Class XII

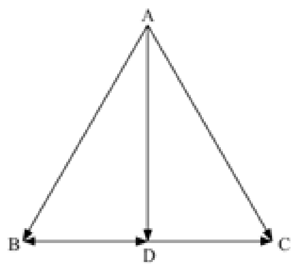
Mathematics

Set-3

Time: 3 hrs **M.M: 100 Marks**

Section A

Solution1: In Δ ABC,



Using the triangle law of vector addition, we have

$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

$= (3\hat{i} - j + 4\hat{k}) - (\hat{j} + \hat{k})$

$= 3\hat{i} - 2j + 3\hat{k}$

(Since AD is the median)

In ΔABD, using the triangle law of vector addition, we have

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= (\hat{j} + \hat{k}) + \left(\frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}\right)$

$= \frac{3}{2}\hat{i} + 0\hat{j} + \frac{5}{2}\hat{k}$

$$\therefore AD=\sqrt{\left(\frac{3}{2}\right)^2+0^2+\left(\frac{5}{2}\right)^2}=\frac{1}{2}\sqrt{34}$$

Hence, the length of the median through A is $\frac{1}{2}\sqrt{34}$ units.

Solution2: Given:

Normal vector, $\hat{n}=2\hat{i}-3\hat{j}+6\hat{k}$

Perpendicular distance, d = 5 units

The vector equation of a plane that is at a distance of 5 units from the origin and has its normal vector $\hat{n}=2\hat{i}-3\hat{j}+6\hat{k}$ is as follows:

$\vec{r}.\hat{n}=d$

$\vec{r}.\ (2\hat{i}-3\hat{j}+6\hat{k})=5$

Solution3: Let $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1 & 1 & 1+\cos\theta\end{vmatrix}$

Applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$,, we get

$\Delta=\begin{vmatrix}1 & 1 & 1\\ 0 & \sin\theta & 0\\ 0 & 0 & \cos\theta\end{vmatrix}$

$=\sin\theta\cos\theta$

$=\frac{\sin 2\theta}{2}$

We know that $-1\le \sin 2\theta\le 1$.

$\therefore$ Maximum value of $\Delta = \frac{1}{2}\times 1=\frac{1}{2}$

Solution4: Given = $(A-I)^3+(A+I)^3-7A$ b

$$= A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$$
$$= 2A^3 + 6AI^2 - 7A$$
$$= 2A.A^2 + 6AI^2 - 7A$$
$$= 8A - 7A = A$$

Solution5: We have

$$A = \begin{vmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{vmatrix}$$
$$A' = \begin{vmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{vmatrix}$$

We know that a matrix is symmetric if A = A'.

Thus,

$$A = \begin{vmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{vmatrix}$$

Now,

2b=3

$$\Rightarrow b = \frac{3}{2}$$

Also,

3a=−2

$$\Rightarrow a = \frac{-2}{3}$$

Therefore, $a = \frac{-2}{3}$ *and* $b = \frac{3}{2}$

Solution6: Let A and B be the points with position vectors $\vec{a} - 2\vec{b}$ *and* $2\vec{a} + \vec{b}$ respectively.

Also, let R divideAB externally in the ratio 2 : 1.

$\therefore$ Position vector of $\dfrac{2\times(2\vec{a}+\vec{b})-1\times(\vec{a}-2\vec{b})}{2-1}=3\vec{a}+4\vec{b}$

Section B

Solution7: Given:

$$\left(1+y^2\right)+\left(x-e^{\tan-1y}\right)\frac{dy}{dx}=0$$

$$Let\,tan^{-1}y=t$$

$$\Rightarrow y=tan\,t$$

$$\Rightarrow \frac{dy}{dx}=sec^2t\frac{dt}{dx}$$

Therefore, the equation becomes

$$\left(1+tan^2t\right)+\left(x-e^t\right)sec^2t\frac{dt}{dx}=0$$

$$\Rightarrow sec^2t+\left(x-e^t\right)\left(sec^2t\right)\frac{dt}{dx}=0$$

$$\Rightarrow 1+\left(x-e^t\right)\frac{dt}{dx}=0$$

$$\Rightarrow \left(x-e^t\right)\frac{dt}{dx}=-1$$

$$\Rightarrow x-e^t=\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt}+1.x=e^t$$

$$IF\quad {}_e\!\int 1.dt=e^t$$

$$\therefore e^t.\left(\frac{dx}{dt}+1.x\right)=e^t.e^t$$

$$e\Rightarrow \frac{d}{dt}\left(xe^t\right)=e^{2t}$$ Integrating both the sides, we get

$xe^t = \int e^{2t} dt$

$\Rightarrow xe^t = \frac{1}{2} e^{2t} + C \quad .....(1)$ Substituting the value of t in (1), we get

$xe^{\tan -1y} = \frac{1}{2} e^{2\tan^{-1y}} + C1$

$\Rightarrow e^{2\tan^{-1y}} = 2xe^{\tan -1y} + C$

It is the required general solution

Solution8: It is given that $\vec{a}, \vec{b}, \vec{b} + \vec{c}$ and are coplanar.

Therefore, Scalar triple product=Volume of the parallelopoid=0

$(\vec{a}+\vec{b}).\left[\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right]=0$

$\Rightarrow \left(\vec{a}+\vec{b}\right).\left[\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}\times\vec{a}\right)+0+\left(\vec{c}\times\vec{a}\right)\right]=0$

$\Rightarrow \vec{a}.\left(\vec{b}\times\vec{c}\right)+\vec{a}.\left(\vec{b}\times\vec{a}\right)+\vec{a}.(\vec{c}\times\vec{a})+\vec{b}.\left(\vec{b}\times\vec{c}\right)+\vec{b}.\left(\vec{b}\times\vec{a}\right)+\vec{b}.(\vec{c}\times\vec{a})=0$

$\Rightarrow \left[abc\right]+0+0+0+0+\left[bca\right]=0$

$\Rightarrow 2[abc]=0$

$\Rightarrow [abc]=0$

Therefore, the vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$ are coplanar.

