MATRICES&DETERMINANTS

1 If A and B are square matrices of order 3 each such that $|A| = -1$, $|B| = 3$, then $|3AB|$ equals.

2 If A = $\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$, B = $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$ find AB . use it to

Solve the system of Equations : $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

3 Solve the following system of equations, using matrices :

$3x - 2y + 3z = 8$, $2x + y - z = 1$, $4x - 3y + 2z = 4$.

4. If $A^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ and B= $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$, find $(AB)^{-1}$

5 If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x, y such that $A^2 + xI = yA$ Hence find A^{-1}.

6

Find A^{-1} if A= $\begin{pmatrix} -1 & 2 & -1 \\ 2 & -3 & 1 \\ 5 & 1 & 1 \end{pmatrix}$

7 Using elementary transformations, find the inverse of the following matrix :

$$\begin{pmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 3 \end{pmatrix}$$

8 Two cricket teams honoured their players for three values,excellent batting,to the point bowling,and unparalled Fielding by giving rs x,y,z per player respectively.The first team paid resp, 2,2,1 players for above values with Total prize money of rs 11 lakhs,while 2nd team paid resp 1,2,2 players for these values wita a total prize Money of rs 9 lakhs.if the total award money of one person eachfor these values amount to Rs 6 lakhs,then express the above situation as a matrix equation and find the award money per person for each value. For which of above values wolud you like to pay more ?

Using propertis of determinants prove the following

1. P. That $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)\times(a+b+c).(a^2+b^2+c^2)$

2. Prove that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$ where p is say any scalar

3 Show that $x=2$ is a root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ and solve it completely

4. If *a, b, c* are positive, unequal, show that value of det. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is always negative

5 . P.T. $\Delta = \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$ or Prove that. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab+bc+ca+abc$

6 Prove that

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

7 . $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

8 If *a, b, c* are all positive and are p^{th}, q^{th} and r^{th} terms of a G.P. then show that

$$\Delta = \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

9. $$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$$

10 $$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

11 Show that : $$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3.$$

12

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

13 Prove that

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc.(a^2+b^2c^2)^3$$

14 Using Properties of Determinants Prove tht $$\begin{vmatrix} .-bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ba & b^2+ba & -ab \end{vmatrix} = (ab+bc+ca)^3$$

1 5 Prove that

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

16 Solve the equation

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

17 If x , y, z are real numbers such that $x + y + z = \pi$ then find the value of ,

$$\begin{vmatrix} \sin(x+y+z) & \sin(x+z) & \cos z \\ -\sin y & 0 & \tan x \\ \cos(x+y) & \tan(y+z) & 0 \end{vmatrix}$$

INVERSE TRIGONOMETRIC FUNCTIONS

1 The value of Sin $\left(\sin^{-1}\frac{1}{3}+\sec^{-1}3\right)+\cos\left(\tan^{-1}\frac{1}{2}+\tan^{-1}2\right)$ is

2 write principle value branch of sine inverse function

3 The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\cos\frac{5\pi}{3}\right)$ is….. OR Evaluate : $\tan^{1}\sqrt{3}-\sec^{-1}(-2)$.

4 If $\sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}x$, then find x .

5 Show that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$.

6 Show that $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}$.

7 Prove $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

8 find value of $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $|x|<1$, $y>0$ and $xy<1$

9 If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x

10 If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of x

11 Solve the Equation : $\tan^{-1}\left(\frac{x-1}{x+1}\right)+\tan^{-1}\left(\frac{2x-1}{2x+1}\right)=\tan^{-1}\frac{23}{36}$

12 Prove $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}$, $x\in\left(0,\frac{\pi}{4}\right)$

13 Write the following function in the simplest form : $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$

14

If $\cos^{-1}\dfrac{x}{a}+\cos^{-1}\dfrac{y}{b}=\lambda$, Prove that $\dfrac{x^2}{a^2}-\left(\dfrac{2xy}{ab}\right)\cos\lambda+\dfrac{y^2}{b^2}=\sin^2\lambda$.

