ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)
path to success

DISTANCE LEARNING PROGRAMME
(Academic Session : 2015 - 2016)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (ADVANCED) 2016

Test Type : **ALL INDIA OPEN TEST** (MAJOR)   Test Pattern : JEE-Advanced

TEST # 12   **TEST DATE : 15 - 05 - 2016**

PAPER-1

PART-1 : PHYSICS   **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,D	C,D	A,C,D	A,D	A,C,D	B	B	A,B,C,D	B,C	A,C
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		R	R	S	P		Q,S,T	P,R,T	P,R,T	Q,S,T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	0	5	5	2	5	1	0	2		

SOLUTION

SECTION–I

1. Ans. (A,D)

Sol. By conservation of linear momentum both will get same velocity of centre of mass

$$\therefore \quad \frac{1}{2}mv_{cm}^2 = mgh$$

$$h_1 = h_2$$

But by conservation of angular momentum about centre of mass

System 2 have angular velocity also

$$\therefore \quad K_1 < K_2$$

$$[\text{where } k = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2]$$

2. Ans. (C,D)

Sol. $$h = \frac{2\times 0.085}{r\times 13.6\times 10^3\times 10}\times\frac{-4}{500} = -1\text{mm}$$

3. Ans. (A,C,D)

Sol. (A) $S_x = -4\cos kx \sin \omega t$

$$= -4\cos\left(\frac{2\pi}{0.40}\times 0.05\right)\sin\left(\frac{2\pi}{0.2}\times 0.05\right) = 2\sqrt{2}$$

(C) $$\frac{\lambda}{T} = v = \frac{0.4}{0.2} = 2\text{m/s}$$

$$v_p = -4\cos\left(\frac{2\pi}{0.4}\times\frac{1}{15}\right)\times\frac{2\pi}{0.2}\times\cos\left(\frac{2\pi}{0.2}\times 0.1\right)$$

$$= +40\pi\times\frac{1}{2} = 20\pi$$

4. Ans. (A, D)

Sol. $$\frac{\frac{1}{2}\times k\in_0 E_1^2}{\frac{1}{2}\times k_2\in_0 E_2^2} = \frac{k_2}{k_1} = \frac{5}{3}$$

$$\sigma\left(1-\frac{1}{3}\right),\ -\sigma\left(1-\frac{1}{5}\right) = \left(\frac{2}{3}-\frac{4}{5}\right)\sigma = \frac{-2\sigma}{15}$$

5. Ans. (A,C,D)

Sol. Parallel ray will pass through focus after passing through lens.

$\therefore$ Slab will shift the image by an amount

$$\text{of} = t\left[1-\frac{1}{\mu}\right] = 3\left[1-\frac{2}{3}\right] = 1\text{ cm}$$

Final image will form at 11 cm

If converging rays meet at the point of image, image is known as real. If diverging rays meet at thepoint of image, image is known as virtual.

6. Ans. (B)

Sol. As tension is always perpendicular to the velocity its speed remain same.

7. Ans. (B)

8. Ans. (A,B,C,D)

Sol. $F_x = -\frac{2cx}{a^2}$

$ma = F_y = -\frac{2cy}{b^2}$

9. Ans. (B,C)

10. Ans. (A,C)

Sol. $\frac{1}{2}kx^2 = \frac{1}{2}m_2 \times 4^2 = \frac{1}{2}m_1 x_2^2$

$\frac{m_2}{m_1} = \frac{4}{16} = \frac{1}{4}$

$m_1v_1 = m_2v_2 \qquad \Rightarrow \frac{v_1}{v_2} = \frac{1}{4}$

$\frac{1}{2}kx^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}\times\frac{m_1}{4}\times(4v_1)^2$

$\frac{1}{2}\times m_1 \times 2^2 = \frac{1}{2}m_1 \times 5v_1^2$

$v_1 = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} \qquad v_2 = \frac{8}{\sqrt{5}}$

$T = \frac{2\pi}{4}\sqrt{\frac{m}{R}}$

SECTION-II

1. Ans. (A)-(R); (B)-(R); (C)-(S); (D)-(P)

Sol. Path diffrence due to slab-1 $= \Delta x = t(u - 1)$

$= 5\left[\frac{1}{2}\right] = \frac{5}{2}\mu m$

$\therefore$ Phase diffrence $\Delta\phi = \frac{2\pi\Delta x}{\lambda}$

$= \frac{2\pi}{500\times10^{-9}}\times\frac{5}{2}\times10^{-6}\,m = 10\pi$

Path diffrence due to slab (2) $= \Delta x = t[\mu - 1]$

$= \frac{3}{2}\left[\frac{3}{3}\right] = \frac{9}{4}\mu m$

$\Delta\phi = \frac{2\pi}{\lambda}\times\frac{9}{4}\times10^{-6} = 9\pi$

Path diffrence due to slab (3) $= \Delta x = t[\mu - 1]$

$= \frac{1}{4}[1] = \frac{1}{4}\mu m$

$\Delta\phi = \frac{2\pi}{500\times10^{-9}}\times\frac{1}{4}\times10^{-6} = \pi$

(A) $10\pi \rightarrow$ fifth Maxima

(B) $(10\pi - \pi) = 9\pi \rightarrow$ fifth minima

(C) $(10\pi - (9\pi + \pi) = 0 \rightarrow$ Central Maxima

(D) $[[10\pi + \pi] - 9\pi] = 2\pi \rightarrow$ first Maxima

2. Ans. (A)-(Q,S,T); (B)-(P,R,T); (C)-(P,R,T); (D)-(Q,S,T)

SECTION-IV

1. Ans. 0

2. Ans. 5

3. Ans. 5

Sol. $\frac{B\omega a^2}{2R} = i$

$= \frac{0.1\times40\times\left(\frac{1}{20}\right)^2}{2\times1} = 2\times\frac{1}{400}$

4. Ans. 2

Sol. $\frac{dm}{dt} = \frac{\rho}{0}\times0.08 = \frac{\rho/0}{1+r\Delta T}\times0.081$

