Review of Mathematical Foundations – Part 3

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Objectives

Objective

Review random variables & their distributions

Discrete Random Variables

- Let x be a discrete random variable that can take any of the m different values in the set $V=\{v_1, v_2, \ldots, v_m\}$ with respective probabilities $\{p_1, p_2, \ldots, p_m\}$, i.e., $p_i=Prob[x= v_i]$.
  - $p_i \geq 0, \quad \sum_{j=1,\ldots,m} p_j = 1.$

- **Probability Mass Function** $P(x)$ is used to represent the set of probabilities $\{p_1, p_2, \ldots, p_m\}$

$$P(x) \geq 0, \quad \sum_{x \text{ in } V} P(x) = 1$$

Expected Value (Means) & Variance

- The **expected value** (mean) of x, E[x], often denoted μ

$$\mu = E[x] = \sum_{x \text{ in } V} xP(x)$$

- The expected value of a function $f(x)$, E[$f(x)$],

$$E[f(x)] = \sum_{x \text{ in } V} f(x)P(x)$$

- E[ ] is linear when viewed as an operator.

$$E[\alpha f(x) + \beta g(x)] =$$

- The **variance** of x, Var[x], often denoted σ^2

$$\sigma^2 = Var(x) = E[(x-\mu)^2] = \sum_{x \text{ in } V} (x-\mu)^2 P(x)$$

Joint Distributions

- Consider a pair of discrete random variables, x and y, taking values in $V=\{v_1, v_2, \ldots, v_m\}$ and $W=\{w_1, w_2, \ldots, w_n\}$ respectively.
  - (x, y) to take a pair of values (v_i, w_j) with probability p_{ij}
  - Or, we consider the **joint probability mass function** $P(x, y)$

Marginal Distributions

- Knowing $P(x, y)$, can we figure out $P_x(x)$ or $P_y(y)$?
  - ➔ The concept of **marginal distribution** for x and y respectively.

Statistical Independence

- Random variables x and y are said to be statistically independent if and only if $P(x, y) = P_x(x)\, P_y(y)$

Covariance

- Cov(x, y), often denoted σ_{xy}

- **Covariance matrix** Σ, $\Sigma = E[(\mathbf{x} - \boldsymbol{\mu})(\mathbf{x} - \boldsymbol{\mu})^t]$

Conditional Density

- $P(x|y) =$

- Similarly, we may write the Bayes Rule in terms of densities.

How about continuous random variables?

- Instead of $P(x)$, we have the **probability density function** (PDF) $p(x)$

- Some properties of $p(x)$:

- The **cumulative distribution function** (CDF) $F(x)$:

Continuous Random Variables

- Mean, variance, etc., are similarly defined, via integrals.

- Joint PDF $p(x,y)$ of two variables
  - Marginal PDFs for x and y
  - If $x \sim p_x(x)$ and $y \sim p_y(y)$ are independent $p(x,y) =$

Continuous Random Variables

- Conditional PDF $p(x|y)$

- Bayes rule for PDF: