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Senior School Certificate Examination

March 2016

Marking Scheme — Mathematics 65/1/N, 65/2/N, 65/3/N

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/N

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Finding $A^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ $\frac{1}{2}$

Getting $\alpha = \frac{\pi}{4}$ or $45°$ $\frac{1}{2}$

2. $k = 27$ 1

3. For a unique solution

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$ $\frac{1}{2}$

$\Rightarrow k \neq 0$ $\frac{1}{2}$

4. Getting equation as $\frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$ $\frac{1}{2}$

Sum of intercepts $\frac{5}{2} + 5 - 5 = \frac{5}{2}$ $\frac{1}{2}$

5. Getting $\lambda = -9$ and $\mu = 27$ $\frac{1}{2}$ each

6. $\vec{a} + \vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ $\frac{1}{2}$

Unit vector parallel to $\vec{a} + \vec{b}$ is $\frac{1}{7}(6\hat{i} - 3\hat{j} + 2\hat{k})$ $\frac{1}{2}$

SECTION B

7. $\tan^{-1}(x - 1) + \tan^{-1}(x + 1) = \tan^{-1} 3x - \tan^{-1} x$ $\frac{1}{2}$

$\Rightarrow \tan^{-1}\left(\frac{2x}{2 - x^2}\right) = \tan^{-1}\left(\frac{2x}{1 + 3x^2}\right)$ $1\frac{1}{2}$

$\frac{2x}{2 - x^2} = \frac{2x}{1 + 3x^2}$ $\frac{1}{2}$

$2x(1 + 3x^2 - 2 + x^2) = 0$ $\frac{1}{2}$

$x = 0, \frac{1}{2}, -\frac{1}{2}$ 1

OR

Let $2x = \tan\theta$ 1

L. H. S $= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right) - \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$ 1

$= \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 2\theta)$ 1

$= 3\theta - 2\theta$

$= \theta$ or $\tan^{-1} 2x$

$\therefore$ L. H. S = R. H. S 1

8. Getting matrix equation as $\begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 145 \\ 180 \end{pmatrix}$ $1\frac{1}{2}$

$\Rightarrow \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}^{-1}\begin{pmatrix} 145 \\ 180 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$

$\Rightarrow$ E = 10, H = 15 $1\frac{1}{2}$

The poor boy was charged ₹ 65 less

Value: Helping the poor 1

9. L.H.L = a + 3 $1\frac{1}{2}$

R.H.L = b/2 $1\frac{1}{2}$

f(x) is continuous at x = 0. So, a + 3 = 2 = b/2 $\frac{1}{2}$

$\Rightarrow$ a = –1 and b = 4 $\frac{1}{2}$

10. $\frac{dx}{dy} = \frac{\sin a}{\cos^2(a+y)}$ $1\frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \frac{-2\cos(a+y)\sin(a+y)}{\sin a}\frac{dy}{dx}$

$= \frac{-\sin 2(a+y)}{\sin a}\frac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow \sin a \frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$ $\frac{1}{2}$

OR

Let $2x = \sin\theta$ 1

$$\therefore\ y = \sin^{-1}\left(\frac{6x - \sqrt{1-4x^2}}{5}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\sin\theta - \frac{4}{5}\cos\theta\right)$$

$$= \sin^{-1}(\cos\alpha\sin\theta - \sin\alpha\cos\theta) \qquad \left[\cos\alpha = \frac{3}{5};\ \sin\alpha = \frac{4}{5}\right]$$ 1

$$= \sin^{-1}(\sin(\theta - \alpha))$$

$$= \theta - \alpha$$ 1

$$= \sin^{-1}(2x) - \alpha$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$ 1

11. Slope of the tangent $= 3x^2 + 2 = 14$ 1

Points of contact (2, 8) and (–2, –16) 1

Equations of tangent

$14x - y - 20 = 0$ 1

and $14x - y + 12 = 0$ 1

12. Let $2x = t$

$$I = \frac{1}{2}\int\frac{(t-5)}{(t-3)^3}e^t\,dt$$ 1

$$= \frac{1}{2}\int\left[\frac{1}{(t-3)^2} - \frac{2}{(t-3)^3}\right]e^t\,dt$$ 2

$$= \frac{1}{2}\frac{1}{(t-3)^2}e^t + C = \frac{1}{2}\frac{1}{(2x-3)^2}e^{2x} + C$$ 1

OR

Writing $\dfrac{x^2+x+1}{(x^2+1)(x+2)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x+2}$

$$\Rightarrow\ A = \frac{2}{5}, B = \frac{1}{5}, C = \frac{3}{5}$$ 1

$$\therefore\ I = \frac{1}{5}\int\frac{2x}{x^2+1}dx + \frac{1}{5}\int\frac{dx}{x^2+1} + \frac{3}{5}\int\frac{dx}{x+2}$$ 2

$$\Rightarrow\ I = \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\log|x+2| + C$$ 1

13. Using property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ 1

$I = \int_{-2}^{2}\left(\frac{x^2}{1+5^x}\right)dx = \int_{-2}^{2}\left(\frac{x^2}{1+5^{-x}}\right)dx$ 1

$2I = \int_{-2}^{2} x^2\,dx$ 1

$2I = \frac{16}{3}$ or $I = \frac{8}{3}$ 1

14. Writing $x + 3 = A(-4 - 2x) + B$

$\Rightarrow \quad A = -\frac{1}{2},\ B = 1$ 1

$\therefore \quad I = -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\,dx + \int\sqrt{\left(\sqrt{7}\right)^2 - (x+2)^2}\,dx$ 1

$I = -\frac{1}{3}(3-4-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}} + C$ 2

15. Writing linear equation $\frac{dy}{dx} + \frac{\cos x}{1+\sin x}y = -\frac{x}{1+\sin x}$ 1

$I.F = e^{\int\frac{\cos x}{1+\sin x}dx} = 1 + \sin x$ 1

General solution is: $y(1 + \sin x) = -\frac{x^2}{2} + C$ 1

Particular solution is: $y\,(1 + \sin x) = 1 - \frac{x^2}{2}$ 1

16. $\frac{dx}{dy} = \frac{2xe^v - y}{2ye^v}$

$\frac{x}{y} = v$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$ 1

$v + y\frac{dv}{dy} = \frac{2vye^v - y}{2ye^v}$

$2\int e^v dv = -\int\frac{dy}{y}$ 1

General solution is: $2e^v = -\log|y| + C$ or $2e^{x/y} = -\log|y| + C$ 1

Particular solution is: $2e^{x/y} + \log|y| = 2$ 1

17. $\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k},\ \overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k},\ \overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$ $1\frac{1}{2}$

