adjoint of unit matrix is unit matrix .
(iii) A(adj A) = (adj A) A .
(iv) Adjoint of a diagonal matrix is a diagonal matrix .

7. Show that every skew-symmetric matrix of odd order is singular.

8. A square matrix A is said to be involutory $A^2 = I$. If a square matrix P is such that $P^2 = P$, then show that $A = 2P - I$ is involutory and $B=\frac{1}{2}(A+I)$ satisfies the condition $B^2 = B$.

9. Show that $\begin{bmatrix} \cos{}^{-} & -\sin{}^{-} \\ \sin{}^{-} & \cos{}^{-} \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{1}{2}{}^{-} \\ \tan\frac{1}{2}{}^{-} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{1}{2}{}^{-} \\ -\tan\frac{1}{2}{}^{-} & 1 \end{bmatrix}^{-1}$.

10. Find all the value of c for which the equations 2x + 3y = 3, (c + 2)x + (c + 4)y = c + 6 and $(c+2)^2x + (c+4)^2y = (c+6)^2$ are constant. Also solve the above equations for these values of 'c'

LEVEL–II

1. If $D = \text{diag}\ (a_1, a_2, a_3, \ldots, a_n)$ where $a_i \neq 0\ \forall i = 1, 2, \ldots n$, then show that $D^{-1} = \text{diag}(a_1^{-1}, a_2^{-1}, \ldots, a_n^{-1})$.

2. If adj. B = A and |P| = |Q| = 1, then prove that adj. $(Q^{-1} BP^{-1}) = PAQ$.

3. Prove that the system of equations $x + y - z = 7, 3x - 5y + 2z = 8, kx - 4y + z = 15$ has infinity many solutions for $k = 4$ and a unique solution for any other value of k.

4. Prove that the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $x^2 - (a + d)x + ad - bc = 0$.

5. If a, b and c are distinct, solve the equation $\begin{vmatrix} x^2 - a^2 & x^2 - b^2 & x^2 - c^2 \\ (x-a)^3 & (x-b)^3 & (x-c)^3 \\ (x+a)^3 & (x+b)^3 & (x+c)^3 \end{vmatrix} = 0$ for x.

6. If $\Delta(n, r) = \begin{vmatrix} {}^{n}C_r & {}^{n}C_{r+1} & {}^{n}C_{r+2} \\ {}^{n+1}C_r & {}^{n+1}C_{r+1} & {}^{n+1}C_{r+2} \\ {}^{n+2}C_r & {}^{n+2}C_{r+1} & {}^{n+2}C_{r+2} \end{vmatrix}$. Show that $\Delta(n, r) = \frac{{}^{n+2}C_3}{{}^{r+2}C_3} \Delta(n-1, r-1)$. Hence or otherwise, prove that $\Delta(n, r) = \frac{{}^{n+2}C_3 . {}^{n+1}C_3 \ldots\ldots\ldots\ldots {}^{n-r+3}C_3}{{}^{r+2}C_3 . {}^{r+1}C_3 \ldots\ldots\ldots\ldots\ldots\ldots {}^{3}C_3}$.

7. If $\Delta(x) = \begin{vmatrix} x^2 - 5x + 3 & 2x - 5 & 3 \\ 3x^2 + x + 4 & 6x + 1 & 9 \\ 7x^2 - 6x + 9 & 14x - 6 & 21 \end{vmatrix} = ax^3 + bx^2 + cx + d$, Then prove that a = 0, b = 0, c = 0.

8. For each x, -1 < x < 1 let A(x) be the matrix $A(x) = \frac{1}{\sqrt{1-x}} \begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ show that $A(x)\,A(y) = \sqrt{1+xy}\ a(z)$ where $x, y \in R$ $-1 < x < 1$ $z = \frac{x+y}{1+xy}$.

9. Suppose f(x) is a function satisfying the following conditions :

(a) $f(0) = 2, f(1) = 1$

(b) f(x) has a minimum value of x = 5/2 and

(c) $f'(x) = \begin{vmatrix} 2ax & 2ax - 1 & 2ax + b + 1 \\ b & b + 1 & -1 \\ 2(ax + b) & 2ax + 2b + 1 & 2ax + b \end{vmatrix}$, where a, b are some constants.

Determine the constants a, b and the function f(x).

