Member *ed*:

$$[k]: \quad 200 \begin{array}{c} \begin{array}{cccccc} u_e & v_e & \theta_{ze} & u_d & v_d & \theta_{zd} \end{array} \\ \left[\begin{array}{cccccc} 0.0048 & 0 & -12.00 & -0.0048 & 0 & -12.00 \\ & 0.800 & 0 & 0 & -0.800 & 0 \\ & & 0.4 \times 10^5 & 12.00 & 0 & 0.2 \times 10^5 \\ & & & 0.0048 & 0 & 12.00 \\ & \text{Sym.} & & & 0.800 & 0 \\ & & & & & 0.4 \times 10^5 \end{array}\right] \end{array}$$

Assembled global stiffness equation:

$$\lfloor P_{xa} \; P_{ya} \; P_{mza} \;|\; P_{xb} \; P_{yb} \; P_{mzb} \;|\; P_{xc} \; P_{yc} \; P_{mzc} \;|\; P_{xd} \; P_{yd} \; P_{mzd} \;|\; P_{xe} \; P_{ye} \; P_{mze} \rfloor^T =$$

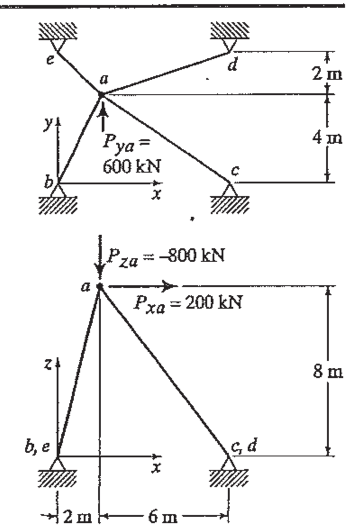
$$200 \begin{array}{c} \begin{array}{ccc|ccc|ccc|ccc|ccc} u_a & v_a & \theta_{za} & u_b & v_b & \theta_{zb} & u_c & v_c & \theta_{zc}\text{ft} & u_d & v_d & \theta_{zd} & u_e & v_e & \theta_{ze} \end{array} \\ \left[\begin{array}{ccc|ccc|ccc|ccc|ccc} 0.0048 & 0 & -12.00 & -0.0048 & 0 & -12.00 & & & & & & & & & \\ & 0.800 & 0 & 0 & -0.800 & 0 & & 0 & & & 0 & & & 0 & \\ & & 0.4 \times 10^5 & 12.00 & 0 & 0.2 \times 10^5 & & & & & & & & & \\ \hline & & & 0.6500 & 0.2591 & 4.969 & -0.6452 & -0.2591 & -7.0313 & & & & & & \\ & & & & 0.9095 & 17.381 & -0.2591 & -0.1095 & 17.381 & & 0 & & & 0 & \\ & & & & & 1.4 \times 10^5 & 7.0313 & -17.381 & 0.5 \times 10^5 & & & & & & \\ \hline & & & & & & 1.1589 & -0.1226 & 14.231 & -0.5137 & 0.3817 & 7.200 & & & \\ & & & & \text{Sym} & & & 0.4006 & -7.781 & 0.3817 & -0.2911 & 9.600 & & 0 & \\ & & & & & & & & 1.4 \times 10^5 & -7.200 & -9.600 & 0.2 \times 10^5 & & & \\ \hline & & & & & & & & & 0.5185 & -0.3817 & 4.80 & -0.0048 & 0 & 12.00 \\ & & & & & & & & & & 1.0911 & -9.60 & 0 & -0.800 & 0 \\ & & & & & & & & & & & 0.8 \times 10^5 & -12.00 & 0 & 0.2 \times 10^5 \\ \hline & & & & & & & & & & & & 0.00480 & 0 & -12.00 \\ & & & & & & & & & & & & & 0.800 & 0 \\ & & & & & & & & & & & & & & 0.4 \times 10^5 \end{array}\right] \end{array} \begin{Bmatrix} u_a \\ v_a \\ \theta_{za} \\ \hline u_b \\ v_b \\ \theta_{zb} \\ \hline u_c \\ v_c \\ \theta_{zc} \\ \hline u_d \\ v_d \\ \theta_{zd} \\ \hline u_e \\ v_e \\ \theta_{ze} \end{Bmatrix}$$