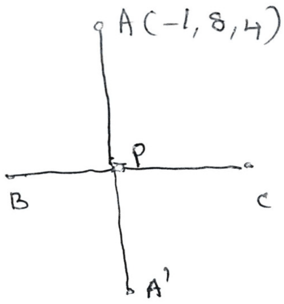
For 4 points to be coplanar, $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

i.e., $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$ $1\frac{1}{2}$

$= -4(12 + 3) + 6(-3 + 24) - 2(1 + 32)$

$= -60 + 126 - 66 = 0$ which is true

Hence, points are coplanar. 1

18. 

Equation of line BC: $\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = r$ 1

General point on BC: $(2r, -2r - 1, -4r + 3)$

$\Rightarrow$ d.r.'s of AP: $(2r + 1, -2r - 9, -4r - 1)$ 1

As AP $\perp$ BC $\Rightarrow r = -1$

$\Rightarrow$ Co-ordinates of P: $(-2, 1, 7)$ 1

Hence, coordinates of Image of A: $(-3, -6, 10)$ 1

19. Let E_1 and E_2 be the events of drawing bag X and bag Y respectively.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

Let A be the event of drawing one white and one black ball from any one of the bag without replacement. Then,

$\Rightarrow$ $P(A/E_1) = \frac{4}{6}\times\frac{2}{5}+\frac{2}{6}\times\frac{4}{5}=\frac{16}{30}$

$P(A/E_2) = \frac{3}{6}\times\frac{3}{5}+\frac{3}{6}\times\frac{3}{5}=\frac{18}{30}$ $1\frac{1}{2}$

Using Bayes' Theorem, we have

$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$ 1

$= \frac{\frac{1}{2}\times\frac{18}{30}}{\frac{1}{2}\times\frac{16}{30}+\frac{1}{2}\times\frac{18}{30}} = \frac{9}{17}$ 1

OR

Let A_i and B_i be the events of throwing 10 by A and B in the respective ith turn. Then,

$P(A_i) = P(B_i) = \frac{1}{12}$ and $P(\overline{A_i}) = P(\overline{B_i}) = \frac{11}{12}$ $1 + \frac{1}{2}$

Probability of winning A, when A starts first

$= \frac{1}{12} + \left(\frac{11}{12}\right)^2 \frac{1}{12} + \left(\frac{11}{12}\right)^4 \frac{1}{12} + \ldots$ 1

$= \frac{1/12}{1-(11/12)^2}$

$= \frac{12}{23}$ 1

Probability of winning of B $= 1 - P(A) = 1 - \frac{12}{23} = \frac{11}{23}$ $\frac{1}{2}$

SECTION C

20. The variate X takes values 3, 4, 5, and 6 — 2

$P(X=3)=\frac{1}{20}; P(X=4)=\frac{3}{20}; P(X=5)=\frac{6}{20}; P(X=6)=\frac{10}{20};$ — 1

Probability distribution is:

X	3	4	5	6
P(X)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

1

Mean $= \Sigma\ XP(X) = \frac{105}{20} = \frac{21}{4}$ — 1

Variance $= \Sigma X^2P(X) - (\Sigma X\ P(X))^2 = \frac{63}{80}$ — 1

21. Proving * is commutative — $1\frac{1}{2}$

Proving * is associative — $1\frac{1}{2}$

Getting identity element as (0, 0) — $1\frac{1}{2}$

Getting inverse of (a, b) as (–a, –b) — $1\frac{1}{2}$

22. Getting $\frac{dy}{d\theta} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$ — 2

Equating $\frac{dy}{d\theta}$ to 0 and getting critical point as cos θ = 0 i.e., $\theta = \frac{\pi}{2}$ — 1

For all $\theta, 0 \le \theta \le \frac{\pi}{2}, \frac{dy}{d\theta} \ge 0$ — 2

Hence, y is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$ — 1

OR

Correct Figure — 1

Writing $V = \frac{1}{3}\pi r^2 h = \frac{\pi}{2} l^3 \sin^2\theta\cos\theta$ — 1

Getting $\frac{dV}{d\theta} = \frac{\pi}{2} l^3[2\sin\theta\cos^2\theta - \sin^3\theta]$ — 1

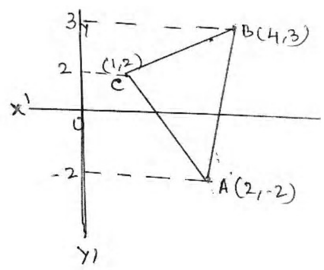
For maxima and minima, $\frac{dV}{d\theta} = 0$ — $\frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$ or $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ — $1\frac{1}{2}$

Getting $\frac{d^2V}{d\theta^2}$ negative — 1

Hence, volume of the cone is maximum when semi-vertical angle is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

23.



