DISTANCE LEARNING PROGRAMME
(Academic Session : 2015 - 2016)

NURTURE TEST SERIES / JOINT PACKAGE COURSE
TARGET : JEE (ADVANCED) 2017

Test Type : **ALL INDIA OPEN TEST (**MAJOR**)**   Test Pattern : JEE-Advanced

TEST # 02   TEST DATE : 07 - 02 - 2016

PAPER-1

PART-1 : PHYSICS   **ANSWER KEY**

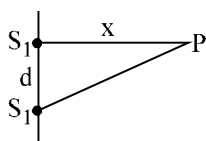| SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	A,B,C,D	A,B,C,D	A,B,C	A,C	A,B,C	A,B,C,D	A,C	A,B,C,D	A,B,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		Q,T	Q,R	P,S	P		P,R,S,T	P,R,S,T	Q,R,T	Q,T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	6	5	4	3	7	1	5	4		

SOLUTION

SECTION-I

1.   **Ans. (A,B,C)**

Sol.   Path difference

$$\Delta x = S_2 P - S_1 P = \sqrt{x^2 + d^2} - x$$

(A) when $x \approx \infty$

$\Delta x = 0$, so phase difference is zero

(B) In some phase so constructive

(C) when path difference $\Delta x = n\lambda$, then constructive interference

when $\Delta x = (2n + 1)\lambda/2$ where n = 0, 1, 2, … destructive

2.   **Ans. (A,B,C,D)**

Sol.   We know that $I = \dfrac{2\pi^2 B}{v} s_0^2 v^2$ and $P = I \times$ area

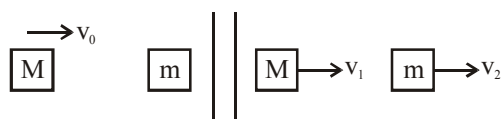also $I = \dfrac{p_0^2}{2\rho v}$ and here $s_0 = \Delta R$, $v = \sqrt{\dfrac{B}{\rho}}$ and $v = f$

3.   **Ans. (A,B,C,D)**

Sol.   Block B & C will perform SHM about COM

$$\therefore \quad \omega = \sqrt{\dfrac{k}{\mu}} = \sqrt{\dfrac{k}{m/2}} = \sqrt{\dfrac{2k}{m}}$$

For collision between M & m

$$Mv_0 = Mv_1 + mv_2 \ \&$$

$$v_2 - v_1 = v_0$$

$$\therefore \quad v_0 = v_1 + \gamma v_2$$

$$v_0 = v_2 - v_1$$

$$2v_0 = v_2 (1 + \gamma)$$

$$\therefore \quad v_2 = \dfrac{2v_0}{1 + \gamma}$$

$$\therefore \quad v_{CM} = \dfrac{mv_2 + m \times 0}{m + m} = \dfrac{v_0}{1 + \gamma}$$

$$\beta = \dfrac{v_{max/COM}}{\omega} = \dfrac{v_0}{\omega(1 + \gamma)}$$

$$v_{max} = \dfrac{2v_0}{1 + \gamma}$$

Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, Indra Vihar, Kota (Rajasthan)-324005
☎ +91-744-5156100 ✉ dlp@allen.ac.in 🌐 www.allen.ac.in

HS-1/17

4. **Ans. (A,B,C)**

Sol. Since the process in chamber 2 is adiabatic

$\therefore \quad P_0 V_0^\gamma = P_2 V_2^\gamma$

$\therefore \quad P_0 V_0^{5/3} = \dfrac{27}{8} P_0 V_2^{5/3}$

$\therefore \quad V_2 = \left(\dfrac{8}{27}\right)^{3/5} V_0$
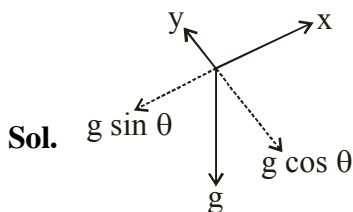
$\therefore$ Volume of chamber

$1 = 2V_0 - V_2 = \left[2 - \left(\dfrac{8}{27}\right)^{3/5}\right] V_0$

$P_0^{1-\gamma} T_0^\gamma = C$

$\therefore \quad T_2 = \left(\dfrac{27}{8}\right)^{2/5} T_0$

Work by the gas $= \dfrac{P_0 V_0 - P_2 V_2}{\gamma - 1}$

5. **Ans. (A,C)**

Sol.

$V_3^2 = V_1^2 - 2g \sin\theta \, x$ so $V_3 < V_1$
$V_4^2 = V_2^2 \, 2g\cos\theta \, (\Delta y) = V_2^2 - 2g\cos\theta(0) = V_2^2$
$V_4 = V_2$

6. **Ans. (A,B,C)**

Sol. $\gamma = \dfrac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \dfrac{\dfrac{n_1 \gamma_1}{\gamma_1 - 1} + \dfrac{n_2 \gamma_2}{\gamma_2 - 1}}{\dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}}$

$(n_1 + n_2) C_V = n_1 C_{V_1} + n_2 C_{V_2}$

$(n_1 + n_2)\dfrac{R}{\gamma - 1} = \dfrac{n_1 R}{\gamma_1 - 1} + \dfrac{n_2 R}{\gamma_2 - 1}$

7. **Ans. (A,B,C,D)**

8. **Ans. (A,C)**

Sol. $F = \dfrac{dV}{dx}$

$\dfrac{dV}{dx} = 0$ at B,C,D

$\therefore$ F = 0 at B,C,D

so a = 0 at B,C,D

speed is maximum at B as potential energy at B is maximum

9. **Ans. (A,B,C,D)**

Sol. For reflection from densor medium, the reflected wave experience a phase difference of π & for refection from rarer medium, there is no phase difference.

10. **Ans. (A,B,D)**

Sol. For same heat flow rate,

$\dfrac{d\theta}{dt}\bigg|_A = \dfrac{d\theta}{dt}\bigg|_B$

$\Rightarrow \dfrac{k_A A_A (100 - 0)}{\ell_A} = \dfrac{k_B A_B (100 - 0)}{\ell_B}$

In figure (ii), both rods are in series

$\therefore \quad R_{eq} = R_1 + R_2$

SECTION-II

1. **Ans.(A)-(Q, T); (B)-(Q, R);(C)-(P, S); (D)-(P)**

Sol. Rate of heat loss $= 80 \times 10.8 = 54 \times 16$ cal/sec.
(A) r = 1.6
$\Rightarrow$ rate of heat supplies by forming steam to water at 0° $= 1.6 \times 640 > 54 \times 16$
$\therefore$ additional ice will melt
(B) Rate of heat loss
$= 54 \times 16 = 64 \times 13.5$ cal/sec.
r = 1.35
= rate of heat supplied for converting steam to water at 0°C $= 1.35 \times 640 = 13.5 \times 64$.
no additional ice will melt or water will fuse.
(C) Rate of heat loss $= 54 \times 16 = 72 \times 12$ cal/sec.
Rate of heat supplied by converting steam to ice at 0°C $= 1.20 \times 720 = 12 \times 72$ cal/sec
no additional ice will melt or water will fuse.
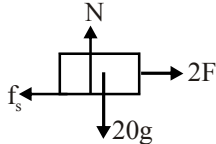(D) Additional water will fuse to ice.

2. **Ans. (A)-(P,R,S,T); (B)-(P,R,S,T); (C)-(Q,R,T); (D)-(Q,T)**

Sol. From phase diagram, in case 'C' & 'D' particle are able to reach at heightest point, while in 'a' & 'b' they don't.

