ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2015 - 2016)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (ADVANCED) 2016

Test Type : **ALL INDIA OPEN TEST** (MAJOR)   Test Pattern : JEE-Advanced

TEST # 03   **TEST DATE : 14 - 02 - 2016**

PAPER-1

PART-1 : PHYSICS   **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,D	A,B	B,C,D	A,B,C	A,C	A,B,C,D	B,C	B,C	A,B,C	C,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		S	R	S	T		T	R	S	P	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	6	1	4	6	2	6	2	6		

SOLUTION

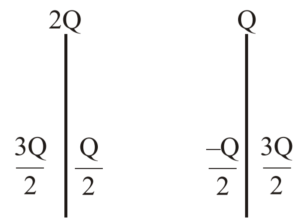
SECTION-I

1. Ans. (B,D)

2. Ans. (A,B)

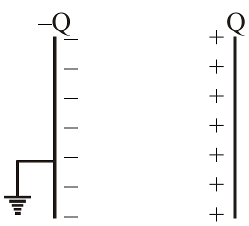
3. Ans. (B,C,D)

Sol. $\Delta V = \dfrac{Q}{2A\in_0} \times d$



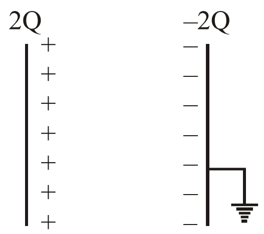
If left plate is earthed,

$\Delta V = \dfrac{Q}{A\in_0} \times d$



If right plate is earthed,

$\Delta V = \dfrac{2Q}{A\in_0} \times d$



4. Ans. (A,B,C)

Sol. $0 = -nmv + m(v_0 - v) \Rightarrow v = \dfrac{v_0}{n+1}$

$\& \ v_m = v_0 - v = v_0 - \dfrac{v_0}{n+1} = \dfrac{nv_0}{n+1}$

$\Delta KE = \dfrac{1}{2} \times nm\left(\dfrac{v_0}{n+1}\right)^2 + \dfrac{1}{2}m\left(\dfrac{nv_0}{n+1}\right)^2$

$= \dfrac{1}{2}mv_0^2\dfrac{n}{n+1}$

5. Ans. (A,C)

Sol. $E_1 = 0 \ \& \ V_1 = 4V_0$

$E_2 = E; \ V_2 = 3V_0$

$E_3 = \sqrt{2}E \ ; \ V_3 = 2\ V_0$

$E_4 = E; \ V_4 = V_0$

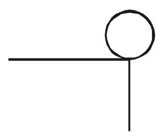
6. Ans. (A,B, C, D)

Sol. In all cases

$\dfrac{\Delta A}{A} \times 100 = 2\dfrac{\Delta P}{P} \times 100$

7. Ans. (B,C)

Sol.



$mg\left(R - R\cos\dfrac{\theta}{2}\right) = \dfrac{1}{2}mv_2^2 + \dfrac{1}{2} \times \dfrac{2}{5}mR^2 \times \dfrac{v_0^2}{R^2}$

$= \frac{7}{10}mv_2^2$

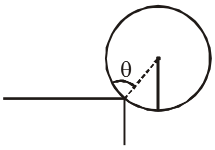
$v_2 = \sqrt{\frac{10}{7}gR(1-\cos\theta_2)}$

$mg\cos\theta_2 = \frac{mv_2^2}{R} = \frac{10g}{7}(1-\cos\theta_2)$

$17\cos\theta_2 = 10$

$\cos\theta_2 = \frac{10}{17}$

$\cos\theta_2 < \cos\theta_1 \Rightarrow \theta_2 > \theta_1$



$mg(R - R\cos\theta_1) = \frac{1}{2}mv_1^2$

$v_1 = \sqrt{2gR(1-\cos\theta_1)}$

$mg\cos\theta_1 = \frac{mv_1^2}{R} = 2mg(1-\cos\theta_1)$

$\cos\theta_1 = \frac{2}{3}$

$v_1 = \sqrt{2gR\left(1-\frac{2}{3}\right)} = \sqrt{\frac{2gR}{3}}$

$v_2 = \sqrt{\frac{10gR}{7}\times\frac{7}{17}}$

$v_1 = \sqrt{\frac{2gR}{3}}$

8. Ans. (B,C)

Sol. Here $N_A = N_0e^{-\lambda t}$

$N_B = N_0(1 - e^{-\lambda t})$

Now $\frac{N_B}{N_A} = e^{\lambda t} - 1$

If $t << T$; then

$\frac{N_B}{N_A} = 1 + \lambda t - 1$ ($e^x = 1 + x$ for small x)

$\frac{N_B}{N_A} = \lambda t$

9. Ans. (A,B,C)

Sol. $2.5A(37 - T_1) = \frac{KA}{\ell}(T_1 - T_2) = 2.5A[T_2 - 7]$

Solving we get

$T_1 = 25$ and $T_2 = 19$

Also heat lost by body with fur coat

$= \frac{dH}{dt} = \frac{5}{2}\times 1[37-25] = 30$

10. Ans. (C,D)

Sol. It is clear that, $I_1 = I'_1$ & $I_2 = I'_2$

But $I_1 > I_2$ & $I'_1 > I'_2$

SECTION-II

1. Ans. (A)→(S); (B)→(R); (C)→(S); (D)→(T)

Sol. (A) $\frac{\frac{4}{3}-1}{-1} = \frac{4}{3V} - \frac{1}{-2} \Rightarrow V = -\frac{8}{5}$

We know, $m = \frac{\mu_1 v}{\mu_2 u} = \frac{3}{5}$

Since $v = -ve$ & $|m| < 1$

Hence image is virtual & diminished.

2. Ans. (A)→(T); (B)→(R); (C)→(S); (D)→(P)

Sol. (A) $U = 3gh_1 + 2gh_2$

$\frac{dU}{dt} = 3g\frac{dh_1}{dt} + 2g\frac{dh_2}{dt}$

$= -3gU + 2gU = -gU = \frac{-g^2}{5}t$

$\frac{d^2U}{dt^2} = \frac{-g^2}{5}$

(B) $U = \frac{1}{2}Li^2 = \frac{1}{2}Li_0^2e^{-\frac{2R}{L}}$

$\frac{dU}{dt} = \frac{1}{2}Li_0^2 \times e^{-\frac{2R}{L}} \times \frac{-2R}{L}$

$\frac{d^2U}{dt^2} = \frac{1}{2}Li_0^2 \times \frac{4R^2}{L^2}e^{-\frac{2R}{L}}$

(C) $U = mgy$

$\frac{dU}{dt} = mg\frac{dy}{dt} = mg\,a_y = 0$

$\frac{d^2U}{dt^2} = mg\,a_y$

(D) $U = \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2\cos^2\omega t$

$\frac{dU}{dt} = \frac{1}{2}kx_0^2 \times 2\sin\omega t\cos\omega t$

$= -\frac{1}{2}kx_0^2\omega\sin 2\omega t$

at $t = 0$ $\frac{du}{dt} = 0$

$\frac{d^2U}{dt^2} = \frac{1}{2}kx_0^2 \times 2\omega^2\cos 2\omega t$

at $t = 0$ $\frac{d^2u}{dt^2} = kx_0^2\omega^2$

$\Rightarrow \frac{d^2u}{dt^2} > 0$

SECTION-IV

1. Ans. 6

Sol. $x = \frac{1}{2} \times 10 \times (0.2)^2 = 0.2\text{m}$

$\therefore\ f = \frac{D^2 - x^2}{4D} = 24\text{cm}$

$\therefore\ \frac{f}{4} = 6\text{cm}$

2. Ans. 1

Sol. $mv\frac{d\theta}{dx} = \mu qvB$

$m\int dv = \mu qB\int dx$

$\frac{0.2 \times 1.5 \times 10^{-15}}{0.3 \times 50 \times 10^{-6} \times 0.2} = x$