Correct Figure 1

Writing equations of three sides in terms of y as

$$x_{AB} = \frac{2}{5}(y+2)+2\,;\; x_{BC} = 3(y-3)+4;\; x_{AC} = \frac{-1}{4}(y+2)+2$$ 1

$$\text{Area} = \int_{-2}^{3}\left(\frac{2}{5}(y+2)+2\right)dy - \int_{-2}^{2}\left(-\frac{1}{4}(y+2)+2\right)dy - \int_{2}^{3}\left(3(y-3)+4\right)dy$$ 1

$$= \left.\frac{2}{10}(y+2)^2+2y\right]_{-2}^{3} - \left[-\frac{1}{8}(y+2)^2+2y\right]_{-2}^{2} - \left[\frac{3}{2}(y-3)^2+4y\right]_{2}^{3}$$ 2

$$= 15-6-\frac{5}{2} \text{ or } \frac{13}{2}$$ 1

24. Required equation of the plane is

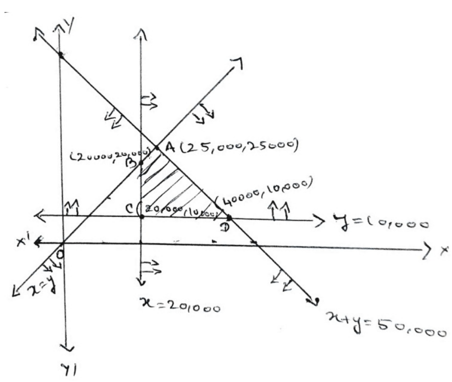
$\vec{r}.[(1-2\lambda)\hat{i}+(-2+\lambda)\hat{j}+(3+\lambda)\hat{k}] = 4-5\lambda$ 2

Intercept of the plane on x-axis = Intercept of the plane on y-axis

$\Rightarrow \frac{4-5\lambda}{1-2\lambda} = \frac{4-5\lambda}{\lambda-2}$ i.e., $\lambda = 1, \frac{4}{5}$ $\left(\text{rejecting } \lambda = \frac{4}{5}\right)$ 2

Required equation of the plane is $\vec{r}.(-\hat{i}-\hat{j}+4\hat{k})+1=0$ 2

25.



Let the investment in bond A be ₹ x and in bond B be ₹ y

Objective function is: $Z = \frac{x}{10}+\frac{9}{100}y$ 1

Subject to constraints

$x+y \geq 50000;\; x \geq 20{,}000;\; y \geq 10{,}000;\; x \geq y$ (*) 2

Correct Figure 2

Vertices of feasible region are A, B, C, and D

Point	$Z = \frac{x}{10}+\frac{9}{100}y$	Value
A(25,000, 25000)	2500 + 2250	4750
B(20,000, 20,000)	2000 + 1800	3800
C(20,000, 10,000)	2000 + 900	2900
D(40,000, 10,000)	4000 + 900	4900

Return is maximum when ₹ 40000 are invested in Bond A and ₹ 10000 in Bond B

Maximum return is ₹ 4900 1

Since there are more than 3 constraints, student may be given full 6 marks even if reaches upto (*).

26. $\Delta = \begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix}$

$R_1 \to zR_1,\ R_2 \to xR_2,\ R_3 \to yR_3$

$$\Delta = \frac{1}{xyz}\begin{vmatrix} z(x+y)^2 & z^2x & z^2y \\ x^2z & x(z+y)^2 & x^2y \\ y^2z & y^2x & y(z+x)^2 \end{vmatrix} \qquad 1$$

$$= \frac{xyz}{xyz}\begin{vmatrix} (x+y)^2 & z^2 & z^2 \\ x^2 & (z+y)^2 & x^2 \\ y^2 & y^2 & (z+x)^2 \end{vmatrix} \qquad 1$$

$C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} (x+y)^2 - z^2 & 0 & z^2 \\ 0 & (z+y)^2 - x^2 & x^2 \\ y^2 - (z+x)^2 & y^2 - (z+x)^2 & (z+x)^2 \end{vmatrix}$$

$$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ y-z-x & y-z-x & (z+x)^2 \end{vmatrix} \qquad 1$$

$R_3 \to R_3 - R_1 - R_2$ we get

$$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ -2x & -2z & 2xz \end{vmatrix} \qquad 1$$

$C_1 \to C_1 + \frac{C_3}{z},\ C_2 \to C_2 + \frac{C_3}{x}$ we get

$$\Delta = (x+y+z)^2\begin{vmatrix} x+y & \frac{z^2}{x} & z^2 \\ \frac{x^2}{z} & z+y & x^2 \\ 0 & 0 & 2xz \end{vmatrix} \qquad 1$$

$$= 2xyz\,(x+y+z)^3 \qquad 1$$

OR

For getting $A^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$ $\qquad 1\frac{1}{2}$

For getting $A^3 = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$ $\qquad 1\frac{1}{2}$

Simplifying $A^3 - 6A^2 + 7A + kI_3$ as $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix}$ 2

Equating $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow \quad k - 2 = 0$

$k = 2$ 1

QUESTION PAPER CODE 65/2/N

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\vec{a}+\vec{b}=6\hat{i}-3\hat{j}+2\hat{k}$ $\frac{1}{2}$

Unit vector parallel to $\vec{a}+\vec{b}$ is $\frac{1}{7}(6\hat{i}-3\hat{j}+2\hat{k})$ $\frac{1}{2}$

2. Getting $\lambda = -9$ and $\mu = 27$ $\frac{1}{2}$ each

3. Getting equation as $\frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5} = 1$ $\frac{1}{2}$

Sum of intercepts $\frac{5}{2}+5-5 = \frac{5}{2}$ $\frac{1}{2}$

4. For a unique solution

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$ $\frac{1}{2}$

$\Rightarrow k \neq 0$ $\frac{1}{2}$

5. $k = 27$ 1

6. Finding $A^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ $\frac{1}{2}$

Getting $\alpha = \frac{\pi}{4}$ or $45°$ $\frac{1}{2}$

SECTION B

7. Let E_1 and E_2 be the events of drawing bag X and bag Y respectively.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

Let A be the event of drawing one white and one black ball from any one of the bag without replacement.

