Introduction to Bayesian Networks Part 1

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The lecture is based on the slides developed by Prof. Yu Zhang from ASU School of Computing and Augmented Intelligence

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Objectives

Objective

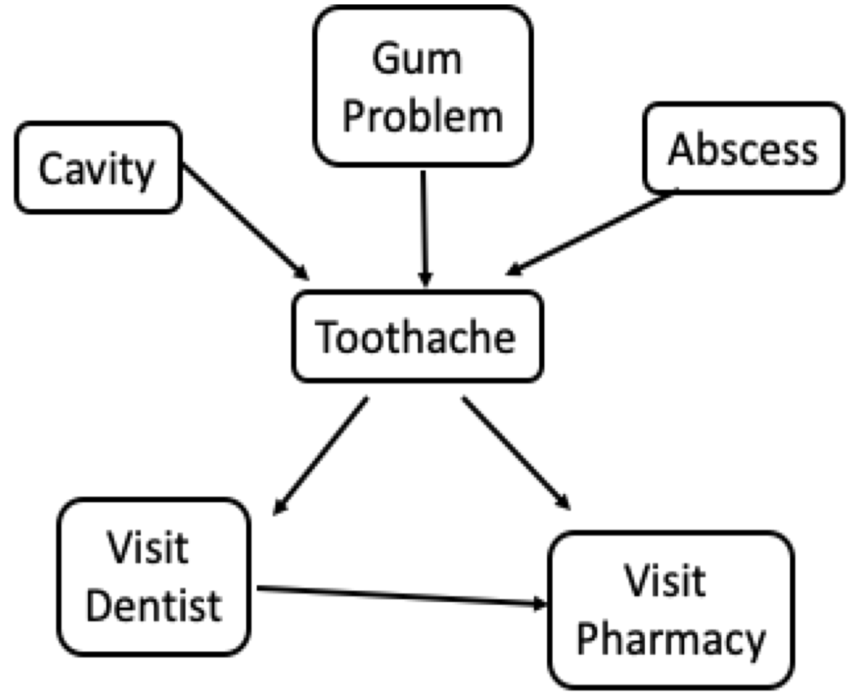
Describe Bayesian Networks

Objective

Illustrate key tasks in implementing Bayesian Networks

Why Do We Use Graphical Models?

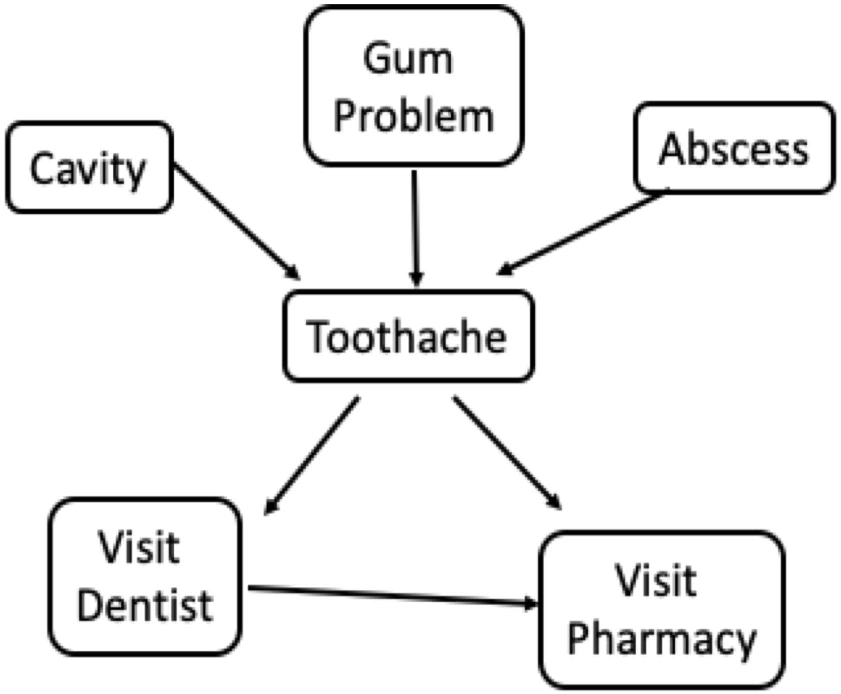
- In machine learning, we are often concerned with joint distributions of many random variables.
- A graph may provide an intuitive way to represent or visualize the relationships of the variables.
  - Much easier for domain experts to build a model.



