Design of Integrated Microrobotic Fish

Presentation 3 - Physical Model (Improving) & COMSOL Simulation (Debugging)

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Contents

Antecedent

Total Impedance Z

In[1]:=

$$R=\frac{Pi*x*(\sqrt{k}+\frac{1}{\sqrt{k}})}{2\sigma*\delta x};$$

$$C_{DL}=\frac{\epsilon*\delta x*\sqrt{k}}{\lambda_D};$$

$$C_{DS}=\frac{\epsilon*\delta x}{\sqrt{k}*\lambda_D};$$

$$Zx=R+\frac{1}{I*\omega*C_{DL}}+\frac{1}{I*\omega*C_{DS}}$$

Out[4]=

$$\frac{(\frac{1}{\sqrt{k}}+\sqrt{k})\ \pi\ x}{2\delta x\ \sigma}-\frac{i\lambda_D}{\sqrt{k}\ \delta x\epsilon\ \omega}-\frac{i\ \sqrt{k}\ \lambda_D}{\delta x\ \epsilon\ \omega}$$

In[10]:= Simplify[Zx]

Out[10]=

$$\frac{(1+k)\ (\pi\ x\ \epsilon\ \omega-2\ i\ \sigma\lambda_D)}{2\ \sqrt{k}\ \delta x\ \epsilon\ \sigma\omega}$$

Total Impedance Z

Remind the formula given in the article

$$Z = \frac{\pi\left(\sqrt{k} + \frac{1}{\sqrt{k}}\right)}{2l\sigma} \frac{\ln A - i\theta}{(\ln A)^2 + \theta^2}$$

where

$$A = \frac{\sqrt{\left[(2\lambda_D\sigma)^2 + (\omega\varepsilon\pi)^2 + x_{min}x_{max}\right]^2 + \left[2\lambda_D\sigma\omega\varepsilon\pi\left(x_{max} - x_{min}\right)\right]^2}}{(2\lambda_D\sigma)^2 + (\omega\varepsilon\pi x_{min})^2}$$

and

$$\theta = \arctan \frac{2\lambda_D\sigma\omega\varepsilon\pi\left(x_{max} - x_{min}\right)}{(2\lambda_D\sigma)^2 + (\omega\varepsilon\pi)^2 x_{min}x_{max}}$$

The author probably made a mistake of A. The correct formula is

Total Impedance Z

In[ ]:= $(*A=\frac{\sqrt{\left((2\lambda_D*\sigma)^2+(\omega*\epsilon*Pi)^2+x_{min}*x_{max}\right)^2+\left(2\lambda_D*\sigma*\omega*\epsilon*Pi\,(x_{max}-x_{min})\right)^2}}{(2\lambda_D*\sigma)^2+(\omega*\epsilon*Pi*x_{min})^2};*)$

$$A = \frac{\left(2\,\lambda_D*\sigma\right)^2+\left(\omega*\epsilon*Pi*x_{max}\right)^2}{\left(2\,\lambda_D*\sigma\right)^2+\left(\omega*\epsilon*Pi*x_{min}\right)^2};$$

$$\theta = \text{ArcTan}\left[\frac{2\,\lambda_D*\sigma*\omega*\epsilon*Pi\,(x_{max}-x_{min})}{\left(2\,\lambda_D*\sigma\right)^2+\left(\omega*\epsilon*Pi\right)^2 x_{min}*x_{max}}\right];$$

反正切

$$Zo = \frac{Pi\left(\sqrt{k}+\frac{1}{\sqrt{k}}\right)}{l*\sigma*\left(\text{Log}[A]+2\,I*\theta\right)}$$

Out[ ]= $$\frac{\left(\frac{1}{\sqrt{k}}+\sqrt{k}\right)\pi}{l\,\sigma\left(2\,i\,\text{ArcTan}\left[\frac{2\,\pi\,\epsilon\,\sigma\,\omega\,(x_{max}-x_{min})\,\lambda_D}{\pi^2\,\epsilon^2\,\omega^2\,x_{max}\,x_{min}+4\,\sigma^2\,\lambda_D^2}\right]+\text{Log}\left[\frac{\pi^2\,\epsilon^2\,\omega^2\,x_{max}^2+4\,\sigma^2\,\lambda_D^2}{\pi^2\,\epsilon^2\,\omega^2\,x_{min}^2+4\,\sigma^2\,\lambda_D^2}\right]\right)}$$

