Circle - Question Bank

LEVEL–I

1. Find the equations to the circle which touches the y-axis at the point (0, 3) and which has intercept 8 on the positive x-axis.

2. A point moves so that the sum of the squares of its distances from the angular points of a triangle is constant. Prove that its locus is a circle.

3. Find the equations of the circles which have radius $\sqrt{13}$ and which touch the line $2x - 3y + 1 = 0$ at (1, 1).

4. Show that the equation of the circle described on the chord $x \cos \alpha + y \sin \alpha = p$ of the circle $x^2 + y^2 = a^2$ as diameter is $x^2 + y^2 - 2p(x \cos \alpha + y \sin \alpha - p) = 0$.

5. Prove that the circle through A (a, c), B (b, c) and C (b, d) is $(x - a)(x - b) + (y - c)(y - d) = 0$. Prove also that AC and BD are diameters where D is (a, d).

6. The square of the distance of a variable point P from the origin is 4 times the distance of P from the line $x = 1$. Prove that the locus of P is either the point-circle (2, 0) or the circle $(x + 2)^2 + y^2 = 8$.

7. Find the tangent of the acute angle between the tangents at (2, – 5) to the circles $x^2 + y^2 = 29$ and $x^2 + y^2 - 8x - 13 = 0$.

8. Prove that $y = mx + b$ is a tangent to the circle $x^2 + y^2 - 6x = 16$ if $(3m + b)^2 = 25(1 + m^2)$.

9. Find the equation of the circle which passes through the origin, has its centre on the line $x + y = 4$, and cuts the circle $x^2 + y^2 - 4x + 2y + 4 = 0$ orthogonally.

10. Prove that for all values of the constant p and q, the circle $(x - a)(x - a + p) + (y - b)(y - b + q) = r^2$ bisects the circumference of circle $(x - a)^2 + (y - b)^2 = r^2$.

LEVEL–II

1. P is a variable point on the circle whose centre is C (1, 2) and which passes through the origin. Prove that the locus of the centroid of triangle OCP is $3(x^2 + y^2) - 4x - 8y + 5 = 0$.

2. The straight line, $\frac{x}{a} + \frac{y}{b} = 1$ meets the axes in the points respectively . A and B. A point P moves so that the angle $APB = 30^0$. Prove that the locus of the point P is a circle.

3. The tangent from P to the circle $x^2 + y^2 = 1$ is perpendicular to the tangent from P to the circle $x^2 + y^2 = 3$. Show that the locus of P is a circle.

4. The circles $x^2 + y^2 = 1$ and $(x - 2)^2 + (y - 4)^2 = 4$ subtend equal angles at P. Prove that the locus of P is $3(x^2 + y^2) + 4x + 8y = 20$.

5. Prove that the locus of the points from which tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y + 12 = 0$ are equal is the line of the common chord AB whose length is 12/5. Prove also that the tangent of the acute angle between the tangents at A to the circles is 3/2.

6. A triangle has two of its sides along the axes. Its 3rd side touches the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. Prove that the locus of the circumcentre of the triangle is $a^2 - 2a(x + y) + 2xy = 0$.

7. **(i)** Show that the locus of a point such that the ratio of its distances from two given points is constant, is a circle. Hence show that the circle cannot pass through the given points.

   (ii) Given the base of a triangle and ratio of the lengths of other two unequal sides, prove that the vertex lies on a fixed circle.

8. **(i)** The centre of the circle S = 0 lies on the line $2x - 2y + 9 = 0$ and S = 0 cuts orthogonally the circle $x^2 + y^2 = 4$. Show that S = 0 passes through two fixed points and find their coordinates.

   (ii) Find the equation of the circle through the points of intersection of the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$ and cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

9. The base of a triangle is fixed. Find the locus of the vertex when one base angle is double the other. Assume the base of the triangle as x-axis with mid point as origin & the length of the base as 2a.

10. Show that the equation of a straight line meeting $x^2 + y^2 = a^2$ the circle in two points at equaldistances 'd' from a point (x_1, y_1) on its circumference is $xx_1 + yy_1 - a^2 + \frac{d^2}{2} = 0$.

IIT JEE PROBLEMS (OBJECTIVE)

(A) Fill in the blanks

1. If A and B are points in the plane such that PA/PB = k (constant) for all P on a given circle, then the value of k cannot be equal to................. **[IIT - 82]**

2. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are ....and........... **[IIT - 83]**

3. The line $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle. The radius of this circle is............... **[IIT - 84]**

4. Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a circle. A pair of tangents from the point (4, 5) with a pair of radii form a quadrilateral of area.......... **[IIT - 85]**

5. From the origin chords are drawn to the circle $(x - 1)^2 + y^2 = 1$. The equation of the locus of the mid–points of these chord is.......... **[IIT - 85]**

6. The equation of the line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$ is .... **[IIT - 86]**

7. From the point A(0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M such that AM = 2AB. The equation of the locus of M is............. **[IIT - 86]**

8. The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their points of contact is......... **[IIT - 87]**

9. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that common chord is maximum length and has a slope equal to 3/4, then the coordinates of the centre C_2 are............ **[IIT - 88]**

10. The area of the triangle formed by the positive x–axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $\left(1, \sqrt{3}\right)$ is, ... **[IIT - 89]**

11. If circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of $\lambda =$ .... **[IIT - 91]**

12. The equation of the locus of the mid points of the chords of the circle $4x^2 + 4y^2 - 12x + 4y + 1 = 0$ that subtend an angle of $\frac{2\pi}{3}$ at its centre is .... **[IIT - 93]**

13. A circle is inscribed in an equilateral triangle of side 'a'. The area of any square inscribed in this circle is .... **[IIT - 94]**

14. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as a diameter is .... **[IIT - 97]**

15. Two vertices of an an equilateral triangle are (–1, 0) and (1, 0) and its third vertex lies above the x–axis.The equation of its circumcircle is........... **[IIT - 97]**

16. The chords of contact of the pair of tangents drawn from each point on the line 2x + y = 4 to the circle $x^2 + y^2 = 1$ pass through the point .... **[IIT - 97]**

