Question Bank - Progression & series

LEVEL–I

1. Find the sum of first 24 terms of the A.P. $a_1, a_2, a_3, \ldots$ if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$.

2. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120° and the common difference is 5. Find the number of sides of the polygon.

3. The r^{th}, s^{th} and t^{th} terms of a certain G..P. are R, S and T respectively. Prove that $R^{s-t} . S^{t-r} . T^{r-s} = 1$.

4. The sum of three numbers in G.P. is 42. If the first two numbers are increased by 2 and third is decreased by 4, the resulting numbers form an A.P. Find the numbers of G.P.

5. If one G.M. G and two arithmetic means p and q be inserted between any given numbers, then show that $G^2 = (2p - q)(2q - p)$.

6. The sum of an infinite geometric series is 162 and the sum of its first n terms is 160. If the reciprocal of its common ratio is an integer, find all possible values of the common ratio, n and the first term of the series.

7. If pth, q^{th}, r^{th} terms of an A.P. be a, b, c respectively, then prove that $p(b - c) + q(c - a) + r(a - b) = 0$.

8. Find S_∞ of the G.P. whose first term is 28 and the fourth term is $\frac{4}{49}$.

9. Find the sum of n terms of the series, the rth term of which is $(2r + 1) 2^r$.

10. After striking the floor a certain ball rebounds $4/5^{th}$ of the height from which it has fallen. Find the total distance that it travels before coming to rest if it gently dropped from a height of 120 meter.

LEVEL–II

1. If $x = 1 + a + a^2 + a^3 + \ldots$ to ∞ $(|a| < 1)$ and $y = 1 + b + b^2 + b^3 + \ldots$ to ∞ $(|b| < 1)$, then prove that $1 + ab + a^2b^2 + a^3b^3 + \ldots$ to ∞ $= \dfrac{xy}{x+y-1}$.

2. If the A.M. of a and b is twice as great as their G.M., then show that a : b = $(2+\sqrt{3}):(2-\sqrt{3})$.

3. An AP and an HP have the same first term, the same last term and the same number of terms ; prove that the product of the r^{th} term from the beginning in one series and the r^{th} term from the end in the other is independent of r.

4. The sum of first ten terms of an A.P. is equal to 155, and the sum of the first two terms of a G.P. is 9, find these progressions, if the first term of A.P. is equal to common ratio of G.P. and the first term of G.P. is equal to common difference of A.P.

5. The series of natural numbers is divided into groups (1); (2, 3, 4); (5, 6, 7, 8, 9); ... and so on. Show that the sum of the numbers in the nth group is $(n-1)^3 + n^3$.

6. Suppose x and y are two real numbers such that the rth mean between x and 2y is equal to the rth mean between 2x and y when n arithmetic means are inserted between them in both the cases. Show that $\dfrac{n+1}{r} - \dfrac{y}{x} = 1$.

7. An A.P. and a G.P. with positive terms have the same number of terms and their first terms as well as last terms are equal. Show that the sum of the A.P. is greater than or equal to the sum of the G.P.

8. Solve the following equations for x and y,

$$\begin{cases} \log_{10} x + \log_{10} x^{1/2} + \log_{10} x^{1/4} + \ldots\ldots\ldots\infty & = y \\ \dfrac{1+3+5+\ldots\ldots\ldots+(2y-1)}{4+7+10+\ldots\ldots\ldots+(3y+1)} & = \dfrac{20}{7\log_{10} x} \end{cases}.$$

9. The first and last terms of an A.P. are a and b. There are altogether (2n + 1) terms. A new series is formed by multiplying each of the first 2n terms by the next term. Show that the sum of new series is $\dfrac{(4n^2-1)(a^2+b^2)+(4n^2+2)ab}{6n}$.

10. Sum the following series to n terms and to infinity :

(i) $\dfrac{1}{1.3.5} + \dfrac{1}{3.5.7} + \dfrac{1}{5.7.9} + \ldots\ldots\ldots\ldots$

(ii) $\dfrac{1}{1.4.7} + \dfrac{1}{4.7.10} + \dfrac{1}{7.10.13} + \ldots\ldots\ldots\ldots$

(iii) $\sum\limits_{r=1}^{n} r(r+1)(r+2)(r+3)$

(iv) $\sum\limits_{r=1}^{n} \dfrac{1}{4r^2-1}$

(v) If $A = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots\ldots\ldots\ldots \dfrac{1}{n} + \dfrac{1}{n+1}$ and

$B = \dfrac{n+2}{2} - \left\{\dfrac{1}{(n+1)n} + \dfrac{2}{n(n-1)} + \dfrac{3}{(n-1)(n-2)} + \ldots\ldots\ldots + \dfrac{(n-1)}{3.2}\right\}$, then show that A = B.

INEQUALITIES

1. **(a)** If $x_i > 0$, $(i = 1, 2, ... n)$, then prove that $(x_1 + x_2 + ... + x_n)\left(\frac{1}{x_1}+\frac{1}{x_2}+...+\frac{1}{x_n}\right) \geq n^2$.

(b) If $a_1, a_2, ...a_n$ are n non-zero real numbers, prove that $\left(a_1^{-2}+........+a_n^{-2}\right) \geq \frac{n^2}{a_1^2+.......+a_n^2}$.

2. **(i)** If $a_1, a_2, ..., a_n$ are n positive real numbers, show that $na_1a_2.......a_n \leq a_1^n + a_2^n + ... + a_n^n$.

(ii) If a, b, c are three distinct positive real numbers. Prove that $\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c} > 6$ or, $bc(b + c) + ca(c + a) + ab(a + b) > 6abc$.

(iii) If a, b, c are three distinct positive real numbers, prove that $a^2(1 + b^2) + b^2(1 + c^2) + c^2(1 + a^2) > 6abc$.

(iv) If a, b, c, d are distinct positive real number, prove that $a^8(1 + b^8) + b^8(1 + c^8) + c^8(1 + d^8) + d^8(1 + a^8) > 8a^3 b^3 c^3 d^3$.

(v) Show that, if a, b, c, d be four positive unequal quantities and $s = a + b + c + d$, then $(s - a)(s - b)(s - c)(s - d) > 81\ abcd$.

