Quesiton Bank -Solutions of Triangles

LEVEL–I

1. In a Δ ABC, if a = 18, b = 24 and c = 30, find

(i) $\sin A, \cos A, \tan A$. **(ii)** $\sin\frac{A}{2}, \cos\frac{A}{2}, \tan\frac{A}{2}$. **(iii)** Area of Δ ABC.

2. In a Δ ABC, prove that

(i) $\frac{a+b}{c} = \frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}$ **(ii)** $a(\cos C - \cos B) = 2(b-c)\cos^2\frac{A}{2}$

(iii) $2a\sin\frac{B}{2}\sin\frac{C}{2} = (b+c-a)\sin\frac{A}{2}$ **(iv)** $2a\cos\frac{B}{2}\cos\frac{C}{2} = (a+b+c)\sin\frac{A}{2}$.

3. In any triangle ABC, show that $\frac{a^2-b^2}{2}\cdot\frac{\sin A\sin B}{\sin(A-B)} = \Delta$.

4. In a triangle ABC, show that $a\cos^2\frac{A}{2} + b\cos^2\frac{B}{2} + c\cos^2\frac{C}{2} = s + \frac{\Delta}{R}$

5. Find the distance between the incentre and excentres of a Δ ABC.

6. The angles of a triangle are in the ratio 1 : 2 : 7. Show that the ratio of the greatest side to the least side is $\sqrt{5}+1 : \sqrt{5}-1$.

7. In a Δ ABC, prove that $(r+r_1)\tan\left(\frac{B-C}{2}\right) + (r+r_2)\tan\left(\frac{C-A}{2}\right) + (r+r_3)\tan\left(\frac{A-B}{2}\right) = 0$.

8. In a Δ ABC, right angled at C, if $\tan A = \sqrt{\frac{\sqrt{5}-1}{2}}$, show that the sides a, b, c are in G.P.

9. In an acute-angled triangle ABC, the circle on the altitude AD as diameter cuts AB at P and AC at Q. show that $PQ = 2R\sin A\sin B\sin C = \frac{\Delta}{R}$.

10. The ex-radii of a triangle are 5 cm, 7.5 cm and 15 cm. Find the sides and the angles of the triangle.

LEVEL–II

1. If A, B, C are the angles of a triangle, prove that $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$.

2. In a circle of radius r, chords of lengths a and b cm, subtends angles θ and 3θ respectively at the centre. Show that $r = a\sqrt{\frac{a}{3a-b}}$ cm.

3. Prove that for a triangle ABC $a\sin\left(\frac{A}{2}+B\right) = (b+c)\sin\frac{A}{2}$.

4. In the triangle ABC, if $(a^2 + b^2)\sin(A - B) = (a^2 - b^2)\sin(A + B)$, prove that the triangle is either isosceles or right angled.

5. In a $\triangle$ ABC, prove $a^2\cos 2B + b^2\cos 2A + 2ab\cos(A - B) = c^2$.

6. **(i)** If $b + c = 3a$, prove that $\cot\frac{B}{2}\cot\frac{C}{2} = 2$.

(ii) If the sides a, b, c are in A.P., prove that $\tan\frac{A}{2} + \tan\frac{C}{2} = \frac{2}{3}\cot\frac{B}{2}$.

7. **(i)** If $a = 2b$, $A = 3B$, find the angles of Δ ABC.

(ii) If three sides of a triangle are 3, 7 and 8 cms, prove that the angles of the triangle are in A.P.

8. In a triangle ABC, if $\cos A + 2\cos B + \cos C = 2$, prove that the sides of the triangle are in A.P.

9. If the line joining the circumcenter to incentre of an acute angled Δ ABC is parallel to the side BC of the triangle ABC, then prove that $\cos B + \cos C = 1$.

10. Tangents parallel to the three sides are drawn to the incircle. If x, y, z are the length of the parts of the tangents within the triangle, then prove that $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

IIT JEE PROBLEMS (OBJECTIVE)

A. Fill in the blanks

1. The set of all real numbers a such that $a + 2a$, $2a + 3$ and $a^2 + 3a + 8$ are the sides of a triangle is .... **[IIT - 85]**

2. In a triangle ABC, if cot A, cot B, cot C are in A. P., then a^2, b^2, c^2, are in .... progression. **[IIT - 85]**

3. A polygon of nine sides, each of length 2, is inscribed in a circle. The radius of the circle is .... **[IIT - 87]**

4. If the angles of a triangle are 30^0 and 45^0 and the included side is $(\sqrt{3}+1)$ cms, then the area of the triangle is .... **[IIT - 88]**

5. If in a triangle ABC $\dfrac{2\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{2\cos C}{c} = \dfrac{a}{bc} + \dfrac{b}{ca}$ then the value of the angle A is .... **[IIT - 93]**

6. In a triangle ABC, AD is the altitude from A. Given b > c, angle C = 23° and $AD = \dfrac{abc}{b^2 - c^2}$ then angle B = ... **[IIT - 94]**

7. A circle is inscribed in an equilateral triangle of side a. The area of any square inscribed in this circle is .... **[IIT - 94]**

8. In a triangle ABC, a : b : c = 4 : 5 : 6. The ratio of the radius of the circumcircle to that of the incircle is .... **[IIT - 96]**