17. For each natural number k, let C_k denote the circle with radius k centrimeteres and centre at the origin. On the circle C_k, α-particle moves k centrimeteres in the counter-clockwise direction. After completing its motion on C_k, the particle moves to C_{k+1} in the radial direction. The motion of the particle continues in this manner. The particle starts at (1, 0). If the particle corsses the positive direction of the x-axis for the first time on the circle C_n then n = ... **[IIT - 97]**

(B) True/False

1. No tangent can be drawn from the point (5/2, 1) to the circumcircle of the triangle with vertices $(1, \sqrt{3})(1, -\sqrt{3})(3, -\sqrt{3})$. **[IIT - 85]**

2. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 - 6x + 2y = 0$. **[IIT - 89]**

(C) Multiple choice with one and more than one correct choice

1. The equations of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$, are
(A) $x = 0$
(B) $y = 0$
(C) $(h^2 - r^2)x - 2rhy = 0$
(D) $(h^2 - r^2)x + 2rhy = 0$ **[IIT - 89]**

2. The number of common tangents to the circle $x^2 + y^2 = 4$ & $x^2 + y^2 - 6x - 8y = 24$ is :
(A) 0 (B) 1 (C) 3 (D) 4 **[IIT - 98]**

3. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then
(A) $x_1 + x_2 + x_3 + x_4 = 0$
(B) $y_1 + y_2 + y_3 + y_4 = 0$
(C) $x_1 x_2 x_3 x_4 = c^4$
(D) $y_1 y_2 y_3 y_4 = c^4$ **[IIT - 98]**

4. Let L_1 be a straight line through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 & L_2 are equal, then which of the following equations can represent L_1 ? **[IIT - 99]**
(A) $x + y = 0$ (B) $x - y = 0$ (C) $x + 7y = 0$ (D) $x - 7y = 0$

(D) Multiple choice with only one correct choice

1. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is **[IIT - 81]**
(A) $x^2 + y^2 - 6x + 4 = 0$
(B) $x^2 + y^2 - 3x + 1 = 0$
(C) $x^2 + y^2 - 4y + 2 = 0$
(D) none of these

2. The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is
(A) $\left(-\frac{16}{5}, \frac{27}{10}\right)$ (B) $\left(-\frac{16}{7}, \frac{53}{10}\right)$ (C) $\left(-\frac{16}{5}, \frac{53}{10}\right)$ (D) none of these **[IIT - 83]**

3. The equation of the circle passing through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is **[IIT - 83]**
(A) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$
(B) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
(C) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$
(D) none of these

4. The locus of the mid–point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is **[IIT - 84]**
(A) $x + y = 2$ (B) $x^2 + y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $x + y = 1$

5. If the circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of centre is **[IIT - 88]**
(A) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
(B) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
(C) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = r^2$
(D) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$

6. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then **[IIT - 89]**

(A) $2 < r < 8$ (B) $r < 2$
(C) $r = 2$ (D) $r > 2$

7. The line $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of area 154 sq. units. Then the equation of this circle is **[IIT - 89]**

(A) $x^2 + y^2 + 2x - 2y = 62$ (B) $x^2 + y^2 + 2x - 2y = 47$
(C) $x^2 + y^2 - 2x + 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 62$

8. The centre of a circle passing through the points (0, 0), (1, 0) & touching the circle $x^2 + y^2 = 9$ is :

(A) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, -\sqrt{2}\right)$ **[IIT - 92]**

9. If the sum of the distance of a point from two perpendicular lines in a plane is 1, then the locus is

(A) square (B) circle (C) straight line (D) two intersecting lines **[IIT - 92]**

10. The locus of the centre of a circle which touches externally the circle, $x^2 + y^2 - 6x - 6y + 14 = 0$ & also touches the y-axis is given by the equation : **[IIT - 93]**

(A) $x^2 - 6x - 10y + 14 = 0$ (B) $x^2 - 10x - 6y + 14 = 0$
(C) $y^2 - 6x - 10y + 14 = 0$ (D) $y^2 - 10x - 6y + 14 = 0$

11. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α .The equation of the locus of the point P is

(A) $x^2 + y^2 + 4x - 6y + 4 = 0$ (B) $x^2 + y^2 + 4x - 6y - 9 = 0$
(C) $x^2 + y^2 + 4x - 6y - 4 = 0$ (D) $x^2 + y^2 + 4x - 6y + 9 = 0$ **[IIT - 96]**

12. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq q$) are bisected by the x-axis, then : **[IIT - 99]**

(A) $p^2 = q^2$ (B) $p^2 = 8q^2$ (C) $p^2 < 8q^2$ (D) $p^2 > 8q^2$

13. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates (3, 4) & (-4, 3) respectively, then $\angle QPR$ is equal to : **[IIT - 2000]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

14. If the circles, $x^2 + y^2 + 2x + 2ky + 6 = 0$ & $x^2 + y^2 + 2ky + k=0$ intersect orthogonally, then 'k' is :

(A) $2 \text{ or } -\frac{3}{2}$ (B) $-2 \text{ or } -\frac{3}{2}$ (C) $2 \text{ or } \frac{3}{2}$ (D) $-2 \text{ or } \frac{3}{2}$

[IIT - 2000]

15. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle PAB as P moves on the circle is **[IIT - 2001]**

(A) a parabola (B) a circle (C) an ellipse (D) a pair of straight lines

16. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals **[IIT - 2001]**

(A) $\sqrt{PQ.RS}$ (B) (PQ + RS)/2

(C) 2PQ.RS/(PQ + RS) (D) $\sqrt{(PQ^2 + RS^2)/2}$

17. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then length of PQ is : **[IIT - 2002]**

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

18. The centre of circle inscribed in formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$

(A) (4, 7) (B) (7, 4) (C) (9, 4) (D) (4, 9) **[IIT - 2003]**

19. If one of the diameters of the circle $x^2 + y^2 – 2x – 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is **[IIT - 2004]**

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) 3 (D) 2

20. A circle is given by $x^2 + (y – 1)^2 = 1$, another circle C touches it externally and also the x–axis, then the locus of its centre is **[IIT-2005]**

