Question Bank - Binomial Theorem

LEVEL–I

1. Show that the middle term in the expansion of $(1 + x)^{2n}$ is $\frac{1.3.5...(2n-1)}{n!}2^n x^n$, where n is a positive integer.

2. Find the terms independent of x, $x \neq 0$, in the expansion of

(i) $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^6$ **(ii)** $\left(x^2 + \frac{1}{x}\right)^{12}$

3. In the binomial expansion of $(1 + x)^n$, the coefficients of the fifth, sixth and seventh terms are in arithmetic progression. Find all the values of n for which this can happen.

4. Show that the coefficient of the middle term of $(1 + x)^{2n}$ is equal to the sum of the coefficients of the two middle terms of $(1 + x)^{2n-1}$.

5. Using Binomial Theorem, prove that $6^n - 5n$ always leaves the remainder 1, when divided by 25.

6. **Using Binomial Theorem, prove each of the following identities :**

(i) $C_0 + 2C_1 + 3C_2 + \ldots + (n + 1)C_n = 2^n + n\,2^{n-1}$

(ii) $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots = \frac{1}{n+1}$.

(iii) $C_0 + \frac{C_2}{3} + \frac{C_4}{5} + \ldots = \frac{2^n}{n+1}$

(iv) $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + n\frac{C_n}{C_{n+1}} = \frac{n(n+1)}{2}$.

(v) $2C_0 + 2^2\frac{C_1}{2} + 2^3\frac{C_2}{3} + \ldots + 2^{n+1}\frac{C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$.

7. If n is a positive integer and if $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then show that

$a_0^2 - a_1^2 + a_2^2 - \ldots + a_{2n}^2 = a_n$.

8. **Prove each of the following identities :**

(i) $(C_0 + C_1)(C_1 + C_2) \ldots (C_{n-1} + C_n) = \frac{(n+1)^n}{n!} C_1C_2 \ldots C_n$.

(ii) $\frac{1}{n!}{}^nC_0 + \frac{n}{(m+1)!}{}^nC_1 + \frac{n(n-1)}{(m+2)!}{}^nC_2 + \ldots + \frac{n(n-1)\ldots 2.1}{(m+n)!}{}^nC_n = \frac{(m+2n)!}{[(m+n)!]^2}$.

(iii) ${}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots + {}^{n+r}C_r = {}^{n+r+1}C_r$.

LEVEL–II

1. If the greatest term has the greatest coefficient in the expansion of $(1 + x)^n$, $n \in \mathbf{N}$, then show that $\frac{n}{n+2} < x < \frac{n+2}{n}$, if n is even and $1 < x < \frac{n+3}{n-1}$, if n is odd.

2. Find the coefficient of x^{n-1} in $(x - {}^nC_0)(x - {}^nC_1)(x - {}^nC_2) \ldots (x - {}^nC_n)$.

3. Prove that : $\sum_{r=1}^{n} {}^nC_r \sin(2r - n) = \sin n$.

4. **Prove each of the following identities :**

(i) $C_0^2 - C_1^2 + C_2^2 - \ldots + (-1)^n C_n^2 = \begin{cases} 0, & \text{if n is odd} \\ \dfrac{(-1)^{n/2} \lfloor n}{(\lfloor n/2)^2}, & \text{if n is even} \end{cases}$

(ii) ${}^nC_0 \cdot {}^{2n}C_n - {}^nC_1\, {}^{2n-2}C_n + {}^nC_2\, {}^{2n-4}C_n - \ldots = 2^n$.

(iii) ${}^nC_0\, {}^nC_0 - {}^{n+1}C_1 \cdot {}^nC_1 + {}^{n+2}C_2 \cdot {}^nC_2 - (n + 3)\, C_3\, {}^nC_3 + \ldots = (-1)^n$

(iv) $C_2 = C_0 C_4 - C_1 C_3 + C_2^2 - C_3 C_1 + C_4 C_0$.

5. For any positive integers m, n (with $n \geq m$), let $\binom{n}{m} = {}^nC_m$. Prove that

$$\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots + \binom{m}{m} = \binom{n+1}{m+1}$$

6. Prove that : $C_0 + C_3 + C_6 + \ldots = \frac{1}{3}\left[2^n + 2\cos\frac{n\pi}{3}\right]$

7. Show that : $25^n - 20^n - 8^n + 3^n$, $n \in \mathbf{I}^+$ is divisible by 85.

8. Find numerically the greatest term in the expansion of :

(a) $(2 + 3x)^9$ when $x = \frac{3}{2}$

(b) Find the index n of the binomial $\left(\frac{x}{5} + \frac{2}{5}\right)^n$ if the 9th term of the expansion has numerically the greatest coefficient $(n \in N)$.

