CONTINUITY OF FUNCTIONS

Exercise - 1 (Objective Questions)

Part : (A) Only one correct option

1. The value of f (0), so that the function, $f(x) = \dfrac{\sqrt{(a^2-ax+x^2)}-\sqrt{(a^2+ax+x^2)}}{\sqrt{(a+x)}-\sqrt{(a-x)}}$ (a > 0) becomes continuous for all x, is given by :

(A) $a\sqrt{a}$ (B) $\sqrt{a}$ (C) $-\sqrt{a}$ (D) $-a\sqrt{a}$

2. The value of R which makes $f(x) = \begin{cases} \sin(1/x) & , x \neq 0 \\ R & , x = 0 \end{cases}$ continuous at x = 0 is:

(A) 8 (B) 1 (C) – 1 (D) None of these

3. A function f(x) is defined as below $f(x) = \dfrac{\cos(\sin x)-\cos x}{x^2}$, $x \neq 0$ and f(0) = a

f(x) is continuous at x = 0 if a equals

(A) 0 (B) 4 (C) 5 (D) 6

4. Let $f(x) = (\sin x)^{\frac{1}{\pi-2x}}$, $x \neq \frac{\pi}{2}$. If f(x) is continuous at $x = \frac{\pi}{2}$ then $f\left(\frac{\pi}{2}\right)$ is

(A) e (B) 1 (C) 0 (D) none of these

5. $f(x) = \begin{cases} \dfrac{\sqrt{(1+px)}-\sqrt{(1-px)}}{x} & , -1 \le x < 0 \\ \dfrac{2x+1}{x-2} & , 0 \le x \le 1 \end{cases}$ is continuous in the interval [– 1, 1], then 'p' is equal to:

(A) – 1 (B) – 1/2 (C) 1/2 (D) 1

6. Let $f(x) = \left|\left(x+\frac{1}{2}\right)[x]\right|$ when $-2 \le x \le 2$. where [.] represents greatest integer function. Then

(A) f(x) is continuous at x = 2 (B) f(x) is continuous at x = 1
(C) f(x) is continuous at x = – 1 (D) f(x) is discontinuous at x = 0

7. The set of all points for which

$$f(x) = \frac{|x-3|}{|x-2|} + \frac{1}{[1+x]}$$ where [.] represents greatest integer function is continuous is

(A) R (B) R – [– 1, 0]
(C) $R - (\{2\} \cup [-1, 0])$ (D) $R - \{(-1, 0) \cup n, n \in I\}$

8. The function $f(x) = [x] \cos\left[\frac{(2x-1)}{2}\right]\pi$, ([.] denotes the greatest integer function) is dicontinuous at:

(A) all x (B) $x = n/2, n \in I - \{1\}$
(C) no x (D) x which is not an integer

9. Let [x] denote the integral part of $x \in R$ and $g(x) = x - [x]$. Let f(x) be any continuous function with $f(0) = f(1)$ then the function $h(x) = f(g(x))$:
(A) has finitely many discontinuities
(B) is continuous on R
(C) is discontinuous at some x = c
(D) is a constant function.

10. The function f(x) is defined by $f(x) = \begin{cases} \log_{(4x-3)}(x^2 - 2x + 5) & \text{if } \frac{3}{4} < x < 1 \,\&\, x > 1 \\ 4 & \text{if } x = 1 \end{cases}$

(A) is continuous at x = 1
(B) is discontinuous at x = 1 since $f(1^+)$ does not exist though $f(1^-)$ exists
(C) is discontinuous at x = 1 since $f(1^-)$ does not exist though $f(1^+)$ exists
(D) is discontinuous since neither $f(1^-)$ nor $f(1^+)$ exists.

