Definite Integrals

LEVEL–I

1. **(i)** $\int_0^{2\pi} \sqrt{1-\sin x}\, dx$ **(ii)** $\int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right]^{\frac{1}{2}} dx$

2. **(i)** $\int_0^{\pi/2} \frac{x \sin 2x}{\sin^4 x + \cos^4 x} dx$ **(ii)** $\int_0^{\pi} \frac{x \cos x}{(1+\sin x)^2} dx$

3. **(i)** $\int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin\left(\frac{\pi}{4}+x\right)} dx$ **(ii)** $\int_0^{\sqrt{3}} \frac{1}{1+x^2} \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$.

4. **(i)** $\int_0^{2\pi} \frac{dx}{2+\sin 2x}$ **(ii)** $\int_0^{\pi} \frac{dx}{(5+4\cos x)^2}$

5. **(i)** $\int_0^{\pi/4} \sqrt{\tan x}\, dx$ **(ii)** $\int_0^{1} (\cos^{-4} x) dx$

6. Evaluate the integral of $\int_0^1 x.\tan^{-1} x\, dx$

7. Using ab initio prove that $\int_a^b \frac{dx}{x^2} = \frac{1}{a} - \frac{1}{b}$

8. Evaluate the limits : $\lim\limits_{x\to\infty} \frac{\left[\int_0^x e^{t^2} dt\right]^2}{\int_0^x e^{2t^2} dt}$.

9. **(i)** $\lim\limits_{n\to\infty} \prod_{r=1}^{n} \frac{(n^2+r^2)^{1/n}}{n^2}$ **(ii)** $\lim\limits_{n\to\infty} \left[\frac{n!}{n^n}\right]^{1/n}$

10. Evaluate: $\int_0^{\pi/4} \sin(x-[x])\, dx$.

11. Show that $\int_0^{\pi/2} \ell n(\sin x) dx = \int_0^{\pi/2} \ell n(\cos x) dx = \int_0^{\pi/2} \ell n(\sin 2x) dx = -\frac{\pi}{2}.\ell n2$.

12. Investigate for maxima and minima of the function, $f(x)=\int_{0}^{x}[2(t-1)(t-2)^{3}+3(t-1)^{2}(t-2)^{2}]dt$.

13. If $p=\int_{-2}^{0}\frac{|\sin x|}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx,\ q=\int_{0}^{2}\frac{|\sin x|}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$, where $[\cdot]$ denotes the greatest integer function, then prove that $p + q = 0$.

14. Evaluate: $I=\int_{-2}^{1}(\tan^{-1}x+\cot^{-1}\frac{1}{x})\,dx$

15. **Show that :**

(i) $$\int_{0}^{\pi}\frac{\sin nx}{\sin x}dx=\begin{cases}\pi & \text{if } n \text{ is an odd integer}\\ 0 & \text{if } n \text{ is an even integer}\end{cases}$$

(ii) $$\int_{0}^{\pi/2}\frac{\sin nx}{\sin x}dx=\begin{cases}\pi/2, & \text{if } n \text{ is an odd integer}\\ 2\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots\ldots(-1)^{1+(n/2)}\frac{1}{n-1}\right), & \text{if } n \text{ is an even integer}\end{cases}$$

LEVEL–II

1. Evaluate ([.] denotes the greatest integer function)

(i) $\int_0^{5\pi/12} [\tan x]dx$ **(ii)** $\int_0^{n^2} \left[\sqrt{x}\right]dx, n \in N$.

2. Evaluate: $\int_0^x \frac{2^t}{2^{[t]}}dt$ (where [.] denotes greatest integer function and $x \in R^+$).

3. If f(y) is a non negative continuous function such that $f(y)+f\left(y+\frac{5}{2}\right)=7\ \forall y \in \left(0, \frac{5}{2}\right)$, then find the value of $\int_0^5 f(y)\,dy$.

4. Prove that $\int_0^x [t]dt = \frac{[x]([x]-1)}{2} + [x](x-[x])$, where [x] denotes greatest integer function.

5. Prove that inequalities : $0 < \int_0^1 \frac{x^7 dx}{(1+x^8)^{\frac{1}{3}}} < \frac{1}{8}$

6. Prove that $\int_{-\pi/2}^{\pi/2} \frac{\log(1+b\sin x)}{\sin x}dx = \pi\sin^{-1}(b)$, where $|b| < 1$.

7. If $\phi(x) = \cos x - \int_0^x (x-t)\phi(t)dt$. Then find the value of $\phi''(x)+\phi(x)$.

8. If $f(\theta) = \frac{d}{d\theta}\left[\int_0^\theta \frac{dx}{1-\cos\theta\cos x}\right]$. Show that $f'(\theta).\sin\theta + 2f(\theta).\cos\theta = \frac{\pi}{2}$.

9. If $U_n = \int_0^\pi \frac{1-\cos nx}{1-\cos x}dx$, where n is a positive integer or zero, then show that $U_{n+2} + U_n = 2\,U_{n+1}$.

Hence deduce that $\int_0^{\pi/2} \frac{\sin^2 n\theta}{\sin^2\theta}d\theta = \frac{n\pi}{2}$.

10. Find the points of extremum of the function(s) :

(i) $F(x) = \int_0^x \frac{\sin t}{t}dt$ in the domain $x > 0$. **(ii)** $F(x) = \int_0^{x^2} \cos t\,dt = 0$

11. Let $f(x) = \begin{bmatrix} 1-x & \text{if} & 0 \le x \le 1 \\ 0 & \text{if} & 1 < x \le 2 \\ (2-x)^2 & \text{if} & 2 < x \le 3 \end{bmatrix}$. Define the function $F(x) = \int_0^{x^2} f(t)dt$ and show that F is continuous in [0, 3] and differentiable in (0, 3).

12. If $U_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$, then show that $U_1, U_2, U_3, ...U_n$ constitute an AP. Hence or otherwise find the value of U_n.

13. Evaluate : $\int_0^{\pi/4} \frac{\sin z + \cos z}{\cos^2 z + \sin^4 z} dz$

14. Evaluate: $A = \lim_{n\to\infty} \left(1+\frac{1}{n^2}\right)^{\frac{2}{n^2}} \left(1+\frac{2^2}{n^2}\right)^{\frac{4}{n^2}} \left(1+\frac{3^2}{n^2}\right)^{\frac{6}{n^2}} ... \left(1+\frac{n^2}{n^2}\right)^{\frac{2n}{n^2}}$

15. Prove that :

$$\lim_{n\to\infty} \frac{1}{n}\left[\cos^{2p}\frac{\pi}{2n} + \cos^{2p}\frac{2\pi}{2n} + \cos^{2p}\frac{3\pi}{2n} + ...+ \cos^{2p}\frac{\pi}{2}\right] = \prod_{r=1}^{p} \frac{p+r}{4r} \quad \text{where} \quad \prod$$

denotes the continued product and $p \in N$.