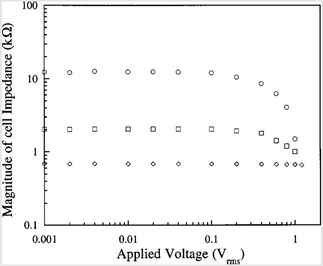
In[ ]:= $$Z = \frac{1}{\int_{x_{min}}^{x_{max}}\frac{2\,\sqrt{k}\,\epsilon\,\sigma\,\omega}{(1+k)\,(\pi\,x\,\epsilon\,\omega-2\,i\,\sigma\,\lambda_D)}\,dx}$$

Out[ ]= $$\left((1+k)\,\pi\right)\Big/\left(\sqrt{k}\,\sigma\left(2\,i\,\text{ArcTan}\left[\frac{\pi\,\epsilon\,\omega\,x_{max}}{2\,\sigma\,\lambda_D}\right]-2\,i\,\text{ArcTan}\left[\frac{\pi\,\epsilon\,\omega\,x_{min}}{2\,\sigma\,\lambda_D}\right]+\text{Log}\left[\pi^2\,\epsilon^2\,\omega^2\,x_{max}^2+4\,\sigma^2\,\lambda_D^2\right]-\text{Log}\left[\pi^2\,\epsilon^2\,\omega^2\,x_{min}^2+4\,\sigma^2\,\lambda_D^2\right]\right)\right)\ \text{if}\ \textit{condition}$$

Total Impedance Z

```
In[ ]:= λD = 4.56 * 10^-9;
σ = 0.018;
k = 6.12;
vL0 = 2.82 * 10^-9;
xmin = 1.6 * 10^-6;
xmax = 12 * 10^-6;
Φ0 = 0.1;
ω = 2 Pi * 1000;
      圆周率
l = 0.235;
c0 = 299 792 458;
μ0 = 4 * Pi * 10^-7;
          圆周率
ϵ0 = 1/(μ0 * c0^2);
ϵ = 80 ϵ0;
Z/l
Zo
Out[ ]= 667.159 - 1265.49 i
Out[ ]= 667.159 - 1265.49 i
```

Total Impedance Z



Here we restore the resistance to three-dimensional by dividing by the total length of the electrodes in the cell $l = 23.5$ cm.

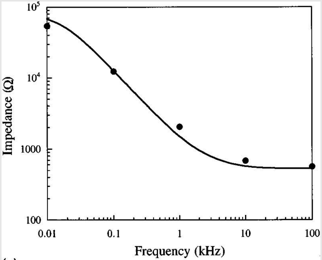
We again made a mistake here last time. What the paper gives in its FIG. 7. is the magnitude of cell impedance, i.e.,

$$\mathrm{Abs}[Z] = 1430.58\,\Omega$$

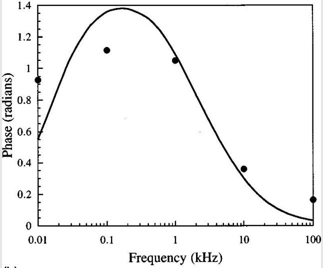
which is closed to the corresponding value in FIG. 7. We can directly found that the frequency is not correlated with the applied voltage.

Next, the left side is FIG. 8. in the paper and the right side is our result.

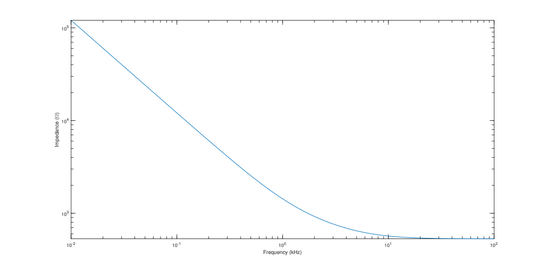
Total Impedance Z

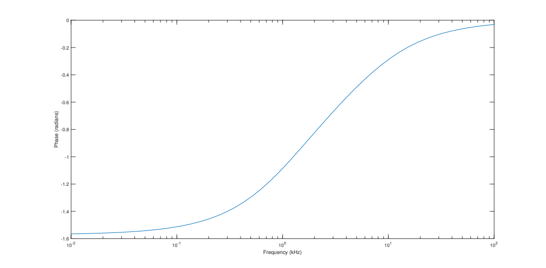


(a)