(vi) If a, b, c, d are distinct positive real numbers such that $3s = a + b + c + d$, then prove that $abcd > 81(s - a)(s - b)(s - c)(s - d)$.

3. If $a_i < 0$ for all $i = 1, 2, ..., n$ prove that

(i) $(a_1+a_2+.........+a_n)\left(\frac{1}{a_1}+\frac{1}{a_2}+.......+\frac{1}{a_n}\right) > n^2$.

(ii) $(1 - a_1 + a_1^2)(1 - a_2 + a_2^2)..........(1 - a_n + a_n^2) > 3^n(a_1 a_2....a_n)$ (where n is even).

4. Prove that $\left[\frac{a^2+b^2}{a+b}\right]^{a+b} > a^a b^b$

5. Prove that $\left[\frac{x^2+y^2+z^2}{x+y+z}\right]^{x+y+z} > x^x y^y z^z > \left[\frac{x+y+z}{3}\right]^{x+y+z}$

6. If none of $b_1, b_2, ...,b_n$ is zero, prove that

$$\left(\frac{a_1}{b_1}+.......+\frac{a_n}{b_n}\right)^2 \leq (a_1^2+........+a_n^2)(b_1^{-2}+.......+b_n^{-2}).$$

7. Show that $\sqrt{1}+\sqrt{2}+........+\sqrt{n} < n\sqrt{\frac{n+1}{2}} < (n+1)^{3/2}$.

8. By considering the sequence 1, a^2, a^4,..........a^{2n},........, where $0 < a < 1$, prove that
(i) $1 - a^{2n} > na^{n-1}(1 - a^2)$ **(ii)** $1 - a^{2n} < n(1 - a^2)$.

9. If x, y, z are postive and x + y + z = 1, prove that $\left(\frac{1}{x}-1\right)\left(\frac{1}{y}-1\right)\left(\frac{1}{z}-1\right)\geq 8$

10. If $n^5 < 5^n$ for a fixed positive integer $n \geq 6$, show that $(n + 1)^5 < 5^{n+1}$.

11. **(i)** If a, b, c are the sides of a triangle , then prove that $a^2 + b^2 + c^2 > ab + bc + ca$.

(ii) In a triangle ABC prove that $\frac{3}{2}\leq\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}<2$.

(iii) If a, b, c be the length of the sides of a scalene triangle, prove that $(a + b + c)^3 > 27 (a + b - c) (b + c - a) (c + a - b)$.

(iv) If a, b, c are positive real numbers representing the sides of a scalene triangle, prove that $ab + bc + ca < a^2 + b^2 + c^2 < 2 (ab + bc + ca)$ or , $1<\frac{a^2+b^2+c^2}{ab+bc+ca}<2$ and hence prove that $3(ab + bc + ca) < (a + b + c)^2 < 4(ab + bc + ca)$ or $3<\frac{(a+b+c)^2}{ab+bc+ca}<4$.

12. If A, B and C are the angles of a triangle, prove that :

(i) $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)\leq\frac{1}{8}$ **(ii)** $\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)\leq\frac{3\sqrt{3}}{8}$

(iii) $\cos A + \cos B + \cos C\leq\frac{3}{2}$. **(iv)** $\tan^2\left(\frac{A}{2}\right)+\tan^2\left(\frac{B}{2}\right)+\tan^2\left(\frac{C}{2}\right)\geq 1$.

13. **(i)** If n is a positive integer, prove that $\{(n+1)!\}^{1/(n+1)}<1+\frac{n}{n+1}(n!)^{1/n}$.

(ii) If n is a positive integer, show that $\left(1-\frac{1}{n}\right)^n<\left(1-\frac{1}{n+1}\right)^{n+1}$.

(iii) For every positive real number $a\neq 1$ and for every positive integer n prove that $\left(\frac{1+na}{1+n}\right)^{n+1}>a^n$.

14. By assigning weights 1 and n to the numbers 1 and 1 + (x/n) respectively, prove that if $x > -n$, then $\left(1+\frac{x}{n+1}\right)^{n+1}\geq\left(1+\frac{x}{n}\right)^n$.

15. Prove that $\frac{1}{\sqrt{2n+1}}>\frac{1.3.5.........(2n-1)}{2.4.6..........2n}>\frac{\sqrt{n+1}}{2n+1}$ $n\in I$.

IIT JEE PROBLEMS (OBJECTIVE)

(A) Fill in the blanks

1. The sum of integers from 1 to 100 that are divisible by 2 or 5 is................ **[IIT - 84]**

2. The solution of the equation $\log_7 \log_5\left(\sqrt{x+5}+\sqrt{x}\right)=0$ is .... **[IIT - 86]**

3. The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$is $n(n+1)^2/2$, when n is even. When n is odd, the sum is........... **[IIT - 88]**

4. Let the harmonic mean and geometric mean of two positive numbers be the ratio 4 : 5. Then the two number are in the ratio........... **[IIT - 92]**

5. Let n be positive integer. If the coefficients of 2nd, 3rd, and 4th terms in the expansion of $(1 + x)^n$ are in A.P., then the value of n is........... **[IIT - 94]**

6. For any odd integer $n \geq 1$, $n^3 - (n-1)^3 + ...+ (-1)^{n-1} 1^3 =$ .... **[IIT - 96]**

7. $x = 1 + 3a + 6a^2 + 10a^3 + ...|a| < 1$ **[REE-96]**
$y = 1 + 4b + 10b^2 + 20b^3 + ... |b| < 1$, find $S = 1 + 3ab + 5(ab)^2 + ...$in terms of x and y.

