Exercise - 1 (Objective Questions)

Part : (A) Only one correct option

1. The domain of the function $f(x) = \dfrac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2+2x+8}}$ is

(A) (1, 4) (B) (– 2, 4) (C) (2, 4) (D) [2, ∞)

2. The function $f(x) = \cot^{-1}\sqrt{(x+3)x} + \cos^{-1}\sqrt{x^2+3x+1}$ is defined on the set S, where S is equal to:

(A) {0, 3} (B) (0, 3) (C) {0, – 3} (D) [– 3, 0]

3. The range of the function $f(x) = \sin^{-1}\left[x^2+\frac{1}{2}\right] + \cos^{-1}\left[x^2-\frac{1}{2}\right]$, where [ ] is the greatest integer function, is:

(A) $\left\{\frac{\pi}{2}, \pi\right\}$ (B) $\left\{0, \frac{\pi}{2}\right\}$ (C) $\{\pi\}$ (D) $\left(0, \frac{\pi}{2}\right)$

4. Range of $f(x) = \log_{\sqrt{5}}\{\sqrt{2}(\sin x - \cos x) + 3\}$ is

(A) [0, 1] (B) [0, 2] (C) $\left[0, \frac{3}{2}\right]$ (D) none of these

5. Range of $f(x) = 4^x + 2^x + 1$ is
(A) (0, ∞) (B) (1, ∞) (C) (2, ∞) (D) (3, ∞)

6. If x and y satisfy the equation y = 2 [x] + 3 and y = 3 [x – 2] simultaneously, the [x + y] is
(A) 21 (B) 9 (C) 30 (D) 12

7. The function $f : [2, \infty) \to Y$ defined by $f(x) = x^2 - 4x + 5$ is both one–one & onto if
(A) Y = R (B) Y = [1, ∞) (C) Y = [4, ∞) (D) Y = [5, ∞)

8. Let S be the set of all triangles and R^+ be the set of positive real numbers. Then the function, $f : S \to R^+$, $f(\Delta)$ = area of the Δ, where $\Delta \in S$ is :
(A) injective but not surjective (B) surjective but not injective
(C) injective as well as surjective (D) neither injective nor surjective

9. Let f(x) be a function whose domain is [– 5, 7]. Let g(x) = |2x + 5|. Then domain of (fog) (x) is
(A) [– 4, 1] (B) [– 5, 1] (C) [– 6, 1] (D) none of these

10. The inverse of the function $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ is

(A) $\frac{1}{2}\log\frac{1+x}{1-x}$ (B) $\frac{1}{2}\log\frac{2+x}{2-x}$ (C) $\frac{1}{2}\log\frac{1-x}{1+x}$ (D) 2 log (1 + x)

11. The fundamental period of the function,
$f(x) = x + a - [x + b] + \sin \pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + \ldots + \sin (2n - 1)\pi x + \cos 2n\pi x$ for every a, b ∈ R is: (where [ ] denotes the greatest integer function)
(A) 2 (B) 4 (C) 1 (D) 0

12. The period of $e^{\cos^4 \pi x + x - [x] + \cos \pi x}$ is ______(where [ ] denotes the greatest integer function)
(A) 1 (B) 2 (C) 3 (D) 4

13. If y = f(x) satisfies the condition $f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $(x \neq 0)$ then f(x) =

(A) $-x^2+2$ (B) $-x^2-2$ (C) x^2+2 (D) x^2-2

14. Given the function $f(x) = \frac{a^x + a^{-x}}{2}$ $(a > 0)$. If f(x + y) + f(x – y) = k f(x). f(y) then k has the value equal to:

(A) 1 (B) 2 (C) 4 (D) 1/2

15. A function f : R → R satisfies the condition, $x^2 f(x) + f(1-x) = 2x - x^4$. Then f(x) is:
(A) $-x^2-1$ (B) $-x^2+1$ (C) x^2-1 (D) $-x^4+1$

