Question Bank - Application of Derivative

LEVEL–I

1. A ladder 16 cm long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 cm away from the wall ?

2. If α, β are the intercepts made on the axes by the tangent at any point of the curve $x = a\cos^3\theta,\ y = b\sin^3\theta$, prove that $\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$.

3. If f(x) differentiable in [1, 5], then show that $f^2(5) - f^2(1) = 8f'(a).f(b)$, where $a, b \in [1, 5]$.

4. Prove that if $(n-1)a_1^2 - 2na_2 < 0$ then the roots of the equation $x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n = 0$ cannot be all real.

5. If $f(x) = 2e^x - a e^{-x} + (2a+1)x - 3$ monotonically increases for every $x \in R$ then find the range of values of ‘a’.

6. The interval to which b may belong so that the function,

$$f(x) = \left(1 - \frac{\sqrt{21 - 4b - b^2}}{b+1}\right)x^3 + 5x + \sqrt{16}$$, increases for all x.

7. Prove the inequality, $\frac{\tan x_2}{\tan x_1} > \frac{x_2}{x_1}$ for $0 < x_1 < x_2 < \frac{\pi}{2}$.

8. Find the polynomial f(x) of degree 6, which satisfies $\lim_{x\to 0}\left(1 + \frac{f(x)}{x^3}\right)^{1/x} = e^2$ and has local maximum at x = 1 and local minimum at x = 0 and 2.

9. A box of maximum volume with top open is to be made by cutting out four equal squares from four corners of a square tin sheet of side length a ft, and then folding up the flaps. Find the side of the square cut off.

10. An air force plane is ascending vertically at the rate of 100 km/h. If the radius of the earth is R Km, how fast the area of the earth, visible from the plane increasing at 3 min after it started ascending.

Take visible area $A = \frac{2\pi R^2 h}{R + h}$ where h is height of the plane in kms above the earth.

LEVEL–II

1. The tangent at a variable point P of the curve $y = x^2 - x^3$ meets it again at Q. Show that the locus of the middle point of PQ is $y = 1 - 9x + 28x^2 - 28x^3$.

2. Let $y = f(x)$ be differentiable in the closed interval [2002, 2004] and $f(2002) = f(2004) = 0$. Show that there exist a point on the curve $y = f(x)$ at which the length of the subtangent is 2003.

3. Show that the function $f(x) = x + \cos x - a$ is an increasing function and hence deduce that the equation $x + \cos x = a$ has no positive root for $a < 1$ and has one positive root for $a > 1$.

4. Find the set of all values of the parameter 'a' for which the function $f(x) = \sin 2x - 8(a+1)\sin x + (4a^2 + 8a - 14)x$ increases for all $x \in R$ and has no critical points for all $x \in R$.

5. Find the cosine of the angle at the vertex of an isosceles triangle having the greatest area for the given constant length L of the median drawn to its lateral side.

6. A ladder is to be carried in a horizontal position round a corner formed by two streets, a feet and b feet wide meeting at right angles. Prove that the length of the longest ladder that will pass round the corner without jamming is, $(a^{2/3} + b^{2/3})^{3/2}$.

7. Two towns located on the same side of the river agree to construct a pumping station and filtration plant at the river's edge, to be used jointly to supply the towns with water. If the distance of the two from the river are 'a' and 'b' and the distance between them is 'c' show that the pipe lines joining them to the pumping station is atleast as great as $\sqrt{c^2 + 4ab}$.

8. A circle of radius 1 unit touches positive x-axis and positive y-axis at P and Q respectively. A variable line 1 passing through origin intersects circle C in two points M and N. Find the equation of the line for which area of triangle MNQ is maximum.

9. Let $f(x) = \begin{cases} 4x - x^3 + \log_e(b^2 - 3b + 3), & 2 \le x < 3 \\ x - 18, & x \ge 3 \end{cases}$. Find all possible real values of b such that f(x) has the smallest value at x = 3.

10. Find the minimum value of $(x_1 - x_2)^2 + \left(\sqrt{2 - x_1^2} - \dfrac{9}{x_2}\right)^2$ where $x_1 \in \left(0, \sqrt{2}\right)$ and $x_2 \in R^+$.

IIT JEE PROBLEMS (OBJECTIVE)

(A) Fill in the blanks

1. The larger of cos (ℓn θ) and ℓn (cos θ) if $e^{-\pi/2} < \theta < \dfrac{\pi}{2}$ is .... **[IIT - 83]**

2. The function $y = 2x^2 - \ell n\,|x|$ is monotonically increasing for values of x ($\neq 0$) satisfying the inequalities .... and monotonically decreasing for values of x satisfying the inequalities .... **[IIT - 83]**

3. The set of all x for which $\ell n\,(1 + x) \le x$ is equal to .... **[IIT - 87]**

4. Let P be a variable point on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with foci F_1 and F_2. If A is the area of the triangle PF_1F_2 then the maximum value of A is .... **[IIT - 94]**

5. If A > 0, B > 0 and A + B = $\pi/3$, then the maximum value of tan A tan B is .... **[IIT - 93]**

6. Let C be curve $y^3 - 3xy + 2 = 0$. If H is the set of points on the curve C where the tangent is horizontal and V is the set of points on the curve C where the tangent is vertical, then H = ...and V = ... **[IIT - 94]**

(B) True or False

1. If x – r is a factor of the polynomial $f(x) = a_nx^4 + ... + a_0$, repeated m times ($1 < m \le n$), then r is a root of $f'(x) = 0$ repeated m times. **[IIT - 83]**

2. For 0 < a < x, the minimum value of the function $\log_a x + \log_x a$ is 2. **[IIT - 84]**

(C) Multiple choice questions with one or more than one correct answer :

1. Let $P(x) = a_0 + a_1x^2 + a_2x^4 + . . . + a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < . . . < a_n$. The function P(x) has **[IIT - 86]**

(A) neither a maximum nor a minimum
(B) only one maximum
(C) only one minimum
(D) only one maximum and only one minimum
(E) none of these

2. If the line ax + by + c = 0 is a normal to the curve xy = 1, then **[IIT - 86]**

(A) a > 0, b > 0
(B) a > 0, b < 0
(C) a < 0, b > 0
(D) a < 0, b < 0
(E) none of these

3. The smallest positive root of the equation, tan x – x = 0 lies in **[IIT - 87]**

(A) $\left(0, \dfrac{\pi}{2}\right)$ (B) $\left(\dfrac{\pi}{2}, \pi\right)$ (C) $\left(\pi, \dfrac{3\pi}{2}\right)$ (D) $\left(\dfrac{3\pi}{2}, 2\pi\right)$

4. Let f and g be increasing and decreasing functions, respectively from $[0, \infty]$ to $[0, \infty]$. Let $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(l)$ is **[IIT - 87]**

(A) always zero
(B) always negative
(C) always positive
(D) strictly increasing
(E) none of these

5. If $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$ then : **[IIT - 93]**

(A) f(x) is increasing on [-1, 2]
(B) f(x) is continuous on [-1, 3]
(C) $f'(2)$ does not exist
(D) f(x) has the maximum value at x = 2.

6. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then : **[IIT - 98]**

(A) h is increasing whenever f is increasing
(B) h is increasing whenever f is decreasing
(C) h is decreasing whenever f is decreasing
(D) nothing can be said in general

7. If $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$, for every real number x, then the minimum value of f **[IIT - 98]**

(A) does not exist because f is unbounded
(B) is not attained even though f is bounded
(C) is equal to 1
(D) is equal to –1

8. The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is **[IIT - 98]**

(A) 0
(B) 1
(C) 2
(D) infinite

9. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t - 1)(t - 2)^3(t - 3)^5 dt$ has a local minimum at x = **[IIT - 99]**

(A) 0
(B) 1
(C) 2
(D) 3

10. f(x) is cubic polynomial which has local maximum at x = –1. If f(2) = 18, f(1) = –1 and $f'(x)$ has local minima at x = 0, then **[IIT - 2006]**

(A) the distance between (–1, 2) and (a, f(a)), where x = a is the point of local minima is $2\sqrt{5}$
(B) f(x) is increasing for $x \in [1, 2\sqrt{5}]$
(C) f(x) has local minima at x = 1
(D) the value of f(0) = 5

11. If $f(x) = \begin{cases} e^x & 0 \le x \le 1 \\ 2 - e^{x-1} & 1 < x \le 2 \\ x - e & 2 < x \le 3 \end{cases}$ and $g(x) = \int_0^x f(t)dt$, $x \in [1, 3]$ then g(x) has **[IIT - 2006]**

(A) local maxima at $x = 1 \ell n 2$ and local minima at x = e
(B) local maxima at x = 1 and local minima at x = 2
(C) no local maxima
(D) no local minima

(D) Multiple choice questions with one correct answer :

1. If $a + b + c = 0$, then the quadratic equation $3ax^2 + 2bx + c = 0$ has **[IIT - 83]**

(A) at least one root in [0, 1]
(B) one root in [2, 3] and the other in [–2, –1]
(C) imaginary roots
(D) none of these

2. AB is a diameter of a circle and C is any point on the circumference of the circle. Then **[IIT - 83]**

(A) the area of triangle ABC is maximum when it is isosceles
(B) the are of triangle ABC is when it is isosceles
(C) the perimeter of triangle ABC is minimum when it is isosceles
(D) none of these

3. The normal to the curve $x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that **[IIT - 83]**

(A) it makes a constant angle with the x–axis
(B) it passes through the origin
(C) it is at constant distance from the origin
(D) none of these

4. If $y = a\ \ell n|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then **[IIT - 83]**

(A) $a = 2, b = -1$
(B) $a = 2, b = -\frac{1}{2}$
(C) $a = -2, b = \frac{1}{2}$
(D) none of these

5. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1. Let the function defined in column 1 have domain, $(-\pi/2, \pi/2)$ **[IIT - 92]**

