Question Bank - Differential Equation

LEVEL–I

1. Find the degree and the order of the differential equations :

(i) $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2 = x\,\ell n\frac{d^2y}{dx^2}$ **(iii)** $\left(\frac{d^2y}{dx^2}\right)^3-3\left(\frac{dy}{dx}\right)^5+2y = x \quad \sin x$

2. (i) Form a differential equation for the family of curves represented by $ax^2 + by^2 = 1$, where a and b are arbitrary constants.

(ii) Obtain the differential equation of the family of circles $x^2 + y^2 + 2gx + 2fy + c = 0$; where g, f and c are arbitrary constants.

3. Solve the following differential equations :

(i) $xydx + (1 + x^2)dy = 0$ **(ii)** $(e^y + 1)\cos x\,dx + e^y \sin x\,dx = 0$

(iii) $(x^2y + x^2)dx + (y^2x – y^2)dy = 0$ **(vi)** $(x + y)(dx – dy) = dx + dy$

4. Solve the following differential equations :

(i) $\frac{dy}{dx}=(4x+y+1)^2, y(0)=1$ **(ii)** $\left(\frac{x+y-a}{x+y-b}\right)\frac{dy}{dx}=\left(\frac{x+y+a}{x+y+b}\right)$

(iii) $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$ **(iv)** $\frac{dy}{dx}-x\tan(y-x)=1$

5. Solve the following differential equations :

(i) $\frac{dy}{dx}=\frac{x^2+3y^2}{3x^2+y^2}$ **(ii)** $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

(iii) $x\frac{dy}{dx}=y-3\sqrt{y^2-x^2}$ **(iv)** $x^3dx-y^3dy=3xy(ydx-xdy)$

6. Solve the following differential equations :

(i) $(3y – 7x + 7)dx + (7y – 3x + 3)dy = 0$

(ii) $\frac{dy}{dx}=\frac{(x-1)^2+(y-2)^2\tan^{-1}\left(\frac{y-2}{x-1}\right)}{(xy-2x-y+2)\tan^{-1}\left(\frac{y-2}{x-1}\right)}$

7. Solve the following differential equations :

(i) $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$ **(ii)** $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$

(iii) $(1 + y + x^2y)dx + (x + x^3)dy = 0$ **(iv)** $(x + \tan y)\,dy = \sin 2y\,dx$

8. **Solve the following differential equations :**

(i) $x\frac{dy}{dx}+y=y^2\log x$ **(ii)** $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$

(iii) $\frac{dy}{dx}=e^{x-y}(e^x-e^y)$ **(iv)** $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$

9. It is known that the decay rate of radium is directly proportional to its quantity at each given instant. Find the law of variation of a mass of radium as a function of time if at $t=0$, the mass of the radius was m_0 and during time $t_0\alpha\%$ of the original mass of radium decay.

10. A hot pie that was cooked at a constant temperature of 325^0 F is taken directly from an oven and placed outdoors in the shade to cool on a day when the air temperature in the shade is 85^0 F. After 5 minutes in the shade, the temperature of the pie had been reduced to 250^0 F.
Determine
(a) the temperature of the pie after 5 minutes and
(b) the time required for the pie to reach 275^0 F.

11. A tank contains 100 lit. of brine made by dissolving $80\,\ell b$ of salt in water. Pure water runs into the tank at the rate of 4 lit./min., and the well–stirred mixture runs out at the rate. Find
(a) the amount of salt in the tank at any time t and
(b) the time required for half the salt to leave the tank.

12. Find the orthogonal trajectories of the family of curves $y=ce^x$.

13. Find the curve for which the square of the intercept cut off from the y-axis by any tangent on the curve is equal to the product of the coordinates of the point of tangency.

14. Let $y=f(x)$ be a curve passing through $(1,\sqrt{3})$ such that tangent at any point P on the curve lying in the first quadrant has positive slope and the tangent and the normal at the point P cut the x-axis at A and B respectively so that the mid point of AB is the origin. Find the differential equation of the curve and hence determine the equation of the curve.

15. The rate of inversion of cane sugar is proportional to its concentration. If the concentration of the of cane sugar is 1/100 at time $t = 0$ and 1/300 at time $t = 10$, find the concentration at any time t.

LEVEL–II

1. **(i)** Find the order and degree of the differential equation $y = \ell n\left\{1 = y' + \frac{(y')^2}{2!} + \frac{(y')^3}{3!} + ...\right\}$.

(ii) Find the order and degree (if defined) of the differential equation $e^{y'''} - xy'' + y = 0$.

2. **(i)** Find the differential equation of the family of circles, having their centers on the x-axis.

(ii) Find the differential equation of all non - vertical lines in a plane.

3. **Solve the differential equations :**

(i) $\sin x.\frac{dy}{dx} = y.\ell ny$ if $y = e$, when $x = \frac{\pi}{2}$

(ii) $e^{(dy/dx)} = x + 1$ given that when $x = 0$, $y = 3$

(iii) $\frac{dy}{dx} + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$

(iv) $xy^2\frac{dy}{dx} = 1 - x^2 + y^2 - x^2y^2$

4. **Solve the differential equations:**

(i) $\cos(x+y)\,dy = dx$

(ii) $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

(iii) $\sqrt{1+x+y}\frac{dy}{dx} = (x+y-1)$

(iv) $\frac{dy}{dx} + 1 = e^{x+y}$

5. **Solve the differential equations:**

(i) $2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$

(ii) $\left(2\sqrt{xy} - x\right)dy + ydx = 0$

(iii) $y^2dx + (x^2 - xy - y^2)dy = 0$

(iv) $\left[x\cos\frac{y}{x} + y\sin\frac{y}{x}\right]y - \left[y\sin\frac{y}{x} - x\cos\frac{y}{x}\right]x\frac{dy}{dx} = 0$

6. **Solve the differential equations:**

(i) $\frac{dy}{dx} = \frac{2(y+2)^2}{(x+y-1)^2}$

(ii) $\frac{dy}{dx} + \frac{\cos x(3\cos y - 7\sin x - 3)}{\sin y(3\sin x - 7\cos y + 7)} = 0$

7. Solve the differential equations:

(i) $\frac{dy}{dx}+\frac{x}{1+x^2}y=\frac{1}{2x(1+x^2)}$ **(ii)** $\frac{dy}{dx}=\frac{1}{x\cos y+\sin 2y}$

8. Solve the differential equations:

(i) $xy - \frac{dy}{dx} = y^3e^{-x^2}$. **(ii)** $(x^3 + y^2 + 2)dx + 2y\,dy = 0$.

(iii) $\frac{dy}{dx}=\frac{e^y}{x^2}-\frac{1}{x}$ **(iv)** $x^2y - x^3\frac{dy}{dx} = y^4\cos x$.

9. A depositor currently has Rs. 6000 and plans to invest it in an account that accrues interest continuously. What interest rate must the bank pay if the depositor need to have Rs.10,000 in four year?

10. A bar of iron, previously heated to 1200°C, is cooled in large bath of water maintained at a constant temperature of 50°C. The bar cools by 200° C in first minute. How much longer it take to cool second 200° C?

11. A tank contain 40 lit. of a chemical solution prepared by dissolving 80g of a soluble substance in fresh water. Fluid containing 2g of this substance per lit. runs in at the rate of 3 lit/min. and the well-stirred mixture runs out at the same rate. Find the amount of substance in the tank after 20 minutes.