$0.8 + 0.8\,r\Delta T = 0.81$

$r\Delta T = \frac{0.01}{0.80}\times10^3$

$$\Delta T = \frac{1}{80} \times 2 = \frac{100}{16}$$

$$\rho = \frac{dms}{dt} \Delta T$$

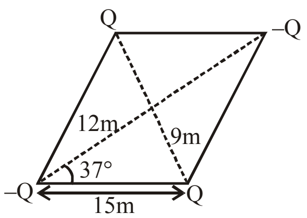
$$= 10^3 \times \frac{0.8 \times 10^{-3}}{10} \times 4000 \times \frac{25}{4}$$

$= 2 \times 10^3$ W

5. **Ans. 5**

Sol. Potential at centre,

$$-\frac{kQ}{12} - \frac{kQ}{12} + \frac{kQ}{9} + \frac{kQ}{9} = kQ\left[\frac{-1}{6} + \frac{2}{9}\right] = kQ\left[\frac{1}{18}\right]$$



$\therefore$ Potential at centre

$$= \frac{9 \times 10^9 \times 0.01 \times 10^{-6}}{18} V = \frac{90}{18} = 5V$$

6. **Ans. 1**

Sol. $\dfrac{\Delta H}{n\Delta T}$

$$= \frac{nC_Q\Delta T + W}{n\Delta T}$$

$= 2R - R = R$

7. **Ans. 0**

8. **Ans. 2**

Sol. Momentum of electron $p_e = \sqrt{2meV}$

Momentum of particle, $p_p = \sqrt{2\frac{m}{4} \cdot eV}$

$$\therefore \frac{\lambda_p}{\lambda_e} = \frac{h/p_p}{h/p_e} = \sqrt{\frac{m}{m/4}} = 2$$

PART-2 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C,D	C,D	A,C,D	B,C,D	A,B,C	A,B	B,D	B,C,D	B,C	A,C,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		P	S	R,T	Q		P,Q,S	P,Q,S	P,T	Q,R	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	8	8	4	2	4	3	2	4		

SOLUTION

SECTION - I

1. **Ans. (C, D)**

All molecules move with different speeds and due to molecular collision, kinetic energy of molecules will change.

2. **Ans. (C,D)**

$$\frac{1}{4}P_4(s) + \frac{3}{2}Cl_2(g) \longrightarrow PCl_3(g) 32 \quad ; \Delta H = 140$$

$$140 = \left(\frac{1}{4} \times 320 + \frac{3}{2} \times 120\right) - 3 \times (B - E)_{P-Cl}$$

$$(B\text{–}E)_{P-Cl} = \frac{120}{3} = 40 kJ/mole$$

$$\frac{1}{4}P_4(s) + \frac{3}{2}H_2(g) \longrightarrow PH_3(g) \quad ; \Delta H = 8$$

$$8 = \left(\frac{1}{4} \times 320 - \frac{3}{2} \times 216\right) - 3 \times (B - E)_{P-H}$$

$(B\text{–}E)_{P-H} = 132$ kJ/mole

$$H_2 \to 2H ; \Delta H = 216 \quad \therefore (\Delta H_f)_H = \frac{216}{2} = 108$$

$$Cl_2 \to 2Cl ; \Delta H = 120 \quad \therefore (\Delta H_f)_{Cl} = \frac{120}{2} = 60$$

3. **Ans. (A,C,D)**

Anode : $2Br^- \rightarrow Br_2 + 2e^-$

Cathode : $2H_2O + 2e^- \rightarrow H_2 + 2OH^-$

K^+ combines with OH^- so KOH will also form.

4. **Ans. (B,C,D)**

5. **Ans. (A,B,C)**

6. **Ans. (A,B)**

7. **Ans. (B,D)**

8. **Ans. (B,C,D)**

9. **Ans. (B,C)**

10. **Ans. (A,C,D)**

SECTION - II

1. **Ans (A)-(P) ; (B)-(S) ; (C)-(R,T); (D)-(Q)**

(A) $pH = 10 + \log\frac{0.1}{0.1} = 10 \Rightarrow (P)$

(B) $pOH = 6 + \log\frac{0.1}{0.1} = 6 \Rightarrow pH = 8\ (S)$

(C) $pH = \frac{1}{2}[14 + 5 - 7] = 6\ (R), (T)$

(D) $[H^+] = \frac{500 \times 0.02}{1000} = 0.01,$

$[OH^-] = \frac{500 \times 0.02}{1000} = 0.01$

so solution is neutral $\therefore$ (Q).

2. **Ans. (A)-(P,Q,S); (B)-(P,Q,S); (C)-(P,T); (D)-(Q,R)**

SECTION - IV

1. **Ans. 8**

$10\ H_2O + SCN^- \rightarrow SO_4^{2-} + CO_3^{2-} + NO_3^- + 2OH^+ + 16\ e^-$

$\therefore$ 'n' factor or $Ba(SCN)_2 = 2 \times 16 = 32$

$\therefore \frac{n}{4} = \frac{32}{4} = 8$

2. **Ans. 125 [OMR Ans. 8]**

$2A(g) + B(g) \rightarrow 2C(g)$

$\Delta ng = 2 - (1 + 2) = -1$

$\Delta H_r = 25.6 + \frac{(-1) \times 2 \times 300}{1000} = 25kcal$

$\Delta S_r = 2 \times 500 - (2 \times 200 + 100) = 500$ cal.

$\Delta G = 25 - \frac{300 \times 500}{1000} = -125kcal$

$\therefore |\Delta G| = 125$

$\therefore$ Ans. 8

3. **Ans. (4)**

4. **Ans. (2)**

5. **Ans. (4)**

i, ii, iv, v

6. **Ans. (3)**

i, ii, vi

7. **Ans. (2)**

ii, iv

8. **Ans. (4)**

i, ii, iv, v

PART-3 : MATHEMATICS

ANSWER KEY

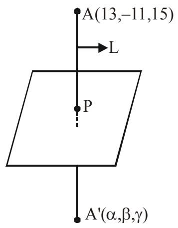
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C,D	A,B,D	A,B,C	B,C,D	A,B,D	A,D	A,C,D	A,B,C,D	B,C,D	A,C
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		Q,R	P,R	Q,T	Q,S		P,Q,R	T	T	S	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	3	4	6	2	0	3	2	1		

