

① Find the linear and quadratic Taylor polynomial of $f(x, y) = x^5 y^3 + \sin^2(\pi x) \cos(\pi y)$ @ the point $(x_0, y_0) = (1, 1)$.

$$① f^{(0,0)}(1, 1) = 1^5 \cdot 1^3 + \sin^2(\pi) \cos(\pi) = 1$$

$$② f^{(1,0)}(x, y) = 5y^3 x^4 + 2\pi \cos(\pi y) \sin(\pi x) \cos(\pi x) = 5y^3 x^4 + \pi \cos(\pi y) \sin(2\pi x)$$

$$f^{(1,0)}(1, 1) = 5 + \pi \cos(\pi) \sin(2\pi) = 5$$

$$③ f^{(0,1)}(x, y) = 3x^5 y^2 - \pi \sin^2(\pi x) \sin(\pi y)$$

$$f^{(0,1)}(1, 1) = 3 + \pi \sin^2(\pi) \sin(\pi) = 3$$

$$④ f^{(2,0)}(x, y) = 20y^3 x^3 + 2\pi^2 \cos(\pi y) \cos(2\pi x)$$

$$f^{(2,0)}(1, 1) = 20 + 2\pi^2 \cos(\pi) \cos(2\pi) = 20 - 2\pi^2$$

$$⑤ f^{(0,2)}(x, y) = 6x^5 y - \pi^2 \sin^2(\pi x) \cos(\pi y)$$

$$f^{(0,2)}(1, 1) = 6 - \pi^2 \sin^2(\pi) \cos(\pi) = 6$$

$$⑥ f^{(1,1)}(x, y) = 15x^4 y^2 - 2\pi^2 \sin(\pi x) \cos(\pi x) \sin(\pi y) = 15x^4 y^2 - \pi^2 \sin(2\pi x) \sin(\pi y)$$

$$f^{(1,1)}(1, 1) = 15 - \pi^2 \sin(2\pi) \sin(\pi) = 15$$

Linear Taylor Polynomial -

$$P_1(1, 1) = 1 + 5(x-1) + 3(y-1)$$

Quadratic Taylor Polynomial -

$$P_2(1, 1) = 1 + 5(x-1) + 3(y-1) + \frac{(20-2\pi^2)(x-1)^2}{2!} + \frac{6(y-1)^2}{2!} + 15(x-1)(y-1)$$