9. Match the following

Sides a, b, c of a triangle ABC are in AP and $\cos\theta_1 = \dfrac{a}{b+c}$, $\cos\theta_2 = \dfrac{b}{a+c}$, $\cos\theta_3 = \dfrac{c}{a+b}$,

then the value of $\tan^2\left(\dfrac{\theta_1}{2}\right) + \tan^2\left(\dfrac{\theta_3}{2}\right)$ is .... **[IIT – 2006]**

B. Multiple choice Questions with one or more than one correct answer :

1. There exists a triangle ABC satisfying the conditions **[IIT - 86]**

(A) $b \sin A = a, A < \dfrac{\pi}{2}$

(B) $b \sin A > a, A > \dfrac{\pi}{2}$

(C) $b \sin A > a, A < \dfrac{\pi}{2}$

(D) $b \sin A < a, A < \dfrac{\pi}{2}, b > a$

2. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P. then the length of the third side can be

(A) $5-\sqrt{6}$ (B) $3\sqrt{3}$ (C) 5 (D) $5+\sqrt{6}$ **[IIT - 87]**

3. If in a triangle PQR, sin P, sin Q, sin R are in A.P., then **[IIT - 98]**
(A) the altitudes are in A.P. (B) the altitudes are in H.P.
(C) the medians are in G.P. (D) the medians are in A.P.

4. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is **[IIT - 98]**

(A) $\frac{3}{4}$ (B) $3\sqrt{3}$ (C) 3 (D) $\frac{3\sqrt{3}}{2}$

5. Internal bisector of $\angle A$ of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent sides of ΔABC then

(A) AE is HM of b and c (B) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$ **[IIT – 2006]**

(C) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$ (D) the triangle AEF is isosceles

C. Multiple choice Questions only one correct answer :

1. From the top of a light-house 60 meters high with its base at the sea-level, the angle of depression of a boat is 15^0. The distance of the boat from the foot of the light house is **[IIT - 83]**

(A) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)60$ metres (B) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)60$ metres (C) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$ metres

(D) none of these

2. In a triangle ABC, angle A is greater than angle B. If the measures of angle A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$, $o < k < 1$, then the measure of angle C is **[IIT - 90]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

3. In a triangle ABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$. Let D divide BC internally in the ratio 1 : 3 then $\frac{\sin\angle BAD}{\sin\angle CAD}$ is equal to **[IIT - 95]**

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{\frac{2}{3}}$

4. In a triangle ABC, $2ac\sin\frac{1}{2}(A - B + C)$ is equal to **[IIT - 2000]**
(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$ (C) $b^2 - c^2 - a^2$ (D) $c^2 - a^2 - b^2$

5. In triangle ABC, Let $\angle C = \frac{\pi}{2}$. If 'r' is the inradius and 'R' is the circumradius of the triangle, then 2(r + R) is equal to **[IIT - 2000]**
(A) a + b – c (B) b + c (C) c + a (D) a + b + c

6. A pole stands vertically inside a triangular park triangle ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in triangle ABC the foot of the pole is at the **[IIT - 2000]**

(A) centroid (B) circumcentre (C) incentre (D) orthocentre

7. A man from the top of a 100 metres high tower sees a car moving towards the tower at an angle of depression of 30^0. After some time, the angle of depression becomes 60^0. The distance (in metres) travelled by the car during this time is **[IIT - 2001]**

(A) $100\sqrt{3}$ (B) $\frac{200\sqrt{3}}{3}$ (C) $\frac{100\sqrt{3}}{3}$ (D) $200\sqrt{3}$

8. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle ABC (R being the radius of the circumcircle) ? **[IIT - 2002]**

(A) a, sin A, sin B (B) a, b, c (C) a, sin B, R (D) a, sin A, R

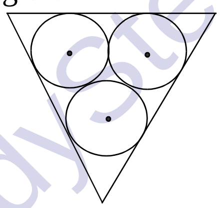
9. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is **[IIT - 2003]**

(A) $\sqrt{3}:(2+\sqrt{3})$ (B) 1 : 6 (C) $1:2+\sqrt{3}$ (D) 2 : 3

10. The sides of a triangle are in the ratio $1:\sqrt{3}:2$, then the angles of the triangle are in the ratio **[IIT - 2004]**

(A) 1 : 3 : 5 (B) 2 : 3 : 4 (C) 3 : 2 : 1 (D) 1 : 2 : 3

11. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is **[IIT - 2005]**



(A) $4:2\sqrt{3}$ (B) $6:4\sqrt{3}$ (C) $12+\frac{7\sqrt{3}}{4}$ (D) $3+\frac{7\sqrt{3}}{4}$

12. In a triangle ABC, a, b, c are the lengths of its sides and A, B, C are the angles of triangle ABC. The correct relation is given by **[IIT - 2005]**

(A) $(b-c)\sin\left(\frac{B-C}{2}\right)=a\cos\frac{A}{2}$ (B) $(b-c)\cos\left(\frac{A}{2}\right)=a\sin\frac{B-C}{2}$

(C) $(b+c)\sin\left(\frac{B+C}{2}\right)=a\cos\frac{A}{2}$ (D) $(b-c)\sin\left(\frac{A}{2}\right)=2a\sin\frac{B+C}{2}$

13. Inradius of a circle which is inscried in a isosceles triangle one of whose angle is $2\pi/3$, is $\sqrt{3}$ then area of the triangle is **[IIT – 2006]**

(A) $4\sqrt{3}$ (B) $12-7\sqrt{3}$ (C) $12+7\sqrt{3}$ (D) none of these

14. a, b, c are the sides of a triangle ABC such that $x^2 - 2(a + b + c)x + 3\lambda (ab + bc + ca) = 0$ has real roots, then **[IIT – 2006]**

(A) $\lambda<\frac{4}{3}$ (B) $\lambda>\frac{5}{3}$ (C) $\lambda\in\left(\frac{4}{3},\frac{5}{3}\right)$ (D) $\lambda\in\left(\frac{1}{3},\frac{5}{3}\right)$

IIT JEE PROBLEMS (SUBJECTIVE)

1. Let the angles A, B, C of a triangle ABC be in A.P. and let $b:c=\sqrt{3}:\sqrt{2}$. Find the angle A. **[IIT - 81]**

2. A vertical pole stands at a point Q on a horizontal ground. A and B are points on the ground, d meters a part. The pole subtends angles α and β at A and B respectively. AB subtends an angle γ at O. Find the height of the pole. **[IIT - 82]**

3. Four ships A, B, C and D are at sea in the following relative positions. B is on the straight line segment AC, B is due North of D and D is due west of C. The distance between B and D is 2 km. $\angle BDA = 40^0$, $\angle BCD = 25^0$. What is the distance between A and D ? **[IIT - 83]**

4. The ex-radii r_1, r_2, r_3 of triangle are in H.P. Show that its sides a, b, c are in A.P. **[IIT - 83]**

5. For a triangle ABC in given that $\cos A+\cos B+\cos C=\frac{3}{2}$. Prove that the triangle is equilateral. **[IIT - 84]**

6. With usual notation, if in a triangle ABC, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then prove that $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$. **[IIT - 84]**

7. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that $a=b\tan\frac{1}{2}(\alpha+\beta)$. **[IIT - 85]**

8. In a triangle ABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ and it divides the angle A into angles 30^0 and 45^0. Find the length of the side BC. **[IIT - 85]**

9. If in a triangle ABC, $\cos A\cos B+\sin A\sin B\sin C=1$, show that $a:b:c=1:1:\sqrt{2}$. **[IIT - 86]**

10. A man observes a tower AB of height h from a point P on the ground. He moves a distance d towards the foot of the tower and finds that the angle of elevation has doubted. He further moves a distance $\frac{3}{4}$ d in the same direction and finds that the angle of elevation is three times that at P. Prove that $36h^2 = 35d^2$. **[IIT - 86]**

11. A 2 meter long object is fired vertically upwards from the midpoint of two locations A and B, 8 meters a part. The speed of the object after t seconds is given by $\frac{ds}{dt} = (2t + 1)$ meters per second Let α and β be the angles subtended by the object at A and B, respectively after one and two seconds. Find the value of $\cos(\alpha - \beta)$. **[IIT - 87]**

12. A sign-post in the form of an isosceles triangle ABC is mounted on a pole of height h fixed to the ground. The base BC of the triangle is parallel to the ground. A man standing on the ground at a distance d from the sign-post finds that the top vertex A of the triangle subtends an angle β and either of the other two vertices subtends the same angle α at his feet. Find the area of the triangle **[IIT - 88]**

13. ABC is a triangular park with AB = AC = 100 m. A television tower stands at the midpoint of BC. The angles of elevation of the top of the tower at A, B, C are 45^0, 60^0, 60^0, respectively. Find the height of the tower. **[IIT - 89]**

14. A vertical tower PQ stands at a point P. Points A and B are located to the South and East of P respectively. M is the midpoint of AB. PAM is an equilateral triangle; and N is the foot of the perpendicular from P on AB. Let AN = 20 metres and the angle of elevation of the top of the tower at N is $\tan^{-1}(2)$. Determine the height of the tower and the angles of elevation of the top of the tower at A and B. **[IIT - 90]**

15. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of the triangle. **[IIT - 91]**

16. In a triangle of base a the ratio of the other two sides is $r(<1)$. Show that the altitude of the triangle is less than or equal to $\frac{ar}{1-r^2}$. **[IIT - 91]**

17. A man notices two objects in a straight line due west. After walking a distance c due north he observes that the objects subtends an angle α at his eye, after walking a further distance 2c due north, an angle β. Show that the distance between the objects is $\frac{8c}{3\cot\beta - \cot\alpha}$. **[IIT - 91]**

18. Three circles touch the one another externally. The tangent at their point of contact meet at a point whose distance from a point of contact is 4. Find the ratio of the product of the radii to the sum of the radii of the circles. **[IIT - 92]**

19. If the sides a, b, c of the triangle are in AP then find the value of $\tan\frac{A}{2} + \tan\frac{C}{2}$ in terms of cot (B/2). **[REE - 93]**

20. An observer at O notices that the angle of elevation of the top of a tower is 30^o. The line joining O to the base of the tower makes an angle of $\tan^{-1}(1/\sqrt{2})$ with north and is inclined Eastwards. The observer travels a distance of 300 metres towards the North to a point A and finds the tower to his east. The angle of elevation or the top of the tower at A is ϕ. Find ϕ and the height of the tower. **[IIT - 93]**

21. A tower AB leans towards west making an angle α with the vertical. The angular elevation of B, the topmost point of the tower is β as observed from a point C due west of A at a distance d due west of A at a distance d from A. If the angular elevation of B from the point D due east of C at a distance 2d from C is γ, then prove that $2 \tan\alpha = \cot\beta - \cot\gamma$. **[IIT - 94]**

22. Let $A_1, A_2, \ldots A_n$ be the vertices of an n-sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$, find the value of n. **[IIT - 94]**

23. Consider the following statement concerning a triangle ABC. **[IIT - 94]**

(i) The sides a, b, c & area Δ are rotational.

