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Senior School Certificate Examination

March 2016

Marking Scheme — Mathematics 65/2/1/F, 65/2/2/F, 65/2/3/F

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/2/1/F

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. 1×1 — 1

2. Expanding we get

$x^3 = -8 \Rightarrow x = -2$ — $\frac{1}{2}+\frac{1}{2}$

3. $P = \frac{1}{2}(A + A') \qquad \therefore \; P = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$ — $\frac{1}{2}+\frac{1}{2}$

4. $(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = 0$ — $\frac{1}{2}$

$\Rightarrow \; |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \; \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}$ — $\frac{1}{2}$

5. $a^2 b^2 \sin^2\theta + a^2 b^2 \cos^2\theta = 400$ — $\frac{1}{2}$

$\Rightarrow \; |\vec{b}| = 4$ — $\frac{1}{2}$

6. $\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 5\sqrt{3}$ or $x + y + z = 15$ $\quad \left[\frac{1}{2} \text{ mark for dc's of normal}\right]$ — 1

SECTION B

7. $\text{LHS} = \cot^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$ — 1+1

$= \cot^{-1}\left(\cot\frac{x}{2}\right)$ — 1

$= \frac{x}{2} = \text{RHS}$ — 1

OR

$\tan^{-1}\left[\frac{\frac{x-2}{x-1}+\frac{x+2}{x+1}}{1-\frac{x-2}{x-1}\cdot\frac{x+2}{x+1}}\right] = \frac{\pi}{4}$ — $1\frac{1}{2}$

$\Rightarrow \; \frac{2x^2-4}{3} = \tan\frac{\pi}{4}$ — $1\frac{1}{2}$

$\Rightarrow \; x = \pm\sqrt{\frac{7}{2}}$ — 1

8. Let each poor child pay ₹ x per month and each rich child pay ₹ y per month.

$\therefore \quad 20x + 5y = 9000$ $\frac{1}{2}$

$5x + 25y = 26000$ $\frac{1}{2}$

In matrix form,

$$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$ 1

$$AX = B \Rightarrow X = A^{-1}B$$

$$A^{-1} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$\Rightarrow \quad x = 200, y = 1000$ 1

Value: Compassion or any relevant value 1

9. $f'_{1-} = 2x + 3 = 5$

$f'_{1+} = b$

$f'_{1-} = f'_{1+} \Rightarrow \boxed{b = 5}$ 1+1

$$\lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$$

$\Rightarrow \quad 4 + a = b + 2$ 1

$\Rightarrow \quad \boxed{a = 3}$ 1

10. Let $\quad u = \tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$

Put $\quad x = \tan\theta \Rightarrow \theta = \tan^{-1}x$ $\frac{1}{2}$

$$\therefore \quad u = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$ 1

$$\Rightarrow \quad \frac{du}{dx} = \frac{1}{2(1+x^2)}$$ 1

$$v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$= 2\tan^{-1}x$$

$$\Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2}$$ 1

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{4}$$ $\frac{1}{2}$

OR

$$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t$$ $\frac{1}{2}$

$$y = \sin pt \Rightarrow \frac{dy}{dt} = p\cos pt$$ $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{p\cos pt}{\cos t}$$ 1

$$\frac{d^2y}{dx^2} = \frac{\cos t\,(-p^2 \sin pt) - p\cos pt\,(-\sin t)}{\cos^2 t}\cdot\frac{dt}{dx}$$

$$= \frac{-p^2 \sin pt \cos t + p\cos pt \sin t}{\cos^3 t}$$ 1

Now $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$ $\left[\text{Substituting values of } y, \frac{dy}{dx} \text{ \& } \frac{d^2y}{dx^2}\right]$ 1

11. Eqn of given curves

$y^2 = 4ax$ and $x^2 = 4by$

Their point of intersections are (0, 0) and $\left(4a^{1/3}b^{2/3}, 4a^{2/3}b^{1/3}\right)$ 1

$$y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y}, \text{ slope} = \frac{a^{1/3}}{2b^{1/3}} \quad \text{...(i)}$$ 1

$$x^2 = 4by \Rightarrow \frac{dy}{dx} = \frac{x}{2b}, \text{ slope} = \frac{2a^{1/3}}{b^{1/3}} \quad \text{...(ii)}$$ 1

At (0, 0), angle between two curves is 90°

or

Acute angle θ between (i) and (ii) is

$$\theta = \tan^{-1}\left\{\frac{3}{2}\left(\frac{a^{1/3}b^{1/3}}{a^{2/3}+b^{2/3}}\right)\right\}$$ 1

12. $$I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin\alpha\sin(\pi - x)}\,dx$$ 1

$$2I = \pi\int_0^{\pi} \frac{dx}{1+\sin\alpha\sin x}$$

$$= 2\pi\int_0^{\pi/2} \frac{dx}{1+\sin\alpha\sin x}$$

$$\Rightarrow I = \pi\int_0^{\pi/2} \frac{dx}{1+\sin\alpha\,\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$ 1

$$I = \pi \int_0^1 \frac{2dt}{1+t^2+2t\sin\alpha} \qquad \text{Put } \tan\frac{x}{2} = t$$ $\frac{1}{2}$

$$\Rightarrow \; I = 2\pi \int_0^1 \frac{dt}{(t+\sin\alpha)^2 + \cos^2\alpha}$$ 1

$$= \frac{2\pi}{\cos\alpha}\left[\tan^{-1}\left(\frac{t+\sin\alpha}{\cos\alpha}\right)\right]_0^1$$

$$\Rightarrow \; I = \frac{\pi}{\cos\alpha}\left(\frac{\pi}{2} - \alpha\right)$$ $\frac{1}{2}$

13. $I = \int (2x+5)\sqrt{10-4x-3x^2}\,dx$

$$= -\frac{1}{3}\int (-4-6x)\sqrt{10-4x-3x^2}\,dx + \frac{11}{3}\int \sqrt{10-4x-3x^2}\,dx$$ 1

$$= -\frac{2}{9}(10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3}\int \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x-\frac{2}{3}\right)^2}\,dx$$ 1 + 1

$$= -\frac{2}{9}(10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3}\left[\frac{\left(x-\frac{2}{3}\right)\sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x-\frac{2}{3}\right)^2}}{2} + \frac{17}{9}\sin^{-1}\frac{3x-2}{\sqrt{34}}\right] + C$$ 1

OR

$x^2 = y$ (say) $\frac{1}{2}$

$$\frac{(y+1)(y+4)}{(y+3)(y-5)} = 1 + \frac{A}{y+3} + \frac{B}{y-5}$$ $\frac{1}{2}$

using partial fraction we get $A = \frac{1}{4}$, $B = \frac{27}{4}$ 1

$$\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}\,dx = \int 1.dx + \frac{1}{4}\int \frac{dx}{x^2+3} + \frac{27}{4}\int \frac{dx}{x^2-5}$$ 1

$$= x + \frac{1}{4\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + C$$ 1

14. $I = \int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

put $\sin^{-1}x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$ $\frac{1}{2} + \frac{1}{2}$

$$= \int t\cdot\sin t\,dt$$ 1

$$= -t\cos t + \sin t + c$$ $1\frac{1}{2}$

$$= -\sqrt{1-x^2}\,\sin^{-1}x + x + c$$ $\frac{1}{2}$

15. $y^2\,dx + (x^2 - xy + y^2)\,dy = 0$

$\Rightarrow \dfrac{dx}{dy} = -\dfrac{(x^2 - xy + y^2)}{y^2}$ $\qquad \frac{1}{2}$

put $x = vy \Rightarrow \dfrac{dx}{dy} = v + y\dfrac{dv}{dy}$ $\qquad \frac{1}{2}$

