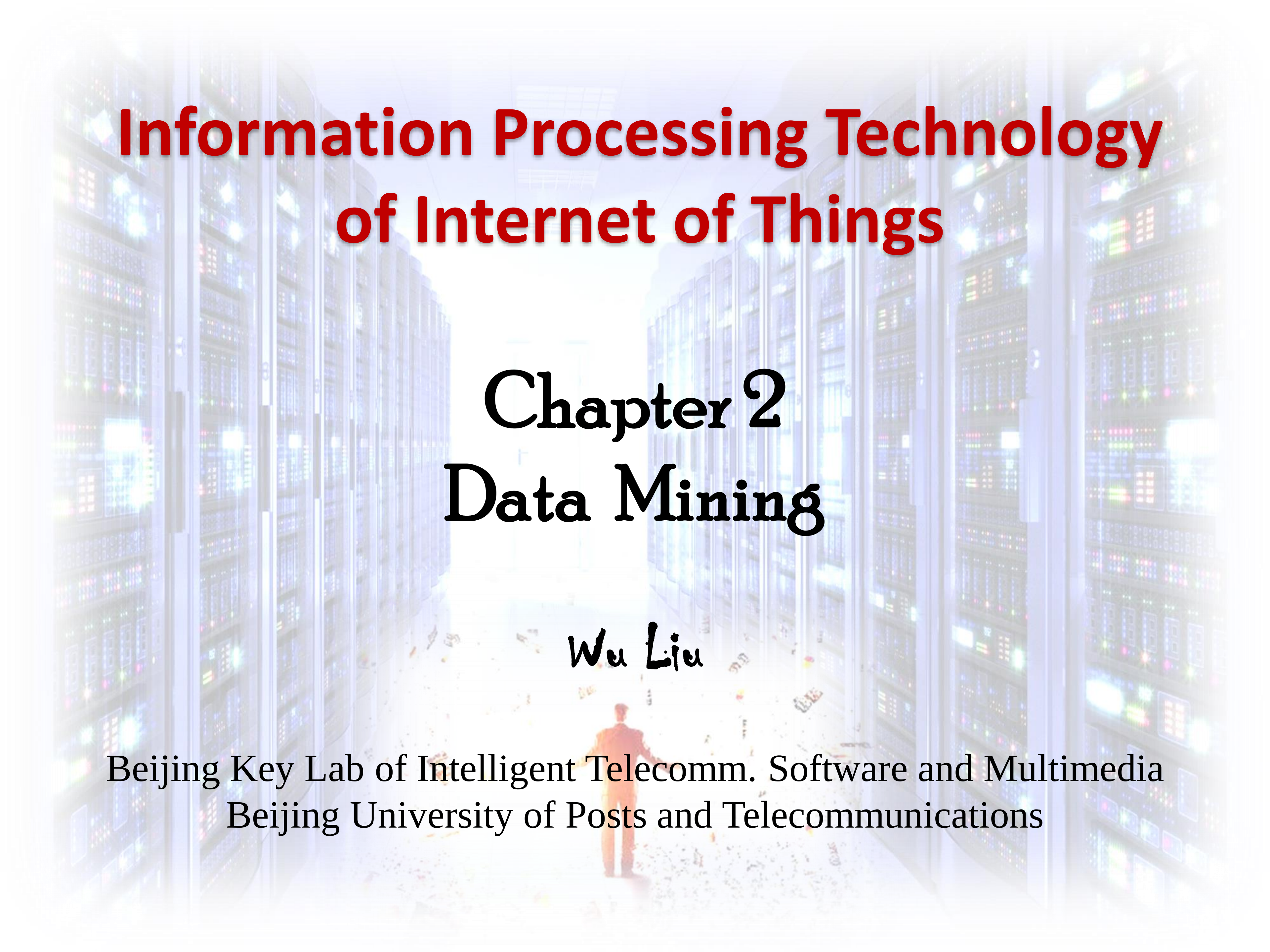

Information Processing Technology of Internet of Things

Chapter 2
Data Mining

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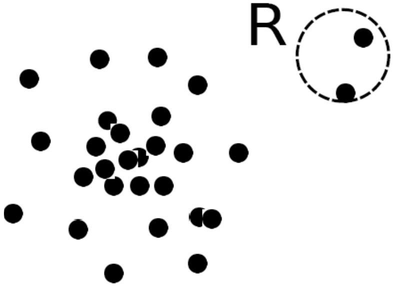
2.4 Outlier Analysis

Outline

- Outlier and Outlier Analysis
- Outlier Detection Methods
- Statistical Approaches
- Proximity-Base Approaches
- Clustering-Base Approaches
- Classification Approaches
- Mining Contextual and Collective Outliers
- Outlier Detection in High Dimensional Data

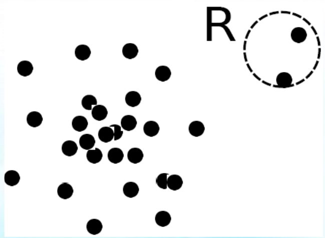
What Are Outliers

- **Outlier**: A data object that **deviates significantly** from the normal objects as if it were **generated by a different mechanism**
  - Ex.: Unusual credit card purchase, sports: Michael Jordon,
- Outliers are different from the noise data
  - Noise is random error or variance in a measured variable
  - Noise should be removed before outlier detection
- Outliers are interesting: It violates the mechanism that generates the normal data
- Outlier detection vs. *novelty detection*: early stage, outlier; but later merged into the model
- Applications:
  - Credit card fraud detection
  - Telecom fraud detection
  - Customer segmentation
  - Medical analysis