SOLUTION

SECTION-I

1. Ans. (A,B,C,D)

$$L \equiv \frac{x-13}{2} = \frac{y+11}{-2} = \frac{z-15}{1} = \lambda$$



$$P \equiv (2\lambda+13, -2\lambda-11, \lambda+15)$$

Putting P in plane $\lambda \equiv -6$

$\equiv F(1,1,9)$

$\alpha + 13 = 2,\ \beta - 11 = 2,\ \gamma + 15 = 18$

$\Rightarrow A' \equiv (-11,13,3)$

Perpendicular distance $= \left|\dfrac{26+22+15-9}{3}\right| = 18$

volume $= \dfrac{1}{6}\times\dfrac{9}{2}\times\dfrac{9}{2}\times 9 = \dfrac{3^5}{2^3}$

2. Ans. (A,B,D)

$$z^4 - z^2 + 1 = 0 \Rightarrow z^2 = \cos\left(\frac{\pi}{3}\right) \; z^2 = \cos\left(\frac{5\pi}{3}\right)$$

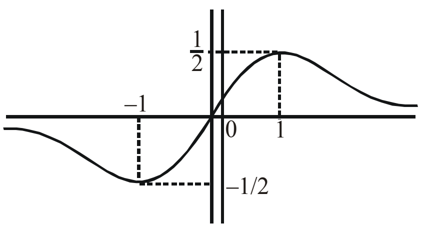
$$z = \cos\left(m\pi + \frac{\pi}{3}\right) z = \cos\left(n\pi + \frac{5\pi}{6}\right)$$

now Doyourself.

3. Ans. (A,B,C)

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{1+x^2} \Rightarrow \frac{d}{dx}(1+x^2)y = 1$$

$$\Rightarrow f(x) = \frac{x}{x^2+1}$$



4. Ans. (B,C,D)

Do yourself

5. Ans. (A,B,D)

LR = 4a = 4

$AB = 4\ \mathrm{cosec}^2\alpha = 16$

$$\frac{2}{t_1+t_2} = \frac{1}{\sqrt{3}}$$

$$2\sqrt{3} = t_1 - \frac{1}{t_1}$$

$$t_1^2 - 2\sqrt{3}t_1 = 1$$

6. Ans. (A,D)

a and $\dfrac{1}{b}$ are the roots of the equation

$6x^2 + 20x + 15 = 0$

$$a + \frac{1}{b} = -\frac{10}{3} \text{ and } \frac{a}{b} = \frac{5}{2}$$

$$\frac{b^3}{ab^2 - 9(ab+1)^3} = \frac{1}{a.\frac{1}{b} - 9\left(a + \frac{1}{b}\right)^3}$$

$$= \frac{1}{\frac{5}{2} + 9.\frac{1000}{27}} = \frac{6}{2015}$$

7. Ans. (A,C,D)

$$I = \int_0^1 f(3^x)dx$$

$$I = \int_0^1 f(3^{1-x})dx$$

$$2I = \int_0^1 f(3)dx$$

Also $f(a.b) = f(a) + f(b)$

$\Rightarrow \quad f(a^2) = 2f(a)$

using this C & D also correct.

8. Ans. (A,B,C,D)

$$f(x)=\int_0^1|x-t|\,dt=\begin{cases}\int_0^1(t-x)\,dt, x\le 0\\ \int_0^x(x-t)\,dt+\int_x^1(t-x)\,dt, 0<x<1\\ \int_0^1(x-t)\,dt, x\ge 1\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\frac{1-2x}{2}, x\le 0\\ \frac{2x^2-2x+1}{2}, 0<x<1\\ \frac{2x-1}{2}, x\ge 1\end{cases}$$

9. Ans. (B,C,D)

$$y=\frac{x-\frac{1}{x}}{x^3-\frac{1}{x^3}+2}=\frac{x-\frac{1}{x}}{\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)+2}$$

$$x-\frac{1}{x}=t\Rightarrow y=\frac{t}{t^3+3t+2}$$

$$y=\frac{1}{t^2+\frac{2}{t}+3}$$

$$t^2+\frac{1}{t}+\frac{1}{t}\ge 3\Rightarrow t^2+\frac{2}{t}+3\ge 6\Rightarrow y\le\frac{1}{6}$$

10. Ans. (A,C)

$f'(x)\le f'(x)+xf''(x)\Rightarrow f'(x)\le(xf'(x))^1$

$$\Rightarrow\int_0^x f'(x)\,dx\le\int_0^x\left(xf'(x)\right)^1dx$$

$$\Rightarrow f(x)\le xf'(x) \qquad ...(1)$$

Now at

$$h(x)=\frac{f(x)}{x}\Rightarrow h'(x)=\frac{xf'(x)-f(x)}{x^2}\ge 0$$

$\forall\, x\in(0,1)$ (from (1))

$\Rightarrow h\uparrow$

Now $g(x)>x$ {As f is concave up in $(0,1)$}

$h(g(x))>h(x)$

$$\frac{f(g(x))}{g(x)}>\frac{f(x)}{x}\Rightarrow f(x)\,g(x)<x^2\ \forall\, x\in(0,1)$$

SECTION – II

1. Ans. (A)→(Q,R); (B)→(P,R); (C)→(Q,T); (D)→(Q,S)

$$P(A)=\frac{{}^6C_4+{}^6C_5+{}^6C_6}{2^6}=\frac{11}{32}=\frac{a}{b}$$

$$P(B)=\frac{1}{2}=\frac{p}{q}$$

$$P\left(\frac{A}{B}\right)=\frac{P(AB)}{P(B)}=2\times\frac{{}^5C_3+{}^5C_4+{}^5C_5}{2^6}=\frac{1}{2}=\frac{u}{v}$$

$$P\left(\frac{B}{A}\right)=\frac{P(BA)}{P(A)}=\frac{32}{11}\times\frac{16}{64}=\frac{8}{11}=\frac{m}{n}$$

2. Ans. (A)→(P,Q,R); (B)→(T); (C)→(T); (D)→(S)

(A) $$\frac{\sum_{k=0}^{499}(2k+1)\,{}^{1000}C_{2k+1}}{100}$$

$$=10\sum_{k=0}^{499}{}^{999}C_{2k}$$

$$=10.2^{998}$$

(B) $\left(\sqrt{3}+1\right)^4=I+f$

$$\left(\sqrt{3}-1\right)^4=f'$$

$$I+f+f'=2\left\{{}^4C_0\left(\sqrt{3}\right)^4+{}^4C_2\left(\sqrt{3}\right)^2+{}^4C_4\right\}$$