16. The domain of the function, $f(x) = \sqrt{\frac{1}{(|x|-1)\cos^{-1}(2x+1)\tan 3x}}$ is:

(A) (– 1, 0) (B) $(-1, 0) - \left\{-\frac{\pi}{6}\right\}$

(C) $(-1, 0] - \left\{-\frac{\pi}{6}, -\frac{\pi}{2}\right\}$ (D) $\left(-\frac{\pi}{6}, 0\right)$

17. If f (x) = 2 [x] + cos x, then f: R → R is: (where [ ] denotes greatest integer function)
(A) one–one and onto (B) one–one and into
(C) many–one and into (D) many–one and onto

18. If $q^2 - 4pr = 0$, p > 0, then the domain of the function, $f(x) = \log(px^3 + (p+q)x^2 + (q+r)x + r)$ is:

(A) $R - \left\{-\frac{q}{2p}\right\}$ (B) $R - \left[(-\infty, -1] \cup \left\{-\frac{q}{2p}\right\}\right]$

(C) $R - \left[(-\infty, -1) \cap \left\{-\frac{q}{2p}\right\}\right]$ (D) none of these

19. If [2 cos x] + [sin x] = – 3, then the range of the function, $f(x) = \sin x + \sqrt{3}\cos x$ in $[0, 2\pi]$ is:
(where [ ] denotes greatest integer function)

(A) [– 2, – 1) (B) (– 2, – 1] (C) (– 2, – 1) (D) $[-2, -\sqrt{3})$

20. The domain of the function $f(x) = \log_{1/2}\left(-\log_2\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$ is:

(A) 0 < x < 1 (B) 0 < x ≤ 1 (C) x ≥ 1 (D) null set

21. The range of the functions $f(x) = \log_{\sqrt{2}}\left(2 - \log_2\left(16\sin^2 x + 1\right)\right)$ is
(A) (– ∞, 1) (B) (– ∞, 2) (C) (– ∞, 1] (D) (– ∞, 2]

22. The domain of the function, $f(x) = \sin^{-1}\left(\frac{1+x^3}{2x^{3/2}}\right) + \sqrt{\sin(\sin x)} + \log_{(3\{x\}+1)}(x^2+1)$,

where {x} represents fractional part function is:
(A) x ∈ {1} (B) x ∈ R – {1, – 1} (C) x > 3, x ≠ I (D) none of these

23. The minimum value of $f(x) = a\tan^2 x + b\cot^2 x$ equals the maximum value of $g(x) = a\sin^2 x + b\cos^2 x$ where $a > b > 0$, when

(A) $4a = b$ (B) $3a = b$ (C) $a = 3b$ (D) $a = 4b$

24. Let $f(2, 4) \to (1, 3)$ be a function defined by $f(x) = x - \left[\frac{x}{2}\right]$ (where [.] denotes the greatest integer function), then $f^{-1}(x)$ is equal to :

(A) $2x$ (B) $x + \left[\frac{x}{2}\right]$ (C) $x + 1$ (D) $x - 1$

25. The image of the interval R when the mapping $f: R \to R$ given by $f(x) = \cot^{-1}(x^2 - 4x + 3)$ is

(A) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (B) $\left[\frac{\pi}{4}, \pi\right)$ (C) $(0, \pi)$ (D) $\left(0, \frac{3\pi}{4}\right]$

26. If the graph of the function $f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$ is symmetric about y-axis, then n is equal to:

(A) 2 (B) 2 / 3 (C) 1 / 4 (D) – 1 / 3

27. If $f(x) = \cot^{-1}x$ $: R^+ \to \left(0, \frac{\pi}{2}\right)$

and $g(x) = 2x - x^2$ $: R \to R$. Then the range of the function $f(g(x))$ wherever define is

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(0, \frac{\pi}{4}\right)$ (C) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{4}\right)$

