Non-rigid Point Set Registration Based on DIS&ANG Descriptor and RANSAC

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Abstract—**Point set registration is a general problem in many domains, such as 3D reconstruction, simulation design, computer vision, and motion tracking. For non-rigid transformation, the local structure of the points reserve relatively complete, and has important significance. But the general point set registration algorithm only considers the global structure information, the local structure information is given the air. In this paper, we propose a local structure descriptor, which not only considers the distance information of the local neighborhood, but also considers the angle information between neighbors. Therefore, the description operator can more accurately describe the local structure information. Combined with the above descriptor, a non-rigid point cloud registration algorithm based on the random sample consensus (RANSAC) and the local structure information is proposed in this paper. Firstly, we use the random sample consensus algorithm to make an initial registration. Secondly, the local structure descriptor and the Gaussian mixture model are used to register exactly. Extensive experiments show that our algorithm has obvious improvement than the state-of-the-art methods under various types of distortions, such as deformation, outliers and rotation.**

Keywords-point set registration; local structure; random sample consensus

I. INTRODUCTION

Registration of point sets is a fundamental problem in computer vision, medical image analysis, and pattern recognition. The goal of point set registration is to assign correspondences between two sets of points and to recover the transformation between one point set and the other. It is also applied in many areas including stereo matching, shape matching, image registration and content-based image retrieval, etc.

The registration problem can be roughly categorized into rigid or non-rigid registration. Rigid-registration research [1], [2] is fairly mature. Because the true underlying non-rigid transformations are often unknown, non-rigid registration is more difficult. In this paper, our research focuses on non-rigid point set registration.

Since decades, many non-rigid registration methods have been proposed. Iterative closest point (ICP) [3] is the source of all registration algorithm. It iteratively determines the correspondence based on L2-distance and obtains spatial transformation related to the point sets. Because of the one-to-one correspondence in each iteration, ICP is easy to get stuck in local minimum. Besides, ICP needs a good initial transformation to assign two point sets. In addition, researchers proposed many methods [4]-[6] based on ICP algorithm.

Robust point matching (RPM) [7] improves correspondence using soft assignment. RPM jointly determines the transformation and correspondence between the sets by employing deterministic annealing for a global-to-local search and soft-assignment to relax the one-to-one correspondence. The fuzzy correspondence provides a gradual improvement without jumps in the space of binary correspondences.

Recently, some non-rigid registration algorithms based on the Gaussian mixture model (GMM) have been proposed. One of the representative method is Kernel correlation (KC) [8] registration approach proposed by Tsin and Kanade, and its cost function is represented as KL-divergence between two distributions. Another representative algorithm is Coherent point drift (CPD) [9], [10]. The method considers one point set as GMM centroids and the other set as sample points of the component of GMM. So the registration problem is represented as a mixture density estimation problem. The CPD method can deal with high dimensional point sets and be robust to outliers. The similarity of the above algorithms is that they only consider the global structure of point sets ignoring the local structure reserved relatively complete.

To preserve local neighborhood structures, Ma *et al.* introduced Robust Point Matching method based on local features and a robust L2E estimator (RPM-L2E) [11], [12]. The RPM-L2E can exploit both global and local structures, but they separately find the correspondences obtained from the shape context (SC) [13] and the transformation..

Therefore, to improve the result of registration between point sets with shape difference, we propose a non-rigid registration method based on the random sample consensus (RANSAC) [14] and the local structure information. The method adopt Gaussian mixture model to simulate the point set, and consider the local structure information by DIS&ANG descriptor.

This paper is organized as follows. Section II introduce a local structure descriptor DIS&ANG and our registration algorithm. Section III is the experimental results and performance evaluation on synthetic data. Finally, Section IV is the conclusion.

II. METHOD DESCRIPTION

In the section, we propose a local structure descriptor DIS&ANG in part A. A non-rigid point set registration method is proposed based on Gaussian mixture model and DIS&ANG descriptor in part B. Finally, considering the defects of our algorithm, we add the RANSAC and propose our final registration algorithm in part C.

A. DIS&ANG Local Structures Descriptor

We present a point set local structure descriptor by summarizing some of the current descriptors [13], [15]-[18] and add them into the registration process.