$v + y\dfrac{dv}{dy} = \dfrac{(v^2y^2 - y^2v + y^2)}{y^2}$

$\Rightarrow \dfrac{dv}{v^2+1} = -\dfrac{dy}{y}$ $\qquad 1\frac{1}{2}$

Integrating both sides

$\tan^{-1} v = -\log y + c$ $\qquad \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \tan^{-1}\dfrac{x}{y} = -\log y + c$ $\qquad \frac{1}{2}$

16. $\dfrac{dx}{dy} - \dfrac{x}{1+y^2} = \dfrac{\cot^{-1} y}{1+y^2}$ $\qquad \frac{1}{2}$

$\text{I.F} = e^{-\int \frac{1}{1+y^2}} = e^{\cot^{-1} y}$ $\qquad 1$

$\Rightarrow \dfrac{d}{dy}\left(x \cdot e^{\cot^{-2} y}\right) = \dfrac{\cot^{-1} t\, e^{\cot^{-1} y}}{1+y^2}$

Integrating, we get

$x \cdot e^{\cot-1y} = \displaystyle\int \dfrac{\cot^{-1} y\, e^{\cot-1y}}{1+y^2}\,dy$ $\qquad 1\frac{1}{2}$

put $\cot^{-1} y = t$

$= -\displaystyle\int t\, e^t\, dt$

$= (1 - t)\, e^t + c$

$\Rightarrow x = (1 - \cot^{-1}y) + ce^{-\cot^{-1} y}$ $\qquad 1$

17. $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ ...(i)

$\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ ...(ii)

$(1) - (2) \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{d} \times (\vec{b} - \vec{c})$ $\qquad 2$

$\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0}$ $\qquad 1$

$\Rightarrow (\vec{a} - \vec{d}) \parallel (\vec{b} - \vec{c})$ $\qquad 1$

18. Equation of line $\overrightarrow{AB}$

$\vec{r} = (-\hat{j}-\hat{k}) + \lambda(4\hat{i}+6\hat{j}+2\hat{k})$ 1

Equation of line $\overrightarrow{CD}$

$\vec{r} = (3\hat{i}+9\hat{j}+4\hat{k}) + \mu(-7\hat{i}-5\hat{j})$ $\frac{1}{2}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i}+10\hat{j}+5\hat{k}$ $\frac{1}{2}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 10\hat{i}-14\hat{j}+22\hat{k}$ 1

$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = 30 - 140 + 110 = 0$ 1

$\Rightarrow$ Lines intersect

19. Let selection of defective pen be considered success

$p = \frac{2}{20} = \frac{1}{10},\ q = \frac{9}{10}$ 1

Reqd probability = P(x = 0) + P(x = 1) + P(x = 2) $1\frac{1}{2}$

$$= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5 + {}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4 + {}^5C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4 + \frac{1}{10}\times\left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^3 \times \frac{34}{25}$$ $1\frac{1}{2}$

OR

$\sum_{i=0}^{4} P(x_i) = 1$ $\frac{1}{2}$

$\Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8}$ $\frac{1}{2}$

(i) $P(x=1) = \frac{1}{8}$ 1

(ii) P(at most 2 colleges) $= P(0) + P(1) + P(2)$

$= \frac{5}{8}$ 1

(iii) P(atleast 2 colleges) $= 1 - [P(x=0) + P(x=1)]$

$= 1 - \frac{1}{8} = \frac{7}{8}$ 1

SECTION C

20. $f(x) = |x| + x, \quad g(x) = |x| - x \quad \forall\ x \in R$

$(fog)\,(x) = f(g(x))$ 1

$\quad = ||x| - 1| + |x| - x$ 1

$(gof)\,(x) = g(f(x))$ $\frac{1}{2}$

$\quad = ||x| + x| - |x| - x$ $\frac{1}{2}$

$(fog)\,(-3) = 6$ 1

$(fog)\,(5) = 0$ 1

$(gof)\,(-2) = 2$ 1

21. $abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow\ abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b}+1 & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$\Rightarrow\ abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$ $1\frac{1}{2}$

$\Rightarrow\ abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = 0$

$\because\ a, b, c, \neq 0$

$\therefore\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$ $1\frac{1}{2}$

OR

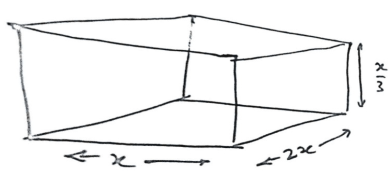
$|A| = 1$ 1

$$\text{adj } A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ 3

$$A(\text{adj } A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$ 1

$$|A|\, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$ 1

22. $S = 6x^2 + 4\pi r^2$



$$\Rightarrow r = \sqrt{\frac{S - 6\pi^2}{4\pi}} \qquad \text{...(i)}$$ $\frac{1}{2}$

$$V = \frac{2x^3}{3} + \frac{4}{3}\pi r^3$$ 1

$$= \frac{2}{3}x^3 + \frac{4}{3}\pi\left(\frac{S - 6x^2}{4\pi}\right)^{3/2}$$

$$= \frac{2x^3}{3} + \frac{(S - 6x^2)^{3/2}}{6\sqrt{\pi}}$$ 1

$$\frac{dV}{dx} = 2x^2 - \frac{3x}{\sqrt{\pi}}\sqrt{S - 6x^2}$$ 1

$$\frac{dV}{dx} = 0 \Rightarrow 2x^2 = \frac{3x\sqrt{S - 6x^2}}{\sqrt{\pi}}$$

$$\Rightarrow r = \frac{x}{3} \text{ [using (i)]}$$ 1

$$\frac{d^2V}{dx^2} = 4x\left[\frac{3x}{\sqrt{\pi}} \cdot \frac{(-12x)}{2\sqrt{S - 6x^2}} + \frac{3}{\sqrt{\pi}}\sqrt{S - 6x^2}\right]$$

$$\left.\frac{d^2V}{dx^2}\right|_{x = \frac{r}{3}} > 0$$ 1

$$\Rightarrow V \text{ is minimum at } x = \frac{r}{3} \text{ i.e. } r = 3x$$

$$\text{Minimum value of sum of volume} = \left(\frac{2x^3}{3} + 36\pi x^3\right) \text{ cubic units}$$ $\frac{1}{2}$

OR

Equatioin of given curve

$y = \cos(x + y)$ ...(i)

$\Rightarrow \dfrac{dy}{dx} = -\sin(x+y)\left(1 + \dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-\sin(x+y)}{1+\sin(x+y)}$ 2

given line $x + 2y = 0$, its slope $= -\dfrac{1}{2}$ $\dfrac{1}{2}$

condition of || lines

$\dfrac{-\sin(x+y)}{1+\sin(x+y)} = -\dfrac{1}{2}$ 1

$\Rightarrow \sin(x+y) = 1$

$\Rightarrow \cos(x+y) = 0 \;\; y = 0$ using (i)

$\Rightarrow \cos x = 0 \Rightarrow x = (2n+1)\dfrac{\pi}{2}, n \in I$

$\therefore\; x = \dfrac{-3\pi}{2}, \dfrac{\pi}{2} \in [-2\pi, 2\pi]$ 1

Thus tangents are || to the line $x + 2y = 0$

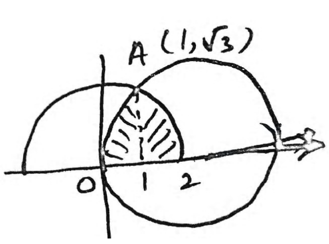
only at pts $\left(-\dfrac{3\pi}{2}, 0\right)$ and $\left(\dfrac{\pi}{2}, 0\right)$ $\dfrac{1}{2}$

