Question Bank - Vector

LEVEL–I

1. Points X and Y are taken on the sides QR and RS, repetitively of a parallelogram PQRS, so that $\overrightarrow{QX} = 4\overrightarrow{XR}$ and $\overrightarrow{RY} = 4\overrightarrow{YS}$. The line XY cuts the line PR at Z. Prove that $\overrightarrow{PZ} = \left(\frac{21}{25}\right)\overrightarrow{PR}$.

2. Show that $\begin{cases} \vec{p} = \vec{a}\times[(\vec{e}\times\vec{b})\times(\vec{d}\times\vec{c})] \\ \vec{q} = \vec{b}\times[(\vec{e}\times\vec{c})\times(\vec{d}\times\vec{a})] \\ \vec{r} = \vec{c}\times[(\vec{e}\times\vec{a})\times(\vec{d}\times\vec{b})] \end{cases}$ form the sides of a triangle, where $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{p}, \vec{q}, \vec{r}$ are non zero vectors.

3. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a}+\vec{b} = \mu\vec{p}$, $\vec{b}.\vec{q} = 0$ and $(\vec{b})^2 = 1$, where μ is a scalar then prove that $|(\vec{a}.\vec{q})\vec{p} - (\vec{p}.\vec{q})\vec{a}| = |\vec{p}.\vec{q}|$.

4. Solve the following equation for the vector $\vec{p}$; $\vec{p}\times\vec{a} + (\vec{p}.\vec{b})\vec{c} = \vec{b}\times\vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are non zero non coplanar vectors and $\vec{a}$ is neither perpendicular to $\vec{b}$ nor to $\vec{c}$, hence show that $\left[\vec{p}\times\vec{a} + \frac{[\vec{a}\,\vec{b}\,\vec{c}]}{\vec{a}.\vec{c}}\vec{c}\right]$ is perpendicular to $\vec{b}-\vec{c}$.

5. ABC is triangle and 'O' any point in the plane of the same AO, BO and CO meet the sides BC, CA and AB in D, E, F respectively, show that $\frac{OD}{AD} + \frac{OE}{BE} + \frac{OF}{CF} = 1$.

6. A straight line $'\ell'$ cuts the lines AB, AC and AD of a parallelogram ABCD at points B, C, D respectively. If AB_1 = $AB_1 = \lambda_1.AB$, $AD_1 = \lambda_2.AD$ and $AC_1 = \lambda_3.AC$ λ, then prove that $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$.

7. The internal bisectors of the angles of a triangles of a triangle ABC meet the opposite sides in D, E, F ; the vectors to prove that the area of the triangle DEF is given by $\frac{(2abc)\Delta}{(a+b)(b+c)(c+a)}$ where Δ is the area of the triangle.

8. The angles between three non zero vectors $\vec{a}, \vec{b}, \vec{c}$ which are not necessarily coplanar are α between $\vec{b}$ and $\vec{c}$, β between $\vec{c}$ and $\vec{a}$, γ between $\vec{a}$ and $\vec{b}$. Vectors $\vec{u}$ and $\vec{v}$ are defined by ; $\vec{u} = (\vec{a}\times\vec{b})\times\vec{c}$, $\vec{v} = \vec{a}\times(\vec{b}\times\vec{c})$. If $\vec{u}$ is perpendicular to $\vec{v}$ then show that either $\vec{a}$ is perpendicular to $\vec{c}$ or $\cos\beta = \cos\alpha.\cos\gamma$. Hence show that $\vec{a}\times\vec{b}$ is perpendicular to $\vec{b}\times\vec{c}$. Now if vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar then show that $[\vec{u}\,\vec{v}\,\vec{a}] = [\vec{u}\,\vec{v}\,\vec{b}] = [\vec{u}\,\vec{v}\,\vec{c}]$.

9. The point D, E, F divide sides BC, CA, AB of a triangle ABC in the ratio 1 : 2. The pairs of lines AD, BE ; BE, CF ; CF, AD meet at P, Q, R respectively. Show that the area of the triangle PQR is (1/ 7) the area of triangle ABC.

10. Let u, v, w be three unit vectors such that $u+v+w=a,\ u\times(v\times w)=b$, $(u\times v)\times w = c,\ a.u = \frac{3}{2},\ a.v = \frac{7}{4}$ and $|a| = 2$. Find u, v and w in terms of a, b and c.

LEVEL–II

1. Let $\vec{a}, \vec{b}, \vec{c}$ be non coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$. Find scalars p, q and r in terms of θ.

2. If $p\vec{x}+(\vec{x}\times\vec{a}) = \vec{b};\ (p\neq 0)$ prove that $\vec{x} = \dfrac{p^2\vec{b}+(\vec{b}.\vec{a})\vec{a}-p(\vec{b}\times\vec{a})}{p(p^2+\vec{a}^2)}$.

3. Let $\hat{x}, \hat{y}$ and $\hat{z}$ be the unit vectors such that $\hat{x}+\hat{y}+\hat{z} = \vec{a},\ \hat{x}\times(\hat{y}\times\hat{z}) = \vec{b},\ (\hat{x}\times\hat{y})\times\hat{z} = \vec{c}$, $\vec{a}.\hat{x} = \frac{3}{2},\ \vec{a}.\hat{y} = \frac{7}{4}$ and $|\vec{a}| = 2$. Find $\hat{x}, \hat{y}$ and $\hat{z}$ in terms of $\vec{a}, \vec{b}$ and $\vec{c}$.

4. Show that the solution of the equation $k\vec{r}+\vec{r}\times\vec{a} = \vec{b}$ where k is a non-zero scalar and $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, is given by $\vec{r} = \dfrac{1}{k^2+a^2}\left(\dfrac{\vec{a}.\vec{b}}{k}\vec{a}+k\vec{b}+\vec{a}\times\vec{b}\right)$.

5. In a quadrilateral PQRS ; $\overrightarrow{PQ} = \vec{a},\ \overrightarrow{QR} = \vec{b},\ \overrightarrow{SP} = \vec{a}-\vec{b}$, M is the midpoint of $\overrightarrow{QR}$ and x is a point on SM such that SX = $\frac{4}{5}$ SM. Prove that P, X and R are collinear.

6. In a triangle ABC, the median CM is perpendicular to the angle bisector AL, and $\frac{CM}{AL} = n$. Using vector method, show that $\cos A = \dfrac{9-4n^2}{9+4n^2}$.

7. Consider the non zero vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ such that no three of which are coplanar then prove that $\vec{a}[\vec{b}\,\vec{c}\,\vec{d}]+\vec{c}[\vec{a}\,\vec{b}\,\vec{d}] = \vec{b}[\vec{a}\,\vec{c}\,\vec{d}]+\vec{d}[\vec{a}\,\vec{b}\,\vec{c}]$. Hence prove that $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ represent the position vectors of the vertices of a plane quadrilateral if and only if $\dfrac{[\vec{b}\,\vec{c}\,\vec{d}]+[\vec{a}\,\vec{b}\,\vec{d}]}{[\vec{a}\,\vec{c}\,\vec{d}]+[\vec{a}\,\vec{b}\,\vec{c}]} = 1$.

8. Show that the circumcenter of the tetrahedron OABC is given by,

$\dfrac{\vec{a}^2(\vec{b}\times\vec{c})+\vec{b}^2(\vec{c}\times\vec{a})+\vec{c}^2(\vec{a}\times\vec{b})}{2[\vec{a}\,\vec{b}\,\vec{c}]}$, where $\vec{a}$, $\vec{b}$, $\vec{c}$ are the pv's of the points A, B, C respectively relative to the origin 'O'.

9. Find the position vector of the point of intersection of the three planes $r \cdot n_1 = q_1$, $r \cdot n_2 = q_2$, $r \cdot n_3 = q_3$ where n_1, n_2 and n_3 are non-coplanar vectors.

10. If the point $R(\vec{r})$ is on the line, which is parallel to the vector, $a\hat{i}+b\hat{j}+c\hat{k}$ (where a, b, c $\neq 0$) and passing through the point $S(\vec{s})$, then prove that, $\vec{r}\times(a\hat{i}+b\hat{j}+c\hat{k}) = \vec{s}\times(a\hat{i}+b\hat{j}+c\hat{k})$. Further if, $T(\vec{t})$ is a point outside the given line then show that the distance of the line from the point $T(\vec{t})$ is given by, $\dfrac{\sqrt{\left[(\vec{t}-\vec{s}).(c\hat{j}-b\hat{k})\right]^2+\left[(\vec{t}-\vec{s}).(a\hat{k}-c\hat{i})\right]^2+\left[(\vec{t}-\vec{s}).(b\hat{i}-a\hat{j})\right]^2}}{\sqrt{a^2+b^2+c^2}}$.

