Submission No. 2: Volatility And Multivariate Analysis

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Code & Excel File

The code of the submission is available at https://github.com/yehjxraymond/wqu_econometrics_group_proj/tree/master/submission2

The R code, broken in two parts, are written in Jupyter Notebook with R Kernel and can be found at:

- ./submission2/Part1.ipynb
- ./submission2/Part2.ipynb

Installation instruction can be found here.

Part 1 - Volatility Modelling Analysis

1.0 Volatility Analysis.

In this assignment, we will do a forecasting of Apple (AAPL) daily stock return by applying volatility analysis using a GARCH Model. We will analyse each GARCH model i.e; ARCH, GARCH-M, IGARCH, EGARCH, TARCH, multivariate GARCH, then we will compare and select the best fit.

Daily OCHL data source of Apple is obtained from Yahoo Finance.

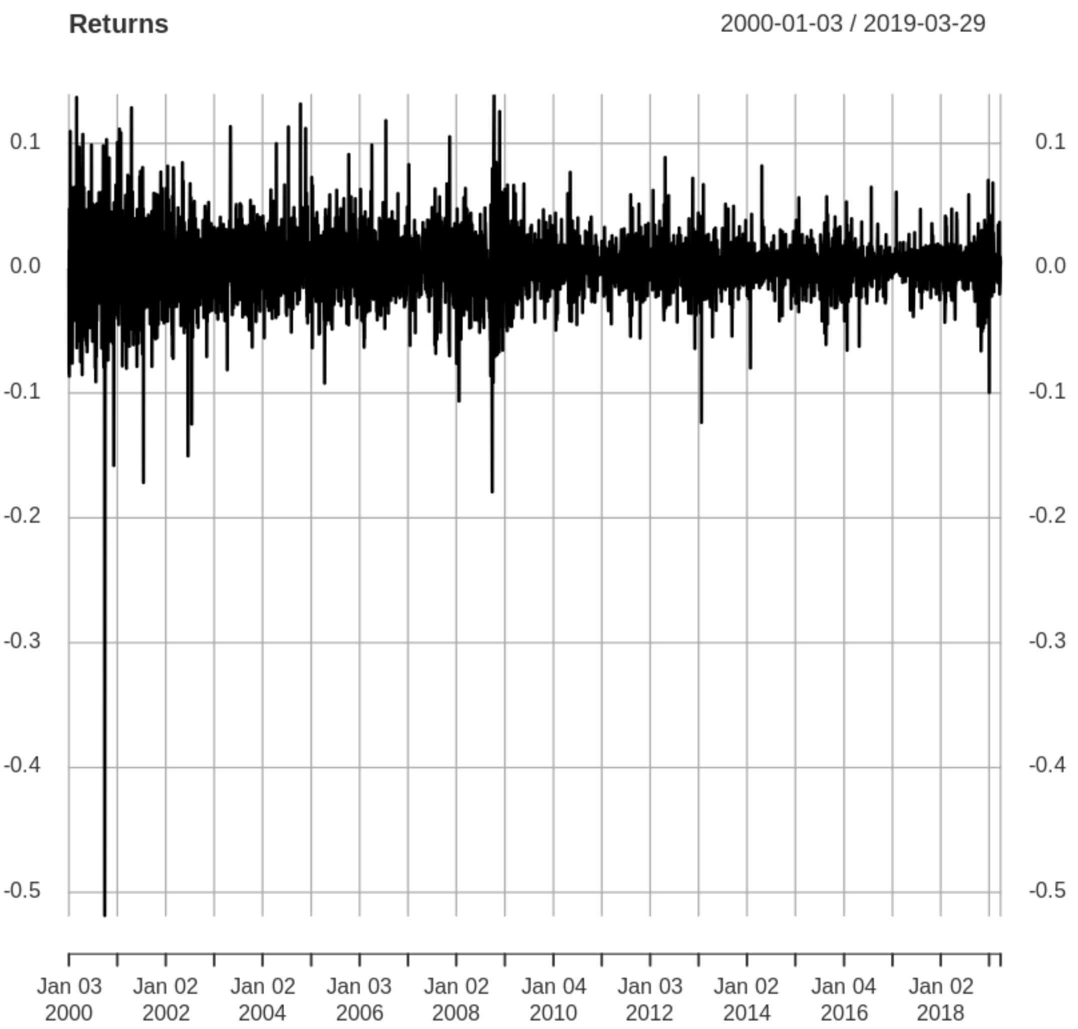
```
# Load necessary libraries

library(quantmod)
library(ggplot2)
library(tseries)
library(rugarch)
library(lmtest)
library(moments)
```

```
# Download Apple data from Yahoo Finance
getSymbols("AAPL", src = "yahoo", from = '2000-01-01', to = '2019-04-01', getSymbols.yahoo.warning=FALSE)

# Show first few rows from the dataset
head(AAPL)
```

'AAPL'

```
           AAPL.Open AAPL.High AAPL.Low AAPL.Close AAPL.Volume AAPL.Adjusted
2000-01-03  3.745536  4.017857 3.631696   3.997768   133949200      2.665724
2000-01-04  3.866071  3.950893 3.613839   3.660714   128094400      2.440975
2000-01-05  3.705357  3.948661 3.678571   3.714286   194580400      2.476697
2000-01-06  3.790179  3.821429 3.392857   3.392857   191993200      2.262367
2000-01-07  3.446429  3.607143 3.410714   3.553571   115183600      2.369532
2000-01-10  3.642857  3.651786 3.383929   3.491071   126266000      2.327857
```

```
# Plotting returns
Price = as.xts(AAPL$AAPL.Adjusted)
names(Price) = c("price")
Returns = dailyReturn(Price)
plot(Returns)
```




```
# Basic normality test on the returns
returnsDf = as.data.frame(Returns)$daily.returns
shapiro.test(returnsDf)
skewness(returnsDf)
kurtosis(returnsDf)
```

```
    Shapiro-Wilk normality test

data:  returnsDf
W = 0.89249, p-value < 2.2e-16
```

-1.62812865907981

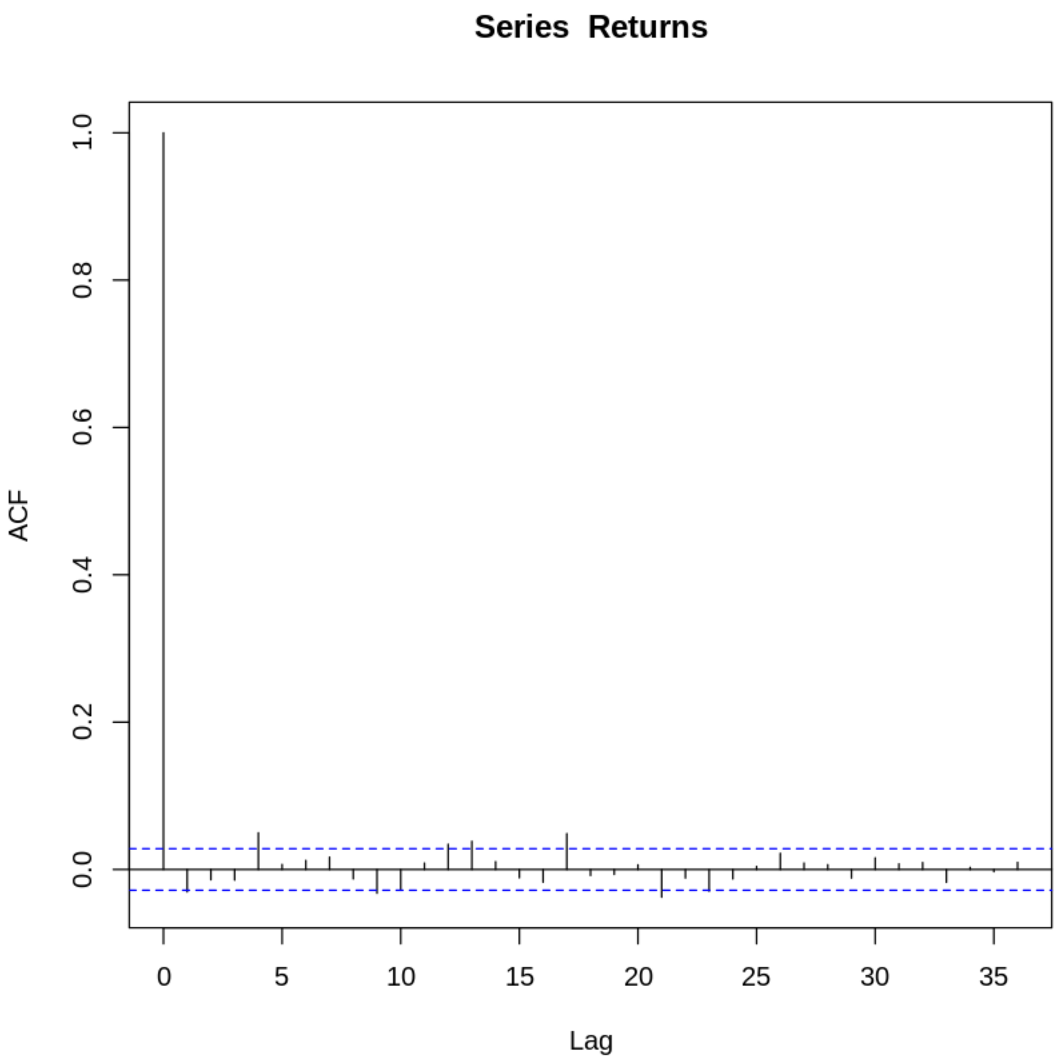
40.7237359235109

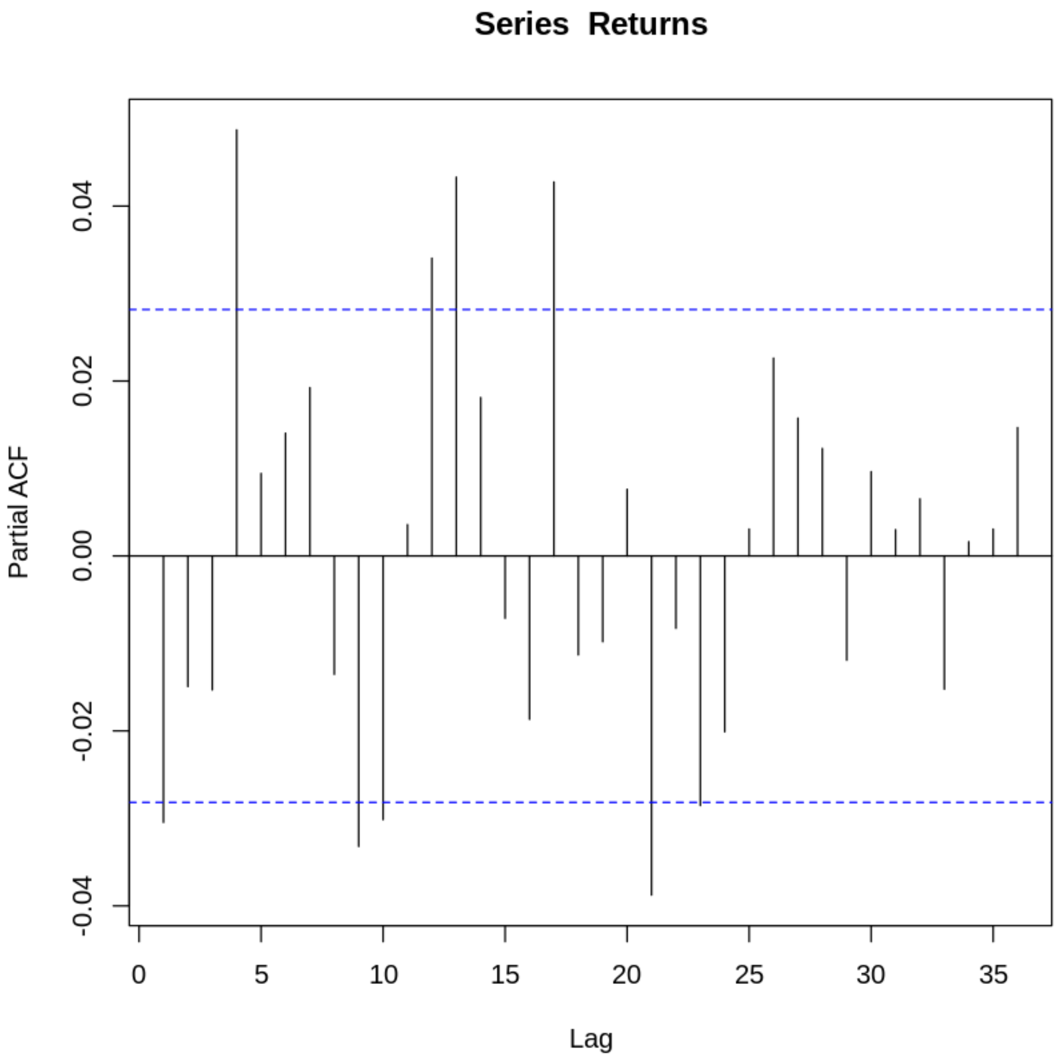
```
# Stationary test on returns
adf.test(Returns)
```

```
Warning message in adf.test(Returns):
“p-value smaller than printed p-value”



    Augmented Dickey-Fuller Test

data:  Returns
```

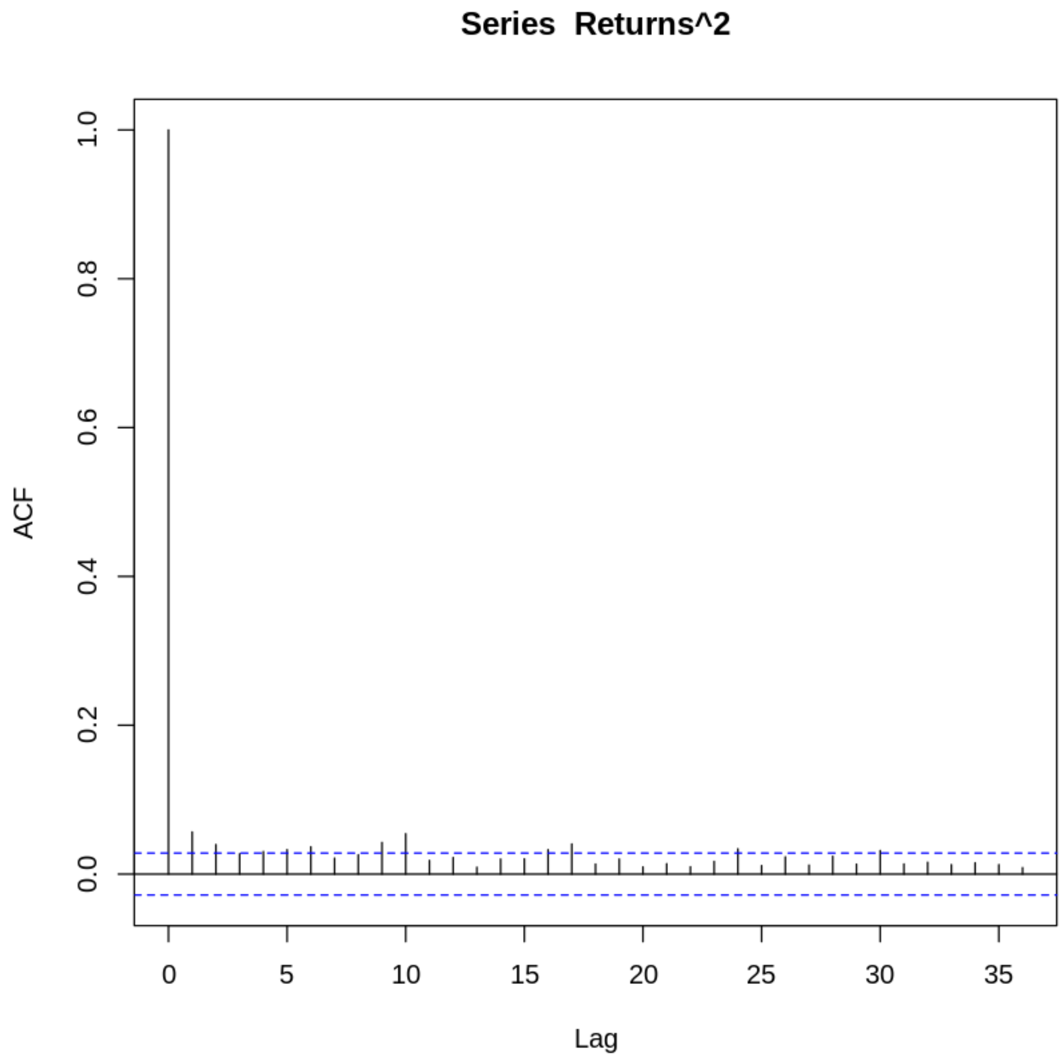
```
Dickey-Fuller = -15.572, Lag order = 16, p-value = 0.01
alternative hypothesis: stationary
```