(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y): y \le 0\}$ (B) $\{(x, y) : x^2 + (y – 1)^2 = 4\} \cup \{(x, y): y \le 0\}$

(C) $\{(x, y) : x^2 = y\} \cup \{(0, y): y \le 0\}$ (D) $\{(x, y) : x^2 = 4y\} \cup \{(0, y): y \le 0\}$

21. Inradius of a circle which is inscribed in a isosceles triangle one of whose angle is $2\pi/3$, is $\sqrt{3}$ then area of the triangle is **[IIT - 2006]**

(A) $\sqrt{3}\,4$ (B) $12 – 7\sqrt{3}$ (C) $12 + 7\sqrt{3}$ (D) none of these

22. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is **[IIT - 2007]**

(A) 3 (B) 2 (C) 3/2 (D) 1

(E) Question based on write-up

☛ Let ABCD be a square of side length 2 units. C_2 is the circle through vertices A, B, C, D and C_1 is the clrcle touching all sides of the square ABCD. L is the line through A **[IIT - 2006]**

1. If P is a point on C_1 and Q is a point on C_2, then $\dfrac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to

(A) 0.75 (B) 1.25 (C) 1 (D) 0.5

2. A circle touches the line L and the circle C_1 extrenally such that botht he circles are ont eh same side of the line, then the locus of the centre of the circle is

(A) ellipse (B) hyperbola (C) parabola (D) parts of straight line

3. A line M through A is drawn parallel to BD. Points S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is

(A) 1/2 sq. untis (B) 2/3 sq. units (C) 1 sq. unit (D) 2 sq. units

(F) Statement & Reason

1. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$ **[IIT - 2007]**

Statement -1 : The tangents are mutually perpendicular.

because

Statement -2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is True, Statment-2 is True. Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True, Statement-2 **is not** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

(G) Match the column

1. Match the statements in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS. **[IIT - 2007]**

	Column I		**Colmnu II**
(A)	Two intersecting circles	(p)	have a common tangent
(B)	Two mutually external circles	(q)	have a common normal
(C)	Two circles, one strictly inside the other	(r)	do not have a common tangent
(D)	Two branches of a hyperbola	(s)	do not have a common normal

IIT JEE PROBLEMS (SUBJECTIVE)

1. Let A be the centre of the circle $x^2 + y^2 - 4x - 20 = 0$. Suppose that the tangents at the points B(1, 7) and D(4, -2) on the circle meet at the point C. Find the area of the quadrilateral ABCD. **[IIT - 81]**

2. Find the equations of the circle passing through (-4, 3) and touching the lines $x + y = 4$, and $x - y = 2$. **[IIT - 82]**

3. Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the midpoints of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$. **[IIT - 83]**

4. The abscissa of the two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$, Find the equation and the radius of the circle with AB as diameter. **[IIT - 84]**

5. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts intercepts of length 8 on these lines. **[IIT - 86]**

6. Let a given line L_1 intersect the x and y axes at P and Q, respectively. Let another line L_2, perpendicular to L_1, cut the x and y axes at R and S, respectively. Show that the locus of the point of intercepts of length 8 on these lines. **[IIT - 87]**

7. The circle $x^2 + y^2 - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcenter of the triangle is $x + y - xy + k(x^2 + y^2)^{1/2} = 0$. Find k. **[IIT - 87]**

8. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a given circle. Find the locus of the foot of the perpendicular drawn from the origin upon any chord of S which subtends a right angle at the origin. **[IIT - 88]**

9. If $\left(m_i, \dfrac{1}{m_i}\right)$, $m_i > 0,\ i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1m_2m_3m_4 = 1$. **[IIT - 89]**

10. Find the equation of the circle having the lines $x^2 + 2xy + 3x + 6y = 0$ as its normal and having size just sufficient to contain the circle $x(x - 4) + y(y - 3) = 0$. **[IIT - 90]**

11. A circle touches the line $y = x$ at a point P such that OP = $4\sqrt{2}$ where O is origin. The circle contains the point (-10, 2) in its interior & the length of its chord on the line, $x + y = 0$ is $6\sqrt{2}$ .Determine the equation of the circle. **[IIT - 90]**

12. Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y = 10$. Find the equations of the circles. **[IIT - 91]**

13. Find the equation of the circle passing through the points A(4, 3) & B(2, 5) & touching the axis of y. Also find the point P on the y-axis such that the angle APB has largest magnitude. **[REE-91]**

14. Find the radius of the smallest circle which touches the straight line 3x - y = 6 at (1, -3) and also touches the line y = x. Compute upto one place of decimal. **[REE-91]**

15. A ball moving around the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ in anti-clockwise direction leaves it tangentially at the point P(-2, -2). After getting reflected from a straight line it passes through the centre of the circle. Find the equation of this straight line if its perpendicular distance from P is $\frac{5}{2}$. You can assume that the angle of incidence is equal to the angle of reflection. **[REE-91]**

16. Let a circle be given by 2x(x - a) + y(2y - b) = 0, $(a \neq 0,\ b \neq 0)$. Find the condition on a & b if two chords, each bisected by the x-axis, can be drawn to the circle from (a, b/2). **[IIT - 92]**

17. The extremities of a diagonal of a rectangle are (-4, 4) & (6, -1). A circle circumscribes the rectangle & cuts an intercept AB on the y-axis. Find the area of the triangle formed by AB & the tangent to the circle at A & B. **[IIT - 92]**

18. From a point P tangents drawn to the circles $x^2 + y^2 + x - 3 = 0$, $3x^2 + 3y^2 - 5x + 3y = 0$ & $4x^2 + 4y^2 + 8x + 7y + 9 = 0$ are of equal lengths. Find the equation of the circle through P which touches the line x + y = 5 at the point (6, -1). **[IIT - 92]**

19. Consider a family of circles passing through two fixed points A(3, 7) & B(6, 5). Show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point. **[IIT - 93]**

20. Find the coordinates of the point at which the circles $x^2 + y^2 - 4x - 2y + 4 = 0$ and $x^2 + y^2 - 12x - 8y + 36 = 0$ touch each other. Also find the equations of common tangents touching the circles in distinct points. **[IIT - 93]**