$\therefore$ Required equation of tangents are

$y - 0 = -\dfrac{1}{2}\left(x + \dfrac{3\pi}{2}\right) \Rightarrow 2x + 4y + 3\pi = 0$ $\dfrac{1}{2}$

$y - 0 = -\dfrac{1}{2}\left(x - \dfrac{1}{2}\right) \Rightarrow 2x + 4y - \pi = 0$ $\dfrac{1}{2}$

23. Their point of intersection $(1, \sqrt{3})$ 1



Correct Figure 1

Required Area $= \int_0^1 \sqrt{(2)^2 - (x-2)^2}\, dx + \int_1^2 \sqrt{2^2 - x^2}\, dx$ 2

$= \left[\dfrac{(x-2)\sqrt{4x - x^2}}{2} + 2\sin^{-1}\dfrac{x-2}{2}\right]_0^1 + \left[\dfrac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\dfrac{x}{2}\right]_1^2$ 1

$= \left(\dfrac{5\pi}{3} - \sqrt{3}\right)$ Sq. units 1

24. Equation of family of planes passing through two given planes

$$(x + 2y + 3z - 4) + k\,(2x + y - z + 5) = 0 \qquad 1$$

$$\Rightarrow \quad (1 + 2k)\,x + (2 + k)\,y + (3 - k)\,z = 4 - 5k \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{x}{\frac{4-5k}{1+2k}} + \frac{y}{\frac{4-5k}{2+k}} + \frac{z}{\frac{4-5k}{3-k}} = 1$$

As per condition

$$\frac{4-5k}{1+2k} = \frac{2(4-5k)}{(3-k)} \qquad 1$$

$$\Rightarrow \quad k = \frac{4}{5} \text{ or } \frac{1}{5} \qquad 1$$

For $k = \frac{1}{5}$, Eqn. of plance is $7x + 11y + 14z = 15$ 1

For $k = \frac{4}{5}$, Eqn. of plane is $13x + 14y + 11z = 0$ $\frac{1}{2}$

Equation of plane passing through (2, 3, –1)

and parallel to the plane is:

$$7(x - 2) + 11(y - 3) + 14(z + 1) = 0$$

$$\Rightarrow \quad 7x + 11y + 14z = 33 \qquad 1$$

Vector form: $\vec{r}\cdot(7\hat{i} + 11\hat{j} + 14\hat{k}) = 33$ $\frac{1}{2}$

25. Let H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 balck ball, transferred

H_3 be the event 2 black and 1 balck ball transferred

E be the event that ball drawn from B is red. 1

$$P(H_1) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28} \qquad P(E/H_1) = \frac{6}{10}$$

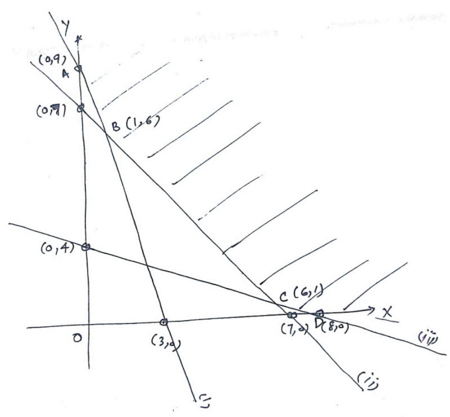
$$P(H_2) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28} \qquad P(E/H_2) = \frac{5}{10}$$

$$P(H_3) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28} \qquad P(E/H_3) = \frac{4}{10} \qquad 1\frac{1}{2} + 1\frac{1}{2}$$

$$P(H_1/E) = \frac{\frac{3}{28}\times\frac{6}{10}}{\frac{3}{28}\times\frac{6}{10} + \frac{15}{28}\times\frac{5}{10} + \frac{10}{28}\times\frac{4}{10}} \qquad 1\frac{1}{2}$$

$$= \frac{18}{133} \qquad \frac{1}{2}$$

26.



Let x tablets of type X and y tablets of type Y are taken

Minimise C = 2x + y 1

subjected to

$$\left.\begin{array}{l} 6x + 2y \geq 18 \\ 3x + 3y \geq 21 \\ 2x + 4y \geq 16 \\ x, y \geq 0 \end{array}\right\}$$ 2

Correct Graph $1\frac{1}{2}$

$C|_{A(0, 9)} = 9$

$C|_{B\,(1, 6)} = 8 \leftarrow$ Minimum value

$C|_{C\,(6, 1)} = 13$

$C|_{D\,(8, 0)} = 16$

$2x + y < 8$ does not pass through unbounded region $\frac{1}{2}$

Thus, minimum value of C = 8 at x = 1, y = 6. 1

QUESTION PAPER CODE 65/2/2/F

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $P = \frac{1}{2}(A + A')$ $\qquad \therefore \; P = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$ $\qquad \frac{1}{2} + \frac{1}{2}$

2. $(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$ $\qquad \frac{1}{2}$

$\Rightarrow \; |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \; \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}$ $\qquad \frac{1}{2}$

3. $a^2 b^2 \sin^2\theta + a^2 b^2 \cos^2\theta = 400$ $\qquad \frac{1}{2}$

$\Rightarrow \; |\vec{b}| = 4$ $\qquad \frac{1}{2}$

4. $\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 5\sqrt{3}$ or $x + y + z = 15$ $\qquad \left[\frac{1}{2}\text{ mark for dc's of normal}\right]$ $\qquad 1$

5. 1×1 $\qquad 1$

6. Expanding we get

$x^3 = -8 \Rightarrow x = -2$ $\qquad \frac{1}{2} + \frac{1}{2}$

SECTION B

7. $f'_{1-} = 2x + 3 = 5$

$f'_{1+} = b$

$f'_{1-} = f'_{1+} \Rightarrow \boxed{b = 5}$ $\qquad 1+1$

$\lim_{x\to 1^-} f(x) = f(1) = \lim_{x\to 1^+} f(x)$

$\Rightarrow \; 4 + a = b + 2$ $\qquad 1$

$\Rightarrow \; \boxed{a = 3}$ $\qquad 1$

8. Let $\quad u = \tan^{-1}\frac{\sqrt{1+x^2} - 1}{x}$

Put $\quad x = \tan\theta \Rightarrow \theta = \tan^{-1} x$ $\qquad \frac{1}{2}$

$\therefore \quad u = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{1 - \cos\theta}{\sin\theta}\right]$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x \qquad 1$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2(1+x^2)} \qquad 1$$

$$v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$= 2\tan^{-1}x$$

$$\Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2} \qquad 1$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{4} \qquad \frac{1}{2}$$

OR

$$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t \qquad \frac{1}{2}$$

$$y = \sin pt \Rightarrow \frac{dy}{dt} = p\cos pt \qquad \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{p\cos pt}{\cos t} \qquad 1$$

$$\frac{d^2y}{dx^2} = \frac{\cos t\,(-p^2\sin pt) - p\cos pt\,(-\sin t)}{\cos^2 t}\cdot\frac{dt}{dx}$$

$$= \frac{-p^2\sin pt\cos t + p\cos pt\sin t}{\cos^3 t} \qquad 1$$

Now $\quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0 \left[\text{Substituting values of } y, \frac{dy}{dx} \;\&\; \frac{d^2y}{dx^2}\right]$ 1