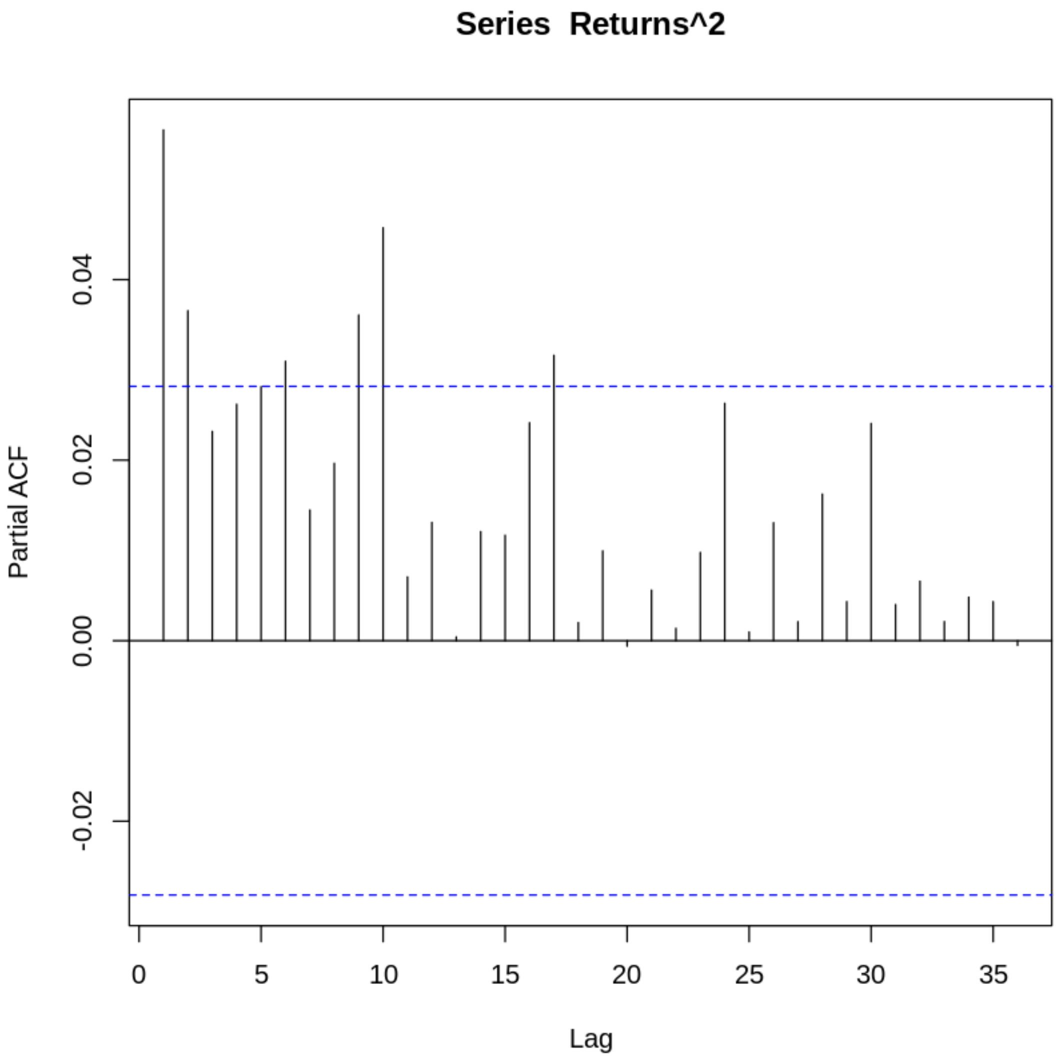
```
# Examining the ACF & PACF of returns
acf(Returns)
pacf(Returns)
```






```
# Examining the ACF & PACF of returns^2
acf(Returns^2)
pacf(Returns^2)
```


Series Returns^2
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
Lag



Intuitions

Looking at the ADF test we can see that the daily return is stationary.

The normality test shows skews and kurtosis. The negative skew of the returns may suggest that negative news impact the innovation of the series more than positive news. This may suggest the need for asymmetric GARCH models.

The ACF & PACF of the series suggests possibility of AR model. We will investigate the impact of AR terms on the GARCH model.

The ACF & PACF of the series suggests possibility of MA terms for the GARCH model. We will investigate further the impact of additional MA terms on the GARCH model.

1.1 GARCH Model

Since the returns is stationary, we will model the volatility with the simplest GARCH(1,1) model first to compare against other models.

```
spec_garch_11 = ugarchspec(mean.model = list(armaOrder = c(0,0)))
show(spec_garch_11)
```

```
*---------------------------------*
*       GARCH Model Spec          *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model        : sGARCH(1,1)
Variance Targeting    : FALSE
```

```
Conditional Mean Dynamics
------------------------------------
Mean Model        : ARFIMA(0,0,0)
Include Mean        : TRUE
GARCH-in-Mean        : FALSE

Conditional Distribution
------------------------------------
Distribution    :  norm
Includes Skew    :  FALSE
Includes Shape    :  FALSE
Includes Lambda    :  FALSE
```

```
fit_garch_11 = ugarchfit(spec_garch_11, Returns, solver='hybrid', out.sample = 30)
```

```
show(fit_garch_11)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : sGARCH(1,1)
Mean Model    : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001957    0.000275   7.1239 0.000000
omega   0.000006    0.000002   2.8611 0.004221
alpha1  0.070646    0.009108   7.7565 0.000000
beta1   0.923010    0.010445  88.3689 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001957    0.000291  6.71943 0.000000
omega   0.000006    0.000007  0.79615 0.425947
alpha1  0.070646    0.032051  2.20413 0.027515
beta1   0.923010    0.035988 25.64783 0.000000

LogLikelihood : 11538.93

Information Criteria
------------------------------------

Akaike       -4.7962
Bayes        -4.7908
Shibata      -4.7962
Hannan-Quinn -4.7943

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.003614  0.9521
Lag[2*(p+q)+(p+q)-1][2]  0.231636  0.8350
Lag[4*(p+q)+(p+q)-1][5]  3.451694  0.3307
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.5901  0.4424
Lag[2*(p+q)+(p+q)-1][5]    1.3949  0.7657
Lag[4*(p+q)+(p+q)-1][9]    2.0510  0.8988
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.4915 0.500 2.000  0.4832
ARCH Lag[5]    1.4486 1.440 1.667  0.6059
ARCH Lag[7]    1.8084 2.315 1.543  0.7578

Nyblom stability test
```

```
------------------------------------
Joint Statistic:  1.75
Individual Statistics:
mu     0.3266
omega  0.2720
alpha1 0.9633
beta1  1.1454

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          1.07 1.24 1.6
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias           2.2301 0.025784  **
Negative Sign Bias  0.4115 0.680716
Positive Sign Bias  3.1981 0.001392 ***
Joint Effect       11.4520 0.009517 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     193.1    8.068e-31
2    30     207.2    8.243e-29
3    40     210.7    2.002e-25
4    50     231.4    1.747e-25


Elapsed time : 0.2939816
```

1.2 Asymmetric GARCH

Noted earlier, we will want to explore the impact of asymmetry on the GARCH model due to the difference in the impact of positive and negative news to the returns process. We will start with the TGARCH model.

TGARCH

```
spec_tgarch_11 = ugarchspec(
    variance.model = list( model = "fGARCH", submodel = "TGARCH", garchOrder = c(1,1)),
    mean.model = list(armaOrder = c(0,0))
)
spec_tgarch_11
```

```
*---------------------------------*
*       GARCH Model Spec          *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model        : fGARCH(1,1)
fGARCH Sub-Model    : TGARCH
Variance Targeting    : FALSE

Conditional Mean Dynamics
------------------------------------
Mean Model        : ARFIMA(0,0,0)
Include Mean        : TRUE
GARCH-in-Mean        : FALSE

Conditional Distribution
------------------------------------
Distribution    :  norm
Includes Skew    :  FALSE
Includes Shape    :  FALSE
Includes Lambda    :  FALSE
```

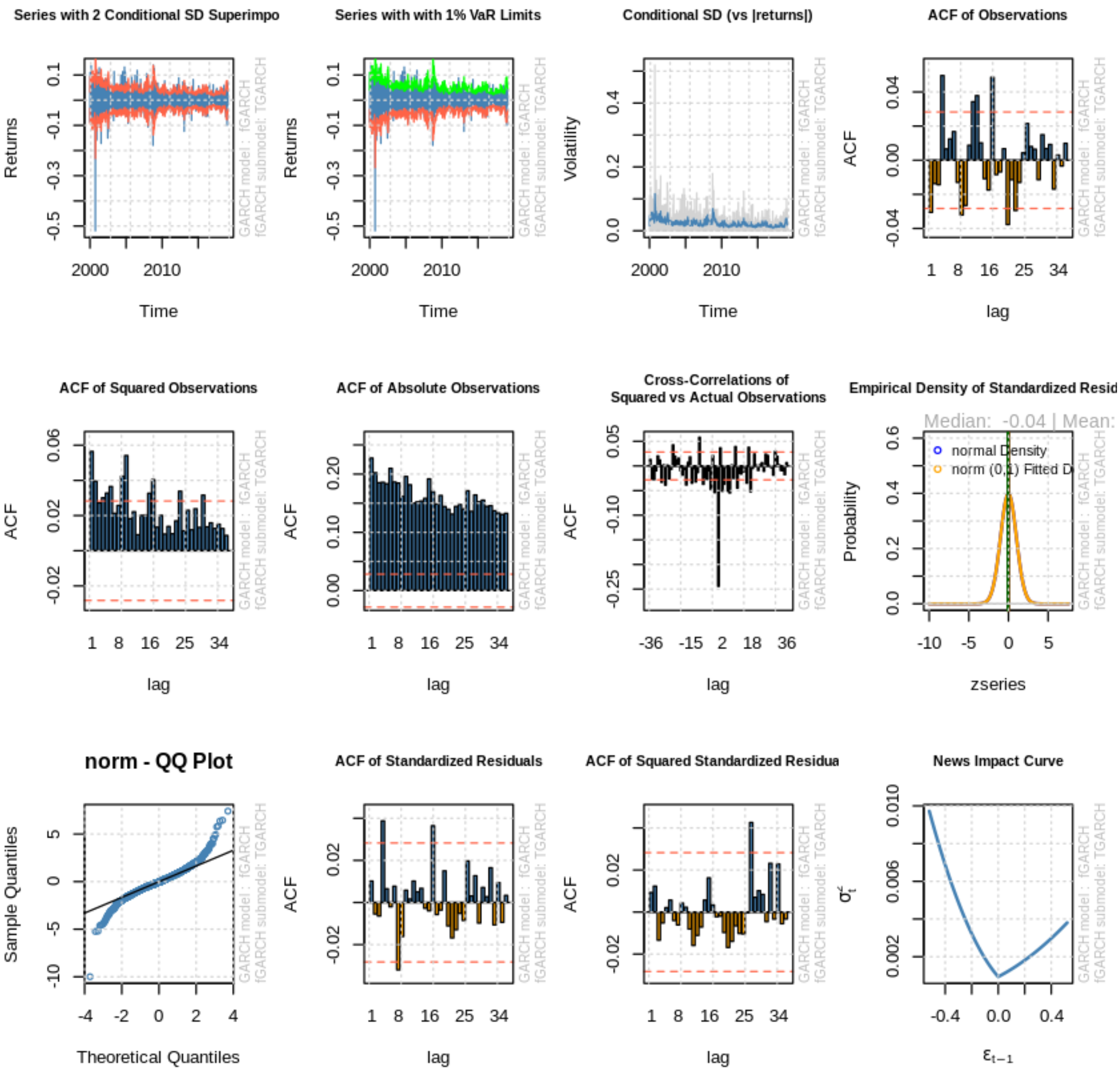
```
fit_tgarch_11 = ugarchfit(spec_tgarch_11, Returns, solver='hybrid', out.sample = 30)
fit_tgarch_11
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : fGARCH(1,1)
```

```
fGARCH Sub-Model    : TGARCH
Mean Model    : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001607    0.000260   6.1896        0
omega   0.000420    0.000076   5.5067        0
alpha1  0.095212    0.009311  10.2257        0
beta1   0.911350    0.009377  97.1886        0
eta11   0.372888    0.048901   7.6254        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001607    0.000313   5.1312 0.000000
omega   0.000420    0.000164   2.5588 0.010503
alpha1  0.095212    0.027032   3.5222 0.000428
beta1   0.911350    0.024959  36.5139 0.000000
eta11   0.372888    0.075080   4.9665 0.000001

LogLikelihood : 11589.88

Information Criteria
------------------------------------

Akaike       -4.8170
Bayes        -4.8103
Shibata      -4.8170
Hannan-Quinn -4.8146

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.4992  0.4798
Lag[2*(p+q)+(p+q)-1][2]    0.5714  0.6610
Lag[4*(p+q)+(p+q)-1][5]    3.6733  0.2977
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.4158  0.5191
Lag[2*(p+q)+(p+q)-1][5]    1.5754  0.7214
Lag[4*(p+q)+(p+q)-1][9]    1.9817  0.9072
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.8617 0.500 2.000  0.3533
ARCH Lag[5]    0.9786 1.440 1.667  0.7393
ARCH Lag[7]    1.1110 2.315 1.543  0.8948

Nyblom stability test
------------------------------------
Joint Statistic:  2.722
Individual Statistics:
mu     0.7083
omega  1.4864
alpha1 1.6176
beta1  1.8628
eta11  0.7872

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          1.28 1.47 1.88
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value      prob sig
Sign Bias           2.4287 0.0151890  **
Negative Sign Bias  0.9311 0.3518294
Positive Sign Bias  3.3517 0.0008093 ***
Joint Effect       12.2649 0.0065288 ***


Adjusted Pearson Goodness-of-Fit Test:
```

```
------------------------------------
  group statistic p-value(g-1)
1    20     160.9    1.725e-24
2    30     180.1    9.558e-24
3    40     190.0    9.609e-22
4    50     207.4    2.173e-21


Elapsed time : 0.7492216
```

```
plot(fit_tgarch_11, which = "all")
```



We can observe that providing asymmetry to the model improves the model's performance. With that, we can explore if the asymmetry is a result of conditional variance with the EGARCH model.