12. Prove that the family of curves $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, where λ is a parameter, is self-orthogonal.

13. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Find the equation of the curve satisfying the above condition and which passes through (1, 1).

14. Find the equation of the curve which is such that area of the rectangle constructed on the abscissa of any point and the intercept of the tangent at the point on y-axis is equal to 4.

15. Solve the following differential equation :

(i) $\left(\frac{dy}{dx}\right)^2-(x+y)\frac{dy}{dx}+xy=0$ **(ii)** $\left(\frac{dy}{dx}\right)^2-\left(e^x+e^{-x}\right)\frac{dy}{dx}+1=0$

(iii) $\left(y\frac{dy}{dx}+2x\right)^2=\left(y^2+2x^2\right)\left[1+\left(\frac{dy}{dx}\right)^2\right],\ f(1)=0$

OBJECTIVE PROBLEMS ASKED IN IIT-JEE

(A) Fill in the blanks

1. A spherical rain drop evaporates at a rate proportional to its surface area at any instant t. The differential equation giving the rate of change of the radius of the rain drop is ______ **[IIT – 97]**

(B) Multiple choice questions with one or more than one correct answer :

1. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x+C_5}$ where C_1, C_2, C_3, C_4, C_5 are arbitary constants, is

(A) 5 (B) 4 (C) 3 (D) 2 **[IIT – 98]**

2. The differential equation representing the family of curves, $y^2 = 2c\left(x+\sqrt{c}\right)$, where c is a positive parameter is of : **[IIT – 99]**

(A) order 1 (B) order 2 (C) degree 3 (D) degree 4

(C) Multiple choice questions with one correct answer :

1. A solution of the differential equation, $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is : **[IIT – 99]**

(A) $y = 2$ (B) $y = 2x$ (C) $y = 2x - 4$ (D) $y = 2x^2 - 4$

2. If $x^2 + y^2 = 1$, then **[IIT - 2000]**

(A) $yy'' - 2(y')^2 + 1 = 0$ (B) $yy'' + (y')^2 + 1 = 0$

(C) $yy'' + (y')^2 - 1 = 0$ (D) $yy'' + 2(y')^2 + 1 = 0$

3. If y(t) is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then y(1) is equal to **[IIT - 2003]**

(A) –1/2 (B) e + 1/2 (C) e – 1/2 (D) 1/2

4. If $y = y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x,\ y(0) = 1$, then find $y\left(\frac{\pi}{2}\right)$ **[IIT - 2004]**

(A) 1/3 (B) 2/3 (C) –1/3 (D) 1

5. If $y = y(x)$ and it follows the relation $x\cos y + y\cos x = \pi$ then $y''(0)$ **[IIT - 2005]**

(A) 1 (B) – 1 (C) π (D) $-\pi$

6. The solution of primitive integral equation $(x^2 + y^2)dy = xy\,dx$ is $y = y(x)$. If $y(1) = 1$ and $(x_0) = e$, then x_0 is equal to **[IIT - 2005]**

(A) $\sqrt{2(e^2-1)}$ (B) $\sqrt{2(e^2+1)}$ (C) $\sqrt{3}e$ (D) $\sqrt{\frac{e^2+1}{2}}$

7. For the primitive integral equation $ydx + y^2dy = xdy;\ x \in R,\ y > 0, y = y(x),\ y(1) = 1$, then y(–3) is **[IIT - 2005]**

(A) 3 (B) 2 (C) 1 (D) 5

8. A curve passes through (1, 1) and any tangent at a point P on the curve is such that it intersect x and y axes respectively at A and B and PA : PB = 3 : 1. Then differential equation of the curve is **[IIT - 2006]**

(A) $xy' - 3y = 0$ (B) $xy' + x^2 = 0$ (C) $xy'' + y = 0$ (D) $3xy' + y = 0$

9. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(A) variable radii and a fixed centre at (0, 1)
(B) variable radii and a fixed centre at (0, –1)
(C) fixed radius 1 and variable centre along the x-axis
(D) fixed radius 1 and variable centre along the y-axis

SUBJECTIVE PROBLEMS ASKED IN IIT-JEE

1. If $(a + bx)\, e^{y/x} = x$, then prove that $x^2 \dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y\right)^2$. **[IIT - 83]**

2. A normal is drawn at a point P(x, y) of a curve. It meets the x-axis at Q. If PQ is of constant length k, then show that the differential equation describing such curves is, $y\dfrac{dy}{dx} = \pm\sqrt{k^2 - y^2}$. Find the equation of such a curve passing through (0, k). **[IIT - 94]**

3. Solve the following differential equation, $x(1 - x^2)dy + (2x^2y - y - 5x^3)dx = 0$ **[REE-94]**

4. If $y + \dfrac{d}{dx}(x\,y) = x(\sin x + \log x)$, find y(x). **[REE-95]**

5. Let y = f(x) be a curve passing through (1, 1) such that the triangle formed by the coordinate axes and the tangent at any point of the curve lies in the first quadrant and has area 2. Form the differ initial equation and determine all such possible curves. **[IIT - 95]**

6. A curve y = f(x) passes through the point P(1, 1). The normal to the curve at P is ; a(y - 1) + (x - 1) = 0. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point, determine the equation of the curve. Also obtain the area bounded by the y-axis, the curve and the normal to the curve at P. **[IIT - 96]**

7. Determine the equation of the curve passing through the origin in the form y = f(x), which satisfies the differential equation $\dfrac{dy}{dx} = \sin(10x + 6y)$. **[IIT - 96]**

8. Solve the differential $\cos^2x\dfrac{dy}{dx} - (\tan 2x)\,y = \cos^4x$, $|x| \le \dfrac{\pi}{4}$ equation, when $y\left(\dfrac{\pi}{6}\right) = \dfrac{3\sqrt{3}}{8}$.

[REE-96]

9. Solve the differential equation $y\cos\dfrac{y}{x}(xdy - ydx) + x\sin\dfrac{y}{x}(xdy + ydx) = 0$, when $y(1) = \dfrac{\pi}{2}$.

[REE-97]

10. A and B are two separate reservoirs of water, capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at the time. One hour after the water is released, the quantity of water in reservoir A is $\dfrac{3}{2}$ times the quantity of water in reservoir B. After how many hours do both the reservoirs have the same quantity of water. **[IIT - 97]**

11. Let u(x) and v(x) satisfy the differential equations $\frac{du}{dx}+p(x)u=f(x)$ and $\frac{dv}{dx}+p(x)v=g(x)$, where p(x), f(x) and g(x) are continuous functions. If $u(x_1) > v(x_1)$ for some x_1 and $f(x) > g(x)$ for all $x > x_1$, prove that any point (x, y) where $x > x_1$ does not satisfy the equations y = u(x) and y = v(x). **[IIT - 97]**

12. A curve C has the property that if the tangent drawn at any point P on C meets the coordinate axes at A and B, the P is the midpoint of AB. The curve passes through the point (1, 1). Determine the equation of the curve. **[IIT - 98]**