IIT JEE PROBLEMS (OBJECTIVE)

(A) Fill in the blanks

1. Let $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$, then $\int_0^{\pi/2} f(x)dx$ = ... **[IIT - 87]**

2. The integral $\int_0^{1.5} [x^2]dx$, where [ ] denotes the greatest integer function, equals........... **[IIT - 88]**

3. The value of $\int_{-2}^{2} |1-x^2|\, dx$ is................. **[IIT - 89]**

4. The value of $\int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi} d\phi$ is .... **[IIT - 93]**

5. The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$ = ... **[IIT - 94]**

6. If for nonzero $x, a\,f(x) + bf\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right) - 5$; where $a \neq b$ then $\int_1^2 f(x)dx =$ .. **[IIT - 95]**

7. For $n > 0$ $\int_0^{2\pi} \frac{x\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx =$ .... **[IIT - 96]**

8. $\lim_{x\to 0} \int_0^{x^2} \frac{\cos t^2 dt}{x \sin x}$ = ... **[IIT - 97]**

9. The value of $\int_1^{e^{37}} \frac{\pi \sin(\pi \ell n\, x)}{x} dx$ is .... **[IIT - 97]**

10. Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = F(k) - F(1)$ then one of the possible values of k is ....

[IIT - 97]

(B) True/False

1. The value of the integral $\int_0^{2\alpha} \frac{f(x)}{\{f(x)+f(2a-x)\}} dx$ is equal to a. **[T/F]**

[IIT - 88]

(C) Multiple choice questions with one or more than one correct answer :

1. If $\int_0^x f(t) = x + \int_x^1 t\, f(t)dt$, then the value of $f(1)$ is **[IIT - 98]**

(A) 1/2 (B) 0 (C) 1 (D) -1/2

2. Let f(x) = x – [x], for every real number x, where [x] is the integral part of x. Then $\int_{-1}^{1} f(x)dx$ is

(A) 1 (B) 2 (C) 0 (D) $\frac{1}{2}$ **[IIT - 98]**

3. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line y = mx equals 9/2 ?

(A) –4 (B) –2 (C) 2 (D) 4 **[IIT - 99]**

4. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5 dt$ has a local minimum at x = **[IIT - 99]**

(A) 0 (B) 1 (C) 2 (D) 3

(D) Multiple choice questions with one correct answer :

1. The value of the definite integral $\int_0^1 (1 + e^{-x^2})\, dx$ is **[IIT - 81]**

(A) -1 (B) 2 (C) 1 + e^{-1} (D) none of these

2. Let a, b, c be non-zero real numbers such that **[IIT - 81]**

$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx$. The quadratic equation $ax^2 + bx + c = 0$ has

(A) no root in (0, 2) (B) at least one root in (0, 2)

(C) a double root in (0, 2) (D) two imaginary roots

3. The area bounded by the curve y = f(x), the x-axis and the ordinates x = 1 and s = b is (b – 1) sin (3b + 4). then f(x) is **[IIT - 82]**

(A) (x – 1)cos (3x + 4) (B) sin (3x + 4)

(C) sin (3x + 4) + 3 (x – 1) cos (3x + 4) (D) none of these

4. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ is **[IIT - 83]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) none of these

5. For any integer n, the integral $\int_0^{\pi} e^{\cos^2 x} \cos^3(2n+1)x \, dx$. has the value **[IIT - 85]**

(A) π (B) 1 (C) 0 (D) none of these

6. Let $f : R \to R$ and $g : R \to R$ be continuous functions. Then the value of the integral

$\int_{-\pi/2}^{\pi/2} [f(x)+f(-x)][g(x)-g(x)]dx$ is **[IIT - 90]**

(A) π (B) 1 (C) -1 (D) 0

7. Let $f : R \to R$ be a differentiable function and f(1) = 4. Then the value of $\lim_{x\to 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is

(A) $8f'(1)$ (B) $4f'(1)$ (C) $2f'(1)$ (D) $f'(1)$ **[IIT - 90]**

8. The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ **[IIT - 93]**

(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

9. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$. Then the constants A and B are respectively : **[IIT - 95]**

(A) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (B) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ (C) 0 and $-\frac{4}{\pi}$ (D) $\frac{4}{\pi}$ and 0

10. The value of $\int_{\pi}^{2\pi} [2\sin x]dx$ where [ ] represents the greatest integer function is : **[IIT - 95]**

(A) $-\frac{5\pi}{3}$ (B) $-\pi$ (C) $\frac{5\pi}{3}$ (D) -2 π

11. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} xf[x(1-x)]dx$, $I_2 = \int_{1-k}^{k} f[x(1-x)]dx$, where $2k-1>0$.

Then $\frac{I_1}{I_2}$ is **[IIT - 97]**

(A) 2 (B) k (C) $\frac{1}{2}$ (D) 1

12. If $g(x) = \int_0^x \cos^4 t \, dt$, then $g(x+\pi)$ equals : **[IIT - 97]**

(A) $g(x) + g(\pi)$ (B) $g(x) - g(\pi)$ (C) $g(x)\ g(\pi)$ (D) $[g(x)/g(\pi)]$

13. $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}$ equals : **[IIT - 97]**

(A) $1+\sqrt{5}$ (B) $-1+\sqrt{5}$ (C) $-1+\sqrt{2}$ (D) $1+\sqrt{2}$

14. If for all real number y, [y] is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2}[2\sin x]dx$ is : **[IIT - 99]**

(A) $-\pi$ (B) 0 (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

15. $\int_{\pi/4}^{3\pi/4}\frac{dx}{1+\cos x}$ is equal to : **[IIT - 99]**

(A) 2 (B) -2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

16. The value of the integral $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx$ is : **[IIT - 2000]**

(A) 3/2 (B) 5/2 (C) 3 (D) 5

17. Let g(x) = $\int_0^x f(t)dt$, where f is such that $\frac{1}{2}\le f(t)\le 1$ for $t\in(0,1]$ and $0\le f(t)\le\frac{1}{2}$ for $t\in(1,2]$. Then g(2) satisfies the inequality : **[IIT - 2000]**

(A) $-\frac{3}{2}\le g(2)<\frac{1}{2}$ (B) $0\le g(2)<2$ (C) $\frac{3}{2}<g(2)\le\frac{5}{2}$ (D) $2<g(2)<4$

18. If $f(x)=\begin{cases} e^{\cos x}.\sin x & \text{for } |x|\le 2 \\ 2 & \text{otherwise}\end{cases}$. Then $\int_{-2}^{3}f(x)dx$: **[IIT - 2000]**

(A) 0 (B) 1 (C) 2 (D) 3

19. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx,\ a>0$ is **[IIT - 2001]**

(A) π (B) $a\pi$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

20. The area bounded by the curve $y=|x|-1$ and $y=-|x|+1$ is **[IIT - 2002]**

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

21. Let $f(x) = \int_1^x \sqrt{2-t^2}\,dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are **[IIT - 2002]**

(A) ± 1 (B) $\pm\frac{1}{\sqrt{2}}$ (C) $\pm\frac{1}{2}$ (D) 0 and 1

22. Let T > 0 be a fixed real number. Suppose f is a continuous function such that for all $x \in R\, f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$ then the value of $\int_3^{3+3T} f(2x)dx$ is **[IIT - 2002]**