(b)





Total Impedance Z

MATLAB Code

```
lambda_D = 4.56*10^(-9);
sigma = 0.018;
k = 6.12;
v_L0 = 2.82*10^(-9);
x_min = 1.6*10^(-6);
x_max = 12*10^(-6); % Psi_0 = 0.1;
l = 0.235;
c_0 = 299792458;
mu_0 = 4*pi*10^(-7);
epsilon_0 = 1/(mu_0*c_0^2);
epsilon = 80 * epsilon_0;
f = logspace(1,5);
omega = 2*pi*f;
A = ((2*lambda_D*sigma)^2+(omega*epsilon*pi*x_max).^2)./((2*lambda_D*sigma)^2+(omega*epsilon*pi*x_min
    ).^2);
% A = sqrt(((2*lambda_D*sigma)^2+(omega*epsilon*pi).^2+x_min*x_max).^2+(2*lambda_D*sigma*omega*epsilon*
    pi*(x_max-x_min)).^2)./((2*lambda_D*sigma)^2+(omega*epsilon*pi*x_min).^2);
theta = atan((2*lambda_D*sigma*omega*epsilon*pi*(x_max-x_min))./((2*lambda_D*sigma)^2+(omega*epsilon*
    pi).^2*x_min*x_max));
Z = pi*(sqrt(k)+1/sqrt(k))./(l*sigma*(log(A)+2i*theta));
figure(1);
loglog(f/1000,abs(Z)); % ylim([100,10^5]);
xlabel('Frequency (kHz)'); ylabel('Impedance (\Omega)');
figure(2);
semilogx(f/1000,angle(Z));
xlabel('Frequency (kHz)'); ylabel('Phase (radians)');
```

Time-Averaged Velocity

Given $v_{DL} = \frac{\lambda_D \rho_{DL} E_{HL}}{\eta}$, how to derive its time-averaged value?

Note that both $\rho_{DL} = \frac{\Psi_{DL}\varepsilon}{\lambda_D}$ and $E_{HL} = -\frac{\Psi}{\sqrt{k}(1+k)} \frac{\frac{i\omega\varepsilon\pi}{2\lambda_D\sigma}}{(1+\frac{i\omega\varepsilon\pi x}{2\lambda_D\sigma})^2}$ have the part $\Psi = \Psi_0 \exp i\omega t$. Remind the method we use in calculating the time-averaged power:

$$< P >= \frac{1}{T}\int_0^T V_0 I_0 \cos{(\omega t + \phi_1)} \cos{(\omega t + \phi_2)} \mathrm{d}t$$

Using the formula

$$\cos{(A + B)} = \cos A \cos B - \sin A \sin B$$

we have

$$< P >= \frac{1}{T}\int_0^T V_0 I_0 (\cos \omega t \cos \phi_1 - \sin \omega t \sin \phi_1)(\cos \omega t \cos \phi_2 - \sin \omega t \sin \phi_2) \mathrm{d}t$$

Time-Averaged Velocity

$$< P >= \frac{1}{T} \int_0^T (\cos^2 \omega t \cos \phi_1 \cos \phi_2 + \sin^2 \omega t \sin \phi_1 \sin \phi_2 \\ - \cos \omega t \sin \omega t (\cos \phi_1 \sin \phi_2 + \cos \phi_2 \sin \phi_1)) \mathrm{d}t$$

Since $\int_0^T \sin^2 \omega t = \frac{T}{2}$, $\int_0^T \cos^2 \omega t = \frac{T}{2}$, and $\int_0^T \sin \omega t \cos \omega t = 0$,

$$< P >= \frac{1}{2} V_0 I_0 \left(\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2\right) = \frac{1}{2} V_0 I_0 \cos\left(\phi_1 - \phi_2\right)$$

Thus,

$$< P >= \frac{1}{2} \mathrm{Re} \left(I_0 V_0 e^{i\phi_1} e^{-i\phi_2} \right) = \frac{1}{2} \mathrm{Re} \left(\tilde{I}_0 \tilde{V}_0^* \right)$$

