Limit of Functions

Exercise - 1 (Objective Questions)

1. $\underset{n\to\infty}{\text{Limit}} \dfrac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}} =$

(A) 5 (B) 3 (C) 1 (D) zero

2. $\underset{x\to -1}{\text{Limit}} \dfrac{\cos 2-\cos 2x}{x^2-|x|} =$

(A) $2\cos 2$ (B) $-2\cos 2$ (C) $2\sin 2$ (D) $-2\sin 2$

3. The value of $\underset{x\to 0}{\text{Limit}} \dfrac{1}{x}\sqrt{\dfrac{1-\cos 2x}{2}}$ is:

(A) 1 (B) -1 (C) 0 (D) none

4. $\underset{x\to 0}{\text{Limit}} \sin^{-1}(\sec x)$.

(A) is equal to $\pi/2$ (B) is equal to 1 (C) is equal to zero (D) none of these

5. $\underset{x\to 5}{\text{Limit}} \dfrac{x^2-9x+20}{x-[x]}$ where [x] is the greatest integer not greater than x:

(A) is equal to 1 (B) 0 (C) 4 (D) none

6. $\underset{x\to -\pi}{\text{Limit}} \dfrac{|x+\pi|}{\sin x}$:

(A) is equal to -1 (B) is equal to 1 (C) is equal to π (D) does not exist

7. $\underset{x\to 3}{\text{Limit}} \dfrac{(x^3+27)\ln(x-2)}{(x^2-9)} =$

(A) -8 (B) 8 (C) 9 (D) -9

8. $\underset{x\to 1}{\text{Limit}} \dfrac{\sum_{k=1}^{100} x^k-100}{x-1} =$

(A) 0 (B) 5050 (C) 4550 (D) -5050

9. $\underset{x\to\infty}{\text{Limit}} \left(\sqrt{(x+a)(x+b)}-x\right) =$

(A) $\sqrt{ab}$ (B) $\dfrac{a+b}{2}$ (C) ab (D) none

10. $\underset{x\to\infty}{\text{Limit}} \dfrac{x^3 \,.\, \sin\frac{1}{x}+x+1}{x^2+x+1} =$

(A) 0 (B) 1/2 (C) 1 (D) none

11. $\underset{n\to\infty}{\text{Limit}} \dfrac{(n+2)!+(n+1)!}{(n+3)!}$, $n \in N =$

(A) 0 (B) 1 (C) 2 (D) – 1

12. $\underset{x\to 0}{\text{Limit}} \, |x|^{\sin x} =$

(A) 0 (B) 1 (C) – 1 (D) none of these

13. $\underset{x\to\infty}{\text{Limit}} \left(\dfrac{x^2-2x+1}{x^2-4x+2}\right)^x =$

(A) 1 (B) 2 (C) e^2 (D) e

14. The values of a and b such that $\underset{x\to 0}{\text{Lim}} \dfrac{x(1+a\cos x)-b\sin x}{x^3} = 1$ are

(A) $\frac{5}{2}, \frac{3}{2}$ (B) $\frac{5}{2}, -\frac{3}{2}$ (C) $-\frac{5}{2}, -\frac{3}{2}$ (D) $-\frac{5}{2}, \frac{3}{2}$

15. $\underset{x\to 0}{\text{Limit}} \dfrac{2\left(\sqrt{3}\sin\left(\frac{\pi}{6}+x\right)-\cos\left(\frac{\pi}{6}+x\right)\right)}{x\sqrt{3}\left(\sqrt{3}\cos x-\sin x\right)} =$

(A) – 1/3 (B) 2/3 (C) 4/3 (D) – 4/3

16. If $f(x) = \begin{cases} x-1, & x \ge 1 \\ 2x^2-2, & x < 1 \end{cases}$, $g(x) = \begin{cases} x+1, & x > 0 \\ -x^2+1, & x \le 0 \end{cases}$ and $h(x) = |x|$

then find $\underset{x\to 0}{\text{Lim}} f(g(h(x)))$

(A) 1 (B) 0 (C) –1 (D) does not exists

17. $\underset{x\to 1}{\text{Limit}} (1 - x + [x - 1] + [1 - x]) =$ where [x] denotes greatest integer function.

(A) 0 (B) 1 (C) – 1 (D) does not exist

18. $\underset{x\to 0}{\text{Lt}} \left[\dfrac{\sin[x-3]}{[x-3]}\right]$, where [.] denotes greatest integer function is :

(A) 0 (B) 1 (C) does not exist (D) sin 1

19. Let $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right) & x \ne 0 \\ 0 & x = 0 \end{cases}$, then $\lim_{x\to\infty} f(x)$ equals

(A) 0 (B) – 1/2 (C) 1 (D) none of these.