Solution9: The equations of the given lines are

$\vec{r}=\left(8\hat{i}-19\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right) \quad .....(1)$

$\vec{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right) \quad .....(2)$

Normal parallel to (1) is $\vec{n_1}=3\hat{i}-16\hat{j}+7\hat{k}$.

Normal parallel to (2) is $\vec{n_2}=3\hat{i}+8\hat{j}-5\hat{k}$.

The required line is perpendicular to the given lines. So, the normal $\vec{n}$

parallel to the required line is perpendicular to $\vec{n}_1$ and $\vec{n}_2$.

$$\therefore \vec{n} = \vec{n}_1 - \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

Thus, the vector equation of the required line is

$$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \gamma\left(24\hat{i} + 36\hat{j} + 72\hat{k}\right)$$

$$\Rightarrow \vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + k\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) \qquad (Where\, k = 12\gamma)$$

Also, the Cartesian equation of the required line is

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

Solution10: Let E_1, E_2 and E_3 be the events denoting the selection of A, B and C as managers, respectively.

$P(E_1)$ = Probability of selection of $A = \frac{1}{7}$

$P(E_2)$ = Probability of selection of B $= \frac{2}{7}$

$P(E_3)$ = Probability of selection of C $= \frac{4}{7}$

Let A be the event denoting the change not taking place.

$P\left(\frac{A}{E_1}\right)$ = *Probability that A does not introduce change* = 0.2

$P\left(\frac{A}{E_2}\right)$ = *Probability that B does not introduce change* = 0.5

$P\left(\frac{A}{E_3}\right)$ = *Probability that C does not introduce change* = 0.7

$\therefore$ Required probability = $P\left(\frac{E_3}{A}\right)$

By Bayes' theorem, we have

$$P\left(\frac{E_3}{A}\right)$$

$$=\frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{B^2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$ v

$$=\frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2+\frac{2}{7}\times 0.5+\frac{4}{7}\times 0.7}$$

$$=\frac{2.8}{0.2+1+2.8}=\frac{2.8}{4}=0.7$$

OR

Total of 7 on the dice can be obtained in the following ways:

(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)

Probability of getting a total of 7 = $\frac{3}{36}=\frac{1}{6}$

Probability of not getting a total of 10 = $1-\frac{1}{12}=\frac{11}{12}$

Total of 10 on the dice can be obtained in the following ways:

(4, 6), (6, 4), (5, 5)

Probability of getting a total of 10 = $\frac{3}{36}=\frac{1}{12}$

Probability of not getting a total of 10 = $1-\frac{1}{12}=\frac{11}{12}$

Let E and F be the two events, defined as follows:

E = Getting a total of 7 in a single throw of a dice

F = Getting a total of 10 in a single throw of a dice

$P(E)\ =\ \frac{1}{6},\ P(\bar{E})=\frac{5}{6}P(F)\ =\frac{1}{12},\ P(\bar{F})=\frac{11}{12}$

A wins if he gets a total of 7 in 1st, 3rd or 5th ... throws.

Probability of A getting a total of 7 in the 1st throw = $\frac{1}{6}$

A will get the 3rd throw if he fails in the 1st throw and B fails in the 2nd throw.

Probability of A getting a total of 7 in the 3rd throw =

$$P(\bar{E})P(\bar{F})P(E)=\frac{5}{6}\times\frac{11}{12}\times\frac{1}{6}$$

Similarly, probability of getting a total of 7 in the 5th throw =

$$P(\bar{E})P(\bar{F})P(\bar{E})P(\bar{F})P(E)=\frac{5}{6}\times\frac{11}{12}\times\frac{5}{6}\times\frac{11}{12}\times\frac{1}{6}$$ and so on

Probability of winning of A $=$ $\frac{1}{6}+\left(\frac{5}{6}\times\frac{11}{12}\times\frac{1}{6}\right)+\left(\frac{5}{6}\times\frac{11}{12}\times\frac{5}{6}\times\frac{11}{12}\times\frac{1}{6}\right)+\ldots$

$$=\frac{\frac{1}{6}}{1-\frac{5}{6}\times\frac{11}{12}}=\frac{12}{17}$$

∴ Probability of winning of B = 1 − Probability of winning of A = $1-\frac{12}{17}=\frac{5}{17}$

Solution11: LHS: $\left(tan^{-1}\frac{1}{5}+tan^{-1}\frac{1}{7}\right)+\left(tan^{-1}\frac{1}{3}+tan^{-1}\frac{1}{8}\right)$

$$=tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right)+tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right)$$

$$\left[\because tan^{-1}A+tan^{-1}B=tan^{-1}\left(\frac{A+B}{1-AB}\right)\right]$$

$$=tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}\right)$$

$$=tan^{-1}1$$

$$=\frac{\pi}{4}$$

OR

$$2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)$$

$$=\tan^{-1}(2\operatorname{cosec}x)$$

$$\left[\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$\Rightarrow \frac{2\cos}{\sin^2 x}=2\operatorname{cosec} x$$

$$\Rightarrow \cos x=\sin x$$

$$\Rightarrow \tan x=1$$

$$\Rightarrow x=\frac{\pi}{4}$$

Solution12: Let the monthly incomes of Aryan and Babban be 3x and 4x, respectively.

Suppose their monthly expenditures are 5y and 7y, respectively.

