FALL 2021. HW 3, PROB 1–5 : DUE ON SEPT/23

1. Theoretical Questions

Problem 1 We consider to find a root of $f : \mathbb{R} \mapsto \mathbb{R}$ under the assumption that the closed interval $[a, b]$ contains x_* such that

$$f(x_*) = 0$$

is given. We shall apply the bisection method to approximately find x_*. In each iteration, the bisection method generates iterate, say x_n, starting at $x_0 = (b+a)/2$. Find a formula of the iteration number n that is required by the bisection method for which x_n can be said to approximate x_* within an absolute error, (i.e., $|x_n - x_*|$) tolerance ε. Your formula should be given in terms of a, b and ε.

2. Computational Questions

Guideline for Computational Homeworks : Here is what you need to submit : (1) Printout of your code (2) Printout of figures if asked in the questions (3) Upload the source code as a single file if asked in the questions so that I can attempt to run and test. Note that the code should not ask any input files. It must be self-contained. More details will be discussed in class if needs arise.

Problem 2 The function $f(x) = \sin(x)$ has a zero on the interval $(3, 4)$, namely, $x_* = \pi$. Perform three iterations of Newton method to approximate this zero, using $x_0 = 4$. Determine the absolute error in each of the computed approximations. What is the apparent order of convergence?

Problem 3 Apply the Newton's method to find the solution to

$$x^3 - x - 3 = 0$$

starting with $x_0 = 0$. Compute $x_1, x_2, x_3, x_4, x_5, x_6$ and x_7 and compare pair of numbers (x_0, x_4), (x_1, x_5), (x_2, x_6) and (x_3, x_7). What can you conclude from this computations (Use your computer code) ?

Problem 4 Find an approximation by the method of false position for the root of function $f(x) = e^x - x^2 + 3x - 2$ accurate to within 10^{-5} (absolute error) on the interval $[0, 1]$. (Use your computer code)

Problem 5 Find an approximation to $\sqrt{3}$ correct to within 10^{-4} using the Bisection method (Hint: Consider $f(x) = x^2 - 3$.) (Use your computer code)