KRR with Uncertainty

LP^{MLN} Relationships to Other Languages

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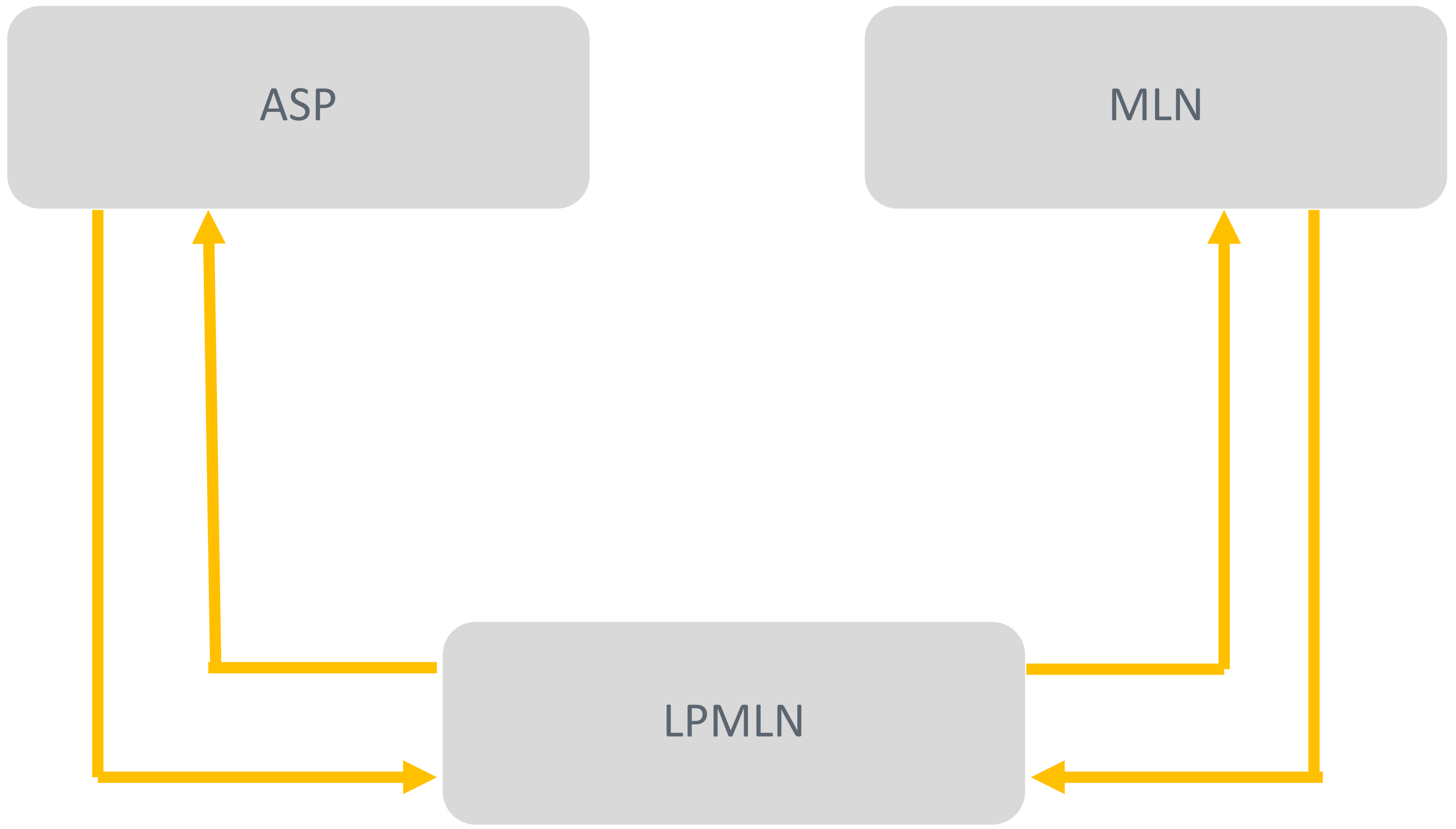
Objectives

