Ontology Languages

Reasoning Problems

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Objectives

Objective

Identify the kinds of reasoning in ontologies

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Apply ontology tools for reasoning problems

Concept Satisfiability

- Checking whether a concept C is satisfiable with respect to a knowledge base K, i.e., whether there exists a model I of K such that $C^I \neq \emptyset$.

- Formally: $K \nvDash C \equiv \bot$

Example: Concept Satisfiability

ParentOfOnlyMaleChildren ≡ Person⊓∃hasChild.Male⊓∀hasChild.Male

Male ⊑ ¬Female

- Is each of the concepts satisfiable w.r.t. K?
  - ParentOfOnlyMaleChildren ⊓ ∃hasChild.Male
  - ParentOfOnlyMaleChildren ⊓ ∃hasChild.Female

Let's Test on Protégé

description logics	OWL in Protégé
concept	class
role	object property
constant/individual	individual
theory	ontology
$\top$	owl:Thing
$\bot$	owl:Nothing
$\exists r.C$	r some C
$\forall r.C$	r only C
$\neg C$	not C
$C_1 \sqcup C_2$	C_1 or C_2
$C_1 \sqcap C_2$	C_1 and C_2

Subsumption

- This is the problem of checking whether C is subsumed by D with respect to a knowledge base K, i.e., whether $C^I \subseteq D^I$ in every model I of K.

- Formally: K |= C ⊑ D

Example: Subsumption

$\text{Parent} \equiv \text{Person} \sqcap \exists \text{hasChild}.\text{Person} \sqcap \forall \text{hasChild}.\text{Person}$

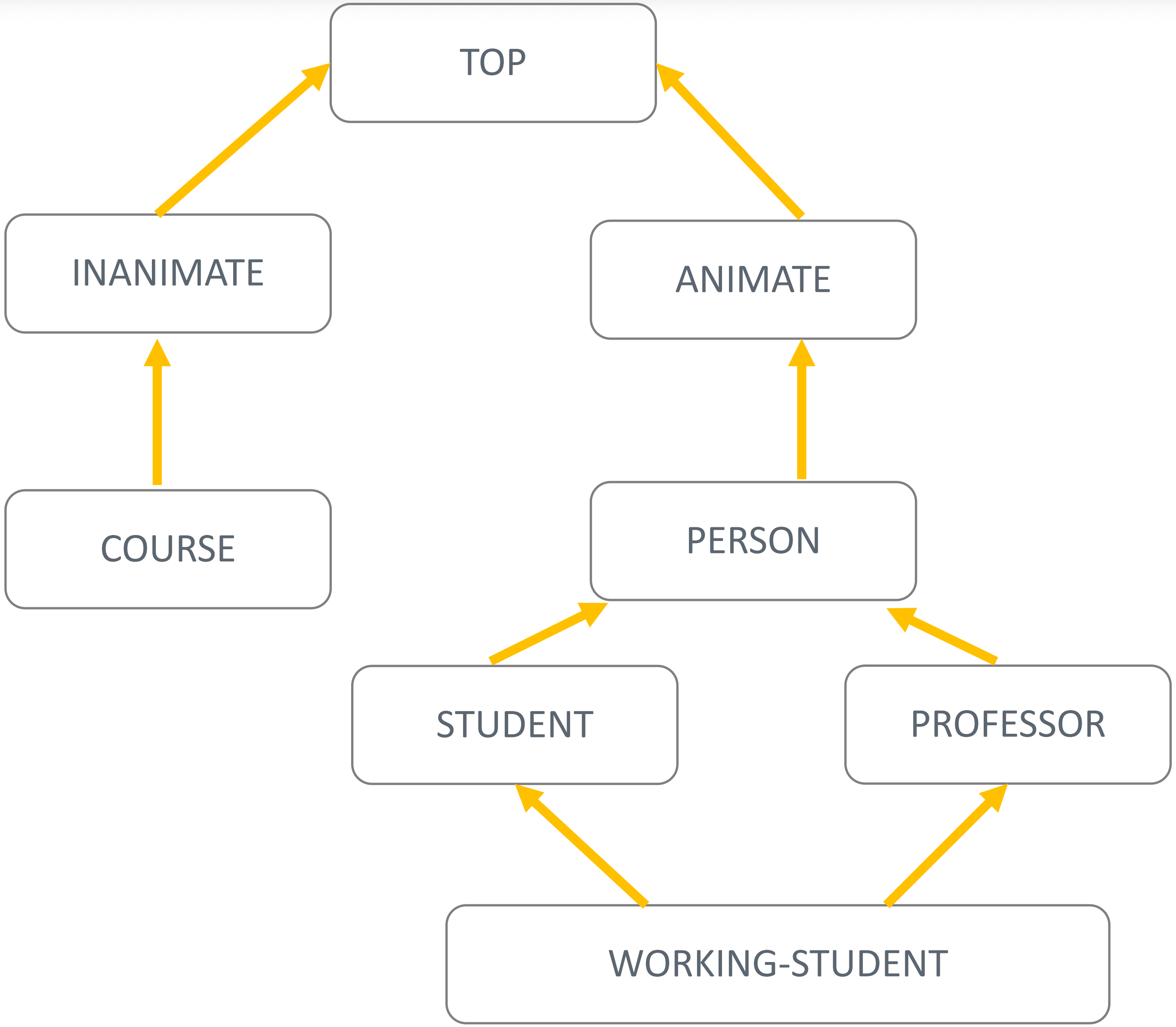
$\text{ParentOfOnlyMaleChildren} \equiv \text{Person} \sqcap \exists \text{hasChild}.\text{Male} \sqcap \forall \text{hasChild}.\text{Male}$