SECTION-IV

1. Ans. 6

Sol. $t_{max} = \mu N = 0.4 \times 20g = 80$

F = 40 t

Sliding starts when

$F = f_{max}$

80t = 80

$\therefore$ t = 1s

$\int 2F\,dt - \int f_k\,dt = mv\ 0$

$\Rightarrow \int\limits_1^3 80t\,dt - 80\int\limits_1^3 dt = 20\ V$

$\therefore$ v = 8m/s

$P = \vec{F} \cdot \vec{v} = 40t \times 2v = 40 \times 3 \times 16 = 1920$

2. Ans. 5

Sol. Since 'B' completes the circle $\Rightarrow v_B$ at lowest point

$= \sqrt{5gR}$

From work energy theorem for rod,

$Mg\dfrac{\ell}{2} = \dfrac{1}{2} \times \dfrac{M\ell^2}{3}\omega^2$

$\Rightarrow \omega = \sqrt{\dfrac{3g}{\ell}}$

For angular momentum conservation before & after collision,

$\dfrac{M\ell^2}{3}.\omega = mv_B\ell$

$\Rightarrow \left(\dfrac{M}{m}\right) = \dfrac{3v_B}{\ell\omega} = \dfrac{3}{\ell}\dfrac{\sqrt{5gR}}{\sqrt{\dfrac{3g}{\ell}}} = N$

$\therefore\ N^2 = \dfrac{9 \times 5gR}{\ell^2 \cdot \dfrac{3g}{\ell}}$ since $\ell = R$(given)

$\therefore\ N^2 = 15$

$\therefore\ \dfrac{N^2}{3} = 5$

3. Ans. 4

Sol. From Bernoulli's equation,

$P_0 + \rho g\,(h_1 - h_2) = \dfrac{1}{2}\rho v^2 + P_0$

[Since cross section Area of beaker >> cross section area of faucet)

$\therefore\ v = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 10 \times \dfrac{8}{100}} = \dfrac{4}{\sqrt{10}}$

4. Ans. 3

Sol. For floating

mg = Buoyant force

Since mass doesnot change with temperature

$F_{B_1} = F_{B_2} \Rightarrow \rho_{w_1}V_1g = \rho_{w_2}V_2g$

$\rho_{w_1}(0.99\ V_0) = \left(\dfrac{\rho_{w_1}}{1 + \gamma_w\Delta T}\right)\left(V_0(1 + 3\alpha\Delta T)\right)$

$\Rightarrow \left(1 - \dfrac{1}{100}\right) = \left(\dfrac{1 + 3\alpha\Delta T}{1 + \gamma_w\Delta T}\right)$

By Binomial approx.

$1 - \dfrac{1}{100} = (1 + (3\alpha - \gamma_w)\Delta T)$

$\dfrac{1}{100} = (\gamma_w - 3\alpha)\Delta T$

$\Rightarrow \Delta T = \dfrac{1}{100(\gamma_w - 3\alpha)} = 40°C$

$\Rightarrow T_f = 60°C$

5. **Ans. 7**

Sol. We have $T_1 = 2\pi\sqrt{\dfrac{m}{k}}$

or

$T_1^2 = 4\pi^2\dfrac{m}{k}$ ... (1)

After more weights Δm are added, we have

$T_2 = 2\pi\sqrt{\dfrac{m+\Delta m}{k}}$, or $T_2^2 = 4\pi^2\dfrac{m+\Delta m}{k}$ ... (2)

By subtracting Eq. (1) from Eq. (2), we get

$T_2^2 - T_1^2 = 4\pi^2\dfrac{\Delta \ell}{g}$, or $\Delta \ell = \dfrac{g}{4\pi^2}\left(T_2^2 - T_1^2\right)$.

Upon inserting the numerical data we obtain $\Delta \ell = 1.75$ cm.

6. **Ans. 1**

Sol. For the semicircular plate of radius ℓ, the centre of mass lies at a distance of $\dfrac{4\ell}{3\pi}$ from the centre.

Taking σ to be the mass per unit area, the position of centre of mass of the remaining piece of the square would be at a distance of $\dfrac{\ell(3\pi-4)}{3(8-\pi)}$ from the centre of the original square plate. Now, taking the centre of the original square to the origin, the centre of mass of the new structure can be determined. This turns out to be at a distance of $\dfrac{\ell}{3}$ to the right of the origin.

7. **Ans. 5**

Sol.

S   M

52 cm   50 cm
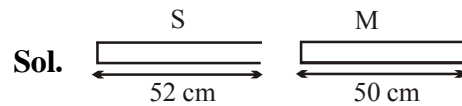
$v = 325$ m/s

$v_0 = \dfrac{V}{4L}$ ∴ $v_{10} = \dfrac{325}{4\times 0.52}$

$v_{20} = \dfrac{325}{4\times 0.50}$

$v_{10} - v_{20} = \dfrac{325}{4}\left(1 - \dfrac{1}{0.264}\right)$

$= \dfrac{325}{4}\left(\dfrac{0.004}{0.260\times 0.264}\right)$

$= \dfrac{0.325}{0.260\times 0.264} = 6.25$ Hz

8. **Ans. 4**
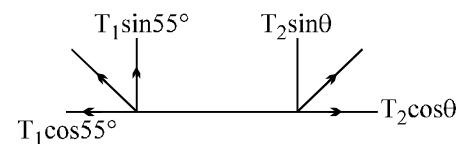
Sol. $T_1 = 8\times 10^8\times 10^{-6} = 800$ N

$T_2 = 3\times 10^8\times 2\times 10^{-6} = 600$ N

$800\times\dfrac{3}{5} = 600\times\cos\theta$

$\Rightarrow \cos\theta = 4/5 \Rightarrow \theta = 37°$

$T_1\sin 55°$ $T_2\sin\theta$

$T_1\cos 55°$ $T_2\cos\theta$

$\Rightarrow 800\sin 53° + 600\sin 37° = F$

$\Rightarrow 800\times\dfrac{4}{5} + 600\times\dfrac{3}{5} = F$
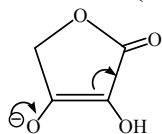
$\Rightarrow F = 1000$ N

PART-2 : CHEMISTRY　　　　　　　　　　　　　　　　　　　　　**ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	B,C	B,D	D	B,D	D	D	A,C,D	A,D

SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		Q,S	P,R,T	Q,R,T	Q,S		Q,S	Q,S,T	R,S	P,S,T	

SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	1	3	4	1	6	1	4	7		

SOLUTION

SECTION–I

1. **Ans. (B)**

2. **Ans. (B)**

3. **Ans. (B, C)**

4. **Ans. (B, D)**

5. **Ans. (D)**

6. **Ans. (B, D)**

7. **Ans. (D)**

DOU = 3 (So, option A & C can not be answer)

(Resonance stabilisation by extended conjugation)

8. **Ans. (D)**
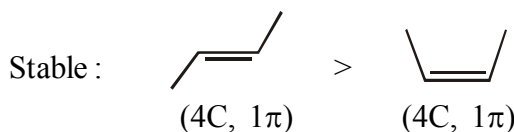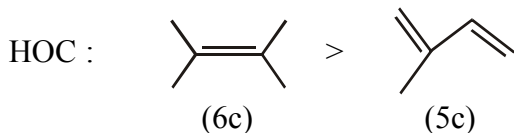
In option (D) both compound has same number of carbon & π-bond. So, stability of compound is deciding factor for heat of hydrogenation as well as for heat of combustion.

9. **Ans. (A,C,D)**

Acid-base reaction favours in formation of weak acid & weak base. Acidic strength order is :

$CH_3COOH > Ph–OH > H_2O > HC≡CH$

10. **Ans. (A,D)**

In option (B) & (C), given pairs are not two different compounds they represent same ion whose resonance energy is fixed.

SECTION–II

1. Ans (A)-(Q,S); (B)-(P,R,T); (C)-(Q, R, T); (D)-(Q, S)

2. Ans (A) - (Q,S); (B) - (Q,S,T); (C) - (R,S); (D) - (P,S,T)

SECTION–IV

1. **Ans. (1)**

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{\sqrt{2m.KE}} = \frac{\sqrt{h\lambda}}{\sqrt{2mC}}$$

$$= \sqrt{\frac{6\times10^{-3}\times9\times10^{-9}}{2\times9\times10^{-31}\times3\times10^{8}}} = 10^{-10} = 1\text{Å}$$

2. **Ans. (3)**

$$Z = \frac{V_{real}}{V_{ideal}}$$

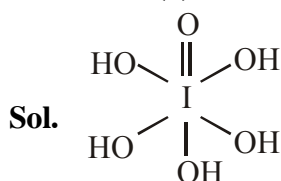$$1.5 = \frac{V_{real}}{2}$$

$$V_{real} = 3$$

3. **Ans. (4)**

4. **Ans. (1)**

5. **Ans. (6)**

Sol.

