QUESTION BANK - COMPLEX NUMBERS

LEVEL–I

1. Locate the complex number $z = x + iy$ for which

(i) $\log_{1/2}|z-2| > \log_{1/2}|z|$ **(ii)** $\log_{\sec\frac{\pi}{4}} \dfrac{|z|^2+|z|+4}{2|z|-1} > 2$

(iii) $\log_{14}\left(13+|z^2-4i|\right)+\log_{196}\dfrac{1}{(13+|z^2+4i|)^2}=0$

2. **(i)** If $|z| = a,\ (z \neq a)$, then find the locus of w, where $w = \dfrac{z-a}{z+a}$.

(ii) Show that the roots of the equation $z^n = (z+1)^n,\ n \in N$ are collinear.

3. **(i)** Prove that $(a_1^2 + b_1^2)(a_2^2 + b_2^2)\ldots\ldots\ldots(a_n^2 + b_n^2)$ can be written as the sum of two squares.

4. **(i)** Prove that :

$$\left(\frac{-1+i\sqrt{3}}{2}\right)^n+\left(\frac{-1-i\sqrt{3}}{2}\right)^n=-1, \text{ where } n \in N \text{ but not a multiple of 3.}$$

(ii) Show that, $\left(\dfrac{pi+1}{pi-1}\right)^m e^{2mi\cot^{-1}(p)} = 1$.

5. Using the concept of complex numbers, prove that

$$\sum_{r=1}^{n}\cos(\theta+r\alpha)=\frac{\sin\frac{n+1}{2}\alpha\cos\left(\theta+n\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

6. Prove that if $|a|<1$, $1+a\sin\theta+a^2\sin 2\theta+a^3\sin 3\theta+\ldots\ldots=\dfrac{a\sin\theta}{1-2a\cos 2\theta+a^2}$

7. If $z_n=\cos\dfrac{n\pi}{2^n}+i\sin\dfrac{n\pi}{2^n}$, then find $\lim\limits_{n\to\infty}(z_1.z_2.\ldots..z_n)$

8. The three points z_1, z_2, z_3 are connected by the relation $az_1 + bz_2 + cz_3 = 0$, where a, b, c are real and $a + b + c = 0$. Prove that the three points are collinear.

9. Solve the $z^7 = 1$, and use this to obtain an equation whose roots are $\cos\dfrac{2\pi}{7}$, $\cos\dfrac{4\pi}{7}$, $\cos\dfrac{6\pi}{7}$.

Hence prove that $\cos\dfrac{2\pi}{7}+\cos\dfrac{4\pi}{7}+\cos\dfrac{6\pi}{7}=-\dfrac{1}{2}$.

10. a, b, c are real numbers and z is a complex number such that $a^2 + b^2 + c^2 = r^2$ and $b + ic = (r + a)z$.

Prove that $\dfrac{a+ib}{r+c}=\dfrac{1+iz}{1-iz}$ and $\dfrac{c+ia}{r+b}=\dfrac{i(1-z)}{1+z}$.

LEVEL–II

1. If $x_1, x_2, x_3, \ldots, x_n$ are the n roots of the equation $x^n + p_1x^{n-1} + p_2x^{n-2} + \ldots + p_{n-1}x + p_n = 0$, ($p_1, p_2, \ldots, p_n$ real), prove that $(1 + x_1^2)(1 + x_2^2)(1 + x_3^2)\ldots(1 + x_n^2) = (1 - p_2 + p_4 - p_6 + \ldots)^2 + (p_1 - p_3 + p_5 - p_7 + \ldots)^2$.

2. Find the roots common the equations $x^5 - x^3 + x^2 - 1 = 0$, $x^4 = 1$.

3. Assume that A_i($i = 1, 2, \ldots, n$) are the vertices of a regular polygon inscribed in a circle of radius unity. Find :

(i) $|A_1A_2|^2 + |A_1A_3|^2 + \ldots + |A_1A_n|^2$ **(ii)** $|A_1A_2||A_1A_3|\ldots|A_1A_n|$

4. If points $A_1, A_2, \ldots A_6$ representing the complex numbers $z_1, z_2, \ldots, z_6$ respectively are the vertices of a regular hexagon and if z_0 be the complex number representing the centroid of the hexagon then prove that $z_1^2 + z_2^2 + z_3^2 = 6z_0^2$.

5. z_1, z_2, z_3 are three non-zero complex numbers such that $z_2 \neq z_1$, and $a = |z_1|$, $b = |z_2|$, $c = |z_3|$. f $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then show that $\arg.\frac{z_3}{z_2} = \arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^2$.

6. Show that the triangle whose vertices are z_1, z_2, z_3 and a, b, c are similar, if $\begin{vmatrix} z_1 & a & 1 \\ z_2 & b & 1 \\ z_3 & c & 1 \end{vmatrix} = 0$.

7. Let A, B, C, D, E be points on the complex plane representing the complex numbers z_1, z_2, z_3, z_4, z_5 respectively. If $(z_3 - z_2)\, z_4 = (z_1 - z_2)\, z_5$, prove that the triangle ABC and DOE are similar.

8. Show that the polynomial $x^{4l} + x^{4m+1} + x^{4n+2} + x^{4p+3}$ is divisible by $x^3 + x^2 + x + 1$, whose ℓ, m, n, p are positive integers.

9. Prove that the polynomial $x^{3n} + x^{3m+1} + x3k + 2$ is exactly divisible by $x^2 + x + 1$ if m, n, k are non negative integers.

10. Two points represented by complex numbers a, b lie on a circle with centre at the origin and radius r. The tangents at 'a' and 'b' intersects at z. Prove that $z = \frac{2ab}{a+b}$.

IIT JEE PROBLEMS (OBJECTIVE)

A. Fill in the blanks

1. If the expression $\dfrac{\left[\sin\left(\dfrac{x}{2}\right)+\cos\left(\dfrac{x}{2}\right)+i\tan(x)\right]}{\left[1+2i\sin\left(\dfrac{x}{2}\right)\right]}$ is real, then the set of all possible values of x is....... **[IIT – 87]**

2. For any two complex numbers z_1, z_2 and any real number a and b. **[IIT – 88]**
$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$ ....