IIT JEE PROBLEMS (OBJECTIVE)

A. Fill In the blanks

1. Let $\vec{A}$, $\vec{B}$, $\vec{C}$ be vectors of length 3, 4, 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}$, $\vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to $\vec{A}+\vec{B}$. Then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is............ **[IIT - 81]**

2. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $\vec{A} = (1, a, a^2)$, $\vec{B} = (1, b, b^2)$, $\vec{C}\,(1, c, c^2)$ are non coplanar, then the product abc = ... **[IIT - 85]**

3. If $\vec{A}$ $\vec{B}$ $\vec{C}$ are three non coplanar vectors, then $\dfrac{\vec{A}.\vec{B}\times\vec{C}}{\vec{C}\times\vec{A}.\vec{B}} + \dfrac{\vec{B}.\vec{A}\times\vec{C}}{\vec{C}.\vec{A}\times\vec{B}} = \ldots$ **[IIT - 85]**

4. If $\vec{A} = (1, 1, 1)$, $\vec{C} = (0, 1, -1)$ are given vectors, than a vector B satisfying the equations $\vec{A}\times\vec{B} = \vec{C}$ and $\vec{A}.\vec{B} = 3$ .... **[IIT - 85]**

5. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}(a \neq b \neq c \neq 1)$ are coplanar, then the value of $\dfrac{1}{(1-a)}+\dfrac{1}{(1-b)}+\dfrac{1}{(1-c)}-$........ **[IIT - 87]**

6. Let $\vec{b} = 4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projection 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, are given by....... **[IIT - 87]**

7. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are .... and ....

respectively. **[IIT - 88]**

8. Given that $\vec{a} = (1,1,1)$, $\vec{c} = (0,1,-1)$, $\vec{a}.\vec{b} = 3$ and $\vec{a}\times\vec{b} = \vec{c}$, then $\vec{b} = ...$ **[IIT - 91]**

9. A unit vector coplanar with $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and perpendicular to $\hat{i}+\hat{j}+\hat{k}$ is .... **[IIT - 92]**

10. The unit vector perpendicular to the plane determined by P(1, –1, 2), Q(2, 0, –1) and R(0, 2, 1) is .... **[IIT - 94]**

11. A non zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is .... **[IIT - 96]**

12. If $\vec{b}, \vec{c}$ are any two non collinear unit vector and $\vec{a}$ is any vector, then

$$(\vec{a}.\vec{b})\vec{b}+(\vec{a}.\vec{c})\vec{c}+\frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}(\vec{b}\times\vec{c}) = ...$$ **[IIT - 96]**

13. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a}\times(\vec{a}\times\vec{c})+\vec{b}=\vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is .... **[IIT - 97]**

14. Let $\vec{OA} = \vec{a}$, $\vec{OB} = 10\vec{a}+2\vec{b}$ and $\vec{OC} = \vec{b}$ where O, A and C are non-collinear points. Let p denote the area of the quadrilateral OABC, and let q denote the area of the parallelogram with OA and OC as adjacent sides. If p = kq, then k = ... **[IIT - 97]**

B. True/False

1. Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ be unit vectors suppose that $\vec{A}.\vec{B} = \vec{A}.\vec{C} = 0$ and that the angle between $\vec{B}$ and $\vec{C}$ is $\frac{\pi}{6}$. Then $\vec{A} = \pm 2(\vec{B}\times\vec{C})$. **[IIT - 81]**

2. If X . A = 0, X . B = 0, X . C = 0 for some non-zero vector X, then [A B C] = 0. **[IIT - 83]**

3. The points with position vectors a + b, a – b and a + kb are collinear for all real values of k. **[IIT - 84]**

4. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $(\vec{a}-\vec{b}).(\vec{b}-\vec{c})\times(\vec{c}-\vec{a}) = 2\vec{a}.\vec{b}\times\vec{c}$. **[IIT - 89]**

C. Multiple Choice Questions with One or More than One Correct Answer

1. Let $\vec{a} = a_1i+a_2j+a_3k$, $\vec{b} = b_1i+b_2j+b_3k$, $\vec{c} = c_1i+c_2j+c_3k$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$,

then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to **[IIT - 86]**

(A) 0 (B) $\frac{1}{4}\left(a_1^2+a_2^2+a_3^2\right)\ \left(b_1^2+b_2^2+b_3^2\right)$

(C) 1 (D) $\frac{3}{4}\left(a_1^2+a_2^2+a_3^3\right)\left(b_1^2+b_2^2+b_3^3\right)\left(c_1^2+c_2^2+c_3^3\right)$

2. The numbers of vectors of unit length perpendicular to vectors $\vec{a}$ =(1, 1,0) and $\vec{b}$ = (0, 1, 1) is
(A) one (B) two (C) three (D) infinite **[IIT - 87]**

3. Let $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$, $\vec{b} = \hat{i}+2\hat{j}-\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is **[IIT - 93]**

(A) $2\hat{i}+3\hat{j}-3\hat{k}$ (B) $2\hat{i}+3\hat{j}+3\hat{k}$ (C) $-2\hat{i}-\hat{j}+5\hat{k}$ (D) $2\hat{i}+\hat{j}+5\hat{k}$

4. If $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|\vec{c}|=\sqrt{3}$ then : **[IIT - 98]**
(A) $\alpha=1,\ \beta=-1$ (B) $\alpha=1,\ \beta=\pm1$
(C) $\alpha=-1,\ \beta=\pm1$ (D) $\alpha=\pm1,\ \beta=1$

5. For three vectors $\vec{u},\ \vec{v},\ \vec{w}$ which of the following expressions is not equal to any of the remaining three ? **[IIT - 98]**
(A) $\vec{u}.(\vec{v}\times\vec{w})$ (B) $(\vec{v}\times\vec{w}).\vec{u}$ (C) $\vec{v}(\vec{u}\times\vec{w})$ (D) $(\vec{u}\times\vec{v}).\vec{w}$

6. Which of the following expressions are meaningful ? **[IIT - 98]**
(A) $\vec{u}.(\vec{v}\times\vec{w})$ (B) $(\vec{u}.\vec{v}).\vec{w}$ (C) $(\vec{u}.\vec{v})\vec{w}$ (D) $\vec{u}\times(\vec{v}.\vec{w})$

7. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u}=\vec{a}-(\vec{a}.\vec{b})\vec{b}$ and $\vec{v}=\vec{a}\times\vec{b}$, then $|\vec{v}|$ is
(A) $|\vec{u}|$ (B) $|\vec{u}|+|\vec{u}.\vec{a}|$ (C) $|\vec{u}|+|\vec{u}.\vec{b}|$ (D) $\vec{u}+\vec{u}.(\vec{a}+\vec{b})$
[IIT - 98]

D. Multiple Choice Questions with One Correct Answer

1. The scalar $\vec{A}.(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$ equals **[IIT - 81]**

(A) 0 (B) $[\vec{A}\ \vec{B}\ \vec{C}]+[\vec{B}\,\vec{C}\,\vec{A}]$

(B) $[\vec{A}\ \vec{B}\ \vec{C}]$ (D) none of these

2. For non-zero vectors $\vec{a},\ \vec{b},\ \vec{c}$, $|(\vec{a}\times\vec{b}).\vec{c}|=|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds if and only if **[IIT - 82]**

(A) $\vec{a}.\vec{b}=0,\ \vec{b}.\vec{c}=0$ (B) $\vec{b}.\vec{c}=0,\ \vec{c}.\vec{a}=0$

(C) $\vec{c}.\vec{a} = 0,\ \vec{a}.\vec{b} = 0$ (D) $\vec{a}.\vec{b} = b\ .\ \vec{c} = \vec{c}\ .\ \vec{a} = 0$

3. The volume of the parallelepiped whose sides are given by
$\overrightarrow{OA} = 2i - 2j,\ \ \overrightarrow{OB} = i + j - k,\ \ \overrightarrow{OC} = 3i - k$ is **[IIT - 83]**

(A) $\frac{4}{13}$ (B) 2 (C) $\frac{2}{7}$ (D) none of these

4. The points with position vectors 60i + 3j, 40i – 8j, ai – 52j are collinear if **[IIT - 83]**
(A) a = – 40 (B) a = 40
(C) a = 20 (D) none of these

5. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations

$\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]},\ \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]},\ \vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, then the value of the expression

$(\vec{a}+\vec{b}).\vec{p} + (\vec{b}+\vec{c}).\vec{q} + (\vec{c}+\vec{a}).\vec{r}$ is equal to **[IIT - 88]**
(A) 0 (B) 1 (C) 2 (D) 3

6. Let a, b, c be distinct nonnegative numbers. If vectors ai + aj + ck, i + k and ci + cj + bk lie in a plane, then c is : **[IIT - 93]**
(A) the AM of a and b (B) the GM of a and b (C) the HM of a and b (D) equal to zero

7. Let $\vec{a} = \hat{i} - \hat{j},\ \vec{b} = \hat{j} - \hat{k},\ \vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}.\vec{d} = 0 = (\vec{b}\ \vec{c}\ \vec{d})$, then $\vec{d}$ equals

(A) $\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$ (B) $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (C) $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (D) $\pm\hat{k}$ **[IIT - 95]**

8. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{\sqrt{2}}(\vec{b}+\vec{c})$ then the angle between

$\vec{a}$ and $\vec{b}$ is : **[IIT - 95]**
(A) $3\pi/4$ (B) $\pi/4$ (C) $\pi/2$ (D) π

9. Let $\vec{u}, \vec{v}, \vec{w}$ be the vectors such that $\vec{u}+\vec{v}+\vec{w} = 0$, if $|\vec{u}| = 3,\ |\vec{v}| = 4$ and $|\vec{w}| = 5$ then the value of $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is : **[IIT - 95]**
(A) 47 (B) -25 (C) 0 (D) 25

10. If $\vec{A}, \vec{B}, \vec{C}$ are three non coplanar vectors, then $(\vec{A}+\vec{B}+\vec{C}).[(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})]$ equals

(A) 0 (B) $[\vec{A}\,\vec{B}\,\vec{C}]$ (C) $2[\vec{A}\,\vec{B}\,\vec{C}]$ (D) $-[\vec{A}\,\vec{B}\,\vec{C}]$ **[IIT - 95]**

