Introduction to Bayesian Networks Part 2

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The lecture is based on the slides developed by Prof. Yu Zhang from ASU School of Computing and Augmented Intelligence

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Objectives

Objective

Describe Bayesian Networks

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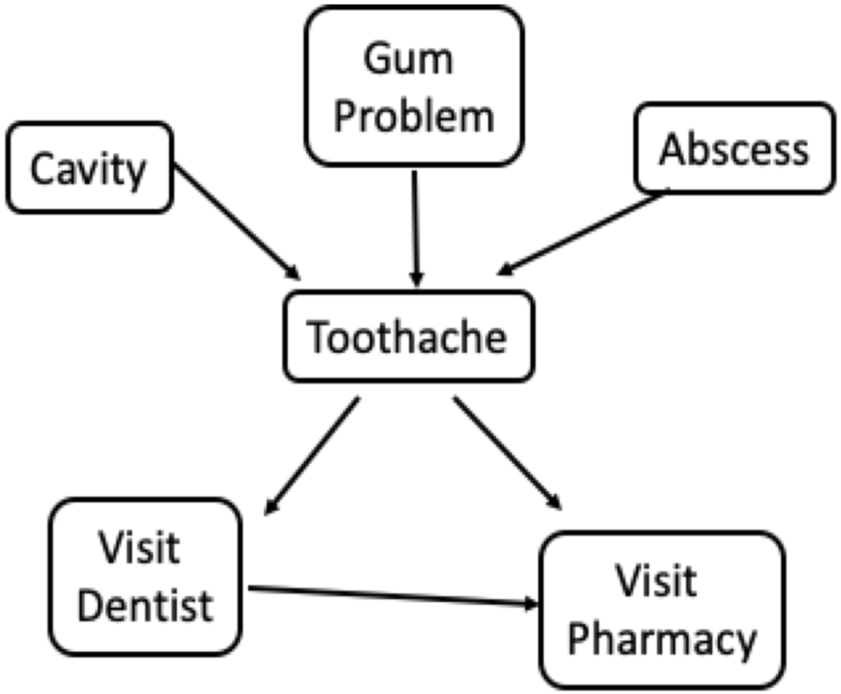
Illustrate key tasks in implementing Bayesian Networks

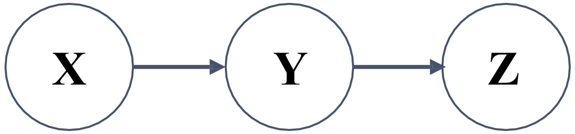
D-Separation

- Study independence properties for simple structures.
- Analyze complex cases in terms of these simple structures.



Causal Chains

- This configuration is a "causal chain":



- Guaranteed X independent of Y?
  - **Answer: No**
  - One example of Conditional Probability Tables (CPTs) for which X is **not** independent of Y is sufficient to show this result.
  - E.g., When x = true (false), y = true (false), z = true (false) are all causally related.

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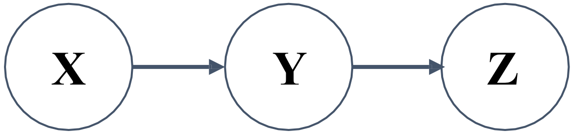


- Guaranteed X independent of Y given Z?

$$P(X|Y,Z) = \frac{P(X,Y,Z)}{P(Y,Z)}$$

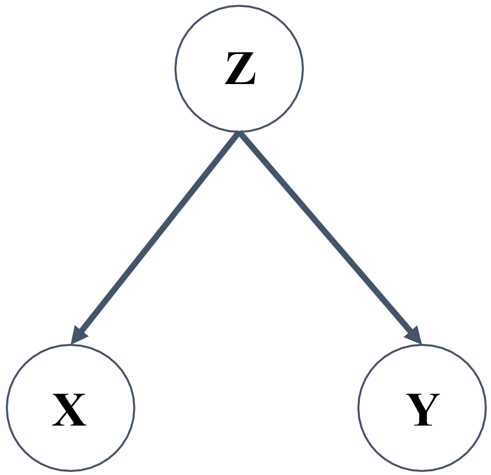
$$= \frac{P(X)P(Z|X)P(Y|Z)}{P(Y|Z)P(Z)} = P(X|Z)$$

  – **Answer: Yes!**

  – Evidence along the chain "blocks" the influence.

Common Cause

- This configuration is a "common cause":



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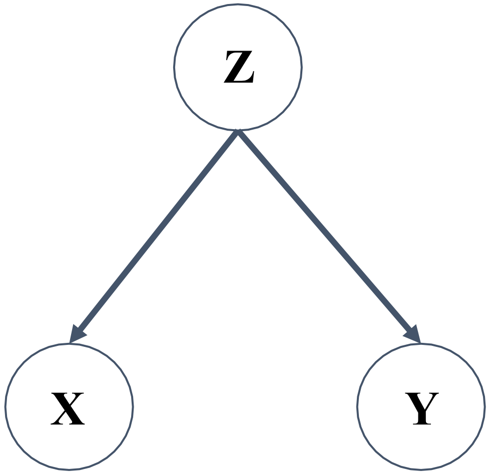


- Guaranteed X and Y independent given Z?

$$P(X|Y,Z) = \frac{P(X,Y,Z)}{P(Y,Z)}$$

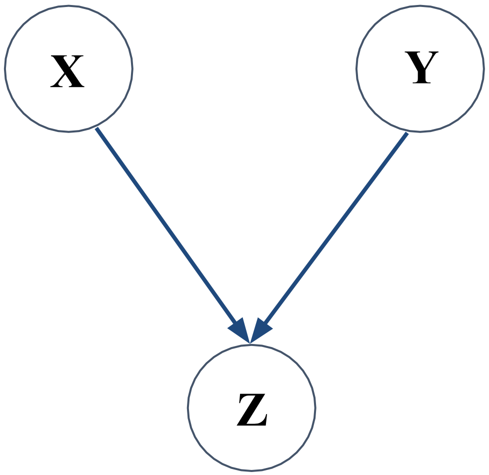
$$= \frac{P(X|Z)P(Y|Z)P(Z)}{P(Y|Z)P(Z)} = P(X|Z)$$

  – **Answer: Yes!**

  – Observing the cause "blocks" influence between effects.