10. Let A be a square matrix of order $n \times n$ and B be its adjoint; then show that $|AB + kI_n| = (|A| + k)^n$.

IIT JEE PROBLEMS (OBJECTIVE)

A. Fill in the blanks

1. Let $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$ be an identity in λ, where p, q, r, s and t are constants. Then, the value of t is .... **[IIT - 1981]**

2. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is .... **[IIT - 1981]**

3. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. the probability that the value of determinant chosen is positive is .... **[IIT - 1982]**

4. Given that x = –9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ the order two roots are .... and .... **[IIT - 1983]**

5. The system of equation $\lambda x + y + z = 0$, $-x + \lambda y + z = 0$, $-x - y + \lambda z = 0$ will have a nonzero solution . If real values of λ are given by .... **[IIT – 1984]**

6. The value of the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ is .... **[IIT– 1988]**

7. For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is .... **[IIT - 1993]**

B. True/False

1. The determinants $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ and $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ are not identically equal. **[IIT - 1983]**

2. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$ then the two triangles with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3), and (a_1, b_1), (a_2, b_2), (a_3, b_3) must be congruent. **[IIT - 1985]**

C. Multiple choice Question with One or More than One Correct Answer

1. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, if

(A) a, b, c are in A.P.
(B) a, b, c are in G.P.
(C) a, b, c are in H.P.
(D) α is a root of the equation $ax^2 + bx + c = 0$
(E) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c = 0$ **[IIT– 1986]**

2. The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$, are **[IIT - 1986]**

(A) $\frac{7\pi}{24}$ (B) $\frac{5\pi}{24}$ (C) $\frac{11\pi}{24}$ (D) $\frac{\pi}{24}$

3. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then **[IIT-1998]**

(A) x = 3, y = 1
(B) x = 1, y = 3
(C) x = 0, y = 3
(D) x = 0, y = 0

D. Multiple choice Question with One Correct Answer

1. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value –1. Then **[IIT - 1981]**

(A) C is empty
(B) B has as many elements as C
(C) $A = B \cup C$
(D) B has twice as many elements as elements as C

2. Let $\omega (\neq 1)$ is a cube root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$ is equal to **[IIT - 1995]**

(A) 0 (B) 1 (C) i (D) ω

3. Let a, b, c be the real numbers. The following system of equations in x, y and z

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has **[IIT - 1995]**

(A) no solution
(B) unique solution
(C) infinitely many solutions
(D) finitely many solutions

4. If A and B are square matrices of equal degree, then which one is correct among the following **[IIT - 1995]**

(A) $A + B = B + A$ (C) $A + B = A - B$ (C) $A - B = B - A$ (D) $AB = BA$

5. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a consistent. Then $\frac{d^3 f(x)}{dx^3}$ at $x = 0$ is **[IIT - 1996]**

(A) p
(B) $p + p^2$
(C) $p + p^3$
(D) independent of p

6. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$ if **[IIT - 1997]**

(A) x, y, z are in A. P.
(B) x, y, z are in G. P.
(C) x, y, z are in H. P.
(D) xy, yz, zx are in A. P.

7. The parameter, on which the value of the determinant

$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d) & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon, is **[IIT - 1997]**

(A) a (B) p (C) d (D) x

8. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$, then f(100) is equal to **[IIT - 1999]**

(A) 0
(B) 1
(C) 100
(D) -100

9. If the system of equations x - ky - z = 0, kx - y - z =0, x + y - z = 0 has a nonzero solution, then the possible values of k are **[IIT-2000]**

(A) -1, 2 (B) 1, 2 (C) 0, 1 (D) -1, 1

10. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(A) 0 (B) 2 (C) 1 (D) 3 **[IIT-2001]**

11. Let $\omega = \dfrac{-1+i\sqrt{3}}{2}$, then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is **[IIT-2002]**

(A) 3ω (B) $3\omega(\omega - 1)$ (C) $3\omega^2$ (D) $3\omega(1 - \omega)$

12. The number of values of k for which the system of equations,
$(k + 1)x + 8y = 4k$; $kx + (k + 3)y = 3k - 1$ has infinitely many solution is **[IIT-2002]**
(A) 0 (B) 1 (C) 2 (D) infinite

13. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$ is **[IIT-2003]**

(A) 1 (B) – 1 (C) 4 (D) no real values

14. Given $2x - y + 2z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ then the value of λ such that the given system of equation has no solution, is
(A) 3 (B) 1 (C) 0 (D) –3