(A) $\frac{3}{2}I$ (B) 2 I (C) 3 I (D) 6 I

23. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left([x]+\ell n\left(\frac{1+x}{1-x}\right)\right)dx$ equals **[IIT - 2002]**

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $2\ell n\left(\frac{1}{2}\right)$

24. If $\ell(m, n) = \int_0^1 t^m(1+t)^n dt$, then the expression for $\ell(m, n)$ in terms of $\ell(m+1, n-1)$ is

(A) $\frac{m}{n+1}\ell(m+1, n-1)$ (B) $\frac{n}{m+1}\ell(m+1, n-1)$

(C) $\frac{2^n}{m+1}+\frac{n}{m+1}\ell(m+1, n-1)$ (D) $\frac{2^n}{m+1}-\frac{n}{m+1}\ell(m+1, n-1)$ **[IIT - 2003]**

25. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}dt$, then f(x) increases in **[IIT - 2003]**

(A) (2, 2) (B) no value of x (C) $(0, \infty)$ (D) $(-\infty, 0)$

26. The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x-axis in the I[st] quadrant is

(A) 9 (B) 27/4 (C) 36 (D) 18 **[IIT - 2003]**

27. $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$, is equal to **[IIT - 2004]**

(A) $\frac{\pi}{4}-1$ (B) $\frac{\pi}{2}-1$ (C) $\frac{\pi}{4}+1$ (D) $\frac{\pi}{2}+1$

28. If f(x) is differentiable and $\int_0^{t^2} x\,f(x)dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals **[IIT - 2004]**

(A) $-\frac{5}{2}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{5}{2}$

29. The value of $\int_{-2}^{0}[x^3+3x^2+3x+3+(x+1)\cos(x+1)]dx$ is **[IIT - 2005]**

(A) 0 (B) 3 (C) 4 (D) 1

30. If $\int_{\sin x}^{1} t^2 f(t)dt = 1-\sin x\ \forall x \in [0, \pi/2)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is **[IIT - 2005]**

(A) 3 (B) $\sqrt{3}$ (C) $\frac{1}{3}$ (D) none of these

31. The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = 1/4$ is **[IIT - 2005]**

(A) 4 sq. units (B) 1/6 sq. units (C) 4/3 sq. units (D) 1/3 sq. units

W I. Read the following passage and answer the question from 32 to 34

For every function f(x) which is twice differentiable, there will be good approximation of $\int_a^b f(x)dx \cong \left(\frac{b-a}{2}\right)$ (f(a) + f(b)). Now if we take $c = \frac{a+b}{2}$, then using above again, we get

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \cong \frac{b-a}{4}\,(f(a)+f(b)+f(c)) \text{ and so on.}$$

We get approximation for value of $\int_a^b f(x)dx$.

32. Good approximation of $\int_0^{\pi/2} \sin x\ dx$ is **[IIT - 2006]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}\left(\sqrt{2}-1\right)$ (C) $\frac{\pi}{8}\left(\sqrt{2}+1\right)$ (D) $\frac{\pi}{8}$

33. $f''(x) < 0\ \forall\ x \in (a, b)$, C(c, f(c)) is point of maxima where $c \in (a, b)$, then $f'(c)$ is **[IIT - 2006]**

(A) $\frac{f(b)-f(a)}{b-a}$ (B) $\frac{f(b)-f(a)}{a-b}$ (C) $2\left(\frac{f(b)-f(a)}{b-a}\right)$ (D) 0

34. If $\lim\limits_{t \to a} \dfrac{\int_a^t f(x)dx - \frac{t-a}{2}(f(t)+f(a))}{(t-a)^3} = 0$, then degree of polynomial function f(x) ut-most is

(A) 0 (B) 1 (C) 3 (D) 2 **[IIT - 2006]**

35. **Match the following** **[IIT - 2006]**

Column I		**Column II**
(A) $\int\limits_0^{\pi/2} (\sin x)^{\cos x}(\cos x \cot x - \log(\sin x)^{\sin x})dx$	**(i)**	$\frac{4}{3}$
(B) $\left\lvert \int\limits_0^1 (1-y^2)dx \right\rvert + \left\lvert \int\limits_1^0 (y^2-1)dy \right\rvert$	**(ii)**	1

36. Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. **[IIT - 2007]**

Column I		**Column II**
(A) $\int\limits_{-1}^{1} \frac{dx}{1+x^2}$	(p)	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int\limits_0^1 \frac{dx}{\sqrt{1-x^2}}$	(q)	$2\log\left(\frac{2}{3}\right)$
(C) $\int\limits_0^3 \frac{dx}{1-x^2}$	(r)	$\frac{\pi}{3}$
(D) $\int\limits_1^2 \frac{dx}{x\sqrt{x^2-1}}$	(s)	$\frac{\pi}{2}$

IIT JEE PROBLEMS (SUBJECTIVE)

1. Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$. **[IIT - 81]**

2. Show that : $\lim\limits_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{6n}\right)=\log 6$. **[IIT - 81]**

3. Evaluate $\int_0^1 (tx+1-x)^n\,dx$, where n is a positive integer and t is a parameter independent of x.

Hence show that $\int_0^1 x^k(1-x)^{n-k}dx = [\,^nC_k(n+1)]^{-1}$, for k = 0, 1,.........., n. **[IIT - 81]**

4. Show that : $\int_0^{\pi} x f(\sin x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$. **[IIT - 82]**

5. Find the value of $\int_{-1}^{3/2} |x \sin \pi x|\,dx$. **[IIT - 82]**

6. Evaluate : $\int_0^{\pi/4} \frac{\sin x + \cos x}{9+16\sin 2x}dx$. **[IIT - 83]**

7. Find the area bounded by the x-axis, parts of the curve $y = \left(1+\frac{8}{x^2}\right)$ and the ordinates at x = 2 and x = 4. If the ordinate at x = a divides the area into two equal parts, find a. **[IIT - 83]**

8. Evaluate : $\int_0^{1/2} \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$. **[IIT - 84]**

9. Find the area of the region bounded by the x-axis and the curves defined by $y = \tan x,\ -\frac{\pi}{3}\le x\le\frac{\pi}{3}$ and $y = \cot x,\ \frac{\pi}{6}\le x\le\frac{3\pi}{2}$ **[IIT - 84]**

10. Given a function f(x) such that it is integrable over every interval on the real line and f(t + x) = f(x), for every x and a real t, then show that the integral $\int_a^{a+t} f(x)dx$ is in dependent of a. **[IIT - 84]**

11. Evaluate the $\int_0^{\pi/2} \frac{x\sin x\cos x}{\cos^4 x+\sin^4 x}dx$. **[IIT - 85]**

12. Sketch the region bounded by the curve $y = \sqrt{5-x^2}$ and $y = |x - 1|$ and find its area. **[IIT - 85]**

13. Find the area bounded by the curve $x^2 + y^2 = 4$, $x^2 = -\sqrt{2}\,y$ and $x = y$. **[IIT - 86]**

14. Evaluate $\int_0^{\pi} \frac{x\,dx}{1+\cos\alpha \sin x}$, $0 < \alpha < \pi$. **[IIT - 86]**

15. Find the area bounded by the curves, $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$ above the x-axis. **[IIT - 87]**

