MATH 3383 : Homework 1 –

① Let $f(x) = \sin x$. Find Taylor Polynomial Approx, $P_n(x)$ @ $X_0 = 0$.
Graph over interval $[\pi/4, \pi/4]$ for $n = 0, 1, 2, 3, 4$.

$$P_n(0) = \frac{f(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f^2(0)}{2!}x^2 + \frac{f^3(0)}{3!}x^3 + \frac{f^4(0)}{4!}x^4 + \dots + \frac{f^n(0)}{4!}$$

$$P_n(0) = \boxed{\sum_{k=0}^{n} \frac{f^k(0)}{k!} x^k} \quad \text{for } f(x) = \sin x$$

$$P_0(0) = \sin(0) = 0$$

$$P_1(0) = 0 + \frac{\cos(0)}{1!}x^1 = x$$

$$P_2(0) = 0 + x - \cancel{\frac{\sin(0)}{2!}}^{\,0} x^2 = x$$

$$P_3(0) = 0 + x + 0 - \frac{\cos(0)}{3!}x^3 = x - \frac{x^3}{3!}$$

$$P_4(0) = 0 + x + 0 - \frac{x^3}{3!} + \cancel{\frac{\sin(0)}{4!}}^{\,0} x^4 = x - \frac{x^3}{3!}$$

② $F(x) = \cos x$ , w/ $x_0 = \pi/2$ over $[\pi/4, 3\pi/4]$

$P_n(\pi/2) = \boxed{\sum_{k=0}^{n} \frac{f^{k}(\frac{\pi}{2})}{k!} (x - \pi/2)^k}$ for $f(x) = \cos x$

$P_0(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) = 0$

$P_1(\frac{\pi}{2}) = 0 - \frac{\sin(\frac{\pi}{2})}{1!}(x - \frac{\pi}{2}) = -x + \frac{\pi}{2}$

$P_2(\frac{\pi}{2}) = 0 - x + \frac{\pi}{2} + \cancel{\frac{-\cos(\frac{\pi}{2})}{2!}}^{\to 0}(x - \frac{\pi}{2})^2 = -x + \frac{\pi}{2}$

$P_3(\frac{\pi}{2}) = 0 - x + \frac{\pi}{2} + \frac{\sin(\frac{\pi}{2})}{3!}(x - \frac{\pi}{2})^3 = -x + \frac{\pi}{2} + \frac{(x - \frac{\pi}{2})^3}{3!}$

$P_4(\frac{\pi}{2}) = 0 - x + \frac{\pi}{2} + \frac{(x - \frac{\pi}{2})^4}{3!} + \cancel{\frac{\cos(\frac{\pi}{2})}{4!}}^{\to 0}(x - \frac{\pi}{2})^4 = -x + \frac{\pi}{2} + \frac{(x - \frac{\pi}{2})^3}{3!}$