21. Find the equation of the circle which touches the circle $x^2 + y^2 - 6x + 6y + 17 = 0$ externally & to which the lines $x^2 - 3xy - 3x + 9y = 0$ are normal. **[REE-94]**

22. Let C be any circle with centre $\left(0, \sqrt{2}\right)$. Prove that at the most two rational points can be there on C. (A rational point is a point both of whose coordinate are rational numbers). **[IIT - 94]**

23. From a point on the line 4x - 3y = 6 tangents are drawn to the circle; $x^2 + y^2 - 6x - 4y + 4 = 0$ which make an angle of $tan^{-1}\left(\frac{24}{7}\right)$ between them. Find the coordinates of all such points & the equations of tangents. **[REE-95, IIT -96]**

24. Find the intervals of values of a for which the line y + x = 0 bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the $2x^2 + 2y^2 - \left(1+\sqrt{2}a\right)x - \left(1-\sqrt{2}a\right)y = 0$ circle. **[IIT - 96]**

25. A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively and the distance between the point A and the mid point of the line segment DC is d, prove that the area of the circle is

$$\frac{\pi d^2 \cos^2\alpha}{\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}.$$ **[IIT - 96]**

26. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line drawn from the point P intersects the curve at points Q and R. If the product PQ. PR is independent of the slope of the line then show that the curve is a circle. **[IIT - 97]**

27. Let C be any circle with centre $(0, \sqrt{2})$, prove that at the most two rational points can be there on C. (A rational point is a point both of whose coordinates are rational numbers.) **[IIT - 97]**

28. A tangent drawn from the point (4, 0) to the circle $x^2 + y^2 = 8$ touches it at a points A in the first quadrant. Find the coordinates of the another point B on the circle such that AB = 4. **[REE-96, IIT-97]**

29. C_1 & C_2 are two concentric circles, the radius of C_2 being twice that of C_1. From a point P on C_2, tangents PA & PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1. **[IIT - 98]**

30. Find the equation of a circle which touches the line x + y = 5 at the point (-2, 7) and cuts the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally. **[REE-98]**

31. Let T_1, T_2 be two tangents drawn from (-2, 0) onto the circle $C : x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to the circles, when taken two at a time. **[IIT - 99]**

32. Extremities of a diagonal of a rectangle are (0, 0) & (4, 3). Find the equation of the tangents to the circumcircle of a rectangle which are parallel to this diagonal. **[REE-2000]**

33. A circle of radius 2 units rolls on the outerside of the circle, $x^2 + y^2 + 4x = 0$, touching it externally. Find the locus of the centre of this outer circle. Also find the equations of the common tangents of the two circles when the line joining the centres of the two circles makes on angle of 60^0 with x-axis. **[IIT - 2001]**

34. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C. **[IIT - 2001]**

35. For the circle $x^2 + y^2 = r^2$ find the value of r for which the area enclosed by the tangents drawn from the point P(6, 8) to the circle and chord of contact is maximum. **[IIT - 2003]**

36. A circle touches the line 2x + 3y + 1 = 0 at the point (1, -1) and is orthogonal to the circle whose one pair of diametrically opposite end points are (3, 0) and (1, -3). Find the equation of the circle. **[IIT - 2004]**

37. Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. **[IIT - 2005]**

SET–I

1. The shortest distance from the point M(-7, 2) to the circle $x^2 + y^2 - 10x - 14y - 151 = 0$ is
(A) 1 (B) 2 (C) 3 (D) none of these

2. The centre of the smallest circle touching the circles $x^2 + y^2 - 2y - 3 = 0$ and $x^2 + y^2 - 8x - 18y + 93 = 0$ is
(A) (3, 2) (B) (4, 4) (C) (2, 7) (D) (2, 5)

3. The area of equilateral triangle inscribed in the circle $x^2 + y^2 - 2x = 0$ is
(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$ (C) $\frac{3\sqrt{3}}{8}$ (D) none of these

4. The equation of the locus of the point of intersection of any two perpendicular tangents to the circle $x^2 + y^2 = 4$ is given by
(A) $x^2 + y^2 = 2$ (B) $x^2 + y^2 = 8$ (C) $x^2 + y^2 = 16$ (D) none of these

5. The radius of the circle passing through the points (1, 2), (5, 2) and (5, -2) is
(A) $5\sqrt{2}$ (B) $2\sqrt{5}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

6. The equations of the tangents to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$, which are perpendicular to the line $4x + 3y = 7$ are
(A) $3x + 4y + 19 = 0$, $3x + 4y + 31 = 0$ (B) $4x - 3y + 19 = 0$, $4x - 3y - 31 = 0$
(C) $3x - 4y + 31 = 0$, $3x - 4y - 19 = 0$ (D) none of these

7. The line joining (5, 0) to $(10 \cos\theta, 10 \sin\theta)$ is divided internally in the ratio 2 : 3 at P. If θ varies then the locus of P is
(A) a pair of straight lines (B) a circle
(C) a straight line (D) none of these

8. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) none of these

9. A point (2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and AP,AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is
(A) $(x + g)(x - 2) + (y + f)(y - 1) = 0$ (B) $(x + g)(x - 2) - (y + f)(y - 1) = 0$
(C) $(x - g)(x + 2) + (y - f)(y + 1) = 0$ (D) none of these

10. Equation of a circle $S(x, y) = 0$, $(S(2, 3) = 16)$ which touches the line $3x + 4y - 7 = 0$ at (1, 1) is given by
(A) $x^2 + y^2 + x + 2y - 5 = 0$ (B) $x^2 + y^2 + 2x + 2y - 6 = 0$
(C) $x^2 + y^2 + 4x - 6y = 0$ (D) none of these

11. The equations of the tangents drawn from the origin to the circle, $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are
(A) $(h^2 - r^2)x + 2rhy = 0$ (B) $y = 0$
(C) $(h^2 - r^2)x - 2rhy = 0$ (D) none of these

12. The equation of a circle with centre (4, 3) and touching the circle $x^2 + y^2 = 1$ is

(A) $x^2 + y^2 - 8x - 6y - 9 = 0$ (B) $x^2 + y^2 - 8x - 6y + 11 = 0$
(C) $x^2 + y^2 - 8x - 6y - 11 = 0$ (D) none of these