Similarly, we can write

$$< v_{DL} >= \frac{1}{2} \mathrm{Re} \{ \frac{\lambda_D \rho_{DL} E_{HL}^*}{\eta} \}$$

Improved Model

Electrical Prolem

Let's start from Poisson's equation:

$$\nabla^2\phi = \frac{\rho}{\varepsilon} = \frac{e(n_- - n_+)}{\varepsilon}$$

and the continuity equation (charge conservation equation):

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$$

Remind in VE320 Chapter 5: Carrier Transport Phenomena 5.2 Carrier Diffusion 5.2.2 Total Current Density Eq. (5.36) is the sum of electric drift and diffusion density:

$$J = en\mu_n E + ep\mu_p E + eD_n \nabla n - eD_p \nabla p$$

Adding drift due to the fluid motion:

$$\boldsymbol{J}_\pm = \mp en_\pm \mu \nabla \phi - eD\nabla n + en_\pm \boldsymbol{u}$$

where $\boldsymbol{u}$ is the liquid velocity, ρ is the (volume) charge density.

Electrical Prolem

Scales and parameters

Here to simplify our calculation, we use a dimensional analysis technique called "nondimensionalization" or "scaling" in order to suggest that certain quantities are better measured relative to some appropriate unit that is especially useful for systems that can be described by differential equations.

Next, we plan to use this method (or not) to calculate the electric field.

Electrical Prolem

The electrical potential in the bulk electrolyte satisfies Laplace's Equation

$$\nabla^2\Phi = 0$$

Φ is the potential just outside the double layer.
Low voltage assumption: the voltage drop across the double layer is linear to the surface charge $\frac{\partial q_s}{\partial t} = -\sigma E_y$. The surface charge conservation equation:

$$\sigma\frac{\partial \Phi}{\partial y} = i\omega C_{DL}(\Phi - V_j)$$

Note that different from previous definition, here C_{DL} is the capacitance per unit of **area** of the total double layer.
Boundary condition at the interface between the electrolyte and the glass:

$$(\sigma + i\varepsilon\omega)\frac{\partial \Phi}{\partial y} = (\sigma_g + i\varepsilon_g\omega)\frac{\partial \Phi_g}{\partial y}$$

Electrical Prolem

Remind the Ampère's circuital law with Maxwell's addition (differential equations SI convention):

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t}$$

Since the electric potential is alternating as $\Phi = \Phi_0 \exp(i\omega t)$,

$$\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{j} + i\varepsilon\omega \frac{\partial \Phi}{\partial y} = (\sigma + i\varepsilon\omega)\frac{\partial \Phi}{\partial y}$$

not to be confused with the electromagnetic wave $E = E_0 \exp(i(\boldsymbol{k} \cdot \boldsymbol{r}) - \omega t)$, which leads to $\sigma - i\varepsilon\omega$.

Electrical Prolem

Material	Conductivity σ at 20 °C (S/m)
Glass	10^{-15} to 10^{-11}
Sea water	4.8
Drinking water	5×10^{-4} to 5×10^{-2}
Deionized water	4.2×10^{-5}

Since $\varepsilon_g \ll \varepsilon$ and $\omega \ll \frac{\sigma}{\varepsilon}$, we can simplify the equation to

$$\frac{\partial \Phi}{\partial y} = 0$$

Besides,

$$\int_{-\frac{L}{2}}^{-\frac{L}{2}} \frac{\partial \Phi}{\partial y} \mathrm{d}x = 0$$

Fluid Dynamic

Debye-Hückel theory: surface capacitance

$$C_{DL} = \frac{\varepsilon}{\lambda_D}$$

Helmholtz-Smoluchowski formula: electro-osmotic slip velocity

$$u = \frac{\varepsilon \Delta\Phi}{\eta} E_x = -\frac{\varepsilon \Delta\Phi}{\eta} \frac{\partial \Phi}{\partial x}$$

Using the same strategy introduced before, the time-averaged horizontal fluid velocity at the interface between the double layer and the bulk is

$$< u >= -\frac{\varepsilon}{2\eta} \mathrm{Re}\left[\Delta\Phi \frac{\partial \Phi^*}{\partial x}\right] = -\frac{\varepsilon}{2\eta} \Lambda \mathrm{Re}\left[\Delta\Phi_{DL} \frac{\partial \Phi^*}{\partial x}\right] = -\frac{\varepsilon}{2\eta} \Lambda \frac{\partial}{\partial x} |\Phi - V_j|^2$$

Φ is the potential just outside the double layer; V_j is the potential applied to electrode j; Φ_{DL} is the total double layer potential drop.

