Linear Algebra: Basic Notations (1/4)

- A d-dimensional column vector $\mathbf{x}$ and its transpose $\mathbf{x}^t$

- n by d matrix M and its d by n transpose M^t

Linear Algebra: Basic Notations (2/4)

- A square matrix M is symmetric if

- Multiplying a vector by a matrix: M**x** = y

- Multiplying two matrices M_1 **and** M_2

Linear Algebra: Basic Notations (3/4)

- The identity matrix $\mathbf{I}$ of d by d

- Inner product of two vectors $\mathbf{x}^t\mathbf{y}$

- Outer product of two vectors $\mathbf{x}\mathbf{y}^t$

Linear Algebra: Basic Notations (4/4)

- The length or Euclidean **norm** of a vector $\mathbf{x}$, denoted $||\mathbf{x}||$

- Normalized vector, $||\mathbf{x}|| = 1$

Matrix: Additional Definitions (1/2)

- Determinant of a matrix M: denoted |M| or det(M)
  - Look at size 2x2
  - What about size 3x3 and above?
- Trace of a matrix

Matrix: Additional Definitions (2/2)

- Matrix inversion M^{-1}

- Eigenvectors and eigenvalues of M

Derivatives Involving Matrices (1/3)

- If the entries of a matrix M depend on a scalar parameter θ, we have

$$\frac{\partial \mathrm{M}}{\partial \theta} =$$

- Derivative of a scalar-valued function $f(\mathbf{x})$ of d variables x_i, i=1,…,d, and $\mathbf{x}=(x_1, \ldots, x_d)^t$, or the gradient w.r.t. $\mathbf{x}$ is

$$\nabla f(\mathbf{x}) = \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} =$$

Derivatives Involving Matrices (2/3)

- If $\mathbf{f}(\mathbf{x})$ is an n-dimensional vector-valued function of d variables x_i, $i=1,\ldots,d$, and $\mathbf{x}=(x_1, \ldots, x_d)^t$, we have the derivative as*

$$\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} =$$

* We could use the Jacobian form too; See "numerator layout" vs "denominator layout" in matrix calculus.

Derivatives Involving Matrices (3/3)

Some useful results

$$\frac{\partial}{\partial \mathbf{x}}[\mathrm{M}\mathbf{x}] =$$

$$\frac{\partial}{\partial \mathbf{x}}[\mathbf{y}^{\mathrm{t}}\mathbf{x}] =$$

$$\frac{\partial}{\partial \mathbf{x}}[\mathbf{x}^{t}\mathrm{M}\mathbf{x}] =$$