3. If α, β, γ are the numbers between 0 and 1 such that the points $z_1 = \alpha + i$, $z_2 = 1 + \beta i$ and $z_3 = 0$ from an equilateral triangle, then $\alpha =$ ....and $\beta =$ .... **[IIT – 89]**

4. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2AC. If the points D and M represent the complex numbers 1 + i and 2 - i respectively, then A represents the complex number ....or .... **[IIT – 93]**

5. Suppose Z_1, Z_2, Z_3 are the vertices of an equilateral triangle inscribed in the circle ; $|Z| = 2$. If $Z_1 = 1+i\sqrt{3}$ then $Z_2 =$ ...., $Z_3 =$ .... **[IIT – 94]**

6. The value of the expression ;
$1.(2-\omega)(2-\omega^2) + 2.(3-\omega)(3-\omega^2) +$ ....$+ (n-1).(n-\omega)(n-\omega^2)$ where ω is an imaginary cube root of unity is .... **[IIT – 96]**

B. True/False

1. For complex number $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we write $z_1 \cap z_2$, if $x_1 \le x_2$ and $y_1 \le y_2$, then for all complex number z with $1 \cap z$ we have $\dfrac{1-z}{1+z} \cap 0$. **[IIT – 81]**

2. If the complex numbers, Z_1, Z_2 and Z_3 represent the vertices of an equilateral triangle such that $|Z_1| = |Z_2| = |Z_3|$ then $Z_1 + Z_2 + Z_3 = 0$. **[IIT – 84]**

3. If three complex numbers are in A.P. then they lie on a circle in the complex plane. **[IIT – 85]**

4. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle. **[IIT – 88]**

C. Multiple Choice Question with One and More than One Correct Answer :

1. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\text{Re}(z_1\bar{z}_2) = 0$, then the pair of complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfies **[IIT – 85]**

(A) $|\omega_1| = 1$ (B) $|\omega_2| = 1$ (C) $\text{Re}(\omega_1\bar{\omega}_2) = 0$ (D) none of these

2. Let z_1 and z_2 be complex numbers such that $z_1 \ne z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\dfrac{z_1 + z_2}{z_1 - z_2}$ may be **[IIT – 86]**

(A) zero (B) real & positive (C) real & negative (D) purely imaginary

3. If z_1 and z_2 are two nonzero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\text{Arg}\, z_1 - \text{Arg}\, z_2$ is equal to **[IIT – 87]**

(A) $-\pi$ (B) $-\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{2}$

4. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$ is **[IIT – 87]**

(A) -1 (B) 0 (C) $-i$ (D) i

5. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals **[IIT – 98]**

(A) 128ω (B) -128ω (C) $128\omega^2$ (D) $-128\omega^2$

6. The value of the sum $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$, where $i = \sqrt{-1}$, equals **[IIT – 98]**

(A) i (B) $i - 1$ (C) $-i$ (D) 0

D. Multiple Choice Question with One Correct Answer :

1. The complex numbers $z = x + iy$ which satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lie on **[IIT – 81]**

(A) the x–axis (B) the straight line $y = 3$
(C) a circle passing through the origin (D) none of these

2. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then **[IIT – 82]**

(A) $\text{Re}(z) = 0$ (B) $\text{Im}(z) = 0$
(C) $\text{Re}(z) > 0, \text{Im}(z) > 0$ (D) $\text{Re}(z) > 0, \text{Im}(z) < 0$

3. The inequality $|z-4| < |z-2|$ represents the region given by **[IIT – 83]**

(A) $\text{Re}(z) \geq 0$ (B) $\text{Re}(z) < 0$ (C) $\text{Re}(z) > 0$ (D) none of these

4. If $z = x + iy$ and $\omega = \frac{1-iz}{z-i}$, then $|\omega| = 1$ implies that, in the complex plane **[IIT – 83]**

(A) z lies on the imaginary axis (B) z lies on the real axis
(C) z lies on the unit circle (D) none of these

5. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if **[IIT – 83]**

(A) $z_1 + z_4 = z_2 + z_3$ (B) $z_1 + z_3 = z_2 + z_4$ (C) $z_1 + z_2 = z_3 + z_4$ (D) none of these

6. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c = (1-r)a + rb$ and $w = (1-r)u + rv$, where r is a complex number, then the two triangles **[IIT – 85]**

(A) have the same area (B) are similar (C) are congruent (D) none of these

7. The complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other, for **[IIT – 88]**

(A) $x = n\pi$ (B) $x = 0$ (C) $x = (n + 1/2)\pi$ (D) no value of x

Complex Numbers

8. If $\omega(\neq 1)$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$, then A and $B\omega$, then A and B are respectively the numbers **[IIT – 95]**
(A) 0, 1 (B) 1, 1 (C) 1, 0 (D) -1, 1

9. Let z and ω be two non zero complex numbers such that $|z| = |\omega|$ and Arg z + Arg $\omega = \pi$, then z equals : **[IIT – 95]**
(A) ω (B) $-\omega$ (C) $\overline{\omega}$ (D) $-\overline{\omega}$

10. Let Z and W be two complex numbers such that $|Z| \le 1, |W| \le 1$ and $|Z + iW| = |Z - i\overline{W}| = 2$. Then Z equals : **[IIT – 95]**
(A) 1 or i (B) i or -i (C) 1 or -1 (D) i or -1

11. For positive integers n_1, n_2 the value of the expression ; **[IIT – 96]**
$(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$,where $i = \sqrt{-1}$ is a real numbers if and only if
(A) $n_1 = n_2 + 1$ (B) $n_1 = n_2 - 1$ (C) $n_1 = n_2$ (D) $n_1 > 0, n_2 > 0$

12. If $i = \sqrt{-1}$, then $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to : **[IIT – 99]**
(A) $1 - i\sqrt{3}$ (B) $-1 + i\sqrt{3}$ (C) $i\sqrt{3}$ (D) $-i\sqrt{3}$

13. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is : **[IIT - 2000]**
(A) equal to 1 (B) less than 1 (C) greater than 3 (D) equal to 3

14. If arg(z) < 0, then arg (-z) - arg (z) = **[IIT - 2000]**
(A) π (B) $-\pi$ (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

15. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is
(A) of area zero (B) right angled isosceles
(C) equilateral (D) obtuse - angled isosceles **[IIT - 2001]**

16. Let z_1 and z_2 be nth roots of unity which subtend a right angle at the origin. Then n must be of the form
(A) 4k + 1 (B) 4k + 2 (C) 4k + 3 (D) 4k **[IIT - 2001]**

17. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is **[IIT - 2002]**
(A) 3ω (B) $3\omega(\omega - 1)$ (C) $3\omega^2$ (D) $3\omega(1-\omega)$