Graphical Models for Causal Relations

- Graphical models arise naturally from, often causal, independence relations of physical events.

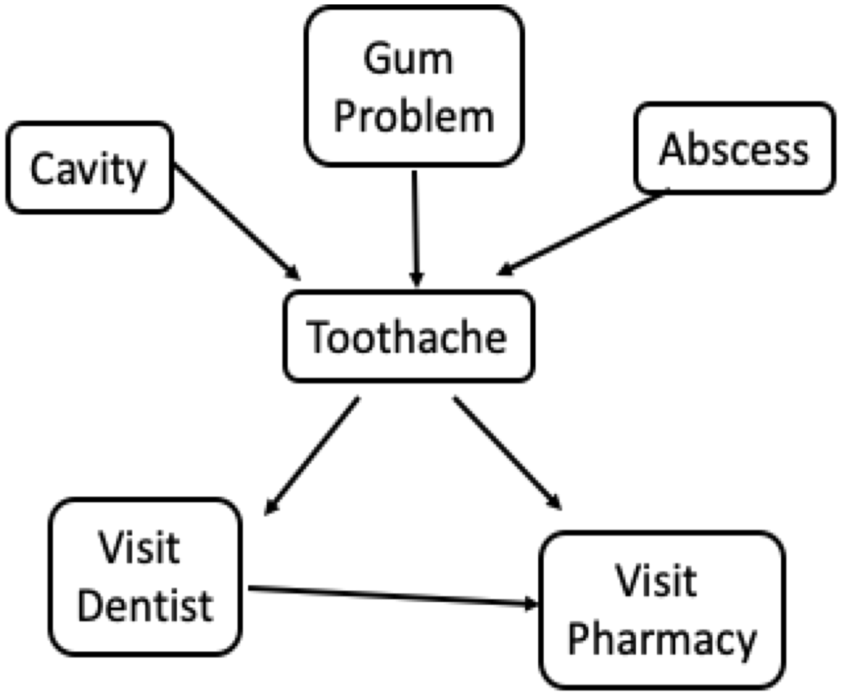
- Caveat: probabilistic relationship does not imply causality.



Representation of Bayesian Network (BN)

A BN is a directed acyclic graph (DAG), where…

- **Nodes** (vertices) represent **random variables**.
    - Can be assigned (observed) or unassigned (unobserved)
- **Directed edges** represent immediate **dependence** of nodes.
    - Indicate "direct influence" between variables.
    - Formally: encode conditional independencies.