28. Let $f: (e^2, \infty) \to R$ be defined by $f(x) = \ln(\ln(\ln x))$, then

(A) f is one one but not onto
(B) f is on to but not one - one
(C) f is one-one and onto
(D) f is neither one-one nor onto

29. Let $f: (e, \infty) \to R$ be defined by $f(x) = \ln(\ln(\ln x))$, then

(A) f is one one but not onto
(B) f is on to but not one - one
(C) f is one-one and onto
(D) f is neither one-one nor onto

30. Let $f(x) = \sin x$ and $g(x) = |\ln x|$ if composite functions fog(x) and gof (x) are defined and have ranges R_1 & R_2 respectively then.

(A) $R_1 = \{u: -1 < u < 1\}$ $R_2 = \{v: 0 < v < \infty\}$
(B) $R_1 = \{u: -\infty < u \le 0\}$ $R_2 = \{v: -1 \le v \le 1\}$
(C) $R_1 = \{u: 0 \le u < \infty\}$ $R_2 = \{v: -1 < v < 1; v \ne 0\}$
(D) $R_1 = \{u: -1 \le u \le 1\}$ $R_2 = \{v: 0 \le v < \infty\}$

31. Function $f: (-\infty, 1) \to (0, e^5]$ defined by $f(x) = e^{-(x^2 - 3x + 2)}$ is

(A) many one and onto (B) many one and into (C) one one and onto (D) one one and into

32. The number of solutions of the equation $[\sin^{-1} x] = x - [x]$, where [.] denotes the greatest integer function is

(A) 0 (B) 1 (C) 2 (D) infinitely many

33. The function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is

(A) an odd function
(B) an even function
(C) neither an odd nor an even function
(D) a periodic function

Part : (B) May have more than one options correct

34. For the function $f(x) = \ln\ (\sin^{-1} \log_2 x)$,

(A) Domain is $\left[\frac{1}{2}, 2\right]$ (B) Range is $\left(-\infty, \ln\frac{\pi}{2}\right]$

(C) Domain is (1, 2] (D) Range is R

35. A function 'f' from the set of natural numbers to integers defined by,

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$$ is:

(A) one-one (B) many-one (C) onto (D) into

36. Domain of $f(x) = \sin^{-1}[2 - 4x^2]$ where [x] denotes greatest integer function is:

(A) $\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right) - \{0\}$ (B) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right] - \{0\}$ (C) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ (D) $\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$

37. If $F(x) = \frac{\sin \pi [x]}{\{x\}}$, then F (x) is:

(A) periodic with fundamental period 1

(B) even

(C) range is singleton

(D) identical to sgn $\left(\operatorname{sgn}\frac{\{x\}}{\sqrt{\{x\}}}\right) - 1$, where {x} denotes fractional part function and [ ] denotes greatest integer function and sgn (x) is a signum function.

38. $D \equiv [-1, 1]$ is the domain of the following functions, state which of them are injective.

(A) $f(x) = x^2$ (B) $g(x) = x^3$ (C) $h(x) = \sin 2x$ (D) $k(x) = \sin(\pi x/2)$

Exercise - 2 (Subjective Questions)

1. Find the domain of the function $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

2. Find the domain of the function $f(x) = \sqrt{1-2x} + 3\sin^{-1}\left(\frac{3x-1}{2}\right)$

3. Find the inverse of the following functions. $f(x) = \ln(x + \sqrt{1+x^2})$

4. Let $f : \left[-\frac{\pi}{3}, \frac{\pi}{6}\right] \to B$ defined by $f(x) = 2\cos^2 x + \sqrt{3}\sin 2x + 1$. Find the B such that f^{-1} exists. Also find $f^{-1}(x)$.

5. Find for what values of x, the following functions would be identical.

$f(x) = \log(x - 1) - \log(x - 2)$ and $g(x) = \log\left(\frac{x-1}{x-2}\right)$.

