Inverse Circular Functions

Exercise - 1 (Objective Questions)

Part : (A) Only one correct option

1. If $\cos^{-1}\lambda + \cos^{-1}\mu + \cos^{-1}v = 3\pi$ then $\lambda\mu + \mu v + v\lambda$ is equal to
(A) – 3 (B) 0 (C) 3 (D) – 1

2. Range of $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ is
(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (C) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (D) none of these

3. The solution of the equation $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$ is
(A) x = 2 (B) x = – 4 (C) x = 4 (D) none of these

4. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos x) + \sin^{-1}(\sin x)\}]$, where $x \in \left(\frac{\pi}{2}, \pi\right)$ is
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}$ (D) $-\frac{\pi}{2}$

5. The set of values of k for which $x^2 - kx + \sin^{-1}(\sin 4) > 0$ for all real x is
(A) {0} (B) (–2, 2) (C) R (D) none of these

6. $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$ is equal to
(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$

7. $\cos^{-1}\left\{\frac{1}{2}x^2 + \sqrt{1-x^2}\cdot\sqrt{1-\frac{x^2}{4}}\right\} = \cos^{-1}\frac{x}{2} - \cos^{-1}x$ holds for
(A) $|x| \le 1$ (B) $x \in R$ (C) $0 \le x \le 1$ (D) $-1 \le x \le 0$

8. $\tan^{-1}a + \tan^{-1}b$, where a > 0, b > 0, ab > 1, is equal to
(A) $\tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (B) $\tan^{-1}\left(\frac{a+b}{1-ab}\right) - \pi$
(C) $\pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (D) $\pi - \tan^{-1}\left(\frac{a+b}{1-ab}\right)$

9. The set of values of 'x' for which the formula $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ is true, is
(A) (– 1, 0) (B) [0, 1] (C) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ (D) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

10. The set of values of 'a' for which $x^2 + ax + \sin^{-1}(x^2 - 4x + 5) + \cos^{-1}(x^2 - 4x + 5) = 0$ has at least one solution is

(A) $(-\infty, -\sqrt{2\pi}\,] \cup [\,\sqrt{2\pi}, \infty)$ (B) $(-\infty, -\sqrt{2\pi}\,) \cup (\,\sqrt{2\pi}, \infty)$
(C) R (D) none of these

11. All possible values of p and q for which $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$ holds, is

(A) $p = 1, q = \frac{1}{2}$ (B) $q > 1, p = \frac{1}{2}$ (C) $0 \le p \le 1, q = \frac{1}{2}$ (D) none of these

12. If $[\cot^{-1}x] + [\cos^{-1}x] = 0$, where [.] denotes the greatest integer function, then complete set of values of 'x' is
(A) (cos1, 1] (B) (cot 1, cos 1) (C) (cot1, 1] (D) none of these

13. The complete solution set of the inequality $[\cot^{-1}x]^2 - 6\,[\cot^{-1}x] + 9 \le 0$, where [.] denotes greatest integer function, is
(A) $(-\infty, \cot 3]$ (B) [cot 3, cot 2] (C) $[\cot 3, \infty)$ (D) none of these

14. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)$, $x \ne 0$ is equal to

(A) x (B) 2x (C) $\frac{2}{x}$ (D) $\frac{x}{2}$

15. If $\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \frac{\pi}{4}$, then $\tan\theta$ is equal to
(A) 1/3 (B) 3 (C) 1 (D) – 1

16. If $u = \cot^{-1}\sqrt{\tan\alpha} - \tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$ is equal to

(A) $\sqrt{\tan\alpha}$ (B) $\sqrt{\cot\alpha}$ (C) $\tan\alpha$ (D) $\cot\alpha$

17. The value of $\cot^{-1}\left\{\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}$, $\frac{\pi}{2} < x < \pi$, is:

(A) $\pi - \frac{x}{2}$ (B) $\frac{\pi}{2} + \frac{x}{2}$ (C) $\frac{x}{2}$ (D) $2\pi - \frac{x}{2}$

18. The number of solution(s) of the equation, $\sin^{-1}x + \cos^{-1}(1-x) = \sin^{-1}(-x)$, is/are
(A) 0 (B) 1 (C) 2 (D) more than 2