6. **Ans. (1)**

Sol. sp^3 hybridised P atom = 3

sp^2 hybridised N atom = 3

$$ratio = \frac{3}{3} = 1$$

7. **Ans. (4)**

(iii) , (iv) , (v) , (vii)
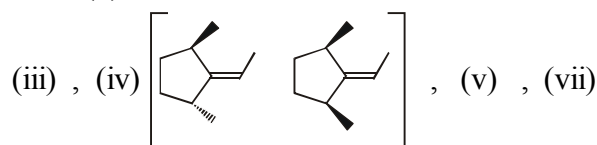
8. **Ans. (7)**

(i) , (ii) , (iii) , (iv) , (vi) , (vii) , (ix)
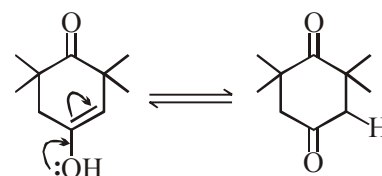
PART-3 : MATHEMATICS　　　　　　　　　　　　　　　　　　**ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,D	A,C	A,B,C,D	A,B	A,B	A,C	B,C	A,C	A,B,C,D	B,C

SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		R	Q	T	T		Q	S,T	Q	T	

SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	7	6	3	7	3	4	8	3		

SOLUTION

SECTION-I

1. **Ans. (A,D)**

$$\left(\sqrt{x} + \frac{1}{2\sqrt[4]{x}}\right)^n$$

$$T_{r+1} = {}^nC_r \cdot \left(\sqrt{x}\right)^{n-r} \left(\frac{1}{2\sqrt[4]{x}}\right)^r$$

$$= \frac{{}^nC_r}{2^r} \cdot x^{\frac{n-r}{2}} \cdot \frac{1}{x^{r/4}} = \frac{{}^nC_r}{2^r} \cdot x^{\frac{2n-3r}{4}}$$

$$\frac{{}^nC_0}{2^0}, \frac{{}^nC_1}{2^1}, \frac{{}^nC_2}{2^2} \text{ are in A.P}$$

$${}^nC_1 = 1 + \frac{{}^nC_2}{4} \Rightarrow n = \frac{1 + n(n-1)}{8}$$

$$n^2 - 9n + 8 = 0$$

$$\Rightarrow n = 1, 8$$

$$\therefore n = 8$$

$$\therefore T_{r+1} = \frac{{}^8C_r}{2^r} \cdot x^{\frac{16-3r}{4}}$$

$$\Rightarrow r = 0, 4, 8$$

2. **Ans. (A,C)**

(A) $S_1 \equiv x^2 + y^2 + 24x - 10y + a = 0$

for real circle, $g^2 + f^2 - c \geq 0$

$$144 + 25 - a \geq 0$$

$$a \leq 169$$

Also $a \geq 0$

∴ Total non-negative integral values of $a = 170$

(B) for no point in common $c_1 c_2 > r_1 + r_2$

$$\& \ c_1 c_2 < |r_1 - r_2|$$

$$c_1 c_2 = 13$$

$$13 > \sqrt{169 - a} + 6$$

$$\Rightarrow \ 169 - a < 49$$

$$a > 120 \ \& \ a \leq 169$$

So in this condition we have 49 integral values of a

But from $c_1 c_2 < |r_1 - r_2|$,

we will get additional values of a.

for orthogonal cut

$$2.12.0 + 2(-5).0 = -36 + a \Rightarrow a = 36$$

If $a = 0$, $c_1 c_2 = 13$ & $r_1 + r_2 = 19$

$$c_1 c_2 < r_1 + r_2$$

No. of common tangent = 2

3. **Ans. (A,B,C,D)**

$$\sin x + \sin y = \frac{96}{65}$$

$$\Rightarrow \ 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{96}{65} \quad \text{.....(1)}$$

Also $\cos x + \cos y = \frac{72}{65}$

$$\Rightarrow \ 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{72}{65} \quad \text{......(2)}$$

(1) ÷ (2), we get

$$\tan\left(\frac{x+y}{2}\right) = \frac{96}{72} = \frac{8}{6} = \frac{4}{3}$$

Now, $\sin(x+y) = \dfrac{2 \cdot \dfrac{4}{3}}{1 + \dfrac{16}{9}} = \dfrac{24}{25}$

$$\cos(x+y) = -\frac{7}{25}$$

Square and add (1) & (2)

$$\cos^2\left(\frac{x-y}{2}\right) \cdot 4 = \frac{8^2(144+81)}{65^2}$$

$$\cos^2\left(\frac{x-y}{2}\right) = \frac{16 \cdot 3^2 \cdot 5^2}{13^2 \cdot 5^2}$$

$$\therefore \ \cos\left(\frac{x-y}{2}\right) = \pm\frac{12}{13}$$

4. **Ans. (A,B)**

$$S = \sum_{k=0}^{2014} {}^{2014}C_k \cdot k = 2014 \cdot 2^{2013}$$

$$= 1007 \cdot 2^{2014} = 19 \times 53 \times 2^{2014}$$

& $E = 53 \times 2^{2014}$

∴ Highest exponent of $2 = 2014$

Number of divisors of $E = 2015 \times 2 = 4030$

5. **Ans. (A,B)**

$$\frac{1}{x-1} - \frac{4}{x-4} - \frac{5}{x-5} + \frac{8}{x-8} = \frac{6x^2 - 27x}{40}$$

Now,

$$\left(\frac{1}{x-1}+1\right) - \left(\frac{4}{x-4}+1\right) - \left(\frac{5}{x-5}+1\right)$$

$$+ \left(\frac{8}{x-8}+1\right) = \frac{6x^2 - 27x}{40}$$

$x = 0$

Clubbing 1ˢᵗ & last & 2ⁿᵈ & 3ʳᵈ, we get

$$\frac{2x-9}{(x-1)(x-8)} - \frac{2x-9}{(x-4)(x-5)} = \frac{3}{40}(2x-9)$$

$$x = \frac{9}{2} \Rightarrow \frac{1}{(x-1)(x-8)} - \frac{1}{(x-4)(x-5)} = \frac{3}{40}$$

Solving, we get $x = 9$

Now, we can verify the options.

6. **Ans. (A,C)**

Put $2\sqrt{2}\,|\sin^3 x| = a, |\tan^3 x| = b, |\cot^3 x| = c$

we get $\dfrac{a}{b+c} + \dfrac{b}{c+a} + \dfrac{c}{a+b} = \dfrac{3}{2}$

$$\Rightarrow \frac{a+b+c}{b+c} + \frac{b+c+a}{c+a} + \frac{c+a+b}{a+b} = \frac{9}{2}$$

$$(a+b+c)\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right) = \frac{9}{2}$$

Using $AM \geq HM$,

$$\frac{(a+b)+(b+c)+(c+a)}{3} \geq \frac{3}{\dfrac{1}{a+b} + \dfrac{1}{b+c} + \dfrac{1}{c+a}}$$

$$(a+b+c)\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right) \geq \frac{9}{2}$$

$a = b = c$

$2\sqrt{2}\,|\sin^3 x| = |\tan^3 x| = |\cot^3 x|$

∴ $x = 2n\pi \pm \dfrac{\pi}{4}$ ($n \in I$)

∴ Number of solution in $[0, 4\pi]$ is 8

(C) $n(A) = 6$; $n(B) = 10$

∴ Number of functions $= 10^6$

(D) Sum of 4 (+)ve solutions

$$\frac{\pi}{4} + \frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4} = 4\pi$$

7. **Ans. (B,C)**

Rationalise

$$f(x) = \frac{1}{2}\left[(x+1)^{1/3} - (x-1)^{1/3}\right]$$

$$f(1) = \frac{1}{2}\left[2^{1/3} - 0^{1/3}\right]$$

$$f(3) = \frac{1}{2}\left[4^{1/3} - 2^{1/3}\right]$$

$$f(5) = \frac{1}{2}\left[6^{1/3} - 4^{1/3}\right]$$

$$f(999) = \frac{1}{2}\left[(1000)^{1/3} - (999)^{1/3}\right]$$

∴ $E = \dfrac{1}{2} \times 10 = 5$

Now, for (c) part

$T_{r+1} \gtrless T_r$

$${}^7C_r \cdot \left(\frac{1}{3}\right)^r \gtrless {}^7C_{r-1} \cdot \left(\frac{1}{3}\right)^{r-1}$$

$$\frac{1}{r} \cdot \frac{1}{3} \gtrless \frac{1}{8-r}$$

$8 - r \gtrless 3r$

$2 \gtrless r$

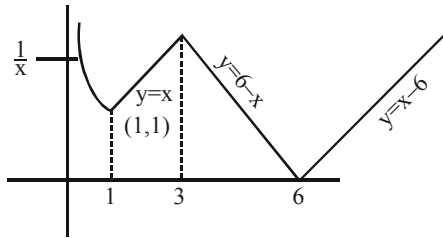∴ Greatest terms are T_2 & T_3.