8. Let p and q be roots of the equation $x^2 - 2x + A = 0$, and let r and s be the roots of the equation $x^2 - 18x + B = 0$. If $p < q < r < s$ are in arithmetic progression, then A =............................, and B =................. **[IIT - 97]**

9. Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. Then $\dfrac{y^3+z^3}{xyz}$ = ... **[IIT - 97]**

(B) Multiple choice questions with one or more than one correct answer :

1. If the first and the $(2n-1)^{st}$ terms of an A.P., a G.P. and an H.P. are equal and their n^{th} terms are a, b and c respectively, then
(A) a = b = c (B) $a \geq b \geq c$ (C) a + c = b (D) $ac - b^2 = 0$ **[IIT - 88]**

2. Indicate the correct alternative(s), for $0 < \phi < \pi/2$, if :

$$x = \sum_{n=0}^{\infty}\cos^{2n}\phi,\ y = \sum_{n=0}^{\infty}\sin^{2n}\phi,\ z = \sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi \text{ then :}$$ **[IIT - 93]**

(A) xyz = xz + y (B) xyz = xy + z (C) xyz = x + y + z (D) xyz = yz + x

3. Let T_r be the r^{th} term of an AP, for r = 1, 2, 3..........If for some positive integers m, n we have $T_m = \dfrac{1}{n}$ and $T_n = \dfrac{1}{m}$, then T_{mn} equals : **[IIT - 98]**

(A) $\dfrac{1}{mn}$ (B) $\dfrac{1}{m}+\dfrac{1}{n}$ (C) 1 (D) 0

4. If $x > 1, y > 1, z > 1$ are in GP, then $\dfrac{1}{1+\ell n\, x}, \dfrac{1}{1+\ell n\, y}, \dfrac{1}{1+\ell n\, z}$ are in : **[IIT - 98]**

(A) AP (B) HP (C) GP (D) none of these

5. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then **[IIT - 98]**

(A) $b_0 = 1, b_1 = 3$
(B) $b_0 = 0, b_1 = n$
(C) $b_0 = -1, b_1 = n$
(D) $b_0 = 0, b_1 = n^2 + 3n + 3$

6. For a positive integer n, let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...\frac{1}{(2^n)-1}$, then **[IIT - 99]**

(A) $a(100) \le 100$
(B) $a(100) > 100$
(C) $a(200) \le 100$
(D) $a(200) > 100$

(C) Multiple choice questions with one correct answer :

1. The third term of geometric progression is 4. The product of the first five terms is
(A) 4^3 (B) 4^5 (C) 4^4 (D) none of these **[IIT - 82]**

2. The rational number, which equals the number $2.\overline{357}$ with recurring decimal is

(A) $\frac{2355}{1001}$ (B) $\frac{2379}{997}$ (C) $\frac{2355}{999}$ (D) none of these **[IIT - 83]**

3. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in
(A) A.P. (B) G.P. (C) H.P. (D) none of these **[IIT - 85]**

4. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval
(A) $(2, \infty)$ (B) (1, 2) (C) (–2, –1) (D) none of these **[IIT - 85]**

5. Sum of the first n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$ is equal to
(A) $2^n - n - 1$ (B) $1 - 2^{-n}$ (C) $n + 2^{-n} - 1$ (D) $2^n + 1$ **[IIT - 88]**

6. The number $\log_2 7$ is
(A) an integer
(B) a rational number
(C) an irrational number
(D) a prime number **[IIT - 90]**

7. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation : **[IIT - 98]**
(A) $0 < M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

8. The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0$ is **[IIT - 99]**
(A) 2 (B) 4 (C) 6 (D) 8

9. Let $a_1, a_2, ...a_{10}$, be in A.P. and $h_1, h_2, ..., h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$ then $a_4 h_7$ is : **[IIT - 99]**
(A) 2 (B) 3 (C) 5 (D) 6

10. Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then : **[IIT - 2000]**

(A) $a = \frac{7}{4}, r = \frac{3}{7}$ (B) $a = 2, r = \frac{3}{8}$ (C) $a = \frac{3}{2}, r = \frac{1}{2}$ (D) $a = 3, r = \frac{1}{4}$

11. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G. P., then the integral values of p and q respectively, are **[IIT - 2000]**

(A) -2, - 32 (B) -2, 3 (C) -6, 3 (D) -6, -32

12. If the sum of the first 2n terms of the A. P. 2, 5, 8, ...is equal to the sum of the first n terms of the A.P. 57, 59, 61,..........., the n equals **[IIT - 2001]**

(A) 10 (B) 12 (C) 11 (D)13

13. Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd and bcd are **[IIT –2001]**

(A) Not in A.P./G.P./H.P. (B) in A.P. (C) in G.P. (D) H.P.

14. The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is **[IIT - 2001]**

(A) 3 (B) 1 (C) 2 (D) 0

15. Suppose a, b, c are in A.P. a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is **[IIT - 2002]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

16. An infinite G.P. has first term 'x' and sum '5', then x belongs to **[IIT - 2004]**

(A) $x < -10$ (B) $-10 < x < 0$ (C) $0 < x < 10$ (D) $x > 10$

17. In the quadratic equation $ax^2 + bx + c = 0$, $\Delta = b^2 - 4ac$ and and $\alpha + \beta,\ \alpha^2 + \beta^2,\ \alpha^3 + \beta^3$, are in G.P. where α, β are the root of $ax^2 + bx + c = 0$, then **[IIT - 2005]**

(A) $\Delta \neq 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

Write Up I **[IIT-2007]**

Let V_r denotes the sum of the first r terms of an arithmetic progression (A. P.) whose first term is r and the common difference is $(2r-1)$. Let

$T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, . . .

