Questions - Area Under Curve

LEVEL–I

1. Prove that the area enclosed between the parabolas $y^2 = 4ax$ and $x^2 = 4$ by is 16 ab/3.

2. Prove that the area included between the parabola $y^2 = 4ax$ and the line $y = mx$ is $8a^2/3m^3$.

3. Find the area bounded on the right by the line $x + y = 2$, on the left by the parabola $y = x^2$ and above by the x-axis.

4. Find the area of the region bounded by the curves, $y = x^2 + 2$; $y = x$; $x = 0$ and $x = 3$.

5. Find the area bounded by the curve $y = x^2 + 1$ and the tangents to it drawn from the origin.

6. Find the area bounded by the curve $y^2 = x$ and $x = |y|$.

7. Find the area of the region bounded by the parabola $y = x^2$ and the rays given by $y = |x|$.

8. Find the area of the figure bounded by the parabola $(y - 2)^2 = x - 1$ the tangent to it at the point with ordinate 3 and the x-axis.

9. Find the area of the circle $4x^2 + 4y^2 = 9$, which is interior to the parabola $y^2 = 4x$.

10. Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$.

11. Find the area of the circle $x^2 + y^2 = 16$, which is exterior to the parabola $y^2 = 6x$.

12. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$.

$(a > 0, b > 0)$

13. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the ordinates $x = ae$ and $x = 0$, where $b^2 = a^2 (1 - e^2)$.

14. Find the value of 'c' for which the area of the figure bounded by the curve, $y = 8x^2 - x^5$, the straight lines $x = 1$ and $x = c$ and the abscissa axis is equal to 16/3.

15. Find the area enclosed by the curves $3x^2 + 5y = 32$ and $y = |x – 2|$.

LEVEL–II

1. Find the area of the region $\{(x, y)\}$: $0 \le y \le x^2 + 1,\ 0 \le y \le x + 1,\ 0 \le x \le 2\}$.

2. The tangent to the parabola $y = x^2$ has been drawn so that the abscissa x_0 of the point of tangency belongs to the interval [1, 2]. Find x_0 for which the triangle bounded by the tangent, the axis of ordinates and the straight line $y = x_0^2$ has the greatest area.

3. Find the area bounded by the curves $y = x^4 - 2x^2$ and $y = 2x^2$.

4. Find the area bounded $y^2 = 4(x + 1)$, $y^2 = -4(x - 1)$ and $y = |x|$ above axis of x.

5. Compute the area of the figure bounded by the curve $y^2 = 4x$ and a normal to it inclined at an angle of 135^0 to the positive x-axis.

6. Compute the area of the figure which lies in the first quadrant inside the curve $x^2 + y^2 = 3a^2$ and is bounded by the parabola $x^2 = 2ay$ and $y^2 = 2ax$ $(a > 0)$.

7. Find the area common to the two curves $y^2 = ax$ and $x^2 + y^2 = 4ax$.

8. Find the area of the region in the first quadrant bounded on the left by the y–axis, below by the curve $x = 2\sqrt{y}$, above left by the curve $y = (\sqrt{x} + 1)$, and above right by the line $x = 3 - y$.

9. Find the value of K for which the area bounded by the parabola $y = x^2 + 2x - 3$ and the line $y = Kx + 1$ is least. Also find the least area.

10. Find the area of the region in the first quadrant enclosed by the x-axis, the line $x = \sqrt{3}\,y$ and the circle $x^2 + y^2 = 4$.

11. **(i)** Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

(ii) Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

12. Prove that whole area of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

13. Show that the area common to the ellipses $a^2x^2 + b^2y^2 = 1$, $b^2x^2 + a^2y^2 = 1$ where $0 < a < b$ is $4(ab)^{-1}\tan^{-1}(a/b)$.

14. Find the values of m$(m > 0)$ for which the area bounded by the line $y = mx + 2$ and $x = 2y - y^2$ is,
(i) 9/2 square units and **(ii)** minimum. Also find the minimum area.

15. The line $3x + 2y = 13$ divides the area enclosed by the curve, $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ into two parts. Find the ratio of the longer area to the smaller area.

IIT JEE PROBLEMS (OBJECTIVE)

(A) Fill in the blanks

1. Match the following

(i) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ is _____.