	Column - 1		**Column - 2**
(i)	$x + \sin x$	**(A)**	increasing
(ii)	$\sec x$	**(B)**	decreasing
		(C)	neither increasing nor decreasing

6. The function $f(x) = \dfrac{\ell n(\pi + x)}{\ell n(e + x)}$ is : **[IIT - 95]**

(A) increasing on $[0, \infty)$
(B) decreasing on $[0, \infty)$
(C) increasing on $[0, \pi/e)$ and decreasing $\left[\frac{\pi}{e}, \infty\right)$
(D) decreasing on $[0, \pi/e)$ and increasing $[\pi/e, \infty)$

7. The function $f(x) = |px - q| + r|x|,\ x \in (-\infty, \infty)$, where $p > 0, q > 0, r > 0$ assume its minimum value only at one point if : **[IIT - 95]**

(A) $p \neq q$
(B) $r \neq q$
(C) $r \neq p$
(D) $p = q = r$

8. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the point **[IIT - 95]**

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

9. The slope of the tangent to a curve $y = f(x)$ at $[x, f(x)]$ is $2x + 1$. If the curve passes through the point (1,2), then the area bounded by the curve, the x-axis and the line $x = 1$ is **[IIT - 95]**

(A) $\frac{5}{6}$ (B) $\frac{6}{5}$ (C) $\frac{1}{6}$ (D) 6

10. If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \le 1$, then in this interval : **[IIT - 97]**

(A) both f(x) and g(x) are increasing functions (B) both f(x) and g(x) are decreasing function
(C) f(x) is an increasing function (D) g(x) is an increasing function

11. $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x, then minimum value of f : **[IIT - 98]**

(A) does not exist because f is unbounded (B) is not attained even though f is bounded
(C) is equal to 1 (D) is equal to -1

12. The number of values of x where the function $f(x) = \cos x + \cos\left(\sqrt{2}x\right)$ attains its maximum is **[IIT - 98]**

(A) 0 (B) 1 (C) 2 (D) infinite

13. The function $f(x) = \sin^4 x + \cos^4 x$ increases if **[IIT - 99]**

(A) $0 < x < \pi/8$ (B) $\pi/4 < x < 3\pi/8$ (C) $3\pi/8 < x < 5\pi/8$ (D) $5\pi/8 < x < 3\pi/4$

14. If the normal to the curve, $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis. Then $f'(3) =$ **[IIT - 2000]**

(A) -1 (B) $-\frac{3}{4}$ (C) $\frac{4}{3}$ (D) 1

15. Let $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \le 2 \\ 1 & \text{for } x = 0 \end{cases}$. Then at $x = 0$, 'f ' has : **[IIT - 2000]**

(A) a local maximum (B) no local maximum
(C) a local minimum (D) no extremum.

16. For all $x \in (0, 1)$: **[IIT - 2000]**

(A) $e^x < 1 + x$ (B) $\log_e(1 + x) < x$ (C) $\sin x > x$ (D) $\log_e x > x$

17. Consider the following statements in S and R : **[IIT - 2000]**

S : Both sin x and cos x are decreasing functions in the interval $(\pi/2, \pi)$.

R : If a differentiable function decreases in an interval (a, b), then its derivative also decreases in (a, b).

Which of the following is true ?

(A) both S and R are wrong
(B) both S and R are correct, but R is not the correct explanation for S
(C) S is correct and R is the correct explanation for S
(D) S is correct and R is wrong.

18. Let $f(x) = \int e^x (x-1)(x-2)dx$. Then f decreases in the interval **[IIT - 2000]**

(A) $(-\infty, -2)$ (B) $(-2, -1)$ (C) $(1, 2)$ (D) $(2, \infty)$

19. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes lies in the first quadrant. If its area is 2, then the value of b is **[IIT - 2000]**

(A) -1 (B) 3 (C) -3 (D) 1

20. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is **[IIT - 2000]**

(A) $3y = 9x + 2$ (B) $y = 2x + 1$ (C) $2y = x + 8$ (D) $y = x + 2$

21. If $f(x) = xe^{x(x-1)}$, then f(x) is **[IIT - 2001]**

(A) increasing on [–1/2, 1] (D) decreasing on R
(C) increasing on R (D) decreasing on [–1/2, 1]

22. Let $f(x) = (1 + b^2) x^2 + 2bx + 1$ and let m (b) be the minimum value of f(x). As b varies, the range of m(b) is **[IIT - 2001]**

(A) [0, 1] (B) (0, 1/2] (C) [1/2, 1] (D) (0, 1]

23. The length of a longest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing, is **[IIT - 2002]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) π

24. The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is/are **[IIT - 2002]**

(A) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (B) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$ (C) (0, 0) (D) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

25. In [0, 1] Langranges Mean Value theorem is NOT applicable to **[IIT - 2003]**

(A) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$ (B) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(C) $f(x) = x\,|x|$ (D) $f(x) = |x|$

Application of Derivative

26. Tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ (where $\theta \in (0, \pi/2)$). Then the value of θ such that sum of intercepts on axes made by this tangent is minimum is **[IIT - 2003]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{4}$

27. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ **[IIT - 2004]**

(A) f(x) is a strictly increasing function (B) f(x) has a local maxima
(C) f(x) is a strictly decreasing function (D) f(x) is bounded

28. If $f(x) = x^\alpha \log x$ (if $x > 0$) and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied to f on [0, 1] is **[IIT - 2004]**

(A) -2 (B) -1 (C) 0 (D) 1/2

29. If f(x) is differentiable and strictly increasing function in a neighbourhood of 0, then the value of $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is **[IIT - 2004]**

(A) 1 (B) 0 (C) -1 (D) 2

30. If y is a function of x and $\log(x + y) - 2xy = 0$, then the value of $y'(0)$ is equal to **[IIT - 2004]**

(A) 1 (B) -1 (C) 2 (D) 0

31. If P(x) is polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that $P(0) = 0$, $P(1) = 1$ and $P'(x) > 0\ \forall\ x \in [0, 1]$, then **[IIT - 2005]**

(A) $S = \phi$ (B) $S = ax + (1 - a)x^2\ \forall\ a \in (0, 2)$
(C) $S = ax + (1 - a)x^2\ \forall\ a \in (0, \infty)$ (D) $S = ax + (1 - a)x^2\ \forall\ a \in (0, 1)$

32. Let f be twice differentiable function such that $f(x) = x^2$ for $x = 1, 2, 3$ then **[IIT - 2005]**

(A) $f''(x) = 2,\ \forall x \in (1, 3)$ (B) $f'(x) = f''(x)$, for some $x \in (2, 3)$
(C) $f''(x) = 2$, for some $x \in (1, 3)$ (D) $f'' = 3\ \forall\ x \in (2, 3)$

33. If $y = f(x)$ is a function of x satisfying the relation, $y\cos x + x\cos y = \pi$, then the value of $f''(0)$ is **[IIT - 2005]**

(A) π (B) $-\pi$ (C) 0 (D) 2π

34. The function $f(x) = ||x| - 1|$, $x \in R$ is differentiable at all $x \in R$ except at the points **[IIT - 2005]**

(A) 1, 0, –1 (B) 1 (C) 1, –1 (D) –1

35. If f(x) is a continuous and differentiable function such that $f\left(\frac{1}{n}\right) = 0\ \forall\ n \in N$, then **[IIT - 2005]**

(A) $f(x) = 0\ x \in N\ (0, 1]$ (B) $f(0) = 0, f'(0) = 0$
(C) $f'(0) = 0, f''(0) = 0$ (D) $f(0) = 0$ and $f'(0)$ may or may not be zero

36. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$

(A) on the left of $x = c$ (B) on the right of $x = c$

(C) at no point (D) at all points **[IIT - 2007]**

(E) Statements & Reasons **[IIT - 2007]**

37. Let $f(x) = 2 + \cos x$ for all real x.

Statement 1 : for each real t, there exists a point c in $[t, t + \pi]$ such that $f'(c) = 0$.

because

Statement -2 : $f(t) = f(t + 2\pi)$ for each real t.

(A) Statement-1 is True, Statment-2 is True. Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True, Statement-2 **IS NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

(F) Write-Ups

W I If a continuous function f defined on the real line **R**, assumes positive and negative values in **R** then the equation $f(x) = 0$ has a root in **R**. For example, if it is known that a continuous function f on **R** is positive at some point and its minimum value is negative then the equation $f(x) = 0$ has a root in **R** Consider $f(x) = ke^x - x$ for all real x where k is a real constant. **[IIT - 2007]**

38. the line $y = x$ meets $y = ke^x$ for $k \le 0$ at

(A) no point (B) one point (C) two points (D) more than two points

39. The positive value of k for which $ke^x - x = 0$ has only one root is

(A) $1/e$ (B) 1 (C) e (D) $\log_e 2$

40. For $k > 0$, the set of all values of k for which $ke^x - x = 0$ has two distinct roots is

(A) $\left(0, \frac{1}{e}\right)$ (B) $\left(\frac{1}{e}, 1\right)$ (C) $\left(\frac{1}{e}, \infty\right)$ (D) $(0, 1)$

IIT JEE PROBLEMS (SUBJECTIVE)

1. Let x and y be two real variable such that $x > 0$ and $xy = 1$. Find the minimum value of $x + y$. **[IIT - 81]**

2. Use the function of $f(x) = x^{\frac{1}{x}}$, $x > 0$, to determine the bigger of the two numbers e^{π} and π^{e}. **[IIT - 81]**

3. For all x in [0, 1], let the second derivative $f''(x)$ of a function f(x) exist and satisfy $|f''(x)| < 1$. If $f(0) = f(1)$, that show that $|f'(x)| < 1$ for all x in [0, 1]. **[IIT - 81]**

4. If f(x) and g(x) are differentiable function for $0 \le x \le 1$ such that $f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2$, then show that there exist c satisfying $0 < c < 1$ and $f'(c) = 2g'(c)$. **[IIT - 82]**