18. The sum of $V_1 + V_2 + \ldots + V_n$ is

(A) $\frac{1}{12} n(n+1)(3n^2 - n + 1)$ (B) $\frac{1}{12} n(n+1)(3n^2 + n + 2)$

(C) $\frac{1}{2} n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

19. T_r is always

(A) an odd number (B) an even number

(C) a prime number (D) a composite number

20. Which one of the following is a correct statement ?

(A) $Q_1, Q_2, Q_3, \ldots$ are in A. P. with common difference 5

(B) $Q_1, Q_2, Q_3, \ldots$ are in A. P. with common difference 6

(C) $Q_1, Q_2, Q_3, \ldots$ are in A. P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = \ldots$

Write Up II **[IIT-2007]**

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

21. Which one of the following statements is correct ?

(A) $G_1 > G_2 < G_3 > \ldots$ (B) $G_1 < G_2 < G_3 < \ldots$

(C) $G_1 = G_2 = G_3 = \ldots$ (D) $G_1 < G_3 < G_5 < \ldots$ and $G_2 > G_4 > G_6 > \ldots$

22. Which one of the following statements is correct ?

(A) $A_1 > A_2 > A_3 > \ldots$ (B) $A_1 < A_2 < A_3 < \ldots$

(C) $A_1 > A_2 > A_3 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$

(D) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

23. Which one of the following statements is correct ?

(A) $H_1 > H_2 > H_3 > \ldots$ (B) $H_1 < H_2 < H_3 < \ldots$

(C) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 < \ldots$

(D) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

IIT JEE PROBLEMS (SUBJECTIVE)

1. If $a_1, a_2, \ldots\ldots, a_n$ are in arithmetic progression, where $a_i > 0$ for all i, show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}.$$ **[IIT - 82]**

2. Does there exist a geometric progression containing 27, 8 and 12 as three of its terms ? If it exits, how many such progressions are possible ? **[IIT - 83]**

3. Find three numbers a, b, c between 2 and 18 such that **(i)** their sum is 25 **(ii)** the numbers 2, a, b sare consecutive terms of an A.P. and **(iii)** the numbers b, c, 18 are consecutive terms of a G.P. **[IIT - 83]**

4. If $1, a_1, a_2, \ldots, a_{n-1}$ are the n roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3)\ldots(1-a_{n-1}) = n$ **[IIT - 84]**

5. If $a > 0, b > 0$ and $c > 0$, prove that $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$. **[IIT - 84]**

6. If n is a natural number such that $n = p_1^{\alpha_1}.p_2^{\alpha_2}.p_3^{\alpha_3}\ldots\ldots p_k^{\alpha_k}$ and $p_1, p_2, \ldots, p_k$ are distinct primes, then show that $\ell n\ n \geq k\ \ell n 2$. **[IIT - 84]**

7. Find the sum of series : $\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}.......\text{up to m terms}\right]$ **[IIT - 85]**

8. The sum of the squres of three distinct real numbers, which are in G.P., is S^2. If their sum is a S, show that $a^2\in\left(\frac{1}{3},1\right)\cup(1,3)$. **[IIT - 86]**

9. Solve for x the following equation :

$$\log_{(2x+3)}(6x^2+23x+21)=4-\log_{(3x+7)}(4x^2+12x+9).$$ **[IIT - 87]**

10. If $\log_3 2 . \log_3(2^x-5)$ and $\log_3\left(2^x-\frac{7}{2}\right)$ are in arithmetic progression, determine the value of x. **[IIT - 90]**

11. If p be the first of n arithmetic means between two numbers and q be the first of n harmonic means between the same two numbers, prove that the value of q cannot be between p and $\left(\frac{n+1}{n-1}\right)^2$ p. **[IIT - 91]**

12. If $S_1, S_2, S_3, \ldots S_n$ are the sums of infinite geometric series whose first terms are 1, 2,3, ..., n and whose common ratios are $\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots,\frac{1}{n+1}$ respectively, then find the value of $S_1^2+S_2^2+S_3^2+\ldots+S_{2n-1}{}^2$. **[IIT - 91]**

13. The sum of the first ten terms of an AP is 155 and the sum of first two terms of a GP is 9. The first term of the AP is equal to the common ratio of the GP and the first term of the GP is equal to the common difference of the AP. Find the two progressions. **[REE-93]**

14. If the $(m + 1)^{th}$, $(n + 1)^{th}$ and $(r + 1)^{th}$ terms of an AP are in GP and m, n, r are in HP. Find that the ratio of the common difference to the first term of the AP. **[REE-94]**

15. The real numbers x_1, x_2, x_3 satisfying the equation $x^3+x^2+\beta x+\gamma=0$ are in A.P. Find the intervals in which β and γ lie. **[IIT - 96]**

16. a, b, c are the first three terms of a geometric series. If the harmonic mean of a and b is 12 and that of b and c is 36, find the first five terms of the series. **[REE-98]**

17. The sum of an infinite geometric series is 162 and the sum of its n terms is 160. If the inverse of its common ratio is an integer, find all possible values of the common ratio, n and the first terms of the series. **[REE-99]**

18. Let a, b, c, d be real numbers in G.P. If u, v, w, satisfy the system of equations **[IIT - 99]**
$u + 2v + 3w = 6$, $4u + 5v + 6w = 12$, $6u + 9v = 4$, then show that the roots of the equation
$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2+[(b-c)^2+(c-a)^2+(d-b)^2]x+u+v+w=0$ are reciprocals of each other.

19. The fourth power of the common difference of an arithmetic progression with integer entries added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of and integer. **[IIT - 2000]**

20. Given that α, γ are roots of the equation, $Ax^2 - 4x + 1 = 0$ and β, δ the roots of the equation, $Bx^2 - 6x + 1 = 0$, find values of A and B, such that α, β, γ and δ are in H.P. **[REE-2000]**

21. The sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals. Find whether bc^2, ca^2 and ab^2 in A.P., G.P. or H.P. ? **[REE-2000]**

22. Solve the following equations for x and y

$$\log_2 x + \log_4 x + \log_{16} x + \ldots\ldots\ldots\ldots = y\ \frac{5+9+13+\ldots\ldots\ldots+(4y+1)}{1+3+5+\ldots\ldots\ldots+(2y-1)} = 4\log_4 x.$$ **[REE-2001]**

23. Let $a_1, a_2, \ldots\ldots\ldots\ldots$ be positive real numbers in G.P. for each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of $a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_n$. Find an expression for the G.M. of $G_1, G_2, \ldots\ldots\ldots G_n$ in terms of $A_1, A_2 \ldots\ldots\ldots\ldots A_n, H_1, H_2, \ldots\ldots\ldots\ldots H_n$. **[IIT - 2001]**

24. Let a, b be positive real numbers. If a, A_1, A_2, b are in arithmetic progression a, G_1, G_2, b are geometric progression and a, H_1, H_2, b are in harmonic progression, show that
$\frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}=\frac{(2a+b)(a+2b)}{9ab}$. **[IIT - 2002]**

