KRR with Uncertainty

Language LP^{MLN}

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Objectives

Objective

Explain the different roles of logic and probability in KR

Objective

Explain the benefits of combining logic and probability in KR

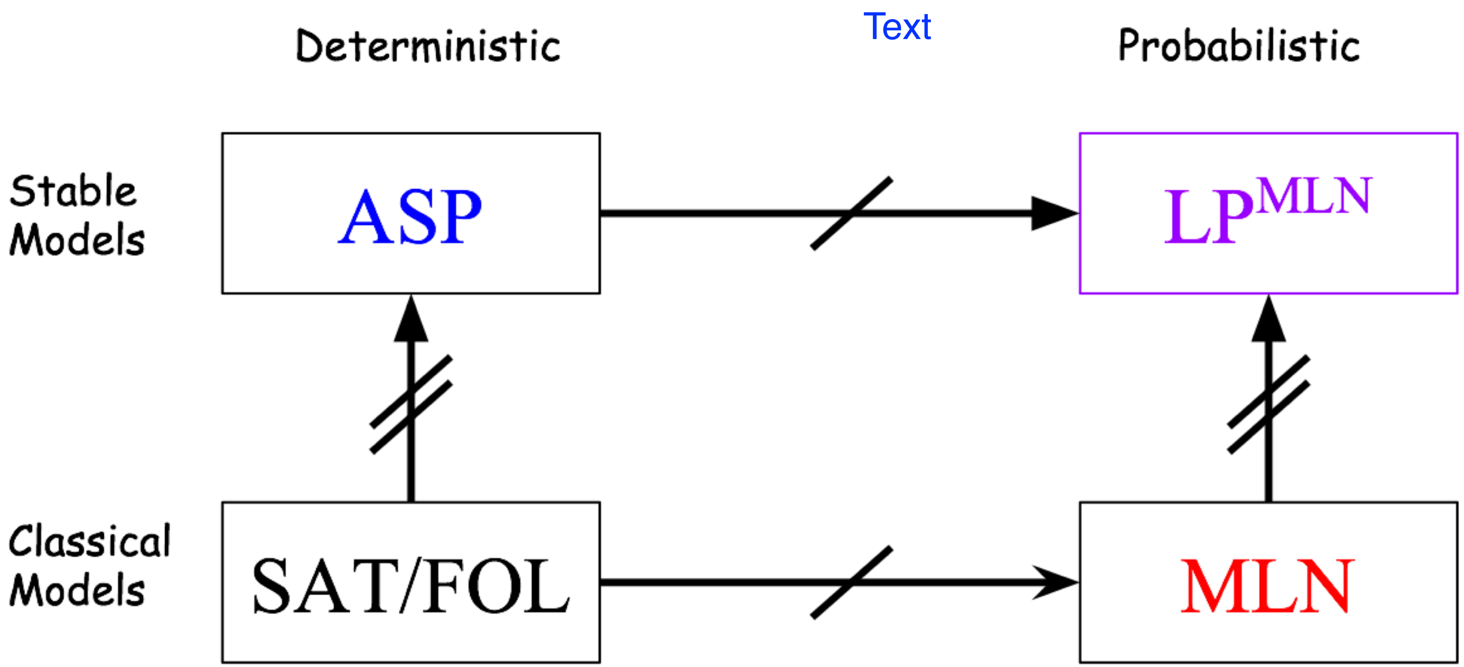
Language LP^{MLN}

- **A probabilistic extension of Answer Set Programs, following the log-linear models of Markov Logic**

- **It provides versatile methods to overcome the deterministic nature of the stable model semantics, such as:**
  - Resolving inconsistencies in answer set programs
  - Defining ranking/probability distribution over stable models
  - Applying methods from machine learning to compute KR formalisms

Language LP^{MLN}

- Overcomes the weakness of ASP in handling uncertainty.
- Overcomes the weakness of MLN in handling expressive commonsense reasoning.