(B) Multiple choice questions with one or more than one correct answer :

1. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2 ? **[IIT - 99]**

(A) - 4 (B) - 2

(C) 2 (D) 4

(C) Multiple choice questions with one correct answer :

1. The area bounded by the curves $y = f(x)$, the x–axis and the ordinates $x = 1$ and $x = b$ is $(b - 1)\sin(3b + 4)$. Then f(x) is **[IIT - 82]**

(A) $(x - 1)\cos(3x + 4)$ (B) $\sin(3x + 4)$

(C) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$ (D) none of these

2. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(A) 1 (B) 2

(C) $2\sqrt{2}$ (D) 4 **[IIT-2002]**

3. The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x–axis in the 1st quadrant is

(A) 9 (B) 27/4

(C) 36 (D) 18 **[IIT-2003]**

4. The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is

(A) 2 (B) 3

(C) 4 (D) 6 **[IIT-2004]**

5. The area enclosed between the curves $y = ax^2$ and $a = ay^2$ $(a > 0)$ is 1 sq. unit, then the value of a is

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$

(C) 1 (D) $\frac{1}{3}$ **[IIT-2004]**

6. The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = 1/4$ is

(A) 4 sq. units (B) 1/6 sq. units **[IIT-2005]**

(C) 4/3 sq. units (D) 1/3 sq. units

IIT JEE PROBLEMS (SUBJECTIVE)

1. Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$. **[IIT – 81]**

2. For any real t, $x = \frac{e^t + e^{-t}}{2}$, $y = \frac{e^t - e^{-t}}{2}$ is a point on the hyperbola $x^2 - y^2 = 1$. Show that the area bounded by this hyperbola and the lines joining its centre to the points corresponding to t_1 and $-t_1$ is t_1. **[IIT – 82]**

3. Find the area bounded by the x-axis, parts of the curve $y = \left(1 + \frac{8}{x^2}\right)$ and the ordinates at $x = 2$ and $x = 4$. If the ordinate at $x = a$ divides the area into two equal parts, find a. **[IIT - 83]**

4. Find the area of the region bounded by the x-axis and the curves defined by

$y = \tan x - \frac{\pi}{3} \le x \le \frac{\pi}{3}$, $y = \cot x$ $\frac{\pi}{6} \le x \le \frac{3\pi}{2}$. **[IIT – 84]**

5. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area. **[IIT – 85]**

6. Find the area bounded by the curves $x^2 + y^2 = 4$, $x^2 = -\sqrt{2}\,y$ and $x = y$. **[IIT – 86]**

7. Find the area bounded by the curves, $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$ above the x – axis. **[IIT – 87]**

8. Find the area of the region bounded by the curve C : $y = \tan x$, tangent drawn to C at $x = \frac{\pi}{4}$ and the x-axis. **[IIT – 88]**

9. Find all maxima and minima of the function $y = x(x-1)^2$, $0 \le x \le 2$. Also determine the area bounded by the curve $y = x(x-1)^2$, the y-axis and the line $y = 2$. **[IIT – 89]**

10. Compute the area of the region bounded by the curves $y = ex\ \lambda n\ x$ and $y = \frac{\lambda n\ x}{ex}$, where in $c = 1$. **[IIT – 90]**

11. Sketch the curves and identify the region bounded by $x = \frac{1}{2}$, $x = 2$, $y = \lambda n\ x$ and $y = 2^x$. Find the area of this region. **[IIT – 91]**

12. Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{(1+x^2)}$. Find the area. **[IIT – 92]**

13. Find the area bounded by the curve $y = x(x - 1)(x - 2)$ and the axis of x. **[REE-93]**

14. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$? **[IIT-94]**

15. Find the ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$. **[REE-95]**

16. Find the area given by $x + y \le 6$, $x^2 + y^2 \le 6y$ and $y^2 \le 8x$. **[REE-95]**

17. Consider a square with vertices at (1, 1), (-1, 1), (-1, -1) and (1, -1). Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area. **[IIT-95]**

18. Find the area of the region formed by $x^2 + y^2 - 6x - 4y + 12 \le 0$, $y \le x$ and $x \le 5/2$. **[REE-96]**

19. Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0$, $y = 0$ and $x = \pi/4$. Prove that for $n > 2$, $A_n + A_{n-2} = 1/(n - 1)$ and deduce that $1/(2n + 2) < A_n < 1/(2n - 2)$. **[IIT-96]**