13. The equation of the circle passing through (1, -3) and the points common to the two circle $x^2 + y^2 - 6x + 8y - 16 = 0$, $x^2 + y^2 + 4x - 2y - 8 = 0$ is

(A) $x^2 + y^2 - 4x + 6y + 24 = 0$ (B) $2x^2 + 2y^2 + 3x + y - 20 = 0$
(C) $3x^2 + 3y^2 - 5x + 7y - 19 = 0$ (D) none of these

14. A circle is concentric with circle $x^2 + y^2 - 2x + 4y - 20 = 0$. If perimeter of the semicircle is 36 then the equation of the circle is :

(A) $x^2 + y^2 - 2x + 4y - 44 = 0$ (B) $(x - 1)^2 + (y + 2)^2 = (126/11)^2$
(C) $x^2 + y^2 - 2x + 4y - 43 = 0$ (D) none of these

15. Two circles $(x + a)^2 + (y + b)^2 = a^2$ and $(x+\alpha)^2+(y+\beta)^2=\beta^2$ intersect orthogonally if

(A) $2a\alpha + 2b\beta = b^2 + \alpha^2$ (B) $a\alpha + b\beta = \alpha^2 + b^2$
(C) $a\alpha + b\beta + \alpha^2 + b^2 = 0$ (D) none of these

16. Equation of the circle passing through the points A(-4, 3) and B(12, 1) and having radius as small as possible is

(A) $x^2 + y^2 - 8x + 4y - 45 = 0$ (B) $x^2 + y^2 + 8x - 4y - 45 = 0$
(C) $x^2 + y^2 - 8x - 4y - 45 = 0$ (D) $x^2 + y^2 - 8x - 4y - 51 = 0$

17. The length of the shortest chord of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ which passes through the point (a, b) inside the circle is

(A) $2(a^2 + b^2 + 2ga + 2fb + c)^{1/2}$ (B) $2[-(a^2 + b^2 + 2ga + 2fb + c)]^{1/2}$
(C) $[-2(a^2 + b^2 + 2ga + 2fb + c)]^{1/2}$ (D) none of these

18. Two circles are represented by the equations $7x^2 + 7y^2 - 7x + 14y + 18 = 0$ and $4x^2 + 4y^2 - 7x + 8y + 20 = 0$. Which of the following is the equation of the radical axis of the above two circles

(A) $3x^2 + 3y^2 + 6y - 6 = 0$ (B) $21x - 68 = 0$
(C) $6y - 2 = 0$ (D) none of these

19. Maximum number of rational points (points having both coordinates rational) on a circle having centre at $\left(\sqrt{2}, \sqrt{3}\right)$ is

(A) 1 (B) 2 (C) 3 (D) none of these

20. A circle touches the lines $y = \frac{x}{\sqrt{3}}$, $y = x\sqrt{3}$ and has unit radius. If the centre of this circle lies in the first quadrant then possible equation of this circle is -

(A) $x^2 + y^2 - 2x(\sqrt{3} + 1) - 2y(\sqrt{3} + 1) + 8 + 4\sqrt{3} = 0$
(B) $x^2 + y^2 - 2x(1 + \sqrt{3}) - 2y(1 + \sqrt{3}) + 5 + 4\sqrt{3} = 0$
(C) $x^2 + y^2 - 2x(1 + \sqrt{3}) - 2y(1 + \sqrt{3}) + 7 + 4\sqrt{3} = 0$
(D) $x^2 + y^2 - 2x(1 + \sqrt{3}) - 2y(1 + \sqrt{3}) + 6 + 4\sqrt{3} = 0$

SET–II

1. The axes are translated so that the new equation of the circle $x^2 + y^2 - 5x + 2y - 5 = 0$ has no first degree terms. Then the new equation is

(A) $x^2 + y^2 = 9$ (B) $x^2 + y^2 = \frac{49}{4}$ (C) $x^2 + y^2 = \frac{81}{16}$ (D) none of these

2. If $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$, represents

(A) equation of a straight line, if θ is constant and r is variable

(B) equation of a circle, if r is constant and θ is a variable

(C) a straight line passing through a fixed point and having a known slope

(D) all of these

3. The equation of a straight line is $ax + by + a^2 + b^2 = 0$ and that of circle is $x^2 + y^2 + ax + by = 0$. Then

(A) the straight line intersects the circle in two distinct points

(B) the straight line passes outside the circle

(C) the straight line is a diameter of the circle

(D) the straight line touches the circle

4. The internal common tangents of the circles $x^2 + y^2 - 4x - 4y + 4 = 0$ and $x^2 + y^2 + 6x + 6y + 9 = 0$ are

(A) $x - y = 2$ and $x + 2y = 3$ (B) $2x - 3y = 4$ and $x + 5 = 5$

(C) $x = 0$ and $y = 0$ (D) $x + 1 = 0$ and $y - 3 = 0$

5. The locus of the mid points of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend and angle of $\frac{\pi}{3}$ radians at its circumference is

(A) $(x - 2)^2 + (y + 3)^2 = 6.25$ (B) $(x + 2)^2 + (y - 3)^2 = 6.25$

(C) $(x + 2)^2 + (y - 3)^2 = 18.75$ (D) $(x + 2)^2 + (y + 3)^2 = 18.75$

6. The two circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} =$

(A) $1/c$ (B) c (C) $|1/c|$ (D) c^2

7. A circle passes through the point $\left(3, \sqrt{\frac{7}{2}}\right)$ and touches the line pair $x^2 - y^2 - 2x + 1 = 0$. The coordinates of the centre of the circle are

(A) (0, 4) (B) (5, 0) (C) (6, 0) (D) none of these

8. Point M moved along the circle $(x - 4)^2 + (y - 8)^2 = 20$. Then it broke away from it and moving along a tangent to the circle, cuts the x-axis at the point (-2, 0). The coordinates of the point on the circle at which the moving point broke away can be

(A) $\left(-\frac{3}{5}, \frac{46}{5}\right)$ (B) (3, 5) (C) (6, 4) (D) none of these