8. **Ans. (A,C)**

$$f(x) = \begin{cases} \dfrac{1}{x} & 0 < x \leq 1 \\ x & 1 < x \leq 3 \\ 6-x & 3 < x \leq 6 \\ x-6 & x > 6 \end{cases}$$

Plotting $f(x)$ we get

Clearly, if a,b,c,d are positive distinct numbers such that $f(a) = f(b) = f(c) = f(d)$, then
$y = t \in (1,3)$ must intersect
the graph of $y = f(x)$ at four points

$\therefore \ a = \dfrac{1}{t}, b = t, c = 6 - t, d = t + 6$

$\therefore \ abcd = 36 - t^2$

$t^2 \in (1,9)$

$\therefore$ Range of abcd is (27,35)

$\therefore$ Number of integral values in the range 7.

9. **Ans. (A,B,C,D)**

(A) $f(x)$ is odd $\therefore a + 2 = -b + 7 \Rightarrow a + b = 5$

(B) Clearly $x^2 \in (0,1) \Rightarrow [x^2] = 0$

$\Rightarrow f(x) = \dfrac{1}{\sqrt{1 - x^2}} \Rightarrow$ even

(C) for sum of coefficients; put $x = 1 \ \therefore f(0) = 1$

10. **Ans. (B,C)**

$\angle D + \angle E + \angle F = \dfrac{15\pi + \pi}{32} = \dfrac{\pi}{2}$

$\therefore \ \tan(D + E + F) = \dfrac{s_1 - s_3}{1 - s_2}$

$\therefore \ s_2 = 1$

$\therefore \ \tan D \tan E + \tan E \tan F + \tan F \tan D = 1$

$\&\ \cot D + \cot E + \cot F = \cot D \cot E \cot F$

SECTION – II

1. **Ans. (A)→(R); (B)→(Q); (C)→(T); (D)→(T)**

(A) $\displaystyle\sum_{n=0}^{\infty} 2^{-n+(-1)^n} = 2 + 2^{-2} + 2^{-1} + 2^{-4} + \ldots\infty$

$= \left(2 + \dfrac{1}{2} + \dfrac{1}{2^3} + \ldots\infty\right) + \left(\dfrac{1}{2^2} + \dfrac{1}{2^4} + \dfrac{1}{2^6} + \ldots\infty\right)$

$= \dfrac{2}{1 - \dfrac{1}{2^2}} + \dfrac{\dfrac{1}{2^2}}{1 - \dfrac{1}{2^2}} = \dfrac{2.4}{3} + \dfrac{1}{3} = 3$

(B) Put $\sin x = t$, we get

$16t^3 = 14 + \sqrt[3]{t + 7}$

$\therefore \ f(t) = f^{-1}(t)$

$\sin x = 1 \Rightarrow x = 2n\pi + \dfrac{\pi}{2}(n \in I)$

$\therefore$ 2 solutions.

(C) $\tan^{-1}(2\sin x) = \cot^{-1}(\cos x)$

$\tan^{-1}(2\sin x) + \tan^{-1}(\cos x) = \dfrac{\pi}{2}$

$\therefore \ \sin x > 0, \cos x > 0 \ \&\ 2\sin x \cos x = 1$

$\therefore \ x = 2n\pi + \dfrac{\pi}{4}$

$\Rightarrow$ 5 solutions

(D) Sum of roots

$= \dfrac{a^2 - a + 4}{a - 1} = a + \dfrac{4}{a - 1} = a - 1 + \dfrac{4}{a - 1} + 1 \geq 5$

Minimum value is 5

2. **Ans.(A)→(Q);(B)→(S,T);(C)→(Q); (D)→(T)**

(A) $\displaystyle\sum_{n=0}^{\infty} \tan^{-1}\left(\dfrac{2}{\left(\sqrt{n+2} + \sqrt{n}\right)\left(1 + \sqrt{n+2}\sqrt{n}\right)}\right)$

$= \displaystyle\sum_{n=0}^{\infty} \tan^{-1}\left(\dfrac{\sqrt{n+2} - \sqrt{n}}{1 + \sqrt{n+2}\sqrt{n}}\right)$

$= \displaystyle\sum_{n=0}^{\infty}\left(\tan^{-1}\sqrt{n+2} - \tan^{-1}\sqrt{n}\right)$

$= \dfrac{3\pi}{4} \qquad \therefore \ \left[\dfrac{3\pi}{4}\right] = 2$

(B) $c > |a - b| \Rightarrow c^2 > a^2 + b^2 - 2ab$
Similarly, $a^2 > b^2 + c^2 - 2bc$
$\qquad\qquad b^2 > a^2 + c^2 - 2ac$
$\Rightarrow \ a^2 + b^2 + c^2 < 2(ab + bc + ca)$

$2\dfrac{(a^2 + b^2 + c^2)}{ab + bc + ca} < 4$

(C) $a^2(b + c) = b^2(a + c)$
$a^2b - b^2a + a^2c - b^2c = 0$
$ab(a - b) + c(a - b)(a + b) = 0$
$a \neq b$
$\therefore \ ab + ac + bc = 0$
Multiply both sides by $(a - c)$,
we get $(a - c)(ab + ac + bc) = 0$
$a^2b + a^2c - ac^2 - bc^2 = 0$
$a^2(b + c) = c^2(a + b) = 2$

(D) $P^2 + Q^2 + R^2 + S^2$

$\quad = \sin^2A\sin^2B + \sin^2C\cos^2A + \sin^2A\cos^2B$
$\quad\quad + \cos^2A\cos^2C$

$\quad = \sin^2A + \cos^2A = 1$

SECTION-IV

1. **Ans. 7**

$\sin(x - y)\sin(x + y) = \sin^2x - \sin^2y$

$\therefore E = \sin^2(\sin^{-1}(0.5)) - \sin^2(\sin^{-1}(0.4)) = 0.09$

2. **Ans. 6**

Interpret the problem geometrically consider n right triangle joined at their vertices with bases $a_1, a_2, a_3, \ldots, a_n$ and heights $1, 3, \ldots, 2n-1$. The sum of their hypotenusses is the value of S_n. The minimum value of S_n, then is the length of the straight line. Connecting the bottom vertex of first right triangle & the top vertex of the last right triangle, so

$$S_n \geq \sqrt{\left(\sum_{k=1}^{n}(2k-1)^2\right) + \left(\sum_{k=1}^{n}a_k\right)^2}$$

$\therefore S_n \geq \sqrt{17^2 + n^4}$

Now $17^2 + n^4 = m^2$ ($m \in I$)