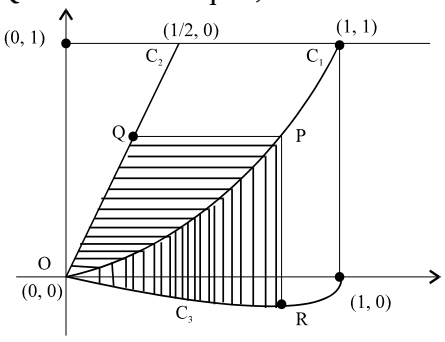
20. Let $f(x) = \text{Maximum } \{x^2, (1 - x)^2, 2x(1 - x)\}$, where $0 \le x \le 1$. Determine the area of the region bounded by the curves $y = f(x)$, x - axis, $x = 0$ and $x = 1$. **[IIT-97]**

21. Indicate the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis and obtain the area enclosed by them. **[REE-97]**

22. Find all the possible values of $b > 0$, so that the area of the bounded region enclosed between the parabolas $y = x - bx^2$ and $y = \frac{x^2}{b}$ is maximum. **[IIT - 97]**

23. Let O (0, 0), A (2, 0) and B(1, $\frac{1}{\sqrt{3}}$) be the vertices of a triangle. Let R be the region consisting of all those points P inside ΔOAB which satisfy $d(P, OA) \le \min \{d(P, OB), d(P, AB)\}$, where d denotes the distance from the point to the corresponding line. Sketch the region R and find its area. **[IIT - 97]**

24. Let C_1 and C_2 be the graph of the functions $y = x^2$ and $y = 2x$, $0 \le x \le 1$ respectively. Let C_3 be the graph of a function $y = f(x)$, $0 \le x \le 1$, $f(0) = 0$. For a point P and C_1, let the lines through P, parallel to the axes, meet C_2 and C_3 at Q and R respectively. If for every position of P (on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function f(x). **[IIT-98]**



25. Indicate the region bounded by the curves $y = x \log x$ and $y = 2x - 2x^2$ and obtain the area enclosed by them. **[REE-98]**

26. Let f(x) be a continuous function given by $f(x) = \begin{cases} 2x & \text{for } |x| \le 1 \\ x^2 + ax + b & \text{for } |x| > 1 \end{cases}$. Find the area of the region in the third quadrant bounded by the curves, $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$. **[IIT-99]**

27. Find the area of the region lying inside $x^2 + (y - 1)^2 = 1$ and outside $c^2 x^2 + y^2 = c^2$ where $c = \sqrt{2} - 1$. **[REE-99]**

28. Find the area enclosed by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at (2, 3) and the x - axis. **[REE-2000]**

29. Let $b \neq 0$ and for j = 0, 1, 2, ...n, let S_j be the area of the region bounded by the y axis and the curve $xe^{ay} = \sin by$, $\frac{j\pi}{b} \le y \le \frac{(j+1)\pi}{b}$. Show that S_0, S_1, S_2, ...S_n are in geometric progression. Also, find their sum for a = -1 and $b = \pi$. **[IIT-2001]**

30. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and y = 2, which lies to the right of the line x = 1. **[IIT-2002]**

31. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$. **[IIT-2005]**

32. If $\begin{bmatrix} 4a^2, & 4a, & 1 \\ 4b^2, & 4b, & 1 \\ 4c^2, & 4c, & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$, f(x) is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of y = f(x) with x–axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by f(x) and chord AB. **[IIT-2005]**