9. Eqn of given curves

$y^2 = 4ax$ and $x^2 = 4by$

Their point of intersections are (0, 0) and $\left(4a^{1/3}b^{2/3}, 4a^{2/3}b^{1/3}\right)$ 1

$$y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y},\ \text{slope} = \frac{a^{1/3}}{2b^{1/3}} \quad \ldots(i) \qquad 1$$

$$x^2 = 4by \Rightarrow \frac{dy}{dx} = \frac{x}{2b},\ \text{slope} = \frac{2a^{1/3}}{b^{1/3}} \quad \ldots(ii) \qquad 1$$

At (0, 0), angle between two curves is 90°

or

Acute angle θ between (i) and (ii) is

$$\theta = \tan^{-1}\left\{\frac{3}{2}\left(\frac{a^{1/3}b^{1/3}}{a^{2/3}+b^{2/3}}\right)\right\} \qquad 1$$

10. $I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin\alpha \sin(\pi - x)}\,dx$ 1

$$2I = \pi \int_0^{\pi} \frac{dx}{1 + \sin\alpha \sin x}$$

$$= 2\pi \int_0^{\pi/2} \frac{dx}{1 + \sin\alpha \sin x}$$

$$\Rightarrow I = \pi \int_0^{\pi/2} \frac{dx}{1 + \sin\alpha \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}}$$ 1

$$I = \pi \int_0^{1} \frac{2dt}{1 + t^2 + 2t\sin\alpha} \qquad \text{Put } \tan\frac{x}{2} = t$$ $\frac{1}{2}$

$$\Rightarrow I = 2\pi \int_0^{1} \frac{dt}{(t + \sin\alpha)^2 + \cos^2\alpha}$$ 1

$$= \frac{2\pi}{\cos\alpha}\left[\tan^{-1}\left(\frac{t + \sin\alpha}{\cos\alpha}\right)\right]_0^1$$

$$\Rightarrow I = \frac{\pi}{\cos\alpha}\left(\frac{\pi}{2} - \alpha\right)$$ $\frac{1}{2}$

11. $I = \int (2x + 5)\sqrt{10 - 4x - 3x^2}\,dx$

$$= -\frac{1}{3}\int (-4 - 6x)\sqrt{10 - 4x - 3x^2}\,dx + \frac{11}{3}\int \sqrt{10 - 4x - 3x^2}\,dx$$ 1

$$= -\frac{2}{9}(10 - 4x - 3x^2)^{3/2} + \frac{11\sqrt{3}}{3}\int \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x - \frac{2}{3}\right)^2}\,dx$$ 1 + 1

$$= -\frac{2}{9}(10 - 4x - 3x^2)^{3/2} + \frac{11\sqrt{3}}{3}\left[\frac{\left(x - \frac{2}{3}\right)\sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x - \frac{2}{3}\right)^2}}{2} + \frac{17}{9}\sin^{-1}\frac{3x - 2}{\sqrt{34}}\right] + C$$ 1

OR

$x^2 = y$ (say) $\frac{1}{2}$

$$\frac{(y+1)(y+4)}{(y+3)(y-5)} = 1 + \frac{A}{y+3} + \frac{B}{y-5}$$ $\frac{1}{2}$

using partial fraction we get $A = \frac{1}{4}$, $B = \frac{27}{4}$ 1

$$\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}\,dx = \int 1.dx + \frac{1}{4}\int \frac{dx}{x^2+3} + \frac{27}{4}\int \frac{dx}{x^2-5}$$ 1

$$= x + \frac{1}{4\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}}\log\left|\frac{x - \sqrt{5}}{x + \sqrt{5}}\right| + C$$ 1

12. $I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

put $\sin^{-1} x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$ $\quad \frac{1}{2}+\frac{1}{2}$

$= \int t \cdot \sin t \, dt$ $\quad 1$

$= -t \cos t + \sin t + c$ $\quad 1\frac{1}{2}$

$= -\sqrt{1-x^2} \sin^{-1} x + x + c$ $\quad \frac{1}{2}$

13. $y^2 dx + (x^2 - xy + y^2) dy = 0$

$\Rightarrow \frac{dx}{dy} = -\frac{(x^2 - xy + y^2)}{y^2}$ $\quad \frac{1}{2}$

put $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $\quad \frac{1}{2}$

$v + y\frac{dv}{dy} = \frac{(v^2y^2 - y^2v + y^2)}{y^2}$

$\Rightarrow \frac{dv}{v^2+1} = -\frac{dy}{y}$ $\quad 1\frac{1}{2}$

Integrating both sides

$\tan^{-1} v = -\log y + c$ $\quad \frac{1}{2}+\frac{1}{2}$

$\Rightarrow \tan^{-1}\frac{x}{y} = -\log y + c$ $\quad \frac{1}{2}$

14. $\frac{dx}{dy} - \frac{x}{1+y^2} = \frac{\cot^{-1} y}{1+y^2}$ $\quad \frac{1}{2}$

$I.F = e^{-\int \frac{1}{1+y^2}} = e^{\cot^{-1} y}$ $\quad 1$

$\Rightarrow \frac{d}{dy}\left(x \cdot e^{\cot^{-2} y}\right) = \frac{\cot^{-1} t \, e^{\cot^{-1} y}}{1+y^2}$

Integrating, we get

$x \cdot e^{\cot{-1}y} = \int \frac{\cot^{-1} y \, e^{\cot{-1}y}}{1+y^2} dy$ $\quad 1\frac{1}{2}$

put $\cot^{-1} y = t$

$= -\int t \, e^t \, dt$

$= (1-t) e^t + c$

$\Rightarrow \quad x = (1 - \cot^{-1}y) + ce^{-\cot^{-1} y}$ $\quad 1$

15. $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ ...(i)

$\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ ...(ii)

$(1)-(2) \Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = \vec{d}\times(\vec{b}-\vec{c})$ 2

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = \vec{0}$ 1

$\Rightarrow (\vec{a}-\vec{d}) \parallel (\vec{b}-\vec{c})$ 1

16. Equation of line $\overrightarrow{AB}$

$\vec{r} = (-\hat{j}-\hat{k}) + \lambda\,(4\hat{i}+6\hat{j}+2\hat{k})$ 1

Equation of line $\overrightarrow{CD}$

$\vec{r} = (3\hat{i}+9\hat{j}+4\hat{k}) + \mu\,(-7\hat{i}-5\hat{j})$ $\frac{1}{2}$

$\vec{a}_2-\vec{a}_1 = 3\hat{i}+10\hat{j}+5\hat{k}$ $\frac{1}{2}$

$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 10\hat{i}-14\hat{j}+22\hat{k}$ 1

$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = 30-140+110 = 0$ 1

$\Rightarrow$ Lines intersect

17. Let selection of defective pen be considered success

$p = \frac{2}{20} = \frac{1}{10}, q = \frac{9}{10}$ 1

Reqd probability $= P(x=0)+P(x=1)+P(x=2)$ $1\frac{1}{2}$

$$= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5 + {}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4 + {}^5C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4 + \frac{1}{10}\times\left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^3 \times \frac{34}{25}$$ $1\frac{1}{2}$

OR

$\sum_{i=0}^{4} P(x_i) = 1$ $\frac{1}{2}$

$\Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8}$ $\frac{1}{2}$

(i) $P(x=1) = \frac{1}{8}$ 1

(ii) P(at most 2 colleges) $= P(0) + P(1) + P(2)$

$$= \frac{5}{8}$$ 1

(iii) P(atleast 2 colleges) $= 1 - [P(x = 0) + P(x = 1)]$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$ 1