9. Given $S_n = 1 + q + q^2 + \ldots\ldots + q^n$ and $S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + \ldots\ldots \left(\frac{q+1}{2}\right)^n$, $q \neq 1$, prove that ${}^{n+1}C_1 + {}^{n+1}C_2 \cdot s_1 + {}^{n+1}C_3 \cdot s_2 + \ldots\ldots + {}^{n+1}C_{n+1} \cdot s_n = 2^n \cdot S_n$.

10. Find the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$.

IIT JEE PROBLEMS (OBJECTIVE)

A. Fill in the blanks

1. The larger of $99^{50} + 100^{50}$ and 101^{50} is .... **[IIT - 82]**

2. The sum of the coefficients of the polynomials $(1 + 3x - 3x^2)^{2163}$ is .... **[IIT - 82]**

3. If $(1 + ax)^n = 1 + 8x + 24x^2 + ...$determine a and n. **[IIT - 83]**

4. The sum of the rational terms in the expansion of $(\sqrt{2} + 3^{1/5})^{10}$ is **[IIT - 97]**

B. Multiple choice questions with one or more than one correct choice.

1. If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - \ldots + + (-1)^n (n+1)C_n^2\right],$$ where n is an even positive integer, is equal to **[IIT - 86]**

(A) 0 (B) $(-1)^{n/2}(n + 1)$ (C) $(-1)^n (n + 2)$ (D) $(-1)^n n$

2. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals **[IIT - 98]**

(A) $(n - 1)a_n$ (B) $n\,a_n$ (C) $\frac{na_n}{2}$ (D) none of these

C. Multiple choice questions with one correct choice.

1. The coefficient of x^4 in $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is **[IIT - 83]**

(A) $\frac{405}{256}$ (B) $\frac{504}{259}$ (C) $\frac{450}{263}$ (D) none of these

2. Given positive integers $r > 1$, $n > 2$, and the coefficients of the $(3r)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + x)^{2n}$ are equal, which, if any, of the following statements is true ? **[IIT-80]**
(A) $n = 2r$ (B) $n = 3r$ (C) $n = 2r + 1$ (D) none of these

3. The expression is a polynomial of degree $\left[x+(x^3-1)^{\frac{1}{2}}\right]^5+\left[x-(x^3-1)^{\frac{1}{2}}\right]^5$ **[IIT - 92]**

(A) 5 (B) 6 (C) 7 (D) 8

4. If n is an odd natural number, then $\sum_{r=0}^{n}\frac{(-1)^r}{{}^nC_r}$ is equal to **[IIT - 98]**

(A) 0 (B) $\frac{1}{n}$ (C) $\frac{n}{2^n}$ (D) none of these

5. If in the expansion of $(1 + x)^m (1 - x)^n$, coefficients of x and x^2 are 3 and - 6 respectively, then m is

(A) 6 (B) 9 (C) 12 (D) 24 **[IIT - 99]**

7. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of the 5^{th} and 6^{th} terms is zero. Then $\frac{a}{b}$ equals **[IIT - 2002]**

(A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$

9. Coefficient of t^{24} in $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ **[IIT - 2003]**

(A) ${}^{12}C_6 + 3$ (B) ${}^{12}C_6 + 1$ (C) ${}^{12}C_6$ (D) ${}^{12}C_6 + 2$

10. If ${}^{n-1}C_r = (k^2 - 3)\, {}^nC_{r+1}$, then k belongs to **[IIT - 2004]**

(A) $\left(-\sqrt{3}, \sqrt{3}\right)$ (B) $(-\infty, -2]$ (C) $[2, \infty)$ (D) $\left(\sqrt{3},\ 2\right]$

11. The value of $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}-...+\binom{30}{20}\binom{30}{30}$ is where $\binom{n}{r}={}^nC_r$

(A) $\binom{30}{10}$ (B) $\binom{30}{15}$ (C) $\binom{60}{30}$ (D) $\binom{31}{10}$ **[IIT – 2005]**

Binomial Theorem

IIT JEE PROBLEMS (SUBJECTIVE)

1. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, determine r. **[IIT - 79]**

2. Given that $C_1 + 2C_2x + 3C_3x^2 + \ldots\ldots\ldots + 2nC_{2n}x^{2n-1} = 2n(1 + x)^{2n-1}$, where $C_r = \dfrac{(2n)!}{r!(2n-r)!}$, prove that $C_1^2 - 2C_2^2 + 3C_3^2 \ldots\ldots\ldots - 2nC^2_{2n} = (-1)^n\, n\, C_n$. **[IIT - 79]**

3. If $(1 + x)^n = c_o + c_1x + c_2x^2 + \ldots\ldots\ldots + c_nx^n$, then show that the sum of the products of the c_i's taken two at a time, represented by $\sum\limits_{0\le i\le j\le n} c_ic_j$, is equal to $2^{2n-1} - \dfrac{(2n)!}{2(n!)^2}$. **[IIT - 83]**