20. $\lim_{x\to a^-} \left(\dfrac{|x|^3}{a}-\left[\dfrac{x}{a}\right]^3\right)$ (a > 0), where [x] denotes the greatest integer less than or equal to x is

(A) a^2+1 (B) a^2-1 (C) a^2 (D) $-a^2$

21. Let α, β be the roots of $ax^2 + bx + c = 0$, where $1 < \alpha < \beta$. Then $\underset{x\to x_0}{\text{Lim}} \dfrac{|ax^2+bx+c|}{ax^2+bx+c} = 1$ then which of the following statements is incorrect

(A) $a > 0$ and $x_0 < 1$
(B) $a > 0$ and $x_0 > \beta$
(C) $a < 0$ and $\alpha < x_0 < \beta$
(D) $a < 0$ and $x_0 < 1$

22. $\underset{n\to\infty}{\text{Limit}} \dfrac{1n+2(n-1)+3(n-2)+\ldots\ldots +n.1}{1^2+2^2+3^2+\ldots\ldots +n^2}$ has the value :

(A) $\frac{1}{2}$
(B) $\frac{1}{3}$
(C) $\frac{1}{4}$
(D) 1

23. $\underset{x\to 0}{\text{Lim}} \left[(1-e^x)\dfrac{\sin x}{|x|}\right]$ is (where [·] represents greatest integral part function)

(A) – 1
(B) 1
(C) 0
(D) does not exist

24. If $l = \lim\limits_{x\to\infty} (\sin\sqrt{x+1} - \sin\sqrt{x}\,)$ and $m = \lim\limits_{x\to-\infty} [\sin\sqrt{x+1} - \sin\sqrt{x}\,]$ where [.] denotes the greatest integer function then :

(A) $l = m = 0$
(B) $l = 0$; m is undefined
(C) l, m both do not exist
(D) $l = 0$, $m \neq 0$ (although m exist)

25. If $f(x) = \sum\limits_{\lambda=1}^{n}\left(x-\dfrac{1}{\lambda}\right)\left(x-\dfrac{1}{\lambda+1}\right)$ then $\lim\limits_{n\to\infty} f(0)$ is.

(A) 1
(B) –1
(C) 2
(D) None

26. The limit $\lim\limits_{\theta\to 0}\left(\left[\dfrac{n\sin\theta}{\theta}\right]+\left[\dfrac{n\tan\theta}{\theta}\right]\right)$, where [x] is the greatest integer function and $n \in I$, is

(A) 2n
(B) 2n + 1
(C) 2n – 1
(D) does not exist

27. The limit $\underset{x\to\infty}{\text{Limit}}\ x - x^2 \ln\left(1+\dfrac{1}{x}\right)$ is equal to :

(A) 1/2
(B) 3/2
(C) 1/3
(D) 1

28. $\lim\limits_{x\to\pi/2}\left[\dfrac{x-\frac{\pi}{2}}{\cos x}\right]$ is : (where [.] represents greatest integer function.

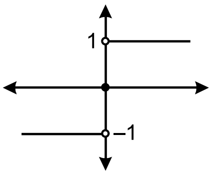
(A) – 1
(B) 0
(C) – 2
(D) does not exist

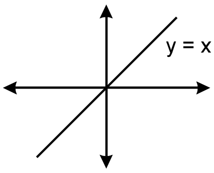
29. If $f(x) = \sin x$, $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \ldots.$
$= 2$, otherwise and
$g(x) = x^2 + 1$, $x \neq 0, 2$
$= 4$, $x = 0$
$= 5$, $x = 2$

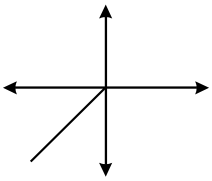
then $\underset{x\to 0}{\text{Limit}}\ g[f(x)]$ is :

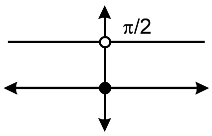
(A) 1
(B) 0
(C) 4
(D) does not exists

30. The graph of the function $f(x) = \underset{t\to 0}{\text{Lim}}\left(\dfrac{2x}{\pi}\cot^{-1}\dfrac{x}{t^2}\right)$, is

(A) 

(B) 

(C) 

(D) 

31. The value of $\lim\limits_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to:

(A) 1/5 (B) 1/6 (C) 1/4 (D) 1/2

32. $\lim\limits_{x\to\infty} \frac{e^x\left(\left(2^{x^n}\right)^{\frac{1}{e^x}} - \left(3^{x^n}\right)^{\frac{1}{e^x}}\right)}{x^n}$, $n \in N$ is equal to :