13. Solve the differential equation $(1 + \tan y)(dx - dy) + 2x\,dy = 0$. **[REE-98]**

14. A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x-axis. Determine the equation of the curve. **[IIT - 99]**

15. Solve the differential equation, $(x^2 + 4y^2 + 4xy)dy = (2x + 4y + 1)dx$. **[REE-99]**

16. A country has a food deficit of 10%. Its population grows continuously at a rate of 3% per year. Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person constant, prove that the country will become self-sufficient in food after 'n' years, where 'n' is the smallest integer than or equal to, $\frac{\ell n10-\ell n9}{\ell n(1.04)-0.03}$. **[IIT - 2000]**

17. A hemispherical tank of radius 2 meters is initially full of water and has an outlet of 12 cm^2 cross sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $V(t) = 0.6\sqrt{2gh(t)}$, where V(t) and h(t) are respectively the velocity of the flow through the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank. **[IIT - 2001]**

18. Let $f(x),\ x \ge 0$, be a non negative continuous function, and let $F(x)=\int_0^x f(t)\,dt,\ x\ge 0$.

If for some $c > 0$, $f(x) \le c\,F(x)$ for all $x \ge 0$, then show that $f(x) = 0$ for all $x \ge 0$. **[IIT - 2001]**

19. Find the equation of curve passes through the origin and the tangent to which at every point (x, y) has slope equal to $\frac{x^4+2xy+1}{1+x^2}$. **[REE-2001]**

20. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = k > 0). Find the time after which the cone is empty. **[IIT - 2003]**

21. If $P(1) = 0$ and $\frac{dP(x)}{dx} > P(x)$ for all $x \geq 1$ then prove that $P(x) > 0$ for all $x > 1$. **[IIT - 2003]**

22. A curve passes through (2, 0) and slope at point P(x, y) is $\frac{(x+1)^2 + (y-3)}{(x+1)}$. Find equation of curve and area between curve and x–axis in 4^{th} quadrant. **[IIT - 2004]**

23. If length of tangent at any point on the curve $y = f(x)$ intercepted between the point and the x–axis is of length 1. Find the equation of the curve. **[IIT - 2005]**

SET–I

1. Solution of $2y \sin x \frac{dy}{dx} = 2 \sin x \cos x - y^2 \cos x$, $x = \frac{\pi}{2}$, $y = 1$ is given by

(A) $y^2 = \sin x$
(B) $y = \sin^2 x$
(C) $y^2 = 1 + \cos x$
(D) none of these

2. If $\phi(x) = \int \{\phi(x)\}^{-2} dx$ and $\phi(1) = 0$ then $\phi(x) =$

(A) $\{2(x - 1)\}^{1/4}$
(B) $\{5(x - 2)\}^{1/5}$
(C) $\{3(x - 1)\}^{1/3}$
(D) none of these

3. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$ is

(A) $x\phi(y/x) = k$
(B) $\phi(y/x) = kx$
(C) $y\phi(y/x) = k$
(D) $\phi(y/x) = ky$

4. The largest value of c such that there exists a differential function h(x) for $-c < x < c$ that is a solution of $y_1 = 1 + y^2$ with $h(0) = 0$ is

(A) 2π
(B) π
(C) $\pi/2$
(D) $\pi/4$

5. If $y = f(x)$ passing through (1, 2) satisfies the differential equation $y(1 + xy)dx - xdy = 0$ then

(A) $f(x) = \frac{2x}{2 - x^2}$
(B) $f(x) = \frac{x+1}{x^2+1}$
(C) $f(x) = \frac{x-1}{4 - x^2}$
(D) $f(x) = \frac{4x}{1 - 2x^2}$

6. The integrating factor of the differential equation $\frac{dy}{dx}(x \log_e x) + y = 2 \log_e x$ is given by

(A) x
(B) e^x
(C) $\log_e x$
(D) $\log_e (\log_e x)$

7. The particular solution of the differential equation $y' + 3xy = x$ which passes through (0, 4) is

(A) $y = 1 - 11e^{-3x^2/2}$
(B) $3y = 1 + 11e^{-3x^2/2}$
(C) $3y = 1 - 11e^{-3x^2/2}$
(D) none of these

8. A differential equation associated with the primitive $y = a + b e^{5x} + c e^{-7x}$ is

(A) $y_3 + 2y_2 - y_1 = 0$
(B) $y_3 + 2y_2 - 35y_1 = 0$
(C) $4y_3 + 5y_2 - 20y_1 = 0$
(D) none of these

9. The solution of $\frac{dy}{dx} = 2^{y-x}$ is

(A) $2^x + 2^y = k$
(B) $2^x - 2.2^y = k$
(C) $2^{-x} - 2^{-y} = k$
(D) $2^{-x} + 2^{-y} = k$

10. A solution of differential equation $x\left(\frac{dy}{dx}\right)^2+(y-x)\frac{dy}{dx}-y=0$ is

(A) $(x - y + c)(xy - c) = 0$
(B) $(x + y + c)(xy - c) = 0$
(C) $(x - y + c)(2xy - c) = 0$
(D) $(y - x + c)(xy - c) = 0$

11. The solution of $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$

(A) $\sin^{-1} x - \sin^{-1} y = 0$
(B) $\sin^{-1} x + \sin^{-1} y = 0$
(C) $\sin^{-1} x = c \sin^{-1} y$
(D) $(\sin^{-1} x)(\sin^{-1} y) = c$

12. If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$ then the value of x when $y = 3$ is

(A) e^5
(B) $e^6 + 1$
(C) $\frac{e^6+9}{2}$
(D) $\ell n 6$

13. The solution of differential equation $\frac{dy}{dx}+\frac{y}{x}=\sin x$ is

(A) $x(y + \cos x) = \cos x + c$
(B) $x(y - \cos x) = \sin x + c$
(C) $x(y + \cos x) = \sin x + c$
(D) none of these

14. Solution of $x\sin\frac{y}{x}dy=\left(y\sin\frac{y}{x}-x\right)dx$ is

(A) $\cos\left(\frac{y}{x}\right)=c$
(B) $\log\left(\frac{y}{x}\right)+\log x=c$
(C) $\log\frac{y}{x}-\log x=c$
(D) none of these

15. Solution of $x\frac{dy}{dx}+y=x^2y^4$ is

(A) $x^2y^2(3 + cx) = 1$
(B) $x^2y^3(3 + cx) = 1$
(C) $x^3y^3(3 + cx) = 1$
(D) none of these

16. Solution $\sec^2 y\frac{dy}{dx}+x\tan y=x^3$ is

(A) $\tan y = x^2+ce^{-x^2}$
(B) $\tan y = x^2-2+ce^{-x^2}$
(C) $\tan y = x^2-2+ce^{-x^2/2}$
(D) none of these

17. The solution of the differential equation $ydx - x\,dy + (\log x)dx = 0$ is

(A) $y = \log x + cx$
(B) $y + 1 + \log x = cx$
(C) $y + cx = \log\frac{1}{x}$
(D) none of these

18. The differential equation $x\frac{dy}{dx} = 5(y\ln x - y\ln y)$ is

(A) homogeneous but not linear (B) linear but not homogeneous

(C) both homogeneous and linear (D) neither homogeneous nor linear

19. Solution of $\left(1+x\sqrt{x^2+y^2}\right)dx+\left(-1+\sqrt{x^2+y^2}\right)y\,dy=0$ is

(A) $x+\frac{1}{3}(x^2+y^2)^{3/2}=c$ (B) $x+\frac{y^2}{2}+\frac{1}{3}(x^2+y^2)^{3/2}=0$