15. Let $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then α equals **[IIT-2004]**

(A) ± 3 (B) ± 5 (C) ± 1 (D) 0

16. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \left[\dfrac{1}{6}(A^2 + cA + dI)\right]$, then (c, d) is

[IIT-2005]

(A) (–6, 11) (B) (–11, 6) (C) (11, 6) (D) (6, 11)

17. If $P = \begin{bmatrix} \dfrac{\sqrt{3}}{2} & \dfrac{1}{2} \\ \dfrac{-1}{2} & \dfrac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2005} P$ is **[IIT-2005]**

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 2005 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

E. Write up **[IIT-2006]**

W I $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, if U_1, U_2 and U_3 are column matrices satisfying.

$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and U is 3×3 matrix whose column are U_1, U_2, U_3 then answer the following questions :

1. The value of $|U|$ is

(A) 3 (B) –3 (C) 3/2 (D) 2

2. The sum of the elements of U^{-1} is

(A) –1 (B) 0 (C) 1 (D) 3

3. The value of $[\,3\ 2\ 0\,]\,U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is

(A) [5] (B) [5/2] (C) [4] (D) [3/2]

IIT JEE PROBLEMS (SUBJECTIVE)

1. Let a, b, c be positive and not all equal. Show that the value of determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

2. Without expanding a determinant at any stage, show that $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA + B,$

where A and B are determinants of order 3 not involving x. **[IIT– 1982]**

3. Show that the system of equations 3x – y + 4z = 3, x + 2y – 3z = – 2, $6x+5y+\lambda z=-3$ has at least one solution for any real number λ. Find the set of solutions if $\lambda = -5$. **[IIT– 1983]**

4. If α be a repeated root of a quadratic equation f(x) = 0 and A(x), B(x) and C(x) be polynomials of degree 3, 4 and 5 respectively, then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by f(x), where prime denotes the derivatives. **[IIT– 1984]**

5. Show that $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$. **[IIT– 1985]**

6. Consider the system of linear equations in x, y, z :
$(\sin 3\theta)x - y + z = 0,\ (\cos 2\theta)x + 4y + 3z = 0,\ 2x + 7y + 7z = 0$. Find the values of θ for which this system has nontrivial solutions. **[IIT– 1986]**

7. Let $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 2n^2-3n \end{vmatrix}$. Show that $\sum_{a=1}^{n} \Delta_a = c$ is a constant. **[IIT– 1989]**

8. Let the three digit numbers A28, 3B9 and 62C, where A, B and C are integers between 0 and 9, be divisible by a fixed integer k. Show that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is divisible by k. **[IIT– 1990]**

9. Evaluate : $\begin{vmatrix} {}^xC_1 & {}^xC_2 & {}^xC_3 \\ {}^yC_1 & {}^yC_2 & {}^yC_3 \\ {}^zC_1 & {}^zC_2 & {}^zC_3 \end{vmatrix}$. **[IIT– 1990]**

10. Suppose three digit number A 28, 3 B 9 and 62 C where A, B, C are integers between 0 and 9, are divisible by a fixed integer k. Prove that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is also divisible by k.

[IIT-1990]

11. If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$. Then find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$.

12. For a fixed integer n, if D = $\begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$, then show that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n.

[IIT-1992]

13. For positive number x, y and z the numerical value of determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$.

[IIT-1993]

14. Let λ and α be real. Find the set of all values of λ for which the system of linear equation $\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$, $x + (\cos\alpha)y + (\sin\alpha)z = 0$, $-x + (\sin\alpha)y - (\cos\alpha)z = 0$ has nontrivial solution for $\lambda = 1$, find the value of α. **[IIT-1993]**

15. For all values of A, B, C and P, Q, R, show that $\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$

[IIT-1994]

16. Let $a > 0, d > 0$. Find the value of the determinant

$$\Delta = \begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{a(a+d)(a+2d)} \\ \frac{1}{a+d} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{a+2d} & \frac{1}{(a+2d)(a=3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

[IIT-1996]

17. Find those values of c for which the equations $2x + 3y = 3$, $(c+2)x + (c+4)y = c+6$, $(c+2)^2 x + (c+4)^2 y = (c+6)^2$ are consistent. Also solve above equations for there values of c. **[IIT-1996]**