5. Find the shortest distance of the point (0, c) form the parabola $y = x^2$ where $0 \le c \le 5$. **[IIT - 82]**

6. If $ax^2 + \frac{b}{x} \ge c$ for all positive x where $a > 0$ and $b > 0$ show that $27ab^2 \ge 4c^3$. **[IIT - 82]**

7. A swimmer S is in the sea at a distance d km from the closest point A on a straight shore. The house of the swimmer is on the shore at a distance L km from A. He can swim at a speed of U km/hr and walk at a speed of V km/hr (V > U). At what point on the shore should he land so that he reaches his house in the shortest possible time ? **[IIT - 83]**

8. Show that $1 + x\,\ell n\left(x + \sqrt{x^2 + 1}\right) \ge \sqrt{1 + x^2}$ for all $x \ge 0$. **[IIT - 83]**

9. Find the coordinates of the point on the curve $y = \frac{x}{1 + x^2}$, where the tangent to the curve has the greatest slope. **[IIT - 84]**

10. Find all the tangents to the curve $y = \cos(x + y), -2\pi \le x \le 2\pi$, that are parallel to the line $x + 2y = 0$. **[IIT - 85]**

11. Let $f(x) = \sin^3 x + \lambda \sin^2 x, -\frac{\pi}{2} < x < \frac{\pi}{2}$. Find the intervals in which λ should lie in order that f(x) has exactly one minimum and exactly one maximum. **[IIT - 85]**

12. Let $A(p^2, -p), B(q^2, q), C(r^2, -r)$ be the vertices of the triangle ABC. A parallelogram AFDE is drawn with vertices D, E and F on the line segments BC, CA and AB respectively. Using calculus, show that maximum area of such a parallelogram is $\frac{1}{4}(p+q)(q+r)(p-r)$. **[IIT - 86]**

13. Find the point on the curve $4x^2 + a^2y^2 = 4a^2$, $4 < a^2 < 8$ that is farthest from the point (0, –2). **[IIT - 87]**

14. Investigate for maxima and minima the function

$$f(x) = \int_1^x [2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2]\, dt.$$ **[IIT - 88]**

15. Find all maxima and minima of the function $y = x(x-1)^2,\ 0 \le x \le 2$. **[IIT - 88]**

16. Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1, 2). **[IIT - 90]**

17. A point P is given on the circumference of a circle of radius r. Chords QR are parallel to the tangent at P. Determine the maximum possible area of the triangle PQR. **[IIT - 90]**

18. Three normals are drawn from the point (c, 0) to the curve $y^2 = x$. Show that c must be greater than 1/2. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other. **[IIT - 91]**

19. A ladder 15m long leans against a wall 7 m high and a portion of the ladder protrudes over the wall such that its projection along the vertical is 3 m. How fast does the bottom start to slip away from the wall if the ladder slides down along the top edge of the wall at 2m/s. **[IIT - 91]**

20. A window of fixed perimeter P (including the base of the arch) is in the form of a rectangle surmounted by a semicircle. The semicircular portion is fitted with coloured glass while the rectangular part is fitted with clear glass. The clear glass transmits three times as much light per square meter as the coloured glass does. What is the ratio of the sides of the rectangle so that window transmits the maximum light? **[IIT - 91]**

21. Find the equation of the straight line which is tangents at one point and normal at another point to the curve, $y = 8t^3 - 1$, $x = 4t^2 + 3$. **[REE-91]**

22. A cord of length 2 L divides a circular area of radius R into two segments. Find the sides of rectangle with largest area that can be inscribed in the smaller segment. **[REE-91]**

23. What normal to the curve : $y = x^2$ forms the shortest chord ? **[IIT - 92]**

24. Town A and B are situated on the same side of a straight road at distances a and b respectively perpendicular drawn from A and B meet the road at the point c and d respectively. The distance between C and D is c. A hospital is to be built at a point P on the road such that the distance APB is minimum. Find the position of P. **[IIT - 92]**

25. Find the equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$. **[IIT - 93]**

26. Tangent at a point P1 [other than (0, 0)] on the curve $y = x^3$ meets the curve again at P2. The tangent at P_2 meets the curve at P_3 and so on. Show that the absisa of $P_1, P_2, P_3, ...P_n$, from a GP.

Also find the ratio $\dfrac{\text{area}(P_1P_2P_3)}{\text{area}(P_2P_3P_4)}$. **[IIT - 93]**

27. Let $f(x) = \begin{cases} -x^3 + \dfrac{(b^3 - b^2 + b - 1)}{(b^2 + 3b + 2)}, & 0 \le x < 1 \\ 2x - 3, & 1 \le x \le 3 \end{cases}$. Find all possible real values of b such that f(x) has the smallest value at x = 1. **[IIT - 93]**

28. Show that the normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at P(0, -3) meets the curve again at two points. **[IIT - 93]**

29. Find the points on the curve $9y^2 = x^3$ where normal to the curve make equal intercepts with the axes. **[REE-93]**

30. Find the values of x for which the function $f(x) = 1 + 2 \sin x + 3 \cos^2 x$, $(0 \le x \le 2\pi/3)$ is maximum or minimum. Also find these values of the function. **[REE-93]**

31. The curve $y = ax^3 + bx^2 + cx + 5$, touches the x-axis at P(-2, 0) and cuts the y-axis at a point Q where its gradient is 3. Find a, b, c. **[IIT - 94]**

32. The circle $x^2 + y^2 = 1$ cuts the x-axis at P and Q. Another circle with centre at Q and variable radius intersects the first circle at R above the x-axis and the line segment PQ at S. Find the maximum area of the triangle QSR. **[IIT - 94]**

33. Find the maximum area of an isosceles triangle inscribed in the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with its vertex at one end of the major axis. **[IIT - 94]**

34. Let (h, k) be a fixed point, where h > 0, k > 0. A straight line passing through this point cuts the positive direction of the coordinate axes at the points P and Q. Find the minimum area of the triangle OPQ, O being the origin. **[IIT - 95]**

35. Find the point (α, β) on the ellipse $4x^2 + 3y^2 = 12$, in the first quadrant, so that the area enclosed by the line $y = x$; $y = \beta$, $x = \alpha$ and the x-axis is maximum. **[IIT - 95]**

36. Find the intervals in which the function $f(x) = 3 \cos^4 x + 10 \cos^3 x + 6 \cos^2 x - 3$, $0 \le x \le \pi$; is monotonically increasing or decreasing. **[REE-95]**

37. Determine the points of maxima and minima of the function; $f(x) = \left(\dfrac{1}{8}\right)\ell n\, x - bx + x^2$, $x > 0$, where $b \ge 0$ is a constant. **[IIT - 96]**

38. Let $f(x) = \begin{cases} xe^{ax}, & x \le 0 \\ x + ax^2 - x^3, & x > 0 \end{cases}$; where 'a' is a positive constant. Find the interval in which $f'(x)$ is increasing. **[IIT - 96]**

39. A curve $y = f(x)$ passes through the point $P(1, 1)$. The normal to the curve at P is $a(y-1)+(x-1)=0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point, determine the equation of the curve. Also obtain the area bounded by the y-axis, the curve and the normal to the curve at P. **[IIT - 96]**

40. A 12 cm long wire is bent to form a triangle with one of its angles as 60^0. Find the sides of the triangle when its area is largest. **[REE-96]**

41. Let $a + b = 4$, where $a < 2$, and let $g(x)$ be a differentiable function. If $\frac{dg}{dx} > 0$ for all x, prove that $\int_0^a g(x)dx + \int_0^b g(x)\,dx$ increases as $(b-a)$ increases. **[IIT - 97]**

42. A conical vessel is to be prepared out of a circular sheet of gold of unit radius. How much sectorial area is to be removed from the sheet so that the vessel has maximum volume. **[REE-97]**

43. Suppose $f(x)$ is a functions satisfying the following conditions : **[IIT - 98]**

(a) $f(0) = 2,\ f(1) = 1$

(b) f has a minimum value at $x = \frac{5}{2}$ and

(c) for all x, $f'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

When a, b are some constants. Determine the constants a, b and the function f(x).

44. A curve C has the property that if the tangent drawn at any point P on C meets the co-ordinate axes at A and B, then P is the mid-point of AB. The curve passes through the point (1, 1). Determine the equation of the curve. **[IIT - 98]**

45. Find the points on the curve $ax^2 + 2bxy + ay^2 = c$; $c > b > a > 0$, whose distance from the origin is minimum. **[REE-98]**

46. Find the acute angles between the curves $y = |x^2 - 1|$ and $y = |x^2 - 3|$ at their point of intersection. **[REE-98]**

47. Find the coordinates of all the points P on the ellipse $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)=1$ for which the area of the triangle PON is maximum, where O denotes the origin and N the foot of the perpendicular from O to the tangent at P. **[IIT - 99]**

48. Find the normal to the ellipse $\left(\frac{x^2}{9}\right)+\left(\frac{y^2}{4}\right)=1$ which are farthest from its centre. **[IIT - 99]**

49. Find the equation of the straight line which is tangent at one point and normal at another point of the curve, $x = 3t^2$, $y = 2t^3$. **[REE-2000]**

50. Find the point on the straight line, $y = 2x + 11$ which nearest to the circle, $16(x^2 + y^2) + 32x - 8y - 50 = 0$. **[REE-2000]**

51. Let $-1 \le p \le 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval [1/2, 1] and identify it. **[IIT - 2001]**

52. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line $x + y = 7$, is minimum. **[IIT - 2003]**

53. Using the relation $2(1 - \cos x) < x^2 .\ x \ne 0$ or otherwise, prove that $\sin x (\tan x) \ge x,\ \forall\ x \in \left[0, \frac{\pi}{4}\right]$. **[IIT - 2003]**

