KRR with Uncertainty

Bayesian Networks

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Objectives

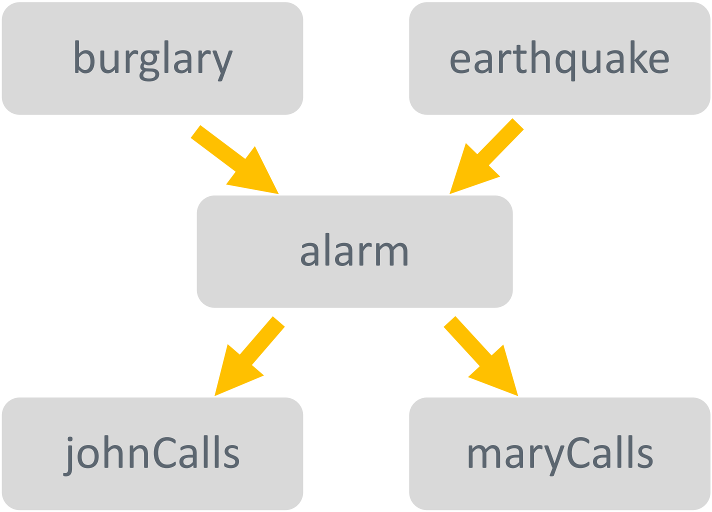
Objective

Explain the concept of Bayesian networks and how Bayesian networks utilize conditional independence

Bayesian Network

- An augmented, directed acyclic graph, where each node corresponds to a random variable X_i and each edge indicates a direct influence among the random variables.

- The influence for each variable X_i is quantified with a conditional probability distribution (CPD) P(X_i | Pa(X_i)), where Pa(X_i) are the parents of X_i in the graph.



Qualitative component: DAG

$\mathbf{P}(burglary)$
$(0.001, 0.999)$

$\mathbf{P}(earthquake)$
$(0.002, 0.998)$

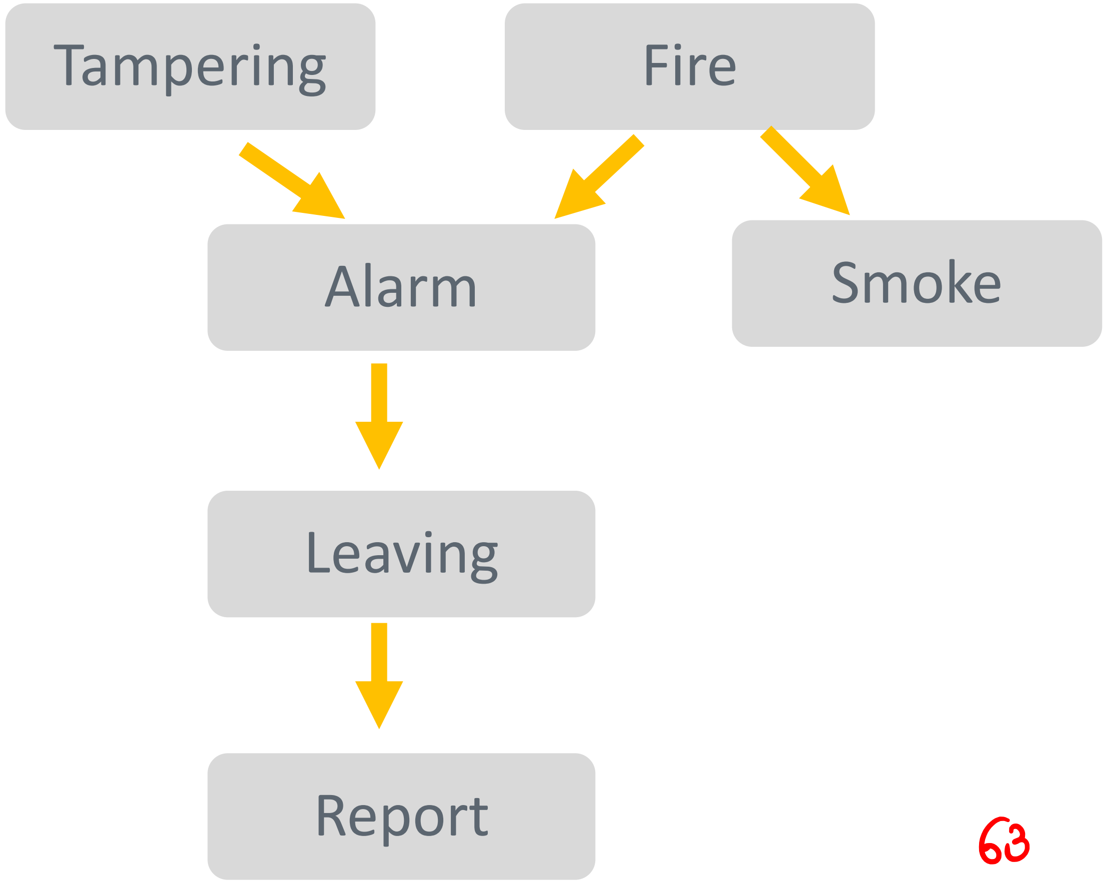
burglary	*earthquake*	$\mathbf{P}(alarm \mid burglary, earthquake)$
true	*true*	$(0.95, 0.05)$
true	*false*	$(0.95, 0.05)$
false	*true*	$(0.29, 0.71)$
false	*false*	$(0.001, 0.999)$