11. If $\vec{p}, \vec{q}, \vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation $\vec{p}\times\left((\vec{x}-\vec{q})\times\vec{p}\right) + \vec{q}\times\left((\vec{x}-\vec{r})\times\vec{q}\right) + \vec{r}\times\left((\vec{x}-\vec{p})\times\vec{r}\right) = \vec{0}$, then $\vec{x}$ is given by **[IIT - 97]**

(A) $\frac{1}{2}(\vec{p}+\vec{q}-2\vec{r})$ (B) $\frac{1}{2}(\vec{p}+\vec{q}+\vec{r})$ (C) $\frac{1}{3}(\vec{p}+\vec{q}+\vec{r})$ (D) $\frac{1}{3}(2\vec{p}+\vec{q}-\vec{r})$

12. Let $\vec{a}=2\vec{i}+\vec{j}-2\hat{k}$ and $\vec{b}=\vec{i}+\vec{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c}=|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is 30^0, then $|(\vec{a}\times\vec{b})\times\vec{c}|=$ **[IIT - 99]**

(A) 2/3 (B) 3/2 (C) 2 (D) 3

13. Let $\vec{a}=2\vec{i}+\mathrm{j}+\vec{k}$, $\vec{b}=\vec{i}+2\vec{j}-\vec{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ perpendicular to $\vec{a}$, then $\vec{c}$ = **[IIT - 99]**

(A) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (B) $\frac{1}{\sqrt{3}}(-\hat{i}-\hat{j}-\hat{k})$ (C) $\frac{1}{\sqrt{5}}(\hat{i}-2\hat{j})$ (D) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

14. If the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ form the sides BC, CA and AB respectively of a triangle ABC, then **[IIT - 2000]**

(A) $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$ (B) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(C) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}$ (D) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

15. Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ be such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{a})=\vec{o}$. Let P_1 and P_2 be planes determined by the pairs of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{d}$ respectively. Then the angle between P_1 and P_2 is: **[IIT - 2000]**

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

16. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a}-\vec{b} \quad 2\vec{b}-\vec{c} \quad 2\vec{c}-\vec{a}]=$

(A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$ **[IIT - 2000]**

17. Let $\vec{a}=\vec{i}-\vec{k}$, $\vec{b}=x\vec{i}+\vec{j}+(1-x)\vec{k}$ and $\vec{c}=y\vec{i}+x\vec{j}+(1+x-y)\vec{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on **[IIT - 2001]**

(A) only x (B) only y (C) Neither x Nor y (D) both x and y

18. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$ does NOT exceed **[IIT - 2001]**

(A) 4 (B) 9 (C) 8 (D) 6

19. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is **[IIT - 2002]**

(A) 45^0 (B) 60^0 (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

20. Let $\vec{V}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{W}=\vec{i}+3\vec{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $\left[\vec{U} \quad \vec{V} \quad \vec{W}\right]$ is **[IIT - 2002]**

(A) -1 (B) $\sqrt{10}+\sqrt{6}$ (C) $\sqrt{59}$ (D) $\sqrt{60}$

21. The value of 'a' so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is **[IIT - 2003]**

(A) –3 (B) 3 (C) $1/\sqrt{3}$ (D) $\sqrt{3}$

22. If $\vec{a} = (\hat{i}+\hat{j}+\hat{k})$, $\vec{a}.\vec{b} = 1$ and $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$, then $\vec{b}$ is **[IIT - 2004]**

(A) $\hat{i}-\hat{j}+\hat{k}$ (B) $2\hat{j}-\hat{k}$ (C) $\hat{i}$ (D) $2\hat{i}$

23. The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is **[IIT - 2004]**

(A) $\dfrac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (B) $\dfrac{2\hat{i}-3\hat{j}}{\sqrt{13}}$ (C) $\dfrac{3\hat{i}-\hat{k}}{\sqrt{10}}$ (D) $\dfrac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

24. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and

$$\vec{b}_1 = \vec{b} - \frac{\vec{a}.\vec{b}}{a^2}\vec{a},\ \vec{b}_2 = \vec{b} + \frac{\vec{a}.\vec{b}}{a^2}\vec{a},\ \ \vec{c}_1 = \vec{c} - \frac{\vec{c}.\vec{a}}{a^2}\vec{a} + \frac{\vec{b}.\vec{c}}{c^2}\vec{b}_1,\ \vec{c}_4 = \frac{\vec{c}.\vec{a}}{c^2}\vec{a} - \frac{\vec{b}.\vec{c}}{c^2}\vec{b}_1,$$

$$\vec{c}_2 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}.\vec{c}}{|\vec{b}_1|^2}\vec{b}_1,\ \vec{c}_3 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1,\ \vec{c}_4 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b}_1$$

then the triplet of pairwise orthogonal vectors is **[IIT - 2005]**

(A) $(\vec{a}, \vec{b}_1, \vec{c}_3)$ (B) $(\vec{a}, \vec{b}_1, \vec{c}_2)$ (C) $(\vec{a}, \vec{b}_1, \vec{c}_1)$ (D) $(\vec{a}, \vec{b}_2, \vec{c}_2)$

25. $\vec{a} = \hat{i}+2\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-\hat{j}+\hat{k}$, $\vec{c} = \hat{i}+\hat{j}-\hat{k}$. A vector coplanar to $\vec{a}$ and $\vec{b}$ has a projection along $\vec{c}$ of magnitude $\dfrac{1}{\sqrt{3}}$, then the vector is **[IIT - 2006]**

(A) $4\hat{i}-\hat{j}+4\hat{k}$ (B) $4\hat{i}+\hat{j}-4\hat{k}$ (C) $2\hat{i}+\hat{j}+\hat{k}$ (D) none of these

26. The number of distance real value of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is

(A) zero (B) one (C) two (D) three

27. Let the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}$, $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement 1 : $\overrightarrow{PQ}\times\left(\overrightarrow{RS}+\overrightarrow{ST}\right)\neq\vec{0}$

because

Statement -2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$.

(A) Statement-1 is True, Statment-2 is True. Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

28. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct ?

(A) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$ (B) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$

(C) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}\neq\vec{0}$ (D) $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{a}$ are mutually perpendicular

IIT JEE PROBLEMS (SUBJECTIVE)

1. $A_1, A_2, ..., A_n$ are the vertices of a regular plane polygon with n sides and O is its centre. Show that $\sum_{i=1}^{n-1}\left(\overrightarrow{OA}_i\times\overrightarrow{OA}_{i+1}\right)=(1-n)\left(\overrightarrow{OA}_2\times\overrightarrow{OA}_1\right)$. **[IIT - 82]**

2. Find all values of λ such that $x, y, z \neq (0, 0, 0)$ and $(\vec{i}+\vec{j}+3\vec{k})x+(3\vec{i}-3\vec{j}+\vec{k})y+(-4\vec{i}+5\vec{j})z=\lambda(\vec{i}x+\vec{j}y+\vec{k}z)$, where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors along the coordinate axes. **[IIT - 82]**

3. If c be a given non-zero scalar, and $\vec{A}$ and $\vec{B}$ be given non-zero vectors such that $\vec{A}\perp\vec{B}$, find the vector $\vec{X}$ which satisfies the equations $\vec{A}.\vec{X}=c$ and $\vec{A}\times\vec{X}=\vec{B}$. **[IIT - 83]**

4. A vector $\vec{A}$ has components A_1, A_2, A_3 in a right handed rectangular cartesian coordinate system oxyz. The coordinate system is rotated about the x-axis through and angle $\frac{\pi}{2}$. Find the components of A in the new coordinate system in terms of A_1, A_2, A_3. **[IIT - 83]**

5. The position vectors of the points A, B, C and D are $3\hat{i}-2\hat{j}-\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$, $-\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, find the value of λ.
[IIT - 86]

6. If A, B, C, D are any four points in space, prove that $\left|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}\right|=4$ (Area of the triangle ABC). **[IIT - 87]**

7. Let OACB be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA. Using vector methods prove that BD and CO intersect in the same ratio. Determine this ratio.
[IIT - 88]

8. If vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, show that $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = \vec{0}$. **[IIT - 89]**

9. In a triangle OAB, E is the midpoint of BO and D is a point on AB such that AD : DB = 2 : 1. If OD and AE intersect at P, determine the ratio OP : PD using vector methods. **[IIT - 89]**

10. Let $\vec{A} = 2\vec{i} + \vec{k}$, $\vec{B} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{C} = 4\vec{i} - 3\vec{j} + 7\vec{k}$. Determine a vector $\vec{R}$. Satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}.\vec{A} = 0$. **[IIT - 90]**

11. Determine the value of 'c' so that for all real x, the vector $cx\hat{i} - 6\hat{j} - 3\hat{k}$ and $x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other. **[IIT - 91]**

12. In a triangle ABC, D and E are points on BC and AC respectively, such that BD = 2DC and AE = 3 EC. Let P be the point of intersection of AD and BE. Find $\frac{BP}{PE}$ using vector method. **[IIT - 93]**

13. Find the distance of the point B(i + 2j + 3k) from the line which is passing through A(4i + 2j + 2k) and which is parallel to the vector $\vec{C} = 2i + 3j + 6k$. **[IIT - 93]**