(C) $x-\frac{y^2}{2}+\frac{1}{3}(x^2+y^2)^{3/2}=c$ (D) none of these

20. Solution of $(xy^4+y)dx - xdy = 0$ is

(A) $\frac{x^4}{4}+\left(\frac{x}{y}\right)^3=c$ (B) $\frac{x^4}{4}+\frac{1}{3}\left(\frac{x}{y}\right)^3=c$

(C) $\frac{x^4}{4}+3\left(\frac{x}{y}\right)^2=c$ (D) none of these

21. The differential equation $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$ has for is solution

(A) $x + e^{x/y} = c$ (B) $x + y\,e^{x/y} = c$ (C) $y\,e^{x/y} = c$ (D) none of these

22. The general solution of the differential equation $\left(\frac{1}{x^2}+\frac{3y^2}{x^4}\right)dx-\frac{2y}{x^3}dy=0$, is

(A) $x^3 + y^3 = cx^2$ (B) $x^2 + y^2 = cx^3$ (C) $x^2 + y^3 = cx^3$ (D) $x^3 + y^2 = cx^2$

23. Solution of the equation $\frac{dy}{dx}+\frac{1}{x}\tan y=\frac{1}{x^2}\tan y\sin y$ is

(A) $2x = \sin y\,(1 + 2\,cx^2)$ (B) $2x = \sin y\,(1 + cx^2)$

(C) $2x + \sin y\,(1 + cx^2) = 0$ (D) none of these

24. Solution of $(x+1)\frac{dy}{dx}+1=e^{x-y}$ is

(A) $e^y(x + 1) = c$ (B) $e^y(x + 1) = e^x + c$

(C) $e^y(x + 1) = c\,e^x$ (D) none of these

25. A particle moves in a straight line with a velocity given by $\frac{dx}{dt}=x+1$ (x is the distance described). The time taken by a particle to traverse a distance of 99 meters is

(A) $\log_{10} e$ (B) $2\log_e 10$ (C) $2\log_{10} e$ (D) $\frac{1}{2}\log_{10} e$

SET–II

1. The order of the differential equation whose general solution is given by

$y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$ is where c_1, c_2, c_3, c_4, c_5 are arbitrary constant is

(A) 2 (B) 3 (C) 4 (D) 5

2. Differential equation of a family of circle which passes through the origin and whose centre lies on the y axis is

(A) $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$ (B) $(x^2 + y^2)\frac{dy}{dx} - 2xy = 0$

(C) $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$ (D) $(x^2 + y^2)\frac{dy}{dx} + 2xy = 0$

3. The differential equation whose solution is $y^2 = 2cx + c^2$ is

(A) $\frac{dy}{dx} + xy = x$ (B) $\left(\frac{dy}{dx}\right)^2 + 2\left(\frac{dy}{dx}\right)\frac{x}{y} = 1$

(C) $\frac{dy}{dx} = xy + x$ (D) $\left(\frac{dy}{dx}\right)^2 + xy^2\left(\frac{dy}{dx}\right) = x$

4. The differential equation whose general solution is $y = Ae^x \cos x + B e^x \sin x$ is

(A) $y^2 = 2y_1 - y$ (B) $y_2 = 2(y_1 - y)$

(C) $y_2 = 2(y_1 + y)$ (D) none of these

5. A function $y = f(x)$ satisfying $f'(x) = x^{-3/2}$, $f'(4) = 2$ and $f(0) = 0$ is

(A) $f(x) = -4\sqrt{x} + 3x$ (B) $f(x) = 4\sqrt{x} + 3x$

(C) $f(x) = \frac{4}{\sqrt{x}} + 3x$ (D) $f(x) = 3x - \frac{4}{\sqrt{x}}$

6. A curve passes through the point (5, 3) and at any point (x, y) on it, the product of its slope and ordinate is equal to its abscissa. Its equation represents

(A) a parabola (B) a circle (C) a hyperbola (D) an ellipse

7. The equation of a curve whose slope at any point is thrice its abscissa and which passes through (–1, –3) is

(A) $y = x^2 - 4$ (B) $3y = 2(x^2 - 4)$

(C) $2y = 3(x^2 - 3)$ (D) none of these

8. The equation of a curve that passes through the point (1, 2) and satisfies the differential equation

$\frac{dy}{dx} = -\frac{2xy}{x^2+1}$ is

(A) $y(x^2 + 1) = 1$ (B) $y(x^2 - 1) = 1$

(C) $y(x^2 + 1) = 4$ (D) none of these

9. The degree of the differential equation of the family of the curves given by

$$\sqrt{(1+x^2)}+\sqrt{(1+y^2)}=A\left(x\sqrt{(1+y^2)}-y\sqrt{(1+x^2)}\right)$$

(A) 2 (B) 3 (C) 4 (D) none of these

10. The solution of the initial value problem $yx^2y'=1-y+x^3-x^3y,\ y(1)=0$ is

(A) $\ell n(1-y)+y=-x+\frac{x^2}{2}+\frac{1}{2}$

(B) $\ell n(1-y)+y=x-\frac{x^2}{2}-\frac{1}{2}$

(C) $\ell n(1-y)-y=-x+\frac{x^2}{2}+\frac{1}{2}$

(D) $\ell n(1-y)-y=x+\frac{x^2}{2}+\frac{1}{2}$

11. Equation of the curve whose slope at the point (x, y) is $-\frac{(x+y)}{x}$ and which passes through the point (2, 1), is

(A) $x^2 + 2xy = 8$ (B) $2y^2 + xy = 4$ (C) $y^2 + xy = 3$ (D) none of these

12. Solution of the equation $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$ is

(A) $e^y+e^x=\frac{x^3}{3}+c$ (B) $e^y-e^x=\frac{x^3}{3}+c$ (C) $e^y+e^x+\frac{x^3}{3}=c$ (D) $e^{x+y}=\frac{x^3}{3}+c$

13. Solution of the equation $\frac{dy}{dx}=1+xy+x+y$ is

(A) $(1 + y)(1 + x) = c$

(B) $\log_e(1+y)=1+x+c$

(C) $\ell n|1+y|=x+\frac{x^2}{2}+c$

(D) $\ell n|1+y|+x+\frac{x^2}{2}+c$

14. If $\frac{dy}{dx}=\frac{x+y}{x}$, then

(A) $y=x\,\ell n|x|+cx$

(B) $y=x\,\ell n|x|+c$

(C) $y=\ell n|x|+cx$

(D) $xy=\ell n|x|+cx$

15. Solution of $x^4\frac{dy}{dx}+x^3y+\operatorname{cosec} xy=0$ is

(A) $2x^2 \cos xy + cx^2 + 1 = 0$

(B) $2x^2 \cos xy = cx^2$

(C) $2x^2 \cos xy = cx^2 + 1$

(D) $x^2 \cos xy + cx^2 = 1$

16. Solution of $\cos^2 x\frac{dy}{dx}+y=\tan x$ is

(A) $y = \tan x + 1 + ce^{-\tan x}$

(B) $y = \tan x - 1 + c\,e^{-\tan x}$

(C) $y\,e^{\tan x} = \tan x\,e^{\tan x} + e^{\tan x} + c$

(D) none of these

17. Solution of $x\dfrac{dy}{dx}+y=y^2\log x$ is

(A) y (log x + cx + 1) = 1
(B) (log x + cx + 1) = y
(C) y(log x + c + x) = 1
(D) y(log x + cx + 1) = 0

18. Find the solution of differential equation $\dfrac{dy}{dx}+100=\dfrac{dy}{dx}+2$

(A) y = x + C
(B) y + x = C
(C) x = const.
(D) y = const.

