Paper Code : 0000CT103115004

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2015 - 2016)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (MAIN) 2016

Test Type : **ALL INDIA OPEN TEST** (MAJOR) Test Pattern : JEE-Main

TEST # 06 **TEST DATE : 20 - 03 - 2016**

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	2	4	2	3	1	1	1	3	3	1	2	2	2	3	1	4	4	3	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	1	3	3	3	4	2	Bonus	2	1	3	1	3	2	3	2	2	3	3	1	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	3	4	3	2	4	1	3	2	4	2	4	1	2	2	4	4	4	3	1
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	1	1	3	3	2	3	2	1	4	3	3	1	2	2	2	3	4	2	1	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	1	3	4	2	2	2	4	3	3										

HINT – SHEET

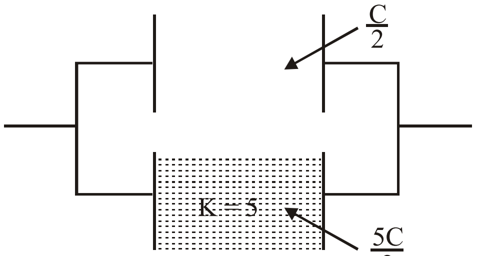
1. Ans. (4)

Sol. $\dfrac{\Delta W}{\Delta Q_{given}} = \left(1 - \dfrac{T_L}{T_M}\right)$

2. Ans. (2)

3. Ans. (4)

Sol. Now



if $C = \dfrac{A\varepsilon_0}{d}$

$C_{eq} = 3C$

$\Delta C = 3C - C$

$\% \ \Delta C = 200\%$

4. Ans. (2)

Sol. $\overline{\overline{A}\,.\,\overline{B}} = \overline{\overline{A}}\,.\,\overline{\overline{B}} = A\,.\,B$ = A and B

5. Ans. (3)

Sol. $(\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 = 0$

$\delta_1 + \delta_2 = 0$

6. Ans. (1)

Sol. $\Delta\ell = \dfrac{\ell T}{AY} = \dfrac{\ell T}{\pi r^2 Y} = \left(\dfrac{T}{\pi Y}\right) \times \dfrac{\ell}{r^2}$

$\dfrac{\ell}{r^2} \rightarrow$ max for L =100 cm

& r = 0.2 mm

7. Ans. (1)

Sol. $6\pi\eta r v = \left(\frac{4}{3}\pi r^3\right)\ell \qquad \Rightarrow v \propto r^2$

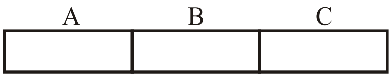
& $\frac{4}{3}\pi r'^3 = 8 \times \left(\frac{4}{3}\pi r^3\right) \Rightarrow r' = 2r$

& $\frac{v}{v'} = \left(\frac{r}{r'}\right)^2$

$\Rightarrow v' = 4v = 40$ cm/sec

8. Ans. (1)

Sol.



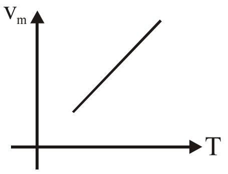
$R_A = \frac{\ell}{K_A A} \quad R_B = \frac{\ell}{\frac{K_A}{2}\times A} \quad R_C = \frac{\ell}{\frac{K_A}{6}\times A}$

$R_A = \frac{3\ell}{kA} = R_A + R_B + R_C$

$= \frac{9\ell}{K_A \times A} \Rightarrow K = \frac{K_A}{3}$

9. Ans. (3)

Sol.

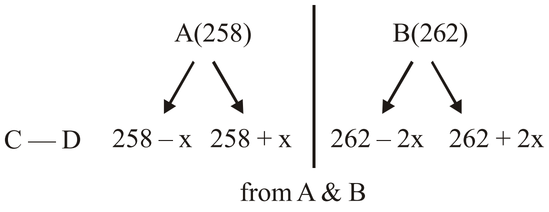


$\therefore \; T\lambda_m = b$

$\Rightarrow T \propto v_m$

10. Ans. (3)

Sol.