14. A rigid body is rotating at 5 radians/sec. about an axis AB, where A and B are the points (2i + j + k) and (8i - 2j + 3k) respectively. Find the velocity of the particle P of the body at the point (5i - j + k). **[REE-93]**

15. Solve the following simultaneous equations for vectors $\vec{x}, \vec{y}$ $\begin{array}{l} \vec{x} + \vec{y} = \vec{a} \\ \vec{x} \times \vec{y} = \vec{b} \\ \vec{x}.\vec{a} = 1 \end{array}$. **[REE-94]**

16. A rigid body is rotating about an axis through the point (3, -1, -2). If the particle at the point (4, 1, 0) has velocity 4i - 4j + 2k and that at the point (3, 2, 1) has velocity 6i - 4j + 4k. Find the magnitude and direction of the angular velocity of the body. **[REE-94]**

17. If the vectors $\vec{b}, \vec{c}, \vec{d}$ are not coplanar, then prove that the vector $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$ is parallel to $\vec{a}$. **[IIT - 94]**

18. Find the scalars α, β if $\vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a}.\vec{b})\vec{b} = (4 - 2\beta - \sin\alpha)\vec{b} + (\beta^2 - 1)\vec{c}$ and $(\vec{c}.\vec{c})\vec{a} = \vec{c}$ while $\vec{b}, \vec{c}$ are non zero non collinear vectors. **[REE-95]**

19. A, B, C and D are four points such that $\vec{AB} = m(2\vec{i} - 6\vec{j} + 2\vec{k})$, $\vec{BC} = (\vec{i} - 2\vec{j})$ and $\vec{CD} = n(-6\vec{i} + 15\vec{j} - 3\vec{k})$. Find the conditions on the scalars m and n so that CD intersects AB at some point E. Also find the area of the triangle BCE. **[REE-95]**

20. The position vectors of the vertices A, B and C of a tetrahedron ABCD are i + j + k, i and respectively. The altitude from vertex D to the opposite face ABC meets the median line through A of the triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron $\frac{2\sqrt{2}}{3}$ is. Find the position vector of the point E for all its possible positions. **[IIT - 96]**

21. Let x, y and z be unit vectors such that $\hat{x} + \hat{y} + \hat{z} = \hat{a}$, $\hat{x} \times (\hat{y} \times \hat{z}) = \vec{b}$, $(\hat{x} \times \hat{y}) \times \hat{z} = \vec{c}$, $\vec{a}.\hat{x} = \frac{3}{2}$, $\vec{a}.\hat{y} = \frac{7}{4}$ and $|\vec{a}| = 2$. Find x, y and z in terms $\vec{a}$, $\vec{b}$, $\vec{c}$. **[REE-96]**

22. The position vectors of two points of A and C are 9i - j + 2k and 7i - 2j + 7k respectively. The point of intersection of vectors $\vec{AB} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{CD} = 2\hat{i} - \hat{j} + 2\hat{k}$ is P. If vector $\vec{PQ}$ is perpendicular to $\vec{AB}$ and $\vec{CD}$ and PQ = 15 units. Find the position vector of Q. **[REE-96]**

23. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are vectors such that $|\vec{B}| = |\vec{C}|$, prove that ;
$[(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} \times \vec{C}).(\vec{B} + \vec{C}) = 0$. **[IIT - 97]**

24. Vectors $\vec{x}$, $\vec{y}$, $\vec{z}$ each of magnitude $\sqrt{2}$, make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}$, $y \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$ then find $\vec{x}$, $\vec{y}$, $\vec{z}$ in terms of $\vec{a}$, $\vec{b}$, $\vec{c}$. **[IIT - 97]**

25. The position vectors of the points P and Q are $5\vec{i} + 7\vec{j} - 2\vec{k}$ and $-3\vec{i} + 3\vec{j} + 6\vec{k}$ respectively. The vector $\vec{A} = 3\vec{i} - \vec{j} + \vec{k}$ passes through the point P and the vector $\vec{B} = -3\vec{i} + 2\vec{j} + 4\vec{k}$ passes through the point Q. A third vectors $2\vec{i} + 7\vec{j} - 5\vec{k}$ intersects vectors $\vec{A}$ and $\vec{B}$. Find the position vectors of the points of intersection. **[REE-97]**

26. Prove, by vector methods or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the midpoints of parallel sides. (You may assume that the trapezium is not a parallelogram). **[IIT - 98]**

27. For any two vectors $\vec{u}$, $\vec{v}$, prove that **[IIT - 98]**

(i) $(\vec{u}.\vec{v})^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2$ and

(ii) $(1 + |\vec{u}|^2)(1 + |\vec{v}|^2) = (1 - \vec{u}.\vec{v}|)^2 + |\vec{u} + \vec{v} + (\vec{u} \times \vec{v})|^2$

28. If $\vec{x}\times\vec{y}=\vec{a},\ \vec{y}\times\vec{z}=\vec{b},\ \vec{x}.\vec{b}=\gamma,\ \vec{x}.\vec{y}=1$ and $\vec{y}.\vec{z}=1$ then find $\vec{x},\ \vec{y},\ \vec{z}$ in terms of $\vec{a},\ \vec{b},\ \gamma$. **[REE-98]**

29. Vectors $\vec{AB}=3\hat{i}-\vec{j}+\vec{k}$ and $\vec{CD}=-3\hat{i}+2\vec{j}+4\vec{k}$ are not coplanar. The position vectors of points A and C are $6\hat{i}+7\vec{j}+4\vec{k}$ and $-9\vec{j}+2\vec{k}$ respectively. Find the position vectors of a point P on the line AB and a point Q on the line CD such that $\vec{PQ}$ is perpendicular to $\vec{AB}$ and $\vec{CD}$ both. **[REE - 98]**

30. Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w}+(\vec{w}\times\vec{u})=\vec{v}$, then prove that $|(\vec{u}\times\vec{v}).\vec{w}|\le\frac{1}{2}$ and the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$. **[REE - 98]**

31. An arc AC of a circle subtends a right angle at the centre O. The point B divides the arc in the ratio 1 : 2. If $\vec{OA}=\vec{a}$ and $\vec{OB}=\vec{b}$, then calculate $\vec{OC}$ in terms of $\vec{a}$ and $\vec{b}$. **[IIT - 99]**

32. If $\vec{a},\ \vec{b},\ \vec{c}$ are non-coplanar vectors and $\vec{d}$ is a unit vector, then find the value of, $|(\vec{a}.\vec{d})(\vec{b}\times\vec{c})+(\vec{b}.\vec{d})(\vec{c}\times\vec{a})+(\vec{c}.\vec{d})(\vec{a}\times\vec{b})|$ independent of $\vec{d}$. **[REE - 99]**

33. Show, by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices. **[IIT - 2001]**

34. Let ABC and PQR be any two triangle in the same plane. Assume that the perpendiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from P, Q, R to BC, CA, AB respectively are also concurrent. **[IIT - 2000]**

35. If $\vec{a}=\hat{i}+\hat{j}-\hat{k},\ \vec{b}=-\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-\hat{k}$, find a unit vector normal to the vectors $\vec{a}+\vec{b}$ and $\vec{b}-\vec{c}$. **[REE-2000]**

36. Given that vectors $\vec{a},\ \vec{b}$ are perpendicular to each other, find vector $\vec{\upsilon}$ in terms of $\vec{a},\ \vec{b}$ satisfying the equations, $\vec{\upsilon}.\vec{a}=0,\ \vec{\upsilon}.\vec{b}=1$ and $[\vec{\upsilon},\ \vec{a},\ \vec{b}]=1$. **[REE-2000]**

37. $\vec{a},\ \vec{b},\ \vec{c}$ are three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}(\vec{b}+\vec{c})$. Find angle between vectors $\vec{a},\ \vec{b}$ given that vectors $\vec{b},\ \vec{c}$ are nonparallel. **[REE-2000]**

38. A particle is placed at a corner P of a cube of side 1 meter. Forces of magnitudes 2, 3 and 5 kg weight act on the particle along the diagonals of the faces passing through the point P. Find the moment of these forces about the corner opposite to P. **[REE-2000]**

39. The diagonals of a parallelogram are given by vectors $2\hat{i}+3\hat{j}-6\hat{k}$ and $3\hat{i}-4\hat{j}-\hat{k}$. Determine its sides and also the area. **[REE - 2001]**

40. Find the value of λ such that a, b, c are all nonzero and
$(-4\hat{i}+5\hat{j})a+(3\hat{i}-3\hat{j}+\hat{k})b+(\hat{i}+\hat{j}+3\hat{k})c=\lambda(a\hat{i}+b\hat{j}+c\hat{k})$. **[REE - 2001]**

41. Find the vector $\vec{r}$ which is perpendicular to $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{r}.(2\hat{i}+\hat{j}+\hat{k})+8=0$. **[REE - 2001]**

42. Two vertices of a triangle are at $-\hat{i}+3\hat{j}$ and $2\hat{i}+5\hat{j}$ and its orthocenter is at $\hat{i}+2\hat{j}$. Find the position vector of third vertex. **[REE - 2001]**

43. Show by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrence in terms of the position vectors of the vertices. **[IIT - 2001]**

44. Find 3-dimension vectors $\vec{v}_1,\ \vec{v}_2,\ \vec{v}_3$ satisfying $\vec{v}_1.\vec{v}_1=4,\ \vec{v}_1.\vec{v}_2=-2,\ \vec{v}_1.\vec{v}_3=6$, $\vec{v}_2.\vec{v}_2=2,\ \vec{v}_2.\vec{v}_3=-5,\ \vec{v}_3.\vec{v}_3=29$. **[IIT - 2001]**