16. The value of the integral $\int_0^{2a} [f(x)/\{f(x) + f(2a - x)\}]\,dx$. **[IIT - 87]**

17. Find the area of the region bounded by the curve $C : y = \tan x$, tangent drawn to C at $x = \frac{\pi}{4}$ and the x-axis. **[IIT - 88]**

18. Evaluate : $\int_0^1 \log\left[\sqrt{(1-x)} + \sqrt{(1+x)}\right]dx$. **[IIT - 88]**

19. If f and g are continuous functions on [0, a] satisfying $f(x) = f(a - x)$ and $g(x) + g(a - x) = 2$, then show that

(a) $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx$. **[IIT - 89]**

(b) $\int_0^a \frac{dx}{1+e^{f(x)}} = \frac{a}{2}$, (if $f(x) + f(a - x) = 0$). **[IIT - 89]**

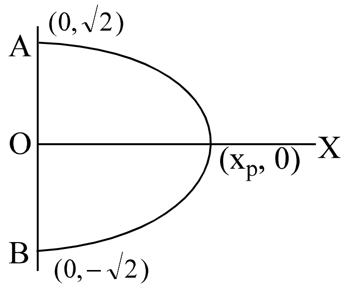
20. Prove that for any positive integer k, $\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + ... + \cos(2k-1)x]$. Hence prove that $\int_0^{\pi/2} \sin 2kx \cot x\,dx = \frac{\pi}{2}$. **[IIT - 90]**

21. Show that : $\int_0^{\pi/2} f(\sin 2x)\sin x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$. **[IIT - 90]**

22. Compute the area of the region bounded by the curves $y = ex \ln x$ and $y = \frac{\ln x}{ex}$ (where $\ln e = 1$). **[IIT - 90]**

23. Sketch the curve and identify the region bounded by $x = \frac{1}{2}$, $x = 2$, $y = \ln x$ and $y = 2^x$. Find the area of this region. **[IIT - 91]**

24. If 'f' is continuous function with $\int_0^x f(t)dt \to \infty$ as $|x| \to \infty$, then show that every line $y = mx$.



intersects the curve $y^2 + \int_0^x f(t)dt = 2$! **[IIT - 91]**

25. Evaluate $\int_0^{\pi} \frac{\sin(2x)\sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}dx$. **[IIT - 91]**

26. Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$. Find the area. **[IIT - 92]**

27. Determine a positive integer $n \le 5$, such that $\int_0^1 e^x (x-1)^n dx = 16 - 6e$. **[IIT - 92]**

28. Evaluate $\int_{-3}^{3} \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)}dx$. **[IIT - 93]**

29. Show that $\int_0^{n\pi+v} |\sin x| dx = 2n + 1 - \cos v$; where n is a positive integer and $0 \le v < \pi$.

[IIT - 94]

30. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$? **[IIT - 94]**

31. Evaluate $\int_0^{\pi} \frac{x}{1+\sin\alpha \sin x}dx$. **[REE - 94]**

32. **(a)** $\int_0^1 \frac{2-x^2}{(1+x)\sqrt{1-x^2}}dx$. **[REE - 95]**

(b) $\int_0^{\pi/2} \frac{\sin 8x.\log(\cot x)}{\cos 2x}dx$. **[REE - 95]**

33. Let $I_m = \int_0^{\pi} \frac{1-\cos mx}{1-\cos x}dx$. Use mathematical induction to prove that $I_m = m\pi$, m = 0, 1, 2, ...

[IIT - 95]

34. Evaluate : $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4}\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx$. **[IIT - 95]**

35. Consider a square with vertices at (1, 1)(–1, 1), (–1, –1) and (1, –1). Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area. **[IIT - 95]**

36. Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0$, $y = 0$ and $x = \frac{\pi}{4}$. Prove that for $n > 2$, $A_n + A_{n-2} = \frac{1}{n-1}$ and deduce $\frac{1}{2n+2} < A_n < \frac{1}{2n-2}$. **[IIT - 96]**

37. Find $\lim\limits_{n\to\infty} S_n$, if : $S_n = \frac{1}{2n} + \frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-4}} + \ldots\ldots\ldots + \frac{1}{\sqrt{3n^2+2n-1}}$. **[REE - 97]**

38. Let a + b = 4, where a < 2 and let g(x) be a differentiable function. If $\frac{dg}{dx} > 0$ for all x, prove that $\int_0^a g(x)dx + \int_0^b g(x)dx$ increases as (b - a) increases. **[IIT - 97]**

39. Determine the value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$. **[IIT - 97]**

40. Integrate the following : $\int_0^{\pi/4} \ell n(1+\tan x)dx$. **[IIT - 97]**

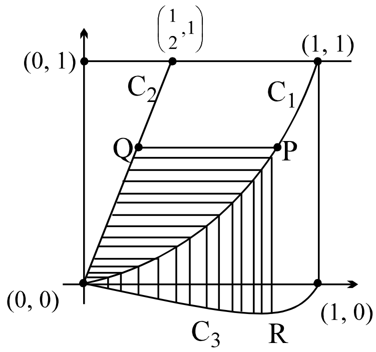
41. Find all the possible values of b > 0, so that the area of the bounded region enclosed between the parabolas $y = x - bx^2$ and $y = \frac{x^2}{b}$ is maximum. **[IIT - 97]**

42. Let O (0, 0), A (2, 0) and B$(1, \frac{1}{\sqrt{3}})$ be the vertices of a triangle. Let R be the region consisting of all those points P inside ΔOAB which satisfy $d(P, OA) \le \min\{d(P, OB), d(P, AB)\}$, where d denotes the distance from the point to the corresponding line. Sketch the region R and find its area. **[IIT - 97]**

43. Let f(x) = maximum $\{x^2, (1-x)^2, 2x(1-x)\}$, where $0 \le x \le 1$. Determine the area of the region bounded by the curve y = f(x), x-axis, x = 0 and x = 1. **[IIT - 97]**

44. Prove that $\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = 2\int_0^1 \tan^{-1} x\,dx$. Hence or otherwise, evaluate the integral $\int_0^1 \tan^{-1}(1-x+x^2)dx$. **[IIT - 98]**

45. Let C_1 and C_2 be the graphs of the function $y = x^2$ and $y = 2x$, $0 \le x \le 1$ respectively. Let C_3 be the graph of a function $y = f(x)$, $0 \le x \le 1$, $f(0) = 0$. For a point P on C_1, let the lines through P, parallel to the axes, meet C_2 and C_3 at Q and R respectively (see figure). If for every position of P (on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function f(x). **[IIT - 98]**



46. Evaluate $\displaystyle\int_0^1 \frac{1}{(5+2x-2x^2)(1+e^{(2-4x)})}dx$. **[REE - 98]**

47. Integrate : $\displaystyle\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$ **[IIT - 99]**

48. Let f(x) be a continuous function given by $f(x) = \begin{cases} 2x, & |x| \le 1 \\ x^2+ax+b, & |x| > 1 \end{cases}$

Find the area of the region in the third quadrant bounded by the curve $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$. **[IIT - 99]**

49. **(i)** Evaluate the integral $\displaystyle\int_0^{\pi/6} \frac{\sqrt{3\cos 2x - 1}}{\cos x}dx$. **[REE - 99]**

(ii) Find the sum of the following infinite series $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n!}\left[\sum_{k=0}^{n}(k+1)\int_0^1 2^{-(k+1)x}dx\right]$. **[REE - 99]**

50. For $x > 0$, let $f(x) = \displaystyle\int_e^x \frac{\ell n\, t}{1+t}dt$. Find the function $f(x) + f(1/x)$ and show that, $f(e) + f(1/e) = 1/2$.

