# KRR with Uncertainty

## $LP^{MLN}$ Relationships to Other Languages

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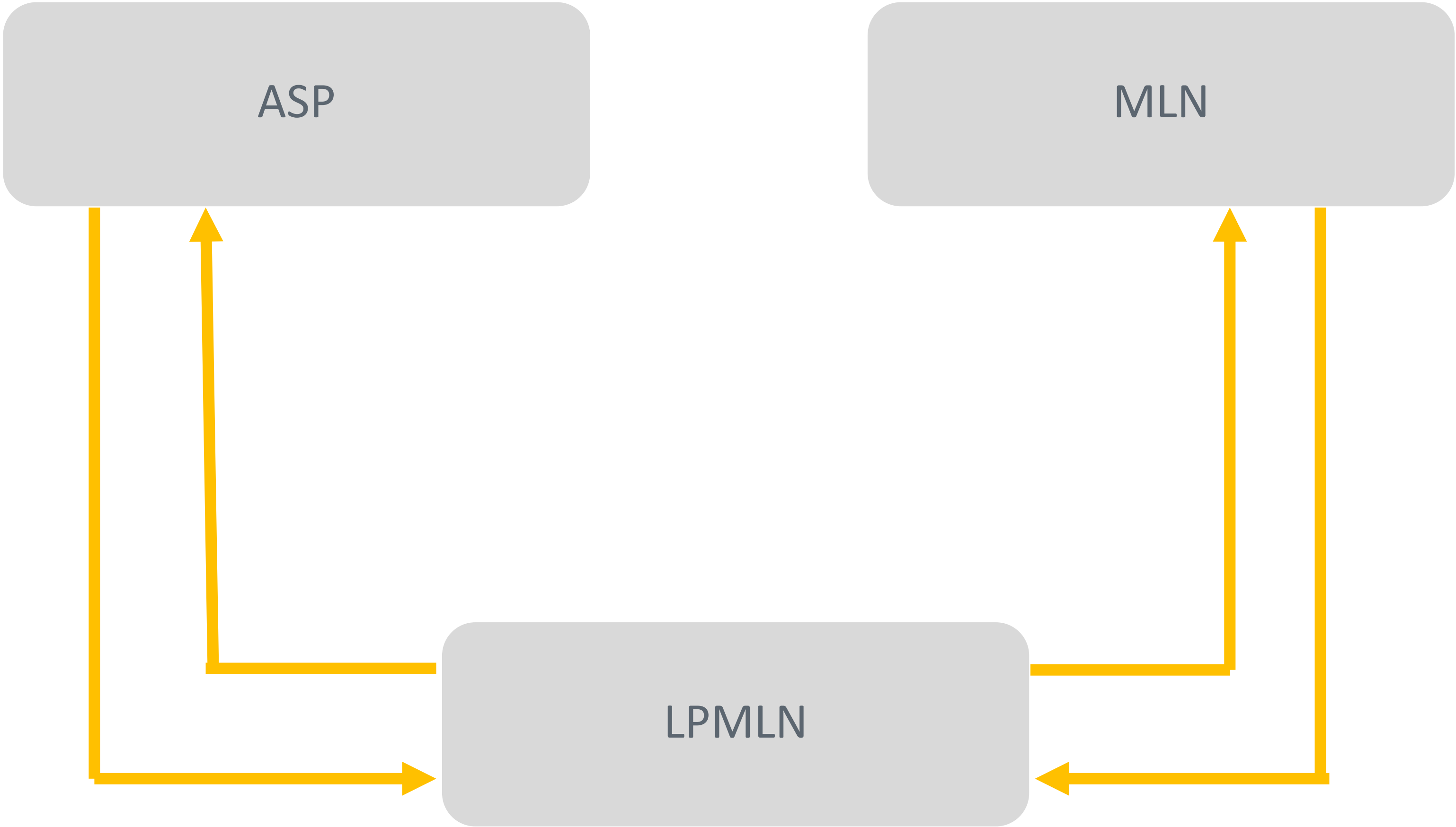
# Objectives