19. The number of solutions of the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is

(A) 0 (B) 1 (C) 2 (D) 3

20. If $\tan^{-1}\frac{1}{1+2} + \tan^{-1}\frac{1}{1+2.3} + \tan^{-1}\frac{1}{1+3.4} + ...+ \tan^{-1}\frac{1}{1+n(n+1)} = \tan^{-1}\theta$, then θ is equal to

(A) $\frac{n}{n+2}$ (B) $\frac{n}{n+1}$ (C) $\frac{n+1}{n}$ (D) $\frac{1}{n}$

21. If $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$, $n \in N$, then the maximum value of 'n' is:

(A) 1 (B) 5 (C) 9 (D) none of these

22. The number of real solutions of (x, y) where, $y|$ = sin x, y = $\cos^{-1}$ (cos x), $-2\pi \le x \le 2\pi$, is:

(A) 2 (B) 1 (C) 3 (D) 4

23. The value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is equal to

(A) 3/4 (B) – 3/4 (C) 1/16 (D) 1/4

Part : (B) May have more than one options correct

24. α, β and γ are three angles given by

$\alpha = 2\tan^{-1}(\sqrt{2} - 1)$, $\beta = 3\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\left(-\frac{1}{2}\right)$ and $\gamma = \cos^{-1}\frac{1}{3}$. Then

(A) $\alpha > \beta$ (B) $\beta > \gamma$ (C) $\alpha < \gamma$ (D) $\alpha > \gamma$

25. $\cos^{-1}x = \tan^{-1}x$ then

(A) $x^2 = \left(\frac{\sqrt{5}-1}{2}\right)$ (B) $x^2 = \left(\frac{\sqrt{5}+1}{2}\right)$

(C) $\sin(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$ (D) $\tan(\cos^{-1}x) = \left(\frac{\sqrt{5}-1}{2}\right)$

26. For the equation $2x = \tan(2\tan^{-1}a) + 2\tan(\tan^{-1}a + \tan^{-1}a^3)$, which of the following is invalid?

(A) $a^2x + 2a = x$ (B) $a^2 + 2ax + 1 = 0$ (C) $a \ne 0$ (D) $a \ne -1, 1$

27. The sum $\sum_{n=1}^{\infty} \tan^{-1}\frac{4n}{n^4 - 2n^2 + 2}$ is equal to:

(A) $\tan^{-1}2 + \tan^{-1}3$ (B) $4\tan^{-1}1$ (C) $\pi/2$ (D) $\sec^{-1}\left(-\sqrt{2}\right)$

28. If the numerical value of $\tan(\cos^{-1}(4/5) + \tan^{-1}(2/3))$ is a/b then

(A) a + b = 23 (B) a – b = 11 (C) 3b = a + 1 (D) 2a = 3b

29. If α satisfies the inequation $x^2 - x - 2 > 0$, then a value exists for

(A) $\sin^{-1}\alpha$ (B) $\cos^{-1}\alpha$ (C) $\sec^{-1}\alpha$ (D) $\text{cosec}^{-1}\alpha$

30. If $f(x) = \cos^{-1}x + \cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3 - 3x^2}\right\}$ then:

(A) $f\left(\frac{2}{3}\right) = \frac{\pi}{3}$ (B) $f\left(\frac{2}{3}\right) = 2\cos^{-1}\frac{2}{3} - \frac{\pi}{3}$

(C) $f\left(\frac{1}{3}\right) = \frac{\pi}{3}$

(D) $f\left(\frac{1}{3}\right) = 2\cos^{-1}\frac{1}{3} - \frac{\pi}{3}$ m

Exercise - 2 (Subjective Questions)

1. Find the value of the following :

(i) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

(ii) $\tan\left[\cos^{-1}\frac{1}{2} + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$

(iii) $\sin^{-1}\left[\cos\left\{\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right]$

2. Solve the equation : $\cot^{-1}x + \tan^{-1}3 = \frac{\pi}{2}$

3. Solve the equation : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

4. Solve the following equations :

(i) $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, $(x > 0)$

(ii) $3\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left(\frac{1}{3}\right)$

5. Find the value of $\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$, if $x > y > 1$.

6. If $x = \sin(2\tan^{-1}2)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$ then find the relation between x and y .