The objects in region *R are outliers.*

Types of Outliers (I)

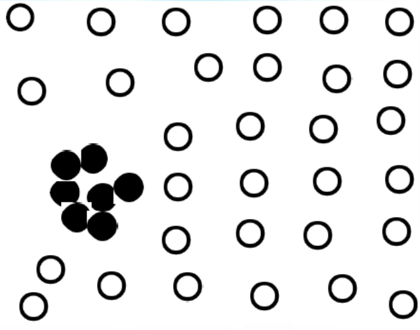


Global Outlier

- Three kinds: *global, contextual* and *collective* outliers
- **Global outlier** (or point anomaly)
  - Object is O_g if it significantly deviates from the rest of the data set
  - Ex. Intrusion detection in computer networks
  - Issue: Find an appropriate measurement of deviation
- **Contextual outlier** (or *conditional outlier*)
  - Object is O_c if it deviates significantly based on a selected context
  - Ex. 80^o F in Urbana: outlier? (depending on summer or winter?)
  - Attributes of data objects should be divided into two groups
    - Contextual attributes: defines the context, e.g., time & location
    - Behavioral attributes: characteristics of the object, used in outlier evaluation, e.g., temperature
  - Can be viewed as a generalization of *local outliers*—whose density significantly deviates from its local area
  - Issue: How to define or formulate meaningful context?

Types of Outliers (II)

- **Collective Outliers**
  - A subset of data objects collectively deviate significantly from the whole data set, even if the individual data objects may not be outliers
  - Applications: E.g., intrusion detection:
    - When a number of computers keep sending denial-of-service packages to each other
  - Detection of collective outliers
    - Consider not only behavior of individual objects, but also that of groups of objects
    - Need to have the background knowledge on the relationship among data objects, such as a distance or similarity measure on objects.
- A data set may have multiple types of outlier
- One object may belong to more than one type of outlier



Collective Outlier

Challenges of Outlier Detection

- Modeling normal objects and outliers properly
  - Hard to enumerate all possible normal behaviors in an application
  - The border between normal and outlier objects is often a gray area
- Application-specific outlier detection
  - Choice of distance measure among objects and the model of relationship among objects are often application-dependent
  - E.g., clinic data: a small deviation could be an outlier; while in marketing analysis, larger fluctuations
- Handling noise in outlier detection
  - Noise may distort the normal objects and blur the distinction between normal objects and outliers. It may help hide outliers and reduce the effectiveness of outlier detection
- Understandability
  - Understand why these are outliers: Justification of the detection
  - Specify the degree of an outlier: the unlikelihood of the object being generated by a normal mechanism

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Outlier Detection I: Supervised Methods

- Two ways to categorize outlier detection methods:
  - Based on whether user-*labeled* examples of outliers can be obtained:
    - Supervised, semi-supervised vs. unsupervised methods
  - Based on *assumptions about normal data and outliers*:
    - Statistical, proximity-based, and clustering-based methods
- **Outlier Detection I: Supervised Methods**
  - Modeling outlier detection as a classification problem
    - Samples examined by domain experts used for training & testing
  - Methods for Learning a classifier for outlier detection effectively:
    - *Model normal objects* & report those not matching the model as outliers, or
    - *Model outliers* and treat those not matching the model as normal
  - Challenges
    - Imbalanced classes, i.e., outliers are rare: Boost the outlier class and make up some artificial outliers
    - Catch as many outliers as possible, i.e., recall is more important than accuracy (i.e., not mislabeling normal objects as outliers)

Outlier Detection II: Unsupervised Methods

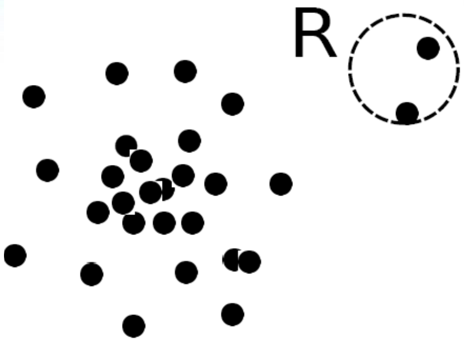
- Assume the normal objects are somewhat “clustered” into multiple groups, each having some distinct features
- An outlier is expected to be far away from any groups of normal objects
- Weakness: Cannot detect collective outlier effectively
  - Normal objects may not share any strong patterns, but the collective outliers may share high similarity in a small area
- Ex. In some intrusion or virus detection, normal activities are diverse
  - Unsupervised methods may have a high false positive rate but still miss many real outliers.
  - Due to the high similarity between outliers, supervised methods can be more effective, e.g., identify attacking some key resources
- Many clustering methods can be adapted for unsupervised methods
  - Find clusters, then outliers: not belonging to any cluster
  - Problem 1: Hard to distinguish noise from outliers
  - Problem 2: Costly since first clustering: but far less outliers than normal objects
    - Newer methods: tackle outliers directly

Outlier Detection III: Semi-Supervised Methods

- Situation: In many applications, the number of labeled data is often small: Labels could be on outliers only, normal objects only, or both
- Semi-supervised outlier detection: Regarded as applications of semi-supervised learning
- If some labeled normal objects are available
  - Use the labeled examples and the proximate unlabeled objects to train a model for normal objects
  - Those not fitting the model of normal objects are detected as outliers
- If only some labeled outliers are available, a small number of labeled outliers many not cover the possible outliers well
  - To improve the quality of outlier detection, one can get help from models for normal objects learned from unsupervised methods

Outlier Detection (1): Statistical Methods

- Statistical methods (also known as model-based methods) assume that the normal data follow some statistical model (a stochastic model)
  - **The data not following the model are outliers**.