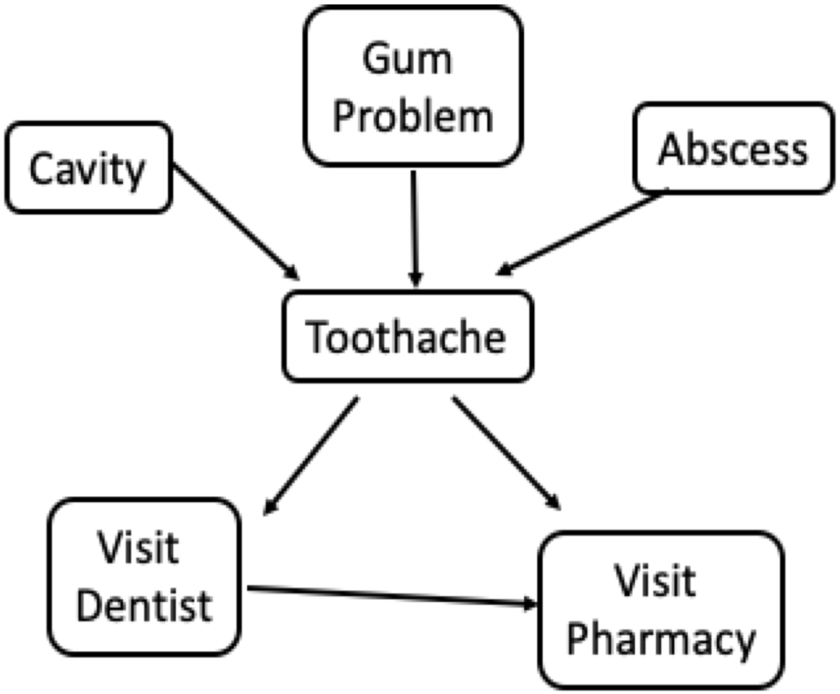


E.g., Toothache (+t) → Cavity (+c)?

Other names: Belief Networks, Bayes Nets, etc.

Size of BN

- **How many values must be stored using BN:**
  - $2 + 2 + 2 + 2^4 + 2^2 + 2^3 = 36$
- **We can save a lot of space!**
  - Space is affected by the largest Conditional Probability Table (CPT) of the BN.



Independence

Bayesian Network:

$$P(x_1, x_2, \ldots x_n) = \Pi_{i=1}^{n} P(x_i | \text{parents}(X_i))$$

The factorization adds additional constraints to $P(X_1, X_2, X_3, \ldots X_n)$.

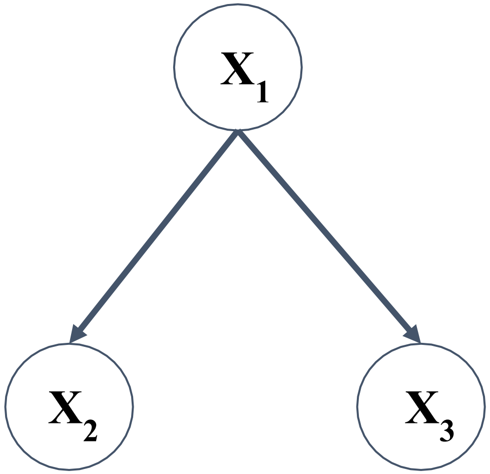
Independence

Bayesian Network:

$$P(x_1,x_2,x_3) = P(x_1)P(x_2|x_1)P(x_3|x_1)$$

Chain rule:

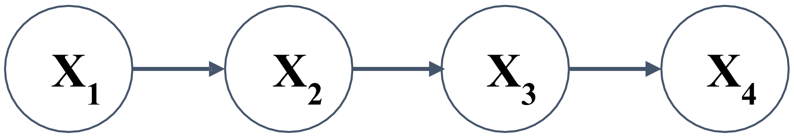
$$P(x_1,x_2,x_3) = P(x_1)P(x_2|x_1)P(x_3|x_1)$$



$$X_3 \perp\!\!\!\perp X_2 \mid X_1$$

In BN, each variable x is independent of all its non-descendants given its parents.

Additional Conditional Independence



- Do we also have $X_1 \perp\!\!\!\perp X_3, X_4 \mid X_2$?
  - Yes! There is a proof for this:

$$P(X_1|X_2, X_3, X_4) = \frac{P(X_1, X_2, X_3, X_4)}{P(X_2, X_3, X_4)}$$

$$= \frac{P(X_1)P(X_2|X_1)P(X_3|X_2)P(X_4|X_3)}{\sum_{x_1} P(X_1)P(X_2|X_1)P(X_3|X_2)P(X_4|X_3)}$$

$$= \frac{P(X_1, X_2)}{P(X_2)} = P(X_1|X_2)$$

Note:

$$\sum_{x_1} P(X_1)P(X_2|X_1) = P(X_2) \; and \; P(X_1)P(X_2|X_1) = P(X_1,X_2)$$