Since each saves Rs 15,000 per month,

Monthly saving of Aryan: 3x-5y =15,000

Monthly saving of Babban: 4x−7y =15,000

The above system of equations can be written in the matrix form as follows:

$$\begin{bmatrix}3 & -5\\ 4 & -7\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}15000\\ 15000\end{bmatrix}$$

or,

AX = B, where A= $A=\begin{bmatrix}3 & -5\\ 4 & -7\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix} \text{ and } B=\begin{bmatrix}15000\\ 15000\end{bmatrix}$

Now,

$$|A| = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix} = -21 - (-20) = -1$$

$$Adj\ A = \begin{bmatrix} -7 & -4 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$$

So, $A^{-1} = \dfrac{1}{|A|} adj\ A = -1\begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$

$$\therefore X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 105000 - 75000 \\ 60000 - 45000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$$\Rightarrow x = 30,000\ and\ \ y = 15,000$$

Therefore,
Monthly income of Aryan $= 3 \times Rs\,30,000 = Rs\,90,000$
Monthly income of Babban $= 4 \times Rs\,30,000 = Rs\,1,20,000$
From this problem, we are encouraged to understand the power of savings. We should save certain part of our monthly income for the future.

Solution13: x = a sin 2t (1 + cos 2t)

y = b cos 2t (1 – cos 2t)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{b - [-2\sin 2t + 2\cos 2t \sin 2t \times 2]}{a[2\cos 2t + 2\cos 4t]}$$

$$\frac{dy}{dx} = \frac{b}{a}\left[\frac{-2\sin 2t + 2\sin 4t}{2\cos 2t + 2\cos 4t}\right]$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}}=\frac{b}{a}\left[\frac{-2+0}{0-2}\right]=\frac{b}{a}$$

and $$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{3}}=\frac{b}{a}\left[\frac{-2\sqrt{3}}{-2}\right]=\frac{\sqrt{3}b}{a}$$

OR

$y=x^x$

Applying logarithm,

log y = x log x

$$\frac{1}{y}\frac{dy}{dx}=\log x+x\times\frac{1}{x}=1+\log x$$

$$\frac{dy}{dx}=X^x[1+\log x]$$

$$\frac{d^2y}{dx^2}=\frac{d(x^x)}{dx}(1+\log x)+x^x\left[\frac{d}{dx}(1+\log x)\right]$$

$$=x^x(1+\log x)(1+\log x)+x^x\left[\frac{1}{x}\right]$$

$$=x^x(1+\log x)^2+x^{x-1}$$

$$\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=x^x(1+\log x)^2+x^{x-1}-\frac{1}{x^2}\left(x^x(1+\log x)^2\right)-\frac{x^x}{x}$$

$$=x^x(1+\log x)^2+x^{x-1}-x^x(1+\log x)^2-x^{x-1}$$

=0

Hence proved.

Solution14: $$f(x)=\begin{cases}\frac{1-\sin^3 x}{3\cos^2 x}, if\ x<\frac{x}{2}\\ P\quad, if\ x=\frac{\pi}{2}\\ \frac{q(1-\sin x)}{(\pi-2x)^2}\ , if\ x>\frac{\pi}{2}\end{cases}$$

for continuity,

$$\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}+} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}-} \left(\frac{1-\sin^3 x}{3\cos^2 x}\right) = \lim_{x\to\frac{\pi}{2}} \frac{(1-\sin x)(1+\sin^2 x+\sin x)}{3\left[1-\sin^2 x\right]}$$

$$\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}} \frac{(1+\sin^2 x+\sin x)}{3(1+\sin x)} = \frac{1+1+1}{3(2)} = \frac{1}{2}$$

$$Let\ \frac{\pi}{2} - x = \theta \Rightarrow x = \frac{\pi}{2} - \theta$$

$$\lim_{x\to\frac{\pi}{2}+} = \lim_{\theta\to 0} \frac{\left[1-\sin(\frac{\pi}{2}-\theta)\right]}{(2\theta)^2} = \frac{q}{4} \lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2}$$

$$= \frac{q}{4} \lim_{\theta\to 0} \frac{2\sin^2\frac{\theta}{2}}{\theta^2} = \frac{q}{2} \lim_{\theta\to 0} \frac{\sin^2\frac{\theta}{2}}{4\times(\theta/2)} = \frac{q}{8}$$

Now,

$$\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}+} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{2} = p = \frac{q}{8}$$

$$\Rightarrow P = \frac{1}{2}\ and\ q = \frac{8}{2} = 4$$

Solution15: Given: $x = 3cost - cos^3 t$
$y = 3sint - sin^3 t$

Slope of the tangent, $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dx}}=\frac{3\cos t-3\sin^2\cos t}{-3\sin t+3\cos^2 t\sin t}$

$=\frac{3\cos t-\left[\cos^2 t\right]}{-3\sin t\left[\sin^2 t\right]}$

$\frac{dy}{dx}=\frac{-\cos^{3t}}{\sin^3 t}$

$\therefore$ Slope of the normal$=\frac{\sin^3 t}{\cos^3 t}$

The equation of the normal is given by

$$\frac{y-(3\sin t-\sin^3 t)}{x-(3\cos t-\cos^3 t)}=\frac{\sin^3 t}{\cos^3 t}$$

$\Rightarrow y\cos^3 t-3\sin t\cos^3 t+\sin^3 t\cos^3 t$

$=x\sin^3 t-3\cos t\sin^3 t+\sin^3 t\cos^3 t$

$\Rightarrow y\cos^3 t-x\sin^3 t=3(\sin t\cos t-\cos t\sin^3 t)$

$\Rightarrow y\cos^3 t-x\sin^3 t=3\sin t\cos t(\cos^2 t-\sin^3 t)$

$\Rightarrow y\cos^3 t-x\sin^3 t=\frac{3}{2}\sin 2t\cos 2t=\frac{3}{4}\sin 4t$

$\Rightarrow y\cos^3 t-x\sin^3 t=3(\sin t\cos t-\cos t\sin^3 t)$

$\Rightarrow 4(y\cos^3 t-x\sin^3 t)=3\sin 4t$

Hence proved.