3. Ans. 4

Sol. $\pi R^2 \times \frac{\sigma^2}{2\in_0} = 8 \times 10^8 \times 2\pi RT$

$\frac{R}{4\in_0 t} \times \left(\frac{Q}{4\pi R^2}\right)^2 = 8 \times 10^8$

$Q = \sqrt{8 \times 10^8 \times 64\pi^2 R^3 \omega t}$

$= \sqrt{\pi} \times \sqrt{\frac{8 \times 10^3 \times 16 \times \frac{1}{8}}{9 \times 10^9} \times 0.09 \times 10^{-3}}$

$= 4\sqrt{\pi} \times 10^{-3}$

4. Ans. 6

Sol. $5 = \frac{320}{3}\left[\frac{330 + v_0}{330 - 10}\right] - \frac{320}{3} \Rightarrow v_0 = 5\text{ m/s}$

$\therefore\ t = \frac{90}{15} = 6\text{ sec}$

5. Ans. 2

Sol. $e = B\ell v = \frac{8}{10} \times 3 \times 5 = 12\text{ Volt}$

$\therefore\ q = CE\,(1 - e^{-t/\tau})$

$\therefore\ 24 = 6 \times 12\,(1 - e^{-t/\tau}) \Rightarrow e^{-t/\tau} = \frac{2}{3}$

$\therefore\ i = \frac{E}{R}e^{-t/\tau} = \frac{12}{4} \times \frac{2}{3} = 2$

6. Ans. 6

Sol. $T_{max} - mg = \frac{mu^2}{\ell}$

$T_{min} + mg = \frac{mu^2}{\ell} = \frac{m}{\ell}\left(u^2 - 4g\ell\right)$

$T_{max} = mg + \frac{mu^2}{\ell}$

$T_{min} = \frac{mu^2}{\ell} - 5mg$

$\therefore$ difference $= 6$ mg

7. Ans. 2

Sol. $I_0 W_0 = IW$

$I_0 \times w_0 \times q_0 = Iwq$

$\sqrt{\frac{m\ell^2}{3}}\sqrt{\frac{mg\frac{\ell}{2}}{\frac{m\ell^2}{3}}} \times \sqrt{\frac{40}{3}}$

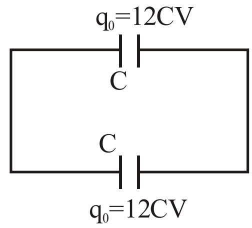
$= \sqrt{\left(mg\frac{\ell}{2} + Mg\ell\right) \times \left(\frac{m\ell^2}{3} + M\ell^2\right)} \times \theta_0$

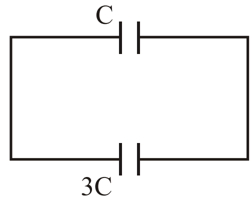
$\Rightarrow \theta_0 = \frac{\sqrt{\frac{40}{3}} \times \sqrt{1.5 \times 10 \times \frac{1}{2} \times \frac{1.5 \times 1}{3}}}{\sqrt{\left(1.5 \times 10 \times \frac{1}{2} + 0.5 \times 10 \times 1\right)\left(\frac{1.5 \times 1^2}{3} + 0.5 \times 1^2\right)}}$

$= \frac{\sqrt{\frac{40}{3} \times \frac{15}{4}}}{\sqrt{\left(\frac{15}{2} + 5\right)(1)}} = \sqrt{\frac{50 \times 2}{25}} = 2$

8. Ans. 6

Sol.





In second case

$V_C = \frac{Q_{total}}{C_{total}}$

$V_C = \frac{24CV}{4C} = 6\text{Volt}$

PART-2 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	B	A,D	A,B,C,D	A,B,C,D	A,C,D	A,B,C	A,C	A,D	A,B
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		R,T	S,T	P,Q,T	P,T		P,R,T	P,R	P,Q,R,S,T	P,Q,R,S	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	2	4	4	9	1	2	6	5		

SOLUTION

SECTION - I

1. **Ans. (A,B,C)**

(A) For solution Z_2 at P_1 pressure

$X_A = 0.25$ $X_B = 0.75$

$Y_A = 0.5$ $Y_B = 0.5$

(B) For solution Z_2 at P_3 pressure $\rightarrow$ solution will not vapourise so

$X_A = 0.4$; $X_B = 0.6$

(C) For sulution Z_1 at P_2 pressure $\rightarrow$ solution will not vapourise so

$X_A = 0.2$; $X_B = 0.8$

2. **Ans. (B)**

For a reaction to occur $\Delta G^o < 0$

$2MnO + 2C \rightarrow 2Mn + 2CO$; $\Delta G = 50$ kJ/mol (not feasible)

$2/3\ Ca_2O_3 + 2C \rightarrow 4/3\ Cr + 2CO$; $\Delta G = -50$ kJ/mol (feasible)

3. **Ans.(A,D)**

(A) Fact

(B) Probability of finding an electron is nearly 90% in an orbital

(C) No of angular nodes are l

(D) For 1s $|\Psi|^2$ is maximum at nucleus

4. **Ans. (A, B, C, D)**
5. **Ans. (A, B, C, D)**
6. **Ans. (A, C, D)**
7. **Ans. (A,B,C)**
8. **Ans. (A,C)**
9. **Ans. (A,D)**
10. **Ans. (A, B)**

SECTION - II

1. **Ans (A)$\rightarrow$(R,T); (B)$\rightarrow$(S,T); (C)$\rightarrow$(P, Q, T); (D)$\rightarrow$(P, T)**

(A) $10^{-6} = \dfrac{x^2}{1-x} \Rightarrow x = 10^{-3}$; pOH = 3; pH = 11

(B) $10^{-7} = \dfrac{x^2}{0.1-x} \Rightarrow x = 10^{-4}$; pOH = 4, pH = 10

(C) $\underset{20 \times 0.5}{BOH} + \underset{2 \times V}{HCl} \rightarrow \underset{20 \times 0.5}{BCl} + H_2O$

$[BCl] = \dfrac{20 \times 0.5}{20+5} = 0.4$

$\dfrac{x^2}{0.4-x} = 4 \times 10^{-5} \Rightarrow x = 4 \times 10^{-3}$

$\Rightarrow$ pH = 2.4

(D) $\dfrac{(10^{-3}+x)(x)}{0.1-x} = \dfrac{10^{-14}}{5 \times 10^{-10}} \Rightarrow x = 10^{-3}$

$[H^+] = 10^{-3} + 10^{-3} = 2 \times 10^{-3} \Rightarrow$ pH = 2.7

2. **Ans. (A) $\rightarrow$ (P,R,T); (B) $\rightarrow$ (P,R); (C) $\rightarrow$ (P,Q,R,S,T); (D) $\rightarrow$ (P,Q,R,S)**

SECTION-IV

1. **Ans. 2**

$\Lambda_m = \dfrac{G.l^2}{n} = \dfrac{K.V}{n}$

$\dfrac{1}{100} \times \dfrac{400}{2} = 2 Scm^2 mol^{-1}$

2. **Ans. 4**

(i) It is enthalpy of hydration

(ii) For atomisation coefficient of NH_3 should be one

(iii) Products should be in gaseous ionic form

(iv) $\Delta H^o_r = 3/2\ \Delta_{P-P}H^o$

3. **Ans. 4**
4. **Ans. 9**
5. **Ans. 1**
6. **Ans. 2**
7. **Ans. 6**
8. **Ans. 5**

