RENT A CAR SERVICE MODEL – OUTPUT ANALYSIS

Output of the system that will be analyzed:

- The average waiting time in both queues (Output – 1).
- Average service times for both services (Output – 2).
- Average time spent in the system (Output – 3).
- The average time between arrivals (Output – 4).

ORIGINAL MODEL

OUTPUT 1 - Average waiting time for both queue:

SEED VALUES	OUTPUT VALUES
1	0.17
4	0.17
7	0.15
10	0.39
13	0.23

Figure-1: The average waiting time in the queue of customers who have reservation

Replication Number: 5

Mean Value: 0.222

Standard Deviation: 0.098 $S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$\sqrt{\frac{\Sigma(0.17-0.222)+(0.17-0.222)+(0.15-0.222)+(0.39-0.222)+(0.23-0.222)^2}{4}}$ = 0.098

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S/\sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$0.222 \pm 2.776 \frac{0.098}{\sqrt{5}} = 0.222 \pm 0.122 \qquad 95\% \text{ CI} = [0.100, 0.344]$$

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$R \geq (\frac{1.64 \ x \ 0.098}{0.1})^2 = R \geq 2.58$

That means, at least 3 replicant is needed.

3) **95% Prediction Interval:** $Y \pm t_{\frac{\alpha}{2},n-1} S \sqrt{1+\frac{1}{n}}$

$t_{0.025,4}$ = 2.776

0.222 ± 2.776 x 0.098$\sqrt{1+1/5}$ = 0.222 ± 0.298 95% Prediction Interval = [-0.076, 0.520]

SEED VALUES	OUTPUT VALUES
1	2.25
4	1.98
7	1.75
10	2.43
13	2.77

Figure-2: The average waiting time in the queue of customers who have no reservation

Replication Number: 5

Mean Value: 2.236

Standard Deviation: 0.395 $S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$\sqrt{\frac{\Sigma(2.25-2.236)+(1.98-2.236)+(1.75-2.236)+(2.43-2.236)+(2.77-2.236)^2}{4}}$ = 0.395

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S / \sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$2.236 \pm 2.776 \frac{0.395}{\sqrt{5}} = 2.236 \pm 0.490$$ 95% CI = [1.746, 2.726]

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$R \geq (\frac{1.64\ x\ 0.395}{0.1})^2$ = R ≥ 41.96

That means, at least 42 replicant is needed.

3) **95% Prediction Interval:** $Y \pm t_{\frac{\alpha}{2},n-1} S \sqrt{1+\frac{1}{n}}$

$t_{0.025,4}$ = 2.776

2.236 ± 2.776 x 0.395 $\sqrt{1+1/5}$ = 2.236 ± 1.201 95% Prediction Interval = [1.035, 3.437]

OUTPUT 2 - Average service times for both services:

SEED VALUES	OUTPUT VALUES
1	4.24
4	4.05
7	4.48
10	4.51
13	4.55

Figure-3: Average service time of service that is for customers who have reservation

Replication Number: 5

Mean Value: 4.366

Standard Deviation: 0.214 $S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$\sqrt{\frac{\Sigma(4.24-4.366)+(4.05-4.366)+(4.48-4.366)+(4.51-4.366)+(4.55-4.366)^2}{4}}$ = 0.214

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S / \sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$4.366 \pm 2.776 \frac{0.214}{\sqrt{5}} = 4.366 \pm 0.266$$ 95% CI = [4.100, 4.632]

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$R \geq (\frac{1.64 \; x \; 0.214}{0.1})^2 = R \geq 12.32$

That means, at least 13 replicant is needed.

3) **95% Prediction Interval:** $Y \pm t_{\frac{\alpha}{2},n-1} S \sqrt{1 + \frac{1}{n}}$

$t_{0.025,4}$ = 2.776

$4.366 \pm 2.776 \text{ x } 0.214 \sqrt{1 + 1/5} = 4.366 \pm 0.650$ 95% Prediction Interval = [3.938, 5.016]

SEED VALUES	OUTPUT VALUES
1	6.63
4	6.38
7	6.38
10	6.46
13	6.64

Figure-4: Average service time of service that is for customers who have <u>no reservation</u>

<u>Replication Number:</u> 5

<u>Mean Value:</u> 6.498

<u>Standard Deviation:</u> 0.129 $S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$\sqrt{\frac{\Sigma(6.63-6.498)+(6.38-6.498)+(6.38-6.498)+(6.46-6.498)+(6.64-6.498)^2}{4}}$ = 0.129