EGARCH

```
spec_egarch_11 = ugarchspec(mean.model = list(armaOrder = c(0,0)), variance.model= list(garchOrder = c(1,1), model="eGARCH"))
show(spec_egarch_11)
```

```
*---------------------------------*
*       GARCH Model Spec          *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model         : eGARCH(1,1)
```

```
Variance Targeting    : FALSE

Conditional Mean Dynamics
-----------------------------------
Mean Model        : ARFIMA(0,0,0)
Include Mean        : TRUE
GARCH-in-Mean        : FALSE

Conditional Distribution
-----------------------------------
Distribution    :  norm
Includes Skew    :  FALSE
Includes Shape    :  FALSE
Includes Lambda    :  FALSE
```

```
fit_egarch_11 = ugarchfit(spec_egarch_11, Returns, solver='hybrid', out.sample = 30)
fit_egarch_11
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : eGARCH(1,1)
Mean Model    : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001671    0.000320   5.2216        0
omega  -0.117904    0.010479 -11.2514        0
alpha1 -0.056031    0.007641  -7.3326        0
beta1   0.983375    0.001327 740.8497        0
gamma1  0.157625    0.013045  12.0832        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001671    0.000542   3.0844 0.002040
omega  -0.117904    0.029518  -3.9944 0.000065
alpha1 -0.056031    0.013044  -4.2954 0.000017
beta1   0.983375    0.003409 288.4905 0.000000
gamma1  0.157625    0.048721   3.2353 0.001215

LogLikelihood : 11591.07

Information Criteria
------------------------------------

Akaike       -4.8175
Bayes        -4.8108
Shibata      -4.8175
Hannan-Quinn -4.8151

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.2875  0.5918
Lag[2*(p+q)+(p+q)-1][2]    0.4484  0.7183
Lag[4*(p+q)+(p+q)-1][5]    3.4212  0.3355
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.6966  0.4039
Lag[2*(p+q)+(p+q)-1][5]    1.3126  0.7858
Lag[4*(p+q)+(p+q)-1][9]    1.7437  0.9333
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.7434 0.500 2.000  0.3886
ARCH Lag[5]    0.9123 1.440 1.667  0.7591
ARCH Lag[7]    1.1462 2.315 1.543  0.8887
```

```
Nyblom stability test
------------------------------------
Joint Statistic:  2.3992
Individual Statistics:
mu     0.39248
omega  1.42038
alpha1 0.43223
beta1  1.30444
gamma1 0.06334

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          1.28 1.47 1.88
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value      prob sig
Sign Bias           2.5725 0.0101269  **
Negative Sign Bias  0.5439 0.5865569
Positive Sign Bias  3.8790 0.0001063 ***
Joint Effect       15.5530 0.0014002 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     165.7    1.952e-25
2    30     179.7    1.121e-23
3    40     187.1    3.024e-21
4    50     200.7    2.954e-20


Elapsed time : 0.5421457
```

We can see the EGARCH(1,1) model performed better than the other models but it is still failing at Ljung-box test for the square residuals. We will attempt to increase the MA term for the EGARCH model to account for the serial correlation of the square of residuals.

EGARCH(1,2)

```
spec_egarch_12 = ugarchspec(mean.model = list(armaOrder = c(0,0)), variance.model= list(garchOrder = c(1,2), model="eGARCH"))
show(spec_egarch_12)
```

```
*---------------------------------*
*       GARCH Model Spec          *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model        : eGARCH(1,2)
Variance Targeting    : FALSE

Conditional Mean Dynamics
------------------------------------
Mean Model        : ARFIMA(0,0,0)
Include Mean        : TRUE
GARCH-in-Mean        : FALSE

Conditional Distribution
------------------------------------
Distribution    :  norm
Includes Skew    :  FALSE
Includes Shape    :  FALSE
Includes Lambda    :  FALSE
```

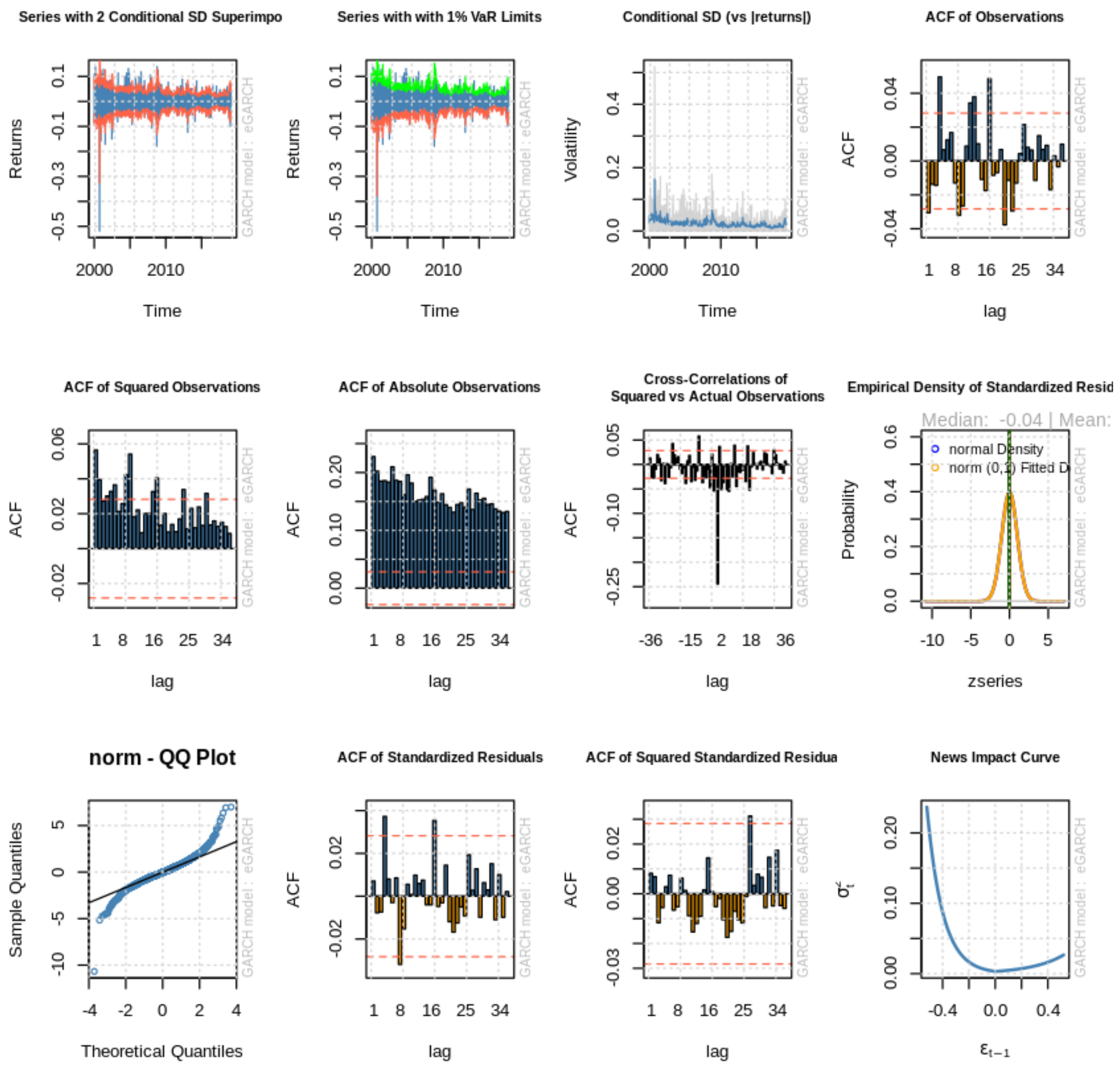
```
fit_egarch_12 = ugarchfit(spec_egarch_12, Returns, solver='hybrid', out.sample = 30)
fit_egarch_12
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : eGARCH(1,2)
Mean Model    : ARFIMA(0,0,0)
Distribution    : norm
```

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001672    0.000292   5.7192        0
omega  -0.127462    0.021907  -5.8182        0
alpha1 -0.061576    0.006700  -9.1906        0
beta1   0.776043    0.001656 468.6130        0
beta2   0.205902    0.001889 109.0295        0
gamma1  0.181361    0.015097  12.0127        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001672    0.000407   4.1102 0.000040
omega  -0.127462    0.074786  -1.7044 0.088314
alpha1 -0.061576    0.016251  -3.7891 0.000151
beta1   0.776043    0.005435 142.7765 0.000000
beta2   0.205902    0.004964  41.4828 0.000000
gamma1  0.181361    0.035165   5.1574 0.000000

LogLikelihood : 11593.1

Information Criteria
------------------------------------

Akaike       -4.8179
Bayes        -4.8098
Shibata      -4.8179
Hannan-Quinn -4.8151

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.2425  0.6224
Lag[2*(p+q)+(p+q)-1][2]    0.3871  0.7491
Lag[4*(p+q)+(p+q)-1][5]    3.3711  0.3435
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.3273  0.5672
Lag[2*(p+q)+(p+q)-1][8]     1.3018  0.9487
Lag[4*(p+q)+(p+q)-1][14]    2.1694  0.9876
d.o.f=3

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[4]    0.1496 0.500 2.000  0.6989
ARCH Lag[6]    0.3603 1.461 1.711  0.9299
ARCH Lag[8]    0.5810 2.368 1.583  0.9750

Nyblom stability test
------------------------------------
Joint Statistic:  2.5969
Individual Statistics:
mu     0.3389
omega  1.2813
alpha1 0.6046
beta1  1.1792
beta2  1.1811
gamma1 0.0644

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          1.49 1.68 2.12
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value      prob sig
Sign Bias           2.4838 0.0130337  **
Negative Sign Bias  0.7355 0.4620856
Positive Sign Bias  3.7090 0.0002104 ***
Joint Effect       14.4159 0.0023904 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
```

```
  group statistic p-value(g-1)
1    20     165.0    2.682e-25
2    30     174.2    1.203e-22
3    40     191.3    5.612e-22
4    50     200.2    3.550e-20


Elapsed time : 0.6495256
```

```
plot(fit_egarch_12, which="all")
```



1.3 Other Models

ARCH

```
spec_garch_10 = ugarchspec(mean.model = list(armaOrder = c(0,0)), variance.model= list(garchOrder = c(1,0)))
show(spec_garch_10)
```

```
*---------------------------------*
*       GARCH Model Spec          *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model         : sGARCH(1,0)
Variance Targeting     : FALSE
```

```
Conditional Mean Dynamics
-----------------------------------
Mean Model        : ARFIMA(0,0,0)
Include Mean        : TRUE
GARCH-in-Mean        : FALSE

Conditional Distribution
-----------------------------------
Distribution    :  norm
Includes Skew    :  FALSE
Includes Shape    :  FALSE
Includes Lambda    :  FALSE
```

```
fit_garch_10 = ugarchfit(spec_garch_10, Returns, solver='hybrid', out.sample = 30)
fit_garch_10
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : sGARCH(1,0)
Mean Model    : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.119601    0.000040 2974.594        0
omega   0.000007    0.000001   10.727        0
alpha1  0.999000    0.000451 2213.723        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.119601    0.000438 272.7793  0.00000
omega   0.000007    0.000006   1.1816  0.23736
alpha1  0.999000    0.004535 220.2867  0.00000

LogLikelihood : 1575.533

Information Criteria
------------------------------------

Akaike       -0.65386
Bayes        -0.64982
Shibata      -0.65386
Hannan-Quinn -0.65244

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      68.26 1.11e-16
Lag[2*(p+q)+(p+q)-1][2]     68.36 0.00e+00
Lag[4*(p+q)+(p+q)-1][5]     69.88 0.00e+00
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.05502  0.8145
Lag[2*(p+q)+(p+q)-1][2]   0.05974  0.9489
Lag[4*(p+q)+(p+q)-1][5]   0.07356  0.9990
d.o.f=1

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[2]   0.00943 0.500 2.000  0.9226
ARCH Lag[4]   0.01949 1.397 1.611  0.9983
ARCH Lag[6]   0.02926 2.222 1.500  0.9999

Nyblom stability test
------------------------------------
Joint Statistic:  13.5592
Individual Statistics:
```

```
mu      0.65768
omega   0.05674
alpha1 11.55546

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          0.846 1.01 1.35
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value       prob sig
Sign Bias            16.79  1.473e-61 ***
Negative Sign Bias   10.39  5.201e-25 ***
Positive Sign Bias   11.56  1.603e-30 ***
Joint Effect        568.26 7.667e-123 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     17703            0
2    30     18560            0
3    40     18944            0
4    50     18963            0


Elapsed time : 0.759702
```

GARCH-M

```
spec_garchm_11 = ugarchspec(mean.model = list(armaOrder = c(0,0), archm = TRUE, archpow = 1))
show(spec_garchm_11)
```

```
*---------------------------------*
*       GARCH Model Spec          *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model        : sGARCH(1,1)
Variance Targeting    : FALSE

Conditional Mean Dynamics
------------------------------------
Mean Model        : ARFIMA(0,0,0)
Include Mean        : TRUE
GARCH-in-Mean        : TRUE

Conditional Distribution
------------------------------------
Distribution    :  norm
Includes Skew    :  FALSE
Includes Shape    :  FALSE
Includes Lambda    :  FALSE
```

```
fitt = ugarchfit(spec_garchm_11, Returns, solver='hybrid', out.sample = 30)
fitt
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : sGARCH(1,1)
Mean Model    : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.002073    0.000881  2.35274 0.018636
archm  -0.006323    0.045476 -0.13903 0.889422
omega   0.000006    0.000002  2.75707 0.005832
alpha1  0.070454    0.009256  7.61133 0.000000
beta1   0.923156    0.010748 85.89406 0.000000

Robust Standard Errors:
```

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.002073    0.001043  1.98751 0.046866
archm  -0.006323    0.053571 -0.11803 0.906046
omega   0.000006    0.000008  0.76041 0.447012
alpha1  0.070454    0.032656  2.15746 0.030970
beta1   0.923156    0.037283 24.76053 0.000000

LogLikelihood : 11538.94

Information Criteria
------------------------------------

Akaike       -4.7958
Bayes        -4.7891
Shibata      -4.7958
Hannan-Quinn -4.7935

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.00345  0.9532
Lag[2*(p+q)+(p+q)-1][2]   0.23168  0.8350
Lag[4*(p+q)+(p+q)-1][5]   3.44977  0.3310
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.6103  0.4347
Lag[2*(p+q)+(p+q)-1][5]    1.4107  0.7618
Lag[4*(p+q)+(p+q)-1][9]    2.0649  0.8971
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.4863 0.500 2.000  0.4856
ARCH Lag[5]    1.4415 1.440 1.667  0.6078
ARCH Lag[7]    1.8001 2.315 1.543  0.7595

Nyblom stability test
------------------------------------
Joint Statistic:  2.0444
Individual Statistics:
mu     0.3510
archm  0.2294
omega  0.2730
alpha1 0.9679
beta1  1.1500

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          1.28 1.47 1.88
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias           2.2132 0.026928  **
Negative Sign Bias  0.4345 0.663942
Positive Sign Bias  3.1969 0.001398 ***
Joint Effect       11.4463 0.009542 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     193.6    6.256e-31
2    30     208.5    4.684e-29
3    40     210.0    2.672e-25
4    50     227.0    9.874e-25


Elapsed time : 0.8942223
```

IGARCH

The IGARCH model will be unsuitable for forecasting the returns process as the process is already stationary and does not benefit from futher integration.