Fluid Dynamic

The solution in the diffuse layer

The slip electro-osmotic velocity

$$< u >= -\frac{\varepsilon}{2\eta}\Lambda\frac{\partial}{\partial x}\left[(\Phi - V_j)(\Phi - V_j)^*\right]$$

where Λ is the ratio of the diffuse double layer impedance to the total double layer impedance given by

$$\Lambda = \frac{(i\omega C_d)^{-1}}{(i\omega C_{DL})^{-1}} = \frac{C_s}{C_s + C_d}$$

where C_d and C_s are the capacitances of the diffuse layer and the Stern or compact layer respectively. Boundary condition:

$$\sigma\frac{\partial\Phi}{\partial y} = i\omega C_{DL}(\Phi - \Phi_g) = i\varepsilon_g\omega\frac{\partial\Phi_g}{\partial y}$$

Fluid Dynamic
The solution in the bulk

Incompressible Navier-Stokes equations (convective form)

$$\overbrace{\rho\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\substack{\text{Unsteady}\\ \text{acceleration}}} + \underbrace{(\mathbf{v}\cdot\nabla)\mathbf{v}}_{\substack{\text{Convective}\\ \text{acceleration}}}\Big)}^{\text{Inertia}} = \underbrace{-\nabla p}_{\substack{\text{Pressure}\\ \text{gradient}}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}} + \underbrace{\mathbf{f}}_{\substack{\text{Other}\\ \text{forces}}}$$

For microsystems, the Reynolds number $\mathrm{Re} = \frac{\rho VD}{\eta} = \frac{VD}{\nu}$ is very small ($D \sim 2\times10^{-5}$m, $V \sim 5\times10^{-4}$m/s, $\nu = \mu/\rho \sim 10^{-6}\,\mathrm{m}^2/\mathrm{s}$, $\mathrm{Re} \sim 10^{-2}$), so inertial terms (and the externally applied body forces) can be neglected:

$$\eta\nabla^2\boldsymbol{u} - \nabla p = 0$$

$$\nabla\cdot\boldsymbol{u} = 0$$

Next, we plan to solve the differential equations.

COMSOL Simulation

Parameters

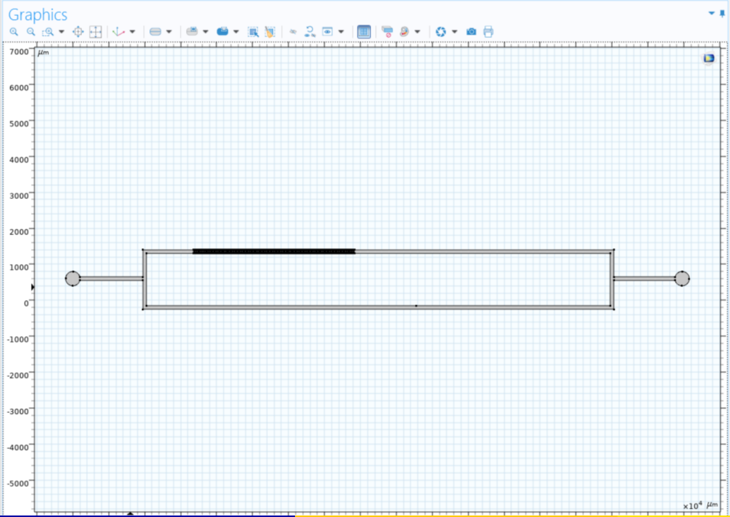
To simulate our model, we mainly referred to a COMSOL sample "Electroosmotic Micromixer". Our parameters are:

Name	Expression	Value	Description
sigma	0.00123[S/m]	0.00123 S/m	Conductivity of the ionic solution
epsilon_r	80.2	80.2	Relative permittivity of the fluid
V0	0.1*sqrt(2)[V]	0.14142 V	Maximum value of the AC poten...
omega	2*pi[rad]*1000[Hz]	6283.2 Hz	Angular frequency of the AC po...
zeta	-0.120596[V]	-0.1206 V	Zeta potential
U0	0[mm/s]	0 m/s	Mean inflow velocity