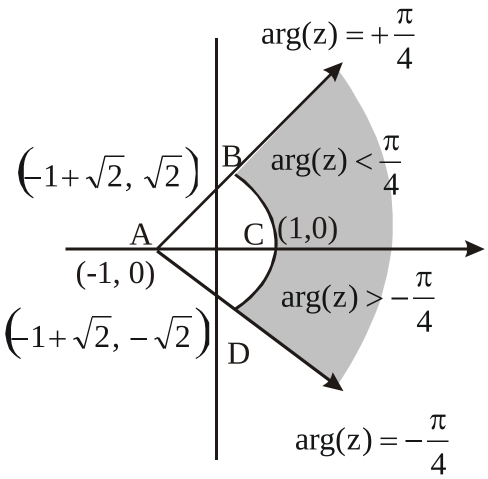
18. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is **[IIT - 2002]**
(A) 0 (B) 2 (C) 7 (D) 17

19. If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then $\text{Re}(\omega)$ is **[IIT - 2003]**
(A) 0 (B) $-\frac{1}{|z+1|^2}$ (C) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ (D) $\frac{\sqrt{2}}{|z+1|^2}$

20. If $\omega(\neq 1)$ be a cube root of unity and $(1+\omega^2)^n = (1+\omega^4)^n$, then the least positive value of n is

(A) 2 (B) 3 (C) 5 (D) 6 **[IIT - 2004]**

21. The locus of z which lies in shaded region (excluding the boundaries) is best represented by



[IIT - 2005]

(A) $z : |z+1| > 2$ and $|\arg(z+1)| < \frac{\pi}{4}$ (B) $z : |z-1| > 2$ and $|\arg(z-1)| < \frac{\pi}{4}$

(C) $z : |z+1| > 2$ and $|\arg(z+1)| < \frac{\pi}{2}$ (D) $z : |z-1| > 2$ and $|\arg(z+1)| < \frac{\pi}{2}$

22. If a, b, c are integers, not all simultaneously equal and ω is cube root of unity $(\omega \neq 1)$, then minimum value of $|a + b\omega + c\omega^2|$ is **[IIT - 2005]**

(A) 0 (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

23. If $\omega = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{\omega - \overline{\omega}z}{1-z}\right)$ is purely real, then the set of values of z is **[IIT - 2006]**

(A) $|z| = 1, z \neq 2$ (B) $|z| = 1$ and $z \neq 1$ (C) $z = \overline{z}$ (D) none of these

24. A man walks a distance of 3 units from the origin towards the north-east (N 45°E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position P in the Argand plane is

(A) $3e^{i\pi/4} + 4i$ (B) $(3-4i)e^{i\pi/4}$ (C) $(4+3i)\,e^{i\pi/4}$ (D) $(3+4i)e^{i\pi/4}$

[IIT - 2007]

25. If $|z| = 1$ and $z \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on

(A) a line not passing through the origin (B) $|z| = \sqrt{2}$

(C) the x-axis (D) the y-axis **[IIT - 2007]**

IIT JEE PROBLEMS (SUBJECTIVE)

1. Let the complex number z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be circumcenter of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$. **[IIT – 81]**

2. Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2 + z_2^2 - z_1z_2 = 0$. **[IIT – 83]**

3. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, iz and $z + iz$ is $\frac{1}{2}|z|^2$. **[IIT – 86]**

4. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C. Show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$. **[IIT – 86]**

5. Let $Z_1 = 10 + 6i$ and $Z_2 = 4 + 6i$. If Z is any complex number such that the argument of $\frac{Z - Z_1}{Z - Z_2}$ is $\frac{\pi}{4}$, then prove that $|Z - 7 - 9i| = 3\sqrt{2}$. **[IIT – 90]**

6. Find the complex numbers Z which simultaneously satisfy the equations : $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}$ and $\left|\frac{z-4}{z-8}\right| = 1$. **[REE-93]**

7. Use De Moiver's theorem to solve equation $2\sqrt{2}x^4 = \left(\sqrt{3}-1\right) + i\left(\sqrt{3}+1\right)$. **[REE-94]**

8. Find all complex numbers z for which $\arg\left[\frac{3z-6-3i}{2z-8-6i}\right] = \frac{\pi}{4}$ and $|z - 3 + i| = 3$. **[REE-95]**

9. If $iz^3 + z^2 - z + i = 0$, the show that $|z| = 1$. **[IIT – 95]**

10. If $|Z| \le 1, |W| \le 1$, show that $|Z - W|^2 \le (|Z| - |W|)^2 + (\operatorname{Arg} Z - \operatorname{Arg} W)^2$. **[IIT – 95]**

11. Find all nonzero complex numbers Z satisfying $\bar{z} = iZ^2$. **[IIT – 96]**

12. Find all complex numbers satisfying the equation $2|z|^2 + z^2 - 5 + i\sqrt{3} = 0$. **[REE-96]**

13. Evaluate : $\sum_{p=1}^{32}(3p+2)\left(\sum_{q=1}^{10}\left(\sin\frac{2q\pi}{11} - i\cos\frac{2q\pi}{11}\right)\right)^p$. **[REE-97]**

14. Let z_1 and z_2 be roots of the equation $z^2 + pz + q = 0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If $\angle AOB = \alpha \ne 0$ and OA = OB, where O is the origin. Prove that $p^2 = 4q\cos^2\left(\frac{\alpha}{2}\right)$. **[IIT – 97]**

15. Prove that $\sum_{k=1}^{n-1}(n-k)\cos\frac{2k\pi}{n} = -\frac{n}{2}$ where $n \ge 3$ is an integer. **[IIT – 97]**

16. Let $\bar{b}z + b\bar{z} = c,\ b \neq 0$, be a line in the complex plane, where $\bar{b}$ is the complex conjugate of b. If a point z_1 is the reflection of a point z_2 through the line, then show that $c = \bar{z}_1 b + z_2 \bar{b}$. **[IIT – 97]**

17. Find all the roots of the equation $(3z - 1)^4 + (z - 2)^4 = 0$ in the simplified form of a + ib. **[REE-98]**

18. For complex numbers z and ω, prove that, $|z|^2\,\omega - |\omega|^2\,z = z - \omega$ if and only if, $z = \omega$ or $z\bar{\omega} = 1$. **[IIT – 99]**

19. If $\alpha = e^{\frac{2\pi i}{7}}$ and f(x) = $A_0 + \sum_{k=1}^{20} A_k x^k$, then find the value of , $f(x) + f(\alpha x) + \ldots\ldots\ldots + f(\alpha^6 x)$ independent of α. **[REE-99]**