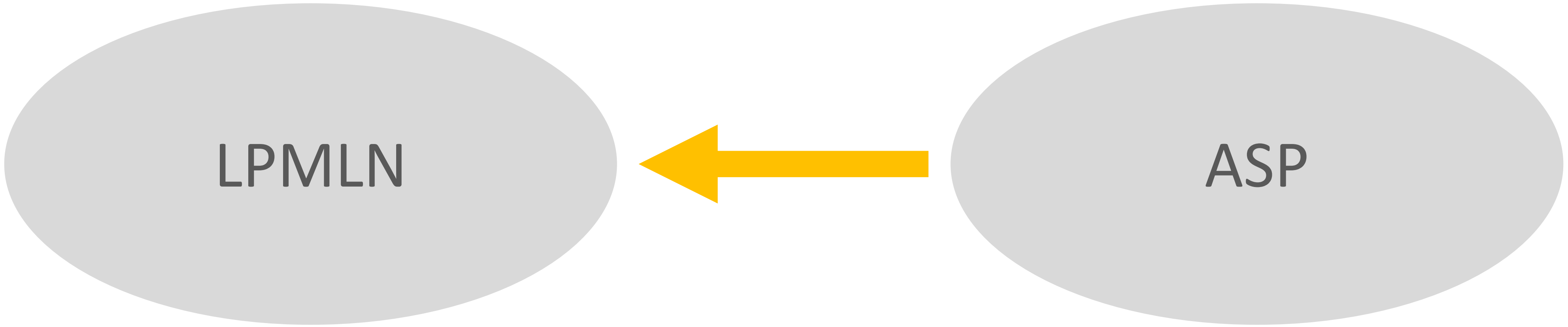
## Objective

Explain the relationships between LPMLN and other languages

# LPMLN vs. ASP vs. MLN



# From ASP to LP$^{MLN}$



# ASP as a Special Case of LP$^{MLN}$

- Any answer set program $\Pi$ can be viewed as a special case of an LP$^{MLN}$ program $P_\Pi$ by assigning the infinite weight to each rule

| $\Pi$ | $p \leftarrow \text{not } q$ | $P_\Pi$ | $\alpha:\ p \leftarrow \text{not } q$ |
|---|---|---|---|
| | $q \leftarrow \text{not } p$ | | $\alpha:\ q \leftarrow \text{not } p$ |

$\{p\}$
$\{q\}$

$P_\pi$

| $I$ | $W(I)$ | $P(I)$ |
|---|---|---|
| $\phi$ | $e^0$ | $0$ |
| $\{p\}$ | $e^{2\alpha}$ | $\frac{1}{2}$ |
| $\{q\}$ | $e^{2\alpha}$ | $\frac{1}{2}$ |
| $\{p, q\}$ | $0$ | $0$ |

- **Theorem:** For any answer set program $\Pi$, the (deterministic) stable models of $\Pi$ are exactly the (probabilistic) stable models of LP$^{MLN}$ program $P_\Pi$ whose weight is $e^{k\alpha}$ , where k is the number of all ground rules in $\Pi$

# Example

- If Π has at least one (deterministic) stable model, then all (probabilistic) stable models of $P_\Pi$ have the same probability, and are thus the stable models of Π as well

- **Q**: What if Π has no stable models?