alarm	$\mathbf{P}(johncalls \mid alarm)$
true	$(0.90, 0.10)$
false	$(0.05, 0.95)$

alarm	$\mathbf{P}(marycalls \mid alarm)$
true	$(0.70, 0.30)$
false	$(0.01, 0.99)$

Quantitative component: CPD

Example BN: Fire Diagnosis

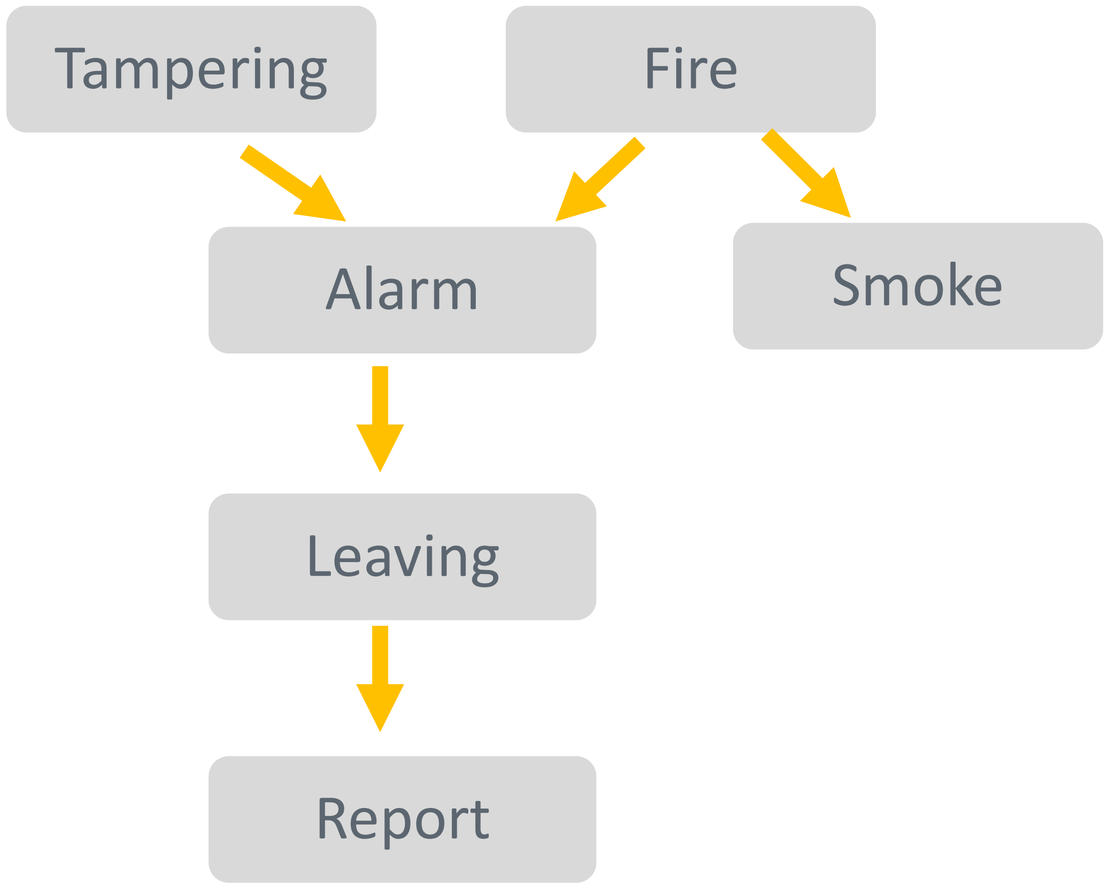


63

- All variables are Boolean.
- **Q:** How many probabilities do we need to specify for this Bayesian network?