Then,

$$\Rightarrow \quad P(A/E_1) = \frac{4}{6}\times\frac{2}{5}+\frac{2}{6}\times\frac{4}{5}=\frac{16}{30}$$

$$P(A/E_2) = \frac{3}{6}\times\frac{3}{5}+\frac{3}{6}\times\frac{3}{5}=\frac{18}{30}$$ $1\frac{1}{2}$

Using Bayes' Theorem, we have

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)}$$ 1

$$= \frac{\frac{1}{2}\times\frac{18}{30}}{\frac{1}{2}\times\frac{16}{30}+\frac{1}{2}\times\frac{18}{30}} = \frac{9}{17}$$ 1

OR

Let A_i and B_i be the events of throwing 10 by A and B in the respective ith turn. Then,

$P(A_i) = P(B_i) = \frac{1}{12}$ and $P(\overline{A_i}) = P(\overline{B_i}) = \frac{11}{12}$ $1 + \frac{1}{2}$

Probability of winning A, when A starts first

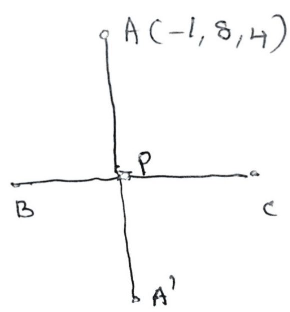
$$= \frac{1}{12} + \left(\frac{11}{12}\right)^2 \frac{1}{12} + \left(\frac{11}{12}\right)^4 \frac{1}{12} + \ldots$$ 1

$$= \frac{1/12}{1-(11/12)^2}$$

$$= \frac{12}{23}$$ 1

Probability of winning of B $= 1 - P(A) = 1 - \frac{12}{23} = \frac{11}{23}$ $\frac{1}{2}$

8.



Equation of line BC: $\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = r$ 1

General point on BC: $(2r, -2r - 1, -4r + 3)$

$\Rightarrow$ d.r.'s of AP: $(2r + 1, -2r - 9, -4r - 1)$ 1

As AP $\perp$ BC $\Rightarrow r = -1$

$\Rightarrow$ Co-ordinates of P: $(-2, 1, 7)$ 1

Hence, coordinates of Image of A: $(-3, -6, 10)$ 1

9. $\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$, $\overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$, $\overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$ $1\frac{1}{2}$

For 4 points to be coplanar, $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

i.e., $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$ $1\frac{1}{2}$

$= -4(12 + 3) + 6(-3 + 24) - 2(1 + 32)$

$= -60 + 126 - 66 = 0$ which is true

Hence, points are coplanar. 1

10. $\frac{dx}{dy} = \frac{2xe^v - y}{2ye^v}$

$\frac{x}{y} = v$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$ 1

$v + y\frac{dv}{dy} = \frac{2vye^v - y}{2ye^v}$

$2\int e^v dv = -\int \frac{dy}{y}$ 1

General solution is: $2e^v = -\log|y| + C$ or $2e^{x/y} = -\log|y| + C$ 1

Particular solution is: $2e^{x/y} + \log|y| = 2$ 1

11. Writing linear equation $\frac{dy}{dx} + \frac{\cos x}{1+\sin x} y = -\frac{x}{1+\sin x}$ 1

I.F $= e^{\int \frac{\cos x}{1+\sin x} dx} = 1 + \sin x$ 1

General solution is: $y(1 + \sin x) = -\frac{x^2}{2} + C$ 1

Particular solution is: $y(1 + \sin x) = 1 - \frac{x^2}{2}$ 1

12. Writing $x + 3 = A(-4 - 2x) + B$

$\Rightarrow \quad A = -\frac{1}{2}, \; B = 1$ 1

$\therefore \quad I = -\frac{1}{2}\int (-4-2x)\sqrt{3-4x-x^2}\, dx + \int \sqrt{\left(\sqrt{7}\right)^2 - (x+2)^2}\, dx$ 1

$I = -\frac{1}{3}(3-4-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}} + C$ 2

13. Let $2x = t$

$I = \frac{1}{2}\int \frac{(t-5)}{(t-3)^3} e^t \, dt$ 1

$= \frac{1}{2}\int \left[\frac{1}{(t-3)^2} - \frac{2}{(t-3)^3}\right] e^t \, dt$ 2

$= \frac{1}{2}\frac{1}{(t-3)^2} e^t + C = \frac{1}{2}\frac{1}{(2x-3)^2} e^{2x} + C$ 1

OR

Writing $\frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$

$\Rightarrow \quad A = \frac{2}{5}, B = \frac{1}{5}, C = \frac{3}{5}$ 1

$\therefore \quad I = \frac{1}{5}\int \frac{2x}{x^2+1} dx + \frac{1}{5}\int \frac{dx}{x^2+1} + \frac{3}{5}\int \frac{dx}{x+2}$ 2

$\Rightarrow \quad I = \frac{1}{5}\log\left|x^2+1\right| + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\log|x+2| + C$ 1

14. Slope of the tangent $= 3x^2 + 2 = 14$ 1

Points of contact $(2, 8)$ and $(-2, -16)$ 1

Equations of tangent

$14x - y - 20 = 0$ 1

and $14x - y + 12 = 0$ 1

15. $\frac{dx}{dy} = \frac{\sin a}{\cos^2 (a + y)}$ $1\frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \frac{-2\cos(a + y)\sin(a + y)}{\sin a}\frac{dy}{dx}$

$= \frac{-\sin 2(a+y)}{\sin a}\frac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow \sin a \frac{d^2y}{dx^2} + \sin 2(a + y)\frac{dy}{dx} = 0$ $\frac{1}{2}$

OR

Let $2x = \sin \theta$ 1

$\therefore\ y = \sin^{-1}\left(\frac{6x - \sqrt{1-4x^2}}{5}\right)$

$= \sin^{-1}\left(\frac{3}{5}\sin\theta - \frac{4}{5}\cos\theta\right)$

$= \sin^{-1} (\cos \alpha \sin \theta - \sin \alpha \cos \theta)$ $[\cos \alpha = \frac{3}{5}; \sin \alpha = \frac{4}{5}]$ 1

$= \sin^{-1}(\sin(\theta - \alpha))$

$= \theta - \alpha$ 1

$= \sin^{-1} (2x) - \alpha$

$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$ 1

16. Using property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ 1

$I = \int_{-2}^{2}\left(\frac{x^2}{1+5^x}\right)dx = \int_{-2}^{2}\left(\frac{x^2}{1+5^{-x}}\right)dx$ 1

$2I = \int_{-2}^{2} x^2\,dx$ 1

$2I = \frac{16}{3}$ or $I = \frac{8}{3}$ 1

17. L.H.L = a + 3 $1\frac{1}{2}$

R.H.L = b/2 $1\frac{1}{2}$

f(x) is continuous at x = 0. So, a + 3 = 2 = b/2 $\frac{1}{2}$

$\Rightarrow a = -1$ and $b = 4$ $\frac{1}{2}$

18. Getting matrix equation as $\begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 145 \\ 180 \end{pmatrix}$ $1\frac{1}{2}$

$$\Rightarrow \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}^{-1}\begin{pmatrix} 145 \\ 180 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$$