7. If arc sinx + arc siny + arc sinz = π then prove that:$(x, y, z > 0)$

(i) $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

(ii) $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

8. Solve the following equations :

(i) $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a$ $a \geq 1; b \geq 1, a \neq b$.

(ii) $\sin^{-1}\sqrt{\frac{x}{1+x}} - \sin^{-1}\frac{x-1}{x+1} = \sin^{-1}\frac{1}{\sqrt{1+x}}$

(iii) Solve for x, if $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$

9. If $\alpha = 2\tan^{-1}\left(\frac{1+x}{1-x}\right)$ & $\beta = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then prove that $\alpha + \beta = \pi$. What the value of α + β will be if $x > 1$?

10. If $X = \operatorname{cosec} \tan^{-1}\cos\cot^{-1}\sec\sin^{-1}a$ & $Y = \sec\cot^{-1}\sin\tan^{-1}\operatorname{cosec}\cos^{-1}a$; where $0 \le a \le 1$. Find the relation between X & Y. Express them in terms of 'a'.

11. Solve the following inequalities:

(i) $\cos^{-1}x > \cos^{-1}x^2$
(ii) $\sin^{-1}x > \cos^{-1}x$
(iii) $\tan^{-1}x > \cot^{-1}x$.
(iv) $\sin^{-1}(\sin 5) > x^2 - 4x$.
(v) $\tan^2(\arcsin x) > 1$
(vi) $\operatorname{arccot}^2 x - 5\operatorname{arccot} x + 6 > 0$
(vii) $\tan^{-1}2x \ge 2\tan^{-1}x$

12. Find the sum of each of the following series :

(i) $\cot^{-1}\frac{31}{12} + \cos^{-1}\frac{139}{12} + \cot^{-1}\frac{319}{12} + \ldots + \cot^{-1}\left(3n^2 - \frac{5}{12}\right)$.

(ii) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \ldots.. + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \ldots\ldots\ldots\ldots\ldots\ldots \infty$

13. Prove that the equation, $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = \alpha\,\pi^3$ has no roots for $\alpha < \frac{1}{32}$.

14. (i) Find all positive integral solutions of the equation, $\tan^{-1}x + \cot^{-1}y = \tan^{-1}3$.
(ii) If 'k' be a positive integer, then show that the equation:
$\tan^{-1}x + \tan^{-1}y = \tan^{-1}k$ has no non–zero integral solution.

Answers

Exercise # 1

1. C **2.** C **3.** C **4.** D **5.** D **6.** C **7.** C
8. C **9.** D **10.** D **11.** C **12.** C **13.** A **14.** C
15. B **16.** A **17.** B **18.** B **19.** B **20.** A **21.** B
22. C **23.** A **24.** BC **25.** AC **26.** BC **27.** AD
28. ABC **29.** CD **30.** AD

Exercise # 2

1. (i) 1 (ii) $\frac{1}{\sqrt{3}}$ (iii) $\frac{\pi}{6}$ **2.** x = 3.

3. $\pm\frac{1}{\sqrt{2}}$ **4.** (i) $x = \frac{1}{\sqrt{3}}$ (ii) x = 2

5. $\frac{1+xy}{x-y}$ **6.** $x = 4y^2$

8. (i) x = ab (ii) x ≥ 0 (iii) x = – 1

9. $-\pi$ **10.** $X = Y = \sqrt{3-a^2}$

11. (i) $[-1, 0)$ (ii) $\left(\frac{\sqrt{2}}{2}, 1\right]$ (iii) $x > 1$
(iv) $2-\sqrt{9-2\pi} < x < 2+\sqrt{9-2\pi}$
(v) $\left(\frac{\sqrt{2}}{2}, 1\right) \cup \left(-1, -\frac{\sqrt{2}}{2}\right)$
(vi) $(-\infty, \cot 3) \cup (\cot 2, \infty)$ (vii) $x \le 0$

12. (i) $\cot^{-1}\frac{18n+13}{12n}$ (ii) $\frac{\pi}{4}$

14. Two solutions (1, 2) (2, 7)