4. If p be a natural number then prove that $p^{n+1} + (p + 1)^{2n-1}$ is divisible by $p^2 + p + 1$ for every positive integer n. **[IIT - 84]**

5. Prove that for every positive integer n, ${}^{n+1}C_1 + {}^{n+1}C_2\, s_1 + {}^{n+1}C_3\, s_2 + \ldots\ldots + {}^{n+1}C_{n+1}\, s_n = 2^n S_n$, where, $s_n = 1 + q + q^2 + \ldots\ldots + q^n$ and $S_n = 1 + \dfrac{q+1}{2} + \left(\dfrac{q+1}{2}\right)^2 + \ldots\ldots + \left(\dfrac{q+1}{2}\right)^n$, where $q \ne 1$. **[IIT - 84]**

6. For $0 \le r \le k \le n$, evaluate $\binom{n}{k} - \binom{n}{k-1} + \binom{n}{k-2} - \binom{n}{k-3} + \ldots\ldots\ldots\ldots\ldots + (-1)^{-r}\binom{n}{k-r}$. what happens if $n = r = k$? **[REE–84]**

7. Find the sum of the series $\sum\limits_{r=0}^{n} (-1)^r\ {}^nC_r\left[\dfrac{1}{2^r} + \dfrac{3^r}{2^{2r}} + \dfrac{7^r}{2^{3r}} + \dfrac{15^r}{2^{4r}} + \ldots\ldots\ldots \text{upto } m \text{ terms}\right]$. **[IIT - 85]**

8. Prove that $\binom{n}{1} + 4\binom{n}{2} + 9\binom{n}{3} + \ldots\ldots\ldots + k^2\binom{n}{k} + \ldots\ldots + n^2\binom{n}{n} = n(n+1)2^{n-2}$ by induction on n or otherwise. **[IIT - 86]**

9. A student is allowed to select at most n books from a collection of (2n + 1) books. If the total number of ways in which he can select at least one book is 63, find the value of n. **[IIT - 87]**

10. Let $R = (5\sqrt{5} + 1)^{2n+1}$ and $f = R - [R]$, where [ ] denotes the greatest integer function. Prove that $Rf = 4^{2n+4}$. **[IIT - 88]**

11. If $C_r = {}^nC_r$, prove that $C_0 - 2^2 C_1 + 3^2 C_2 - \ldots\ldots\ldots + (-1)^n (n + 1)^2 C_n = 0$. **[IIT - 89]**

12. Prove that $\frac{n^7}{7}+\frac{n^5}{5}+\frac{2n^3}{3}-\frac{n}{105}$ is an integer for every positive integer n. **[IIT - 90]**

13. If n is a positive integer and $C_k = {}^nC_k$, find the value of $\sum_{k=1}^{n} k^3\left(\frac{C_k}{C_{k-1}}\right)^2$. **[REE-91]**

14. Find the sum of the following series upto infinity. **[REE-91]**

$$1+\frac{\sqrt{2}-1}{2\sqrt{2}}+\frac{3-2\sqrt{2}}{12}+\frac{5\sqrt{2}-7}{24\sqrt{2}}+\frac{17-12\sqrt{2}}{80}.$$

15. Determine the value of 'x' in the expression $(x + x^t)^5$, if the third terms in the expression is 10,00,000. Where $t = \log_{10} x$. **[REE-92]**

16. Sum the following series : $9+\frac{16}{2!}+\frac{27}{3!}+\frac{42}{4!}+.........\infty$. **[REE-92]**

17. $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{i=0}^{2n} b_r (x-3)^r$ and a $K = 1$ for all $K \ge n$, then show that $b_n = {}^{2n+1}C_{n+1}$. **[IIT - 92]**

18. Find the value of 'x' for which the sixth term of $\left[\left[2^{\log(10-3^x)}\right]^{1/2}+\left[2^{(x-2)\log 3}\right]^{1/5}\right]^m$ is equal to 21 and binomial coefficients of second, third and fourth terms are the first, third and fifth terms of an arithmetic progression. [Take every where base of log as 10]. **[REE-93]**

19. Find the sum of $a\left(x^2+\frac{1}{x^2}\right)-\frac{a^2}{2}\left(x^4+\frac{1}{x^4}\right)+\frac{a^3}{3}\left(x^6+\frac{1}{x^6}\right)-.......$ and determine the values of a and x for which it is valid. **[REE-93]**

20. Prove that $\sum_{r=1}^{K} (-3)^{r-1}\, {}^{3n}C_{2r-1} = 0$, where $K = \frac{3n}{2}$ and n is an even positive integer. **[IIT - 93]**