PART-3 : MATHEMATICS

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B	A,C	A,B,D	A,C	A,B,C	A,B	A,C	A,B,C,D	A,B	A,C,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		P,S	P	P,R	T		P,Q	P	R	S,T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	9	6	4	4	6	3	3	4		

SOLUTION

SECTION-I

1. Ans. (A,B)

$$I_n=\int_0^{\pi/2} x(\sin x+\cos x)^n\,dx=2^{n/2}\int_0^{\pi/2} x\cos^n\left(x-\frac{\pi}{4}\right)dx$$

Put $x-\dfrac{\pi}{4}=t$

$$\Rightarrow I_n=2^{n/2}\int_{-\pi/4}^{\pi/4}\left(\frac{\pi}{4}+t\right)\cos^n t\,dt$$

$$\Rightarrow I_n=\pi 2^{\frac{n}{2}-1}\int_0^{\pi/4}(\cos t)^n\,dt$$

$$\Rightarrow I_n=\pi 2^{\frac{n}{2}-1}\left\{\left((\cos t)^{n-1}\sin t\right)_0^{\pi/4}+(n-1)\int_0^{\pi/4}(\cos t)^{n-2}\left(1-\cos^2 t\right)dt\right\}$$

$$\Rightarrow \frac{nI_n-\pi/2}{I_{n-2}}=2(n-1)\ \{\text{After simplification}\}$$

2. Ans. (A,C)

Let $g(x)=ax^2+bx+c$

$\because [f(x)]=[g(x)]\ \forall\ x\in R$

$\therefore -1<f(x)-g(x)<1\ \forall\ x\in R$

$\therefore -1<(1-a)x^2-x(1+b)+(1-c)<1\ \forall\ x\in R$

$\Rightarrow a=1,\ b=-1$

$\Rightarrow [x^2-x+1]=[x^2-x+c]\Rightarrow[1]=[c]$

Let $x^2-x+1=I+f$ $\qquad c=1+f'$

$[I+f]=[I-1+f+c]$

$0=[f+c]-1$

$1=[c+f]$

$1=[1+f'+f]$

$[f'+f]=0\ \forall\ f\in(0,1)$

$f'=0$

$\Rightarrow\quad c=1$

$\Rightarrow f(x)=g(x)\ \forall\ x\in R$

3. Ans. (A,B,D)

$f(x)=(x-1)\,|x-3|-4x+12$

$$f(x)=\begin{cases}9-x^2 & x<3\\ x^2-8x+15 & x\ge 3\end{cases}$$

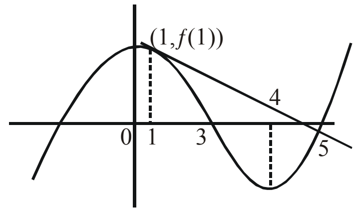
$$f'(x)=\begin{cases}-2x & x<3\\ 2x-8, & x>3\end{cases}$$

$f'(3^-)=-6$

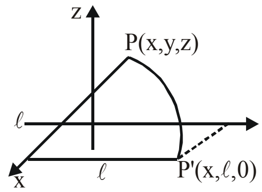
$f'(3^+)=-2$

$f'(0)=0$

$f'(4)=0$



4. Ans. (A,C)



Obviously 'P' lies on plane $y=z$

To see the curve rotate $y=z$ about x-axis to become $z=0$. New co-ordiante of point corresponding to P in new plane is

$\left(4\cos t,4\sqrt{2}\sin t,0\right)$

$\Rightarrow$ Locus is $\dfrac{x^2}{16}+\dfrac{y^2}{32}=1;z=0$

5. Ans. (A,B,C)

$P(2^2.5^2)=2^2.5^2\left(1-\dfrac{1}{2}\right)\left(1-\dfrac{1}{5}\right)=2.5.4$

$\because$ no. of numbers multiple of $2=2.5^2$

no. of numbers multiple of $5=2^2.5.$

no. of numbers multiple of both $=2.5$

$\therefore P(2^2.5^2)=2^2.5^2-2.5^2-2^2.5+2\,.5$

$$=2^2 5^2\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)$$

similarly check other options.

6. Ans. (A,B)

E : Both die shows face 6

A_1 : Both are biased

A_2 : Both are fair

A_3 : One is biased & other is fair.

$$P\left(\frac{A_1}{E}\right)=\frac{\frac{1}{6}\times\frac{1}{8}\times\frac{1}{8}}{\frac{1}{6}\times\frac{1}{8}.\frac{1}{8}+\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}+\frac{4}{6}.\frac{1}{6}.\frac{1}{8}}=\frac{9}{73}$$

$$P\left(\frac{A_2}{E}\right)=\frac{16}{73}$$

7. Ans. (A,C)

$$P(X)=\frac{2}{3}\times\frac{3}{5}+\frac{1}{3}\times\left\{\frac{1}{2}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}\right\}$$

$$=\frac{6}{15}+\frac{1}{15}=\frac{7}{15}$$

$$P\left(\frac{Y}{X}\right)=\frac{6/15}{\frac{6}{15}+\frac{1}{15}}=\frac{6}{7}$$

8. Ans. (A,B,C,D)

No. of roots common to

$z^{n_1}=1$ & $z^{n_2}=1$ is HCF(n_1,n_2)

9. Ans. (A,B)

Do yourself

10. Ans. (A,C,D)

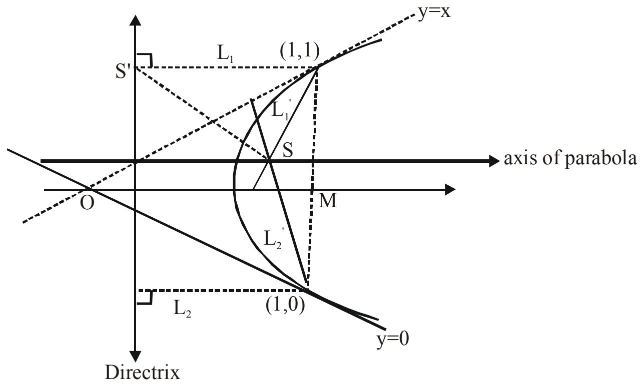
$$I_1=\int_{-100}^{100}\left[t^3\right]dt$$

$$2I_1=\int_{-100}^{100}(-1)\,dt \quad \{\text{king \& odd}\}$$

$$I_1=-100$$

SECTION – II

1. Ans. (A)→(P,S); (B)→(P); (C)→(P,R); (D)→(T)



In the figure ; line MO is parallel to axis of parabola.

$\Rightarrow$ Slope of axis $=\frac{1}{2}$. L_1 & L_2 shown in figure are parallel to axis of parabola and image of L_1 & L_2 in corresponding tangents pass through focus.

$L_1 \equiv x - 2y + 1 = 0$

$L_1' \equiv -2x + y + 1 = 0$

$L_2 \equiv x - 2y - 1 = 0$

& $L_2' \equiv x + 2y - 1 = 0$

$$S\equiv\left(\frac{3}{5},\frac{1}{5}\right)$$

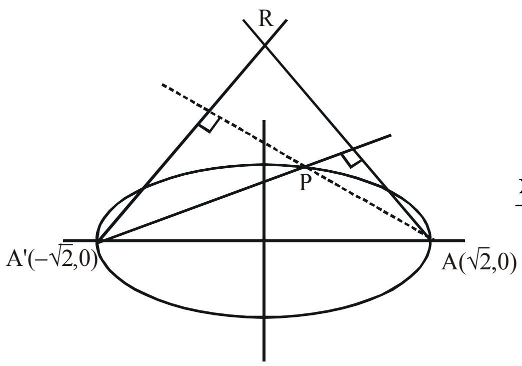
Now image of S lies on directrix

$$\Rightarrow\ S'=\left(\frac{1}{5},\frac{3}{5}\right)$$

$\Rightarrow$ Equation of directrix $2x + y = 1$

now check options.

2. Ans. (A)→(P,Q); (B)→(P); (C)→(R); (D)→(S,T)



$$\frac{x^2}{2}+\frac{y^2}{1}=1$$

Equation of AP $\Rightarrow$ $y=-\frac{a}{b}\cot\frac{\theta}{2}(x-a)$

Equation of A'P $\Rightarrow$ $y=\frac{a}{b}\tan\frac{\theta}{2}(x+a)$

$$\Rightarrow\ y^2=-\frac{a^2}{b^2}\left(x^2-a^2\right)\Rightarrow-\frac{y^2}{a^2}=\frac{x^2}{b^2}-\frac{a^4}{b^2}$$

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=\frac{a^4}{b^2}$$

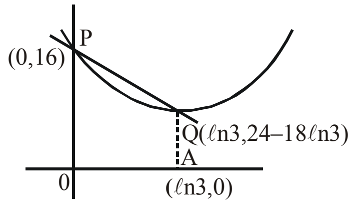
$$x^2+\frac{y^2}{2}=4$$

$$\frac{x^2}{4}+\frac{y^2}{8}=1$$

Now check options.

SECTION – IV

1. Ans. 9



Area of trap OAQP = $\frac{1}{2}\left(40-18\ell n3\right)\ell n3$

Req. area $=\frac{1}{2}\left(40-18\ell n3\right)-\int_0^{\ell n3}\left(e^{2x}-18x+15\right)dx$

$=5\ell n3-4$

2. Ans. 6

$f(x) = x^4 + px^2 + qx + r \begin{cases}\alpha\\ \alpha\\ \alpha\\ \beta\end{cases}$

$f'(x) = 4x^3 + 2px + q \begin{cases}\alpha\\ \alpha\\ \gamma\end{cases}$

$f''(x)=12x^2+2p\begin{cases}\alpha\\ \delta\end{cases} \Rightarrow f''(x) = 0$

$\Rightarrow \alpha^2=-\frac{p}{6}$

Now

(1) $\alpha^4 + p\alpha^2 + q\alpha + r = 0$

(2) $4\alpha^3 + 2p\alpha + q = 0$

(1) - (2) × α

$\Rightarrow -3\alpha^4 - p\alpha^2 + r = 0$

$-\frac{3p^2}{36}+\frac{p^2}{6}+r=0$

$\Rightarrow p^2 + 12r = 0$

3. Ans. 4

$2x=u\Rightarrow I=\frac{1}{2}\int_{-8}^{16}f\left(u\right)du\Rightarrow I=\frac{8}{2}.\int_0^3 f\left(u\right)du=4$

4. Ans. 4

$f(x) = 0$ has 3 distinct real roots

$\Rightarrow f'(x) = 0$ has atleast 2 distinct real roots.

$\Rightarrow (f(x).f'(x))' = 0$ has atleast 4 distinct real roots.

5. Ans. 6

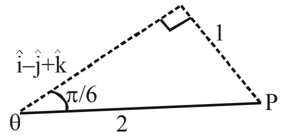
$$N=\sum_{k=1}^{22}\frac{z^{8k}-1}{z^{24k}-1}=\sum_{k=1}^{22}\frac{z^{8k}-1}{z^k-1}=\sum_{k=1}^{22}\left(1+z^k+z^{2k}+....+z^{7k}\right)$$

$= 22 + (0–1)\times 7 = 15$

6. Ans. 3

$\left(\hat{i}-\hat{j}+\hat{k}\right).\left(a\hat{i}+b\hat{j}+c\hat{k}\right)=3$

$\cos\theta=\frac{\sqrt{3}}{2}$

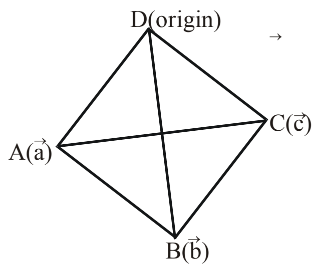


$\sqrt{3}.2\cos\theta=3$ $\{\because a^2 + b^2 + c^2 = 4\}$

locus of 'P' is circle with radius A = π

[A] = 3

7. Ans. 3



$\left|\vec{a}-\vec{b}\right|=\left|-\vec{c}\right|$

$\Rightarrow$ $\vec{a}^2+\vec{b}^2-\vec{c}^2-2\vec{a}.\vec{b}=0$

$\vec{b}^2+\vec{c}^2-\vec{a}^2-2\vec{b}.\vec{c}=0$

$\vec{c}^2+\vec{a}^2-\vec{b}^2-2\vec{a}.\vec{c}=0$

$\vec{a}^2+\vec{b}^2+\vec{c}^2-2\vec{a}.\vec{b}-2\vec{b}.\vec{c}-2\vec{c}.\vec{a}=0$ ...(1)

Also $\left|\vec{a}+\vec{b}-\vec{c}\right|^2\ge 0$

$\vec{a}^2+\vec{b}^2+\vec{c}^2+2\vec{a}.\vec{b}-2\vec{a}.\vec{c}-2\vec{b}.\vec{c}\ge 0$ ....(2)

(1) & (2) similarly

$\vec{a}.\vec{b}>0$

$\vec{b}.\vec{c}>0$

$\vec{c}.\vec{a}>0$

equality cannot hold as points becomes collinear.

8. Ans. 4

Let $\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|=\left|\vec{d}\right|=x$

Consider

$\left(\vec{a}\times\vec{c}\right).\left(\vec{b}\times\vec{d}\right)=\left(\vec{c}.\vec{d}\right)\left(\vec{a}.\vec{b}\right)-\left(\vec{c}.\vec{b}\right)\left(\vec{a}.\vec{d}\right)=0$

$\left\{\because \vec{a}.\vec{b}=\vec{c}.\vec{d}=\vec{b}.\vec{c}=\vec{a}.\vec{d}=\frac{x^2}{2}\right\}$

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (ADVANCED) 2016

Test Type : **ALL INDIA OPEN TEST** (MAJOR)   Test Pattern : JEE-Advanced

TEST # 03   **TEST DATE : 14 - 02 - 2016**

PAPER-2

PART-1 : PHYSICS   **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,D	A,D	B,C	A,B,C	B,C	A,D	A,C	C	A,D	C,D
	Q.	11	12								
	A.	A	A,D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	6	8	5	4	2	7	4	2		

SOLUTION

SECTION-I

1. Ans. (A,D)

Sol. $nR = \frac{1}{25}\times 10^2 = 4 \Rightarrow \frac{m}{32} = \frac{4}{25}\times 3$

$m = \frac{96\times 4}{25} = 15.36\text{gm}$

$V = \frac{nRT}{p}$

V_{max} at 3

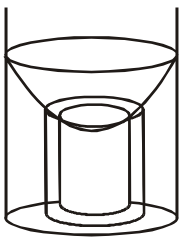
$nR = \frac{16\times 40}{400}$

$V_1 = \frac{1}{25}\times\frac{300}{1}$

$V_2 = \frac{1}{25}\times\frac{400}{2}$

2. Ans. (A,D)

Sol. $2h = h + \frac{\omega^2R^2}{2g}$



$\sqrt{\frac{2gh}{R^2}} = \omega$

$dK = \frac{1}{2}\times dm\omega^2x^2$

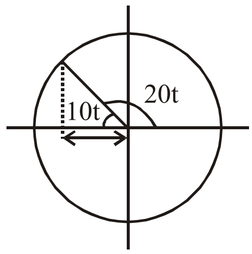
$= \frac{1}{2}\rho\times 2\pi x dx\left(h + \omega^2\frac{x^2}{2g}\right)\times\omega^2x^2$

$K = \pi\rho\omega^2\left[\frac{R^4}{2}h + \frac{\omega^2}{2g}\times\frac{R^6}{6}\right]$

$= \pi\rho\omega^2\left[\frac{R^4}{2}h + \frac{2gh\times R^4}{12g}\right] = \frac{5\pi\rho\omega^2R^4h}{6}$

3. Ans. (B,C)

Sol. $\omega_1 = \sqrt{\frac{100}{1}} = 10\text{ rad/s}$



$\omega_2 = \sqrt{\frac{400}{1}} = 20\text{ rad/s}$

$30\,t = \pi$

$t = \frac{\pi}{30}$ sec

4. Ans. (A,B,C)

Sol. (A) $t = 0$

$i_L = 0$

$V = \frac{V}{2}$

(B) $t = \infty$

$i = \frac{2V}{3R}\quad i_2 = \frac{V}{3R} \Rightarrow V_A = \frac{V}{3}$

(C) after opening

$i_A = \frac{V}{3R}$

$V_L = V_A + V_R = 2R\times\frac{V}{3R} = \frac{2V}{3}$

$V_A = \frac{V}{3}$

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5. Ans. (B,C)

Sol. P_{max} at resonance $\Rightarrow \omega = \frac{1}{\sqrt{LC}}$

$$f = \frac{1}{2\pi}\sqrt{10 \times 40 \times 10^{-6}} \quad = \frac{25}{\pi}\text{Hz}$$

$$Q_{factor} = \frac{\omega}{2\Delta\omega} = \frac{\omega L}{R} \Rightarrow \Delta\omega = \frac{R}{2L} = \frac{100}{20} = 5$$

$$\Delta f = \frac{45}{2\pi}$$

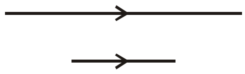
6. Ans. (A, D)

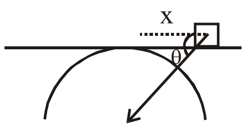
Sol. $\tau_{mg} = 0.5g \times 1\hat{i} = 5\hat{i}$

$$= \tau_B = \left(-0.5 \times 4\hat{i}\right) \times \vec{B}$$

Option A $\tau_B = -5\hat{i} + 6\hat{k}$ those $\vec{B}$ for which $\vec{\tau}$ has $-5\hat{i}$ are useful.

7. Ans. (A,C)

Sol. (B)  unstable equilibrium

(C) 

$$F = \frac{GMm}{R^2} \times \cos\theta = \frac{GMmx}{R^3} \text{ so SHM.}$$

(D) F proportional to x^3

8. Ans. (C)

Sol. Pipe fixed

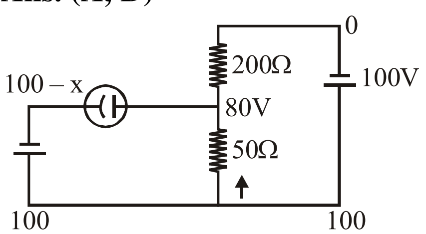
$$S_1 \int p dt = m_1 v$$

$$S_2 \int p dt = m_2 v_2$$

$$v_2 = \frac{m_1 v}{S_1} \times \frac{S_2}{m_2}$$

Pipe free same effect

9. Ans. (A, D)

Sol. 

$$i = \frac{100}{250} = 0.4A$$

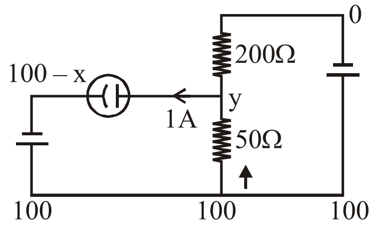
$V_s = 1.5$ V

no light

for current to flow initially

$80 > 100 - x$

light $x > 20V$



$$1 + \frac{y-0}{200} + \frac{y-100}{50} = 0$$

$$\frac{y}{200} + \frac{y}{50} = 1$$

$$\Rightarrow \frac{y+4y}{200} = 1$$

$\Rightarrow y = 40$

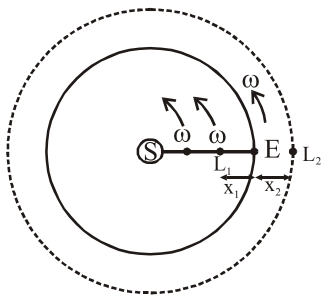
$40 > 100 - x$

$x > 60V$

10. Ans. (C,D)

Sol. $\lambda < \frac{\lambda}{h} = \frac{hC}{2.5} = \frac{1240}{5} \times 2 = 496\text{nm}$

11. Ans. (A)

Sol. 

$$\omega^2 = \frac{GM_s}{R^3}$$

$$\frac{GM_s}{(R-x_1)^2} - \frac{GM_e}{x_1^2} = \omega^2\left(R - x_1\right)$$

$$\Rightarrow \frac{M_s}{(R-x_1)^2} - \frac{M_e}{x_1^2} = \frac{M_s}{R^3}\left(R - x_1\right)$$

$$\therefore x_1^3 = \frac{M_e R^3}{3M_s}$$

$$\frac{GM_s}{(R+x_2)^2} + \frac{GM_e}{x_2^2} = \frac{GM_s}{R^3}\left(R + x_2\right)$$

$$\frac{M_s}{R^2}\left[\frac{3x_2}{R}\right] = \frac{M_e}{x_2^2}$$

12. Ans. (A, D)

SECTION-IV

1. **Ans. 6**

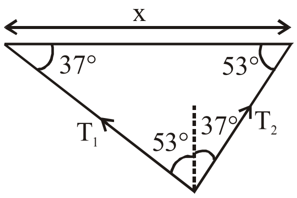
Sol. ${}^{14}_{7}N \rightarrow {}^{10}_{5}B + {}^{4}_{2}He$

$Q = (0.00307 - 0.01294 - 0.00260)\ 931$

$= 0.01247 \times 931$

2. **Ans. 8**

Sol.



$T_2 \sin 37 = T_1 \sin 53$

$3T_2 = 4T_1$

$\ell_2 = x\cos 53° = \frac{3x}{5}$

$\ell_1 = x\cos 37° = \frac{4x}{5}$

$$t_2 = \frac{\frac{3x}{5}}{\sqrt{\frac{T_2}{\mu}}} = 3\sqrt{3}\times 10^{-3}$$

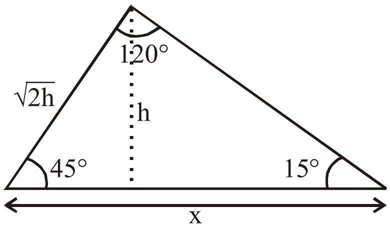
$$\Rightarrow \frac{3x}{5}\sqrt{\frac{3\mu}{4T_1}} = \sqrt{3}\times 10^{-3}$$

$$t_1 = \frac{\frac{4x}{5}}{\sqrt{\frac{T_1}{x}}} = \frac{4}{5}x\sqrt{\frac{\mu}{T_1}} = \frac{4}{5}\times 80\times 10^{-3}$$

$= 8 \times 10^{-3}$ sec

3. **Ans. 5**

Sol.



$$\frac{2\sqrt{2}\times\sqrt{2}\times 1.05}{\sqrt{3}-1} = \frac{2x}{\sqrt{3}}$$

$$\frac{2.1\sqrt{3}}{\sqrt{3}-1} = x$$

$$x = \frac{2.1}{1-0.577} = \frac{2.10}{0.42}$$

4. **Ans. 4**

Sol. $\frac{kQ}{r^2} - \frac{kQ^2}{4r^2} = \frac{mv^2}{r}$

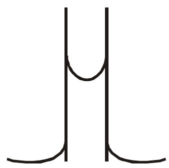
$$mv^2 = \frac{3kQ^2}{4r}$$

$$\Rightarrow E = 2\times\frac{1}{2}mv^2 + \frac{kQ^2}{2r} - \frac{2kQ^2}{r}$$

$$= \frac{3kQ^2}{4r} + \frac{kQ^2}{2r} - \frac{2kQ^2}{r} = -\frac{3kQ^2}{4r}$$

5. **Ans. 2**

Sol.



$2\pi r \times S \times 2 = 0.56\ \pi \times 10^{-3}$

$2\pi r \times 0.07 = 0.56\pi \times 10^{-3}$

$r = 2$mm

6. **Ans. 7**

Sol. $\frac{1}{5} = 1 - \frac{Q_L}{Q_g} \Rightarrow Q_L = \frac{4}{5}Q_g$

$$\frac{1}{10} = 1 - \frac{Q'_L}{Q_L}$$

$$Q'_L = \frac{9}{10}Q_L = \frac{36}{50}Q_g$$

$$\eta = 1 - \frac{Q'_L}{Q_g} = 1 - \frac{36}{50} = \frac{14}{15}$$

$\Rightarrow 28\%$

7. **Ans. 4**

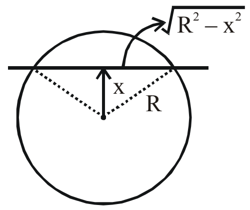
Sol. $R_{approx} = \frac{120}{2} = 60\Omega$

$$i_x = i - i_v = 2 - \frac{120}{960} = \frac{15}{8}A$$

$$\Rightarrow R = \frac{120}{\frac{15}{8}} = 64\Omega \Rightarrow \Delta R = 4\Omega$$

8. **Ans. 2**

Sol.



$$2\sqrt{R^2 - x^2}\times\mu_0 ni\times i = ma$$

$$= mv\frac{dv}{dx}$$

$$\int v\,dv = \frac{2\mu_0 ni^2}{m}\int_0^R \sqrt{R^2 - x^2}\,dx$$

$$\frac{v^2}{2} = \frac{2\times 4\pi\times 10^{-7}\times 4000\times i^2}{m}$$

$$\int_0^R \sqrt{R^2 - x^2}\,dx$$

$x = R \sin \theta$
$dx = R \cos \theta\, d\theta$

$$\int_0^{\pi/2} R^2 \cos^2\theta\, d\theta = \frac{R^2}{2}\int [1+\cos 2\theta]\, d\theta$$

$$= \frac{R^2}{2}\left[\frac{\pi}{2} + \frac{1}{2}(\sin \pi - \sin 0)\right]$$

$$= \frac{R^2\pi}{4}$$

$$\frac{v^2}{2} = \frac{8\pi\times 10^{-7}\times 4000\times i^2}{0.02}\times\frac{\pi}{4}\times 2$$

$$\frac{v^2}{2} = \frac{8\pi\times 10^{-7}\times 4000\times i^2}{4\times 10^{-3}}\times 1^2\times\frac{\pi}{4}$$

$v = 2\text{m/s}$

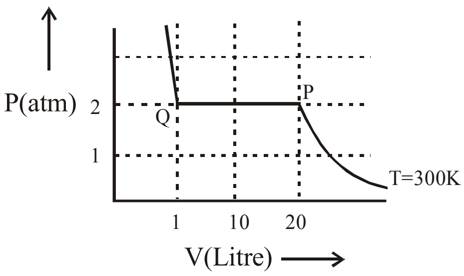
PART-2 : CHEMISTRY — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,C,D	D	B,D	C,D	A,C,D	A,B,D	B	A,C,D	B,C	D
	Q.	11	12								
	A.	A,B,D	B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	2	1	2	0	6	4	6	3		

SOLUTION

SECTION-I

1. Ans. (A,C,D)



(A) A real gas condense if external pressure is just greater than vapour pressure

(B,C,D)

At point 'P' whole real gas is in gaseous phase

so density of gas $= \dfrac{1000}{20\times 1000} = 0.05\text{gm/ml}$

At point 'Q' whole substance is in liquid phase

so density of liquid $= \dfrac{1000}{1\times 1000} = 1\text{gm/ml}$

Note : during condensation at 300K density of gas and liquid remains constant, only amount varies.

2. Ans.(D)

In terms of edge length 'a'

$$X = \frac{\sqrt{2}}{2}a$$

$$Y = \frac{a}{2}$$

$$Z = \frac{\sqrt{3}a}{4}$$

3. Ans. (B, D)

4. Ans. (C, D)

5. Ans. (A,C,D)

6. Ans. (A,B,D)

7. Ans. (B)

8. Ans. (A,C,D)

9 Ans. (B, C)

If n/p ratio is smaller than required possible emission is

$$_{53}P^{135} \longrightarrow {}_{54}Q^{135} + {}_{-1}e^0$$

If n/p is smaller than required possible emission are

$${}_{54}Q^{135} \longrightarrow {}_{52}R^{131} + {}_{2}He^{4}$$

or

$${}_{54}Q^{135} \longrightarrow {}_{53}R^{135} + {}_{+1}e^{0}$$

10. Ans.(D)

For 'Q' using steady state concept

Rate of appereance of Q = Rate of disappeareance of Q

$K_1[P] = K_2[Q]$

Number of nuclei of Q

$$= \frac{k_1}{k_2} \times \text{ no. of nuclei of P}$$

$$= \frac{10/60}{1000} \times 6 \times 10^{23} = 10^{20}$$

For 'R'

Since rate constant is very high for second step than first step

Number of nuclei of R = Number of nuclei of P disintegrated = 6×10^{23}

11. Ans. (A,B,D)

12. Ans. (B)

SECTION-IV

1. Ans. 200 [OMR Ans. 2]

At freezing point vapour pressure of solid

= vapour pressure if liquid

$$10 - \frac{3000}{T} = 5 - \frac{2000}{T}$$

$$5 = \frac{1000}{T}$$

T = 200 K

2. Ans. (T_5 = 100) OMR ANS (1)

Let heat involved in step 5 is Q_5

$q_{total} = - W_{total}$

$500 + 800 + Q_5 = 700$

$Q_5 = - 600$ J

Since ΔS total = 0

$$\frac{500}{200} + \frac{800}{200} - \frac{600}{T_5} = 0$$

$2 + 4 - 6 = 0.$

$T_5 = 100$ K

3. Ans. 2

4. Ans. 0

Sol. $FeO.Cr_2O_3 + Na_2O_2 \rightarrow 2NaFeO_2 + 4Na_2CrO_4 + 2Na_2O$

(A) (B)

$2NaFeO_2 + H_2O \rightarrow NaOH + Fe_2O_3$

(A) (C)

red brown ppt.