1) **<u>95% Confidence Interval:</u>** $\bar{Y} \pm t_{\alpha/2,n-1} S / \sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$6.498 \pm 2.776 \frac{0.129}{\sqrt{5}} = 6.498 \pm 0.160 \qquad \text{95\% CI} = [6.338, 6.658]$$

2) **<u>Total Number of Replications Needed to Estimate Mean Output:</u>** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

   Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

   $R \geq (\frac{1.64 \; x \; 0.129}{0.1})^2 = R \geq 4.47$

   That means, at least 5 replicant is needed.

3) **<u>95% Prediction Interval:</u>** $Y \pm t_{\frac{\alpha}{2},n-1} S \sqrt{1+\frac{1}{n}}$

   $t_{0.025,4}$ = 2.776

   $6.498 \pm 2.776 \text{ x } 0.129 \sqrt{1 + 1/5} = 6.498 \pm 0.392$ 95% Prediction Interval = [6.106,6.890]

OUTPUT 3 - Average time spent in the system:

SEED VALUES	OUTPUT VALUES
1	5.16
4	4.71
7	5.06
10	5.17
13	4.69

Figure-5: The average time spent in the system

Replication Number: 5

Mean Value: 4.958

Standard Deviation: 0.239 $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$\sqrt{\frac{\Sigma(5.16-4.958)+(4.71-4.958)+(5.06-4.958)+(5.17-4.958)+(4.69-4.958)^2}{4}}$ = 0.239

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S / \sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$4.958 \pm 2.776 \frac{0.239}{\sqrt{5}} = 4.958 \pm 0.297$ 95% CI = [4.661, 5.255]

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$R \geq (\frac{1.64\ x\ 0.239}{0.1})^2 = R \geq 15.36$

That means, at least 16 replicant is needed.

3) **95% Prediction Interval:** $Y \pm t_{\frac{\alpha}{2},n-1} S \sqrt{1 + \frac{1}{n}}$

$t_{0.025,4}$ = 2.776

$4.958 \pm 2.776 \text{ x } 0.239\sqrt{1 + 1/5} = 4.958 \pm 0.727$ 95% Prediction Interval = [4.231, 5.685]

OUTPUT 4 - Average time between arrivals:

SEED VALUES	OUTPUT VALUES
1	5.82
4	5.63
7	6.16
10	5.88
13	5.84

Figure-6: The average time between arrivals

Replication Number: 5

Mean Value: 5.866

Standard Deviation: 0.190 $s = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}$

$$\sqrt{\frac{\Sigma(5.82-5.866)+(5.63-5.866)+(6.16-5.866)+(5.88-5.866)+(5.84-5.866)^2}{4}} = 0.190$$

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S / \sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$5.866 \pm 2.776 \frac{0.190}{\sqrt{5}} = 5.866 \pm 0.236 \qquad 95\%\ \text{CI} = [5.630, 6.102]$$

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$$R \geq (\frac{1.64\ x\ 0.190}{0.1})^2 = R \geq 9.70$$

That means, at least 10 replicant is needed.

3) **95% Prediction Interval:** $Y \pm t_{\frac{\alpha}{2},n-1} S \sqrt{1+\frac{1}{n}}$

$t_{0.025,4}$ = 2.776

$$5.866 \pm 2.776 \text{ x } 0.190 \sqrt{1+1/5} = 5.866 \pm 0.578 \qquad 95\%\ \text{Prediction Interval} = [5.288, 6.444]$$

ALTERNATIVE MODEL

OUTPUT 1 - Average waiting time for both queue:

SEED VALUES	OUTPUT VALUES
1	0.20
4	0.17
7	0.14
10	0.03
13	0.24

Figure-7: The average waiting time in the queue of customers who have reservation

Replication Number: 5

Mean Value: 0.156

Standard Deviation: 0.079 $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$\sqrt{\frac{\Sigma(0.20-0.156)+(0.17-0.156)+(0.14-0.156)+(0.03-0.156)+(0.24-0.156)^2}{4}}$ = 0.079

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S/\sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$0.156 \pm 2.776\frac{0.079}{\sqrt{5}} = 0.156 \pm 0.098 \qquad 95\%\ CI = [0.058, 0.254]$$

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$R \geq (\frac{1.64\ x\ 0.079}{0.1})^2 = R \geq 1.68$

That means, at least 2 replicant is needed.

