Ontology Languages

Beyond ALC

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Objectives

Objective

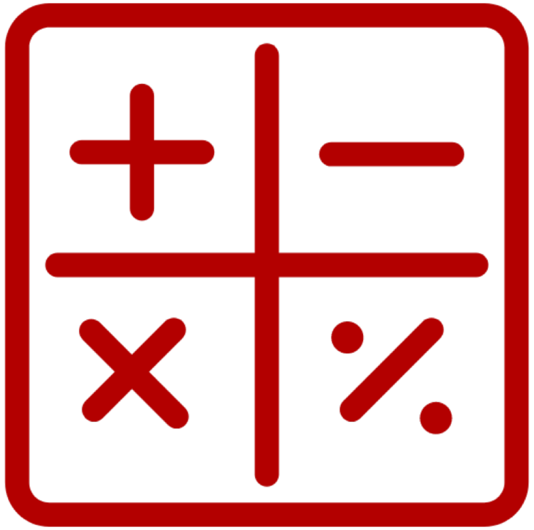
Explain additional constructs in description logics and how they add expressivity

Number Restrictions

- Role restriction cannot express that a teacher teaches at least 3 (or at most 5) courses

- Number restrictions can express arithmetic constraints on the number of fillers of a role

- Examples:
  - $\text{BusyTeacher} \equiv \text{Teacher} \sqcap (\geq 3\ \text{teaches})$
  - $\text{ConsciousTeacher} \equiv \text{Teacher} \sqcap (\leq 5\ \text{teaches})$



Number Restrictions, cont'd

Semantics:

- $(\geq n\, R)^I = \{x \in \Delta^I \mid |\{y \mid (x,y) \in R^I\}| \geq n\}$
- $(\leq n\, R)^I = \{x \in \Delta^I \mid |\{y \mid (x,y) \in R^I\}| \leq n\}$

$\pi_x(\geq 2\, R)$

$= \exists y_1 y_2 (R(x,y_1) \wedge R(x,y_2) \wedge y_1 \neq y_2)$

$\pi_x(\leq 2\, R)$

$= \forall y_1 y_2 y_3 (R(x,y_1) \wedge R(x,y_2) \wedge R(x,y_3) \rightarrow (y_1 = y_2 \vee y_2 = y_3 \vee y_3 = y_1))$

Observation: $(\geq 1\, R) \equiv \exists R.\top$

The above number restrictions are called unqualified.

- $\pi_x(\geq n\, R) = \exists y_1 \ldots y_n (\bigwedge_{k=1}^{n} R(x,y_k) \wedge \bigwedge_{i<j} y_j \neq y_i)$
- $\pi_x(\leq n\, R) = \forall y_1 \ldots y_{n+1} (\bigwedge_{k=1}^{n+1} R(x,y_k) \rightarrow \bigvee_{i<j} y_j = y_i)$

Notation: The description logic which extends $\mathcal{ALC}$ with number restrictions is denoted by ALCN.

Qualified Number Restrictions

Examples:

- $\text{BusyTeacher} \equiv \text{Teacher} \sqcap (\geq 3\ \text{teaches.Course})$
- $\text{ConsciousTeacher} \equiv \text{Teacher} \sqcap (\leq 5\ \text{teaches.Course})$

Semantics:

- $(\geq n\ R.C)^I = \{x \in \Delta^I \mid \text{at least } n\ R^I \text{successors of } x \text{ are in } C^I\}$
- $(\leq n\ R.C)^I = \{x \in \Delta^I \mid \text{at most } n\ R^I \text{successors of } x \text{ are in } C^I\}$

Notation: The description logic which extends ALC with qualified number restrictions is denoted by ALCQ.

Enumerations - Nominals

- **Sometimes it is useful to define a concept that contains exactly the individuals $I_1, \ldots, I_m$. This concept is written as $\{I_1, \ldots, I_m\}$.**

- **Examples:**
  - Weekday ≡ {MON, TUE, WED, THU, FRI, SAT, SUN}
  - Citizen ≡ Person ⊓ ∃hasCountry.Country
  - German ≡ Citizen ⊓ ∃hasCountry.{Germany}

Nominals

- A **nominal** is a concept that contains exactly one individual.
- If we have the ability to define nominals, then using $\sqcup$, we can define concepts containing more than one individual.
- Example:
  - Weekend $\equiv$ {SAT, SUN}

Nominals, cont'd

- If we have nominals and we do not want to make the UNA, then we can explicitly state whether two individuals are the same or different.

- Examples:
  - $\{US\} \equiv \{USA\}$, $\{US\} \sqcap \{Mexico\} \sqsubseteq \bot$

- **Notation:** If we add nominals to DL ALCQ, we get the DL ALCQO.

Example

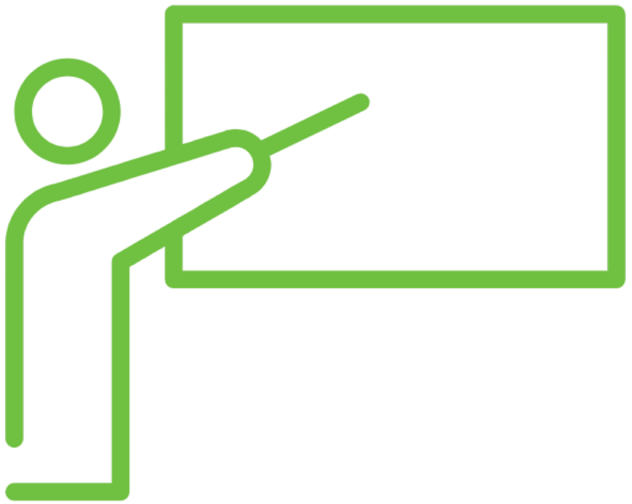
Let a knowledge base K be:

$BusyTeacher \equiv Teacher \sqcap (\geq 3\ teaches)$

$ConsciousTeacher \equiv Teacher \sqcap (\leq 5\ teaches)$

$Teacher(MARY),$

$teaches(MARY, AI), teaches(MARY, KR),$
$teaches(MARY, DB)$



Questions:

- $K \models BusyTeacher(MARY)$?
- $K \models ConsciousTeacher(MARY)$?

Riddle: Two Fathers and Two Sons

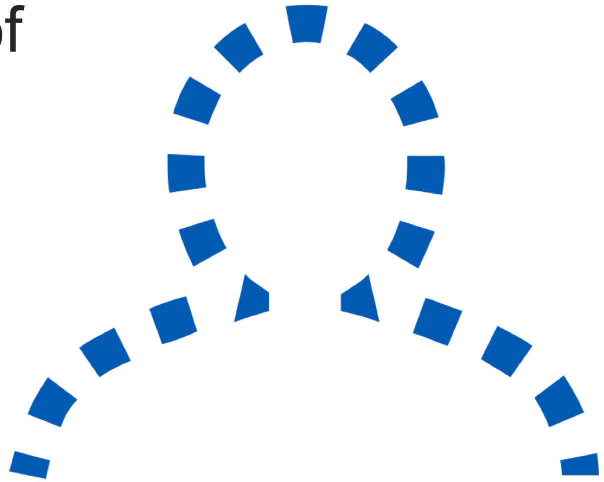
Two fathers and two sons go fishing together in the same boat. They all catch a fish but the total catch for the day is three fish. How is this possible?

OWS and CWA

- **Contrary to databases, DLs make the Open World Assumption.**
  - Absence of information is not interpreted as presence of negative information but simply as lack of knowledge.

- **Thus in the previous example:**

- **$K \not\models \mathrm{ConsciousTeacher}(\mathrm{MARY})$**



Wrap-Up