Question Bank - Probability

LEVEL–I

1. A ten digit number is formed using the digits from 0 to 9, every digit being used exactly once. Find the probability that the number is divisible by 4.

2. In a box, there are 8 alphabets cards with the letters : S, S, A, A, A, H, H, H. Find the probability that the word 'ASH' will form if :

(i) the three cards are drawn one by one and placed on the table in the same order that they are drawn.

(ii) the three cards are drawn simultaneously.

3. Three persons A, B and C in order cut a pack of playing cards, replacing them after each cut, on the condition that the first who cuts a card of spade shall win a prize. Find their respective chances.

4. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Find the probability that out of four face values obtained, the minimum face value is not less than 2 & the maximum face value is not greater than 5.

5. Three of the six vertices of a regular hexagon are chosen at random. Find the probability that the triangle with these three vertices is equilateral.

6. A cube with all six faces coloured is cut into 64 cubical blocks of the same size which are thoroughly mixed. Find the probability that the 2 randomly chosen blocks have 2 coloured faces each.

7. Consider the following events for a family with children
A = { of both sexes } ; {at most one boy }
In which of the following (are/is) the events A and B are independent.
Assume that the birth of a boy or a girl is equally likely mutually exclusive and exclusive.

8. In a game of skill between two players A and B, the probability of A winning a game is 2/5, if he looses the previous game and 3/5 if he wins the previous game. Find the probability that in middle of series of games, A wins the next two games in succession.

9. For any two events A and B, prove that $P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$. Let A and B be two events such that P(A) = $\frac{3}{4}$ and P(B) = $\frac{5}{8}$. Show that

(i) $P(A \cup B) \ge \frac{3}{4}$ **(ii)** $\frac{3}{8} \le P(A \cap B) \le \frac{5}{8}$ **(iii)** $\frac{1}{8} \le P(A \cap \bar{B}) \le \frac{3}{8}$.

10. Each of the 'n' passengers sitting in a bus may get down from it at the next stop with probability p. Moreover, at the next stop either no passenger or exactly one passenger boards the bus. The probability of no passenger boarding the bus at the next stop being p_o. Find the probability that when the bus continues on its way after the stop, there will again be 'n' passengers in the bus.

LEVEL–II

1. There are 2 groups of subjects, one of which consists of 5 science subjects and 3 engineering subjects and other consists of 3 science and 5 engineering subjects. An unbiased die is cast. If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from second group. Find the probability that an engineering subject is selected.

2. A certain drug, manufactured by a Company is tested chemically for its toxic nature. Let the event "*THE DRUG IS TOXIC*" be denoted by H and the event "*THE CHEMICAL TEST REVEALS THAT THE DURG IS TOXIC*" be denoted by S.

Let P(H) = a. P(S/H) = $P\left(\bar{S}/\bar{H}\right)$ = 1 – a. Show that probability that the drug is not toxic given that the chemical test reveals that it is toxic, is free from 'a'.

3. Suppose a sample space consists of the integers 1, 2, 3, . . ., 2n. The probability of choosing an integer k is proportional to log k. Show that the conditional probability of choosing the integer 2, given that an even integer is chosen, is $\frac{\log 2}{n\log 2+\log(n!)}$.

4. A box contains three coins. Two of them are fair and one two – headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.

(i) Find the probability that head appears twice.

(ii) If the same coin is tossed twice, find the probability that it was two headed coin.

(iii) Find the probability that tail appears twice.

5. A special die is so constructed that the probabilities of throwing 1, 2, 3, 4, 5 and 6 are $\frac{1-k}{6},\frac{1+2k}{6},\frac{1-k}{6},\frac{1+k}{6},\frac{1-2k}{6}$ and $\frac{1+k}{6}$ respectively. If two such dice are thrown and the probability of getting a sum equal to 9 lies between $\frac{1}{9}$ and $\frac{2}{9}$, find the set of integral values of k.

6. **(a)** Two natural numbers x and y are chosen at random. Find the probability that $x^2 + y^2$ is divisible by 10.

(b) Two numbers x and y are chosen at random from the set {1, 2, 3, 4, . . . , 3n}. Find the probability that $x^2 - y^2$ is divisible by 3.

7. A box contains 5 radio tubes of which 2 are defective. The tubes are tested one after the other until the 2 defective tubes are discovered. Find the probability that the process stopped on the

(i) Second test : **(ii)** Third test.

(iii) If the process stopped on the third test, find the probability that the first tube is non defective.

8. Sixteen players $S_1, S_2, \ldots, S_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

(i) Find the probability that the player S_1 is among the eight winners.

(ii) Find the probability that exactly one of the two players S_1 and S_2 is among the eight winners.

9. Three points P, Q and R are selected at random from the circumference of a circle. Find the probability that the points lie on a semicircle.

10. A bag contains 'a' white and 'b' black balls. Two players A and B alternately draw a ball from the bag, replacing the ball each time after the draw till one of them draws a white ball and wins the game. If A begins the game and the probability of A winning the game is three times that of B, show that $a : b = 2 : 1$.

IIT JEE PROBLEMS (OBJECTIVE)

A. Fill in the blanks

1. For a biased die the probabilities for the different faces to turn up are given below :

Face	**1**	**2**	**3**	**4**	**5**	**6**
Probability	**0.1**	**0.32**	**0.21**	**0.15**	**0.05**	**0.17**

This die is tossed and you are told that either face 1 or 2 has turned up. Then the probability that it is face 1 is ..... **[IIT - 81]**

2. $P(A \cup B) = P(A \cap B)$ if and only if the relation between P(A) and P(B) is..... **[IIT - 85]**

3. A box contains 100 tickets numbered 1, 2,....., 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The maximum number on them is 5 with probability..... **[IIT - 85]**

4. If $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is..... **[IIT - 86]**

5. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. If one bail is now drawn at random from urn A, the probability that it is found to be red is ..... . **[IIT - 88]**

6. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is ..... **[IIT - 89]**

7. Let A and B be two events such that P(A) = 0.3 and $P(A \cup B) = 0.8$. If A and B are independent events, then P(B) = ... **[IIT - 90]**