3) **Additional Replications Needed to Reduce the Half-width of the Confidence Interval by 10% for the Differences of the Estimated Values:** $R \geq (\frac{Z_{0.1}\, S_0}{\varepsilon})^2$

Confidence Interval: 80%, 1-α = 0.2, α/2 = 0.1, $Z_{0.1}$ = 1.28

$R \geq (\frac{1.28\ x\ 0.079}{0.1})^2 = R \geq 1.02$

That means, at least 2 replicant is needed.

SEED VALUES	OUTPUT VALUES
1	81.68
4	117.12
7	114.63
10	111.72
13	136.19

Figure-8: The average waiting time in the queue of customers who have no reservation

Replication Number: 5

Mean Value: 112.268

Standard Deviation: 19.60 $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

$$\sqrt{\frac{\Sigma(81.68-112.268)+(117.12-112.268)+(114.63-112.268)+(111.72-112.268)+(136.19-112.268)^2}{4}} = 19.60$$

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S/\sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$112.268 \pm 2.776 \frac{19.60}{\sqrt{5}} = 112.268 \pm 24.332 \qquad 95\%\ CI = [87.936,\ 136.600]$$

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$$R \geq (\frac{1.64\ x\ 19.60}{0.1})^2 = R \geq 103323.67$$

That means, at least 103324 replicant is needed.

3) **Additional Replications Needed to Reduce the Half-width of the Confidence Interval by 10% for the Differences of the Estimated Values:** $R \geq (\frac{Z_{0.1}\, S_0}{\varepsilon})^2$

Confidence Interval: 80%, 1-α = 0.2, α/2 = 0.1, $Z_{0.1}$ = 1.28

$$R \geq (\frac{1.28\ x\ 19.60}{0.1})^2 = R \geq 62940{,}77$$

That means, at least 62941 replicant is needed.

OUTPUT 2 - Average service times for both services:

SEED VALUES	OUTPUT VALUES
1	4.61
4	4.63
7	4.53
10	4.50
13	4.39

Figure-9: Average service time of service that is for customers who have reservation

Replication Number: 5

Mean Value: 4.532

Standard Deviation: 0.096 $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

$\sqrt{\frac{\Sigma(4.61-4.532)+(4.63-4.532)+(4.53-4.532)+(4.50-4.532)+(4.39-4.532)^2}{4}}$ = 0.096

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S/\sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$4.532 \pm 2.776 \frac{0.096}{\sqrt{5}} = 4.532 \pm 0.120 \qquad 95\% \text{ CI} = [4.412, 4.652]$$

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$R \geq (\frac{1.64\ x\ 0.096}{0.1})^2 = R \geq 2.48$

That means, at least 3 replicant is needed.

3) **Additional Replications Needed to Reduce the Half-width of the Confidence Interval by 10% for the Differences of the Estimated Values:** $R \geq (\frac{Z_{0.1}\, S_0}{\varepsilon})^2$

Confidence Interval: 80%, 1-α = 0.2, α/2 = 0.1, $Z_{0.1}$ = 1.28

$R \geq (\frac{1.28\ x\ 0.096}{0.1})^2 = R \geq 1.50$

That means, at least 2 replicant is needed.

SEED VALUES	OUTPUT VALUES
1	7.33
4	7.43
7	7.38
10	7.44
13	7.39

Figure-10: Average service time of service that is for customers who have no reservation

Replication Number: 5

Mean Value: 7.394

Standard Deviation: 0.044 $S = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

$$\sqrt{\frac{\Sigma(7.33-7.394)+(7.43-7.394)+(7.38-7.394)+(7.44-7.394)+(7.39-7.394)^2}{4}} = 0.044$$

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S/\sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$7.394 \pm 2.776\,\frac{0.044}{\sqrt{5}} = 7.394 \pm 0.055 \qquad \text{95\% CI} = [7.339, 7.449]$$

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$R \geq (\frac{1.64\ x\ 0.044}{0.1})^2 = R \geq 0.52$

That means, at least 1 replicant is needed.

3) **Additional Replications Needed to Reduce the Half-width of the Confidence Interval by 10% for the Differences of the Estimated Values:** $R \geq (\frac{Z_{0.1}\, S_0}{\varepsilon})^2$

Confidence Interval: 80%, 1-α = 0.2, α/2 = 0.1, $Z_{0.1}$ = 1.28

$R \geq (\frac{1.28\ x\ 0.044}{0.1})^2 = R \geq 0.31$

That means, at least 1 replicant is needed.

