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ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

DISTANCE LEARNING PROGRAMME

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NURTURE TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (MAIN) 2017

Test Type : **ALL INDIA OPEN TEST** (MAJOR) Test Pattern : JEE-Main

TEST # 01 **TEST DATE : 31 - 01 - 2016**

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	4	1	3	3	4	4	2	4	4	2	2	1	3	4	2	2	2	2	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	3	2	1	2	3	2	2	2	1	4	1	2	1	3	2	3	1	2	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	1	3	1	4	3	1	2	4	4	4	3	3	2	4	3	2	3	2	1
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	1	2	1	1	4	3	2	4	1	2	4	3	2	3	4	2	2	3	3
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	1	1	3	1	3	4	1	1	3										

HINT – SHEET

1. Ans. (4)

Sol. $f = \frac{V}{\lambda_1} = \frac{1}{\lambda_1}\sqrt{\frac{T_1}{\mu}}$

$f = \frac{V}{\lambda_1} = \frac{1}{\lambda_1}\sqrt{\frac{T_2}{\mu}}$

$\frac{1}{\lambda_1}\sqrt{\frac{T_1}{\mu}} = \frac{1}{\lambda_2}\sqrt{\frac{T_2}{\mu}}$

$\sqrt{\frac{T_1}{\mu}} = \left(\frac{\lambda_1}{\lambda_2}\right)\sqrt{\frac{T_2}{\mu}}$

$T_1 = (2.2)^2 \cdot T_2$

$Mg = 4.84\left[Mg - \frac{M}{(S\cdot G)}\cdot g\right]$

$1 = 4.84 - \frac{4.84}{S.G}$

$\frac{4.84}{S.G} = 3.84 \Rightarrow S.G = \frac{4.84}{3.84}$

$\Rightarrow S.G = 1.26$

2. Ans. (4)

Sol. Area under the velocity-time graph is displacement.

If area X = area Y then it implies that magnitudes of displacements carresponding to areas X and Y are equal.

3. Ans. (1)

Sol. $\Delta KE = 0$, since V = constant

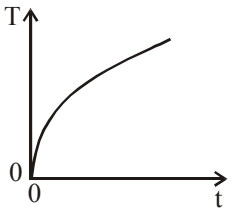
$KE = \frac{1}{2}mv^2 = \text{constant}$

Mass is falling down so potential energy decreases at rate

$= mg\left(\frac{-dx}{dt}\right) = -mgV$ [decreases]

4. Ans. (3)

Sol.



$$\frac{dq}{dt} = \text{constant} = i \text{ (lets say)}$$

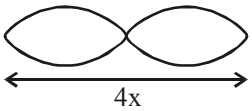
$$i = aT^3 \frac{dT}{dt}, \quad \frac{dT}{dt} = \frac{i}{aT^3}$$

$$dT = \frac{i}{aT^3} dt$$

5. Ans. (3)

Sol. Middle one paper does not fall.

Hence there must be a node.



$$4x = 2 \times \frac{\lambda}{2} \Rightarrow \lambda = 4x$$

6. Ans. (4)

Sol. $V = \omega R$

ω = constant

$$\frac{dV}{dt} = \omega \cdot \frac{dR}{dt}$$

$$\frac{dR}{dt} = \text{constant}$$

$$\Rightarrow \frac{dV}{dt} = \text{constant}$$

slope is constant

v-t graph will be straight line.

7. Ans. (4)

Sol. Speed is maximum at the mean position

Distance between extreme position = 2A.

2A = 70

A = 35 cm

8. Ans. (2)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

given graph : $RT_2 = \frac{\left(2 \times 10^5\right) \times 2}{n}$

$$RT_1 = \frac{\left(1 \times 10^5\right) \times 1}{n}$$

n : number of moles

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}} = 2:1 \Rightarrow 2:1$$

9. Ans. (4)

Sol. Acceleration is maximum at extreme position. Velocity is maximum at mean position.