(ii) a, $\tan\frac{B}{2}$, $\tan\frac{C}{2}$ are rational .

(iii) a, sin A, sin B, sin C are rotational Prove that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (i).

24. A cyclic quardilateral ABCD of area $3\sqrt{3}/4$ is inscribed in a unit circle. If one of its sides AB = 1 & the diagonal BD = $\sqrt{3}$, find lengths of the other sides . **[REE - 95]**

25. A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively and the distance between the point A and the mid point of the line segment DC is d, prove that the area of the circle is $\frac{\pi d^2 \cos^2\alpha}{\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}$. **[IIT - 96]**

26. In a ΔABC, $\angle C = 60°$ & $\angle A = 75°$. If D is a point on AC such that the area of the ΔBAD is $\sqrt{3}$ times the area of the ΔBCD, find the $\angle$ABD. **[REE - 96]**

27. If in a ΔABC, a = 6 , b = 3 and cos (A – B) = 4/5 then find its area. **[REE - 97]**

28. A semicircular arch AB of length 2L and a vertical tower PQ are situated in the same vertical plane. The feet A and B of the arch and the base Q of the tower are at the same horizontal level, with B between A and Q. A man at A finds the tower hidden from his view due to the arch. He starts crawling up the arch and just sees the topmost point P of the tower after covering a distance $\frac{L}{2}$ along the arch. He crawls further to the topmost point of the arch and notes the angle of elevation of P to be θ. Compute the height of the tower in terms of L and θ. **[IIT - 97]**

29. Let A, B, C be three angles such that $A = \frac{\pi}{4}$ and tan B tan C = p. Find all possible values of p such that A, B, C are the angles of triangle. **[IIT - 97]**

30. Two sides of a triangle are of $\sqrt{6}$ lengths and 4 and the angle opposite to smaller side is 30°. How many such triangles are possible ? Find the length of their third side and area. **[IIT - 98]**

31. C_1 and C_2 are two concentric circles, the radius of C_2 being twice that of C_1. From a point P on C_2, tangents PA and PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1. **[IIT - 98]**

32. Prove that a triangle ABC is equilateral if and only if $\tan A + \tan B + \tan C = 3\sqrt{3}$. **[IIT - 98]**

33. Let ABC be a triangle having 'O' and 'I' as its circumcenter and incentre respectively. If R and r are the circumradius and the inradius respectively, then prove that $(IO)^2 = R^2 - 2Rr$. Further show that the triangle BIO is a right triangle if and only if b is the arithmetic means of a and c. **[IIT - 99]**

34. The radii r_1, r_2, r_3 of described circles of a triangle ABC are in harmonic progression. If its area is 24 sq. cm and its perimeter is 24 cm, find the lengths of its sides. **[IIT - 99]**

35. In any triangle ABC, prove that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$. **[IIT - 2000]**

36. Let ABC be a triangle with incentre 'I' and inradius 'r'. Let D, E, F be the feet of the perpendiculars from I to the sides BC, CA & AB respectively. If r_1, r_2 & r_3 are the radii of circles inscribed in the quadrilaterals AFIE, BDIF & CEID respectively, prove that

$$\frac{r_1}{r-r_1} + \frac{r_2}{r-r_2} + \frac{r_3}{r-r_3} = \frac{r_1r_2r_3}{(r-r_1)(r-r_2)(r-r_3)}.$$ **[IIT - 2000]**

37. If Δ is the area of a triangle with side lengths a, b, c then show that $\Delta \le \frac{1}{4}\sqrt{(a+b+c)abc}$. Also show that equality occurs in the above inequality if and only if a = b = c. **[IIT - 2001]**

38. If I_n is the area of n sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given circle, prove that $I_n = \frac{O_n}{2}\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right)$. **[IIT - 2003]**

SET–I

1. In a triangle the length of the two larger sides are 24 and 22 respectively. If the angles are in A.P., then the third side is

(A) $12 + 2\sqrt{3}$ (B) $12 - 2\sqrt{3}$

(C) $2\sqrt{3} + 2$ (D) $2\sqrt{3} - 2$.

2. In a triangle ABC, $\dfrac{b^2 \sin 2C + c^2 \sin 2B}{\Delta}$ is always equal to

(A) 1 (B) 2

(C) 3 (D) 4

3. If p_1, p_2, p_3 are the altitudes of the triangle ABC from the vertices A, B, C and Δ the area of the triangle, then $p_1^{-2} + p_2^{-2} + p_3^{-2}$ is equal to

(A) $\dfrac{a+b+c}{\Delta}$ (B) $\dfrac{a^2+b^2+c^2}{4\Delta^2}$

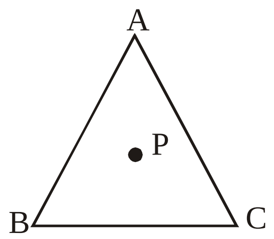
(C) $\dfrac{a^2+b^2+c^2}{\Delta^2}$ (D) none of these.

4. If A_1, A_2, A_3 denote respectively the areas of an inscribed polygon of 2n sides, inscribed polygon of n sides and circumscribed polygon of n sides, then A_2, A_1, A_3 are in

(A) A.P. (B) G.P.