8. If the mean and the variance of a binomial variable X are 2 and 1 respectively, then the probability that X takes a value greater than one is equal to..... **[IIT - 91]**

9. Three faces of a fair die are yellow, two faces red and one blue. The die is tossed three times the probability that the colours, yellow, red and blue, appear in the first, second and the third tosses respectively is..... **[IIT - 92]**

10. If two events A and B are such that $P(A^C) = 0.3$, (B) = 0.4 and $(P(AB^C) = 0.5$, then $P(B/(A \cup B^C)) =$ ..... . **[IIT - 94]**

11. Three numbers are chosen at random without replacement from {1, 2, ... , 10}. The probability that the minimum of the chosen number is 3 , or their maximum is 7, is ..... . **[IIT - 97]**

B. True or False

1. If the letters of the word "ASSASSIN" are written down at random in a row, the probability that no two S's occur together is 1/35. **[IIT - 83]**

2. The probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is 0.5. **[IIT - 89]**

C. Multiple choice questions with one and more than one correct answer.

1. If M and N are any two events, the probability that exactly one of them occurs is **[IIT - 84]**

(A) $P(M) + P(N) - 2P(M \cap N)$ (B) $P(M) + P(N) - P(M \cap N)$

(C) $P(M^c) + P(N^c) - 2P(M^c \cap N^c)$ (D) $P(M \cap N^c) - P(M^c \cap N)$

2. A student appears for tests I, II and III. The student is successful if he passes either in test I and II or test I and III. The probabilities of the students passing test I, II and III are p, q and 1/2 respectively. If the probability that the student is successful is 1/2, then **[IIT - 86]**

(A) p = q = 1 (B) p = q = 1/2

(C) p = 1, q = 0 (D) p = 1, q = 1/2 (E) none of these

3. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$ is **[IIT - 87]**

(A) 0.4 (B) 0.8 (C) 1.2 (D) 1.4 (E) none of these

4. For two given events A and B, $P(A \cap B)$ **[IIT - 88]**

(A) not less than P(A) + P(B) – 1 (B) not greater than P(A) + P(B)

(C) equal to P(A) + P(B) – $P(A \cup B)$ (D) equal to P(A) + P(B) + $P(A \cup B)$

5. If E and F are independent events such that 0 < P(E) < 1 and 0 < P(F) < 1, then **[IIT - 89]**

(A) E and F are mutually exclusive

(B) E and F^c (the complement of the event F) are independent

(C) E^c and F^c are independent (D) $P(E \mid F) + P(E^c \mid F) = 1$

6. For any events A and B in a sample space **[IIT - 91]**

(A) $P(A/B) \geq \frac{P(A) + P(B) - 1}{P(B)}$, $P(B) \neq 0$ is always true

(B) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$ does not hold

(C) $P(A \cup B) = 1 - P(\overline{A})P(\overline{B})$, if A and B are independent

(D) $P(A \cup B) = 1 - P(\overline{A})P(\overline{B})$, if A and B are disjoint

7. Let E and F be two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2. Then **[IIT - 93]**

(A) p(E) = 1/3, p(F) = 1/4 (B) p(E) = 1/2, p(F) = 1/6

(C) p(E) = 1/6, p(F) = 1/2 (D) p(E) = 1/4, p(F) = 1/3

8. If from each of the 3 boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls , one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is:

(A) 13/32 (B) 1/4 (C) 1/32 (D) 3/16 **[IIT - 98]**

9. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if 0 < P(F) < 1, then :

(A) $P(E \mid F) + P(\overline{E} \mid F) = 1$ (B) $P(E \mid F) + P(E \mid \overline{F}) = 1$ **[IIT - 98]**

(C) $P(\overline{E} \mid F) + P(E \mid \overline{F}) = 1$ (D) $P(E \mid \overline{F}) + P(\overline{E} \mid \overline{F}) = 1$

10. There are 4 machines and it is known that exactly 2 of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only 2 testes are needed is : **[IIT - 98]**

(A) 1/3 (B) 1/6 (C) 1/2 (D) 1/4

11. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then **[IIT - 98]**

(A) occurrence of $E \Rightarrow$ occurrence of F (B) occurrence of $F \Rightarrow$ occurrence of E

(C) non–occurrence of $E \Rightarrow$ non–occurrence of F (D) none of these

12. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals **[IIT - 98]**

(A) $\frac{1}{2}$ (B) $\frac{1}{32}$ (C) $\frac{31}{32}$ (D) $\frac{1}{5}$

13. The probability that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two, which of the following relations are true ? **[IIT - 98]**

(A) $p + m + c = \frac{19}{20}$ (B) $p + m + c = \frac{27}{20}$ (C) $pmc = \frac{1}{10}$ (D) $pmc = \frac{1}{4}$

D. Multiple choice questions with one correct answer.

1. Two fair dice are tossed. Let x be the event that the first die shows an even number and y be the event that the second die shows an odd number. The two events x and y are : **[IIT - 79]**

(A) mutually exclusive (B) independent and mutually exclusive

(C) dependent (D) none of these

2. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is **[IIT - 80]**

(A) 0.39 (B) 0.25 (C) 0.11 (D) none of these

3. The probability that an event A happens in one trial of an experiments is 0.4. Three independent trials of the experiments are performed. The probability that the event A happens at least once is

(A) 0.936 (B) 0.784 (C) 0.904 (D) none of these **[IIT - 80]**

4. If A and B are two events such that P(A) > 0, and $P(B) \neq 1$, then $P(\overline{A}/\overline{B})$ is equal to

(A) 1 – P(A/B) (B) $1 - P(\overline{A}/\overline{B})$ (C) $\frac{P(\overline{A})}{P(\overline{B})}$ (D) $\frac{1-P(A \cup B)}{P(\overline{B})}$ **[IIT - 82]**

5. Fifteen coupons are numbered 1,2,.....15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is **[IIT - 83]**

(A) $\left(\frac{9}{16}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$ (C) $\left(\frac{3}{5}\right)^7$ (D) none of these

6. One hundred identical coins, each with probability, p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is **[IIT - 88]**

(A) $\frac{1}{2}$ (B) $\frac{49}{101}$ (C) $\frac{50}{101}$ (D) $\frac{51}{101}$

7. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting points 0, 1 and 2 are 0. 45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is **[IIT - 92]**

