Ontology Languages

ALC Semantics

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Objectives

Objective

Explain the semantics of description logics

Recall: ALC Syntax of Concepts

The set of concept expressions or just concepts is defined inductively as follows:

- Every concept name is a concept.
- $\top$ (top concept) and $\bot$ (bottom concept) are concepts.
- If C and D are concepts and R is a role name then the following are concepts:
  - $\neg C$ (complement of C)
  - $C \sqcap D$ (conjunction of C and D)
  - $C \sqcup D$ (disjunction of C and D)
  - $\forall R.C$ (universal restriction)
  - $\exists R.C$ (existential restriction)

Ex: $\forall \mathbf{hasChild}.\mathbf{Male}$

$\exists \mathbf{hasChild}.\mathbf{Male}$

Recall: ALC Syntax of Terminological Axioms

- **Let A be a concept name and C, D be concepts**

- **A terminological axiom is a statement in any of the following forms:**
  - Concept definitions: $A \equiv D$ which is read “A is defined to be equivalent to D”
  - Concept inclusions: $C \sqsubseteq D$ which is read “C is subsumed by D”

Defining ALC Semantics (1 of 2)

ALC Semantics: An interpretation I is a pair $(\Delta^I, \cdot^I)$ which consists of:

- **a nonempty set Δ^I (the universe of the interpretation)**
- **a function $\cdot^I$ (the interpretation function) which maps**
  - every individual name a to an element a^I of Δ^I
  - every concept name C to a subset C^I of Δ^I
  - every role name R to a subset R^I of $\Delta^I \times \Delta^I$

Defining ALC Semantics (2 of 2)

- I is extended to arbitrary concepts as follows:
  - $\top^I = \Delta^I$
  - $\bot^I = \emptyset$
  - $(\neg C)^I = \Delta^I \setminus C^I$
  - $(C \sqcap D)^I = C^I \cap D^I$
  - $(C \sqcup D)^I = C^I \cup D^I$
  - $(\forall R.C)^I = \{ x \in \Delta^I \mid \text{all } R^I \text{ successors of } x \text{ are in } C^I \}$
  - $(\exists R.C)^I = \{ x \in \Delta^I \mid \text{some } R^I \text{ succssor of } x \text{ is in } C^I \}$

$\forall hasChild.Male$

$\exists hasChild.Male$

(We call $b \in \Delta^I$ an R^I successor of a in I if $(a, b) \in R^I$)

TBox: Semantics

- **Satisfaction.** Let $I = (\Delta^I, \cdot^I)$ be an interpretation
  - I satisfies the statement $C \sqsubseteq D$ if $C^I \subseteq D^I$
  - I satisfies the statement $C \equiv D$ if $C^I = D^I$
- **Model.** An interpretation I is a **model** for a TBox T if I satisfies all the statements in T
- **Satisfiability.** A TBox T is **satisfiable** if it has a model

ABox: Semantics

Satisfaction. Let $I = (\Delta^I, \cdot^I)$ be an interpretation

- I satisfies $C(a)$ if $a^I \in C^I$
- I satisfies $R(a,b)$ if $(a^I, b^I) \in R^I$

Model. An interpretation I is a **model** of an ABox A if it satisfies every assertion of A

Satisfiability. An ABox A is **satisfiable** if it has a model

Knowledge Bases: Semantics

Satisfaction. An interpretation $I = (\Delta^I, \cdot^I)$ **satisfies** a knowledge base $K = (T, A)$ if I satisfies both T and A

Model. An interpretation $I = (\Delta^I, \cdot^I)$ is a **model** of a knowledge base $K = (T, A)$ if I is a model of T and A

Satisfiability. A knowledge base K is **satisfiable** if it has a model

Entailment

Definition: Let K be a knowledge base and F a terminological axiom or an assertion. We say that K **entails** F (denoted by $K \vDash F$) if every model of K is a model of F

Example:

TBox $\boldsymbol{T}$:

$$\text{Female} \sqsubseteq \text{Person}$$

ABox A :

$$\text{Female(ANNA)}.$$

If $K = (T, A)$ then $K \vDash$ Person(ANNA).

Example 1

TBox T:

$$\exists \text{teaches.Course} \sqsubseteq \neg \text{Undergrad} \sqcup \text{Professor}$$

ABox A:

$$\text{teaches(JOHN, CSE579), Course(CSE579), Undergrad(JOHN).}$$

If $K = (T, A)$ **then** $K \vDash \text{Professor(JOHN)}$.

There is nothing wrong with the entailment. Q: Why?

Example 1 - Revisited

TBox T:

$\exists$teaches.Course $\sqsubseteq$ Undergrad $\sqcup$ Professor

ABox A:

teaches(JOHN, CSE579), Course(CSE579), Undergrad(JOHN).

$K = (T, A)$ **which one of the following holds?**

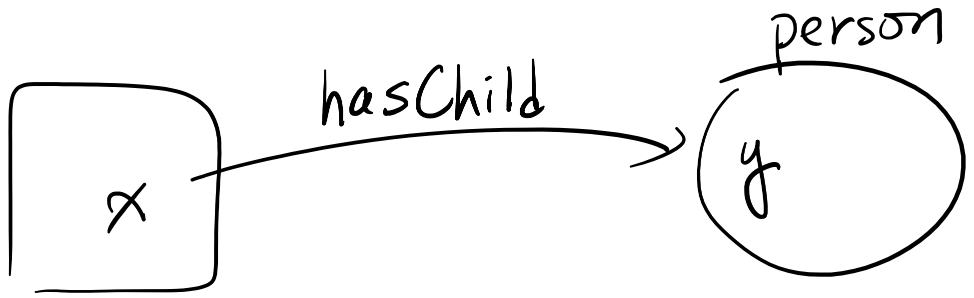
$K \models$ **Professor(JOHN)**, $K \models$ **$\neg$Professor(JOHN)**

Example 2

TBox $\mathcal{T}$:

∃hasChild.⊤ ⊑ Parent

⊤ ⊑ ∀hasChild.Person



ABox $\mathcal{A}$:

hasChild(ANNA, JOHN).

If $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ then

$\mathcal{K} \models$ **Parent(ANNA)** and $\mathcal{K} \models$ **Person(JOHN).**

Validity

Definition: Let φ be a terminological axiom or assertion. We will say that φ is **valid** if every interpretation is a model of φ.

Examples:

- $A \sqcap B \sqsubseteq A$,
- $A \sqcap B \sqcap C \sqsubseteq A \sqcap B$,
- $\forall R.(A \sqcap B) \sqsubseteq \forall R.A$
- $\top(\mathrm{ANNA})$,
- $\neg\bot(\mathrm{ANNA})$

Wrap-Up