25. If a,b,c are in A.P. a^2,b^2,c^2 are in H.P. Then prove that either a = b = c or a , b, -c/2 form a G..P.
[IIT - 2003]

26. Prove that $(a + 1)^7 (b + 1)^7 (c + 1)^7 > 7^7 a^4 b^4 c^4$, where $a, b, c \in R^+$. **[IIT - 2004]**

27. An infinite G.P has first term x and sum 5, then find the exhaustive range of x ? **[IIT - 2004]**

28. For n = 1, 2, 3, . . . , let

$$A_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n, \text{ and } B_n = 1 - A_n.$$

Find the smallest natural number n_0 such that $B_n > A_n$ for all $n \geq n_0$. **[IIT - 2006]**

SET–I

1. If a_1, a_2, a_3,............are in AP then a_p, a_q, a_r are in AP if p, q, r are in
(A) AP (B) GP (C) HP (D) none of these

2. The product of n positive numbers is unity. Then their sum is
(A) a positive integer (B) divisible by n (C) equal to $n + \frac{1}{n}$ (D) never less than n

3. If $p, q, r, s \in N$ and they are four consecutive terms of an AP then the $p^{th}, q^{th}, r^{th}, s^{th}$ terms of a GP in
(A) AP (B) GP (C) HP (D) none of these

4. If in a progression a_1, a_2, a_3,........, etc., $(a_r - a_{r+1})$ bears a constant ratio with $a_r.a_{r+1}$ then the terms of the progression are in
(A) AP (B) GP (C) HP (D) none of these

5. Let x, y, z be three positive prime numbers. The progression in which $\sqrt{x}, \sqrt{y}, \sqrt{z}$ can be three terms (not necessarly consecutive) is
(A) AP (B) GP (C) HP (D) none of these

6. Let f(x) = 2x + 1. Then the number of real number of real values of x for which the three unequal numbers f(x), f(2x), f(4x) are in GP is
(A) 1 (B) 2 (C) 0 (D) none of these

7. If $a_r > 0, r \in N$, $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_{2n}$ are in AP then

$$\frac{a_1 + a_{2n}}{\sqrt{a_1} + \sqrt{a_2}} + \frac{a_2 + a_{2n-1}}{\sqrt{a_2} + \sqrt{a_3}} + \frac{a_3 + a_{2n-2}}{\sqrt{a_3} + \sqrt{a_4}} + \ldots\ldots\ldots + \frac{a_n + a_{n+1}}{\sqrt{a_n} + \sqrt{a_{n+1}}}$$ is equal to

(A) (n - 1) (B) $\frac{n(a_1 + a_{2n})}{\sqrt{a_1} + \sqrt{a_{n+1}}}$ (C) $\frac{n-1}{\sqrt{a_1} + \sqrt{a_{n+1}}}$ (D) none of these

8. If $a_1, a_2, a_3, \ldots\ldots\ldots, a_{2n+1}$ are in AP then $\frac{a_{2n+1} - a_1}{a_{2n+1} + a_1} + \frac{a_{2n} - a_2}{a_{2n} + a_2} + \ldots\ldots\ldots\ldots + \frac{a_{n+2} - a_n}{a_{n+2} + a_n}$ is equal to

(A) $\frac{n(n+1)}{2} . \frac{a_2 - a_1}{a_{n+1}}$ (B) $\frac{n(n+1)}{2}$ (C) $(n + 1)(a_2 - a_1)$ (D) none of these

9. Let a_1, a_2, a_3,........ be in AP and a_p, a_q, a_r be in G.P. Then $a_q : a_p$ is equal to

(A) $\frac{r-p}{q-p}$ (B) $\frac{q-p}{r-q}$ (C) $\frac{r-q}{q-p}$ (D) none of these

10. In an AP, the pth term is q and the (p + q)th term is 0. Then the qth term is
(A) - p (B) p (C) p + q (D) p - q

Progression & series

11. In a sequence of (4n + 1) terms the first (2n + 1) terms as in AP whose common difference is 2, and the last (2n + 1) terms are in GP whose common ratio is 0.5. If the middle terms of the AP and GP are equal then the middle term of the sequence is

(A) $\frac{n.2^{n+1}}{2^n - 1}$ (B) $\frac{n.2^{n+1}}{2^{2n} - 1}$ (C) $n . 2^n$ (D) none of these

12. If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$ then x, y, z in

(A) AP (B) GP (C) HP (D) none of these

13. If a, b, c, d and p are distinct real numbers such that
$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$ then a, b, c, d are in
(A) AP (B) GP (C) HP (D) none of these

14. The largest term common to the sequences 1, 11, 21, 31,.........to 100 terms and 31, 36, 41, 46,....to 100 terms is
(A) 381 (B) 471 (C) 281 (D) none of these

15. The interior angles of a convex polygon are in AP, the common difference being 5^0. If the smallest angle is $\frac{2\pi}{3}$ then the number of sides is

(A) 9 (B) 16 (C) 7 (D) none of these

16. In the value of 100! the number of zeros at the end is
(A) 11 (B) 22 (C) 23 (D) 24

17. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8,..............., where k consecutive terms have the value k (k = 1, 2, 4, 8,...), the 1025^{th} term is
(A) 2^9 (B) 2^{10} (C) 2^{11} (D) 2^8

18. Let $\{t_n\}$ be a sequence of integers in GP in which $t_4 : t_6 = 1 : 4$ and $t_2 + t_5 = 216$. Then t_1 is
(A) 12 (B) 14 (C) 16 (D) none of these

19. If $\log\left(\frac{5c}{a}\right)$, $\log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ are in AP, where a, b, c are in G.P., then a, b, c are the lengths of sides of
(A) an isosceles triangle (B) an equilateral triangle
(C) a scalene triangle (D) none of these

20. If x, 2y, 3z are in AP, where the distinct numbers x, y, z are in GP, then the common ratio of the GP is

(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

SET–II

1. If three numbers are in HP then the numbers obtained by subtracting half of the middle number from each of them are in

(A) AP (B) GP (C) HP (D) none of these

2. a, b, c, d, e are five numbers in which the first three are in AP and the last three are in HP. If the three numbers in the middle are in GP then the numbers in the odd places are in