$Na_2CrO_4 + H_2O \rightarrow CrO_4^{-2} + 2Na^+$

(B) (D)

$CrO_4^{-2} + 2H^+ \rightarrow Cr_2O_7^{2-} + H_2O$

(D) (E)

(orange)

5. Ans. 6

6. Ans. 4

7. Ans. 6

8. Ans. 3

PART-3 : MATHEMATICS — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,B,C	B,D	A,B	A,C	B,C	A,D	A,C	A,C	C	B
	Q.	11	12								
	A.	C	C								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	0	0	4	9	2	3	5	5		

SOLUTION

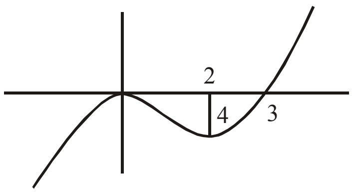
SECTION-I

1. Ans. (A,B,C)

$$\int_0^1 f(xt)dt = 0 \Rightarrow \int_0^x f(t)dt = 0 \Rightarrow f(x) = 0$$

(A) $y = 0$ & $y = x^3 - 3x^2 + P$ intersect at 3 distinct points.

$x^3 - 3x^2 + P = 0 \Rightarrow \quad P = 3x^2 - x^3$



$\therefore\ P \in (-4,0)\ \therefore\ P = -3,-2,-1$

(B) $f(x) = 0$ is a periodic function

(C) for minimum area $f(3) = 0$

$81 - 12 - a = 0 \Rightarrow a = 69$

(D) $f(x) = 0$ is even as well as odd.

2. Ans. (B,D)

For $k_1 > k_2 \quad \Rightarrow k_1\cos^2x > k_2\cos^2x$

$\Rightarrow -k_1\cos^2x < -k_2\cos^2x$

$$\Rightarrow \sqrt{1-k_1\cos^2 x} < \sqrt{1-k_2\cos^2 x}$$

$$\Rightarrow \frac{1}{\sqrt{1-k_1\cos^2 x}} > \frac{1}{\sqrt{1-k_2\cos^2 x}}$$

$\therefore\ f(k_1) > f(k_2)$

Hence $f(x)$ is increasing function.

3. Ans. (A,B)

$AB = B^T$

$\Rightarrow A = B^TB^{-1}$

$|A| = |B^T|.\ |B^{-1}| = 1 \qquad ...(1)$

Also $\text{Adj } A = \text{Adj}(B^T B^{-1})$

$= \text{Adj}(B^{-1})\ \text{Adj}(B^T)$

$\text{Adj}A = (\text{Adj}B)^{-1}\ \text{Adj}(B^T)$

$\Rightarrow (\text{Adj}B)^T = (\text{Adj}B)\ \text{Adj}A$

$= (\text{Adj}B)\ B$

$\Rightarrow (\text{Adj}B)^T = |B|.I$

$\Rightarrow \text{Adj}B = |B|.I$

$\Rightarrow \text{Adj}(\text{Adj}A) = |\text{Adj}A|\ I$

$\therefore\ |A|^{n-2}A = |A|^{n-1}I$ (order n)

$\Rightarrow A = |A|I \Rightarrow A = I$ (By (1))

$\therefore\ B = I$

4. Ans. (A,C)

If $f(x)$ touches x-axis at $\left(\sqrt{2},0\right)$

it also touches at $\left(-\sqrt{2},0\right)$

$\therefore$ Roots are $\sqrt{2},\sqrt{2},-\sqrt{2},-\sqrt{2}$

5. Ans. (B,C)

$$I(n) = \int_0^{\frac{1}{n}}(2016+x)\cos^2 x\,dx$$

$$\therefore\ I\left(\frac{1}{\pi}\right) = \int_0^{\pi}(2016+x)\cos^2 x dx$$

$$I\left(\frac{1}{\pi}\right) = \int_0^{\pi}(2016+\pi-x)\cos^2 x dx$$

$$\therefore\ 2I\left(\frac{1}{\pi}\right) = (\pi+2016)\int_0^{\pi}\cos^2 x dx$$

$$\Rightarrow I\left(\frac{1}{\pi}\right) = \frac{(\pi+4032)\pi}{4}$$

$$\lim_{n\to\infty} nI(n) = \lim_{n\to\infty}\frac{\int_0^{1/n}(x+2016)\cos^2 x dx}{1/n}$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n}+2016\right)\cos^2\frac{1}{n}\times\left(-\frac{1}{n^2}\right)}{\left(-1/n^2\right)} = 2016$$

6. Ans. (A,D)

$z_1 = \omega, \quad z_2 = \omega^2$

$$\therefore\ z_1^n + z_2^n = \left(e^{\frac{i2\pi}{3}}\right)^n + \left(e^{-\frac{i2\pi}{3}}\right)^n$$

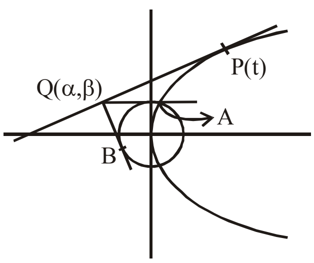
$$= 2\cos\left(\frac{2n\pi}{3}\right)$$

$$z_1^{2016} + z_2^{2016} = 2$$

$$z_1^{2015} + z_2^{2015} = 2\cos\left(\frac{4030\pi}{3}\right)$$

$$= 2\cos\left(1343\pi + \frac{\pi}{3}\right) = 2\times\left(-\frac{1}{2}\right) = -1$$

7. Ans. (A,C)



Equation of chord of contact

$\alpha x + \beta y = 4$ ...(1)

Also, Equation of tangent at P(t)

is $ty = x + 2t^2$

Also $t\beta = \alpha + 2t^2$ ...(2)

from (1) & (2)

$(t\beta - 2t^2)x + \beta y = 4$

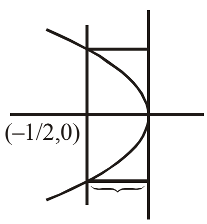
$\beta(tx + y) - (2t^2x + 4) = 0$

$\therefore\ tx + y = 0\ \ \&\ 2t^2x + 4 = 0$

$$t = -\frac{y}{x}$$

$$\therefore\ 2.\frac{y^2}{x^2}\times x + 4 = 0$$

$y^2 + 2x = 0$



Area bounded by S = 0 & the line 2x + 1 = 0

$$\frac{2}{3}\times\frac{1}{2}\times 2 = \frac{2}{3}$$

$2x - y + 1 = 0$ is a focal chord.

Hence angle is 90°.