ARMA(1,0)-GARCH(1,1)

```
spec_arma_10_garch_11 = ugarchspec(mean.model = list(armaOrder = c(1,0)))
show(spec_arma_10_garch_11)
```

```
*---------------------------------*
*       GARCH Model Spec          *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model        : sGARCH(1,1)
Variance Targeting    : FALSE

Conditional Mean Dynamics
------------------------------------
Mean Model        : ARFIMA(1,0,0)
Include Mean        : TRUE
GARCH-in-Mean        : FALSE

Conditional Distribution
------------------------------------
Distribution    :  norm
Includes Skew    :  FALSE
Includes Shape    :  FALSE
Includes Lambda    :  FALSE
```

```
fit_arma_10_garch_11 = ugarchfit(spec_arma_10_garch_11, Returns, solver='hybrid', out.sample = 30)
show(fit_arma_10_garch_11)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : sGARCH(1,1)
Mean Model    : ARFIMA(1,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001958    0.000275  7.125310 0.000000
ar1    -0.000214    0.015543 -0.013791 0.988997
omega   0.000006    0.000002  2.821438 0.004781
alpha1  0.070379    0.009165  7.678730 0.000000
beta1   0.923267    0.010546 87.547567 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001958    0.000291  6.717125 0.000000
ar1    -0.000214    0.014478 -0.014805 0.988188
omega   0.000006    0.000007  0.774343 0.438728
alpha1  0.070379    0.032616  2.157831 0.030941
beta1   0.923267    0.036875 25.037722 0.000000

LogLikelihood : 11538.93

Information Criteria
------------------------------------

Akaike       -4.7958
Bayes        -4.7891
Shibata      -4.7958
Hannan-Quinn -4.7934

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.005214  0.9424
Lag[2*(p+q)+(p+q)-1][2]  0.233655  0.9978
Lag[4*(p+q)+(p+q)-1][5]  3.455068  0.3259
d.o.f=1
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
```

```
                        statistic p-value
Lag[1]                     0.6017  0.4379
Lag[2*(p+q)+(p+q)-1][5]    1.3992  0.7647
Lag[4*(p+q)+(p+q)-1][9]    2.0534  0.8985
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.4862 0.500 2.000  0.4856
ARCH Lag[5]    1.4380 1.440 1.667  0.6088
ARCH Lag[7]    1.7986 2.315 1.543  0.7599

Nyblom stability test
------------------------------------
Joint Statistic:  2.0208
Individual Statistics:
mu     0.3242
ar1    0.2934
omega  0.2696
alpha1 0.9688
beta1  1.1491

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          1.28 1.47 1.88
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias           2.2304 0.025765  **
Negative Sign Bias  0.4165 0.677049
Positive Sign Bias  3.2050 0.001360 ***
Joint Effect       11.4975 0.009318 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     193.2    7.564e-31
2    30     207.2    8.199e-29
3    40     210.8    1.962e-25
4    50     232.5    1.106e-25


Elapsed time : 0.8379197
```

1.4 Summary (Volatility Analysis)

Comparison of Viable Models

Model	Log Likelihood	AIC
ARCH(1)	1575.533	-0.65386
GARCH(1,1)	11538.93	-4.7962
ARMA(1,0)-GARCH(1,1)	11538.93	-4.7962
TGARCH(1,1)	11589.88	-4.8170
EGARCH(1,1)	11591.07	-4.8175
EGARCH(1,2)	11593.1	-4.8179
GARCH-M(1,1)	11538.94	-4.7958

Comparing the different models, we can see that the asymmetric GARCH model, EGARCH(1,2), appears to be the best model for AAPL daily returns, matching our initial intuition that negative news impact the returns more significantly than positive news.

The EGARCH(1,2) model's coefficients, mu, omega, alpha1, beta1, beta2 & gamma1 are all significant.

We can also observe that adding AR terms to the model does not improve the model. This can be seen from the ARMA(1,0)-GARCH(1,1) model where the coefficient of the ar1 term is tested to be insignificant.

1.5 Forecasting with EGARCH(1,2)

Now that we can be reasonably sure that our risk model works properly, we can produce VaR forecasts as well. We will use the rolling forecast method of the package to generate the next (15) period returns.

```
forecast_egarch_12 = ugarchforecast(fit_egarch_12, n.ahead=15, n.roll=5)
forecast_egarch_12
```

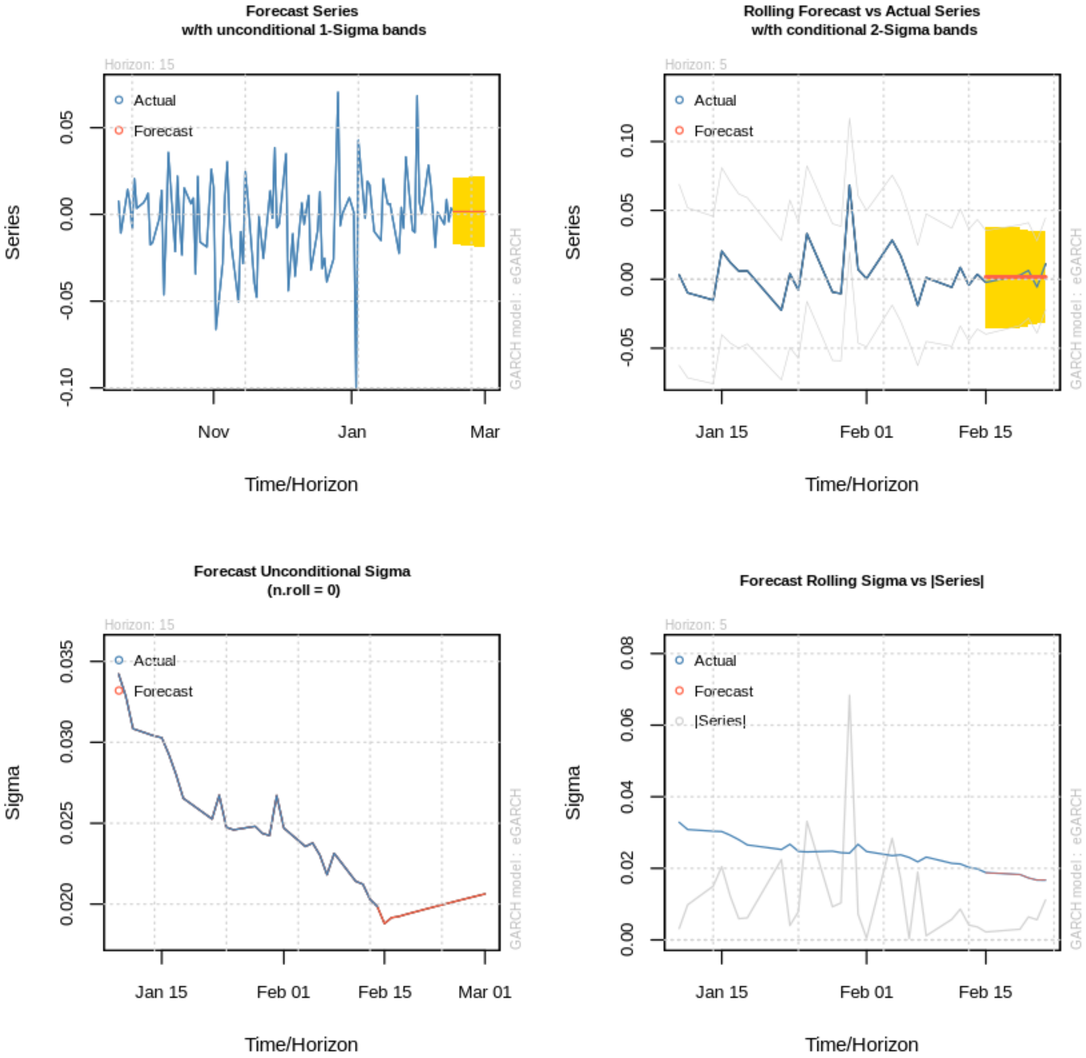
```
*------------------------------------*
*       GARCH Model Forecast         *
*------------------------------------*
Model: eGARCH
Horizon: 15
Roll Steps: 5
Out of Sample: 15

0-roll forecast [T0=2019-02-14]:
       Series   Sigma
T+1  0.001672 0.01880
T+2  0.001672 0.01917
T+3  0.001672 0.01924
T+4  0.001672 0.01937
T+5  0.001672 0.01949
T+6  0.001672 0.01961
T+7  0.001672 0.01972
T+8  0.001672 0.01984
T+9  0.001672 0.01996
T+10 0.001672 0.02007
T+11 0.001672 0.02019
T+12 0.001672 0.02030
T+13 0.001672 0.02041
T+14 0.001672 0.02052
T+15 0.001672 0.02063
```

1.6 Conclusion

Using EGARCH(1,2) to forecast we get the next period daily return to be 0.001672. The one-period ahead forecast for the volatility (sigma) is 0.01880. Since we assume a normal distribution, the 99% VaR can be calculated using the 99% quantile (type in qnorm(0.99)) of the standard normal distribution. The one-month 99% VaR estimatefor the next period is hence qnorm(0.99)*0.01880 = 0.04373. Hence, with 99% probability the monthly return is above -4.4%.

The chart of the forecast is as followed:

```
plot(forecast_egarch_12, which="all")
```



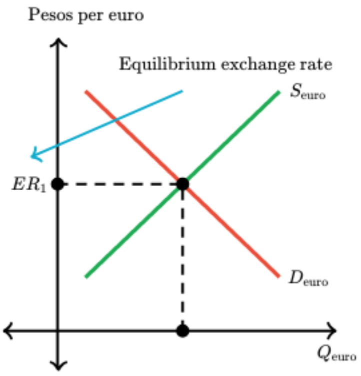
1.7 Reference

1. Daróczi, Puhle, Berlinger (2013) *"Introduction to R for Quantitative Finance"*, Packt Publishing
2. Berlinger, Illés, Badics (2015) *"Mastering R for Quantitative Finance"*, Packt Publishing

Part 2 - Calculating Equilibrium FX (Multivariate Analysis)

2.1 Economic theories and models for calculating equilibrium FX.

The equilibirum foreign exchange (FX) rate is determined by the demand and supply of one currency given another. This value can be determined by the intersection between the demand curve and the supply curve of the currency pair.



In the example given by Khan Academy[1], we can see that the exchange rate between Peso and Euro is defined by:

- Different amount of Euros supplied at different rate of Peso per Euro, forming the supply curve
- Different amount of Euros demanded at different rate of Peso per Euro, forming the demand curve

The intersection of these two curves determine the equilibrium FX rate between Peso and Euro:

- A rate higher than the equilibrium will result in the supply of Euros to exceed the demand, applying a downward pressure in the rate, towards short-term equilibrium.
- A rate lower than the equilibirum will result in the demand of Eruos to exceed the supply, applying a upward pressure in the rate, towards short-term equilibirum.

2.2 Macroeconomic variables used for calculating equilibrium FX.

Macroeconomic variables which affects the demand or supply curve of the one currency, expressed in another will affect the equilibrium FX rate.

Interest Rates

One such example is the interest rate of a country. When interest rates increases, savers from other countries will be encouraged to buy financial assets in that country[1]. That will require foreign savers to first buy the country's currency to buy the financial assets. This results in higher demand for the country's currency, resulting in increased exhange rate.

Factors increasing interest rate can include the country's budget deficit.

Money Supply

The money supply of the country affects the supply of the currency directly. An increase in money supply results in a decrease in equilibrium FX[2].

One metrics that is available is M1 which directly measures physical currency and coin, demand deposits, travelers checks, other checkable deposits and negotiable order of withdrawal (NOW) accounts[3]. Basically, the most liquid form of money in a country.

Inflation

Typically, a country with a consistently lower inflation rate exhibits a rising currency value, as its purchasing power increases relative to other currencies.[4]

Industrial Production Index

The Index of Industrial Production (IIP) has shown to be dependent on the exchange rate[5].

Political Stability

Political stability of a country also affects the exchange rate positively. Investors are encouraged to invest in countries with higher political stability, resulting in higher demand for the currency[4].

2.3 The connection between linear regression and Vector Error Correction (VEC).

The Vector Auto-Regressive(VAR) model extends the linear regression model by allowing different processes to be grenerated from their mutual histories.

$$\mathbf{x}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{x}_{t-1} + \cdots + \mathbf{A}_p\mathbf{x}_{t-p} + \boldsymbol{\varepsilon}_t$$

The VAR equation is produce the generating process based on previous period values, producing a short term forecasting result.

The Vector Error Correction (VEC) model, also known as cointegration model, extends the VAR model. It introduces the long-term relationships between variables, through their cointegration.[6]

$$\Delta \mathbf{x}_t = \mathbf{A}_0 + \mathbf{\Pi} \mathbf{x}_{t-1} + \mathbf{C}_1 \Delta \mathbf{x}_{t-1} + \cdots + \mathbf{C}_p \Delta \mathbf{x}_{t-p} + \boldsymbol{\varepsilon}_t$$

The VECM integrates information of the cointegration of the variables to allow them to return to long-run equilibirum. This can be seen from the PI term in the equation above. We can see that the VECM is a linear model, forming a linear relationship between the first difference of the result and the difference(s) of the previous results of the process, with it's constant and error terms.

2.4 Calculate equilibrium FX using VEC. The Behavioural Equilibrium Exchange Rate (BEER) approach will be applied to the analysis.

USD/SGD is used for this analysis

Models

The VECM model will be used with the macroeconomic data below.

Data Used

- USD/SGD (https://fred.stlouisfed.org/series/EXSIUS)
- US M1 (https://fred.stlouisfed.org/series/M1NS)
- SG M1 (https://secure.mas.gov.sg/msb-xml/Report.aspx?tableSetID=I...I...)
- US Inflation (https://fred.stlouisfed.org/series/FPCPITOTLZGUSA)
- SG Inflation (https://fred.stlouisfed.org/series/FPCPITOTLZGSGP)
- US Interest rate (https://fred.stlouisfed.org/series/FEDFUNDS)
- SG Interest rate (https://secure.mas.gov.sg/msb/InterestRatesOfBanksAndFinanceCompanies.aspx)

Variables Used

- USD/SGD
- US M1 Increase (log)
- SG M1 Increase (log)
- Inflation differential (US-SG)
- Interest rate differential (US-SG)

```
library(quantmod)
library(readxl)
library(vars)
library(timeSeries)
library(urca)
library(tsDyn)
```

We will first download the data from the sources. From the data source, we can observe that the maximum date range that has data for all the dataset is from Jan 1992 to Jan 2017. We will make use of the monthly data.

For the dataset of M1_US and M1_SG, the dataset is of annual frequency. We will resample these data using linear interpolation to yield the monthly data.

```
# Read data from CSV
USDSGD = read.table("data/USDSGD.csv", header=TRUE, sep = ",", colClasses = c("Date", "numeric"))
INTEREST_SG = read.table("data/INTEREST-SG.csv", header=TRUE, sep = ",", colClasses = c("Date", "numeric"))
INTEREST_US = read.table("data/INTEREST-US.csv", header=TRUE, sep = ",", colClasses = c("Date", "numeric"))
M1_SG = read.table("data/M1-SG.csv", header=TRUE, sep = ",", colClasses = c("Date", "numeric"))
M1_US = read.table("data/M1-US.csv", header=TRUE, sep = ",", colClasses = c("Date", "numeric"))
INF_US = read.table("data/INFLATION-US.csv", header=TRUE, sep = ",", colClasses = c("Date", "numeric"))
INF_SG = read.table("data/INFLATION-SG.csv", header=TRUE, sep = ",", colClasses = c("Date", "numeric"))
```