Example

KB_1

```
bird(x) <- residentBird(x).
bird(x) <- migratoryBird(x).
<- residentBird(x), migratoryBird(x).
```

KB_2

```
residentBird(Jo).
```

KB_3

```
migratoryBird(Jo).
```

Example

KB1

```
bird(x) <- residentBird(x).
bird(x) <- migratoryBird(x).
<- residentBird(x), migratoryBird(x).
```

Unsatisfiable!

no answer set, no information

KB2

```
residentBird(Jo).
```

KB3

```
migratoryBird(Jo).
```

LP^{MLN} (1 of 3)

- **Syntactically, it's a simple extension of answer set programs where each rule is prepended by weights**
  - infinite weight (∞) tells the rule expresses a definite knowledge
- **Each stable model gets weights from the rules that are true in the stable model**
  - a stable model does not have to satisfy all rules
  - the more rules true, the more likely the stable model

LP^{MLN} (2 of 3)

- **Adopting the log-linear models of MLN, language LP^{MLN} provides a simple and intuitive way to incorporate the concept of weights into the stable model semantics**
  - While MLN is an undirected approach, LP^{MLN} is a directed approach, where the directionality comes from the stable model semantics
- **Probabilistic answer set computation can be reduced to sampling and optimization problems**

Syntax of LP^{MLN}

- $w: R$ **where**
  - w is a real number or α for denoting the infinite weight
  - R is an ASP rule
- **Variables are understood in terms of grounding same as in MLN**

Semantics of LP^{MLN}

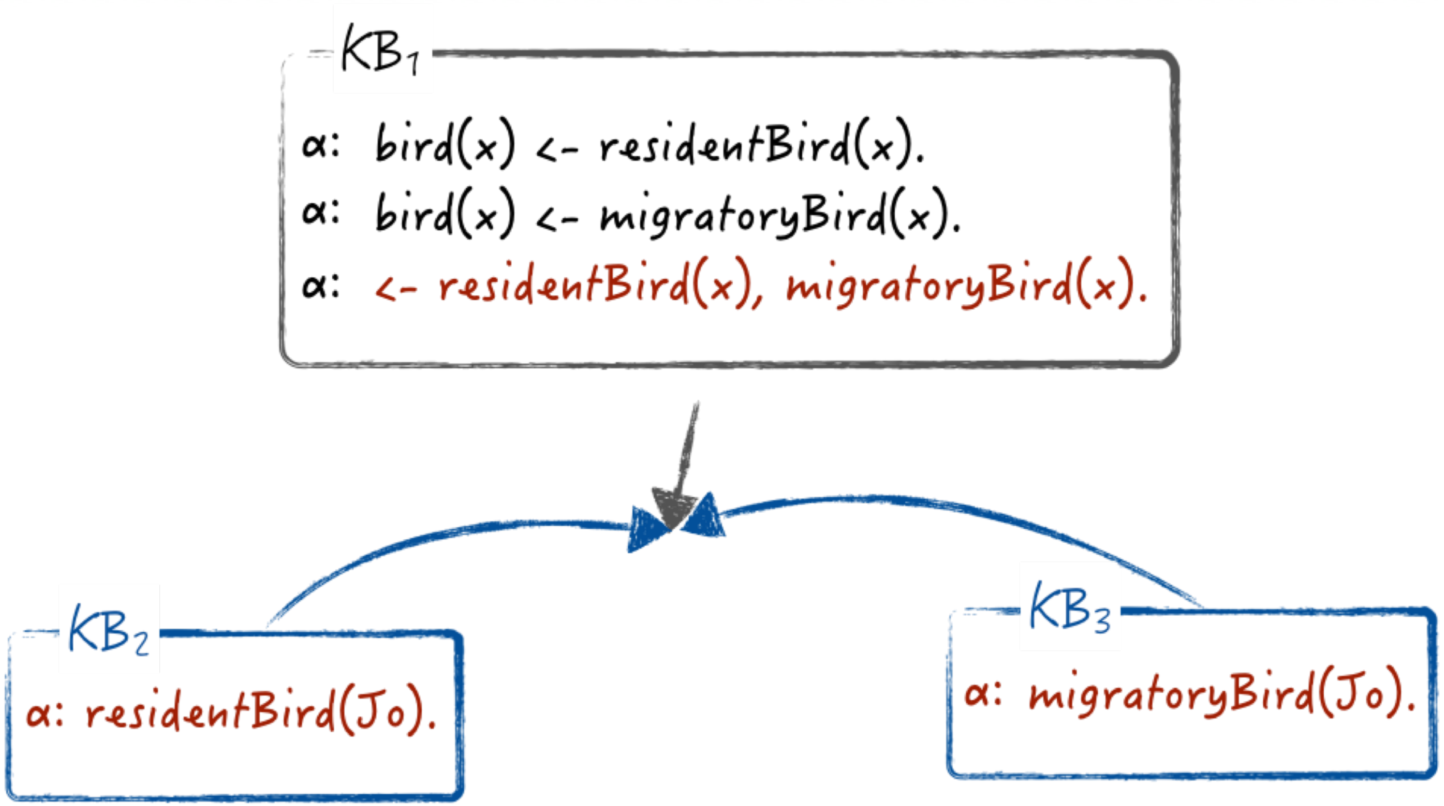
- Π_I denotes the set of rules $w : R$ in Π such that $I \models R$
- I is a **soft stable model** of Π if I is a (standard) stable model of Π_I
- The unnormalized weight of an interpretation I under Π is defined as