$I+1=2\{9+18+1\}$

$=56$

(C) Do yourself

(D) No. of 4 digit numbers

= Dearrangment of 4 objects

SECTION – IV

1. **Ans. 3**
Do yourself

2. **Ans. 4**
Do yourself

3. **Ans. 6**
Use |PS – PS'| = 2b

4. **Ans. 2**

$$\log^2 y+2\left(2^x+\frac{1}{2^x}\right)\log y+2\left(2^{2x}+\frac{1}{2^{2x}}\right)=0$$

$$\left(\log y+2^x+\frac{1}{2^x}\right)^2+2\left(2^{2x}+\frac{1}{2^{2x}}\right)=2^{2x}+\frac{1}{2^{2x}}+2$$

$$\underbrace{\left(\log y+2^x+\frac{1}{2^x}\right)^2}_{\ge 0}=\underbrace{-\left(2^{2x}+\frac{1}{2^{2x}}\right)+2}_{\le 0}$$

$\Rightarrow$ x = 0; y = e^{-2}

5. **Ans. 0**

$\vec{c}=\vec{a}\times\vec{c}+\vec{a}\times\vec{b}$ ...(1)

$\Rightarrow \vec{a}.\vec{c}=0$ ...(2)

Also $\vec{a}\times\vec{c}=\vec{a}\times\left(\vec{a}\times\vec{c}\right)+\vec{a}\times\left(\vec{a}\times\vec{b}\right)$

{Taking cross with $\vec{a}$}

$$\vec{c}-\vec{a}\times\vec{b}=-\vec{a}^2\vec{c}+\left(\vec{a}.\vec{b}\right)\vec{a}-\vec{a}^2\vec{b}$$

$$\left(1+\vec{a}^2\right)\vec{c}+\vec{b}\times\vec{a}+\left(\vec{a}\times\vec{b}\right)\times\vec{a}=0$$

6. **Ans. 3**

$$\lim_{x\to\infty}\frac{1}{x+1}\cot\left(\frac{\pi}{2}-\frac{\pi x+1}{2x+2}\right)$$

$$=\lim_{x\to\infty}\frac{1}{x+1}\cot\left(\frac{2\pi-2}{4\left(x+1\right)}\right)$$

$$=\lim_{x\to\infty}\frac{\frac{\pi-1}{2\left(x+1\right)}}{\tan\frac{\left(\pi-1\right)}{2\left(x+1\right)}}=\frac{2}{\pi-1}$$

7. **Ans. 2**
Do yourself

8. **Ans. 1**

$f(1^-)\ge f(1)$

$2\ge 1+k$

$k\le 1$

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Test Type : **ALL INDIA OPEN TEST** (MAJOR)   Test Pattern : JEE-Advanced

TEST # 12   **TEST DATE : 15 - 05 - 2016**

PAPER-2

PART-1 : PHYSICS   **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,D	A,B,D	C,D	B,C	A,B	A,C	B	A,B,D	B,D	A,D
	Q.	11	12								
	A.	D	A								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	5	5	4	9	4	4	9	6		

SOLUTION

SECTION-I

1. Ans. (A,D)

Sol. $i = \frac{V}{R} = Q_{max}\omega = CV_1 \times \frac{1}{\sqrt{LC}}$

$$\frac{V}{R} = \sqrt{\frac{C}{L}} \times 2V$$

$$R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{2}{8} \times 10^6} = 250\ \Omega$$

$$\Delta H = V \times \int_0^\infty idt - \frac{1}{2}Li^2$$

$$= V \times \frac{V}{R}\int_0^\infty e^{-\frac{Rt}{L}}dt - \frac{1}{2}Li^2$$

$$= -\frac{V^2}{R} \times \frac{L}{R}\left[e^{-\frac{Rt}{L}}\Big|_0^\infty\right] - \frac{1}{2}Li^2$$

$$= \frac{V^2L}{R^2} - \frac{1}{2}Li^2$$

$$= \frac{V^2L}{R^2} - \frac{1}{2}L \times \frac{V^2}{R^2} = \frac{1}{2}\frac{V^2L}{R^2}$$

2. Ans. (A,B,D)

Sol. $\sqrt{\frac{3RT_A}{M}} = \sqrt{\frac{8RT_C}{M\pi}} = \sqrt{\frac{2RT_B}{M}}$

$$3T_A = \frac{8T_C}{\pi} = 2T_B$$

$$T_C = \frac{280 \times 3\pi}{8} = 105\pi \ \& \ T_B = 420$$

$$\frac{P_B}{P_A} = \frac{T_B}{T_A} = \frac{3}{2}$$

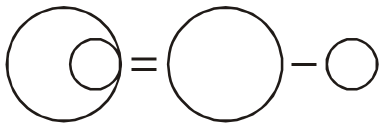
$$\frac{V_C}{V_A} = \frac{T_C}{T_A} = \frac{3\pi}{8}$$

$$\omega_{B-C} = -\frac{3}{2}R\left(T_C - T_B\right)$$

$$= \frac{3}{2}R\left(420 - 105\pi\right)$$

$$\omega_{CA} = (280 - 105\pi)R$$

3. Ans. (C,D)

Sol. (A) 

$$\frac{\mu_0 J}{2} \times 0 - \frac{\mu_0 J \times \frac{R}{2}}{2}$$

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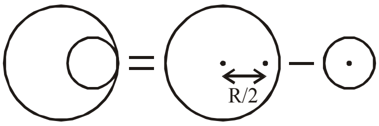
$$=\frac{\mu_0 R}{4}\times\frac{I}{\left(\pi R^2-\frac{\pi R^2}{4}\right)}$$

$$=\frac{\mu_0 R}{4}\times\frac{I}{\frac{3\pi R^2}{4}}$$

(B) 

$$\frac{\mu_0 J\times\frac{R}{2}}{2}-0$$

$$=\frac{\mu_0 R}{4}\times\frac{I}{\frac{3\pi R^3}{4}}$$

$$=\frac{\mu_0 I}{3\pi R}$$

4. Ans. (B, C)

Sol. (A) $\frac{kQ}{a^2}+\frac{kQ}{9a^2}-\frac{kQ}{\left(\frac{3a}{\sqrt{5}}\right)^2}\neq 0$

