

Electric Charges And Fields

Basic Properties of Electric Charges

- The total electric charge on an object is equal to the algebraic sum of all the electric charges distributed on the different parts of the object
- The total charge of an isolated system remains constant with time.
- All observable charges are always some integral multiple of elementary charge, e ($= \pm 1.6 \times 10^{-19} \text{ C}$)

Coulomb's Law

- Two point charges attract or repel each other with a force which is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.
- $F = K \frac{q_1 q_2}{r^2}$, $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$

Principle of Superposition

- It is based on the property that the forces with which two charges attract or repel each other are not affected by the presence of a third *or more* additional charges.
- The total force on a given charge due to number of charges is equal to the vector sum of the individual forces exerted on the given charge by all the other charges.

Electric Field

- It is the space around a charge, in which any other charge experiences an electrostatic force.

Electric Field Intensity

- The electric field intensity at a point due to a source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.
- Electric field due to a point charge at distance r from it is $E = \frac{q}{4\pi\epsilon_0 r^2}$
- Electric field due to a number of charges is found by adding the individual electric fields vectorially.

Electric Field Lines

- An electric line of force is the path along which a unit positive charge would move, if it is free to do so.

- Properties of electric field lines
 - They are continuous curves without any breaks
 - They cannot cross each other.
 - They cannot form closed loops.

Continuous Charge Distribution

- **Linear charge density** – When charge is distributed along a line then charge density is given by λ .
 - $\lambda = \frac{q}{L}$
- **Surface charge density** – When charge is distributed along a surface, the charge density is given by σ .

- $\sigma = \frac{q}{A}$

- **Volume charge density** – When charge is distributed along a volume, the charge density is given by δ .

$$\delta = \frac{q}{V}$$

Electric Dipole

- System of two equal and opposite charges separated by a certain small distance.

Electric Dipole Moment

- It is a vector quantity, with magnitude equal to the product of either of the charges and the length of the electric dipole and direction from the negative charge to the positive charge.

$$\vec{p} = q(2\vec{a})$$

Electric Field on Axial Line of an Electric Dipole

$$E = \frac{\vec{2p}}{4\pi\epsilon_0 r^3}$$

Electric Field for Points on the Equatorial Plane

$$E = \frac{-\vec{p}}{4\pi\epsilon_0 r^3}$$

Dipole in a Uniform External Field

In a uniform electric field E , a dipole experiences a torque t , due to two equal and unlike parallel forces acting on dipole

$\tau = \text{Force} \times \text{Perpendicular distance between the two forces}$

$$\tau = pE \sin\theta$$

$$\therefore \vec{\tau} = \vec{p} \times \vec{E}$$

Electric flux

- It is the total number of electric field lines of force crossing the unit surface area in a direction normal to the surface.
- The flux Φ of an electric field E , through a small area element ds is given by

$$\phi = \int_s \vec{E} \cdot \vec{ds} = \int_s \vec{E}_n ds$$

Gauss's law

- It states that the total electric flux through a closed surface enclosing a charge q is equal to $\frac{1}{\epsilon_0}$ times the magnitude of the charge enclosed.
- $\phi = \frac{q}{\epsilon_0}$
- $\phi = \oint_s \vec{E} \cdot \vec{ds}$

Applications of Gauss's Law

- Electric field intensity due to an infinitely long straight wire of linear charge density λ at a point which is at a perpendicular distance r from the wire is given by $\frac{\lambda}{2\pi\epsilon_0 r} \hat{n}$, where \hat{n} is the radial unit vector in the plane normal to the wire passing
- Electric field intensity due to a uniformly charged infinite plane sheet of surface charge density s is given by $\frac{\sigma}{2\epsilon_0} \hat{n}$, where \hat{n} is a unit vector normal to the plane, acting outward on either side.
- Electric field intensity due to a total charge q distributed along its surface is given by $\frac{q}{4\pi\epsilon_0 r^2} \left(r \geq R \right), 0 \left(r < R \right)$

Here, r is the distance of the point from the centre of the shell and R is the radius of the shell.

- Mechanical force per unit area of a charged conductor, $f = \frac{1}{2} \epsilon E^2$.
- Energy density per unit volume of a charged conductor, $\frac{du}{dv} = \frac{1}{2} \epsilon E^2$.