21. Let n be the positive integer. If the coefficient of 2nd, 3nd, 4th terms in the expansion of $(1 + x)^n$ are in A.P. then find the value of n. **[IIT - 94]**

22. Let n be a positive integer and $(1 + x + x^2)^n = a_0 + a_1 x + \ldots + a_{2n}x^{2n}$

Show that $a_0^{\,2} - a_1^{\,2} + a_2^{\,2} - a_3^{\,2} + \ldots\ldots\ldots\ldots - a^2_{2n-1} + a_{2n}^{\,2} = a_n$ **[IIT - 94]**

23. Given that the 4^{th} term in the expansion of $\left(2+\frac{3x}{8}\right)^{10}$ has the maximum numerical value, find the range of values of x for which this will be true. **[REE-94]**

24. Find the sum of the infinite series $a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots$, where $a_n = (\log_e 3)^n \sum_{k=1}^{n}\frac{2k+1}{k!(n-k)!}$ for each positive integer n. **[REE-95]**

25. Let $(1 + x^2)^2 . (1 + x)^n = \sum_{k=0}^{n+4} a_k.x^k$. If a_1, a_2 and a_3 are in AP, find n. **[REE-96]**

26. In the expansion of the expression $(x + a)^{15}$, if the eleventh term is the geometric mean of the eighth and twelfth terms, which term in the expansion is the greatest ? **[REE-96]**

27. Prove that : . **[IIT - 97]**

28. Find the sum of series $\frac{3}{1!}+\frac{5}{2!}+\frac{9}{3!}+\frac{15}{4!}+\frac{23}{5!}+..........\infty$. **[REE-98]**

29. Let n be any positive integer. Prove that **[IIT - 99]**

$$\sum_{k=0}^{m}\frac{\binom{2n-k}{k}}{\binom{2n-k}{n}}\cdot\frac{(2n-4k+1)}{(2n-2k+1)}2^{n-2k}=\frac{\binom{n}{m}}{\binom{2n-2m}{n-m}}2^{n-2m}.$$

for each non negative integer $m \le n$ $\left(\text{Here }\binom{p}{q}={}^pC_q\right)$

30. For any positive integers m, n (with $n \ge m$), let $\binom{n}{m} = {}^nC_m$. Prove that **[IIT - 2000]**

$\binom{n}{m}+\binom{n-1}{m}+\binom{n-2}{m}+..........+\binom{m}{m}=\binom{n+1}{m+1}$. Hence or otherwise prove that,

$$\binom{n}{m}+2\binom{n-1}{m}+3\binom{n-2}{m}+..........+(n-m+1)\binom{m}{m}=\binom{n+2}{m+2}.$$

31. Find the largest coefficient in the expansion of $(1 + x)^n$, given that the sum of coefficients of the terms in its expansion is 4096. **[REE-2001]**

32. Find the coefficient of x^{49} in the polynomial

$\left(x-\frac{C_1}{C_0}\right)\left(x-2^2.\frac{C_2}{C_1}\right)\left(x-3^2.\frac{C_3}{C_2}\right)............\left(x-50^2.\frac{C_{50}}{C_{49}}\right)$ where $C_r = {}^{50}C_r$. **[REE-2001]**

33. Prove that $2^n\binom{n}{0}\binom{n}{k}-2^{k-1}\binom{n}{1}\binom{n-k}{k-1}+2^{k-2}\binom{n}{2}\binom{n-2}{k-2}........(-1)^k\binom{n}{k}\binom{n-k}{0}=\binom{n}{k}$

[IIT - 2003]

SET–I

1. The value of x in the expression $\left[x+x^{\log_{10}(x)}\right]^5$ if the third terms in the expansion in the expansion is 10,00,000

(A) 10 (B) 11 (C) 12 (D) none of these

2. If $\frac{T_2}{T_3}$ in the expansion of $(a+b)^n$ and $\frac{T_3}{T_4}$ in the expansion of $(a+b)^{n+3}$ are equal, then n is equal to

(A) 3 (B) 4 (C) 5 (D) 6

3. If number of terms in the expansion of $(x - 2y + 3z)^n$ are 45, then n is equal to

(A) 7 (B) 8 (C) 9 (D) none of these

4. In the binomial $(2^{1/3} + 3^{-1/3})^n$, if the ratio of the seventh term from the beginning of the expansion to the seventh term from its end is 1/6, then n is equal to

(A) 6 (B) 9 (C) 12 (D)

5. If $(1 + ax)^n = 1 + 8x + 24x^2 + \ldots\ldots\ldots\ldots$then the value of a and n is

(A) 2, 4 (B) 2, 3 (C) 3, 6 (D) 1, 2

6. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is

(A) ${}^nC_4$ (B) ${}^nC_4 + {}^nC_2$

(C) ${}^nC_4 + {}^nC_2 + {}^nC_4 . {}^nC_2$ (D) ${}^nC_4 + {}^nC_2 + {}^nC_1 . {}^nC_2$