  - Take into account that probability tables have to sum to *1*.

Ex. for BN construction: Fire Diagnosis (1 of 3)

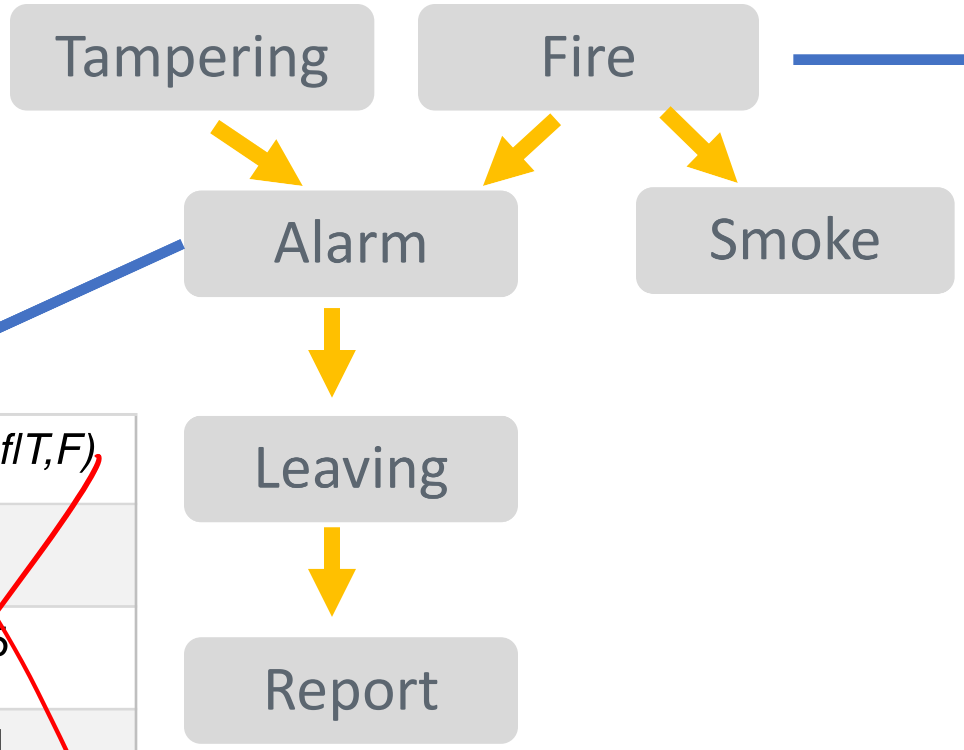
P(Tampering=t)	~~P(Tampering=f)~~
0.02	~~0.98~~



We don’t need to store P(Tampering=f) since probabilities sum to 1

Ex. for BN construction: Fire Diagnosis (2 of 3)