Case-I $\qquad 258 - x = 262 - 2x \Rightarrow x = 4$

or

Case-II $\qquad 258 + x = 262 - 2x \Rightarrow x = \frac{4}{3}$

$f_C = 258 - x = 262 - 2x$ where $x = y$

$= 254$ Hz

11. Ans. (1)

Sol. $600 = \frac{\sqrt{\frac{T}{\mu}}}{2\ell} = f_0$

Now $\qquad T' = \frac{T}{9}$ & $\ell = 2\ell$

$\mu' = \frac{\mu}{4}$

$f'_0 = \frac{\sqrt{\frac{T'}{\mu'}}}{2\ell'} = \frac{\sqrt{\frac{\frac{T}{4}}{\frac{u}{4}}}}{2(2\ell)} = \frac{1}{3}f_0 = 200$ Hz

12. Ans. (2)

Sol. $h \sin v_C = 1$

$h = \frac{5}{3}$ & $\tan\theta_P = h$

$O_P = \tan^{-1}(n)$

13. Ans. (2)

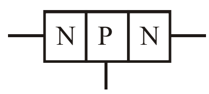
Sol. $I = neAv_d$

$20 = 10^{24} \times 1.6 \times 10^{-19} \times (1 \times 10^{-3})^2 \times v_d$

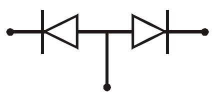
$\Rightarrow v_d = 12.5 \times 10^{-4}$ m/sec

14. Ans. (2)

Sol.



$\Rightarrow$



15. Ans. (3)

Sol. $\frac{\Delta u}{u}\times 100 = \left(\frac{2\Delta d}{d} + \frac{\Delta v}{v} + \frac{\Delta\ell}{\ell}\right)\times 100$

$\frac{\Delta\ell}{\ell} = \frac{30}{1800} = \left[2\times\frac{0.04}{1-6} + \left(\frac{30}{1800}\right)\right]\times 100$

$= 8.166\% \approx 8.2\%$

16. Ans. (1)

17. Ans. (4)

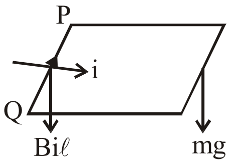
Sol. $A = 2$

$v_{max} = 6$

$$\begin{cases} v_{max} = A\omega \\ \Rightarrow \omega = 3 \\ \& \ T = \dfrac{2\pi}{\omega} \end{cases}$$

18. Ans. (4)

Sol. $Bi\ell \times d_1 = mg \times d_2$



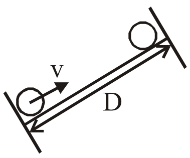
19. Ans. (3)

Sol. at $t = T$

$N_x = N_y$ (one half life)

20. Ans. (3)

Sol.



time of collision

$$t = \frac{D}{v_0}$$

$$v_0 = \sqrt{2gD\sin\theta}$$

$$AC = v_0 t - \frac{1}{2}\ \text{gin}\sin\theta\ t^2$$

$$BC = \frac{1}{2} g\sin\theta\ t^2$$

$$\frac{AC}{BC} = D - \frac{1}{2} g\sin\theta \frac{D^2}{2gD\sin\theta} = \frac{3D}{4}$$

$$\frac{1}{2}\frac{g\sin\theta D^2}{2gD\sin\theta} = 3$$

21. Ans. (1)

Sol. $2\ mg\sin\theta = mg$

$$\sin\theta = \frac{1}{2} \quad \theta = 30°$$

22. Ans. (3)

Sol. $U = \rho_{lig}Adg$

$\& \ F_{net} = mg - \rho_{lig}Adg$

& mg = constant

23. Ans. (3)

24. Ans. (3)

25. Ans. (4)

Sol. Energy conservation

& Linear mometum conservation

26. Ans. (2)

27. Ans. (Bonus)

28. Ans. (2)

29. Ans. (1)

Sol. Optical path diff $\Delta x = (\mu - 1)t$

Phase diff $\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x$

$$= \frac{2\pi}{600\times10^{-9}}\times0.4\times5\times10^{-6}$$

$$\Delta\phi = \frac{20\pi}{3}$$

$$I_{res} = I_0\cos^2\left(\frac{\Delta\phi}{2}\right) = I_0\cos^2\left(\frac{10\pi}{3}\right)$$

$$\left[I_{res} = \frac{I_0}{4}\right]$$

30. Ans. (3)

31. Ans.(1)

32. Ans. (3)

Sol. Colloidal particles are positively charged.