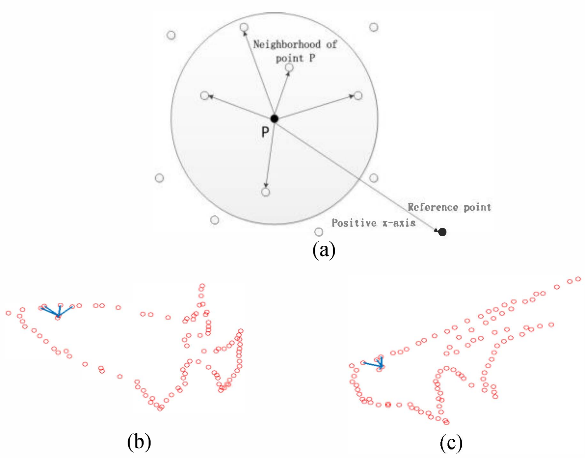


Figure 1. The distance and angle computation. (a) Graphical representation of the distance and angle computation. (b) A point $x_i \in X$ from the fish shape with its distance and angle sets $DIS(x_i)=\{0.242, 0.258, 0.187, 0.044, 0.232\}$ and $ANG(x_i)=\{2.79, 3.01, 1.91, 4.80, 1.20\}$ of its five adjacent points in . (c) The point $y_j \in Y$ in the deformed fish shape which has four adjacent points and its distance and angle sets $DIS(y_j)=\{0.225, 0.203, 0.156, 0.067\}$ and $ANG(y_j)=\{2.62, 1.45, 1.79, 4.09\}$.

As shown in Fig. 1(a), we describe the local structure information by the distance from the point P to its neighbors and the angle information. Let $D_i(x_i, a_{x_i}^j)$ be the distance between origin point x_i and its neighbor points, which $a_{x_i}^j$ represents the neighbor points of the point x_i , j represents the number of neighbors. Then the distance set of an origin point x_i is defined as $DIS(x_i)=\{D(x_i,a_{x_i}^1), D(x_i,a_{x_i}^2),\ldots,D(x_i,a_{x_i}^j)\}$; When we calculate the angle between the original point and the neighbor point, we must set a reference point. In this paper, we use the centroid of point set as a reference point. The direction from a point to the center of mass is set as the positive x-axis of the descriptor. Let $A_i(x_i, a_{x_i}^j)$ be the angle between origin point x_i and its neighbor points. The angle set of an origin point x_i is defined as $ANG(x_i)=\{A_i(x_i,a_{x_i}^1), A_i(x_i,a_{x_i}^2),\ldots,A_i(x_i,a_{x_i}^j)\}$. Any point in the point set model can be described by the distance and angle descriptor. As shown in Fig. 1 (b) and (c), we show the descriptor with $DIS(x_i)$ and $ANG(x_i)$.

The purpose of obtaining the local structure information is to obtain the similarity matrix between the two point set. It is not difficult to find a problem by observing the descriptor in Fig. 1(b) and (c). For the descriptor of the same point in the original model and deformed model, the dimension is inconsistent. This is difficulty to the comparison between descriptors. Therefore, we compare descriptor of the point pair by adding trailing zeros to the shorter one, which ensures same dimension. We define the similarity between the point set X and point set Y using the following formula:

$$W_{ij} = \exp(-dis / 2\xi^2) \tag{1}$$

where $dis = \sqrt{\| DIS(x_i) - DIS(y_j) \|^2} * \sqrt{\| ANG(x_i) - ANG(y_j) \|^2}$, ξ is a smooth coefficient, we set it to 2 by experiment.

B. Registration Using DIS&ANG Descriptor

Related definitions:

$X_{N\times D} = (x_i,\ldots,x_N)^T$: Point set X (Data point set).

$Y_{M\times D} = (y_i,\ldots,y_M)^T$: Point set Y (Centroid of the GMM), where $x_n, y_m \in R^{D\times 1}$.

$T(Y,\theta)$: Coordinate transformation of Y , θ represent transformation parameters.

A general methodology for non-rigid point set registration using Gaussian mixture models is that we consider the alignment of two point sets as a probability density estimation problem. Where one point set represents the Gaussian mixture model (GMM) centroids, and the other one represents the data points. At the optimum, two point sets become aligned and the correspondence [19], [20] is obtained using the maximum of the GMM posterior probability for a given data point [9], [10].