(B) $\frac{4kQ}{a^2}+\frac{kQ}{a^2}-\frac{kQ}{\left(\frac{a}{\sqrt{5}}\right)^2}=0$

(C) $\frac{kQ}{9a^2}+\frac{kQ}{25a^2}-\frac{kQ(34)}{225a^2}=0$

(D) $\frac{kQ^2}{16a^2}+\frac{kQ^2}{4a^2}-\frac{kQ^2}{\left(\frac{3a}{\sqrt{15}}\right)^2}\neq 0$

5. Ans. (A,B)

Sol. $\frac{1}{2}MR^2\omega=\frac{1}{2}\frac{5}{4}MR^2\omega'$

$$\omega'=\frac{4}{5}\omega$$

$$k=\frac{1}{2}\times\frac{5}{8}MR^2\left(\frac{4}{5}\omega\right)^2=\frac{1}{5}MR^2\omega^2$$

6. Ans. (A,C)

Sol. $\frac{1}{\sqrt{\lambda_z}}=C(z-1)$

$$\frac{1}{\sqrt{\lambda_1}}=C(z_1-1)$$

$$\frac{z_1-1}{z-1}=\sqrt{\frac{\lambda_z}{\lambda_1}}=2$$

$$z_1=2z-1$$

$$\frac{z_2-1}{z-1}=\frac{1}{2}$$

$$z=\frac{z}{2}+\frac{1}{2}$$

7. Ans. (B)

Sol. $T=\frac{T_1T_2}{T_1+T_2}=\frac{10\times 30}{40}=7.5\text{days}$

$\Rightarrow t = 15$ days

eliminated = $5\left(1-\frac{1}{\sqrt{2}}\right)$

Total driving till then = $5e^{-\frac{\ell n2}{10}\times 15}$

$$=5\left(\frac{1}{2}\right)^{\frac{3}{2}}=\frac{5}{2\sqrt{2}}\mu g$$

decayed = $5\left(1-\frac{1}{2\sqrt{2}}\right)$

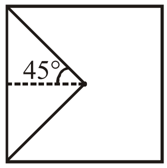
Remaining in body = $\frac{5}{\sqrt{2}}$

$\Rightarrow$ decayed in body = $\frac{5}{\sqrt{2}}\left(1-\frac{1}{\sqrt{2}}\right)$

8. Ans. (A,B,D)

Sol. Let magnetic field at centre of square due to current in square is B_1

$$\left(\sqrt{3}B\right)^2 = B_1^2 + B^2$$



$$B_1 = \sqrt{2}B$$

$$\frac{\mu_0 i}{4\pi\frac{a}{2}}[\sin 45] \times 2 \times 4 = \sqrt{2}B$$

$$\Rightarrow \frac{2\sqrt{2}\mu_0 i}{\pi a} = \sqrt{2}B$$

$$\Rightarrow i = \frac{\pi a B}{2\mu_0}$$

$$\tau = M \times B = ia^2 B = \frac{\pi a^3 B^2}{2\mu_0}$$

(B) $U = MB \cos 90 = 0$

(C) $B_{net} = \sqrt{2}B + B = B\left(\sqrt{2}+1\right)$

(D) $B_{net} = \sqrt{2}B - B = B\left(\sqrt{2}-1\right)$

9. Ans. (B,D)

Sol. $A \to B$,

$$mg(4) + \frac{1}{2}k\left(2^2 - 0\right) - 11 = \frac{1}{2}mv_B^2$$

$B \to C$,

$$mg(3) - 4 = \frac{1}{2}m\left(v_C^2 - v_B^2\right)$$

10. Ans. (A,D)

Sol. $N_B = \dfrac{mv_B^2}{R}$

$$N_C - mg = \frac{mv_C^2}{R}$$

11. Ans. (D)

Sol. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{\sqrt{2mqV}}$ where E = kinetic energy

12. Ans. (A)

Sol. $\lambda = \sqrt{\dfrac{150}{V}} = \sqrt{\dfrac{150}{150}}\text{Å} = 1\text{Å}$

SECTION-IV

1. Ans. 5

Sol. At equilibrium

$$8g = k \times 0.2$$

$$k = \frac{8g}{0.2}$$

When mass get turn then new equilibrium shift by

$$\Delta x = \frac{1g}{k} = \frac{1g \times 0.2}{8g}$$

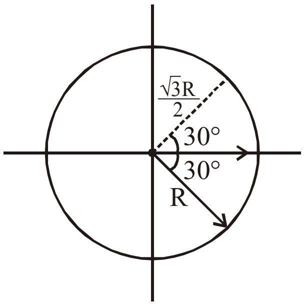
$\Delta x = 2.5$ cm

If will be amplitude for SHM maximum height $= 2A = 5$ cm

2. Ans. 5

Sol. $\omega = \sqrt{\dfrac{GM}{R^3}} = \sqrt{\dfrac{4}{3}\pi G\rho}$

$$\theta = \frac{\pi}{3} = \omega t$$



$$t = \frac{\pi}{3\omega} = \frac{\pi}{3}\sqrt{\frac{1}{\frac{4\pi}{3}G\rho}}$$

$$= \frac{\pi}{3}\sqrt{\frac{1}{\frac{4\pi}{3}\times\frac{20}{3}\times10^{-11}\times800\pi}} = 1250\,\text{sec}$$