(A) 0.8750 (B) 0.0875 (C) 0.0625 (D) 0.0250

8. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is then : **[IIT - 93]**

(A) 16/81 (B) 1/81 (C) 80/81 (D) 65/81

9. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A).P(B)$, then **[IIT - 95]**

(A) P(B/A) = P(B) - P(A) (B) $P(A^c \cup B^c) = P(A^c) + P(B^c)$

(C) $P((A \cup B)^c) = P(A^c) + P(B^c)$ (D) P(A/B) = P(A)

10. The probability of India wining a test match against West Indies is 1/2. Assuming independence from match the probability that in a 5 match series India's second win occurs at the third test is **[IIT - 95]**

(A) 1/8 (B) 1/4 (C) 1/2 (D) 1/3

11. Three of the six vertices of a regular hexagon arc chosen at random. The probability that the triangle with these three vertices is equilateral, equals **[IIT - 95]**

(A) 1/2 (B) 1/5 (C) 1/10 (D) 1/20

12. For the three events A, B and C, P(exactly one of the events A or B occurs) = P(exactly one of the events B or C occurs) = P(exactly one of the events C or A occurs) = p and P(all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. Then the probability of at least one of three events A, B and C occurring is **[IIT - 97]**

(A) $\frac{3p + 2p^2}{2}$ (B) $\frac{p + 3p^2}{4}$ (C) $\frac{p + 3p^2}{2}$ (D) $\frac{3p + 2p^2}{4}$

13. 7 white balls and 3 black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals : **[IIT - 98]**

(A) 1/2 (B) 7/15 (C) 2/15 (D) 1/3

14. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals **[IIT - 99]**

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{49}$

15. Two number is selected randomly from the set S = {1, 2, 3, 4, 5, 6}without replacement one by one. The probability that minimum of the two numbers is less than 4 is **[IIT - 2003]**

(A) $\frac{1}{15}$ (B) $\frac{14}{15}$ (C) $\frac{1}{5}$ (D) $\frac{4}{5}$

16. If $P(B) = \frac{3}{4}$, $P\left(A \cap B \cap \overline{C}\right) = \frac{1}{3}$ and $P\left(\overline{A} \cap B \cap \overline{C}\right) = \frac{1}{3}$ and, then $P(B \cap C)$ is : **[IIT - 2003]**

(A) 1/12 (B) 1/6 (C) 1/15 (D) 1/9

17. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three are divisible by both 2 and 3 is **[IIT - 2004]**

(A) 4/25 (B) 4/35 (C) 4/33 (D) 4/1155

18. A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even no. of trials is **[IIT - 2005]**

(A) 5/11 (B) 5/6 (C) 6/11 (D) 1/6

W I There are n urns each containing n + 1 balls such that the i^{th} urn contains i white balls and (n + 1 – i) red balls. Let u_i be the event of selecting i^{th} urn, i = 1, 2, 3, . . . n and 'w' denotes the event of getting a white balls. **[IIT - 2006]**

19. If $P(u_i) \propto i$, where i = 1, 2, 3, . . . n, then $\lim_{n\to\infty} P(w)$ is equal to

(A) 1 (B) 2/3 (C) 3/4 (D) 1/4

20. If $P(u_i) = c$, where c is a constant then $P(u_n/w)$ is equal to

(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

21. If n is even and E denotes the event of choosing even numbered urn $\left(P(u_i) = \frac{1}{n}\right)$, then the value of P(w/E) is

(A) $\frac{n+2}{2n+1}$ (B) $\frac{n+2}{2(n+1)}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{n+1}$

22. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is **[IIT - 2007]**

(A) 1/2 (B) 1/3 (C) 2/5 (D) 1/5

23. Let E^c denotes the complement of an event E. Let E, F, G be pairwise independent events with P(G) > 0 and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c \mid G)$ equals : **[IIT - 2007]**

(A) $P(E^c) + P(F^c)$ (B) $P(E^c) - P(F^c)$ (C) $P(E^c) - P(F)$ (D) $P(E) - P(F^c)$

24. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$ i = 1, 2, . . . , n. Let E be any other event with $0 < P(E) < 1$ **[IIT - 2007]**

Statement -1 : $P(H_i \mid E) > P(E \mid H_i).P(H_i)$ for i = 1, 2, . . . , n

Statement -2 : $\sum_{i-1}^{n} P(H_i) = 1$.

(A) Statement-1 is True, Statment-2 is True. Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True, Statement-2 **is not** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

IIT JEE PROBLEMS (SUBJECTIVE)

1. Six boys and six girls set in a row randomly. Find the probability that the boys and girls set alternately **[IIT - 79]**

2. Set A has 3 elements, and set B has 6 elements. What can be the minimum number of elements in the set $A \cup B$. **[IIT - 80]**

3. An antiaircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane ? **[IIT - 81]**

4. A and B are two candidates seeking admission in IIT. The probability that A is selected is 0.5 and the probability that both A and B are selected is almost 0.3.Is it possible that the probability of B getting selected is 0.9 ? **[IIT - 82]**

5. Cards are drawn one by one at random from a well shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then show that

$P_r\{N = n\} = \dfrac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13}$, where $2 \le n \le 50$. **[IIT - 83]**

6. A, B, C are events such that

$P_r(A) = 0.3$, $P_r(B) = 0.4$, $P_r(C) = 0.8$

$P_r(AB) = 0.08$, $P_r(AC) = 0.28$, $P_r(ABC) = 0.09$

If $P_r(A \cup B \cup C) \ge 0.75$, then show that $P_r(BC)$ lies in the interval $0.23 \le x \le 0.48$. **[IIT - 83]**

7. In a certain city only two news papers A and B are published, it is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisements and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population reads an advertisement ? **[IIT - 84]**

8. A and B are two independent events . The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. Find the probability of the occurrence of A. **[IIT - 84]**

9. In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers candidates decide two tick the answer at random, if he is allowed upto three chances to answer the questions, find the probability that he will get marks in the questions. **[IIT - 85]**

10. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one day by one without replacement and are tested till all defective articles are found. What is probability that the testing procedure ends at the twelfth testing. **[IIT - 86]**