54. If the function $f : [0, 4] \to R$ is differentiable then show that
(i) For $a, b \in (0, 4)$, $(f(4))^2 - (f(0))^2 = 8f'(a) f(b)$. **[IIT - 2003]**
(ii) $\int_0^4 f(t)dt = 2[\alpha f(\alpha^2) + \beta f(\beta^2)] \forall 0 < \alpha, \beta < 2$

55. If $P(1) = 0$ and $\frac{dP(x)}{dx} > P(x)$ for all $x \ge 1$ then prove that $P(x) > 0$ for all $x > 1$. **[IIT - 2003]**

56. Tangents are drawn from P(6, 8) to the circle $x^2 + y^2 = r^2$. Find the radius of the circle such that the area of the triangle formed by tangents and chord of contact is maximum. **[IIT - 2003]**

57. Prove by Roll's theorem that $p(x) = 51x^{101} - 2323x^{100} - 45x + 1035$ has a root in the interval $\left((45)^{\frac{1}{100}}, 46\right)$ **[IIT - 2004]**

58. Prove that $\sin x + 2x \ge \frac{3x(x+1)}{\pi}$ for all $x \in \left[0, \frac{\pi}{2}\right]$ justified the inequality used. **[IIT - 2004]**

59. If p(x) be cubic polynomial and $p(-1) = 10$, $p(1) = -6$ and p(x) has local maxima at $x = -1$ and $p'(x)$ has minima at $x = 1$. Find distance between the points, of local maxima and minima. **[IIT - 2005]**

60. If $|f(x_2) - f(x_1)| < (x_2 - x_1)^2$ for all $x_1, x_2 \in R$ then find the equation of tangent at the point (1, 2) to the curve $y = f(x)$. **[IIT - 2005]**

61. f(x) is a differentiable function and g(x) is double differentiable function such that $|f(x)| \le 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 9$. Prove that there exists some $c \in (-3, 3)$ such that $g(c).g''(c) < 0$. **[IIT - 2005]**

62. If f(x) is a twice differentiable function such that $f(a) = 0$, $f(b) = 2$, $f(c) = 1$, $f(d) = 2$, $f(e) = 0$, where $a < b < c < d < e$, then the minimum number of zeroes of $g(x) = (f'(x))^2 + f''(x), f(x)$ in the interval [a, e] is ____________ . **[IIT - 2006]**

SET–I

1. Angle of intersection of $x^2 + y^2 - 6x - 2y - 10 = 0$ and $y = 2x - 5$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

2. Total number of parallel tangents of $f_1(x) = x^2 - x + 1$ and $f_2(x) = x^3 - x^2 - 2x + 1$ is equal to

(A) 2 (B) 3 (C) 4 (D) none of these

3. Tangents are drawn to $y = \cos x$ from the point P(0, 0). Points of contact of these tangents will always lie on

(A) $\frac{1}{x^2} = \frac{1}{y^2} + 1$ (B) $\frac{1}{x^2} = \frac{1}{y^2} - 1$ (C) $x^2 + y^2 = 1$ (D) $x^2 - y^2 = 1$

4. The curve $x^2 - 4y^2 + c = 0$ and $y^2 = 4x$ will intersect orthogonally for

(A) $c \in (0, 16)$ (B) $c \in (-3, 4)$ (C) $c \in (3, 4)$ (D) none of these

5. If the line joining the points (0, 3) and (5, -2) is a tangent to the curve $y = \frac{ax}{1+x}$, then

(A) $a = 1 \pm \sqrt{3}$ (B) $a = \phi$ (C) $a = -1 \pm \sqrt{3}$ (D) $a = -2 \pm 2\sqrt{3}$

6. If the line $ax + by + c = 0$ is a tangent to the curve $xy + 2 = 0$ then

(A) $a > 0, b > 0$ (B) $a > 0, b < 0$ (C) $a > 0, c > 0$ (D) $a > 0, c < 0$

7. ℓ_1 and ℓ_2 are the side lengths of two variable squares S_1 and S_2 respectively. If $\ell_1 = \ell_2 + \ell_2^3 + 6$ then rate of change of the area of S_2 with respect to rate of change of the area of S_1 when $\ell_2 = 1$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{1}{32}$

8. Total number of values of 'x' where the function $f(x) = \cos x + \cos\sqrt{2}x$ attains its maximum value is

(A) 1 (B) 2 (C) 4 (D) none of these

9. If $A + B = \frac{2\pi}{3}$ where $A, B > 0$, then minimum value of $\sec A + \sec B$ is equal to

(A) 4 (B) 8 (C) 6 (D) none of these

10. If $x = a$ is the point of local maxima for $y = f(x)$, then which of the following is always true

(A) $f'(A) = 0$ (B) $f'(A) = 0,\ f''(A) < 0$

(C) $f(A) = 0,\ f'(A) > 0$ (D) none of these

11. Let $f(x) = \{x\}$, where $\{.\}$ denotes the fractional part. For $f(x)$, $x = 5$ is

(A) a point of local maxima (B) a point of local minima

(C) neither a point of local minima nor maxima (D) a stationary point

Application of Derivative

12. $f(x) = \begin{cases} 6, & x \le 1 \\ 7 - x, & x > 1 \end{cases}$ then for f(x), x = 1 is

(A) a point of local maxima
(B) a point of local minima
(C) neither a point of local minima nor maxima
(D) a stationary point

13. $f(x) = \begin{cases} \cos\frac{\pi x}{2}, & x > 0 \\ x + a, & x \le 0 \end{cases}$. Then x = 0 will be point of local maxima for f(x) if

(A) $a \in (-1, 1)$
(B) $a \in (0, 1)$
(C) $a \le 0$
(D) $a \ge 1$

14. $f(x) = x + \frac{1}{x},\ x \ne 0$, then

(A) f(x) has no point of local maxima
(B) f(x) has no point local minima
(C) f(x) has exactly one point of local minima
(D) f(x) has exactly two points of local minima

15. If $f(x) = x^3 + ax^2 + bx + c$ attains its local minima at certain negative real number then

(A) $a^2 - 3b > 0,\ a < 0,\ b < 0$
(B) $a^2 - 3b > 0,\ a < 0,\ b > 0$
(C) $a^2 - 3b > 0,\ a > 0,\ b < 0$
(D) $a^2 - 3b > 0,\ a > 0,\ b > 0$

16. Let $f(x) = ax^3 + bx^2 + cx + d,\ a \ne 0$. If x_1 and x_2 are the real and distinct roots of $f'(x) = 0$ then $f(x) = 0$ will have three real and distinct roots if

(A) $x_1 . x_2 < 0$
(B) $f(x_1) . f(x_2) > 0$
(C) $f(x_1) . f(x_2) < 0$
(D) $x_1 . x_2 > 0$

17. A rectangle is inscribed in an equilateral triangle of side length '2a' units. Maximum area of this rectangle can be

(A) $\sqrt{3}a^2$
(B) $\frac{\sqrt{3}a^2}{4}$
(C) a^2
(D) $\frac{\sqrt{3}a^2}{2}$

18. If the equation $3ax^2 + 2bx + c = 0$ has its coefficients such that $a + b + c = 0$ where $a, b, c \in R$ then the equation has at least one real root in the interval

(A) (–1, 1)
(B) (1, 2)
(C) $\left(\frac{1}{2}, \frac{3}{2}\right)$
(D) none of these

19. If $f''(x) < 0\ \forall x \in R$ and $g(x) = f(x^2 - 2) + f(6 - x^2)$ then

(A) g(x) is an increasing in [–2, 0]
(B) g(x) is an increasing in $[2, \infty)$
(C) g(x) has a local minima at x = – 2
(D) g(x) has a local maxima at x = 2

20. $f(x) = \int_0^x \left|\log_2(\log_3(\log_4(\cos t + a)))\right| dt$. If f(x) is increasing for all real values of x then

(A) $a \in (-1, 1)$
(B) $a \in (1, 5)$
(C) $a \in (1, \infty)$
(D) $a \in (5, \infty)$

21. Let 'P' be a point on $x^2 = 4y$ that is nearest to the point A(0, 4) then coordinates of 'P' are

(A) (4, 4)
(B) (0, 0)
(C) $(\sqrt{8}, 2)$
(D) (2, 1)

22. Let $f''(x) > 0 \ \forall x \in R$ and $g(x) = f(2 - x) + f(4 + x)$. Then g(x) is increasing in

(A) $(-\infty, -1)$ (B) $(-\infty, 0)$ (C) $(-1, \infty)$ (D) none of these

23. Let $f : [0, \infty) \to [0, \infty)$ and $g : [0, \infty) \to [0, \infty)$ be non-increasing and non-decreasing functions, $h(x) = g(f(x))$. If 'f' and 'g' are differentiable for all points in their respective domains and $h(0) = 0$ then h(x) will always be

(A) an increasing function (B) a decreasing function
(C) identically zero (D) none of these

24. If $xy = 10$ then minimum value of $12x^2 + 13y^2$ is equal to

(A) 15 (B) $40\sqrt{39}$ (C) $3\sqrt{13}$ (D) $30\sqrt{13}$

25. If $9 - x^2 > |x - a|$ has atleast one negative solution, where then complete set of values of a is

(A) $\left(-\frac{25}{2}, 9\right)$ (B) $\left(-\frac{35}{4}, 9\right)$ (C) $\left(-\frac{37}{2}, 9\right)$ (D) $\left(-\frac{37}{4}, 9\right)$

26. f(x) be a differentiable function such that $f'(x) = \dfrac{1}{(\log_3(\log_{1/4}(\cos x + a)))}$. If f(x) is increasing for all values of x then

(A) $a \in (5, \infty)$ (B) $a \in \left(1, \frac{5}{4}\right)$ (C) $a \in \left(\frac{5}{4}, 5\right)$ (D) none of these

27. Let f(x) be a function such that $f'(x) = \log_{1/3}(\log_3(\sin x + a))$. If f(x) is decreasing for all real values of x then

(A) $a \in (1, 4)$ (B) $a \in (4, \infty)$ (C) $a \in (2, 3)$ (D) $a \in (2, \infty)$