33. Ans.(2)

Sol. Let normality of $KMnO_4^-$ solution is N

$\therefore \quad N \times 10 = 50 \times 1 \quad \Rightarrow \quad N = 5$

molecular equation of MnO_4^- = molecular equation of oxalic acid

$$5 \times 100 = \frac{W}{E} \times 1000 \Rightarrow \left(\frac{W}{90/2}\right) \times 1000$$

W = 22.5 gm Ans.

34. Ans. (3)

35. Ans. (2)

36. Ans. (2)

37. Ans. (3)

38. Ans. (3)

39. Ans (1)

40. Ans.(4)

41. Ans.(3)

Sol.

$$Z = \frac{PV}{nRT}$$

$$\Rightarrow \frac{V}{n} = \frac{0.9 \times 0.0821 \times 273}{9} = 2.24 \text{ litre/mol}$$

$\therefore$ Volume of 1 millimole of gas $\Rightarrow$ 2. 24 ml

42. Ans. (3)

Sol. Angular nodes = ℓ

Radial nodes = n – ℓ - 1

Orbital	Angular nodes	Radial nodes
2s	0	2–0–1 = 1
2p	1	2–1–1 = 0
3p	1	3–1–1=1
3d	2	3–2–1=0

43. Ans. (4)

44. Ans. (3)

45. Ans. (2)

46. Ans. (4)

47. Ans. (1)

48. Ans. (3)

49. Ans.(2)

Sol. $C_A = C_{A_0} - C_B - C_C$

$$x = C_{A_0} - x - x$$

or $\quad 3x = C_{A_0} \Rightarrow x = \dfrac{C_{A_0}}{3}$

$\because \quad C_A = C_{A_0} \, e^{-(\lambda_1+\lambda_2)t}$

or $\quad \dfrac{C_{A_0}}{3} = C_{A_0} e^{-(\lambda_1+\lambda_2)t}$

or $\quad 3 = e^{(\lambda_1+\lambda_2)t}$

or $\quad t = \dfrac{\ln 3}{2\ln 3} = \dfrac{1}{2} \text{ hr} = 30 \text{ min}$

50. Ans. (4)

Sol. Theory based

51. Ans. (2)

$$\underset{0.1-X}{HCOOH} \rightleftharpoons \underset{X}{HCOOH^-} + \underset{X}{H^+}$$

$\Delta T_f = 0.2046 = 1.86 \times m$

$$m = \frac{0.2046}{1.86} = 0.1 + X = 0.11$$

$$K_C = \frac{K_W}{K_a}$$

$$\because K_a = \frac{X^2}{0.1 - X} = \frac{10^{-2}}{9}$$

$\Rightarrow K_b\,(HCOO^-) = 9 \times 10^{-12}$ M

52. Ans. (4)

53. Ans. (1)

54. Ans. (2)

55. Ans. (2)

56. Ans. (4)

57. Ans.(4)

58. Ans. (4)

59. Ans. (3)

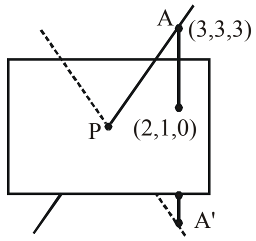
60. Ans. (1)

61. Ans. (1)

PA' is desired line

P is (2,1,0)