Area Under Curve

SET–I

1. The area (in square units) bounded by the curve $y = x^3$, the x-axis and the ordinate at $x = -2$ and $x = 1$ is

(A) $\frac{9}{2}$ (B) $\frac{15}{2}$ (C) $\frac{15}{4}$ (D) $\frac{17}{4}$

2. The area bounded by parabola $y^2 = x$, straight line $y = 4$ and y-axis is

(A) $\frac{16}{3}$ (B) $\frac{64}{3}$ (C) $7\sqrt{2}$ (D) none of these

3. Area bounded by parabola $y^2 = x$ and straight line $2y = x$ is

(A) $\frac{4}{3}$ (B) 1 (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

4. Area bounded by lines $y = 2 + x$, $y = 2 - x$ and $x = 2$ is

(A) 3 (B) 4 (C) 8 (D) 16

5. The area of the curve $x^2 + y^2 = 2ax$ is

(A) πa^2 (B) $2\pi a^2$ (C) $4\pi a^2$ (D) $\frac{1}{2}\pi a^2$

6. The area enclosed by the parabola $y^2 = 8x$ and the line $y = 2x$ is

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

7. The area bounded by the curve $y = \sin x$ and x-axis from $x = 0$ to 2π is

(A) 0 (B) 1 (C) 2 (D) 4

8. The area between x-axis and curve $y = \cos x$ when $0 \le x \le 2\pi$ is

(A) 0 (B) 2 (C) 3 (D) 4

9. The area common to the parabola $y = 2x^2$ and $y = x^2 + 4$ is

(A) $\frac{2}{3}$ sq. units (B) $\frac{3}{2}$ sq. units (C) $\frac{32}{3}$ sq. units (D) $\frac{3}{32}$ sq. units

10. The area bounded by the curve $y = x^3$, x-axis and two ordinates $x = 1$ to $x = 2$ is equal to

(A) $\frac{15}{2}$ sq. unit (B) $\frac{15}{4}$ sq. unit (C) $\frac{17}{2}$ sq. unit (D) $\frac{17}{4}$ sq. unit

11. The area bounded by the curve $y = 4x - x^2$ and the x-axis is

(A) $\frac{30}{7}$ sq. unit (B) $\frac{31}{7}$ sq.unit (C) $\frac{32}{3}$ sq.unit (D) $\frac{34}{3}$ sq.unit

12. The area formed by triangular shaped region bounded by the curves y = sin x, y = cos x and x = 0 is

(A) $\sqrt{2}-1$ (B) 1 (C) $\sqrt{2}$ (D) $1+\sqrt{2}$

13. The area bounded by $y=1+\frac{8}{x^2}$ and the ordinates x = 2 and x = 4 is

(A) 2 (B) 4 (C) log 2 (D) log 4

14. The area bounded by $y = 2 - x^2$ and x + y = 0 is

(A) $\frac{7}{2}$ sq. unit (B) $\frac{9}{2}$ sq. unit (C) 9 sq.unit (D) none of these

15. The area bounded by the curve x-axis and $y = x^4 - 2x^3 + x^2 + 3$ with x-axis and ordinates corresponding to the minima of y is

(A) 1 (B) $\frac{91}{30}$ (C) $\frac{30}{9}$ (D) 4

16. The area bounded by the curves y = | x | – 1 and y = – | x | + 1 is

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

17. The area bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is

(A) $\frac{8a^3}{3}$ (B) $\frac{16a^2}{3}$ (C) $\frac{32a^2}{3}$ (D) $\frac{64a^2}{3}$

18. The area bounded by the parabola $x = 4 - y^2$ and the y-axis, in square units, is

(A) $\frac{3}{32}$ (B) $\frac{32}{3}$ (C) $\frac{33}{3}$ (D) $\frac{16}{3}$

19. The area of the region bounded by the limits $x = 0, x = \frac{\pi}{2}$ and f(x) = sin x, g(x) = cos x is

(A) $2\left(\sqrt{2}+1\right)$ (B) $\sqrt{3}-1$ (C) $2\left(\sqrt{3}-1\right)$ (D) $2\left(\sqrt{2}-1\right)$

20. The area of the region bounded by the parabola $y = x^2 + 1$ and the straight line x + y = 3 is given by

(A) $\frac{45}{7}$ (B) $\frac{25}{4}$ (C) $\frac{\pi}{18}$ (D) $\frac{9}{2}$

SET–II

1. The area enclosed by the curves $x^2 = y$, $y = x + 2$ and x-axis is

(A) $\frac{5}{6}$ (B) $\frac{5}{4}$ (C) $\frac{5}{2}$ (D) none of these

2. The ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$, is

(A) 15 : 49 (B) 13 : 48 (C) 12 : 37 (D) none of these

3. The area of figure bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to it at the point with the ordinate 3 and the x-axis is