15 If $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\dfrac{5\Pi^2}{8}$ then find x .

16 Prove that

$$\tan^{-1}\left(\frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

CONTINUITY & DIFFERENTIATION

1 If $x^p . y^q = (x+y)^{p+q}$, prove that (1) $\frac{dy}{dx} = \frac{y}{x}$ and (2) $\frac{d^2y}{dx^2} = 0$.

2 If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f^1(0)$

3 If $x = \sin\left(\frac{1}{a}\log y\right)$, show that $(1-x^2)y_2 - xy_1 - a^2 y = 0$

4 If $y = (x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$.

5 If $x = \left(\cos\theta + \log\tan\frac{\theta}{2}\right)$, $y = \sin\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$

6. If $x = \sqrt{\sin^{-1} t}$, $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = \frac{-y}{x}$

7 IF F (x) is continuous $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0 \\ \frac{2x+1}{x-2}, & \text{if } 0 \le x \le 1 \end{cases}$ at x=0 Find value of k

8 Find the value of k, such that the function 'f defined by $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & x \ne \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

9 If $\sin x = y\sin(x+b)$, show that $\frac{dy}{dx} = \frac{\sin b}{\sin^2(x+b)}$.

10 . If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

11 Determine the values of a, b, c for which the following function is continuous at x = 0 :

$$F(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & \text{for } x < 0 \\ c, & \text{for } x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{for } x > 0 \end{cases}$$

12 Discuss differentiability of $f(x) = |x-2|$ at $x = 2$

13 Differentiate the following function with respect to x

(1) $\tan^{-1}\left(\dfrac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ (2) $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$ (3) $\cos^{-1}\left(\dfrac{3\cos x - 4\sin x}{5}\right)$ (4) $\tan^{-1}\left(\dfrac{4x}{1-4x^2}\right)$

(5) $\sin^2\left(\cot^{-1}\sqrt{\dfrac{1+bx}{1-bx}}\right)$ (6) $\cos^{-1}\left(x^{3/2} + \sqrt{1-x-x^2+x^3}\right)$

14 Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\dfrac{2x}{1-x^2}$

15 If $x^y = e^{x-y}$, $x > 0$, Prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$ or $\dfrac{\log x}{[\log(xe)]^2}$

16 If $y = \sin(m\sin^{-1}x)$, Prove that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + m^2y = 0$

17 Find $\dfrac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

18 . If $(x-a)^2 + (y-b)^2 = c^2$, Prove that $\dfrac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{3/2}}{\dfrac{d^2y}{dx^2}}$ is a constant Independent of a and b.

19 Using Rolle's theorem, find the points on the curve $y = x^2$, $x \in [-2, 2]$, where the tangent is parallel to x – axis .

20 Using Langrange's Mean Value Theorem find a point on the parabola $y = (x-3)^2$, where tangent is parallel to the chord joining (3, 0), (5, 4)

APPLICATION OF MAXIMA AND MINIMA

1 If the sum of a side and the hypotenuse of a right – angled triangle be given , show that the area of the triangle will be maximum if the angle between the given side and the hypotenuse be 60^0.

2 A wire of length 36 cm is cut into two pieces . One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle . Find the length of each piece so that the sum of the areas of the two be minimum .

3 Show that the semi – vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.

4 If the length of three sides of a trapezium other than the base are equal to 10 cm, then find the maximum are of the trapezium .

5 Let AP and BQ be two vertical poles at points A and B respectively . If AP = 16 m, PQ = 22 m, AB = 20 Then find the distance of a point R on AB from A such that $RP^2 + RQ^2$ is minimum .

6 An open box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides . Find the volume of the largest such box .

.7 An open box with a square base is to be made out of a given quantity of sheet of area a^2 . Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.

8 Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi – vertical angle 30^0 is $\frac{4}{81}\pi h^3$.

9 Show that the height h of a right circular cylinder of maximum total surface area, including the two ends that can be inscribed in a sphere of radius ' r ' is given by $h^2 = 2r^2\left(1-\frac{1}{\sqrt{5}}\right)$.

10 Prove that the volume of the largest cone that can be inscribed in a sphere of radius r is $\frac{8}{27}$ of the volume of the sphere .

11 A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semi – circle . Find the dimensions of the rectangle, so that its area is maximum . Also find maximum area .

12 An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water . Show that the cost of the material will be the least when the depth of the tank is half of its width .

13 A square tank of capacity of 250 cu. Meters has to be dug out . The cost of land is Rs. 50 per sq .meter . The cost of digging increases with the depth and cost for the whole tank is 400 (depth $)^2$ rupees

14 Show that the height of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h is $\frac{h}{3}$

15 Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, - 8) .

16 A window is in the form of a rectangle above which there is a semicircle . If the perimeter of the window is p cm . Show that the window will allow the maximum possible light only when the radius of the semicircle is $\frac{p}{\pi+4}$ cm .

17 Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex coinciding with one extremity of the major axis .