Considering the external points in the point cloud model, the probability density function of the Gaussian mixture model needs to be defined as follows:

$$p(x) = \sum_{m=1}^{M+1} P(m) p(x \mid m) \tag{2}$$

In this case, $p(x \mid m) = \frac{1}{(2\pi\sigma^2)^{D/2}} \exp(-\frac{\| x - y_m \|^2}{2\sigma^2})$. An additional uniform distribution $p(x \mid M+1) = \frac{1}{N}$ is added, $P(m)$ is the weight of each Gauss Single Model. The M Gaussian single model and the uniform distribution are composed of Gaussian mixture model. The Gaussian mixed model takes the form:

$$P(x) = \tau \frac{1}{a} + (1-\tau) \sum_{m=1}^{M} \frac{S_{mn}}{(2\pi\sigma^2)^{D/2}} \exp(-\frac{\| x - y_m \|^2}{2\sigma^2}) \tag{3}$$

where S_{mn} is the membership probability distribution of GMM, and $\sum_{n=1}^{N} S_{mn} = 1$; τ is the outliers rate.

Consider the Gaussian mixture model in Eq. (3), we need to set the membership probability S_{mn}. It is a prior and is typically assumed to be equal for all GMM components [9], [10], i.e., $S_{mn} = 1/M$. In this paper, we use the descriptor similarity matrix W_{ij} to initialize S_{mn}. Therefore, in our point set registration algorithm, the membership probability S_{mn} is set by the point set data, it is not a prior.

The purpose of the point set registration is to obtain the transformation and the correspondence of the point pair. In this paper, we define the non-rigid transformation as the initial position plus a displacement function $T(Y,v) = Y + v(Y)$, so that the registration problem is transformed into a question of estimating the displacement function $v(Y)$. In order to ensure the smoothness of the displacement function, we need to introduce a Hilbert space regular term [21] $\lambda \| Lv \|^2$. We obtain the objective function of the complete data by considering the negative likelihood function $E(\theta,\sigma^2) = -\sum_{n=1}^{N} \ln \sum_{m=1}^{M+1} P(m) p(x_n \mid m)$. The formula is as follows:

$$\psi(\theta,\theta^{old}) = \frac{1}{2\sigma^2}\sum_{n=1}^{N}\sum_{m=1}^{M} P^{old}(m \mid x_n) \| x_n - T(y_m) \|^2 + \frac{DN^{'}}{2}\log\sigma^2 + \frac{\lambda}{2}\| Lv \|^2 \quad (4)$$

where $N^{'} = \sum_{n=1}^{N}\sum_{m=1}^{M} P^{old}(m \mid x_n) \le N$ only $\tau = 0$ and $N = N^{'}$.

Next, what we need to do is how to estimate the parameters in the mixed model. Because the EM algorithm is simple and easy to understand, we estimate parameters by the EM algorithm in the mixed model.

C. *Algorithm Procedure*

As parts of the synthesized data get poor registration result using RANSAC [14] to do the initial registration, so we set the label $flag$ to control execution of the initial registration. The specific process is shown below:

Algorithm1. Our Non-rigid Point Set Registration Algorithm

1.**Initialization:** $U = 0$, $\sigma^2 = \frac{1}{DMN}\sum_{n=1}^{N}\sum_{m=1}^{M} \| x_n - y_m \|^2$, free parameters $0 \le \tau < 1$, $\gamma = 2$, $\beta = 2$, $\xi = 2$, $flag = 1$ $error = 0$.

2.**If** flag=1
Initial registration: Using the RANSAC algorithm to obtain a transformation ϑ for the two point sets X and Y; Update the coordinates of model Y: $Y = Y\vartheta$;

3. **End**

4. **Repeat**

5. E-step:
Using the above local structure descriptor to obtain the similarity matrix W_{ij} between the two point set;
Initialize S_{mn} by the W_{ij}, update posterior probability P;

6. M-step:
Update $T(Y)$ and σ^2;

7. **Until** convergence.

8. Get the transformation and the Correspondence C;

9. Calculate error1 by the RMSE and reset flag=0, execute step2 to step8 calculate error2;

Output: minimum error.

III. EXPERIMENTAL RESULTS

In order to evaluate the performance of our algorithm, we tested with the same synthesized data as in [9]-[12]. The experiments were performed on a laptop with 2.60 GHz Intel(R) Pentium(R) CPU G620, 16 GB memory and matlab Code.

A. *Evaluation Criterions*

The registration error between the two point sets is usually evaluated by the Euclidean distance of the two models.