(A) 0 (B) ln(2/3) (C) ln(3/2) (D) none

33. $\lim\limits_{y\to0}\left(\lim\limits_{x\to\infty}\frac{\exp\left(x\ln\left(1+\frac{ay}{x}\right)\right)-\exp\left(x\ln\left(1+\frac{by}{x}\right)\right)}{y}\right) =$

(A) a + b (B) a – b (C) b – a (D) – (a + b)

Exercise - 2 (Subjective Questions)

1. Evaluate the following limits, where [.] represents greatest integer function and { . } represents fractional part function

(i) $\lim\limits_{x\to\frac{\pi}{2}} [\sin x]$ (ii) $\lim\limits_{x\to2}\left\{\frac{x}{2}\right\}$ (iii) $\lim\limits_{x\to\pi} \operatorname{sgn}[\tan x]$

2. If f(x) = $\begin{cases} x^2+2 & , x \ge 2 \\ 1-x & , x < 2 \end{cases}$ and g(x) = $\begin{cases} 2x & , x > 1 \\ 3-x & , x \le 1 \end{cases}$, evaluate $\lim\limits_{x\to1} f(g(x))$.

3. Evaluate each of the following limits, if exists

(i) $\lim\limits_{x\to2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$ (ii) $\lim\limits_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$, $a \ne 0$

4. Evaluate the following limits, if exists

(i) $\lim\limits_{x\to0}\frac{\tan 3x-2x}{3x-\sin^2 x}$ (ii) $\lim\limits_{x\to a}\frac{(x+2)^{\frac{5}{2}}-(a+2)^{\frac{5}{2}}}{x-a}$

(iii) $\lim\limits_{x\to0}\frac{x\left(e^{2+x}-e^2\right)}{1-\cos x}$

5. Evaluate the following limits, if exist :

(i) $\lim\limits_{x\to\infty}\sqrt{x^2+x-1}-x$ (ii) $\lim\limits_{x\to\infty}\left(\frac{1}{x^2}+\frac{2}{x^2}+\ldots.+\frac{x}{x^2}\right)$

(iii) $\lim\limits_{x\to\infty}\left\{\cos\left(\sqrt{x+1}\right)-\cos\left(\sqrt{x}\right)\right\}$ (iv) $\lim\limits_{x\to\infty}\sqrt{x^2-8x}+x$

6. Evaluate the following limits using expansions :

(i) $\lim\limits_{x\to 0} \dfrac{e^x - 1 - \sin x - \dfrac{\tan^2 x}{2}}{x^3}$

(ii) If $\lim\limits_{x\to 0} \dfrac{a + b\sin x - \cos x + ce^x}{x^3}$ exists, then find values of a, b, c. Also find the limit

7. Evaluate $\lim\limits_{x\to\infty} \dfrac{[1.\,2x] + [2\,.\,3x] + ... + [n\,.\,(n+1)\,x]}{n^3}$ where [.] denotes greatest integer function

8. If f(x) = $\mathop{\text{Lim}}\limits_{n\to\infty} \dfrac{x^{2n} - 1}{x^{2n} + 1}$, find range of f(x).

9. Evaluate the following limits

(i) $\mathop{\text{Limit}}\limits_{x\to 1} \dfrac{(\ln(1+x) - \ln 2)(3.4^{x-1} - 3x)}{[(7+x)^{\frac{1}{3}} - (1+3x)^{\frac{1}{2}}].\sin(x-1)}$ (ii) $\mathop{\text{Limit}}\limits_{x\to 4} \dfrac{(\cos\alpha)^x - (\sin\alpha)^x - \cos 2\alpha}{x-4}, \alpha \in \left(0, \dfrac{\pi}{2}\right)$

10. Evaluate the following limits

(i) $\mathop{\text{Limit}}\limits_{x\to\infty} x^3 \left\{\sqrt{x^2 + \sqrt{1 + x^4}} - x\sqrt{2}\right\}$ (ii) $\mathop{\text{Limit}}\limits_{x\to -\infty} \dfrac{x^5 \tan\left(\dfrac{1}{\pi x^2}\right) + 3|x|^2 + 7}{|x|^3 + 7|x| + 8}$

11. Evaluate the following limits

(i) $\mathop{\text{Limit}}\limits_{x\to 0} \left[\sin^2\left(\dfrac{\pi}{2 - ax}\right)\right]^{\sec^2\left(\frac{\pi}{2 - bx}\right)}$

(ii) $\mathop{\text{Limit}}\limits_{x\to\infty} \left(\dfrac{a_1^{1/x} + a_2^{1/x} + a_3^{1/x} + ... + a_n^{1/x}}{n}\right)^{nx}$, where $a_1, a_2, a_3, ..., a_n > 0$.

12. Find the values of a & b so that:

(i) $\mathop{\text{Limit}}\limits_{x\to 0} \dfrac{(1 + ax\,\sin x) - (b\,\cos x)}{x^4}$ may find to a definite limit.