Solution16: $I=\int\frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta$

$\Rightarrow I=\int\frac{(3\sin\theta-2)\cos\theta}{5-(1-\sin^2\theta)-4\sin\theta}d\theta$

$\Rightarrow I=\int\frac{(3\sin\theta-2)\cos\theta}{\sin^2\theta-4\sin\theta+4}d\theta$

No, let $\sin\theta=t.$

$$\Rightarrow \cos\theta\, d\theta = dt$$

$$\therefore I = \int \frac{(3t-2)dt}{t^2-4t+4}$$

$$\Rightarrow 3t-2 = A\frac{d}{dx}\left(t^2-4t+4\right)+B$$

$$\Rightarrow 3t-2 = A\left(2t-4\right)+B$$

$$\Rightarrow 3t-2 = \left(2A\right)t+B-4A$$

Comparing the coefficients of the like powers of t, we get

$$2A = 3 \Rightarrow A = \frac{3}{2}$$

And B−4 A=−2

$$\Rightarrow B-4\times\frac{3}{2} = -2$$

$$\Rightarrow B = -2+6 = 4$$

Substituting the values of A and B, we get

$$3t-2 = \frac{3}{2}(2t-4)+4$$

$$\therefore I = \int \frac{(3t-2)dt}{t^2-4t+4}$$

$$= \int \left(\frac{\frac{3}{2}(2t-4)+4}{t^2-4t+4}\right)dt$$

$$= \frac{3}{2}\int\left(\frac{2t-4}{t^2-4t+4}\right)dt + 4\int\frac{dt}{t^2-4t+4}$$

$$= \frac{3}{2}I_1 + 4I_2 \qquad \text{......(1)}$$

Here,

$$I_1 = \int \frac{(2t-4)dt}{t^2-4t+4} \quad and \ I_2 = \int \frac{dt}{t^2-4t+4}$$

Now

$$I_1 = \int \frac{(2t-4)dt}{t^2-4t+4}$$

$$Let\ t^2-4t+4 = p$$

$$\Rightarrow (2t-4)dt = dp$$

$$I_1 = \int \frac{(2t-4)dt}{t^2-4t+4}$$

$$= \int \frac{dp}{p} = \log|p| + C_1$$

$$= \log\left|t^2-4t+4\right| + C_1 \qquad \text{..........(2)}$$

And $I_2 = \int \frac{dt}{t^2-4t+4}$

$$= \int \frac{dt}{(t-2)^2}$$

$$= \int (t-2)^{-2} dt$$

$$= \frac{(t-2)^{-2+1}}{-2+1} + C_2$$

$$= \frac{-1}{t-2} + C_2 \qquad \text{....(3)}$$

From (1), (2) and (3), we get

$$I = \frac{3}{2} log\ |t^2-4t+4\ | + 4 \times \frac{-1}{t-2} + C_1 + C_2$$

$$= 32\, log\ |sin^2\theta - 4\, sin\,\theta + 4\ | + \frac{4}{2-t} + C \qquad (where\ C = C_1 + C_2)$$

$$= \frac{3}{2}\log\left|(\sin\theta - 2)^2\right| + \frac{4}{2-\sin\theta} + C$$

$$= \frac{3}{2} \times 2\log\left|(\sin\theta - 2)\right| + \frac{4}{2-\sin\theta} + C$$

$$= 3\log\left|(2-\sin\theta\right| + \frac{4}{2-\sin\theta} + C$$

OR

$$Let\ I = \int_0^{\pi} e^{2x}\sin(\frac{\pi}{4}+x)dx$$

Integrating by parts, we get

$$\Rightarrow I = \frac{1}{2}\left[e^{2x}\,sin\left(\frac{\pi}{4}+x\right)_0^{\pi}\right] - \frac{1}{2}\left\{\left[\frac{1}{2}e^{2x}\cos\left(\frac{\pi}{4}+x\right)_0^{\pi}\right] + \frac{1}{2}\int_0^{\pi} e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx\right\}$$

$$\Rightarrow I = \frac{1}{2}\left[e^{2x}\,sin\left(\frac{\pi}{4}+x\right)_0^{\pi}\right] - \frac{1}{4}\left[e^{2x}\cos\left(\frac{\pi}{4}+x\right)_0^{\pi}\right] - \frac{1}{4}I$$

$$\Rightarrow \frac{5}{4}I = \frac{1}{2}\left[e^{2x}\,sin\left(\pi+\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\right] - \frac{1}{4}\left[e^{2\pi}\cos\left(\pi+\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \frac{5}{4}I = \frac{1}{2}\left[-e^{2x}\times\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] - \frac{1}{4}\left[e^{2\pi}\times\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow I = -\frac{1}{5\sqrt{2}}(e^{2\pi}+1)$$

Solution17: $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}dx$

$$Let:\ x^{\frac{3}{2}} = t$$

$$\Rightarrow \frac{3}{2}x^{\frac{1}{2}}dx = dt$$

$$x^{\frac{1}{2}}dx = \frac{2}{3}dt$$

Putting the values in I, we get

$$I=\int\frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$$
$$=\frac{2}{3}\int\frac{1}{\sqrt{a^3-t^2}}dt$$

Using the following formula of integration, we get

$$\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)$$

$$\therefore\frac{2}{3}\int\frac{1}{\sqrt{a^3-t}}dt=\frac{2}{3}sin^{-1}\left(\frac{t}{a^{\frac{3}{2}}}\right)+C$$

Again, putting the value of t, we get

$$\frac{2}{3}\int\frac{1}{\sqrt{a^3-t^2}}dt=\frac{2}{3}sin^{-1}\left(\frac{t}{a^{\frac{3}{2}}}\right)+C$$

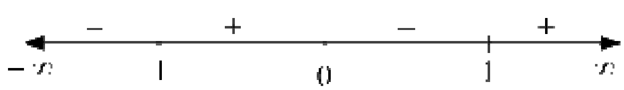
$$=\frac{2}{3}sin^{-1}\left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}}\right)+C$$

Here, C is constant of integration.

