Question Bank - Quardratic Equation

LEVEL – I

1. **Solve the following for real values of x :**

(a) $3|x^2-4x+2|=5x-4$ **(b)** $|x^2+4x+3|+2x+5=0$

(c) $(x+3).|x+2|+|2x+3|+1=0$ **(d)** $(x-1)|x^2-4x+3|+2x^2+3x-5=0$

(e) $|(x+3)|(x+1)+|2x+5|=0$ **(f)** $|x^3+1|+x^2-x-2=0$

(g) $2^{|x+2|}-|2^{x+1}-1|=2^{x+1}+1$

2. **Solve the following equations / inequations for real x :**

(a) $3^x.8^{x/(x+2)}=6.$ **(b)** $\log_{[(x+6)/3]}[\log_2\{(x-1)/(2+x)\}]>0$

(c) $\dfrac{1}{\log_4[(x+1)/(x+2)]}<\dfrac{1}{\log_4(x+3)}$ **(d)** $x^{1/\log_{10}x}.\log_{10}x<1$

(e) $\log_{1/2}(x+1)>\log_2(2-x).$ **(f)** $\log_x 2 . \log_{2x} 2 . \log_2 4x >1.$

(g) $\log_{1/2}x+\log_3 x>1.$ **(h)** $\log_x\dfrac{4x+5}{6-5x}<-1.$

(i) $(\log_{|x+6|}2)\log_2(x^2-x-2)\geq 1.$ **(j)** $\dfrac{\log_5(x^2-4x+11)^2-\log_{11}(x^2-4x-11)^3}{\sqrt{2-5x-3x^2}}\geq 0$

3. Solve the equations for $a^2-b=1$:

(i) $(a+\sqrt{b})^x+(a-\sqrt{b})^x=2a$ **(ii)** $(a+\sqrt{b})^{x^2-15}+(a-\sqrt{b})^{x^2-15}=2a.$

4. **(a)** If α be a root of the equation $4x^2+2x-1=0$ then prove that $4\alpha^3-3\alpha$ is the other root

(b) If α,β are the roots of the equation $x^2-2x+3=0$ obtain the equation whose roots are $\alpha^3-3\alpha^2+5\alpha-2,\ \beta^3-\beta^2+\beta+5.$

5. **(a)** If $\alpha,\ \beta$ be the roots of the equation, $\lambda^2(x^2-x)+2\lambda x+3=0$ and $\lambda_1,\ \lambda_2$ be the two values of λ for which α and β are connected by the relation, $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{4}{3}$, then find the quadratic equation whose roots are $\dfrac{\lambda_1^2}{\lambda_2}$ and $\dfrac{\lambda_2^2}{\lambda_1}$.

(b) If α, β are the roots of $ax^2 + bx + c = 0$ & $\alpha', -\beta$ are the roots of $a'x^2 + b'x + c' = 0$, show that α, α' are the roots of $\left[\frac{b}{a} + \frac{b'}{a'}\right]^{-1} x^2 + x + \left[\frac{b}{c} + \frac{b'}{c'}\right]^{-1} = 0$.

6. Find the range of values of a, such that $f(x) = \frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.

7. **(a)** If the ratio of the roots of $\ell x^2 + nx + n = 0$ is p : q, then prove that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} - \sqrt{\frac{n}{\ell}} = 0$, where ℓ, n, p, $q \in R^+$.

(b) If the roots of the equation $[1/(x + p)] + [1/(x + q)] = 1/r$ are equal in magnitude but opposite in sign, show that $p + q = 2r$ and that the product of the roots is equal to $(-1/2)(p^2 + q^2)$.

(c) Show that if p, q, r and s are real numbers and $pr = 2(q + s)$, then at least one of the equations $x^2 + px + q = 0$, $x^2 + rx + s = 0$ has real roots.

(d) If by eliminating x between the equation $x^2 + ax + b = 0$ and $xy + \ell(x + y) + m = 0$, a quadratic in y is formed whose roots are the same as those of the original quadratic in x. Then prove either $a = 2\ell$ and $b = m$ or $b + m = a\ell$.

(e) Prove that if both roots of the equation $x^2 + px + q = 0$ are positive then the roots of the equation $qy^2 + (p - 2rq)y + 1 - pr = 0$ are positive for all $r \geq 0$. Discuss the case when $r < 0$.

8. If x_1, x_2 be the roots of the equation $x^2 - 3x + A = 0$ & x_3, x_4 be those of the equation $x^2 - 12x + B = 0$ and x_1, x_2, x_3, x_4 are in G.P. Find A and B.

9. Show that the function $z = 2x^2 + 2xy + y^2 - 2x + 2y + 2$ is not smaller than -3. $\forall x, y \in R$.

10. **(a)** Prove that the function $y = (x^2 + x + 1)/(x^2 + 1)$ cannot have values greater than 3/2 and values smaller than 1/2 for $\forall\ x \in R$.

(b) Find the least value of $(6x^2 - 22x + 21)/(5x^2 - 18x + 17)$ for all real values of x, using the theory of quadratic equations.

(c) Find the minimum value of the expression $2 . \log_{10} x - \log_x 0.01$; where $x > 1$.

(d) Find the values of 'a' for which $-3 < [(x^2 + ax - 2)/(x^2 + x + 1)] < 2$ is valid for all real x.

11. **(a)** If the quadratic equation $x^2 + bx + ac = 0$ and $x^2 + cx + ab = 0$ have a common root, prove that the equation containing their other roots is $x^2 + ax + bc = 0$, where $a \neq 0$.

(b) If $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have a common root and a/a_1, b/b_1, c/c_1 are in AP, show that a_1, b_1 and c_1 are in G.P.

(c) If the equations $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ with rational coefficients have one and only one root in common then prove that $b^2 - ac$ and $b_1^2 - a_1c_1$ will be both perfect squares.

12. If $(ax^2 + bx + c)y + a'x^2 + b'x + c' = 0$, find the condition that x may be a rational function of y.

13. $x^2 - (a - 5)x + 4a = 0$ $(a \in R)$ be a quadratic equation. Find the value of 'a' for which

(a) both roots are real and distinct

(b) both roots are equal

(c) roots are not real

(d) roots are opposite in sign

(e) roots are equal in magnitude but opposite in sign

(f) both roots are positive

(g) both roots are negative

(h) atleast one root is positive

(i) one root is smaller than 2, the other root is greater than 2

(j) both roots are greater than 2

(k) both roots are smaller than 2

(l) exactly one of the roots lie in the interval (1, 2)

(m) both roots lie in the interval (1, 2)

(n) atleast one root lie in the interval (1, 2)

(o) one root is greater than 2, the other roots is smaller than 1

(p) atleast one root is greater than 2.

14. (a) If α, β are the two distinct roots of $x^2 + 2(k - 3) . x + 9 = 0$, then find the values of k such that $\alpha, \beta \in (-6, 1)$.

(b) For what real values of 'a' the equation $ax^2 + x + a - 1 = 0$ posses two distinct real roots α and β satisfying the inequality $\left|\frac{1}{\alpha} - \frac{1}{\beta}\right| > 1$.

15. If α is a root of $ax^2 + bx + c = 0$, β is a root of $-ax^2 + bx + c = 0$, where $0 < \alpha < \beta$, show that the equation $ax^2 + 2bx + 2c = 0$ has a root γ satisfying $0 < \alpha < \gamma < \beta$.