18. Solve for x : $\begin{vmatrix} -a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0.$ **[IIT-97]**

19. If the p^{th}, q^{th} and r^{th} terms of an H.P. be a, b, c respectively, then prove that $\Delta = \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0.$

[IIT-1997]

20. Prove that for all values of $^-$, $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(2\theta + \frac{4\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0.$ **[IIT-2000]**

21. Find the real values of r for which the following system of linear equations has a nontrivial solutions. Also find the non-trival solutions : $2rx - 2y + 3z = 0$, $x + ry + 2z = 0$, $2x + 0y + rz = 0$.
[IIT-2000]

22. Let a, b, c be real numbers with $a^2+b^2+c^2 = 1$. Show that the equation

$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$, represents a straight line. **[IIT-2001]**

. . . If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a, b, c real positive numbers, abc = 1 and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$. **[IIT-2003]**

24. If M is a 3×3 matrix, where det M = 1 and $MM^T = I$, where 'I' is an identity matrix, prove that det $(M - I) = 0$.

25. Let $A = \begin{bmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{bmatrix}$, $B = \begin{bmatrix} a & 1 & 1 \\ 0 & c & d \\ f & g & h \end{bmatrix}$, $U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$ and $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$.

If there is a vector matrix, X, so that AX = U has infinitely many solutions, prove that BX = V can not have a unique solution. If $afd \neq 0$, then that BX = V has no solution. **[IIT-2004]**

PROBLEMS ASKED IN PET–CET

Q.1 The equations $2x + y = 5$, $x + 3y = 5$, $x - 2y = 0$ have **[P.E.T. 1985]**

(A) no solution (B) one solution
(C) two solution (D) infinity many solutions

Q.2 If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined. Then B is of the type **[P.E.T. 1986]**

(A) 3×4 (B) 3×3 (C) 4×4 (D) 4×3

Q.3 For the equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$ **[P.E.T. 1987]**

(A) there is only one solution (B) there exists infinitely many solutions
(C) there is no solution (D) none of these

Q.4 If $A = [a, b]$, $B = [-b\ -a]$ and $C = \begin{bmatrix} a \\ -a \end{bmatrix}$, then the correct statement is **[P.E.T. 1987]**

(A) $A = -B$ (B) $A + B = A - B$ (C) $AC = BC$ (D) $CA = CB$

Q.5 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to **[C.E.T. 1990]**

(A) A (B) – A (C) null matrix (D) I

Q.6 Consider the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$.

If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the system has **[Roorkee 1990]**

(A) more than two solutions (B) one trivial and one nontrivial solutions
(C) no solution (D) only trivial solution (0, 0, 0)

Q.7 Form the matrix equation $AB = AC$ we can conclude $B = C$ provided **[C.E.T. 1992]**

(A) A is singular (B) A is non-singular
(C) A is symmetric (D) A is square

Q.8 If k is a scalar and A is an $n\times n$ square matrix, then $|kA| =$ **[C.E.T. 1992]**

(A) $k|A|^n$ (B) $k|A|$ (C) $k^n|A^n|$ (D) $k^n|A|$

Q.9 A and B are two square matrices of same order and A' denotes the transpose of A, then **[C.E.T. 1992]**

(A) $(AB)' = B'A'$ (B) $(AB)' = A'B'$
(C) $AB = 0 \Rightarrow |A| = 0$ or $|B| = 0$ (D) $AB = 0 \Rightarrow A = 0$ or $B = 0$

Q.10 Matrix A is the such that $A^2 = 2A - I$, where I is the identity matrix. Then for $n \ge 2$, A^n is equal to **[EAMCET-92]**

(A) $nA - (n - 1)I$ (B) $nA - I$ (C) $2^{n-1}A - (n - 1)I$ (D) $2^{n-1}A - I$

Q.11 If A is a singular matrix, then Adj A is **[C.E.T. 1993]**
(A) singular (B) non-singular (C) symmetric (D) not defined

Q.12 A and B are two nonzero square matrices such that AB = 0. Then **[C.E.T. 1993]**
(A) both A and B are singular (B) either of them is singular
(C) neither matrix is singular (D) none of these

Q.13 The order of $[x\ y\ z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is **[EAMCET 1994]**

(A) 3×1 (B) 1×1 (C) 1×3 (D) 3×3

Q.14 If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ is equal to
[EAMCET 1994]
(A) 2AB (B) 2BA (C) A + B (D) AB

