Review of Mathematical Foundations – Part 4

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Objectives

Objective

Discuss common densities useful for machine learning applications

Common Distributions

- Uniform Distribution

- Normal (Gaussian) Distribution

The Uniform Distribution, *U*(*a*, *b*)

- 1-D example, with PDF

$$p(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{o.}w. \end{cases}$$

The Uniform Distribution, *U*(*a*, *b*)

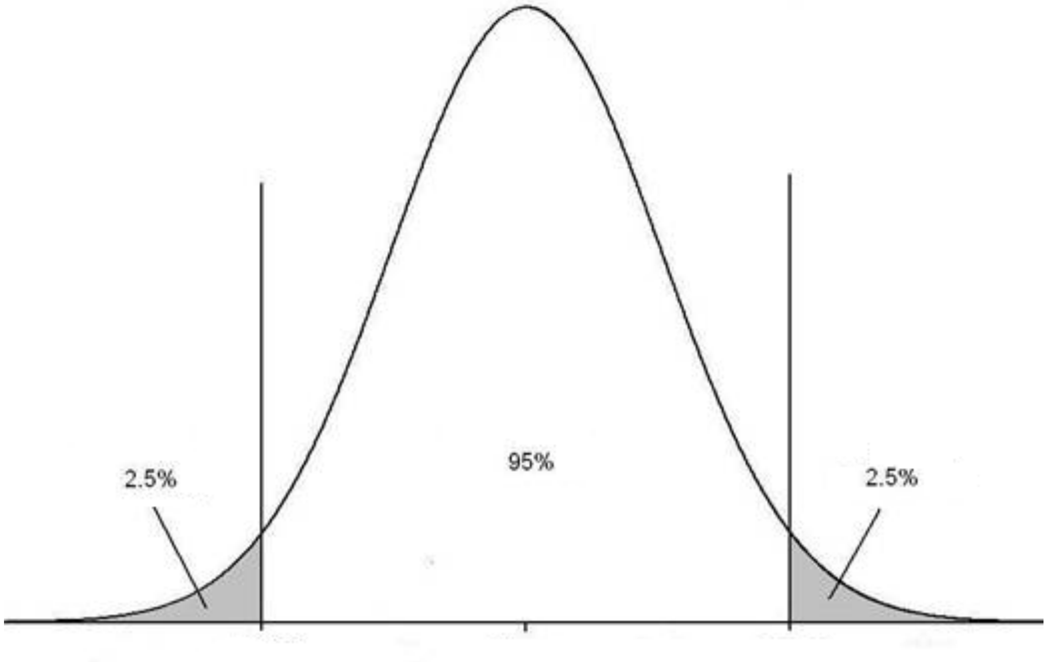
$$p(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{o}.w. \end{cases}$$

What is the CDF of *p*(*x*)?

The Normal Distribution, $N(\mu, \sigma^2)$

- 1-D example, with PDF

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



The Normal Distribution, $N(\mu, \sigma^2)$

- 1-D example, with PDF

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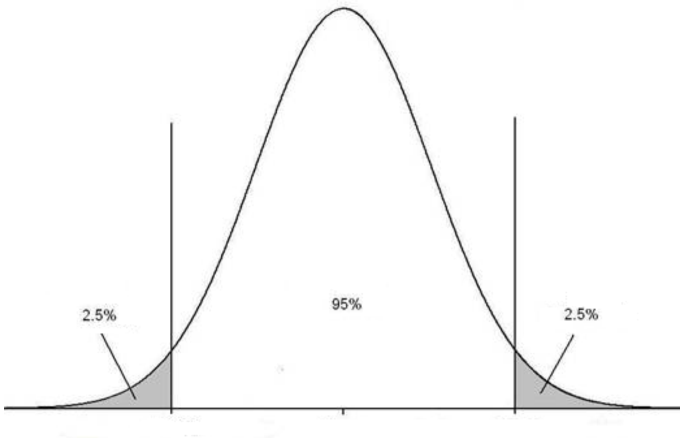


- What is the mean and variance ?

Standardized Normal Distribution

- 1-D example, with PDF $p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

- What is the CDF?

- The error function

$$\mathrm{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-x^2} dx$$

CDF for General Normal Distribution

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

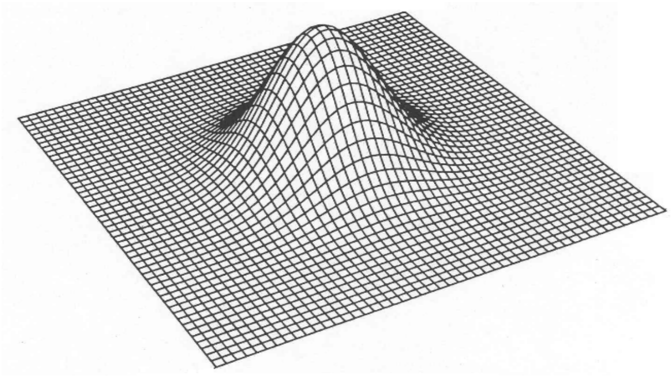
- What is the CDF for $N(\mu, \sigma^2)$?

Multivariate Normal Distribution

- *d*-dimensional vector **x** is said to be of multivariate normal distribution if its PDF is of the form

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp[-\frac{1}{2}(\mathbf{x}-\mu)^t\Sigma^{-1}(\mathbf{x}-\mu)]$$

- Visualization of a 2-*d* example



Whitening Transformation

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp[-\frac{1}{2}(\mathbf{x}-\mu)^t\Sigma^{-1}(\mathbf{x}-\mu)]$$

- Given some data **x** distributed according to the above density, we may apply some transformation to **x**, so that the covariance matrix of the transformed data is diagonal.
  - The transformation can be formed by the eigenvectors of Σ