(A) 3 (B) 6 (C) 9 (D) none of these

4. The area of the region enclosed by the curves $y = x \log x$ and $y = 2x - 2x^2$ is

(A) $\frac{7}{12}$ (B) $\frac{1}{2}$ (C) $\frac{5}{12}$ (D) none of these

5. If the area bounded by the curves $y = x - bx^2$ and $y = \frac{1}{b}x^2$, where $b > 0$ is maximum, then $b =$

(A) 0 (B) 1 (C) 2 (D) none of these

6. The area of the region formed by $x^2 + y^2 - 6x - 4y + 12 \le 0$, $y \le x$ and $x \le \frac{5}{2}$ is

(A) $\frac{\pi}{6} - \frac{\sqrt{3}+1}{8}$ (B) $\frac{\pi}{6} + \frac{\sqrt{3}-1}{8}$ (C) $\frac{\pi}{6} - \frac{\sqrt{3}-1}{8}$ (D) none of these

7. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line $x = 2$ and x-axis is

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$

8. The area bounded by the x-axis and the curve $y = 4x - x^2 - 3$ is

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) 7 (D) $\frac{3}{2}$

9. The area of the region bounded by $y = |x - 1|$ and $y = 1$ is

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) none of these

10. The area between the curve $y = |x|$ and x-axis is

(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) $\frac{1}{3}$

11. Area bounded by the curves $y = x \sin x$ and x-axis between $x = 0$ and $x = 2\pi$ is

(A) 2π (B) 3π (C) 4π (D) none of these

12. Area bounded by the curve $xy^2 = a^2(a - x)$ and y-axis is

(A) $\frac{\pi a^2}{2}$ (B) πa^2 (C) $3\pi a^2$ (D) $\frac{3\pi a^2}{2}$

13. Area bounded by the curves $x = 1$, $x = 3$, $xy = 1$ and x-axis is

(A) $\log 2$ (B) $\log 3$ (C) $\log 5$ (D) none of these

14. Area between the curve $y = 4 + 3x - x^2$ and x-axis is

(A) $\frac{125}{3}$ sq. units (B) $\frac{125}{4}$ sq. units (C) $\frac{125}{6}$ sq. units (D) none of these

15. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2 sq.units (B) $\frac{9}{4}$ sq.units (C) $6\sqrt{3}$ sq. units (D) none of these

16. Area common to the curves $y = x^3$ and $y = \sqrt{x}$ is

(A) $\frac{5}{3}$ (B) $\frac{5}{4}$ (C) $\frac{5}{12}$ (D) none of these

17. The larger of the area bounded by $y = \cos x$, $y = x + 1$ and $y = 0$ is

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 1 (D) none of these

18. The area of the region bounded by the curve $y = x - x^2$ between $x = 0$ and $x = 1$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$

19. The area of the region bounded the curve $y = 2x - x^2$ and the line $y = x$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

20. The area bounded by the curves $y = \sin x$ between the ordinates $x = 0$, $x = \pi$ and the x-axis is

(A) 2 sq. units (B) 4 sq. units (C) 3 sq. units (D) 1 sq. units

SET–III

A. Multiple choice Questions with one correct answer

1. If A is the area lying between the curve y= sinx and x-axis between x = 0 and x = $\frac{\pi}{2}$. Area of the region between the curves y = sin2x and x-axis in the same interval is given by

(A) $\frac{A}{2}$ (B) A (C) 2A (D) none of these

2. The area of the triangle formed by the positive x-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $2\sqrt{3}$ (D) none of these

3. The area of the region bounded by the two parabolas $y = x^2$ and $y^2 = x$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

4. The area bounded by y = |x + 3|, x-axis and the two ordinates x = –6 and x = 0

(A) 3 (B) 6 (C) 9 (D) 12

B. Multiple choice Questions with one or more than one correct answer

5. If a curve $y = a\sqrt{x} + bx$ passes through the point (1, 2) and the area bounded by the curve, line x = 4 and x-axis is 8 square units, then

(A) a = 3 (B) a = –3 (C) b = 1 (D) b = –1

6. Area laying between the curves y = tan x, y = cot x and x-axis, $x \in [0, \pi/2]$ is

(A) $\frac{1}{2}\log 4$ (B) log2 (C) $2\log\frac{1}{\sqrt{2}}$ (D) none of these

C. Fill in the blanks

7. If the ordinate x = a divides the area bounded by x-axis, part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates x = 2, x = 4, into two equal parts, then the value of 'a' is _____ .