18 A given quantity of metal is to be cast into a half cylinder with rectangular base and semicircular ends.show that in order that the total surface area may be minimum, the ratio of the length of cylinder to the diameter of its circular ends is $\Pi : (\Pi + 2)$

19 A point on the hypotenuse of a right – angled triangle is at distances a and b from the sides of the triangle . Show that the minimum length of the hypotenuse is

$$(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}} .$$

INCREASING & DECREASING FUNCTIONS , TANGENT & NORMAL ,APPROXIMATION, RATE OF CHANGE OF QUANTITIES

1 Determine the interval , where $f(x) = \sin(x) - \cos(x)$, $0 \le x \le 2\Pi$ is strictly increasing or Decreasing .

2 Find the interval (s) for which the function $f(x) = \log(2 + x) - \frac{2x}{2+x}$ is increasing or decreasing .

3 Prove that curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at (a, b) for all values of n .

4 Using differentials, evaluate $\left(\frac{17}{81}\right)^{\frac{1}{4}}$ approximately .

5 A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y – Coordinates is changing 8 times as fast as x – coordinate .

6 Show that $y = \log(1 + x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x, throughout its domain .

7 Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles, if $k^2 = 8$.

8 A man 2 meters tall walks at a uniform speed of 5 km / hr away from a lamppost which is 6 meters high . Find the rate at which the length of his shadow increases .

9 Find the equations of tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$ that are parallel to the line $x + 2y = 0$.

10 Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$; $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from origin .

11 If $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$. Find intervals of Increasing or Decreasing

Linear Programming

1A toy company manufactures two types of dolls, A and B . Market tests and available resources have indicated that the combined production level should not exceed 1200 dools per week and the demand for dools of type B is at most half of that for dolls of type A . Further the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units . If the company makes profit of Rs. 12 and Rs. 16 per doll respectively on dolls A and B how many of each should be produced weekly in order to maximize the profit ?

2 An aeroplane can carry a maximum of 200 passengers . A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket . The airline reserves at least 20 seats for executive class . However, at least 4 times as many passengers prefer to travel by economy class than by executive class . Determine how many tickets of each type must be sold in order to maximize profit for the airline ? What is the maximum profit ?

3 A farmer mixes two brands P and Q of cattle feed . Brand P costing Rs 250 per bag, contains 3 units of nutritional elements A, 2 . 5 unit of element B and 2 units of element C . Brand Q costing Rs 200 per bag contains 1 . 5 units of nutritional element A, 11 . 25 units of element B,
and 3 units of element C . The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively . Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag ? What is the minimum cost of the mixture per bag ?

4 A merchant plans to sell two types of personal computers – a desk top model and a portable model that will cost Rs . 25000 and Rs. 40000 respectively . He estimates that the total monthly demand of computers will not exceed 250 units . Determine the number of units of each type of computers which the merchant should stock to get a maximum profit if he does not want to invest more than 70 lakh and if his profit on the desktop model is Rs. 4500 and the portable is Rs. 5000 .

5 A retired person wants to invest up to an amount of Rs. 30000 in the fixed income securities . His broker recommends investing in two bonds, bond A yielding 7 % per annum and bond B yielding 10% per annum . After some consideration, he decides to invest at the most Rs. 12000 in bond B and at least Rs. 6000 in bound A . He also wants that the amount invested in bond A should be more than the amount invested in bound B . What should the broker recommend if the investor wants to maximize his return investments .

6 Every gram of wheat provides 0 . 1 g of proteins and 0 . 25 g of carbohydrates . The corresponding values of rice are 0 . 05 g and 0 . 5 respectively . Wheat costs Rs . 2 per kg. and rice Rs. 8 . The minimum daily requirements of protein and carbohydrates for an average child are 50 g and 200 g respectively . In what quantities should whet and rice be mixed in the daily diet to provide the maximum daily requirements of protein and carbohydrates at minimum cost ?

7 If a person rides his motor – cycle at 25 km Per hour, he has to spend Rs. 2 per km on petrol ; if he rides it at a faster speed of 4o km . per hour, the petrol cost increases to Rs. 5 per km . He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour . Expres this as a linear programming problem and then solve it .

8 A garden er has a supply of fertilizer of type I which consists of 10 % nitrogen and 6 % phosphoric acid and type 2 fertilizer which consists of 5 % nitrogen and 10 % phosphoric acid. After testing the soil condtions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type I fertilizer costs 60 paise per kg and type 2 fertilizer costs 40 paise per kg, determine how many kilograms of each fertilizer should be used so that nutrient requirements are met at a minimum cost ? what is the minimum cost ?

9 Sudha wants to invest Rs. 12000 in Saving Certificates and in National Saving Bands . According to rules, she has to invest at least Rs. 1000 in Saving Certificates and at least Rs . 2000 in National Saving Bonds . If the rate of interest on Saving Certificates is 8 % p . a. and the rate of interest on the National Saving Bonds is 10 % p. a.how should she invest her money to earn maximum yearly income ? What is the maximum yearly income ?