Solution18: Let: ${}^{2}\int_{-1}|x^3-x|dx.$

$$f(x)=x^3-x$$
$$f(x)=x^3-x=x(x-1)(x+1)$$

The signs of f(x) for the different values are shown in the figure given belo w:



$$f(x)>0\ for\ all\ x\in(-1,0)\cup(1,2)$$
$$f(x)<0\ for\ all\ x\in(0,1)$$

Therefore,

$$|x^3-x|=\{x^3-x,\ x\in(-1,0)\cup(1,2)-(x^3-x),\ x\in(0,1)$$

$$\therefore I = \int_{-1}^{2} x^3 - x \, | dx$$

$$= \int_{-1}^{0}(x^3 - x)dx - \int_{0}^{1}(x^3 - x)dx + \int_{1}^{2}(x^3 - x)dx$$

$$= \int_{-1}^{0} | x^3 - x \, | dx + \int_{0}^{1} | x^3 - x \, | dx + \int_{1}^{2} | x^3 - x \, | dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{10}^{0} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{16}{4} - \frac{4}{4}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{3}{4} + (4 - 2) = \frac{11}{4}$$

Solution19: Given:

$$\left(1 - y^2\right)\left(1 + logx\right) dx + 2xydy = 0$$

$$\Rightarrow \left(1 - y^2\right)\left(1 + logx\right) dx = -2xydy$$

$$\Rightarrow \left(\frac{1 - \log x}{2x}\right) dx = -\left(\frac{y}{1 - y^2}\right) dy \qquad .....(1)$$

Let: $1 + logx = t$

and $\left(1 - y^2\right) = p \Rightarrow \frac{1}{x} dx = dt \, and \, -2ydy = dp$

Therefore, (1) becomes

$$\int \frac{t}{2} dt = \int \frac{1}{2p} dp$$

$$\Rightarrow \frac{t^2}{4} = \frac{\log p}{2} + C \quad ....(2)$$

Substituting the values of t and p in (2), we get

$$\frac{(1 + \log x^2)}{4} = \frac{\log(1 - y^2)}{2} + C \qquad .....(3)$$

At x=1 and y=0, (3) becomes $C=\frac{1}{4}$

Substituting the value of C in (3), we get

$$\frac{(1+\log x^2)}{4}=\frac{\log(1-y^2)}{2}+\frac{1}{4}$$

$$\Rightarrow \left(1+logx^2\right)=2log\left(1-y^2\right)+1$$

Or

$$(logx)^2+logx^2=log\left(1-y^2\right)^2$$

It is the required particular solution.

SECTION C

Solution20: The equation of the plane passing through three given points can be given by

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ x-3 & y-0 & z-1 \\ x-4 & y+1 & z-0 \end{vmatrix}=0$$

Performing elementary row operations $R2\rightarrow R1-R2\ and\ R3\rightarrow R1-R3,$ we get

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 3-2 & 0-2 & 0 \\ 4-2 & -1-2 & -1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix}=0$$

Solving the above determinant, we get

$$\Rightarrow (x-2)(2-0)-(y-2)(-1-0)+(z-1)(-3+4)=0$$
$$\Rightarrow (2x-4)+(y-2)+(z-1)=0$$
$$\Rightarrow 2x+y+z-7=0$$

Therefore, the equation of the plane is $2x+y+z-7=0$.

Now, the equation of the line passing through two given points is

$$\frac{x-3}{2-3}=\frac{y+4}{-3+4}=\frac{z+5}{1+5}=\lambda$$

$$\Rightarrow \frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=\lambda$$

$$\Rightarrow x=(-\lambda+3), y=(\lambda-4), z=(6\lambda-5)$$

At the point of intersection, these points satisfy the equation of the plane $2x+y+z-7=0$.

Putting the values of x, y and z in the equation of the plane, we get the value of λ .

$$2(-\lambda+3)+(\lambda-4)+(6\lambda-5)-7=0$$
$$\Rightarrow -2\lambda+6+\lambda-4+6\lambda-5-7=0$$
$$\Rightarrow 5\lambda=10$$
$$\Rightarrow \lambda=2$$

Thus, the point of intersection is P(1, −2, 7).

Now, let P divide the line AB in the ratio m : n.

By the section formula, we have

$$1=\frac{2m+3m}{m+n}$$
$$\Rightarrow m+2n=0$$
$$\Rightarrow m=-2n$$
$$\Rightarrow mn=\frac{-2}{1}$$

Hence, P externally divides the line segment AB in the ratio 2 : 1.

Solution21: Let X denote the total number of red balls when four balls are drawn one by one with replacement.

P (getting a red ball in one draw) = 23

P (getting a white ball in one draw) = 13

x	0	1	2	3	4
P(x)	$\left(\frac{1}{3}\right)^4$	$\frac{2}{3}\left(\frac{1}{3}\right)^3 \cdot {}^4C_1$	$\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 \cdot {}^4C_2$	$\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right) \cdot {}^4C_3$	$\left(\frac{2}{3}\right)^4$

	$\frac{1}{81}$	$\frac{8}{81}$	$\frac{24}{81}$	$\frac{32}{81}$	$\frac{16}{81}$

Using the formula for mean, we have

$$\bar{X} = \sum P_i X_i$$

$$Mean\left(\bar{X}\right)=\left(0\times\frac{1}{81}\right)+1\left(\frac{8}{81}\right)+2\left(\frac{24}{81}\right)+3\left(\frac{32}{81}\right)+4\left(\frac{16}{81}\right)$$

$$=\frac{1}{81}(8+48+96+64)$$

$$=\frac{216}{81}=\frac{8}{3}$$

Using the formula for variance, we have

$$Var\left(X\right)=\sum P_i X_i^2-\left(\sum P_i X_i\right)^2$$

$$Var\left(X\right)=\left\{\left(0\times\frac{1}{81}\right)+1\left(\frac{8}{81}\right)+4\left(\frac{24}{81}\right)+9\left(\frac{32}{81}\right)+16\left(\frac{16}{81}\right)\right\}-\left(\frac{8}{3}\right)^2$$

$$=\frac{64}{81}-\frac{64}{9}=\frac{8}{9}$$

Hence, the mean of the distribution is $\frac{8}{3}$ and the variance of the distribution is $\frac{8}{9}$.