3. Ans. 4

Sol. $T = c\,s^x\rho^y r^z$

$[T] = [s]^x\,[\rho]^y\,[r]^z$

$= [MT^{-2}]^x\,[ML^{-3}]^y\,[L]^z$

$\Rightarrow [T] = M^{x+y}\,L^{-3y+z}\,T^{-2x}$

$x + y = 0$

$-2x = 1 \Rightarrow x = -\frac{1}{2}$, $y = \frac{1}{2}$

$-3y + z = 0$

$z = \frac{3}{2}$

$$T = c\sqrt{\frac{\rho r^3}{s}}$$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta\rho}{\rho} + \frac{3}{2}\frac{\Delta r}{r} + \frac{1}{2}\frac{\Delta s}{s}$$

$$= \frac{1}{2} + 3 + \frac{1}{2} = 4$$

4. Ans. 9

Sol. $\frac{d\rho}{dT} = \frac{68}{9}\times10^{-3}\,\Omega m/K$

Radius of wire r = 2mm

$$\frac{i^2\rho\ell}{\pi r^2} = e2\pi r\ell\sigma\,(T^4 - T_0^4)$$

$$\frac{i^2\rho\ell}{\pi r^2} \approx e2\pi r\ell\sigma\,(T^4)$$

$$i = \sqrt{\frac{2e\pi^2 r^3\sigma T^4}{\rho}} \qquad ....(i)$$

As $\rho = \frac{68}{9}\times10^{-3}\,T$

Put $\rho = \frac{68}{9}\times10^{-3}\,T$ in equation (i) and after solving we will get i = 0.18 A

5. Ans. 4

Sol. $v = \sqrt{\mu Rg}$

$$\pi^2 = \frac{1}{8}R\times10$$

$$R = \frac{4\pi^2}{5}$$

$$T = \frac{\frac{\pi R}{2}}{v} = \frac{\pi}{2\pi}\times\frac{4\pi^2}{5} = 4\,\text{sec}$$

6. Ans. 4

Sol. $330 = \frac{330}{300 - v_1}\times300$

$\Rightarrow v_1 = 30$ m/s

$$360 = \frac{330}{330 - v_2}\times300$$

$$330 - v_2 = \frac{330}{12}\times10 = 275$$

$v_2 = 55$ m/s

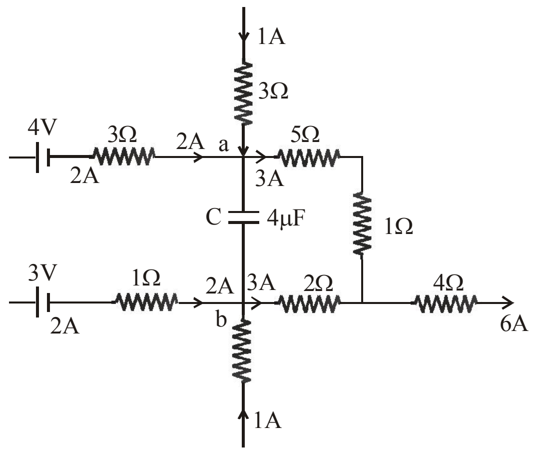
$\Delta V = 25$ m/s

$100 = \Delta V \times T$

$\Rightarrow T = 4$ sec

7. Ans. 9

Sol. Using Kirchhoff's first law at junctions a and b, we have found the current in other wires of the circuit on which currents were not shown.



Now, to calculate the energy stored in the capacitor we will have to first find the potntial difference V_{ab} across it.

$$V_a - 3 \times 5 - 3 \times 1 + 3 \times 2 = V_b$$

$\therefore$ $V_a - V_b = V_{ab} = 12$ volt

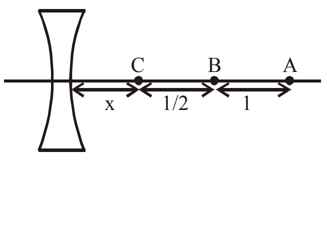
$\therefore$ $U = \frac{1}{2}CV^2_{ab}$

$$= \frac{1}{2} \times (0.125 \times 10^{-6})\,(12)^2 \text{ J}$$

$= 9$ mJ

8. Ans. 6

Sol. From situation, it is clear that it is diverging lens to left of C.



$$\frac{1}{-\left(x+\frac{1}{2}\right)} + \frac{1}{\left(x+\frac{3}{2}\right)} = \frac{1}{f}$$

$$\frac{1}{-x} + \frac{1}{\left(x+\frac{1}{2}\right)} = \frac{1}{f}$$

$$\frac{-2}{2x+1} + \frac{2}{2x+3} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{9x+2-4x-6}{(2x+3)(2x+1)} = \frac{-4}{\left(4x^2+8x+3\right)}$$

$$-f = x^2 + 2x + \frac{3}{4}$$

$$-\frac{1}{x} + \frac{2}{2x+1} = \frac{1}{f}$$

$$\frac{2x-(2x+1)}{(2x+1)x} = \frac{1}{f}$$

$$-f = 2x^2 + x = x^2 + 2x + \frac{3}{4}$$

$$x^2 - x - \frac{3}{4} = 0$$

$$\frac{1 \pm \sqrt{1+3}}{2} = \frac{3}{2} \text{ or } -\frac{1}{2}$$

$$-f = 2x^2 + x$$

$$= 2\times\frac{9}{4} + \frac{3}{2} = 6\text{cm}$$

PART-2 : CHEMISTRY

ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,B,D	A,B	B,D	A,D	B,C	A,C	C	A,B,C,D	A	A
	Q.	11	12								
	A.	C,D	B,D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	8	5	3	3	4	2	3	3		

SOLUTION

SECTION–I

1. Ans. (A, B, D)

(a) Packing fraction of,

$$\eta = \frac{\frac{1}{6}\times 3\times \pi R^2}{\frac{\sqrt{3}}{4}\times 4R^2} = \frac{\pi}{3\sqrt{3}} = 0.91$$

(b) C.N. = 6 ; it is a hexagonal 2D lattice

(c) $\frac{r}{R} = 0.155$ so, diameter = 2r = 0.31R

(d)Distance between 2 layers is $\frac{2\sqrt{2}}{\sqrt{3}}R$

2. Ans. (A, B)

For the equal mix, $Y_{ben} = x_{tol}$ & $y_{tol} = x_{Ben}$

$\Rightarrow y_{ben} - x_{Ben}$ = minimum, which occurs at

$P_{total} = \sqrt{P^{\circ}_{Ben}.P^{\circ}_{Tol}}$

Hence (a), (b) are correct

(C) At this, instant V.P. of equilibrium mixture corresponds to V'

(D) If external pressure is increased then condensation occurs and not vaporisation.

3. Ans. (B, D)

As_2S_3 sol is negatively charged so the D.M. moves to cathode while sol particles do not move in either direction.