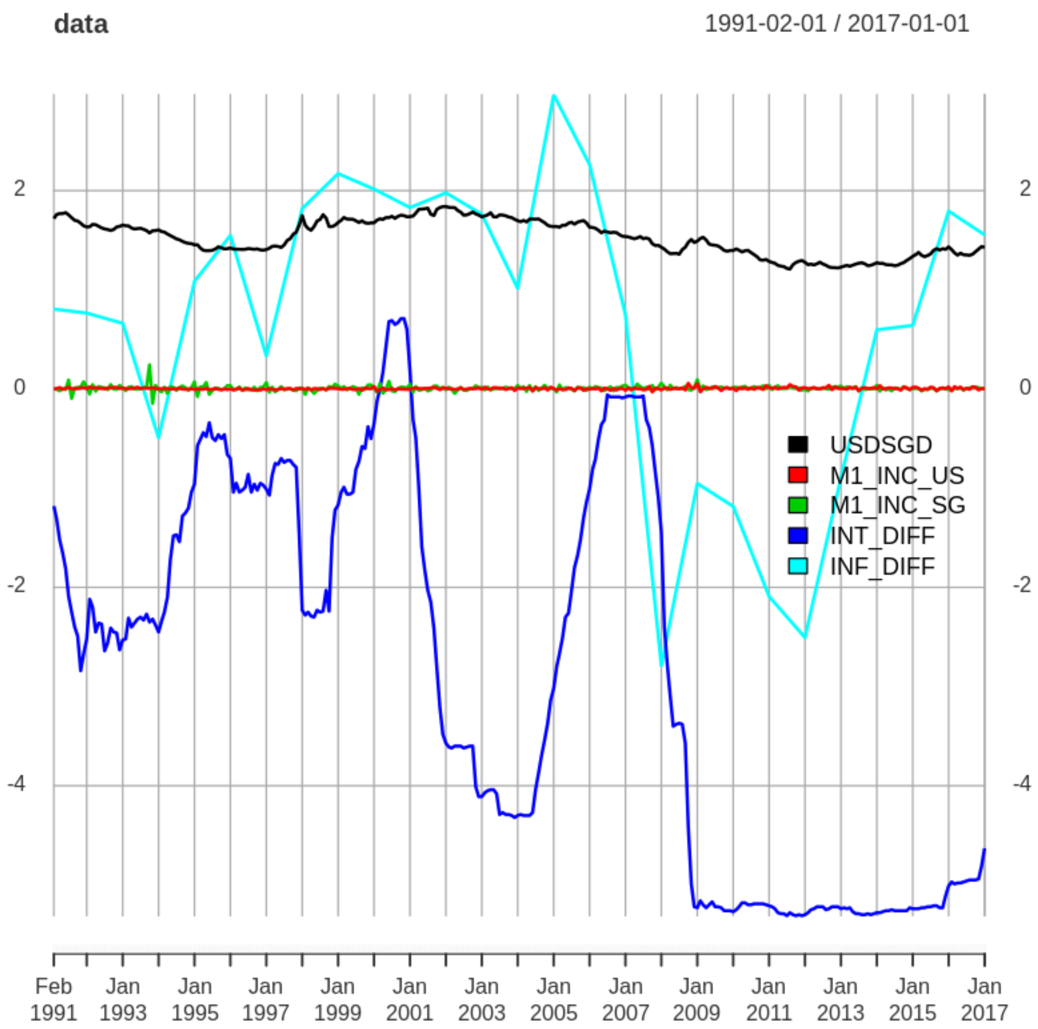
```
# Date Range: Jan 1991 - Jan 2017
DATE_RANGE = "199101/201701"

# Convert to xts
usdsgd = xts(USDSGD, order.by = USDSGD$DATE, x=USDSGD$EXSIUS)
interest_us = xts(INTEREST_US, order.by = INTEREST_US$DATE, x=INTEREST_US$FEDFUNDS)
interest_sg = xts(INTEREST_SG,  order.by = INTEREST_SG$Month, x = INTEREST_SG$Prime.Lending.Rate)
m1_us = xts(M1_US, order.by = M1_US$DATE, x=M1_US$M1)
m1_sg = xts(M1_SG, order.by = M1_SG$Month, x=M1_SG$S..MILLION)
inf_us = xts(INF_US, order.by = INF_US$DATE, x=INF_US$FPCPITOTLZGUSA)
inf_sg = xts(INF_SG, order.by = INF_SG$DATE, x=INF_SG$FPCPITOTLZGSGP)

# Resampling m1 to daily before converting to monthly data
seq_daily = seq(as.Date("1991-01-01"), as.Date(end(data), frac = 1), by = "day")
m1_us_daily = na.approx(m1_us, x = as.Date, xout = seq_daily)
m1_sg_daily = na.approx(m1_sg, x = as.Date, xout = seq_daily)
m1_us_monthly = merge(data, m1_us_daily, join = "left")
m1_us_monthly_pct = monthlyReturn(m1_us_monthly$m1_us_daily, type = "log")
m1_sg_monthly = merge(data, m1_sg_daily, join = "left")
m1_sg_monthly_pct = monthlyReturn(m1_sg_monthly$m1_sg_daily, type = "log")
data = merge(usdsgd[DATE_RANGE], m1_us_monthly_pct[DATE_RANGE], join = "left")
data = merge(data, m1_sg_monthly_pct[DATE_RANGE], join = "left")
```

```r
# Interpolating inflation to monthly range
seq_monthly = seq(as.Date(start(data)), as.Date(end(data), frac = 1), by = "month")
inf_us_monthly = na.approx(inf_us, x = as.Date, xout = seq_monthly)
inf_sg_monthly = na.approx(inf_sg, x = as.Date, xout = seq_monthly)
# data = merge(data, inf_us_monthly[DATE_RANGE], join = "left")
# data = merge(data, inf_sg_monthly[DATE_RANGE], join = "left")

# Fill in any missing values in case we miss them
data = na.approx(data)

# Calculate interest rate & inflation rate differential
INT_DIFF = interest_us[DATE_RANGE] - interest_sg[DATE_RANGE]
INF_DIFF = inf_us_monthly[DATE_RANGE] - inf_sg_monthly[DATE_RANGE]
data = merge(data, INT_DIFF[DATE_RANGE], join = "left")
data = merge(data, INF_DIFF[DATE_RANGE], join = "left")

# Renaming columns
names(data) = c("USDSGD", "M1_INC_US", "M1_INC_SG", "INT_DIFF", "INF_DIFF")

# Show data after transformation
head(data)
nrow(data)
```

```
           USDSGD    M1_INC_US    M1_INC_SG INT_DIFF  INF_DIFF
1991-01-01 1.7455  0.000000000  0.000000000    -0.54 0.8092622
1991-02-01 1.7180  0.001516092  0.008959708    -1.18 0.8055665
1991-03-01 1.7589  0.004397424 -0.002767862    -1.31 0.8022285
1991-04-01 1.7688  0.014311606 -0.018541698    -1.52 0.7985328
1991-05-01 1.7688 -0.003334098  0.003669354    -1.65 0.7949563
1991-06-01 1.7782  0.006943358 -0.005656008    -1.81 0.7912607
```

Once the data has been transformed to the xts series, we can now plot the all the different dataset.

```r
# Plot the data
plot(data, legend.loc = "right")
```




```r
# Print correlation of the data
cor(data)
```

	USDSGD	M1_INC_US	M1_INC_SG	INT_DIFF	INF_DIFF
USDSGD	1.0000000	-0.12923933	-0.041475697	0.497684576	0.6454498
M1_INC_US	-0.1292393	1.00000000	0.072252470	-0.308143046	-0.2167271
M1_INC_SG	-0.0414757	0.07225247	1.000000000	-0.003533487	-0.1081419
INT_DIFF	0.4976846	-0.30814305	-0.003533487	1.000000000	0.4237391
INF_DIFF	0.6454498	-0.21672710	-0.108141863	0.423739134	1.0000000

From the correlation table, we can see that the exchange rate (USDSGD) is most highly correlated to the inflation differential followed by the interest rate differential. It also has the least correlation to the M1 money supply of Singapore.

```r
# VAR
VAR_model = VAR(data, lag.max=12, type = "none", ic = "AIC")
summary(VAR_model)
```

```
VAR Estimation Results:
=========================
Endogenous variables: USDSGD, M1_INC_US, M1_INC_SG, INT_DIFF, INF_DIFF
Deterministic variables: none
Sample size: 310
Log Likelihood: 3198.1
```

```
Roots of the characteristic polynomial:
0.9989 0.9816 0.9511 0.9511 0.5136 0.3817 0.3817 0.114 0.114 0.03086
Call:
VAR(y = data, type = "none", lag.max = 12, ic = "AIC")


Estimation results for equation USDSGD:
=======================================
USDSGD = USDSGD.l1 + M1_INC_US.l1 + M1_INC_SG.l1 + INT_DIFF.l1 + INF_DIFF.l1 + USDSGD.l2 + M1_INC_US.l2 + M1_INC_SG.l2 +
INT_DIFF.l2 + INF_DIFF.l2

              Estimate Std. Error t value Pr(>|t|)
USDSGD.l1     1.251514   0.056469  22.163  < 2e-16 ***
M1_INC_US.l1  0.157608   0.113593   1.387   0.1663
M1_INC_SG.l1  0.061030   0.043307   1.409   0.1598
INT_DIFF.l1   0.003049   0.006924   0.440   0.6600
INF_DIFF.l1   0.025011   0.012052   2.075   0.0388 *
USDSGD.l2    -0.252847   0.056332  -4.489 1.02e-05 ***
M1_INC_US.l2  0.117548   0.112881   1.041   0.2986
M1_INC_SG.l2 -0.007003   0.043435  -0.161   0.8720
INT_DIFF.l2  -0.002524   0.006815  -0.370   0.7114
INF_DIFF.l2  -0.023824   0.012230  -1.948   0.0523 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.01978 on 300 degrees of freedom
Multiple R-Squared: 0.9998,    Adjusted R-squared: 0.9998
F-statistic: 1.865e+05 on 10 and 300 DF,  p-value: < 2.2e-16


Estimation results for equation M1_INC_US:
==========================================
M1_INC_US = USDSGD.l1 + M1_INC_US.l1 + M1_INC_SG.l1 + INT_DIFF.l1 + INF_DIFF.l1 + USDSGD.l2 + M1_INC_US.l2 + M1_INC_SG.l2 +
INT_DIFF.l2 + INF_DIFF.l2

               Estimate Std. Error t value Pr(>|t|)
USDSGD.l1    -0.0621156  0.0287234  -2.163  0.03137 *
M1_INC_US.l1 -0.2784374  0.0577803  -4.819  2.3e-06 ***
M1_INC_SG.l1  0.0236080  0.0220285   1.072  0.28472
INT_DIFF.l1  -0.0108804  0.0035218  -3.089  0.00219 **
INF_DIFF.l1   0.0037977  0.0061301   0.620  0.53605
USDSGD.l2     0.0627929  0.0286537   2.191  0.02919 *
M1_INC_US.l2 -0.0488249  0.0574178  -0.850  0.39581
M1_INC_SG.l2 -0.0002042  0.0220936  -0.009  0.99263
INT_DIFF.l2   0.0090680  0.0034665   2.616  0.00935 **
INF_DIFF.l2  -0.0047408  0.0062207  -0.762  0.44661
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.01006 on 300 degrees of freedom
Multiple R-Squared: 0.3098,    Adjusted R-squared: 0.2868
F-statistic: 13.47 on 10 and 300 DF,  p-value: < 2.2e-16


Estimation results for equation M1_INC_SG:
==========================================
M1_INC_SG = USDSGD.l1 + M1_INC_US.l1 + M1_INC_SG.l1 + INT_DIFF.l1 + INF_DIFF.l1 + USDSGD.l2 + M1_INC_US.l2 + M1_INC_SG.l2 +
INT_DIFF.l2 + INF_DIFF.l2

              Estimate Std. Error t value Pr(>|t|)
USDSGD.l1    -0.137542   0.073980  -1.859   0.0640 .
M1_INC_US.l1  0.251549   0.148819   1.690   0.0920 .
M1_INC_SG.l1 -0.364381   0.056737  -6.422 5.25e-10 ***
INT_DIFF.l1   0.016057   0.009071   1.770   0.0777 .
INF_DIFF.l1  -0.030390   0.015789  -1.925   0.0552 .
USDSGD.l2     0.147407   0.073801   1.997   0.0467 *
M1_INC_US.l2  0.189969   0.147886   1.285   0.1999
M1_INC_SG.l2 -0.110045   0.056905  -1.934   0.0541 .
INT_DIFF.l2  -0.015028   0.008928  -1.683   0.0934 .
INF_DIFF.l2   0.026653   0.016022   1.664   0.0973 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.02591 on 300 degrees of freedom
Multiple R-Squared: 0.2232,    Adjusted R-squared: 0.1973
F-statistic:  8.62 on 10 and 300 DF,  p-value: 2.159e-12
```

```
Estimation results for equation INT_DIFF:
=========================================
INT_DIFF = USDSGD.l1 + M1_INC_US.l1 + M1_INC_SG.l1 + INT_DIFF.l1 + INF_DIFF.l1 + USDSGD.l2 + M1_INC_US.l2 + M1_INC_SG.l2 +
INT_DIFF.l2 + INF_DIFF.l2

             Estimate Std. Error t value Pr(>|t|)
USDSGD.l1    -1.29657    0.40647  -3.190  0.00157 **
M1_INC_US.l1  0.48631    0.81765   0.595  0.55245
M1_INC_SG.l1  0.25171    0.31173   0.807  0.42004
INT_DIFF.l1   1.47154    0.04984  29.527  < 2e-16 ***
INF_DIFF.l1   0.12081    0.08675   1.393  0.16475
USDSGD.l2     1.25519    0.40548   3.096  0.00215 **
M1_INC_US.l2 -0.58025    0.81252  -0.714  0.47570
M1_INC_SG.l2  0.55703    0.31265   1.782  0.07582 .
INT_DIFF.l2  -0.48305    0.04906  -9.847  < 2e-16 ***
INF_DIFF.l2  -0.09353    0.08803  -1.062  0.28890
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.1424 on 300 degrees of freedom
Multiple R-Squared: 0.9984,    Adjusted R-squared: 0.9983
F-statistic: 1.854e+04 on 10 and 300 DF,  p-value: < 2.2e-16


Estimation results for equation INF_DIFF:
=========================================
INF_DIFF = USDSGD.l1 + M1_INC_US.l1 + M1_INC_SG.l1 + INT_DIFF.l1 + INF_DIFF.l1 + USDSGD.l2 + M1_INC_US.l2 + M1_INC_SG.l2 +
INT_DIFF.l2 + INF_DIFF.l2

             Estimate Std. Error t value Pr(>|t|)
USDSGD.l1    -0.04190    0.12283  -0.341   0.7333
M1_INC_US.l1 -0.45111    0.24709  -1.826   0.0689 .
M1_INC_SG.l1 -0.04827    0.09420  -0.512   0.6088
INT_DIFF.l1  -0.01597    0.01506  -1.060   0.2900
INF_DIFF.l1   1.90523    0.02621  72.677   <2e-16 ***
USDSGD.l2     0.04537    0.12253   0.370   0.7114
M1_INC_US.l2 -0.46398    0.24554  -1.890   0.0598 .
M1_INC_SG.l2 -0.06234    0.09448  -0.660   0.5099
INT_DIFF.l2   0.01482    0.01482   1.000   0.3183
INF_DIFF.l2  -0.91089    0.02660 -34.241   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.04303 on 300 degrees of freedom
Multiple R-Squared: 0.9993,    Adjusted R-squared: 0.9992
F-statistic: 4.119e+04 on 10 and 300 DF,  p-value: < 2.2e-16



Covariance matrix of residuals:
              USDSGD  M1_INC_US  M1_INC_SG   INT_DIFF   INF_DIFF
USDSGD     3.912e-04  3.292e-05  1.660e-05 -0.0004693  4.540e-05
M1_INC_US  3.292e-05  1.012e-04  2.824e-05 -0.0001411  2.560e-05
M1_INC_SG  1.660e-05  2.824e-05  6.716e-04 -0.0004112  8.841e-06
INT_DIFF  -4.693e-04 -1.411e-04 -4.112e-04  0.0202727 -1.091e-03
INF_DIFF   4.540e-05  2.560e-05  8.841e-06 -0.0010910  1.851e-03

Correlation matrix of residuals:
            USDSGD M1_INC_US M1_INC_SG INT_DIFF  INF_DIFF
USDSGD     1.00000   0.16543  0.032390 -0.16663  0.053347
M1_INC_US  0.16543   1.00000  0.108315 -0.09847  0.059127
M1_INC_SG  0.03239   0.10831  1.000000 -0.11143  0.007929
INT_DIFF  -0.16663  -0.09847 -0.111430  1.00000 -0.178083
INF_DIFF   0.05335   0.05913  0.007929 -0.17808  1.000000
```

Using a basic VAR model, we can see that the significant coefficients for the equation is:

- Interest rate differential
- M1 of US

We can also note that there is a total of 300 degree of freedom. This number is way too high as there are only 313 rows on the dataset. We will need to estimate with fewer variables later.

```
plot(VAR_model)
```