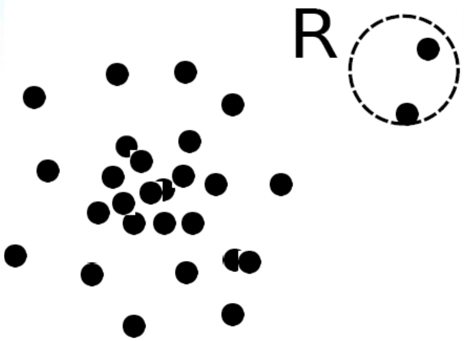
- Example (right figure): First use Gaussian distribution to model the normal data
  - For each object y in region R, estimate $g_D(y)$, the probability of y fits the Gaussian distribution
  - If $g_D(y)$ is very low, y is unlikely generated by the Gaussian model, thus an outlier

- Effectiveness of statistical methods: highly depends on whether the assumption of statistical model holds in the real data

- There are rich alternatives to use various statistical models
  - E.g., parametric vs. non-parametric

Outlier Detection (2): Proximity-Based Methods

- An object is an outlier if the nearest neighbors of the object are far away, i.e., the proximity of the object is significantly deviates from the proximity of most of the other objects in the same data set
- Example (right figure): Model the proximity of an object using its 3 nearest neighbors
  - Objects in region R are substantially different from other objects in the data set.
  - Thus the objects in R are outliers

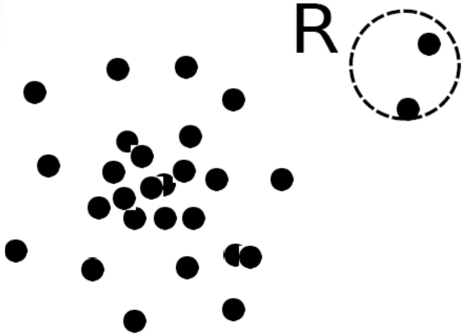


- The effectiveness of proximity-based methods highly relies on the proximity measure.
- In some applications, proximity or distance measures cannot be obtained easily.
- Often have a difficulty in finding a group of outliers which stay close to each other
- Two major types of proximity-based outlier detection
  - Distance-based vs. density-based

Outlier Detection (3): Clustering-Based Methods

- Normal data belong to large and dense clusters, whereas outliers belong to small or sparse clusters, or do not belong to any clusters

- Example (right figure): two clusters
  - All points not in R form a large cluster
  - The two points in R form a tiny cluster, thus are outliers



- Since there are many clustering methods, there are many clustering-based outlier detection methods as well

- Clustering is expensive: straightforward adaption of a clustering method for outlier detection can be costly and does not scale up well for large data sets

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Statistical Approaches

- Statistical approaches assume that the objects in a data set are generated by a stochastic process (a generative model)
- **Idea**: learn a generative model fitting the given data set, and then identify the objects in low probability regions of the model as outliers
- Methods are divided into two categories: *parametric* vs. *non-parametric*
- Parametric method
  - Assumes that the normal data is generated by a parametric distribution with parameter θ
  - The probability density function of the parametric distribution $f(x, \theta)$ gives the probability that object x is generated by the distribution
  - The smaller this value, the more likely x is an outlier
- Non-parametric method
  - Not assume an a-priori statistical model and determine the model from the input data
  - Not completely parameter free but consider the number and nature of the parameters are flexible and not fixed in advance
  - Examples: histogram and kernel density estimation

Parametric Methods I: Detection Univariate Outliers Based on Normal Distribution

- Univariate data: A data set involving only one attribute or variable
- Often assume that data are generated from a normal distribution, learn the parameters from the input data, and identify the points with low probability as outliers
- Ex: Avg. temp.: {24.0, 28.9, 28.9, 29.0, 29.1, 29.1, 29.2, 29.2, 29.3, 29.4}
  - Use the maximum likelihood method to estimate μ and σ

$$\ln \mathcal{L}(\mu, \sigma^2) = \sum_{i=1}^{n} \ln f(x_i|(\mu, \sigma^2)) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2$$

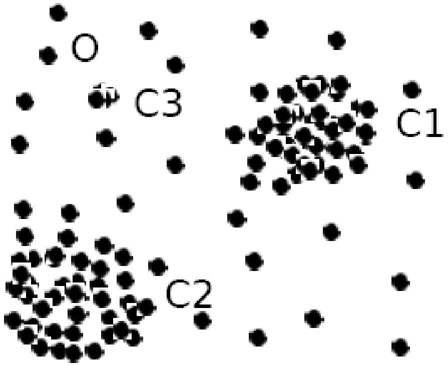
  - Taking derivatives with respect to μ and σ^2, we derive the following maximum likelihood estimates

$$\hat{\mu} = \overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

  - For the above data with n = 10, we have $\hat{\mu} = 28.61$ $\hat{\sigma} = \sqrt{2.29} = 1.51$
  - Then (24 – 28.61) /1.51 = – 3.04 < –3, 24 is an outlier since $\mu \pm 3\sigma$ region contains 99.7% data