10. Ans. (4)

Sol. $V \propto R^3$, $R \to 2R$ then $V \to 8V$

Work = P (8V – V) = 7PV [isobaric process]

11. Ans. (2)

Sol. As point R, S and T have zero intensity

$\Rightarrow$ Destructive interference will occur at these points.

$$\frac{\lambda}{2} = 1.5$$

$\lambda = 3m$

$v = \lambda f$

f = 100 Hz

12. Ans. (2)

Sol. Soap bubble has two surface

Total length = $2(2\pi r)$

$$T = \frac{F}{4\pi r} \Rightarrow F = \left(4\pi r\right) \times T$$

13. Ans. (1)

Sol. If n' represents the apparent frequency and n the actual one, then use the relation $n' = n\left[\frac{\left(v \pm w\right)}{\left(v \pm w\right) - v_s}\right]$ where v is the velocity of sound with respect to wind, w is the velocity of wind and v_s that of the source. Note that in this case the observer at rest.

14. Ans. (3)

Sol. For particle A

$y = x$

$$\frac{dy}{dt} = \frac{dx}{dt} \Rightarrow v_{y_A} = v_{x_A}$$

$\& \; v_B = 3\hat{i}$

$\& \; v_{x_A} = v_B$

$\Rightarrow v_{x_A} = 3\hat{i} \Rightarrow v_{y_A} = 3\hat{j}$

$\Rightarrow \vec{v}_A = 3\hat{i} + 3\hat{j} \Rightarrow \left|\vec{v}_A\right| = 3\sqrt{2}$ m/s

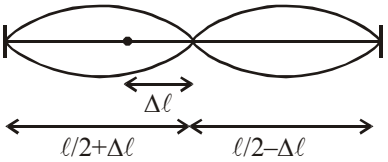
15. Ans. (4)

16. Ans. (2)

Sol. Note that the beat frequency is $(n_1 - n_2)$ and the corresponding vibrating lengths are $(\ell/2 - \Delta)$ and $(\ell/2 + \Delta\ell)$.

Given : $n_0 = \frac{V}{2\ell}$

$\Rightarrow \frac{V}{\ell} = 2n_0$



$$n_1 = \frac{V}{2\left(\frac{\ell}{2}+\Delta\ell\right)} = \frac{V}{\ell+2\Delta\ell}$$

$$n_2 = \frac{V}{2\left(\frac{\ell}{2}-\Delta\ell\right)} = \frac{V}{\ell-2\Delta\ell}$$

Beat frequency $= n_2 - n_1 = \frac{V}{\ell-2\Delta\ell} - \frac{V}{\ell+2\Delta\ell}$

$$= \frac{V}{\ell\left(1-\frac{2\Delta\ell}{\ell}\right)} - \frac{V}{\ell\left(1+\frac{2\Delta\ell}{\ell}\right)}$$

$$= \frac{V}{\ell}\left(1-2\frac{\Delta\ell}{\ell}\right)^{-1} - \frac{V}{\ell}\left(1+2\frac{\Delta\ell}{\ell}\right)^{-1}$$

$$= \frac{V}{\ell}\left(1+2\frac{\Delta\ell}{\ell}\right) - \frac{V}{\ell}\left(1-2\frac{\Delta\ell}{\ell}\right) \quad [(1+x)^n = 1+nx]$$

$$= \frac{V}{\ell} + \frac{2V}{\ell}\frac{\Delta\ell}{\ell} - \frac{V}{\ell} + \frac{2V}{\ell}\frac{\Delta\ell}{\ell}$$

$$= 2\cdot 2n_0 \cdot \frac{\Delta\ell}{\ell} + 2\cdot 2n_0 \cdot \frac{\Delta\ell}{\ell} = 8n_0\left(\frac{\Delta\ell}{\ell}\right)$$

17. Ans. (2)

Sol. New reading = weigth of water (X) + weight of water displaced by the object (Z)

$\Rightarrow$ New reading = X + Z

18. Ans. (2)

Sol. Net force and net torque should be zero.

19. Ans. (2)

Sol. $\frac{d\theta}{dt} = \frac{dm}{dt} \times S \times dT$

$$\Rightarrow \frac{6.7\times10^9}{60} = \frac{dm}{dt} \times 4200 \times (14-6)$$

$$\Rightarrow \frac{dm}{dt} = \frac{6.7\times10^9}{4200\times8\times60}$$

20. Ans. (4)

Sol. Note that pV^γ = constant and that ρ is inversely proportional to V.