EXAMPLE 5.4

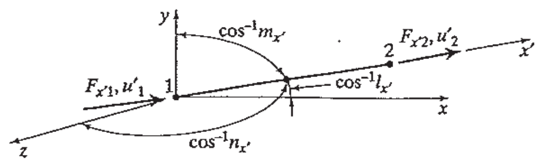
A pin-jointed space truss is supported and loaded as shown. $E = 200{,}000$ MPa. Bar areas are:

$$A_{ab} = 20 \times 10^3 \text{ mm}^2$$
$$A_{ac} = 30 \times 10^3 \text{ mm}^2$$
$$A_{ad} = 40 \times 10^3 \text{ mm}^2$$
$$A_{ae} = 30 \times 10^3 \text{ mm}^2$$

1. Calculate the displacement at a.
2. Calculate the reactions.



Develop the three-dimensional stiffness matrix for an axial force member. Stating Equation 4.34 in the local coordinates shown and eliminating all irrelevant degrees of freedom,



$$\begin{Bmatrix} F_{x'1} \\ F_{x'2} \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = [\mathrm{k}']\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix}$$

In Equation 5.7 delete all columns except those corresponding to F_{x1}, F_{y1}, F_{z1}, F_{x2}, F_{y2}, and F_{z2} and all rows except those corresponding to $F_{x'1}$ and $F_{x'2}$ (compare Example 5.1) yielding

$$[\Gamma] = \begin{bmatrix} l_{x'} & m_{x'} & n_{x'} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{x'} & m_{x'} & n_{x'} \end{bmatrix}$$

Using Equation 5.16, $[\mathrm{k}] = [\Gamma]^T[\mathrm{k}'][\Gamma]$,

$$[\mathrm{k}] = \frac{EA}{L}\begin{array}{c} \begin{array}{cccccc} u_1 & v_1 & w_1 & u_2 & v_2 & w_2 \end{array} \\ \left[\begin{array}{ccc|ccc} l_{x'}^2 & l_{x'}m_{x'} & l_{x'}n_{x'} & -l_{x'}^2 & -l_{x'}m_{x'} & -l_{x'}n_{x'} \\ & m_{x'}^2 & m_{x'}n_{x'} & -l_{x'}m_{x'} & -m_{x'}^2 & -m_{x'}n_{x'} \\ & & n_{x'}^2 & -l_{x'}n_{x'} & -m_{x'}n_{x'} & -n_{x'}^2 \\ \hline & & & l_{x'}^2 & l_{x'}m_{x'} & l_{x'}n_{x'} \\ & \text{Sym} & & & m_{x'}^2 & m_{x'}n_{x'} \\ & & & & & n_{x'}^2 \end{array}\right] \end{array}$$

1. ***Displacements.*** Record direction cosines for each member. Locate local origin for each member at a.

Member	x	y	z	L	$l_{x'}$	$m_{x'}$	$n_{x'}$
ab	−2	−4	−8	9.165	−0.2182	−0.4364	−0.8729
ac	6	−4	−8	10.770	0.5571	−0.3714	−0.7428
ad	6	2	−8	10.198	0.5883	0.1961	−0.7845
ae	−2	2	−8	8.485	−0.2357	0.2357	−0.9428

Record direction cosine products multiplied by A/L. (Values of A/L are in mm.)

Member	(A/L)	×	$l_{x'}^2$	$m_{x'}^2$	$n_{x'}^2$	$l_{x'}m_{x'}$	$l_{x'}n_{x'}$	$m_{x'}n_{x'}$
ab	2.182		0.1034	0.4156	1.663	0.2078	0.4156	0.8312
ac	2.785		0.8643	0.3841	1.537	−0.5762	−1.1525	0.7683
ad	3.922		1.3574	0.1508	2.414	0.4524	−1.8101	−0.6034
ae	3.536		0.1964	0.1964	3.143	−0.1964	0.7858	−0.7858
	Σ		2.5215	1.1469	8.757	−0.1124	−1.7612	0.2103

Referring to the [k] matrix and adding stiffnesses in the usual way, the global stiffness equations for the nonzero degrees of freedom are