$\text{Male} \sqsubseteq \text{Person}$

Q: $\text{ParentOfOnlyMaleChildren} \sqsubseteq \text{Parent}$?

Q: $\text{ParentOfOnlyMaleChildren} \sqsubseteq \text{Male}$?

Taxonomies (1 of 3)

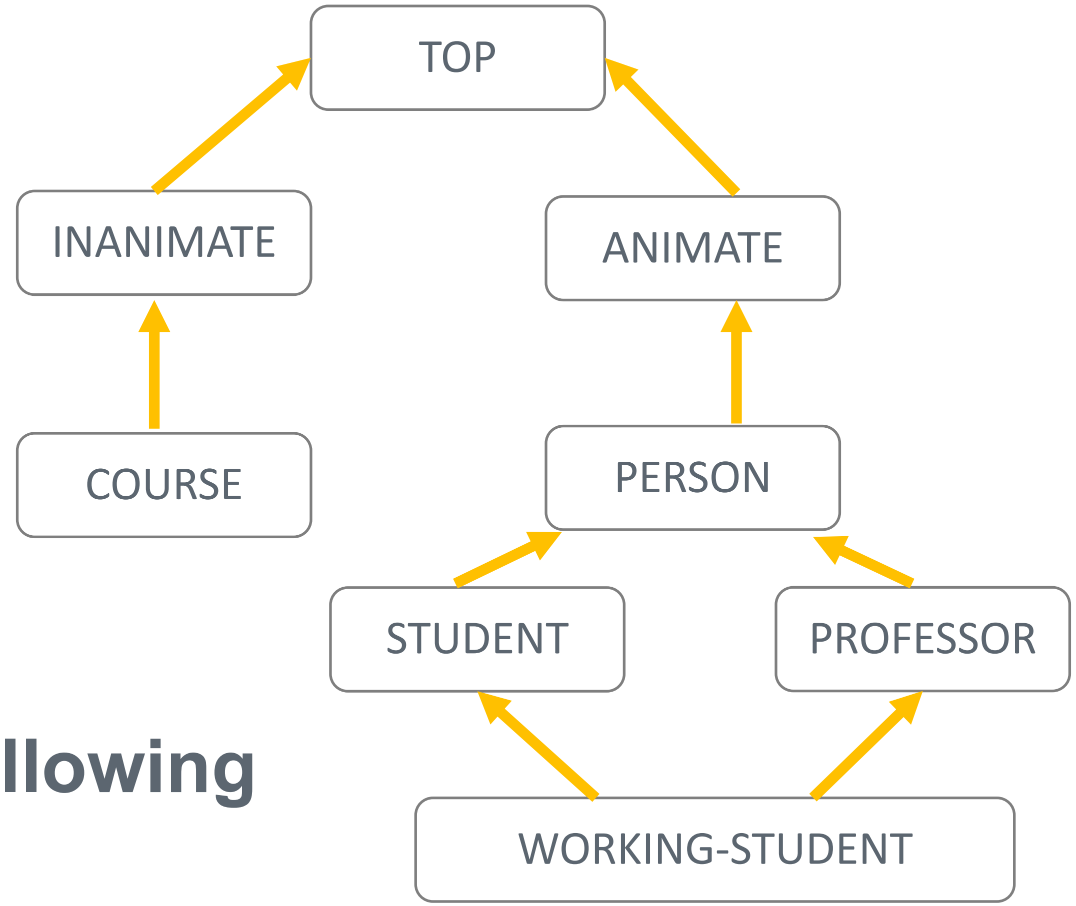


Taxonomies (2 of 3)

- The subsumption relationship between concepts defined by $\sqsubseteq$ is a partial order (i.e., it is reflexive, antisymmetric and transitive).

- Subsumption induces a taxonomy such as the one on the previous slide where only direct subsumptions have been explicitly drawn.