(C) H.P (D) none of these.

5. In the adjacent figure P is any interior point of the equilateral triangle ABC of side length 2 units. If x_a, x_b and x_c represent the distance of P from the sides BC, CA and AB respectively then $x_a + x_b + x_c$ is equal to



(A) 6 (B) $\sqrt{3}$

(C) $\dfrac{\sqrt{3}}{2}$ (D) $2\sqrt{3}$

6. If radius of the circumcircle of a Δ ABC is 4 cm and D, E, F are the feet of perpendiculars drawn from the vertices to the opposite sides, then radius of the circumcircle of Δ DEF is

(A) 3 cm (B) 1 cm

(C) 2 cm (D) none of these.

7. In a triangle ABC if $\dfrac{r}{r_1} = \dfrac{1}{2}$, then the value of $\tan\dfrac{A}{2}\left(\tan\dfrac{B}{2} + \tan\dfrac{C}{2}\right)$ is equal to

(A) 2 (B) $\dfrac{1}{2}$

(C) 1 (D) none of these

8. In triangle ABC the value of the expression $r_1r_2 + r_2r_3 + r_3r_1$ is always equal to

(A) $\frac{1}{2}(a+b+c)^2$ (B) $(a+b+c)^2$

(C) $\frac{1}{16}(a+b+c)^2$ (D) $\frac{1}{4}(a+b+c)^2$

9. In triangle ABC, $A = \frac{\pi}{2}$ and c, sinB, cosB are given to be rational numbers, then

(A) a, b are irrational (B) a, b are rational

(C) a is rational and b is irrational (D) b is rational and a is irrational

10. If in triangle ABC, $r_1 > r_2 > r_3$, then

(A) $a < b < c$ (B) $a < c < b$

(C) $a > b > c$ (D) $a > c > b$

11. If in a $\triangle$ ABC, $a \tan A + b \tan B = (a + b) \tan\left(\frac{A+B}{2}\right)$, then

(A) A = B (B) A = – B

(C) A = 2B (D) B = 2A.

12. In triangle ABC, $a^2 + b^2 + c^2 - ac - \sqrt{3}\,ab = 0$ then triangle is necessarily

(A) isoscales (B) right angled

(C) obtuse angled (D) equilateral

13. If p_1, p_2, p_3 are respectively the perpendiculars from the vertices of a triangle to the opposite sides, then $\frac{\cos A}{p_1} + \frac{\cos B}{p_2} + \frac{\cos C}{p_3}$ is equal to

(A) $\frac{1}{r}$ (B) $\frac{1}{R}$

(C) $\frac{1}{\Delta}$ (D) none of these.

14. If p is the product of the sines of the angles of a triangle, and q the product of their cosines, the tangents of the angles are roots of the equation

(A) $qx^3 - px^2 + (1 + q)x - p = 0$ (B) $px^3 - qx^2 + (1 + p)x - q = 0$

(C) $(1 + q)x^3 - px^2 + qx - p = 0$ (D) none of these.

15. If P is a point on the altitude AD of the triangle ABC such that $\angle CBP = B/3$, then AP is equal to

(A) $2a\sin\frac{C}{3}$ (B) $2b\sin\frac{C}{3}$

(C) $2c\sin\frac{B}{3}$ (D) $2c\sin\frac{C}{3}$

16. If in triangle ABC, $(a + b + c)(a + b - c) = \lambda ab$ then exhaustive range of 'λ' is

(A) (2, 4) (B) (0, 4)

(C) (0, 2) (D) (1, 4)

17. In triangle ABC, $(r_1 + r_2 + r_3 - r)$ is equal to

(A) $2a \sin A$ (B) $2a \operatorname{cosec} A$

(C) $2a \sin \frac{A}{2}$ (D) $2a \operatorname{cosec} \frac{A}{2}$

18. In triangle ABC, internal angle bisector of $\angle A$ makes an angle θ with side BC. Value of $\sin\theta$ is equal to

(A) $\left|\sin\left(\frac{B-C}{2}\right)\right|$ (B) $\left|\sin\left(\frac{B}{2}-C\right)\right|$

(C) $\cos\left(\frac{B-C}{2}\right)$ (D) $\cos\left(\frac{B}{2}-C\right)$

19. In triangle ABC, AD is the altitude. If $b > c$, $C = 18^0$ and $AD = \frac{abc}{b^2 - c^2}$, then B is equal to

(A) 72^0 (B) 36^0

(C) 108^0 (D) 54^0

20. In any triangle minimum value of $\frac{r_1 r_2 r_3}{r^3}$ is equal to

(A) 1 (B) 9

(C) 27 (D) none of these

SET–II

1 The sides of a triangle ABC are x, y, $\sqrt{x^2+y^2+xy}$ respectively. The size of the greatest angle in radians is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) none of these

2 The sides of a triangle are 7, $4\sqrt{3}$ and $\sqrt{13}$. The smallest angle is equal to

(A) 45^0 (B) 30^0

(C) 60^0 (D) none of these

3 Given $A=\frac{\pi}{3}$ in a triangle ABC. Then the value of $\left(1+\frac{a}{c}+\frac{b}{c}\right)\left(1+\frac{c}{b}-\frac{a}{b}\right)$ is equal to