28. Tangents are drawn to $x^2 + y^2 = 16$ from the point P(0, h). These tangents meet the x-axis at A and B. If the area of triangle PAB is minimum then

(A) $h = 12\sqrt{2}$ (B) $h = 6\sqrt{2}$ (C) $h = 8\sqrt{2}$ (D) $h = 4\sqrt{2}$

29. Tangents PA and PB are drawn to $y = x^2 - x + 1$ from the point $P\left(\frac{1}{2}, h\right)$. If the area of triangle PAB is maximum then

(A) $h = -\frac{1}{4}$ (B) $h = -\frac{1}{2}$ (C) $h = -2$ (D) none of these

30. The curve $C_1 : y = 1 - \cos x,\ x \in (0, \pi)$ and $C_2 : y = \frac{\sqrt{3}}{2}|x| + a$ will touch each other if

(A) $a = \frac{3}{2} - \frac{\pi}{\sqrt{3}}$ (B) $a = \frac{3}{2} - \frac{\pi}{2\sqrt{3}}$ (C) $a = \frac{1}{2} - \frac{\pi}{\sqrt{3}}$ (D) $a = \frac{3}{4} - \frac{\pi}{\sqrt{3}}$

SET–II

1. The parabolas $y^2 = 4ax$ and $x^2 = 4by$ intersect orthogonally at point $P(x_1, y_1)$ where $x_1.y_1 \neq 0$ provided

(A) $b = a^2$ (B) $b = a^3$ (C) $b^3 = a^2$ (D) none of these

2. Two variable curves $C_1 : y^2 = 4a(x - b_1)$ and $C_2 : x^2 = 4a(y - b_2)$ where 'a' is a given positive real number and b_1 and b_2 are variables, touch each other. Locus of the point of contact is

(A) $xy = a^2$ (B) $xy = 2a^2$ (C) $xy = 4a^2$ (D) none of these

3. Point on $y^2 = 4x$ that is nearest to the circle $x^2 + (y - 12)^2 = 1$, is

(A) (4, -4) (B) (4, 4) (C) (9, 6) (D) (9, -6)

4. The function $f(x) = \left|\frac{x^2-2}{x^2-4}\right|$ has

(A) no point of local minima
(B) no point of local maxima
(C) exactly one point of local minima
(D) exactly one point of local maxima

5. The function $f(x) = x(x^2 - 4)^n (x^2 - x + 1)$, $n \in N$ assumes a local minima at $x = 2$ then

(A) 'n' can be any odd number
(B) 'n' can only be an odd prime number
(C) 'n' can be any even number
(D) 'n' can only be a multiple of four

6. $f(x) = \begin{cases} \tan^{-1} x, & |x| < \frac{\pi}{4} \\ \frac{\pi}{2} - |x|, & |x| \geq \frac{\pi}{4} \end{cases}$, then

(A) f(x) has no point of local maxima
(B) f(x) has no point of local minima
(C) f(x) has exactly one point of local maxima
(D) f(x) has exactly two points of local minima

7. $f(x) = e^x.\cos x$, $x \in [0, 2\pi]$. The slope of tangent of the function is minimum for

(A) $x = \pi$ (B) $x = \frac{\pi}{4}$ (C) $x = \frac{3\pi}{4}$ (D) $x = \frac{3\pi}{2}$

8. If $f(x) = a\,\ell n|x| + bx^2 + x$ has extremes at $x = 1$ and $x = 3$ then

(A) $a = \frac{3}{4}, b = -\frac{1}{8}$ (B) $a = \frac{3}{4}, b = \frac{1}{8}$ (C) $a = -\frac{3}{4}, b = -\frac{1}{8}$ (D) $a = -\frac{3}{4}, b = \frac{1}{8}$

9. Total number of critical points of $f(x) = \frac{|2-x|}{x^2}$ are equal to

(A) 3 (B) 2 (C) 1 (D) 4

10. $f(x) = \int_0^x (t^2 - 1)\cot t\, dt$, $x \in (0, 2\pi)$.f(x) attains local maximum value at

(A) $x = \frac{\pi}{2}$ (B) $x = 1$ (D) $x = \frac{3\pi}{2}$ (D) none of these

11. $f(x)=\int_0^x (e^t-1)(t-1)(\sin t-\cos t)\sin t\,dt$, $\forall x \in \left(-\frac{\pi}{2}, 2\pi\right)$ then f(x) is

(A) Decreasing in $\left(-\frac{\pi}{2}, 0\right)$, Decreasing in $\left(\frac{\pi}{4}, 1\right)$, Decreasing in $\left(\pi, \frac{\pi}{4}\right)$

(B) Decreasing in $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$, Decreasing in $(1, \pi)$, Decreasing in $\left(\frac{5\pi}{4}, 2\pi\right)$

(C) Decreasing in $\left(\frac{\pi}{4}, 1\right)$, Decreasing in $\left(\pi, \frac{5\pi}{4}\right)$

(D) Decreasing in $\left(0, \frac{\pi}{4}\right)$, Decreasing in $(1, \pi)$, Decreasing in $\left(\frac{5\pi}{4}, 2\pi\right)$

12. $f: R \to R$ be a differentiable function $\forall x \in R$. If tangent drawn to the curve at any point $x \in (a, b)$ always lie below the curve then

(A) $f'(x)>0, f''(x)<0 \ \forall \ x \in (a, b)$
(B) $f'(x)<0, f''(x)<0 \ \forall \ x \in (a, b)$
(C) $f'(x)>0, f''(x)>0 \ \forall \ x \in (a, b)$
(D) none of these

13. A spherical balloon is pumped at the constant rate of 3 m^3/min. The rate of increase of its surface area at certain instant is found to be 5 m^2/min. At this instant its radius is equal to

(A) $\frac{1}{5}$m (B) $\frac{3}{5}$m (C) $\frac{6}{5}$m (D) $\frac{2}{5}$m

14. The abscissa of points P and Q on the curve $y = e^x + e^{-x}$ such that tangents at P and Q make 60^0 with x-axis

(A) $\ell n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$ and $\ell n\left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)$
(B) $\ell n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$
(C) $\ell n\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)$
(D) $\pm \ell n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$

15. A lamp of negligible height, is placed on the ground 'ℓ_1' m away from a wall. A man 'ℓ_2' m tall is walking at a speed of $\frac{\ell_1}{10}$ m/sec from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of his shadow on the wall is

(A) $-\frac{5\ell_2}{2}$ m/sec (B) $-\frac{2\ell_2}{5}$ m/sec (C) $-\frac{\ell_2}{2}$ m/sec (D) $-\frac{\ell_2}{5}$ m/sec

16. Let f(x) and g(x) be real valued functions such that f(x) . g(x) = 1 $\forall$ x, $y \in R$. If $f''(x)$ and $g''(x)$ exist for all values of x, and $f'(x)$ and $g'(x)$ are never zero, then $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)}$ is equal to

(A) $\frac{2g'(x)}{f(x)}$ (B) $\frac{2g'(x)}{g(x)}$ (C) $\frac{2f'(x)}{g(x)}$ (D) $\frac{2f'(x)}{f(x)}$

17. Consider the parabola $y^2 = 4x$. $A \equiv (4, -4)$ and $B \equiv (9, 6)$ be two fixed points on the parabola. Let 'C' be a moving point on the parabola between A and B such that the area of triangle ABC is maximum, then coordinate of 'C' is

(A) $\left(\frac{1}{4}, 1\right)$ (B) (4, 4) (C) $\left(3, 2\sqrt{3}\right)$ (D) $\left(3, -2\sqrt{3}\right)$

18. The equation $x^3 - 3x + a = 0$ will have exactly one real root if

(A) (0, 2) (B) (-2, 2)
(C) $(-\infty, -2) \cup (2, \infty)$ (D) (-2, 0)

19. The inequality $x^2 - 4x > \cot^{-1} x$ is true in

(A) [0, 4] (B) (4, 5)
(C) $(-\infty, -1] \cup [5, \infty)$ (D) (-1, 4)

20. Total number of critical points of f(x) = max. {sin x, cos x} $\forall\ x \in (-2\pi, 2\pi)$ is equal to

(A) 5 (B) 7 (C) 4 (D) 3

21. The equation $x^3 - 3x + [a] = 0$, where [.] denotes the greatest integer function, will have real and distinct roots if

(A) $a \in (-\infty, 2)$ (B) $a \in (0, 2)$
(C) $a \in (\infty, -2) \cup (0, \infty)$ (D) $a \in [-1, 2)$

22. y = f(x) is parabola, having its axis parallel to y-axis. If the line y = x touches this parabola at x = 1, then

(A) $f''(1) - f'(0) = 1$ (B) $f''(0) - f'(1) = 1$ (C) $f''(1) + f'(0) = 1$ (D) $f''(0) + f'(1) = 1$

23. Let $g'(x) > 0$ and $f'(x) < 0\ \forall\ x \in R$ then

(A) $g(f(x+1)) > g(f(x-1))$ (B) $f(g(x-1)) > f(g(x+1))$
(C) $g(f(x+1)) < g(f(x-1))$ (D) $g(g(x+1)) < g(g(x-1))$

24. If $f''(x) > 0\ \forall\ x \in R$ then for any two real numbers x_1 and x_2, $(x_1 \neq x_2)$

(A) $f\left(\frac{x_1 + x_2}{2}\right) > \frac{f(x_1) + f(x_2)}{2}$ (B) $f\left(\frac{x_1 + x_2}{2}\right) < \frac{f(x_1) + f(x_2)}{2}$

(C) $f'\left(\frac{x_1 + x_2}{2}\right) > \frac{f'(x_1) + f'(x_2)}{2}$ (D) $f'\left(\frac{x_1 + x_2}{2}\right) < \frac{f'(x_1) + f'(x_2)}{2}$