9. Angle between tangents drawn to $x^2 + y^2 - 2x - 4y + 1 = 0$ at the points where it is cut by the line $y = 2x + c$, is $\frac{\pi}{2}$ then

(A) $|c| = \sqrt{5}$ (B) $|c| = 2\sqrt{5}$ (C) $|c| = \sqrt{10}$ (D) $|c| = 2\sqrt{10}$

10. From the point A(0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn and extended to a point M such that AM = 2 AB. The equation of the locus of M is
(A) $x^2 + 8x + y^2 = 0$ (B) $x^2 + 8x + y^2 + (y - 3)^2 = 0$
(C) $(x - 3)^2 + 8x + y^2 = 0$ (D) $x^2 + 8x + 8y^2 = 0$

11. From (3, 4) chords are drawn to the circle $x^2 + y^2 - 4x = 0$. The locus of the mid points of the chords is
(A) $x^2 + y^2 - 5x + 4y + 6 = 0$ (B) $x^2 + y^2 + 5x - 4y + 6 = 0$
(C) $x^2 + y^2 - 5x + 4y + 6 = 0$ (D) $x^2 + y^2 - 5x - 4y - 6 = 0$

12. The value of 'c' for which the set, $\{(x, y) \mid x^2 + y^2 + 2x \le 1\} \cap \{(x, y) \mid x - y + c \ge 0\}$ contains only one point in common is
(A) $(-\infty, -1] \cup [3, \infty)$ (B) {-1, 3}
(C) {-3} (D) {-1}

13. The distance between the chords of contact of tangents to the circle, $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and the point (g, f) is
(A) $\sqrt{g^2 + f^2}$ (B) $\frac{\sqrt{g^2 + f^2 - c}}{2}$ (C) $\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$ (D) $\frac{\sqrt{g^2 + f^2 + c}}{2\sqrt{g^2 + f^2}}$

14. If $f(x, y) = x^2 + y^2 + 2ax + 2by + c = 0$ represents a circle. If f(x, 0) = 0 has equal roots, each being 2 and f(0, y) = 0 has 2 and 3 as its roots, then centre of circle is -
(A) $\left(2, \frac{5}{2}\right)$ (B) $\left(3, \frac{7}{9}\right)$ (C) $\left(-2, -\frac{5}{2}\right)$ (D) Data are inconsistent

15. The equation of the circle having normal at (3, 3) as the straight line y = x and passing through the point (2, 2) is
(A) $x^2 + y^2 - 5x + 5y + 12 = 0$ (B) $x^2 + y^2 + 5x - 5y + 12 = 0$
(C) $x^2 + y^2 - 5x - 5y - 12 = 0$ (D) $x^2 + y^2 - 5x - 5y + 12 = 0$

16. If two distinct chords drawn from the point (a, b) of the circle $x^2 + y^2 - ax - by = 0$ (where $ab \ne 0$) are bisected by the x-axis, then the roots of the quadratic equation $bx^2 - ax + 2b = 0$ are necessarily.
(A) imaginary (B) real and equal (C) real and unequal (D) rational

17. A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. The locus of the centre of the circle drawn on this chord as diameter is
(A) $x^2 + y^2 - ax = 0$ (B) $x^2 + y^2 + ax = 0$ (C) $x^2 + y^2 - ay = 0$ (D) $x^2 + y^2 - ay = 0$

18. Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y - 10 = 0$ then equation of one such circle is
(A) $x^2 + y^2 - 6x + 2y - 15 = 0$ (B) $x^2 + y^2 - 10x - 10y + 25 = 0$
(C) $x^2 + y^2 + 6x - 2y - 15 = 0$ (D) $x^2 + y^2 - 10x - 10y - 25 = 0$

19. A point 'P' moves in such a way that $\frac{PA}{PB} = \lambda$, where $\lambda \in (0, 1)$ is a constant and A, B are fixed points such that AB = a. Locus of P is a circle whose diameter is equal to
(A) $\frac{a\lambda}{1-\lambda^2}$ (B) $\frac{a\lambda}{2(1-\lambda^2)}$ (C) $\frac{2a\lambda}{1-\lambda^2}$ (D) none of these

20. A circle passes through the points A(1, 0), B(5, 0) and C(0, h). If $\angle ACB$ is maximum then
(A) $h = \sqrt{5}$ (B) $h = 2\sqrt{5}$ (C) $h = \sqrt{10}$ (D) $h = 2\sqrt{10}$

SET–III

Question based on write-up

☞ Let S = 0 be a circle, L = 0 be a line, then $S+\lambda L=0$ will represent the family of co–axial circle i.e. every radical axis being L = 0 for each pair of circle of the system $S+\lambda L=0$ is the same. Point circles of the system is called limiting points of the system.

1. If S = 0 and L = 0 are intersecting each other than limiting points are

(A) lying on S = 0 but not on L = 0
(B) points of intersection of S = 0 and L = 0
(C) real, not lying on S = 0 and L = 0
(D) imaginary

2. If S = 0 and L = 0 are non–intersecting each other than limiting points are

(A) real and coincident
(B) real and distinct
(C) imaginary
(D) none of these

3. If S = 0 and L = 0 are touching each other than limiting points are

(A) real and coincident
(B) real and distinct
(C) imaginary
(D) none of these

4. Limiting point of the system $S+\lambda L=0$ always

(A) lies on orthogonal circle
(B) lies on director circle of the system of S = 0
(C) lies on auxiliary circle of S = 0
(D) point of intersection of S = 0 and L = 0

5. The polar of the limiting point of a coaxial system w.r.t. any circle of the system is

(A) same for all the circles of the system
(B) different for all the circles of the system
(C) may or may not be same for all the circles of the system
(D) none of these

☞ A ball is moving around the circle $14x^2 + 14y^2 + 216x - 69y + 432 = 0$ in clockwise direction leaves it tangentially at the point P(–3, 6). After getting reflected from a straight line L = 0 it passes through the center of the circle. The perpendicular distance of this straight line L = 0 from the point P is $\frac{11}{13}\sqrt{130}$. You can assume that the angle of incidence is equal to the angle of reflection.