45. Let $\vec{A}(t)=f_1(t)\hat{i}+f_2(t)\hat{j}$ and $\vec{B}(t)=g_1(t)\hat{i}+g_2(t)\hat{j},\quad t\in[0,1]$, where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are nonzero vectors for all t and $\vec{A}(0)=2\hat{i}+3\hat{j}$, $\vec{A}(1)=6\hat{i}+2\hat{j},\ \ \vec{B}(0)=3\hat{i}+2\hat{j}$ and $\vec{B}(1)=2\hat{i}+6\hat{j}$, then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t. **[IIT - 2001]**

46. Let V be the volume of the parallelepiped formed by the vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If a_r, b_r, c_r, where r = 1, 2, 3, are non negative real numbers and $\sum_{r=1}^{3}(a_r,b_r,c_r)=3L$, show that $V\le L^3$. **[IIT - 2002]**

47. If $\hat{a},\hat{b},\hat{c}$ are three non-coplanar unit vectors and $\alpha,\ \beta,\ \gamma$ are the angles between $\hat{a}$ and $\hat{b}$, $\hat{b}$ and $\hat{c}$, $\hat{c}$ and $\hat{a}$ respectively and $\vec{x},\vec{y},\vec{z}$ are unit vectors along the bisectors of the angles $\alpha,\ \beta,\ \gamma$ respectively. Prove that $\left[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}\right]=\dfrac{1}{16}[a\ b\ c]^2\ \sec^2\dfrac{\alpha}{2}\ \sec^2\dfrac{\beta}{2}\ \sec^2\dfrac{\gamma}{2}$. **[IIT - 2003]**

48. Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1). If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in **(i)** and mid-point of PQ lies on it. **[IIT - 2003]**

49. If $\vec{u},\ \vec{v},\ \vec{w}$ are three non-coplanar unit vectors and α,β,γ are the angles between $\vec{u}$ and $\vec{v}$ and $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x},\ \vec{y},\ \vec{z}$ are unit vectors along the bisectors of the α,β,γ respectively. Prove that $[\vec{x}\times\vec{y}\ \vec{y}\times\vec{z}\ \vec{z}\times\vec{x}]=\dfrac{1}{16}[\vec{u}\ \vec{v}\ \vec{w}]\sec^2\dfrac{\alpha}{2}\sec^2\dfrac{\beta}{2}\sec^2\dfrac{\gamma}{2}$. **[IIT - 2003]**

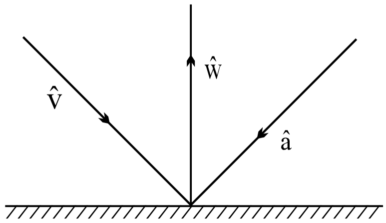
50. If $\vec{a},\ \vec{b},\ \vec{c}$ and $\vec{d}$ are four distinct vectors satisfying $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $\vec{a}.\vec{b}-\vec{a}.\vec{c}\ne\vec{d}.\vec{b}-\vec{d}.\vec{c}$. **[IIT - 2004]**

51. A plane containing two lines with direction ratios (1, –1, 0) and (–1, 0, 1) passes through the point

(1, 1, 1). Find the volume of the tetrahedron whose vertices are origin and the points where the coordinates axes meet the plane. **[IIT - 2004]**

52. $\hat{u}$ is incident on a plane whose unit vector normal to the plane is $\hat{a}$. If $\hat{v}$ is the reflected ray. Find $\hat{v}$ in terms of $\hat{u}$ and $\hat{a}$. **[IIT - 2004]**

53. If the incident ray on a surface is along the unit vector $\hat{v}$, the reflected ray is along the unit vector $\hat{w}$ and the normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$. **[IIT - 2005]**



SET - I

1. The value of $|a \times \hat{i}|^2 + |a \times \hat{j}|^2 + |a \times \hat{k}|^2$ is

(A) a^2 (B) $2a^2$ (C) $3a^2$ (D) none of these

2. Let $P = 3i + 4j$, $q = 5i$, $r = \frac{1}{4}(p+q)$ and $2s = p - 1$. Then

(A) $|p + r| = |q + s|$
(B) $|r + \lambda s| = |r - \lambda s|$, $\lambda \in R$
(C) $|p + q| = |p - q|$
(D) r is $\perp$ to s

3. Distance of $P(\vec{p})$ from the plane $\vec{r} = \vec{a} + \lambda\vec{b}$ is

(A) $\left|(\vec{a}-\vec{p}) + \frac{((\vec{p}-\vec{a}).\vec{b})\vec{b}}{|\vec{b}|^2}\right|$

(B) $\left|(\vec{b}-\vec{p}) + \frac{((\vec{p}-\vec{a}).\vec{b})\vec{b}}{|\vec{b}|^2}\right|$

(C) $\left|(\vec{a}-\vec{p}) + \frac{((\vec{p}-\vec{a}).\vec{b})\vec{b}}{|\vec{b}|^2}\right|$

(D) none of these

4. If the projection of point $P(\vec{p})$ on the plane $\vec{r}.\vec{n} = q$ is the point $S(\vec{s})$

(A) $\vec{s} = \frac{(q - \vec{p}.\vec{n})\vec{n}}{|\vec{n}|^2}$

(B) $\vec{s} = \vec{p} + \frac{(q - \vec{p}.\vec{n})\vec{n}}{|\vec{n}|^2}$

(C) $\vec{s} = \vec{p} - \frac{(\vec{p}.\vec{n})\vec{n}}{|\vec{n}|^2}$

(D) $\vec{s} = \vec{p} - \frac{(\vec{q} - \vec{p}.\vec{n})\vec{n}}{|\vec{n}|^2}$

5. Angle between $\hat{i}$ and the line of intersection of the planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 0$ and $\vec{r}.(3\hat{i} + 3\hat{j} + \hat{k}) = 0$ is equal to

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (C) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (D) none of these

6. $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors. $\vec{r}$ is a vector satisfying $\vec{r}.\vec{a}=0,\ \vec{r}.\vec{b}=1$ and $[\vec{r},\vec{a},\vec{b}]=1$ then $\vec{r}$ is equal to

(A) $\vec{a}+(\vec{a}\times\vec{b})$ (B) $\vec{b}+(\vec{a}\times\vec{b})$ (C) $\vec{a}+\vec{b}+(\vec{a}\times\vec{b})$ (D) $\vec{a}-\vec{b}+(\vec{a}\times\vec{b})$

7. A vector $\vec{a}=(x,y,z)$ makes an obtuse angle with y-axis, equal angles with $\vec{b}=(y,\,-2z,\,3x)$ and $\vec{c}=(2z,\,3x,\,-y)$ and $\vec{a}$ is perpendicular to $\vec{d}=(1,\,-1,\,2)$. If $|\vec{a}|=2\sqrt{2}$, then vector $\vec{a}$ is

(A) $\left(-\frac{2\sqrt{2}}{3},\frac{2\sqrt{2}}{3},\frac{2\sqrt{2}}{3}\right)$ (B) (-2, -2, 2)

(C) (1, 1, 1) (D) $\left(\frac{\sqrt{2}}{3},-\frac{\sqrt{2}}{3},\sqrt{2}\right)$

8. Let $\vec{\alpha}=a\hat{i}+b\hat{j}+c\hat{k},\ \vec{\beta}=b\hat{i}+c\hat{j}+a\hat{k}$ and $\vec{\gamma}=c\hat{i}+a\hat{j}+b\hat{k}$ be three coplanar vectors with $a\neq b$, and $\vec{v}=\hat{i}+\hat{j}+\hat{k}$. Then $\vec{v}$ is perpendicular to

(A) $\vec{\alpha}$ (B) $2\vec{\beta}$ (C) $5\vec{\gamma}$ (D) none of these

9. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$, then the value of $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is

(A) $6\vec{b}\times\vec{c}$ (B) $2(\vec{a}\times\vec{b})$ (C) $\vec{c}\times\vec{a}$ (D) $\vec{0}$

10. If $|\vec{A}|=2,\ |\vec{B}|=4,\ |\vec{C}|=5$ and $\vec{A}(\vec{B}+\vec{C})=\vec{B}(\vec{C}+\vec{A})=\vec{C}(\vec{A}+\vec{B})=0$ then $|\vec{A}+\vec{B}+\vec{C}|$ is equal to

(A) 3/2 (B) $5\sqrt{2}$ (C) $3\sqrt{5}$ (D) 0

11. If the nonzero vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then the solution of the equation $\vec{r}\times\vec{a}=\vec{b}$ is given by

(A) $\vec{r}=x\vec{a}+\frac{1}{\vec{a}.\vec{a}}(\vec{a}\times\vec{b})$ (B) $\vec{r}=x\vec{b}-\frac{1}{\vec{b}.\vec{b}}(\vec{a}\times\vec{b})$

(C) $\vec{r}=x\vec{a}\times\vec{b}$ (D) none of these

12. If a, b, c are three unit vectors, b $\|$ c and $a\times(b\times c)=\frac{1}{2}b$, then angle between a and c is

(A) $\pi/6$ (B) $\pi/2$ (C) $\pi/3$ (D) none of these

13. The vector a = xi - 3j - k and b = 2xi + xj - k include an acute angle, and b and positive y-axis include an obtuse angle. Then values of x may be

