Definition of eigenvalues and eigenvectors of a matrix

Let **A** be any square matrix. A non-zero vector $\bar{\mathbf{x}}$ is an **eigenvector** of **A** if

$A\bar{x} = \lambda\bar{x}$

for some number λ, called the corresponding **eigenvalue**.

Steps to find the eigenvalues and eigenvectors of a nxn matrix

1. Multiply a nxn identity matrix by the scalar λ. $\lambda\mathbf{I}$
2. Subtract the identity matrix multiple from the matrix A. $(\mathbf{A} - \lambda\mathbf{I})$
3. Find the determinant of the matrix and the difference. Det $(\mathbf{A} - \lambda\mathbf{I})$
4. Solve for the values of λ that satisfy the equation $|\mathbf{A} - \lambda\mathbf{I}| = 0$
5. Solve for the corresponding vector to each λ. $\bar{x}\ (\mathbf{A} - \lambda\mathbf{I}) = \bar{0}$

Is it correct?

We can check by substituting:

$$A\bar{x} = \lambda\bar{x}$$

Here is an example:

Example of Eigen value and Eigen vector:

2x2 matrix(A)

$$A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$$

step 1 $\lambda I \Rightarrow \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

step 2 $A - \lambda I = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{bmatrix}$

Step 3 $\det \begin{bmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{bmatrix} \Rightarrow (7-\lambda)(-1-\lambda) - (3)(3) \Rightarrow$

$$-7 - 7\lambda + \lambda + \lambda^2 - 9 \Rightarrow \lambda^2 - 6\lambda - 14 =$$

$$\lambda^2 - 6\lambda - 16 = 0 \rightarrow (\lambda - 8)(\lambda + 2) = 0$$

$\boxed{\lambda = 8}$ $\boxed{\lambda = -2}$

Step 4

eigen values

step 5 $\begin{bmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{bmatrix}$

we use $\boxed{\lambda = 8}$ to find corresponding eigen vector (X

$$\begin{bmatrix} 7-8 & 3 \\ 3 & -1-8 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix}}_{B}$$

Solve: $B\bar{X} = \bar{0}$

$$\begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -X_1 + 3X_2 = 0$$

$$3X_2 = X_1$$

$$\boxed{X_2 = 1}$$

$$\boxed{X_1 = 3}$$

So, with $\lambda = 8$ corresponding vector is $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

check $\Rightarrow$ $A\bar{x} = \lambda \bar{x}$

$$\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 8 \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \end{bmatrix}$$

we can repeat step 5 for $\lambda = -2$ and corresponding vector is $X_1 = 1$ $X_2 = -3$