Common Effect

- This configuration is a "common effect" (v-structure):



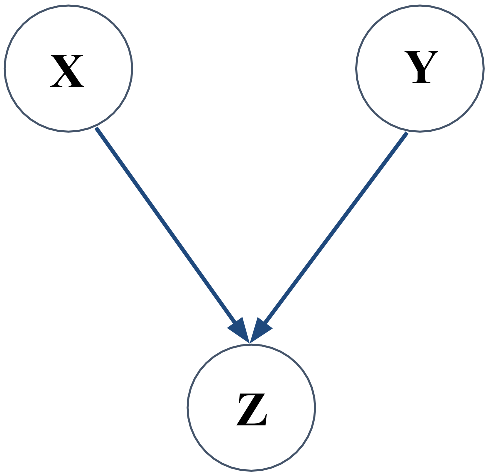
- Are X and Y independent?

$$P(X,Y) = \sum_z P(X,Y,z) = \sum_z P(X)P(Y)P(z|X,Y) = P(X)P(Y)$$

  – Answer: Yes!

Common Effect

- This configuration is a "common effect" (v-structure):



- Are X and Y independent given Z?
  - **Answer: No**
- This is backwards from the other cases.
  - Observing an effect "activates" influence between possible causes.

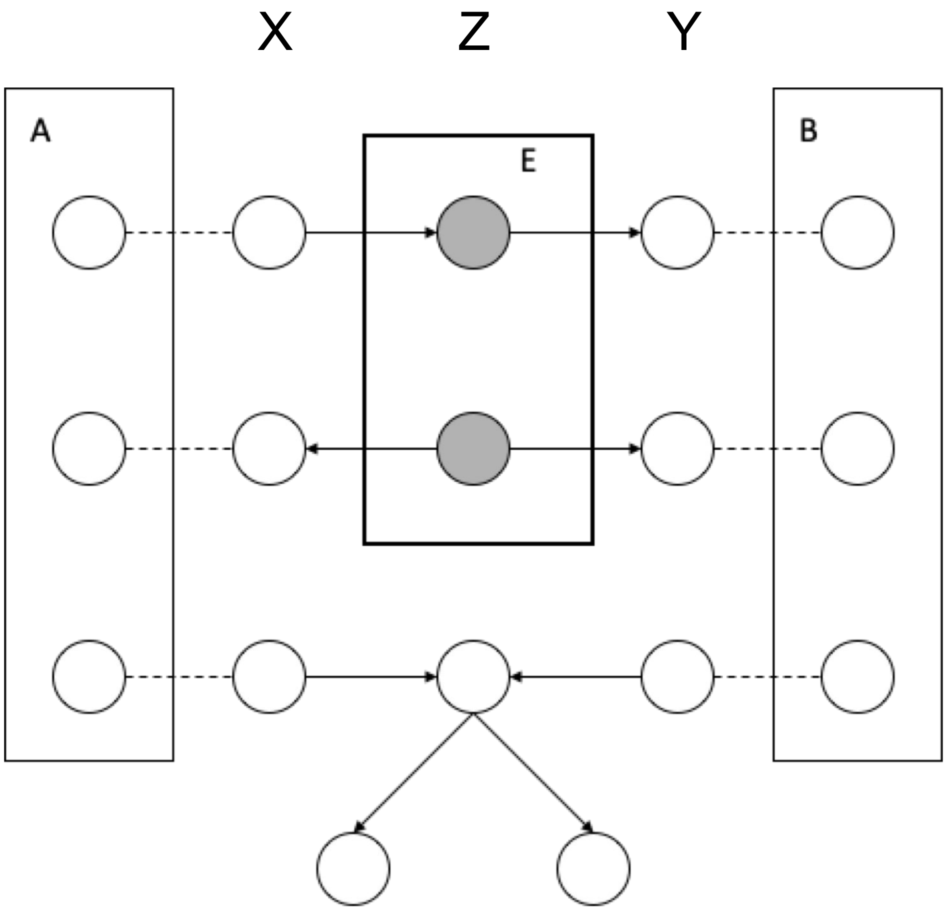
Active / Inactive Path

$A \perp\!\!\!\perp B \mid E$?

- Consider any **undirected** path from A to B.
- A path is **active** if **each triple** on the path is **active**:
  - Causal chain X → Z → Y where Z is unobserved (either direction) (not in E)
  - Common cause X ← Z → Y where Z is unobserved (not in E)
  - Common effect X → Z ← Y where Z is observed or one of its descendants is observed (in E)
- All it takes to block a path is a **single inactive segment** — D-separation.

D-Separation

- If **every** undirected path from a node in A to a node in B is D-separated by E, then A and B are conditionally independent given E.



D-Separation Example

- Cavity and Visit Pharmacy are **independent** given Toothache.
- Cavity and Abscess are **independent** given no evidence about Toothache, Visit Dentist, or Visit Pharmacy.
  - Otherwise, they are **dependent**.