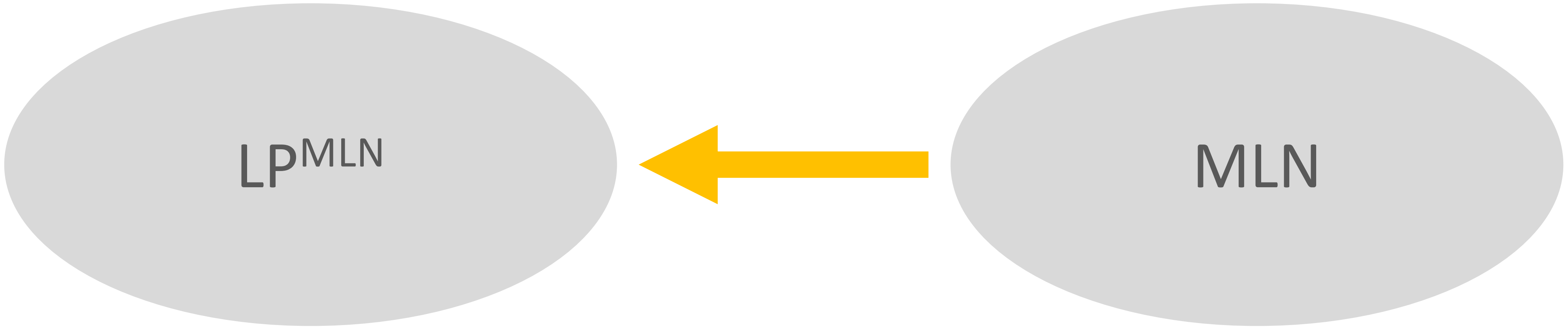
$\Pi$ Bird(Jo) ← ResidentBird(Jo)
Bird(Jo) ← MigratoryBird(Jo)
⊥ ← ResidentBird(Jo), MigratoryBird(Jo)
ResidentBird(Jo)
MigratoryBird(Jo)

$P_\Pi$ $\alpha$: Bird(Jo) ← ResidentBird(Jo)
$\alpha$: Bird(Jo) ← MigratoryBird(Jo)
$\alpha$: ⊥ ← ResidentBird(Jo), MigratoryBird(Jo)
$\alpha$: ResidentBird(Jo)
$\alpha$: MigratoryBird(Jo)

- **Q**: What are the stable models $P_\Pi$?

{B(Jo), R(Jo)}, {B(Jo), M(Jo)}, {B(Jo)}

# From MLN to LP$^{MLN}$



# Embedding Propositional Logic in ASP

- **Theorem. For any propositional formula F of a finite signature $\sigma$, X is a model of $\boxed{\text{F}}$ iff X is a stable model of $\boxed{F \wedge Ch}$ where $Ch$ is the conjunction of the choice rules $\{\sigma\}^{ch}$.**
  - The effect of adding the choice rules is to exempt A from minimization under the stable model semantics

$$F = p \leftarrow \neg q$$

**models of $F$:** $\boxed{\{p\},\ \{q\},\ \{p, q\}}$

**stable models of $F$:** $\{p\}$

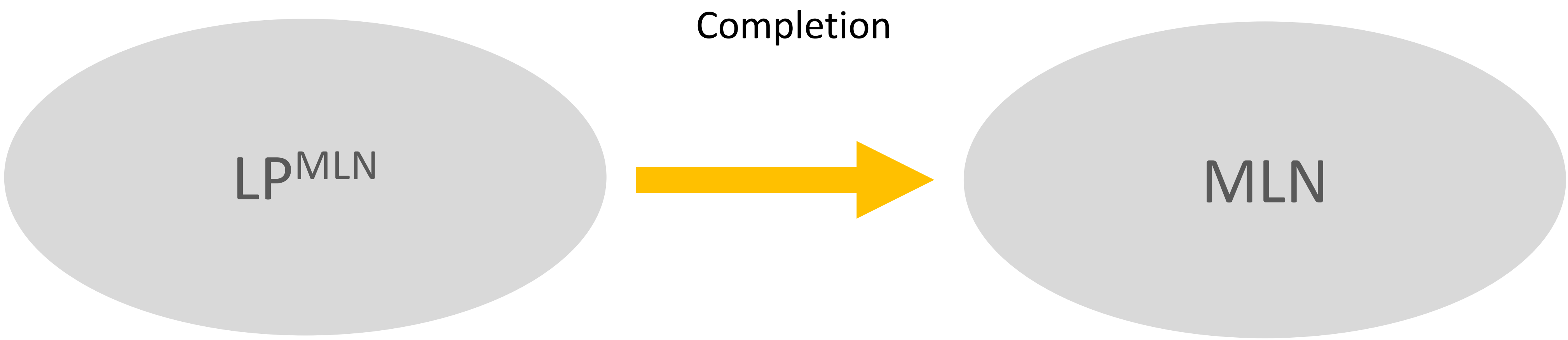
**stable models of $F \wedge \{p; q\}^{ch}$:** $\boxed{\{p\},\ \{q\},\ \{p, q\}}$

