Inference in Bayesian Networks Part 1

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The lecture is based on the slides developed by Prof. Yu Zhang from ASU School of Computing and Augmented Intelligence

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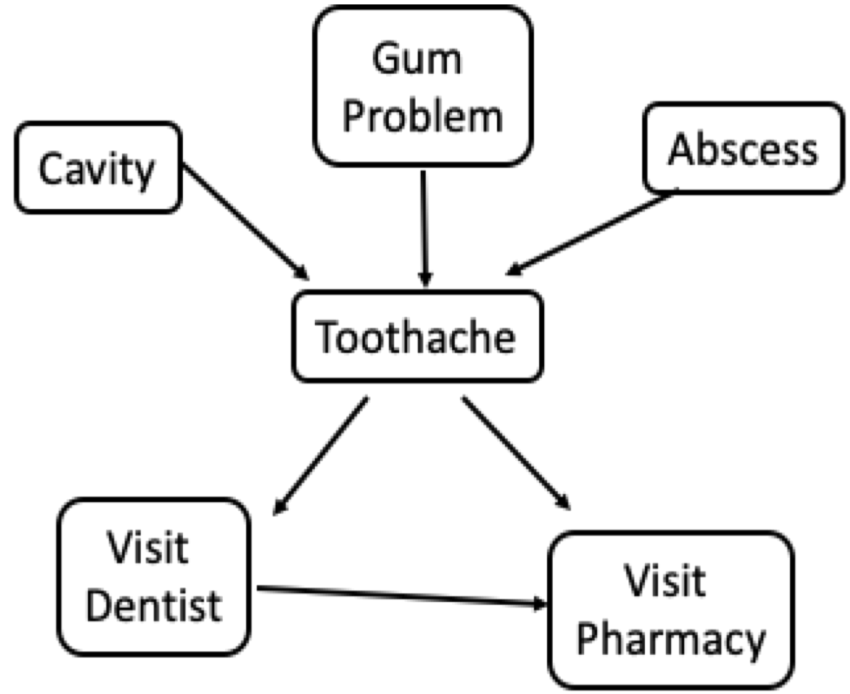
Inference in Bayesian Networks

- Given a model and some data ("evidence"), how do we update our belief?

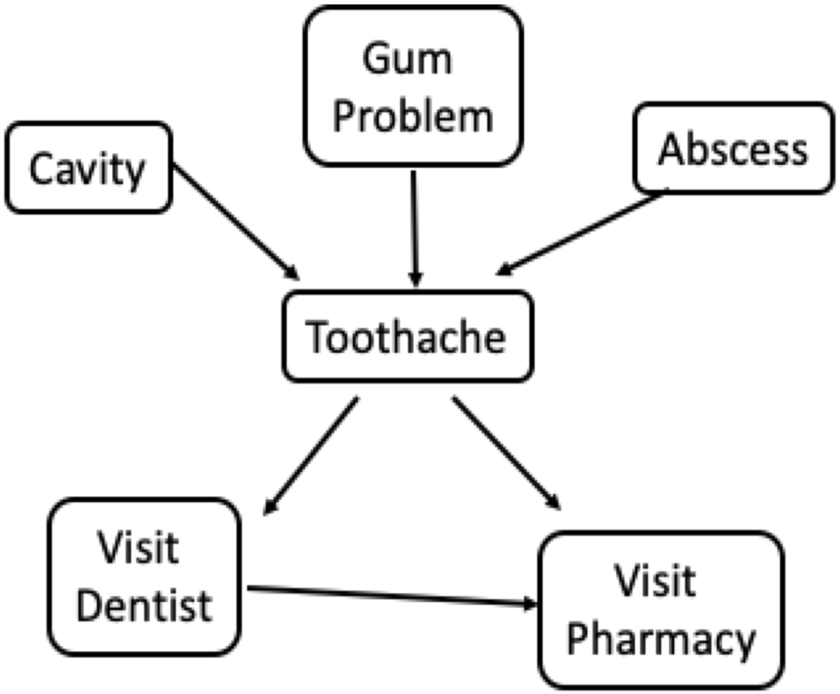
- What are the model parameters?



Inference in Bayesian Networks (cont'd)