6. The equation of tangent to the circle at P is

(A) $2x - y + 12 = 0$
(B) $4x + 3y - 6 = 0$
(C) $3x - 2y + 21 = 0$
(D) $2x + 5y - 24 = 0$

7. Radius of the circle is

(A) $\frac{165}{14}$
(B) $\frac{165}{46}$
(C) $\frac{165}{28}$
(D) none of these

8. If angle between the tangent at P and the line through 'P' perpendicular to the line L = 0 is θ, then $\tan\theta$ is

(A) $\frac{2}{11}$
(B) $\frac{3}{11}$
(C) $\frac{4}{11}$
(D) none of these

9. Slope of the line L = 0 is

(A) $\frac{11}{7}$ (B) $\frac{7}{11}$ (C) $\frac{8}{7}$ (D) $\frac{9}{7}$

10. Equation of the line L = 0 is
(A) $7y - 9x + 41 = 0$
(B) $7y - 8x - 41 = 0$
(C) $11y - 7x - 41 = 0$
(D) $7y - 11x - 41 = 0$

Multiple choice question with one and more than one

11. The length of the tangent drawn from any point of the circle $x^2 + y^2 + 2gx + 2fy + \lambda = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + \mu = 0$ is

(A) $\sqrt{\mu-\lambda}$ (B) $\sqrt{\lambda+\mu}$ (C) $\sqrt{\lambda-\mu}$ (D) none of these

12. The equation of a tangent to the circle $x^2 + y^2 = 25$ passing through (–2, 11) is
(A) $4x + 3y = 25$
(B) $7x - 24y = 320$
(C) $3x + 4y = 38$
(D) $24x - 7y + 125 = 0$

13. A tangent drawn from the point (4, 0) to the circle $x^2 + y^2 = 8$ touches it at a point A in the first quadrant. The coordinates of another point B on the circle such that AB = 4, are
(A) (2, –2) (B) (–2, 2) (C) (2, 2) (D) (–2, –2)

14. The equation of tangents to the circle $x^2 + y^2 - 6x - 6y + 9 = 0$ drawn from the origin are
(A) x = y (B) x = 0 (C) y = 0 (D) x + y = 0

15. If the circle $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other, then α is equal to

(A) 0 (B) 1 (C) $-\frac{4}{3}$ (D) $\frac{4}{3}$

16. The equation of tangent to the circle $x^2 + y^2 = 25$, which is inclined at an angle of 30° to the axis of x, is
(A) $x\sqrt{3} + y + 10 = 0$
(B) $x\sqrt{3} - y + 10 = 0$
(C) $x - y\sqrt{3} + 10 = 0$
(D) $x - y\sqrt{3} - 10 = 0$

17. Equations of the circles concentric with the circle $x^2 - 2x + y^2 - 4y = 0$ and touching the circle $x^2 + y^2 + 2x = 1$, are
(A) $x^2 + y^2 - 2x - 4y = 0$
(B) $x^2 + y^2 - 2x - 4y + 3 = 0$
(C) $x^2 + y^2 - 2x - 4y - 13 = 0$
(D) $x^2 + y^2 - 2x - 4y - 1 = 0$

18. **True And False :**

(i) Circle on which the coordinates of any point are $(2+4\cos\theta,\ -1+4\sin\theta)$ where θ is parameter is $(x - 2)^2 + (y + 1)^2 = 16$.

(ii) The locus of the point of intersection of the lines $x = a\frac{1+t^2}{1-t^2},\ y = \frac{2at}{1-t^2}$ is a circle of radius a, t being parameter is it true of false ?

(iii) The equation $x^2 + y^2 + 2x - 10y + 30 = 0$ represents a circle.

(iv) The equation of the circle which passes through the point (4, 5) and has its centre at (2, 2) is $(x - 2)^2 + (y - 2)^2 = 13$.

(v) A circle has radius 3 units and its centre lies on $y = x - 1$. If it passes through the point (7, 3) it's equation is $x^2 + y^2 - 6x - 8y + 14 = 0$.

19. Fill In The Blanks :

(i) The parametric equation of the circle $x^2 + y^2 + x + \sqrt{3}\,y = 0$ are ....

(ii) The radical centre of three circles described on the three sides of a triangle as diameter is.......

(iii) The extremities of the diameter of a circle are (1, 2) and (3, 4). Then its centre is........., radius ....and equation is .... Also the tangents parallel to the diameter are..........

(iv) If the two circles $x^2 + y^2 - 3x + ky - 5 = 0$ are $4x^2 + 4y^2 - 12x - y - 9 = 0$ are concentric, then $k =$ ....

(c) The locus of a point which divides the joining A(–1, 1) and a variable point on the circle $x^2 + y^2 = 4$ in the ratio 3 : 2 is..........

20. Match the following :

Let $S \equiv x^2 + y^2 + 2gx + 2fy + c$, $T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$ and $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$, then match the following :

	A		**B**
(i)	Equation of tangent	**(A)**	$T = 0$
(ii)	Equation of chord of contact	**(B)**	$T = S_1$
(iii)	Equation of chord with mid–point (x_1, y_1)	**(C)**	$T = 0$, when points lies on the circle
(iv)	Equation of pair of tangents	**(D)**	$\sqrt{S_1}$
(v)	Centre of the circle $S = 0$	**(E)**	$x = \left(-g + \sqrt{g^2 + f^2 - c}\cos\theta\right)$ $y = \left(-f + \sqrt{g^2 + f^2 - c}\sin\theta\right)$
(vi)	Radius of the circle $S = 0$	**(F)**	S_1
(vii)	Length of the tangent	**(G)**	$SS_1 = T^2$
(viii)	Power of point $P(x_1, y_1)$	**(H)**	$\sqrt{g^2 + f^2 - c}$
(ix)	Parametric form of the circle $S = 0$	**(I)**	$(-g, -f)$