16. If the quadratic equation $ax^2 + bx + c = 0$ has real roots, of opposite signs in the interval (-2, 2) then prove that $1 + \frac{c}{4a} - \left|\frac{b}{2a}\right| > 0$.

17. **(a)** If $(x - 3a)(x - a - 3) < 0$ for all $x \in [1, 3]$ find a.

(b) Find all numbers a for each for which the least value of quadratic trinomial $4x^2 - 4ax + a^2 - 2a + 2$ on the interval $0 \le x \le 2$ is equal to 3.

18. **(a)** Find the set of values of p for which the equation $p.2^{\cos^2 x} + p.2^{-\cos^2 x} - 2 = 0$ has real roots.

(b) Solve the equation $9^{-|x-2|} - 4.3^{-|x-2|} - a = 0$ for every real number a.

19. **(a)** The quadratic equation $x^2 + px + q = 0$ where p and q are integers has rational roots. Prove that the roots are all integral.

(b) If the coefficients of the quadratic equation $ax^2 + bx + c = 0$ are odd integers then prove that the roots of the equation cannot be rational number.

(c) If $a, b, c \in I$ and $ax^2 + bx + c = 0$ has an irrational root. Prove that $|f(\lambda)| \ge \frac{1}{q^2}$, where $\lambda \in Q = \frac{p}{q}$ and $f(x) = ax^2 + bx + c$.

(d) Let a, b and c be integers with a > 1 and let p be a prime number. Show that if $ax^2 + bx + c$ is equal to 'p' for two distinct integral values of x, then it can't be equal to '2p' for any integral value of x.

20. **(a)** How many roots does the equation $\sqrt{x^2+1} - \frac{1}{\sqrt{x^2 - \frac{5}{3}}} = x$ posses ? Find the them.

(b) Find the a < 0 for which the inequalities $2\sqrt{ax} < 3a - x$ and $x - \sqrt{\frac{x}{a}} > \frac{6}{a}$ have solutions in common.

LEVEL – II

1. Find the values of a for which the equation $x^4 + (1 - 2a)x^2 + a^2 - 1 = 0$
(a) has no solutions **(b)** has one solution
(c) has two solutions **(d)** has three solutions.
(e) has four distinct real solutions

2. Find all real values of a for which the equation $x^4 + (a - 1)x^3 + x^2 + (a - 1)x + 1 = 0$ possesses at least two distinct negative roots.

3. Find the real values of 'm' for which the equation, $\left(\frac{x}{1+x^2}\right)^2 - (m-3)\left(\frac{x}{1+x^2}\right) + m = 0$ has real roots ?

4. Find all values of 'a' for which the equation
$(x^2 + x + 2)^2 - (a - 3)(x^2 + x + 2)(x^2 + x + 1) + (a - 4)(x^2 + x + 1)^2 = 0$ has at least one real roots.

5. If the equation $x(x + 1)(x + a)(x + 1 + a) = a^2$ has four real roots,
prove that $a \in \left(-\infty, -\sqrt{5}-2\right] \cup \left[-\sqrt{5}+2, \sqrt{5}-2\right] \cup \left[\sqrt{5}+2, +\infty\right)$.

6. Prove that the minimum value of $[(a+x)(b+x)]/(c+x), x > -c$ is $\left(\sqrt{a-c}+\sqrt{b-c}\right)^2$. $\forall\ a > c$ and $b > c$.

7. Find all values of k for which the inequality $\frac{x^2+k^2}{k(6+x)} \geq 1$ is satisfied for all x such that $-1 < x < 1$.

8. For what real value of 'a' do the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ lie between the roots of the equation $x^2 - 2(a + 1)x + a(a - 1) = 0$?

9. A quadratic trinomial $f(x) = ax^2 + bx + c$ is such that the equation $f(x) = x$ has no real roots. Prove that in this case the equation $f(f(x)) = x$ has no real roots either.

10. Find all real values of a for each of which the equation $\sqrt{x-a}\,(x^2+(1+2a^2)x+2a^2) = 0$ has only two distinct roots. Write the roots.

11. **(a)** If $3x^2 - 5x + 9 = y^2$ for $x, y \in Q$, show that $x = \frac{6m+5}{3-m^2}$, $m \in Q$.

(b) Find all nonnegative integral solutions of $y^2 + 6xy - 8x = 0$. Does it contain a positive integral solution which is a multiple of 3 ?

12. **(a)** If the equation $ax^2 - bx + c = 0$ has two distinct real roots between 1 and 2, where $a, b, c \in N$, show that $a \geq 5$ and $b \geq 11$.

(b) If $ax^2 - bx + c = 0$ have two distinct roots lying in the interval (0, 1), where $a, b, c \in N$ then prove that $\log_5 abc \geq 2$.

13. **(a)** Find the integral values of 'a' for which $(a + 2)x^2 + 2(a + 1)x + a = 0$ will have both roots integers.

(b) Find the integral values of 'm' for which the roots of the equation $mx^2 + (2m - 1)x + m - 2 = 0$ are rational.

(c) Find the values of a so that $x^2 - x - a = 0$ has integral roots, where $a \in N$, and $6 \le a \le 100$.

(d) If a, b – 1 and c are odd prime numbers and $ax^2 + bx + c = 0$ has rational roots then, prove that one root of the equation will be independent of a, b and c.

(e) Show that the quadratic equation $x^2 + 7x - 14(q^2 + 1) = 0$, where q is an integer, has no integral roots.

(f) Let $x^2 - px + q = 0$ and $x^2 - qx + p = 0$ both have unequal integral roots, where $p, q \in N$. Prove that the possible number of solutions of the ordered point (p, q) is 2. Find them.

14. Find the integral values of x and y satisfying the system of inequalities $y - |x^2 - 2x| + (1/2) > 0$ & $y + |x - 1| < 2$.

15. **(a)** Find the value of a for which inequality $ax^2 + 4x + 10 \le 0$ has atleast one real solution and every solution of the inequality $x^2 - x - 2 < 0$ is larger than any solution of the inequality $ax^2 + 4x + 10 \le 0$.

(b) Find all values of the parameter 'k' for which the solution set of the inequation $x^2 + 3k^2 - 1 \ge 2k(2x - 1)$ is a subset of the solution set of the inequation $x^2 - (2x - 1)k + k^2 \ge 0$.

(c) Find all values of k for which there is at least one common solution of the inequalities $x^2 + 4kx + 3k^2 > 1 + 2k$ and $x^2 + 2kx \le 3k^2 - 8k + 4$.

(d) Find all values of 'k' for which any real x is a solution of at least one of the inequalities $x^2 + 5k^2 + 8k > 2(3kx + 2)$ and $x^2 + 4k^2 \ge k(4x + 1)$.

16. **(a)** Find all the value's of the parameters c for which the inequality has at least one solution

$$1 + \log_2\left(2x^2 + 2x + \frac{7}{2}\right) \ge \log_2\left(cx^2 + c\right).$$

(b) Find the value of 'b' for which the equation $2\log_{\frac{1}{25}}(bx + 28) = -\log_5\left(12 - 4x - x^2\right)$ has

(i) only one solution (ii) two different solutions (iii) no solution

17. **Solve the following for real values of x (depending upon the real parameter if any)**

(a) $|x| < \frac{a}{x}$

(b) $x + \sqrt{a + \sqrt{x}} = a$.

(c) $x^3 + 1 = 2\sqrt[3]{2x - 1}$.

(d) $x^2 - \sqrt{a - x} = a$.