- **Given a model and some data ("evidence"), how do we update our belief?**

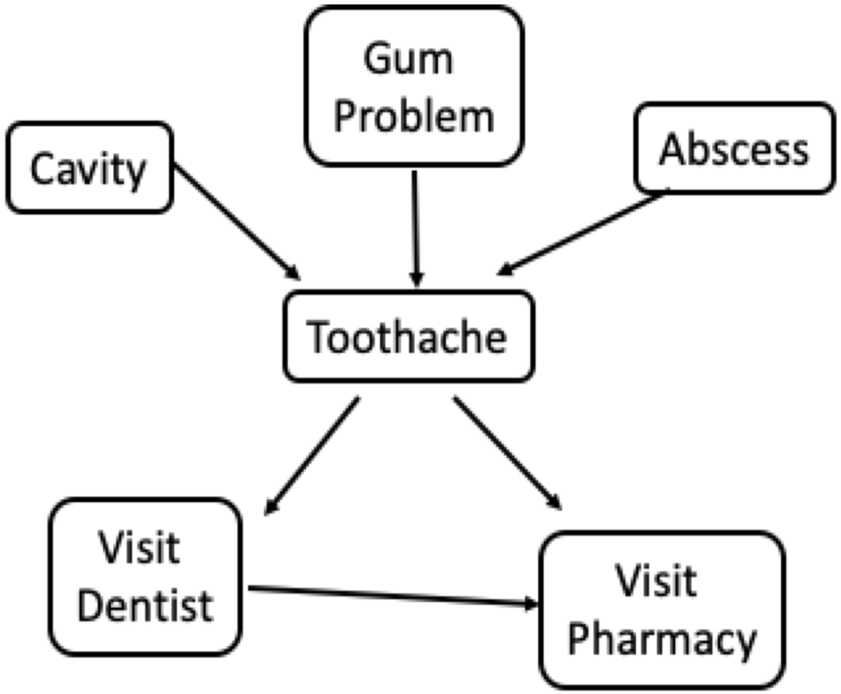
- **Example: For a patient with a history of gum problems who has visited both the dentist and the pharmacy:**
  - What is probability that the patient has a **Toothache**?



Inference in Bayesian Networks (cont'd)

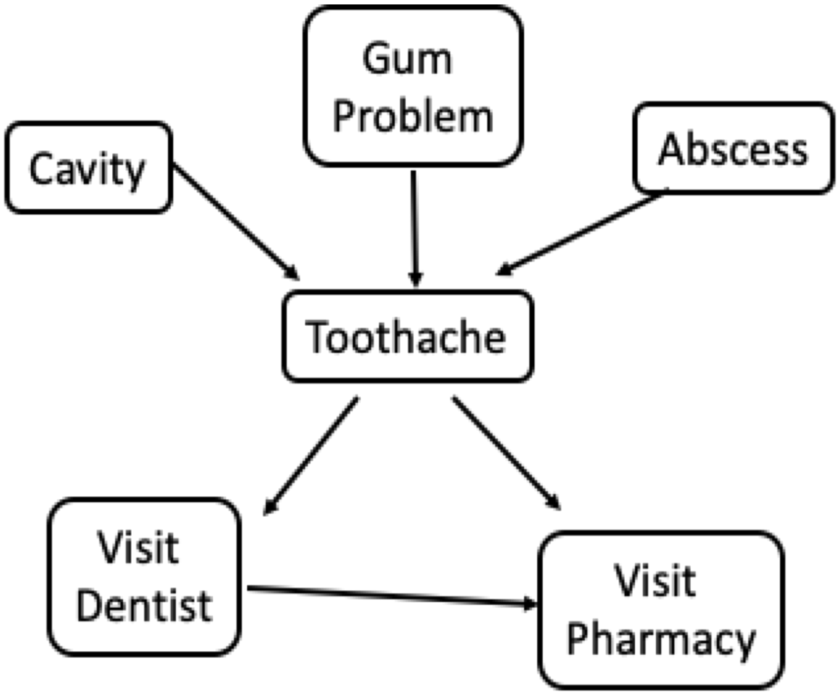
- In a simple BN like this, we can compute the exact probabilities.

- In general, for a tree-structured BN, we may use belief propagation for the inference problem.



Inference in Bayesian Networks (cont'd)

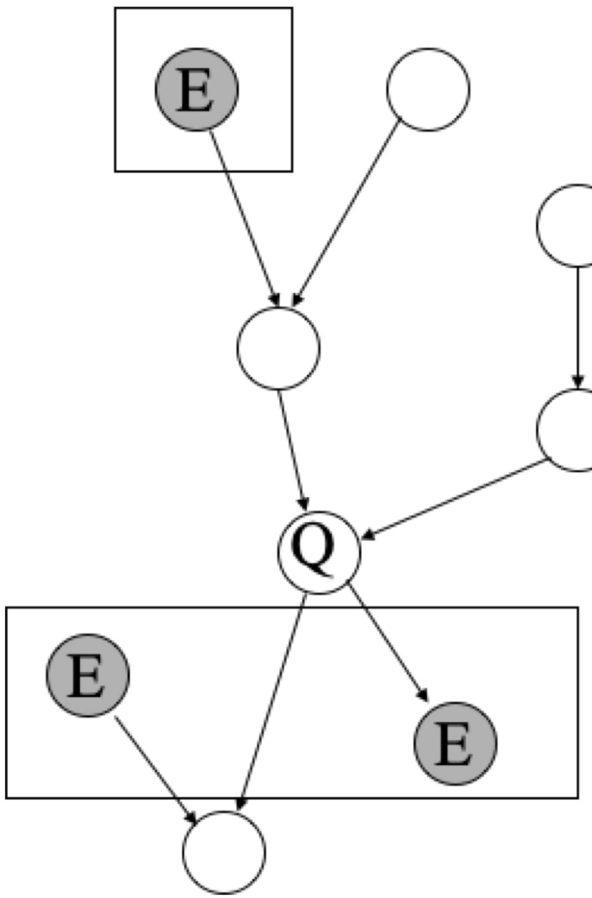
- **For general structures, sometimes it is possible to generalize this method (e.g., the junction tree algorithm).**
- **More often, we must resort to approximation methods.**
  - Examples:
    - Variational methods
    - Sampling (Monte Carlo) methods



Inference

- Inference: Calculating some useful quantity from a joint probability distribution.