10 An oil company has two depots A and B with capacities of 7000 L(litres) and 4000 litres respectively.The Company is to supply oil to three petrol pumps D, E and F whose requirements are 4500 L , 3000 L and 3500L Respectively The distance (in km) between the depots and petrol pumps is given in the following table

FROM \ TO	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is rupees 1 per km,how should the delivery be scheduled in order that the transportation cost is minium ? Find the minimum cost ?.

INTEGRATION (ASSIGNMENT 1)

1 $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}$ 2 $\int \sin^4 x dx$ 3 $\int \sin x \,.\, \sin 2x \sin 3x \, dx$ 4 $\int \frac{\sin x}{1+\sin x}\, dx$

5 $\int \frac{\tan x}{\sec x + \tan x}\, dx$ 6 $\int \tan^{-1}(\sec x + \tan x)\, dx$ 7 $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}\, dx$ 8 $\int \frac{\cos^9 x}{\sin x}\, dx$

9 $\int \frac{\sqrt{\tan x}}{\sin x.\cos x}\, dx$ 10 $\int \tan^5 x.\sec^3 x \, dx$ 11 $\int \frac{1}{1+\tan x} dx$ **12** $\int \frac{dx}{3\sin x + \sqrt{3}\cos x}$

13 $\int \frac{1}{x(x^5+1)}$ **dx** **14** $\int x^{99}.(x^{50}+1)^{5}\, dx$ 15 $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}$ 16 $\int 5^{5^{5^x}}\ 5^{5^x}\ .\ 5^x$

17 $\int \sqrt{1+2\tan x(\tan x + \sec x}\, dx$ 18 $\int \tan x.\ \tan 2x\ .\ \tan 3x\, dx$ 19 $\int \sqrt{\frac{x}{a^3 - x^3}}\, dx$ 20 $\int \sqrt{-2x^2 - 3x + 5}\, dx$

21 $\int (x+3)\sqrt{3-4x-x^2}\, dx$ 22 $\int \frac{1}{1+\sin x + \cos x} dx$ 23 $\int \frac{2\sin 2\theta - \cos\theta}{6-\cos^2\theta - 4\sin\theta} d\theta$

24 $\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$. 25 $\int \sqrt{e^x - 1}\, dx$ 26 $\int \frac{1}{\cos(x+a)\cos(x+b)}\, dx$.

27 $\int \frac{dx}{\cos^2 x + \sin 2x}$. 28 $\int \frac{dx}{e^x - 1}\, dx$. 29 $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$ 30 $\int \frac{3x+5}{x^3 - x^2 - x + 1} dx$

MOST IMPORTTANT PROBLEMS (ASSIGNMENT 2)

1 $\int \frac{1}{\sin^4 x + \cos^4 x} dx$ 2 $\int \sqrt{\tan x}\, dx$ 3 $\int \left(\sqrt{\tan\theta} + \sqrt{\cot\theta}\right) d\theta$ 4 $\int \frac{dx}{1+x^4}$.

5 $\int \frac{1}{x^4 + x^2 + 1} dx$ 6 $\int \frac{x^2}{(x^2+1)(x^2+4)}\, dx$. 7 $\int \frac{x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$ 8 $\int \frac{x}{x^3 + x^2 + x + 1} dx$

9 $\int \frac{dx}{\sin^2 x + \sin 2x}$ 10 $\int \frac{\tan\phi + \tan^3\phi}{1+\tan^3\phi} d\phi$ 11 $\int (\sin^{-1} x)^2\, dx$ 12 $\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$.

13 $\int \frac{dx}{\sin x + \sin 2x}$ 14 $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx$. 15 $\int e^{\tan^{-1} x}\left(\frac{1+x+x^2}{1+x^2}\right)$

16 $\int \frac{(x^2+1)e^x}{(x+1)^2}$ 17 $\int e^{2x}\left(\frac{\sin 4x-2}{1-\cos 4x}\right)$ dx . 18 $\int e^x \frac{x^3-x+2}{(x^2+1)^2}$ dx

19 $\int e^{2x}\left(\frac{\sin 4x-2}{1-\cos 4x}\right)dx$ 20 $\int \frac{Logx}{(1+Logx)^2}dx$ 21 $\int \frac{dx}{x\sqrt{ax-x^2}}$ 22 $\int \frac{x^{1/2}}{x^{1/3}+x^{1/2}}$ dx

23 $\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}$ dx 24 $\int_0^{2\pi} e^{x/2}.\sin(\frac{x}{2}+\frac{\pi}{4})$ dx 25 $\int_0^{\pi/4}\sqrt{\tan x}dx$ 26 $\int_0^{\pi/2}\sin 2x.\tan^{-1}(\sin x)$ dx

27 $\int_0^{\pi/4} \frac{\sin x.\cos x}{\cos^2 x+\sin^4 x}$ dx 28 $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x+4\sin^2 x}$ 29 dx $\int_0^{\pi/2} (\sqrt{\tan x}+\sqrt{\cot x})$ dx $=\pi\sqrt{2}$.