(e) $x + \frac{x}{\sqrt{x^2 - 1}} > \frac{35}{12}$.

18. **(a)** Find the greatest value of function $f(x) = \frac{1}{2bx^2 - x^4 - 3b^2}$ on the interval [-2, 1] depending on the parameter b.

(b) Find the greatest value of the function $f(x) = x^4 - 6bx^2 + b^2$ on the interval [-2, 1] depending on the parameter b.

19. **(a)** Find all the values of $a \in R$ such that the equality $a^3 + a^2 |a + x| + |a^2x + 1| = 1$ has atleast four integer solutions for x.

(b) Find all the values of $a \neq 0$ such that the inequality $a^2\left|a + \frac{x}{a^2}\right| + |1 + x| \leq 1 - a^3$ has atleast five integer solutions for x.

20. **(a)** For what real values of a does the range of the function $y = \frac{x - 1}{a - x^2 + 1}$ not contain any values belonging to the interval [-1, -1/3] ?

(b) For what real values of a does the range of the function $y = \frac{x - 1}{1 - x^2 - a}$ not contain any value from the interval [-1, 1] ?

(c) Let S be the range of the function $f(x) = \frac{x + 1}{x^2 + a}$ $\forall x \in R$. Find a so that

(i) $[0, 1] \subset S$ **(ii)** $[0, 1] \cap S = \phi$

IIT JEE PROBLEMS (OBJECTIVE)

(A) Fill in the blanks :

1. If $2+i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) = (----). **[IIT - 82]**

2. If the product of the roots of the equation $x^3 - 3kx + 2e^{2\ell nk} - 1 = 0$ is 7, then the roots are real for k = ... **[IIT - 84]**

3. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq b)$ have a common root, then the numerical value of a + b is............. **[IIT - 86]**

4. The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is............... **[IIT - 97]**

(B) True or False :

1. The equation $2x^2 + 3x + 1 = 0$ has an irrational root. **[IIT - 83]**

2. If a < b < c < d, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are real and distinct. **[IIT - 84]**

3. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then P(x) Q(x) = 0 has at least two real roots. **[IIT - 85]**

(C) Multiple choice questions with one or more than one correct answer :

1. the equation $x^{3/4(\log_2 x)^2 + \log_2 x - 5/4} = \sqrt{2}$ has **[IIT - 91]**

(A) at least one real solution
(B) exactly three solutions
(C) exactly one irrational solution
(D) complex roots

(D) Multiple choice questions with one correct answer :

1. If ℓ, m, n are real, $\ell \neq m$, then the roots by the equation $(\ell - m)x^2 - 5(\ell + m)x - 2(\ell - m) = 0$ are

(A) real and equal
(B) complex
(C) real and unequal
(D) none of these **[IIT - 79]**

2. The equation $2\cos^2\left(\frac{1}{2}x\right)\sin^2 x = x^2 + x^{-2},\ 0 < x \leq \frac{\pi}{9}$ has

(A) no real solution
(B) one real solution
(C) more than one real solution
(D) none of these **[IIT - 80]**

3. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has **[IIT - 84]**

(A) no root
(B) one root
(C) two equal roots
(D) infinitely many roots

4. If a and b are the roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always **[IIT - 89]**

(A) two real roots
(B) two positive roots
(C) two negative roots
(D) one positive and one negative roots

5. Let a, b, c be real number , $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is the root of $a^2x^2 - bx - 2c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies **[IIT - 89]**

(A) $\gamma = \frac{\alpha+\beta}{2}$
(B) $\gamma = \alpha + \frac{\beta}{2}$
(C) $\gamma = \alpha$
(D) $\alpha < \gamma < \beta$

6. Let f(x) be a quadratic expression which is positive for all real values of x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x, **[IIT - 90]**

(A) $g(x) < 0$
(B) $g(x) > 0$
(C) $g(x) = 0$
(D) $g(x) \geq 0$

7. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ in the variable x, has real roots. Then p can take any value in the interval **[IIT - 91]**

(A) $(0, 2\pi)$
(B) $(-\pi, 0)$
(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(D) $(0, \pi)$

8. Let α, β be the roots of the equation $(x-a)(x-b) = c$, $c \neq 0$, then the roots of the equation $(x-\alpha)(x-\beta) + c = 0$ are **[IIT - 92]**

(A) a, c
(B) b, c
(C) a, b
(D) a + c, b + c

9. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has **[IIT - 97]**

(A) no solution
(B) one solution
(C) two solutions
(D) more than two solutions

10. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ & $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ then : **[IIT - 99]**

(A) a + b = c
(B) b + c = a
(C) a + c = b
(D) b = c

11. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real & less than 3 then **[IIT - 99]**

(A) $a < 2$ (B) $2 \le a \le 3$ (C) $3 < a \le 4$ (D) $a > 4$

12. For the equation, $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to **[IIT - 2000]**

(A) 1/3 (B) 1 (C) 3 (D) 2/3

13. If $\alpha, \beta(\alpha < \beta)$, are the roots of the equation, $x^2 + bx + c = 0$, where $c < 0 < b$, then **[IIT - 2000]**

(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$ (C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

14. If $b > a$, then the equation, $(x - a)(x - b) - 1 = 0$ has :

(A) both roots in [a, b] (B) both roots in $(-\infty, a)$

(C) both root in $[b, \infty)$ (D) one root in $(-\infty, a)$ & other in $(b, +\infty)$

15. The number of integer values of m, for which the x coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is **[IIT - 2001]**

(A) 2 (B) 0 (C) 4 (D) 1

16. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is

(A) $(-\infty, -2) \cup (2, \infty)$ (B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

(C) $(-\infty, -1) \cup (1, \infty)$ (D) $(\sqrt{2}, \infty)$ **[IIT - 2002]**

17. Let f(x) = $ax^2 + bx + c$, $a \ne 0$ and $\Delta = b^2 - 4ac$. If $\alpha + \beta$, $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ are in G.P., then

(A) $\Delta \ne 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $bc \ne 0$

[IIT - 2005]

18. a, b, c are the sides of a triangle ABC such that $x^2 - 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ has real roots **[IIT - 2006]**

(A) $\lambda < \frac{4}{3}$ (B) $\lambda > \frac{5}{3}$ (C) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$ (D) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$

19. Let α, β the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is **[IIT - 2007]**

(A) $\frac{2}{9}(p - q)(2q - p)$ (B) $\frac{2}{9}(q - p)(2p - q)$

(C) $\frac{2}{9}(q - 2p)(2q - p)$ (D) $\frac{2}{9}(2p - q)(2q - p)$

IIT JEE PROBLEMS (SUBJECTIVE)

1. If α, β are the roots of $x^2 + px + q = 0$ and r, δ are the roots of the $x^2 + rx + s = 0$, evaluate $(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)$ in terms of p, q, r and s. Deduce the condition that the equations have a common root. **[IIT - 79]**

2. Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution. **[IIT - 82]**

3. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then show that $(ac^n)^{\frac{1}{n+1}} + (a^n 1)^{\frac{1}{n+1}} + b = 0$. **[IIT - 83]**

4. For $a \le 0$, determine all real roots of the equation $x^2 - 2a|x-a| - 3a^2 = 0$. **[IIT - 86]**

5. Let $\alpha_1, \alpha_2\ \beta_1, \beta_2$ be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equation $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a nontrivial solution, then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$.
[IIT - 87]

6. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$. **[IIT - 88]**

7. Let $p \ge 3$ be an integer and α, β be the roots of $x^2 - (p+1)x + 1 = 0$. Using mathematical induction show that $\alpha^n + \beta^n$ **(i)** is a integer **(ii)** is not divisible by p **[IIT - 92]**

8. If α, β are the roots of the equation $x^2 - px + q = 0$, then find the quadratic equation the roots of which are $(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)$ & $\alpha^3\beta^2 + \alpha^2\beta^3$. **[REE - 94]**

9. Let a, b, c be real, If $ax^2 + bx + c = 0$ has two real roots α & β, where $\alpha < -1$ & $\beta > 1$ then show that $1 + c/a + |b/a| < 0$. **[IIT - 95]**

10. Prove that the values of the function $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ do not lie between $\frac{1}{3}$ & 3 for any real x.
[IIT - 96]

11. If α, β are the roots of the equation $x^2 - bx + c = 0$, then find the equation whose roots are, $(\alpha^2 + \beta^2)(\alpha^3 + \beta^3)$ and $\alpha^5\beta^3 + \alpha^3\beta^5 - 2\alpha^4\beta^4$. **[REE - 98]**

12. If α, β are the roots of the equation, (x - a) (x - b) + c = 0, find the roots of the equation, $(x-\alpha)(x-\beta) = c$. **[REE - 2000]**

13. If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta, \beta + \delta$, are the roots of, $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then prove that, $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$. **[IIT - 2000]**

14. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. **[IIT - 2001]**

15. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of 'a' which equation has unequal real roots for all values of 'b'. **[IIT - 2003]**

Quadratic Equation

SET – I

1. The set of values of p for which the roots of the equation $3x^2 + 2x + p(p-1) = 0$ are of opposite signs, is

(A) $(-\infty, 0)$ (B) $(0, 1)$ (C) $(1, \infty)$ (D) $(0, \infty)$

2. If $x^2 - 4x + \log_{1/2} a = 0$ does not have two distinct real roots, then maximum value of a is

(A) 1/4 (B) 1/16 (C) –1/4 (D) none of these

3. If a_1, a_2, a_3 $(a_1 > 0)$ are in G.P. with common ratio r, then the value of r, for which the inequality $9a_1 + 5a_3 > 14a_2$ holds, can not lie in the interval

(A) $[1, \infty)$ (B) $[1, 9/5]$ (C) $[4/5, 1]$ (D) $[5/9, 1]$

4. If $x^2 - 2x + \sin^2\alpha = 0$, then

(A) $x \in [-1, 1]$ (B) $x \in [0, 2]$ (C) $x \in [-2, 2]$ (D) None of these

5. Consider the equation $x^2 + x - n = 0$, where a is an integer lying between 1 to 100. Total number of different values of 'n' so that the equation has integral roots, is

(A) 6 (B) 4 (C) 9 (D) None of these

6. Let $p(x) = 0$ be a polynomial equation of least possible degree, with rational coefficients, having $\sqrt[3]{7} + \sqrt[3]{49}$ as one of its roots. Then the product of all the roots of $p(x) = 0$ is

(A) 7 (B) 49 (C) 56 (D) 63

7. The least value of the expression $x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$ is

(A) 0 (B) 1 (C) no least value (D) none of these

8. If α and β are the roots of the equation $2x^2 - 3x - 6 = 0$, then the equation whose roots are $\alpha^2 + 2, \beta^2 + 2$ is

(A) $4x^2 + 49x + 118 = 0$ (B) $4x^2 - 49x + 118 = 0$

(C) $4x^2 - 49x - 118 = 0$ (D) $x^2 - 49x + 118 = 0$

9. If the roots of the equation $x^2 - px + q = 0$ differ by unity, then

(A) $p^2 = 1 - 4q$ (B) $p^2 = 1 + 4q$ (C) $q^2 = 1 - 4p$ (D) $q^2 = 1 + 4p$

10. If a, b, c are in G. P, then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

(A) A.P. (B) G. P. (C) H.P. (D) none of these

11. If the expression $\left(mx - 1 + \frac{1}{x}\right)$ is non-negative for all positive real x, then the minimum value of m must be
(A) –1/2 (B) 0 (C) 1/4 (D) 1/2

12. Number of positive integers n for which $n^2 + 96$ is a perfect square is
(A) 4 (B) 8 (C) 12 (D) infinite

13. If both the roots of the equation $x^2 – (p – 4)x + 2e^{2lnp} – 4 = 0$ are negative then p belongs to
(A) $\left(-\sqrt{2}, 4\right)$ (B) $\left(\sqrt{2}, 4\right)$ (C) $\left(-4, \sqrt{2}\right)$ (D) none of these

14. The value of 'p' for which the sum of the square of the roots of $2x^2 – 2(p – 2)x – p – 1 = 0$ is least, is
(A) 1 (B) 3/2 (C) 2 (D) –1

15. The equations $ax^2 + bx + a = 0$ and $x^3 – 2x^2 + 2x – 1 = 0$ have two roots in common. Then a + b must be equal to
(A) 1 (B) –1 (C) 0 (D) none of these

16. The value of the biquadratic expression, $x^4 - 8x^3 + 18x^2 - 8x + 2$ when $x = 2 + \sqrt{3}$ is
(A) 1 (B) 2 (C) 0 (D) none of these

17. If b > a, the equation (x - a) (x - b) + 1 = 0, has
(A) must be in (a, b) (B) must be in [a, b]
(C) one root in $(-\infty, a)$ and other in (b, ∞) (D) none of these

18. If $a, b \in R$, $a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then a + b + 1 is
(A) positive (B) negative
(C) zero (D) depends on the sign b

19. The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, then the ordered pair (x_1, x_2) is
(A) (-5, -7) (B) (1, -1) (C) (-1, 1) (D) (5, 7)

20. If α, β are the roots of the equation $2x^2 + 4x - 5 = 0$, the equation whose roots are the reciprocals of $2\alpha - 3$ and $2\beta - 3$ is
(A) $x^2 + 10x - 11 = 0$ (B) $x^2 + 10x + 11 = 0$
(C) $11x^2 + 10x + 1 = 0$ (D) $11x^2 - 10x + 1 = 0$

21. If x satisfies $|x - 1| + |x + 2| + |x - 3| \geq 6$ then
(A) $0 \leq x \leq 4$ (B) $x \leq -2$ or $x \geq 4$
(C) $x \leq 0$ or $x \geq 4$ (D) $0 < x < 4$

22. If a, b, c are real numbers satisfying the condition $a + b + c = 0$ then the roots of the quadratic equation $3ax^2 + 5bx + 7c = 0$ are

(A) positive (B) negative (C) real and distinct (D) imaginary

23. The simultaneous equations, $y = x + 2|x|$ and $y = 4 + x - |x|$ have the solution set given by

(A) $\left(\frac{4}{3}, \frac{4}{3}\right)$ (B) $\left(4, \frac{4}{3}\right)$ (C) $\left(-\frac{4}{3}, \frac{4}{3}\right)$ (D) none of these

24. If the roots of the given equation, $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ are real if

(A) $p \in (-\pi, 0)$ (B) $p \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (C) $p \in (0, \pi)$ (D) $p \in (0, 2\pi)$

25. If a and b are the odd integers, then the roots of the equation, $2ax^2 + (2a + b)x + b = 0,\ a \neq 0$, will be

(A) rational (B) irrational (C) non-real (D) equal

26. The real values of ‘a’ for which the quadratic equation, $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite signs is given by

