Question Bank - Parabola

LEVEL–I

1. Find the equation of the parabola whose axis is parallel to the y-axis and which passes through the points (0, 4), (1, 9) and (–2, 6) and determine its latus rectum.

2. Show that the locus of the intersection of the tangents to the parabola $y^2 = 4ax$ at $(at^2, 2at)$, $(a/t^2, 2a/t)$ is a straight line parallel to the y-axis.

3. Two equal parabola have the same vertex and their axes are at right angles. Prove that their tangent touches each at the end of a latus rectum.

4. Find the coordinates of the points in which the line $12x – 9y – 4 = 0$ cuts the parabola $3y^2 = 4x$. Show that the tangents at these points meet on the line $3x + 1 = 0$.

5. The normals at the points (4a, – 4a), (9a, – 6a) of the parabola $y^2 = 4ax$ meet in P. Find the equation of the third normals from P.

6. Prove that the locus of the middle points of all chords of the parabola $y^2 = 4ax$ passing through the vertex is the parabola $y^2 = 2ax$.

7. The tangent at the point P on the parabola $y^2 = 4ax$ whose abscissa is equal to the latus rectum meets the axis in T and the normal at P cuts the parabola again in Q. Prove that PT : PQ = 4 : 5.

8. Tangents drawn to the parabola $y^2 = 4ax$ at the points P and Q intersect at T. If triangle TPQ is equilateral then find the side length of this triangle.

9. Three normal to $y^2 = 4x$ pass through the point (15, 12). Show that one of the normal is given by $y = x - 3$ and find the equations of the others.

10. If from the vertex of a parabola a pair of chords be drawn at right angles to one another, with these chords as adjacent sides a rectangle be constructed, then find the locus of the outer corner of the rectangle.

LEVEL–II

1. Prove that the normal at $(am^2, -2am)$ to the parabola $y^2 = 4ax$ intersects the parabola again at an angle $\tan^{-1}\left|\frac{m}{2}\right|$.

2. Two perpendicular lines $y = mx$, $my = -x$ are drawn through the origin. Find the points, other than the origin, where the lines cut the parabola $y^2 = 4ax$.

3. Prove that the feet of the perpendiculars from the point $(a, 0)$ to the normals of the parabola all lie on the curve whose equation is $y^2 = a(x - a)$.

4. The tangents at the extremities of a normal chord of the parabola $y^2 = 4ax$ meet in a point T. Show that the locus of T is the curve $(x + 2a)\,y^2 + 4a^2 = 0$

5. Prove that the locus of the middle points of chords of a parabola passing through a fixed point is a parabola whose latus rectum is half that of the given parabola.

6. Prove that the locus of the middle points of normal chords of the parabola $y^2 = 4ax$ is $\frac{y^2}{2a} + \frac{4a^3}{y^2} = x - 2a$.

7. Show that if the normals at the points P, Q, R on the parabola $y^2 = 4ax$ meet in (α, β) the orthocenter of the triangle PQR will be $(a - 6\alpha, -1/2\beta)$.

8. Prove that the locus of the midpoints of all the tangents drawn from points on the directrix to the parabola $y^2 = 4ax$ is $y^2(2x + a) = a(3x + a)^2$.

9. Prove that the locus of intersections of tangents to the parabola $y^2 = 4ax$ which intercept a fixed length 'l' on the directrix is $(y^2 - 4ax)(x + a)^2 = l^2x^2$.

10. A variable tangent to the parabola $y^2 = 4ax$ meets the circle $x^2 + y^2 = r^2$ at P and Q. Prove that the locus of the mid point of PQ is $x(x^2 + y^2) + ay^2 = 0$.

IIT JEE PROBLEMS (OBJECTIVE)

A. Fill in the blanks

1. The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is .... **[IIT - 94]**

B. Multiple choice questions with one or more than one correct answer :

1. Equation of common tangent of $y = x^2$, $y = -x^2 + 4x - 4$ is **[IIT-2006]**
(A) $y = 4(x - 1)$ (B) $y = 0$ (C) $y = -4(x - 1)$ (D) $y = -30x - 50$

C. Multiple choice questions with one correct answer :

1. The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is
(A) $\left(-\frac{16}{5}, \frac{27}{10}\right)$ (B) $\left(-\frac{16}{7}, \frac{53}{10}\right)$ (C) $\left(-\frac{16}{5}, \frac{53}{10}\right)$ (D) none of these **[IIT - 83]**

2. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and parabola is **[IIT - 95]**
(A) $\left(\frac{p}{2}, p\right)$ or $\left(\frac{p}{2}, -p\right)$ (B) $\left(\frac{p}{2}, -\frac{p}{2}\right)$
(C) $\left(-\frac{p}{2}, p\right)$ (D) $\left(-\frac{p}{2}, -\frac{p}{2}\right)$

3. The curve described parametrically by $x = t^2 + t + 1$, $y = t^2 - t + 1$ represents **[IIT - 2000]**
(A) a pair of straight lines (B) an ellipse
(C) a parabola (D) a hyperbola

4. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of 'k' is :
(A) 1/ 8 (B) 8 (C) 4 (D) 1/4 **[IIT - 2000]**

5. If $x + y = k$ is normal to $y^2 = 12x$, then 'k' is : **[IIT - 2000]**
(A) 3 (B) 9 (C) - 9 (D) - 3

6. The locus of the midpoint of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix **[IIT - 2000]**
(A) $x = -a$ (B) $x = -\frac{a}{2}$ (C) $x = 0$ (D) $= \frac{a}{2}$

7. The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x -axis is **[IIT - 2001]**
(A) $\sqrt{3}y = 3x + 1$ (B) $\sqrt{3}y = -(x + 3)$ (C) $\sqrt{3}y = x + 3$ (D) $\sqrt{3}y = -(3x + 1)$

8. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is **[IIT - 2001]**
(A) $x = -1$ (B) $x = 1$ (C) $x = -\frac{3}{2}$ (D) $x = \frac{3}{2}$

9. The equation of the common tangent to the curve $y^2 = 8x$ and $xy = -1$ is **[IIT - 2002]**
(A) $3y = 9x + 2$ (B) $y = 2x + 1$ (C) $2y = x + 8$ (D) $y = x + 2$

10. The locus of the midpoint of the line segment joining the focus to a moving on the parabola $y^2 = 4ax$ is another parabola with directrix **[IIT - 2002]**

(A) $x = -a$ (B) $x = -\frac{a}{2}$ (C) $x = 0$ (D) $x = \frac{a}{2}$

11. The focal chord to $y^2 = 16x$ is tangents to $(x-6)^2 + y^2 = 2$, then the possible value of the slope of this chord are **[IIT - 2003]**

(A) $\{-1, 1\}$ (B) $\{-2, 2\}$ (C) $\{-2, 1/2\}$ (D) $\{2, -1/2\}$

12. The area enclosed between the curves $y = ax^2$ and $x = ay^2 (a > 0)$ is 1 sq. unit, then the value of a is **[IIT - 2003]**

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1}{3}$

13. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ **[IIT - 2004]**

14. Tangent to the curve $y = x^2 + 6$ at a point (1, 70 touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q, then the coordinates of Q are **[IIT - 2005]**

(A) (–6, –11) (B) (–9, –13) (C) (–10, –15) (D) (–6, –7)

15. Axis of a parabola is $y = x$ and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ respectively from the origin, then equation of the parabola is **[IIT-2006]**

(A) $(x - y)^2 = 8(x + y - 2)$ (B) $(x + y)^2 = 2(x + y - 2)$
(C) $(x - y)^2 = 4(x + y - 2)$ (D) $(x + y)^2 = 2(x - y + 2)$

(D)

1. **Statement 1 :** The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line

because

Statement -2 : A parabola is symmetric about its axis.

(A) Statement-1 is True, Statment-2 is True. Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True, Statement-2 **IS NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True **{IIT-2007}**

(E) Linked Comprehension Type

Paragraph for Question Nos. 1 to 3

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

1. The ratio of the areas of the triangles PQS and PQR is

(A) $1 : \sqrt{2}$ (B) 1 : 2 (C) 1 : 4 (D) 1 : 8

2. The radius of the circumcircle of triangle PRS is

(A) 5 (B) $3\sqrt{3}$ (C) $3\sqrt{2}$ (D) $2\sqrt{3}$

3. The radius of the incircle of the triangle PQR is

(A) 4 (B) 3 (C) 8/3 (D) 2

[IIT-2007]

(F) Match the following **[IIT-2006]**

1. Normals are drawn at points P, Q and R lying on the parabola $y^2 = 4x$ which intersect at (3, 0). Then

	Column I		Column II
(i)	Area of ΔPQR	(a)	2
(ii)	Radius of circumcircle of ΔPQR	(b)	5/2
(iii)	Centroid of ΔPQR	(c)	(5/2, 0)
(iv)	Circumcenter of ΔPQR	(d)	(2/ 3, 0)