(A) 2 (B) 5 (C) all x < 0 (D) all x > 0

14. The straight lines whose direction cosines are given by $a\ell+bm+cn=0,\ fmn+gn\ell+h\ell m=0$ are perpendicular if

(A) $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$ (B) $\sqrt{(a/f)}+\sqrt{(b/g)}+\sqrt{(c/h)}=0$

(C) $\sqrt{(af)}=\sqrt{(bg)}=\sqrt{(ch)}$ (D) $\sqrt{(a/f)}=\sqrt{(b/g)}=\sqrt{(c/h)}$

15. The area of the parallelogram constructed on the vectors $\vec{a}=\vec{p}+2\vec{q}$ & $\vec{b}=2\vec{p}+\vec{q}$ where $\vec{p}$ & $\vec{q}$ are unit vectors forming an acute angle of 30º is :

(A) 3/2 (B) 5/2 (C) 7/2 (D) none of these

16. If the vector $\vec{b}$ is collinear with the vector $\vec{a}=\left[2\sqrt{2},-1,4\right]$ & $|\vec{b}|=10$, then :

(A) $\vec{a}\pm\vec{b}=0$ (B) $\vec{a}\pm 2\vec{b}=0$ (C) $2\vec{a}\pm\vec{b}=0$ (D) none of these

17. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ & $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ then the vectors $\vec{a}-\vec{d}$ & $\vec{b}-\vec{c}$ are :

(A) collinear (B) linearly independent

(C) perpendicular (D) parallel

18. The point B divides the arc AC of the quadrant of a circle with centre 'O' as the origin in the ratio 1 : 2 . If $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$, then the vector $\overrightarrow{OC}$ in terms of $\vec{a}$ & $\vec{b}$ is :

(A) $2\vec{b}-\sqrt{3}\,\vec{a}$ (B) $\sqrt{3}\,\vec{b}-2\vec{a}$ (C) $2\vec{b}+\sqrt{3}\,\vec{a}$ (D) none of these

19. The angle between the vectors $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$, given $|\vec{a}|=2$, $|\vec{b}|=1$, and angle between $\vec{a}$ & $\vec{b}$ is π/3, is :

(A) $\tan^{-1}\frac{2}{\sqrt{3}}$ (B) $\tan^{-1}\sqrt{\frac{2}{3}}$ (C) $\tan^{-1}\sqrt{\frac{3}{7}}$ (D) none of these

20. For non–zero vectors $\vec{a},\vec{b},\vec{c}$, $|\vec{a}\times\vec{b}.\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$ holds if and only if ;

(A) $\vec{a}.\vec{b}=0$, $\vec{b}.\vec{c}=0$ (B) $\vec{c}.\vec{a}=0$, $\vec{a}.\vec{b}=0$

(C) $\vec{a}.\vec{c}=0$, $\vec{b}.\vec{c}=0$ (D) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

SET - II

1. If $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$, then the angle between $\vec{a}$ & $\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{2\pi}{3}$ (C) $\frac{5\pi}{3}$ (D) $\frac{\pi}{3}$

2. Given the points A(–2, 3, –4), B(3, 2, 5), C(1, –1, 2) & D(3, 2, –4). The projection of the vector $\overrightarrow{AB}$ on the vector $\overrightarrow{CD}$ is :

(A) $\frac{22}{3}$ (B) $-\frac{21}{4}$ (C) $-\frac{47}{7}$ (D) none of these

3. Given the vertices $A(2, 3, 1)$, $B(4, 1, -2)$, $C(6, 3, 7)$ & $D(-5, -4, 8)$ of a tetrahedron. The length of the altitude drawn from the vertex D is :

(A) 7 (B) 9 (C) 11 (D) none of these

4. Four coplanar forces are applied at a point O. Each of them is equal to k, & the angle between two consecutive forces equals 45^o. Then the resultant has the magnitude equal to :

(A) $k\sqrt{2+2\sqrt{2}}$ (B) $k\sqrt{3+2\sqrt{2}}$ (C) $k\sqrt{4+2\sqrt{2}}$ (D) none of these

5. The force determined by the vector $\vec{r} = (1, -8, -7)$ is resolved along three mutually perpendicular directions, one of which is the direction of the vector $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$. Then the vector component of the force $\vec{r}$ in the direction of the vector $\vec{a}$ is :

(A) $-14\hat{i} - 14\hat{j} - 7\hat{k}$

(B) $-\frac{14}{3}\hat{i} - \frac{14}{3}\hat{j} - \frac{7}{3}\hat{k}$

(C) $-\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$

(D) none of these

6. A point taken on each median of a triangle divides the median in the ratio $1:3$, reckoning from the vertex. Then the ratio of the area of the triangle with vertices at these points to that of the original triangle is :

(A) 5:13 (B) 25:64 (C) 13:32 (D) none of these

7. If $\vec{a}, \vec{b}, \vec{c}$ represents the vectors $\overrightarrow{BC}, \overrightarrow{CA}, \overrightarrow{AB}$ respectively, then which one is correct

(A) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ (B) $\vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ (C) $\vec{c} \times \vec{a} = \vec{a} \times \vec{b}$ (D) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

8. The volume of the parallelepiped whose edges are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$ is :

(A) 7 (B) 5 (C) 4 (D) none of these

9. If $\vec{e}_1$ & $\vec{e}_2$ are two unit vectors and θ is the angle between them, then $\sin\left(\frac{\theta}{2}\right)$ is :

(A) $\frac{1}{2}|\vec{e}_1 + \vec{e}_2|$ (B) $\frac{1}{2}|\vec{e}_1 - \vec{e}_2|$ (C) $\frac{\vec{e}_1 \cdot \vec{e}_2}{2}$ (D) $\frac{|\vec{e}_1 \times \vec{e}_2|}{2|\vec{e}_1||\vec{e}_2|}$

10. Constant forces $2\hat{i} - 5\hat{j} + 6\hat{k}$, $-\hat{i}+2\hat{j}-\hat{k}$ & $2\hat{i} + 7\hat{j}$ act on a particle which is displaced from the position $4\hat{i} - 3\hat{j} - 2\hat{k}$ to the position $6\hat{i} + \hat{j} - 3\hat{k}$. Then the total work done is :

(A) 15 units (B) 17 units (C) 20 units (D) none of these

11. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}$, $\vec{b}$ to $\vec{c} + \vec{a}$ & $\vec{c}$ to $\vec{a} + \vec{b}$. Then $|\vec{a} + \vec{b} + \vec{c}|$ is :

(A) $2\sqrt{5}$ (B) $2\sqrt{2}$ (C) $10\sqrt{5}$ (D) $5\sqrt{2}$

12. Given the vectors $\vec{a}$ & $\vec{b}$ the angle between which equals 120°. If $|\vec{a}| = 3$ & $|\vec{b}| = 4$, then the length of the vector, $2\vec{a} - \frac{3}{2}\vec{b}$ is :

(A) $6\sqrt{3}$ (B) $7\sqrt{2}$ (C) $4\sqrt{5}$ (D) none of these

13. The scalar $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 =$

(A) $\vec{a}^2 + \vec{b}^2$ (B) $\vec{a}^2 - \vec{b}^2$ (C) $\vec{a}^2\ \vec{b}^2$ (D) none of these

14. The acute angle between the medians drawn from the acute angles of an isosceles right angled triangle is :

(A) $\cos^{-1}\frac{2}{3}$ (B) $\cos^{-1}\frac{3}{4}$ (C) $\cos^{-1}\frac{4}{5}$ (D) none of these

15. Given a parallelogram ABCD. If $|\overrightarrow{AB}| = a$, $|\overrightarrow{AD}| = b$ & $|\overrightarrow{AC}| = c$, then $\overrightarrow{DB} \cdot \overrightarrow{AB}$ has the value:

(A) $\frac{3a^2 + b^2 - c^2}{2}$ (B) $\frac{a^2 + 3b^2 - c^2}{2}$ (C) $\frac{a^2 - b^2 + 3c^2}{2}$ (D) none of these

16. Given a parallelogram OACB . The lengths of the vectors $\vec{OA}$, $\vec{OB}$ & $\vec{AB}$ are a, b & c respectively . The scalar product of the vectors $\vec{OC}$ & $\vec{OB}$ is :

(A) $\frac{a^2 - 3b^2 + c^2}{2}$ (B) $\frac{3a^2 + b^2 - c^2}{2}$ (C) $\frac{3a^2 - b^2 + c^2}{2}$ (D) $\frac{a^2 + 3b^2 - c^2}{2}$

17. Given three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ which satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$. If $|\vec{a}| = 3$, $|\vec{b}| = 1$ and $|\vec{c}| = 4$, then $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} =$

(A) -11 (B) -13 (C) -15 (D) none of these

18. If $\vec{a}$, $\vec{b}$, $\vec{c}$ be the unit vectors such that $\vec{b}$ is not parallel to $\vec{c}$ and $\vec{a} \times (2\vec{b} \times \vec{c}) = \vec{b}$ then the angle that $\vec{a}$ makes with $\vec{b}$ & $\vec{c}$ are respectively :

(A) $\frac{\pi}{3}$ & $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ & $\frac{2\pi}{3}$ (C) $\frac{\pi}{2}$ & $\frac{2\pi}{3}$ (D) $\frac{\pi}{2}$ & $\frac{\pi}{3}$

19. P, Q have position vectors $\vec{a}$ & $\vec{b}$ relative to the origin 'O' & X, Y divide $\vec{PQ}$ internally and externally respectively in the ratio 2 : 1 . Vector $\vec{XY}$ =