P(Tampering=t)
0.02



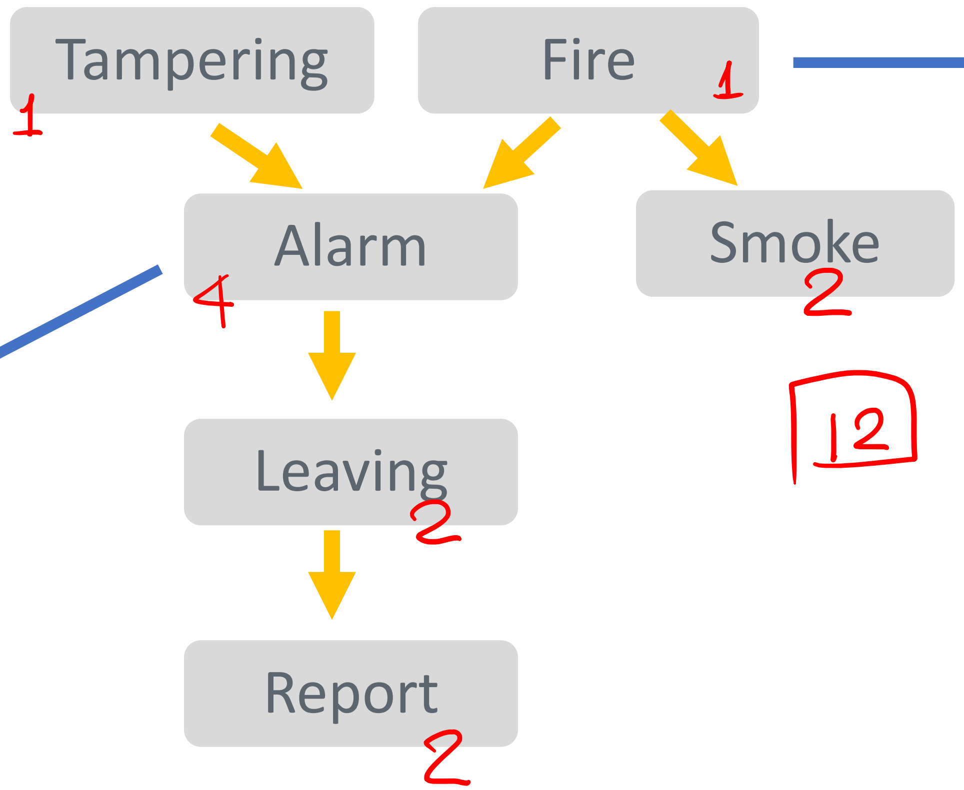
P(Fire=t)
0.01

Tampering T	*Fire F*	*P(Alarm=t|T,F)*	~~*P(Alarm=f|T,F)*~~
t	t	0.5	~~0.5~~
t	f	0.85	~~0.15~~
f	t	0.99	~~0.01~~
f	f	0.0001	~~0.9999~~

Each row of this table is a conditional probability distribution.

Ex. for BN construction: Fire Diagnosis (3 of 3)