18. LHS $= \cot^{-1}\left[\dfrac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) + \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) - \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right]$ 1+1

$$= \cot^{-1}\left(\cot\frac{x}{2}\right)$$ 1

$$= \frac{x}{2} = \text{RHS}$$ 1

OR

$$\tan^{-1}\left[\frac{\frac{x-2}{x-1} + \frac{x+2}{x+1}}{1 - \frac{x-2}{x-1}\cdot\frac{x+2}{x+1}}\right] = \frac{\pi}{4}$$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{2x^2 - 4}{3} = \tan\frac{\pi}{4}$$ $1\frac{1}{2}$

$$\Rightarrow \quad x = \pm\sqrt{\frac{7}{2}}$$ 1

19. Let each poor child pay ₹ x per month and each rich child pay ₹ y per month.

$\therefore \quad 20x + 5y = 9000$ $\frac{1}{2}$

$5x + 25y = 26000$ $\frac{1}{2}$

In matrix form,

$$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$ 1

$$AX = B \Rightarrow X = A^{-1}B$$

$$A^{-1} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

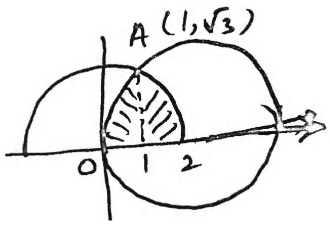
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$\Rightarrow \quad x = 200, y = 1000$ 1

Value: Compassion or any relevant value 1

SECTION C

20. Their point of intersection $(1, \sqrt{3})$ 1



Correct Figure 1

$$\text{Required Area} = \int_0^1 \sqrt{(2)^2 - (x-2)^2}\, dx + \int_1^2 \sqrt{2^2 - x^2}\, dx$$ 2

$$= \left[\frac{(x-2)\sqrt{4x - x^2}}{2} + 2\sin^{-1}\frac{x-2}{2}\right]_0^1 + \left[\frac{x\sqrt{4 - x^2}}{2} + 2\sin^{-1}\frac{x}{2}\right]_1^2$$ 1

$$= \left(\frac{5\pi}{3} - \sqrt{3}\right) \text{Sq. units}$$ 1

21. Equation of family of planes passing through two given planes

$(x + 2y + 3z - 4) + k\,(2x + y - z + 5) = 0$ 1

$\Rightarrow \quad (1 + 2k)\,x + (2 + k)\,y + (3 - k)\,z = 4 - 5k \qquad \ldots(i)$

$$\Rightarrow \quad \frac{x}{\frac{4-5k}{1+2k}} + \frac{y}{\frac{4-5k}{2+k}} + \frac{z}{\frac{4-5k}{3-k}} = 1$$

As per condition

$$\frac{4-5k}{1+2k} = \frac{2(4-5k)}{(3-k)}$$ 1

$$\Rightarrow \quad k = \frac{4}{5} \text{ or } \frac{1}{5}$$ 1

For $k = \frac{1}{5}$, Eqn. of plance is $7x + 11y + 14z = 15$ 1

For $k = \frac{4}{5}$, Eqn. of plane is $13x + 14y + 11z = 0$ $\frac{1}{2}$

Equation of plane passing through (2, 3, –1)

and parallel to the plane is:

$7(x - 2) + 11(y - 3) + 14(z + 1) = 0$

$\Rightarrow \quad 7x + 11y + 14z = 33$ 1

Vector form: $\vec{r} \cdot (7\hat{i} + 11\hat{j} + 14\hat{k}) = 33$ $\frac{1}{2}$

22. Let H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 balck ball, transferred

H_3 be the event 2 black and 1 balck ball transferred

E be the event that ball drawn from B is red. 1

$$P(H_1) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28} \qquad P(E/H_1) = \frac{6}{10}$$

$$P(H_2) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28} \qquad P(E/H_2) = \frac{5}{10}$$

$P(H_3) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28}$ $\qquad P(E/H_3) = \frac{4}{10}$ $\qquad 1\frac{1}{2} + 1\frac{1}{2}$

$$P(H_1/E) = \frac{\frac{3}{28} \times \frac{6}{10}}{\frac{3}{28} \times \frac{6}{10} + \frac{15}{28} \times \frac{5}{10} + \frac{10}{28} \times \frac{4}{10}}$$ $1\frac{1}{2}$

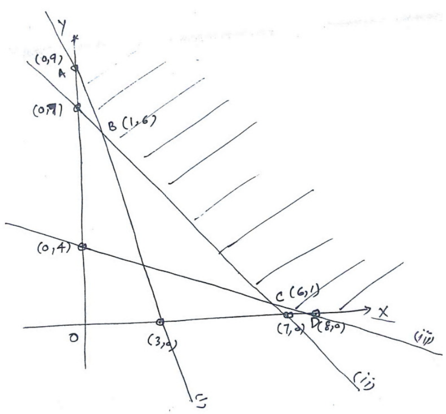
$$= \frac{18}{133}$$ $\frac{1}{2}$

23. Let x tablets of type X and y tablets of type Y are taken

Minimise $C = 2x + y$ 1

subjected to

$$\left.\begin{array}{l} 6x + 2y \geq 18 \\ 3x + 3y \geq 21 \\ 2x + 4y \geq 16 \\ x, y \geq 0 \end{array}\right\}$$ 2



Correct Graph $1\frac{1}{2}$

$C|_{A(0,\,9)} = 9$

$C|_{B\,(1,\,6)} = 8 \leftarrow$ Minimum value

$C|_{C\,(6,\,1)} = 13$

$C|_{D\,(8,\,0)} = 16$

$2x + y < 8$ does not pass through unbounded region $\frac{1}{2}$

Thus, minimum value of C = 8 at x = 1, y = 6. 1

24. $f(x) = |x| + x, \quad g(x) = |x| - x \quad \forall\ x \in R$

$(fog)(x) = f(g(x))$ 1

$= ||x| - 1| + |x| - x$ 1

$(gof)(x) = g(f(x))$ $\frac{1}{2}$

$= ||x| + x| - |x| - x$ $\frac{1}{2}$

$(fog)(-3) = 6$ 1

$(fog)(5) = 0$ 1

$(gof)(-2) = 2$ 1

25. $$abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$$ $1\frac{1}{2}$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \ abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b}+1 & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$$ $1\frac{1}{2}$

$$\Rightarrow \ abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ $1\frac{1}{2}$

$$\Rightarrow \ abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = 0$$

$\because \ a, b, c, \neq 0$

$\therefore \ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$ $1\frac{1}{2}$

OR

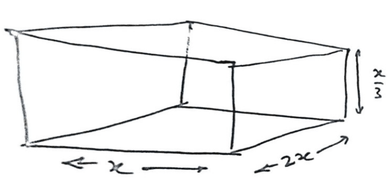
$|A| = 1$ 1

$$\text{adj}\, A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ 3

$$A(\text{adj}\, A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$ 1

$$|A|\, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$ 1

26.