[IIT - 2000]

51. **(a)** $S_n = \dfrac{1}{1+\sqrt{n}} + \dfrac{1}{2+\sqrt{2n}} + \ldots\ldots\ldots\ldots + \dfrac{1}{n+\sqrt{n^2}}$. Find $\displaystyle\lim_{n\to\infty} S_n$. **[REE - 2000]**

(b) Given $\displaystyle\int_0^1 \frac{\sin t}{1+t}dt = \alpha$, find the value of $\displaystyle\int_{4\pi-2}^{4\pi} \frac{\sin\frac{t}{2}}{4\pi+2-t}dt$ in terms of α. **[REE - 2000]**

52. Let $b \ne 0$ and for $j = 0, 1, 2, \ldots n$, let S_j be the area of the region bounded by the y-axis and the curve $xe^{ay} = \sin by$, $\dfrac{j\pi}{b} \le y \le \dfrac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \ldots\ldots\ldots S_n$ are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$. **[IIT - 2001]**

53. Evaluate $\int_{0}^{\pi/2} \frac{\cos^9 x}{\cos^3 x + \sin^3 x} dx.$ **[REE - 2001]**

54. Evaluate $\int_{0}^{\pi} \frac{x\, dx}{1 + \cos\alpha \sin x}$. **[REE - 2001]**

55. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$, which lies to the right of the line $x = 1$. **[IIT - 2002]**

56. If f is an even function then prove that $\int_{0}^{\pi/2} f(\cos 2x)\cos x\, dx = \sqrt{2}\int_{0}^{\pi/4} f(\cos 2x)\cos x\, dx$. **[IIT - 2003]**

57. If the function $f : [0, 4] \to R$ is differentiable then show that $\int_{0}^{4} f(t)dt = 2\left[\alpha f(\alpha^2) + \beta f(\beta^2)\right]$ for some $\alpha, \beta \in [0, 2]$. **[IIT - 2003]**

58. If $y(x) = \int_{-\pi^2/16}^{x^2} \frac{\cos x \cos\sqrt{\theta}}{1 + \sin^2\sqrt{\theta}} d\theta$. Find $\frac{dy}{dx}$ at $x = \pi$. **[IIT - 2004]**

59. Evaluate : $\int_{-\pi/3}^{\pi/3} \frac{4x^3 + \pi}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} dx$. **[IIT - 2004]**

60. Evaluate : $\int_{0}^{\pi} e^{|\cos x|}\left\{2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right\}\sin x\, dx$. **[IIT - 2005]**

61. Find the area bounded by the curve $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$. **[IIT - 2005]**

62. f(x) is a differentiable function and g(x) is a double differentiable function such that $|f(x)| \le 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 9$. Prove that there exists some $c \in (-3, 3)$ such that $g(c).\, g''(c) < 0$. **[IIT - 2005]**

63. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$, f(x) is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by f(x) and chord AB. **[IIT - 2006]**

64. $\frac{5050\int_{0}^{1}(1 - x^{50})^{100} dx}{\int_{0}^{1}(1 - x^{50})^{101} dx}$ **[IIT - 2006]**

SET – I

1. The value of $\int_0^{2\pi} \cos^5 x \ \ dx$ is equal to

(A) 2 (B) π (C) 0 (D) none of these

2. The value of the integral $\int_a^b \frac{|x|}{x} dx (a < b)$ is

(A) b - a (B) a - b (C) b + a (D) none of these

3. If $\int_0^1 \frac{e^t dt}{t+1} = a$, then $\int_{b-1}^{b} \frac{e^{-t}}{t-b-1} dt$ is equal to

(A) ae^{-b} (B) $-ae^{-b}$ (C) $-be^{-a}$ (D) ae^{b}

4. The value of $\int_{-4}^{4} \left(\sqrt{1+x+x^2} - \sqrt{1-x+x^2}\right) dx$ is

(A) > 0 (B) < 0 (C) 0 (D) none of these

5. The value of $\sum_{r=1}^{n} \int_0^1 f(r-1+x) dx$ is equal to

(A) $\int_0^1 f(x) dx$ (B) $n\int_0^1 f(x) dx$ (C) $(n-1)\int_0^1 f(x) dx$ (D) $\int_0^n f(x) dx$

6. $\lim_{n\to\infty} \left[\frac{1}{2n} + \frac{1}{2n+1} + ... + \frac{1}{6n}\right]$ is equal to

(A) $\ell n 3$ (B) $\ell n 6$ (C) $\ell n 2$ (D) $2\ell n 2$

7. The value of the integral $\int_0^1 \frac{x^3}{1+x^8} dx$ is

(A) $\frac{\pi}{16}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{8}$ (D) none of these

8. If f and it's first two derivative are continuous over [a, b] then the value of $\int_a^b x \ f''(x) dx$ is

(A) $[bf'(b) - af'(a)] - [af(b) - bf(a)]$ (B) $[bf'(b) - af'(a)] - [f(b) - f(a)]$
(C) $[af'(b) - bf'(a)] - [f(b) - f(a)]$ (D) none of these

9. The value of integral $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$ is equal to

(A) 0 (B) 1 (C) $\pi/2$ (D) none of these

10. If f (x) is continuous and $n \in N$, then the value of $\int_{-2}^{2}[f(x)-f(-x)]x^{4n}dx$ is

(A) n + 2 (B) n + 4 (C) 2n (D) none of these

11. The value of integral $\int_{-1}^{3}\left(\tan^{-1}\left(\frac{x}{1+x^2}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)dx$

(A) $\frac{3\pi}{2}$ (B) π (C) 2π (D) none of these

12. If $\int_{0}^{1}\left(\frac{\tan^{-1}x}{x}\right)dx=\lambda\int_{0}^{\pi/2}\frac{x}{\sin x}dx$ then the value if λ is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{2}{\pi}$ (D) $\frac{\pi}{2}$