11. Let $f(x) = \dfrac{1-\sin x}{(\pi - 2x)^2} \cdot \dfrac{\ln(\sin x)}{\ln(1 + \pi^2 - 4\pi x + 4x^2)}$ $x \neq \dfrac{\pi}{2}$. The value of $f\left(\dfrac{\pi}{2}\right)$ so that the function is continuous at $x = \pi/2$ is:
(A) 1/16
(B) 1/32
(C) –1/64
(D) 1/128

12. Let $f(x) = \begin{bmatrix} x^2 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{bmatrix}$ then:

(A) f(x) is discontinuous for all x
(B) discontinuous for all x except at x = 0
(C) discontinuous for all x except at x = 1 or – 1
(D) none of these

13. Let $f(x) = [x^2] - [x]^2$, where [.] denotes the greatest integer function. Then
(A) f(x) is discontinuous for all integeral values of x
(B) f(x) is discontinuous only at x = 0, 1
(C) f(x) is continuous only at x = 1
(D) none of these

14. Let f(x) be a continuous function defined for $1 \le x \le 3$. If f(x) takes rational values for all x and f(2) = 10 then the value of f(1.5) is
(A) 7.5
(B) 10
(C) 8
(D) none of these

15. Let f(x) = Sgn(x) and $g(x) = x(x^2 - 5x + 6)$. The function f(g(x)) is discontinuous at
(A) infinitely many points
(B) exactly one point
(C) exactly three points
(D) no point

16. The function $f(x) = \left[x^2\left[\dfrac{1}{x^2}\right]\right]$, $x \ge 0$, is [.] represents the greatest integer less than or equal to x
(A) continuous at x = 1
(B) continuous at x = – 1
(C) discontinuous at infinitely many points
(D) continuous at x = – 1

17. The function f defined by $f(x) = \lim\limits_{t\to\infty} \cdot \left\{\dfrac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1}\right\}$ is
(A) everywhere continuous
(B) discontinuous at all integer values of x
(C) continuous at x = 0
(D) none of these

18. If [x] and {x} represent integral and fractional parts of a real number x, and $f(x) = \dfrac{a^{2|x| + \{x\}} - 1}{2[x] + \{x\}}$, $x \neq 0$, $f(0) = \log_e a$, where $a > 0$, $a \neq 1$, then
(A) f(x) is continuous at x = 0
(B) f(x) has a removable discontinuity at x = 0
(C) $\lim\limits_{x\to 0}$ f(x) does not exist
(D) none of these

Part : (B) May have more than one options correct

19. If f(x) = $\sqrt{x}$ and g(x) = x – 1, then

(A) fog is continuous on $[0, \infty)$
(B) gof is continuous on $[0, \infty)$
(C) fog is continuous on $[1, \infty)$
(D) none of these

20. The function f(x) = $\begin{cases} x^m \sin\frac{1}{x} & , \quad x > 0 \\ 0 & , \quad x = 0 \end{cases}$ is continuous at x = 0 if

(A) $m \geq 0$
(B) $m > 0$
(C) $m < 1$
(D) $m \geq 1$

21. Let f(x) = $\frac{1}{[\sin x]}$ ([.] denotes the greatest integer function) then

(A) domain of f(x) is $(2n\pi + \pi, 2n\pi + 2\pi) \cup \{2n\pi + \pi/2\}$
(B) f(x) is continuous when $x \in (2n\pi + \pi, 2n\pi + 2\pi)$
(C) f(x) is continuous at $x = 2n\pi + \pi/2$
(D) f(x) has the period 2π

22. Let f(x) = [x] + $\sqrt{x - [x]}$, where [x] denotes the greatest integer function. Then

(A) f(x) is continuous on R^+
(B) f(x) is continuous on R
(C) f(x) is continuous on R – I
(D) discontinuous at x = 1

23. Let f(x) and g(x) be defined by f(x) = [x] and g(x) = $\begin{cases} 0 & , \quad x \in I \\ x^2 & , \quad x \in R - I \end{cases}$ (where [.] denotes the greatest integer function) then

(A) $\lim_{x \to 1}$ g(x) exists, but g is not continuous at x = 1
(B) $\lim_{x \to 1}$ f(x) does not exist and f is not continuous at x = 1
(C) gof is continuous for all x
(D) fog is continuous for all x

24. Which of the following function(s) defined below has/have single point continuity.

(A) f(x) = $\begin{bmatrix} 1 & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{bmatrix}$
(B) g(x) = $\begin{bmatrix} x & \text{if } x \in Q \\ 1-x & \text{if } x \notin Q \end{bmatrix}$

(C) h(x) = $\begin{bmatrix} x & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{bmatrix}$
(D) k(x) = $\begin{bmatrix} x & \text{if } x \in Q \\ -x & \text{if } x \notin Q \end{bmatrix}$

Exercise - 2 (Subjective Questions)

1. Discuss the continuity of the function, f (x) at x = 3, if

$f(x) = \begin{cases} x[x] & , \text{ if } \quad 0 \le x < 3 \\ (x-1)[x] & , \text{ if } \quad 3 \le x \le 4 \end{cases}$ where [.] denotes greatest integer function.