4. Ans. (A,D)

5. Ans. (B,C)

6. Ans. (A,C)

7. Ans. (C)

8. Ans. (A,B,C,D)

9. Ans. (A)

H_2O_2 decomposes by 1st order kinetics

$K \times 5 = \ln \frac{20}{15}$

$K = 0.06\ min^{-1}$; $t_{1/2} = \frac{\ln 2}{0.06} = 11.67\ min$

10. Ans.(A)

$N_1V_1 = N_2V_2$; $\frac{10}{11.35/2} \times \frac{11.35}{2} = 0.1 \times 5 \times V$

11. Ans. (C,D)

12. Ans. (B,D)

SECTION–IV

1. Ans. (8)

$n_1E_1^0 + n_2E_2^0 = n_3E_3^0$

($n_1 = x - y$, $n_2 = y - z$, $n_3 = x - z$)

$(x - y) \times 2 + (y - z) \times 3 = (x - z) \times 10$

$2x - 2y + 3y - 3z = 10x - 10z$

$y - 8x + 7z = 0$

$\frac{y + 7z}{x} = 8$

2. Ans. (5)

$n - l - 1 = 2$; $l = 3 \Rightarrow n = 6$

$13.6\ \frac{Z^2}{6^2} = 13.6 \times \frac{3^2}{3^2} \Rightarrow Z = 6$; oxidation number = +5.

3. Ans. (3)

4. Ans. (3)

5. Ans. (4)

6. Ans. (2)

7. Ans. (3)

8. Ans. (3)

PART-3 : MATHEMATICS — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,B,D	B,D	A,B,C,D	B,C,D	A,C	A,B,C,D	A,B,C,D	B,C	B	A
	Q.	11	12								
	A.	D	B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	0	2	9	3	1	4	2	6		

SOLUTION

SECTION-I

1. Ans. (A,B,D)

$$\left.\frac{dy}{dx}\right|_{(h,k)}=\frac{\sqrt{1+k^2}}{3k}$$

$$\therefore\ \left(\frac{k-0}{h-2}\right)=\frac{\sqrt{1+k^2}}{3k}\Rightarrow\sqrt{1+k^2}=\frac{3k^2}{h-2}\quad ...(1)$$

Also, $\sqrt{1+k^2}=\frac{h}{3}\Rightarrow 1+k^2=\frac{h^2}{9}$ ...(2)

$\therefore$ From (1) and (2), we get

$$\frac{3\left(\frac{h^2}{9}-1\right)}{h-2}=\frac{h}{3}\Rightarrow h=\frac{9}{2},k=\pm\frac{\sqrt{5}}{2}$$

$$\Rightarrow A\left(\frac{9}{2},\frac{\sqrt{5}}{2}\right)\ \&\ \Rightarrow B\left(\frac{9}{2},-\frac{\sqrt{5}}{2}\right)$$

2. Ans. (B,D)

We have

$$|a+b|=|a-b|\Rightarrow\left|\frac{a}{b}+1\right|=\left|\frac{a}{b}-1\right|$$

$\Rightarrow \frac{a}{b}$ lies on perpendicular bisector of (–1,0) and (1,0)

so, $\frac{a}{b}$ lies on imaginary axis.

$$\Rightarrow \arg\left(\frac{a}{b}\right)=\pm\frac{\pi}{2}$$

$$\therefore\ |\arg(a)-\arg.(b)|=\frac{\pi}{2}$$

3. Ans. (A,B,C,D)

Let T(h,k) where $h=t_1t_2$, $k=t_1+t_2$.

Also $t_1^2=16t_2^2$

$\therefore$ On eliminating t_1 & t_2, we get locus of T(h,k) is $y^2=\frac{25}{4}x$

4. Ans. (B,C,D)

$$r=\frac{\Delta}{s}=\frac{\frac{\sqrt{3}}{4}a^2}{\frac{3a}{2}}=\frac{a}{2\sqrt{3}},$$

where a is length of side of triangle

As, $r\in Q\Rightarrow a$ is irrational and multiple of $\sqrt{3}$.

Also, R = 2r $\Rightarrow$ $R=\frac{a}{\sqrt{3}}\in Q$.

Also $\Delta=\frac{\sqrt{3}}{4}a^2\notin Q$ and $r_1=r_2=r_3=\frac{\sqrt{3}}{2}a\in Q$.

5. Ans. (A,C)

$$f(x)=\sum_{n=2}^{\infty}\left(\frac{n-1}{e^{(n-1)x}}-\frac{n}{e^{nx}}\right)$$

$$=\lim_{n\to\infty}\left(\left(\frac{1}{e^x}-\frac{2}{e^{2x}}\right)+\left(\frac{2}{e^{2x}}-\frac{3}{e^{3x}}\right)+....+\left(\frac{n-1}{e^{(n-1)x}}-\frac{n}{e^{nx}}\right)\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{e^x}-\frac{n}{e^{nx}}\right)=e^{-x}$$

6. Ans. (A,B,C,D)

we have

$\Rightarrow(\sin2x+\cos3y)^2+(\cos3y+\tan4z)^2+(\tan4z+\sin2x)^2\le 0$

$\therefore$ sin2x = cos3y = tan4z = 0

$$\Rightarrow x=\frac{\pi}{2};y=\frac{\pi}{6},\frac{\pi}{2};z=\frac{\pi}{4},\frac{\pi}{2}$$

Now verify alternatives

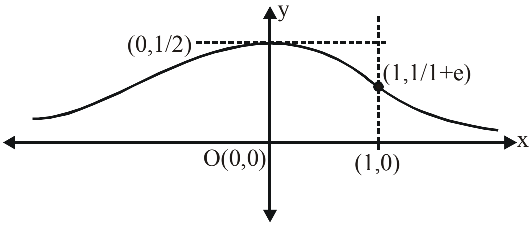
7. Ans. (A,B,C,D)

$$-\int\frac{dy}{y^2}=\int 2x\,e^{x^2}dx$$

$\Rightarrow\frac{1}{y}=e^{x^2}+c$, As $f(0)=\frac{1}{2}$

$\Rightarrow c = 1$

$\therefore\ y = \dfrac{1}{1+e^{x^2}}$



$\therefore\ \dfrac{1}{1+e}(1-0) < \int_0^1 f(x)\,dx < \dfrac{1}{2}(1-0)$

8. Ans. (B,C)

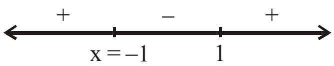
$\lim\limits_{x\to-\infty} f(x) = -1$

$f'(x) = 0 \ \Rightarrow\ x = -1$

$\therefore\ f(-1) = -\dfrac{1}{\sqrt{2}}$

Also, $\lim\limits_{x\to1^+} f(x) = \infty, \lim\limits_{x\to1^-} f(x) = -\infty$

Range of $f(x) = \left(-\infty, \dfrac{-1}{\sqrt{2}}\right] \cup (1,\infty)$



sign of $f'(x)$

$\Rightarrow$ x = –1 is point of local maximum of $f(x)$.