13. The value of $\int_{-\pi/4}^{\pi/4}\frac{dx}{\sec^2 x(1+\sin x)}$ is ,

(A) $\frac{\pi}{4}$ (B) π (C) $\frac{\pi}{2}$ (D) 2π

14. The value of $\int_{0}^{1}|\sin 2\pi x|\,dx$

(A) 0 (B) $\pm\frac{1}{\pi}$ (C) $-\frac{2}{\pi}$ (D) $\frac{1}{\pi}$

15. If f(x) is a continuous function for all real value of x and satisfies $\int_{n}^{n+1}f(x)dx=\frac{n^2}{2}\ \forall n\in I$, then $\int_{-3}^{5}f(|x|)\,dx$ is equal to

(A) $\frac{19}{2}$ (B) $\frac{35}{2}$ (C) $\frac{17}{2}$ (D) none of these

16. If f is a continuous function then $\frac{1}{k}\int_{ak}^{bk}f\left(\frac{x}{k}\right)dx$ is

(A) $\frac{1}{k}\int_{a}^{b}f(x)dx$ (B) $\int_{a}^{b}f(x)dx$ (C) $k\int_{a}^{b}f(x)dx$ (D) none of these

17. The value of $\int_{0}^{1}\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

(A) 1 (B) 0 (C) - 1 (D) none of these

18. If $I_1 = \int_0^{n\pi} f(|\cos x|)dx$ and $I_2 = \int_0^{5\pi} f(|\cos x|)dx$ then $(n \in N)$

(A) $n I_1 = 5 I_2$ (B) $I_1 + I_2 = n + 5$ (C) $\frac{I_1}{n} = \frac{I_2}{5}$ (D) none of these

19. $\int_{50}^{100} \frac{\ln x}{\ln x + \ln(150 - x)} dx$ is equal to;

(A) $\pi/4$ (B) $\pi/2$ (C) 25 (D) 50

20. The value of $\int_{-\pi}^{\pi} (a \sin^3 x + b \tan x^3 + c^2)dx$ is

(A) dependent on a, b, c (B) dependent on a and b
(C) dependent on b and c (D) dependent on c only

Definite Integrals

SET–II

1. The value of $\int_{-\pi}^{\pi}(\cos px-\sin qx)^2\,dx$, where p and q are integers, is

(A) π (B) 0 (C) 4π (D) 2π

2. If $I_n=\int_0^{\pi/4}\tan^n\theta\,d\theta$, then for any positive integer the value of $n(I_{n-1}+I_{n+1})$ is

(A) 1 (B) 2 (C) $\pi/4$ (D) π

3. The value of $I=\int_0^3\left([x]+\left[x+\frac{1}{3}\right]+\left[x+\frac{2}{3}\right]\right)dx$, where [.] denotes the greatest integer function, is equal to

(A) 10 (B) 11 (C) 12 (D) none of these

4. The value of $\int_{-10}^{10}\frac{3^x}{3^{[x]}}dx$ is equal to

(A) 20 (B) $-\frac{40}{\ell n3}$ (C) $\frac{20}{\ell n3}$ (D) none of these

5. If f(x) and g(x) are real valued functions such that $f(x)>0\ \forall\ x\in R$ and g(x) is differentiable every where, and $h(x)=\int_0^{g(x)}f(t)dt$, then

(A) h(x) is increasing function when g(x) is decreasing function
(B) h(x) is increasing function when g(x) is increasing function
(C) h(x) is decreasing when g(x) is decreasing
(D) Nothing can be said in general about the behavior of h(x)

6. If $f(x)=\begin{vmatrix} x & \cos x & e^{x^2} \\ \sin x & x^2 & \sin^2 x \\ \tan x & 1 & 2 \end{vmatrix}$ then the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f(x)dx$ is equal to

(A) 1 (B) 0 (C) 2 (D) none of these

7. The value of $\int_0^{\pi/2}|\cos x-\sin x|\,dx$ is equal to

(A) $2\sqrt{2}-1$ (B) 2 (C) 4 (D) $4\sqrt{2}-1$

8. Let f(x) be a function satisfying $f'(x) = f(x)$ with f(0) = 1 and g be the function satisfying } $f(x) + g(x) = x^2$. The value of integral $\int_0^1 f(x)g(x)\,dx$ is

(A) $\frac{1}{4}(e-7)$ (B) $\frac{1}{4}(e-2)$ (C) $\frac{1}{2}(e-3)$ (D) none of these

9. Let f(x) be a continuous function such that f(a - x) + f(x) = 0 for all $x \in [0, a]$, then $\int_0^a \frac{dx}{1+e^{f(x)}}$ is equal to

(A) a (B) a/2 (C) f(a) (D) $\frac{f(a)}{2}$

10. Let $f : R \to R$ and $g : R \to R$ be continuous functions. Then the value of

$\int_{-\pi/2}^{\pi/2} (f(x) + f(-x))\,(g(x) - g(x-)dx$ is

(A) π (B) 1 (C) - 1 (D) 0

11. The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\tan^2 x}$ is

(A) 1 (B) $\frac{\pi}{12}$ (C) $\frac{\pi}{6}$ (D) none of these

12. If $I = \int_{-1}^{1} \left(\frac{x^2 + \sin x}{1+x^2}\right) dx$ then

(A) 0 (B) 2 (C) $\frac{\pi}{2}$ (D) $2 - \frac{\pi}{2}$

13. If f(x) be quadratic polynomial such that $f(0) = 2,\ f'(0) = -3,$ and $f''(0) = 4$, then $\int_{-1}^{1} f(x)\,dx$ is

(A) –3 (B) $\frac{16}{3}$ (C) 0 (D) $\frac{3}{16}$

14. If f is an odd function, then $I = \int_{-a}^{a} \frac{f(\sin x)}{f(\cos x) + f(\sin^2 x)} dx$

(A) can’t be evaluated (B) I = 0

(C) $I = \frac{\pi}{2}$ (D) none of these

15. The value of $\int_0^{100} \{\sqrt{x}\} dx$ (where $\{x\}$ is the fractional part of x) is

(A) 50 (B) 1 (C) 100 (D) none of these

16. $\int_0^{\pi} x\, f(\sin x) dx$ is equal to

(A) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ (B) $\pi \int_0^{\pi/2} f(\sin x)dx$ (C) $2\pi \int_0^{\pi/2} f(\sin x)dx$ (D) none of these

17. If $(a + b - x) = f(x)$ then $\int_a^b x\, f(x)dx$ is equal to

(A) $\frac{a-b}{2}\int_a^b f(x)dx$

(B) $\left(\frac{a+b}{2}\right)\int_a^b f(x)dx$

(C) 0

(D) none of these

18. For an integer n, the integral $\int_0^{\pi} e^{\cos^2 x} \cos^3(2n+1)x\, dx$ has the value

(A) π (B) 1 (C) 0 (D) none of these

19. $\int_0^1 [f(x).g''(x) - f''(x).g(x)]dx$ is equal to

(A) $f'(1)g(1) - f(1)g'(1)$

(B) $f(1)g'(1) + f'(1)g(1)$

(C) $f(1)g'(1) - f'(1)g(1)$

(D) none of these

20. If $f(x) = \begin{cases} 3[x] - 5\frac{|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$, then $\int_{-3/2}^{2} f(x)dx$ is equal to ([.] denote the greatest integer function)

(A) $-\frac{11}{2}$ (B) $-\frac{7}{2}$ (C) -6 (D) $-\frac{17}{2}$

SET–III

I TRUE OR FALSE

1. **(i)** $\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x)+f(-x)]dx$

(ii) The value of integral $\int_{0}^{2\pi} e^{\sin^2 nx} \tan nx \, dx$ is π.