Taxonomies (3 of 3)



Question: What is the place of the following concept in the example taxonomy?

$$N \equiv ANIMATE \sqcap (STUDENT \sqcup PROFESSOR)$$

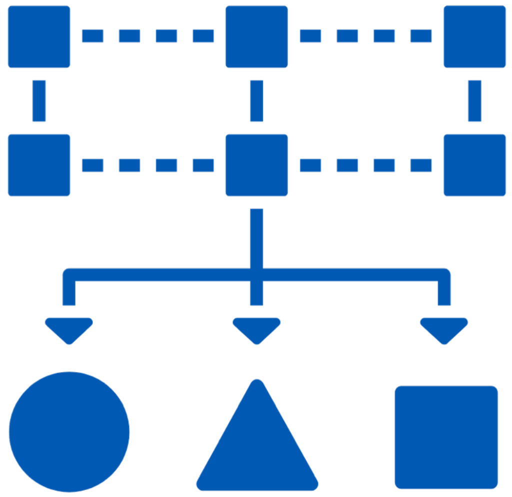
Classification

- **The problem of classification:** Given a concept C and a TBox T, for all concepts D, determine whether D subsumes C, or D is subsumed by C

- Intuitively, this amounts to finding the “right place” for C in the taxonomy implicitly present in T

- **Classification** is the task of inserting new concepts in a taxonomy. It is sorting in partial orders

- What is the solution to the classification problem posed in the previous slide?



Knowledge Base Satisfiability

- Checking whether K is satisfiable, i.e., whether it has a model.

- Is the following KB satisfiable?

$$\text{ParentOfOnlyMaleChildren} \equiv \text{Person} \sqcap \exists \text{hasChild.Male} \sqcap \forall \text{hasChild.Male}$$

$$\text{Male} \sqsubseteq \text{Person},\ \text{Female} \sqsubseteq \text{Person},\ \text{Male} \sqsubseteq \neg\text{Female}$$

Male(JOHN), Male(NICK), Female(ANNA),

hasChild(JOHN, NICK), hasChild(JOHN, ANNA),

ParentOfOnlyMaleChildren(JOHN)

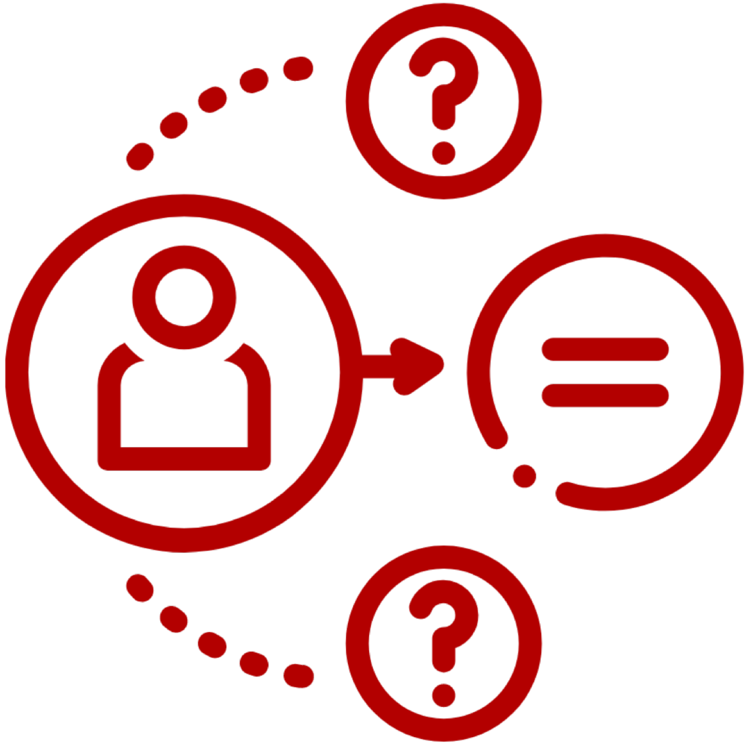
Instance Checking

- The problem of checking whether the assertion C(a) is satisfied in every model of K

- Formally: $K \models C(a)$

Answering Concept Queries

- Find all a such that {a | K |= C(a)}.



Reduction to Satisfiability

- Some of the previous reasoning problems can be solved by reducing them to the problem of knowledge base satisfiability:

$\Gamma \models F$ iff
$\Gamma \cup \{\neg F\}$ is unsat.

- Concept Satisfiability: K $\not\models$ C ≡ ⊥ iff there exists an x such that K ∪ {C(x)} is satisfiable

- Subsumption: K |= C ⊑ D iff there exists an x such that K∪{(C⊓¬D)(x)} is not satisfiable

- Instance Checking: K |= C(a) iff K ∪ {¬C(a)} is not satisfiable

Reasoning Algorithms

- Terminating, complete and efficient algorithms for deciding satisfiability, and all the other reasoning problems mentioned earlier, are available for ALC
- These algorithms are based on tableaux-calculi techniques
- Completeness is important for the usability of description logics in real applications
- Such algorithms have been shown to be efficient for real knowledge bases, even if the problem in the corresponding logic is in PSPACE or EXPTIME

Wrap-Up