2. Find the values of ' a ' & ' b ' so that the function, f (x) = $\begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & , x < \pi/2 \\ a & , x = \pi/2 \\ \frac{b(1-\sin x)}{(\pi - 2x)^2} & , x > \pi/2 \end{cases}$ is continuous at x = $\pi/2$.

3. Discuss the continuity of the function, f (x) = $\begin{cases} \frac{e^x - 1}{\ell n(1+2x)} & , x \neq 0 \\ 7 & , x = 0 \end{cases}$ at x = 0. If discontinuous, find the nature of discontinuity ?

4. If f(x) = x + {– x} + [x], where [x] is the integral part & {x} is the fractional part of x. Discuss the continuity of f in [– 2, 2]. Also find nature of each discontinuity.

5. Let f(x) = $\begin{cases} 1+x & ,0\le x\le 2 \\ 3-x & ,2<x\le 3 \end{cases}$. Determine the form of g(x) = f (f(x)) & hence find the point of discontinuity of g, if any.

6. Examine the continuity at x = 0 of the sum function of the infinite series:

$$\frac{x}{x+1}+\frac{x}{(x+1)(2x+1)}+\frac{x}{(2x+1)(3x+1)}+\ldots\ldots\ldots\ldots\infty .$$

7. If f(x) = $\frac{\sin 3x + A\sin 2x + B\sin x}{x^5}$ (x ≠ 0) is continuous at x = 0. Find A & B. Also find f (0).

8. Let [x] denote the greatest integer function & f(x) be defined in a neighbourhood of 2 by

$$f(x) = \begin{cases} \dfrac{\exp\left((x+2)\frac{1}{4}[x+1]\ln 4\right)-16}{4^x-16} & ,x<2 \\ A\dfrac{1-\cos(x-2)}{(x-2)\tan(x-2)} & ,x>2 \end{cases}.$$

Find the values of A & f(2) in order that f(x) may be continuous at x = 2.

9. Discuss the continuity of the function f (x) = $\underset{n\to\infty}{\text{Limit}}\ \frac{(1+\sin x)^n+\ln x}{2+(1+\sin x)^n}$.

10. Let f(x + y) = f(x) + f(y) for all x, y and if the function f(x) is continuous at x = 0, then show that f(x) is continuous at all x.

11. If f(x · y) = f(x)· f(y) for all x, y and f(x) is continuous at x = 1. Prove that f(x) is continuous for all x except at x = 0. Given f(1) ≠ 0.

12. If f $\left(\frac{x+2y}{3}\right)=\frac{f(x)+2f(y)}{3}$ ∀ x, y ∈ R and f(x) is continuous at x = 0. Prove that f (x) is continuous for all x ∈ R.

13. If f(x) = sin x and g(x) = $\begin{cases} \max^m \{f(t)\ ; 0\le t\le x, 0\le x\le 2 \\ 3x-4 \quad ; x>2 \end{cases}$, then discuss the continuity of g(x) ∀ x ≥ 0 .

Answers

Exercise # 1

1. C **2.** D **3.** A **4.** B **5.** B **6.** D **7.** D

8. B **9.** B **10.** D **11.** C **12.** C **13.** D **14.** B

15. C **16.** C **17.** B **18.** C **19.** BC **20.** BD

21. ABD **22.** ABC **23.** ABC **24.** BCD

Exercise # 2

1. continuous at x = 3

2. a = $\frac{1}{2}$, b = 4

3. Removable isolated point

4. discontinuous at all integral values in [– 2, 2]

5. g(x)= 2 + x ; 0 ≤ x ≤ 1,
= 2 – x ; 1 < x ≤ 2,
= 4 – x ; 2 < x ≤ 3,
g is discontinuous at x = 1 & x = 2

6. Discontinuous **7.** A = – 4, B = 5, f(0) = 1

8. A = 1; f(2) = 1/2

9. f (x) is discontinuous at natural multiples of π

13. continuous for all x ≥ 0 except at x = 2