(A) 4 (B) 5

(C) 3 (D) none of these

4 If the area of a triangle is 100 sq.cms, $r_1 = 10$ cms and $r_2 = 50$ cms, then the value of $b-a$ is equal to

(A) 6 cms (B) 10 cms

(C) 20 cms (D) 8 cms

5 In triangle ABC, if b sinC (b cosC + c cosB) = 42, then the area of the triangle ABC is

(A) 21 sq.units (B) 25 sq.units

(C) 41 sq.units (D) none of these

6 In a triangle ABC, if $\frac{a^3+b^3+c^3}{\sin^3 A+\sin^3 B+\sin^3 C}=343$, the diameter of the circle circumscribing the triangle is

(A) 14 units (B) 7 units

(C) 21 units (D) none of these

7. In a triangle ABC, right angles at B, the inradius is

(A) $\frac{AB+BC-AC}{2}$ (B) $\frac{AB+AC-BC}{2}$

(C) $\frac{AB+BC+AC}{2}$ (D) none of these

8 If in a triangle ABC, $b\cos^2\frac{A}{2}+a\cos^2\frac{B}{2}=\frac{3}{2}c$ then a, b, c are

(A) in A.P. (B) in G.P.

(C) in H.P. (D) none of these

9 If in a triangle ABC angle B = 90° then $\tan^2 A/2$ is

(A) $\frac{b-c}{a}$ (B) $\frac{b-c}{b+c}$

(C) $\frac{b+c}{b-c}$ (D) none of these

10. Angles A, B and C of a triangle ABC are in AP. If $\frac{b}{c} = \sqrt{\frac{3}{2}}$ $\angle A$ is equal to

(A) $\pi/6$ (B) $\pi/4$

(C) $5\pi/12$ (D) $\pi/2$

11. With the usual notation in any Δ ABC

(A) $\frac{a+b+c}{\sin A + \sin B + \sin C} = \frac{1}{2R}$ (B) $\frac{\cos A}{\sqrt{4R^2 - a^2}} = \frac{\cos B}{\sqrt{4R^2 - b^2}} = \frac{\cos C}{\sqrt{4R^2 - c^2}}$

(C) $\frac{a \sec A + b \sec B + c \sec C}{\tan A \tan B \tan C} = 2R$ (D) $\Delta = \sqrt{s(s+a)(s+b)(s+c)}$

12. In a triangle ABC, points D and E are taken on side BC such that BD = DE = EC. If angle ADE = angle AED = θ then

(A) $\tan\theta = \tan B$ (B) $3\tan\theta = \tan C$

(C) $\frac{6\tan\theta}{\tan 2\theta - 9}$ (D) none of these

13. In a triangle ABC, CH and CM are the lengths of the altitude and median to the base AB. If a = 10, b = 26, c = 32 then length (HM)

(A) 5 (B) 7

(C) 9 (D) none of these

14. The product of the arithmetic mean of the lengths of the sides of a triangle and harmonic mean of the lengths of the altitudes of the triangle is equal to

(A) Δ (B) 2Δ

(C) 3Δ (D) 4Δ

15. In a triangle ABC, CD is the bisector of the angle C. If $\cos\frac{C}{2}$ has the value $\frac{1}{3}$ and l(CD) = 6, then $\left(\frac{1}{a} + \frac{1}{b}\right)$ has the value equal to

(A) $\frac{1}{9}$ (B) $\frac{1}{12}$

(C) $\frac{1}{6}$ (D) none of these

16. If the median of a triangle ABC through A is perpendicular to AB then $\frac{\tan A}{\tan B}$ has the value equal to

(A) $\frac{1}{2}$

(B) 2

(C) -2

(D) $-\frac{1}{2}$

17. Let ABC be a triangle on a horizontal plane. If the elevation of the top of a tower on the plane at each of the angular points A, B and C be θ, then the height of the tower is

(A) $\frac{1}{2}a \cdot \tan\theta \cdot \operatorname{cosec} A$

(B) $a \cdot \tan\theta \cdot \operatorname{cosec} A$

(C) $2a \cot\theta \cdot \operatorname{cosec} A$

(D) $2a \cdot \tan\theta \cdot \sin A$

18. In a Δ ABC if $r : r_1 : R = 2 : 12 : 5$ where all symbols have their usual meaning then

(A) Δ ABC is an acute angled triangle

(B) Δ ABC is an obtuse angled triangle

(C) Δ ABC is right angled which is not isosceles

(D) Δ ABC is isosceles which is not right angled

19. The sides of a Δ ABC satisfy the equation, $2a^2 + 4b^2 + c^2 = 4ab + 2ac$. Then

(A) the triangle is equilateral

(B) the triangle is obtuse.

(C) $A = \cos^{-1}\frac{1}{4}$

(D) none of these

20. If in a Δ ABC, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then the triangle is

(A) right angled

(B) isosceles

(C) equilateral

(D) obtuse

SET–III

More than one

1. In a triangle ABC, let D be the the mid point of BC. If AB = 2, BC = 4 and CA = 3, then

(A) $\cos B = \frac{11}{16}$ (B) $\cos B = \frac{7}{8}$ (C) AD =2 .4 (D) AD = 1.58

2. A triangle ABC has side $c = 2\sqrt{2}$ and $\angle A = 30^o$. If the radius of its circumcircle is 2, then

(A) a = 2 (B) $a = 2\sqrt{2}$ (C) $\angle C = 45^o$ (D) $\angle C = 60^o$

3. If in a triangle ABC, CD is the angular bisector of the angle ACB then CD is equal to

(A) $\frac{a+b}{2ab}\cos\frac{C}{2}$ (B) $\frac{a+b}{ab}\cos\frac{C}{2}$ (C) $\frac{2ab}{a+b}\cos\frac{C}{2}$ (D) $\frac{b\sin A}{\sin\left(B+\frac{C}{2}\right)}$