11. A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point. **[IIT - 87]**

12. An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white is not replaced into the urn. Otherwise it is replaced along with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black. **[IIT - 87]**

13. A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number $N(\geq 2)$ of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise. **[IIT - 88]**

Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a " best of 3 games " or a " best of 5 games " match against B, which option should choose so that the probability of his winning the match is higher ? (No game ends in a draw). **[IIT - 89]**

15. A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen at random. Find the probability that P and Q have no common elements. **[IIT - 90]**

16. In a test an examine either guesses or copies of knows the answer to a multiple choice question with four choices. The probability that he make a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered it. **[IIT - 91]**

17. A lot contains 50 defective and 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as A = (the first bulb is defective), B = (the second bulb is nondefective), C = (the two bulbs are both defective or both nondefective). Determine whether
(i) A, B, C are pairwise independent **(ii)** A, B, C are independent **[IIT - 92]**

18. Numbers are selected at random, one at a time, from the two digit numbers 00, 01, 02, ... ,99 with replacement. An event E occurs if and only if the product of the two digits of a selected number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times. **[IIT - 93]**

19. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4,.....,12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8. **[IIT - 94]**

20. In how many ways 3 girls and 9 boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back ? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats ? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats ? **[IIT - 96]**

21. If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}with replacement. Determine the probability that the roots of the equation $x^2 + px + q = 0$ are real. **[IIT - 97]**

22. 3 players A, B and C toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B,.....) till a head shows. Let p be the probability that the coin shows a head. Let α, β and γ be respectively the probabilities that A, B and C gets the first head. Prove that $\beta = (1-p)\alpha$.
Determine α, β and γ (in terms of p). **[IIT - 98]**

23. Eight players P_1, P_2, P_3,.....,P_8 play a knockout tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the players P_4 reaches the final. **[IIT - 99]**

24. A coin has probability 'p' of showing head when tossed. It is tossed 'n' times. Let P_n denote the probability that no two (or more) consecutive heads occur. Prove that , $p_1 = 1$, $p_2 = 1 - p^2$ and $p_n = (1 - p)p_{n-1} + p(1 - p)p_{n-2}$, for all $n \geq 3$. **[IIT - 2000]**

25. An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white. **[IIT - 2001]**

26. An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6 only three numbers appear in the list. **[IIT - 2001]**

27. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tal. What is the probability that the coin drawn is fair ? **[IIT - 2002]**

28. For a student to qualify, he must pass at least two out of three exams. The probability that he will pass the 1st exam is p. It he fails in one of the exams then the probability of his passing in the next exam is $\frac{p}{2}$ otherwise it remains the same. Find the probability that he will qualify. **[IIT - 2003]**

29. A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$ respectively. If A is hit then find the probability that B hits the target and C does not. **[IIT - 2003]**

30. If A and B are two independent events, then show that $P(A \cap B).P(\overline{A} \cup \overline{B}) \leq P(C)$, where C is the event that exactly one of A and B occurs. **[IIT - 2004]**

31. There are 18 balls, 12 red and 6 white. Six balls are drawn one by one without replacement. If at least 4 are white, find the probability that next two draw will result in one red and one white ball. **[IIT - 2004]**

32. A person goes to office either by car, bus or train, the probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reach office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car. **[IIT - 2005]**

Probability

SET–I

1. A binary operation is chosen at random from the set of all binary operations on a set A containing n elements. The probability that the binary operation is commutative is

(A) $\frac{n^2}{n^{n^2}}$ (B) $\frac{n^{n/2}}{n^{n^2}}$ (C) $\frac{n^{n/2}}{n^{n^2/2}}$ (D) none of these

2. If in a distribution each x is replaced by corresponding value of f(x) (where f is an one to one function) , then the probability of getting $f(x_i)$ when the probability of getting x_i is p_i is

(A) p_i (B) $f(p_i)$ (C) $f\left(\frac{1}{P_i}\right)$ (D) none of these

3. A letter is known to have come either from "KRISHNAGIRI" or "DHARMAPURI". On the post mark only the two consecutive letters "RI" are visible. Then the chance that it came from Krishnagiri is :

(A) 3/5 (B) 2/3 (C) 9/14 (D) none of these

4. Four persons are selected from a group of 4 men, 2 women and 3 children. The probability that exactly two of them are men is

(A) 9/11 (B) 10/23 (C) 11/24 (D) 10/21

5. A and B toss a coin alternatively till one of them gets a head and wins the game. If A begins the game, the probability B wins the game is

(A) 1/2 (B) 1/3 (C) 1/4 (D) 2/3

6. Two different digits are chosen at random from the set 1, 2, 3, ...8. Then the probability that

(A) their sum is equal to 9 is 1/7 (B) their sum will exceed 13 is 1/14

(C) both digits exceeding 5 is 3/28 (D) all of these

7. For two events A and B ; $P(A \cup B) = P(A \cap B)$, then

(A) P(A) = P(B) (B) P(A) – P(B) = 2(A) P(B/A)

(C) P(A) + P(B) = 2P(B) P(A/B) (D) none of these

8. Two points P and Q are taken at random on a line segment OA of length a. The probability that PQ > b, where 0 < b < a, is

(A) $\frac{b}{a}$ (B) $\frac{b^2}{a^2}$ (C) $\left(\frac{a-b}{a}\right)^2$ (D) $\left(\frac{a-2b}{a-b}\right)^2$

9. The probability that atleast one of the events A and B occur is 0.6 . If A and B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$ is

(A) 0.4 (B) 0.8 (C) 1.2 (D) 1.4

10. Probability of getting heads in all four trials when a coin is tossed four times, is equal to

(A) 1/4 (B) 15/16 (C) 1/16 (D) none of these

11. A natural number 'n' is selected at random from the set of first 100 natural number the probability that $n + \frac{100}{n} \leq 50$ is equal to

(A) 9/10 (B) 39/50 (C) 9/20 (D) none of these

12. Let 'A', 'B' and 'C' be three independent events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P(C) = \frac{1}{4}$. The probability of exactly 2 of these events occurring, is equal to