(A) $a > 5$ (B) $0 < a < 4$ (C) $a > 0$ (D) $a > 7$

27. If α, β are the roots of $x^2 - ax + b = 0$ and $\alpha^n + \beta^n = V_n$, then

(A) $V_{n+1} = a V_n + b V_{n-1}$ (B) $V_{n+1} = a V_n + a V_{n-1}$
(C) $V_{n+1} = a V_n - b V_{n-1}$ (D) $V_{n+1} = a V_{n-1} + b V_n$

28. The number of values of k for which the equation, $x^2 - 3x + k = 0$ has two real and distinct roots lying in the interval (0, 1) are

(A) 0 (B) 2 (C) 3 (D) infinitely many

29. The sum of the roots of a equation is 2 and sum of their cubes is 98, then the equation is

(A) $x^2 + 2x + 15 = 0$ (B) $x^2 + 15x + 2 = 0$
(C) $15x^2 - 2x + 15 = 0$ (D) $x^2 - 2x - 15 = 0$

30. If the roots of the equation, $Ax^2 + Bx + C = 0$ are α, β and the roots of the equation, $x^2 + px + q = 0$ are α^2, β^2, then value of ‘p’ will be

(A) $\frac{B^2 - 2AC}{A^2}$ (B) $\frac{2AC - B^2}{A^2}$ (C) $\frac{B^2 - 4AC}{A^2}$ (D) none of these

SET – II

1. Sum of the real roots of the equation $x^2 + 5|x| + 6 = 0$

(A) equals to 5 (B) equals to 10 (C) equals to –5 (D) does not exist

2. If $c > 0$ and $4a + c < 2b$, then $ax^2 - bx + c = 0$ has a root in the interval

(A) (0, 2) (B) (2, 4) (C) (0, 1) (D) –2, 0)

3. If the equation $(a - 5)x^2 + 2(a - 10)x + a + 10 = 0$ has real roots of the same sign, then

(A) $a > 10$ (B) $-5 < a < 5$

(C) $a < -10$ and $5 < a \le 6$ (D) none of these

4. If the equation $\frac{x^2}{3} - 4x + 13 = \sin\frac{a}{x}$ has a solution, then a is equal to $(n \in I)$

(A) $(2n+1)\frac{\pi}{2}$ (B) $3(4n+1)\frac{\pi}{2}$ (C) $3(1 + 4n)\pi$ (D) none of these

5. The largest negative integer which satisfies $\frac{x^2 - 1}{(x-2)(x-3)} > 0$, is

(A) –4 (B) –3 (C) –1 (D) –2

6. Equation $x^2 + x + a = 0$ will have exactly one root in the interval (0, 1] if

(A) $-2 \le a < 0$ (B) $-2 < a < -1$ (C) $-1 \le a < 0$ (D) $0 \le a < 1$

7. The constant term of the quadratic expression $\sum_{k=1}^{n}\left(x - \frac{1}{k+1}\right)\left(x - \frac{1}{k}\right)$ as $n \to \infty$ is

(A) –1 (B) 0 (C) 1 (D) none of these

8. The set of values of 'a' for which $x^2 - ax + \sin^{-1}(\sin 4) > 0 \; \forall x \in R$ is

(A) R (B) (–2, 2) (C) ϕ (D) none of these

9. If the product of the roots of the equation $2x^2 + ax + 4\sin a = 0$ is 1, then roots will be imaginary, if

(A) $a \in R$ (B) $a \in \left\{-\frac{7\pi}{6}, \frac{\pi}{6}\right\}$ (C) $a \in \left\{-\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ (D) none of these

10. If α, β, γ are the roots of the equation, $x^3 + P_0x^2 + P_1x + P_2 = 0$, then $(1-\alpha^2)(1-\beta^2)(1-\gamma^2)$ is equal to

(A) $(1 + P_1)^2 - (P_0 + P_2)^2$ (B) $(1 + P_1)^2 + (P_0 + P_2)^2$

(C) $(1 - P_1)^2 - (P_0 - P_2)^2$ (D) none of these

Quadratic Equation

11. If x = 2 + $2^{2/3}$ + $2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is
(A) 3 (B) 2 (C) 1 (D) none of these

12. The solution set of the inequation, $\log_{1/2}(2^{x+2} - 4^x) \geq -2$ is
(A) $(-\infty, 2-\sqrt{13})$ (B) $(-\infty, 2+\sqrt{13})$ (C) $(-\infty, 2)$ (D) none of these

13. If α, β are the roots of the quadratic equation $6x^2 - 6x + 1 = 0$, then

$$\frac{1}{2}(a+b\alpha+c\alpha^2+d\alpha^3)+\frac{1}{2}(a+b\beta+c\beta^2+d\beta^3) =$$

(A) $\frac{12d+6c+4b+a}{12}$ (B) 12a + 6b + 4c + 9d

(C) $\frac{1}{12}(12a + 6b + 4c + 3d)$ (D) none of these

14. Let α, β be the roots of the equation (x – a) (x – b) = c, c $\neq$ 0. Then the roots of the equation $(x-\alpha)(x-\beta) + c = 0$ are
(A) a, c (B) b, c (C) a, b (D) a + c, b + c

15. If both the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are less than 3, then
(A) a < 2 (B) $2 \leq a \leq 3$ (C) $3 < a \leq 4$ (D) a > 4

16. The equation $\frac{a(x-b)(x-c)}{(a-b)(a-c)}+\frac{b(x-c)(x-a)}{(b-c)(b-a)}+\frac{c(x-a)(x-b)}{(c-a)(c-b)} = x$ is satisfied by
(A) no value of x (B) exactly two values of x
(C) exactly three values of x (D) all values of x

17. The number of real roots of the equation, $e^{\sin x} - e^{-\sin x} - 4 = 0$ are
(A) 1 (B) 2 (C) infinite (D) none of these

18. If both roots of the quadratic equation (2 - x) (x + 1) = p are distinct and positive then p must lie in the interval
(A) p > 2 (B) $2 < p < \frac{9}{4}$ (C) p < - 2 (D) $-\infty < p < \infty$

19. If x is real, the function, $\frac{(x-a)(x-b)}{(x-c)}$ will assume all real values, provided
(A) a > b > c (B) a < b < c (C) a > c > b (D) none of these

20. The value of p for which both the roots of the quadratic equation, $4x^2 - 20px + (25p^2 + 15p - 66)$ are less than 2 lies in

(A) (4/5, 2) (B) $(2, \infty)$ (C) (-1, 4/5) (D) $(-\infty, -1)$

21. The number of real roots of $\left(x+\frac{1}{x}\right)^3 + x + \frac{1}{x} = 0$ is

(A) 0 (B) 2 (C) 4 (D) 6

22. If $x = 2 + 2^{2/3} + 2^{1/3}$, then $x^3 - 6x^2 + 6x =$

(A) 3 (B) 2 (C) 1 (D) none of these

23. The set of real value(s) of p for which the equation, $|2x + 3| + |2x - 3| = px + 6$ has more than two solutions is

(A) [0, 4) (B) (-4, 4) (C) R - {4, -4, 0} (D) {0}

24. Number of quadratic equations with real roots which remain unchanged even after squaring their roots, is

(A) 1 (B) 2 (C) 3 (D) 4

25. The quadratic equation $ax^2 + bx + c = 0$ has imaginary roots if

(A) $a < -1, 0 < c < 1, b > 0$
(B) $a < -1, -1 < c < 0, 0 < b < 1$
(C) $a < -1, c < 0, b > 1$
(D) none of these