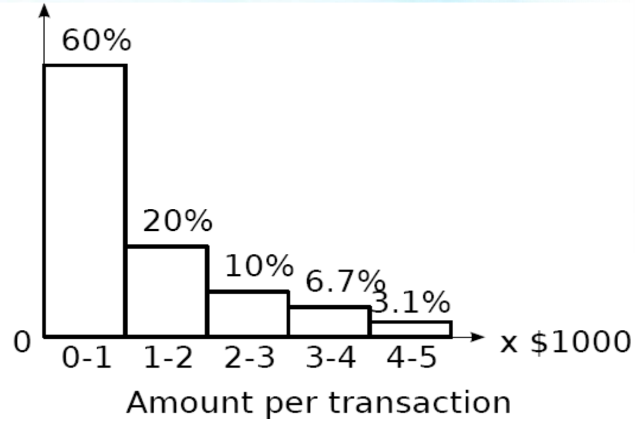
Parametric Methods II:

- Detection of Multivariate Outliers
  - Multivariate data: A data set involving two or more attributes or variables
  - Transform the multivariate outlier detection task into a univariate outlier detection problem
- Using Mixture of Parametric Distributions
  - Assuming data generated by a normal distribution could be sometimes overly simplified
  - Example (right figure): The objects between the two clusters cannot be captured as outliers since they are close to the estimated mean



  - To overcome this problem, assume the normal data is generated by two normal distributions.
  - Then use Expectation-Maximization (EM) algorithm to learn the parameters μ1, σ1, μ2, σ2 from data
  - An object o is an outlier if it does not belong to any cluster

Non-Parametric Methods: Detection Using Histogram



- The model of normal data is learned from the input data without any *a priori* structure.
- Often makes fewer assumptions about the data, and thus can be applicable in more scenarios
- Outlier detection using histogram:
  - Figure shows the histogram of purchase amounts in transactions
  - A transaction in the amount of $7,500 is an outlier, since only 0.2% transactions have an amount higher than $5,000
- Two steps:
  - Histogram construction: equal width or equal depth; number of bins in the histogram or the size of each bin
  - Outlier detection: check if the object falls in one of the histogram's bins, or assign an outlier score to the object
- Problem: Hard to choose an appropriate bin size for histogram
  - Too small bin size → normal objects in empty/rare bins, false positive
  - Too big bin size → outliers in some frequent bins, false negative

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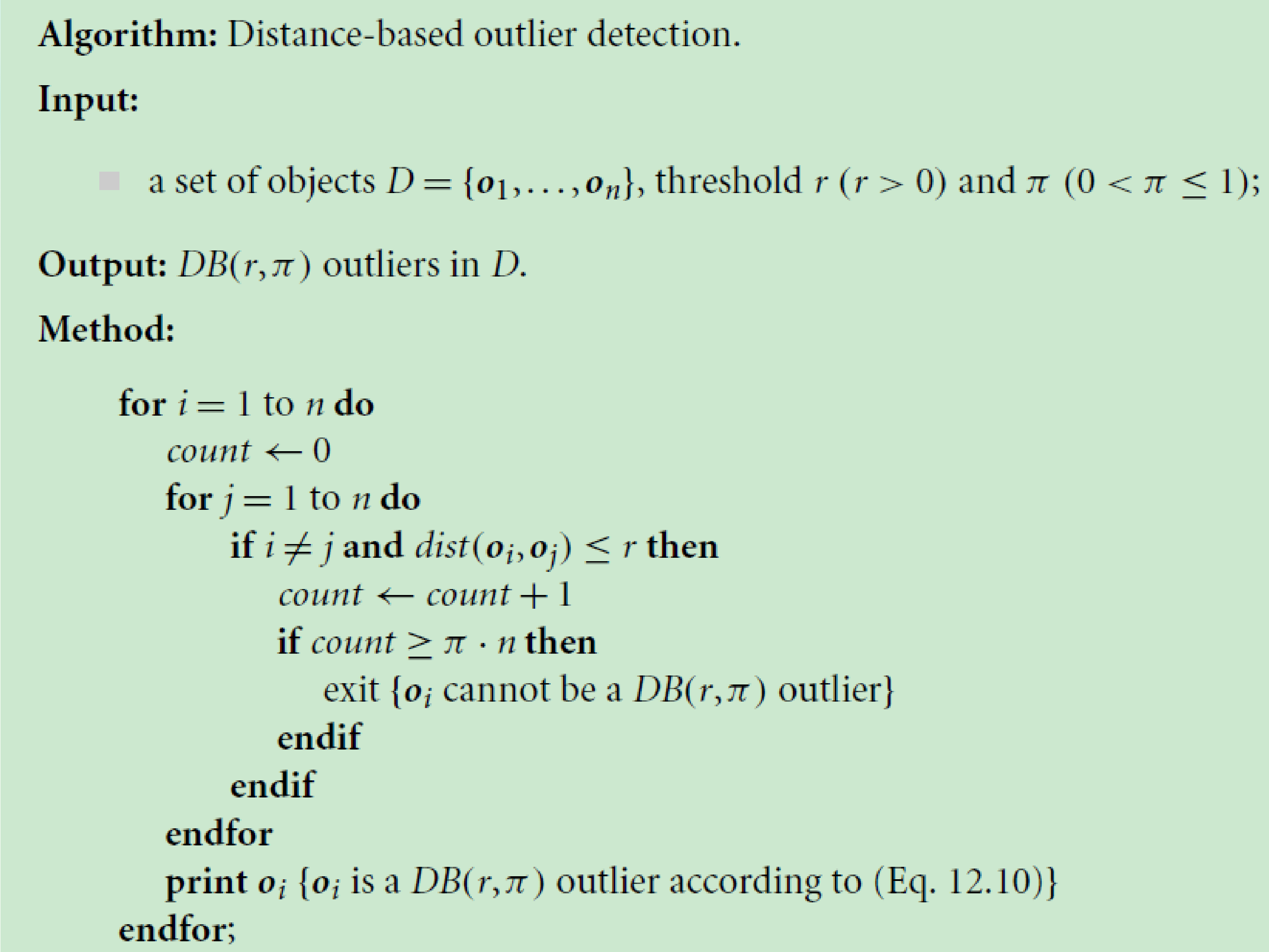
Proximity-Based Approaches: Distance-Based vs. Density-Based Outlier Detection