# Embedding MLN in LP$^{MLN}$

- For any MLN $L$, LP$^{MLN}$ program $\Pi_L$ is obtained from $L$ by adding $w : \{A\}^{ch}$ for every ground atom A of σ and any weight w

- Theorem: Any MLN $L$ and its LP$^{MLN}$ counterpart $\Pi_L$ have the same probability distribution over all interpretations

# From LP$^{MLN}$ to MLN



# Turning LP$^{\text{MLN}}$ into MLN (1 of 2)

- We first consider how to turn an ASP program into a propositional formula.

- **Completion** is a process that turns an ASP program $\Pi$ into a propositional formula $F$ so that the stable models of $\Pi$ are precisely the models of $F$.

# Turning LP$^{\text{MLN}}$ into MLN (2 of 2)

- The process works only for "tight" ASP programs (defined later).

- The method can be generalized to turning an LP$^{\text{MLN}}$ program $\Pi$ into an MLN program $L$ so that the probabilistic answer sets of $\Pi$ are precisely the models of $L$ with the same probability distribution.

# Completion

- For any ground ASP program $\Pi$ that consists of rules of the form
  - $A \leftarrow Body$
  - where $A$ is an atom and $Body$ is a formula,
- The completion of $\Pi$ is defined as the union of $\Pi$ and

$$A \rightarrow \bigvee_{A \leftarrow Body \in \Pi} Body$$

for each ground atom A

**Theorem:** For any "tight" answer set program $\Pi$, the stable models of $\Pi$ are exactly the models of the completion of $\Pi$ .

# Example 1

## Stable models of

$$p \leftarrow \neg q$$
$$q \leftarrow \neg p$$

$\{p\}$
$\{q\}$

$A \rightarrow \bigvee$ Body

$A \leftarrow$ Body

## Models of completion

$p \leftarrow \neg q$
$q \leftarrow \neg p$
$p \rightarrow \neg q$
$q \rightarrow \neg p$

$\boxed{p \leftrightarrow \neg q}$

$\{p\}$
$\{q\}$

# Example 2

## Stable models of

$$p \leftarrow \neg q$$
$$q \leftarrow \neg r$$

$\{q\}$

$A \rightarrow \bigvee \text{Body}$

$A \leftarrow \text{Body}$

## Models of completion

$$p \leftarrow \neg q$$
$$q \leftarrow \neg r$$
$$p \rightarrow \neg q$$
$$q \rightarrow \neg r$$
$$r \rightarrow \bot$$

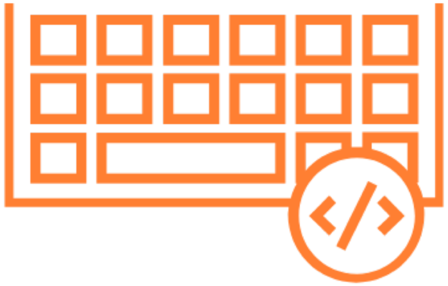
$$p \leftrightarrow \neg q$$
$$q \leftrightarrow \neg r$$
$$\neg r$$

$\{q\}$

# Tight Programs

**Theorem:** For any "tight" answer set program $\Pi$, the answer sets of $\Pi$ are exactly the models of the completion of $\Pi$

What would go wrong if $\Pi$ is non-tight?