8. The area of the region bounded by $x^2 + y^2 - 2x \leq 1; x + y \leq 1; y \geq 0$ is _____ .

9. The area bounded by the curves y = sin x and y = cos x between two consecutive points of their intersection is _____ .

10. The area of the region enclosed by the line x = 0, y = 3 and curve $x = 2y^2$ is _____ .

11. The area of the region enclosed by the line $4x = y + 16$ and curve $y^2 - 4x = 4$ is _____ .

D. Match the column

12.

	Column I	**Column II**
(a)	The area between the curves $y = 2x^4 - x^2$, the x-axis and the ordinates of two minimum of the curve is	(P) 2 sin 1
(b)	The area bounded by the curve $x = at^2$, $y = 2at$, and the x-axis $(1 \le t \le 3)$ is	(Q) $\frac{7}{120}$
(c)	The area of a circle centred at (1, 2) and passing through (4, 6) is	(R) $\frac{104a^2}{3}$
(d)	The area of the bounded by $y = \cos x$, $y = 0$, $\lvert x \rvert = 1$ is given by	(S) 25π

13.

	Column I	**Column II**
(a)	The area bounded by the curves $y = 2 \cos x$ and the x-axis from $x = 0$ to $x = 2\pi$ is	(P) $\frac{(\pi^3 - 8)}{4}$
(b)	The area in the first quadrant between $x^2 + y^2 = \pi^2$ and $y = \sin x$ is	(Q) 1
(c)	The measurement of the area bounded by the coordinate axes and the curve $y = \log_e x$ is	(R) 2a
(d)	The area of the loop between the curve $y = a \sin x$ and x-axis is	(S) 8

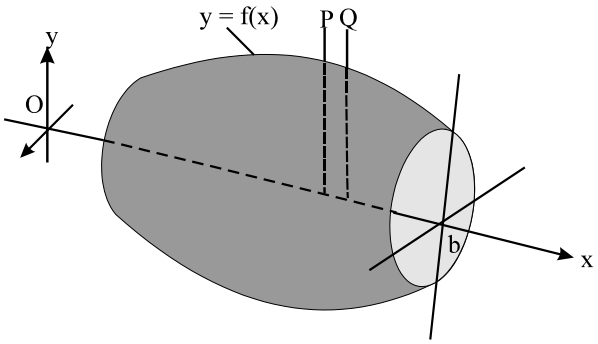
14.

	Column I	**Column II**
(a)	The area between the parabolas $y^2 = 4ax$ and $x^2 = 8ay$ is	(P) 27
(b)	Area bounded by the curve $y = (x - 1)(x - 2)(x - 3)$ and x-axis lying between the ordinates $x = 0$ and $x = 3$ is equal to	(Q) 2π
(c)	The area cut off the parabola $4y = 3x^2$ by the straight line $2y = 3x + 12$ in sq. units is	(R) $\frac{32}{3}a^2$
(d)	The area of the region satisfying $\lvert x \rvert + \lvert y \rvert \le \sqrt{\pi}$ is	(S) $\frac{11}{4}$

Area Under Curve

W I **Read the following passage :**

If the function $f(x) \geq 0$ is smooth on [a, b], the area of the surface generated by revolving the curve y = f(x) about the x–axis is



$$S = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_a^b 2\pi \ f(x)\sqrt{1+(f'(x))^2}\, dx \qquad \text{.....(i)}$$

If $x = g(y) \geq 0$ is smooth on [c, d], the area of the surface generated by revolving the curve x = g(y) about the y–axis is

$$S = \int_c^d 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy = \int_c^d 2\pi\, g(y)\sqrt{1+(g'(y))^2}\, dy \qquad \text{.....(ii)}$$

15. The area of the surface generated by revolving the curve $x = 2\sqrt{x},\ 1 \leq x \leq 2$ about x – axis is

(A) $\frac{8\pi}{3}\left(2\sqrt{2}-3\sqrt{2}\right)$ (B) $\frac{8\pi}{3}\left(2\sqrt{2}-3\sqrt{3}\right)$