8. Ans. (A,C)

Equation of family of planes

$(2x - y + z - 2) + \lambda(x + 2y - z - 3) = 0$

It must satisfy (3, 2, 1)

$(6 - 2 + 1 - 2) + \lambda(3 + 4 - 1 - 3) = 0$

$3 + 3\lambda = 0 \Rightarrow \lambda = -1$

$x - 3y + 2z + 1 = 0$

Equation of acute angle bisector

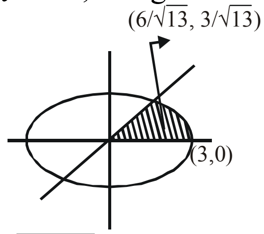
$$\frac{2x - y + z - 2}{\sqrt{6}} = \frac{x + 2y - z - 3}{\sqrt{6}}$$

$x - 3y + 2z + 1 = 0$

Paragraph for Question 9 to 10

Solving $y = \frac{x}{2}$ & $\frac{x^2}{9} + y^2 = 1$, we get.

$$x = \frac{6}{\sqrt{13}}\ \&\ y = \frac{3}{\sqrt{13}}$$



Shaded region

$$= \frac{1}{2}\times\frac{6}{\sqrt{13}}\times\frac{3}{\sqrt{13}} + \int_{6/\sqrt{13}}^{3}\sqrt{1 - \frac{x^2}{9}}\,dx$$

$$= \frac{9}{\sqrt{13}} + \frac{1}{3}\left(\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right)\Bigg|_{6/\sqrt{13}}^{3}$$

$$= \frac{9}{\sqrt{13}} + \frac{1}{3}\left(\frac{9}{2}.\frac{\pi}{2} - \frac{3}{\sqrt{13}}\frac{9}{\sqrt{13}} - \frac{9}{2}\sin^{-1}\frac{2}{\sqrt{13}}\right)$$

$$= \frac{3\pi}{4} - \frac{3}{2}\sin^{-1}\frac{2}{\sqrt{13}}$$

$$= \frac{3}{2}\sin^{-1}\left(\frac{3}{\sqrt{13}}\right)$$

similarly calculate the other area

as $\frac{3}{2}\sin^{-1}\left(\frac{1}{\sqrt{1 + 9m^2}}\right)$

9. Ans. (C)

10. Ans. (B)

$16\Delta^2 = (a+b+c)\ (a+b-c)\ (a+c-b)\ (b+c-a)$

Now, b = 2c & a = 3, we get

$16\Delta^2 = (3c + 3)\ (3 + c)\ (3 - c)\ (3c - 3)$

$= 9(9 - c^2)\ (c^2 - 1)$

Using A.M $\geq$ G.M, we get

$(9 - c^2) + (c^2 - 1) \geq 2\ ((9 - c^2)\ (c^2 - 1))^{1/2}$

$\therefore$ Maximum value will be attained

for $9 - c^2 = c^2 - 1$

$c = \sqrt{5}$

$\therefore\ 16\Delta^2 = 9.4.4$

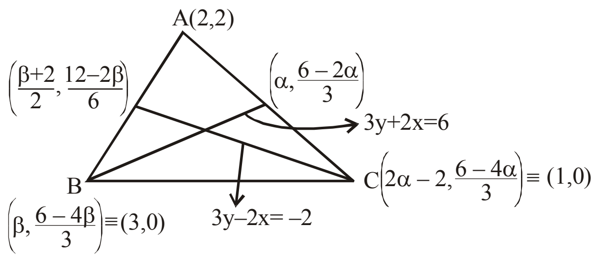
$\therefore\ D_{max} = 3$

11. Ans. (C)

12. Ans. (C)

SECTION – IV

1. Ans. 0



$\Rightarrow\ 6 - 4\alpha - 4\alpha + 4 = -2$

$\Rightarrow\ \alpha = \frac{3}{2}$

Also $\frac{3.(12-2\beta)}{6} - (\beta+2) = -2$

$12 - 2\beta - 2\beta - 4 = -4$

$4\beta = 12 \Rightarrow \beta = 3$

$\therefore$ Slope of BC = 0

2. Ans. 0

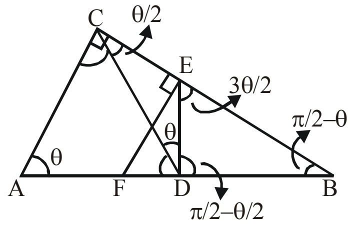
$(1 + x^2 + x^4 + ... + x^{100} - x\ (1 + x^2 + ... + x^{98}))$

$(1 + x^2 + ... + x^{100} - x(1 + x^2 + ... + x^{98}))$

$= (1 + x^2 + x^4 + ... + x^{100})^2 - x^2(1 + x^2 + ... + x^{98})$

which implies there is no term having odd power of x.

3. Ans. 4



Using sine law in ΔBDE

$$\frac{BE}{\cos\frac{\theta}{2}} = \frac{BD}{\sin\left(\frac{3\theta}{2}\right)} \qquad ...(1)$$

Also sine law in ΔBDC

$$\frac{BC}{\cos\frac{\theta}{2}} = \frac{BD}{\sin\frac{\theta}{2}} \qquad ....(2)$$

Dividing we get

$$\frac{BE}{BC} = \frac{\sin\frac{\theta}{2}}{\sin\frac{3\theta}{2}} = EF$$

$\therefore\ \lim\limits_{\theta\to 0} EF = \frac{1}{3} \qquad \therefore\ p + q = 4$

4. Ans. 9

Using Baye's theorem

$$\frac{\frac{2}{6}\times 1}{\frac{3}{6}\times 0 + \frac{1}{6}\times\frac{1}{2} + \frac{2}{6}\times 1}$$

$= \frac{4}{5}$

$\therefore\ a + b = 9.$

5. Ans. 2

We have to calculate

$$\left(\frac{a^2+1}{a}\right)\left(\frac{b^2+1}{b}\right)\left(\frac{c^2+1}{c}\right)$$

Now $x^3 - 2x^2 + 3x - 4 = 0 \begin{cases} a \\ b \\ c \end{cases}$

$\therefore$ abc = 4

Also $x^3 - 2x^2 + 3x - 4 = (x - a)(x - b)(x - c)$

Put $x = i$ & $-i$ and multiply, we get

$\therefore (a^2 + 1)(b^2 + 1)(c^2 + 1) = 8$

$$\therefore \frac{(a^2+1)(b^2+1)(c^2+1)}{abc} = 2$$

6. Ans. 3

$V_{DABC} = \frac{1}{6} = \frac{1}{3} \times$ (area of base) $\times$ height

Also $\frac{1}{6} \le \frac{1}{3} \times \frac{1}{2} \times AC \times BC \times \sin 45° \times AD$

$$\therefore \left(\frac{AC}{\sqrt{2}}\right).BC.AD \ge 1$$

Now, $\frac{AC}{\sqrt{2}} + BC + AD \ge 3.\sqrt[3]{\frac{AC}{\sqrt{2}}.BC.AD}$

(using A.M.- G.M.)

$$\therefore \frac{AC}{\sqrt{2}}.BC.AD \le 1$$

$\therefore$ Equality must hold

$$\frac{AC}{\sqrt{2}} = BC = AD = 1$$

$\therefore$ AD must be perpendicular

$\therefore CD^2 = AD^2 + AC^2 = 3$

7. Ans. 5

Given $|z| = 1$ & $\left|a + z + \frac{1}{z}\right| = 1$

$\therefore$ 'a' moves on a circle with centre $\left(-z - \frac{1}{z}\right)$ and radius 1.

$-z - \frac{1}{z}$ can take any value in the interval $[-2,2]$, so the set S consists of the locus of unit circles centered at a point on the line segment with end points at -2 & 2 on the argand plane. Therefore, the area S is the area of two semi circles and area of rectangle which is $\pi + 8$.

8. Ans. 5

$$S = \frac{1}{2^0 + \sqrt{2^{2015}}} + \frac{1}{2^1 + \sqrt{2^{2015}}} + \ldots + \frac{1}{2^{2015} + \sqrt{2^{2015}}}$$

$$S = \frac{1}{2^{2015} + \sqrt{2^{2015}}} + \frac{1}{2^{2014} + \sqrt{2^{2015}}} + \ldots + \frac{1}{2^0 + \sqrt{2^{2015}}}$$

$$2S = \frac{1}{\sqrt{2^{2015}}} + \frac{1}{\sqrt{2^{2015}}} + \ldots\ldots.2016 \text{ times}$$

$$\therefore \quad S = \frac{2016}{\sqrt{2^{2017}}}$$

$\therefore \quad k = 5$