IIT JEE PROBLEMS (SUBJECTIVE)

1. Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point (h, k). Show that $h > 2$. **[IIT – 81]**

2. A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B. If AB subtends a right angle at the vertex of the parabola. Find the slope of AB. **[IIT – 82]**

3. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2). **[IIT - 84]**

4. Three normals are drawn from the point (c, 0) to the curve $y^2 = x$. Show that c must be greater than $\frac{1}{2}$. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other. **[IIT – 91]**

5. What normal to the curve $y = x^2$ forms the shortest chord ? **[IIT – 92]**

6. Through the vertex O of a parabola $y^2 = 4x$, chords OP and OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ. **[IIT - 94]**

7. From the point (-1, 2) tangent lines are drawn to the parabola $y^2 = 4x$. Find the equation of the chord of contact. Also find the area of the triangle formed by the chord of contact and the tangents. **[REE - 94]**

8. Consider a square with vertices at (1, 1), (–1, 1), (–1, –1) and (1, –1). Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area. **[IIT - 95]**

9. Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1 : 2 is a parabola. Find the vertex of this parabola. **[IIT - 95]**

10. A ray of light is coming along the line y = b from the positive direction of x - axis and strikes a concave mirror whose intersection with the x y plane is a parabola $y^2 = 4ax$. Find the equation of the reflected ray and show that it passes through the focus of the parabola. Both a and b are positive. **[REE- 95]**

11. Points A, B and C lie on the parabola $y^2 = 4ax$. The tangents to the parabola at A, B and C, taken in pairs, intersect at points P, Q and R. Determine the ratio of the areas of the triangles ABC and PQR. **[IIT - 96]**

12. From a point A common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ and parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola. **[IIT - 96]**

13. Find the locus of the point of intersection of those normals to the parabola $x^2 = 8y$ which are at right angles to each other. **[REE -97]**

14. The angle between a pair of tangents drawn from a point P to the parabola $y^2 = 4ax$ is 45^0. Show that the locus of the point P is a hyperbola. **[IIT - 98]**

15. The ordinates of points P and Q on the parabola $y^2 = 12x$ are in the ratio 1 : 2. Find the locus of the point of intersection of the normals to the parabola at P and Q. **[REE -98]**

16. Find the equations of the common tangents of the circle $x^2 + y^2 - 6y + 4 = 0$ and the parabola $y^2 = x$. **[REE -99]**

17. Let C_1 and C_2 be respectively, the parabolas $x^2 = y - 1$ and $y^2 = x - 1$. Let P be any point on C_1 and Q be any point on C_2. Let P_1 and Q_1 be the reflections of P and Q, respectively, with respect to the line y = x. Prove that P_1 lies on C_2, Q_1 lies on C_1 and $PQ \geq \min\{PP_1, QQ_1\}$. Hence or otherwise determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (P, Q) with P on C_1 and Q on C_2. **[IIT - 2000]**

18. Find the locus of the points of intersection of tangents drawn at the ends of all normals chords of the parabola $y^2 = 8(x - 1)$. **[REE -2001]**

19. Three normals with slopes $m_1 m_2$ and m_3 are drawn from a point P not on the axis of the parabola $y^2 = 4x$. If $m_1 m_2 = \alpha$, results in the locus of P being a part of the parabola, find the value of α. **[IIT - 2003]**

20. At any point P on the parabola $y^2 - 2y - 4x + 5 = 0$, a tangent is drawn which meets the directrix at Q. Find the locus of point R which divides QP externally in the ratio $\frac{1}{2}:1$. **[IIT - 2004]**

SET–I

1. If $y^2 - 2x - 2y + 5 = 0$ is
(A) a circle with centre (1, 1)
(B) a parabola with vertex (1, 2)
(C) a parabola with directrix $x = 3/2$
(D) a parabola with directrix $x = -1/2$

2. The focus of the parabola $x^2 - 8x + 2y + 7 = 0$ is
(A) $\left(0, -\frac{1}{2}\right)$
(B) $\left(4, \frac{9}{2}\right)$
(C) (4, 4)
(D) $\left(-4, -\frac{9}{2}\right)$

3. Equation of a common tangent to the circle, $x^2 + y^2 = 50$ and the parabola, $y^2 = 40x$ can be
(A) $x + y - 10 = 0$
(B) $x - y + 10 = 0$
(C) $x - y - 10 = 0$
(D) none of these

