Practice of Answer Set Programming

Elaborations of Sudoku Puzzle

Ira A. Fulton Schools of Engineering
Arizona State University

Objectives

Objective

Solve grid puzzle in ASP

Objective

Apply the concept of elaboration tolerance to representing knowledge

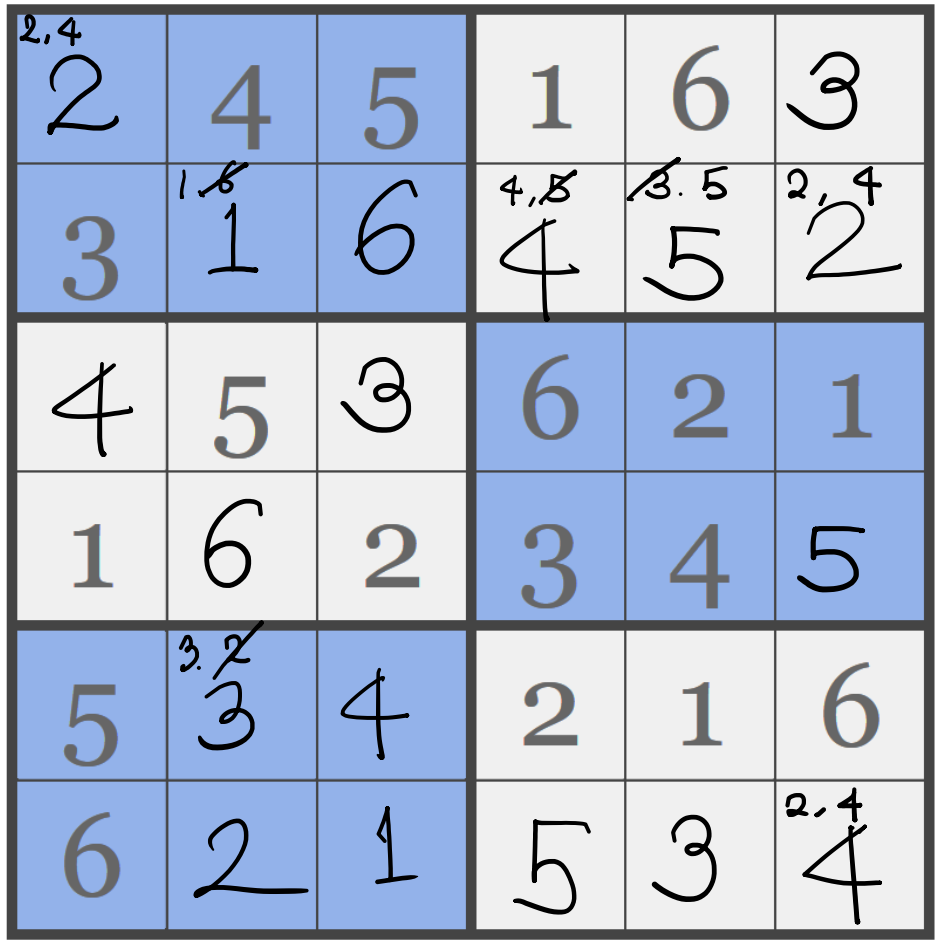
Sudoku in ASP: Elaboration Tolerance

Introduction

9				8		3		
			2	5		7		
	2		3					4
	9	4						
			7	3		5	6	
7		5		6		4		
		7	8		3	9		
		1						3
3								2

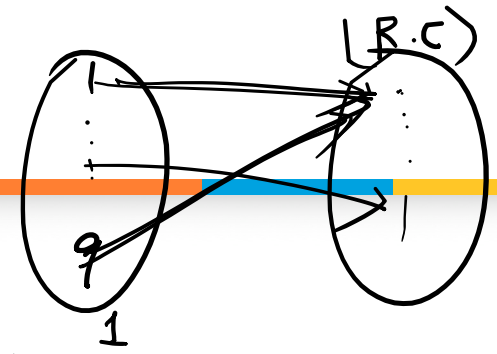
9	7	6	4	8	1	3	2	5
1	4	3	2	5	9	7	8	6
5	2	8	3	7	6	1	9	4
6	9	4	5	1	8	2	3	7
8	1	2	7	3	4	5	6	9
7	3	5	9	6	2	4	1	8
4	6	7	8	2	3	9	5	1
2	5	1	6	9	7	8	4	3
3	8	9	1	4	5	6	7	2