Let A' is reflection of A(3,3,3), then A' is



$$\frac{x-3}{2}=\frac{y-3}{1}=\frac{z-3}{1}=\frac{-2(6+3+3-5)}{6}$$

$$\Rightarrow A'\left(3-\frac{14}{3},3-\frac{7}{3},3-\frac{7}{3}\right)$$

$$\Rightarrow A'\left(-\frac{5}{3},\frac{2}{3},\frac{2}{3}\right)$$

$$PA':\ \frac{x-2}{11}=\frac{y-1}{1}=\frac{z}{-2}$$

62. Ans. (1)

$$\begin{vmatrix}1 & 1 & 2\\ 0 & 1 & -1\\ 1 & 0 & 2\end{vmatrix}\left[\vec{a}\ \vec{b}\ \vec{c}\right]=-\left[\vec{a}\ \vec{b}\ \vec{c}\right]$$

63. Ans. (3)

$$\int\left[e^{x^2-\frac{1}{x}}+x\left(2x+\frac{1}{x^2}\right)e^{x^2-\frac{1}{x}}\right]dx$$

$$=xe^{x^2-\frac{1}{x}}+C$$

$$\because \int\left(xf'(x)+f(x)\right)dx=xf(x)+C$$

64. Ans. (3)

Let m is the slope of common tangent

$$\frac{1}{m}=\pm\sqrt{4m^2-3}$$

$$\Rightarrow\left(4m^2-3\right)m^2=1\Rightarrow 4m^4-3m^2-1=0$$

$$m=\pm 1 \qquad m^2=1,-\frac{1}{4}$$

$y = x + 1$ and $y = -x - 1$

$$\frac{xx_1}{4}-\frac{yy_1}{3}=1$$

point of contact are $(-4,-3)$ or $(-4,3)$

65. Ans. (2)

a, ar, ar^2, $\quad r > 1, a > 0$

$2a, 3ar, ar^2 \rightarrow AP$

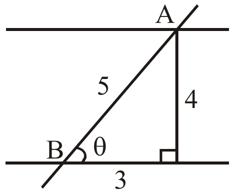
$6r = 2 + r^2 \Rightarrow r^2 - 6r + 2 = 0$

$$r=3\pm\sqrt{7}$$

$$3+\sqrt{7}$$

66. Ans. (3)

$$\tan\theta=\frac{4}{3}$$



$$\frac{m-\left(-\frac{12}{5}\right)}{1-m\frac{12}{5}}=\pm\frac{4}{3}$$

$$\Rightarrow\frac{5m+12}{5-12m}=\pm\frac{4}{3}$$

$\Rightarrow (15m + 36) = \pm(20 - 48m)$

$\Rightarrow (+)\ 63m = -16$

$(-)\ 33m = 56$

$\therefore$ Lines are $\quad (y-1)=-\frac{16}{63}(x-1)$

and $(y-1)=\frac{56}{33}(x-1)$

67. Ans. (2)

$r_3 + c_3 = x,\ r_2 + c_5 = y$

Also $r_2 + c_3 = 0,\ r_3 + c_5 = 0$

$\Rightarrow\ x + y = 0$

68. Ans. (1)

$$\text{Let } B=\begin{bmatrix}3 & 1\\ -9 & -3\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$= A + I,\ A^2 = 0$

$B^{100} = (I + A)^{100}$

$= I + 100A + {}^{100}C_2A^2.........+A^{100}$

$= I + 100A$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}300 & 100\\ -900 & -300\end{bmatrix}$$

$$=\begin{bmatrix}301 & 100\\ -900 & -299\end{bmatrix}$$

69. Ans. (4)

Total ways $-$ (No of ways in which no two consecutive are selected)

$= {}^{20}C_7 - {}^{14}C_7$

70. Ans. (3)

$$\lim_{x\to 2}\frac{(60+x^2)^{1/3}-4}{x-2}=\frac{x-2}{\sin(x-2)}$$

$$\lim_{x\to 2}\frac{\frac{1}{3}(60+x^2)^{-2/3}(2x)}{1}=\frac{4}{3.4^2}=\frac{1}{12}$$