(iii) The value of $\int_{0}^{2\pi} (x\cos x)\, dx$ is zero.

2. **(i)** $\int_{a}^{b} (f(x))^n \, dx = \left[\int_{a}^{b} f(x)\, dx\right]^n$

(ii) The value of the integral $\int_{0}^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx$ is equal to a.

(iii) The value of $\int_{3}^{9} \frac{\sqrt{x}}{\sqrt{12-x}+\sqrt{x}} dx$ is 9.

II. FILL IN THE BLANKS

3. **(i)** If $I_1 = \int_{1}^{2} \frac{e^x}{x} dx$ and $I_2 = \int_{e}^{e^2} \frac{1}{\log x} dx$, then $\frac{I_1}{I_2}$ = ...

(ii) The value of $\int_{0}^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx$ is ....

(iii) The value of the integral $\int_{0}^{\infty} e^{-2x}(\sin 2x+\cos 2x)dx$ is ....

4. **(i)** If $\int_{0}^{a-b} f(x+b)dx = k \int_{b}^{a} f(x)dx$, then the value of k is ....

(ii) The greater of $\int_{0}^{\pi/2} \frac{\sin x}{x} dx$ and $\pi/2$ is ....

(iii) The least value of $f(x) = \int_{x}^{2} \log_{1/3} t \, dt$ for $x \in \left(\frac{1}{10}, 4\right)$ is at x = ...

W I. Read the following passage and answer the questions :

If $I_n = \int\limits_0^{\pi/4} \tan^n x\,dx$, then

5. The value of $I_n + I_{n-2}$ is

(A) $\frac{1}{n-1}$ (B) $\frac{1}{n-3}$ (C) $\frac{1}{n-2}$ (D) none of these

6. The value of $I_{n-1} + I_{n+1}$ is

(A) $\frac{1}{2n}$ (B) $\frac{1}{n}$ (C) $\frac{1}{3n}$ (D) none of these

7. Which of the following statement is correct for all n = 2, 3, 4, ...

(A) $\frac{1}{n+1} < 3I_n < \frac{1}{n-1}$ (B) $\frac{1}{n+1} < I_n < \frac{1}{n-1}$

(C) $\frac{1}{n+1} < 2\,I_n < \frac{1}{n-1}$ (D) none of these

W II. Read the following passage and answer the questions :

Let $g(x) = \int\limits_0^x f(t)dt$ and $f(x+2) = f(x)\,\forall\, x \in D_f$, then

8. Which of the following statement is correct

(A) g is an even function (B) g is an odd function

(C) g is neither even nor odd function (D) none of these

9. The period of the function g is

(A) 3 (B) 2 (C) 4 (D) none of these

10. The value of g(2n) for all $n \in Z$ is

(A) 0 (B) 1 (C) 2 (D) none of these

11. Which of the following statement is correct

(A) g(x + 2) = g(x) + g(2) (B) g(x + 2) = g(x) – g(2)

(C) g(x + 2) = g(x) + 2g(2) (D) none of these

W III. Read the following passage and answer the questions :

In the following questions an Assertion (A) is given followed by a Reason (R).

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion

(B) Both Assertion and Reason are true and Reason is not the correct explanation of Assertion

(C) Assertion is true but Reason is false

(D) Assertion is false but Reason is true

12. **Assertion (A) :** $\int\limits_0^{10} \{x-[x]\}dx = 5$, where [ ] denotes greatest integer function

Reason (R) : $\int\limits_0^{na} f(x)dx = n\int\limits_0^{a} f(x)dx$, if $f(x+a) = f(x)$

13. **Assertion (A) :** $\int_{-\pi/2}^{\pi/2} |\sin x|\,dx = 2$

Reason (R) : $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

14. **Assertion (A) :** $\int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right)dx = 0$

Reason (R) : If f in an odd function $\int_{-a}^{a} f(x)dx = 0$

15. **Assertion (A) :** $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|}$ dx is same as $\int_0^1 \frac{-2x^2}{3-|x|}dx$

Reason (R) : Since $\frac{\sin x}{3-|x|}$ is an odd function so that $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} = \int_0^1 \frac{-2x^2}{3-|x|}dx$

Ì Éï’ **Read the following passage and answer the questions :**

Consider $I_n = \int_0^a \frac{\sin^{\lambda} nx}{\sin^{\mu} x}dx$, where $a \in R^+$ and $n, \ \mu, \ \lambda \in N$.

16. If $a = \frac{\pi}{2}, \lambda = 2, \mu = 1$, then $I_{n+1} - I_n$ is equal to

(A) $\frac{1}{n+1}$ (B) $\frac{1}{2(n+1)}$ (C) $\frac{1}{2n+1}$ (D) none of these

17. If $a = \frac{\pi}{2}, \lambda = 2, \mu = 2$, then $I_{n+1} + I_{n-1} - 2I_n$ is equal to

(A) 0 (B) 1 (C) 2 (D) none of these

18. If $a = \pi, \lambda = 1, \mu = 1$ and n is an even number, then I_n is

(A) π (B) 0 (C) 2π (D) none of these

19. If $a = \pi/2, \lambda = 1, \mu = 1$ and n is an odd number, then I_n is

(A) 0 (B) $\frac{\pi}{2}$ (C) π (D) none of these

20. If $a = \frac{\pi}{2}, \ \lambda = 1, \ \mu = 1$ and n is an even number, then $I_{n+2} - I_n$ is equal to

(A) $2\frac{(-1)^{\frac{n}{2}}}{n-1}$ (B) $\frac{(-1)^{\frac{n}{2}}}{n-1}$ (C) $2\frac{(-1)^{\frac{n}{2}}}{n+1}$ (D) none of these

LEVEL–I ANSWER – KEY

1. **(i)** $4\sqrt{2}$ **(ii)** $4\log\frac{4}{3}$ **2. (i)** $\frac{\pi^2}{8}$ **(ii)** $2-\pi$

3. **(i)** $\frac{\pi(a+b)}{2\sqrt{2}}$ **(ii)** $\frac{7\pi^2}{72}$ **4. (i)** $\frac{2\pi}{\sqrt{3}}$ **(ii)** $\frac{5\pi}{27}$

5. **(i)** $\frac{16\pi}{3}-2\sqrt{3}$ **(ii)** $\frac{\pi^3}{2}-12\pi+24$ **6.** $\frac{\pi}{4}-\frac{1}{2}$

8. 0 **9. (i)** $2e^{\left(\frac{\pi-4}{e}\right)}$ **(ii)** e^{-1} **10.** $1-\frac{1}{\sqrt{2}}$

12. f(x) is max at x = 1, neither max nor min at x = 2, f(x) is min at x = $\frac{7}{5}$