19. Solution of $\dfrac{dy}{dx}=\dfrac{y\phi'(x)-y^2}{\phi(x)}$ is

(A) $y=\phi(x)+c$
(B) $y=\dfrac{\phi(x)}{x+c}$
(C) $y=(x+2)\phi(x)$
(4) none of these

20. Through any point (x, y) of a curve which passes through the origin, lines are drawn parallel to the coordinate axes. The curve, given that it divides the rectangle formed by the two lines and the axes into two areas, one of which is twice the other, represents a family of

(A) circles
(B) parabolas
(C) hyperbola
(D) straight lines

21. The equation of curve not passing through origin and having the portion of the tangent included between the coordinate axes is bisected at the point of contact is

(A) a parabola
(B) an ellipse or a straight line
(C) a circle or an ellipse
(D) a hyperbola

22. If for the differential equation $y'=\dfrac{y}{x}+\phi\left(\dfrac{x}{y}\right)$ the general solution is $y=\dfrac{x}{\log|Cx|}$ then $\phi(x/y)$ is given by

(A) $-x^2/y^2$
(B) y^2/x^2
(C) x^2/y^2
(D) $-y^2/x^2$

23. A spherical rain drop evaporates at a rate proportional to it's surface area at that instant. The radius of the drop initially is 3 mm and after one hour it is found to be 2 mm. If r(t) represents the radius of the drop at time 't' then

(A) r(t) = 3 - t
(B) $r(t) = 3 + t^3 - 2t$
(C) $r(t) = 3 + t^2 - 2t$
(D) $r(t) = 3 - t^3$

24. Let $f:R\to R$ and $g:R\to R$ be twice differentiable functions satisfying $f''(x)=g''(x)$, $2f'(1)=g'(1)=4$ and $3f(2) = g(2) = 9$. The value of f(4) - g(4) is equal to

(A) -6
(B) -16
(C) -10
(D) -8

25. Solution of the differential equation $(xy^2-e^{1/x^3})dx=x^2y\,dy$, is

(A) $\dfrac{y^2}{2x^2}+\dfrac{1}{3}e^{1/x^3}=c$
(B) $\dfrac{y^2}{2x^2}-\dfrac{1}{3}e^{1/x^3}=c$
(C) $\dfrac{x^2}{2y^2}+\dfrac{1}{3}e^{1/x^3}=c$
(D) $\dfrac{x^2}{2y^2}-\dfrac{1}{3}e^{1/x^3}=c$

SET–III

W I Read the following passage and answer the question from 1 to 4

DIFFERENTIAL EQUATION OF FIRST ORDER BUT NOT FIRST DEGREE

The most general form of a first order and higher degree differential equation is $p^n + P_1 p^{n-1} + P_2 p^{n-2} + \ldots + P_n = 0$, where $P_1, P_2, \ldots P_n$ are function of x, y and p = dy/dx. If a 1st order any degree equation can be resolved into differential equation (involving p) of first degree and 1st order, in such case we say that the equation is solvable for p.

Let their solution be $g_1(x,y,c_1) \times g_2(x, y, c_2) \times \ldots \times g_n(x, y, c_n) = 0$, (where $c_1, c_2, \ldots c_n$, are arbitrary constant) we take $c_1 = c_2 = \ldots = c_n = c$ because the differential equation of 1st order 1st degree contain only one arbitrary constant. so solution is

$g_1(x,y,c) \times g_2(x, y, c) \times \ldots \times g_n(x, y, c) = 0$

Example :

Solve $p^2(x + 2y) + 3p^2(x + y) + (y + 2x)p = 0$.

Solution:

The equations is equivalent to $p(p + 1) \{(x + 2y)p + (y + 2x)\} = 0$

First factor equated to zero gives $\frac{dy}{dx} = 0$, which has the solution $y = c$

Second factor equated to zero gives $dy + dx = 0$, which has the solution $y + x = c$

Third factor equated to zero gives $(x + 2y)dy + (y + 2x)dx = 0$

or $d\{xy + x^2 + y^2\} = 0$, which has the solution is $(y - c)(y + x - c)(xy + x^2 + y^2 - c) = 0$

1. Solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - (2x + y)\frac{dy}{dx} + x^2 + xy = 0$, is

(A) $y = \frac{x^2}{2} + c$ or $y = ce^x + x + 1$

(B) $y = \frac{x^2}{2} + c$ or $y = ce^x - x - 1$

(C) $y = x^2 - x - 1$ or $y = \frac{ce^x}{2} + c$

(D) $y = ce^x + x + 1$ or $y = -\frac{x^2}{2} + c$

2. Solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6 = 0$, is

(A) $y = 3x + c$ or $y = 2x - c$

(B) $y = -3x + c$ or $y = 2x + c$

(C) $y = 3x + c$ or $y = 2x + c$

(D) $y = -3x - c$ or $y = -2x - c$

3. Solution of the differential equation $x + y\left(\frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)(1 + xy)$ is

(A) $y = \frac{x^2 + c}{2}$ or $y = \pm\sqrt{2x + c}$

(B) $y = \frac{x^2 + c}{2}$ or $y = \pm\sqrt{2x - c}$

(C) $y = \frac{x^2 - c}{2}$ or $y = \pm\sqrt{2x + c}$

(D) $y = \frac{x^2 - c}{2}$ or $y = \pm\sqrt{2x - c}$

4. Solution of the differential equation $\frac{dy}{dx}\left(\frac{dy}{dx}+y\right)=x(x+y)$ is

(A) $y=\frac{x^2-c}{2}$ or $y=x-1-ce^{-x}$ (B) $y=\frac{x^2+c}{2}$ or $y=1+ce^{-x}-x$

(C) $y=\frac{x^2-c}{2}$ or $y=1+ce^{-x}-x$ (D) none of these

W II Read the following passage and answer the question from 5 to 8

The most general form of a first order and higher degree differential equation is $p^n + P_1 p^{n-1} + P_2 p^{n-2} + \ldots + P_n = 0$, where $P_1, P_2, \ldots P_n$ are function of x, y and p = dy/dx. If differential equation is expressible in the form $y = f(x, p)$ then

Step 1 : Differentiate w.r.t. x, we get p or $\frac{dy}{dx}=f\left(x,p,\frac{dp}{dx}\right)$.

Step 2: Solving this we obtain $\phi(x,p,c)=0$.

Step 3: The solution of differential equation is obtained by eliminating p.

Example :

Solve $y = 2px + p^4x^2$.