$\Rightarrow E = 10,\ H = 15$ $1\frac{1}{2}$

The poor boy was charged ₹ 65 less

Value: Helping the poor 1

19. $\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1} 3x - \tan^{-1} x$ $\frac{1}{2}$

$\Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\left(\frac{2x}{1+3x^2}\right)$ $1\frac{1}{2}$

$\frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$ $\frac{1}{2}$

$2x(1 + 3x^2 - 2 + x^2) = 0$ $\frac{1}{2}$

$x = 0, \frac{1}{2}, -\frac{1}{2}$ 1

OR

Let $2x = \tan\theta$ 1

L. H. S $= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right) - \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$ 1

$= \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 2\theta)$ 1

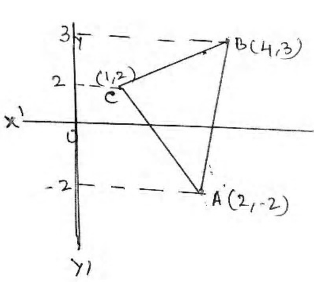
$= 3\theta - 2\theta$

$= \theta$ or $\tan^{-1} 2x$

$\therefore$ L. H. S = R. H. S 1

SECTION C

20. Correct Figure 1



Writing equations of three sides in terms of y as

$x_{AB} = \frac{2}{5}(y+2)+2$; $x_{BC} = 3(y-3)+4$; $x_{AC} = \frac{-1}{4}(y+2)+2$ 1

Area $= \int_{-2}^{3}\left(\frac{2}{5}(y+2)+2\right)dy - \int_{-2}^{2}\left(-\frac{1}{4}(y+2)+2\right)dy - \int_{2}^{3}\left(3(y-3)+4\right)dy$ 1

$= \left.\frac{2}{10}(y+2)^2+2y\right]_{-2}^{3} - \left[-\frac{1}{8}(y+2)^2+2y\right]_{-2}^{2} - \left[\frac{3}{2}(y-3)^2+4y\right]_{2}^{3}$ 2

$= 15 - 6 - \frac{5}{2}$ or $\frac{13}{2}$ 1

21. $\Delta = \begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix}$

$R_1 \to zR_1,\ R_2 \to xR_2,\ R_3 \to yR_3$

$\Delta = \dfrac{1}{xyz}\begin{vmatrix} z(x+y)^2 & z^2x & z^2y \\ x^2z & x(z+y)^2 & x^2y \\ y^2z & y^2x & y(z+x)^2 \end{vmatrix}$ 1

$= \dfrac{xyz}{xyz}\begin{vmatrix} (x+y)^2 & z^2 & z^2 \\ x^2 & (z+y)^2 & x^2 \\ y^2 & y^2 & (z+x)^2 \end{vmatrix}$ 1

$C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$\Delta = \begin{vmatrix} (x+y)^2 - z^2 & 0 & z^2 \\ 0 & (z+y)^2 - x^2 & x^2 \\ y^2 - (z+x)^2 & y^2 - (z+x)^2 & (z+x)^2 \end{vmatrix}$

$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ y-z-x & y-z-x & (z+x)^2 \end{vmatrix}$ 1

$R_3 \to R_3 - R_1 - R_2$ we get

$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ -2x & -2z & 2xz \end{vmatrix}$ 1

$C_1 \to C_1 + \dfrac{C_3}{z},\ C_2 \to C_2 + \dfrac{C_3}{x}$ we get

$\Delta = (x+y+z)^2\begin{vmatrix} x+y & \dfrac{z^2}{x} & z^2 \\ \dfrac{x^2}{z} & z+y & x^2 \\ 0 & 0 & 2xz \end{vmatrix}$ 1

$= 2xyz\,(x+y+z)^3$ 1

OR

For getting $A^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$ $1\frac{1}{2}$

For getting $A^3 = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$ $1\frac{1}{2}$

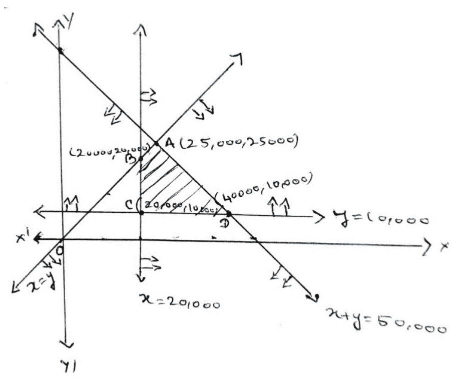
Simplifying $A^3 - 6A^2 + 7A + kI_3$ as $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix}$ 2

Equating $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow \quad k - 2 = 0$

$k = 2$ 1

22. Let the investment in bond A be ₹ x and in bond B be ₹ y



Objective function is: $Z = \frac{x}{10} + \frac{9}{100}y$ 1

Subject to constraints

$x + y \geq 50000$; $x \geq 20,000$; $y \geq 10,000$; $x \geq y$ (*) 2

Correct Figure 2

Vertices of feasible region are A, B, C, and D

Point	$Z = \frac{x}{10} + \frac{9}{100}y$	Value
A(25,000, 25000)	2500 + 2250	4750
B(20,000, 20,000)	2000 + 1800	3800
C(20,000, 10,000)	2000 + 900	2900
D(40,000, 10,000)	4000 + 900	4900

Return is maximum when ₹ 40000 are invested in Bond A and ₹ 10000 in Bond B

Maximum return is ₹ 4900 1

Since there are more than 3 constraints, student may be given full 6 marks even if reaches upto (*).