COMSOL Simulation

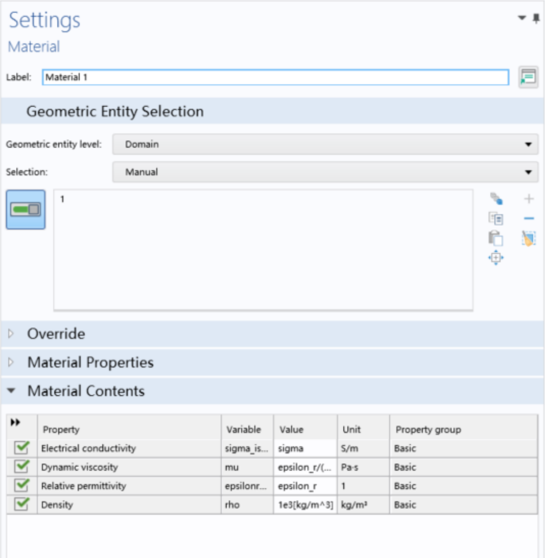
Geometry

Our geometry is:



COMSOL Simulation

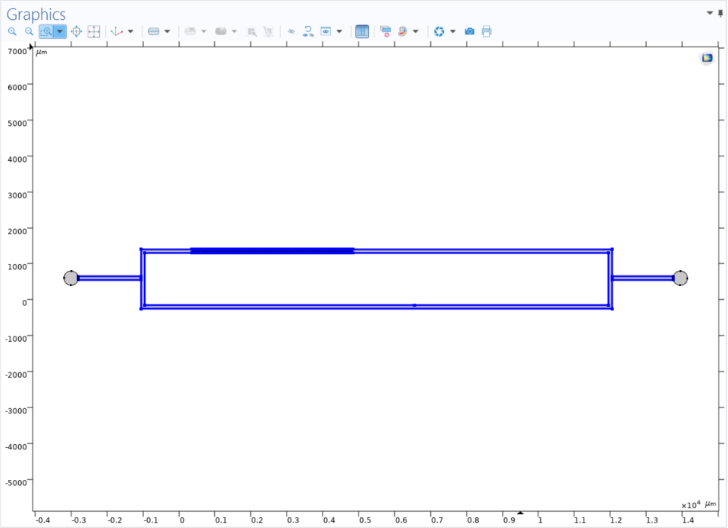
Material



Property	Variable	Value	Unit	Property group
Electrical conductivity	sigma_is...	sigma	S/m	Basic
Dynamic viscosity	mu	epsilon_r/(...	Pa·s	Basic
Relative permittivity	epsilonr...	epsilon_r	1	Basic
Density	rho	1e3[kg/m^3]	kg/m³	Basic

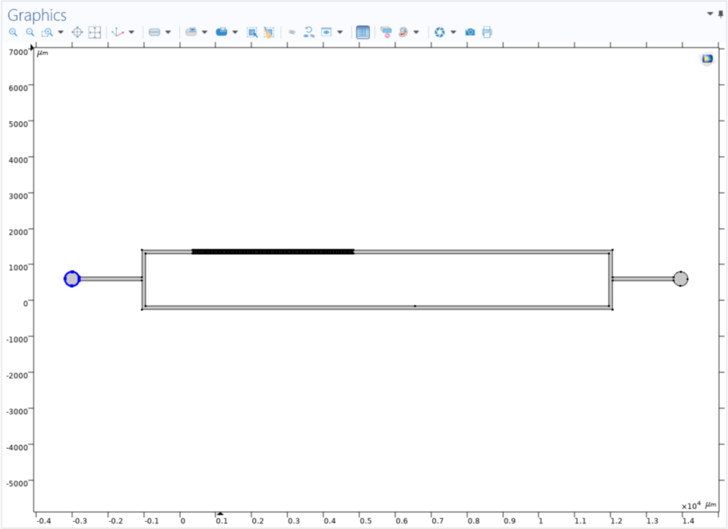
Creeping Flow

Wall ($U_{av} = U0$)