Differential equation

1 Find order &Degree of D.Eqn (1) $5\frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$ (2) $y^{v}+y^2+e^{y'}=0$

2 Form D . E. of all circle touching the (1) x – axis at the origin.

3 Form the eq″ . of all the circles in the first quad. which touch the coordinates axes .

4 Solve the D.Eqn. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

5 Solve the D.Eqn $2ye^{x/y}\,dx + (y-2xe^{x/y})\,dy = 0$.

6 Solve $(x+y)^2\frac{dy}{dx} = a^2$

7 Solve the D.Eqn $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

8 Solve the D.Eqn $\frac{dy}{dx} = -\left(\frac{x+y\cos x}{1+\sin x}\right)$

9 Solve the D.Eqn $(x^2+1)y^1 - 2xy = (x^4+2x^2+1)\cos x$ $y(0) = 0$

10 Solve the D.Eqn $(x-\sin y)\,dy + (\tan y)\,dx = 0$

11 Solve the D.Eqn $\left(x\cos\frac{y}{x}+y\sin\frac{y}{x}\right)y - \left(y\sin\frac{y}{x}-x\cos\frac{y}{x}\right)x\frac{dy}{dx} = 0$

12 Solve $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

13 Show that $y = a\cos(\text{Log}\,x) + b\sin(\text{Log}\,x)$ is a soln. of D . E . $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

14 Solve the Initial value Problem : (1) $e^{(dy/dx)} = x+1$; $y(0) = 5$ (2) $\sin\left(\frac{dy}{dx}\right) = k$; $y(0) = 1$

15 Solve $\frac{dy}{dx}$ tan y = sin (x + y) + sin (x – y)

16 Solve x (x dy – y dx) = y dx, y (1) = 1

17 The population of a village increases at the rate proportional to the number of its inhabitants present at any time . If the population of the village was 20, 000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009 ?

18 In a bank principal increases at the rate of r % per year . Find the value of r if Rs 100 double itself in 10 years (log 2 = 0 . 6931)

Question based on properties of definite integration(important)

1 $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$

2 $\int_0^1 \cot^{-1} (1-x + x^2)dx$

3 $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx$.

4 $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x}$

5 $\int_0^{\pi/2}$ Log sin x dx

6 $\int_0^{\pi/2} \frac{x \sin x . \cos x}{\cos^4 x . + \sin^4 x} dx$

7 $\int_0^{\pi} Log\ (1 + \cos x) dx$

8 $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

9 $\int_{-a}^{a} \sqrt{\frac{a - x}{a + x}} dx$

10 $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$

11 $\int_{\frac{\pi}{6}}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

12 $\int_{-1}^{3/2} | x \sin \pi x | dx$

13 $\int_1^4 (| x - 1 | + | x - 2 | + | x - 3 |) dx$

14 $\int_0^2 | x^2 + 2x - 3 | dx$

15 Prove that : $\int_0^a f(x) dx = \int_0^a f(a - x) dx$ and hence, prove that $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$.

16 Prove that : $\int_0^{\frac{\pi}{4}} \log (1 + \tan \theta) d\theta = \frac{\pi}{8} \log 2$.

17 $\int_0^{\pi} \frac{x}{1 + \cos \alpha . \sin x} dx$

18 $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$

19 $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1 + e^{\tan x}} dx$

Application of integration (AREA)

1 Draw the rough sketch of the region $\{ (x, y) : y^2 \le 4x,\ 4x^2 + 4y^2 \le 25 \}$ and find the area enclosed by the region using the method of integration .

2 Find the area of the region bounded by parabola $y = x^2$ and $y = |x|$.

3 A O B is a part of the ellipse $9x^2 + y^2 = 36$, in the first quadrant, such that OA = 2 and OB = 6 . Find the area between the arc AB and the chord AB . A and B are points on x – axis and y – axis respectively .

4 Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts .

5 Make a rough sketch of the region given below and find its area using integration .
$\{ (x, y) : 0 \le y \le x^2 + 3 ; 0 \le y \le 2x + 3, 0 \le x \le 3)$.