In this paper, we use RMSE to measure the error. The formula of RMSE is as follows:

$$Error = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(x_n - y_n)^2} \quad (5)$$

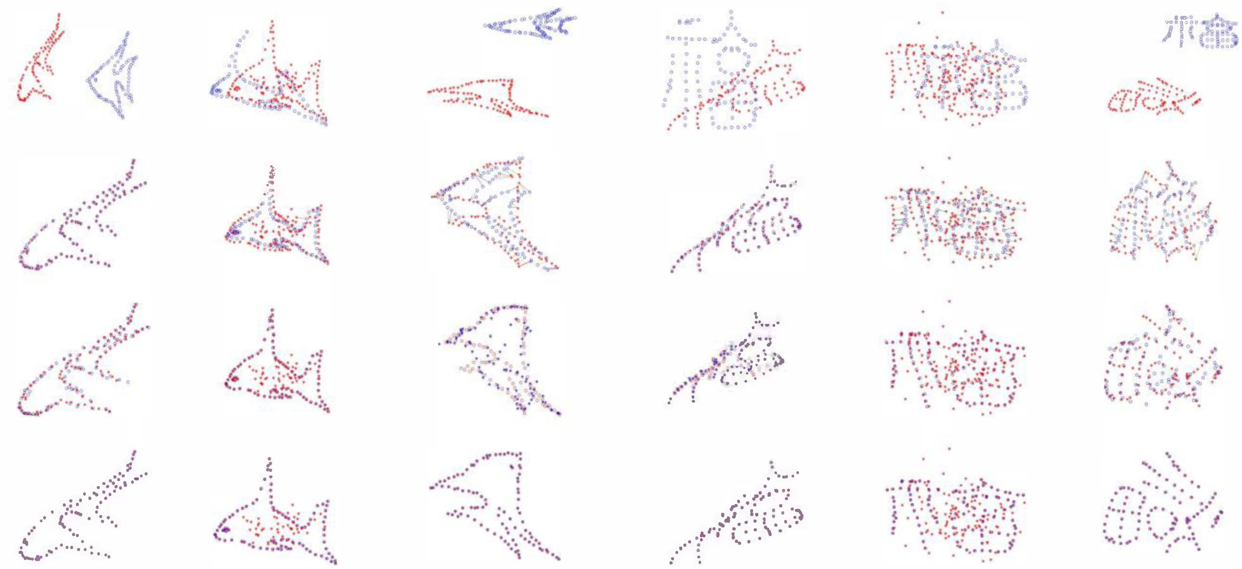
where N represent the number of correspondence of the two point sets.

B. *Experiments on Synthetic Data*

We use Chui-Rangarajan synthetic data to test our algorithm, which contains two different shapes of the model. One is the 98-points fish model, the other is 105-points Chinese character model. For each type of model, there are different degrees of deformation, rotation, outliers in the sub-level classification and each classification has 100 groups of models. Specific classification is as shown in Table I.

In our experiment, there are some free parameters need to be set in advance. According to experience, in this paper, the free parameters set as follows: First, we refer to the article [9], [10], set $\beta = 2$, $\lambda = 2$. Then, we set the remainder parameters based on different degree of deformation, for the deformation and rotation, set $\tau = 0.3$; for the outliers, according to the number of outlier points we set $\tau = 0.3, 0.5, 0.8, 0.9, 0.9$.

For the deformation, outliers and rotation, the registration result of the Fish model and the Chinese character was shown in Fig. 2. As shown in Fig. 2(a) shows the registration results for CPD algorithm, L2E algorithm, and our algorithm for deformation under the same model in sequence. From the registration results shown in Fig. 2, we can visually see that our algorithm has a greater improvement compared with



(a) (b) (c) (d) (e) (f)

Figure 2. Registration examples on the Fish and Chinese point set. From left to right are the six degradations: Fish-deformation (0.08), Fish-outlier (0.5), Fish-rot(120), Chinese-deformation (0.065), Chinese-outlier (1.0), and Chinese-rot (120). The goal is to align the model point set (blue pluses) onto the scene point set (red circles),from top to bottom respectively are registration result of Initial,CPD,L2E-RPM and Ours.

TABLE I. SPECIFIC CLASSIFICATION

Degree of deformation	0.020	0.350	0.050	0.065	0.080
Degree of rotation	30	60	90	120	180
Outlier-to-data ratio	0.1	0.3	0.5	1.0	1.25

the other two algorithms for deformation, outliers and rotation.