$$S = 6x^2 + 4\pi r^2$$

$$\Rightarrow r = \sqrt{\frac{S - 6\pi^2}{4\pi}} \qquad \text{...(i)}$$ $\frac{1}{2}$

$$V = \frac{2x^3}{3} + \frac{4}{3}\pi r^3$$ 1

$$= \frac{2}{3}x^3 + \frac{4}{3}\pi\left(\frac{S-6x^2}{4\pi}\right)^{3/2}$$

$$= \frac{2x^3}{3} + \frac{(S-6x^2)^{3/2}}{6\sqrt{\pi}}$$ 1

$$\frac{dV}{dx} = 2x^2 - \frac{3x}{\sqrt{\pi}}\sqrt{S-6x^2}$$ 1

$$\frac{dV}{dx} = 0 \Rightarrow 2x^2 = \frac{3x\sqrt{S-6x^2}}{\sqrt{\pi}}$$

$$\Rightarrow r = \frac{x}{3} \text{ [using (i)]}$$ 1

$$\left.\begin{aligned} &\frac{d^2V}{dx^2} = 4x\left[\frac{3x}{\sqrt{\pi}}\cdot\frac{(-12x)}{2\sqrt{S-6x^2}} + \frac{3}{\sqrt{\pi}}\sqrt{S-6x^2}\right] \\ &\left.\frac{d^2V}{dx^2}\right|_{x=\frac{r}{3}} > 0 \end{aligned}\right\}$$ 1

$$\Rightarrow V \text{ is minimum at } x = \frac{r}{3} \text{ i.e. } r = 3x$$

Minimum value of sum of volume $= \left(\frac{2x^3}{3} + 36\pi x^3\right)$ cubic units $\frac{1}{2}$

OR

Equatioin of given curve

$y = \cos(x+y)$ ...(i)

$$\Rightarrow \frac{dy}{dx} = -\sin(x+y)\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$$ 2

given line $x + 2y = 0$, its slope $= -\frac{1}{2}$ $\frac{1}{2}$

condition of || lines

$$\frac{-\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2}$$ 1

$\Rightarrow \sin(x+y) = 1$

$\Rightarrow \cos(x+y) = 0 \ \ y = 0$ using (i)

$\Rightarrow \cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}, n \in I$

$\therefore x = \frac{-3\pi}{2}, \frac{\pi}{2} \in [-2\pi, 2\pi]$ 1

Thus tangents are || to the line $x + 2y = 0$

only at pts $\left(-\frac{3\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 0\right)$ $\frac{1}{2}$

$\therefore$ Required equation of tangents are

$y - 0 = -\frac{1}{2}\left(x + \frac{3\pi}{2}\right) \Rightarrow 2x + 4y + 3\pi = 0$ $\frac{1}{2}$

$y - 0 = -\frac{1}{2}\left(x - \frac{1}{2}\right) \Rightarrow 2x + 4y - \pi = 0$ $\frac{1}{2}$

QUESTION PAPER CODE 65/2/3/F

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = 0$ $\frac{1}{2}$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -\frac{3}{2}$ $\frac{1}{2}$

2. $a^2 b^2 \sin^2\theta + a^2 b^2 \cos^2\theta = 400$ $\frac{1}{2}$

$\Rightarrow |\vec{b}| = 4$ $\frac{1}{2}$

3. $\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 5\sqrt{3}$ or $x + y + z = 15$ $\left[\frac{1}{2}\text{ mark for dc's of normal}\right]$ 1

4. 1×1 1

5. Expanding we get

$x^3 = -8 \Rightarrow x = -2$ $\frac{1}{2}+\frac{1}{2}$

6. $P = \frac{1}{2}(A + A')$ $\therefore P = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$ $\frac{1}{2}+\frac{1}{2}$

SECTION B

7. Let $u = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$ $\frac{1}{2}$

$\therefore u = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$ 1

$\Rightarrow \frac{du}{dx} = \frac{1}{2(1+x^2)}$ 1

$v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= 2\tan^{-1} x$

$$\Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2}$$ 1

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{4}$$ $\frac{1}{2}$

OR

$$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t$$ $\frac{1}{2}$

$$y = \sin pt \Rightarrow \frac{dy}{dt} = p\cos pt$$ $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{p\cos pt}{\cos t}$$ 1

$$\frac{d^2y}{dx^2} = \frac{\cos t\,(-p^2 \sin pt) - p\cos pt\,(-\sin t)}{\cos^2 t} \cdot \frac{dt}{dx}$$

$$= \frac{-p^2 \sin pt \cos t + p\cos pt \sin t}{\cos^3 t}$$ 1

Now $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$ $\left[\text{Substituting values of } y, \frac{dy}{dx} \,\&\, \frac{d^2y}{dx^2}\right]$ 1

8. Eqn of given curves

$y^2 = 4ax$ and $x^2 = 4by$

Their point of intersections are (0, 0) and $\left(4a^{1/3} b^{2/3}, 4a^{2/3} b^{1/3}\right)$ 1

$$y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y}, \text{ slope} = \frac{a^{1/3}}{2b^{1/3}} \quad ...(i)$$ 1

$$x^2 = 4by \Rightarrow \frac{dy}{dx} = \frac{x}{2b}, \text{ slope} = \frac{2a^{1/3}}{b^{1/3}} \quad ...(ii)$$ 1

At (0, 0), angle between two curves is 90°

or

Acute angle θ between (i) and (ii) is

$$\theta = \tan^{-1}\left\{\frac{3}{2}\left(\frac{a^{1/3} b^{1/3}}{a^{2/3} + b^{2/3}}\right)\right\}$$ 1

9. $$I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin\alpha \sin(\pi - x)}\,dx$$ 1

$$2I = \pi\int_0^{\pi} \frac{dx}{1+\sin\alpha \sin x}$$

$$= 2\pi\int_0^{\pi/2} \frac{dx}{1+\sin\alpha \sin x}$$

$$\Rightarrow I = \pi\int_0^{\pi/2} \frac{dx}{1+\sin\alpha \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$ 1

$$I = \pi \int_0^1 \frac{2dt}{1 + t^2 + 2t \sin \alpha} \qquad \text{Put } \tan \frac{x}{2} = t$$ $\frac{1}{2}$

$$\Rightarrow I = 2\pi \int_0^1 \frac{dt}{(t + \sin \alpha)^2 + \cos^2 \alpha}$$ 1

$$= \frac{2\pi}{\cos \alpha} \left[\tan^{-1} \left(\frac{t + \sin \alpha}{\cos \alpha} \right) \right]_0^1$$

$$\Rightarrow I = \frac{\pi}{\cos \alpha} \left(\frac{\pi}{2} - \alpha \right)$$ $\frac{1}{2}$

10. $I = \int (2x + 5) \sqrt{10 - 4x - 3x^2} \, dx$

$$= -\frac{1}{3} \int (-4 - 6x) \sqrt{10 - 4x - 3x^2} \, dx + \frac{11}{3} \int \sqrt{10 - 4x - 3x^2} \, dx$$ 1

$$= -\frac{2}{9} (10 - 4x - 3x^2)^{3/2} + \frac{11\sqrt{3}}{3} \int \sqrt{\left(\frac{\sqrt{34}}{3} \right)^2 - \left(x - \frac{2}{3} \right)^2} \, dx$$ 1 + 1

$$= -\frac{2}{9} (10 - 4x - 3x^2)^{3/2} + \frac{11\sqrt{3}}{3} \left[\frac{\left(x - \frac{2}{3} \right) \sqrt{\left(\frac{\sqrt{34}}{3} \right)^2 - \left(x - \frac{2}{3} \right)^2}}{2} + \frac{17}{9} \sin^{-1} \frac{3x - 2}{\sqrt{34}} \right] + C$$ 1

OR

$x^2 = y$ (say) $\frac{1}{2}$

$$\frac{(y + 1)(y + 4)}{(y + 3)(y - 5)} = 1 + \frac{A}{y + 3} + \frac{B}{y - 5}$$ $\frac{1}{2}$

using partial fraction we get $A = \frac{1}{4}, B = \frac{27}{4}$ 1

$$\int \frac{(x^2 + 1)(x^2 + 4)}{(x^2 + 3)(x^2 - 5)} dx = \int 1.dx + \frac{1}{4} \int \frac{dx}{x^2 + 3} + \frac{27}{4} \int \frac{dx}{x^2 - 5}$$ 1

$$= x + \frac{1}{4\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}} \log \left| \frac{x - \sqrt{5}}{x + \sqrt{5}} \right| + C$$ 1