(A) AP (B) GP (C) HP (D) none of these

3. If a, b, c are in AP then $a+\frac{1}{bc}, b+\frac{1}{ca}, c+\frac{1}{ab}$ are in

(A) AP (B) GP (C) HP (D) none of these

4. The AM of two given positive numbers is 2. If the larger number is increased by 1, the GM of the numbers becomes equl to the AM of the given numbers. Then the HM of the given numbers is

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) none of these

5. Let $a_1 = 0$ and $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be real numbers such that $|a_i| = |a_{i-1} + 1|$ for all i then the AM of the numbers $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ has the value A where

(A) $A < -\frac{1}{2}$ (B) $A < -1$ (C) $A \geq -\frac{1}{2}$ (D) $A = -\frac{1}{2}$

6. Let there be a GP whose first term is a and the common ratio is r. If A and H are the arithmetic mean and the harmonic mean respectively for the first n terms of the GP, A. H is equal to

(A) $a^2 r^{n-1}$ (B) $a r^n$ (C) $a^2 r^n$ (D) none of these

7. $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the HM between a and b if n is

(A) 0 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 1

8. If p, q, r be three positive real numbers, then the value of $(p + q)\ (q + r)\ (r + p)$ is

(A) > 8 pqr (B) < 8 pqr (C) 8 pqr (D) none of these

9. Let $t_r = r.(r!)$. Then $\sum_{r=1}^{15} t_r$ is equal to

(A) 15 ! - 1 (B) 15 ! + 1 (C) 16 ! - 1 (D) none of these

10. If n arithmatic mean are inserted between 2 and 38 , then the sum of the resulting series is obtained as 200 , then the value of n is

(A) 6 (B) 8 (C)9 (D)10

11. The sum of three consecutive terms in a geometric progression is 14. If 1 is added to the first and the second term and 1 is subtracted from the third term. The resulting new terms are in arithmetic progression. Then the lowest of the original terms is

(A) 1 (B) 2 (C) 4 (D) 8

12. The angles of a triangle are in A. P. and the ratio of the greatest to the smallest angle is 3 : 1. Then the smallest angle is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) none of these

13. The length of the side of a square is 'a' meter. A second square is formed by joining the middle points of the sides of the square. Then a third square is formed by joining the middle points of the sides of the second square and so on.Then the sum of the area of squares which carried up to infinity is

(A) a (B) $2a^2$ (C) $3a^2$ (D) $4a^2$

14. If $S_n = nP + \frac{n}{2}(n-1)Q$, where S_n denotes the sum of the first n terms of an A.P., then common difference is

(A) P + Q (B) 2P + 3Q (C) 2Q (D) Q

15. The three sides of a right angled triangle are in G. P. The tangents of the two acute angles are

(A) $\sqrt{\frac{5+1}{2}}$ and $\sqrt{\frac{5-1}{2}}$ (B) $\sqrt{\frac{\sqrt{5}+1}{2}}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$ (C) $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$

(D) none of the foregoing pairs of numbers

16. If x, y, z are positive then the minimum value of $x^{\log y - \log z} + y^{\log z - \log x} + z^{\log x - \log y}$ is

(A) 3 (B) 1 (C) 9 (D) 16

17. a, b, c are three positive numbers and abc^2 has the greatest value $\frac{1}{64}$. Then

(A) $a = b = \frac{1}{2},\ c = \frac{1}{4}$ (B) $a = b = \frac{1}{4},\ c = \frac{1}{2}$ (C) $a = b = c = \frac{1}{3}$

(D) none of then

18. The sum of all the numbers between 200 and 400 which are divisible by 7 is

(A) 9872 (B) 7289 (C) 8729 (D) 8279

19. The sum of the series : $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots - 100^2$ is

(A) –10100 (B) –5050 (C) –2525 (D) –5500

20. Suppose that $F(n + 1) = \frac{2F(n)+1}{2}$ for n = 1, 2, 3, . . . and F(1) = 2. Then F(101) equals

(A) 50 (B) 52 (C) 54 (D) none of these

SET–III

1. If x_1, x_2, ...x_n are n non-zero real numbers such that $(x_1^2 + x_2^2, ... + x^2_{n-1})(x_2^2 + x_2^3 + ... + x^2_n) \le (x_1 x_2 + x_2 x_3 + ... + x_{n-1} x_n)^2$ then $x_1, x_2, ..., x_n$ are in

(A) A.P. (B) G.P. (C) H . P. (D) none of these

2. If a, b and c are three positive real numbers, which one of the following hold ?

(A) $a^2 + b^2 + c^2 \ge bc + ca + ab$ (B) $a^3 - b^3 + c^3 \ge 3abc$
(C) $(b - c)(c - a)(a - b) \ge 8\ abc$ (D) none of these

3. If $n \in N$ and $n > 1$, which one of the following holds ?

(A) $n^n > 1 . 3 . 5 ....(2n - 1)$ (B) $2^n > 1 + n\sqrt{2^{n-1}}$

(C) $\frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n} > \frac{1}{2}$ (D) all of these

4. If x, y and z are positive real numbers, such that $x + y + z = a$, then

(A) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} < \frac{9}{a}$ (B) $(a - x)(a - y)(a - z) \ge 8xyz$

(C) $(a - x)(a - y)(a - z) \ge \frac{8}{27}a^3$ (D) all of these

5. If a, b and c are three positive real numbers, then the minimum value of the expression $\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c}$ is

(A) 1 (B) 2 (C) 3 (D) 6

W I **Finding the number of shot arranged in a complete pyramid the base of which is an equilateral triangle are as follows :**

Suppose that each side of the base contains n shot, then the number of shot in the lowest layer is $n + (n - 1) + (n - 2) + ... + 1$;

i.e. $\frac{n(n+1)}{2}$ or $\frac{1}{2}(n^2 + n)$.

If we write $n - 1$, $n - 2$, ...for n and obtain the shot in the 2nd, 3rd,.......layer. If the base of pyramid is a rectangle, then number of shot arranged in a complete pyramid can be find out as follow.