$$W_\Pi(I) = \begin{cases} exp\left(\sum\limits_{w:R \,\in\, \Pi_I} w\right) & \text{if } I \text{ is a soft stable model of } \Pi \\ 0 & \text{otherwise} \end{cases}$$

- The normalized weight (probability) of an interpretation I under Π, denotes $P_\Pi(I)$, is defined as

$$P_\Pi(I) = \lim_{\alpha \to \infty} \frac{W_\Pi(I)}{\sum_J W_\Pi(J)}.$$

Example 1



KB1
α: bird(x) <- residentBird(x).
α: bird(x) <- migratoryBird(x).
α: <- residentBird(x), migratoryBird(x).
KB2
α: residentBird(Jo).
KB3
α: migratoryBird(Jo).

Example 1

KB_1	$\alpha :$	$Bird(x) \leftarrow ResidentBird(x)$	$(r1)$
	$\alpha :$	$Bird(x) \leftarrow MigratoryBird(x)$	$(r2)$
	$\alpha :$	$\leftarrow ResidentBird(x), MigratoryBird(x)$	$(r3)$
KB_2	$\alpha :$	$ResidentBird(Jo)$	$(r4)$
KB_3	$\alpha :$	$MigratoryBird(Jo)$	$(r5)$

I	Π_I	$W_\Pi(I)$	$P_\Pi(I)$
$\emptyset$	$\{r_1, r_2, r_3\}$	$e^{3\alpha}$	0
$\{R(Jo)\}$	$\{r_2, r_3, r_4\}$	$e^{3\alpha}$	0
$\{M(Jo)\}$	$\{r_1, r_3, r_5\}$	$e^{3\alpha}$	0
$\{B(Jo)\}$	$\{r_1, r_2, r_3\}$	0	0
$\{R(Jo), B(Jo)\}$	$\{r_1, r_2, r_3, r_4\}$	$e^{4\alpha}$	1/3
$\{M(Jo), B(Jo)\}$	$\{r_1, r_2, r_3, r_5\}$	$e^{4\alpha}$	1/3
$\{R(Jo), M(Jo)\}$	$\{r_4, r_5\}$	$e^{2\alpha}$	0
$\{R(Jo), M(Jo), B(Jo)\}$	$\{r_1, r_2, r_4, r_5\}$	$e^{4\alpha}$	1/3