6. If $f(x) = \dfrac{4^x}{4^x + 2}$, then show that $f(x) + f(1 - x) = 1$

7. Let f(x) be a polynomial function satisfying the relation $f(x).\,f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)\ \forall\, x \in R - \{0\}$ and $f(3) = -26$. Determine $f'(1)$.

8. Find the domain of definitions of the following functions.

(i) $f(x) = \sqrt{3 - 2^x - 2^{1-x}}$ (ii) $f(x) = \sqrt{\dfrac{x-2}{x+2}} + \sqrt{\dfrac{1-x}{1+x}}$

(iii) $f(x) = \log_{10}(1 - \log_{10}(x^2 - 5x + 16))$

9. Find the range of the following functions.

(i) $f(x) = \dfrac{x^2 - 2x + 4}{x^2 + 2x + 4}$ (ii) $f(x) = \sin \log\left(\dfrac{\sqrt{4 - x^2}}{1 - x}\right)$

(iii) $f(x) = x^4 - 2x^2 + 5$ (iv) $f(x) = \sin^2 x + \cos^4 x$

10. Solve the following equation for x (where [x] & {x} denotes integral and fractional part of x)
$2x + 3[x] - 4\{-x\} = 4$

11. Draw the graph of following functions where [.] denotes greatest integer function and { .} denotes fractional part function.
(i) $y = \{\sin x\}$ (ii) $y = [x] + \sqrt{\{x\}}$

12. Draw the graph of the function $f(x) = \left|x^2 - 4|x| + 3\right|$ and also find the set of values of 'a' for which the equation f(x) = a has exactly four distinct real roots.

13. Examine whether the following functions are even or odd or none.

(i) $f(x) = \dfrac{(1+2^x)^7}{2^x}$ (ii) $f(x) = \begin{cases} x|x|, & x \le -1 \\ [1+x] + [1-x], & -1 < x < 1 \\ -x|x|, & x \ge 1 \end{cases}$

(iii) $f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x + 2\pi}{\pi}\right] - 3}$, where [ ] denotes greatest integer function.

14. Find the period of the following functions.

(i) $f(x) = 1 - \dfrac{\sin^2 x}{1+\cot x} - \dfrac{\cos^2 x}{1+\tan x}$

(ii) $f(x) = \tan\dfrac{\pi}{2}[x]$, where [.] denotes greatest integer function.

(iii) $f(x) = \dfrac{1}{2}\left(\dfrac{|\sin x|}{\cos x} + \dfrac{\sin x}{|\cos x|}\right)$

(iv) $f(x) = \dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

15. If $f(x) = \begin{cases} 1+x^2 & x \le 1 \\ x+1 & 1 < x \le 2 \end{cases}$ and $g(x) = 1 - x$; $-2 \le x \le 1$ then define the function fog(x).

16. Find the set of real x for which the function, $f(x) = \dfrac{1}{[|x-1|] + [|12-x|] - 11}$ is not defined, where [x] denotes the greatest integer not greater than x.

17. Given the functions $f(x) = e^{\cos^{-1}\left(\sin\left(x+\frac{\pi}{3}\right)\right)}$, $g(x) = \operatorname{cosec}^{-1}\left(\dfrac{4-2\cos x}{3}\right)$ & the function h(x) = f(x) defined only for those values of x, which are common to the domains of the functions f(x) and g(x). Calculate the range of the function h(x).

18. Let 'f' be a real valued function defined for all real numbers x such that for some positive constant 'a' the equation $f(x+a) = \dfrac{1}{2} + \sqrt{f(x) - (f(x))^2}$ holds for all x. Prove that the function f is periodic.

19. If $f(x) = -1 + |x-2|$, $0 \le x \le 4$

$g(x) = 2 - |x|$, $-1 \le x \le 3$

Then find fog (x), gof (x), fof(x) & gog(x). Draw rough sketch of the graphs of fog (x) & gof (x).