Solution:

On differentiation w.r.t. x

$$p=2p+2x\frac{dp}{dx}+2xp^4+4p^3x^2\frac{dp}{dx}$$

or $$0=p(1+2xp^3)+2x\frac{dp}{dx}(1+2p^3x)$$

or $$0=\left(p+2x\frac{dp}{dx}\right)(1+2xp^3)$$

Therefore $p+2x\frac{dp}{dx}=0*$

or $$\frac{2dp}{p}=-\frac{dx}{x}$$

On integration $2\log p+\log x=\text{constant}$

or $p^2x = c$

or $$p=\sqrt{\frac{c}{x}}$$

Substituting this value in the given equation, we get $y=2\sqrt{ex}+c^2$

5. Solution of the differential equation $y = px + p^2$, $p = \left(\frac{dy}{dx}\right)$ is

(A) $y = cx + c^2$ or $y = -\frac{x^2}{4}$

(B) $y = cx + c^2$ or $y = \frac{x^2}{4}$

(C) $y = c^2 - cx$ or $y = -x^2$

(D) none of these

6. Solution of the differential equation $y = \frac{dy}{dx}\tan\frac{dy}{dx} + \log\cos\frac{dy}{dx}$ is

(A) $x = \tan\frac{dy}{dx} + c$ or $y = \frac{dy}{dx}\tan\frac{dy}{dx} + \log\cos\frac{dy}{dx}$

(B) $x = \frac{dy}{dx}\tan\frac{dy}{dx} + \log\cos\frac{dy}{dx}$ or $y = \tan\frac{dy}{dx} + c$

(C) $x = \frac{dy}{dx}\tan\frac{dy}{dx} - \log\cos\frac{dy}{dx}$ or $y = \tan x + c$

(D) none of these

7. Solution of the differential equation $y = 2x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$ is

(A) $(xy + c)^2 = 4(x^2 + y)(y^2 - cx)$

(B) $(xy - c)^2 = 4(x^2 - y)(y^2 - cx)$

(C) $(xy + c)^2 = 4(x^2 - y)(y^2 + cx)$

(D) $(xy - c)^2 = 4(x^2 + y)(y^2 + cx)$

8. Solution of the differential equation $y = 2x\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2$ is

(A) $y = 2c\sqrt{x} - c^2$

(B) $y = 2c\sqrt{x} + c^2$

(C) $y = -2c^2\sqrt{x} + c$

(D) none of these

WIII Read the following passage and answer the question from 9 to 12

The most general form of a first order and higher degree differential equation is $p^n + P_1 p^{n-1} + P_2 p^{n-2} + \dots + P_n = 0$, where $P_1, P_2, \dots P_n$ are function of x, y and p = dy/dx. If differential equation is expressible in the form $y = f(y, p)$, then

Step 1 : Differentiate w.r.t. y, we get p or $\frac{dx}{dy} = f\left(y, p, \frac{dp}{dy}\right)$.

Step 2: Solving this we obtain $\phi(x, p, c) = 0$.

Step 3: The solution of differential equation is obtained by eliminating p.

Example :

Solve $x = y + a\log p$.

Solution:

Differentiating w.r.t. y

$$\frac{1}{p} = 1 + \frac{a}{p}\frac{dp}{dy}$$

or $$(1 - p) = a\frac{dp}{dy}$$

or $$dy = a\frac{dp}{1-p}$$

On integration $y = c - a\log(1 - p)$

This and the equation $x = c + a\log\left(\frac{p}{1-p}\right)$.

9. Solution of the differential equation $p^2y + 2px + y = 0$, where $p = \left(\frac{dy}{dx}\right)$ is

(A) $y = cp^{\frac{1}{3}}\left(p^2+3\right)^{-\frac{2}{3}}$ (B) $y = cp^{\frac{1}{3}}\left(p^2-3\right)^{-\frac{2}{3}}$

(C) $y = cp^{\frac{1}{3}}\left(p^2+3\right)^{\frac{2}{3}}$ (D) none of these

10. Solution of the differential equation $x = y + p^2$, where $p = \left(\frac{dy}{dx}\right)$ is

(A) $c - \{p^2 + 2p + 2\log(p - 1)\}$, $x = c - \{2p + 2\log(p - 1)\}$
(B) $c + \{p^2 + 2p + 2\log(p - 1)\}$, $x = c + \{2p + 2\log(p - 1)\}$
(C) $c + \{p^2 - 2p - 2\log(p - 1)\}$, $x = c + \{2p + 2\log(p - 1)\}$
(D) $c + \{p^2 + 2p + 2\log(p + 1)\}$, $x = c + \{2p + 2\log(p - 1)\}$

11. Solution of the differential equation $y^2\log y = xyp + p^2$, where $p = \left(\frac{dy}{dx}\right)$ is

(A) $\log y = cx - c^3$ (B) $\log y = cx + c^3$
(C) $\log y = c^3x + c$ (D) none of these

12. Solution of the differential equation $x + py = p^3$, where $p = \left(\frac{dy}{dx}\right)$ is

(A) $y = \frac{c}{\sqrt{p^2+1}} + p^2 + 2$ (B) $y = \frac{c}{\sqrt{p^2+1}} - p^2 + 2$

(C) $y = \frac{c}{\sqrt{p^2+1}} + p^2 - 2$ (D) none of these

W IV Read the following passage and answer the question from 13 to 16

The most general form of a first order and higher degree differential equation is $p^n + P_1 p^{n-1} + P_2 p^{n-2} + \ldots + P_n = 0$, where $P_1, P_2, \ldots P_n$ are function of x, y and p = dy/dx. If the given differential equation is of the form $y = px + f(p)$

Step 1 : Differentiate w.r.t. x, we get $\frac{dp}{dx}\{x + f'(p)\} = 0$

Step 2: Either $\frac{dp}{dx} = 0$ or $x + f'(p) = 0$

Now, $\frac{dp}{dx} = 0 \Rightarrow p = C$. Putting p = C in (i) we obtain y = Cx + f(C)]

The above solution containing one arbitrary constant is the general solution of the given differential equation. If we consider $x + f'(p) = 0$ and if we eliminate p between this relation and the given differential equation, we will obtain a relation between x and y containing no arbitrary constant. This relation will satisfy the given differential equation. So this is a solution of the given differential equation.

Remark:

It cannot be obtained by giving a particular value to the arbitrary constant in the general solution. Also, it does not contain any arbitrary constant so it is neither a general solution nor a particular solution. Such a solution is called singular solution.

Example :

Solve $xy(y - px) = x + py$.

Solution:

Put $x^2 = U, y^2 = V$.

Therefore $p = \sqrt{\frac{U}{V}}\frac{DV}{dU} = \sqrt{\frac{U}{V}}p$

Therefore, the given equation reduces to

$$\sqrt{UV}\left\{\sqrt{V} - \sqrt{\frac{U}{V}}.\sqrt{U}.P\right\} = \sqrt{U} + \sqrt{V}.\sqrt{\frac{U}{V}}p$$

or $V = UP + (1 + P)$

Its solution

$$V = cU + (1 + c)$$

Therefore, the solution of the given equation is

$$y^2 = cx^2 + (1 + c)$$

13. Solution of the differential equation yp^2-2xp+y, where $p = \left(\frac{dy}{dx}\right)$ is

(A) $y^2 = cx - \frac{c^2}{2}$

(B) $y^2 = cx - \frac{c^2}{2}$

(C) $y^2 = cx + \frac{c^2}{4}$

(D) $y^2 = cx - \frac{c^2}{4}$

14. Solution of the differential equation $y = xp + a/p$, where $p = \left(\frac{dy}{dx}\right)$ is

(A) $y = cx - \frac{a}{c}$ (B) $y = cx + \frac{a}{c}$

(C) $y = -cx - \frac{a}{c}$ (D) none of these

15. Solution of the differential equation $y = 3xp + 6y^2p^2$, where $p = \left(\frac{dy}{dx}\right)$ is

(A) $y^2 = 3cx - 6c^2$ (B) $y^2 = 3cx + 6c^2$

(C) $y^2 = 3cx + 6c$ (D) none of these

16. Solution of the differential equation $x^2(y - px) = yp^2$, where $p = \left(\frac{dy}{dx}\right)$ is

(A) $y^2 = c^2 - cx^2$ (B) $y^2 = c - cx^2$

(C) $y^2 = c^2 + cx^2$ (D) none of these

V. Read the following passage and answer the question 17 to 20

FALLING BODY PROBLEMS

Consider a vertically falling body of mass m that is being influenced only by gravity g and an air resistance that is proportional to the velocity of the body. Assume that both gravity and mass remain constant and, for convenience, choose the downward direction as the positive direction.