Diagram of fit and residuals for USDSGD

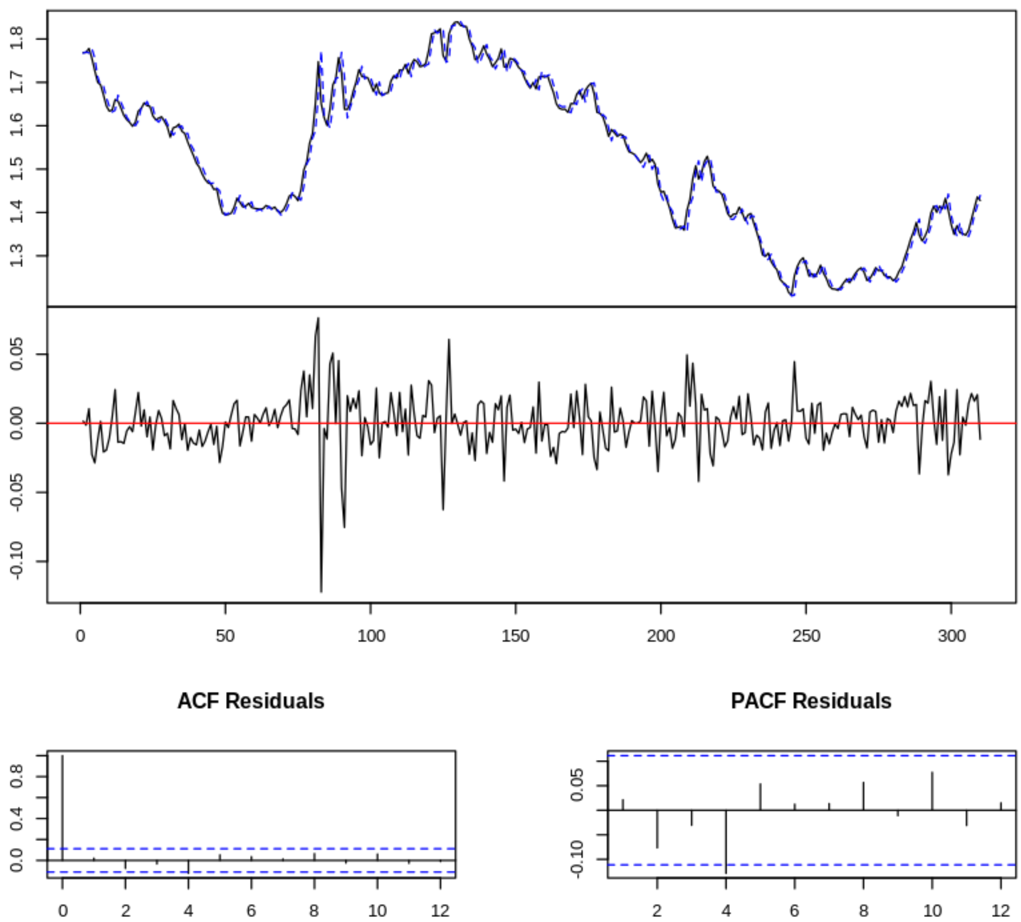


Diagram of fit and residuals for M1_INC_US

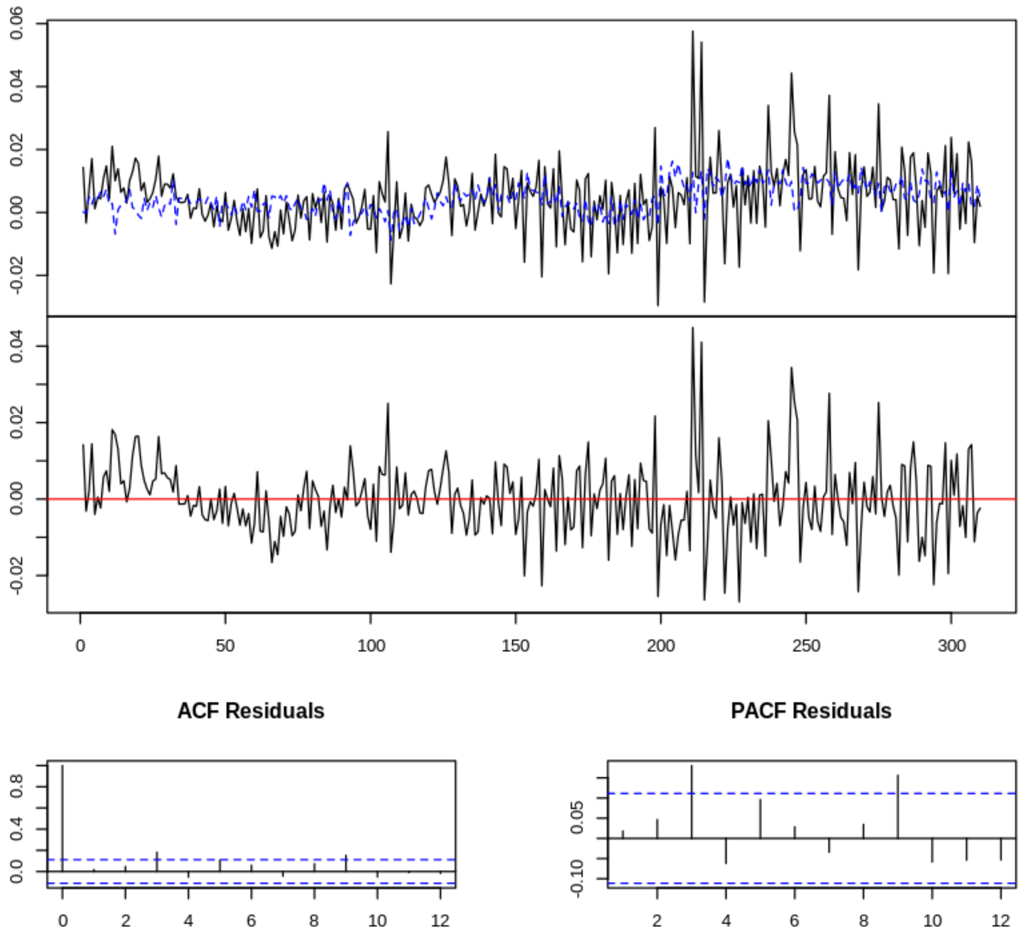
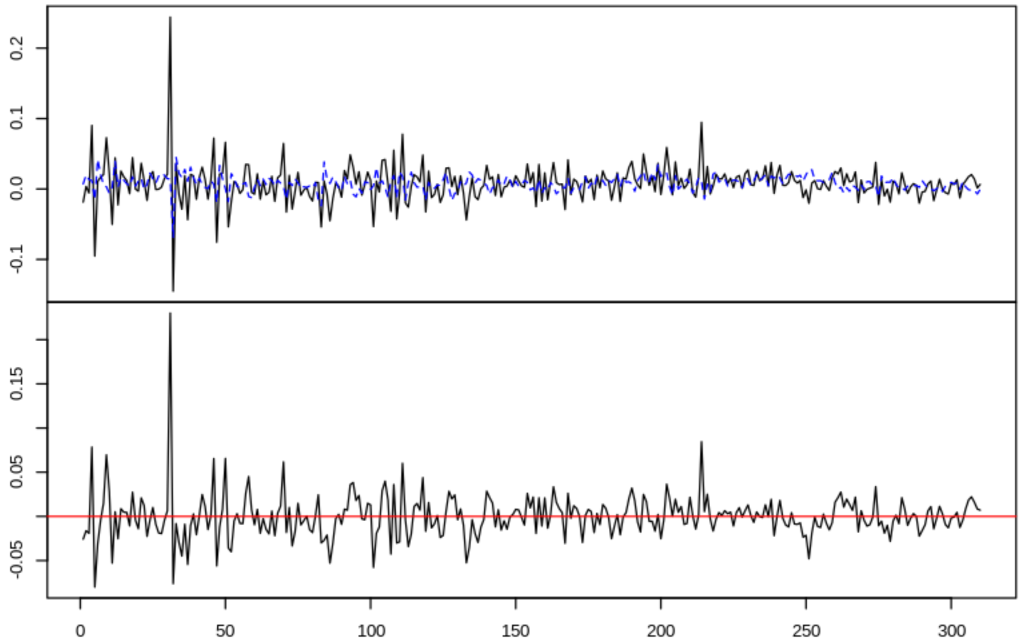
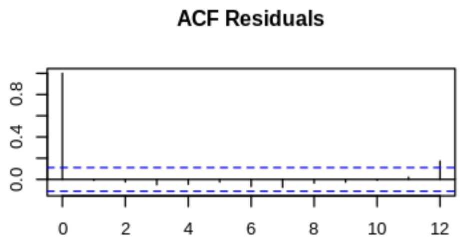


Diagram of fit and residuals for M1_INC_SG





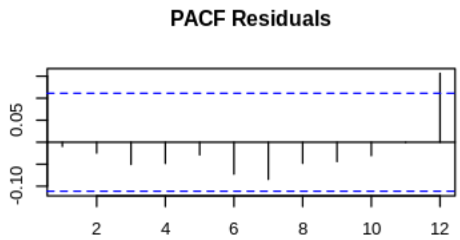
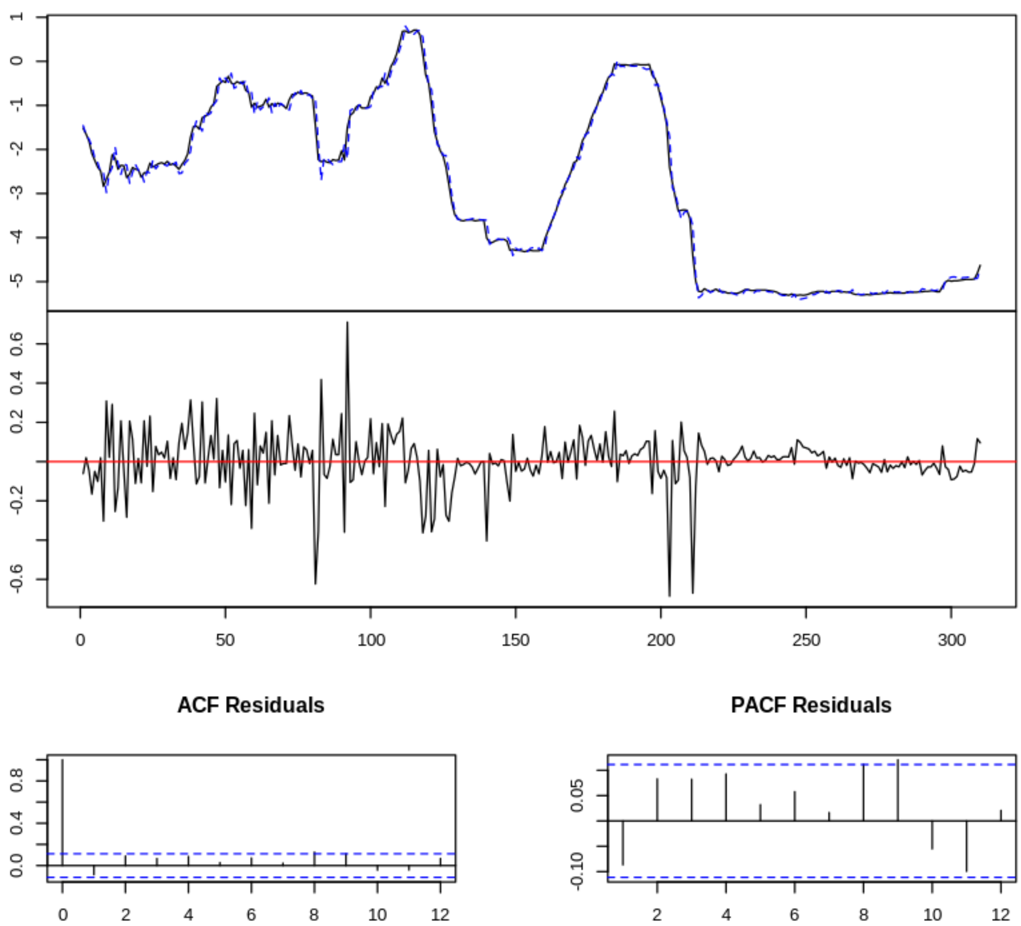
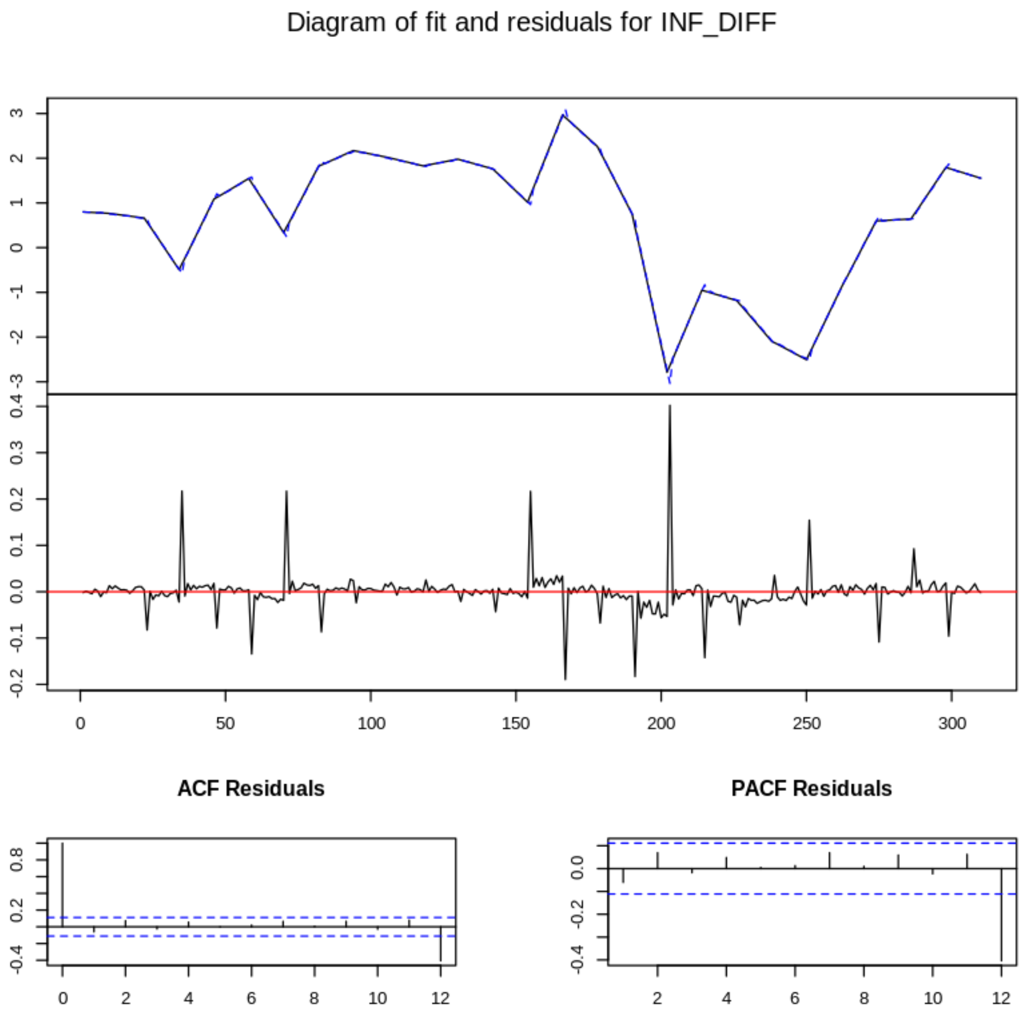