(A) $\frac{1}{4}$ (B) $\frac{7}{24}$ (C) $\frac{3}{4}$ (D) $\frac{17}{24}$

13. For two events A and B, $P(A) = P(A \mid B) = \frac{1}{4}$ and $P(B \mid A) = \frac{1}{2}$, then which of the following is not correct

(A) $P(A' \mid B) = \frac{3}{4}$ (B) $P(B' \mid A') = \frac{1}{2}$ (C) $P(A \cup B) = \frac{3}{4}$ (D) $P(A \cap B) = \frac{1}{8}$

14. Three identical dice are rolled.The probability that the same number will appear on each of them is

(A) $\frac{1}{6}$ (B) $\frac{1}{36}$ (C) $\frac{1}{18}$ (D) $\frac{3}{28}$

15. A bag contains 5 brown and 4 white socks. A man pulls out two socks. The probability that they are of the same colour is

(A) $\frac{5}{10.8}$ (B) $\frac{1}{6}$ (C) $\frac{5}{18}$ (D) $\frac{4}{9}$

16. A pack of cards contains 4 aces, 4 kings, 4 queens and 4 jacks. Two cards are drawn at random. The probability that at least one of them is an ace is

(A) $\frac{1}{5}$ (B) $\frac{9}{20}$ (C) $\frac{1}{6}$ (D) $\frac{1}{9}$

17. Three letters are written to different persons, and addresses on three envelopes are also written. Without looking at the addresses, one letter is put in each envelope, then the probability that the letters go into right envelope is

(A) $\frac{1}{27}$ (B) $\frac{1}{6}$ (C) $\frac{1}{9}$ (D) none of these

18. There are two urns. There are m white & n black balls in the first urn & p white & q black balls in the second urn. One ball is taken from the first urn & placed into the second. Now, the probability of drawing a white ball from the second urn is :

(A) $\frac{pm + (p+1)n}{(m+n)(p+q+1)}$ (B) $\frac{(p+1)m + pn}{(m+n)(p+q+1)}$ (C) $\frac{qm + (q+1)n}{(m+n)(p+q+1)}$ (D) $\frac{(q+1)m + qn}{(m+n)(p+q+1)}$

19. The probability that a leap year selected at random contains 53 Sundays is

(A) $\frac{7}{366}$ (B) $\frac{26}{183}$ (C) $\frac{1}{7}$ (D) $\frac{2}{7}$

20. The probability that a leap year selected at random contains 53 Sundays or 53 Mondays is

(A) $\frac{2}{7}$ (B) $\frac{4}{7}$ (C) $\frac{3}{7}$ (D) $\frac{1}{7}$

Probability

SET–II

1. In a single throw of three dice, the probability of getting a total of atleast 5 is

(A) $\frac{1}{3}$ (B) $\frac{53}{54}$ (C) $\frac{1}{54}$ (D) none of these

2. The letters of word 'SOCIETY' are placed at random in a row. The probability that three vowels come together, is

(A) $\frac{6}{7}$ (B) $\frac{1}{7}$ (C) $\frac{3}{7}$ (D) none of these

3. The probability of getting the sum as a prime number when two dice are thrown together, is

(A) $\frac{1}{2}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) none of these

4. A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. If $P(B) = \frac{3}{2} P(A)$ and $P(C) = \frac{1}{2} P(B)$, then $P(A) =$

(A) $\frac{4}{13}$ (B) $\frac{6}{13}$ (C) $\frac{8}{13}$ (D) none of these

5. Two dice are thrown. The probability that the number appeared have a sum of 8, if it is known that the second dice always exhibits 4, is

(A) $\frac{5}{6}$ (B) $\frac{1}{6}$ (C) $\frac{2}{3}$ (D) none of these

6. If the probabilities that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is

(A) $p + q$ (B) $p + q - 2pq$ (C) $p + q - pq$ (D) $p + q + pq$

7. Let $f : A \to A$ be a function, where $A = \{1, 2, 3, 4, 5\}$. If such a function f is chosen randomly, the probability that it is one-one, is

(A) $\frac{1}{25}$ (B) $\frac{1}{5!}$ (C) $\frac{1}{125}$ (D) $\frac{24}{625}$

8. There are 4 envelopes corresponding to 4 letters. If the letters are placed in the envelopes at random, the probability that all the letters are not placed in the right envelopes, is

(A) $\frac{18}{24}$ (B) $\frac{23}{24}$ (C) $\frac{17}{24}$ (D) none of these

9. A book contains 1000 pages. A page is chosen at random. The probability that the sum of the digits of the marked number on the page is equal to 9, is

(A) $\frac{23}{500}$ (B) $\frac{11}{200}$ (C) $\frac{7}{100}$ (D) none of these

10. Two unbiased dice are rolled. The probability that the sum of the numbers on the two faces is either divisible by 3 or divisible by 4, is

(A) $\frac{5}{9}$ (B) $\frac{7}{17}$ (C) $\frac{9}{17}$ (D) none of these

11. A student takes his examination in four subjects α, β, γ and δ. He estimates his chance of passing in α as $\frac{4}{5}$, in β as $\frac{3}{4}$, in γ as $\frac{5}{6}$ and in δ as $\frac{2}{3}$. To qualify he must pass in α and at least two other subjects. The probability that he qualifies is

(A) $\frac{34}{90}$ (B) $\frac{61}{90}$ (C) $\frac{53}{90}$ (D) none of these

12. One die and one coin are tossed simultaneously. The probability of getting 6 on die and head on coin is

(A) $\frac{1}{2}$ (B) $\frac{1}{6}$ (C) $\frac{1}{12}$ (D) none of these

13. If $E = E_1\ E_2\ E_3\ E_4\ E_5$ and $P(E_1) = \frac{95}{100}$, $P(E_2 / E_1) = \frac{94}{99}$, $P(E_3 / E_1E_2) = \frac{93}{98}$, $P(E_4 / E_1E_2E_3) = \frac{92}{97}$ and $P(E_5 / E_1E_2E_3E_4) = \frac{91}{96}$ (Here $E_1\ E_2$ means $E_1 \cap E_2$ etc.), then P(E) is equal to