- Examples:
  - Posterior probability
    $P(Q|E_1 = e_1, \ldots E_k = e_k)$
  - Most likely explanation:
    $argmax_q P(Q = q \mid E_1 = e_1, \ldots)$



Inference by Enumeration

General case:

<table>
<tr><td>Evidence variables:</td><td>$E_1,\ldots E_k = e_1,\ldots e_k$</td><td rowspan="3">$X_1, X_2,\ldots X_n$
All variables</td></tr>
<tr><td>Query variable:</td><td>Q</td></tr>
<tr><td>Hidden variables:</td><td>$H_1,\ldots H_r$</td></tr>
</table>

We want:

- $P(Q|E_1 = e_1,\ldots E_k = e_k)$

Inference by Enumeration (cont'd)

We want:

- $P(Q|E_1 = e_1, \ldots E_k = e_k)$

Step 1:

– Select the entries consistent with the evidence:

- Gives us the probabilities for these entries:

  $P(Q, e_1, \ldots e_k, H_1, \ldots H_r)$

Inference by Enumeration (cont'd)

We want:

- $P(Q|E_1 = e_1, \ldots E_k = e_k)$

Step 2:

– Sum out H to get joint of Query and evidence
$P(Q, e_1, \ldots e_k) = \sum h_1, \ldots h_r \, P(Q, h_1, \ldots h_r, e_1, \ldots e_k)$

Inference by Enumeration (cont'd)

We want:

- $P(Q|E_1 = e_1, \ldots E_k = e_k)$

Step 3:

- Normalize $\times \frac{1}{Z}$

When normalized by dividing by $P(e_1,\ldots e_k)$, it produces the distribution $P(Q \mid e_1,\ldots e_k)$

$$P(Q|e_1, \ldots e_k) = \frac{1}{Z} P(Q, e_1, \ldots e_k)$$

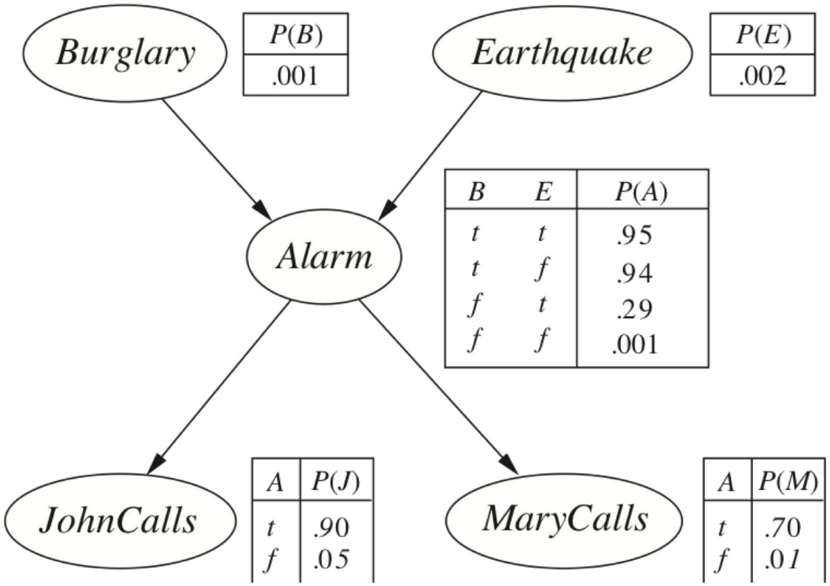
Normalization delay to the last step since no need to compute $P(e_1,\ldots e_k)$

Inference by Enumeration (cont'd)

Evidence variables:	$+j,\ +m$	A, B, E, J, M *All variables*
Query variable:	B	
Hidden variables:	A, E	

We want:

- $P(B \mid +j, +m)$



Artificial Intelligence: A Modern Approach *3rd Edition*.

Inference by Enumeration (cont'd)

$P(B|+j,+m)$

$\propto P(B,+j,+m)$

$= \sum_{a,e} P(B,+j,+m,a,e)$

$= \sum_{a,e} P(B)P(e)P(a|B,e)P(+j|a)P(+m|a)$

$= P(B)P(+e)P(+a|B,+e)P(+j|+a)P(+m|+a)$

$+P(B)P(+e)P(-a|B,+e)P(+j|-a)P(+m|-a)$

$+P(B)P(-e)P(+a|B,-e)P(+j|+a)P(+m|+a)$

$+P(B)P(-e)P(-a|B,-e)P(+j|-a)P(+m|-a)$