Adiabatic process

pV^γ = constant

$$p\left(\frac{M}{\rho}\right)^\gamma = \text{constant}$$

$p\cdot M^\gamma \cdot \rho^{-\gamma}$ = constant (M^γ is constant)

$p\cdot\rho^{-\gamma}$ = constant

21. Ans. (2)

Sol. Lets say the displacement of cylinder from its equilibrium position is 'x'.

$\Rightarrow$ ma = $-\rho$gAx

$$\Rightarrow a = -\left(\frac{\rho gA}{m}\right)x$$

$$\Rightarrow \omega_{cylinder} = \sqrt{\frac{\rho gA}{m}} \quad \text{... (i)}$$

$$\Rightarrow \omega_{spring\text{-}block} = \sqrt{\frac{k}{m}} \quad \text{... (ii)}$$

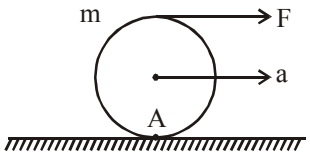
$\Rightarrow \omega_{cylinder} = \omega_{spring\text{-}block}$ ... (iii) [given]

$\Rightarrow$ K = ρgA [from (i), (ii), (iii)];

K = 27 N/m

22. Ans. (3)

Sol.



Writting torque equation about an axis passing through A, perpendicular to the plane of paper.

$$\Rightarrow F \times 2R = \left(\frac{2}{5}mR^2 + mR^2\right) \times \propto \quad \text{... (i)}$$

$a = R\propto$

$$\Rightarrow F = \frac{7}{10}ma$$

23. Ans. (2)

Sol. During SHM speed will be minimum in 1 and 5 region.

Hence probability of greatest number of hit will increase.

24. Ans. (1)

Sol. $\Delta x = \frac{\lambda}{2}$ Δx path difference

$\Delta\phi = k\Delta x$

$= \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$

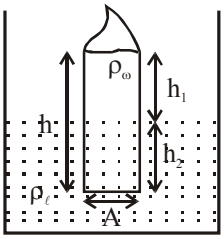
Hence destructive interference will occur at point P.

$A_{resultant} = \sqrt{(2a)^2 + a^2 - 4a^2} = a.$

It will remain constant with time.

25. Ans. (2)

Sol.



$\frac{dh}{dt} = 4$ cm/hr given $\rho_\ell = 2\rho_w$

$F_{boyant} = \rho_\omega(Ah)g = (Ah_2)\rho_\ell \cdot g$

$\rho_\omega \cdot Ahg = \rho_\ell \cdot Ah_2 g$

$\rho_\omega \cdot h = \rho_\ell \cdot h_2 = 2\rho_\omega \cdot h_2$

$h = 2h_2$

$\frac{dh}{dt} = \frac{2dh_2}{dt} \Rightarrow 4 = \frac{2dh_2}{dt} \Rightarrow \frac{dh_2}{dt} = 2$ cm/hr

Therefore net fall of upper end $= \frac{dh}{dt} - \frac{dh_2}{dt}$

= 4cm/hr – 2cm/hr

= 2cm/hr [rate of fall of upper end of candle]

26. Ans. (3)

Sol. $0 = 10 \log\left(\frac{I_0}{10^{-12}}\right) \Rightarrow I_0 = 10^{-12}$

$120° = 10 \log\left(\frac{I}{10^{-12}}\right)$

$\Rightarrow \frac{T}{10^{-12}} = 10^{12} \Rightarrow I = 1 \Rightarrow \frac{I_0}{I} = 10^{-12}$

27. Ans. (2)

Sol. $2 \times 40 \times 10^{-3} \times 10 = 16$ t

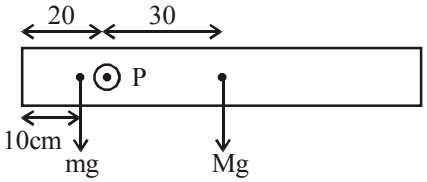
50×10^{-3} s = t

28. Ans. (2)

Sol. Amount of energy intercepted $\propto$ [Amplitude]^2 × [surface area]

29. Ans. (2)

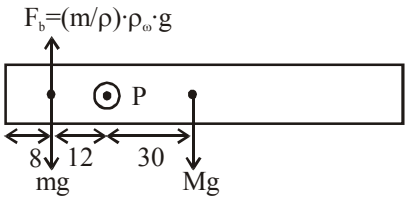
Sol.