(A) $\frac{91.92.93.94}{97.98.99.100}$ (B) $\frac{91.92.93.94.95}{96.97.98.99.100}$ (C) $\frac{94.95.96}{98.99.100}$ (D) none of these

14. 15 coupons are numbered 1, 2, 3, . . . 14, 15. Seven coupons are selected at random, one at a time with replacement. The probability that 9 would be the largest number appearing on the selected coupons, is

(A) $\left(\frac{1}{6}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$ (C) $\left(\frac{3}{5}\right)^7$ (D) $\frac{9^7 - 8^7}{15^7}$

15. Four tickets marked 00, 01, 10 and 11 are placed in a bag. A ticket is drawn at random five times, ticket being replaced each time. The probability that the sum of the numbers on the ticket is 15, is

(A) $\frac{3}{1024}$ (B) $\frac{5}{1024}$ (C) $\frac{7}{1024}$ (D) none of these

16. One mapping is selected at random from all the mappings from the set S = {1, 2, 3, . . ., n} into itself. The probability that the selected mapping is one-to-one is

(A) $1/n^2$ (B) $1/n!$ (C) $(n-1)!/n^{n-1}$ (D) none of these

17. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 and 1 only. The probability that the value of the determinant chosen is non-zero, is

(A) $\frac{3}{16}$ (B) $\frac{3}{8}$ (C) $\frac{1}{4}$ (D) none of these

18. A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw, is

(A) $\frac{5}{64}$ (B) $\frac{27}{32}$ (C) $\frac{5}{32}$ (D) $\frac{1}{2}$

19. Two numbers b and c are chosen at random (with replacement) from the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9. The probability that $x^2 + bx + c > 0$ for all $x \in R$, is

(A) $\frac{32}{81}$ (B) $\frac{44}{81}$ (C) $\frac{31}{81}$ (D) none of these

20. An elevator starts with m passengers and stops at n floors $(m \le n)$. The probability that no two passengers alight at the same floor, is

(A) $\frac{{}^nP_m}{m^n}$ (B) $\frac{{}^nP_m}{n^m}$ (C) $\frac{{}^nC_m}{m^n}$ (D) $\frac{{}^nC_m}{n^m}$

SET–III

Multiple choice question with one or more than one correct choice.

1. If A and B are two events such that $P(A) = \frac{3}{5}$ and $P(B) = \frac{7}{10}$, then

(A) $\ldots\ldots \cap B) \geq \frac{3}{10}$ (B) $P(A \cap B) \leq \frac{7}{10}$

(C) $\frac{3}{5} < P(A \cup B) < \frac{7}{10}$ (D) $\frac{3}{5} < P(A \cup B) \leq \frac{7}{10}$

2. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{2}{3}$, then

(A) $P(A \cup B) \geq \frac{2}{3}$ (B) $\frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$

(C) $\frac{2}{5} \leq P(A./B) \leq \frac{9}{10}$ (D) $P(A \cap B') \leq \frac{1}{3}$

3. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occur is $\frac{1}{3}$. the probability of occurrence of A is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) none of these

4. If A and B are two events such that P(A) = 1/2, P(B) = 2/3, then

(A) $P(A + B) \geq 2/3$ (B) $P(AB') \leq 1/3$

(C) $1/6 \leq P(AB) \leq 1/2$ (D) $1/6 \leq P(A'B) \leq 1/2$

5. An urn contains four tickets with numbers 112, 121, 211, 222 and one ticket is drawn. Let A_i be the event that the i^{th} digit of the number drawn is 1. Then

(A) A_1 and A_2 are independent (B) A_2, A_3 are independent

(C) A_1, A_3 are independent (D) A_1, A_2, A_3 are independent

☞ **Read the passage given below and answer the questions :**

If the number of points in the sample space is infinite, then we can not apply the classical definition of probability. For instance, if we are interested to find the probability that a point selected at random in a circle of radius r, is nearer to the centre then the circumference, we can not apply the classical definition of probability. In this case we define the probability as follows :

$$P = \frac{\text{Measure of the favourable region}}{\text{Measure of the sample space}},$$

where measure stands for length, area or volume depending upon whether S is one-dimensional, two – dimensional or three – dimensional region.

Thus the probability, that the chosen point is nearer to the centre than the circumference

$$= \frac{\text{area of the region of the favourable point}}{\text{area of the circle}} = \frac{\pi(r/2)^2}{\pi r^2} = \frac{1}{4}.$$

6. Two numbers $x \in R$ and $y \in R$ are selected such that $x \in [0, 4]$ and $y \in [0, 4]$, then the probability that the selected numbers satisfy $y^2 \leq x$ is

(A) 1/3 (B) 1/2 (C) 1/5 (D) none of these

7. A bar of unit length is broken into three parts x, y and z, then the probability that a triangle can be formed from the resulting parts is

(A) 1/3 (B) 1/4 (C) 1/5 (D) none of these

8. Two points are taken at random on the given straight line segment of length a, then the probability for the distance between them to exceed a given length c, where $0 < c < a$, is

(A) $\left(1+\frac{c}{a}\right)^2$ (B) $\left(1-\frac{c}{a}\right)^2$ (C) $\left(1-\frac{a}{c}\right)^2$ (D) none of these

9. Three points A, B, C are chosen at random on the circumference of a circle, then the probability for these points to lie on the semicircle is

(A) 1/2 (B) 2/3 (C) 3/4 (D) none of these

10. With the vertices of a square as centres four ellipse are drawn to pass through its centre and the middle points of its sides, then the probability that a point of the square chosen at random will lie within any two of the ellipse is

(A) $\frac{2\pi}{3\sqrt{3}}-1$ (B) $\frac{3\pi}{4\sqrt{3}}-1$ (C) $\frac{\pi}{3}-1$ (D) none of these

☞ **Read the passage given below and answer the questions :**

Let S be the sample space associated with a given random experiment. Then, a real valued function X which assigns to each event $w \in S$ to a unique real number X(w) is called a random variable.

If a random variable X takes values $x_1, x_2, ..., x_n$ with respective probabilities $p_1, p_2, ..., p_n$, then

X :	x_1	x_2	x_3	.....	x_n
P(X) :	p_1	p_2	p_3	.....	p_n

is known as the probability distribution of X.