23. Required equation of the plane is

$\vec{r}.[(1-2\lambda)\hat{i} + (-2+\lambda)\hat{j} + (3+\lambda)\hat{k}] = 4 - 5\lambda$ 2

Intercept of the plane on x-axis = Intercept of the plane on y-axis

$\Rightarrow \frac{4-5\lambda}{1-2\lambda} = \frac{4-5\lambda}{\lambda-2}$ i.e., $\lambda = 1, \frac{4}{5}$ $\left(\text{rejecting } \lambda = \frac{4}{5}\right)$ 2

Required equation of the plane is $\vec{r}.(-\hat{i} - \hat{j} + 4\hat{k}) + 1 = 0$ 2

24. Getting $\frac{dy}{d\theta} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$ 2

Equating $\frac{dy}{d\theta}$ to 0 and getting critical point as $\cos\theta = 0$ i.e., $\theta = \frac{\pi}{2}$ 1

For all $\theta, 0 \le \theta \le \frac{\pi}{2}, \frac{dy}{d\theta} \ge 0$ 2

Hence, y is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$ 1

OR

Correct Figure 1

Writing $V = \frac{1}{3}\pi r^2 h = \frac{\pi}{2} l^3 \sin^2\theta\cos\theta$ 1

Getting $\frac{dV}{d\theta} = \frac{\pi}{2} l^3 [2\sin\theta\cos^2\theta - \sin^3\theta]$ 1

For maxima and minima, $\frac{dV}{d\theta} = 0$ $\frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$ or $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $1\frac{1}{2}$

Getting $\frac{d^2V}{d\theta^2}$ negative 1

Hence, volume of the cone is maximum when semi-vertical angle is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

25. Proving * is commutative $1\frac{1}{2}$

Proving * is associative $1\frac{1}{2}$

Getting identity element as (0, 0) $1\frac{1}{2}$

Getting inverse of (a, b) as (–a, –b) $1\frac{1}{2}$

26. The variate X takes values 3, 4, 5, and 6 2

$P(X=3) = \frac{1}{20}; P(X=4) = \frac{3}{20}; P(X=5) = \frac{6}{20}; P(X=6) = \frac{10}{20};$ 1

Probability distribution is:

X	3	4	5	6
P(X)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

1

Mean = $\Sigma XP(X) = \frac{105}{20} = \frac{21}{4}$ 1

Variance = $\Sigma X^2P(X) - (\Sigma X P(X))^2 = \frac{63}{80}$ 1

QUESTION PAPER CODE 65/3/N

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. For a unique solution

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$ $\frac{1}{2}$

$\Rightarrow k \neq 0$ $\frac{1}{2}$

2. $\vec{a} + \vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ $\frac{1}{2}$

Unit vector parallel to $\vec{a} + \vec{b}$ is $\frac{1}{7}(6\hat{i} - 3\hat{j} + 2\hat{k})$ $\frac{1}{2}$

3. Getting $\lambda = -9$ and $\mu = 27$ $\frac{1}{2}$ each

4. Getting equation as $\frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$ $\frac{1}{2}$

Sum of intercepts $\frac{5}{2} + 5 - 5 = \frac{5}{2}$ $\frac{1}{2}$

5. Finding $A^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ $\frac{1}{2}$

Getting $\alpha = \frac{\pi}{4}$ or $45°$ $\frac{1}{2}$

6. $k = 27$ 1

SECTION B

7. Writing $x + 3 = A(-4 - 2x) + B$

$\Rightarrow A = -\frac{1}{2},\ B = 1$ 1

$\therefore\ I = -\frac{1}{2}\int(-4 - 2x)\sqrt{3 - 4x - x^2}\,dx + \int\sqrt{\left(\sqrt{7}\right)^2 - (x+2)^2}\,dx$ 1

$I = -\frac{1}{3}(3 - 4 - x^2)^{3/2} + \frac{x+2}{2}\sqrt{3 - 4x - x^2} + \frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}} + C$ 2

8. Using property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ 1

$I = \int_{-2}^{2}\left(\frac{x^2}{1+5^x}\right)dx = \int_{-2}^{2}\left(\frac{x^2}{1+5^{-x}}\right)dx$ 1

$2I = \int_{-2}^{2} x^2\,dx$ 1

$2I = \frac{16}{3}$ or $I = \frac{8}{3}$ 1

9. Slope of the tangent = $3x^2 + 2 = 14$ 1

Points of contact (2, 8) and (–2, –16) 1

Equations of tangent

$14x - y - 20 = 0$ 1

and $14x - y + 12 = 0$ 1

10. L.H.L = a + 3 $1\frac{1}{2}$

R.H.L = b/2 $1\frac{1}{2}$

f(x) is continuous at x = 0. So, a + 3 = 2 = b/2 $\frac{1}{2}$

$\Rightarrow$ a = –1 and b = 4 $\frac{1}{2}$

11. $\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1} 3x - \tan^{-1} x$ $\frac{1}{2}$

$\Rightarrow \quad \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\left(\frac{2x}{1+3x^2}\right)$ $1\frac{1}{2}$

$\frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$ $\frac{1}{2}$

$2x(1 + 3x^2 - 2 + x^2) = 0$ $\frac{1}{2}$

$x = 0, \frac{1}{2}, -\frac{1}{2}$ 1

OR

Let $2x = \tan\theta$ 1

L. H. S $= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right) - \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$ 1

$= \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 2\theta)$ 1

$= 3\theta - 2\theta$

$= \theta$ or $\tan^{-1} 2x$

$\therefore$ L. H. S = R. H. S 1

12. $\frac{dx}{dy} = \frac{\sin a}{\cos^2(a+y)}$ $1\frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \frac{-2\cos(a+y)\sin(a+y)}{\sin a}\frac{dy}{dx}$

$= \frac{-\sin 2(a+y)}{\sin a}\frac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow \sin a \dfrac{d^2y}{dx^2} + \sin 2(a+y)\dfrac{dy}{dx} = 0$ $\qquad \frac{1}{2}$

OR

Let $2x = \sin\theta$ $\qquad 1$

$\therefore\ y = \sin^{-1}\left(\dfrac{6x - \sqrt{1-4x^2}}{5}\right)$

$= \sin^{-1}\left(\dfrac{3}{5}\sin\theta - \dfrac{4}{5}\cos\theta\right)$

$= \sin^{-1}(\cos\alpha\sin\theta - \sin\alpha\cos\theta)$ $\qquad [\cos\alpha = \frac{3}{5};\ \sin\alpha = \frac{4}{5}]$ $\qquad 1$

$= \sin^{-1}(\sin(\theta - \alpha))$

$= \theta - \alpha$ $\qquad 1$

$= \sin^{-1}(2x) - \alpha$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-4x^2}}$ $\qquad 1$

13. Let E_1 and E_2 be the events of drawing bag X and bag Y respectively.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$ $\qquad \frac{1}{2}$

Let A be the event of drawing one white and one black ball from any one of the bag without replacement.