26. If both the roots of the equation, $(3a + 1)x^2 - (2a + 3b)x + 3 = 0$ are infinite then

(A) $a = \infty;\ b = 0$
(B) $a = 0;\ b = \infty$
(C) $a = -1/3;\ b = 2/9$
(D) $a = \infty;\ b = \infty$

27. If p and q are the roots of the equation, $x^2 + px + q = 0$ then

(A) p = 1 (B) p = 1 or 0 (C) p = -2 (D) p = -2 or 0

28. If S is the set of all real x such that $(2x - 1) / (2x^3 + 3x^2 + x)$ is positive, then S contains

(A) $(-\infty, -3/2)$
(B) (-3/2, -1/4)
(C) (-1/4, 1/2)
(D) none of these

29. The inequalities $y(-1) \ge -4,\ y(1) \le 0$ and $y(3) \ge 5$ are known to hold for $y = ax^2 + bx + c$ then the least value of 'a' is

(A) -1/4 (B) -1/3 (C) 1/4 (D) 1/8

30. The roots of the equation, $(x - a)(x - b) = a^2 - 2b^2$ are real and distinct for $a \in R - \{0\}$ provided

(A) $-1 \le \frac{b}{a} < \frac{5}{7}$ (B) $-1 < \frac{b}{a} < \frac{5}{7}$ (C) $-1 < \frac{b}{a} \le \frac{5}{7}$ (D) $-2 < \frac{b}{a} < \frac{7}{5}$

SET III

Multiple choice questions with one or more than one correct answers :

1. If both the roots of equation $ax^2 - 4x + 6 = 0$ lies between -2 and 0, then a can be

(A) $a > 0$ & $a \le \frac{2}{3}$ (B) $a = 1/2$ (C) $a \ge \frac{2}{3}$ (D) $a = \frac{1}{4}$

2. A quadratic equation with real roots is formed such that, its roots remain unchanged even after squaring them. The root can be

(A) 0, 0 (B) 1, 0 (C) 1, 1 (D) –1, –1

3. A two digit number is 4 times the sum and three times the product of its digits. The number is

(A) 42 (B) 24 (C) 12 (D) 21

4. For $a > 0, \ne 1$, the roots of the equation

$\log_{ax}(A) + \log_x a^2 + \log_{a^2x} a^3 = 0$ are given by

(A) $a^{-3/4}$ (B) $a^{-4/3}$ (C) $a^{-1/2}$ (D) none of these

5. The equation $x^{\frac{3}{4}(\log_2 x)^2 + (\log_2 x) - \frac{5}{4}} = \sqrt{2}$ has

(A) at least one real solution (B) exactly 3 real solutions

(C) exactly one irrational solutions (D) complex roots

6. Values of m for which the expression $2x^2 + mxy + 3y^2 - 5y - 2$ can be factorized into two linear factors

(A) 7 (B) –7 (C) $\frac{1}{7}$ (D) $-\frac{1}{7}$

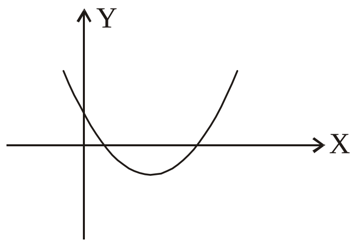
7. If roots of equation $x^2 - (2n + 18)x - n - 11 = 0$ $(n \in I)$ are rational, then n is

(A) 8 (B) –8 (C) 10 (D) –11

8. From the following graphs it can be interpreted that

(A) $c > 0$ (B) $c < 0$

(C) $a > 0$, (D) $abc < 0$



9. If the difference of the roots of the equation $x^2 + kx + 7 = 0$ is 6, then possible values of k are

(A) 4 (B) –4 (C) 8 (D) –8

10. If $a < b < c < d$, then for any real non–zero λ, the quadratic equation,

$(x - a)(x - c) + \lambda(x - b)(x - d) = 0$ has

(A) non–real roots (B) one real root between a and c

(C) one real root between b and d (D) irrational roots

11. If $a < 0$, then roots of the equation $x^2 - 2a\,|\,x - a\,| - 3a^2 = 0$ is

(A) $a(-1 - \sqrt{6})$ (B) $a(1 - \sqrt{2})$ (C) $a(-1 + \sqrt{6})$ (D) $a(1 + \sqrt{2})$

12. The roots of $ax^2 + bx + c = 0$. Where $a \neq 0$ and coefficients are real, are nonreal complex and $a + c < b$. Then

(A) $4a + c > 2b$ (B) $4a + c < 2b$ (C) $a + 4c > 2b$ (D) $a + 4c < 2b$

13. Which of the following is correct for the quadratic equation $x^2 + 2(a - 1)x + a + 5 = 0$

(A) the equation have positive roots, if $a \in (-5, -1)$
(B) the equation have roots of opposite sign, if $a \in (-\infty, -5)$
(C) the equation have negative roots, if $a \in [4, \infty)$
(D) none of these

14. All solutions of the equations $x^2 + y^2 - 8x - 8y = 20$ and $xy + 4x + 4y = 40$ satisfy the following equation(s)

(A) $x + y = 10$ (B) $|\,x + y\,| = 10$ (C) $|\,x - y\,| = 10$ (D) none of these

15. Which of the following statement(s) is/are false

(A) The only integral value of x for which $x^2 + 19x + 92$, is a perfect square is -8.
(B) The only integral value of x for which $x^2 + 19x + 92$, is a perfect square is -11.
(C) The number of integral values of x for which $x^2 + 19x + 92$ is a perfect square are two.
(D) The number of integral values of x for which $x^2 + 19x + 92$ is infinite.

16. Which of the following statement(s) is/are True

(A) The values of m for which the expression $2x^2 + mxy + 3y^2 - 5y - 2$ be expressed as the product of two linear factors are ± 1.
(B) If the expression $ax^2 + by^2 + cz^2 + 2ayz + 2bzx + 2cxy$ can be resolved into rational factors, then $a^3 + b^3 + c^3 + 3abc = 0$.
(C) If $a \in R$ and $a \neq -2$, then the equation $x^2 + a\,|\,x\,| + 1 = 0$ has either four real roots or no real root.
(D) The equation $x^4 - 4x - 1 = 0$ has exactly two real roots

W I **Read the passage and answer the questions from 17 to 21.**

For a polynomial equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots a_n = 0 \ (a_0 \neq 0)$

Sum of roots $= -\dfrac{\text{Coeff. of } x^{n-1}}{\text{Coeff. of } x^n}$; Product of roots $= (-1)^n \dfrac{\text{Constant term}}{\text{Coeff. of } x^n}$

17. If a, b are the roots of the equation $\ell n^2 + mx + n = 0$ then equation whose roots are $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$, is

(A) $n\ell x^2 - (m^2 - 2n\ell)x + n\ell = 0$ (B) $m\ell x^2 - (\ell^2 - 2m\ell)x + n\ell = 0$
(C) $n\ell x^2 + (m^2 + 2n\ell)x + n\ell = 0$ (D) none of these

18. The equation whose roots are 2, –3, $\frac{7}{5}$, is

(A) $5x^3 + 2x^2 + 37x + 42 = 0$ (B) $5x^3 - 2x^2 - 37x - 42 = 0$
(C) $5x^3 - 2x^2 - 37x + 42 = 0$ (D) $-5x^3 - 2x^2 + 37x - 42 = 0$