6 x 6 Sudoku



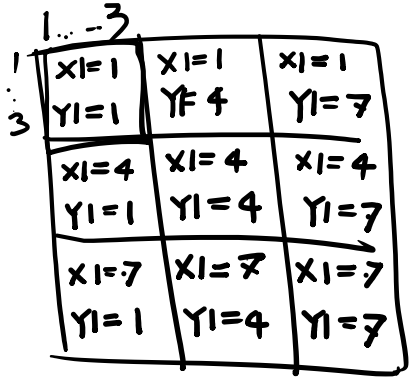
2	4	5	1	6	3
3	1	6	4	5	2
4	5	3	6	2	1
1	6	2	3	4	5
5	3	4	2	1	6
6	2	1	5	3	4

Sudoku in ASP



% Each number 1..9 is assigned to one cell in each box

```
1 { a(X,Y,N):X=1..9,Y=1..9,X1<=X,X<=X1+2,Y1<=Y,Y<=Y1+2 } 1
                    :- N=1..9, X1 = 3*(0..2)+1, Y1 = 3*(0..2)+1.
```



Global variables: $N \in \{1, \ldots, 9\}$, $X1 \in \{1, 4, 7\}$, $Y1 \in \{1, 4, 7\}$

The rule expands into

When N=1, X1=1, Y1=1:

```
1{a(X,Y,1):1<=X,X<=3,1<=Y,Y<=3}1.
```

N=2 //

N=3 //

⋮

N=9 //

Sudoku in ASP

% Each number 1..9 is assigned to one cell in each box

```
1 { a(X,Y,N):X=1..9,Y=1..9,X1<=X,X<=X1+2,Y1<=Y,Y<=Y1+2 } 1
                    :- N=1..9, X1 = 3*(0..2)+1, Y1 = 3*(0..2)+1.
```

Global variables: $N \in \{1, \ldots, 9\}$, $X1 \in \{1, 4, 7\}$, $Y1 \in \{1, 4, 7\}$

The rule expands into

When N=5, X1=4, Y1=7: `1{a(X,Y,5):4<=X,X<=6,7<=Y,Y<=9}1.`

Sudoku in ASP

```
% Each number 1..9 is assigned to one cell in each box
1 { a(X,Y,N):X=1..9,Y=1..9,X1<=X,X<=X1+2,Y1<=Y,Y<=Y1+2 } 1
                  :- N=1..9, X1 = 3*(0..2)+1, Y1 = 3*(0..2)+1.

% no two different numbers given a row and a column
:- a(X,Y,N), a(X,Y,N1), N!=N1.

% no two different columns given a row and a number
:- a(X,Y,N), a(X,Y1,N), Y!=Y1.

% no two different rows given a column and a number
:- a(X,Y,N), a(X1,Y,N), X!=X1.
```

Instance:

```
a(1,1,9).   a(1.5,8).  a(1,7,3). …
```

World’s Hardest Sudoku?

World's hardest sudoku: can you crack it?

Readers who spend hours grappling in vain with the Telegraph's daily sudoku puzzles should look away now.

211 0 in 5 216 Email

The Everest of numerical games was devised by Arto Inkala, a Finnish mathematician, and is specifically designed to be unsolvable to all but the sharpest minds.