$\tau_p = 0$

$Mg \times 30 = mg \times 10$

$\Rightarrow m = 3M$ ... (1)



$\tau_{p\ net} = 0$

$\Rightarrow Mg \times 30 + \frac{m}{\rho} \cdot \rho_\omega \cdot g \times 12 = mg \times 12$

$M \times 30 + \frac{3M}{\rho_r} \times 12 = (3M) \times 12$

$\rho_r = 6$

30. Ans. (1)

Sol. A question of units and dimensions.

Only the dimension of option (A) $\sqrt{g\lambda}$ matches the dimesion of speed.

31. Ans.(4)

$14ClO_3^- + 6N_2H_4 \rightarrow 12NO_3^- + 14Cl^-$

n-factor 6 × 14

32. Ans.(1)

$40 \times 0.1 = 30 \times m \times 2$

$M = \frac{4}{15 \times 4} = \frac{1}{15}$

$0.1 \times V = \frac{1}{15} \times 60$

V = 40 ml

33. Ans.(2)

$\frac{124 \times 0.15}{\frac{1240}{640} \times 1.6 \times 10^{-19}} = \frac{124 \times 640}{124 \times 16} \times 10^{19} \times 0.15 = 6 \times 10^{19}$

34. Ans.(1)

m + 2

$l = 2$

n = 3

$2\pi r = n\lambda$

$\lambda = \frac{2\pi \times 0.529 \times 9}{3}$

= 9.97 Å

35. Ans.(3)

36. Ans.(2)

37. Ans.(3)

38. Ans.(1)

PM = ZdRT

PV = ZRT

$V = \frac{1.5 \times 0.08 \times 400}{2}$ = 24 litre

39. Ans.(2)

40. Ans.(4)

$2NO_2(g) \rightleftharpoons N_2O_4$ at 300K

P_1 P_2

$kp = \frac{P_2}{P_1^2} = 2$ ..... (i)

$P_1 + P_2 = 10$ .....(ii)

P_2 = 8 atm P_1 = 2 atm

$2NO_2 \rightleftharpoons N_2O_4$

(1 + 2P) (4 – P)

$P_T = (5 + P)$

$K_p = \frac{(4-P)}{(1+2P)^2} = 2$

P = 0.19 atm

P_T = 5.19 atm

41. Ans.(3)

$$\frac{100\times0.8}{16}+\frac{100\times1.2}{32}=\frac{80\times2+120}{32} = \frac{280}{32}$$

$$X_A = \frac{\frac{100\times0.8}{16}}{\frac{280}{32}} = \frac{80\times2}{280} = \frac{4}{7}$$

42. Ans.(1)

$\frac{18}{90} = \frac{1}{5}$ mole

$1000 \times \frac{50}{400} \times \frac{1}{5} \times 2 = 0.1 \times V$

V = 500 ml

43. Ans.(3)

$$\frac{15200}{\frac{1}{\lambda}} = \frac{R_H \times \left[\frac{1}{4}-\frac{1}{9}\right]}{R_H \times 9\left[\frac{1}{1}-\frac{1}{4}\right]} = \frac{\frac{5}{9\times4}}{\frac{9\times3}{4}} = \frac{5}{9\times9\times3}$$

$\frac{1}{\lambda} = \frac{15200}{5} \times 9\times9\times3$ = 738720

44. Ans.(1)

45. Ans. (4)

46. Ans. (3)

47. Ans. (1)

48. Ans. (2)

49. Ans. (4)

50. Ans. (4)

51. Ans. (4)

52. Ans. (3)

53. Ans. (3)

54. Ans. (2)

55. Ans. (4)

56. Ans. (3)

57. Ans. (2)

58. Ans. (3)

59. Ans. (2)

60. Ans. (1)

61. Ans. (3)

Shift origin at (102,–4)