Solution22: Let the numbers of units of products A and B to be produced be x and y, respectively.

Product	Machine	
	I (h)	II (h)
A	3	3
B	2	1

Total profit: $Z = 7x + 4y$

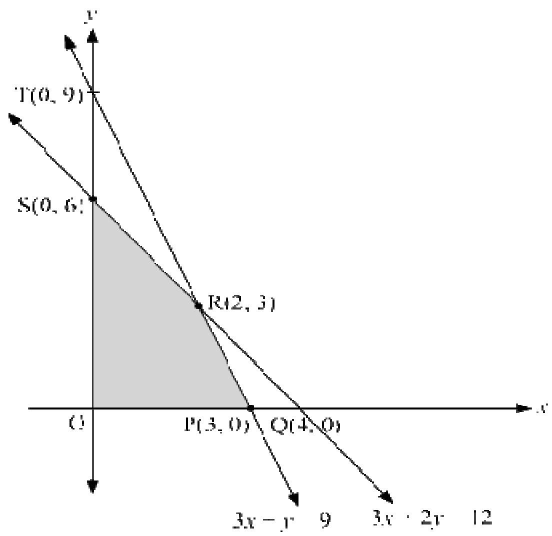
We have to maximise Z = 7x + 4y, which is subject to constraints.

$3x+2y\le 12$ (Constraint on machine I)

$3x + y \le 9$ (Constraint on machine II)

$\Rightarrow x \ge 0 \, and \; y \ge 0$

The given information can be graphically expressed as follows:



Values of Z = 7x + 4y at the corner points are as follows

Corner Point	Z = 7x + 4y
(0, 6)	24
(2, 3)	**26**
(3, 0)	21

Therefore, the manufacturer has to produce 2 units of product A and 3 units of product B for the maximum profit of Rs 26.

Solution23: $Given: f(x) = 9x^2 + 6x - 5$

$Let \; y = 9x^2 + 6x - 5$

$\Rightarrow y = (3x+1)^2 - 1 - 5 = (3x+1)^2 - 6$

$\Rightarrow (3x+1)^2 = y + 6$

$\Rightarrow 3x + 1 = \sqrt{y+6}$

$\Rightarrow x = \dfrac{\sqrt{y+6}-1}{3}\ as\ x \in N$

$\Rightarrow \sqrt{y+6}-1>0$

$\Rightarrow y+6>1$

$\Rightarrow y>-5\ and\ y \in N$

So, the function is invertible if the range of the function f(x) is {1, 2, 3, ...}.
Therefore, the inverse of the function f(x) is $f^{-1}(y),\ i.e. x.$

Now,

$f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3}$

$f^{-1}(43) = \dfrac{\sqrt{43+6}-1}{3} = 2$

$f^{-1}(163) = \dfrac{\sqrt{163+6}-1}{3} = 4$

Solution24: $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} xy+yz+zx-x^2-y^2-z^2 & zx-y^2 & xy-z^2 \\ xy+yz+zx-x^2-y^2-z^2 & xy-z^2 & xy-x^2 \\ xy+yz+zx-x^2-y^2-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \left(xy+yz+zx-x^2-y^2-z^2\right)\begin{vmatrix} 1 & zx-y^2 & xy-z^2 \\ 1 & xy-z^2 & yz-x^2 \\ 1 & yz-x^2 & zx-y^2 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1\ and\ R_3 \to R_3 - R_1$, we get

$$\Delta=\left(xy+yz+zx-x^2-y^2-z^2\right)\begin{vmatrix}1 & zx-y^2 & xy-z^2\\0 & (x+y+z)(y-z) & (x+y+z)(z-x)\\0 & (x+y+z)(y-x) & (x+y+z)(z-y)\end{vmatrix}$$

$$\Rightarrow \Delta=(x+y+z)^2\left(xy+yz+zx-x^2-y^2-z^2\right)\left[\{(y-z)(z-y)-(z-x)(y-x)\}-0+0\right]$$

$$\Rightarrow \Delta=(x+y+z)^2\left(xy+yz+zx-x^2-y^2-z^2\right)^2$$

So, Δ is divisible by (x + y + z).
The quotient when Δ is divisible by
$(x+y+z)$ *is* $(x+y+z)\left(xy+yz+zx-x^2-y^2-z^2\right)$.