4. The line $y = mx + c$ touches the parabola $x^2 = 4ay$ if
(A) $c = -am$
(B) $c = -\frac{a}{m}$
(C) $c = -am^2$
(D) $\frac{a}{m^2}$

5. The length of the latus rectum of the parabola $4y^2 + 2x - 20y + 17 = 0$ is
(A) 3
(B) 6
(C) 1/2
(D) 9

6. In a parabola semi-latus rectum is the harmonic mean of the
(A) segments of a focal chord
(B) segments of the directrix
(C) segments of a chord
(D) none of these

7. If $(t^2, 2t)$ is one end of a focal chord of the parabola, $y^2 = 4x$ then the length of the focal chord will be :
(A) $\left(t + \frac{1}{t}\right)^2$
(B) $\left(t + \frac{1}{t}\right)\sqrt{\left(t^2 + \frac{1}{t^2}\right)}$
(C) $\left(t - \frac{1}{t}\right)\sqrt{\left(t^2 + \frac{1}{t^2}\right)}$
(D) none of these

8. The equation of the parabola with its vertex at (1, 1) and focus at (3, 1) is
(A) $(x - 3)^2 = 8(y - 1)$
(B) $(y - 1)^2 = 8(x - 1)$
(C) $(y - 1)^2 = 8(x - 3)$
(D) $(x - 1)^2 = 8(y - 1)$

9. Equation of the tangent at $(-4, -4)$ on $x^2 = -4y$ is
(A) $2x - y + 4 = 0$
(B) $2x + y - 4 = 0$
(C) $2x - y - 12 = 0$
(D) $2x + y + 4 = 0$

10. The locus of the midpoint of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix
(A) $x = -a$
(B) $x = a$
(C) $x = 0$
(D) $x = \frac{a}{2}$

11. The point of intersection of the tangents to the parabola at the points t_1 and t_2 is
(A) $(a\,t_1 t_2,\ a(t_1 + t_2))$
(B) $(2a\,t_1 t_2,\ a(t_1 + t_2))$
(C) $(2a\,t_1 t_2,\ 2a(t_1 + t_2))$
(D) none of these

12. The tangent drawn at any point P to the parabola $y^2 = 4ax$ meets the directrix at the point K. Then the angle which KP subtends at the focus is

(A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

13. The equation of the parabola whose focus is the point (0, 0) and the tangent at the vertex is $x - y + 1 = 0$ is

(A) $x^2 + y^2 - 2xy - 4x + 4y - 4 = 0$ (B) $x^2 + y^2 - 2xy + 4x - 4y - 4 = 0$
(C) $x^2 + y^2 + 2xy - 4x + 4y - 4 = 0$ (D) $x^2 + y^2 + 2xy - 4x - 4y + 4 = 0$

14. If (x_1, y_1), (x_2, y_2), (x_3, y_3) be three points on the parabola $y^2 = 4ax$, the normals at which meet in a point, then

(A) $y_1 + y_2 + y_3 = 0$ (B) $x_1 + x_2 + x_3 = 0$
(C) $y_1 + y_2 + y_3 = 2a$ (D) $x_1 + x_2 + x_3 = 4a$

15. The normal at a point on $y^2 = 4x$ passes through (5, 0). There are three such normals one of which is the axis. The feet of the three normals from a triangle whose centroid is

(A) (2, 0) (B) (0, 2) (C) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (D) (5, 0)

16. The locus of the point of intersection of the perpendicular tangents to the parabola $x^2 = 4ay$ is

(A) $y = a$ (B) $y = -a$ (C) $x = a$ (D) $x = -a$

17. The normal at the point $P(ap^2, 2ap)$ meets the parabola $y^2 = 4ax$ again at $Q(aq^2, 2aq)$ such that the lines joining the origin to P and Q are at right angle: Then

(A) $p^2 = 2$ (B) $q^2 = 2$ (C) $p = 2q$ (D) $q = 2p$

18. If $2x + y + k = 0$ is a normal to the parabola $y^2 = -8x$, then the value of k is

(A) –16 (B) –8 (C) –24 (D) 24

19. The angle between the tangents drawn from the origin to the parabola $y^2 = 4a(x-a)$ is

(A) 90° (B) 45° (C) 60° (D) $\tan^{-1}2$

20. The parametric coordinates of any point on the parabola $y^2 = x$ can be

(A) $(\sec^2\theta, \sec\theta)$ (B) $(\sin^2\theta, \sin\theta)$ (C) $(\cos^2\theta, \cos\theta)$ (D) none of these