4. In a triangle ABC, the minimum values of $\cot^2 A + \cot^2 B + \cot^2 C$ is equal to

(A) 0 (B) 1 (C) $\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2\,dt}{x\sin x}$ (D) none of these

5. If a and b be the lengths of the sides and c the length of hypotenuse of a right angled triangle, then

(A) a + b > c (B) $a^2 + b^2 = c^2$ (C) $a^3 + b^3 < c^3$ (D) $a^n + b^n < c^n$

W I In an acute angled trinagle ABC altitude are drawn Feet of the altitude (L,M,N) are joined. ΔLMN is called the orthic tringale of ΔABC.

6. Side NM will be

(A) a cos B (B) a sin B (C) a sin A (D) a cos A

7. $\frac{\text{Perimeter of } \Delta\, LNB}{\text{Perimeter of } \Delta\, ABC}$ will be

(A) cos B (B) cos B cos C (C) sin B (D) none of these

8. $\frac{\text{Area of } \Delta LMC}{\text{Area of } \Delta ABC}$ will be

(A) cos C (B)$\sin^2 C$ (C) $\cos^2 C$ (D) none of these

9. If perimeter of ΔLMN is diameter of circum circle of ΔABC then sin A . sin B . sin C is

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{16}$

10. Incentre of ΔLMN is

(A) centroid of ΔABC (B) circumcentre of ΔABC

(C) orthocentre of ΔABC (D) incentre of ΔABC

W II The radius (R) of the circle, which passes through the angular points of the triangle ABC, is $R = \frac{abc}{4S}$.

The radius (r) of the incircle, is given by $r = \frac{S}{s} \Rightarrow r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$.

The radii of escribed circles, which are opposite to A, B, C are given by

$r_1 = \frac{S}{(s-a)}, r_2 = \frac{S}{(s-b)}, r_3 = \frac{S}{(s-c)} \Rightarrow r_1 = s\tan\frac{A}{2}, r_2 = s\tan\frac{B}{2}, r_3 = s\tan\frac{C}{2}$ where S = area of triangle and s = semi perimeter.

11. The value of $r_1r_2r_3$ is

(A) r^2s (B) rs^2 (C) r^3s^3 (D) none of these

12. The value of $(r_1+r_2)(r_2+r_3)(r_3+r_1)$ is

(A) $4R^2s$ (B) $4Rs^2$ (C) $4R^3s^3$ (D) none of these

13. Two sides of a triangle are 2 and $\sqrt{3}$ and the included angle is 30° then the inradius r of the triangle is

(A) $\frac{(\sqrt{3}-1)}{4}$ (B) $\frac{(\sqrt{3}+1)}{2}$ (C) $\frac{(\sqrt{3}-1)}{2}$ (D) $\frac{(\sqrt{3}+1)}{4}$

14. If p_1, p_2, p_3 are respectively the perpendicular from the vertices of the triangle to the opposite sides, then the value of $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}$ is

(A) $\frac{1}{r}$ (B) $\frac{1}{2r}$ (C) $\frac{1}{3r}$ (D) none of these

W III Let ABC be a triangle such that the three ex - radii (radii of the described circles), r_1, r_2 and r_3 are the roots of the cubic $x^3 - px^2 + qx - q = 0$.

15. In radius of the triangle is

(A) 1 (B) p (C) q (D) q / p

16. Circum radius of ΔABC is

(A) equal to 2 (B) greater than to 2 (C) depends on P (D) less than 2

17. What should be the least possible value of $p(q \in R)$

(A) 8 (B) 9 (C) 1 (D) 0

18. Fill in the blanks

(i) In triangle ABC, a = 5, b = 4, c = 3. 'G' is the centroid of triangle, then circumradius of triangle GAB is ________ .

(ii) The product of the distances of the incentre from the angular points of a Δ ABC is _____.

(iii) In a Δ ABC if $b + c = 3a$ then the value of $\cot\frac{B}{2}\cdot\cot\frac{C}{2}$ is _______ .

(iv) If in a triangle $\tan\frac{A}{2}, \tan\frac{B}{2}, \tan\frac{C}{2}$ are in H.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in _____ progression.

(v) In any triangle ABC, the value of $a^3\sin(B-C) + b^3\sin(C-A) + c^3\sin(A-B)$ is _____.

19. True/False

(i) In triangle ABC the medians AA_1 and BB_1 are mutually perpendicular then cos C is $\frac{2c^2}{ab}$.

(ii) In a triangle the angles A, B, C are in AP then $2\cos\frac{A-C}{2} = \frac{a+c}{\sqrt{a^2-ac+c^2}}$.

(iii) If $a\tan A + b\tan B = (a+b)\tan\frac{A+B}{2}$, then triangle ABC is equilateral.

(iv) If the bisector of angle C of triangle ABC meets AB in D and the circumcircle in E then $\frac{CE}{DE} = \frac{(a+b)^2}{c^2}$.

(v) In any triangle ABC, the value of $(b-c)\cot\frac{A}{2} + (c-a)\cot\frac{B}{2} + (a-b)\cot\frac{C}{2} = 1$.