Z

$P(R(Jo)) = \frac{2}{3}$

$P(B(Jo)) = 1$

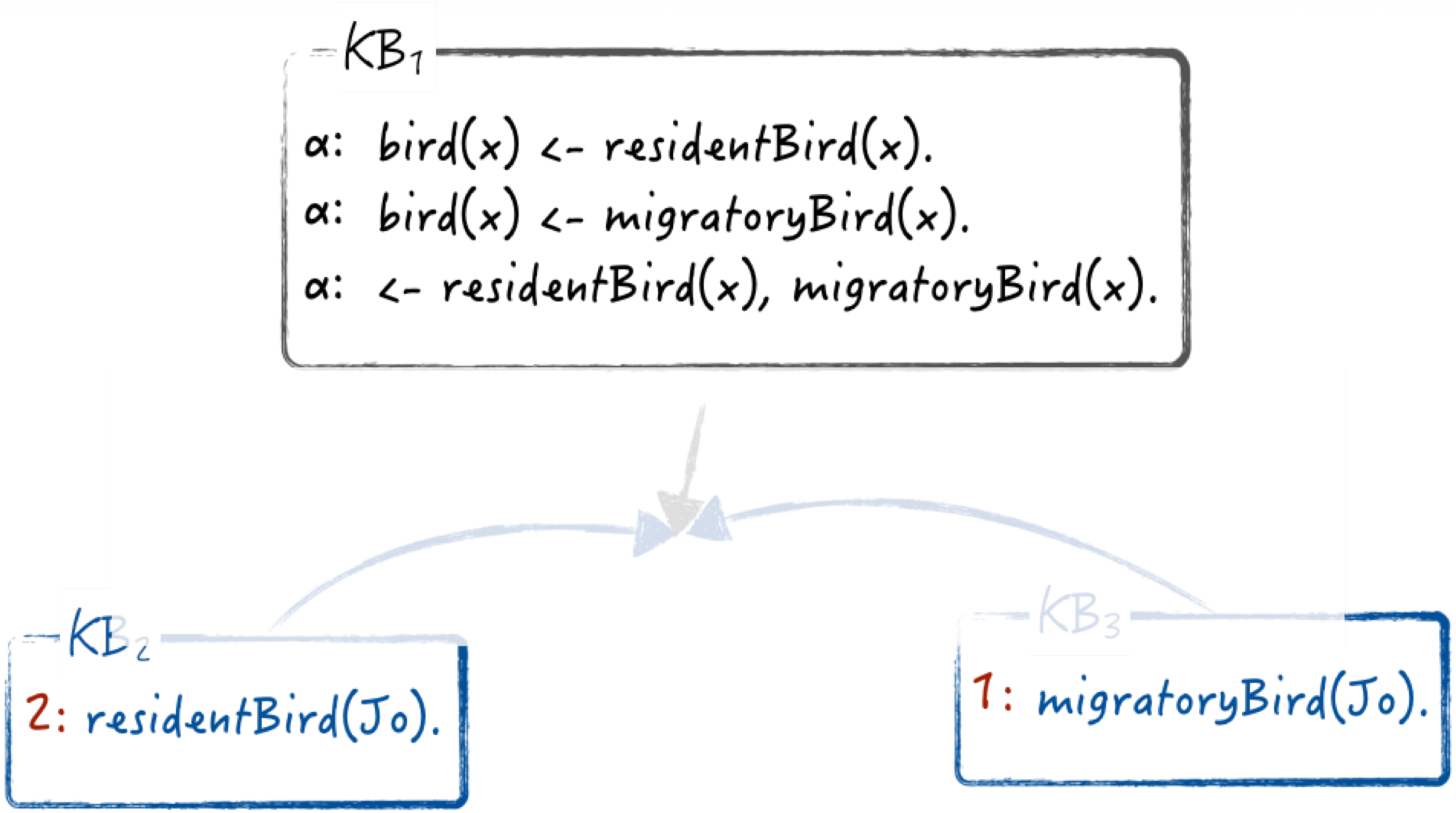
$P(B(Jo) \mid R(Jo)) = 1$

$P(R(Jo) \mid B(Jo)) = \frac{2}{3}$

$P(R(Jo) \wedge M(Jo)) = \frac{1}{3}$

$\lim_{\alpha \to \infty} \frac{e^{3\alpha}}{3 \cdot e^{3\alpha} + 3 \cdot \underline{e^{4\alpha}} + e^{2\alpha}} = 0$