Let m and n be the number of shot in the long and short side respectively of the base. The top layer consist of single row of $m - (n - 1)$ or $m - n + 1$ shot ;
in the next layear the number is $2(m - n + 2)$
in the next layear the number is $3(m - n + 3)$ and so on.
in the lowest layear the number is $n(m - n + n)$

$S = (m - n + 1) + 2(m - n + 2) + 3(m - n + 3) + ... + n(m - n + n)$
$= (m - n)(1 + 2 + 3 + ... + n)(1^2 + 2^2 + ... + n^2)$

$$= \frac{(m-n)n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6} \qquad = \frac{n(n+1)}{6}\{3(m-n)+2n+1\}$$

$$= \frac{n(n+1)(3m-n+1)}{6}$$

6. The number of shot arranged in a complete pyramid square base of side n is

(A) $\frac{n(n+1)(n+2)}{6}$ (B) $\frac{n(n-1)(2n-1)}{6}$ (C) $\frac{n(n+1)(2n+1)}{6}$

(D) none of these

7. The number of shot arranged in an incomplete pyramid, the base of which is rectangle, where a, b denotes the number of shots in the two side of the top layer, and n the number of layers

(A) $\frac{n}{6}[6ab-3(a+b)(n-1)+(n+1)(2n-1)]$ (B) $\frac{n}{6}[6ab+3(a+b)(n-1)+(n-1)(2n-1)]$

(C) $\frac{n}{6}[6ab-3(a+b)(n+1)+(n+1)(2n-1)]$ (D) none of these

8. The number of shot in an incomplete square pile of 27 courses, having 40 shot in each side of the base is

(A) 21000 (B) 21321 (C) 21800 (D) none of these

9. The number of shot in a complete rectangular pile of 15 courses, having 20 shot in the longer side of the its base is

(A) 1800 (B) 1850 (C) 1840 (D) 1810

10. The number of shot required to complete a rectangular pile having 15 and 6 shot in the longer and shorter side, respectively, of its upper course is

(A) 180 (B) 185 (C) 184 (D) 190

W II The sum of n terms of a series each term of which is composed of r factors in arithmetical progression, the first factors of the several terms being in the same arithmetical progression. Let the series be denoted by $u_1 + u_2 + u_3 + \ldots\ldots + u_n$, where

$u_n = (a+nb)(a+\overline{n+1}.b)(a+\overline{n+2}.b)\ldots\ldots(a+\overline{n+r-1}.b)$. Replacing n by n – 1, we have

$u_{n-1} = (a+\overline{n-1}.b)(a+nb)(a+\overline{n+1}.b)\ldots\ldots(a+\overline{n+r-2}.b)$

$\therefore \quad (a+\overline{n-1}.b)\, u_n = \left(a+\overline{n+r-1}.b\right)u_{n-1} = v_n$, say

Replacing n by n + 1 we have

$$(a+\overline{n+r}.b)\, u_n = v_{n+1}$$

Therefore by subtraction ;

$$(r+1)\, b \,.\, u_n = v_{n+1} - v_n.$$

Similarly, $(r+1)\, b \,.\, u_{n-1} = v_n - v_{n-1}$,

....

$(r+1)\, b \,.\, u_2 = v_3 - v_2$,

$(r+1)\, b \,.\, u_1 = v_2 - v_1$.

By addition, $(r + 1)\, b\, .\, S_n = v_{n+1} - v_1$;

that is, $S_n = \dfrac{v_{n+1} - v_1}{(r+1)b} = \dfrac{(a+\overline{n+r}.b)u_n}{(r+1)b} + C$, say ;

where C is a quantity independent of n, which may be founded by ascribing to n some particular value. The above result gives us the following convenient rule :
Write down the nth term, affic the next factor at the end divide by the number of factors thus increased and by the common difference.

11. The sum of n terms of series 1 . 3 . 5 + 3 . 5 . 7 + 5 . 7 . 9 + ... is
(A) $(2n^3 + 8n^2 - 7n - 2)$
(B) $n(2n^3 + 8n^2 + 7n - 2)$
(C) $n(2n^3 - 8n^2 + 7n - 2)$
(D) none of these

12. The sum of n terms of series 1 . 5 . 9 + 2 . 6 . 10 + 3 . 7 . 11 + ...
(A) $\frac{n}{4}(n+1)(n+8)(n+9)$
(B) $\frac{n}{4}(n-1)(n+8)(n+9)$
(C) $\frac{n}{4}(n+1)(n-8)(n+9)$
(D) none of these

13. The sum of n terms of series 1 . 2 . 3 . 4 + 2 . 3 . 4 . 5 + 3 . 4 . 5 . 6 + ...
(A) $\frac{n}{5}n(n-1)(n-2)(n-3)(n-4)$
(B) $\frac{n}{5}n(n+1)(n-2)(n-3)(n-4)$
(C) $\frac{n}{5}n(n+1)(n+2)(n+3)(n+4)$
(D) none of these

14. The sum of n terms of series 1 . 4 . 7 + 4 . 7 . 10 . + 7 . 10 . 13 + ...
(A) $\frac{n}{4}(27n^3 + 90n^2 + 45n - 50)$
(B) $\frac{n}{4}(27n^3 - 90n^2 + 45n - 50)$
(C) $\frac{n}{4}(27n^3 - 90n^2 - 45n - 50)$
(D) none of these

15. The sum of n terms of series 1 . 4 . 7 + 2 . 5 . 8 + 3 . 6 . 9 + ...
(A) $\frac{n}{4}(n-1)(n+6)(n+7)$
(B) $\frac{n}{4}(n+1)(n+6)(n+7)$
(C) $\frac{n}{4}(n+1)(n-6)(n+7)$
(D) none of these

W III To find the greatest value of $a^m b^n c^p$ ....when $a + b + c +$ ....is constant ; m, n, p,....being positive integers. Since m, n, p,........are constants, the expression $a^m b^n c^p$...... will be greatest when $\left(\frac{a}{m}\right)^m \left(\frac{b}{n}\right)^n \left(\frac{c}{p}\right)^p$..... is greatest. But this last expression is the product of m + n + p +...

factors whose sum is $m\left(\frac{a}{m}\right)+n\left(\frac{b}{n}\right)+\left(\frac{c}{p}\right)+.....$, or a + b + c + ..., and therefore constant.