20. Given, z = $\cos\frac{2\pi}{2n+1} + i\sin\frac{2\pi}{2n+1}$, 'n' a positive integer, find the equation whose roots are, $\alpha = z + z^3 + \ldots\ldots + z^{2n-1}$ and $\beta = z^2 + z^4 + \ldots\ldots\ldots z^{2n}$. **[REE -2000]**

21. Find all those of the equation $z^{12} - 56z^6 - 512 = 0$ whose imaginary part is positive. **[REE -2000]**

22. Let a complex umber $\alpha, \alpha \neq 1$ be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$ where p, q are distinct primes. Show that either $1 + \alpha + \alpha^2 + \ldots\ldots\ldots\ldots + \alpha^{p-1} = 0$ or $1 + \alpha + \alpha^2 + \ldots\ldots\ldots\ldots + \alpha^{q-1} = 0$, but not both together. **[IIT – 2002]**

23. Let z_1 & z_2 are two complex numbers such that $|z_1| < 1 < |z_2|$. Prove that $\left|\frac{1 - z_1\bar{z}_2}{z_1 - z_2}\right| < 1$. **[IIT – 2003]**

24. Let a_i, i = 1, 2, 3, ... be the complex numbers such that $|a_r| < 2$. Prove that there is no complex number z such that $|z| < 1/3$ and $\sum_{r=1}^{n} a_r z^r = 1$. **[IIT – 2003]**

25. If $z = x + iy$, $x_1 = \alpha_1 + i\alpha_2$, $x_2 = \beta_1 + i\beta_2$ satisfying $\left|\frac{z - x_1}{z - x_2}\right| = k, (k \neq 1)$, then show that the locus of z is a circle. Find the radius and centre of the circle. **[IIT – 2004]**

26. If one the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}\,i$. Find the other vertices of the square. **[IIT – 2005]**

PROBLEMS ASKED IN AIEEE

1. The inequality $|z-4| < |z-2|$ represents the following region **[AIEEE - 2002]**

(A) $Re(z) > 0$ (B) $Re(z) < 0$ (C) $Re(z) > 2$ (D) none of these

2. Let z and ω are two non-zero complex numbers such that $|z| = |\omega|$ and $\arg z + \arg \omega = \pi$, then z equal to **[AIEEE - 2002]**

(A) ω (B) $-\omega$ (C) $\overline{\omega}$ (D) $-\overline{\omega}$

3. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex, Further, assume that the origin, z_1 and z_2 form an equilateral triangle. Then **[AIEEE- 2003]**

(A) $a^2 = b$ (B) $a^2 = 2b$ (C) $a^2 = 3b$ (D) $a^2 = 4b$

4. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then **[AIEEE- 2003]**

(A) x = 4n, where n is any positive integer (B) x = 2n, where n is any positive integer

(C) x = 4n + 1, where n is any positive integer (D) x = 2n + 1, where n is any positive integer

5. If $|z^2 - 1| = |z|^2 + 1$, then z lies on **[AIEEE- 2004]**

(A) the real axis (B) the imaginary axis (C) a circle (D) a ellipse

6. If z_1 and z_2 are two non zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $\arg z_1 - \arg z_2$ is equal to- **[AIEEE - 2005]**

(A) $-\pi$ (B) $\frac{\pi}{2}$ (C) $-\frac{\pi}{2}$ (D) 0

7. If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$ then z lies on **[AIEEE - 2005]**

(A) a circle (B) an ellipse (C) a parabola (D) a straight line

SET–I

1. If $|z - i| \leq 2$ and $z_0 = 5 + 3i$, the maximum value of $|iz + z_0|$ is

(A) $2+\sqrt{31}$ (B) $\sqrt{31}-2$ (C) 7 (D) - 7

2. If $z_1 = a + ib$, $z_2 = p + iq$ be two unimodular complex numbers such that $\text{Im}(z_1\bar{z}_2) = 1$ and $\omega_1 = a + ip$, $\omega_2 = b + iq$,

(A) $\text{Re}(\omega_1\omega_2) = 1$ (B) $\text{Im}(\omega_1\omega_2) = 1$ (C) $\text{Re}(\omega_1\bar{\omega}_2) = 0$ (D) $\text{Im}(\omega_1\bar{\omega}_2) = 1$

3. If n be an odd positive integer and $1, \alpha_1, \alpha_2, \ldots\ldots\ldots\alpha_{n-1}$ are the nth roots of unity, $(2+\alpha_1)(2+\alpha_2)\ldots\ldots\ldots(2+\alpha_{n-1})$ equals

(A) $2^n - 1$ (B) $2^n + 1$ (C) $\dfrac{2^n+1}{3}$ (D) none of these

4. Given α, β, γ are cube roots of real number $p < 0$, for any x, y, z the value of $\dfrac{\alpha x + \beta y + \gamma z}{\beta x + \gamma y + \alpha z}$ equals

(A) ω (B) ω^2 (C) p (D) $-\omega^2$

5. Let z be a complex number such that $z = (1-t) + i\sqrt{t^2 + t + 2}$ where t is a real parameter then locus of z on Argand Plane is

(A) parabola (B) ellipse (C) hyperbola (D) straight line

6. If $x^2 - x + 1 = 0$, $\sum_{n=1}^{5}\left(x^n + \dfrac{1}{x^n}\right)^2$ equals

(A) 8 (B) 10 (C) 12 (D) 14

7. If the quadratic equation $z^2 + (a + ib)z + c + id = 0$, where a, b, c, d are non-zero real numbers, has a root, then

(A) $abd = b^2c + d^2$ (B) $abc = bc^2 + d^2$ (C) $abd = bc^2 + ad^2$ (D) none of these

8. If $z \neq 0$, $\int_0^{100} \arg(-|z|)dx$ equals

(A) 0 (B) not defined (C) 100 (D) $100\,\pi$

9. Let z_1 and z_2 be two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$ then which of the following is correct ?

(A) $z_1 = z_2$ (B) $z_1 = - z_2$ (C) $z_1 = \bar{z}_2$ (D) $z_1 = -\bar{z}_2$

10. If α is a non-real complex number and $x^2 + \alpha x + \bar{\alpha} = 0$ has a real root γ, then

(A) $\gamma = \alpha + \bar{\alpha}$ (B) $\gamma = 2[\alpha + \bar{\alpha}]$ (C) $\gamma = 1$ (D) none of these

11. If $|z| = 2$ and $\dfrac{z_1 - z_2}{z_1 - z_3} = \dfrac{z-2}{z+2}$ then z_1, z_2, z_3 will be the vertices of

(A) equilateral triangle
(B) right angled triangle
(C) acute angled triangle
(D) none of these