P(Tampering=t)
0.02

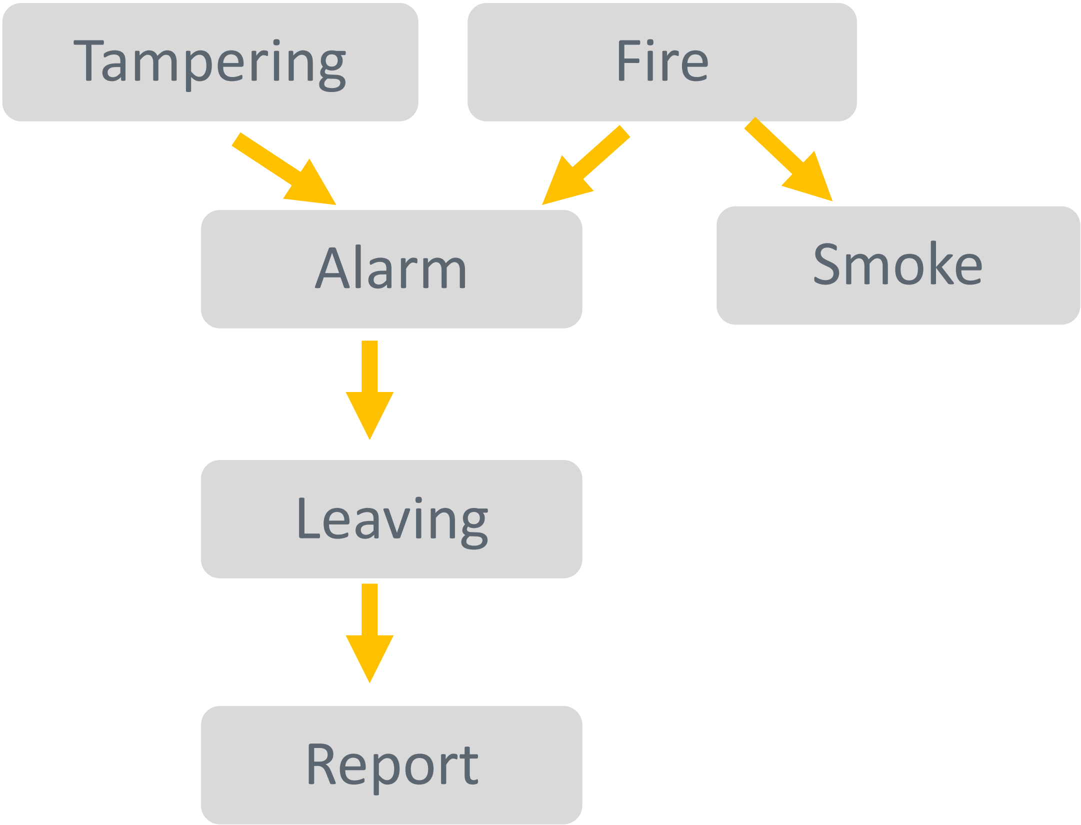


P(TFire=t)
0.01

Tampering T	*Fire F*	*P(Alarm=t|T,F)*
t	t	0.5
t	f	0.85
f	t	0.99
f	f	0.0001

Independence Assumption in Bayesian Networks

- **The Bayesian network represents a set of independence assumptions:**
- **Each variable is independent of its non-descendents given its parents** $(X_i \perp NonDescentdant(X_i) \mid Pa(X_i))$



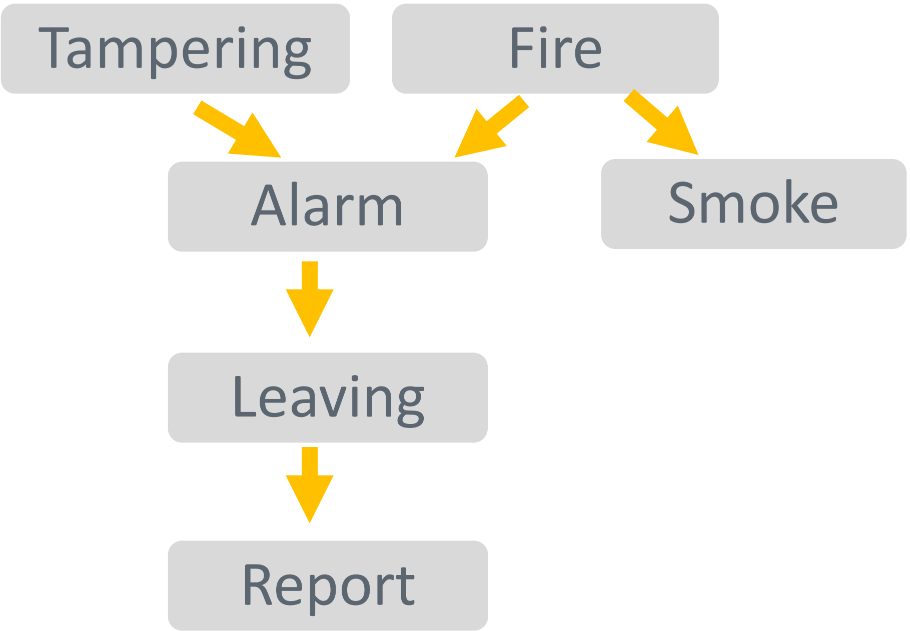
Compact Representation due to Independence Assumption

By the chain rule

$$P(T, F, A, S, L, R) = P(T)\, P(F|\cancel{T})\, P(A|\, T, F)\, P(S \mid \cancel{T}, F, \cancel{A})\, P(L \mid \cancel{T}, \cancel{F}, A, \cancel{S})\, P(R \mid \cancel{T}, \cancel{F}, \cancel{A}, \cancel{S}, L)$$

BN assumption: **each variable is independent of its non-descendents given its parents**

$$P(T, F, A, S, L, R) = P(T)\, P(F \underline{\quad})\, P(A|\, T, F)\, P(S \mid F)\, P(L \mid A)\, P(R \mid L)$$



$$\mathbf{P}(X_1, \ldots, X_n) = \prod_{i=1}^{n} \mathbf{P}(X_i \mid \mathbf{Pa}(X_i))$$

Example

$P(Tampering{=}t, Fire{=}f, Alarm{=}t, Smoke{=}f, Leaving{=}t, Report{=}t) =$

$$P(Tampering{=}t) \times P(Fire{=}f) \times P(Alarm{=}t \mid Tampering{=}t, Fire{=}f) \times$$

$$\times P(Smoke{=}f \mid Fire{=}f) \times P(Leaving{=}t \mid Alarm{=}t)$$

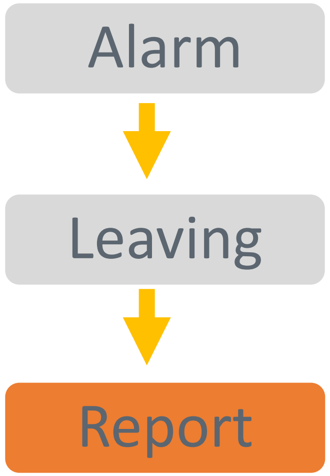
$$\times P(Report{=}t \mid Leaving{=}t) =$$

$$0.2 \text{ x } (1\text{-}0.01) \text{ x } 0.85 \text{ x } (1\text{-}0.01) \text{ x } 0.88 \text{ x } 0.75 = 0.126$$

Bayesian Networks: Types of Inference

Diagnostic

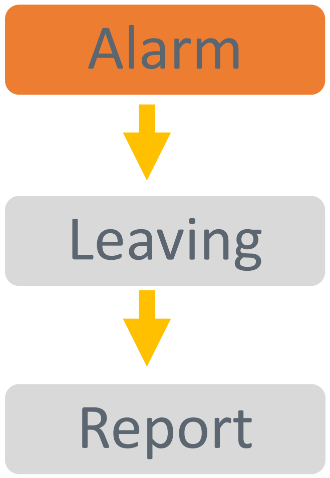
P(A | R=t) ?



$$\frac{P(A=t \wedge R=t)}{P(R=t)}$$

Predictive

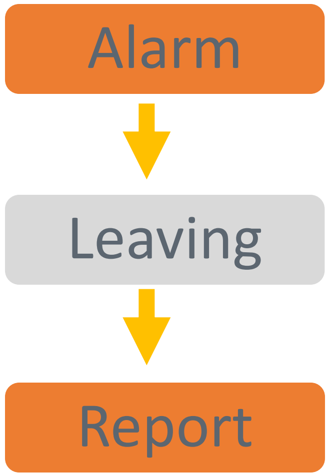
P(R | A=t) ?



$$P(A=t \wedge R=t) = \sum_{L=v} P(A=t \wedge L=v \wedge R=t)$$

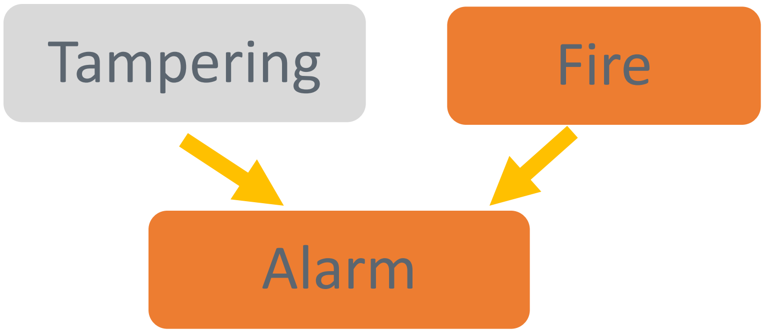
Mixed

P(L | A=f, R=t)?



Intercausal

P(T | A=t, F=t)



Wrap-Up