SET–II

1. If the normals at points t_1 and t_2 meet on the parabola, then

(A) $t_1t_2 = -1$ (B) $t_2 = -t_1 - \frac{2}{t_1}$ (C) $t_1t_2 = 2$ (D) none of these

2 Which one of the following equations represented parametrically, represents equation to a parabolic profile?

(A) $x = 3\cos t$; $y = 4\sin t$ (B) $x^2 - 2 = -2\cos t$; $y = 4\cos^2\frac{t}{2}$

(C) $\sqrt{x} = \tan t$; $\sqrt{y} = \sec t$ (D) $x = \sqrt{1 - \sin t}$; $y = \sin\frac{t}{2} + \cos\frac{t}{2}$

3. The condition that the two tangents to the parabola $y^2 = 4ax$ become normal to the circle $x^2 + y^2 - 2ax - 2by + c = 0$ is given by

(A) $a^2 > 4b^2$ (B) $b^2 > 2a^2$ (C) $a^2 > 2b^2$ (D) $b^2 > 4a^2$

4. The locus of the middle points of focal chords of a parabola is

(A) $y^2 = 2a(x - a)$ (B) $y^2 = 2a(x + a)$ (C) $x^2 = 2a(y - a)$ (D) $x^2 > 2a(y + a)$

5. The normal chord of a parabola $y^2 = 4ax$ at (x_1, x_1) subtends a right angle at the

(A) focus (B) vertex

(C) end of the latusrectum (D) none of these

6. If the chord $y = mx + c$ subtends a right angle at the vertex of the parabola $y^2 = 4ax$, then the value of c is

(A) $-4am$ (B) $4\ am$ (C) $-2am$ (D) $2\ am$

7. If P(–3, 2) is one end of the focal chord PQ of the parabola $y^2 + 4x + 4y = 0$, then the slope of the normal at Q is

(A) $-\frac{1}{2}$ (B) 2 (C) $\frac{1}{2}$ (D) –2

8. The angle between tangents to the parabola $y^2 = 4ax$ at the points where it intersects with the line $x-y-a = 0$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

9. If the lines $(y - b) = m_1(x + a)$ and $(y - b) = m_2(x + a)$ are the tangents of $y^2 = 4ax$ then

(A) $m_1 + m_2 = 0$ (B) $m_1m_2 = 1$ (C) $m_1m_2 = -1$ (D) $m_1 + m_2 = 1$

10. The parabola $y^2 = 4x$ and the circle $(x - 6)^2 + y^2 = r^2$ will have no common tangent if 'r' is equal to

(A) $r > \sqrt{20}$ (B) $r < \sqrt{20}$ (C) $r > \sqrt{18}$ (D) $r \in (\sqrt{20}, \sqrt{28}$

11. Parabolas $y^2 = 4a(x - c_1)$ and $x^2 = 4a(y - c_2)$, where c_1 and c_2 are variable, are such that they touch each other. Locus of their point of contact is

(A) $xy = 2a^2$ (B) $xy = 4a^2$ (C) $xy = a^2$ (D) none of these

12. Minimum distance between the curve $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$ is equal to

(A) $\sqrt{21}$ (B) $\sqrt{26} - \sqrt{5}$ (C) $\sqrt{21} - \sqrt{5}$ (D) $\sqrt{28} - \sqrt{5}$

13. The locus of the foot of the perpendicular from the focus upon a tangent to the parabola $y^2 = 4ax$ is

(A) the directrix (B) tangent at the vertex

(C) $x = a$ (D) none of these

14. If (x_r, y_r); r = 1, 2, 3, 4 be the points of intersection of the parabola $y^2 = 4ax$ and the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, then

(A) $y_1 + y_2 + y_3 + y_4 = 0$ (B) $y_1 + y_2 - y_3 - y_4 = 0$

(C) $y_1 - y_2 + y_3 - y_4 = 0$ (D) $y_1 - y_2 - y_3 + y_4 = 0$

15. The length of a focal chord of the parabola $y^2 = 4ax$ making an angle θ with the axis of the parabola is

(A) $4a \operatorname{cosec}^2\theta$ (B) $4a \sec^2\theta$ (C) $a \operatorname{cosec}^2\theta$ (D) none of these

16. Three normals to the parabola $y^2 = x$ are drawn through a point (C, 0) then

(A) $C = \frac{1}{4}$ (B) $C = \frac{1}{2}$ (C) $C > \frac{1}{2}$ (D) none of these

17. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord . The modulus of the difference of the ordinates of the points Q and R is :

(A) $\frac{A}{2a}$ (B) $\frac{A}{a}$ (C) $\frac{2A}{a}$ (D) $\frac{4A}{a}$

18. The tangent and normal at the point $P(at^2, 2at)$ to the parabola $y^2 = 4ax$ meet the x-axis in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the parabola is inclined to the tangent at P to the circle through P, T, G is

(A) $\tan^{-1}(t^2)$ (B) $\cot^{-1}(t^2)$ (C) $\tan^{-1}(t)$ (D) $\cot^{-1}(t)$

19. From an external point P, pair of tangent lines are drawn to the parabola, $y^2 = 4x$. If θ_1 & θ_2 are the inclinations of these tangents with the axis of x such that, $\theta_1 + \theta_2 = \frac{\pi}{4}$, then the locus of P is :

(A) $x - y + 1 = 0$ (B) $x + y - 1 = 0$ (C) $x - y - 1 = 0$ (D) $x + y + 1 = 0$

20. Length of the chord of contact of the pair of tangents drawn from on the point (x_1, y_1) the parabola, $y^2 = 4ax$ is :

(A) $\frac{1}{a}\sqrt{\left(y_1^2 + 4ax_1\right)\left(y_1^2 - 4a^2\right)}$ (B) $\frac{1}{2a}\sqrt{\left(y_1^2 - 4ax_1\right)\left(y_1^2 + 4a^2\right)}$

(C) $\frac{1}{a}\sqrt{\left(y_1^2 - 4ax_1\right)\left(y_1^2 + 4a^2\right)}$ (D) $\frac{1}{a}\left(y_1^2 - 4ax_1\right)\left(y_1^2 + 4a^2\right)$

SET–III

1. Equation $x^2 - 2x - 2y + 5 = 0$ represents
(A) a circle with centre (1, 1)
(B) a parabola with vertex (1, 2)
(C) a parabola with directrix $y = 5/2$
(D) a parabola with directrix $y = -1/3$

2. The normals to the parabola $y^2 = 4ax$ from the point (5a, 2a) are
(A) $y = x - 3a$ (B) $y = -2x + 12a$ (C) $y = -3x + 33a$ (D) $y = x + 3a$

3. The equation of the lines joining the vertex of the parabola $y^2 = 6x$ to the points on it whose abscissa is 24, is
(A) $y \pm 2x = 0$ (B) $2y \pm x = 0$ (C) $x \pm 2y = 0$ (D) $2x \pm y = 0$

4. The equation of the tangent to the parabola $y^2 = 9x$ which goes through the point (4, 10) is
(A) $x + 4y + 1 = 0$ (B) $9x + 4y + 4 = 0$ (C) $x - 4y + 36 = 0$ (D) $9x - 4y + 4 = 0$

5. Consider the equation of a parabola $y^2 + 4ax = 0$, where $a > 0$. Which of the following is false ?
(A) tangent at the vertex is $x = 0$
(B) directrix of the parabola is $x = 0$
(C) vertex of the parabola is at the origin
(D) focus of the parabola is at (a, 0)

Question based on write-up

☞ Normally, the various propositions you study, e.g. equation of tangent, normal, chord, focal chord, formula for focal distance etc, are derived for the parabola $y^2 = 4ax$. However, all the results with slight transformation are valid for any parabola. Suppose we represent the equation of parabola $y^2 - 4ax = 0$ by S (x, y, a) = 0 and any equation derived for this parabola by P(x, y, a) = 0. Now, if the given parabola is $y^2 = -4ax$, $y^2 + 4ax = 0$ we can write, if S(x, y, – a) = 0, so the corresponding equation of P will be P(x, y, – a) = 0. Similarly for $x^2 = 4ay$ can be written as S(x, y, a) and corresponding transformation is P(x, y, a) (i.e. interchange x and y).