$(m - n^2)(m + n) = 289.1$

$\therefore n^2 = 144$ $\therefore n = 12$

3. **Ans. 3**
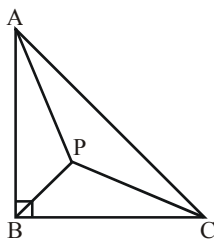
using pythogoreas theorem

$AB^2 + BC^2 = CA^2$

$10^2 + 6^2 + 10.6 + 6^2 + PC^2 + 6PC$

$= 10^2 + PC^2 + 10PC$

$\Rightarrow 4PC = 132 \Rightarrow PC = 33$

4. **Ans. 7**

$$\sum_{n=1}^{2015}(-1)^n\left(\frac{n}{(n-1)!} + \frac{n+1}{n!}\right)$$

$$= \left(-1 - \frac{2}{1!}\right) + \left(\frac{2}{1!} + \frac{3}{2!}\right) - \left(\frac{3}{2!} + \frac{4}{3!}\right) + \ldots$$

$$\quad\quad - \left(\frac{2015}{(2014)!} + \frac{2016}{(2015)!}\right)$$

$$= -1 - \frac{2016}{(2015)!} \quad\quad \therefore \ a+b+c = 4032$$

5. **Ans. 3**

Order at A,B,C is one

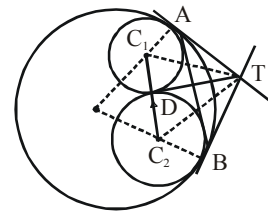$^6C_3 \cdot 3! = \dfrac{6!}{3!}$ $\quad \therefore n = 3$

6. **Ans. 4**

Let $x = k\sin\theta$ & $y = k\cos\theta$

$\therefore \dfrac{\cos^4\theta}{\sin^4\theta} + \dfrac{\sin^4\theta}{\cos^4\theta} = \dfrac{194\sin\theta\cos\theta}{\sin\theta\cos\theta(\cos^2\theta + \sin^2\theta)} = 194$

$\therefore t = \dfrac{x}{y} + \dfrac{y}{x}$, then $\left(t^2 - 2\right)^2 - 2 = 194$

$\therefore t = 4$

7. **Ans. 8**

Let D be the point of tangency of C_1 and C_2. T will be radical center of 3 circles.

$\therefore TD = 4$

Now, $\tan\left(\dfrac{\angle ATD}{2}\right) = \dfrac{1}{2}$ & $\tan\left(\dfrac{\angle BTD}{2}\right) = \dfrac{3}{4}$

$\therefore$ Radius of $C_3 = TA\tan\left(\dfrac{\angle ATB}{2}\right) = 8$

8. **Ans. 3**

$^nC_m \cdot {}^mC_p = \dfrac{n!}{m!(n-m)!} \times \dfrac{m!}{(m-p)!p!} \times \dfrac{(n-p)!}{(n-p)!}$

$\quad\quad = {}^nC_p \cdot {}^{n-p}C_{m-p}$

$\therefore \sum_{p=1}^{n}\sum_{m=p}^{n} {}^nC_p {}^{n-p}C_{m-p} = \sum_{p=1}^{n} {}^nC_p\left({}^{n-p}C_0 + {}^{n-p}C_1 + \ldots + {}^{n-p}C_{n-p}\right)$

$= \sum_{p=1}^{n} {}^nC_p \cdot 2^{n-p} = \sum_{p=0}^{n} {}^nC_p 2^{n-p} - 2^n$

$= 3^n - 2^n = 19$

$\therefore n = 3$

NURTURE TEST SERIES / JOINT PACKAGE COURSE
TARGET : JEE (ADVANCED) 2017

Test Type : **ALL INDIA OPEN TEST (**MAJOR**)**   Test Pattern : JEE-Advanced

TEST # 02   TEST DATE : 07 - 02 - 2016

PAPER-2

PART-1 : PHYSICS   **ANSWER KEY**

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C,D	B,D	A,B	A,D	B,C	B,C	A,D	B,D	A,B	B,D
	Q.	11	12								
	A.	A,B,D	A,C,D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	4	6	2	2	2	4	3	2		

SOLUTION

SECTION-I

1.   Ans. (C,D)

Sol. $\dfrac{R}{L} = \dfrac{\rho}{\rho_w} \rightarrow$ same

$B = \rho V g_{eff}$

$p = p_0 + h\rho g_{eff}$

2.   Ans. (B,D)

Sol. (A) $\mu mg = m\omega^2 r$

$= \alpha^2 r t^2$

$t = \sqrt{\dfrac{\mu g}{\alpha^2 r}}$

(C) $N = mr\alpha$

$\mu N = m\omega^2 r$

$\mu mr\alpha = m\omega^2 r$

$= \alpha^2 t^2$

$t = \sqrt{\dfrac{\mu}{\alpha}}$

3.   Ans. (A,B)

Sol. $PV^\gamma = C \Rightarrow P \times \left(R^3\right)^{\frac{5}{3}} = C$

$TV^{2/3} = $ constant

$T \times R^{3 \times 2/3} = $ constant

4.   Ans. (A,D)

Sol. (A) $\dfrac{4\pi R_S^2 \sigma T_S^4}{4\pi d^2} \times \pi R_p^4 = 4\pi R_p^2 \sigma T_p^4$

$T_p = T_s \sqrt{\dfrac{R_s}{2d}}$

(C) $\dfrac{4\pi R_p^2 \times \sigma T_p^4}{4\pi d^2} \times \pi R_S^2 = \dfrac{dE}{dt}$

$\dfrac{\pi R_S^2 \sigma T_S^4}{d^2} \times \dfrac{R_S^2}{4d^2}$

5.   Ans. (B,C)

Sol. (A) $2\pi a = \dfrac{\lambda}{4} \Rightarrow \lambda = \dfrac{340}{85} = 4m$

$\Rightarrow 2\pi a = 1$

$a = \dfrac{1}{2\pi} m$

(B) closed organ pipe

(C) Sealed end is PAN

$2\pi a = \left(2\dfrac{1}{2}\right)\dfrac{\lambda}{2}$

$$2\pi a = \frac{5\lambda}{4}$$

$$\lambda = \frac{8\pi a}{5}$$

$$\theta = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$$

$$\Rightarrow \theta = \frac{4\pi}{5}, \frac{8\pi}{5}$$

6. **Ans. (B,C)**

Sol. $Dx = 0.3 = n\lambda$

$$= n \times \frac{330}{f}$$

$$f = \frac{3300n}{0.3}$$
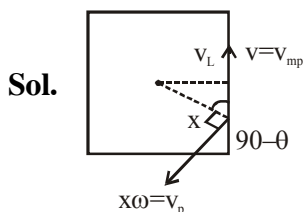
$$f_{max} = 20000$$

$$n = \frac{20000}{1100}$$

$$\Delta x = 0.3 = \left(n + \frac{1}{2}\right)\lambda$$

$$f = 1100\left(n + \frac{1}{2}\right)$$

$$n + \frac{1}{2} = 18.22$$

$$\Rightarrow n = 0 \text{ to } 17$$

7. **Ans. (A,D)**

Sol.

$$v_m = v_{mp} + v_p$$

$$= \sqrt{v^2 + x^2\omega^2 + 2v\omega \cos(90 + \theta)}$$

$$= \sqrt{v^2 + x^2\omega^2 - 2vx\omega \sin\theta}$$

$$L_i = 0 = L_f$$

$$= -\frac{m}{12}\left(a^2 + a^2\right)\omega + m\left(v - x\omega \sin\theta\right)\frac{\alpha}{2}$$

$$\Rightarrow \frac{a^2}{6}\omega + \frac{a^2}{4}\omega = \frac{va}{2} \Rightarrow \omega = \frac{6v}{5a}$$

8. **Ans. (B,D)**

Sol.
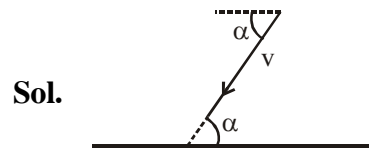
Observer moving away from source.

$$v = \frac{C - v\sin\alpha}{C}v_0$$
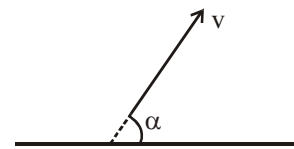
Observer moving towards source.

$$v_1 = \frac{C - v\sin\alpha}{C}v_0$$

9. **Ans. (A,B)**

10. **Ans. (B,D)**

11. **Ans. (A,B,D)**

Sol. $p_0 + h_1\rho_1 g = p_0 + h_2\rho_1 g + h_3\rho_3 g$

12. **Ans. (A,C,D)**

Sol. $p_{in} = 10^5 + 5 \times 0.8 \times 10^3 \times 10 + 10 \times 10^3 \times 10$

$$= 10^5 + 4 \times 10^4 + 10^5$$

$$= 2.4 \times 10^5$$

$p_{out} = 10^5 + 0.8 \times 15 \times 10^3 \times 10 = 2.2 \times 10^5$

$$p_{out} + \frac{1}{2}\rho v^2 = p_{in}$$

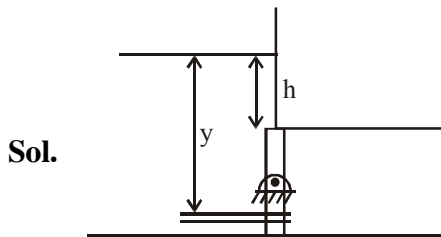$$\frac{1}{2} \times 10^3 \times v^2 = 0.2 \times 10^5$$

$v = \sqrt{40}$ m/s

$\sqrt{40} \times 10^{-6} = 100 \times 10^{-4} v_s$

$v_s = \sqrt{40} \times 10^{-4}$ m/s

$= \sqrt{\dfrac{40}{10}}$ mm/s $= \sqrt{\dfrac{40}{100}}$ mm/s $= \sqrt{\dfrac{2}{5}}$ mm/s

SECTION-IV

1. **Ans. 4**

Sol.

