ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2015 - 2016)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (ADVANCED) 2016

Test Type : **ALL INDIA OPEN TEST** (MAJOR)   Test Pattern : JEE-Advanced

TEST # 11   **TEST DATE : 08 - 05 - 2016**

PAPER-1

PART-1 : PHYSICS   **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B,C	C,D	B	A,C	A,C,D	A,D	A,D	B,D	B,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		R	P,Q	P,Q	S,T		R,S	P,Q,T	P,R,S,T	Q,R,T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	4	5	3	2	8	4	4	2		

SOLUTION

SECTION-I

1. Ans. (A)

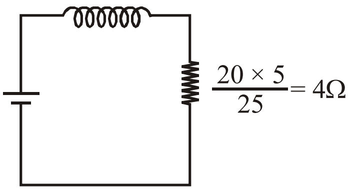
Sol. $F = \frac{dU}{dr} = k = \frac{mv^2}{r}$

$K = \frac{kr}{2}$

$E = U + K = kr + \frac{kr}{2} = \frac{3kr}{2}$

2. Ans. (B, C)

Sol.



$i = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = \frac{10}{4}\left(1 - e^{-\frac{4\times0.25}{1}}\right)$

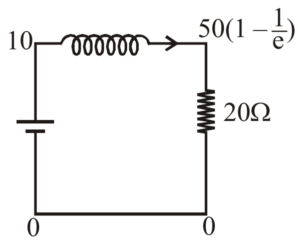
$= 2.5\left(1 - \frac{1}{e}\right)$

$V_L + 4i = 10$

$V_L + 10\left(1 - \frac{1}{e}\right) = 10$

$V_L = \frac{10}{e}$

after



$V_R = 2.5 \times 20\,(1 - \frac{1}{e})$

$= 50(1 - \frac{1}{e})$

3. Ans. (C, D)

Sol. Magnetic field due to sheet is $\frac{\mu_0 Jt}{2}$ & is constant.

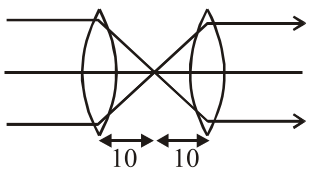
4. Ans. (B)

Sol. Total energy of hydrogen atom is ground state = – 13.6 eV

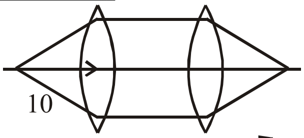
$\therefore M_H = M_P + M_e - \frac{13.6eV}{C^2}$

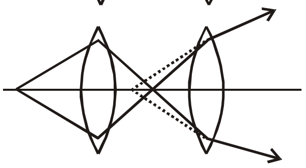
5. Ans. (A, C)

Sol.



$f_1 = f_2 = 10$ cm

(A) 

(C) 

$\frac{1}{v} - \frac{1}{-30} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30}$

$v = 15$

$\frac{1}{v} - \frac{1}{-5} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{5} = \frac{1-2}{10}$

$v = -10$

6. Ans. (A, C, D)

Sol. $E = \frac{1240}{248} eV$

$E_1 = 5 - 2.2 = 2.8$ eV

$E_2 = 5 \times 0.8 - 2.2 = 1.8$ eV

$E_3 = 5 \times 0.8^2 - 2.2 = 1$ eV

$E_4 = 5 \times 0.8^3 - 2.5 = 0.36$ eV

7. Ans. (A, D)

Sol. $v = \frac{J}{m} = A\omega = A\sqrt{\frac{k}{m}}$

$A = \frac{J}{\sqrt{mk}}$

$x = \frac{J}{\sqrt{mk}} \sin\left(\sqrt{\frac{k}{m}} t\right)$

$v = \frac{J}{\sqrt{mk}} \times \sqrt{\frac{k}{m}} \cos\left(\sqrt{\frac{k}{m}} t\right)$

$= \frac{J}{m} \cos\left(\sqrt{\frac{k}{m}} t\right)$

$a = -\frac{J}{\sqrt{mk}} \frac{k}{m} \sin\left(\sqrt{\frac{k}{m}} t\right)$

8. Ans. (A, D)

Sol. (A) $T_1^2 = C(R)^3$

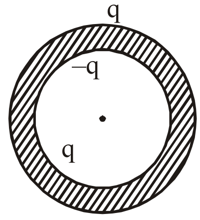
$T_2^2 = C(4R)^3 \quad \Rightarrow \frac{T_1}{T_2} = \frac{1}{8}$

(C) $\frac{A}{T} = \frac{L}{2m}$ can't be commented

(D) $E_1 = -\frac{GMm}{2R}$

$E_2 = \frac{-GMm}{2 \times 4R}$

9. Ans. (B, D)

Sol. 

$V_0 = \frac{kq}{d} - \frac{kq}{R_1} + \frac{kq}{R_2}$

$V_{shell} = \frac{kq}{R_2}$

$V_{out} = \frac{kq}{R}$

10. Ans. (B, D)

SECTION-II

1. Ans. (A)→(R) ; (B)→(P, Q) ; (C)→(P, Q) ; (D)→(S, T)

Sol. For process-1,

Gas undergoes compression work done by gas is –ve

for DU & DQ nothing can be comented.

For process-2,

Gas undergoes expansion Þ work done by gas is +ve

Also temperature of gas increases.

$\therefore$ ΔQ is +ve & ΔU is +ve

For process-3

Process is isobaric

For process-4

Process is iso-choric

2. Ans. (A) → (R,S) ; (B) → (P, Q, T) ; (C) → (P,R,S,T) ; (D) → (Q, R,T)

Sol. For P,

$u = +\frac{f}{2} \qquad \therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For Q,

$u = -15cm \qquad \therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{-10}$

For R

$u = -50$ cm $\qquad \therefore$

$\frac{1.5}{v} - \frac{1}{(-50)} = \frac{1.5-1}{100}$

For S

Object is real $\Rightarrow$ image will be virtual

For T

$u = +\frac{f}{2}$ $\quad\quad$ $\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{-f}$

SECTION-IV

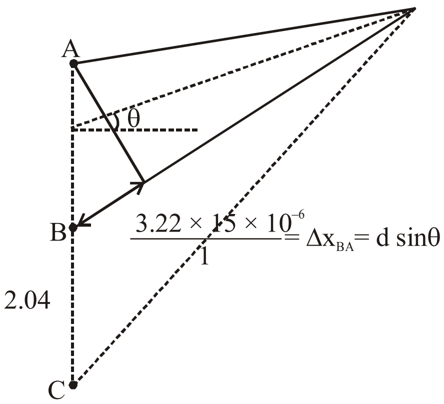
1. Ans. 4

Sol. $E \times 2\pi r = \frac{d\phi}{d\tau} = \pi r^2 \times 0.4$

$E = \frac{r}{2} \times 0.4 = 4 \times 10^{-3}$

2. Ans. 5

Sol.



