Ontology Languages

DL to FOL

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Objectives

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Explain the semantics of ALC via translation to FOL

Introduction

Translation of ALC statements to FOL statements

- shows that ALC (and DLs in general) are subsets of FOL
- provides us with an alternative semantics for DLs.

A function π translates any axiom of an ALC knowledge base into an FOL statement

Resulting FOL statement will contain a

- unary predicate for each concept name
- binary predicate for each role name in the ALC axiom.

Translating ALC Concepts into FOL

- **Viewing role names as binary relations and concept names as unary relations, we define two translation functions, π_x and π_y , that inductively map ALC-concepts into first order formulae with one free variable, x or y:**

$$\pi_x(A) = A(x)$$
$$\pi_x(\neg C) = \neg\pi_x(C)$$
$$\pi_x(C \sqcap D) = \pi_x(C) \ \wedge \pi_x(D)$$
$$\pi_x(C \sqcup D) = \pi_x(C) \ \vee \pi_x(D)$$
$$\pi_x(\forall r.C) = \forall y(r(x,y) \rightarrow \ \pi_y(C))$$
$$\pi_x(\exists r.C) = \exists y(r(x,y) \wedge \pi_y(C))$$

$$\pi_y(A) = A(y)$$
$$\pi_y(\neg C) = \neg\pi_y(C)$$
$$\pi_y(C \sqcap D) = \pi_y(C) \wedge \pi_y(D)$$
$$\pi_y(C \sqcup D) = \pi_y(C) \vee \pi_y(D)$$
$$\pi_y(\forall r.C) = \forall x(r(y,x) \rightarrow \pi_x(C))$$
$$\pi_y(\exists r.C) = \exists x(r(y,x) \wedge \pi_x(C))$$

Translating TBox and Abox into FOL

We can translate a TBox T and an ABox A as follows, where ψ[x/a] denotes the formula obtained from ψ by replacing all free occurrences of x with a:

$$\pi(T) = \bigwedge_{C \sqsubseteq D \in T} \forall x \left(\pi_x(C) \rightarrow \pi_x(D)\right),$$

$$\pi(A) = \bigwedge_{C(a) \in A} \pi_x(C)[x/a] \ \wedge \bigwedge_{r(a,b) \in A} r(a,b).$$

Example 1

The FOL expression for concept inclusion

$$\text{Male} \sqsubseteq \neg\text{Female}$$

is

$$\forall x\,(\pi_x(\text{Male}) \rightarrow \pi_x(\neg\text{Female}))$$

$$\Longleftrightarrow \forall x\,(\text{Male}(x) \rightarrow \neg\pi_x(\text{Female}))$$

$$\Longleftrightarrow \forall x\,(\text{Male}(x) \rightarrow \neg\text{Female}(x))$$

$$\Longleftrightarrow \forall x\,(\text{Female}(x) \rightarrow \neg\text{Male}(x))$$

$$\Longleftrightarrow \text{Female} \sqsubseteq \neg\text{Male}$$

Example 2

$\pi_x(\text{Man}(a) \sqcap \exists \text{hasChild.Female}) =$

$$\pi_x(\text{Man}(a)) \wedge \pi_x(\exists \text{hasChild.Female})$$

$$\Leftrightarrow \text{Man}(a) \wedge \exists y(\text{hasChild}(\underline{x}, y) \wedge \pi_y(\underline{\text{Female}}))$$

$$\Leftrightarrow \text{Man}(a) \wedge \exists y(\text{hasChild}(x, y) \wedge \text{Female}(y))$$

Example 3

$\pi(\text{HappyFather} \sqsubseteq \text{Man} \sqcap \exists \text{hasChild.Female}) =$

$$\forall x \left(\pi_x(\text{HappyFather}) \rightarrow \pi_x(\text{Man} \sqcap \exists \text{hasChild.Female}) \right)$$

$$\Leftrightarrow \forall x \left(\text{HappyFather}(x) \rightarrow \left(\text{Man}(x) \wedge \underbrace{\pi_x(\exists \text{hasChild.Female})}_{\exists y(\text{hasChild}(x,y) \wedge \text{Female}(y))} \right) \right)$$

Wrap-Up