Newton's second law of motion : The net force acting on a body is equal to the time rate of change of the momentum of the body ; or, for constant mass, $F = m\frac{dv}{dt}$ **....(iii)**

where F is the net force on the body and v is the velocity of the body, both at time t.

For the problem at hand, there are two forces acting on the body : (1) the force due to gravity given by the weight w of the body, which equals mg, and (ii) the force due to air resistance given by –kv, where $k \geq 0$ is a constant of proportionality. The minus sign is required because this force opposes the velocity ; that is, it acts the upward, or negative direction. The net force F on the body is, therefore, F = mg – kv, substituting this result into equation (iii)

$$mg - kv = m\frac{dv}{dt} \quad \text{or} \quad \frac{dv}{dt} + \frac{k}{m}v = g \quad \text{........(i)}$$

as the equation of motion for the body. If air resistance is negligible or nonexistent, then k = 0 and equation (i) simplifies to

$$\frac{dv}{dt} = g \quad \text{........(ii)}$$

When k > 0, the limiting velocity v_1 is defined by

$$v_1 = \frac{mg}{k} \quad \text{.......(iii)}$$

Caution : Equations (i), (ii) and (iii) are valid only if the given conditions are satisfied. These equations are not valid if, for example, air resistance is not proportional to velocity but to the velocity squared, or if the upward direction is taken to be the positive direction.

Example :

A body of mass 2 slugs is dropped with no initial velocity and encounters an air resistance that is proportional to the square of its velocity. Find an expression for the velocity of the body at any time t.

Solution:

The force due to air resistance is $-kv^2$, so that Newton's second law of motion becomes

$$m\frac{dv}{dt} = mg - kv^2 \qquad \text{or} \qquad 2\frac{dv}{dt} = 64 - kv^2$$

Rewriting this equation in differential form, we have $\dfrac{2}{64 - kv^2}dv - dt = 0$ ....(i)

which is separable, by partial fractions, $\dfrac{2}{64 - kv^2} = \dfrac{2}{(8-\sqrt{k}v)(8+\sqrt{k}\,v)}$

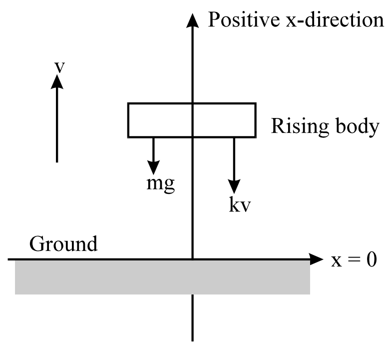
Hence (i) can be rewritten as $\dfrac{1}{8}\left(\dfrac{1}{8-\sqrt{k}v} + \dfrac{1}{8+\sqrt{k}v}\right)dv - dt = 0$

This last equation has as its solution $\dfrac{1}{8}\int\left(\dfrac{1}{8-\sqrt{k}v} + \dfrac{1}{8+\sqrt{k}v}\right)dv - \int dt = c$

Which can be rewritten as $\ell n\left|\dfrac{8+\sqrt{k}v}{8-\sqrt{k}v}\right| = 8\sqrt{k}t + 8\sqrt{k}c$

At t = 0, we are given that v = 0. This implies $c_1 = 1$, and the velocity is given by $\dfrac{8+\sqrt{k}v}{8-\sqrt{k}v} = e^{8\sqrt{k}t}$

A body of mass m is thrown vertically into the air with an initial velocity v_0. If the body encounters an air resistance proportional to its velocity



17. The equation of motion in the coordinate system in figure, is

(A) $\dfrac{dv}{dt} + \dfrac{k}{m}v = g$ (B) $\dfrac{dv}{dt} + \dfrac{k}{m}v = -g$

(C) $\dfrac{dv}{dt} - \dfrac{k}{m}v = -g$ (D) none of these

18. The expression for the velocity of the body any time t is

(A) $v = \left(v_0 - \frac{mg}{k}\right)e^{-(k/m)t} + \frac{mg}{k}$

(B) $v = \left(v_0 - \frac{mg}{k}\right)e^{-(k/m)t} - \frac{mg}{k}$

(C) $v = \left(v_0 + \frac{mg}{k}\right)e^{-(k/m)t} - \frac{mg}{k}$

(D) none of these

19. The time at which the body reaches its maximum height is

(A) $\frac{m}{k}\ell n\left(1 - \frac{v_0 k}{mg}\right)$

(B) $\frac{m}{k}\ell n\left(1 + \frac{v_0 k}{mg}\right)$

(C) $-\frac{m}{k}\ell n\left(1 + \frac{v_0 k}{mg}\right)$

(D) none of these

20. The velocity of the body at t = 1, is

(A) $\left(v_0 + \frac{mg}{k}\right)e^{-\left(\frac{k}{m}\right)} - \frac{mg}{k}$

(B) $\left(v_0 + \frac{mg}{k}\right)e^{-\left(\frac{k}{m}\right)} + \frac{mg}{k}$

(C) $\left(v_0 - \frac{mg}{k}\right)e^{-\left(\frac{k}{m}\right)} - \frac{mg}{k}$

(D) none of these

LEVEL–I ANSWER–KEY

1. **(i)** order = 2, degree = not defined **(iii)** order = 2, degree = 3

2. **(i)** $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$ **(ii)** $\left[1+(y')^2\right].y'''-3y'(y'')^2=0$

3. **(i)** $y^2(1+x^2)=c$ **(ii)** $(e^y+1)\sin x=c$

(iii) $(x+1)^2+(y-1)^2+2\log(x-1)(y+1)=c$ **(iv)** $(x-y+c)=\log(x+y)$

4. **(i)** $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right)=x+c$ **(ii)** $y+\left(\frac{b-a}{2}\right)\log\left|(x+y)^2-ab\right|=x+c$

(iii) $\ell n\left[1+\tan\frac{x+y}{2}\right]=x+c$ **(iv)** $\log\left|\sin(y-x)\right|=\frac{x^2}{2}+c$