6 Sketch the graph $y = |x - 1|$. Evaluate $\int_{-2}^{4} |x - 1|\, dx$. what does the value of this integral represent on the graph

7 Find the area of the region bounded by parabola $y = x^2 + 1$ and the Lines $y = x$, $x = 0$ and $x = 2$.

8 Calculate the area of Region bounded by the two parabolas $y = x^2$ and $y^2 = x$

9 Find the area bounded by the curve $x^2 = 4y$ and the straight Line $x = 4y - 2$.

10 Find the area of the region $\{ (x, y) : y^2 \le 4x,\ 4x^2 + 4y^2 \le 9 \}$

11 Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the straight Line $\frac{x}{3} + \frac{y}{2} = 1$

12 Find the area of the region enclosed bet . the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$

13 Find the area of the circle $4x^2 + 4y^2 = 9$ which is Interior to the parabola $y^2 = 4x$.

14 using Integration find the area of the triangle A B C, coordinates of whose vertices A (2, 0) B (4, 5) and C (6, 3) .

15 Find the area of the region given by $\{ (x, y) / x^2 + y^2 \le 2ax, y^2 \ge ax, x \ge 0, y \ge 0$

16 Find the area of the region $\{ (x, y) : x^2 + y^2 \le 1 \le x + y \}$

17 Find the area $\{ (x, y) : x^2 \le y \le x \}$

18 using Integration, find the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$

19 using Integration, find the area of the region bounded by curves , after making a rough sketech :
$y = 1 + |x + 1|$, $x = -3$, $x = 3$, $y = 0$

20 Draw a Rough sketech of the curves $y = \sin x$ and $y = \cos x$ as x varies from 0 to $\pi/2$ and find the area of the region enclosed by them and x – axis .

PROBABILITY (BAY`S THEOREM)

1 A laboratory blood test is 99 % effective in detecting a certain disease when it is , in fact, present . However, test also yields a false positive result for 0 . 5 % of the healthy person tested (i. e. if a healthy person is tested, then, with probability 0 . 005, the test will imply he has the disease) . If 0 . 1 percent of the population actually has the disease , what is the probability that a person has the disease given that his test result is positive ?

2 A doctor is to visit a patient . From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}, \frac{1}{12}$ if he comes by train, bus and scooter respectively, but he comes by other means of transport, then he will not be late . When he arrivers, he is late . what is the probability that he comes by train ?

3 A card from a pack of 52 cards is lost . From the remaining cards of the pack, two cards are drawn and found to be both diamond . Find the probability of the lost card being a diamond

4 In an examination, an examinee either guesses or copies or knows the answer of multiple choise questions with four choices . The probability that he makes a guess is $\frac{1}{3}$ and probability that he copies the answer. is $\frac{1}{6}$. The probability that his answers is correct, given that he copied it, is $\frac{1}{8}$ .Find the probability that he knew the answer to the question, given that he correctly answere it

5 Suppose that the reliability of a HIV test is specified as follows : of people having HIV, 90 % of the test detect the disease but 10 % go undetected . Of the people free of HIV, 99 % of the tests are judged HIV – ve but 1 % are diagnosed as showing HIV + ve . From a large population of which only 0 . 1 % have HIV, one person is selected at random, given the HIV test, and the pathologist reports him / her as HIV + ve . what is the probability that the person actually has HIV ?

6 Assume that the chance, of a patient having a heart attack 40 % . It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30 % and prescription of a certain drug reduces its chance by 25 % . At a time a patient can choose any one of the two options with equal probabilities . It is given that after going through one of the two options the patient selected at random suffers heart attack . Find the probability that the patient followed a course of meditation and yoga ?

7 There are three coins . One is a two headed coin (having head on both faces), another is a biased coin that comes up head 75 % of the

time and third is an unbiased coin . one of the three coins is chosen at random and tossed, it shows head, what is the probability that it was the two headed coin . ?

8 Suppose a girl throws a die . If she gets a 5 or 6, she tosses a coin three times and notes the number of heads . If she gets 1, 2, 3, or 4 she tosses a coin once and notes whether a head or tail is obtained . If she obtained exactly one head, what is the probability the threw 1, 2, 3 or 4 with the die ?

9 Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed . A reports that head appears . What is probability that actually there was head

10 Bag I contains 3 red and 4 black balls not bag 2 contains 4 red and 5 black balls . One ball is transferred from Bag 1 to Bag 2, then a ball is drawn from Bag 2, the ball so drawn is found to be Red in colour . Find the probability that the transferred ball is black .

11 By examining the chest X – ray, the probability that T . B . is detected when a person is actually suffering from it is 0 . 99 . The probability that the doctor diagnosis incorrectly that a person has T . B . on the basis of X – ray is 0 . 001 . In a certain city, 1 in 1000 persons suffers from T . B .A person is selected at random and is diagnosed to have T . B . what is the chance that he actually has T B .