$\displaystyle\int\limits_{h}^{h+2} \rho g y \, dy \times 6 \times \big(y-(h+1)\big)$

$= 10^4 \times 6 \left[\dfrac{y^3}{3} - (h+1)\dfrac{y^2}{2} \Big|_{h}^{h+2} \right]$

$= 6 \times 10^4 \left[\dfrac{1}{3}\big((h+2)^3 - h^3\big) - \dfrac{(h+1)}{2}\big((h+2)^2 - h^2\big) \right]$

$= 6 \times 10^4 \left[\dfrac{1}{3}\left[6h^2 + \dfrac{6h}{12} + 8 - \dfrac{(h+1)}{2}\big[4^2 h + 4^2\big] \right] \right]$

$= 6 \times 10^4 \left[\dfrac{1}{3}\big(6h^2 + 12h + 8\big) - 2\big(h^2 + 2h + 1\big) \right]$

$= 6 \times 10^4 \left[\dfrac{8}{3} - 2 \right] = 4 \times 10^4 \, \text{Nm}$

2. **Ans. 6**

Sol.

$p = p_0 + \dfrac{25}{r} + h\rho g$
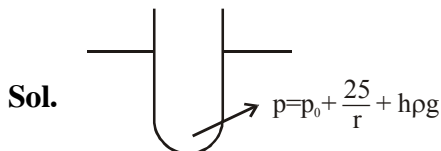
$\Delta p = \dfrac{25}{r} + h\rho g$

$= \dfrac{2 \times 7 \times 10^{-3}}{0.35 \times 10^{-3}} + 2 \times 10^{-2} \times 10^4 = 240$

$\dfrac{\Delta p}{40} = 6$

3. **Ans. 2**

Sol. $t_1 = \dfrac{2u\sin\theta}{g} \Rightarrow u = \dfrac{10}{2\sin\theta}$

$-h = d\tan\theta - \dfrac{1}{2} g \dfrac{d^2}{u^2 \cos^2\theta}$

$-10 = 10\tan\theta - \dfrac{1}{2} \times \dfrac{10 \times 10}{25} \tan^2\theta$

$2\tan^2\theta - \tan\theta - 1 = 0$

$\tan\theta = \dfrac{1 \pm \sqrt{1+8}}{4} = \dfrac{1+3}{4}, \dfrac{1-3}{4} = 1, -\dfrac{1}{2}$

4. **Ans. 2**

Sol $T_a = 2\pi \sqrt{\dfrac{2mR^2}{mgR}}$

$T_b = 2\pi \sqrt{\dfrac{\frac{3}{2}mR^2}{mgR}} = \sqrt{3}$

$\Rightarrow 2\pi \sqrt{\dfrac{mR^2}{2mgR}} = 1$

$T_a = 2$ sec

5. **Ans. 2**

Sol. $W = \Delta KE + \Delta H_{friction}$

as seen from belt frame

$\Delta H_{friction} = \dfrac{1}{2} m\big(0^2 - v^2\big)$

$\Rightarrow W = mv^2$

$= 0.5 \times 2^2 = 2\,\text{J}$

6. **Ans. 4**

Sol. $mg - N = ma = m\dfrac{\ell}{2}\alpha$

$mg\dfrac{\ell}{2} = \dfrac{m\ell^2}{3}\alpha$

$\alpha = \dfrac{3g}{2\ell}$

$mg - \dfrac{m\ell}{2} \times \dfrac{3g}{2\ell} = N$

$N = \dfrac{mg}{4}$

7. **Ans. 3**

Sol. $mv_0 = 2mv_2 + mv_1$

$e = 1 = \dfrac{v_2 - v_1}{v_0}$

$v_2 - v_1 = v_0$

$2v_2 + v_1 = v_0$

$3v_2 = 2v_0$

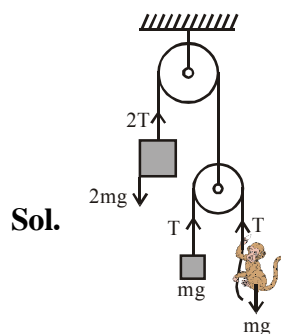$v_2 = \dfrac{2v_0}{3}$

$\dfrac{1}{2} 2mv_2^2 = \dfrac{1}{2} kd^2$

$v_2 = \sqrt{\dfrac{k}{2m}} d$

$v_0 = \dfrac{3}{2} d\sqrt{\dfrac{k}{2m}}$

$= \dfrac{3}{2} \times 0.1 \sqrt{\dfrac{400}{0.5 \times 2}}$

$v_0 = 3$

8. **Ans. 2**

Sol.

$T - mg = ma_b = ma_m$

$2T - 2mg = 2ma_B$

$\Rightarrow a_B = a_b = a_m \Rightarrow v_B = v_b = v_m = v$

$v_p = -v$

$v_R - v_b = 2v_p$

$v_R = -2v - v = -3v$

$v_{mR} = v - (-3v) = 4v = 8$

PART-2 : CHEMISTRY   ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	B	A,B,D	A,D	A	A,C,D	B	A,D	C	D
	Q.	11	12								
	A.	C	C								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	1	6	4	9	2	1	3	5		

SOLUTION

SECTION - I

1. **Ans. (C)**

$282 \times \dfrac{33.33}{100} = 96 \Rightarrow 3$ 'S' atoms per molecule.

2. **Ans. (B)**

$\begin{array}{cccc} H_2 & + & Br_2 \rightleftharpoons & 2HBr \\ aM & & aM & - \\ a\text{-}x & & a\text{-}x & 2x \end{array}$
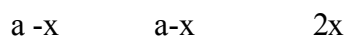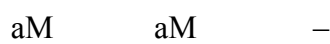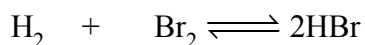
$= \dfrac{25.92/81}{20} = \dfrac{0.32}{20} = 0.016\,M$

$64 = \dfrac{(2x)^2}{(a-x)^2}$

$8a - 0.064 = 0.016$

$8a = 0.08$

$a = 0.01\,M$

$n_{H_2} = 0.01 \times 20 = 0.2\,mol \equiv 0.4\,gm$

$n_{Br_2} = 0.01 \times 20 = 0.2\,mol \equiv 32\,gm$

3. **Ans. (A, B, D)**
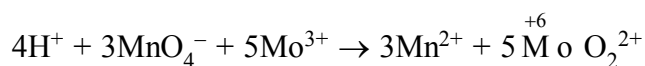
(A) $P \times 1 = 5 \times 0.08 \times 320 \Rightarrow P = 128$ atm

(B) $\left(P + \dfrac{4 \times 5^2}{1^2}\right)(1 - 0) = 5 \times 0.08 \times 320$

$\Rightarrow P = 28$ atm

(C) $\left(P + \dfrac{4 \times 5^2}{12}\right)(1 - 0.04 \times 5) = 128 \Rightarrow P = 60$ atm