12. If z_1, z_2, z_3, z_4 are four distinct complex numbers representing the vertices of a quadrilateral taken in order such that $z_1 - z_4 =$ $z_2 - z_3$ and $\arg\left(\dfrac{z_4 - z_1}{z_2 - z_1}\right) = \dfrac{\pi}{2}$ then the quadrilateral is a

(A) rectangle
(B) rhombus
(C) square
(D) trapezium

13. If $\dfrac{5z_2}{11z_1}$ is purely imaginary, then $\left|\dfrac{2z_1 + 3z_2}{2z_1 - 3z_2}\right|$ is

(A) $\dfrac{37}{33}$
(B) $\dfrac{11}{5}$
(C) 1
(D) $\dfrac{5}{11}$

14. If $(\sin\theta_1 + i\cos\theta_1)(\cos\theta_2 + i\sin\theta_2).........(\sin\theta_n + i\cos\theta_n) = a + ib,\ a^2 + b^2 =$

(A) 1
(B) -1
(C) $(-1)^n$
(D) $\sqrt{2}$

15. If $(\cos\theta + i\sin\theta)\ (\cos 3\theta + i\sin 3\theta).........\{\cos(2n-1)\theta\} + i\sin(2n-1)\theta\} = 1$, then $\theta =$

(A) $\dfrac{r\pi}{n^2}$
(B) $\dfrac{(n-1)\pi}{r^2}$
(C) $\dfrac{2r\pi}{n^2}$
(D) $\dfrac{(2n+1)\pi}{r}$

16. If $\left(\sqrt{3} + i\right)^n = 2^n$, where n is an integer, then

(A) n is a multiple of 5
(B) n is a multiple of 6
(C) n is a multiple of 10
(D) none of these

17. The number of values of z which satisfies both the equations $|z - 1 - i| = \sqrt{2}$ & $|z + 1 + i| = 2$ is

(A) 1
(B) 2
(C) 0
(D) infinitely many

18. If P is a multiple of n, then the sum of the p^{th} power of n^{th} roots of unity is

(A) p
(B) 1
(C) 0
(D) n

19. The complex numbers z_1, z_2, z_3 are collinear if

(A) $\arg\left(\dfrac{z_1 - z_2}{z_2 - z_3}\right) = \dfrac{\pi}{2}$
(B) $\left(\dfrac{z_1 - z_3}{z_2 - z_3}\right)$ is purely imaginary
(C) $\dfrac{z_1 - z_3}{z_2 - z_3}$ is real
(D) $|z_1 - z_3| = |z_2 - z_3|$

20. The number of integral solutions of the equation $(1 - i)^x = 2^x$ are

(A) 0
(B) 1
(C) 2
(D) 3

SET–II

1. The locus of the point z for which $2\arg\left(\dfrac{z-i+3}{z+3i-1}\right)=\pi$ is

(A) a straight line passing through the point 3 - i and -1 + 3i
(B) a straight line passing through the point -3 + i and 1 - 3i
(C) a circle passing through the points -3 + i and 1 - 3i
(D) a circle with its centre at the points -1 - i and radius $2\sqrt{2}$

2. The trigonometric from the complex number $z=1+i\tan\alpha$ where $\dfrac{\pi}{2}<\alpha<\pi$ is

(A) $\dfrac{1}{\cos\alpha}(\cos\alpha+i\sin\alpha)$
(B) $\dfrac{1}{\cos\alpha}(\cos\alpha-i\sin\alpha)$
(C) $\dfrac{1}{\cos\alpha}(-\cos\alpha-i\sin\alpha)$
(D) $-\dfrac{1}{\cos\alpha}[\cos(\pi+\alpha)+i\sin(\pi+\alpha)]$

3. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in that order if

(A) $z_1+z_4=z_2+z_3$ (B) $z_1+z_3=z_2+z_4$ (C) $z_1+z_2=z_3+z_4$ (D) none of these

4. If $z=x+iy$, and $\omega=\dfrac{1-iz}{z-i}$, then $|\omega|=1$ implies that in the complex plane

(A) z lies on the real axis
(B) z lies on the imaginary axis
(C) z lies on the unit circle
(D) none of these

5. The curve represented by $\text{Im}(z^2)=\lambda(\lambda\neq 0)$ is

(A) Rectangular hyperbola
(B) circle
(C) parabola
(D) none of these

6. $Z_1\neq Z_2$ are two points in an Argand plane. If $a|Z_1|=b|Z_2|$, then the point $\dfrac{aZ_1-bZ_2}{aZ_1+bZ_2}$ is

(A) in the I quadrant (B) in the III quadrant (C) on the real axis (D) on the imaginary axis

7. If $z=x+iy$ then the equation $\left|\dfrac{2z-i}{z+1}\right|=m$ does not represents a circle when

(A) $m=\dfrac{1}{2}$ (B) $m=1$ (C) $m=2$ (D) $m=3$

8. If z be complex number such that equation $|z-a^2|+|z-2a|=3$ always represents an ellipse then range of $a(\in R^+)$ is

(A) $(1, \sqrt{2})$ (B) $[1, \sqrt{3}]$ (C) (-1, 3) (D) (0, 3)

Complex Numbers

9. The roots of the cubic equation $(z+\alpha\beta)^3 = \alpha^3(\alpha \neq 0)$, represent the vertices of a triangle, which

(A) is scalene
(B) is equilateral
(C) is isosceles but not equilateral
(D) depends on β

10. Let $P(e^{i\theta_1})$, $Q(e^{i\theta_2})$ and $R(e^{i\theta_3})$ be the vertices of a triangle PQR in the Argand Plane. The orthocenter of the triangle PQR is

(A) $e^{i(\theta_1+\theta_2+\theta_2)}$
(B) $\frac{2}{3}e^{i(\theta_1+\theta_2+\theta_2)}$
(C) $e^{i\theta_1}+e^{i\theta_2}+e^{i\theta_3}$
(D) none of these

11. Let z_1, z_2 and z_3 be three points on $|z| = 1$. If θ_1, θ_2 and θ_3 be the arguments of z_1, z_2, z_3 respectively then $\cos(\theta_1-\theta_2)+\cos(\theta_2-\theta_3)+\cos(\theta_3-\theta_1)$

(A) $\geq -\frac{3}{2}$
(B) $\leq -\frac{3}{2}$
(C) $\geq \frac{3}{2}$
(D) none of these