OR

A = IA
i.e.

$$\begin{bmatrix}8 & 4 & 3\\2 & 1 & 1\\1 & 2 & 2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A$$

Applying $R_1 \leftrightarrow R_3,$ *we get*

$$\begin{bmatrix}1 & 2 & 2\\2 & 1 & 1\\8 & 4 & 3\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix}A$$

Applying $R_2 \to R_2-2R_1,$ *we get*

$$\begin{bmatrix}1 & 2 & 2\\0 & -3 & -3\\8 & 4 & 3\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\0 & 1 & -2\\1 & 0 & 0\end{bmatrix}A$$

Applying $R_3 \to R_3-8R_1,$ *we get*

$$\begin{bmatrix}1 & 2 & 2\\0 & -3 & 1\\0 & -12 & -13\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\0 & 1 & -2\\1 & 0 & -8\end{bmatrix}A$$

Applying $R_3 \to \frac{R_2}{-3}$, *we get*

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & -12 & -13 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -\frac{1}{3} & \frac{2}{3} \\ 1 & 0 & -8 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 2R_2$, *we get*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -12 & -13 \end{bmatrix} = \begin{bmatrix} 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{2}{3} \\ 1 & 0 & -8 \end{bmatrix} A$$

Applying $R_3 \to R_3 + 12R_2$, *we get*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{2}{3} \\ 1 & -4 & 0 \end{bmatrix} A$$

Applying $R_3 \to -R_3$ *and* $R_2 \to R_2 - R_3$, *we get*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{2}{3} & -\frac{1}{3} \\ 1 & -\frac{13}{3} & \frac{2}{3} \\ -1 & 4 & 0 \end{bmatrix} A$$

Thus, we have

$$A^{-1}=\begin{bmatrix}0 & \frac{2}{3} & -\frac{1}{3}\\ 1 & -\frac{13}{3} & \frac{2}{3}\\ -1 & 4 & 0\end{bmatrix}$$

The given system of equations is

$8x+4y+3z=19$

$2x+y+z=5$

$x+2y+2z=7$

This system of equations can be written as AX = B, where

$$A=\begin{bmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\ and\ B=\begin{bmatrix}19\\ 5\\ 7\end{bmatrix}$$

$\therefore\ X=A^{-1}B$

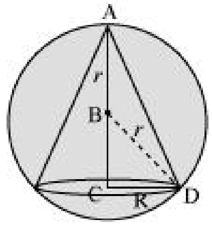
$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0 & \frac{2}{3} & -\frac{1}{3}\\ 1 & -\frac{13}{3} & \frac{2}{3}\\ -1 & 4 & 0\end{bmatrix}\begin{bmatrix}19\\ 5\\ 7\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0+\frac{10}{3}-\frac{7}{3}\\ 19-\frac{65}{3}+\frac{14}{3}\\ -19+20+0\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$$

$\therefore\ x=1,\ y=2\ and\ z=1$

Solution25: A sphere of fixed radius (r) is given.

Let R and h be the radius and the height of the cone, respectively.



The volume (V) of the cone is given by,

$$V = \frac{1}{3}\pi R^2 h$$

Now, from the right triangle BCD, we have:

$$BC = \sqrt{r^2 - R^2}$$

$$\therefore h = r + \sqrt{r^2 - R^2}$$

$$\therefore V = \frac{1}{3}\pi R^2\left(r + \sqrt{r^2 - R^2}\right) = \frac{1}{3}\pi R^2 r + \frac{1}{3}\pi R^2\sqrt{r^2 - R^2}$$

$$\therefore \frac{dV}{dR} = \frac{2}{3}\pi Rr + \frac{2}{3}\pi R\sqrt{r^2 - R^2} + \frac{\pi R^2}{3}\cdot\frac{(-2R)}{2\sqrt{r^2 - R^2}}$$

$$= \frac{2}{3}\pi Rr + \frac{2}{3}\pi R\sqrt{r^2 - R^2} - \frac{\pi R^3}{3\sqrt{r^2 - R^2}}$$

$$= \frac{2}{3}\pi Rr + \frac{2\pi R\left(r^2 - R^2\right) - \pi R^3}{3\sqrt{r^2 - R^2}}$$

$$= \frac{2}{3}\pi Rr + \frac{2\pi Rr^2 - 3\pi R^3}{3\sqrt{r^2 - R^2}}$$

$$\text{Now, } \frac{dV}{dR^2} = 0$$

$$\Rightarrow \frac{2\pi rR}{3} = \frac{3\pi R^3 - 2\pi Rr^2}{3\sqrt{r^2 - R^2}}$$

$$\Rightarrow 2r\sqrt{r^2 - R^2} = 3R^2 - 2r^2$$

$$\Rightarrow 4r^2\left(r^2 - R^2\right) = \left(3R^2 - 2r^2\right)^2$$

$$\Rightarrow 4r^4 - 4r^2R^2 = 9R^4 + 4r^4 - 12R^2r^2$$

$$\Rightarrow 9R^4 - 8r^2R^2 = 0$$

$$\Rightarrow 9R^2 = 8r^2$$

$$\Rightarrow R^2 = \frac{8r^2}{9}$$

$$\text{Now, } \frac{d^2V}{dR^2} = \frac{2\pi r}{3} + \frac{3\sqrt{r^2-R^2}\left(2\pi r^2 - 9\pi R^2\right) - \left(2\pi R r^2 - 3\pi R^3\right)(-6R)\frac{1}{2\sqrt{r^2-R^2}}}{9\left(r^2-R^2\right)}$$

$$= \frac{2\pi r}{3} + \frac{3\sqrt{r^2-R^2}\left(2\pi r^2 - 9\pi R^2\right) + \left(2\pi R r^2 - 3\pi R^3\right)(3R)\frac{1}{2\sqrt{r^2-R^2}}}{9\left(r^2-R^2\right)}$$

Now, when $R^2 = \frac{8r^2}{9}$, it can be shown that $\frac{d^2V}{dR^2} < 0$.

∴ The volume is the maximum when $R^2 = \frac{8r^2}{9}$.

When $R^2 = \frac{8r^2}{9}$, height of the cone $= r + \sqrt{r^2 - \frac{8r^2}{9}} = r + \sqrt{\frac{r^2}{9}} = r + \frac{r}{3} = \frac{4r}{3}$.