PROBABILITY (Problems based on Conditional probability Probability distribution,binomial distribution, other imp. Problems)

1 A black and red die are rolled Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5 .

2 Assume that each child born is equally Likely to be a boy or a girl .If a family has two children, what is the conditional a probability that both are girls given that (1) the youngest is a girl (2)at least one is a girl ?

3 Consider the experiment of throwing a die, if a multiple of 3 comes up throw the die again and if any other numbers comes toss a coin . Find the conditional probability of the event ‘ the coin shows tail’, given that at least one die shows a 3 .

4 Three persons A,B and C fire a target in turn, starting with A. Their probability of hitting the target are 0.5, 0.3 and 0.2 respectively. Find the probabilty of at most one hit. In life we must set a target/goal. To achieve the target we need to follow some values/qualities.Mention any two such qualities.(ans .75 hard work,team work,concern for target)

5 A husband and wife appear in an Interviewfor two vacancies in the same post . The probability of husband selection is $\frac{1}{7}$ and that of wife' s selection is $\frac{1}{5}$ What is the probability that (1) Both of them will be selected (2) only one of them will be selected 3 None of them will be select 4 atleast one of them will be selected

6 A coin is biased (problem in coin) so that the head is 3 times as likely to occur as a tail . If the coin is tossed twice, find the prob . dist for the no. of tails

7 . Two bad eggs are mixed accidently with 10 good ones . Find the prob . dist . of the no. of bad eggs in 3, drawn at random one by one without replacement, from this Lot .

8 In a game, 3 coins are tossed . A person is Paid Rs. 5 if he gets all heads or all tails, and he is supposed to

pay Rs . 3 if he gets one head or two heads . What can he expect to win on an average per game ?

9 Two numbers are selected at random (without replacement) from the first six positive integers . Let x denote the larger of the two numbers obtained . Find E (X)

10 The probability that a bulb produced by a factory will fuse after 150 days of use is 0 . 0 5 . Find the probability that out of 5 such bulbs (1) none (2) not more than one (3) more than one (4) at least one will fuse after 150 days of use .

11 Three rotten apples are mixed with 7 fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Rotten apples leads to wastage of food resources and as we know lots of food material gets wasted because of mismanagement. Give two suggestions to stop the wastage of food.

.

VECTORS

1 If a until vector $\hat{a}$ that makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence the components of $\vec{a}$

2 A girl walks 4 km towards west, then she walks 3 km in a direction 30^0 east of north and stops . determine the girl's displacement from her initial point of departure

3 Find a unit vector in the plane of the vectors $\vec{a} = \hat{i}+2\hat{j}$ and $\vec{b} = \hat{j}+2\hat{k}$ and ⊥ lar to the vector $\vec{c} = 2\hat{i}+\hat{j}+2\hat{k}$

4 P . that for any two vectors $|\vec{a}\times\vec{b}| = \sqrt{\vec{a}^2\vec{b}^2-(\vec{a}\cdot\vec{b})^2}$

5 Show that the points A (1, - 2. – 8), B (5, 0, - 2) and C (11, 3, 7) are collinear and find the ratio in which B divides AC .

6 Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d} = 15$.

7 The scalar product of vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 8 .Find the value of λ.

8 Express the vector $\vec{a} = s\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to $\vec{b} = 3\hat{i} + \hat{k}$ and the other is perpendicular to $\vec{b}$.

9 If the sum of two unit vectors is also a unit vector, Show that magnitude of their difference is $\sqrt{3}$

10 If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude . P . that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$. Hence prove that equal inclination is $\cos^{-1}(\frac{1}{\sqrt{3}})$

11 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplaner vectors then prove that $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{a}+\vec{c}$ are also coplaner.

THREE DIMENSIONAL GEOMETRY

1 Find the equation of the plane passing through the intersection of the planes 4x – y + z = 10 & x + y – z =4 and parallel to the line with direction ratios 2, 1, 1 . Find also the perpendicular distance of (1, 1, 1) from this plane

2 A variable plane which remains at a constant distance 3p from origin, cuts the coordinate axes in A, B Show that locus of the centroid of triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

3 Show that the equation of plane which meets the axes in A, B, C, where centroid of triangle ABC is given as (α,β,γ) is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.

4 Define the line of shortest distance between two skew lines . Find the shortest distance and the vector equation of the line of shortest distance between lines given by $\vec{r} = (-\hat{i}+5\hat{j}) + \lambda(-\hat{i}+\hat{j}+\hat{k})$ and $\vec{r} = (-\hat{i}-3\hat{j}+2\hat{k}) + \mu(3\hat{i}+2\hat{j}+\hat{k})$.