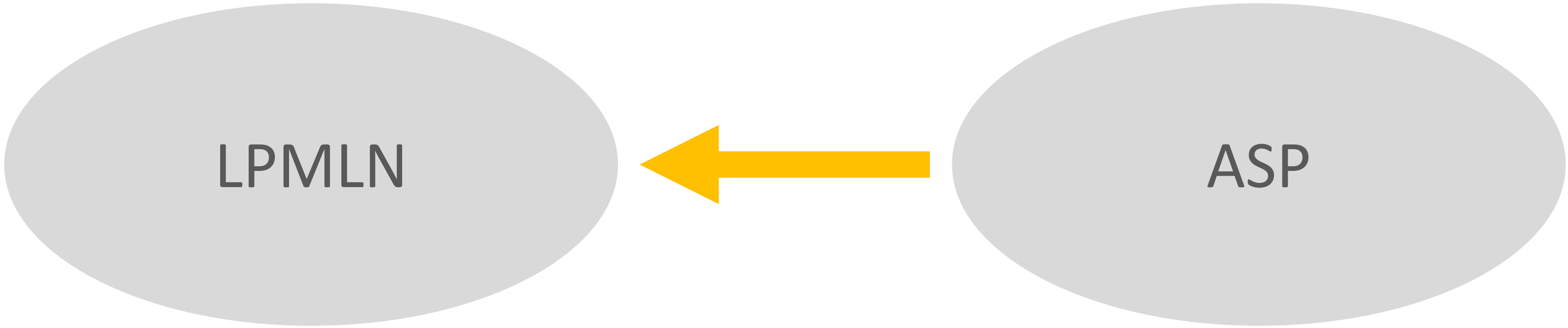
Objective

Explain the relationships between LPMLN and other languages

LPMLN vs. ASP vs. MLN



From ASP to LP^{MLN}



ASP as a Special Case of LP^{MLN}

- **Any answer set program Π can be viewed as a special case of an LP^{MLN} program P_Π by assigning the infinite weight to each rule**

Π	$p \leftarrow \text{not } q$	P_Π	$\alpha:\ p \leftarrow \text{not } q$
	$q \leftarrow \text{not } p$		$\alpha:\ q \leftarrow \text{not } p$

$\{p\}$
$\{q\}$

P_Π

I	$W(I)$	$P(I)$
$\emptyset$	e^0	0
$\{p\}$	$e^{2\alpha}$	$\frac{1}{2}$
$\{q\}$	$e^{2\alpha}$	$\frac{1}{2}$
$\{p, q\}$	0	0

- **Theorem:** **For any answer set program Π, the (deterministic) stable models of Π are exactly the (probabilistic) stable models of LP^{MLN} program P_Π whose weight is $e^{k\alpha}$, where k is the number of all ground rules in Π**

Example

- If Π has at least one (deterministic) stable model, then all (probabilistic) stable models of P_Π have the same probability, and are thus the stable models of Π as well

- **Q**: What if Π has no stable models?