8. The number 7^{1995} when divided by 100 leaves the remainder

(A) 43 (B) 45 (C) 50 (D) none of these

9. Number of terms in the expansion of $(1 - 4x)^{-30}$ are

(A) 29 (B) 30

(C) 31 (D) infinitely many

10. If the sum of the coefficients in the expansion of $(p^2x^2 - 2px + 1)^{51}$ vanishes, then the value of p is

(A) 2 (B) -1 (C) 1 (D) -2

11. If $(1 + x + 2x^2)^{20} = a_0 + a_1 x + a_2 x^2 + \text{-----} + a_{40} x^{40}$ then $a_1 + a_3 + a_5 + \text{--} + a_{39}$ equals

(A) $2^{19}(2^{20} – 1)$ (C) $2^{20}(2^{19} – 19)$ (C) $2^{19}(2^{20} + 21)$ (D) none of these

12. The last digit of $(3^P + 2)$, where $P = 3^{4n+2}$ is

(A) 1 (B) 9 (C) 4 (D) 5

13. If $n \in N$ and n is even, then $\frac{1}{1.(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots\ldots + \frac{1}{(n-1)!1!}$ is equal to

(A) 2^n (B) $\frac{2^{n-1}}{n!}$ (C) $2^n n!$ (D) none of these

14. If $C_0, C_1, C_2, \text{-------} C_{15}$ are the binomial coefficients in the expansion of $(1 + x)^{15}$, then

$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ------- + 15\frac{C_{15}}{C_{14}}$

15. The expression $\frac{1}{\sqrt{4x+1}}\left[\left[\frac{1+\sqrt{4x+1}}{2}\right]^7-\left[\frac{1-\sqrt{4x+1}}{2}\right]^7\right]$ is a polynomial in x of degree

(A) 7 (B) 5 (C) 4 (D) 3

16. $\sum_{r=1}^{n} r.\,{}^{2n}C_r$ is equal to

(A) $n.2^{n-1}$ (B) 2^{2n-1} (C) $2^{n-1}+1$ (D) $n.2^{2n-1}$

17. If $C_0, C_1, C_2, ...C_n$ denote the coefficients in the expansion of $(1 + x)^n$, then the value of $\sum_{r=0}^{n}(r+1)C_r$ is

(A) $n.2^n$ (B) $(n + 1)2^{n-1}$ (C) $(n + 2)2^{n-1}$ (D) $(n+2)2^{n-2}$

18. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + ...+ C_n x^n$, then $C_0 + 5C_1 + 9C_2 + ...+(4n + 1)C_n$ is equal to

(A) $n.2^n$ (B) $(n + 1)2^n$ (C) $(2n + 1)2^n$ (D) $(4n + 1)2^n$

19. The co-efficient of x^9 in the polynomial given by,
$(x + 1)(x + 2) ...... (x + 10) + (x + 2)(x + 3) .... (x + 11) + ... +$
$(x + 11)(x + 12) .... (x + 20)$ is :

(A) 5511 (B) 5151 (C) 1515 (D) 1155

20. If $(1 - x + x^2)^n = a_0 + a_1 x + a_2 x^2 + ...+ a_{2n} x^{2n}$, then $a_0 + a_2 + a_4 +.....+a_{2n}$ equals

(A) $\frac{3^n+1}{2}$ (B) $\frac{3^n-1}{2}$ (C) $\frac{1-3^n}{2}$ (D) $3^n+\frac{1}{2}$

SET–II

1. For $1 \le r \le n$, the value of ${}^{n}C_r + {}^{n-1}C_r + {}^{n-2}C_r + \dots\dots\dots + {}^{r}C_r$ is

(A) ${}^{n}C_{r+1}$ (B) ${}^{n+1}C_r$ (C) ${}^{n+1}C_{r+1}$ (D) none of these

2. The coefficient of x^{53} in the expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$ is

(A) ${}^{100}C_{47}$ (B) ${}^{100}C_{53}$ (C) $-{}^{100}C_{53}$ (D) $-{}^{100}C_{100}$

3. If n be a positive integer such that $n \ge 3$, then the value of the sum to n terms of

$1 \cdot n - \frac{(n-1)}{1!}(n-1) + \frac{(n-1)(n-2)}{2!}(n-2) - \frac{(n-1)(n-2)(n-3)}{3!}(n-3) + \dots\dots$ is