In order to compare with the state-of-the-art algorithm more intuitive, we tested all the models in the test set of the deformation, outliers and rotation using CPD, RPM-L2E and our method and intuitively represent by the root-mean-square error in Fig. 3 and Fig. 4.

As shown in Fig. 3 and Fig. 4, the figure is a comparison of the root-mean-square error of different methods for the Fish and Chinese model deformation, outliers and rotation. The bar of different colors in the graph represents the standard deviation of the registration error, and the center of the bar represents the mean of the registration error. We can see that the registration result is better than the other two methods for the deformation in Fig. 3(a) and Fig. 4(a). We can find that our algorithm is slightly better than RPM-L2E except the outlier ratio= 1.25 for the Fish model and outlier ratio=1/1.25 for the Chinese model, but its registration result is better than CPD algorithm for outliers in Fig. 3(b) and Fig. 4(b). Observing Fig. 3(c) and Fig. 4(c) can be found that the registration results of our method far better than the other methods.

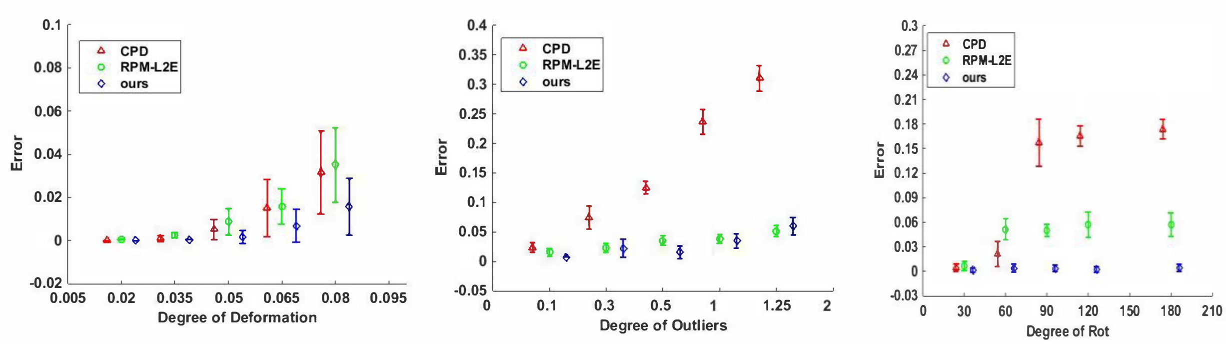
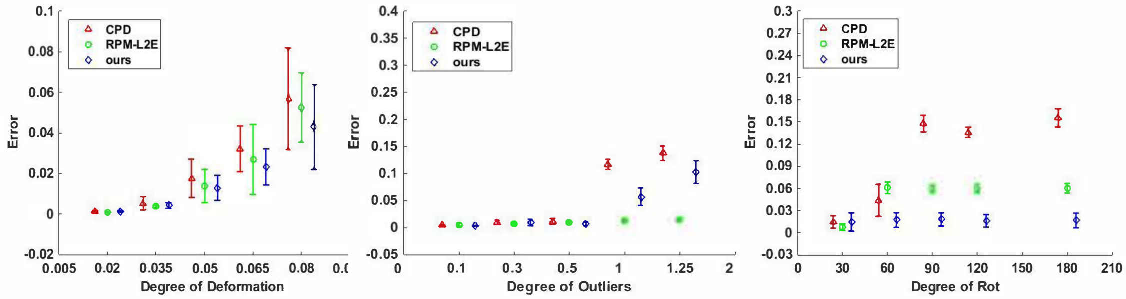


Figure 3. Performances of the registration methods on the fish point set. The registration performance of our method compared with the performances of CPD and RPM-L2E on the fish point set. Each error bar indicates the registration error mean and the standard deviation over 100 trials.



(a) (b) (c)

Figure 4. Performances of the registration methods on the Chinese point set. The registration performance of our method compared with the performances of CPD and RPM-L2E on the Chinese point set. Each error bar indicates the registration error mean and the standard deviation over 100 trials.

IV. CONCLUSION

In this paper, we propose a point set registration algorithm introducing the local structure and the RANSAC. On the one hand, we propose a local structure descriptor and introduce it into Gaussian mixture model. On the other hand, we introduce the RANSAC algorithm to do an initial registration. Experiments on public datasets for Fish and Chinese model demonstrate that our algorithm has a significant improvement compared with the state-of-the-art methods, when there are deformations, outliers and rotations in the data.

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