Hence, it can be seen that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Let volume of the sphere be $V_s = \frac{4}{3}\pi r^3$.

$$r = 3\sqrt{\frac{3V_s}{4\pi}}$$

∴ Volume of cone, $V = \frac{1}{3}\pi R^2 h$

$$\Rightarrow R = \frac{2\sqrt{2}}{3}r$$

$$V = \frac{1}{3}\pi\left(\frac{2\sqrt{2}}{3}r\right)^2 \times \frac{4r}{3}$$

$$\Rightarrow V = \frac{1}{3}\pi\frac{8r^3}{9} \times \frac{4r}{3}$$

$$V = \frac{32\pi r^3}{81} = \frac{32}{81}\pi\left[\frac{3V_s}{4\pi}\right]$$

∴ Volume of cone in terms of sphere = $\frac{8V_8}{27}$

OR

Consider the function $f(x) = sin3x - cos3x.$

f'(x)= 3cos3x+3sin3x

=3(sin3x + cos3x)

$$= 3\sqrt{2}\left\{sin3x\,cos\left(\frac{\pi}{4}\right) + cos3x\,sin\left(\frac{\pi}{4}\right)\right\}$$

$$= 3\sqrt{2}\left\{sin\left(3x + \frac{\pi}{4}\right)\right\}$$

For the increasing interval $f'(x) > 0.$

$$3\sqrt{2}\left\{sin\left(3x + \frac{\pi}{4}\right)\right\} > 0$$

$$sin\left(3x + \frac{\pi}{4}\right) > 0$$

$$\Rightarrow 0 < 3x + \frac{\pi}{4} < \pi$$

$$\Rightarrow 0 < 3x + \frac{3\pi}{4}$$

$$\Rightarrow 0 < x < \frac{\pi}{4}$$

Also,

$$sin\left(3x + \frac{\pi}{4}\right) > 0$$

when, $2\pi < 3x + \frac{\pi}{4} < 3\pi$

$$\Rightarrow \frac{7\pi}{4} < 3x < \frac{11\pi}{44}$$

$$\Rightarrow \frac{7\pi}{12} < x < \frac{11\pi}{12}$$

Therefore, intervals in which function is strictly increasing in

$0 < x < \frac{\pi}{4} and \frac{7\pi}{12} < x < \frac{11\pi}{12}$.

Similarly, for the decreasing interval $f'(x) < 0$.

$$3\sqrt{2}\left\{sin\left(3x+\frac{\pi}{4}\right)\right\}<0$$

$$sin\left(3x+\frac{\pi}{4}\right)<0$$

$$\Rightarrow \pi < 3x+\frac{\pi}{4} < 2\pi$$

$$\Rightarrow \frac{3\pi}{4} < 3x < \frac{7\pi}{4}$$

$$\Rightarrow \frac{\pi}{4} < x < \frac{7\pi}{12}$$

Also,

$$sin\left(3x+\frac{\pi}{4}\right)<0$$

$$When\, 3\pi < 3x+\frac{\pi}{4} < 4\pi,$$

$$\Rightarrow \frac{11\pi}{4} < 3x < \frac{15\pi}{4}$$

$$\Rightarrow \frac{11\pi}{12} < x < \frac{15\pi}{12}$$

The function is strictly decreasing in $\frac{\pi}{4} < x < \frac{7\pi}{12} and \frac{11\pi}{12} < x < \pi.$

Solution26: Given: $x^2 + y^2 \le 2ax, y^2 \ge ax, x, y \ge 0$

$$\Rightarrow x^2 + y^2 - 2ax \le 0,\; y^2 \ge ax, x, y \ge 0$$

$$\Rightarrow x^2 + y^2 - 2ax + a^2 - a^2 \le 0,\; y^2 \ge ax, x, y \ge 0$$

$$\Rightarrow (x-a)^2 + y^2 \le a^2,\; y^2 \ge ax, x, y \ge 0$$

To find the points of intersection of the circle $\left[(x-a)^2 + y^2 = a^2\right]$

and the parabola $\left[y^2 = ax\right]$, we will substitute $y^2 = ax\; in (x-a)^2 + y^2 = a^2$.

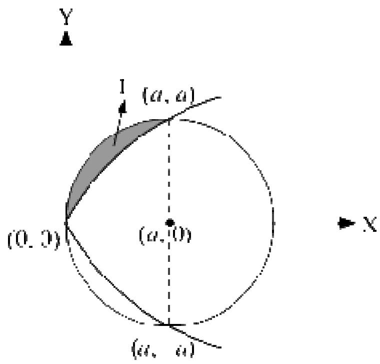
$$(x-a)^2 + ax = a^2$$

$\Rightarrow x^2 + a - 2ax + ax = a^2$

$\Rightarrow x\left(x-a\right) = 0$

$\Rightarrow x = 0,\ a$

Therefore, the points of intersection are (0, 0), (a, a) and (a, −a).



Now,

Area of the shaded region= I

Area of I from x=0 to x=a

$=\left[\int_0^a\left(\sqrt{a^2-(x-a)^2}\right)dx - \int_0^a\sqrt{ax}\,dx\right]$ Let x−a=t for the first part of the integral

$=\int_0^a\left(\sqrt{a^2-(x-a)^2}\right)dx.$

$\Rightarrow dx = dt$

$\therefore A_I = \int_{-a}^{0}\sqrt{a^2-t^2}\,dt - 2\frac{\sqrt{a}}{3}\left|x^{\frac{3}{2}}\right|_0^a$

$=\left|\frac{t}{2}\sqrt{a^2-t^2}+\frac{1}{2}a^2\sin^{-1}\frac{t}{a}\right|_{-a}^{0} - \frac{2a^2}{3}$

$=0-\left(-\frac{\pi a^2}{4}\right)-\frac{2a^2}{3}$

$A_I = \left(\frac{\pi}{4}-\frac{2}{3}\right)a^2$

$\therefore$ Area of the shaded region$=\left(\frac{\pi}{4}-\frac{2}{3}\right)a^2$ square units