(A) 0 (B) 1 (C) - 1 (D) none of these

4. The number of integral terms in the expansion of $(5^{1/2} + 7^{1/6})^{642}$ is

(A) 106 (B) 108 (C) 103 (D) 109

5. The ratio of the coefficient of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ will be

(A) 1 : 2 (B) 2 : 1 (C) 3 : 1 (D) 1 : 3

6. The number of terms in the expansion of $(x+1)^{2n+1} - (x-1)^{2n+1}$, is

(A) 2n (B) n (C) n + 1 (D) 2n + 1

7. If the sum of the coefficients in the expansion of $\left(x^2 + \frac{2}{x^3}\right)^n$ is 243, then the term independent of x, is equal to

(A) 11 (B) $\frac{7}{2}$ (C) 15 (D) 40

8. The value of $4\,\{{}^{n}C_1 + 4 \cdot {}^{n}C_2 + 4^2 \cdot {}^{n}C_3 + \dots\dots + 4^{n-1}\}$ is

(A) 0 (B) $5^n + 1$ (C) 5^n (D) $5^n - 1$

9. If A and B respectively denote the sum of the odd terms and sum of the even terms in the expansion of $(x+y)^n$, then the value of $(x^2 - y^2)^n$, is equal to

(A) $A^2 + B^2$ (B) $A^2 - B^2$ (C) 4AB (D) $(A - B)^2$

10. The value of $\sum_{r=0}^{n} \frac{1}{(2r)!(2n-2r)!}$, is equal to

(A) $\frac{2^{2n}}{(2n)!}$ (B) $\frac{2^{2n-1}}{(2n)!}$ (C) $\frac{2^{n-1}}{n!}$ (D) $\frac{2^n}{n!}$

11. The value of the sum of the series $3C_0 - 7C_1 + 11C_2 - 15C_3 + \ldots$ upto $(n + 1)$ terms, where $C_r = {}^nC_r$, is equal to

(A) 3^n (B) 4^n (C) 0 (D) $4^n - 3^n$

12. The value of the expression $C_0C_2 + C_1C_3 + C_2C_4 + \ldots + C_{n-2}C_n$, is equal to

(A) ${}^{2n}C_{n-1}$ (B) ${}^{2n}C_{n-2}$ (C) ${}^{2n}C_n$ (D) none of the above

13. Co-efficient of α^t in the expansion of,
$(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2 + \ldots\ldots (\alpha + q)^{m-1}$, where $\alpha \neq -q$ and $p \neq q$ is

¨À¨ $\dfrac{{}^mC_t\left(p^t - q^t\right)}{p-q}$ ¨à¨ $\dfrac{{}^mC_t\left(p^{m-t} - q^{m-t}\right)}{p-q}$

¨Å¨ $\dfrac{{}^mC_t\left(p^t + q^t\right)}{p-q}$ ¨å¨ $\dfrac{{}^mC_t\left(p^{m-t} + q^{m-t}\right)}{p-q}$

14. In the expansion of $(1 + x)^n (1 + y)^n (1 + z)^n$, the sum of the co-efficients of the terms of degree 'r' is

¨À¨ ${}^{n^3}C_r$ ¨à¨ ${}^nC_{r^3}$ ¨Å¨ .ãÅø ¨å¨ ‚ ’ ,ãÅø

15. The value of the expression $\sum_{r=0}^{n}(-1)^r\left(\dfrac{{}^nC_r}{{}^{r+3}C_r}\right)$ is

(A) $\dfrac{n(n+1)}{2}$ (B) $\dfrac{n+3}{3}$ (C) $\dfrac{3}{n+3}$ (D) $\dfrac{n+2}{2}$

16. The coefficient of x^m in the expansion of $\sum_{r=0}^{n-m}(1+x)^{m+r}$, $m \leq n$, is equal to

(A) ${}^{n+1}C_{m-1}$ (B) ${}^{n+1}C_{m+1}$ (C) ${}^nC_m$ (D) n^m

17. The sum of the last n coefficients in the expansion of $(1 + x)^{2n-1}$ when expanding in ascending power of x, is equal to

(A) 2^{2n-1} (B) 2^{2n-2} (C) 2^{2n} (D) $2^{2n} - 2^n$

18. The integral part of $\left(\sqrt{3}+1\right)^{2n+1}$, is of the form

(A) $2k + 1, k \in \mathbf{I}$ (B) $(2k + 1)\dfrac{3}{2}, k \in \mathbf{I}$

(C) $2k, k \in \mathbf{I}$ (D) none of these

19. The fractional part of $\dfrac{3^{2n}}{8}$, is equal to

(A) $\dfrac{3}{8}$ (B) $\dfrac{7}{8}$ (C) $\dfrac{1}{8}$ (D) none of these.

20. If the unit digit of $13^n + 7^n - 3^n$, $n \in \mathbf{N}$, is 7, then the value of n is of the form

(A) $4k + 1, k \in I$ (B) $4k + 2, k \in I$ (C) $4k + 3, k \in I$ (D) $4k, k \in I$

SET–III

Multiple choice questions with one or more than one correct option.

1. If $|x| < 1$, then the coefficient of x^n in the expansion of $\log_e(1 + x + x^2 + \ldots)$ is

(A) $\frac{1}{n!}$ (B) $ne^{\log_e n!}$ (C) $\frac{1}{n}$ (D) $n\, e^{-\log_e n^2}$

2. In the expansion of $\left(x - \frac{\alpha}{x}\right)^n$ and $\left(x + \frac{\beta}{x^2}\right)^n$ in powers of x have one term independent of x, then n is divisible by

(A) 2 (B) 3 (C) 4 (D) 6

3. In the expansion of $(a + b + c)^{10}$

(A) total number of terms is 66
(B) coefficient of $a^8 bc$ is 90
(C) coefficient of $a^4 b^5 c^3$ is 0
(D) none of these

4. Let $(1 + x^2)^2 (a + x)^n = \sum_{k=0}^{n+4} \alpha_k x^k$. If a_1, a_2, a_3 are in A. P., then n is equal to

(A) 6 (B) 4 (C) 3 (D) 2

5. In the expansion of $\left(x + \frac{\alpha}{x^2}\right)^n$, $a \neq 0$, if no term is independent of x, then n is

(A) 10 (B) 12 (C) 16 (D) 20

Question based on write-up

☞ If 'O' be the sum of terms at odd position and 'E' that of terms at the even position in the expansion $(x + a)^n$.

6. The value of $(x + a)^n$ in terms of O, E is

(A) OE (B) O + E (C) O – E (D) none of these

7. The value of $(x - a)^n$ in terms of

(A) OE (B) O + E (C) O – E (D) none of these

8. The value of $(x^2 - a^2)^n$ is equal to

(A) $O^2 + E^2$ (B) O + E (C) $O^2 - E^2$ (D) none of these

9. The value of $(x + a)^{2n} - (x - a)^{2n}$ is equal to

(A) OE (B) 2 OE (C) 3 OE (D) 4 OE

10. The value of $(x + a)^{2n} + (x - a)^{2n}$ is equal to

(A) $(O^2 + E^2)$ (B) $2(O^2 + E^2)$ (C) $(O^2 - E^2)$ (D) $2(O^2 - E^2)$

☛ Let n be a positive integer such that $(1 + x)^n = C_0 + C_1x + C_2x^2 + ...+ C_n x_n$.

11. The value of $\sum_{r=0}^{s}\sum_{s=1}^{n} {}^nC_s . {}^sC_r$ (when $r \le s$ is)

(A) 3^n (B) $n . 3^{n-1}$ (C) $n . 3^{n-1} - 1$ (D) $3^n - 1$

12. The value of $\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)C_rC_s$, is

(A) $n . 2^{2n}$ (B) $n(n - 1)2^{n-2}$ (C) $n(n + 1)2^n$ (D) 2^{2n}

13. The value of $\sum_{0\le r}^{n}\sum_{<s\le n}(r+s)\ \ C_rC_s$, is

(A) $n[2^{2n-1}]$ (B) $n[2^{2n-1} - {}^{2n-1}C_{n-1}]$
(C) $n[2^{n-1} - {}^{2n-1}C_{n-1}]$ (D) $[2^{2n-1} - {}^{2n-1}C_{n-1}]$

14. The value of $\sum_{0\le r}\sum_{<s\le n}(r+s)\ \ (C_r+C_s)$, is

(A) $n^2 . 2^{n-1}$ (B) $n(n - 1)2^{n-2}$ (C) $n^2 . 2^{n-2}$ (D) $n^2 . 2^n$

15. The value of $\sum_{0\le r}^{n}\sum_{<s\le n}(r+s)(C_r+C_s+C_rC_s)$, is

(A) $n^2.2^n - \frac{n}{2}[2^{2n} - {}^{2n}C_n]$ (B) $n^2.2^n + \frac{n}{2}[2^{2n} - {}^{2n}C_n]$

(C) $n^2.2^n - \frac{n}{2}[2^{2n} + {}^{2n}C_n]$ (D) $n^2.2^n + \frac{n}{2}[2^n - 1]$

16. The value of $\sum_{r=0}^{n}\sum_{s=0}^{n}\sum_{t=0}^{n}\sum_{u=0}^{n}(1)$, is

(A) ${}^nC_4$ (B) ${}^{n+1}C_4$ (C) ${}^{4n+1}C_4$ (D) $(n + 1)^4$

17. The value of $\sum_{0\le r}\sum_{<s<t}\sum_{<u}\sum_{\le n}(2)$, is

(A) $2 . {}^nC_4$ (B) $2 . {}^{n+1}C_4$ (C) $2 . {}^{4n+1}C_4$ (D) $2(n +1)^4$

18. **True and False :**

(i) The ratio of the coefficient of x^{10} in $(1 - x^2)^{10}$ and the term independent of x in $\left(x-\frac{2}{x}\right)^{10}$ is 1 : 32.