(from www.telegraph.co.uk)

16 X 16 Sudoku

9	14				3		5	15		2				7	1
6	12				14					10				5	11
4			7	6			13	16			1	2			9
	15	16		9	7					11	6		3	14	
	7	15											2	16	
5		13		14		15			10		3		1		8
	8		10		9	4	11	13	6	15		14		3	
16				5		3			14		9				6
15				16		10			9		13				14
	9		6		5	13	3	1	15	4		7		12	
2		8		15		14			16		12		5		13
	13	12											9	11	
	5	3		2	16					13	10		12	9	
8			4	12			1	6			7	15			3
10	1				15					16				6	2
11	2				8		14	3		1				10	7

Elaboration Tolerance [McCarthy, 2003]

> "A formalism is elaboration tolerant to the extent that it is convenient to modify a set of facts expressed in the formalism to take into account new phenomena or changed circumstances."

> "Representations of information in natural language have good elaboration tolerance when used with human background knowledge."

> "Human-level AI will require representations with much more elaboration tolerance than those used by present AI programs, because human-level AI needs to be able to take new phenomena into account."

Offset Sudoku

A region is represented by the same color. In addition to the requirement of Sudoku, every region must contain all the digits 1 through 9.

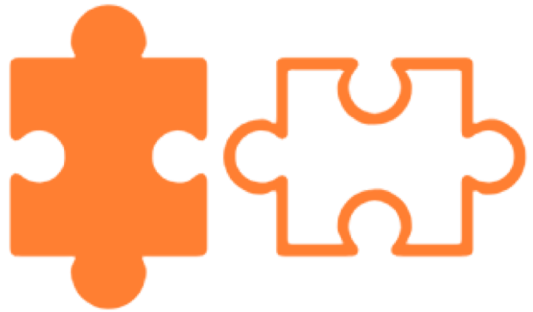
Offset Sudoku in ASP

Add to the basic program:

```
:- a(R,C,N), a(R1,C1,N),
   R\3 == R1\3, C\3 == C1\3,
   1{R != R1; C != C1}.
```

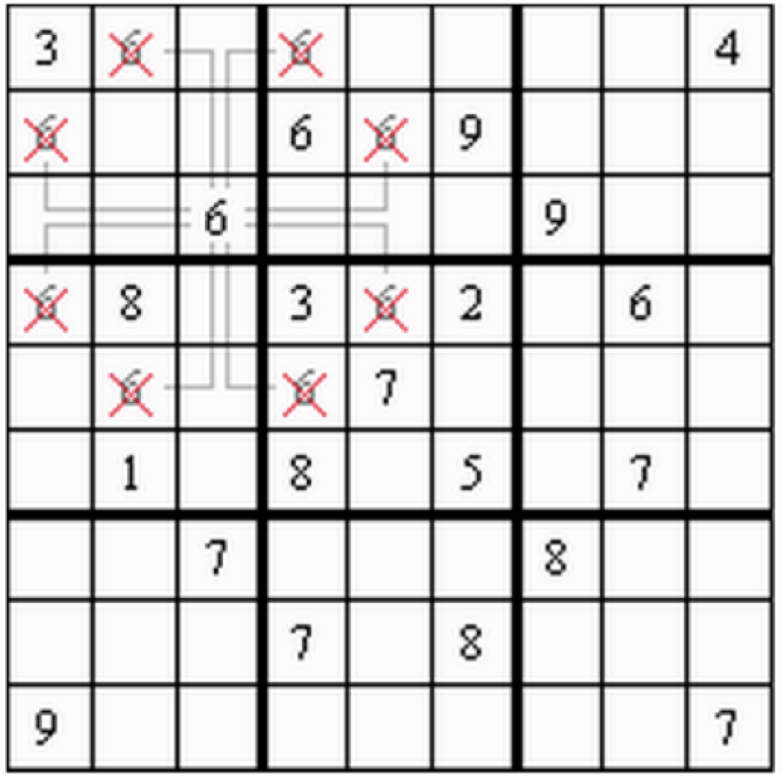
or

```
:- a(R,C,N), a(R1,C1,N),
   R\3 == R1\3, C\3 == C1\3,
   R != R1, C != C1.
```



Anti-Knight Sudoku

Cells that are a chess knight's move away from each other cannot hold equal values:



3	9	1	5	2	7	6	8	4
4	2	5	6	8	9	7	3	1
8	7	6	4	3	1	9	5	2
7	8	9	3	1	2	4	6	5
6	5	3	9	7	4	2	1	8
2	1	4	8	6	5	3	7	9
1	4	7	2	5	3	8	9	6
5	6	2	7	9	8	1	4	3
9	3	8	1	4	6	5	2	7

Anti-Knight Sudoku in Clingo

Add to the basic program

```
:- a(R,C,N), a(R-2,C-1,N).    :- a(R,C,N), a(R-2,C+1,N).
:- a(R,C,N), a(R-1,C-2,N).    :- a(R,C,N), a(R-1,C+2,N).
:- a(R,C,N), a(R+1,C-2,N).    :- a(R,C,N), a(R+1,C+2,N).
:- a(R,C,N), a(R+2,C-1,N).    :- a(R,C,N), a(R+2,C+1,N).
```

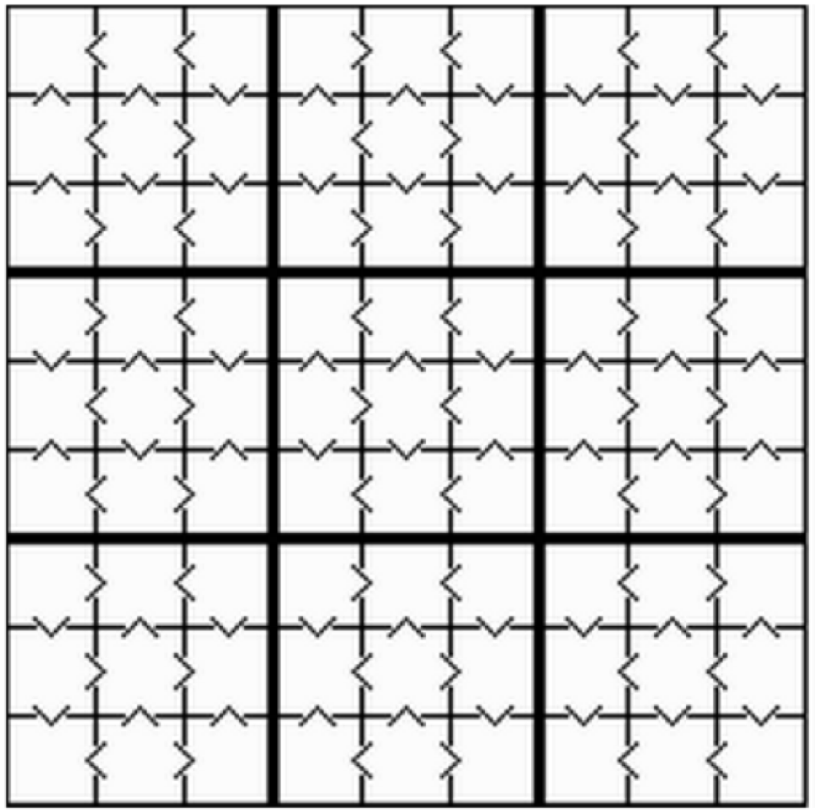
or simply,

```
:- a(R,C,N), a(R1,C1,N), |R1-R|+|C1-C|==3.
```

Q: Why does the simple rule work?

Greater-Than Sudoku

No numerical clues; Only greater-than relationships



2	3	9	5	1	4	6	7	8
4	7	6	8	9	3	1	2	5
8	1	5	7	6	2	9	3	4
9	6	7	3	4	8	5	1	2
1	8	2	9	5	6	7	4	3
3	5	4	1	2	7	8	9	6
7	2	8	6	3	9	4	5	1
6	4	1	2	7	5	3	8	9
5	9	3	4	8	1	2	6	7

Greater-Than Sudoku in ASP

Let gt(R,C,R1,C1) represent that the number in (R,C) is greater than the number in (R1,C1)

Add to the basic program

```
:- a(R,C,N), a(R1,C1,N1), gt(R,C,R1,C1), N <= N1.
```

Together with input data:

```
gt(1,2,1,1).   gt(1,3,1,2).   gt(2,1,1,1).
…
```

Wrap-Up