(D) $P(1 - 0.04 \times 5) = 128 \Rightarrow P = 160$ atm

4. **Ans.(A, D)**
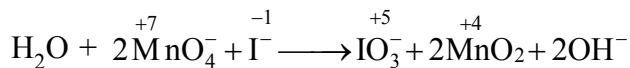
$$4H^+ + 3MnO_4^- + 5Mo^{3+} \rightarrow 3Mn^{2+} + 5\overset{+6}{M}o\,O_2^{2+}$$

20 ml	$n_f = 3$
0.2M	0.5 M
(nf = 5)	10 ml

$$H_2O + 2\overset{+7}{M}nO_4^- + \overset{-1}{I^-} \longrightarrow \overset{+5}{IO_3^-} + 2\overset{+4}{MnO_2} + 2OH^-$$

0.1N

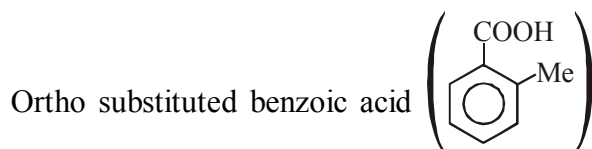| 1 mmol	0.5 mmol
$(n_f = 3)$	$n_f = 6$
	V = 30 ml

5. **Ans. (A)**

6. **Ans. (A,C,D)**

7. **Ans. (B)**
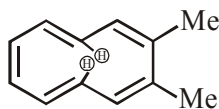
Non-planar, therefore conjugation over complete ring isn't possible.

8. **Ans. (A,D)**

Ortho substituted benzoic acid $\left(\text{} \right)$

is most acidic

$\overset{4}{C}H_3 - \overset{3}{C}H_2 - \overset{2}{C}H_2 - \overset{1}{C}ONH_2$ & $\overset{3}{C}H_3 - \overset{2}{C}H - CH_3$: Chain isomer
$|$
${}_1CONH_2$

9. **Ans.(C)**

$$Fe^{3+}(aq) + 3H_2C_2O_4 \rightleftharpoons [Fe(C_2O_4)_3]^{3-} + 6H^+$$

$$Hg^{2+} + 4SCN^- \rightleftharpoons [Hg(SCN)_4]^{2-}$$
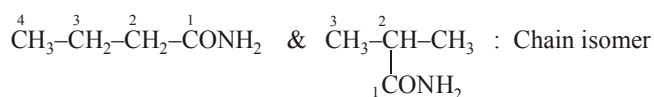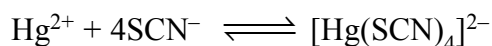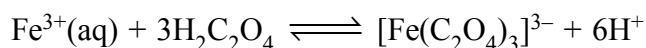
10. **Ans.(D)**

For exothermic equilibrium on increasing temperature Keq decreases.

11. **Ans. (C)**

12. **Ans. (C)**

Phenol, Alcohol , 2°-amine, Ketone, Cyclic amide

SECTION - IV

1. **Ans. (307)**

OMR ANS (1)

$$2C_{10}H_{17}N_3O_6\overset{-2}{S} \longrightarrow C_{20}H_{32}N_6O_{12}\overset{-1}{S} + 2H^+ + 2e^-$$
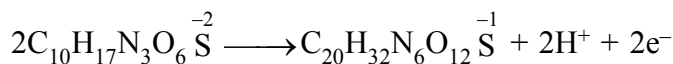
$(n_f = 1)$

2. **Ans. (6)**
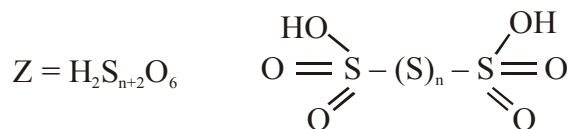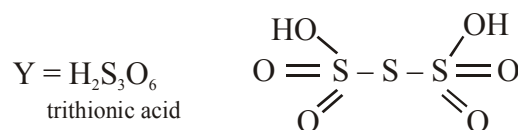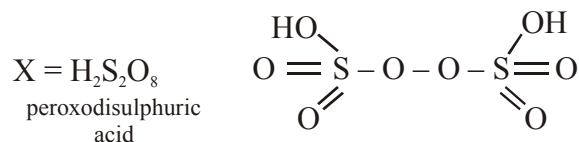
3. **Ans. (4)**

Except 3, 5 & 7 all are correct

4. **Ans. (9)**

$$\frac{6.4}{1.6 \times 10^{-19}} = \frac{1240}{310} \times N \Rightarrow N = 10^{19}.$$

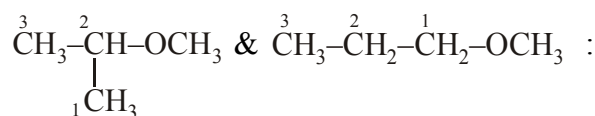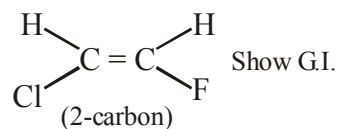Q. E. $= \dfrac{9 \times 10^{17}}{10^{19}} \times 100 = 9\%$

5. **Ans. (2)**

6. **Ans. (1)**

X = $H_2S_2O_8$ peroxodisulphuric acid

Y = $H_2S_3O_6$ trithionic acid

Z = $H_2S_{n+2}O_6$

7. **Ans. (3)**

8. **Ans. (5)**

b , c , e , f , h

 Show G.I.

(2-carbon)

$\overset{3}{C}H_3 - \overset{2}{C}H - OCH_3$ & $\overset{3}{C}H_3 - \overset{2}{C}H_2 - \overset{1}{C}H_2 - OCH_3$:
$|$
${}_1CH_3$

Position isomer

PART-3 : MATHEMATICS

ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	B,C	A,B,C	A,B,C	A,B,C	A,B,C,D	A,B,C,D	A,B,D	A,B	B	A,B
	Q.	11	12								
	A.	A,C	B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	5	7	0	7	2	2	2	3		

SOLUTION

SECTION-I

1. **Ans. (B,C)**

$\cos^2 x = \sin x$

$\cos^4 x = 1 - \cos^2 x$

$\cos^4 x + \cos^2 x - 1 = 0$

$\cos^2 x = \dfrac{-1 + \sqrt{5}}{2}$

$\dfrac{1}{\sin x} + \cos^4 x = \dfrac{1}{\cos^2 x} + \cos^4 x$

$= \dfrac{2}{\sqrt{5} - 1} + \dfrac{3 - \sqrt{5}}{2} = \dfrac{\sqrt{5} + 1}{2} + \dfrac{3 - \sqrt{5}}{2} = 2$

(B) & (C) are correct.

2. **Ans. (A,B,C)**

$f(n) = \displaystyle\sum_{n=1}^{n} \tan^{-1}\left(\dfrac{2 \cdot 3^{n-1}}{1 + 3^n \cdot 3^{n-1}} \right)$

$= \displaystyle\sum_{n=1}^{n} \tan^{-1}(3^n) - \tan^{-1}(3^{n-1})$

$f(n) = \tan^{-1} 3^n - \dfrac{\pi}{4}$

Now check option.

3. **Ans. (A,B,C)**

$f(x) = \cos^{-1}\left(\dfrac{1 - (x+1)^2}{1 + (x+1)^2} \right)$

$x + 1 = \tan\theta \quad \left(-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2} \right)$

$y = \cos^{-1} \cos 2\theta$

$= \begin{cases} -2\tan^{-1}(x+1), \; x < -1 \\ 2\tan^{-1}(x+1), \;\; x \geq -1 \end{cases}$

4. **Ans. (A,B,C)**

5. **Ans. (A,B,C,D)**

$r_n = \dfrac{n}{r^{n-1}}$

$f(r) = 1 + \dfrac{2}{r} + \dfrac{3}{r^2} + \dfrac{4}{r^3} + \ldots\ldots\infty \qquad \ldots\ldots(1)$

$\dfrac{f(r)}{r} = \dfrac{1}{r} + \dfrac{2}{r^2} + \dfrac{3}{r^3} + \ldots\ldots\infty \qquad \ldots\ldots(2)$

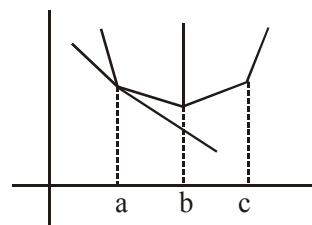$\Rightarrow f(r) = \left(\dfrac{r}{1-r} \right)^2$

Now check options.