20. Match the column

(I) Consider two circles S_1 and S_2 having their centers at B and C with radii r_1 and r_2 respectively, touching each other externally. A circle S with radius r and centre at A touches both of them externally as shown in the figure.

	Column I		**Column II**
(a)	If ΔABC is right angled at A, then $\frac{r_2+r}{r_2-r}$ is	(P)	$r \ge 1$
(b)	If ΔABC is right angled at A and $r_1 . r_2 = 3+2\sqrt{2}$, then	(Q)	$r_1 = r_2$
(c)	If the common tangent of S_1 and S_2 at their point of contact passes through the centre of circle S, then	(R)	$\frac{R}{4}$
(d)	If $r_1 = r_2 = R$ and the circles S_1, S_2 and S have the same external common tangent then r is	(S)	$\frac{r_1}{r}$

(II) ABC is a triangle in which $\cos(A-B) = \frac{4}{5}$ and BC = 6, AC = 3. AD is the median through A, $\angle BAD = \alpha$, CL is perpendicular to AD.

	Column I		**Column II**
(a)	The value of $\sin\alpha$ is	(P)	$\frac{3\sqrt{5}}{2}$
(b)	Lenth of the median AD is	(Q)	$\frac{1}{\sqrt{10}}$
(c)	Radius of circumcircle of the triangle ABC is	(R)	$3\sqrt{2}$
(d)	The value of $\cot\angle ADC$ is	(S)	1

Solutions of Triangles

LEVEL–I ANSWER

1. **(i)** $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}$ **(ii)** $\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}, \frac{1}{3}$ **(iii)** 216 sq. units

5. $II_1 = 4R \sin A/2$, $II_2 = 4R \sin B/2$, $II_3 = 4R \sin C/2$

10. $a = 7.5$, $b = 10$, $c = 12.5$; $A = 2\tan^{-1}\left(\frac{1}{3}\right)$, $B = 2\tan^{-1}\left(\frac{1}{2}\right)$ $C = \frac{\pi}{2}$

LEVEL–II

7. **(i)** $A = 90^0$, $B = 30^0$, $C = 60^0$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. $(5, \infty)$ **2.** arithmetic **3.** $\operatorname{cosec}\frac{\pi}{9}$ **4.** $\frac{\sqrt{3}+1}{2}$ sq. unit

5. 90^o **6.** 113^o **7.** $\frac{a^2}{6}$ sq. unit **8.** 16 : 7 **9.** 2/3

(B)

1. AD **2.** AD **3.** B **4.** C **5.** **ABCD**

(C)

1.	B	**2.**	C	**3.**	A	**4.**	B	**5.**	A
6.	B	**7.**	B	**8.**	D	**9.**	A	**10.**	A
11.	B	**12.**	B	**13.**	C	**14.**	A		

IIT JEE PROBLEMS (SUBJECTIVE)

1. 75^0 **2.** $\frac{d}{\sqrt{\cot^2\alpha + \cot^2\beta - \cot\alpha\cot\beta\cot\gamma}}$ **3.** 4.28km.

8. 2 units **11.** $\frac{5}{\sqrt{26}}$ **12.** $\frac{d\tan\beta - h}{\sqrt{h^2\cot^2\alpha - d^2}}$ **13.** $50\sqrt{3}$

14. $40\sqrt{3}$, 60^0, 45^0 **15.** 4, 5, 6 **19.** $\frac{2}{3}\cot\frac{B}{2}$

20.	45°, $150\sqrt{2}$ meter	**22.**	7	**24.**	90
26.	angle $ABD = 30^0$	**27.**	9 sq. units	**28.**	$\frac{2L}{\pi}\left[\frac{\sqrt{2}-\cot\theta}{1-\cot\theta}\right]$
29.	$p \in (-\infty, 0] \cup [3 + 2\sqrt{2}, \infty)$				
30.	$2, \left(2\sqrt{3}-\sqrt{2}\right), \left(2\sqrt{3}+\sqrt{2}\right), \left(2\sqrt{3}-\sqrt{2}\right), \left(2\sqrt{3}+\sqrt{2}\right)$ sq. unit			**34.**	6, 8, 10 cm

SET–I

1. A	**2.** D	**3.** B	**4.** B	**5.** B
6. C	**7.** B	**8.** D	**9.** B	**10.** C
11. A	**12.** B	**13.** B	**14.** A	**15.** C
16. B	**17.** B	**18.** C	**19.** C	**20.** B

SET–II

1. A	**2.** B	**3.** C	**4.** D	**5.** A
6. B	**7.** A	**8.** D	**9.** B	**10.** C
11. C	**12.** C	**13.** C	**14.** B	**15.** A
16. C	**17.** A	**18.** C	**19.** C	**20.** C

SET–III

1. AD	**2.** AC	**3.** CD	**4.** BC	**5.** ABCD
6. D	**7.** C	**8.** C	**9.** C	**10.** C
11. B	**12.** B	**13.** C	**14.** A	**15.** A
16. B	**17.** B			

18.	**(i)**	$\frac{5}{12}\sqrt{13}$	**(ii)**	$4\,Rr^2$	**(iii)**	2	**(iv)**	Harmonic.	**(v)**	0
19.	**(i)**	T	**(ii)**	T	**(iii)**	F	**(iv)**	T	**(v)**	F
20.	**(I)**	a-S, b-P, c-Q, d-R			**(II)**	a-Q, b-R, c-P, d-S				