Q.15 If $\begin{bmatrix} x & 0 \\ 1 & y \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 2 & 1 \end{bmatrix}$, then **[P.E.T. 1994]**
(A) $x = -3, y = -2$ (B) $x = 3, y = -2$
(C) $x = 3, y = 2$ (D) $x = -3, y = 2$

Q.16 If A', B' are transpose matrices of the square matrices A, B respectively, then $(AB)'$ is equal to
[P.E.T. 1994]
(A) $A'B'$ (B) $B'A'$ (C) AB' (D) BA'

Q.17 If A is a 3×3 matrix and det (3A) = k {det (A) }, k is equal to **[EAMCET 1996]**
(A) 9 (B) 6 (C) 1 (D) 27

Q.18 For what real values of k, the system of equations $x + 2y + z = 1$; $x + 3y + 4z = k$; $x + 5y + 10z = k^2$ has solution ? Find the solution in each case. **[Roorkee 1997]**

Q.19 Using matrix method find the value of λ and μ so that the system of equations $2x - 3y + 5z = 12$, $3x + y + \lambda z = \mu$, $x - 7y + 8z = 17$ has (i) a unique solution (ii) infinite solutions and (iii) no solution. **[Roorkee 1998]**

Q.20 The matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$ is **[EAMCET 1998]**

(A) non-singular (B) singular
(C) skew–symmetric (D) symmetric

SET - I

1. If a, b, and c are complex numbers then $z = \begin{vmatrix} 0 & -b & -c \\ \bar{b} & 0 & -a \\ \bar{c} & \bar{a} & 0 \end{vmatrix}$ is

(A) real (B) purely imaginary (C) 0 (D) none of these

2. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2 =$

(A) 2AB (B) 2BA (C) A + B (D) AB

3. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, which of the following results is true?

(A) $A^2 = I$ (B) $A^2 = -I$ (C) $A^2 = 2I$ (D) none of these

4. If A is an orthogonal matrix, then A^{-1} equals

(A) A (B) A' (C) A^2 (D) none of these

5. If a, b and c are positive integers, then the coefficient of x in $\Delta = \begin{vmatrix} 1+x & (1+x)^a & (1+x)^{bc} \\ 1+x & (1+x)^b & (1+x)^{ca} \\ 1+x & (1+x)^c & (1+x)^{ab} \end{vmatrix}$ is

(A) a + b + c (B) abc (C) $a^2 + b^2 + c^2$ (D) 0

6. If A is a square matrix of order n then adj (adj A) is equal to

(A) $|A|^n A$ (B) $|A|^{n-1} A$ (C) $|A|^{n-2} A$ (D) $|A|^{n-3} A$

7. If a is a square matrix, then $\text{adj}(A') - (\text{adj}A)'$ is equal to

(A) 2|A| (B) 2|A|I (C) null matrix (D) unit matrix

8. If $A = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x)$, then A^{-1} is equal to

(A) f (–x) (B) f(x) (C) –f(x) (D) –f(–x)

9. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $A \,\text{adj}\, A = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the value of k is

(A) sin x cos x (B) 1 (C) 2 (D) 3

10. Which of the following is incorrect

(A) adj (adj A) = A
(B) $(A^T)^T = A$
(C) $(A^{-1})^T = (A^T)^{-1}$
(D) $(A - I)(I + A) = 0 \Leftrightarrow A^2 = I$

11. For the equations : x + 2y + 3z = 1, 2x + y + 3z = 2, 5x + 5y + 9z = 4

(A) there is only one solution
(B) there exists infinitely many solutions
(C) there is no solution
(D) none of there

12. If I_3 is the identity matrix of order 3, then $(I_3)^{-1}$ is equal to
(A) 0 (B) $3\ I_3$ (C) I_3 (D) not necessarily exists

13. If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 56$, then n equals
(A) 4 (B) 6 (C) 8 (D) 7

14. $\begin{vmatrix} x+1 & x+2 & x+\lambda \\ x+2 & x+3 & x+\mu \\ x+3 & x+4 & x+v \end{vmatrix} = 0$, where λ, μ, v are in A.P., is
(A) an equation whose all roots and real (B) an identity in x
(C) an equation with only one root is real (D) none of these

15. Let A and B be two 3×3 matrices, then $AB = 0$ implies
(A) A = 0 or B = 0 (B) |A| = 0 and |B| = 0
(C) |A| = 0 or |B| = 0 (D) A = 0 and B = 0