25. Let $f'(\sin x) < 0$ and $f''(\sin x) > 0 \,\forall\, x \in \left(0, \frac{\pi}{2}\right)$ and g(x) = f(sin x) + f(cos x), then g(x) is decreasing in

(A) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (B) $\left(0, \frac{\pi}{4}\right)$ (C) $\left(0, \frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

26. $f(x) = 2\sin^3 x - 3\sin^2 x + 12\sin x + 5 \quad \forall\ x \in \left(0, \frac{\pi}{2}\right)$ then

(A) 'f ' is increasing in $\left(0, \frac{\pi}{2}\right)$

(B) 'f ' is decreasing in $\left(0, \frac{\pi}{2}\right)$

(C) 'f ' is increasing in $\left(0, \frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(D) 'f ' is decreasing in $\left(0, \frac{\pi}{4}\right)$ and increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

27. Let $f(x) = \begin{cases} 4x - x^3 + \ell n(a^2 - 3a + 3), & 0 \le x < 3 \\ x - 18, & x \ge 3 \end{cases}$. Complete set of 'a' such that f(x) has a local minima at x = 3, is

(A) [-1, 2] (B) $(-\infty, 1) \cup (2, \infty)$ (C) [1, 2] (D) $(-\infty, -1) \cup (2, \infty)$

28. The equation x + cos x = a has exactly one positive root. Complete set of values of 'a' is

(A) (0, 1) (B) $(-\infty, 1)$ (C) (-1, 1) (D) $(1, \infty)$

29. If the function $f(x) = x(x+4).e^{-\frac{x}{2}}$ has its local maxima at x = a then

(A) $a = 2\sqrt{2}$ (B) $a = -2\sqrt{2}$ (C) a = 4 (D) a = – 4

30. If the curve $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^2 = 16x$ intersect at right angle then

(A) $a = \pm 1$ (B) $a = \pm\sqrt{3}$ (C) $a = \pm\frac{1}{\sqrt{3}}$ (D) $a = \pm\sqrt{2}$

SET–III

1. If f is twice differentiable at $x = a$; then which of the following is True

(A) If $f(a)$ is an extreme value of $f(x)$, then $f'(a) = 0$

(B) If $f'(a) = 0$, then $f(a)$ is an extreme value of $f(x)$

(C) If $f'(a) = 0$ and $f''(a) > 0$ then function has a local minima at $x = a$

(D) none of these

2. The line $ax - by + c = 0$ is normal to the curve $xy = -1$ then which one of the following is/are is not true

(A) $a > 0, b > 0$ (B) $a < 0, b < 0$ (C) $a > 0, b < 0$ (D) $a < 1, b > 1$

3. Let $p(x) = a_0 + a_1x^2 + a_2x^4 + \ldots\ldots + a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \ldots\ldots < a_n$. The function p(x) has

(A) Neither a maximum nor a minimum (B) only one maximum

(C) only one minimum (D) none of these

4. At $x = a$, there is minimum for a given function $f(x)$, then

(A) $\lim_{x \to a^-} f'(x) = \lim_{x \to a^+} f'(x)$ (B) $\lim_{x \to a^-} f'(x) > 0$, $\lim_{x \to a^+} f'(x) < 0$

(C) $\lim_{x \to a^-} f'(x) < 0$, $\lim_{x \to a^+} f'(x) < 0$ (D) nothing can be said

5. Let f be a twice differentiable function satisfying $f(1) = e; f(2) = e^2; f(3) = e^3$, then which of the following is/are false

(A) $f(x) = e^x \; \forall \, x \in [1, 3]$

(B) $f'(x) = e^x$ has atleast three solution in [1, 3]

(C) $f''(x) = e^x$ has atleast two solution in [1, 3]

(D) $f''(x) = e^x$ has a solution in [0, 4]

6. Let f(x) and g(x) are defined and differentiable for $x \geq x_0$ and $f(x_0) = g(x_0)$, $f'(x) > g'(x)$ for $x > x_0$, then which of the following is/are not true

(A) $f(x) > g(x)$ for some $x > x_0$ (B) $f(x) = g(x)$ for some $x > x_0$

(C) $f(x) > g(x)$ for all $x > x_0$ (D) $f(x) > g(x)$ for no $x > x_0$

7. If the function $f(x)$ increases in the interval (a, b) then the function $\phi(x) = [f(x)]^2$

(A) increases in (a, b)

(B) decreases in (a, b)

(C) we cannot say that $\phi(x)$ increases or decreases in (a, b)

(D) none of these

8. Let $f(x) = \begin{cases} \sin\frac{\pi x}{2} & , \ 0 \le x < 1 \\ 3 - 2x & , \ x \ge 1 \end{cases}$, then

(A) $f(x)$ has local maxima at $x = 1$
(B) $f(x)$ has local minima at $x = 1$
(C) $f(x)$ does not have any local extrema at $x = 1$
(D) $f(x)$ has a global minima at $x = 1$

9. Among the following statements which one is/are true

(A) $\ell n(1+x) < x$ in $(0, \infty)$
(B) $x < \ell n(1+x)$ in $(0, \infty)$
(C) $\tan x > x$ in $(0, \pi/2)$
(D) $\tan x < x$ in $(0, \pi/2)$

10. If $a < b < c < d$ and $x \in R$ then the least value of the function,
$f(x) = |x-a| + |x-b| + |x-c| + |x-d|$ is

(A) $a + c - b - d$
(B) $a + b + c + d$
(C) $c + d - a - b$
(D) $a + b - c - d$

11. Let f(x) be a differentiable function upto any order such that $f(x).f''(x) \le 0 \quad \forall \quad x \in R$. If α and β be the two consecutive real roots of $f(x) = 0$, then

(A) $f''(x)$ must be equal to zero for atleast one $x \in (\alpha, \beta)$
(B) $f'''(x)$ must be equal to zero for atleast one $x \in (\alpha, \beta)$
(C) $f'(x) \ne 0 \ \forall x \in (\alpha, \beta)$
(D) none of these

12. Among the following statements which one is/are true

(A) The cubic equation $x^3 + 2x^2 + x + 5 = 0$ has three real roots.
(B) The cubic equation $x^3 + 2x^2 + x + 5 = 0$ has only one real root.
(C) The cubic equation $x^3 + 2x^2 + x + 5 = 0$ has only real root α, such that $[\alpha] = -3$.
(D) The cubic equation $x^3 + 2x^2 + x + 5 = 0$ has three real roots α, β, γ, such that $[\alpha] = -3, \ [\beta] = -2, \ [\gamma] = -1$, (where [.] denotes the greatest integer function)

13. Let $f(x) = \begin{cases} |x-1| + a, & x < 1 \\ 2x + 3, & x \ge 1 \end{cases}$. If f(x) has a local minima at x = 1, then a is not

(A) less than 5
(B) greater than or equal to 5
(C) less than or equal to 5
(D) none of these

14. A car is driven at speed of x km/hr., where $x \in (20, 120)$ and its mileage is given by $f(x) = \frac{\ell n(g(x))}{g(x)}$, where $g(x) = \left(\frac{e-1}{50}\right)x + 1$, then the best economical speed is

(A) 70 km./hr.
(B) 49 + e km./hr.
(C) 50 km./hr.
(D) 59 + e km./hr.

W I Read the following passage and give the answer of question 15 to 17

If a function f(x) is :

(a) continuous is closed interval [a, b],

(b) differentiable in open interval (a, b), then exists at least one c between a and b such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

15. Suppose $f(x) = \begin{cases} 2x - x^2, & 0 \le x \le 2 \\ 2x + x^2, & -2 \le x < 0 \end{cases}$, then in the interval [–2, 2]

(A) both LMVT and Rolle's theorem can be applied
(B) only LMVT can be applied
(C) only Rolle's theorem can be applied
(D) neither Rolle's theorem nor LMVT can be applied

16. By Lagrange's Mean Value Theorem, which of the following is true for $x > 1$

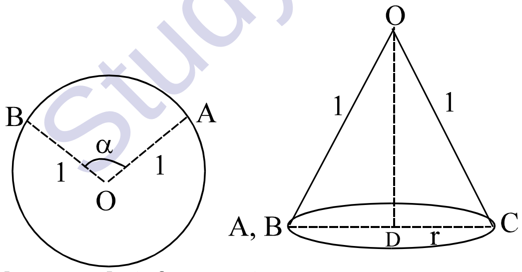
(A) $1 + x\, \ell n\, x < x < 1 + \ell n\, x$
(B) $1 + \ell n\, x < x < 1 + x\, \ell n\, x$
(C) $x < 1 + x\, \ell n\, x < 1 + \ell n\, x$
(D) $1 + \ell n\, x < 1 + x\, \ell n\, x < x$

17. If f(x) and g(x) satisfy the conditions of Mean Value Theorem on the interval [a, b], then which of the following function satisfies the conditions of Rolle's Theorem on [a, b]

(A) g(a) f(x) + g(b) g(x)
(B) (g (a) + g(b)) f(x) + (f(a) + f(b)) g(x)
(C) (g(b) – g(a)) f(x) + (f(a) – f(b)) g(x)
(D) none of these

W II Read the following passage and answer the question 18 to question 21

A conical vessel is to be prepared out of a circular sheet of copper of unit radius as shown in the figure where α be the angle of the sector removed (i.e. $\angle AOB$), then



18. The volume of the vessel. (If $\alpha = \pi$)

(A) $\dfrac{\pi}{24}$
(B) $\dfrac{\sqrt{3}\,\pi^2}{6}$
(C) $\dfrac{\sqrt{3}\,\pi}{24}$
(D) none of these

19. The value of 'r' for which volume is maximum (when α is variable)

(A) $\dfrac{\sqrt{2}}{3}$
(B) $\dfrac{2}{\sqrt{3}}$
(C) $\sqrt{\dfrac{2}{3}}$
(D) none of these

20. The value of 'α' for which volume is maximum (when α is variable)

(A) $\dfrac{2\sqrt{2}\pi}{\sqrt{3}}$
(B) $2\pi - 2\sqrt{\dfrac{2}{3}}\,\pi$
(C) 2π
(D) none of these

21. The sectorial area is to be removed from the sheet so that vessel has the maximum volume, is

(A) $\pi\left(\sqrt{3}-\sqrt{2}\right)$ (B) $\pi\left(\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)$ (C) $\pi\sqrt{\dfrac{2}{3}}$ (D) none of these

W III Read the following passage and give the answer of question 22 to 24

Let f be continuous and differentiable on an interval I. Then f is increasing or decreasing on I if and only if $f'(x) \geq 0$ or $f'(x) \leq 0$ respectively for all x in I. Answer the following questions from 5 to 8.