.5 Find the distance of the point (2, 3, 4) from the plane 3x + 2y + 2z + 5 = 0, measured parallel to the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$.

6 Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ Also find the length of the perpendicular .

7 Find the equation of the line through the point (- 1, 2, 3) which is perpendicular to the lines $\frac{x}{2} = \frac{y-1}{-3} = \frac{z+2}{-2}$ $\frac{x+3}{-1} = \frac{y+2}{2} = \frac{z-1}{3}$.

8 Find the coordinates of the point where the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ meets the plane x + y + 4z = 6

9 Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z + 8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0 .

10 If the sum of two unit vectors is also a unit vector, Show that magnitude of their difference is $\sqrt{3}$

11 A line makes angles $\alpha, \beta, \gamma and \delta$ with the four diagonals of a cube . Prove that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3} .$$

12 A variable plane at a constant distance p from origin meets the coordinates axes in points A, B and C respectively . Through these points, planes are drawn parallel to the coordinates planes, show that locus of point of intersection is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

13 Find the coordinates of the point where the line joining (- 1, - 3, 2) and (1, 3, - 2) Cuts the plane 3 x + 4 y + 5 z = 0

14 Find the Equation of the plane throught the points (2, - 3, 1) and (5, 2, -1) and perpendicular to plane x – 4y + 5z + 2 = 0

15 Find the Eq^n. of the plane passing through the points (2, 3, 4), (- 3, 5, 1) and (4, - 1, 2) and find also the angles which the normal to this plane makes with the axes.

16 Find the distance of the point (3, 8, 2) from the line $\vec{r}=\hat{i}+3\hat{j}+2\hat{k}+\lambda(2\hat{i}+4\hat{j}+3\hat{k})$ measured parallel to the plane $\vec{r}\cdot(3\hat{i}+2\hat{j}-2\hat{k})+15=0$

RELATION & FUNCTION

1 In N × N the Relation defined by *(a,b) R (c,d)* $\Leftrightarrow$ *a d (b + c) = bc (a+d)* …(1) check whether R is an equivalence Relation on N × N

2 Show that R is an Equivalence Relation for $A=\{x\in z: 0\le x\le 12\}$, R = {(a, b) : $|a-b|$ is a multiple of 4}

3 Define a binary operation * on the set $\{0,1,2,3,4,5\}$ as a * b = $\{a+b, if\ a+b<6$ and a+b-6 ,if a+b ≥ 6
Show that zero is identity for this operation and each element a of set is invertible with 6-a being theInverse of a

4 Let $f: N\to N$ be defined by $f(n)=\begin{cases}\frac{n+1}{2}, \text{if n is odd}\\ \frac{n}{2}, \text{if n is even}\end{cases}$ *for* all n ∈ N State whether the function f is onto, one-oneor bijective. Justify your answer.

5 Let $f : Z \to Z$ and $g : Z \to Z$ be defined by $f(n) = 3n$ and $g(n) = \begin{cases} \frac{n}{3}, \text{if n is multiple of 3} \\ 0, \text{if n is not multiple of 3} \end{cases}$ $\forall n \in z$ Show that

$g o f = I_z$, $f o g \neq I_z$

6 $f : R_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$ Show that f *is invertible with* $f^{-1}(y) = \frac{(\sqrt{y+6}) - 1}{3}$

7 Show that fof^{-1} is an identity function if $f(x) = \frac{x-1}{x+1}(x \neq 1, -1)$

8 Let A be a non-empty set and let * be a binary operation on P(A), the power set of A defined by X*Y = (X – Y) $\cup$ (Y – X) For X,Y $\in$ P(A). Show that (i) $\phi \in$ P(A) is the identity of element of (P(A), *)

(ii)X is invertible $\forall X \in P(A)$ and $X = X^{-1}$ i.e. X is inverse of X itself

9 Let A = Q×Q. Let * be a binary operation on A defined by : (a, b) * (c,d) = (ac, ad+b) Find : (i) The identity element of (A, *) (ii) The invertible element of (A, *).

10 Let A = N×N and let * be a binary operation a defined by (a,b) * (c,d) = (a+c, b+d) Show (i) (A, *) is associative. (ii) (A, *) is commutative (iiiFind the identity element .

11 Show that the function f : R $\to \{x \in R : -1 < x < 1\}$ defined by f(x) = $\frac{x}{1+|x|}$, x $\in R$ is one-one and onto function.

12 Let f : $W \to W$ be define as f(n) = n-1,if n is odd and f(n) = n+1,if n is even.show that f is invertible. Find the Inverse of f. Here W is set of all whole numbers.

13 Let f : $R \to R$ defined by f(x) = 10x + 7.Find the function g : $R \to R$ such that gof = fog = I_R.