(C) $\frac{8\pi}{3}\left(3\sqrt{3}-2\sqrt{2}\right)$ (D) $\frac{8\pi}{3}\left(3\sqrt{2}-2\sqrt{3}\right)$

16. The line segment x = 1 – y, $0 \leq y \leq 1$, is revolved about the y–axis to generate the cone then its lateral surface area is

(A) $\pi\sqrt{3}$ (B) $\frac{\pi}{\sqrt{2}}$ (C) π (D) $\pi\sqrt{2}$

17. The area of the surface generated by revolving the curve $x = 2\sqrt{4-y},\ 0 \leq y \leq 15/4$ about y–axis is

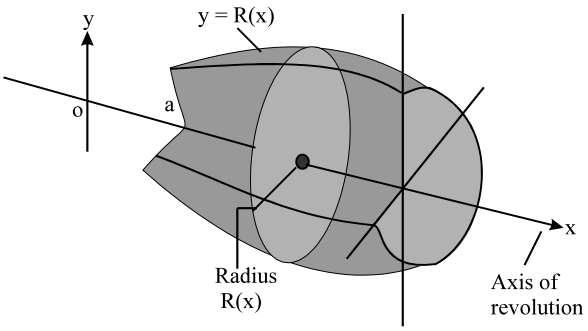
(A) $\frac{5\pi\sqrt{35}}{3}$ (B) $\frac{35\pi\sqrt{5}}{3}$ (C) $\frac{35\pi\sqrt{3}}{5}$ (D) $\frac{3\pi\sqrt{35}}{5}$

18. The surface area of a sphere of radius 'a' by using **equation (i)** to find the area of the surface generated by revolving the curve $y = \sqrt{a^2-x^2},\ -a \leq x \leq a$ about x–axis is

(A) πa^2 (B) $4\pi a^2$ (C) $\frac{4\pi a^2}{3}$ (D) $\frac{3\pi a^2}{4}$

W II **Read the following passage :**

The volume of the solid generated by revolving about the x–axis the region between the x–axis and the graph of the continuous function $y = R(x),\ a \le x \le b$, is



$$V = \int_a^b \pi[\text{radius}]^2 dx = \int_a^b \pi[R(x)]^2 dx \quad \text{.....(i)}$$

The volume of the solid generated by revolving about the y–axis the region between they–axis and the graph of the continuous function

$x = R(y),\quad c \le y \le d$, is

$$V = \int_c^d \pi(\text{radius})^2 dx = \int_c^d \pi[R(y)]^2 dy \quad \text{.....(ii)}$$

19. The region between the curve $y = \sqrt{x},\ 0 \le x \le 4$, and the x–axis is revolved about the x–axis to generate a solid. Then its volume is

(A) 4π (B) 3π (C) 8π (D) 5π

20. The volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$ is

(A) $\frac{6\pi}{7}$ (B) $\frac{3\pi}{7}$ (C) $\frac{5\pi}{6}$ (D) $\frac{7\pi}{6}$

21. The volume of the solid generated by revolving the region between the y–axis and the curve $x = 2/y,\ 1 \le y \le 4$, about the y–axis is

(A) 4π (B) 5π (C) 3π (D) 6π

22. The volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$

(A) $\frac{66\pi\sqrt{2}}{15}$ (B) $\frac{64\pi\sqrt{2}}{15}$ (C) $\frac{6\pi\sqrt{2}}{15}$ (D) $\frac{15\pi\sqrt{2}}{6}$

Area Under Curve

LEVEL–I ANSWER KEY

3. 5/6 sq. units **4.** 21/2 sq. units **5.** $\frac{2}{3}$ **6.** 1/3

7. 1/3 **8.** 9 **9.** $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}+\frac{\sqrt{2}}{6}$

10. $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$ **11.** $\frac{4}{3}(8\pi-\sqrt{3})$ **12.** $\frac{43}{4}(\pi-2)$

13. $ab\left[e\sqrt{1-e^2}+\sin^{-1}e\right]$ **14.** C = -1 or $(8-\sqrt{17})^{1/3}$ **15.** $\frac{33}{2}$ sq. units