12. If $A(z_1)$, $B(z_2)$, $C(z_3)$ are the vertices of an equilateral triangle ABC, value of

$$\arg\left(\frac{z_2+z_3-2z_1}{z_3-z_2}\right) \text{ is equal to :}$$

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{6}$

13. If $A(z_1)$, $B(z_2)$, $C(z_3)$ are three points in the Argand Plane where $|z_1 + z_2| = ||z_1| - |z_2||$ and $|(1 - i)z_1 + i z_3| = |z_1| + |z_3 - z_1|$, then

(A) A, B and C lie on a fixed circle with center $\frac{z_1+z_2}{2}$
(B) A, B and C are collinear points
(C) ABC form an equilateral triangle
(D) ABC from an obtuse angle triangle

14. Number of common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$, z being a complex number, is

(A) 0
(B) 1
(C) 2
(D) 3

15. If $x^6 = (4 - 3i)^5$, then the product of all of its roots is (where $\theta = -\tan^{-1}(3/4)$)

(A) $5^5(\cos 5\theta + i\sin 5\theta)$
(B) $-5^5(\cos 5\theta + i\sin 5\theta)$
(C) $5^5(\cos 5\theta - i\sin 5\theta)$
(D) $-5^5(\cos 5\theta - i\sin 5\theta)$

16. The point in the complex plane at which the curves $\arg(z - 3i) = \frac{3\pi}{4}$ and $\arg(2z + 1 - 2i) = \frac{\pi}{4}$ intersect, is

(A) $\left(\frac{3}{4}, \frac{9}{4}\right)$
(B) $\left(\frac{3}{4}, \frac{7}{4}\right)$
(C) $\left(\frac{2}{4}, \frac{7}{4}\right)$
(D) $\left(\frac{6}{7}, \frac{4}{7}\right)$

17. If ω be an imaginary nth root of unity, then $\sum_{r=1}^{n}(ar+b)\omega^{r-1}$ is equal to

(A) $\frac{n(n+1)a}{2\omega}$ (B) $\frac{nb}{1-n}$ (C) $\frac{na}{\omega-1}$ (D) none of these

18. If $|z-i|\le 2$ and $z_0 = 5+3i$, then the maximum value of $|iz+z_0|$ is

(A) $2+\sqrt{41}$ (B) $\sqrt{41}-2$ (C) 7 (D) none of these

19. Value of $(\sin(\log i^i))^3 + (\cos(\log i^i))3$ is

(A) 1 (B) -1 (C) 2 (D) 2i

20. On the Argand plane the complex number $\frac{(1+2i)}{1-i}$ lies in the

(A) first quadrant (B) 2nd quadrant
(C) 3rd quadrant (D) 4th quadrant

SET–III

More than one

1. If the vertices of an equilateral triangle are situated at $z = 0, z = z_1, z = z_2$, then which of the following is/are true

(A) $|z_1| = |z_2|$ (B) $|z_1 - z_2| = |z_1|$

(C) $|z_1 + z_2| = |z_1| + |z_2|$ (D) $|\arg z_1 - \arg z_2| = \dfrac{\pi}{3}$

2. Value(s) of $(-i)^{1/3}$ is/are

(A) $\dfrac{\sqrt{3}-i}{2}$ (B) $\dfrac{\sqrt{3}+i}{2}$ (C) $\dfrac{-\sqrt{3}-i}{2}$ (D) $\dfrac{-\sqrt{3}+i}{2}$

3. If a and b are real numbers between 0 and 1 and points representing the complex numbers $z_1 = a + i, z_2 = 1 + ib$ along with origin from an equilateral triangle, then

(A) $a = 2-\sqrt{3}$ (B) $b = 2-\sqrt{3}$ (C) $a = \sqrt{3}-1$ (D) $b = \dfrac{\sqrt{3}}{2}$

4. The centre of square ABCD is at $z = 0$. If affix of vertex A is z_1, centroid of triangle ABC is/are

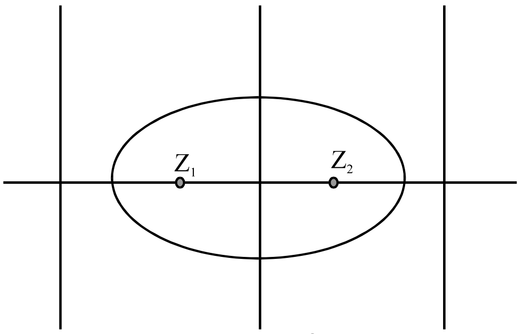
(A) $z_1(\cos\pi + i\sin\pi)$ (B) $z_1[(\cos\pi/2) - i\sin(\pi/2)]$

(C) $\dfrac{z_1}{3}[(\cos\pi/2) + i\sin(\pi/2)]$ (D) $\dfrac{z_1}{3}[(\cos\pi/2) - i\sin(\pi/2)]$

5. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

(A) $x = 0$ (B) $x = 1$ (C) $y = 3$ (D) $y = 0$

W I If Z_1, Z_2 are two complex numbers representing the points S_1 and S_2 and Z is any general complex number then $|Z - Z_1| + |Z - Z_2| = 2a$, $(|Z_2 - Z_1| < 2a)$, represents an ellipse with S_1 and S_2 as foci and 2a being major axis on the complex plane.



6. The complex number representing the centre of the ellipse is

(A) $\dfrac{Z_1 - Z_2}{2}$ (B) $\dfrac{Z_1 + Z_2}{2}$ (C) $Z_1 - Z_2$ (D) none of these

7. Complex equation of ditrectrices if Z_1 and Z_2 lie on real axis and $|Z_1| = |Z_2|$, are

(A) $Z - \bar{Z} = \pm\dfrac{4a^2}{|Z_1 - Z_2|}$ (B) $Z + \bar{Z} = \pm\dfrac{4a^2}{|Z_1 + Z_2|}$

(C) $Z + \bar{Z} = \pm\dfrac{4a^2}{|Z_1 - Z_2|}$ (D) none of these

8. For the ellipse in **Q.7**, complex equation of tangent at the extremity of minor axis are

(A) $\dfrac{Z - \bar{Z}}{i} = \pm\sqrt{4a^2 - (|Z_1 - Z_2|)^2}$ (B) $\dfrac{Z + \bar{Z}}{i} = \pm\sqrt{4a^2 - (|Z_1 - Z_2|)^2}$

(C) $\dfrac{Z + \bar{Z}}{i} = \pm\sqrt{4a^2 + (|Z_1 - Z_2|)^2}$ (D) none of these

9. For any ellipse, complex equation of major axis is

(A) $\dfrac{Z - Z_1}{Z_1 - Z_2} = \dfrac{\bar{Z} - \bar{Z}_1}{\bar{Z}_1 - \bar{Z}_2}$ (B) $\dfrac{Z - Z_1}{Z_1 - Z_2} = \dfrac{\bar{Z} - \bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2}$

(C) $\dfrac{Z + Z_1}{Z_1 - Z_2} = \dfrac{\bar{Z} - \bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2}$ (D) none of these

W II The equation of a straight line in the complex plane is given by

$z\bar{a} + \bar{z}a + b = 0$ ....(i)

where a is a constant complex number and b is a constant real number.