(ii) $\lim\limits_{x\to\infty} \left(\sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 - cx^2 + 3x - d}\right) = 4$

13. Find the limits using expansion : $\mathop{\text{Limit}}\limits_{x\to 0} \left[\dfrac{\lambda n(1+x)^{(1+x)}}{x^2} - \dfrac{1}{x}\right]$

14. Let f(x) = $\dfrac{\sin^{-1}(1 - \{x\}).\cos^{-1}(1 - \{x\})}{\sqrt{2\{x\}}\,.\,(1 - \{x\})}$ then find $\mathop{\text{Limit}}\limits_{x\to 0^+}$ f(x) and $\mathop{\text{Limit}}\limits_{x\to 0^-}$ f(x), where {.} denotes the fractional part function.

15. Let $f(x) = \underset{m\to\infty}{\text{Limit}} \left\{ \underset{n\to\infty}{\text{Limit}} \left(\cos^{2m}(n!\pi x) \right) \right\}$ where $x \in R$. Prove that

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}.$$

16. Evaluate $\underset{x\to 0^+}{\text{Limit}} \left\{ \underset{n\to\infty}{\text{Limit}} \left(\dfrac{[1^2(\sin x)^x] + [2^2(\sin x)^x] + \ldots\ldots + [n^2(\sin x)^x]}{n^3} \right) \right\}$,

where [.] denotes the greatest integer function.

17. Evaluate the following limits

(i) $\underset{n\to\infty}{\text{Limit}} \cos\dfrac{x}{2} \cos\dfrac{x}{4} \cos\dfrac{x}{8} \ldots\ldots\ldots \cos\dfrac{x}{2^n}$

(ii) $\underset{n\to\infty}{\text{Limit}} \dfrac{1}{2}\tan\dfrac{x}{2} + \dfrac{1}{2^2}\tan\dfrac{x}{2^2} + \dfrac{1}{2^3}\tan\dfrac{x}{2^3} + \ldots\ldots + \dfrac{1}{2^n}\tan\dfrac{x}{2^n}$.

(iii) $\underset{x\to\infty}{\text{Limit}} \log_{x-1}(x).\log_x(x+1).\log_{x+1}(x+2).\log_{x+2}(x+3).....\log_k(x^5)$; where $k = x^5 - 1$.

(iv) Let $P_n = \dfrac{2^3-1}{2^3+1}.\dfrac{3^3-1}{3^3+1}.\dfrac{4^3-1}{4^3+1}\ldots\ldots\ldots\dfrac{n^3-1}{n^3+1}$. Prove that $\underset{n\to\infty}{\text{Limit}} P_n = \dfrac{2}{3}$.

Answers

Exercise # 1

1. D **2.** C **3.** D **4.** D **5.** D **6.** D **7.** C

8. B **9.** B **10.** C **11.** A **12.** B **13.** C **14.** C

15. C **16.** B **17.** C **18.** C **19.** C **20.** C **21.** D

22. A **23.** A **24.** B **25.** A **26.** C **27.** A **28.** C

29. A **30.** C **31.** B **32.** B **33.** B

Exercise # 2

1. (i) 0 (ii) Limit does not exists
(iii) Limit does not exists

2. 6 **3.** (i) (-8) (ii) $\dfrac{2}{3\sqrt{3}}$

4. (i) 1/3 (ii) $\dfrac{5}{2}(a+2)^{3/2}$ (iii) $2e^2$

5. (i) 1/2 (ii) 1/2 (iii) zero (iv) ∞

6. (i) $\dfrac{1}{3}$ (ii) a = 2, b = 1, c = –1 and value = $-\dfrac{1}{3}$

7. $\dfrac{x}{3}$ **8.** $\{-1, 0, 1\}$

9. (i) $-\dfrac{9}{4}\ln\dfrac{4}{e}$
(ii) $\cos^4\alpha \ln(\cos\alpha) - \sin^4\alpha \ln(\sin\alpha)$

10. (i) $\dfrac{1}{4\sqrt{2}}$ (ii) $-\dfrac{1}{\pi}$

11. (i) $e^{-\frac{a^2}{b^2}}$ (ii) $(a_1\, a_2\, a_3 \ldots\ldots\, a_n)$

12. (i) $a = -\dfrac{1}{2}$, b = 1 (ii) a = 2, $b \in R$, c = 5, $d \in R$

13. $\dfrac{1}{2}$ **14.** $\dfrac{\pi}{2}, \dfrac{\pi}{2\sqrt{2}}$ **16.** $\dfrac{1}{3}$

17. (i) $\dfrac{\sin x}{x}$ (ii) $\dfrac{1}{x} - \cot x$ (iii) 5