71. Ans. (3)

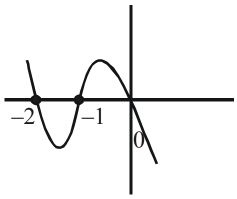
$$I=\int_0^{\pi/2}2\cos 2x 2\cos^2 x dx$$

$$=\int_0^{\pi/2}4\left\{2\cos^4 x-\cos^2 x\right\}dx$$

$$8\frac{3}{4.2}\frac{\pi}{2}-4.\frac{\pi}{4}=\frac{\pi}{4}=\frac{\pi}{2}$$

72. Ans. (1)

$f(x) = -x(x+1)(x+2)+a$



$\Rightarrow f(x)$ is decreasing in $(0,\infty)$

and x^2+1, $2x^2+2x+3$ are always positive

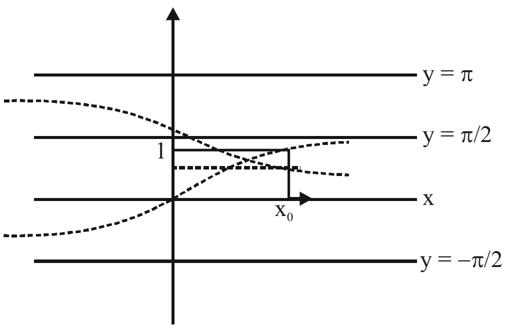
so inequality will be true when

$x^2+1 < 2x^2+2x+3$

$\Rightarrow x^2+2x+2>0$

$\Rightarrow x \in R$

73. Ans. (2)



$\alpha \in (x_0,\infty)$

$[\tan^{-1}\alpha] = 1$ & $[\cot^{-1}\alpha] = 0$

$\therefore [\tan^{-1}\alpha] + [\cot^{-1}\alpha] = 1$

74. Ans. (2)

$$\lim_{x\to 0}\frac{\int_0^{x^2}\frac{\ell n(1+4t)}{1+t^2}dt}{x^4\left(\frac{\sin x}{x}\right)^4}=\lim_{x\to 0}\frac{\frac{\ell n(1+4x^2)}{(1+x^4)}2x}{4x^3}$$

$$=\lim_{x\to 0}\frac{\ell n(1+4x^2)}{4x^2}.\frac{8x^3}{4x^3}.\frac{1}{1+x^4}=2$$

75. Ans. (2)

$${}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3\frac{1}{4}=\frac{10.27}{4^6}=\frac{135}{2^{11}}$$

76. Ans. (3)

$u_{10} = 1 = u_{13} = u_{16}$ .... $= u_{3k+1}$ for $k \geq 3$

$u_{11} = 9 = u_{14} = u_{17}$ .... $= u_{3k+2}$

$u_{12} = 3 = u_{15} = u_{18}$ .... $= u_{3k}$

$u_{500} = 9$

$u_{400} = 1$

$u_{300} = 3$

77. Ans. (4)

as $(a^2+b^2)^2 > (b^2+c^2)^2$

and $(b^2+c^2)^2 > (c^2+a^2)^2$

$\therefore (a^2+b^2)^2 > (c^2+a^2)^2 \Rightarrow$ roots are imaginary

78. Ans. (2)

	x	y
x'	$1/\sqrt{2}$	$-1/\sqrt{2}$
y'	$1/\sqrt{2}$	$1/\sqrt{2}$

$\Rightarrow$ $x=\frac{x'+y'}{\sqrt{2}}$

$y=\frac{-x'+y'}{\sqrt{2}}$

$$\frac{1}{9}\left(\frac{x'+y'}{\sqrt{2}}\right)^2+\frac{1}{4}\left(\frac{x'-y'}{\sqrt{2}}\right)^2=1$$

$\Rightarrow 4[(x')^2+(y')^2+2x'y']+9[(x')^2+(y')^2-2x'y']$

$= 72$

$13(x')^2 + 13(y')^2 - 10x'y' = 72$

$a + b + c = 16$

79. Ans. (1)

Let $\vec{a}=(x_1,y_1,z_1)$

$\vec{b}=(x_2,y_2,z_2)$

$\vec{c}=(x_3,y_3,z_3)$,

then $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\Rightarrow \Sigma x_1x_2 = \Sigma x_2x_3 = \Sigma x_3x_1 = 0$

which is not possible.