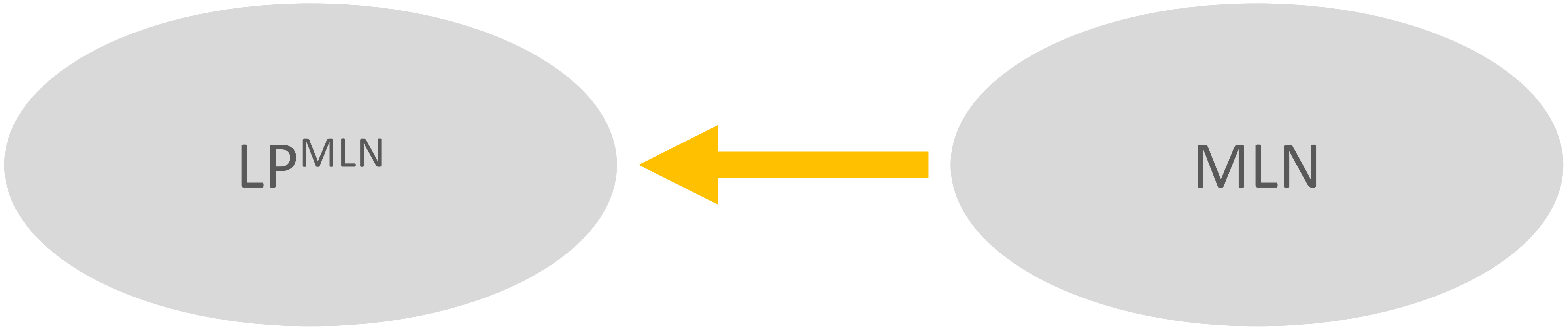
$$
\begin{array}{ll}
\Pi & \text{Bird(Jo)} \leftarrow \text{ResidentBird(Jo)} \\
 & \text{Bird(Jo)} \leftarrow \text{MigratoryBird(Jo)} \\
 & \bot \leftarrow \text{ResidentBird(Jo), MigratoryBird(Jo)} \\
 & \text{ResidentBird(Jo)} \\
 & \text{MigratoryBird(Jo)}
\end{array}
\qquad
\begin{array}{ll}
P_\Pi & \alpha:\ \text{Bird(Jo)} \leftarrow \text{ResidentBird(Jo)} \\
 & \alpha:\ \text{Bird(Jo)} \leftarrow \text{MigratoryBird(Jo)} \\
 & \alpha:\ \bot \leftarrow \text{ResidentBird(Jo), MigratoryBird(Jo)} \\
 & \alpha:\ \text{ResidentBird(Jo)} \\
 & \alpha:\ \text{MigratoryBird(Jo)}
\end{array}
$$

Correction note: There is a typo in the annotation below; the last stable model should be "{B(Jo), R(Jo), M(Jo)}".

- **Q**: What are the stable models P_Π?

{B(Jo), R(Jo)}, {B(Jo), M(Jo)}, {B(Jo)}

From MLN to LP^{MLN}



Embedding Propositional Logic in ASP

- **Theorem. For any propositional formula F of a finite signature σ, X is a model of $\boxed{\text{F}}$ iff X is a stable model of $\boxed{F \wedge Ch}$ where Ch is the conjunction of the choice rules $\{\sigma\}^{ch}$.**
  - The effect of adding the choice rules is to exempt A from minimization under the stable model semantics

$$F \;=\; p \leftarrow \neg q$$

models of F: $\boxed{\{p\},\quad \{q\},\quad \{p, q\}}$

stable models of F: $\{p\}$

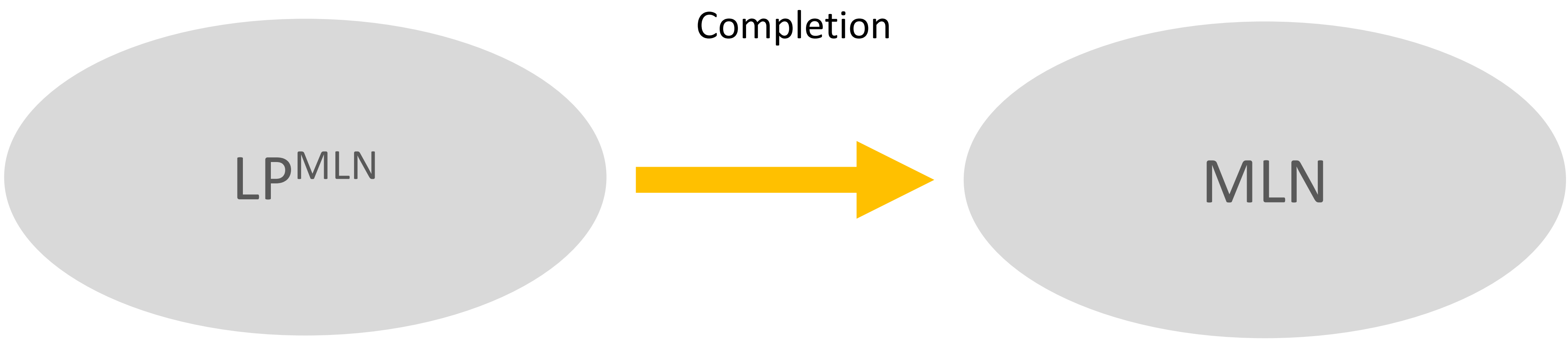
stable models of $F \wedge \{p; q\}^{ch}$: $\boxed{\{p\},\ \{q\},\ \{p, q\}}$