Example 2



KB1
α: bird(x) <- residentBird(x).
α: bird(x) <- migratoryBird(x).
α: <- residentBird(x), migratoryBird(x).
KB2
2: residentBird(Jo).
KB3
1: migratoryBird(Jo).

Example 2

$$
\begin{array}{llll}
KB_1 & \alpha: & Bird(x) \leftarrow ResidentBird(x) & (r1) \\
 & \alpha: & Bird(x) \leftarrow MigratoryBird(x) & (r2) \\
 & \alpha: & \leftarrow ResidentBird(x), MigratoryBird(x) & (r3)
\end{array}
$$

$$KB'_2 \quad 2: \quad ResidentBird(Jo) \quad (r4')$$

$$KB'_3 \quad 1: \quad MigratoryBird(Jo) \quad (r5')$$

I	Π_I	$W_\Pi(I)$	$P_\Pi(I)$	
$\emptyset$	$\{r_1, r_2, r_3\}$	$e^{3\alpha}$	$\frac{e^0}{e^2+e^1+e^0}$	$= 0.09$
$\{R(Jo)\}$	$\{r_2, r_3, r'_4\}$	$e^{2\alpha+2}$	0	
$\{M(Jo)\}$	$\{r_1, r_3, r'_5\}$	$e^{2\alpha+1}$	0	
$\{B(Jo)\}$	$\{r_1, r_2, r_3\}$	0	0	
$\{R(Jo), B(Jo)\}$	$\{r_1, r_2, r_3, r'_4\}$	$e^{3\alpha+2}$	$\frac{e^2}{e^2+e^1+e^0}$	$= 0.67$
$\{M(Jo), B(Jo)\}$	$\{r_1, r_2, r_3, r'_5\}$	$e^{3\alpha+1}$	$\frac{e^1}{e^2+e^1+e^0}$	$= 0.24$
$\{R(Jo), M(Jo)\}$	$\{r'_4, r'_5\}$	e^3	0	
$\{R(Jo), M(Jo), B(Jo)\}$	$\{r_1, r_2, r'_4, r'_5\}$	$e^{2\alpha+3}$	0	

$P(R(Jo)) = 0.67$

$P(M(Jo)) = 0.24$

$P(\neg R(Jo) \wedge \neg M(Jo)) = 0.09$

$P(B(Jo)) = 0.67 + 0.24 = 0.91$

$P(R(Jo) \mid B(Jo)) = \frac{0.67}{0.67+0.24}$

$= 0.74.$

Reward-Based Weight

REWARD-BASED WEIGHT

$$W_{\Pi}(I) = exp\Big(\sum_{w:R\in\Pi, I\models R} w\Big)$$

Probability

$$P_{\Pi}(I) = \lim_{\alpha\to\infty} \frac{W_{\Pi}(I)}{\sum_{J} W_{\Pi}(J)}.$$

Penalty-Based Weight

PENALTY-BASED WEIGHT

$$W_{\Pi}^{pnt}(I) = exp(-\sum_{w:R\in\Pi,I\not\models R} w)$$

Probability

$$P_{\Pi}^{pnt}(I) = \lim_{\alpha\to\infty} \frac{W_{\Pi}^{pnt}(I)}{\sum_J W_{\Pi}^{pnt}(J)}$$

Reward vs. Penalty based Weights

Theorem. For any LPMLN program Π and any interpretation I,

$$W_\Pi(I) \propto W_\Pi^{pnt}(I)$$

$$P_\Pi(I) = P_\Pi^{pnt}(I)$$

Wrap-Up