11. $I = \int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx$

put $\sin^{-1} x = t \Rightarrow \frac{dx}{\sqrt{1 - x^2}} = dt$ $\frac{1}{2} + \frac{1}{2}$

$$= \int t \cdot \sin t \, dt$$ 1

$$= -t \cos t + \sin t + c$$ $1\frac{1}{2}$

$$= -\sqrt{1 - x^2} \sin^{-1} x + x + c$$ $\frac{1}{2}$

12. $y^2\,dx + (x^2 - xy + y^2)\,dy = 0$

$$\Rightarrow \frac{dx}{dy} = -\frac{(x^2 - xy + y^2)}{y^2} \qquad \frac{1}{2}$$

put $x = vy \Rightarrow \dfrac{dx}{dy} = v + y\dfrac{dv}{dy}$ $\qquad \frac{1}{2}$

$$v + y\frac{dv}{dy} = \frac{(v^2y^2 - y^2v + y^2)}{y^2}$$

$$\Rightarrow \frac{dv}{v^2+1} = -\frac{dy}{y} \qquad 1\frac{1}{2}$$

Integrating both sides

$$\tan^{-1} v = -\log y + c \qquad \frac{1}{2}+\frac{1}{2}$$

$$\Rightarrow \tan^{-1}\frac{x}{y} = -\log y + c \qquad \frac{1}{2}$$

13. $\dfrac{dx}{dy} - \dfrac{x}{1+y^2} = \dfrac{\cot^{-1} y}{1+y^2}$ $\qquad \frac{1}{2}$

$$\text{I.F} = e^{-\int\frac{1}{1+y^2}} = e^{\cot^{-1} y} \qquad 1$$

$$\Rightarrow \frac{d}{dy}\left(x \cdot e^{\cot^{-2} y}\right) = \frac{\cot^{-1} t\, e^{\cot^{-1} y}}{1+y^2}$$

Integrating, we get

$$x \cdot e^{\cot-1y} = \int \frac{\cot^{-1} y\, e^{\cot-1y}}{1+y^2}\,dy \qquad 1\frac{1}{2}$$

put $\cot^{-1} y = t$

$$= -\int t\, e^t\, dt$$

$$= (1-t)\,e^t + c$$

$$\Rightarrow \quad x = (1 - \cot^{-1}y) + ce^{-\cot^{-1} y} \qquad 1$$

14. $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ ...(i)

$\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ ...(ii)

$(1)-(2) \Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = \vec{d}\times(\vec{b}-\vec{c})$ $\qquad 2$

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = \vec{0}$ $\qquad 1$

$\Rightarrow (\vec{a}-\vec{d})\,||\,(\vec{b}-\vec{c})$ $\qquad 1$

15. Equation of line $\overrightarrow{AB}$

$\vec{r} = (-\hat{j} - \hat{k}) + \lambda(4\hat{i} + 6\hat{j} + 2\hat{k})$ 1

Equation of line $\overrightarrow{CD}$

$\vec{r} = (3\hat{i} + 9\hat{j} + 4\hat{k}) + \mu(-7\hat{i} - 5\hat{j})$ $\frac{1}{2}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 10\hat{j} + 5\hat{k}$ $\frac{1}{2}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 10\hat{i} - 14\hat{j} + 22\hat{k}$ 1

$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 30 - 140 + 110 = 0$ 1

$\Rightarrow$ Lines intersect

16. Let selection of defective pen be considered success

$p = \frac{2}{20} = \frac{1}{10}, q = \frac{9}{10}$ 1

Reqd probability = P(x = 0) + P(x = 1) + P(x = 2) $1\frac{1}{2}$

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 + {}^5C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4 + \frac{1}{10} \times \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^3 \times \frac{34}{25}$$ $1\frac{1}{2}$

OR

$\sum_{i=0}^{4} P(x_i) = 1$ $\frac{1}{2}$

$\Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8}$ $\frac{1}{2}$

(i) $P(x = 1) = \frac{1}{8}$ 1

(ii) P(at most 2 colleges) = P(0) + P(1) + P(2)

$= \frac{5}{8}$ 1

(iii) P(atleast 2 colleges) = 1 – [P(x = 0) + P(x = 1)]

$= 1 - \frac{1}{8} = \frac{7}{8}$ 1

17. LHS $= \cot^{-1}\left[\dfrac{\left(\cos\dfrac{x}{2}+\sin\dfrac{x}{2}\right)+\left(\cos\dfrac{x}{2}-\sin\dfrac{x}{2}\right)}{\left(\cos\dfrac{x}{2}+\sin\dfrac{x}{2}\right)-\left(\cos\dfrac{x}{2}-\sin\dfrac{x}{2}\right)}\right]$ 1+1

$= \cot^{-1}\left(\cot\dfrac{x}{2}\right)$ 1

$= \dfrac{x}{2} = \text{RHS}$ 1

OR

$\tan^{-1}\left[\dfrac{\dfrac{x-2}{x-1}+\dfrac{x+2}{x+1}}{1-\dfrac{x-2}{x-1}\cdot\dfrac{x+2}{x+1}}\right] = \dfrac{\pi}{4}$ $1\frac{1}{2}$

$\Rightarrow \dfrac{2x^2-4}{3} = \tan\dfrac{\pi}{4}$ $1\frac{1}{2}$

$\Rightarrow x = \pm\sqrt{\dfrac{7}{2}}$ 1

18. Let each poor child pay ₹ x per month and each rich child pay ₹ y per month.

$\therefore \quad 20x + 5y = 9000$ $\frac{1}{2}$

$5x + 25y = 26000$ $\frac{1}{2}$

In matrix form,

$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$ 1

$AX = B \Rightarrow X = A^{-1}B$

$A^{-1} = \dfrac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$

$\Rightarrow x = 200,\ y = 1000$ 1

Value: Compassion or any relevant value 1

19. $f'_{1-} = 2x + 3 = 5$

$f'_{1+} = b$

$f'_{1-} = f'_{1+} \Rightarrow \boxed{b = 5}$ 1+1

$\lim\limits_{x\to 1^-} f(x) = f(1) = \lim\limits_{x\to 1^+} f(x)$

$\Rightarrow 4 + a = b + 2$ 1

$\Rightarrow \boxed{a = 3}$ 1

SECTION C

20. Equation of family of planes passing through two given planes

$(x + 2y + 3z - 4) + k\,(2x + y - z + 5) = 0$ 1

$\Rightarrow \quad (1 + 2k)\,x + (2 + k)\,y + (3 - k)\,z = 4 - 5k$ ...(i)

$$\Rightarrow \quad \frac{x}{\frac{4-5k}{1+2k}} + \frac{y}{\frac{4-5k}{2+k}} + \frac{z}{\frac{4-5k}{3-k}} = 1$$

As per condition

$$\frac{4-5k}{1+2k} = \frac{2(4-5k)}{(3-k)}$$ 1

$\Rightarrow \quad k = \frac{4}{5}$ or $\frac{1}{5}$ 1

For $k = \frac{1}{5}$, Eqn. of plance is $7x + 11y + 14z = 15$ 1

For $k = \frac{4}{5}$, Eqn. of plane is $13x + 14y + 11z = 0$ $\frac{1}{2}$

Equation of plane passing through $(2, 3, -1)$

and parallel to the plane is:

$7(x - 2) + 11(y - 3) + 14(z + 1) = 0$

$\Rightarrow \quad 7x + 11y + 14z = 33$ 1

Vector form: $\vec{r} \cdot (7\hat{i} + 11\hat{j} + 14\hat{k}) = 33$ $\frac{1}{2}$