Then,

$\Rightarrow P(A/E_1) = \frac{4}{6}\times\frac{2}{5}+\frac{2}{6}\times\frac{4}{5} = \frac{16}{30}$

$P(A/E_2) = \frac{3}{6}\times\frac{3}{5}+\frac{3}{6}\times\frac{3}{5} = \frac{18}{30}$ $\qquad 1\frac{1}{2}$

Using Bayes' Theorem, we have

$P(E_2/A) = \dfrac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$ $\qquad 1$

$= \dfrac{\frac{1}{2}\times\frac{18}{30}}{\frac{1}{2}\times\frac{16}{30}+\frac{1}{2}\times\frac{18}{30}} = \dfrac{9}{17}$ $\qquad 1$

OR

Let A_i and B_i be the events of throwing 10 by A and B in the respective ith turn. Then,

$P(A_i) = P(B_i) = \frac{1}{12}$ and $P(\overline{A_i}) = P(\overline{B_i}) = \frac{11}{12}$ $\qquad 1 + \frac{1}{2}$

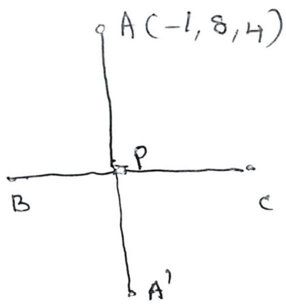
Probability of winning A, when A starts first

$= \frac{1}{12} + \left(\frac{11}{12}\right)^2\frac{1}{12} + \left(\frac{11}{12}\right)^4\frac{1}{12} + \ldots$ $\qquad 1$

$$= \frac{1/12}{1-(11/12)^2}$$

$$= \frac{12}{23}$$ 1

Probability of winning of B $= 1 - P(A) = 1 - \frac{12}{23} = \frac{11}{23}$ $\frac{1}{2}$

14. 

Equation of line BC: $\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = r$ 1

General point on BC: $(2r, -2r-1, -4r+3)$

$\Rightarrow$ d.r.'s of AP: $(2r+1, -2r-9, -4r-1)$ 1

As AP $\perp$ BC $\Rightarrow r = -1$

$\Rightarrow$ Co-ordinates of P: $(-2, 1, 7)$ 1

Hence, coordinates of Image of A: $(-3, -6, 10)$ 1

15. $\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$, $\overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$, $\overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$ $1\frac{1}{2}$

For 4 points to be coplanar, $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

i.e., $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$ $1\frac{1}{2}$

$= -4(12+3) + 6(-3+24) - 2(1+32)$

$= -60 + 126 - 66 = 0$ which is true

Hence, points are coplanar. 1

16. Getting matrix equation as $\begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 145 \\ 180 \end{pmatrix}$ $1\frac{1}{2}$

$\Rightarrow \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}^{-1}\begin{pmatrix} 145 \\ 180 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$

$\Rightarrow E = 10, H = 15$ $1\frac{1}{2}$

The poor boy was charged ₹ 65 less

Value: Helping the poor 1

17. $\frac{dx}{dy} = \frac{2xe^v - y}{2ye^v}$

$\frac{x}{y} = v$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$ 1

$v + y\frac{dv}{dy} = \frac{2vye^v - y}{2ye^v}$

$2\int e^v dv = -\int \frac{dy}{y}$ 1

General solution is: $2e^v = -\log|y| + C$ or $2e^{x/y} = -\log|y| + C$ 1

Particular solution is: $2e^{x/y} + \log|y| = 2$ 1

18. Writing linear equation $\frac{dy}{dx} + \frac{\cos x}{1+\sin x} y = -\frac{x}{1+\sin x}$ 1

I.F = $e^{\int \frac{\cos x}{1+\sin x} dx} = 1 + \sin x$ 1

General solution is: $y(1 + \sin x) = -\frac{x^2}{2} + C$ 1

Particular solution is: $y(1 + \sin x) = 1 - \frac{x^2}{2}$ 1

19. Let $2x = t$

$I = \frac{1}{2}\int \frac{(t-5)}{(t-3)^3} e^t \, dt$ 1

$= \frac{1}{2}\int \left[\frac{1}{(t-3)^2} - \frac{2}{(t-3)^3}\right] e^t \, dt$ 2

$= \frac{1}{2}\frac{1}{(t-3)^2} e^t + C = \frac{1}{2}\frac{1}{(2x-3)^2} e^{2x} + C$ 1

OR

Writing $\frac{x^2 + x + 1}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$

$\Rightarrow \quad A = \frac{2}{5}, B = \frac{1}{5}, C = \frac{3}{5}$ 1

$\therefore \quad I = \frac{1}{5}\int \frac{2x}{x^2+1} dx + \frac{1}{5}\int \frac{dx}{x^2+1} + \frac{3}{5}\int \frac{dx}{x+2}$ 2

$\Rightarrow \quad I = \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\log|x+2| + C$ 1

SECTION C

20. Getting $\frac{dy}{d\theta} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$ 2

Equating $\frac{dy}{d\theta}$ to 0 and getting critical point as $\cos\theta = 0$ i.e., $\theta = \frac{\pi}{2}$ 1

For all $\theta, 0 \le \theta \le \frac{\pi}{2}, \frac{dy}{d\theta} \ge 0$ 2

Hence, y is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$ 1

OR

Correct Figure 1

Writing $V = \frac{1}{3}\pi r^2 h = \frac{\pi}{2} l^3 \sin^2\theta\cos\theta$ 1

Getting $\frac{dV}{d\theta} = \frac{\pi}{2} l^3 [2\sin\theta\cos^2\theta - \sin^3\theta]$ 1

For maxima and minima, $\frac{dV}{d\theta} = 0$ $\frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$ or $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $1\frac{1}{2}$

Getting $\frac{d^2V}{d\theta^2}$ negative 1

Hence, volume of the cone is maximum when semi-vertical angle is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