62. Ans. (1)

Divide by $-x^2$

$\left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right) - 2 = 0$

$m^2 + m - 2 = 0$

Replace m by $-\frac{1}{m}$

$\frac{1}{m^2} - \frac{1}{m} - 2 = 0$

$2m^2 + 2m - 1 = 0$

$\frac{2y^2}{x^2} + \frac{2y}{x} - 1 = 0$

$x^2 - xy - 2y^2 = 0$

63. Ans. (2)

x + y – 1 = 0 (R.A)

$A = \frac{1}{2}$

$\frac{1}{A} = 2$

64. Ans. (1)

$y = (x-1)^3 - 1$

$x = (y-1)^3 - 1$

$f^{-1}(x) = 1 + \sqrt[3]{x+1}$

65. Ans. (1)

$x^2 + x + 1 = 3 \; ; \; x \neq 1$

$x^2 + x - 2 = 0$

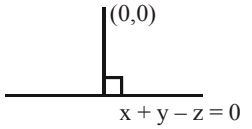
$(x + 2)(x - 1) = 0$

$-2, 1$ (rejected)

66. Ans. (4)

$E + 1 = (2^a)^3 + (2b)^3 + 1 - 3.2^a.2^b.1$

$E_{min} = p = -1; \; 2^a = 2^b = 1 \quad \Rightarrow \quad a = b = 0$



$L = \sqrt{2}$

67. Ans. (3)

$1 > 95 - 30\pi > 63 - 20\pi$

$\sin 1 > \sin 95 > \sin 63$

68. Ans. (2)

$\cos x + \sin x \geq 0$ & $\cos x - \sin x \leq 0$

$\Rightarrow \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

69. Ans. (4)

$n(A \times A) = 9$

Relation containing (1,2), (2,1), (a,a) $= 2^3$

Relation containing

(1,2), (2,1), (1,3), (3,1),(a,a) $= 2^3$

Relation containing

(1,2), (2,1), (2,3), (3,2),(a,a) $= 2^3$

Relation containing

(1,2), (2,1), (1,3), (3,1),(2,3), (3,1)(a,a) $= 2^3$

$\Rightarrow$ Total number of relation = 32.

70. Ans. (1)

$2\left(n(A) - n(AB)\right) = n(B) - n(AB)$

$2n(A) - n(B) = n(AB)$

$n(A) + 3n(B) = 5n(AB)$

$\Rightarrow \; n(A) + 3n(B) = 10n(A) - 5n(B)$

$8n(B) = 9n(A)$

$\frac{n(A)}{n(B)} = \frac{8}{9}$

$n(A) = 8k; n(B) = 9k$

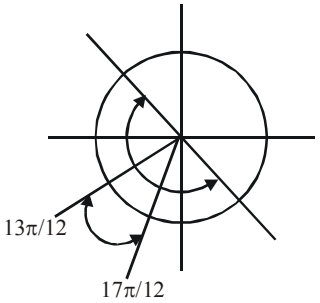
$\Rightarrow n(AB) = 7k$

$n(A \cup B) = 10k \leq 10$

$k = 1$

71. Ans. (2)

$\sin\theta + \cos\theta < 0$



$\left(\sin\theta + \cos\theta\right)^2 > \frac{3}{2}$

$\sin 2\theta > \frac{1}{2}$

$2n\pi + \frac{\pi}{6} < 2\theta < 2n\pi + \frac{5\pi}{6}$

$n\pi + \frac{\pi}{12} < \theta < n\pi + \frac{5\pi}{12}$

72. Ans. (4)

$k \geq \frac{2x}{\underbrace{x^2 + 1}_{[-1,1]}}$

$\Rightarrow k \geq -1$

73. Ans. (3)

Circumcentre of ΔABC

$\Rightarrow$ Orthocentre of ΔPQR

74. Ans. (2)

$x^{10} + x^2 + \frac{1}{x^{12}} \geq 3$ (AM.GM)