$\Delta\phi_{BA} = \frac{2\pi}{\lambda}\Delta x$

$= \frac{2\pi}{600 \times 10^{-9}} \times 3.22 \times 15 \times 10^{-6} = 161\ \pi \equiv \pi$

A & B cancel $\Rightarrow I = I_C = 5\ \omega/m^2$

3. Ans. 3

Sol. $3 - 0.1 \times 2 \times 10 = 2a$

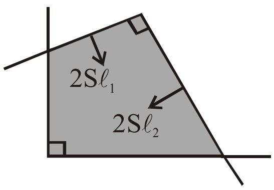
$a = 0.5\ m/s^2$

$v^2 = 2as = 2 \times 0.5 \times 9$

$v = 3\ m/s$

4. Ans. 2

Sol.



$F_{net} = \sqrt{(2S\ell_1)^2 + (2S\ell_2)^2} = ma$

$a = \frac{2S\sqrt{\ell_1^2 + \ell_2^2}}{m} = \frac{2 \times 0.1 \times \sqrt{1.5^2 + 2^2}}{0.25}$

$= 2\ m/s^2$

5. Ans. 8

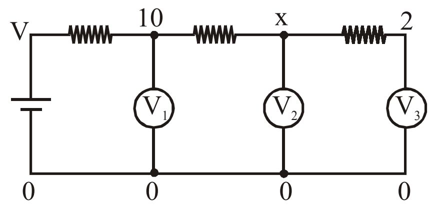
Sol. $B_1 = \rho_{air}Vg - \rho_{H_2}Vg = Vg\,(1.08)$

$B_2 = \rho_{air}Vg - \rho_{H_2}Vg = Vg\,(1.00)$

$\frac{B_1 - B_2}{B_1} = 8\%$

6. Ans. 4

Sol.



$\frac{x-10}{R} + \frac{x-2}{R} + \frac{x-0}{r} = 0$ .....(1)

$\frac{2-x}{R} + \frac{2-0}{r} = 0$ ....(2)

$\frac{2}{r} = \frac{x-2}{R}$

$\frac{R}{r} = \frac{x}{2} - 1$

from (1) $2x - 12 + \frac{xR}{r} = 0$

$(2x - 12) + x\left(\frac{x}{2} - 1\right) = 0$

$4x - 24 + x^2 - 2x = 0$

$x^2 + 2x - 24 = 0$

$x = -6, 4 \Rightarrow x = 4$

7. Ans. 4

Sol. $\Delta\ell_1 = \alpha\ell\,\Delta T = 4 \times 0.01 = \alpha \times 2.44 \times 20$

$\alpha = \frac{1}{244 \times 5}$

$\Delta\ell_2 = \frac{1}{244 \times 5} \times 2.44 \times 50 = 0.1\text{cm}$

$L = (2.44 + 0.1)$ cm $= 2.54$ cm

Therefore division coinciding at 50°C is 4th division

8. Ans. 2

Sol. $n_1C_{v_1}dT + n_2C_{v_2}dT + PdV = 0$

$\frac{1}{2} \times 2RdT + 4 \times \frac{7R}{4}dT + 4RT\frac{dV}{V} = 0$

$$2\int\frac{dT}{T}+\int\frac{dV}{V}=0$$

$$2\ell n\left(\frac{T}{300}\right)+\ell n\left(\frac{1}{4}\right)=0$$

$$\ell n\frac{T}{300}=\ell n2$$

$T = 600$ K

$$\Delta U=\frac{1}{2}\times 2R\times(600-300)+4\times\frac{7R}{4}(600-300)$$

$= 8R \times 300$

$= 8\times\frac{25}{3}\times 300 = 2\times 10^4 J$

PART-2 : CHEMISTRY — ANSWER KEY

<table>
<tr><td rowspan="2">SECTION-I</td><td>Q.</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>A.</td><td>A,B,D</td><td>A,B,C</td><td>B,C</td><td>A,B,C,D</td><td>B,D</td><td>B</td><td>A,B,C,D</td><td>B,D</td><td>A,B,D</td><td>A,B,C,D</td></tr>
<tr><td rowspan="2">SECTION-II</td><td rowspan="2">Q.1</td><td>A</td><td>B</td><td>C</td><td>D</td><td rowspan="2">Q.2</td><td>A</td><td>B</td><td>C</td><td>D</td><td rowspan="2"></td></tr>
<tr><td>P,Q,S</td><td>Q,R,S</td><td>P,S</td><td>P,Q,S,T</td><td>Q,T</td><td>P</td><td>R,T</td><td>S,T</td></tr>
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<tr><td>A.</td><td>7</td><td>5</td><td>3</td><td>5</td><td>0</td><td>3</td><td>6</td><td>5</td></tr>
</table>

SOLUTION

SECTION-I

1. **Ans. (A,B,D)**
2. **Ans. (A,B,C)**

$$A(H_2O)_8(s) \rightleftharpoons A(g) + 8H_2O(g) \;;\; Kp_1$$
$$\qquad (x+y)\; 8x$$

$$A(H_2O)_8(s) \rightleftharpoons A(g) + 8H_2O(s) \;;\; Kp_2$$
$$\qquad (x+y)$$

$x + y = 0.2$

$8x = 0.001$

$Kp_1 = (x + y)(8x)^8 = 0.2 \times (10^{-3})^8 = 2 \times 10^{-25}$

$Kp_2 = (x + y) = 0.2$

$$H_2O(s) \rightleftharpoons H_2O(g) \;;\; \left(\frac{Kp_1}{Kp_2}\right)^{1/8}$$

$$V.P. = \left(\frac{2\times10^{-25}}{2\times10^{-1}}\right)^{1/8} = 10^{-3}\text{bar}$$

3. **Ans. (B,C)**
4. **Ans. (A,B,C,D)**
5. **Ans. (B, D)**
6. **Ans. (B)**
7. **Ans. (A,B,C,D)**
8. **Ans. (B,D)**
9. **Ans. (A,B,D)**
10. **Ans. (A,B,C,D)**

SECTION-II

1. **Ans. (A) → (P,Q,S); (B) → (Q,R,S); (C) → (P,S); (D) → (P,Q,S,T)**
2. **Ans. (A) → (Q,T); (B) → (P); (C) → (R,T); (D) → (S,T)**

SECTION-IV

1. **Ans. (10^{-7}M) OMR ANS (7)**

$$k_1\times k_2 = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$$

$$10^{-20} = \frac{(10^{-3})^2[S^{2-}]}{0.1}$$

$[S^{2-}] = 10^{-15}$

$Ksp = [Mn^{2+}][S^{2-}] = [Mn^{2+}][10^{-15}]$

$[Mn^{2+}] = 10^{-7}$

2. **Ans. (5)**

$$\bar{v} = R_H z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\bar{v}_1 = R_H\times1\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=R_H\times\frac{3}{4}$$

$$\bar{v}_2 = R_H\times9\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=R_H\times\frac{9\times5}{36}$$

$$\frac{\bar{v}_1}{\bar{v}_2}=\frac{3}{4}\times\frac{36}{9\times5}=\frac{3}{5}$$

3. **Ans(3)**

$$2CH_3COOH(g) \longrightarrow (CH_3COOH)_2(g)$$

$\Delta H^0 = -2 \times 7.5 = -15$ kcal

$\Delta G^0 = -RT \ln Keq = -2 \times 300 \ln e^{10} = -6000$ cal.