# Completion and Non-tight programs

$$p \leftarrow q$$

$$q \leftarrow p$$

**Models:** $\emptyset$ , $\{p, q\}$

**Stable models:** $\emptyset$

**Completion:**

$$\left.\begin{array}{l} p \leftarrow q \\ q \leftarrow p \\ p \rightarrow q \\ q \rightarrow p \end{array}\right\} \Leftrightarrow \quad p \Leftrightarrow q$$

Models of Comp: $\emptyset$ , $\{p, q\}$

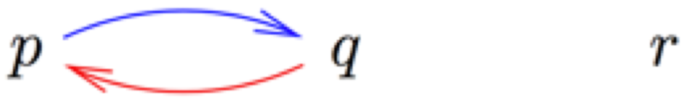
# Positive Dependency Graph

- **A program is a finite set of rules of the form**

$$a \leftarrow \underbrace{a_1, \ldots, a_m}_{P}, \underbrace{\textit{not } a_{m+1}, \ldots, \textit{not } a_n}_{N} .$$

- **The positive dependency graph of $\Pi$ is the directed graph such that**
  - its vertices are the atoms occurring in $\Pi$, and
  - for each $a \leftarrow P, N$ in $\Pi$, its edges go from $a$ to each atom in $P$.

$$\begin{aligned} \Pi_1 : \quad & p \leftarrow q \\ & q \leftarrow p \\ & r \leftarrow \textit{not } p \end{aligned}$$



# Loop

- A nonempty set L of atoms is called a loop of $\Pi$ if, for every pair $a_1, a_2$ of atoms in L, there exists a path of non-zero length from $a_1$ to $a_2$ in the positive dependency graph of $\Pi$ such that all vertices in this path belong to L.

$$\begin{tikzcd} p \arrow[r, bend left] & q \arrow[l, bend left] & & r \end{tikzcd}$$

- $\Pi_1$ has only one loop: $\{p, q\}$.

- A program is called **tight** if it has no loops.

# Which of these Examples is a Tight Program?

A.

$$\begin{aligned} p &\leftarrow \neg q \\ q &\leftarrow \neg p \end{aligned}$$

p q

B.

$$\begin{aligned} p &\leftarrow \neg q \\ q &\leftarrow \neg r \end{aligned}$$

p q r

C.

$$\begin{aligned} p &\leftarrow q, r \\ q &\leftarrow r, s \\ s &\leftarrow \neg p \end{aligned}$$

$$\begin{tikzcd} p \arrow[r, bend left] \arrow[d] & q \arrow[dl] \arrow[d] \\ r & s \end{tikzcd}$$

~~D.~~

$$\begin{aligned} p &\leftarrow q \\ q &\leftarrow p \\ p &\leftarrow \neg r \end{aligned}$$

# Completion: Turning LP$^{MLN}$ to MLN

- For any ground LPMLN program that consists of rules of the form
  - w: $A \leftarrow Body$
  - where $A$ is an atom and $Body$ is a formula,

- The completion of $\Pi$ is defined as the union of $\Pi$ and hard rules

$$\alpha : \quad A \rightarrow \bigvee_{w:A \leftarrow Body \in \Pi} Body$$

for each ground atom A

- **Theorem:** "Tight" LP$^{MLN}$ program $\Pi$ under the stable model semantic has the same probability distribution over all interpretations with the completion of $\Pi$ under the MLN semantics

# Example

- $\Pi$: under LP$^{MLN}$

  $2:\ p \leftarrow \neg q$

  $1:\ q \leftarrow \neg p$

- Comp($\Pi$): under MLN

  $2:\ p \leftarrow \neg q$

  $1:\ q \leftarrow \neg p$

  $\alpha:\ p \rightarrow \neg q$

  $\alpha:\ q \rightarrow \neg p$

# Wrap-Up