To analyse the equation (i) completely. We can put z = x + iy. Let a = a = $\alpha + i\beta$, where , α, β are constant real numbers then the equation (i) becomes

$(x + iy)\,((\alpha - i\beta) + (x - iy)\,(\alpha + i\beta) + b = 0 \Rightarrow 2\alpha x + 2\beta y + b = 0$ or $\alpha x + \beta y + \dfrac{b}{2} = 0$ ....(ii)

The equation (ii) is the Cartesian form of the line given by the equation (i). With the known properties of an equation of straight line in the Cartesian plane we can derive different charcterstics of the line given by the equation (i).

For example : The slope of the line given by equation (ii) is

$$-\frac{\alpha}{\beta} = -\frac{\mathrm{Re}(a)}{\mathrm{Im}(a)} = \frac{\mathrm{Re}(a)}{\mathrm{Im}(\bar{a})} = -\frac{\bar{a} + a}{\bar{a} - a}$$

10. The intercept of the straight line given by equation (i) on the imaginary axis is

(A) $-\dfrac{b}{a + \bar{a}}$ (B) $\dfrac{ib}{\bar{a} - a}$ (C) $-\dfrac{\bar{a} + a}{b}$ (D) $\dfrac{\bar{a} - a}{2ib}$

11. The straight line given by the equation (i) represents a line parallel to real axis it

(A) Re(a) = 0 (B) Im(a) = 0

(C) Re(a) = Im(a) (D) b = 0

12. Locus of points with constant real part is of the form

(A) $(z - \bar{z})i\alpha + b = 0$ (B) $(z - \bar{z})\alpha + b = 0$

(C) $(z + \bar{z})i\alpha + b = 0$ (D) $(z + \bar{z})\alpha + b = 0$

13. Two straight lines represented by complex equations $z\bar{a}_1 + \bar{z}a_1 + b_1 = 0$ and $z\bar{a}_2 + \bar{z}a_2 + b_2 = 0$ are parallel if

(A) $\frac{a_1}{a_2}$ is purely real
(B) $\frac{a_1}{a_2}$ is purely imaginary
(C) a_1a_2 is purely real
(D) a_1a_2 is purely imaginary

W III Let $z = \alpha + i\beta = R \text{ cis } \theta = r \text{ cis}(\theta + 2n\pi) = re^{i(\theta+2n\pi)}$

$\log z = \log(re^{i(\theta+2n\pi)}) = \log r + i(\theta + 2n\pi) = \log|z| + i \text{ Arg } z + 2n\pi i$

if we put $n = 0$, we get principal value of $\log z$.

i.e. Principal value of $\log z = \log|z| + i \text{ Arg } z$.

14. $\log(1 + i)$ can be expressed as

(A) $\frac{1}{2}\log 2 - i(2n\pi + \frac{\pi}{4})$
(B) $\frac{1}{4}\log 2 + i(n\pi + \frac{\pi}{2})$
(C) $\frac{1}{2}\log 2 + i(2n\pi + \frac{\pi}{4})$
(D) none of these

15. $\log(1 + i)^{1-i}$ can be expressed as

(A) $\left(\frac{1}{2}\log 2 + \frac{\pi}{4}\right) + i(\frac{\pi}{4} - \frac{1}{2}\log 2)$
(B) $\left(\frac{1}{4}\log 2 - \frac{\pi}{2}\right) + i(\frac{\pi}{2} - \frac{1}{4}\log 2)$
(B) $\left(\frac{1}{2}\log 2 - \frac{\pi}{2}\right) + i(\frac{\pi}{2} - \frac{1}{2}\log 2)$
(D) none of these

16. If $\sin(\log i^i) = a + ib$, then value of a, b is
(A) $a = 1$ $b = 0$
(B) $a = -1$ $b = 0$
(C) $a = 0$ $b = 1$
(D) none of these

17. The value of i^i is

(A) $e^{\frac{\pi}{2}}$
(B) $e^{\frac{\pi}{4}}$
(C) $e^{-\frac{\pi}{2}}$
(D) $e^{-\frac{\pi}{4}}$

Fill in the Blanks

18. **(i)** The region represented by the inequality $|2z - 3i| < |3z - 2i|$ is ....

(ii) If ω is the imaginary cube root of unity, then the value of $\tan\{(\omega^{100} + \omega^{101})\pi + \pi/4\}$ is ....

(iii) $p + iq > r + it$ exists only when p = ... and t = ...

(iv) The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other, for x = ...

(v) The area of the triangle on the Argand plane formed by the complex numbers z, iz and z + iz is ....

True or False

19. **(i)** If $|z_1| = |z_2| = ... = |z| = 1$, then $|z_1 + z_2 + z_3 + ... + z_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + ... + \frac{1}{z_n}\right|$.

(ii) If $z^{14} + \frac{1}{z^{14}} = -1$, where z^2 is a root of the equation $z + \frac{1}{z} = 1$.

(iii) If $|a| < 1$, then $1 + a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + ... = \frac{1 - a\cos\theta}{1 - 2a\cos 2\theta + a^2}$

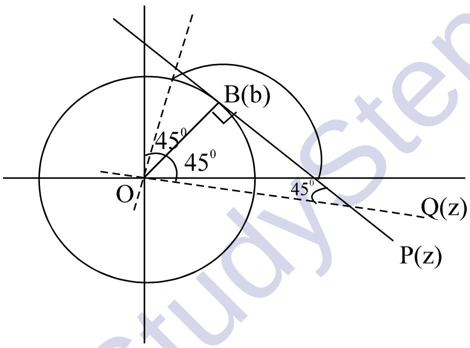
(iv) All the roots of the equation $z^n\cos\theta_0 + z^{n-1}\cos\theta_1 + ... + \cos\theta_n = 2$, where $\theta_0, \theta_1, \theta_2, ...\theta_{n \in R}$ lie out side the circle $|z| = 1/2$.