19. The equation whose roots are 0, ±a, $\frac{c}{b}$, is

(A) $bx^4 - cx^3 - a^2bx^2 + a^2cx = 0$ (B) $bx^4 + cx^3 + a^2bx^2 + a^2cx = 0$
(C) $bx^4 - cx^3 + a^2bx^2 + a^2cx = 0$ (D) $-bx^4 - cx^3 - a^2bx^2 + a^2cx = 0$

20. The equation whose one root is $2 + \sqrt{3}$, is

(A) $x^2 + 4x + 1 = 0$ (B) $x^2 - 4x + 1 = 0$ (C) $x^2 - 4x - 1 = 0$ (D) none of these

21. The equation whose roots are the squares of the sum and of the difference of the roots of $2x^2 + 2(m + n)x + m^2 + n^2 = 0$

(A) $x^2 + 4mnx + (m^2 - n^2)^2 = 0$ (B) $x^2 - 4mnx - (m^2 - n^2)^2 = 0$
(C) $x^2 - 4mnx + (m^2 - n^2)^2 = 0$ (D) $x^2 + 4mnx - (m^2 - n^2)^2 = 0$

W II Read the passage and answer the questions from 22 to 24.

Each question has a conditional statement followed by a result statements.
If condition $\Rightarrow$ result, then condition is sufficient and
If result $\Rightarrow$ condition, then condition is necessary
If condition is necessary as well as sufficient for the result, **mark (A)**
If condition is necessary but not sufficient for the result, **mark (B)**
If condition is sufficient but not necessary for the result, **mark (C)**
If neither necessary nor sufficient for the result, **mark (D)**

Consider the following example :

Condition : $a > 0, b > 0$
Result : $a + b > 0$

Here, if $a > 0$ and $b > 0$, then it always implies that $a + b$ is positive but if $a + b$ is positive, then a and b both need not to be positive. So condition $\Rightarrow$ result but result does not always implies condition hence condition is sufficient but not necessary for the result to be hold. So answer is 'C'.

22. Condition : Let $f(x) = x^2 + bx + c$, $f(2) > 0$ and $b^2 - 4c > 0$
Result : Both roots of the quadratic equation $x^2 + bx + c = 0$ are distinct and more than 2

23. Condition : $x^2 + bx + c = 0$ has integral roots
Result : For the quadratic equation $x^2 + bx + c = 0$, $b^2 - 4c$ is a perfect square of an integer and $b, c \in$ Integer

24. Condition : One of the root of the quadratic equation $ax^2 + bx + c = 0, (a, b, c \in R)$ is $2 + \sqrt{3}$
Result : Other root of the quadratic equation $ax^2 + bx + c = 0, (a, b, c \in R)$ is $2 - \sqrt{3}$

W III Read the passage and answer the questions from 26 to 30

Let $f(x) = ax^2 + bx + c$, where $a, b, c \in R$ and $a \neq 0$, then $f(x) \geq 0$, $f(x) < 0$, $f(x) \leq 0$, $f(x) > 0$ an quadratic inequations, The set of all real values of n which satisfy an equation is called its solution set.

(a) If $(n-\alpha)(n-\beta) > 0$, then x lies outside the interval $[\alpha, \beta]$ i.e. $x \in (-\infty, \alpha) \cup (\beta, \infty)$.

(b) If $(n-\alpha)\ (n-\beta) \geq 0$, then x lies on end outside the interval $[\alpha, \beta]$ i.e. $(-\infty, \alpha] \cup [\beta, \infty)$.

(c) If $(n-\alpha)\ (n-\beta) < 0$, then x lies inside the interval $[\alpha, \beta]$ i.e. (α, β).

(d) If $(n-\alpha)\ (n-\beta) \leq 0$, then x lies inside the interval $[\alpha, \beta]$ i.e. $[\alpha, \beta]$.

25. The values of 'a' for which the inequality $a - x < x^2$ is satisfied by all $x \in R$

(A) $\left(-\frac{1}{4}, \infty\right)$ (B) $\left(\frac{1}{4}, \infty\right)$ (C) $\left(-\infty, -\frac{1}{4}\right)$ (D) $\left(-\infty, -\frac{1}{4}\right]$

26. The values of 'x' for which the inequality $\frac{x+1}{(x-1)^2} < 1$ is

(A) $(-\infty, 0) \cup (3, \infty)$
(B) $(-\infty, -3) \cup (0, \infty)$
(C) $(0, 3)$
(D) none of these

27. The values of 'x' for which the inequality $\frac{x^2 - 7x + 12}{2x^2 + 4x + 5} > 0$ is

(A) $(-\infty, -4) \cup (-3, \infty)$
(B) $(-\infty, 3) \cup (4, \infty)$
(C) $(3, 4)$
(D) none of these

28. The values of 'x' for which the inequality $\frac{x^2 + 6x - 7}{x^2 + 1} \leq 2$ is

(A) $(-\infty, \infty)$ (B) $(-\infty, 0)$ (C) $(0, \infty)$ (D) none of these

29. The values of 'x' for which the inequality $\frac{1 + x^2}{x^2 - 5x + 6} < 0$ is

(A) $(1, 3)$ (B) $(-\infty, 2) \cup (3, \infty)$ (C) $(2, 3)$ (D) none of these

30. The solution set of the equation $\frac{(x^2 + 2x + 1)(x^3 - 1)}{(x^2 - x)} \geq 0$, is

(A) R (B) $R - \{0, 1\}$ (C) $(0, 1) \cup (1, \infty)$ (D) none of these

ANSWER LEVEL–I

1. **(a)** $x = 2$ or 5 **(b)** $x = -4$ or $-(1+\sqrt{3})$ **(c)** $x = \frac{(-7-\sqrt{17})}{2}$

(d) $x = 1$ **(e)** $x = -2$ or -4 or $-(1+\sqrt{3})$ **(f)** $x = -1$ or 1

(g) $x \geq -1$ or $x = -3$

2. **(a)** $x = 1$ or $-2\log_3 6$ **(b)** $(-6, -5) \cup (-3, -2)$ **(c)** $(-1, \infty)$

(d) $(0, 1) \cup (1, 10^{1/10})$ **(e)** $-1 < x < \frac{1-\sqrt{5}}{2}$ or $\frac{1+\sqrt{5}}{2} < x < 2$

(f) $2^{-\sqrt{2}} < x < 2^{-1}; 1 < x < 2^{\sqrt{2}}$ **(g)** $0 < x < 3^{1/1-\log 3}$ (where base of log is 2)

(h) $\frac{1}{2} < x < 1$ **(i)** $x < -7, \quad -5 < x \leq -2, \quad x \geq 4$ **(j)** $(-2, 2-\sqrt{15})$

3. **(i)** $x = \pm 1$ **(ii)** $x = \pm 4, \pm\sqrt{14}$ **4. (b)** $x^2 - 3x + 2 = 0$

5. **(a)** $3x^2 + 68x - 18 = 0$ **6.** $a \in \left(-\infty, -\frac{1}{2}\right)$

8. A = 2 or -18, B = 32 or -288 **10.** **(b)** 1 **(c)** 4 **(d)** $-2 < a < 1$

12. $(ac^1 - a^1c)^2 = (ab^1 - a^1b)(bc^1 - b^1c)$

13. **(a)** $(-\infty, 1) \cup (25, \infty)$ **(b)** $\{1, 25\}$ **(c)** $(1, 25)$ **(d)** $-\infty, 0$
(e) f **(f)** $[25, \infty)$ **(g)** $(0, 1]$
(h) $(-\infty, 0] \cup [25, \infty)$ **(i)** $(-\infty, -7)$ **(j)** $[25, \infty)$ **(k)** $(-7, 1]$
(l) $(-7, -2)$ **(m)** f **(n)** $(-7, -2)$
(o) $(-\infty, -7) \cup [25, \infty)$ **(p)** $(-\infty, -7) \cup [25, \infty)$

