placements in cases in which internal forces have already been calculated. The requirement of compatibility of displacements is satisfied through the simultaneous solution of two equations. Note that superscripts are used for defining unambiguously the element on which the force component acts.

Example 2.2 is an elementary application of the displacement method of analysis for forces and displacements.

In Example 2.3 stiffness equations are used to determine directly the force needed to obtain a desired displacement.

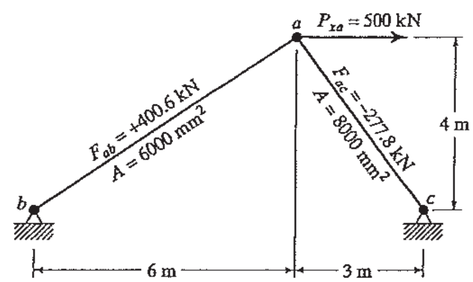
Example 2.4 is an elementary application of flexibility equations to the solution of a thermal loading problem.

Example 2.5 illustrates that structural equations may sometimes be used advantageously in the solution of small displacement kinematic problems.

In Example 2.6 some of the many different ways in which element stiffness equations are combined to form force-displacement relationships are illustrated.

EXAMPLE 2.1

A statically determinate truss is subjected to the load and resulting bar forces shown. What is the displacement of a? $E = 200{,}000$ MPa.



Consider ab:

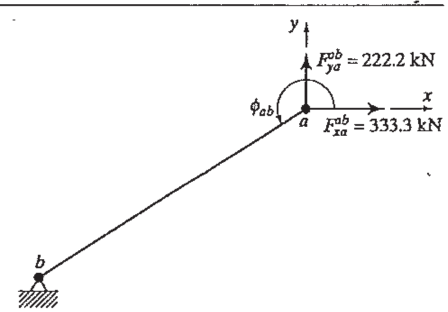
$$\left(\frac{EA}{L}\right)_{ab} = \frac{200 \times 6 \times 10^3}{\sqrt{6^2 + 4^2} \times 10^3} = 166.4 \text{ kN/mm}$$

$$\phi_{ab} = \tan^{-1}\left(\frac{-4}{-6}\right) = 213.69^\circ$$

From the first part of Equation 2.5 (with $u_b = v_b = 0$),

$$F_{xa}^{ab} = 166.4(\cos^2 \phi_{ab} \cdot u_a + \sin \phi_{ab} \cos \phi_{ab} \cdot v_a)$$

$$333.3 = 166.4(0.6923u_a + 0.4615v_a) \qquad \text{(a)}$$



Consider ac:

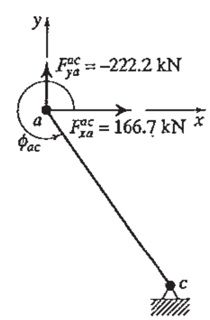
$$\left(\frac{EA}{L}\right)_{ac} = \frac{200 \times 8 \times 10^3}{5 \times 10^3} = 320.0 \text{ kN/mm}$$

$$\phi_{ac} = \tan^{-1}\left(-\frac{4}{3}\right) = 306.87^\circ$$

From the first part of Equation 2.5 (with $u_c = v_c = 0$),

$$F_{xa}^{ac} = 320.0(\cos^2 \phi_{ac} \cdot u_a + \sin \phi_{ac} \cos \phi_{ac} \cdot v_a)$$

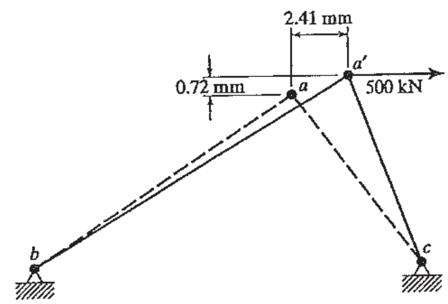
$$166.7 = 320.0(0.3600u_a - 0.4800v_a) \qquad \text{(b)}$$



Solve Equations a and b simultaneously:

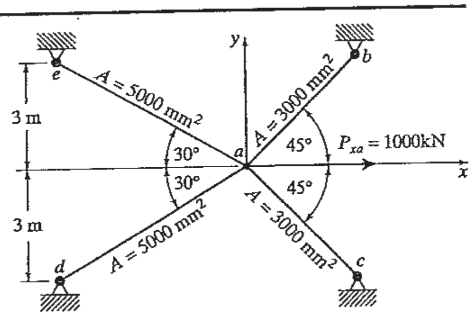
$$\begin{aligned}
0.6923u_a + 0.4615v_a &= 2.003 \quad \text{(a)}\\
0.3600u_a - 0.4800v_a &= 0.5209 \quad \text{(b)}
\end{aligned}$$

$$\begin{aligned}
u_a &= 2.41 \text{ mm} \rightarrow\\
v_a &= 0.72 \text{ mm} \uparrow\\
\overline{aa'} &= 2.52 \text{ mm} \nearrow
\end{aligned}$$



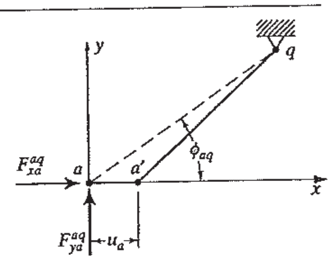
EXAMPLE 2.2

A statically indeterminate truss is subjected to the load shown. What are the bar forces and the displacement of a? $E = 200{,}000$ MPa.



By symmetry, joint a must displace horizontally. Therefore, there is only one unknown degree of freedom, u_a. From the first part of Equation 2.5, using the nomenclature indicated, and denoting q as the typical support point

$$F_{xa}^{aq} = \left(\frac{EA}{L}\right)_{aq} \cdot (\cos\phi_{aq})^2 u_a$$



thus, for $q = b$

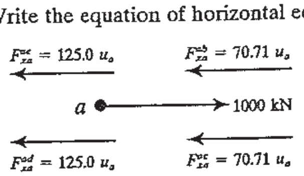
$$F_{xa}^{ab} = \frac{200 \times 3 \times 10^3(\cos 45°)^2}{3\sqrt{2} \times 10^3}u_a = 70.71u_a \text{ kN}$$

$$F_{xa}^{ac} = \frac{200 \times 3 \times 10^3(\cos 315°)^2}{3\sqrt{2} \times 10^3}u_a = 70.71u_a \text{ kN}$$

$$F_{xa}^{ad} = \frac{200 \times 5 \times 10^3(\cos 210°)^2}{6 \times 10^3}u_a = 125.00u_a \text{ kN}$$

$$F_{xa}^{ae} = \frac{200 \times 5 \times 10^3(\cos 150°)^2}{6 \times 10^3}u_a = 125.00u_a \text{ kN}$$

Write the equation of horizontal equilibrium of joint a with u_a as an unknown:



$$\Sigma H_a = 1000 - 2(70.71 + 125.0)u_a = 0$$

Solving for u_a,

$$u_a = 2.55 \text{ mm}$$