Embedding MLN in LP$^{\text{MLN}}$

- For any MLN L, LP$^{\text{MLN}}$ program Π_L is obtained from L by adding w : $\{A\}^{ch}$ for every ground atom A of σ and any weight w

- Theorem: Any MLN L and its LP$^{\text{MLN}}$ counterpart Π_L have the same probability distribution over all interpretations

From LP^{MLN} to MLN



Turning LP^{MLN} into MLN (1 of 2)

- We first consider how to turn an ASP program into a propositional formula.

- **Completion** is a process that turns an ASP program Π into a propositional formula F so that the stable models of Π are precisely the models of F.

Turning LP^{MLN} into MLN (2 of 2)

- The process works only for "tight" ASP programs (defined later).

- The method can be generalized to turning an LP^{MLN} program Π into an MLN program L so that the probabilistic answer sets of Π are precisely the models of L with the same probability distribution.

Completion

For any ground ASP program Π that consists of rules of the form

- $A \leftarrow Body$
- where A is an atom and $Body$ is a formula,

The completion of Π is defined as the union of Π and

$$A \rightarrow \bigvee_{A \leftarrow Body \in \Pi} Body$$

for each ground atom A

Theorem: For any "tight" answer set program Π, the stable models of Π are exactly the models of the completion of Π .

Example 1

Stable models of

$$p \leftarrow \neg q$$
$$q \leftarrow \neg p$$

$\{p\}$
$\{q\}$

$A \rightarrow \bigvee$ Body

$A \leftarrow$ Body

Models of completion

$p \leftarrow \neg q$
$q \leftarrow \neg p$
$p \rightarrow \neg q$
$q \rightarrow \neg p$

$\boxed{p \leftrightarrow \neg q}$

$\{p\}$
$\{q\}$

Example 2

Stable models of

$$p \leftarrow \neg q$$
$$q \leftarrow \neg r$$

$\{q\}$

$A \rightarrow \bigvee \text{Body}$

$A \leftarrow \text{Body}$

Models of completion

$p \leftarrow \neg q$
$q \leftarrow \neg r$
$p \rightarrow \neg q$
$q \rightarrow \neg r$
$r \rightarrow \bot$

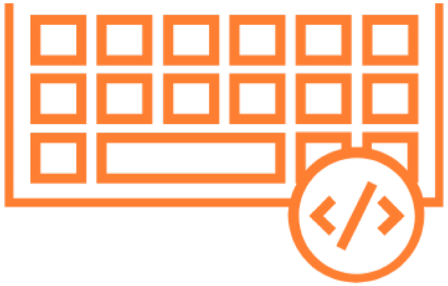
$p \leftrightarrow \neg q$
$q \leftrightarrow \neg r$
$\neg r$

$\{q\}$

Tight Programs

Theorem: For any "tight" answer set program Π, the answer sets of Π are exactly the models of the completion of Π

What would go wrong if Π is non-tight?