Hence $a_m b_n c_p$.....will be greatest when the factors $\frac{a}{m}, \frac{b}{n}, \frac{c}{p}$,..... are all equal, that is, when

$$\frac{a}{m}=\frac{b}{n}=\frac{c}{p}=.....=\frac{a+b+c+.....}{m+n+p+......}$$

Thus the greatest value is

$$m^m n^n p^p........\left(\frac{a+b+c+.....}{m+n+p+......}\right)^{m+n+p+.......}$$

16. The greatest value of $(a + x)^3 (a - x)^4$ for any real value of x numerically less than a is

(A) $\frac{3^3.4^4}{7^7}a^7$ (B) $\frac{5^3.8^4}{7^7}a^7$ (C) $\frac{6^3.10^4}{7^7}a^7$ (D) none of these

17. An odd integer is divided into two integral parts whose product is a maximum, then these two parts are

(A) n, n – 1 (B) n, n + 1 (C) n, 2n + 1 (D) none of these

18. The minimum value of $\frac{(a+x)(b+x)}{c+x}$, $(x > -c, a > c, b > c)$ is

(A) $\left(\sqrt{(a-c)}+\sqrt{b-c}\right)^2$ (B) $\left(\sqrt{a-c}-\sqrt{b-c}\right)^2$

(C) $\frac{ab}{c}$ (D) none of these

19. The maximum value of $(7 - x)^4 (2 + x)^5$ when x lies between 7 and – 2

(A) $4^5 . 5^4$ (B) $4^4 . 5^5$ (C) $3^4 . 5^4$ (D) none of these

20. The minimum value of $\frac{(5+x)(2+x)}{1+x}$, $(x > -1)$ is

(A) 7 (B) 8 (C) 9 (D) none of these

LEVEL–I ANSWER

1. 900 **2.** 9 **4.** 6, 12, 24 OR 24, 12, 6

6. $r=\frac{1}{3},\frac{1}{9}$ or $\frac{1}{81}$ n = 4, 2 or 1 and a = 108, 144 or 160

8. 7/6 **9.** $n2^{n+2}-2^{n+1}+2$ **10.** 1080 m

LEVEL–II

4. A.P. is $2+5+8+11+$ & G.P is $3+6+12+24+$ or A.P. is $\frac{25}{2}+\frac{79}{6}+\frac{83}{6}$

G.P. is $\frac{2}{3}+\frac{25}{3}+\frac{625}{6}+$ ...

8. $x=10^5$, $y=10$

10. **(i)** $S_n=(1/12)-[1/\{4(2n+1)(2n+3)\}]$; $S_\infty=1/12$

(ii) $S_n=(1/24)-[1/\{6(3n+1)(3n+4)\}]$; $S_\infty=1/24$

(iii) $(1/5)n(n+1)(n+2)(n+3)(n+4)$ **(iv)** $n/(2n+1)$

OBJECTIVE PROBLEMS ASKED IN IIT-JEE

(A)

1. **3050** **2.** **4** **3.** $n^2\left(\frac{n+1}{2}\right)$

4. **4 : 1 & 1 : 4** **5.** **7** **6.** $\frac{1}{4}(2n-1)(n+1)^2$

7. **S =** $\frac{1+ab}{(1-ab)^2}$ **Where a = 1 - $x^{-1/3}$ and b = 1 - $y^{-1/4}$** **8.** **-3, 77** **9.** **2**

(B)

1. BD **2. BC** **3. C** **4. B** **5. B** **6. AD**

(C)

1. B	**2. C**	**3. A**	**4. A**	**5. C**	**6. C**	**7. A**
8. B	**9. D**	**10. D**	**11. A**	**12. C**	**13. D**	**14. B**
15. D	**16. B**	**17. C**	**18. B**	**19. D**	**20. B**	**21. C**
22. A	**23. B**					

SUBJECTIVE PROBLEMS ASKED IN IIT-JEE

2. Yes, infinite **3.** 5, 8, 12 **7.** $\dfrac{2^{mn}-1}{2^{mn}(2^n-1)}$ **9.** $-\dfrac{1}{4}$

10. 3 **12.** $\dfrac{n(2n+1)(4n+1)-3}{3}$

13. (3 + 6 + 12 + ...) ; (2/3 + 25/3 + 625/6 + ...) ; (2, 5, 8, ...); $\left(\dfrac{25}{2}, \dfrac{79}{6}, \ldots\ldots\ldots\ldots\right)$

14. $-\dfrac{2}{n}$ **15.** $\beta \in \left(-\infty, \dfrac{1}{3}\right], \gamma \in \left[-\dfrac{1}{27}, \infty\right)$ **16.** 8, 24, 72, 216, 648

17. $r \neq 1/9$; $n = 2$; $a = 144$ **OR** $r \neq 1/3$; $n = 4$; $a = 36$ **OR** $r = 1/81$; $n = 1$; $a = 160$

20. $A = 3$; $B = 8$ **21.** A.P. **22.** $x = 2\sqrt{2}$ and $y = 3$

23. $\left[(A_1, A_2, \ldots\ldots\ldots\ldots A_n)(H_1, H_2, \ldots\ldots\ldots\ldots H_n)\right]^{\frac{1}{2n}}$ **28.** least value of $n_0 = 6$

SET–I

1. A	2. D	3. B	4. C	5. D	6. C	7. B
8. A	9. C	10. B	11. A	12. C	13. B	14. D
15. A	16. D	17. B	18. A	19. D	20. B	

SET–II

1. B	2. B	3. A	4. A	5. C	6. A	7. A
8. A	9. C	10. B	11. B	12. A	13. B	14. D
15. B	16. A	17. B	18. C	19. B	20. B	

SET–III

1. B	2. A	3. D	4. B	5. D	6. C	7. B
8. B	9. C	10. D	11. B	12. A	13. C	14. A
15. B	16. A	17. B	18. A	19. B	20. C	