$\Delta G^0 = \Delta H^0 - T\Delta S^0$

$-6000 = -15000 - 300\,\Delta S^0$

$$\Delta S^0 = \frac{-9000}{300} = -30\text{cal./k}$$

4. **Ans. 5**
5. **Ans. 0**
6. **Ans. 3**
7. **Ans. 6**
8. **Ans. 5**

PART-3 : MATHEMATICS — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C,D	A,C,D	C,D	B,C,D	A,C,D	A,B	B,D	A,B,C,D	A,B	C,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		T	R	Q	P		Q,T	P	R	Q,T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	1	0	2	7	3	2	5	6		

SOLUTION

SECTION-I

1. Ans. (B,C,D)

$$I=\int_{-1}^{1}\left(e^{x^3}+e^{-x^3}\right)dx=2\int_{0}^{1}\left(e^{x^3}+e^{-x^3}\right)dx>2$$

Since $e^{x^3}+e^{-x^3}$ is concave upwards

$I < f(0) + f(1)$

$$I<2+e+\frac{1}{e}\cdot$$

2. Ans. (A,C,D)

$a^3 + b^3 + (-2c)^3 - 3ab(-2c)$

$= (a + b - 2c)\,(a + b\omega - 2c\omega^2)$

$(a + b\omega^2 - 2c\omega) = 0$

3. Ans. (C,D)

(A) $(ABA^T)^T = (A^T)^T B^T A^T = AB^T A^T$

Need not be symmetric

(B) $(AB - BA)^T = B^T A^T - A^T B^T$ Need not be skew symmetric

(C) $B = |A|\ \dfrac{\text{adj}A}{|A|} \Rightarrow B = \text{adj}A$

Now, $c = \text{adj}(A^T) - \text{adj}A$

$\therefore\ c^T = (\text{adj}\,A^T)^T - (\text{adj}A)^T$

$= \text{adj}A - \text{adj}A^T = -c$

$\therefore$ skew, symmetric matrix.

(D) $A^T = -B\ \&\ A^T = -A$

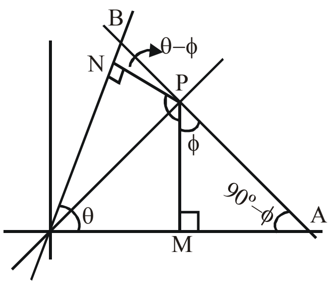
$\therefore A = -B$

Now $B^{15} = -A^{15}$

Let $c = B^{15}$

$\therefore\quad C^T = (B^T)^{15} = (-A)^{15} = -A^{15} = B^{15}$

4. Ans. (B,C,D)



$$PA.PB=\frac{PM.PN}{\cos\phi\cos(\theta-\phi)}$$

$$\Delta=\frac{2PM.PN}{\cos\theta+\cos(2\phi-\theta)}$$

for PA. PB to be

minimum $2\phi - \theta = 0 \Rightarrow \theta = 2\phi$

$\therefore$ OA = OB $\Rightarrow \Delta$OAB is issosceles

slope of PA $= \tan(90^\circ + \phi) = -\cot\phi = -\cot\dfrac{\theta}{2}$

$$\tan\theta=3\Rightarrow\frac{2t}{1-t^2}=3\Rightarrow 3-3t^2=2t$$

$3t^2 + 2t - 3 = 0$

$$t=\frac{-2\pm\sqrt{4+36}}{6}=\frac{-1\pm\sqrt{10}}{3}$$

5. Ans. (A,C,D)

$$f(x)=e^{\frac{-1}{x^2}}+\int_{0}^{\frac{\pi x}{2}}\sqrt{1+\sin t}\,dt$$

$$f'(x)=e^{-\frac{1}{x^2}}.\frac{2}{x^3}+\frac{\pi}{2}\sqrt{1+\sin\frac{\pi x}{2}}$$

Now $\displaystyle\lim_{x\to0^+}\frac{e^{-1/x^2}}{x^3}=\lim_{x\to\infty}\frac{x^3}{e^{x^2}}=0$ (By L'Hôpitals Rule)

hence $f'(x)$ exists and continuous for all $x \in (0,\infty)$

$f'(x)$ is bounded

Also, $\displaystyle\lim_{x\to\infty} f(x)=\infty\ \Rightarrow\ f(x)$ is unbounded

$\Rightarrow \exists\ \alpha > 0$ such that $|f(x)| > |f'(x)|\ \forall\ x \in (\alpha,\infty)$

and $$f''(x)=\frac{6e^{-1/x^2}}{x^4}+\frac{4}{x^6}e^{-\frac{1}{x^2}}+\frac{\pi^2}{8}\frac{\cos\frac{\pi x}{2}}{\sqrt{1+\sin\frac{\pi x}{2}}}$$

Clearly $f''(x)$ does not exist at $x = 3, 7, 11, \ldots.$

6. Ans. (A,B)

Put $x = y + \frac{1}{2}$

$\Rightarrow \quad 8y^4 + 4y^2 + a - \frac{3}{2} = 0$

Put $y^2 = z$

$\therefore \quad 8z^2 + 4z + \left(a - \frac{3}{2}\right) = 0$

Case- I If $a > \frac{3}{2} \Rightarrow$ All roots are non real

$\therefore$ sum of roots = 2

Case-II If $a = \frac{3}{2}$

$\Rightarrow \quad z = 0$ or $-\frac{1}{2}$

$\therefore \quad y = 0,0$ or $r = \frac{1}{2}, \frac{1}{2}$

If $z = -\frac{1}{2} \Rightarrow 2$ non real roots

$\therefore \; z = \frac{1}{2} + \frac{1}{2} + \alpha + \beta \Rightarrow \alpha + \beta = 1$

Case III: If $a < \frac{3}{2}$

$\Rightarrow z \to x_1, -x_2$

$y = \pm\sqrt{x_1} \qquad \therefore \; x = \frac{1}{2} \pm \sqrt{x_1}$

Again $\alpha + \beta = 1$

7. Ans. (B,D)

$$\lim_{n\to\infty} \frac{\left(\sum_{x=1}^{n} x^4\right)\left(\sum_{x=1}^{n} x^5\right)}{\left(\sum_{x=1}^{n} x^t\right)\left(\sum_{x=1}^{n} x^{9-t}\right)} = \frac{\frac{1}{5}\cdot\frac{1}{6}}{\frac{1}{t+1}\cdot\frac{1}{10-t}} = \frac{4}{5}$$

$\therefore \; t = 2,7$

8. Ans. (A,B,C,D)

Either the line L is parallel to the angle bisector of given lines.

$\therefore \; m = \pm 1$ or $\tan\theta_1 = \tan\theta_2$ (in same sense)

$\therefore \; m = -\frac{11}{2}$ or $-\frac{2}{11}$

9. Ans. (A,B)

$$\frac{\Delta}{s(s-a)} \cdot \frac{\Delta}{s(s-c)} = \frac{1}{3} \Rightarrow \frac{s(s-a)(s-b)(s-c)}{s^2(s-a)(s-c)} = \frac{1}{3}$$

$\Rightarrow 3s - 3b = s$

$2s = 3b \Rightarrow a + c = 2b$

$\therefore$ a,b,c are in A.P

Also $a + c \geq 2\sqrt{ac}$

Also $\Sigma \tan\frac{A}{2}\tan\frac{B}{2} = 1$

$\Rightarrow \tan\frac{B}{2}\left(\tan\frac{A}{2} + \tan\frac{C}{2}\right) = \frac{2}{3}$

10. Ans. (C,D)

Put $\tan^2\alpha = x$ & $\tan^2\beta = y$

$$\therefore \; \frac{(x+1)^2}{y} + \frac{(y+1)^2}{x} = \frac{x^2}{y} + \frac{y^2}{x} + \frac{1}{y} + \frac{1}{x} + 2\left(\frac{x}{y} + \frac{y}{x}\right)$$

Apply A.M. $\geq$ G.M to get minimum value as 8.

SECTION – II

1. Ans. (A)$\to$(T); (B)$\to$(R); (C)$\to$(Q); (D)$\to$(P)

(A) Give sum

$$= \tan\frac{\pi}{15} + \tan\frac{4\pi}{15} + \tan\frac{7\pi}{15} + \tan\frac{10\pi}{15} + \tan\frac{13\pi}{15}$$

$$= (\tan 12° + \tan 48°) + (\tan 84° - \tan 24°) - \sqrt{3}$$

$$= \frac{\sin 60°}{\cos 12° \cos 48°} + \frac{\sin 60°}{\cos 24° \cos 84°} - \sqrt{3}$$

$$= \sqrt{3}\left[\frac{1}{\cos 60° + \cos 36°} + \frac{1}{\cos 108° + \cos 60°} - 1\right]$$

$$= \sqrt{3}\left[\frac{1}{\frac{1}{2} + \frac{\sqrt{5}+1}{4}} + \frac{1}{\frac{1}{2} - \frac{\sqrt{5}-1}{4}} - 1\right] = 5\sqrt{3}$$

$\Rightarrow \quad k = 3$

(B) Give sum $= \sum_{r=0}^{n-1} \frac{(n+1)-(r+1)}{(r+1)(r+2)(r+3)}$

$$= (n+1)\sum_{r=0}^{n-1} \frac{1}{(r+1)(r+2)(r+3)} - \sum_{r=0}^{n-1} \frac{1}{(r+2)(r+3)}$$

$$= \frac{(n+1)}{2}\sum_{r=0}^{n-1}\left(\frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)}\right)$$

$$-\sum_{r=0}^{n-1}\left(\frac{1}{r+2} - \frac{1}{r+3}\right)$$

$= \frac{55}{24}$ for $n = 10 \qquad \therefore \quad k = 8$

(C) tangent of slope 2 of ellipse is

$y = 2x \pm \sqrt{4a^2 + b^2}$ which passes through (–2, 0)

$\Rightarrow \quad 4a^2 + b^2 = 16$

Now using A.M. - G.M.

$4a^2 + b^2 \geq 4ab$

$\Rightarrow \quad ab \leq 4 \quad \Rightarrow \quad \pi ab < 4\pi$

$\Rightarrow \quad k = 4$

(D) $\tan k = \frac{2\sqrt{7+5}}{4} = \sqrt{3} \quad \Rightarrow \quad k = \frac{\pi}{3}$

$\therefore \quad \tan^2 k = \tan^2\frac{\pi}{3} = 3$

2. Ans. (A)→(Q,T); (B)→(P); (C)→(R); (D)→(Q,T)

Do yourself by using simple properties

SECTION – IV

1. Ans. 1

$$\frac{1}{a_{n+1}} = \frac{1}{a_n(a_n+1)} = \frac{1}{a_n} - \frac{1}{a_n+1}$$

$$\therefore \frac{1}{a_n+1} = \frac{1}{a_n} - \frac{1}{a_{n+1}}$$

$$\therefore s = \frac{1}{a_1} - \frac{1}{a_{101}} = z - \frac{1}{a_{101}} \text{ also } a_{101} > 1$$

$\therefore [s] = 1$

2. Ans. 0

$$g(x) = \int\left(1 - \frac{f'(x)+f''(x)+f'''(x)}{f(x)+f'(x)+f''(x)+f'''(x)}\right)dx$$

$$= x - \ell n\left|f(x)+f'(x)+f''(x)+f'''(x)\right| + c$$

$= x - 3\ell n|x| + c$

Now, $g(1) = 1 \Rightarrow c = 0$

$\therefore g(e) = e - 3 \Rightarrow |g(e)| = 3 - e$

$[|g(e)|] = 0$

3. Ans. 2

Let $2^x > 3x \Rightarrow 2^{x+1} > 2^x + 3x$

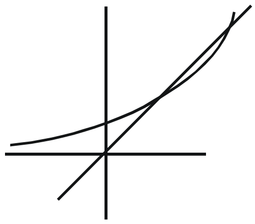
$\therefore (x-2) + 2\log_2(2^x + 3x) < (x-2) + 2\log_2(2^{x+1})$

$= x - 2 + 2x + 2 = 3x$

$\therefore 2^x < 3x$

which is contradiction

$\therefore 2^x = 3x$

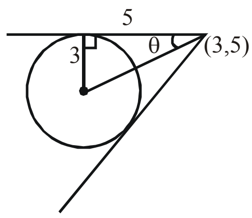


$\therefore$ Two solutions

4. Ans. 7

Clearly (3,5) is focus of parabola

$\therefore \tan\theta = \frac{3}{5}$



$2\tan 2\theta = \frac{15}{8}$

$\therefore a - b = 7$

5. Ans. 3

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right) = \frac{\pi}{2}$$

$$\beta = \tan^{-1}\left(\frac{12}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\frac{63}{16}$$

$$= \pi - \tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\frac{63}{16} = \pi$$

$\therefore 2\sin\alpha = 2 \quad \& \quad \cos\pi = -1$

$\therefore$ equation $x^2 - x - 2 = 0$

$p = 1 \;\&\; q = -2 \Rightarrow p - q = 3$

6. Ans. 2

$$\sum_{i=1}^{\infty} a_i = \frac{2\cot x}{1-\sin^2 x} = \frac{2}{\cos x \sin x}$$

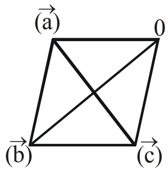
$$\sum_{j=1}^{\infty} b_j = \frac{\sin 2x}{1-\sin^2 x} = \frac{2\sin x}{\cos x}$$

$$\therefore \sum_{i=1}^{\infty} a_i - \sum_{j=1}^{\infty} b_j = \frac{2}{\cos x . \sin x} - \frac{2\sin x}{\cos x}$$

$$= \frac{2\left(1-\sin^2 x\right)}{\cos x \sin x} = 2\cot x$$

$\therefore$ minimum value $= 2$

7. Ans. 5



$$\vec{a}.\vec{b} = \frac{1}{2} \quad \vec{b}.\vec{c} = \frac{1}{2} \quad \vec{c}.\vec{a} = \frac{1}{2}$$

Now, $\vec{c} = \alpha\left(\vec{a}+\vec{b}\right) + \beta\left(\vec{a}\times\vec{b}\right)$

Take dot product with $\vec{a}$

$$\frac{1}{2} = \alpha\left(1+\frac{1}{2}\right)$$

$$\therefore \alpha = \frac{1}{3}$$

Now, take dot with $\vec{a}\times\vec{b}$

$$\therefore \left[\vec{a}\ \vec{b}\ \vec{c}\right] = \beta \left|\vec{a}\times\vec{b}\right|^2 = \beta.\frac{3}{4}$$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = \frac{3}{4} - \frac{1}{8} - \frac{1}{8} = \frac{1}{2}$$

$$\therefore \beta = \frac{4}{3\sqrt{2}}$$

$$\therefore \alpha^2\beta^2 = \frac{1}{9}.\frac{16}{9.2} = \frac{8}{81}$$

$$\therefore \left[\frac{81}{16}\right] = 5$$

8. Ans. 6

Use $\int_a^b f(x)dx + \int_c^d g(y)dy = bd - ac$

where f & g are inverse of each other.

LEADER TEST SERIES / JOINT PACKAGE COURSE
TARGET : JEE (ADVANCED) 2016

Test Type : **ALL INDIA OPEN TEST** (MAJOR) Test Pattern : JEE-Advanced

TEST # 11 **TEST DATE : 08 - 05 - 2016**

PAPER-2

PART-1 : PHYSICS **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C,D	A,B	A,C	B,C	A,D	B,C	A,B,C	A,D	A	B,D
	Q.	11	12								
	A.	B,D	A,B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	6	2	4	4	5	5	3	2		

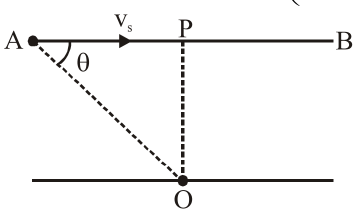
SOLUTION

SECTION-I

1. Ans. (C,D)

Sol. The graph shows the situation shown in figure below. The observed frequency will initially be more than the natural frequency. When the source is at P, observed frequency is equal to its natural frequency i.e., 2000 Hz.

For region AP : $f = f_0\left(\dfrac{v}{v - v_s\cos\theta}\right)$



For PB : $f = f_0\left(\dfrac{v}{v + v_s\cos\theta}\right)$

Minimum value of f will be:

$$f_{min} = f_0\left(\frac{v}{v+v_s}\right) \quad \text{when } \cos\theta = 1$$

$$\text{or } 1800 = 2000\left(\frac{300}{300+v_s}\right)$$

Solving this we get, v_s = 33.33 m/s

and maximum value of f can be

$$f_{max} = f_0\left(\frac{v}{v-v_s}\right) \quad \text{when } \cos\theta = 1$$

$$\text{or } f_{max} = 2000\left(\frac{300}{300-33.33}\right) = 2250 \text{ Hz}$$

2. Ans. (A, B)

Sol. $y = x\tan\theta - \dfrac{1}{2}\dfrac{gx^2}{u^2\cos^2\theta}$

(A) $2 = 4 \times 1 - \dfrac{1}{2}\times\dfrac{10^2\times 4^2}{16\times 5\times\dfrac{1}{2}}$

(B) $2 = 4 \times 3 - \dfrac{1}{2}\times\dfrac{10^2\times 4^2}{16\times 5\times\dfrac{1}{10}} = 2$

(C) $2 = 4 \times 1 - \dfrac{1}{2}\times\dfrac{10^2\times 4^2}{4\times 5\times 1} = 0$

(D) $2 = 4 \times 3 - \dfrac{1}{2}\times\dfrac{10^5\times 16}{16\times 5\times\dfrac{1}{10}} = 2$

3. Ans. (A,C)

Sol. From conservation of linear momentum, velocity of 1 kg block just after the collision is 2m/sec.

After the collision relative velocity of approach = 6 m/sec,

Relative velocity of separation = 6 m/sec

4. Ans. (B,C)

5. Ans. (A, D)

Sol. Net force towards centre of earth

$= mg' = \frac{mgx}{R}$

Normal force N = mg′ sin θ

Thus pressing face $N = \frac{mgx}{R}\frac{R}{2X}$

$N = \frac{mg}{2}$ Constant and independent of X

tangential force F = ma = mg′ cos θ

$a = g' \cos\theta = \frac{gx}{R}\frac{\sqrt{\frac{R^2}{4}-X^2}}{X}$

$a = \frac{gx}{R}\sqrt{R^2-4x^2}$

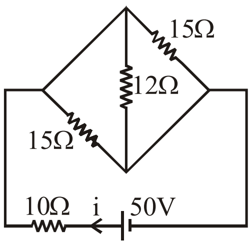
curve is parabolic and at X = R/2, a = 0

6. Ans. (B, C)

7. Ans. (A,B,C)

8. Ans. (A,D)

Sol.



Just after closing of switch S

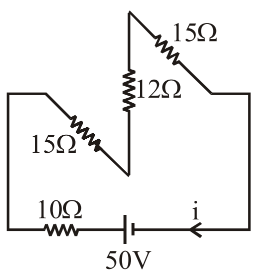
$\frac{1}{R_{eq}} = \frac{1}{15}+\frac{1}{15}+\frac{1}{12}$

$\frac{1}{R_{eq}} = \frac{2}{15}+\frac{1}{12}$

$= 4.61 + 10$

$= 14.61$

$\therefore i = \frac{50}{14.61} = 3.42$ A



After long time of closing of switch S

$i = \frac{50}{10+15+12+15} = 0.962$ Amp

9. Ans. (A)

Sol. $f = \frac{n}{2L}\times C$

$\& \ n = \frac{2Lf}{C}$

$n + dn = \frac{2L(f+df)}{C}$

$dn = \frac{2L}{C}df$

10. Ans. (B, D)

Sol. $P = \int_0^\infty \frac{hf}{\ell^{\frac{hf}{T}}-1}\times\frac{2L}{C}df$

Take $\frac{hf}{kT} = x$

$P = \int_0^\infty \frac{xkT}{\ell^x-1}\times\frac{2L}{C}\times\frac{kT}{h}dx = \frac{2k^2T^2L}{Ch}\times\frac{\pi^2}{6}$

$= \frac{\pi^2}{3}\frac{k^2T^2L}{Ch}$

11. Ans. (B, D)

12. Ans. (A, B)

SECTION-IV

1. Ans. 6

Sol. According to conservation of energy

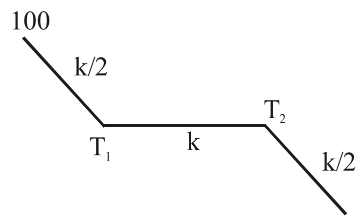
$\frac{hc}{\lambda_1}+\frac{hc}{\lambda_2} = RchZ^2\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$

n = 6.03

quantum number = 6

2. Ans. 2

Sol. $R_{TH} = \frac{2L}{kA}+\frac{L}{kA}+\frac{2L}{kA} = \frac{5L}{kA}$



$\frac{dH}{dt} = \frac{100}{5L}kA = 20\frac{kA}{L}$

$100 - T_1 = 40$

$\frac{dH}{dt} = \frac{T_2-0}{\frac{2L}{kA}} = 20\frac{kA}{L} \Rightarrow T_2 = 40$

$T_B = T_2, T_D = T_1 \Rightarrow \Delta T = 20$

3. Ans. 4

Sol. As the voltage across the charging battery,

$V = E + Ir = 30 + 15 \times 0 = 30V$

So the potential difference across the resistance

$V_R = 120 - 30 = 90$ V

So the power wasted in heating the circuit

$P = VI = 90 \times 15 = 1350$ W

So the energy wasted as heat in time t

$H = P \times t = (1350 \times t)$ joule $= \dfrac{1350}{4.2} \times t$ calorie

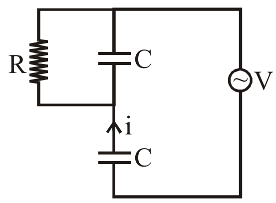
Now if this heat changes the temperature of 1 kg of water from 15ºC to 100ºC

$$\frac{1350t}{4.2} = mc\Delta\theta = 1 \times 10^3 \times 1 \times (100 - 15)$$

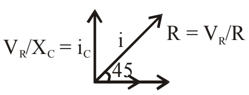
i.e., $t = \dfrac{85 \times 4.2 \times 100}{135} = 264.4$ s $= 4.4$ minute

4. Ans. 4

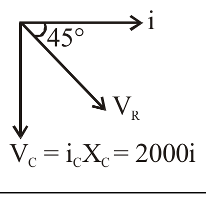
Sol.



$$X_C = \frac{10}{500} = 2000$$



$$i = \frac{V_R}{2000} \times \sqrt{2} \Rightarrow V_R = 1000\sqrt{2}\ i$$



$$V = \sqrt{(2000i)^2 + \left(1000\sqrt{2}i\right)^2}$$

$$+2x\ 2000\ i \times 1000\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$= i \times 10^3\sqrt{4+2+4}$$

$$\Rightarrow i = \frac{200 \times 10^{-3}}{\sqrt{16}} = \frac{1}{5\sqrt{10}} A$$

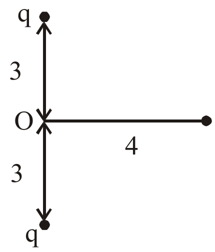
$$i_R = i \cos 45 = \frac{1}{5\sqrt{20}}$$

$$P = \frac{1}{25 \times 20} \times 2000 = 4W$$

5. Ans. 5

6. Ans. 5

Sol.



$$\frac{2k \times 25 \times 10^{-10}}{5}$$

$+1.5$

$$= \frac{2k \times 25 \times 10^{-10}}{\sqrt{x^2 + 3^2}}$$

$$+7.5 = +\frac{5 \times 9}{\sqrt{x^2 + 9}}$$

$$\sqrt{x^2 + 9} = \frac{45}{7.5}$$

$x^2 + 9 = 36$

$x^2 = 27$

$x = 3\sqrt{3} = 5.1$

7. Ans. 3

8. Ans. 2

Sol. $B = \dfrac{\mu_0 i}{2\pi a}$

$$\phi = \frac{\mu_0 i}{2\pi a} \times A \times N$$

$$\varepsilon = \frac{\mu_0 i_0 \omega AN}{2\pi a}\cos\omega t$$

$$P = \left(\frac{\mu_0 i_0 \omega AN}{2\pi a}\right)^2 \times \frac{1}{2} \times \frac{1}{R} = \frac{2 \times 10^{-8}}{R}$$

PART-2 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	C,D	B,C,D	A,B	A	A,B,C,D	B,D	B,C,D	A,C,D	B
	Q.	11	12								
	A.	A	A,B,C,D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	2	2	3	0	0	3	3	3		

SOLUTION

SECTION-I

1. **Ans. (A,C)**

2. **Ans. (C,D)**

3. **Ans. (B,C,D)**

4. **Ans. (A,B)**

5. **Ans. (A)**

6. **Ans. (A,B,C,D)**

7. **Ans. (B,D)**

8. **Ans. (B,C,D)**

9. **Ans. (A, C, D)**

Cell reaction is -

$$2A^+ + B \longrightarrow 2A + B^{2+}$$

(0.1m) (0.1m)

$$E^o_{cell} = E^{\circ}_{A^+/A} - E^{\circ}_{B^{+2}/B}$$

$$0.7 = 0.4 - E^{\circ}_{B^{+2}/B}$$

$$E^{\circ}_{B^{+2}/B} = -0.3V$$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.06}{2}\log\frac{(0.1)}{(0.1)^2}$$

10. **Ans. (B)**

$$2H^+ + B \longrightarrow 2A + B^{2+}$$

10^{-1} 10^{-1}

$$E = 0.3 - \frac{0.06}{2}\log\frac{10^{-1}}{(10^{-1})^2}$$

$$E = 0.27\ V$$

11. **Ans. (A)**

12. Ans. **(A,B,C,D)**

SECTION-IV

1. **Ans. 0.2 [OMR Ans. 2]**

$$\frac{\Delta P}{P_A^0} = \frac{n_B}{n_A + n_B} = \frac{15/60}{\frac{18}{18} + \frac{15}{60}} = 0.2$$

2. **Ans. (2×10^{-1}) OMR ANS (2)**

$$r_{A_2} = K[A]^2$$

$$K = \frac{r_{A_2}}{[A]^2} = \frac{10^{-5}}{(10^{-2})^2} = 10^{-1}$$

$$\frac{K_A}{2} = K \Rightarrow K_A = 2K = 2 \times 10^{-1}$$

3. **Ans. 3**

Sol. (i) $Mn^{2+} + BiO_3^- \xrightarrow{+H^{\oplus}} MnO_4^- + Bi + H_2O$

(iv) $Mn^{2+} + PbO_2 \xrightarrow{+H^{\oplus}} MnO_4^- + Pb^{2+} + H_2O$

(vi) $Mn^{2+} + S_2O_8^{2-} + H_2O \longrightarrow MnO_4^- + 2SO_4^{2-} + 2OH^-$

4. **Ans. 0**

5. **Ans. 0**

6. **Ans. 3**

7. **Ans. 3**

8. Ans. 3

PART-3 : MATHEMATICS — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,D	A,C,D	A,B,C	A,D	A,D	A,D	A,B,C,D	A,C	B,C,D	A,D
	Q.	1	2								
	A.	B,C	A,B,D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	5	6	2	5	3	3	4	6		

SOLUTION

SECTION-I

1. Ans. (A,D)

$$\int_1^{\sqrt[3]{2}} \frac{\left(\frac{1}{x^{10}}+\frac{1}{x^7}\right)dx}{\left(\frac{2}{x^9}+\frac{3}{x^6}+1\right)^{1/3}} \quad \frac{2}{x^9}+\frac{3}{x^6}+1=t$$

$$-18\left(\frac{1}{x^{10}}+\frac{1}{x^7}\right)dx = dt$$

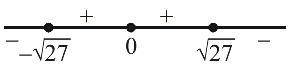
$$= -\frac{1}{18}\int_6^2 \frac{dt}{t^{1/3}} = -\frac{1}{12}\left(t^{2/3}\right)_6^2 = \frac{1}{2}\left[\sqrt[3]{36}-\sqrt[3]{4}\right]$$

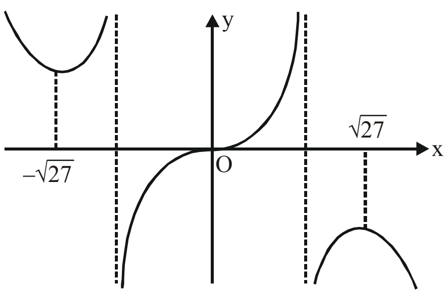
2. Ans. (A,C,D)

$$y=\frac{x^3}{9-x^2} \Rightarrow y' = \frac{\left(9-x^2\right)3x^2+2x.x^3}{\left(9-x^2\right)^2}$$

$$y' = \frac{x^2\left[27-3x^2+2x^2\right]}{\left(9-x^2\right)^2}$$

$$= -\frac{x^2\left(x-\sqrt{27}\right)\left(x+\sqrt{27}\right)}{\left(9-x^2\right)^2}$$





$f\left(-\sqrt{27}\right) = 7.8$

$\Rightarrow n = 8$

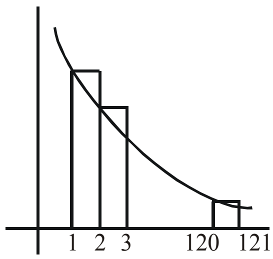
$\ell = 9$

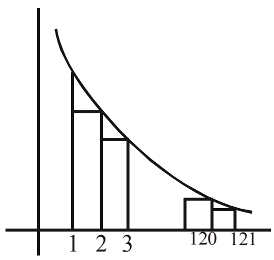
$m = 3$

3. Ans. (A,B,C)

$$A > \int_1^{121} \frac{1}{\sqrt{x}}dx = 20$$

$$B < \int_1^{121} \frac{1}{\sqrt{x}}dx = 20$$



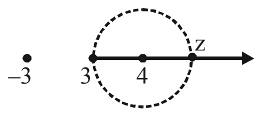


$A - 20 > 20 - B$

$A + B > 40$

4. Ans. (A,D)

$|z + 3| - |z - 3| = 6 \Rightarrow$ ray



5. Ans. (A,D)

(A) $\dfrac{2x^4+2y^4+4z^4+1}{4} \geq 2xyz$

$\Rightarrow\ 2x^4 + 2y^4 + 4z^4 - 8yxz \geq -1$

$\Rightarrow\ \ell_1 = -1$

$\Rightarrow$ at $x = y = \left(\frac{1}{2}\right)^{1/4}, z = \frac{1}{\sqrt{2}}$

(C) $x^4y + xy^4 + \dfrac{4}{x^2y^3} + \dfrac{1}{x^3y^2} + 8 \geq 5.2$

but $x^4y = xy^4 = \dfrac{4}{x^2y^3} = \dfrac{1}{x^3y^2} = 8$

is not possible

$\therefore\ \ell_2 > 10$

6. Ans. (A,D)

$\cos x dy + y\sin x dx = \cos^4 x$

$$\frac{dy}{dx} + y\tan x = \cos^3 x$$

$$y\sec x = \int \cos^2 x dx$$

$$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C$$

by (0, 0), C = 0

$$\Rightarrow \quad y = \frac{x}{2}\cos x + \frac{1}{4}\sin 2x\cos x$$

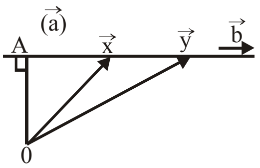
(A) $f(x) = \frac{x}{2}\cos x + 1 \Rightarrow \sin 2x\cos x = 4$ not possible.

(B) $f\left(\frac{\pi}{2}\right) = 0$

(C) $f'\left(\frac{5\pi}{2}\right) = -\frac{5\pi}{4}$

(D) $f'(0) = 1$

7. Ans. (A,B,C,D)



$\vec{x}.\vec{a} = 1 = \vec{y}.\vec{a}$

$\vec{b} = \vec{x} - \vec{y}$, $3\vec{c} = \vec{x} + \vec{y}$

$L : \vec{r} = \vec{a} + \lambda\vec{b}$

8. Ans. (A,C)

$||z|^2 - 1| \le |z^2 - 1| \le |z|^2 + 1$

$||z^2| - 1| \le |z| + 2 \le |z|^2 + 1$

I. $|z|^2 - |z| - 1 \ge 0 \Rightarrow |z| \in \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

II. $|z|^2 - |z| - 3 \le 0 \Rightarrow |z| \in \left[0, \frac{1+\sqrt{13}}{2}\right]$

III. $|z|^2 + |z| + 1 \ge 0 \Rightarrow$ always true

$$\Rightarrow |z| \in \left[\frac{1+\sqrt{5}}{2}, \frac{1+\sqrt{13}}{2}\right]$$

Paragraph for Question 9 and 10

P(4,3) $\quad 4x - 3y = 7$

$$\frac{x-4}{\frac{3}{5}} = \frac{y-3}{\frac{4}{5}} = -\frac{5}{2} \Rightarrow Q\left(\frac{5}{2}, 1\right)$$

$$\frac{x-4}{\frac{3}{5}} = \frac{y-3}{\frac{4}{5}} = -5 \Rightarrow R(1, -1)$$

Let $S_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

& $S_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, A<B

for $S_1 : \frac{5x}{2a^2} - \frac{y}{b^2} = 1$

$$\Rightarrow \frac{5}{2a^2} = \frac{4}{7}, \frac{1}{b^2} = \frac{3}{7}$$

$$S_1 : \frac{x^2}{35/8} - \frac{y^2}{7/3} = 1 \Rightarrow e_1^2 : \frac{7/3}{35/8} + 1 \Rightarrow e_1^2 = \frac{23}{15}$$

for $S_2 : \frac{x}{A^2} - \frac{y}{B^2} = 1 \Rightarrow A^2 = \frac{7}{4}, B^2 = \frac{7}{3}$

$$\frac{x^2}{7/4} + \frac{y^2}{7/3} = 1 \Rightarrow e_2^2 = 1 - \frac{7/4}{7/3} \Rightarrow e_2^2 = \frac{1}{4} \Rightarrow e_2 = \frac{1}{2}$$

$$S_1 \ \& \ S_2 \Rightarrow x^2\left(\frac{4}{7} + \frac{8}{35}\right) = 2 \Rightarrow y \notin R$$