Completion and Non-tight programs

$$p \leftarrow q$$
$$q \leftarrow p$$

Models: $\emptyset$, $\{p, q\}$

Stable models: $\emptyset$

Completion:

$$\left.\begin{array}{l} p \leftarrow q \\ q \leftarrow p \\ p \rightarrow q \\ q \rightarrow p \end{array}\right\} \Leftrightarrow \quad p \Leftrightarrow q$$

Models of Comp: $\emptyset$, $\{p, q\}$

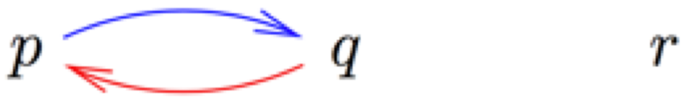
Positive Dependency Graph

- A program is a finite set of rules of the form

$$a \leftarrow \underbrace{a_1, \ldots, a_m,}_{P} \underbrace{\textit{not } a_{m+1}, \ldots, \textit{not } a_n}_{N} .$$

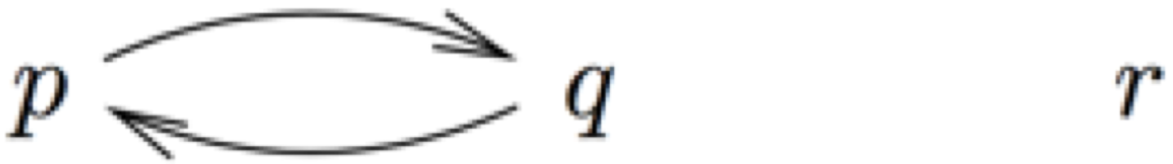
- The positive dependency graph of Π is the directed graph such that
  - its vertices are the atoms occurring in Π, and
  - for each $a \leftarrow P, N$ in Π, its edges go from a to each atom in P.

$$\begin{aligned} \Pi_1 : \quad & p \leftarrow q \\ & q \leftarrow p \\ & r \leftarrow \textit{not } p \end{aligned}$$



Loop

- A nonempty set L of atoms is called a loop of Π if, for every pair a_1, a_2 of atoms in L, there exists a path of non-zero length from a_1 to a_2 in the positive dependency graph of Π such that all vertices in this path belong to L.



- Π_1 has only one loop: $\{p, q\}$.

- A program is called **tight** if it has no loops.

Which of these Examples is a Tight Program?

A.

$$\begin{aligned} p &\leftarrow \neg q \\ q &\leftarrow \neg p \end{aligned}$$

p q

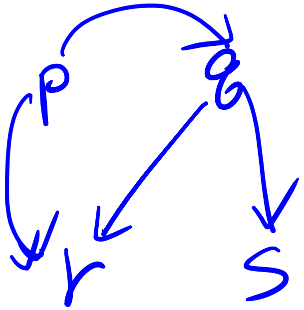
B.

$$\begin{aligned} p &\leftarrow \neg q \\ q &\leftarrow \neg r \end{aligned}$$

p q r

C.

$$\begin{aligned} p &\leftarrow q, r \\ q &\leftarrow r, s \\ s &\leftarrow \neg p \end{aligned}$$



~~**D.**~~

$$\begin{aligned} p &\leftarrow q \\ q &\leftarrow p \\ p &\leftarrow \neg r \end{aligned}$$

Completion: Turning LP$^{\text{MLN}}$ to MLN

- For any ground LPMLN program that consists of rules of the form
  - w: $A \leftarrow Body$
  - where A is an atom and $Body$ is a formula,
- The completion of Π is defined as the union of Π and hard rules

$$\alpha: \quad A \rightarrow \bigvee_{w:A\leftarrow Body\in\Pi} Body$$

for each ground atom A

- **Theorem:** "Tight" LP$^{\text{MLN}}$ program Π under the stable model semantic has the same probability distribution over all interpretations with the completion of Π under the MLN semantics

Example

- Π: under LP^{MLN}

  2: $p \leftarrow \neg q$

  1: $q \leftarrow \neg p$

- Comp(Π): under MLN

  2: $p \leftarrow \neg q$

  1: $q \leftarrow \neg p$

  α: $p \rightarrow \neg q$

  α: $q \rightarrow \neg p$

Wrap-Up