14. $\frac{5\pi}{2}-4\tan^{-1}2+\ln\frac{5}{2}$

LEVEL–II

1. (i) $\frac{\pi}{4}$ **(ii)** $\frac{1}{6}n(n-1)(4n+1)$ **2.** $\frac{[x]+2^{\{x\}}-1}{\ln 2}$ **3.** $\frac{35}{2}$ **7.** $-\cos x$

10. (i) $x = n\pi,\ n \in N$ ...& max. if n is odd and min. if n is even **(ii)** min. at -2, 0, 2 and max. at x=± 1

11. $$f(x)=\begin{cases} x-\frac{x^2}{2} & \text{if } 0\le x\le 1 \\ \frac{1}{2} & \text{if } 1<x\le 2 \\ \frac{(x-2)^3}{3}+\frac{1}{2} & \text{if } 2<x\le 3 \end{cases}$$ **12.** $U_n = \frac{n\pi}{2}$

13. $\frac{\pi}{4}-\frac{1}{\sqrt{3}}\log\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$ **14.** $\frac{4}{e}$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. $-\left(\frac{15\pi+32}{60}\right)$ **2.** $2-\sqrt{2}$ **3.** 4

4. $\pi\left(\sqrt{2}-1\right)$ **5.** $\frac{1}{2}$ **6.** $\frac{1}{a^2-b^2}\left[a(\log 2-5)+\frac{7b}{2}\right]$

7. π^2 **8.** 1 **9.** 2 **10.** 16

(B)

1. T

(C)

1. A **2.** A **3.** BD **4.** BD

(D)

1. D	**2.** B	**3.** C	**4.** A	**5.** C	**6.** D
7. A	**8.** D	**9.** D	**10.** B	**11.** C	**12.** A
13. B	**14.** C	**15.** A	**16.** B	**17.** B	**18.** C
19. C	**20.** B	**21.** A	**22.** C	**23.** A	**24.** A
25. A	**26.** D	**27.** B	**28.** A	**29.** C	**30.** A
31. D	**32.** C	**33.** D	**34.** D	**35.** (A-ii, B-i)	

36. A-s, B-s, C-p, D-r

IIT JEE PROBLEMS (SUBJECTIVE)

1. $\frac{9}{8}$ sq. unit

3. $\frac{t^{n+1}-1}{(t-1)(n+1)}$

5. $\frac{3}{\pi}+\frac{1}{\pi^2}$

6. $\frac{1}{20}\ell n3$

7. $a = 2\sqrt{2}$

8. $\frac{6-\pi\sqrt{3}}{12}$

9. $\log\frac{3}{2}$ sq. units

11. $\frac{\pi^2}{16}$

12. $\frac{5\pi-2}{4}$ sq. units

13. $\pi+\frac{1}{3}$ sq. units

14. $\frac{\pi\alpha}{\sin\alpha}$

16. $4+25\sin^{-1}\frac{4}{5}$

16. a

17. $\frac{1}{2}\left[\log 2-\frac{1}{2}\right]$ sq. units

18. $\frac{1}{2}\left[\log 2-\frac{\pi}{2}-1\right]$

22. $\frac{e^2-5}{4e}$

23. $\frac{4-\sqrt{2}}{\log 2}-\frac{5}{2}\log 2+\frac{3}{2}$

25. $\frac{8}{\pi^2}$

26. $\left(\pi-\frac{2}{3}\right)$ sq. units

27. $n = 3$

28. $\frac{3}{2}\log 2-\frac{1}{10}$

29. $2n + 1 - \cos\gamma$

30. 121 : 4

31. $\frac{2\pi}{\cos\alpha}\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

32. (a) $\frac{\pi}{2}$ **(b)** zero

34. $\frac{\pi}{12}[\pi+3\log_e(2+\sqrt{3})-4\sqrt{3}]$

35. $\frac{16\sqrt{2}-20}{3}$

37. $\frac{\pi}{6}$ **39.** π^2 **40.** $\frac{\pi}{8}\ln 2$ **41.** b = 1

42. $2-\sqrt{3}$ **43.** $\frac{17}{27}$ sq. units **44.** log 2 **45.** $f(x)=x^3-x^2$

46. $\frac{1}{2\sqrt{11}}\ell n\frac{\sqrt{11}+1}{\sqrt{11}-1}$ **47.** $\frac{\pi}{2}$ **48.** $\frac{257}{192}$ sq. units

49. (i) $\sqrt{\frac{2}{3}}\pi-2\tan^{-1}\sqrt{2}$ **(ii)** $\frac{1}{\ell n2}\left(\frac{\sqrt{e}}{2}+e\right)$ **51.** **(a)** $2\ell n2$ **(b)** $-\alpha$

52. $\frac{n(1+e)}{1+\pi^2}\left(\frac{e^{n+1}-1}{e-1}\right)$ **53.** $\frac{1}{8}\left[\frac{5\pi}{4}-\frac{1}{3}\right]$ **54.** $I=\begin{cases}\frac{\pi\alpha}{\sin\alpha} & \text{if } \alpha\in(0,\pi)\\ \frac{\pi}{\sin\alpha}(\alpha-2\pi) & \text{if } \alpha\in(0,2\pi)\end{cases}$

55. $\left(\frac{20}{3}-4\sqrt{2}\right)$ sq. units **58.** 2π **59.** $\frac{4\pi}{\sqrt{3}}\left[\tan^{-1}3-\frac{\pi}{4}\right]$

60. $\frac{24}{5}\left[e\cos\left(\frac{1}{2}\right)+\frac{1}{2}e\sin\left(\frac{1}{2}\right)-1\right]$ **61.** $\frac{1}{3}$ sq. units **63.** $\frac{125}{3}$ sq. units

64. $\frac{5051}{5050}$

SET – I

1.	C	**2.**	D	**3.**	B	**4.**	C	**5.**	D
6.	A	**7.**	A	**8.**	B	**9.**	B	**10.**	D
11.	B	**12.**	B	**13.**	C	**14.**	C	**15.**	B
16.	B	**17.**	B	**18.**	C	**19.**	C	**20.**	D

SET – II

1.	D	**2.**	A	**3.**	C	**4.**	B	**5.**	BC
6.	B	**7.**	D	**8.**	D	**9.**	B	**10.**	D
11.	B	**12.**	D	**13.**	B	**14.**	B	**15.**	D
16.	AB	**17.**	B	**18.**	C	**19.**	C	**20.**	A

SET – III

I. True or False

1.	**(i)**	T	**(ii)**	F	**(iii)**	T
2.	**(i)**	F	**(ii)**	T	**(iii)**	F

II. Fill in the blanks

3.	**(i)**	1	**(ii)**	a	**(iii)**	$\frac{1}{2}$
4.	**(i)**	1	**(i)**	$\frac{\pi}{2}$	**(iii)**	1

5. A **6.** B

7.	C	**8.**	A	**9.**	B	**10.**	A	**11.**	A
12.	A	**13.**	A	**14.**	A	**15.**	A	**16.**	C
17.	A	**18.**	B	**19.**	B	**20.**	C		