16. Let $f(x) = x^2 - 5x + 6$ and $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then f(A) is equal to

(A) $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & -1 & -3 \\ 0 & -1 & -10 \\ -5 & 4 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & -1 & -3 \\ 0 & -1 & -10 \\ -5 & 4 & 0 \end{bmatrix}$

17. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix. Then for $n \geq 2$, $A^n =$
(A) $nA - (n-1)I$ (B) $nA - I$ (C) $2^{n-1}\ A - (n-1)1$ (D) $2^{n-1}\ A - I$

18. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to
(A) A (B) –A (C) null matrix (D) I

19. Consider the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$. If $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = 0$, then the system has
(A) infinite solutions (B) one trivial and one non-trivial solutions
(C) no solution (D) only trivial solution (0, 0, 0)

20. Let $A = [a_{ij}]_{n \times n}$ be a square matrix, and let c_{ij} be cofactor of a_{ij} in A. If $C = [c_{ij}]$, then
(A) $|C| = |A|$ (B) $|C| = |A|^{n-1}$ (C) $|C| = |A|^{n-2}$ (D) none of these

SET - II

1. If A is a matrix such that $A^2 =$ I and I is unit matrix of same order, then (I – A) (I + A) is
(A) Zero matrix (B) A (C) I (D) 2A

2. If A = dig (2, –1, 3), B = dig (–1, 3, 2), then A^2B is equal to
(A) dig (5, 4, 11) (B) dig (–4, 3, 18) (C) dig (3, 1, 8) (D) B

3. If A and B are two matrices and $(A + B)(A - B) = A^2 - B^2$, then :
(A) AB = BA (B) $A^2 + B^2 = A^2 - B^2$
(C) $A^{-1}B^{-1} = AB$ (D) None of the above

4. Adj.(AB) – (Adj.B)(Adj.A) is equal to
(A) Adj. A – Adj. B (B) I (C) O (D) none of these

5. Let $A = \begin{bmatrix} 4 & 6 & -1 \\ 3 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$. The expression/s which is/are not defined is :
(A) $B'B$ (B) CAB (C) $A + B'$ (D) $A^2 + A$

6. If matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$ and its inverse is denoted by $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then the value of $a_{23} =$
(A) $\frac{21}{20}$ (B) $\frac{1}{5}$ (C) $-\frac{2}{5}$ (D) $\frac{2}{5}$

7. From the matrix equation AB = AC, we conclude B = C provided
(A) A is singular (B) A is non-singular (C) A is symmetric (D) A is a square

8. If A is a square matrix of order m^n where m is odd, then the true statement is (where I is unit matrix) .
(A) det (- A) = - det A (B) det A = 0
(C) det (A + I) = I + det A (D) det 2A = 2 det A

9. If A & B are the symmetric matrices of same order. Then which of the following statements are true
(A) AB – BA is a symmetric matrix (B) A + B is a symmetric matrix
(C) $A^2 - B^2$ is a skew-symmetric matrix (D) none of these

10. Which of the following relations is incorrect
(A) $(A + B + \ldots + l)^T = A^T + B^T + \ldots + l^T$ (B) $(AB \ldots\ldots l)^T = A^TB^T \ldots\ldots l^T$
(C) $(kA)^T = kA^T$ (D) $(A^T)^T = A$

11. Which of the following statement is true :
(A) Non singular square matrix does not have a unique inverse
(B) Determinant of a singular matrix is not also always zero
(C) If $|A| \neq 0$ then $|A \text{ adj } (A)| = |A|^{(n-1)}$ where $A = [a_{ij}]_{n\times n}$
(D) none of these

12. If A and B are two square matrices such that AB = A and BA = B, then
(A) only B is idempotent
(B) A, B are idempotent
(C) only A is idempotent
(D) none of these

13. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^{100} =$
(A) 2^{100} A
(B) 2^{99} A
(C) 2^{101} I
(D) none of these

14. If $f(x) = \begin{vmatrix} 1 & 3\cos x & 1 \\ \sin x & 1 & 3\cos x \\ 1 & \sin x & 1 \end{vmatrix}$, then the maximum value of f(x) is
(A) 5
(B) 10
(C) 15
(D) 20