If X is a discrete random variable which assumes values $x_1, x_2, x_3, ..., x_n$ with respective probabilities $p_1, p_2, ..., p_n$, then the mean $\overline{X}$ of X is defined as

$\overline{X} = p_1 x_1 + p_2 x_2 + ...+ p_n x_n$ or, $\overline{X} = \sum_{i=1}^{n} p_i x_i$

11. Three cards are drawn from a pack of 52 playing cards, then the probability distribution of the number of aces is

(B)

X :	0	1	2	3
P(X) :	$\frac{4324}{5525}$	$\frac{1128}{5525}$	$\frac{72}{5525}$	$\frac{1}{5525}$

(C)

X :	0	1	2	3
P(X) :	$\frac{3424}{5525}$	$\frac{1218}{5525}$	$\frac{72}{5525}$	$\frac{1}{5525}$

(D) none of these

Probability

12. An unbiased die is thrown twice, then the probability distribution of the number of sixes is

(A)

X :	0	1	2
P(X) :	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

(B)

X :	0	1	2
P(X) :	$\frac{23}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

(C)

X :	0	1	2
P(X) :	$\frac{25}{36}$	$\frac{7}{18}$	$\frac{1}{36}$

(D) none of these

13. Two cards are drawn without replacement from a well–shuffled deck of 52 cards. Determine the probability distribution of the number of honour cards (i.e. Jack, Queen, King and Ace) is

(A)

X :	0	1	2
P(X) :	$\frac{105}{221}$	$\frac{96}{221}$	$\frac{20}{221}$

(B)

X :	0	1	2
P(X) :	$\frac{103}{221}$	$\frac{96}{221}$	$\frac{20}{221}$

(C)

X :	0	1	2
P(X) :	$\frac{105}{221}$	$\frac{92}{221}$	$\frac{20}{221}$

(D) none of these

14. Two cards are drawn successively with replacement from a well–shuffled deck of 52 cards, then the mean of the number of aces is

(A) $\frac{2}{13}$ (B) $\frac{1}{13}$ (C) $\frac{3}{13}$ (D) none of these

☞ **Read the passage given below and answer the questions :**

If p be the probability of success of a person in any venture and M be the sum of money which he will receive in any case of success, the sum of money denoted by pM is called his expectation.

15. A and B throw with one die for a stake of Rs. 11 which is to be won by the player who first throw 6. If A has the first throw, then their respective expectations are

(A) Rs. 6 and Rs. 5 (B) Rs. 7 and Rs. 4
(C) Rs. 4 and Rs. 7 (D) none of these

16. A and B throw with a fair coin for a stake of Rs. 27 alternatively which is to be won by the player who throw head two times. If A has the first throw, then their respective expectations are

(A) Rs. 16 and Rs. 11 (B) Rs. 15 and Rs. 12
(C) Rs. 17 and Rs. 10 (D) none of these

17. A and B throw with a pair of dies for a stake of Rs. 20 which is to be won by the player A if he throws sum 4 and by the player B if he throws sum 5. If A has the first throw, then their respective expectations are

(A) Rs. 9 and Rs. 11 (B) Rs. 12 and Rs. 8
(C) Rs. 10 and Rs. 10 (D) none of these

18. True And False :

(i) 6 fair dice are thrown simultaneously . The probability that all of them show different faces is 5/324.

(ii) If $P(A \cap B) = \frac{1}{4}$, $P(A' \cap B') = \frac{1}{5}$ & P(A) = P(B) = p, then p = 21/40.

(iii) A cubical die is biased such that the probability of a number appearing when the die is tossed is proportional to the given number . Let A denotes the event of occurrence of an even number and B denotes the event of occurrence of a prime number, then $P(A \cup B^c) = 13/42$.

(iv) A fair coin is tossed 10 times . The probability that two heads do not occur continuously is 9/64 .

(v) Of all the seven digit numbers the sum of whose digits is 59 , a number is chosen at random. The probability that the number is divisible by 11 is 2/21.

19. Fill in the blanks :

(i) 2 men A & B draw one ball each (with replacement) from a box containing 2 white & 4 red balls . If A is allowed to begin followed by B, the probability that B draws a white ball before A, is _____ .

(ii) In a box, there are 8 alphabet cards with the letters E, E, E, P, P, T, T, T . The probability that "PET" will form if the three cards are drawn one by one & placed on the table in the same order that they are drawn is _____ & the probability of forming the word PET is _____ if the 3 cards are drawn simultaneously .

(iii) An unbiased cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times . The probability of getting a total score of 4 or 6 is _____ .

(iv) When the ignition is switched on, the engine picks up with probability 3/4 . The probability that the ignition has to be switched on atleast 3 times for the engine to begin working is _____ & exactly 3 times is _____ .

(v) In a batch of 10 articles 4 are defective . 6 articles are taken from the batch for inspection. If more than 2 articles in this batch are defective, the whole batch is rejected . The probability that the batch will be rejected is _____ .

(vi) There are 'm' radar units, each of which detects a target during one cycle with probability p (independently of other cycles & other units) . Each radar takes n cycle. 'A' be the event that the target is detected by atleast one unit . 'B' be the event that the target is detected by all the units. Then P(A) = _____ and P(B) = _____ .

(vii) 18 teams participate in a basket ball championship, out of which 2 groups each consisting of 9 teams are formed at random . 5 of the teams are first class . Let A denote the event "all are first class teams get into the same group", then P(A) = _____ .

(viii) There is a group of k targets, each of which independently of the other targets, can be detected by a radar unit with probability p . Each of 'm' radar units detects the targets independently of other units . The probability that not all the targets in the group will be detected is _____ .

(ix) If 2 squares are chosen at random on a chess board then the chance that they have contact at a corner is _____ (express your answer as a rational fraction in the lowest form) .

(x) 2 positive real numbers x & y satisfy $x \le 1$ & $y \le 1$ are chosen at random . The probability that $x + y \le 1$, given that $x^2 + y^2 \ge 1/4$ is _____ .