(v) All the roots of the equation $(3z - 1)^4 + (z - 2)^4 = 0$ in the simplified form are

$$z = \frac{5\sqrt{2}-7}{10\sqrt{2}-6} - \frac{5i}{10\sqrt{2}-6}, \frac{5\sqrt{2}+7}{10\sqrt{2}+6} - \frac{5i}{10\sqrt{2}+6}, \frac{5\sqrt{2}+7}{10\sqrt{2}+6} + \frac{5i}{10\sqrt{2}+6}, \frac{5\sqrt{2}-7}{10\sqrt{2}-6} + \frac{5i}{10\sqrt{2}-6}$$

Match the following

20. Let z be a complex number lying on a circle $|z| = \sqrt{2}\,a$ and $b = b_1 + ib_2$ (any complex number).



(a)	The equation of tangent at point 'b' is	(P) $z = \pm\frac{ib^2}{2a^2}\bar{z}$
(b)	The length of perpendicular from z_0 (any point on the circle) on the tangent at 'b' is	(Q) $z\bar{b} + \bar{z}b = 0$
(c)	The equation of straight line parallel to the tangent and passing through centre circle is	(R) $\frac{\lvert z_0\bar{b} + \bar{z}_0 b - 4a^2\rvert}{2\sqrt{2}\,a}$
(d)	The equation of lines passing through the centre of the circle and making an angle $\frac{\pi}{4}$ with the normal at 'b' are	(S) $z\bar{b} + \bar{z}b = 4a^2$

LEVEL–I ANSWER-KEY

1. **(i)** all points towards the right of x = 1 except the point (2, 0)
(ii) region outside the circle of radius 1/2 with centre (0, 0)
(iii) all real and purely imaginary number

2. **(i)** imaginary axis **7.** 1 **9.** $8x^3 + 4x^2 - 4x - 1 = 0$

LEVEL–II

2. 1, –1 **3.** **(i)** 2n **(ii)** n

IIT JEE PROBLEMS (OBJECTIVE)

A.

1. $2n\pi,\ n\pi + \frac{\pi}{4}$ **2.** $(a^2 + b^2)\,(|z_1^2| + |z_2^2|)$ **3.** $2-\sqrt{3},\ 2-\sqrt{3}$

4. $\left(3-\frac{1}{2}i\right)$ or $\left(1-\frac{3}{2}i\right)$ **5.** $-2 + 0\,i$ and $1 - i\sqrt{3}$ **6.** $\frac{1}{4}n(n-1)(n^2+3n+4)$

B.

1. T **2.** T **3.** F **4.** T

C.

1. A, B, C **2.** D **3.** C **4.** D **5.** D **6.** B

D.

1. A	**2.** B	**3.** D	**4.** B	**5.** B	**6.** B	**7.** D
8. B	**9.** D	**10.** C	**11.** D	**12.** C	**13.** A	**14.** B
15. D	**16.** D	**17.** B	**18.** B	**19.** A	**20.** B	**21.** A
22. B	**23.** A	**24.** D	**25.** D			

IIT JEE PROBLEMS (SUBJECTIVE)

6. 6 + 8i or 6 + 17 i **7.** $\cos\frac{r\pi}{48} + i\sin\frac{r\pi}{48}$ where r = 5, 29, 53 and 77

8. $4\left(1+\frac{1}{\sqrt{5}}\right)+i\left(1-\frac{2}{\sqrt{5}}\right)$ and $\left[4\left(1-\frac{1}{\sqrt{5}}\right)+i\left(1+\frac{2}{\sqrt{5}}\right)\text{ is to be rejected}\right]$

11. $\frac{\sqrt{3}}{2}-\frac{i}{2},\ -\frac{\sqrt{3}}{2}-\frac{i}{2},\ i$ **12.** $\pm\left(\frac{\sqrt{6}}{2}-\frac{1}{\sqrt{2}}i\right);\ \pm\left(\frac{1}{\sqrt{6}}-\frac{3}{\sqrt{2}}i\right)$

13. 48(1 - i)

17. $Z = \frac{\left(29+20\sqrt{2}\right)+i\left(\pm 15+25\sqrt{2}\right)}{82}, \frac{\left(29-20\sqrt{2}\right)+i\left(\pm 15-25\sqrt{2}\right)}{82}$

19. $7A_0 + 7A_7 x^7 + 7A_{14} x^{14}$

20. $z^2 + z + \frac{\sin^2 n\theta}{\sin^2\theta} = 0$ where $\theta = \frac{2\pi}{2n+1}$

21. $\pm 1 + i\sqrt{3}, \frac{\left(\pm\sqrt{3}+i\right)}{\sqrt{2}}, \sqrt{2}i$

25. centre $= \frac{\alpha - k^2\beta}{1-k^2}$, radius $= \frac{k}{|1-k^2|}|\alpha-\beta|$

26. $\left(1-\sqrt{3}\right)+i, -i\sqrt{3}, \left(\sqrt{3}+1\right)-i$

PROBLEMS ASKED IN AIEEE

1.	D	**2.**	D	**3.**	C	**4.**	A	**5.**	B
6.	D	**7.**	D						

SET–I

1.	C	**2.**	D	**3.**	C	**4.**	B	**5.**	C
6.	A	**7.**	A	**8.**	D	**9.**	D	**10.**	C
11.	B	**12.**	A	**13.**	C	**14.**	A	**15.**	C
16.	B	**17.**	B	**18.**	D	**19.**	C	**20.**	B

SET–II

1.	D	**2.**	D	**3.**	B	**4.**	A	**5.**	A
6.	D	**7.**	C	**8.**	D	**9.**	B	**10.**	C
11.	A	**12.**	B	**13.**	A	**14.**	C	**15.**	B
16.	A	**17.**	C	**18.**	C	**19.**	B	**20.**	B

SET–III

1.	ABD	**2.**	AC	**3.**	AB	**4.**	CD	**5.**	AD
6.	B	**7.**	C	**8.**	A	**9.**	A	**10.**	B
11.	A	**12.**	D	**13.**	B	**14.**	C	**15.**	A
16.	B	**17.**	C						

18. **(i)** the exterior of the unit circle with its centre at $z = 0$ **(ii)** 1

(iii) 0, 0 **(iv)** no value of x. **(v)** $\frac{1}{2}|z|^2$

19. **(i)** T **(ii)** F **(iii)** T **(iv)** T **(v)** T

20. a-S; b-R; c-Q; d-P