21. Let H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 balck ball, transferred

H_3 be the event 2 black and 1 balck ball transferred

E be the event that ball drawn from B is red. 1

$P(H_1) = \frac{^3C_2}{^8C_2} = \frac{3}{28}$ $\qquad P(E/H_1) = \frac{6}{10}$

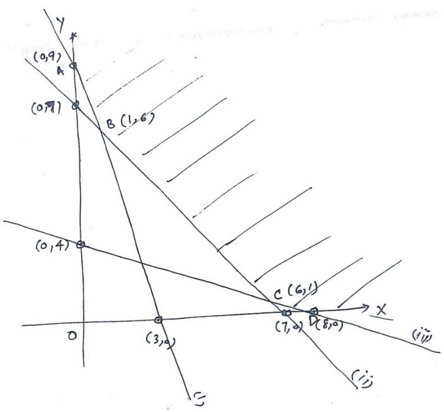
$P(H_2) = \frac{^3C_1 \times {}^5C_1}{^8C_2} = \frac{15}{28}$ $\qquad P(E/H_2) = \frac{5}{10}$

$P(H_3) = \frac{^5C_2}{^8C_2} = \frac{10}{28}$ $\qquad P(E/H_3) = \frac{4}{10}$ $\qquad 1\frac{1}{2} + 1\frac{1}{2}$

$$P(H_1/E) = \frac{\frac{3}{28} \times \frac{6}{10}}{\frac{3}{28} \times \frac{6}{10} + \frac{15}{28} \times \frac{5}{10} + \frac{10}{28} \times \frac{4}{10}}$$ $1\frac{1}{2}$

$$= \frac{18}{133}$$ $\frac{1}{2}$

22. Let x tablets of type X and y tablets of type Y are taken



Minimise $C = 2x + y$ — 1

subjected to

$$\left.\begin{array}{l} 6x + 2y \geq 18 \\ 3x + 3y \geq 21 \\ 2x + 4y \geq 16 \\ x, y \geq 0 \end{array}\right\}$$ — 2

Correct Graph — $1\frac{1}{2}$

$C|_{A(0,\,9)} = 9$

$C|_{B\,(1,\,6)} = 8 \leftarrow$ Minimum value

$C|_{C\,(6,\,1)} = 13$

$C|_{D\,(8,\,0)} = 16$

$2x + y < 8$ does not pass through unbounded region — $\frac{1}{2}$

Thus, minimum value of $C = 8$ at $x = 1, y = 6$. — 1

23. $f(x) = |x| + x, \quad g(x) = |x| - x \quad \forall\ x \in R$

$(fog)(x) = f(g(x))$ — 1

$\qquad = ||x| - 1| + |x| - x$ — 1

$(gof)(x) = g(f(x))$ — $\frac{1}{2}$

$\qquad = ||x| + x| - |x| - x$ — $\frac{1}{2}$

$(fog)(-3) = 6$ — 1

$(fog)(5) = 0$ — 1

$(gof)(-2) = 2$ — 1

24. $$abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$$ — $1\frac{1}{2}$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow\ abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b}+1 & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$$ — $1\frac{1}{2}$

$$\Rightarrow \ abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ $\quad 1\frac{1}{2}$

$$\Rightarrow \ abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = 0$$

$\because$ a, b, c, $\neq 0$

$\therefore \ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$ $\quad 1\frac{1}{2}$

OR

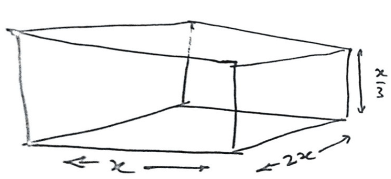
$|A| = 1$ $\quad 1$

$$\text{adj } A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 3$$

$$A(\text{adj } A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad 1$$

$$|A|\, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad 1$$

25. $S = 6x^2 + 4\pi r^2$



$\Rightarrow r = \sqrt{\frac{S - 6\pi^2}{4\pi}}$ ...(i) $\quad \frac{1}{2}$

$V = \frac{2x^3}{3} + \frac{4}{3}\pi r^3$ $\quad 1$

$= \frac{2}{3}x^3 + \frac{4}{3}\pi\left(\frac{S-6x^2}{4\pi}\right)^{3/2}$

$= \frac{2x^3}{3} + \frac{(S-6x^2)^{3/2}}{6\sqrt{\pi}}$ $\quad 1$

$\frac{dV}{dx} = 2x^2 - \frac{3x}{\sqrt{\pi}}\sqrt{S-6x^2}$ $\quad 1$

$\frac{dV}{dx} = 0 \Rightarrow 2x^2 = \frac{3x\sqrt{S-6x^2}}{\sqrt{\pi}}$

$\Rightarrow r = \frac{x}{3}$ [using (i)] $\quad 1$

$$\frac{d^2V}{dx^2} = 4x\left[\frac{3x}{\sqrt{\pi}}\cdot\frac{(-12x)}{2\sqrt{S-6x^2}} + \frac{3}{\sqrt{\pi}}\sqrt{S-6x^2}\right]$$ 1

$$\left.\frac{d^2V}{dx^2}\right|_{x=\frac{r}{3}} > 0$$

$\Rightarrow$ V is minimum at $x = \frac{r}{3}$ i.e. $r = 3x$

Minimum value of sum of volume $= \left(\frac{2x^3}{3} + 36\pi x^3\right)$ cubic units $\frac{1}{2}$

OR

Equatioin of given curve

$y = \cos(x+y)$ ...(i)

$$\Rightarrow \frac{dy}{dx} = -\sin(x+y)\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$$ 2

given line $x + 2y = 0$, its slope $= -\frac{1}{2}$ $\frac{1}{2}$

condition of || lines

$$\frac{-\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2}$$ 1

$\Rightarrow \sin(x+y) = 1$

$\Rightarrow \cos(x+y) = 0 \quad y = 0$ using (i)

$\Rightarrow \cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}, n \in I$

$\therefore x = \frac{-3\pi}{2}, \frac{\pi}{2} \in [-2\pi, 2\pi]$ 1

Thus tangents are || to the line $x + 2y = 0$

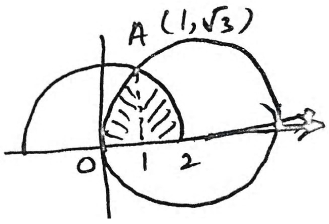
only at pts $\left(-\frac{3\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 0\right)$ $\frac{1}{2}$

$\therefore$ Required equation of tangents are

$y - 0 = -\frac{1}{2}\left(x + \frac{3\pi}{2}\right) \Rightarrow 2x + 4y + 3\pi = 0$ $\frac{1}{2}$

$y - 0 = -\frac{1}{2}\left(x - \frac{1}{2}\right) \Rightarrow 2x + 4y - \pi = 0$ $\frac{1}{2}$

26. Their point of intersection $\left(1, \sqrt{3}\right)$ 1



Correct Figure 1

$$\text{Required Area} = \int_0^1 \sqrt{(2)^2 - (x-2)^2}\, dx + \int_1^2 \sqrt{2^2 - x^2}\, dx$$ 2

$$= \left[\frac{(x-2)\sqrt{4x - x^2}}{2} + 2\sin^{-1}\frac{x-2}{2}\right]_0^1 + \left[\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\frac{x}{2}\right]_1^2$$ 1

$$= \left(\frac{5\pi}{3} - \sqrt{3}\right) \text{Sq. units}$$ 1