Equality occurs of $x = \pm 1$

But $\sec^{-1}x$ takes maximum at $x = -1$

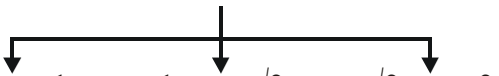
$f_{min}(x) = f(-1) = 3 + \frac{1}{\pi + 1}$

75. Ans. (3)

$x^2 - 1 < [x^2] \leq x^2$

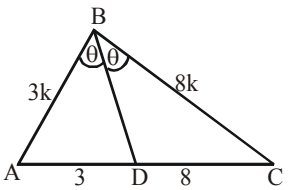
$[x^2] = 2x - 1$

$\Rightarrow x^2 < 2x \Rightarrow 0 < x < 2$



$0 < x < 1$	$1 \leq x < \sqrt{2}$	$\sqrt{2} \leq x < 2$
$\Rightarrow x = 1/2$	$x=1$	$x = 3/2$
	sum = 3	

76. Ans. (4)



$3k + 8k > 11; \; 8k$

$k > 1; \; 11 + 3k > 8k$

$1 < k < \frac{11}{5}$

$k = 2$

Perimeter = 33

77. Ans. (2)

$$\sum_{n=1}^{\infty}\frac{1}{2^n}\left(\frac{1}{2^1}+\frac{2}{2^2}+\frac{3}{2^3}+....+\frac{n-1}{2^{n-1}}\right)$$

$$\sum_{n=1}^{\infty}\frac{1}{2^n}\left(2-\frac{n+1}{2^{n-1}}\right)$$

$$\sum_{n=1}^{\infty}\frac{1}{2^{n-1}}-\frac{n+1}{2^{2n-1}}=\frac{4}{9}$$

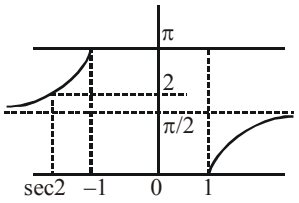
78. Ans. (2)

$$\sin^{-1}\sqrt{1-x^2}=\sin^{-1}\sqrt{\frac{2-x}{2}}$$

$2 - 2x^2 = 2 - x$

$$x=\frac{1}{2}$$

79. Ans. (3)



$$Df \to (-\infty,\sec 2]\cup[1,\infty)$$

80. Ans. (3)

$$x-\frac{1}{x}=1 \text{ or } x-\frac{1}{x}=-1$$

$x^2 - x - 1 = 0$ $\qquad$ $x^2 + x - 1 = 0$

$$\frac{1+\sqrt{5}}{2} \qquad \frac{\sqrt{5}-1}{2}$$

required sum = $\sqrt{5}$

81. Ans. (2)

82. Ans. (1)

Graph is

symmetric about x = 6

$–2b = 6 \Rightarrow b = –3$

83. Ans. (1)

84. Ans. (3)

$0 \le \text{LHS} < 1$

{as it is absolute difference of distance of point P(x,0) from A(2,0) & B(3,0)} $0 \le 2 - a \le 1$

$\Rightarrow 1 \le a \le 2$ $\quad$ sum = 3

85. Ans. (1)

Put $x = \sin\theta$

$$y=\frac{|\sin\theta|}{1+|\cos\theta|}$$

86. Ans. (3)

coefficient of x^{100}

non-negative integral solution of

$a + b + c = 100$

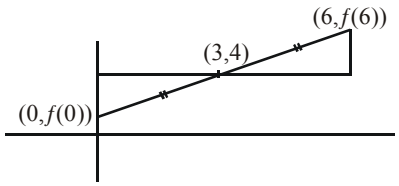
$^{102}C_2$

87. Ans. (4)

$\because$ 'f' is odd f^{-1} is also odd

$\Rightarrow$ sum = 0

88. Ans. (1)



$f(0) + f(0) = 8$

similarly every pair has sum 8.

89. Ans. (1)

$^{10}C_2$

90. Ans. (3)

6n, 7e, 2d, 2s, 2t,1i

number of selections = $7 \times 8 \times 3 \times 3 \times 3 \times 2 - 1$

= 3023