22. Let $f(x) = \cos x - x$. Then the equation $f(x) = 0$ has

(A) Unique solution in $(0, ® / 6)$
(B) Unique solution in $(® / 6, ® / 3)$
(C) infinitely many solutions in $(0, ® / 4)$
(D) infinitely many solutions in $(0, ® / 2)$

23. Let f be continuous and differentiable function such that $f(x)$ and $f'(x)$ have opposite signs everywnere. Then

(A) f is increasing
(B) f is decreasing
(C) $|f|$ is increasing and decreasing
(D) $|f|$ is decreasing

24. Let $f(x) = \int e^x (x-1)(x-2)\, dx$. Then f decreases in the interval :

(A) $(-\infty, -2)$ (B) $(-2, -1)$ (C) $(1, 2)$ (D) $(2, \infty)$

W IV Consider the following function and answer the question 25 to question 29

$f(x) = 2x^3 - 3(a-3)x^2 + 6ax + a + 2$

25. The value of 'a' for which f(x) has exactly one point of local maxima and one point of local minima

(A) $(-\infty, 1) \cup (9, \infty)$
(B) $(-\infty, 1] \cup [9, \infty)$
(C) $[1, 9]$
(D) $(1, 9)$

26. The value of 'a' for which f(x) has local minima at some negative real x

(A) $(-\infty, 1) \cup (9, \infty)$
(B) $(-\infty, 1] \cup [9, \infty)$
(C) $(0, 1)$
(D) $(1, 9)$

27. The values of 'a' for which f(x) has local maxima at some negative and local minima at positive real x

(A) $(-\infty, 0] \cup [9, \infty)$
(B) $(9, \infty)$
(C) $(0, 1]$
(D) $(-\infty, 0)$

28. The values of 'a' for which f(x) has no point of extrema

(A) $[1, 9]$ (B) ϕ (C) $(-\infty, 0)$ (D) $(1, 9)$

29. The values of 'a' for which f(x) is increasing in $[2, \infty)$

(A) $[1, 9]$ (B) $(1, 9)$ (C) $(-\infty, 9]$ (D) $(-\infty, 1]$

W V Consider the following function and answer the question 30 to question 33

Consider the curve $x = 1 - 3t^2, y = t - 3t^3$. If tangent at point $(1 - 3t^2, t - 3t^3)$ inclined at an angle θ to positive x–axis and tangent at point P(–2, 2) cuts the curve again at Q.

30. The curve is symmetrical about

(A) $y - x = 0$ (B) $y + x = 0$ (C) $y = 0$ (D) $x = 0$

31. $\tan\theta + \sec\theta$ is equal to

(A) t (B) 3t (C) $t + 3t^2$ (D) none of these

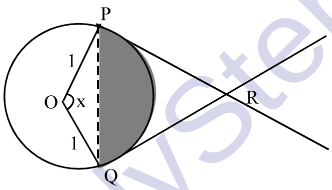
32. The point Q will be

(A) $\left(-\frac{1}{3}, -\frac{2}{9}\right)$ (B) (1, –2) (C) (–2, 1) (D) none of these

33. The angle between the tangents at P and Q will be

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

W VI Read the following passage and give the answer of question 34 to 38



A circular arc PQ of radius 1 subtends an angle of x radian at its centre O, ($0 < x < \pi$) as shown in the figure. The point R is the intersection of the two tangents at points P and Q of the arc. Let us define the following function

S(x) = area of the sector OPQ

T(x) = area of the triangle PQR

U(x) = area of the shaded region

34. $S'\left(\frac{\pi}{4}\right)$ has value

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{8}$ (C) $\frac{1}{2}$ (D) none of these

35. The expression for T(x) is

(A) $\frac{1}{2}\sin x$ (B) $\tan\left(\frac{x}{2}\right) - \frac{\sin x}{2}$

(C) $\frac{1}{2}\tan^2\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$ (D) none of these

36. If $0 < x < \pi$, then the function U(x) is
(A) always increasing
(B) always decreasing
(C) increases in $\left(0, \frac{\pi}{2}\right)$ and decrease in $\left(\frac{\pi}{2}, \pi\right)$
(D) decreases in $\left(0, \frac{\pi}{2}\right)$ and increases in $\left(\frac{\pi}{2}, \pi\right)$

37. For the domain $0 < x < \pi$, the root of the equation $\frac{x}{2} - U(x) = T(x)$ is
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) none of these

38. The value of the limit $\lim\limits_{x \to 0^+} \frac{U(x)}{T(x)}$, is equal to
(A) 1 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) none of these

W VII Consider the following function and answer the question 39 to question 41
Suppose $f(x)$ is continuous on an interval I, and a and b are two points of I. Then if y_0 is a number between $f(a)$ and $f(b)$, there exists a number c between a and b such that $f(c) = y_0$. In particular if $f(a)$ and $f(b)$ possess opposite signs, then there exists atleast one solution of the equation $f(x) = 0$ in the open interval (a, b).

39. Let $f(x) = \frac{1}{x-1} + \frac{1}{x+3} + \frac{1}{x-2} + 5$, then $f(x) = 0$ has
(A) no real roots (B) at most one real roots
(C) exactly 3 real roots (D) exactly 2 real roots

40. $f(x) = ax^2 + bx + c$ and $3a + b + 3c = 0$, then $f(x) = 0$ has
(A) two real and distinct roots
(B) two real and equal roots
(C) non real roots
(D) real roots as well as non real roots depending upon a, b and c

41. Let $f : [0, \infty) \to R$ be a continuous function such that $23f(1) + 10f(2) + 2005f(3) = 0$, then $f(x) = 0$ has always at least one real root in
(A) $[2, 3]$ (B) $[1, 2]$ (C) $(1, 2)$ (D) $(0, 4)$

LEVEL–I ANSWER

1. $\frac{2}{\sqrt{15}}$ cm/sec

5. $a \geq 0$

6. $b \in (-7, -1) \cup (2, 3)$

8. $2x^4 - 12/5\, x^5 + 2/3\, x^6$

9. max. at a/6

10. $\frac{200\pi R^3}{(R+5)^2}$ km²/h

LEVEL–II

5. $\cos A = 0.8$

8. $y = \frac{1}{\sqrt{3}}x$

10. $2\sqrt{2}$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. $\cos(\ell n\, \theta)$

2. $x \in \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$ **;** $x \in \left(-\infty, -\frac{1}{2}\right) \cup \left(0, \frac{1}{2}\right)$

3. $x \geq 0$

4. abc

5. $\frac{1}{3}$

6. ϕ ; (–1 , 1)

(B)

1. F **2.** F

(C)

1. C	**2.** BC	**3.** C	**4.** A	**5.** ABC	
6. AC	**7.** D	**8.** A	**9.** BD	**10.** BC	**11.** AB

(D)

1. A	**2.** A	**3.** C	**4.** B	**5.** i-a, ii-c	**6.** B	**7.** C
8. B	**9.** A	**10.** C	**11.** D	**12.** A	**13.** B	**14.** D
15. D	**16.** B	**17.** D	**18.** C	**19.** C	**20.** D	**21.** A
22. D	**23.** A	**24.** D	**25.** A	**26.** A	**27.** A	**28.** D
29. C	**30.** A	**31.** B	**32.** C	**33.** A	**34.** A	**35.** B
36. A						

(E)

37. B

(F)

38. B **39.** A **40.** A

IIT JEE PROBLEMS (SUBJECTIVE)

1. 2

2. e^{π}

5. $\sqrt{c-\frac{1}{4}}$

7. $\left(\frac{Ud}{\sqrt{V^2-U^2}}\right)$km

9. (0, 0)

10. $2x+4y-\pi=0$, $2x+4y+3\pi=0$

11. $\lambda\in\left(-\frac{3}{2},\ 0\right)\cup\left(0,\ \frac{3}{2}\right)$

13. (0, 2)

14. f is minimum at $x=\frac{7}{5}$

15. $\frac{10}{3}$ sq. units

16. $2\sqrt{3}\,x-y=\left(2\sqrt{3}-2\right)$ **or** $2\sqrt{3}\,x+y=\left(2\sqrt{3}+2\right)$

17. $\left(3\sqrt{3}/4\right)r^2$

18. c = 3/4

19. $\frac{6}{\sqrt{5}}\,m/s$

20. $\frac{6+\pi}{6}$

21. $\sqrt{2}\,x-y=\left(89\sqrt{2}/27\right)+1$ or $\sqrt{2}\,x+y=\left(89\sqrt{2}/27\right)-1$

22. $2R\sin\theta,\ R\cos\theta-\sqrt{R^2-L^2}$ where $\cos\theta=\frac{\sqrt{R^2-L^2}+\sqrt{9R^2-L^2}}{4R}$

23. $x+\sqrt{2}\,y-\sqrt{2}=0$ and $x-\sqrt{2}\,y+\sqrt{2}=0$

24. P is at a distance of ac / (a + b) from C

25. x + y – 1 = 0

26. $\frac{1}{16}$

27. $b\in(-2,\ -1)\cup(1,\ \infty)$

29. (4, 8/3)

30. maxima at x = arc sin(1/3) and max. value = 13/3, minima at x = $\pi/2$ and min. value = 3