LEVEL–II

1. 23/6 sq. units **2.** $x_0 = 2, A(x_0) = 8$ **3.** 128/15 sq. units

4. $\frac{8}{3}-\frac{8}{3}(3-2\sqrt{2})^{3/2}-(2\sqrt{2}-2)^2$ **5.** 64/3 sq. units

6. $\left(\sqrt{4}+\frac{3}{2}.\arcsin\frac{1}{3}\right)a^2$ sq. units **7.** $a^2\left(3\sqrt{3}+\frac{4\pi}{3}\right)$

9. K = 2, A = 32/3 **10.** $\frac{\pi}{3}$

11.(i) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$ **(ii)** 4π **14.(i)** m = 1 **(ii)** $m=-\frac{1}{2}$, area = 0

15. $\frac{3\pi+2}{\pi-2}$

IIT JEE PROBLEMS (OBJECTIVE)

(A) **1.** $\frac{4}{3}$ **(B)** **1.** B, D

(C) **1.** C **2.** B **3.** D **5.** A **6.** D

IIT JEE PROBLEMS (SUBJECTIVE)

1. $\frac{9}{8}$ sq.units **3.** a = $2\sqrt{2}$ **4.** $\ln 3/2$ sq.units

5. $\frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}\right]-\frac{3}{2}$ sq. units **6.** $\frac{5}{3}$ sq.units

7. $2+\frac{25}{2}\sin^{-1}\frac{4}{5}$ sq.units **8.** $\lambda n\sqrt{2}+\frac{\pi}{4}-\frac{1}{2}$ sq.units **9.** $\frac{10}{3}$ sq.units

10. $\frac{e^2-5}{4e}$ sq.units **11.** $-\frac{1}{2}+\left(4-\sqrt{2}\right)\frac{1}{\lambda n\,2}+\frac{3}{2}\lambda n\,2$ sq.units

12. $\pi-\frac{2}{3}$ sq. units **13.** $\frac{1}{2}$ **14.** 4 : 121 **15.** 49 : 15

16. $(9\pi/4)-(1/6)$ **17.** $\frac{1}{3}\left(16\sqrt{2}-20\right)$ **18.** $\frac{\pi}{6}-\frac{\sqrt{3}-1}{8}$ **20.** 17/27 sq. units

21. 5/6 sq. units **22.** b = 1 **23.** $2-\sqrt{3}$

24. $f(x) = x^3 - x^2$ **25.** 7/12 **26.** 257/192 ; a = 2 ; b = -1

27. $\left(\frac{\pi-2}{2\sqrt{2}}\right)$ sq. units **28.** 9 sq. units

29. $\frac{S_j}{S_{j+1}}=e^{\frac{\pi a}{b}};S_0=\frac{b\left(e^{\frac{a\pi}{b}}+1\right)}{a^2+b^2}$ for a = -1, b = π, $S_0=\frac{\pi(e+1)}{\pi^2+1}$ and r = π

30. $\left(\frac{20}{3}-4\sqrt{2}\right)$ sq. units **31.** $\frac{1}{3}$ sq. units **32.** $\frac{125}{3}$ sq. units

Area Under Curve

SET–I

1.	C	**2.**	B	**3.**	A	**4.**	B	**5.**	A
6.	A	**7.**	D	**8.**	D	**9.**	C	**10.**	B
11.	C	**12.**	A	**13.**	B	**14.**	B	**15.**	B
16.	B	**17.**	B	**18.**	B	**19.**	D	**20.**	D

SET–II

1.	A	**2.**	A	**3.**	C	**4.**	A	**5.**	B
6.	C	**7.**	B	**8.**	A	**9.**	A	**10.**	A
11.	C	**12.**	B	**13.**	B	**14.**	C	**15.**	B
16.	C	**17.**	B	**18.**	A	**19.**	D	**20.**	A

SET–III

1.	B	**2.**	C	**3.**	B	**4.**	C	**5.**	AD
6.	AB	**7.**	$2\sqrt{2}$	**8.**	$\frac{\pi}{4}$	**9.**	$2\sqrt{2}$		
10.	18	**11.**	243/8	**12.**	a-Q, b-R, c-S, d-P				
13.	a-S, b-P, c-Q, d-R			**14.**	a-R, b-S, c-P, d-Q				
15.	C	**16.**	D	**17.**	B	**18.**	B	**19.**	C
20.	D	**21.**	C	**22.**	B				