Design of Integrated Microrobotic Fish

Presentation 2 - Physical Model (Preliminary)

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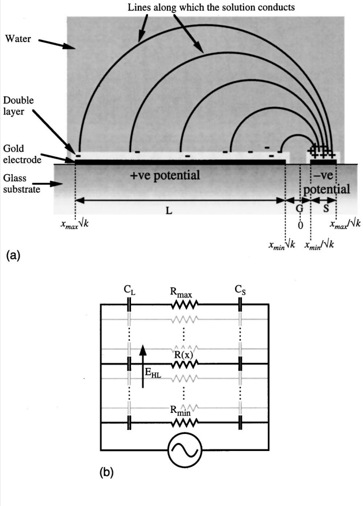
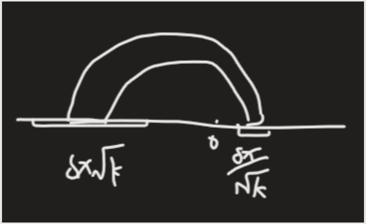
Basic Frame



- The double layers at the electrode surfaces ->Capacitance
  The capacitance of the double layer at each end of the tube per unit length of the electrode C_{DL} and at the large electrode C_{DS}
- The bulk water ->Resistance
  The resistance of the tube per unit length of electrode $R(x)$

Resistance



Different form common three-dimensional resistance given by $R = \frac{\rho L}{S}$, here the resistance is two-dimensional given by $R = \frac{\rho L}{d} = \frac{L}{\sigma d}$.
We can take infinitesimals of x (δx) to calculate $R(x)$. Here we can assume the electric field lines are semicircles, so $L = \pi r$ where the radius $r = \frac{\sqrt{k}+\frac{1}{\sqrt{k}}}{2}$.
Therefore, we have $R(x) = \frac{\pi x\left(\sqrt{k}+\frac{1}{\sqrt{k}}\right)}{2\sigma\delta x}$.

Capacitance

Different form common three-dimensional capacitance given by $C = \frac{\varepsilon S}{d}$, here the resistance is two-dimensional given by $R = \frac{\varepsilon L}{d}$. Then, we can derive that $C_{DL} = \frac{\varepsilon \delta x \sqrt{k}}{\lambda_D}$ and $C_{DL} = \frac{\varepsilon \delta x}{\sqrt{k}\lambda_D}$.

Electric Field

Now, there are three parts of impedance: C_{DL}, C_{DS}, and $R(x)$ in total, and the total difference of electric potential is $\Psi = \Psi_0 \exp i\omega t$. Based on two equations:

$$\Psi = I \cdot R_{total} = I(R + \frac{1}{i\omega C_{DL}} + \frac{1}{i\omega C_{DS}})$$

$$\Psi_{DL}(x) = I \cdot \frac{1}{i\omega C_{DL}}$$

We have

$$\Psi_{DL}(x) = \Psi - IR - \frac{I}{i\omega C_{DS}} = \frac{\Psi}{1+k} \frac{1}{1 + \frac{i\omega\varepsilon\pi x}{2\lambda_D \sigma}}$$

Electric Field

Then, by taking the derivative of $\Psi_{DL}(x)$, we have

$$E_{HL} = \frac{1}{\sqrt{k}}\frac{\mathrm{d}\Psi_{DL}}{\mathrm{d}x} = -\frac{\Psi}{\sqrt{k}(1+k)}\frac{\frac{i\omega\varepsilon\pi}{2\lambda_D\sigma}}{(1+\frac{i\omega\varepsilon\pi x}{2\lambda_D\sigma})^2}$$

Velocity

Based on the principle of the fluid dynamics, we have the velocity of the ions and the fluid v_{DL} is

$$v_{DL} = \frac{\lambda_D \rho_{DL} E_{HL}}{\eta}$$

where

$$\rho_{DL} = \frac{\Psi_{DL} C_{DL}}{\delta x \sqrt{k}} = \frac{\Psi_{DL} \varepsilon}{\lambda_D}$$

Velocity

In[∘]:= $(*V_{L0}=\frac{\epsilon}{2\eta\sqrt{k}\,(1+k)^2};*)$

$(*\omega_0=\frac{2\lambda_D*\sigma}{\epsilon*Pi};*)$

$$V_{DL} = -\frac{V_{L0}}{x} * \Psi_0^2 * \frac{\left(\frac{\omega*x}{\omega_0}\right)^2}{\left(1+\left(\frac{\omega*x}{\omega_0}\right)^2\right)^2}$$

Out[∘]= $-\frac{x\,\omega^2\,V_{L0}\,\Psi_0^2}{\left(1+\frac{x^2\omega^2}{\omega_0^2}\right)^2\omega_0^2}$

In[∘]:= $V_{ave} = \frac{\int_{x_{min}}^{x_{max}} V_{DL}\,dx}{x_{max}-x_{min}}$

Out[∘]= $\frac{\epsilon\,\omega^2\,\Psi_0^2\,\omega_0^2\left(-\frac{1}{2\omega^4 x_{max}^2+2\omega^2\omega_0^2}+\frac{1}{2\omega^4 x_{min}^2+2\omega^2\omega_0^2}\right)}{2\sqrt{k}\,(1+k)^2\,\eta\,(x_{max}-x_{min})}$ if condition

In[∘]:= $V_{ave} = \frac{\int_{x_{min}}^{x_{max}} V_{DL}\,dx}{x_{max}-x_{min}}$

Out[∘]= $-\frac{\omega^2\,V_{L0}\,\Psi_0^2\,\omega_0^2\left(-\frac{1}{2\omega^4 x_{max}^2+2\omega^2\omega_0^2}+\frac{1}{2\omega^4 x_{min}^2+2\omega^2\omega_0^2}\right)}{x_{max}-x_{min}}$ if condition