Diagram of fit and residuals for INT_DIFF

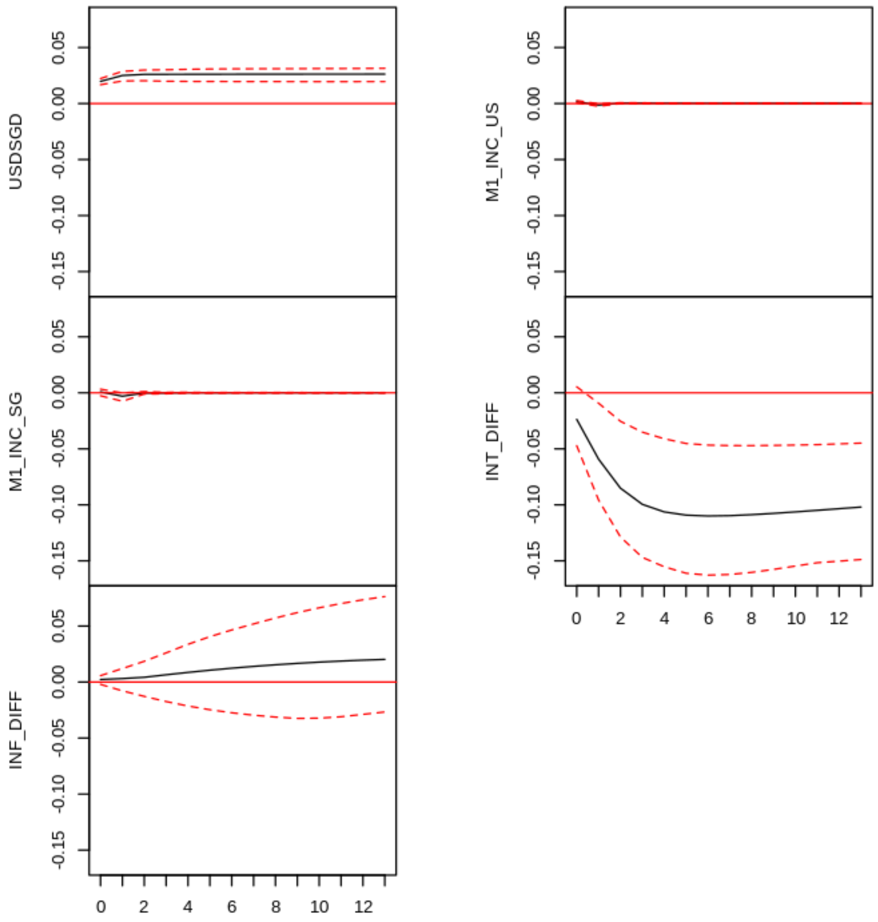




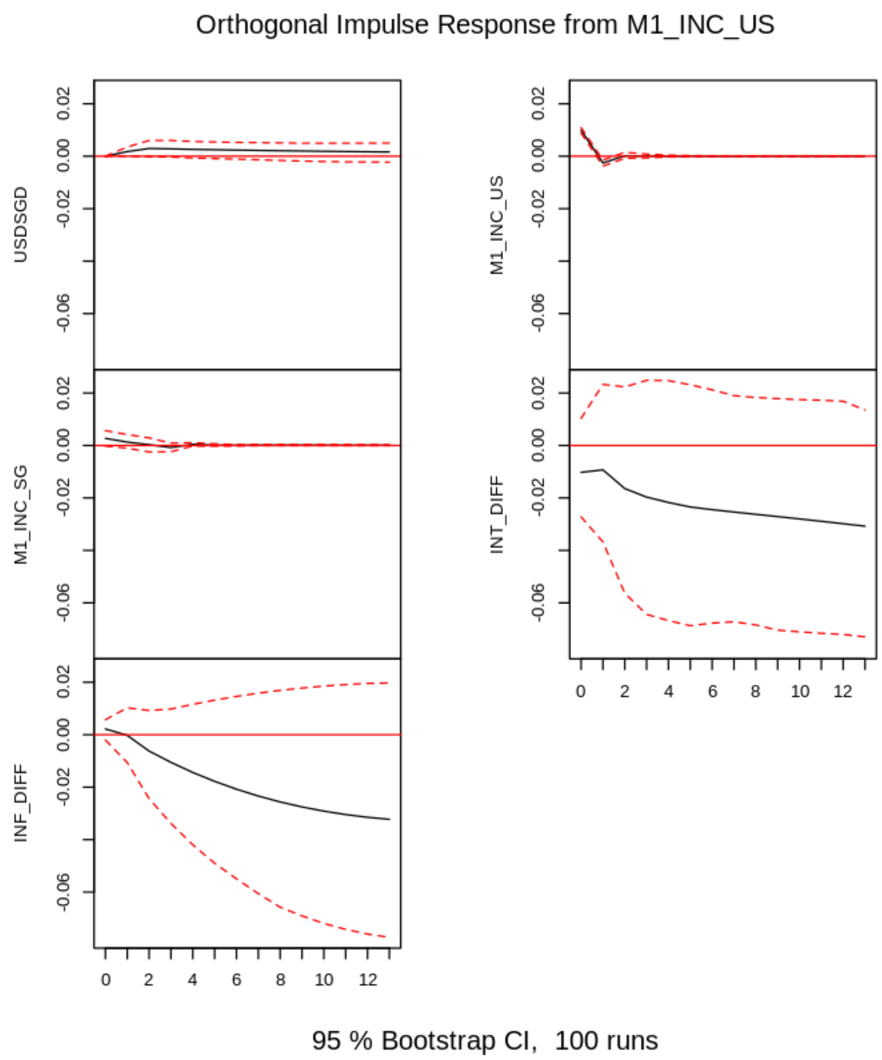
The ACF and PACF of the USDSGD dataset seemed to be stationary and are made up of white noises.

```
# compute and plot the impulse response functions
VAR_irf = irf(VAR_model, n.ahead = 13,boot = TRUE, ci = 0.95)
plot(VAR_irf)
```

Orthogonal Impulse Response from USDSGD

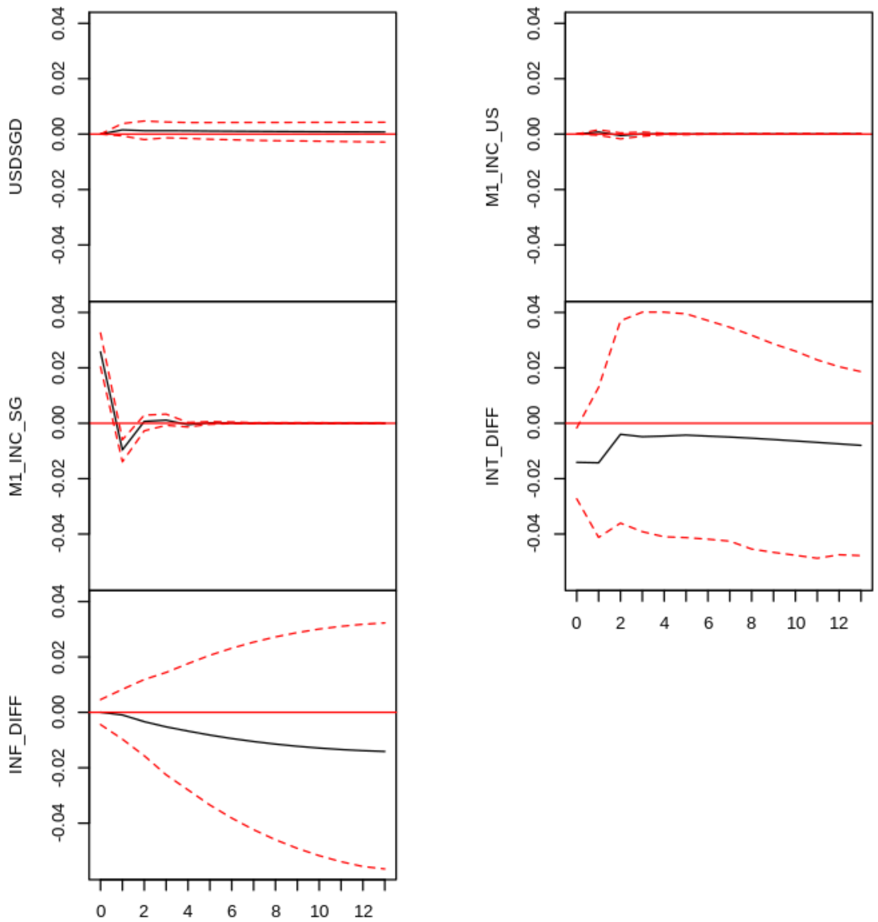


95 % Bootstrap CI, 100 runs

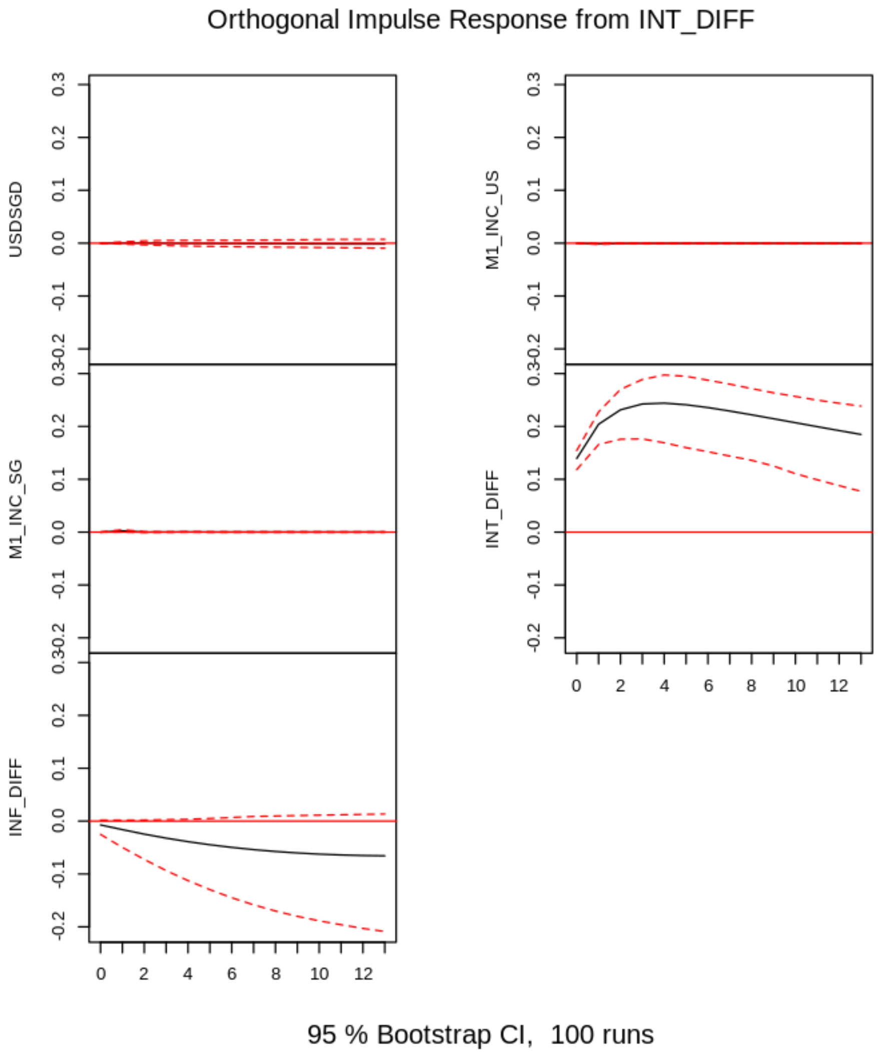


Orthogonal Impulse Response from M1_INC_US
USDSGD
M1_INC_US
M1_INC_SG
INT_DIFF
INF_DIFF
0.02
0.00
-0.02
-0.06
0
2
4
6
8
10
12
95 % Bootstrap CI, 100 runs

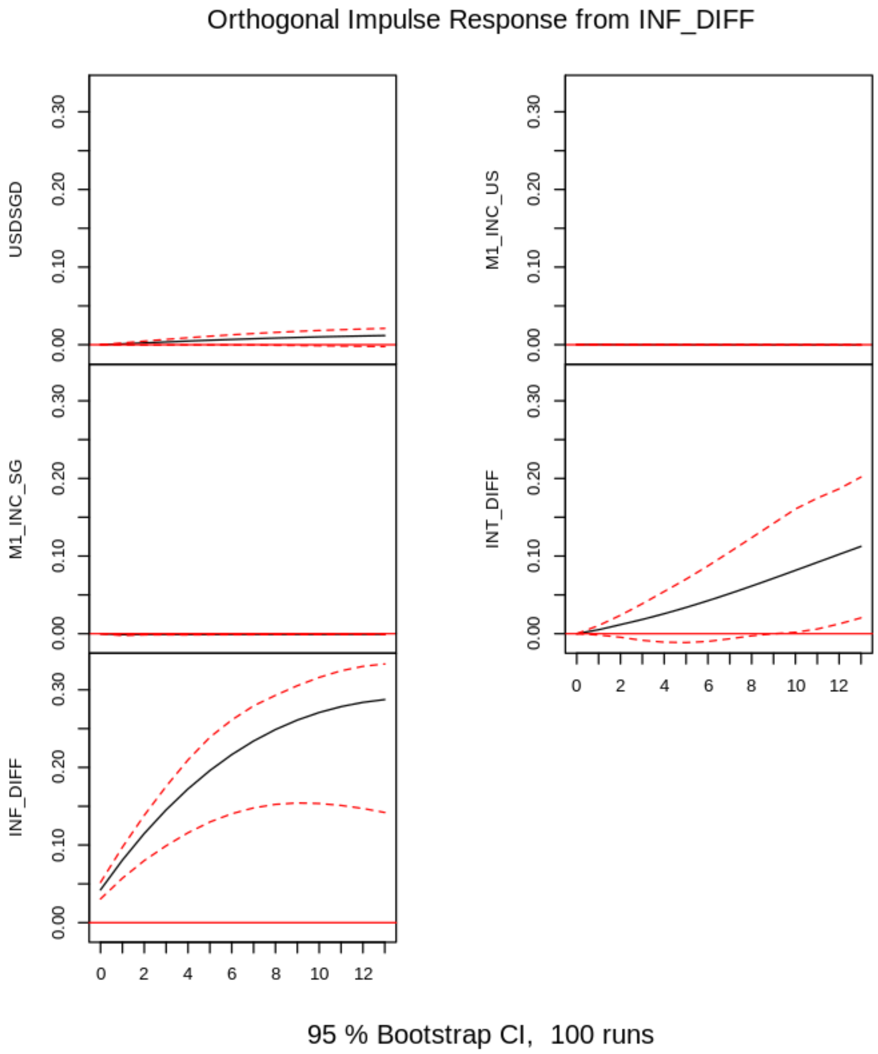
Orthogonal Impulse Response from M1_INC_SG



95 % Bootstrap CI, 100 runs

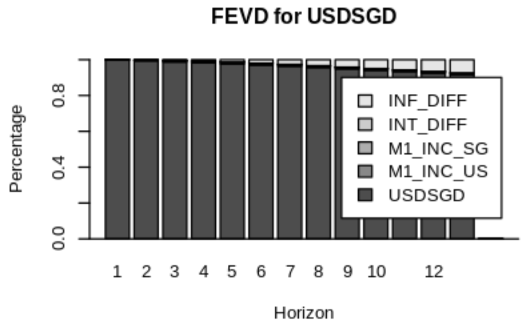


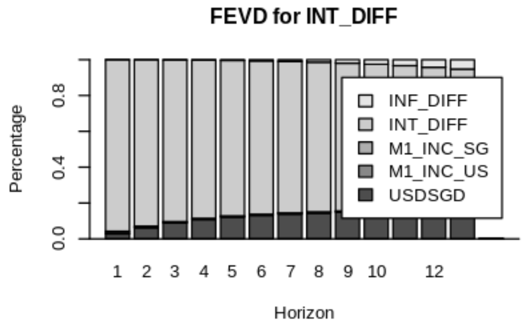
Orthogonal Impulse Response from INT_DIFF
USDSGD
M1_INC_US
M1_INC_SG
INT_DIFF
INF_DIFF
0.3
0.2
0.1
0.0
-0.1
-0.2
0
2
4
6
8
10
12
95 % Bootstrap CI, 100 runs

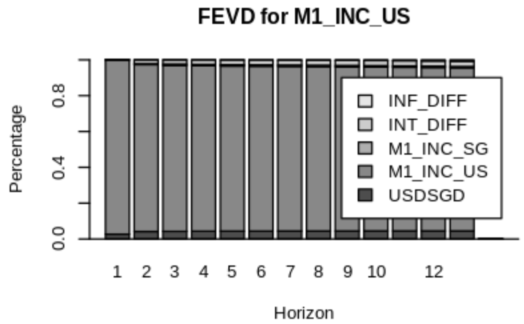


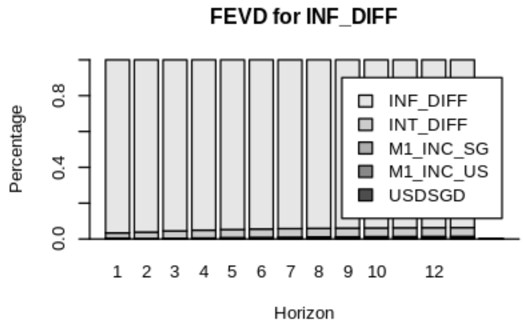
From the impulse function of USDSGD, we can see that both M1_US and M1_SG quickly return to the mean, interest rate differential also seemed to be returning to the mean, but very slowly.