6. The focal distance of the point (x, y) on the parabola $x^2 - 8x + 16y = 0$ is
(A) $|y - 4|$ (B) $|y - 5|$ (C) $|y - 2|$ (D) $|x - 4|$

7. Normals are drawn from the point (7, 14) to the parabola $x^2 - 8x - 16y = 0$. The slopes of these normals are
(A) 1 (B) –2 (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

8. The coordinates of the feet of the normals obtained in previous problem are
(A) (0, 8) (B) (4, 3) (C) (16, 8) (D) (–8, 4)

9. The points on the axis of the parabola $x^2 + 2x + 4y + 13 = 0$ from the where three distinct normals can be drawn are given by
(A) $(-1, k)$, $k \in (-1, 2)$
(B) (–1, 6)
(C) $(-1, k)$, $k \in (-\infty, -5)$
(D) (–1, 1)

10. The line $x\cos\alpha + y\sin\alpha = p$ touches the parabola $x^2 + 4a(y + a) = 0$, if
(A) $a = p\sec\alpha$
(B) $a\cos 2\alpha = p\sin\alpha$
(C) $a^2\cos\alpha + p^2\sin\alpha = 0$
(D) $a\tan\alpha = p\sec\alpha$

Three normals can be drawn from a point (x_1,y_1) to the parabola $y^2 = 4ax$. The points where these normals meet the parabola are called the feet of the normals or conormal points. The sum of the slopes of the normals is zero and the sum of the ordinates of the feet of the normals is also zero.

$$m_1 + m_2 + m_3 = 0 \quad \Rightarrow \quad y_1 + y_2 + y_3 = -2a(m_1 + m_2 + m_3) = 0$$

$$m_1 m_2 + m_2 m_3 + m_3 m_1 = \frac{2a - x_1}{a}$$

$$m_1 m_2 m_3 = -\frac{y_1}{a},$$ where m_1, m_2, m_3 are slopes and y_1, y_2, y_3 are ordinates.

11. The locus of the point of intersection of the three normals to the parabola $y^2 = 4ax$, two of which are inclined at right angles to each other is

(A) $y\{y^2 + (3a + x)a\} = 0$ (B) $y\{y^2 + (3a - x)a\} = 0$
(C) $y\{y^2 - (3a - x)a\} = 0$ (D) none of these

12. The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h, k). The centroid of triangle PQR lies on

(A) $x = 0$ (B) $y = 0$
(C) $x = -a$ (D) none of these

13. The number of distinct normals that can be drawn from $\left(\frac{11}{4}, \frac{1}{4}\right)$ to the parabola $y^2 = 4x$ is

(A) 1 (B) 2 (C) 3 (D) none of these

14. Three normals to the parabola $y^2 = x$ are drawn through a point (C, 0), then

(A) $C = \frac{1}{4}$ (B) $C = \frac{1}{2}$ (C) $C > \frac{1}{2}$ (D) none of these

15. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissa are in AP, then the slopes of the tangents at the three conormal points are in

(A) G.P (B) A.P.
(C) H.P. (D) none of these

16. The locus of the points such that two of the three normals from them to the parabola $y^2 = 4ax$ coincide is :

(A) $27ay^2 + 4(x + 2a)^3 = 0$ (B) $27ay^2 + 4(x - 2a)^3 = 0$
(C) $27ay^2 = 4(x - 2a)^3$ (D) none of these

17. **Assertion :** Solpe of tangents drawn from (4, 10) to parabola $y^2 = 9x$ are $\frac{1}{4}, \frac{9}{4}$.

Reason : Every parabola is symmetric about its directrix.

(A) both Assertion and Reason are true and Assertion is the correct explanation of 'Reason'
(B) both Assertion and Reason are true and Assertion is not the correct explanation of 'Reason'
(C) Assertion is true but Reason is false
(D) Assertion is false but Reason is true

18. True/False :

(i) The equation of the parabola whose focus is at the origin is $y^2 = 4a(x + a)$.

(ii) The locus of the mid–points of the chords of the parabola $y^2 = 4ax$ which pass through the vertex, is the parabola $y^2 = 2ax$

(iii) The focus of the parabola $x^2 + 8y = 0$ is at (0, 2)

(iv) The line $y = mx + c$ is a tangent to the parabola $y^2 = 4a(x + a)$, then $c = ma + \frac{a}{m}$

(v) The points $x = 2at,\ y = at^2$ are lies on the parabola $x^2 = 4ay$.

19. Fill in the blanks :

(i) The equation of the parabola whose focus is the point (2, 3) and directrix is the line $x - 4y + 3 = 0$ is ....and the length of its latus rectum is.............

(ii) For the parabola $y^2 + 4x - 6y + 13 = 0$, the vertex is ...., focus is......... directrix is.........L.R. is.........

(iii) The length of the latus rectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is ....

(iv) The focus of the parabola $y = 2x^2 + x$ is ....

(v) The vertex of the parabola $(y - 2)^2 = 16(x - 1)$ is ....