Paragraph for Question 9 to 10

$C_1 : x^2 + y^2 - 10x + 24 = 0$ is circle whose centre is (5,0) and radius is 1. Also, $C_2 : x^2 + y^2 - 10x + 21 = 0$ is circle whose centre is (5,0) and radius is 2.

9. Ans. (B)

10. Ans. (A)

Paragraph for Question 14 to 16

$$P(B_1) = \frac{1}{10}, P(B_2) = \frac{2}{10}, P(B_3) = \frac{3}{10}, P(B_4) = \frac{4}{10}$$

11. Ans. (D)

$$P(E_1) = \frac{1}{10}\times(1) + \left(\frac{2}{10}\times\frac{1}{2}\right) + \left(\frac{3}{10}\times\frac{1}{3}\right) + \left(\frac{4}{10}\times\frac{1}{4}\right)$$

$$= \frac{4}{10} = \frac{2}{5}$$

12. Ans. (B)

$$P(B_3/E_2) = \frac{P(B_3 \cap E_2)}{P(E_2)}$$

$$= \frac{\left(\frac{3}{10}\times\frac{1}{3}\right)}{\frac{1}{10}\times(0) + \left(\frac{2}{10}\times\frac{1}{2}\right) + \left(\frac{3}{10}\times\frac{1}{3}\right) + \left(\frac{4}{10}\times\frac{1}{4}\right)}$$

$$= \frac{\frac{1}{10}}{\frac{3}{10}} = \frac{1}{3}$$

SECTION – IV

1. Ans. 0

$$M = \begin{bmatrix} \sqrt{3} & 1 & 0 \\ 1 & -\sqrt{3} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

As, $MM^T = 4I \Rightarrow 2M^T + adj.M = 0$

$\Rightarrow |2M^T + ady.M| = 0$

2. Ans. 2

Let $P(x) = 12(x-1)(x-3)(x-5) + (2x+1)$

3. Ans. 9

$$P = {}^nC_6 \cdot \left(3^{\frac{1}{3}}\right)^{n-6} \cdot \left(4^{-\frac{1}{3}}\right)^{6}$$

$$Q = {}^nC_{n-6} \cdot \left(3^{\frac{1}{3}}\right)^{6} \cdot \left(4^{-\frac{1}{3}}\right)^{n-6}$$

$$\therefore\ \frac{Q}{P} = 12 \Rightarrow (12)^{\frac{n-6}{3}} = (12)^1$$

$$\Rightarrow \frac{n-6}{3} = 1 \Rightarrow n = 9$$

4. Ans. 3

we have

$$\frac{1}{a.b.c}\begin{vmatrix} a(a^3+1) & a^3b & a^3c \\ ab^3 & b(b^3+1) & b^3c \\ ac^3 & bc^3 & c(c^3+1) \end{vmatrix} = 11$$

$$\Rightarrow \begin{vmatrix} a^3+1 & a^3 & a^3 \\ b^3 & b^3+1 & b^3 \\ c^3 & c^3 & c^3+1 \end{vmatrix} = 11$$

apply $C_1 \to C_1 + C_2 + C_3$

& solving

$\Rightarrow a^3 + b^3 + c^3 + 1 = 11$

$\Rightarrow a^3 + b^3 + c^3 = 10$

$\therefore$ possibilities are

(1,1,2), (1,2,1), (2,1,1)

$\Rightarrow$ Number of triplets = 3

5. Ans. 1

$\frac{x^2}{9} - \frac{y^2}{4} = 1$; P(3secθ, 2tanθ)

$\therefore$ Equation of chord of contact AB with respect to P is T = 0

$\Rightarrow 3x\sec\theta + 2y\tan\theta = 9$ ...(1)

Also, equation of chord whose mid-point is (h,k) is $T = S_1$

$\Rightarrow hx + ky - 9 = h^2 + k^2 - 9$ ...(2)

$\therefore$ On comparing (1) and (2), we get

$$\frac{3\sec\theta}{h} = \frac{2\tan\theta}{k} = \frac{9}{\left(h^2+k^2\right)}$$

as, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow$ locus of (h,k) is

$$\left(\frac{x^2}{9} - \frac{y^2}{4}\right) = 1\left(\frac{x^2+y^2}{9}\right)^2$$

$\Rightarrow \lambda = 1$

6. Ans. 4

$\underset{\alpha\ \ \beta}{x^2} - 6x + 12 = 0$; Here $(\beta - 6) = -\alpha$

$$\therefore \left(\alpha-2\right)^{24} - \frac{\left(\beta-6\right)^8}{\alpha^8} + 1 = \left(\alpha-2\right)^{24}$$

$= 2^{24}$.

7. Ans. 2

Normal vector of required plane is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -3 \\ 4 & 3 & 5 \end{vmatrix} = 14\hat{i} - 27\hat{j} + 5\hat{k}$$

$\therefore$ Equation of required plane is

$14(x-1) - 27(y-2) + 5(z-3) = 0$

$\Rightarrow 14x - 27y + 5z + 25 = 0$

8. Ans. 6

$\frac{dy}{dx} + \left(-\frac{1}{x}\right)y = \left(x - \frac{2}{x}\right)$ (Linear differential equation)

$\Rightarrow f(x) = (x-1)^2 + 1$

$\therefore$ Required area

$$= \int_0^3 \left(\left(x-1\right)^2 + 1\right)dx = 3 + 3 = 6$$