OUTPUT 3 - Average time spent in the system:

SEED VALUES	OUTPUT VALUES
1	16.09
4	26.44
7	23.11
10	16.68
13	22.70

Figure-11: The average time spent in the system

Replication Number: 5

Mean Value: 21.004

Standard Deviation: 4.464 $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

$$\sqrt{\frac{\Sigma(16.09-21.004)+(26.44-21.004)+(23.11-21.004)+(16.68-21.004)+(22.70-21.004)^2}{4}} = 4.464$$

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S / \sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$21.004 \pm 2.776 \frac{4.464}{\sqrt{5}} = 21.004 \pm 5.542 \qquad 95\%\ CI = [15.462, 26.546]$$

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$$R \geq (\frac{1.64\ x\ 4.464}{0.1})^2 = R \geq 5359.64$$

That means, at least 5360 replicant is needed.

3) **Additional Replications Needed to Reduce the Half-width of the Confidence Interval by 10% for the Differences of the Estimated Values:** $R \geq (\frac{Z_{0.1}\, S_0}{\varepsilon})^2$

Confidence Interval: 80%, 1-α = 0.2, α/2 = 0.1, $Z_{0.1}$ = 1.28

$$R \geq (\frac{1.28\ x\ 4.464}{0.1})^2 = R \geq 3264.88$$

That means, at least 3265 replicant is needed.

OUTPUT 4 - Average time between arrivals:

SEED VALUES	OUTPUT VALUES
1	4.97
4	4.68
7	4.95
10	5.11
13	4.88

Figure-12: The average time between arrivals

Replication Number: 5

Mean Value: 4.918

Standard Deviation: 0.157 $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

$$\sqrt{\frac{\Sigma(4.97-4.918)+(4.68-4.918)+(4.95-4.918)+(5.11-4.918)+(4.88-4.918)^2}{4}} = 0.157$$

1) **95% Confidence Interval:** $\bar{Y} \pm t_{\alpha/2,n-1} S / \sqrt{n}$

1-α = 0.05, α/2 = 0.025, $t_{0.025,4}$ = 2.776

$$4.918 \pm 2.776 \frac{0.157}{\sqrt{5}} = 4.918 \pm 0.195 \qquad 95\% \text{ CI} = [4.723, 5.113]$$

2) **Total Number of Replications Needed to Estimate Mean Output:** $R \geq (\frac{Z_{0.05}\, S_0}{\varepsilon})^2$

Confidence Interval: 90%, 1-α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.64

$$R \geq (\frac{1.64 \ x \ 0.157}{0.1})^2 = R \geq 6.62$$

That means, at least 7 replicant is needed.

3) **Additional Replications Needed to Reduce the Half-width of the Confidence Interval by 10% for the Differences of the Estimated Values:** $R \geq (\frac{Z_{0.1}\, S_0}{\varepsilon})^2$

Confidence Interval: 80%, 1-α = 0.2, α/2 = 0.1, $Z_{0.1}$ = 1.28

$$R \geq (\frac{1.28 \ x \ 0.157}{0.1})^2 = R \geq 4.04$$

That means, at least 5 replicant is needed.

Comparison:

Outputs	Original Model	Alternative Model
Output – 1 (Reservation)	[0.100, 0.344]	[0.058, 0.254]
Output – 1 (No Reservation)	[1.746, 2.726]	[87.936, 136.600]
Output - 2 (Reservation)	[4.100, 4.632]	[4.412, 4.652]
Output – 2 (No Reservation)	[6.338, 6.658]	[7.339, 7.449]
Output 3	[4.661, 5.255]	[15.462, 26.546]
Output 4	[5.630, 6.102]	[4.723, 5.113]

Figure-13: 95% Confidence Interval Comparison

All outputs are different from point of view of 95% confidence interval for the two systems. Average waiting time in no reservation queue (Output – 1) and average time spent system (Output – 2) are the main difference between original and alternative models because there are more non-reserved customers in the alternative system and service time of non-reserved service is in the alternative model slower than original model. So, interarrival times, probability of non-reserved customer arrival and service time of no reservation service make alternative model inefficient. In conclusion, importance of reservation in the real systems can be seen with these two models.

Hasan Şenyurt

150120531

No group