(ii) The coefficient of $(r + 1)^{th}$ term in the expansion of $(1 + x)^{n+1}$ is equal to the sum of the coefficients of r^{th} and $(r + 1)^{th}$ terms in the expansion of $(1 + x)^n$.

(iii) The coefficient of x^n in the expansion of $(1 + x)^{2n}$ is double the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

19. Fill In The Blanks :

(i) The coefficient of x^5 in expansion of $(1 + x^2)^5 (1 + x)^4$ is ....

(ii) If $(1 + x + x^2)^{6n} = a_0 + a_1x + a_2x^2 + ...$, then $a_0 + a_3 + a_6 + ... =$ ....

(iii) If the coefficients of $(2r + 1)^{th}$ and $(r + 5)^{th}$ terms in the expansion of $(1 + x)^{25}$ are equal, then value of r is ....

(iv) The largest term in the expansion of $(3 + 2x)^{50}$ where $x = 1/5$, is ....

(v) If $(1 + x - 2x^2)^6 = 1 + a_1 x + a_2 x^2 + ... + a_{12} x^{12}$, then the expression $a_2 + a_4 + a_6 + ... + a_{12}$ has the value, is ....

20. Match the column

Column I	Column II
(a) If the coefficients of x^7 and x^8 in are equal, then n is	(P) 2
(b) If the coefficient of x in the expansion of $\left(x - \frac{1}{ax^2}\right)^{10}$ is –15, then the value of a, is	(Q) 01
(c) The remainder when 2^{2003} in divided by 17 is	(R) 55
(d) The last two digits of 3^{400}, are	(S) 8

LEVEL–I ANSWER

2.(i) $\frac{5}{12}$ **(ii)** 495 **3.** n = 7, 14

LEVEL–II

8. **(a)** $T_7 = \frac{7.3^{13}}{2}$ **(b)** $\frac{5}{8} < x < \frac{20}{21}$ **10.** $\frac{17}{54}$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. $(101)^{50}$ **2.** 0 **3.** a = 2, n = 4 **4.** 41

(B)

1. C **2.** C

(C)

1.	**A**	**2.**	A	**3.**	C	**4.**	A	**5.**	C
6.	D	**7.**	B	**8.**	C	**9.**	D	**10.**	D
11.	A								

IIT JEE PROBLEMS (SUBJECTIVE)

1. r = 3 **8.** $\frac{2^{mn}-1}{2^{mn+n}-2^{mn}}$ **10.** n = 3

14. $\frac{n(n+1)^2(n+2)}{(2n+1)}$ **15.** $2 - \frac{1+\sqrt{2}}{2}\ln 2$ **16.** x = 10 or $10^{-5/2}$

19. x = 2 or 0 **20.** $S = \ln\left(1+\frac{a}{x^2}+ax^2+a^2\right)$ where -1 < a < 1 and | x | > 1

22. n = 7 **24.** $x \in \left(-\frac{64}{21}, -2\right) \cup \left(2, \frac{64}{21}\right)$

25. $6\ln(27e)$ **26.** n = 3 or 4 **27.** T_8

29.	4e - 3	**32.** $^{12}C_6$		**33.** -22100	

SET–I

1.	A	**2.**	C	**3.**	B	**4.**	B	**5.**	A
6.	D	**7.**	B	**8.**	A	**9.**	D	**10.**	C
11.	A	**12.**	D	**13.**	B	**14.**	A	**15.**	D
16.	D	**17.**	C	**18.**	C	**19.**	D	**20.**	A

SET–II

1.	C	**2.**	C	**3.**	A	**4.**	B	**5.**	B
6.	C	**7.**	D	**8.**	D	**9.**	B	**10.**	B
11.	C	**12.**	B	**13.**	B	**14.**	C	**15.**	C
16.	B	**17.**	B	**18.**	C	**19.**	C	**20.**	A

SET–III

1.	CD	**2.**	AB		**3.**	ABC	**4.**	BCD	**5.**	ACD
6.	B	**7.**	C		**8.**	C	**9.**	D	**10.**	B
11.	D	**12.**	A		**13.**	B	**14.**	D	**15.**	B
16.	D	**17.**	B		**18.**	**(i)** T	**(ii)**	T	**(iii)**	T
19.	**(i)**	60	**(ii)**	3^{6n-1}	**(iii)**	4 OR 7	**(iv)**	6^{th}, 7^{th}	**(v)**	31
20.	a-R, b-P, c-S, d-Q									