20. Match the column

Column I		**Column II**
(a)	The latus rectum of the parabola $y^2 = 5x + 4y + 1$ is	(P) $x = \frac{3}{2}$
(b)	Two perpendicular tangents to $y^2 = 4ax$ always intersect on the line	(Q) 0
(c)	The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is	(R) $\frac{3\sqrt{2}}{4}$
(d)	The number of tangent(s) to the parabola $y^2 = 8x$ through (2, 1) is	(S) $x = -a$
(e)	If two different tangents of $y^2 = 4ax$ are the normals to $x^2 = 4by$ then $\|b\|$ is less than	(T) 5
(f)	Minimum distance between the curves $y^2 = x - 1$ and $x^2 = y - 1$ is	(U) $\frac{1}{2\sqrt{2}}$

LEVEL–I ANSWER

1.	$y = 2x^2 + 3x + 4$, LR = 2 unit	**4.**	$\left(\frac{1}{12}, -\frac{1}{3}\right), \left(\frac{4}{3}, \frac{4}{3}\right)$
5.	$5x + y = 135a$	**8.**	$4a\sqrt{3}$ unit
9.	$y = -4x + 72$, $y = 3x - 33$	**10.**	$y^2 = 4a(x - 8a)$

LEVEL–II

2. $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$, $(4am^2, -4am)$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. (–1, 0)

(B)

1. AB

(C)

1.	C	**2.**	A	**3.**	C	**4.**	C	**5.**	B
6.	C	**7.**	C	**8.**	D	**9.**	D	**10.**	C
11.	A	**12.**	A	**13.**	C	**14.**	D	**15.**	A

(D)

1. A

(E)

1. C **2.** B **3.** D

(F)

1. i-a, ii-b, iii-d, iv-c

IIT JEE PROBLEMS (SUBJECTIVE)

2. $\pm\sqrt{2}$

3. $x + y - 3 = 0$

4. $\frac{3}{4}$

5. $y = \frac{x}{\sqrt{2}} + 1,\ y = -\frac{x}{\sqrt{2}} + 1$

6. $(4, 0)$; $y^2 = 2(x - 4)$

7. $x - y = 1$; $8\sqrt{2}$ sq. units

9. $\left(\frac{4}{9}, x - \frac{2}{9}\right) = \left(y - \frac{8}{9}\right)^2$ vertex $\left(\frac{2}{9}, \frac{8}{9}\right)$

10. $4abx + (4a^2 - b^2)y - 4a^2b = 0$

11. 2 : 1

12. $15a^2/4$

13. $x^2 - 2y + 12 = 0$

15. $x = 3\left[7\left(\frac{y}{18}\right)^{\frac{2}{3}} + 2\right]$

16. $x - 2y + 1 = 0$; $y = mx + \frac{1}{4m}$, where $m = \frac{-5 \pm \sqrt{30}}{10}$

18. $(x + 3)y^2 + 32 = 0$

19. $\alpha = 2$

20. $(x - 1)(y - 1)^2 + 4$

SET–I

1.	C	**2.**	C	**3.**	B	**4.**	C	**5.**	C
6.	A	**7.**	A	**8.**	C	**9.**	A	**10.**	C
11.	A	**12.**	D	**13.**	C	**14.**	A	**15.**	A
16.	B	**17.**	A	**18.**	D	**19.**	A	**20.**	D

SET–II

1.	C	**2.**	B	**3.**	D	**4.**	A	**5.**	A
6.	A	**7.**	A	**8.**	D	**9.**	C	**10.**	B
11.	B	**12.**	C	**13.**	B	**14.**	A	**15.**	A
16.	C	**17.**	C	**18.**	C	**19.**	C	**20.**	C

SET–III

1.	BC	**2.**	AB	**3.**	BC	**4.**	CD	**5.**	BD
6.	B	**7.**	C	**8.**	C	**9.**	C	**10.**	B
11.	B	**12.**	B	**13.**	C	**14.**	C	**15.**	C
16.	C	**17.**	C						

18. **(i)** T **(ii)** T **(iii)** **F** **(iv)** **T** **(v)** **T**

19. **(i)** $16x^2 + y^2 + 8xy - 74x - 78y + 212, \dfrac{14}{\sqrt{17}}$ **(ii)** $(-1, 3), (-2, 3), x = 0, -4$

(iii) 8 **(iv)** $\left(-\dfrac{1}{4}, 0\right)$ **(v)** $(1, 2)$

20. a-T, b-S, c-P, d-Q, e-U, f-R