(A) $\frac{3}{2}(\vec{b} - \vec{a})$ (B) $\frac{4}{3}(\vec{a} - \vec{b})$ (C) $\frac{5}{6}(\vec{b} - \vec{a})$ (D) $\frac{4}{3}(\vec{b} - \vec{a})$

20. The vectors $\vec{p}$ & $\vec{q}$ satisfy the system of equations , $2\vec{p}+\vec{q}=\vec{a}$, $\vec{p}+2\vec{q}=\vec{b}$ and the angle between $\vec{p}$ & $\vec{q}$ is θ . If it is known that in the rectangular system of coordinates the vectors $\vec{a}$ & $\vec{b}$ have the forms $\vec{a} = (1, 1)$ & $\vec{b} = (1, -1)$, then $\cos\theta =$

(A) $\frac{4}{5}$ (B) $-\frac{4}{5}$ (C) $-\frac{3}{5}$ (D) none of these

SET - III

Multiple Choice Questions with One or More Than One Correct Answer

1. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}$. The value of x are

(A) –2/3 (B) 2/3 (C) 1/3 (D) 2

2. If in a right angle triangle ABC, the hypotenious AB = p, then $\vec{AB}.\vec{AC} + \vec{BC}.\vec{BA} + \vec{CA}.\vec{CB}$ is equal to

(A) $2p^2$ (B) $p^2/2$ (C) p^2 (D) $AC^2 + BC^2$

3. If $|\vec{a}| = 2$, $|\vec{b}|=3$, $|2\vec{a}-\vec{b}| = 5$, then

(A) $\vec{a}.\vec{b}=1$ (B) $\vec{a}.\vec{b}=0$ (C) $|2\vec{a}+\vec{b}|=5$ (D) $|2\vec{a}+\vec{b}|=\sqrt{5}$

4. If $\vec{a}, \vec{b}, \vec{c}$ are any three vectors and $(\vec{c}\times\vec{a})\times\vec{b} = 0$ then which of the following is/are possible?

(A) $\vec{a}, \vec{b}$ or $\vec{c}$ may be zero vector

(B) $\vec{b}$ may be perpendicular to both $\vec{a}$ and $\vec{c}$

(C) $\vec{a}$ and $\vec{c}$ may be collinear

(D) $\vec{a}$ and $\vec{c}$ are non collinear

5. The vector $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$ and $\vec{a}-k\vec{b}$ (k scalar) are collinear for

(A) k = 0 (B) k = 1 (C) k = –1 (D) k = 2

W I. In a rhombus OABC, vector $\vec{a}, \vec{b}, \vec{c}$ are respectively the position vectors of vertices A, B, C with reference to O as origin. A point E is taken on the side BC which divides it in the ratio of 2 : 1. Also, the line segment AE intersects the line bisecting the angle O internally in point P. If CP, when extended meets AB in point F, then

6. The position vector of point P, is

(A) $\frac{3}{5}(\vec{a}+\vec{c})$ (B) $\frac{1}{5}(\vec{a}+\vec{c})$ (C) $(\vec{a}+\vec{c})$ (D) none of these

7. The position vector of point F, is

(A) $\vec{a}+\frac{1}{3}\vec{c}$ (B) $\vec{a}+\vec{c}$ (C) $\vec{a}-\frac{1}{3}\vec{c}$ (D) none of these

8. The vector $\overrightarrow{AF}$, is given by

(A) $\frac{1}{3}\vec{c}$ (B) $\vec{c}$ (C) $\frac{1}{2}\vec{c}$ (D) none of these

W II In the following questions an **Assertion (A)** is given followed by a **Reason (R)**.
(A) both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'
(B) both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'
(C) Assertion is true but Reason is false
(D) Assertion is false but Reason is true

9. **Assertion (A) :** In ΔABC $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0$

Reason (R) : If $\overrightarrow{OA}=\vec{a}.\overrightarrow{OB}=\vec{b}$, then $\overrightarrow{AB}=\vec{a}+\vec{b}$ (triangle law of addition)

10. **Assertion (A) :** If I is the incentre of ΔABC then $|\overrightarrow{BC}|\overrightarrow{IA}+|\overrightarrow{CA}|\overrightarrow{IB}+|\overrightarrow{AB}|\overrightarrow{IC}=0$

Reason (R) : The position vector of centroid of ΔABC is $\frac{\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}}{3}$.

11. **Assertion (A) :** $\vec{a}=i+pj+2k$ and $\vec{b}=2i+3j+qk$ are parallel vectors if $p=\frac{3}{2}$, $q=4$

Reason (R) : If $\vec{a}=a_1i+a_2j+a_3k$ and $\vec{b}=b_1i+b_2j+b_3k$ are parallel $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$

12. **Assertion (A) :** If $\vec{a}=2i+k$, $\vec{b}=3j+4k$ and $\vec{c}=8i-3j$ are coplanar then $\vec{c}=4\vec{a}-\vec{b}$

Reason (R) : A set vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3...\vec{a}_n$ is said to be linearly independent if every relation of the form $\ell_1\vec{a}_1+\ell_2\vec{a}_2+\ell_3\vec{a}_3+...+\ell_n\vec{a}_n=0$ implies that $\ell_1=\ell_2=\ell_3...=\ell_n=0$ (scalar).

W III A new operation * is defined between two non antiparallel vectors $\overline{\alpha}$ and $\overline{\beta}$ as $\overline{\alpha}*\overline{\beta}=|\overline{\alpha}||\overline{\beta}|\tan\left(\frac{\theta}{2}\right)$, where θ is the angle between $\overline{\alpha}$ and $\overline{\beta}$.

13. The condition for which $\overline{\alpha}$ and $\overline{\beta}$ are perpendicular is

(A) $\overline{\alpha} * \overline{\beta} = 0$ (B) $\frac{\overline{\alpha} * \overline{\beta}}{|\overline{\alpha}||\overline{\beta}|} = 1$ (C) $\frac{\overline{\alpha} * \overline{\beta}}{|\overline{\alpha}||\overline{\beta}|} = -1$ (D) none of these

14. $\overline{\alpha} * \overline{\alpha}$ is

(A) $|\overline{\alpha}|^2$ (B) not define (C) 0 (D) none of these

15. For $\overline{\alpha} * \overline{\beta} = \overline{\alpha}.\overline{\beta}$

(A) $|\overline{\alpha}| = 0$ is a necessary condition

(B) $|\overline{\alpha}|.|\overline{\beta}| = 0$ is a necessary condition

(C) $t^3 + t^2 + t = 1$ is a sufficient condition, where $t = \tan\frac{\theta}{2}$

(D) none of these

16. Let $\overline{\alpha}$ and $\overline{\beta}$ be two linearly independent vector such that $|\overline{\alpha} \times \overline{\beta}| = |\overline{\alpha} * \overline{\beta}|$, then

(A) $\overline{\alpha} \perp \overline{\beta}$ (B) $\overline{\alpha} \parallel \overline{\beta}$

(C) $\overline{\alpha}$ and $\overline{\beta}$ are inclined at $\frac{\pi}{4}$ (D) at least one of the $\overline{\alpha}$ and $\overline{\beta}$ is null vector

17. Projection of $\overline{\alpha}$ on $\overline{\beta}$ is

(A) $\frac{\overline{\alpha} * \overline{\beta}}{|\overline{\beta}|}$ (B) $\frac{\overline{\alpha} * \overline{\beta}}{|\overline{\alpha}|}$

(C) $|\overline{\alpha}|\left(\frac{|\alpha|^2|\beta|^2 - (\overline{\alpha} * \overline{\beta})^2}{|\alpha|^2|\beta|^2 + (\overline{\alpha} * \overline{\beta})^2}\right)$ (D) $|\overline{\beta}|\left(\frac{|\alpha|^2|\beta|^2 - (\overline{\alpha} * \overline{\beta})^2}{|\alpha|^2|\beta|^2 + (\overline{\alpha} * \overline{\beta})^2}\right)$

18. **True/False**

(i) A triangle is formed by the vertices A(1, 1, 2), B(3, 4, 2) and C(5, 6, 4). The exterior angle of the triangle at the vertex B is $\cos^{-1}(5/\sqrt{31})$.

(ii) If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$, then $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 2(\vec{a} \times \vec{b})$

(iii) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non coplanar vectors, then the vectors $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ and $\vec{a} + \vec{b} + \vec{c}$ are perpendicular to each other.

(iv) In the triangle OAB, $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$. A point P is taken on OA such that $\frac{OP}{PA}$ =3 and a point Q is taken on OB such that $\frac{OQ}{QB} = \frac{1}{2}$. If the lines AQ and BP are perpendicular then $9a^2 + 4b^2 = 15\,\vec{a}.\vec{b}$.

(v) If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors and $\vec{d}.\vec{a} = \vec{d}.\vec{b} = \vec{d}.\vec{c} = 0$, then $\vec{d}$ is a zero vector.

19. Fill in the blanks

(i) The system reciprocal to $\vec{a}$, $\vec{b}$ and $\vec{a} \times \vec{b}$ is ....

(ii) For given vectors $\vec{a}$ (2, 1, –1), $\vec{b}$ (1, 2, 1,) $\vec{c}$ (2, –1, 3) and $\vec{d}$ (3, –1, 2) the projection of the vector $\vec{a}+\vec{c}$ on the vector $(\vec{b}-\vec{d})\times\vec{c}$ is ....