- **Intuition**: Objects that are far away from the others are outliers
- **Assumption** of proximity-based approach: The proximity of an outlier deviates significantly from that of most of the others in the data set
- Two types of proximity-based outlier detection methods
  - Distance-based outlier detection: An object o is an outlier if its neighborhood does not have enough other points
  - Density-based outlier detection: An object o is an outlier if its density is relatively much lower than that of its neighbors

Distance-Based Outlier Detection

- For each object o, examine the number of other objects in the *r*-neighborhood of o, where *r* is a user-specified **distance threshold**
- An object o is an outlier if most (taking π as a **fraction threshold**) of the objects in D are far away from o, i.e., not in the r-neighborhood of o
- An object o is a DB(r, π) outlier if $\frac{\|\{o' | dist(o, o') \leq r\}\|}{\|D\|} \leq \pi$
- Equivalently, one can check the distance between *o* and its *k*-th nearest neighbor o_k, where $k = \lceil \pi \|D\| \rceil$. *o* is an outlier if dist(*o*, o_k) > r
- Efficient computation: Nested loop algorithm
  - For any object o_i, calculate its distance from other objects, and count the # of other objects in the r-neighborhood.
  - If π·n other objects are within r distance, terminate the inner loop
  - Otherwise, o_i is a DB(r, π) outlier
- Efficiency: Actually *CPU time* is not $O(n^2)$ but *linear* to the data set size since for most non-outlier objects, the inner loop terminates early

Distance-Based Outlier Detection



Algorithm: Distance-based outlier detection.

Input:

- a set of objects $D = \{\mathbf{o}_1, \ldots, \mathbf{o}_n\}$, threshold r ($r > 0$) and π ($0 < \pi \leq 1$);

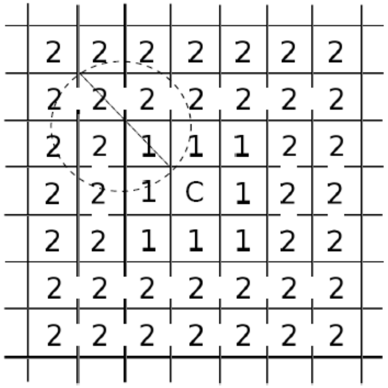
Output: $DB(r, \pi)$ outliers in D.

Method:

```
for $i = 1$ to $n$ do
    $count \leftarrow 0$
    for $j = 1$ to $n$ do
        if $i \neq j$ and $dist(\mathbf{o}_i, \mathbf{o}_j) \leq r$ then
            $count \leftarrow count + 1$
            if $count \geq \pi \cdot n$ then
                exit {$\mathbf{o}_i$ cannot be a $DB(r, \pi)$ outlier}
            endif
        endif
    endfor
    print $\mathbf{o}_i$ {$\mathbf{o}_i$ is a $DB(r, \pi)$ outlier according to (Eq. 12.10)}
endfor;
```

Distance-Based Outlier Detection: A Grid-Based Method (1)

- Why efficiency is still a concern? When the complete set of objects cannot be held into main memory, cost I/O swapping
- The major cost: (1) each object tests against the whole data set, why not only its close neighbor? (2) check objects one by one, why not group by group?
- Grid-based method (CELL): Data space is partitioned into a multi-D grid. Each cell is a hyper cube with diagonal length r/2
- The neighboring cells of C can be divided into two groups.
  - level-1 cells: The cells immediately next to C the
  - level-2 cells: The cells one or two cells away from C in any direction

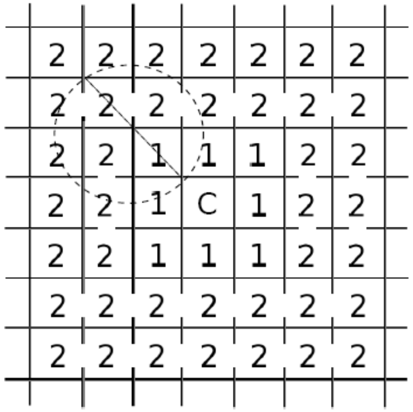


2	2	2	2	2	2	2
2	2	2	2	2	2	2
2	2	1	1	1	2	2
2	2	1	C	1	2	2
2	2	1	1	1	2	2
2	2	2	2	2	2	2
2	2	2	2	2	2	2

- **Level-1 cell property**: Given any possible point, x of C, and any possible point, y, in a level-1 cell, then dist(x,y)<= r.
- **Level-2 cell property**: Given any possible point, x of C, and any point, y, such that dist(x,y)>= r, then y is in a level-2 cell.