$\Rightarrow$ 4 common tangents

9. Ans. (B,C,D)

10. Ans. (A,D)

Paragraph for Question 11 and 12

11. Ans. (B,C)

$$\sum_{k=1}^{n} P(E_i) = 1 \quad \Rightarrow \quad k(1^2 + 2^2 \ldots\ldots n^2) = 1$$

$$\Rightarrow \quad k = \frac{6}{n(n+1)(2n+1)}$$

$P(w) = P(E_1 w \cup E_2 w \cup \ldots\ldots E_n w)$

$$= k\frac{1}{2n} + k2^2\frac{2}{2n} + k3^2\frac{3}{3n} ...... + kn^2\frac{n}{2n}$$

$$= \frac{k}{2n}[1^3 + 2^3 + ... + n^3] = \frac{3}{n^2(n+1)(2n+1)}.\left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{3}{4}\frac{(n+1)}{(2n+1)}$$

$$\lim_{n\to\infty} P(w) = \frac{3}{8},\ \lim_{n\to\infty} nP(E_n) = nkn^2$$

$$= n^3.\frac{6}{n(n+1)(2n+1)} = 3$$

12. Ans. (A,B,D)

$$P(E_i) = ki,\ k = \frac{2}{n(n+1)}$$

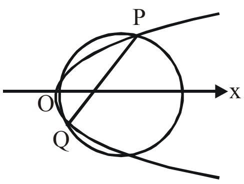
$$P(E_i / w) = \frac{P(E_i w)}{P(w)} = \frac{ki\frac{i}{2n}}{k\frac{1}{2n} + 2k.\frac{2}{2n} + ... + nk.\frac{n}{2n}}$$

$$= \frac{6i^2}{n(n+1)(2n+1)}$$

SECTION – IV

1. Ans. 5

$P(t_1^2, 2t_1)$, $Q(t_2^2, 2t_2)$, $t_1t_2 = -4$



$$\left|\frac{1}{2}\left(2t_1t_2^2 - 2t_2t_1^2\right)\right| = 20$$

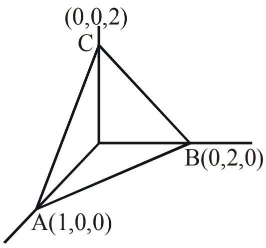
$t_1 - t_2 = 5 \Rightarrow t_1 = 1, 4$

$$\left.\begin{array}{ll} \text{I.} & P(1,2), Q(16,-8) \\ \text{II.} & P(16,8), Q(1,-2) \end{array}\right\} \Rightarrow d = \sqrt{325}$$

$$\frac{d^2}{65} = 5$$

2. Ans. 6

$$\Delta = \frac{1}{2}\sqrt{4+16+4} = \sqrt{6}$$



3. Ans. 2

$x = \pi \Rightarrow y = \pi \Rightarrow p(\pi, \pi)$

diff. xcosy y' + siny + 1 = y' $\Rightarrow y'(\pi) = \dfrac{1}{\pi+1}$

diff. once again

$$\cos y\, y' - x\sin y (y')^2 + x\cos y\, y'' + \cos y\, y' = y''$$

at p, $-\dfrac{1}{\pi+1} - 0 - \pi y'' - \dfrac{1}{\pi+1} = y''$

$$\Rightarrow\ y''(1+\pi) = -\frac{2}{(\pi+1)} \Rightarrow y'' = -\frac{2}{(\pi+1)^2}$$

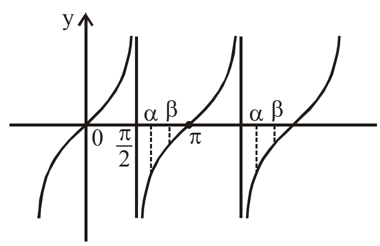
4. Ans. 5

$$\tan\theta = \frac{-8 \pm \sqrt{64-36}}{6} = \frac{-8+\sqrt{28}}{6}$$

both negative $\tan\alpha \tan\beta = 1$

$\tan\beta = \cot\alpha$

$$\beta = \frac{3\pi}{2} - \alpha,\ \frac{7\pi}{2} - \alpha$$



$$\Rightarrow\ \alpha + \beta = \frac{3\pi}{2}, \frac{7\pi}{2}$$

$$\Rightarrow\ \text{sum} = 5\pi$$

5. Ans. 3

Let $x^2 + y^2 + 2gx + 2fy = 0$ is the circle

$2g(0) + 2f(-4) = 12 \quad \Rightarrow \quad f = -\frac{3}{2}$

$2g(-2) + 2f(-3) = -3 \quad \Rightarrow \quad g = 3$

$\Rightarrow \; x^2 + y^2 + 6x - 3y = 0$

$$R = \frac{3\sqrt{5}}{2}$$

6. Ans. 3

$$n = \frac{\sin 40^\circ}{2\cos 20^\circ - \cos 40^\circ}$$

$$n = \frac{\sin 40^\circ}{\cos 20^\circ + \sin 10^\circ}$$

$$= \frac{\sin 40^\circ}{\sin 70^\circ + \sin 10^\circ} = \frac{\sin 40^\circ}{2\sin 40^\circ \cos 30^\circ} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{n^2} = 3$$

7. Ans. 4

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left|\vec{r}.\frac{2\hat{i}+\hat{j}}{\sqrt{5}}\right| = \left|\vec{r}.\frac{\left(-2\hat{i}+\hat{j}\right)}{\sqrt{5}}\right| = \left|\vec{r}.\hat{k}\right|$$

$$\Rightarrow |2x + y| = |-2x + y| = \sqrt{5}\,|z|$$

$$\Rightarrow \vec{r} = \left(0, \frac{\sqrt{5}}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right), \left(0, \frac{\sqrt{5}}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right), \left(\frac{\sqrt{5}}{3}, 0, \frac{2}{3}\right), \left(\frac{\sqrt{5}}{3}, 0, -\frac{2}{3}\right)$$

8. Ans. 6

$x = 1,\; y = 5^3 - 4^3 = 61$