15. Let $\{A_1, A_2, A_3, \ldots, A_k\}$ be the set of all the third order matrices that can be made with the distinct nonzero real numbers $a_1, a_2, a_3, \ldots, a_9$ (repetition of element in a matrix is allowed). Then
(A) k = 9!
(B) k = 9{9!)
(C) $\sum_{i=1}^{k} |A_i| = 0$
(D) none of these

16. If A = [a b], B = [-b -a] and $C = \begin{bmatrix} a \\ -a \end{bmatrix}$, then the correct statement is
(A) A = - B
(B) A + B = A - B
(C) AC = BC
(D) CA = CB

17. If A and B are two matrices such that A + B and AB are both defined, then
(A) A and B are two matrices not necessarily of same order
(B) A and B are square matrices of same order
(C) Number of columns of A = number of rows of B
(D) None of these

18. The order of [x y z] $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is
(A) 3 × 1
(B) 1 × 1
(C) 1 × 3
(D) 3 × 3

19. $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then x =
(A) 3
(B) 5
(C) 2
(D) 4

20. If k is a scalar and A is an n × n square matrix , then $|kA| =$
(A) $k|A|^n$
(B) $k|A|$
(C) $k^n|A^n|$
(D) $k^n|A|$

SET - III

More than One

1. Let $\Delta = \begin{vmatrix} a & a+d & a+2d \\ a+d & a+2d & a \\ a+2d & a & a+d \end{vmatrix}$, then

(A) Δ depends on a (B) Δ depends on d (C) Δ is a constant (D) all above

2. Let a, b > 0 and $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$, then

(A) $a + b - x$ is a factor of Δ
(B) $x^2 + (a + b)x + a^2 + b^2 - ab$ is a factor of Δ
(C) $\Delta = 0$ has two real roots if $a = b$
(D) none of these

3. If A and B are square matrices of the same order such that $A^2 = A$, $B^2 = B$, $AB = BA = 0$, then

(A) $A(B)^2 = 0$ (B) $(A + B)^2 = A + B$
(C) $(A - B)^2 = A - B$ (D) All above

4. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, (where $b \neq c$) satisfies the equation $x^2 + k = 0$, then

(A) $a + d = 0$ (B) $k = -|A|$ (C) $k = |A|$ (D) none of these

5. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then

(A) Adj A is a zero matrix (B) Adj $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

(C) $A^{-1} = A$ (D) $A^2 = I$

W I If $f(x) = \begin{vmatrix} 5+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 5+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 5+\sin 2x \end{vmatrix}$, then

6. Domain of f(x) is

(A) $(0, \infty)$ (B) $(-\infty, \infty)$ (C) $(-\infty, 0)$ (D) $(1, \infty)$

7. Range of function f(x) is

(A) [50, 100] (B) [–50, 0] (C) [–50, 50] (D) [50, 250]

8. Period of function f(x) is

(A) π (B) 2π (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

9. The value of $\lim_{x\to 0}\frac{f(x)-150}{x}$ is

(A) 0 (B) 150 (C) 200 (D) 250

W II If a, b > 0 and $\Delta(x) = \begin{vmatrix} x & a & a \\ b & x & a \\ b & b & x \end{vmatrix}$, then

10. $\Delta(x)$ is increasing in

(A) $(-\sqrt{ab}, \sqrt{ab})$ (B) $(-\infty, -\sqrt{ab}) \cup (\sqrt{ab}, \infty)$

(C) $(-\sqrt[3]{ab}, \sqrt[3]{ab})$ (D) none of these

11. $\Delta(x)$ is decreasing in

(A) $(-\sqrt{ab}, \sqrt{ab})$ (B) $(-\sqrt[3]{ab}, \sqrt[3]{ab})$

(C) $(-\infty, -\sqrt[3]{ab}) \cup (\sqrt[3]{ab}, \infty)$ (D) none of these

12. $\Delta(x)$ has a local minimum, at :

(A) $x = \sqrt[3]{ab}$ (B) $x = -\sqrt[3]{ab}$ (C) $x = \sqrt{ab}$ (D) $x = -\sqrt{ab}$

W III If abc = p and $A = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, where A is an orthogonal matrix. Then

13. The value of a + b + c is

(A) 2 (B) p (C) 2p (D) ±1

14. The value of ab + bc + ca is

(A) 1 (B) p (C) 2p (D) 3p

15. The value of $a^2 + b^2 + c^2$ is

(A) 1 (B) p (C) 2p (D) 3p

16. The value of $a^3 + b^3 + c^3$ is

(A) p (B) 2p (C) 3p (D) none of these

17. The equation whose roots are a, b, c is

(A) $x^3 - 2x^2 + p = 0$ (B) $x^3 - px^2 + px + p = 0$

(C) $x^3 - 2x^2 + 2px + p = 0$ (D) $x^3 \pm x^2 - p = 0$

18. **TRUE FALSE**

(i) There is unique unit matrix

(ii) There exists an algebra of matrices similar to algebra of numbers.