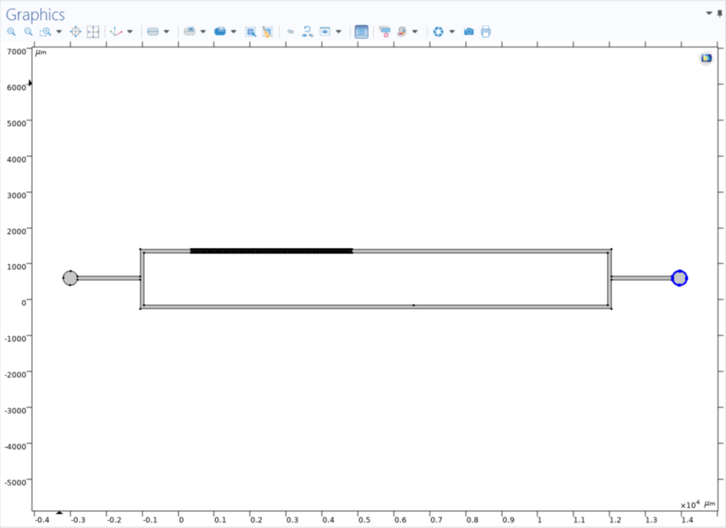
Creeping Flow

Inlet



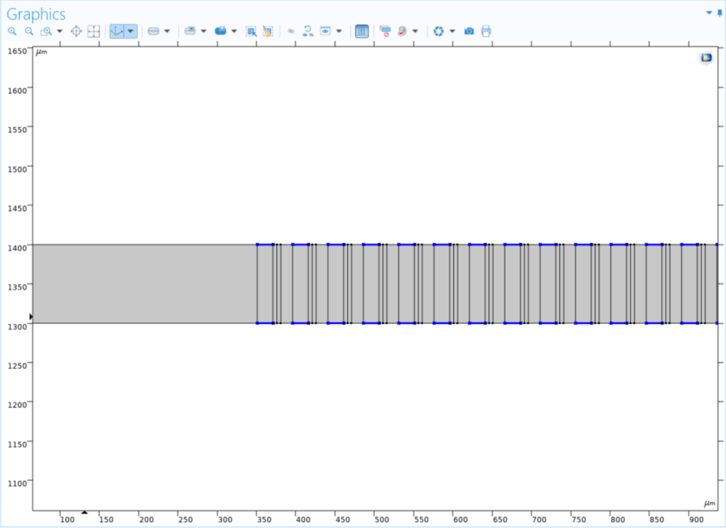
Creeping Flow

Outlet



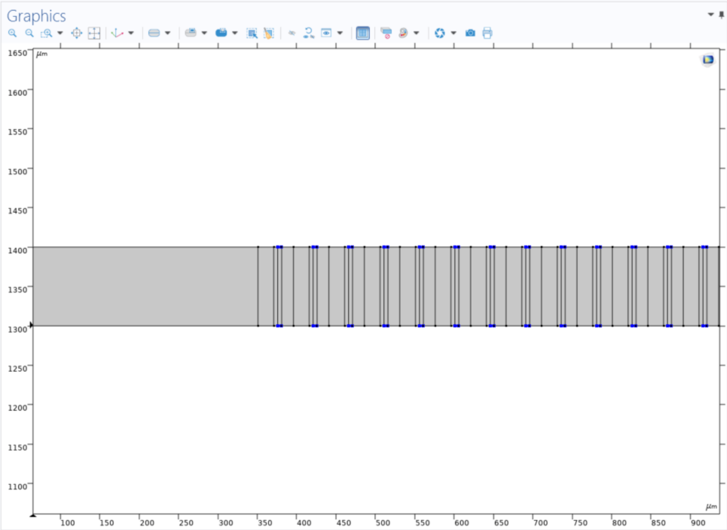
Electric Currents

Electric Potential 1 ($V_0 = V0$)



Electric Currents

Electric Potential 2 ($V_0 = -V0$)



COMSOL Simulation

Material

Then, we set up a study using time-dependent solver to solve the fluid flow. After computing, we can derive the results including:

- Velocity, streamlines
- Electric potential

Next, we plan to modify the original layout in GDS file in AutoCAD to fit the COMSOL geometry requirements.

Thanks!