20. Find the integral solutions to the equation [x] [y] = x + y. Show that all the non-integral solutions lie on exactly two lines. Determine these lines. Here [.] denotes greatest integer function.

Answers

Exercise # 1

1. D **2.** C **3.** C **4.** B **5.** B **6.** C **7.** B

8. B **9.** C **10.** A **11.** A **12.** B **13.** D **14.** B

15. B **16.** D **17.** C **18.** B **19.** D **20.** D **21.** D

22. D **23.** D **24.** C **25.** D **26.** D **27.** C **28.** A

29. C **30.** D **31.** D **32.** B **33.** B **34.** BC

35. AC **36.** B **37.** ABCD **38.** BD

Exercise # 2

1. $[-2, 0) \cup (0, 1)$ **2.** $\left[-\frac{1}{3}, \frac{1}{2}\right]$

3. $f^{-1} = \dfrac{e^x - e^{-x}}{2}$

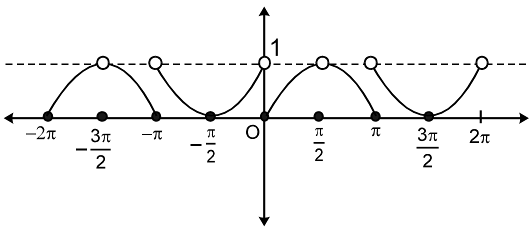
4. $B = [0, 4]$; $f^{-1}(x) = \dfrac{1}{2}\left(\sin^{-1}\left(\dfrac{x-2}{2}\right) - \dfrac{\pi}{6}\right)$

5. $(2, \infty)$ **7.** -3 **8.** (i) $[0, 1]$ (ii) ϕ (iii) $(2, 3)$

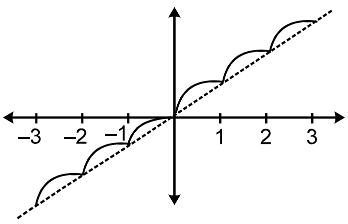
9. (i) $\left[\frac{1}{3}, 3\right]$ (ii) $[-1, 1]$ (iii) $[4, \infty)$ (iv) $\left[\frac{3}{4}, 1\right]$

10. $\left\{\frac{3}{2}\right\}$

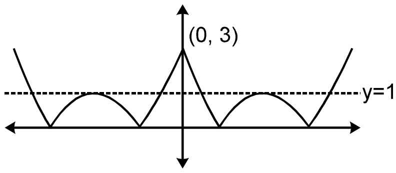
11. (i)



(ii)



12.



$a \in (1, 3) \cup \{0\}$

13. (i) neither even nor odd (ii) even (iii) odd

14. (i) π (ii) 2 (iii) 2π (iv) π

15. $f(g(x)) = \begin{cases} 2 - 2x + x^2 & 0 \le x \le 1 \\ 2 - x & -1 \le x < 0 \end{cases}$

16. (0, 1) U {1, 2,........, 12} U (12, 13) **17.** $\left[e^{\frac{\pi}{6}}, e^{\pi}\right]$

18. Period 2 a

19. $fog(x) = \begin{cases} -(1+x) & , & -1 \le x \le 0 \\ x - 1 & , & 0 < x \le 2 \end{cases}$;

$gof(x) = \begin{cases} x+1 & , & 0 \le x < 1 \\ 3-x & , & 1 \le x \le 2 \\ x-1 & , & 2 < x \le 3 \\ 5-x & , & 3 < x \le 4 \end{cases}$

$fof(x) = \begin{cases} x & , & 0 \le x \le 2 \\ 4-x & , & 2 < x \le 2 \end{cases}$;

$gog(x) = \begin{cases} -x & , & -1 \le x \le 0 \\ x & , & 0 < x \le 2 \\ 4-x & , & 2 < x \le 3 \end{cases}$

20. Integral solution (0, 0); (2, 2). x + y = 6, x + y = 0