(iii) If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\vec{c}=0$ and $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to ....

(iv) If $(\vec{a}\times\vec{b}).\,(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = [\vec{a}.(\vec{b}\times\vec{c})]^k$, then k is ....

(v) The value of c for which the vector $\vec{p} = (c\log_2 x, -6, 3)$ and $\vec{q} = (\log_2 x, 2, 2c\log_2 x)$ make an obtuse angle for any $x \in (0, \infty)$ are ....

20. Match the column

(a)	The value of α for which the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+\alpha\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$ are coplanar is	(P)	4
(b)	The area of a parallelogram having diagonals $\vec{a} = 3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b} = \hat{i}-3\hat{j}+4\hat{k}$ is	(Q)	–3
(c)	$\vec{r}.\vec{a}=0$, $\vec{r}.\vec{b}=0$ and $\vec{r}.\vec{c}=0$ for some non zero vector $\vec{r}$, then the value of $[\vec{a}\,\vec{b}\,\vec{c}]$ is	(R)	$10\sqrt{3}$
(d)	The volume of parallelopiped whose sides are given OA = $2\hat{i}-3\hat{j}$, OB = $\hat{i}+\hat{j}-\hat{k}$ and OC = $3\hat{i}-\hat{k}$ is	(S)	0

ANSWER LEVEL – I

10. $u = a+\frac{4}{3}b+\frac{8}{3}c$, $v = -4c$, $w = \frac{4}{3}(c-b)$

LEVEL–II

1. $p = -\frac{1}{\sqrt{1+2\cos\theta}}$; $q = \frac{2\cos\theta}{\sqrt{1+2\cos\theta}}$; $r = -\frac{1}{\sqrt{1+2\cos\theta}}$ **or** $p = \frac{1}{\sqrt{1+2\cos\theta}}$;

$q = -\frac{2\cos\theta}{\sqrt{1+2\cos\theta}}$; $r = \frac{1}{\sqrt{1+2\cos\theta}}$

3. $\hat{x} = \frac{1}{3}\left(3\vec{a}+4\vec{b}+8\vec{c}\right)$, $\hat{y} = -4\vec{c}$, $\hat{z} = \frac{4}{3}(\vec{c}-\vec{b})$

9. $r = \frac{1}{[n_1n_2n_3]}[q_3(n_1\times n_2)+q_1(n_2\times n_3)+q_2(n_3\times n_1)]$

IIT JEE PROBLEMS (OBJECTIVE)

(A)

1. $5\sqrt{2}$ **2.** -1 **3.** 0 **4.** $\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$

5. 1 **6.** $2\hat{i}-\hat{j}$ **7.** $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}, \vec{a}-\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

8. $\frac{5\hat{i}-2\hat{j}+2\hat{k}}{3}$ **9.** $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$ or $\frac{-\hat{j}+\hat{k}}{\sqrt{2}}$ **10.** $-\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$

11. $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ **12.** $\vec{a}$ **13.** $\frac{\pi}{6}$ **14.** 6

(B)

1.	T	**2.**	T	**3.**	T	**4.**	F

(C)

1.	C	**2.**	B	**3.**	AC	**4.**	D
5.	C	**6.**	AC	**7.**	AC		

(D)

1.	A	**2.**	D	**3.**	D	**4.**	A	**5.**	D
6.	B	**7.**	A	**8.**	A	**9.**	C	**10.**	D
11.	B	**12.**	B	**13.**	A	**14.**	B	**15.**	A
16.	A	**17.**	C	**18.**	B	**19.**	B	**20.**	C
21.	C	**22.**	C	**23.**	C	**24.**	B	**25.**	A
21.	C	**22.**	C	**23.**	B				

IIT JEE PROBLEMS (SUBJECTIVE)

2. {-1, 0}

3. $\vec{x} = \frac{c}{|\vec{A}|^2}\vec{A} - \frac{1}{|\vec{A}|^2}\left(\vec{A}\times\vec{B}\right)$

4. $\vec{A} = A_2\hat{i}' - A_1\hat{j}' + A_3\hat{k}'$

5. $\lambda = -\frac{146}{13}$

8. 2 : 1

10. $\vec{R} = -\vec{i} - 8\vec{j} + 2\vec{k}$

11. $-\frac{4}{3} < c < 0$

12. 8 : 3

13. $\sqrt{10}$ units

14. $\vec{V} = \frac{5}{7}(4i + 6j - 3k)$ and $|\vec{V}| = \frac{5\sqrt{61}}{7}$ units/sec

15. $\vec{x} = \frac{\vec{a} + (\vec{a}\times\vec{b})}{(\vec{a}.\vec{a})}$ $\vec{y} = \vec{a} - \frac{\left[\vec{a} + (\vec{a}\times\vec{b})\right]}{(\vec{a}.\vec{a})}$

16. $\vec{w} = \frac{4}{3}i + \frac{2}{3}j - \frac{4}{3}k, \quad |\vec{w}| = 2$

18. $\alpha = n\pi + \frac{(-1)^n\pi}{2}, \ n \in I$ and $\beta = 1$

19. $m \ge \frac{1}{2};\ n \ge \frac{1}{3};\ \frac{\sqrt{6}}{2}$ sq units

20. $\vec{E} = 3\hat{i} - \hat{j} - \hat{k}$ or $-\hat{i} + 3\hat{j} + 3\hat{k}$

22. $6\hat{i} - 9\hat{j} - 9\hat{k}$ or $-4\hat{i} + 11\hat{j} + 11\hat{k}$

24. $\vec{x} = \vec{a}\times\vec{c};\ \vec{y} = \vec{b}\times\vec{c};\ \vec{z} = \vec{b} + \vec{a}\times\vec{c}$ or $\vec{b}\times\vec{c} - \vec{a}$

25. (2, 8, -3) ; (0, 1, 2)

28. $$x = \frac{\frac{\vec{a}\times\vec{b}}{\gamma} - \vec{a}\times\frac{\vec{a}\times\vec{b}}{\gamma}}{\left(\frac{\vec{a}\times\vec{b}}{\gamma}\right)^2};\ y = \frac{\vec{a}\times\vec{b}}{\gamma};\ z = \frac{\frac{\vec{a}\times\vec{b}}{\gamma} + \vec{b}\times\frac{\vec{a}\times\vec{b}}{\gamma}}{\left(\frac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$$

29. $P \equiv (3,8,3)$ and $Q \equiv (-3,-7,6)$

31. $\vec{c} = -\sqrt{3}\vec{a} + 2\vec{b}$ **32.** $\left[\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right]$ **35.** $\pm\hat{i}$ **36.** $\dfrac{\vec{b}}{\vec{b}^2} + \dfrac{\vec{a}\times\vec{b}}{(\vec{a}\times\vec{b})^2}$

37. $\dfrac{2\pi}{3}$ **38.** $|\vec{M}| = \sqrt{7}$

39. $\frac{1}{2}(5\hat{i} - \hat{j} - 7\hat{k}), \frac{1}{2}(-\hat{i} + 7\hat{j} - 5k); \frac{1}{2}\sqrt{1274}$ sq. units

40. $\lambda = -2 \pm \sqrt{29}$ **41.** $\vec{r} = -13\hat{i} + 11\hat{j} + 7\hat{k}$ **42.** $\frac{5}{7}\hat{i} + \frac{17}{7}\hat{j} + \lambda\hat{k}$ where $\lambda \in R$

44. $\vec{v}_1 = 2\hat{i},\ \vec{v}_2 = -\hat{i} \pm \hat{j},\ \vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$ **48.** $x + y - 2z = 3$; (6, 5, –2)

51. $\dfrac{9}{2}$ **53.** $\hat{\omega} = \hat{v} - 2(\hat{a}, \hat{v})\hat{a}$

SET–I

1.	B	**2.**	B	**3.**	A	**4.**	B	**5.**	A
6.	B	**7.**	A	**8.**	A	**9.**	B	**10.**	C
11.	A	**12.**	C	**13.**	C	**14.**	A	**15.**	A
16.	C	**17.**	A	**18.**	A	**19.**	A	**20.**	D

SET–II

1.	D	**2.**	C	**3.**	C	**4.**	C	**5.**	B
6.	B	**7.**	D	**8.**	A	**9.**	B	**10.**	B
11.	D	**12.**	A	**13.**	C	**14.**	C	**15.**	A
16.	D	**17.**	B	**18.**	D	**19.**	D	**20.**	B

SET–III

1.	AD	**2.**	CD	**3.**	BC	**4.**	ABC	**5.**	ABCD
6.	A	**7.**	A	**8.**	C	**9.**	C	**10.**	B

11.	A	**12.**	B	**13.**	B	**14.**	C	**15.**	C
16.	A	**17.**	C						
18.	**(i) F**		**(ii) T**		**(iii) F**		**(iv) T**		**(v) T**

19. **(i)** $\frac{\vec{b}\times(\vec{a}\times\vec{b})}{(\vec{a}\times\vec{b})^2}, \frac{(\vec{a}\times\vec{b})\times\vec{a}}{(\vec{a}\times\vec{b})^2}, \frac{\vec{a}\times\vec{b}}{(\vec{a}\times\vec{b})^2}$ **(ii)** $\sqrt{6}$ **(iii)** $-\frac{3}{2}$

(iv) 2 **(v)** $\left(-\frac{4}{3}, 0\right]$

20. a-Q, b-R, c-S, d-P