LEVEL–I ANSWER

1. $x^2 + y^2 - 10x - 6y + 9 = 0$

3. $x^2 + y^2 - 6x + 4y = 0,\ x^2 + y^2 + 2x - 8y + 4 = 0$

7. $\frac{20}{21}$

9. $x^2 + y^2 - 4x - 4y = 0$

LEVEL–II

8. **(ii)** $x^2 + y^2 + 16x + 14y - 12 = 0$

9. $3x^2 - y^2 + 2ax - a^2 = 0$, $(2bx - 2ay)^2$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. $k \in R - \{0\}$

2 $(4, 2), (-2, -6)$

3. $\frac{3}{4}$

4. 8 sq. units

5. $x^2 + y^2 - x = 0$

6. $10x - 3y - 18 = 0$

7. $x^2 + y^2 + 8x - 6y + 9 = 0$

8. $\frac{192}{25}$

9. $\left(-\frac{9}{5}, \frac{12}{5}\right)$ **or** $\left(\frac{9}{5}, -\frac{12}{5}\right)$

10. $2\sqrt{3}$ sq. units

11. 2

12. $16x^2 + 16y^2 - 48x + 16y + 31 = 0$

13. $\frac{a^2}{6}$ sq. units

14. $x^2 + y^2 - x - y = 0$

15. $x^2 + y^2 - \frac{2y}{\sqrt{3}} - 1 = 0$

16. $\left(\frac{1}{2}, \frac{1}{4}\right)$

17. 7

(B)

1.	T	**2.**	T

(C)

1.	AC	**2.**	B	**3.**	ABCD	**4.**	BC

(D)

1.	B	**2.**	C	**3.**	B	**4.**	C	**5.**	A
6.	A	**7.**	C	**8.**	D	**9.**	A	**10.**	D
11.	B	**12.**	D	**13.**	C	**14.**	A	**15.**	B
16.	A	**17.**	C	**18.**	A	**19.**	C	**20.**	D
21.	C	**22.**	B						

(E)

1.	A	**2.**	C	**3.**	C

(F)

1. A

(G)

1. a-pq, b-pq, c-qr, d-qr

IIT JEE PROBLEMS (SUBJECTIVE)

1. 75 **2.** $x^2+y^2+2\left(10\pm\sqrt{54}\right)x+55\pm\sqrt{54}=0$

4. $x^2+y^2+2ax+2py-b^2-q^2=0,\ \sqrt{a^2+p^2+b^2+q^2}$ **5.** $x^2+y^2-10x-4y+4=0$

7. $k=1$ **8.** $x^2+y^2+gx+fy+c/2=0$ **10.** $x^2+y^2+6x-3y-45=0$

11. $x^2+y^2+18x-2y+32=0$

12. $x^2+y^2+6x+2y-15=0;\ x^2+y^2-10x-10y+25=0$

13. $x^2+y^2-4x-6y+9=0$ **OR** $x^2+y^2-20x-22y+121=0,\ P(0,3),\ \theta=45^0$

14. $10\sqrt{2}-4\sqrt{10}\cong1.5$ **15.** $\left(4\sqrt{3}-3\right)x-\left(4+3\sqrt{3}\right)y-\left(39-2\sqrt{3}\right)=0$

16. $(a^2>2b^2)$ **17.** 1331/8 sq. units **18.** $x^2+y^2-7x+7y+12=0$

19. $\left(2,\frac{23}{3}\right)$ **20.** $\left(\frac{14}{5},\frac{8}{5}\right)$; $y=0$ & $24x-7y-16=0$

21. $x^2+y^2-6x-2y+1=0$

23. (0, -2), (6, 6); from (0, -2) equation of pair of tangents is $7x^2-24xy-48x=0$ & from (6, 6) it is $7x^2-24xy+60x+144y-612=0$

24. $(-\infty,-2)\cup(2,\infty)$ **28.** (2, -2) or (-2, 2)

30. $x^2+y^2+7x-11y+38=0$ **31.** $c_1:(x-4)^2+y^2=9;\ c_2:\left(x+\frac{4}{3}\right)^2+y^2=\frac{1}{9}$ common tangent between c & c_1 : $T_1=0;\ T_2=0$ and $x-1=0$; common tangent between c & c_2 : $T_1=0;\ T_2=0$ and $x+1=0$; common tangent between c_1 & c_2 : $T_1=0;\ T_2=0$ and $y=\pm\frac{5}{\sqrt{39}}\left(x+\frac{4}{5}\right)$ where

$T_1:\ x-\sqrt{3}y+2=0$ *and* $T_2:\ x+\sqrt{3}y+2=0$

32. $6x-8y+25=0$ & $6x-8y-25=0$

33. locus : $x^2+y^2+4x-12=0$, common tangents : $\sqrt{3}x-y\pm4+2\sqrt{3}=0$ **34.** ellipse

35. 5 **36.** $2x^2+2y^2-10x-5y+1=0$ **37.** $\sqrt{5}$

SET–I

1. B	**2.** D	**3.** A	**4.** B	**5.** D
6. C	**7.** B	**8.** C	**9.** A	**10.** A
11. C	**12.** C	**13.** B	**14.** A	**15.** A
16. C	**17.** B	**18.** B	**19.** B	**20.** C

SET–II

1. C	**2.** C	**3.** D	**4.**C	**5.** B
6. C	**7.** A	**8.** C	**9.** C	**10.** B
11. A	**12.** B	**13.** C	**14.** D	**15.** D
16. D	**17.** A	**18.** B	**19.** C	**20.** A

SET–III

1. D	**2.** B	**3.** A	**4.** A	**5.** A
6. B	**7.** C	**8.** B	**9.** D	**10.** A
11. AC	**12.** AD	**13.** AB	**14.** BC	**15.** CD
16. CD	**17.** BC			
18. **(i)** T	**(ii)** T	**(iii)** F	**(iv)** T	**(v)** F

19. **(i)** $x = -\frac{1}{2} + \cos\theta,\ y = -\frac{5}{2} + \sin\theta$ **(ii)** Orthocentre

(iii) $(2, 3),\ \sqrt{2},\ y = x + 3,\ y = x - 1$ **(iv)** $k = -\frac{1}{4}$

(v) $25(x^2 + y^2) + 20(x - y) - 28 = 0$

20. (i, c), (ii, a), (iii, b), (iv, g), (v, i), (vi, h), (vii, d), (viii, f), (ix, e)