(iii) If A and B are matrices such that A + B and AB are both defined, then A and B are square matrices of the same order.

(iv) A diagonal matrix commutes with every other matrix of the same order

(v) The determinant of the sum of two matrices is equal to the sum of the determinants of the matrices.

19. If A and B are 3×3 symmetric matrices, then

(a) Symmetric	(P) AB + BA
(b) Anti-symmetric	(Q) A + B
	(R) AB – BA
	(S) A – B

20. Using n distinct real numbers, matrices each having distinct elements and of all possible orders are to be made, then the possible arrangements are :

(a) $n = 3$	(P) 72 possible matrices
(b) $n = 4$	(Q) 12 possible matrices
(c) $n = 5$	(R) 240 possible matrices

ANSWER LEVEL–I

2. $x = 16$

10. $c = 0, -10$; when $c = 0$, $x = 3$, $y = -3$, when $c = -10$; $x = -1/2$, $y = 4/3$

LEVEL–II

3. **(i)** $\lambda = 3,\ \mu \neq 10$ **(ii)** $\lambda \neq 3,\ \mu \in R$ **(iii)** $\lambda = 3,\ \mu = 10$ **5.** 2

5. $x = 0,\ \pm\sqrt{\frac{1}{3}(bc + ca + ab)}$ **9.** $\begin{bmatrix} 1069 & 1558 \\ 2337 & 3046 \end{bmatrix}$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1.	0	**2.**	–1, 2	**3.**	3/16		
4.	2, 7	**5.**	0	**6.**	0	**7.**	0

(B)

1. F **2.** F

(C)

1. B, E **2.** A, C **3.** D

(D)

1.	B	**2.**	A	**3.**	B	**4.**	A	**5.**	D
6.	B	**7.**	B	**8.**	A	**9.**	D	**10.**	C
11.	B	**12.**	B	**13.**	D	**14.**	D	**15.**	A
16.	A	**17.**	A						

(E)

1. A **2.** B **3.** A

IIT JEE PROBLEMS (SUBJECTIVE)

6. $\theta = \frac{n\pi}{3} + \frac{\alpha}{3}\left[(-1)^n - 1\right]$, where $\alpha = \tan^{-1} 2$ **9.** $\frac{xyz(x-y)(y-z)(z-x)}{12}$

11. 2 **13.** 0 **14.** $2\alpha = 2\alpha\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$

16. $\frac{4d^4}{a(a+d)^2(a+2d)^2(a+3d)^2(a+4d)}$ **17.** $c = 0, x = -3, y = 3$ $c = -10, x = -\frac{1}{2}, y = \frac{4}{3}$

18. $x = n\pi,\ 2n\pi \pm \cos^{-1}\frac{1-a^2}{2a}$ **21.** $r = 2, x = k, y = \frac{k}{2}, z = -k$

23. 4

SET–I

1. B	**2.** C	**3.** B	**4.** B	**5.** D	**6.** C
7. C	**8.** A	**9.** B	**10.** A	**11.** A	**12.** C
13. D	**14.** B	**15.** C	**16.** A	**17.** A	**18.** D
19. A	**20.** B				

SET–II

1. A	**2.** B	**3.** A	**4.** C	**5.** C	**6.** D
7. B	**8.** A	**9.** B	**10.** B	**11.** D	**12.** B
13. B	**14.** B	**15.** C	**16.** C	**17.** B	**18.** B
19. B	**20.** D				

SET–III

1. AB	**2.** ABC	**3.** AB	**4.** ABC	**5.** BC	**6.** B
7. D	**8.** A	**9.** C	**10.** B	**11.** A	**12.** C
13. B	**14.** A	**15.** A	**16.** D	**17.** D	
18. **(i)** False	**(ii)** False	**(iii)** True	**(iv)** False	**(v)** False, True	

19. A-PQR, B-R **20.** A-Q, B-P, C-R