21. $\Delta = \begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix}$

$R_1 \to zR_1,\ R_2 \to xR_2,\ R_3 \to yR_3$

$$\Delta = \frac{1}{xyz}\begin{vmatrix} z(x+y)^2 & z^2x & z^2y \\ x^2z & x(z+y)^2 & x^2y \\ y^2z & y^2x & y(z+x)^2 \end{vmatrix}$$ 1

$$= \frac{xyz}{xyz}\begin{vmatrix} (x+y)^2 & z^2 & z^2 \\ x^2 & (z+y)^2 & x^2 \\ y^2 & y^2 & (z+x)^2 \end{vmatrix}$$ 1

$C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} (x+y)^2 - z^2 & 0 & z^2 \\ 0 & (z+y)^2 - x^2 & x^2 \\ y^2 - (z+x)^2 & y^2 - (z+x)^2 & (z+x)^2 \end{vmatrix}$$

$$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ y-z-x & y-z-x & (z+x)^2 \end{vmatrix}$$ 1

$R_3 \to R_3 - R_1 - R_2$ we get

$$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ -2x & -2z & 2xz \end{vmatrix}$$ 1

$C_1 \to C_1 + \frac{C_3}{z},\ C_2 \to C_2 + \frac{C_3}{x}$ we get

$$\Delta = (x+y+z)^2 \begin{vmatrix} x+y & \frac{z^2}{x} & z^2 \\ \frac{x^2}{z} & z+y & x^2 \\ 0 & 0 & 2xz \end{vmatrix}$$ 1

$= 2xyz\ (x + y + z)^3$ 1

OR

For getting $A^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$ $1\frac{1}{2}$

For getting $A^3 = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$ $1\frac{1}{2}$

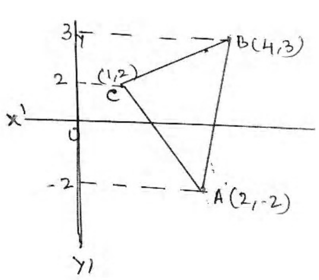
Simplifying $A^3 - 6A^2 + 7A + kI_3$ as $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix}$ 2

Equating $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow\ k - 2 = 0$

$k = 2$ 1

22. Correct Figure 1



Writing equations of three sides in terms of y as

$x_{AB} = \frac{2}{5}(y+2)+2$; $x_{BC} = 3(y-3)+4$; $x_{AC} = \frac{-1}{4}(y+2)+2$ 1

$$\text{Area} = \int_{-2}^{3}\left(\frac{2}{5}(y+2)+2\right)dy - \int_{-2}^{2}\left(-\frac{1}{4}(y+2)+2\right)dy - \int_{2}^{3}\left(3(y-3)+4\right)dy$$ 1

$$= \left.\frac{2}{10}(y+2)^2+2y\right]_{-2}^{3} - \left[-\frac{1}{8}(y+2)^2+2y\right]_{-2}^{2} - \left[\frac{3}{2}(y-3)^2+4y\right]_{2}^{3}$$ 2

$= 15 - 6 - \frac{5}{2}$ or $\frac{13}{2}$ 1

23. Proving * is commutative $1\frac{1}{2}$

Proving * is associative $1\frac{1}{2}$

Getting identity element as (0, 0) $1\frac{1}{2}$

Getting inverse of (a, b) as (–a, –b) $1\frac{1}{2}$

24. The variate X takes values 3, 4, 5, and 6 2

$P(X=3)=\frac{1}{20}$; $P(X=4)=\frac{3}{20}$; $P(X=5)=\frac{6}{20}$; $P(X=6)=\frac{10}{20}$; 1

Probability distribution is:

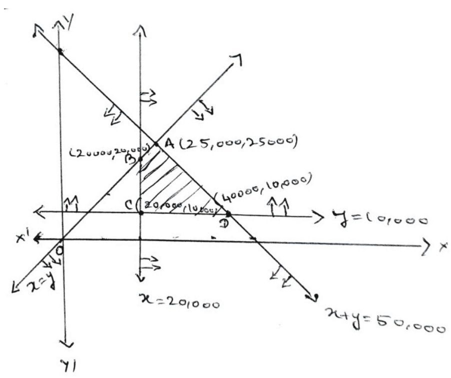
X	3	4	5	6
P(X)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

1

Mean $= \Sigma\ XP(X) = \frac{105}{20} = \frac{21}{4}$ 1

Variance $= \Sigma X^2P(X) - (\Sigma X\ P(X))^2 = \frac{63}{80}$ 1

25. Let the investment in bond A be ₹ x and in bond B be ₹ y



Objective function is: $Z = \frac{x}{10} + \frac{9}{100}y$ 1

Subject to constraints

$x + y \geq 50000$; $x \geq 20{,}000$; $y \geq 10{,}000$; $x \geq y$ (*) 2

Correct Figure 2

Vertices of feasible region are A, B, C, and D

Point	$Z = \frac{x}{10} + \frac{9}{100}y$	Value
A(25,000, 25000)	2500 + 2250	4750
B(20,000, 20,000)	2000 + 1800	3800
C(20,000, 10,000)	2000 + 900	2900
D(40,000, 10,000)	4000 + 900	4900

Return is maximum when ₹ 40000 are invested in Bond A and ₹ 10000 in Bond B

Maximum return is ₹ 4900 1

Since there are more than 3 constraints, student may be given full 6 marks even if reaches upto (*).

26. Required equation of the plane is

$\vec{r}.[(1-2\lambda)\hat{i} + (-2+\lambda)\hat{j} + (3+\lambda)\hat{k}] = 4 - 5\lambda$ 2

Intercept of the plane on x-axis = Intercept of the plane on y-axis

$\Rightarrow \frac{4-5\lambda}{1-2\lambda} = \frac{4-5\lambda}{\lambda-2}$ i.e., $\lambda = 1, \frac{4}{5}$ $\left(\text{rejecting } \lambda = \frac{4}{5}\right)$ 2

Required equation of the plane is $\vec{r}.(-\hat{i} - \hat{j} + 4\hat{k}) + 1 = 0$ 2