Distance-Based Outlier Detection: A Grid-Based Method (2)

- Let a be the number of objects in cell C, b1 be the total number of objects in the level-1 cells, and b2 be the total number of objects in the level-2 cells.
- Pruning using the level-1 & level 2 cell properties:
  - Level-1 cell pruning rule: Based on the level-1 cell property, if a+b1 > $\lceil \pi n \rceil$, then every object o in C is not a DB(r,π)-outlier
  - Level-2 cell pruning rule: Based on the level-2 cell property, if a+b1+b2 < $\lceil \pi n \rceil$ +1, then all objects in C are DB(r,π)-outliers

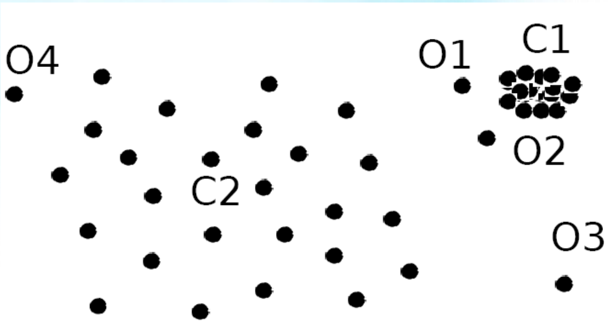


2	2	2	2	2	2	2
2	2	2	2	2	2	2
2	2	1	1	1	2	2
2	2	1	C	1	2	2
2	2	1	1	1	2	2
2	2	2	2	2	2	2
2	2	2	2	2	2	2

- CELL method organizes objects into groups using a grid
  - We can determine that either all objects in a cell are outliers or nonoutliers, and thus do no need to check those objects one by one.
  - We need only check a limited number of cells close to a target cell instead of the whole data set.
- Thus we only need to check the objects that cannot be pruned, and even for such an object o, only need to compute the distance between o and the objects in the level-2 cells (since beyond level-2, the distance from o is more than r)

Density-Based Outlier Detection

- Local outliers: Outliers comparing to their local neighborhoods, instead of the global data distribution
- In Fig., o_1 and o2 are local outliers to C_1, o_3 is a global outlier, but o_4 is not an outlier. However, proximity-based clustering cannot find o_1 and o_2 are outlier (e.g., comparing with O_4).



- **Intuition** (density-based outlier detection): The density around an outlier object is significantly different from the density around its neighbors
- **Method**: Use the relative density of an object against its neighbors as the indicator of the degree of the object being outliers
- k-distance of an object o, distk(o): distance between o and its k-nearest neighbor
- k-distance neighborhood of o, $N_k(o) = \{o'| \ o' \text{ in } D, dist(o, o') \leq distk(o)\}$
  - $N_k(o)$ may contain more than k objects since multiple objects may have identical distance to o

Local Outlier Factor: LOF

- Reachability distance from *o'* to *o*:

$$reachdist_k(o \leftarrow o') = \max\{dist_k(o), dist(o, o')\}$$

where k is a user-specified parameter. k specifies the minimum neighborhood to be examined to determine the local density of an object.

- Local reachability density of *o*:

$$lrd_k(o) = \frac{\|N_k(o)\|}{\sum_{o' \in N_k(o)} reachdist_k(o' \leftarrow o)}$$

- LOF (Local outlier factor) of an object o is the **average of the ratio of local reachability of o and those of o's k-nearest neighbors**

$$LOF_k(o) = \frac{\sum_{o' \in N_k(o)} \frac{lrd_k(o')}{lrd_k(o)}}{\|N_k(o)\|} = \sum_{o' \in N_k(o)} lrd_k(o') \cdot \sum_{o' \in N_k(o)} reachdist_k(o' \leftarrow o)$$

- The lower the local reachability density of o, and the higher the local reachability density of the k-nearest neighbors of o, the higher LOF
- This captures a local outlier whose local density is relatively low comparing to the local densities of its k-nearest neighbors

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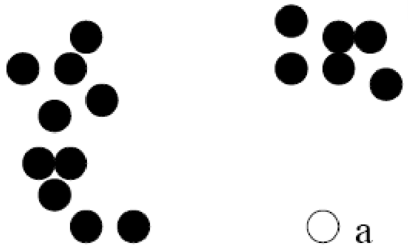
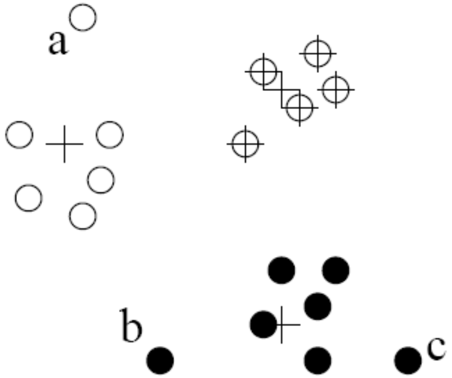
Clustering-Based Outlier Detection (1 & 2): Not belong to any cluster, or far from the closest one

- An object is an outlier if (1) it does not belong to any cluster, (2) there is a large distance between the object and its closest cluster , or (3) it belongs to a small or sparse cluster

- **Case I: Not belong to any cluster**
  - Identify animals not part of a flock: Using a density-based clustering method such as DBSCAN

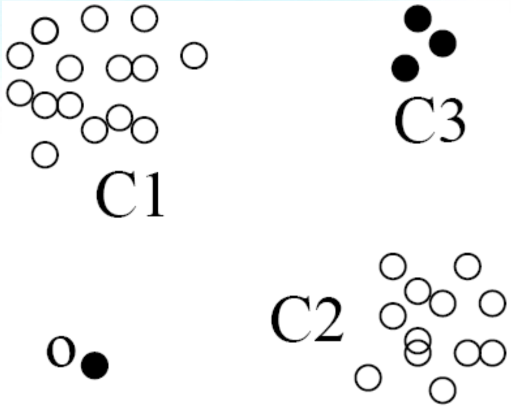


- **Case 2: Far from its closest cluster**
  - Using k-means, partition data points into clusters
  - For each object o, assign an outlier score based on its distance from its closest center
    - If dist(o, c_o)/avg_dist(c_o) is large, likely an outlier