20. Match the following :

	Column I		**Column II**
(a)	If A and B are two mutually exclusive, then	(P)	$\sum p_i x_i^{\,2} - \mu^2$
(b)	If A and B are two subsets of a universal set U, then	(Q)	$P(A \cap B) = P(A).P(B)$
(c)	If A and B are two independent events, then	(R)	$P(A \cup B) = P(A) + P(B)$
(d)	Baye's theorem	(S)	$(A \cup B)^c = A^c \cap B^c$
(e)	Variance of a random variable is given by	(T)	$P(B_i / A) = \dfrac{P(B_i).P(A/B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$

ANSWER LEVEL–I

1. $\frac{20}{81}$ **2.** **(i)** $\frac{3}{56}$ **(ii)** $\frac{9}{28}$ **3.** $\frac{16}{37}, \frac{12}{37}, \frac{9}{37}$

4. $\frac{16}{81}$ **5.** $\frac{1}{10}$ **6.** $\frac{^{24}C_2}{^{64}C_2}$ or $\frac{23}{168}$

7. Independent in (a) and not independent in (b) **8.** 3/10

10. $(1 - p)^{n-1} . [p_o (1 - p) + np (1 - p_o)]$

LEVEL–II

2. $\frac{13}{24}$ **4.** **(i)** $\frac{1}{2}$ **(ii)** $\frac{1}{2}$ **(iii)** $\frac{1}{12}$ **5.** {0}

6. **(a)** $\frac{9}{50}$ **(b)** $\frac{5n-3}{9n-3}$ **7.** **(i)** $\frac{1}{10}$ **(ii)** $\frac{3}{10}$, $\frac{2}{3}$

8. **(i)** $\frac{1}{2}$ **(ii)** $\frac{8}{15}$ **9.** $\frac{3}{4}$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. $\frac{5}{21}$ **2.** $P(A) = P(B) = P(A \cap B) = P(A \cup B)$ **3.** $\frac{4}{45}$

4. $\frac{1}{3} \le p \le \frac{1}{2}$ **5.** $\frac{32}{55}$ **6.** $\frac{2}{5}$ **7.** $\frac{5}{7}$

8, $\frac{11}{16}$ [Hint : Mean = np, variance = npq, p + q = 1, n = number of trial] **9.** $\frac{1}{36}$

10. 1/4 **11.** 11/40

(B)

1. T **2.** F (correct answer 0.45)

(C)

1. A, C, D **2.** E **3.** C **4.** A, B, C **5.** B, C, D

6. A, C **7.** A, D **8.** A **9.** A, D **10.** A

11. D **12.** A **13.** B, C

(D)

1. D	**2.** A	**3.** B	**4.** D	**5.** D	
6. D	**7.** B	**8.** A	**9.** D	**10.** B	
11. C	**12.** A	**13.** B	**14.** A	**15.** D	
16. A	**17.** D	**18.** A	**19.** B	**20.** A	**21.** B
22. C	**23.** C	**24.** B			

IIT JEE PROBLEMS (SUBJECTIVE)

1. $\frac{1}{462}$ **2.** 6 **3.** 0.6976 **4.** No correct ans $P(B) \le 0.8$

7. 13.9 **8.** $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ **9.** $\frac{1}{5}$

10. 0.052 **11.** $462(0.24)^5$ **12.** $\frac{23}{30}$ **13.** $1-\frac{10(N+2)}{N+{}^7C_5}$

14. Best of 3 games 0.353, Best of 5 games 0.317 **15.** $\frac{1}{2^r}$ where r is number of elements of P

16. $\frac{24}{29}$ **17.(i)** True **(ii)** False **18.** $97/(25)^4$ **19.** 193/792

20. ${}^{14}P_{12}$, $4\times 3!\times {}^{11}P_9$, $1/91$ **21.** 0.62

22. $\alpha=\frac{p}{1-(1-p)^3}, \beta=\frac{(1-p)p}{1-(1-p)^3}, \gamma=\frac{(1-p)^2 p}{1-(1-p)^3}$ **23.** 4/35 **25.** $\frac{m}{m+n}$

26. $\frac{{}^6C_3(3^n-3.2^n+3)}{6^n}$ **27.** $\frac{9m}{m+8n}$ **28.** $2p^2-p^3$ **29.** $\frac{1}{2}$

31. $\frac{{}^{10}C_1\times{}^2C_1}{{}^{12}C_2}\times\frac{{}^{12}C_2\times{}^6C_4}{{}^{18}C_6}+\frac{{}^{11}C_1\times{}^1C_1}{{}^{12}C_2}\times\frac{{}^{12}C_1\times{}^6C_5}{{}^{18}C_6}$ **32.** $\frac{1}{7}$

SET–I

1. C	**2.** A	**3.** C	**4.** D	**5.** B
6. D	**7.** C	**8.** C	**9.** C	**10.** C
11. C	**12.** A	**13.** C	**14.** B	**15.** D
16. B	**17.** B	**18.** B	**19.** D	**20.** C

SET–II

1. B	**2.** B	**3.** C	**4.** A	**5.** B
6. B	**7.** D	**8.** B	**9.** B	**10.** A
11. B	**12.** C	**13.** B	**14.** D	**15.** B
16. C	**17.** B	**18.** C	**19.** A	**20.** B

SET–III

1.	AB	**2.**	ABCD	**3.**	AB	**4.**	ABCD	**5.**	ABC
6.	A	**7.**	B	**8.**	B	**9.**	C	**10.**	A
11.	A	**12.**	A	**13.**	A	**14.**	A	**15.**	A
16.	A	**17.**	A						

18.

(i)	T	**(ii)**	T	**(iii)**	F	**(iv)**	T	**(v)**	F

19.

(i)	$\frac{2}{5}$	**(ii)**	$\frac{3}{56}, \frac{9}{28}$	**(iii)**	$\frac{25}{108}$	**(iv)**	$\frac{1}{16}, \frac{3}{64}$	**(v)**	$\frac{19}{42}$

(vi) $P(A) = 1 - (1-p)^{mn}$, $P(B) = [1-(1-p)^n]^m$ **(vii)** $\frac{1}{34}$

(viii) $1 - \{1-(1-p)^m\}^k$ **(ix)** $\frac{7}{144}$ **(x)** $\frac{8-\pi}{16-\pi}$

20. (a, R), (b, S), (c, Q), (d, T), (e, P)