14. **(a)** $\left(6, \frac{27}{4}\right)$ **(b)** $a \in (0, 1) \cup (1, 6/5)$

17. **(a)** $a \in (0, 1/3)$ **(b)** $a = \{1-\sqrt{2}, 5+\sqrt{10}\}$

18. **(a)** $\left[\frac{4}{5}, 1\right]$ **(b)** $x_{1,2} = 2 \pm \log_3(2-\sqrt{4+a})$ where $-3 \leq a < 0$

20. **(a)** one, x = -4/3 **(b)** $a \in (-2/3, \quad 0)$

LEVEL–II

1. **(a)** for $a \in (-\infty, -1) \cup (5/4, +\infty)$ **(b)** for a = -1
(c) $a \in (-1, 1) \cup \{5/4\}$ **(d)** for a = 1 **(e)** $a \in [1, 5/4)$

2. $a \in \left(\frac{5}{2}, \infty\right)$ **3.** $\left[-\frac{7}{2}, \frac{5}{6}\right]$ **4.** $a \in \left(5, \frac{19}{3}\right)$

7. $k \in [7 + 3\sqrt{5}\ )/2,\ \infty)$ **8.** (-1/4, 1)

10. {a, -1} for $a \in (-\infty, -1)$,{a, -2a^2} for $a \in (-1/2, 0)$

11. **(b)** (x, y) = (0, 0), No.

13. **(a)** $a \in \{-4, -3, -1, 0\}$ **(b)** $m = k(k + 1) k \in I$
(c) $a \in \{6, 12, 20, 30, 42, 56, 72, 90\}$ **(f)** (5, 6), (6, 5)

14. (0, 0); (1, 1); (2, 0)

15. **(a)** $a \in \left(0, \frac{2}{5}\right]$ **(b)** $k \in [-1,\ \infty)$
(c) all $k \in (-\infty,\ 1/2) \cup (3/2,\ \infty)$ **(d)** $k \in (-\infty, 0] \cup \{1\}$

16. **(a)** $(0, 8]$ **(b) (i)** $(-\infty,\ -14) \cup \{4\} \cup \left[\frac{14}{3},\ \infty\right)$ **(ii)** $\left(4, \frac{14}{3}\right)$ **(iii)** [–14, 4)

17. **(a)** $\left(-\sqrt{-a}, 0\right)$ for $a < 0$, $\left(0, \sqrt{a}\right)$ for $a > 0$, ϕ for $a = 0$

(b) $x \in \phi$ if $a \in (-\infty,\ 0) \cup (0,\ 1)$; $x = \{0\}$ if $a = 0$; $x = \left\{\left(2a - 1 - \sqrt{4a-3}\right)/2\right\}$ if $a \in [1, \infty)$

(c) $\left\{\left(-1-\sqrt{5}\right)/2, \left(\sqrt{5}-1\right)/2, 1\right\}$

(d) $x \in \phi$ if $a \in (-\infty, -1/4)$, $x = \left\{\left(-1 \pm \sqrt{4a+1}\right)/2\right\}$ if $a \in [-1/4, 0]$, $x = \left\{\left(-1-\sqrt{4a+1}\right)/2\right\}$
if $a \in (0,\ 1)$, $x = \left\{\left(-1-\sqrt{4a+1}\right)/2,\ \left(1+\sqrt{4a-3}\right)/2\right\}$ if $a \in [1,\ \infty)$

(e) $\left(1, \frac{5}{4}\right) \cup \left(\frac{5}{3}, \infty\right)$

18.(a) $[-2,1]\max f(x) = f(-2) = -1/(3b^2 - 8b + 16)$ for $b \in (-\infty, 2]$, $[-2,1]\max f(x) = f(0) = -\frac{1}{3b^2}$ for $b \in [2, \infty)$

(b) $[-2,1]\max f(x) = f(-2) = 16 - 24b + b^2$ for $b \in (-\infty, 2/3]$; $[-2,1]\max f(x) = f(0) = b^2$ for $b \in [2/3, \infty)$

19. **(a)** $(-\infty, -3] \cup \left[-\frac{\sqrt{3}}{3}, \frac{1}{2}\right]$ **(b)** $a \in (-\infty, -\sqrt[3]{4}\,] \cup [\,\sqrt[3]{5}, \infty)$

20. **(a)** $a \in \left(-\infty, -\frac{1}{4}\right)$ **(b)** ϕ

(c) (i) $a \in (-\infty, -1) \cup (-1, 5/4]$ **(ii)** for no value of a

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. (–4, 7) **2.** 2 **3.** –1 **4.** 4

(B)

1. F **2. T** **3. T**

(C)

1. ABC

(D)

1. C	**2. A**	**3. A**	**4. A**	**5. D**	**6. B**	
7.	**8. C**	**9. A**	**10. A**	**11. A**	**12. C**	
13.B	**14. D**	**15. A**	**16. B**	**17. C**	**18. A**	**19. D**

IIT JEE PROBLEMS (SUBJECTIVE)

1. $(s - q)^2 + q(r - q)^2 - p(s - q)\,.\,(r - p)$, $(q - s)^2 = (r - p)\,.\,(ps - rq)$ **4.** $(a \pm a\sqrt{2}, -a \pm a\sqrt{6})$

6. $(-4, -1 - \sqrt{3})$ **8.** $x^2 - p(p^4 - 5p^2q + 5q^2)x + p^2q^2(p^2 - q^2)(p^2 - q) = 0$

11. $x^2 - (x_1 + x_2)x + x_1x_2 = 0$ where $x_1 = (b^2 - 2c)(b^3 - 3cb)$; $x^2 - c^3(b^2 - 4c)$

12. (a, b)

14. $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$ **15.** $a > 1$

SET– I

1. B	**2. B**	**3. B**	**4. B**	**5. C**	**6. C**
7. B	**8. B**	**9. B**	**10. A**	**11. C**	**12. A**
13. D	**14. B**	**15. C**	**16. A**	**17. D**	**18. A**
19. A	**20. C**	**21. C**	**22. C**	**23. C**	**24. C**
25. A	**26. B**	**27. C**	**28. A**	**29. D**	**30. B**

SET–II

1. D	**2. A**	**3. C**	**4. C**	**5. D**	**6. A**
7. C	**8. C**	**9. B**	**10. A**	**11. B**	**12. C**
13. C	**14. C**	**15. A**	**16. D**	**17. D**	**18. B**
19. C	**20. D**	**21. A**	**22. B**	**23. D**	**24. C**
25. D	**26. C**	**27. B**	**28. A**	**29. D**	**30. B**

SET–III

1. AB	**2. BC**	**3. B**	**4. BC**	**5. ABC**	**6. AB**
7. BD	**8. ACD**	**9. CD**	**10. BC**	**11. ABCD**	**12. B**
13. AB	**14. AB**	**15. ABD**	**16. CD**	**17. A**	**18. C**
19. A	**20. B**	**21. B**	**22. B**	**23. A**	**24. D**
25. C	**26. A**	**27. B**	**28. A**	**29. C**	**30. C**