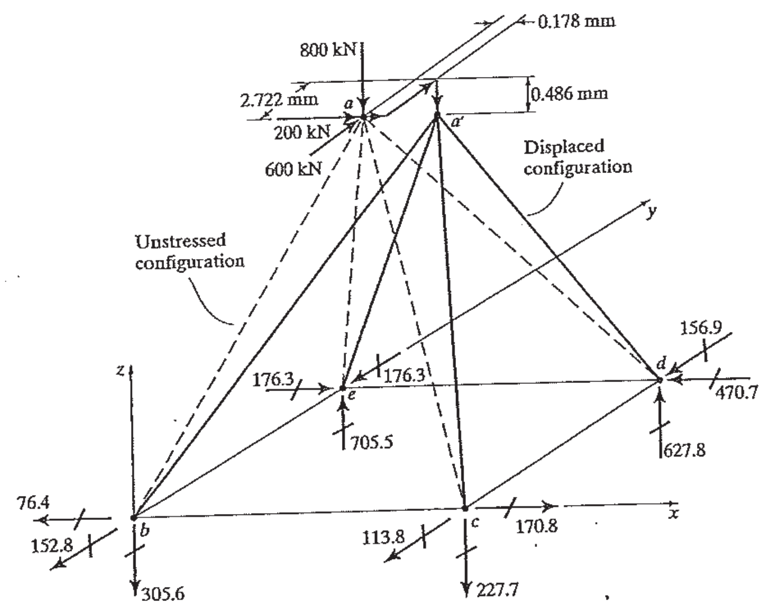
$$\begin{Bmatrix} P_{xa} \\ P_{ya} \\ P_{za} \end{Bmatrix} = \begin{Bmatrix} 200 \\ 600 \\ -800 \end{Bmatrix} = 200\begin{bmatrix} 2.522 & -0.1124 & -1.7612 \\ & 1.147 & 0.2103 \\ \text{Sym.} & & 8.757 \end{bmatrix}\begin{Bmatrix} u_a \\ v_a \\ w_a \end{Bmatrix}$$

Solving for the displacements,

$$\lfloor \Delta \rfloor = \lfloor u_a \quad v_a \quad w_a \rfloor = \lfloor 0.1783 \quad 2.722 \quad -0.4863 \rfloor \cdot \text{mm}$$

2. ***Reactions.*** Using the properties of the element stiffness matrices (see above table), the global components of the reactions are

$$
\begin{Bmatrix} R_{xb} \\ R_{yb} \\ R_{zb} \\ R_{xc} \\ R_{yc} \\ R_{zc} \\ R_{xd} \\ R_{yd} \\ R_{zd} \\ R_{xe} \\ R_{ye} \\ R_{ze} \end{Bmatrix} = -200
\begin{array}{c}
\begin{array}{ccc} \quad u_a & \quad v_a & \quad w_a \end{array} \\
\begin{bmatrix}
0.1034 & 0.2078 & 0.4156 \\
0.2078 & 0.4156 & 0.8312 \\
0.4156 & 0.8312 & 1.663 \\
\hline
0.8643 & -0.5762 & -1.1525 \\
-0.5762 & 0.3841 & 0.7683 \\
-1.1525 & 0.7683 & 1.537 \\
\hline
1.3574 & 0.4524 & -1.8101 \\
0.4524 & 0.1508 & -0.6034 \\
-1.810 & -0.6034 & 2.414 \\
\hline
0.1964 & -0.1964 & 0.7858 \\
-0.1964 & 0.1964 & -0.7858 \\
0.7858 & -0.7858 & 3.143
\end{bmatrix}
\end{array}
\begin{Bmatrix} 0.1783 \\ 2.722 \\ -0.4863 \end{Bmatrix} =
\begin{Bmatrix} -76.4 \\ -152.8 \\ -305.6 \\ \hline 170.8 \\ -113.8 \\ -227.7 \\ \hline -470.7 \\ -156.9 \\ 627.8 \\ \hline 176.3 \\ -176.3 \\ 705.5 \end{Bmatrix} \text{kN}
$$



EXAMPLE 5.5

Find the direction cosines of the member shown. x and z are horizontal; y is vertical. The x' axis is along the member. The $x'y'$ plane makes a dihedral angle of 30° with a vertical plane through the member axis. Demonstrate that the matrix of direction cosines $[\gamma]$ can be generated by three successive rotations.