Clustering-Based Outlier Detection (3): Detecting Outliers in Small Clusters

- *FindCBLOF*: Detect outliers in small clusters
  - Find clusters, and sort them in decreasing size
  - To each data point, assign a *cluster-based local outlier factor* (CBLOF):
    - If object p belongs to a large cluster, CBLOF = cluster_size ×similarity between p and cluster
    - If p belongs to a small one, CBLOF = cluster size × similarity between p and the closest large cluster



- CBLOF defines the similarity between a point and a cluster in a statistical way that represents the probability that the point belongs to the cluster. The larger the value, the more similar the point and the cluster are. The points with the lowest CBLOF scores are suspected outliers.
- Ex. In the figure, o is outlier since its closest large cluster is C1, but the similarity between o and C1 is small. For any point in C3, its closest large cluster is C2 but its similarity from C2 is low, plus |C3| = 3 is small

Clustering-Based Method: Strength and Weakness

- Detect outliers without requiring any labeled data
- Clusters can be regarded as summaries of the data
- Once the cluster are obtained, need only compare any object against the clusters to determine whether it is an outlier (fast)
- Effectiveness depends highly on the clustering method used—they may not be optimized for outlier detection
- High computational cost: Need to first find clusters
- A method to reduce the cost: Fixed-width clustering
  - A point is assigned to a cluster if the center of the cluster is within a pre-defined distance threshold from the point
  - If a point cannot be assigned to any existing cluster, a new cluster is created and the distance threshold may be learned from the training data under certain conditions

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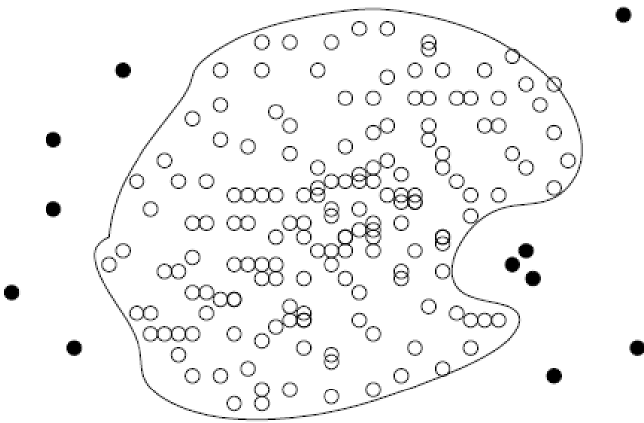
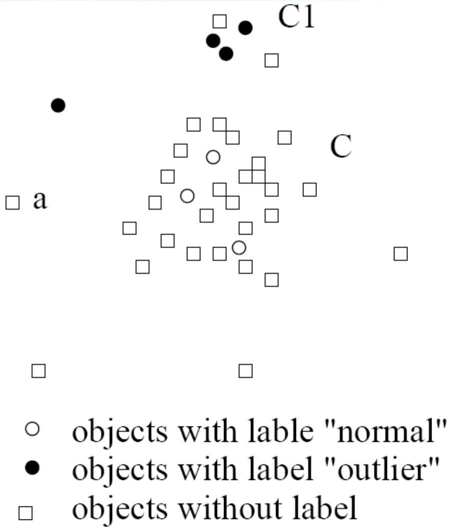
Classification-Based Method I: One-Class Model



- **Idea**: Train a classification model that can distinguish "normal" data from outliers
- A brute-force approach: Consider a training set that contains samples labeled as "normal" and others labeled as "outlier"
    - But, the training set is typically heavily biased: # of "normal" samples likely far exceeds # of outlier samples
    - Cannot detect unseen anomaly
- One-class model: A classifier is built to describe only the normal class.
    - Learn the decision boundary of the normal class using classification methods
    - Any samples that do not belong to the normal class (not within the decision boundary) are declared as outliers
    - Adv: can detect new outliers that may not appear close to any outlier objects in the training set

Classification-Based Method II: Semi-Supervised Learning

- Semi-supervised learning: Combining classification-based and clustering-based methods
- Method
  - Using a clustering-based approach, find a large cluster, C, and a small cluster, C_1
  - Since some objects in C carry the label “normal”, treat all objects in C as normal
  - Use the one-class model of this cluster to identify normal objects in outlier detection
  - Since some objects in cluster C_1 carry the label “outlier”, declare all objects in C_1 as outliers
  - Any object that does not fall into the model for C (such as *a*) is considered an outlier as well



- Comments on classification-based outlier detection methods
  - Strength: Outlier detection is fast
  - Bottleneck: Quality heavily depends on the availability and quality of the training set, but often difficult to obtain representative and high-quality training data

Outline

- Outlier and Outlier Analysis
- Outlier Detection Methods
- Statistical Approaches
- Proximity-Base Approaches
- Clustering-Base Approaches
- Classification Approaches
- Mining Contextual and Collective Outliers
- Outlier Detection in High Dimensional Data

Mining Contextual Outliers I: Transform into Conventional Outlier Detection

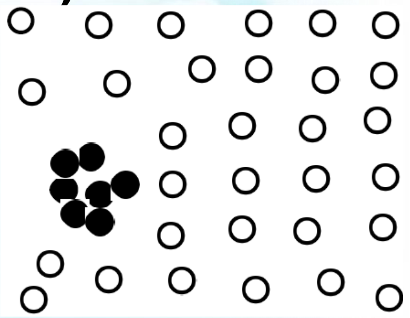
- If the contexts can be clearly identified, transform it to conventional outlier detection
- Steps:
  1. Identify the context of the object using the contextual attributes
  2. Calculate the outlier score for the object in the context using a conventional outlier detection method
- Ex. Detect outlier customers in the context of customer groups
  - **Contextual attributes**: *age group*, *postal code*
  - **Behavioral attributes**: # *of trans/yr*, *annual total trans. amount*

- We can still group customers on age and postal code, and then mine outliers in each group.

Mining Contextual Outliers II: Modeling Normal Behavior with Respect to Contexts

- In some applications, one cannot clearly partition the data into contexts
  - Ex. if a customer suddenly purchased a product that is unrelated to those he recently browsed, it is unclear how many products browsed earlier should be considered as the context
- Model the "normal" behavior with respect to contexts
  - Using a training data set, train a model that predicts the expected behavior attribute values with respect to the contextual attribute values
  - An object is a contextual outlier if its behavior attribute values significantly deviate from the values predicted by the model
- Using a prediction model that links the contexts and behavior, these methods avoid the explicit identification of specific contexts
- Methods: A number of classification and prediction techniques can be used to build such models, such as regression, Markov Models, and Finite State Automaton

Mining Collective Outliers I: On the Set of Structured Objects



- Collective outlier if objects as a group deviate significantly from the entire data
- Need to examine the *structure* of the data set, i.e, the relationships between multiple data objects
- Each of these structures is inherent to its respective type of data
  - For temporal data (such as time series and sequences), we explore the structures formed by time, which occur in segments of the time series or subsequences
  - For spatial data, explore local areas
  - For graph and network data, we explore subgraphs
- Difference from the contextual outlier detection: the structures are often not explicitly defined, and have to be discovered as part of the outlier detection process.
- Collective outlier detection methods: two categories
  - **Reduce the problem to conventional outlier detection**
    - Identify *structure units*, treat each structure unit (e.g., subsequence, time series segment, local area, or subgraph) as a data object, and extract features
    - Then outlier detection on the set of "structured objects" constructed as such using the extracted features

Mining Collective Outliers II: Direct Modeling of the Expected Behavior of Structure Units

- Predefining the structure units for collective outlier detection can be difficult or impossible.
- **Models the expected behavior of structure units directly**
- Ex. Detect collective outliers in temporal sequences
  - Learn a Markov model from the sequences
  - A subsequence can then be declared as a collective outlier if it significantly deviates from the model
- Collective outlier detection remains **a challenging direction** that calls for further research and development.

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Challenges for Outlier Detection in High-Dimensional Data

- Interpretation of outliers
  - Detecting outliers without saying why they are outliers is not very useful in high-D due to many features (or dimensions) are involved in a high-dimensional data set
- Data sparsity
  - Data in high-D spaces are often sparse
  - The distance between objects becomes heavily dominated by noise as the dimensionality increases
- Data subspaces
  - Outlier should be model appropriately in data subspace
  - Adaptive to the subspaces signifying the outliers
  - Capturing the local behavior of data
- Scalable with respect to dimensionality
  - # of subspaces increases exponentially
  - An exhaustive combinatorial exploration of the search space, which contains all possible subspaces, is not a scalable choice.

Approach I: Extending Conventional Outlier Detection

- **Method 1**: Detect outliers in the full space, e.g., HilOut Algorithm
  - Find distance-based outliers, but use the ranks of distance instead of the absolute distance in outlier detection
  - For each object *o*, find its k-nearest neighbors: $\mathrm{nn}_1(\mathrm{o}), \ldots, \mathrm{nn}_k(\mathrm{o})$
  - The weight of object o: $$w(\boldsymbol{o}) = \sum_{i=1}^{k} dist(\boldsymbol{o}, nn_i(\boldsymbol{o}))$$
  - All objects are ranked in weight-descending order
  - Top-*l* objects in weight are output as outliers (*l*: user-specified parm)
- **Method 2**: Dimensionality reduction
  - Works only when in lower-dimensionality, normal instances can still be distinguished from outliers
  - PCA: Heuristically, the principal components with low variance are preferred because, on such dimensions, normal objects are likely close to each other and outliers often deviate from the majority

Approach II: Finding Outliers in Subspaces

- Extending conventional outlier detection: Hard for outlier interpretation
- Find outliers in much lower dimensional subspaces: easy to interpret *why* and *to what extent* the object is an outlier
  - E.g., find outlier customers in certain subspace: *average transaction amount* >> *avg*. and *purchase frequency* << avg.
- Ex. A grid-based subspace outlier detection method
  - Project data onto various subspaces to find an area whose density is much lower than average
  - Discretize the data into a grid with φ equi-depth regions
  - Search for regions that are significantly sparse
    - Consider a k-d cube: k ranges on k dimensions, with n objects
    - If objects are independently distributed, the expected number of objects falling into a k-dimensional region is $(1/ \varphi)^k n = f^k n$,the standard deviation is $\sqrt{f^k(1-f^k)n}$
    - The sparsity coefficient of cube C: $S(C) = \dfrac{n(C) - f^k n}{\sqrt{f^k(1-f^k)n}}$
    - If S(C) < 0, C contains less objects than expected
    - The more negative, the sparser C is and the more likely the objects in C are outliers in the subspace