80. Ans. (2)

$S \equiv (\sim p \wedge r) \vee (\sim p \vee p) \equiv (\sim p \wedge r) \vee t \equiv (\sim p \wedge r)$

81. Ans. (2)

xRx as sinx = sinx $\quad\therefore$ Reflexive

xRy $\Rightarrow$ yRx $\quad\therefore$ Symmetric

xRy & yRz $\Rightarrow$ xRz $\quad\therefore$ transitive

$\Rightarrow$ Relation is equivalence

82. Ans. (1)

$$\mu = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{125}{25} = 5$$

$$\sigma^2 = \frac{\Sigma f_i (x_i - \mu)^2}{\Sigma f_i}$$

$$= \frac{16+0+4+7+0+4+4+9+16}{25} = \frac{60}{25} = 2.4$$

83. Ans. (3)

$$(4+5\omega+6\omega^2)^{n^2+2}[1+\omega^{n^2+2}+\omega^{2n^2+4}] = 0$$

$\Rightarrow \quad n = 3\lambda,\ n \neq 3\lambda + 1,\ 3\lambda + 2$

84. Ans. (4)

$(1 - x)^{10}(1 + 3x)^{10}$

$3^3.{}^{10}C_3 - {}^{10}C_1 \,.\, 3^2 \,.\, {}^{10}C_2 + {}^{10}C_2 \,.\, 3.{}^{10}C_1 - {}^{10}C_3$

$= 3240 - 4050 + 1350 - 120 = 420$

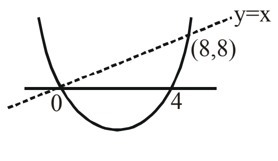
85. Ans. (2)

$$\frac{dy}{dx} + \frac{2}{x}y = x - 3$$

$$yx^2 = \int x^2(x-3)dx \Rightarrow yx^2 = \frac{x^4}{4} - x^3 + c$$

by (4,0) $\Rightarrow$ c = 0

$$\Rightarrow y = \frac{x^2}{4} - x$$



so area $\int_0^8 \left(x - \left(\frac{x^2}{4} - x \right) \right) dx$

$$\left(x^2 - \frac{x^3}{12} \right)_0^8 = 64 - \frac{2}{3}.64 = \frac{1}{3}.64$$

86. Ans. (2)

$$\lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{x}(2^x + 2^{-x})\sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3} = \sqrt{\pi}$$

87. Ans. (2)

$\phi(x) = f(x) - \sin x^2$

$\phi(1) = \phi(2) = \phi(3) = 0$

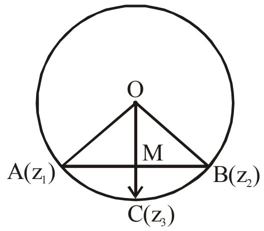
$\Rightarrow \quad \phi'(x) = 0$ at least twice

$\Rightarrow \quad f'(x) = 2x\cos x$ at least twice.

88. Ans. (4)

$$\frac{z_1 + z_2}{2} = \frac{2}{3} z_3$$

$$OM = \frac{2}{3} OC$$



$$AB = 2AM = 2\sqrt{1 - \frac{4}{9}} = \frac{2\sqrt{5}}{3}$$

89. Ans. (3)

$x^2 + (y - 1)^2 = r^2 \quad \Rightarrow \quad 4[r^2 - (y - 1)^2] + y^2 = 4$

$\Rightarrow \quad 3y^2 - 8y + 8 - 4r^2 = 0$

$D = 0 \quad \Rightarrow \quad 8^2 - 4.3.(8 - 4r^2) = 0$

$\Rightarrow \quad 4 - 3(2 - r^2) = 0$

$$r = \sqrt{\frac{2}{3}}$$

90. Ans. (3)

$f(5) + f(1) = 2$