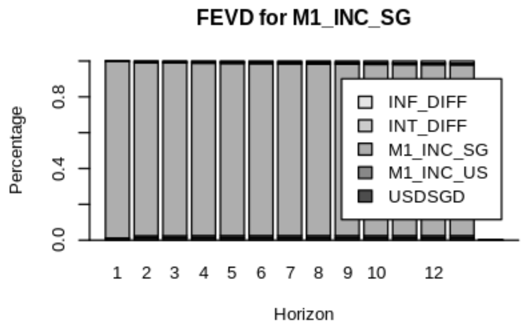
```
# compute and plot the forecast error variance decomposition
VAR_fevd <- fevd(VAR_model,n.ahead = 13)
plot(VAR_fevd)
```











Looking at the forecast error variance decomposition, we can see that the USDSGD does not have components of other variables in the short term but seemed to be explained by the interest rate differential in the long term. We can make use of this information to generate a more parsimonious model.

```
jotest1=ca.jo(data, type="eigen", K=9, ecdet="none", spec="longrun")
summary(jotest1)
```

```
######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
[1] 0.183749082 0.083705728 0.061071661 0.034668610 0.009592389

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 4 |  2.92  6.50  8.18 11.65
r <= 3 | 10.69 12.91 14.90 19.19
r <= 2 | 19.09 18.90 21.07 25.75
r <= 1 | 26.49 24.78 27.14 32.14
r = 0  | 61.52 30.84 33.32 38.78

Eigenvectors, normalised to first column:
(These are the cointegration relations)
```

```
                 USDSGD.l9 M1_INC_US.l9 M1_INC_SG.l9 INT_DIFF.l9  INF_DIFF.l9
USDSGD.l9      1.000000000   1.00000000   1.00000000     1.00000  1.000000000
M1_INC_US.l9   5.569691818 -72.99149460  -0.30710287  2396.09226  5.538829262
M1_INC_SG.l9 -67.074770837   3.75049801   1.08636131  -515.22389  0.707298625
INT_DIFF.l9    0.009467883  -0.07750398  -0.05346909    13.33348  0.045939215
INF_DIFF.l9   -0.223331885  -0.20596255  -0.10581722   -13.84998 -0.001714244

Weights W:
(This is the loading matrix)

               USDSGD.l9 M1_INC_US.l9 M1_INC_SG.l9   INT_DIFF.l9   INF_DIFF.l9
USDSGD.d    -0.003047337 -0.007320024 -0.031007321  1.833804e-05 -0.0026701940
M1_INC_US.d  0.001376064  0.010111033 -0.008308665 -4.064152e-05 -0.0001376793
M1_INC_SG.d  0.031589665 -0.004039068 -0.007448002  6.855564e-06  0.0006929855
INT_DIFF.d  -0.021474798 -0.084573816  0.058440536 -1.145220e-03  0.0071447850
INF_DIFF.d   0.005782302  0.010124143  0.015801041  2.744807e-05 -0.0183578898
```

```
jotest2=ca.jo(data, type="trace", K=9, ecdet="none", spec="longrun")
summary(jotest2)
```

```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.182334987 0.084197049 0.059799870 0.034633135 0.009283369

Values of teststatistic and critical values of test:

           test 10pct  5pct  1pct
r <= 4 |   2.84  6.50  8.18 11.65
r <= 3 |  13.55 15.66 17.95 23.52
r <= 2 |  32.30 28.71 31.52 37.22
r <= 1 |  59.03 45.23 48.28 55.43
r = 0  | 120.23 66.49 70.60 78.87

Eigenvectors, normalised to first column:
(These are the cointegration relations)

                 USDSGD.l9 M1_INC_US.l9 M1_INC_SG.l9 INT_DIFF.l9  INF_DIFF.l9
USDSGD.l9      1.000000000   1.00000000   1.00000000     1.00000  1.000000000
M1_INC_US.l9   7.052974270 -70.91007192   1.03908196  7819.06677  4.506471954
M1_INC_SG.l9 -67.122919617   3.11734181   1.24902002 -1726.33262  0.660890598
INT_DIFF.l9    0.008493893  -0.07830929  -0.05113518    44.34699  0.047017679
INF_DIFF.l9   -0.221603913  -0.20428627  -0.10330311   -46.12759 -0.002472603

Weights W:
(This is the loading matrix)

               USDSGD.l9 M1_INC_US.l9 M1_INC_SG.l9   INT_DIFF.l9   INF_DIFF.l9
USDSGD.d    -0.003161389 -0.008010878 -0.030354994  5.816952e-06 -2.505915e-03
M1_INC_US.d  0.001096862  0.010434602 -0.008490675 -1.202138e-05 -9.601853e-05
M1_INC_SG.d  0.031349973 -0.003467716 -0.007935882  2.266422e-06  7.728191e-04
INT_DIFF.d  -0.018842301 -0.086719776  0.058823572 -3.457795e-04  6.390796e-03
INF_DIFF.d   0.006027834  0.010978225  0.014329304  8.647183e-06 -1.783039e-02
```

Both Johansen-Procedure test shows that we can use r = 2.

```
fit = VECM(data, 3, r = 2, include = "const",estim = "ML", LRinclude = "none")
summary(fit)
```

```
#############
###Model VECM
#############
Full sample size: 313     End sample size: 309
Number of variables: 5     Number of estimated slope parameters 90
AIC -10631.59     BIC -10273.19     SSR 6.834194
Cointegrating vector (estimated by ML):
          USDSGD    M1_INC_US  M1_INC_SG    INT_DIFF     INF_DIFF
r1  1.000000e+00 3.552714e-15 -77.429157 0.018740912 -0.281552410
r2 -5.269906e-19 1.000000e+00  -1.016069 0.001882071 -0.001415143


                   ECT1               ECT2               Intercept
Equation USDSGD    -0.0086(0.0037)*   0.5421(0.2829).    0.0102(0.0053).
Equation M1_INC_US 0.0140(0.0019)***  -1.0870(0.1426)*** -0.0204(0.0027)***
```

```
Equation M1_INC_SG 0.0133(0.0049)**   0.5746(0.3731)     -0.0035(0.0070)
Equation INT_DIFF  -0.0500(0.0270).   3.0332(2.0597)     0.0600(0.0387)
Equation INF_DIFF  0.0290(0.0080)***  -2.0541(0.6090)*** -0.0400(0.0114)***
                   USDSGD -1          M1_INC_US -1       M1_INC_SG -1
Equation USDSGD    0.2623(0.0598)***   -0.3017(0.2522)     -0.0750(0.1001)
Equation M1_INC_US -0.0812(0.0301)**   -0.1540(0.1271)     -0.0047(0.0505)
Equation M1_INC_SG -0.1268(0.0788)     -0.3435(0.3327)     0.2423(0.1320).
Equation INT_DIFF  -1.3804(0.4351)**   -2.2698(1.8366)     -0.6315(0.7289)
Equation INF_DIFF  -0.0957(0.1286)     1.7216(0.5430)**    0.1077(0.2155)
                   INT_DIFF -1         INF_DIFF -1         USDSGD -2
Equation USDSGD    -0.0065(0.0084)     0.0303(0.0270)      -0.0751(0.0622)
Equation M1_INC_US -0.0100(0.0042)*    0.0027(0.0136)      0.0049(0.0314)
Equation M1_INC_SG 0.0179(0.0110)      -0.0362(0.0355)     -0.0863(0.0821)
Equation INT_DIFF  0.3961(0.0609)***   0.1979(0.1962)      -0.1496(0.4530)
Equation INF_DIFF  -0.0142(0.0180)     0.8746(0.0580)***   0.1100(0.1339)
                   M1_INC_US -2        M1_INC_SG -2        INT_DIFF -2
Equation USDSGD    -0.0691(0.1964)     -0.0707(0.0752)     0.0199(0.0089)*
Equation M1_INC_US -0.1304(0.0990)     -0.0124(0.0379)     0.0048(0.0045)
Equation M1_INC_SG -0.1123(0.2590)     0.0928(0.0992)      -0.0197(0.0117).
Equation INT_DIFF  -1.8675(1.4298)     -0.0612(0.5474)     0.1467(0.0648)*
Equation INF_DIFF  1.1445(0.4227)**    -0.0039(0.1618)     0.0017(0.0192)
                   INF_DIFF -2         USDSGD -3           M1_INC_US -3
Equation USDSGD    -0.0163(0.0357)     -0.0225(0.0596)     0.0011(0.1185)
Equation M1_INC_US -0.0084(0.0180)     -0.0199(0.0300)     0.0529(0.0597)
Equation M1_INC_SG -0.0205(0.0471)     -0.0732(0.0786)     0.0516(0.1563)
Equation INT_DIFF  0.0272(0.2599)      0.4621(0.4339)      -0.7459(0.8631)
Equation INF_DIFF  0.1121(0.0769)      -0.0099(0.1283)     0.7835(0.2552)**
                   M1_INC_SG -3        INT_DIFF -3         INF_DIFF -3
Equation USDSGD    -0.0280(0.0443)     -0.0096(0.0079)     0.0109(0.0272)
Equation M1_INC_US -0.0207(0.0223)     0.0035(0.0040)      0.0177(0.0137)
Equation M1_INC_SG 0.0420(0.0585)      0.0117(0.0104)      0.0363(0.0358)
Equation INT_DIFF  0.1714(0.3228)      0.0950(0.0577)      -0.0438(0.1979)
Equation INF_DIFF  -0.1676(0.0955).    0.0034(0.0170)      -0.0758(0.0585)
```

```
predict(fit, n.ahead=3)
```

	USDSGD	M1_INC_US	M1_INC_SG	INT_DIFF	INF_DIFF
314	1.422764	0.000365743	-0.004518124	-4.534573	1.532782
315	1.420645	0.005404270	0.003997137	-4.459303	1.526594
316	1.418958	0.003037646	0.005517923	-4.407052	1.520535

VECM with fewer variables

Using the information above, we can create a more parsimonious model. It can be seen that the significant coefficients of the previous model is the interest rate differential and the M1 money supply of US.

```
data_simp = data$USDSGD
data_simp = merge(data_simp, data$INT_DIFF)
data_simp = merge(data_simp, data$M1_INC_US)
```

```
jotest3=ca.jo(data_simp, type="eigen", K=9, ecdet="none", spec="longrun")
summary(jotest3)
```

```
######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
[1] 0.067385652 0.041820252 0.004837982

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 2 |  1.47  6.50  8.18 11.65
r <= 1 | 12.99 12.91 14.90 19.19
r = 0  | 21.21 18.90 21.07 25.75

Eigenvectors, normalised to first column:
(These are the cointegration relations)

               USDSGD.l9 INT_DIFF.l9 M1_INC_US.l9
USDSGD.l9      1.0000000    1.000000   1.00000000
INT_DIFF.l9    0.1800576   -0.159094   0.04458363
M1_INC_US.l9 193.9029368  -26.387258   2.29898383
```

```
Weights W:
(This is the loading matrix)

              USDSGD.l9  INT_DIFF.l9  M1_INC_US.l9
USDSGD.d    -0.001852385 -0.010897085 -0.0046122959
INT_DIFF.d   0.015492788  0.118265325 -0.0210486186
M1_INC_US.d -0.003589055  0.001925471 -0.0001587515
```

```
The Johansen-Procedure suggest to use r = 1
```

```
fit2 = VECM(data_simp, 2, r = 1, include = "const",estim = "ML", LRinclude = "none")
summary(fit2)
```

```
#############
###Model VECM
#############
Full sample size: 313     End sample size: 310
Number of variables: 3     Number of estimated slope parameters 24
AIC -6488.479     BIC -6391.328     SSR 6.448042
Cointegrating vector (estimated by ML):
   USDSGD   INT_DIFF M1_INC_US
r1      1 -0.6478023 -356.7588


                   ECT                 Intercept           USDSGD -1
Equation USDSGD    -0.0009(0.0007)     0.0009(0.0016)      0.2859(0.0573)***
Equation INT_DIFF  -0.0002(0.0048)     -0.0036(0.0119)     -1.0529(0.4187)*
Equation M1_INC_US 0.0028(0.0003)***   -0.0052(0.0008)***  -0.0754(0.0286)**
                   INT_DIFF -1         M1_INC_US -1        USDSGD -2
Equation USDSGD    -0.0063(0.0078)     -0.1650(0.1817)     -0.0748(0.0582)
Equation INT_DIFF  0.4365(0.0572)***   0.4367(1.3283)      0.1152(0.4258)
Equation M1_INC_US -0.0126(0.0039)**   -0.2476(0.0908)**   -0.0200(0.0291)
                   INT_DIFF -2        M1_INC_US -2
Equation USDSGD    0.0169(0.0076)*    -0.0105(0.1122)
Equation INT_DIFF  0.1687(0.0556)**   -0.0987(0.8205)
Equation M1_INC_US 0.0037(0.0038)     -0.2121(0.0561)***
```

Conclusion

Using VECM we tend to face the curse of dimensionality as shown in the first example with 5 variables. Using the combinations of correlation & forecast error variance decompisition we may deduce a simpler model. The SSR for the first model improved from 6.834194 to 6.448042 and the number of coefficients to estimate reduced from 300 to 24 by using only three variables and lower r.

We can also observe that the second model has great proportion of coefficients that are significant compared to the first model.

While the AIC and BIC suffered when the dimensionality is reduced, we can attribute it to the overfiting of data when higher dimensions are used, and should still opt for the more parismonious model.

```
# Last interest rate in the model
usdsgd["201701"]

# Actual interest rate to be forcasted
usdsgd["201702/201704"]
```

```
             [,1]
2017-01-01 1.4276



             [,1]
2017-02-01 1.4137
2017-03-01 1.4049
2017-04-01 1.3983
```

```
predict(fit2, n.ahead=3)
```

	USDSGD	INT_DIFF	M1_INC_US
314	1.422295	-4.525287	0.003174703
315	1.421568	-4.448128	0.007283518
316	1.421708	-4.398968	0.005159924

Using the second model, we can create a prediction of the quarter interest rate. We can see that the prediction is pretty close to the actual interest rate where it shows a downward trend from 1.4276.

References

[1] https://www.khanacademy.org/economics-finance-domain/ap-macroeconomics/every-graph-used-in-ap-macroeconomics/a/the-foreign-exchange-market-model

[2] https://eml.berkeley.edu/~obstfeld/182_sp06/c14.pdf

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[5]https://www.researchgate.net/publication/228258887_Causal_Relationships_between_Industrial_Production_Interest_Rate_and_Exchange_Rate_Evidence

[6] https://stats.stackexchange.com/questions/77791/why-use-vector-error-correction-model