5. **(i)** $\frac{2xy}{(x-y)^2}-\log(x-y)=c$ **(ii)** $x^2-y^2=(x^2+y^2)^2c^2$

(iii) $x^2\left(y+\sqrt{y^2-x^2}\right)=C$ **(iv)** $(x^2+y^2)^2=c(y^2-x^2)$

6. **(i)** $(y-x+1)^2(y+x-1)^5=c$

(ii) $[(x-1)^2+(y-2)^2]\tan^{-1}\left(\frac{y-2}{x-1}\right)=(x-1)(y-2)+2(x-1)^2\ln c(x-1)$

7. **(i)** $y=\tan^{-1}x-1+ce^{-\tan^{-1}x}$ **(ii)** $2ye^{-\tan^{-1}x}=e^{2\tan^{-1}x}+c$

(iii) $xy=c-\arctan x$ **(iv)** $x\sqrt{\cot y}=c+\sqrt{\tan y}$

8. **(i)** $\frac{1}{xy}=\frac{(\log x+1)}{x}+c$ **(ii)** $e^{x^2}\tan y=\frac{1}{2}(x^2-1)e^{x^2}+c$

(iii) $e^y=ce^{-e^x}+e^x-1$ **(iv)** $\frac{1}{\log y}.\frac{1}{x}=\frac{1}{2x^2}+c$

9. $m = m_0 e^{-kt}$ where $k = -\frac{1}{t_0}\ell n\left(1-\frac{\alpha}{100}\right)$

10. **(a)** 138.6^oF **(b)** 3.12 min

11. **(a)** $Q = 80e^{-0.04t}$ **(b)** 17.3 min

12. $y^2 = -2x + k$

13. $x = ce^{\pm 2\sqrt{\frac{y}{x}}}$

15. $\left(\frac{1}{100}\right)3^{-t/10}$

LEVEL–II

1. **(i)** order = 1, degree = 1 **(ii)** order = 3, degree not defined

2. **(i)** $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$ **(ii)** $\frac{d^2y}{dx^2}=0$

3. **(i)** $y = e^{\tan(x/2)}$ **(ii)** $y = (x+1).\,\ell n(x+1)-x+3$

(iii) $\ell n\left|\tan\frac{y}{4}\right| = c-2\sin\frac{x}{2}$ **(iv)** $y-\tan^{-1}y = \log x-\frac{x^2}{2}+c$

4. **(i)** $-\operatorname{cosec}(x+y)+\cot(x+y)+y=c$ **(ii)** $x = \tan(x+y)-\sec(x+y)+c$

(iii) $2\sqrt{x+y+1}+\frac{2}{3}\log\left(\sqrt{x+y+1}-1\right)-\frac{8}{3}\log\left(\sqrt{x+y+1}+2\right)=x+c$

(iv) $(x+c)e^{x+y}+1$

5. **(i)** $xy^2 = c(x-y)^2$ **(ii)** $\log y+\sqrt{\frac{x}{y}}=c$

(iii) $(x-y)y^2 = c(x+y)$ **(iv)** $xy\cos\frac{y}{x}=c$

6. **(i)** $e^{-2\tan^{-1}\frac{y+2}{x-3}} = c.(y+2)$ **(ii)** $(\cos y - \sin x - 1)^2(\cos y + \sin x - 1)^5 = c$

7. **(i)** $y\sqrt{1+x^2} = c+\frac{1}{2}\ell n\left[\tan\frac{1}{2}\operatorname{arc}\tan x\right]$ Another form is $y\sqrt{1+x^2} = c+\frac{1}{2}\ell n\frac{\sqrt{1+x^2}-1}{x}$

(ii) $x = ce^{\sin y}-2(1+\sin y)$

8. **(i)** $\frac{1}{y^2}e^{x^2} = 2x+c$ **(ii)** $y^2 = 3x^2 - 6x - x^3 + ce^{-x} + 4$

(iii) $cx^2 + 2xe^{-y} = 1$ **(iv)** $x^3y^3 = 3\sin x + c$

9. 12.78 % **10.** an additional 1.24 min. **11.** 80 g

13. $x^2 + y^2 - 2x = 0$ **14.** $y = \pm\frac{2}{x} + cx$

15. **(i)** $y = ce^x$; $y = c + x^2/2$ **(ii)** $(y - e^x - c)(y + e^{-x} - c) = 0$

(iii) $\sqrt{2}\, x^{\pm\frac{1}{\sqrt{2}}} = \frac{y}{x} + \sqrt{\frac{y^2 + 2x^2}{x^2}}$

OBJECTIVE PROBLEMS ASKED IN IIT-JEE

(A)

1. $\frac{dr}{dt} = -k$

(B)

1. C **2.** A, C

(C)

1. C **2.** B **3.** A **4.** A

5. C **6.** C **7.** A **8.** D **9.** C

SUBJECTIVE PROBLEMS ASKED IN IIT-JEE

2. $x^2 + y^2 = k^2$ **3.** $y - 5x = cx\sqrt{1 - x^2}$

4. $y \,.\, x^2 = -x^2 \,.\, \cos x + 2x \,.\, \sin x + 2\cos x + \ell nx.\frac{x^3}{3} - \frac{x^2}{9} + c$

5. $x + y = 2$ and $xy = 1$, $x, y > 0$ **6.** $e^{a(x-1)}, \frac{1}{a}\left[a - \frac{1}{2} + e^{-a}\right]$ sq. unit

7. $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4 - 3\tan 4x}\right) - \frac{5x}{3}$ **8.** $y = \frac{1}{2}\tan 2x.\cos^2 x$

9. $xy\sin\frac{y}{x} = \frac{\pi}{2}$ **10.** $\frac{\log 2}{\log(4/3)}$ hr **12.** $xy = 1$

13. $x\,e^{y}(\cos y + \sin y) = e^{y}\sin y + C$ **14.** $x^2 + y^2 - 2x = 0$

15. $y = \ell n\left((x+2y)^2 + 4(x+22y) + 2\right) - \dfrac{3}{2\sqrt{2}}\,\ell n\left(\dfrac{x+2y+2-\sqrt{2}}{x+2y+2+\sqrt{2}}\right) + c$

17. $\dfrac{14\pi}{27\sqrt{g}}(10)^5$ units **19.** $y = (x - 2\tan^{-1}x)(1 + x^2)$ **20.** $\dfrac{H}{K}$

22. $\dfrac{4}{3}$ sq. units **23.** $\log\left|\dfrac{1-\sqrt{1-y^2}}{y}\right| + \sqrt{1-y^2} = \pm x + c$

SET–I

1. A	**2.** C	**3.** B	**4.** C	**5.** A
6. C	**7.** B	**8.** B	**9.** C	**10.** A
11. B	**12.** C	**13.** C	**14.** C	**15.** B
16. C	**17.** B	**18.** B	**19.** C	**20.** B
21. B	**22.** B	**23.** A	**24.** B	**25.** D

SET–II

1. B	**2.** C	**3.** B	**4.** B	**5.** A
6. C	**7.** C	**8.** D	**9.** D	**10.** C
11. A	**12.** C	**13.** C	**14.** C	**15.** C
16. C	**17.** A	**18.** B	**19.** C	**20.** B
21. D	**22.** D	**23.** D	**24.** A	**25.** C

SET–III

1. B	**2.** C	**3.** B	**4.** B	**5.** A
6. A	**7.** A	**8.** B	**9.** A	**10.** A
11. B	**12.** C	**13.** D	**14.** B	**15.** B
16. C	**17.** B	**18.** C	**19.** B	**20.** A