Inspired by the method we calculate the average power $P = UI*$, we can take the time average of the velocity by

$$< v_{DL} >= \frac{1}{2}\mathrm{Re}\{\frac{\lambda_D \rho_{DL} E_{HL}^*}{\eta}\}$$

Velocity

In[∘]:= Refine [精化]

$$v_{ave} = \frac{\int_{x_{min}}^{x_{max}} v_{DL}\, dx}{x_{max} - x_{min}},$$

```
{Element[ω0, Reals], Element[ω, Reals], Element[xmax, Reals],
 属于      实数域        属于     实数域        属于        实数域
 Element[xmin, Reals], xmax > xmin, Element[x, Reals], ω0 > 0, ω > 0}]
 属于        实数域                   属于     实数域
```

Out[∘]=

$$-\frac{\omega^2 v_{L0} \Psi_0^2 \omega_0^2 \left(-\frac{1}{2\omega^4 x_{max}^2 + 2\omega^2 \omega_0^2} + \frac{1}{2\omega^4 x_{min}^2 + 2\omega^2 \omega_0^2}\right)}{x_{max} - x_{min}}$$

In[∘]:=

$\lambda_D = 30 * 10^{-9};$

$\sigma = 0.00123;$

$k = 6.12;$

$v_{L0} = 2.82 * 10^{-9};$

$\omega_0 = 0.03318;$

$x_{min} = 1.6 * 10^{-6};$

$x_{max} = 12 * 10^{-6};$

$\Psi_0 = 0.8;$

$$\omega = \frac{\omega_0}{\sqrt{x_{min} * x_{max}}}$$

v_{ave}

Out[∘]= 7572.26

Out[∘]= −0.0000663529

Unresolved Problem

According to previous calculation of total impedance,

$$Z = \frac{\pi\left(\sqrt{k} + \frac{1}{\sqrt{k}}\right)}{2\sigma \ln \frac{x_{max}}{x_{min}}} + \frac{\varepsilon\left(x_{max} - x_{min}\right)}{\lambda_D\left(\sqrt{k} + \frac{1}{\sqrt{k}}\right)}$$

However, the article gives a different result that

$$Z = \frac{\pi\left(\sqrt{k} + \frac{1}{\sqrt{k}}\right)}{2l\sigma} \frac{\ln A - i\theta}{(\ln A)^2 + \theta^2}$$

where

$$A = \frac{\sqrt{\left[\left(2\lambda_D\sigma\right)^2 + (\omega\varepsilon\pi)^2 + x_{min}x_{max}\right]^2 + \left[2\lambda_D\sigma\omega\varepsilon\pi\left(x_{max} - x_{min}\right)\right]^2}}{\left(2\lambda_D\sigma\right)^2 + \left(\omega\varepsilon\pi x_{min}\right)^2}$$

Velocity

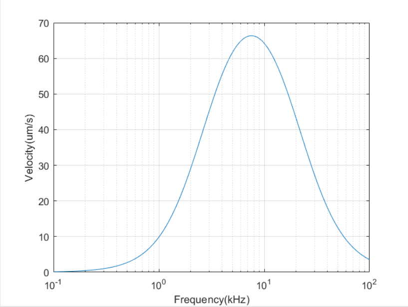
velocity.m

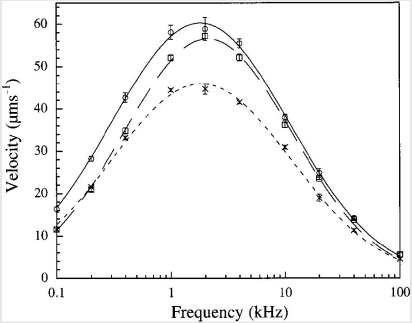
```
psi = 0.8; %potential between two electrodes
vL0 = 2.82 * 10^(-9); %velocity of the large electrode
w0 = 0.03318; % ω0
xmin = 1.6 * 10^(-6);
xmax = 12 * 10^(-6);
w = 0.1*1000:1:100*1000; % x
a = sqrt(xmin.*xmax)/w0;
c = xmax/xmin;
d = xmin/xmax;
Vave = ((psi.^2 .* vL0)./(2.*(xmax-xmin))).*(((w.*a).^2.*(c-d))./(((w.*a).^2 + d).*((w.*a).^2+ c))); %y
semilogx(w./1000,Vave./(10^(-6)));

grid on;
xlabel('Frequency(kHz)');
ylabel('Velocity(um/s)');

```

Velocity


Velocity(um/s)
Frequency(kHz)
70
60
50
40
30
20
10
0
10^{-1}
10^{0}
10^{1}
10^{2}


Velocity (μms^{-1})
Frequency (kHz)
60
50
40
30
20
10
0
0.1
1
10
100

Unresolved Problem

and

$$\theta = \arctan \frac{2\lambda_D \sigma \omega \varepsilon \pi \left(x_{max} - x_{min}\right)}{\left(2\lambda_D \sigma\right)^2 + (\omega \varepsilon \pi)^2 x_{min} x_{max}}$$

Unresolved Problem

```
In[•]:= λD = 4.56*10^-9;
        σ = 0.018;
        k = 6.12;
        vL0 = 2.82*10^-9;
        xmin = 1.6*10^-6;
        xmax = 12*10^-6;
        Φ0 = 0.1;
        ω = 2 Pi*1000;
             圆周率
        l = 0.235;
        ϵ = 80;
        Z
        θ
        A
        Zo

Out[•]= 63519.5

Out[•]= 5.63092×10^-11

Out[•]= 3.90625×10^11

Out[•]= 40.0423 - 8.4476×10^-11 i
```

However, this does not match its figures.