6. **Ans. (A,B,C,D)**

$f(x) = \dfrac{(x^2 - x + 1)(x^2 + x + 1)}{(x^2 + x + 1)^2}$

$f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

Now check options.

7. **Ans. (A,B,D)**

By comparing the slopes, condition in problem is satisfied for $(a, b+c-2a)$ to lie on the line. Putting in line we get

$b + c = 2003$

$b_{max} = 1004$

$c_{min} = 1002$

8. **Ans. (A,B)**

Let P denotes total number of ways of selecting even number of red balls and Q denotes total number ob ways of selecting odd number of red balls.

$(2 - 1)(4 - 1)(6 - 1)(8 - 1)(10 - 1) = P - Q$

$(2 + 1)(4 + 1)(6 + 1)(8 + 1)(10 + 1) = P + Q$

Now solve to get P & Q.

Paragraph for Question 9 to 10

$N = 2^a 3^b 5^c 7^d$

$a = \sum_{m=2}^{10} \sum_{n=1}^{\infty} \left[\dfrac{m}{2^n} \right] = 38$

similarly $b = 17$, $c = 7$, $d = 4$

$N = 2^{38} \, 3^{17} \, 5^7 \, 7^4$

obviously $P = 7$

$Q = 13 \times 6 \times 3 \times 2$

$\quad = 2^2 . 3^2 \times 13'$

9. **Ans. (B)**

10. **Ans. (A,B)**

Paragraph for Question 11 to 12

$y = \sqrt[3]{x + \sqrt{x^2 + 1}} + \sqrt[3]{x - \sqrt{x^2 + 1}}$

Image in $y = x$ & then cubing

$x^3 = \left(\sqrt[3]{y + \sqrt{y^2 + 1}} + \sqrt[3]{y - \sqrt{y^2 + 1}} \right)^3$

$x^3 = 2y + 3(-1)x$

$f^{-1}(x) = \dfrac{x^3 + 3x}{2}$

11. **Ans. (A,C)**

12. **Ans. (B)**

SECTION-IV

1. **Ans. 5**

Replace x by $1 - x$ f solve to get $f(x)$

2. **Ans. 7**

$H_1 - H_2 - H_3 - H_4 = \lfloor 4 = 24$

$\boxed{H_1 H_2} - H_3 - H_4 = {}^4C_2 \lfloor 3 = 36$

$\boxed{H_1 H_2} - \boxed{H_3 H_4} = \dfrac{{}^4C_2}{2} \lfloor 2 = 6$

$\boxed{H_1 H_2 H_3} - H_4 = {}^4C_3 \lfloor 2 = 8$

$H_1 H_2 H_3 H_4 = 1 \Rightarrow 75$

3. **Ans. 0**

$s = \sum_{r=1}^{3} \dfrac{\cos(2.3^{r-1}\theta)}{\sin 3^r \theta} \qquad \left(\theta = \dfrac{\pi}{28} \right)$
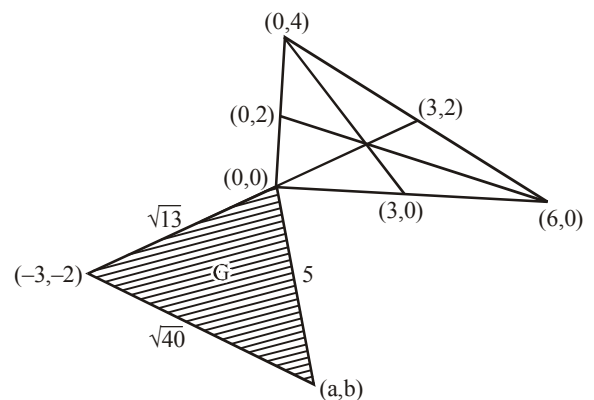
$\Rightarrow \quad s = \sum_{r=1}^{3} \dfrac{\sin(3^{r-1}\theta) \cos(2.3^{r-1}\theta)}{\sin(3^r \theta) \sin(3^{r-1}\theta)}$

$\quad = \dfrac{1}{2} \sum_{r=1}^{3} \dfrac{\sin 3^r \theta - \sin 3^{r-1}\theta}{\sin 3^r \theta \sin 3^{r-1}\theta}$

$S = \sum_{r=1}^{3} \mathrm{cosec}(3^{r-1}\theta) - \mathrm{cosec}(3^r\theta)$

$\quad = \mathrm{cosec}\,\theta - \mathrm{cosec}\,27\theta = 0 \ \{\because \theta + 27\theta = \pi\}$

4. **Ans. 7**

$a^2 + b^2 = 25 \qquad ....(1)$

$(a + 3)^2 + (b + 2)^2 = 40$

$6a + 4b = 2 \qquad ....(2)$

Solving (1) & (2)

possible value of a is 3

$\Rightarrow \quad b = 4$

$\dfrac{G}{S} = \dfrac{3}{4}$

5. **Ans. 2**

$2f(x) + f\left(\dfrac{x+1}{x-1} \right) = x$

$x \to \dfrac{x+1}{x-1}$

$\Rightarrow f(x) + 2f\left(\dfrac{x+1}{x-1} \right) = \dfrac{x+1}{x-1}$

Solving

$3f(x) = 2x - \dfrac{x+1}{x-1}$

$3f(x) + \dfrac{x+1}{x-1} = 2x$

$\Rightarrow \lambda = 2$

6. **Ans. 2**

$y = \sin^2 \alpha (1 - \sin^2 \alpha)^3$

$y = t(1-t)^3 \qquad 0 < t \le 1$

$$\frac{t + \dfrac{3(1-t)}{3}}{4} \ge \sqrt[4]{\frac{t(1-t)^3}{27}}$$

$$\frac{t(1-t)^3}{27} \le \frac{1}{4^4}$$

$$t(1-t)^3 \le \frac{3^3}{256}$$

7. **Ans. 2**

$$\frac{5}{2\sin\theta} = \frac{5}{\sqrt{2}}$$

$\theta = 45°$

$\Rightarrow$ BOA = 90°
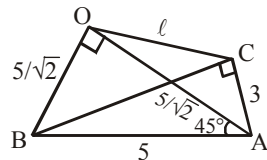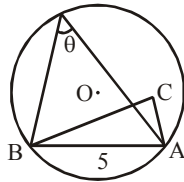
ACO = 135°

$$-\frac{1}{\sqrt{2}} = \frac{9 + \ell^2 - \dfrac{25}{2}}{6\ell}$$

$$-3\sqrt{2}\ell = \ell^2 - \frac{7}{2}$$

$$\ell^2 + 3\sqrt{2}\ell - \frac{7}{2} = 0$$

$$\ell = \frac{-3\sqrt{2} + \sqrt{18+14}}{2} = \frac{1}{\sqrt{2}}$$

8. **Ans. 3**

$s = (1 + x)^{1000} + 2x(1 + x)^{999} + 3x^2(1 + x)^{998} +$

$\qquad\qquad .... + 1001x^{1000}(1 + x)^0$

$$\frac{xs}{(1+x)} = \quad x(1+x)^{999} + 2x^2(1+x)^{998} + ...$$

$$\qquad\qquad .... + 1001x^{1000} + \frac{1001 + x^{1001}}{(1+x)}$$

$$\frac{s.1}{(1+x)} = (1+x)^{1000} + x(1+x)^{999} + ...$$

$$.... + x^{1000}(1+x)^0 - \frac{1001x^{1001}}{(1+x)}$$

$$\frac{s}{(1+x)} = (1+x)^{1000} \left\{ \frac{\left(\dfrac{x}{1+x}\right)^{1001} - 1}{\dfrac{x}{1+x} - 1} \right\} - \frac{1001x^{1001}}{(1+x)}$$

$$\frac{s}{(1+x)} = \frac{x^{1001} - (1+x)^{1001}}{-1} - \frac{1001x^{1001}}{(1+x)}$$

$s = (1 + x)^{1002} - x^{1001}(1 + x) - 1001x^{1001}$

coeff. of $x^{50} = {}^{1002}C_{50}$