31. $a=-\frac{1}{12},\ b=-\frac{3}{4},\ c=3$

32. $\frac{4\sqrt{3}}{9}$

33. $\frac{3\sqrt{3}}{4}ab$

34. 2kh

35. (3/2, 1)

36. increase in $\left[0,\ \frac{\pi}{2}\right]$, decrease in $\left[\frac{\pi}{2},\ \frac{2\pi}{3}\right]$, decrease in $\left[\frac{2\pi}{3},\ \pi\right]$

37. min at $x=\frac{1}{4}(b+\sqrt{b^2-1})$, max at $x=\frac{1}{4}(b-\sqrt{b^2-1})$

38. [-2/a, a/3]

39. $y=e^{a(x-1)}$; 1 sq. unit

40. 4, 4, 4

42. $\pi\sqrt{\frac{2}{3}}$ sq. units

43. $a = \frac{1}{4}$; $b = -\frac{5}{4}$; $f(x) = \frac{1}{4}\left(x^2 - 5x + 8\right)$

44. $\log xy = 1$

45. (2, 1)

46. $\theta = \tan^{-1}\left|\frac{4\sqrt{2}}{7}\right|$

47. $\left(\pm\frac{a^2}{\sqrt{a^2+b^2}}, \pm\frac{b^2}{\sqrt{a^2+b^2}}\right)$

48. $\pm\sqrt{3}\,x \pm \sqrt{2}\,y = 5$

49. $\sqrt{2}\,x + y - 2\sqrt{2} = 0$ or $\sqrt{2}\,x - y - 2\sqrt{2} = 0$

50. (2, 15)

52. (2, 1)

56. 5 units

59. $4\sqrt{65}$

60. $y = 2$

62. 6

SET–I

1.	D	**2.**	D	**3.**	B	**4.**	D	**5.**	B
6.	B	**7.**	D	**8.**	A	**9.**	A	**10.**	D
11.	B	**12.**	C	**13.**	D	**14.**	C	**15.**	D
16.	C	**17.**	D	**18.**	A	**19.**	D	**20.**	D
21.	C	**22.**	C	**23.**	C	**24.**	B	**25.**	D
26.	D	**27.**	B	**28.**	D	**29.**	D	**30.**	A

SET–II

1.	D	**2.**	C	**3.**	B	**4.**	D	**5.**	C
6.	C	**7.**	A	**8.**	C	**9.**	D	**10.**	A
11.	C	**12.**	C	**13.**	C	**14.**	D	**15.**	B
16.	D	**17.**	A	**18.**	C	**19.**	C	**20.**	B
21.	D	**22.**	C	**23.**	C	**24.**	B	**25.**	B
26.	A	**27.**	C	**28.**	D	**29.**	A	**30.**	D

SET–III

1.	AC	**2.**	ABD	**3.**	C	**4.**	D	**5.**	ABC
6.	ABD	**7.**	C	**8.**	A	**9.**	AC	**10.**	C
11.	B	**12.**	BC	**13.**	AC	**14.**	C	**15.**	A
16.	B	**17.**	C	**18.**	C	**19.**	C	**20.**	B
21.	B	**22.**	B	**23.**	D	**24.**	C	**25.**	A
26.	C	**27.**	D	**28.**	A	**29.**	C	**30.**	C
31.	B	**32.**	A	**33.**	D	**34.**	C	**35.**	B
36.	A	**37.**	C	**38.**	C	**39.**	C	**40.**	A
41.	D								

Application of Derivative

1. A man observes that, when he has walked C metres up an inclined plane the angular depression of an object in a horizontal plane through the foot of the slope is α and that when he has walked a further distance of C metres the depression is β. Prove that the inclination of the slope to the horizon is the angle whose cotangent is
$2 \cot \beta - \cot \alpha$.

2. If $xy(y - x) = 2a^3$, show that y has a minimum when $x = a$,. Determine this minimum value. Show that y has a second value at $x = a$ which is less than the minimum. How do you explain this paradox?

3. Evaluate the following integrals

(a) $\int (\log x)^2 \sqrt{x}\, dx$ (b) $\int \frac{x^{1/2}}{x^{1/2} - x^{1/3}} dx$

4. A missile is launched with velocity v from point A and inclined at an angle b to the horizontal. It strikes the plane inclined at a to the horizontal at B. Show that

$$AB = \frac{2v^2 \sin(\beta - \alpha)}{g \cos \alpha} \{\cos(\beta - \alpha) - \tan \alpha \sin(\beta - \alpha)\}$$

5. A train starts from rest and for the first kilometer moves with constant acceleration, for the next 3 kilometers it has constant velocity and for another 2 kilometers it moves the constant retardation to come to rest after 10 min. Find the maximum velocity and the three time intervals in three types of motions.

6. Ship A leaves port X and sails at 25 km/hour due west. Ship B leaves half an hour later from port Y, which is 30 km south-west of X, and sails at 20 km/hours north-west. At what time after A leaves port will they be nearest together.

7. A heavy uniform rod of length 2a lies over a rough peg with one end leaning against a rough vertical wall if C be the distance of the peg from the wall and μ the co-efficient of the fraction both at the peg and the wall is above the peg, then the rod is on the point of sliding downwards when $c = (1 + \mu^2)\, a \sin^3 \theta$.

8. A lot of 100 bulbs from a manufacturing process is known to contain 10 defective and 90 non-defective bulbs. If a sample of 8 bulbs is selected at random, what is the probability that
(a) the sample has 3 defective and 5 non-defective bulbs ?
(b) the sample has at least one defective bulb ?

9. Find the volume of water replaced by a sphere of 20 cm radius when dropped in a vertically standing conical vessel with vertex downward and full of water. The radius at top of the vessel is 40 cm and the height is 30 cm.

10. Find the volume of the material removed out of the solid sphere of 50 cm radius when a solid cylinder of 25 cm radius and 100 cm length is penetrated across the sphere centrally.

11. A river with width equal to 1000 m flows with velocity of 5m/sec. A girl starts from one bank towards another by propelling a boat with a absolute velocity of 10 m/sec perpendicular to the flow. A body standing 500 m downstream from the girl on the same bank starts swimming the same instant to catch the boat. Derive the answers for the following if the boy succeeds in catching the boat exactly in midstream :
(i) After what time the boy succeeded in meeting the boat ?
(ii) what was the absolute velocity and direction of swimming of the boy ?
(iii) What was the distance along the stream from the initial position of the boy when he was on the boat ?

1983

12. A spherical balloon is being inflated so that its volume increase uniformly at the rate of 40 cm^3/min. How fast is its surface area increasing when the radius is 8 cm. Find approximately, how much the radius will increase during the next 1/2 minute.

13. Inside a fixed hollow cylinder of radius R, whose generators are horizontal, are placed symmetrically two cylinders each of the radius 1 m. A third cylinder of radius equal to 1 m is placed symmetrically over the two inside cylinder. Find the minimum value of R for which equilibrium is just possible.

14. A regular hexagon of side a stands in vertical plane with one side on horizontal ground. A particle is projected such that it touches the four upper vertices of the hexagon before returning to the ground. Find the ratio of the velocity of the particle on reaching the ground to its minimum velocity.

15. A cylinder vessel of volume $25\frac{1}{7}$ cubic meters, which is open at top, is to be manufactured from a sheet of metal. Find the dimensions of the vessel so that the amount of sheet used in manufacturing ti is the least possible.

16. A many observes that when he moves up a distance c metres on a slope, the angel depression of a point on the horizontal plane from the base of the slope is 30º; and when he moves up further a distance c metres the angel of depression of the point is 45º. Obtain the angle of inclination of the slope with the horizontal.

17. Tangents are drawn from the origin to the curve y = sin x. Prove that their points of contact lie on
$$x^2y^2 = x^2 – y^2$$

18. How should a wire 20 cms long be divided into two parts, if one part is to be bent into a circle, the other part is to be bent into a square and the two plane figure are to have areas the sum of which is maximum ?

19. In the curve $x^ay^b - k^{a+b}$, prove that he portion of the tangent intercepted between the co-ordinate axis is divided at its points of contact into segments which are in a constant ratio (all the constant being positive). **[1988]**

Application of Derivative

20. Find the vertical angle of a right circular cone of minimum curved surface that circumscribes a given sphere. **[1988]**

21. Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1, 2). **[1990]**

22. A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine were carrying a current of 5.0×10^{-3} ampere. Calculate the magnetic field intensity on the axis at the middle and at the end of the solenoid. **[1992]**

23. A very small circular loop of the area 5×10^{-4} m^2 resistance 2 ohm and negligible inductance is initially coplanar and concentric with a much larger fixed circular loop of the radius 0.1 m. a constant current of 1 ampere is passed in the bigger loop and the smaller loop is rotated with angular velocity ω rad/sec about a diameter. Calculate (i) the flux linked with the smaller loop, (ii) induced e.m.f. and (iii) induced current in the smaller loop, as a function of time.**[1992]**

24. Find the points on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes. **[1993]**

25. Find the values of x for which the function

$$f(x) = 1 + 2\sin x + 3\cos^2 x, \left(0 \le x \le \frac{2\pi}{3}\right)$$

is maximum or minimum. Also find these values of the function. **[1993]**

26. Find the intervals in which the function $f(x) = 3\cos^4 x + 10\cos^3 x + 6\cos^2 x - 3, 0 \le x \le \pi$; is monotonically increasing or decreasing. **[1995]**

27. Find the point (1, b) on the ellipse $4x^2 + 3y^2 = 12$, in the first quadrant, so that the area encloses by the lines $y = x$, $y = \beta$, $x = \alpha$ and the x – axis is maximum. **[1995]**

28. A 12 cm long wire is bent to form a triangle with one of its angles as 60º. Find the sides of the triangle when its area is largest. **[1996]**

29. A conical vessel is to be prepared out of a circular sheet of gold of unit radius. How much sectorial area is to be removed from the sheet so that the vessel has maximum volume ? **[1997]**