Unsupervised Deep Learning for Fast Imaging: From DAE to Generative Model

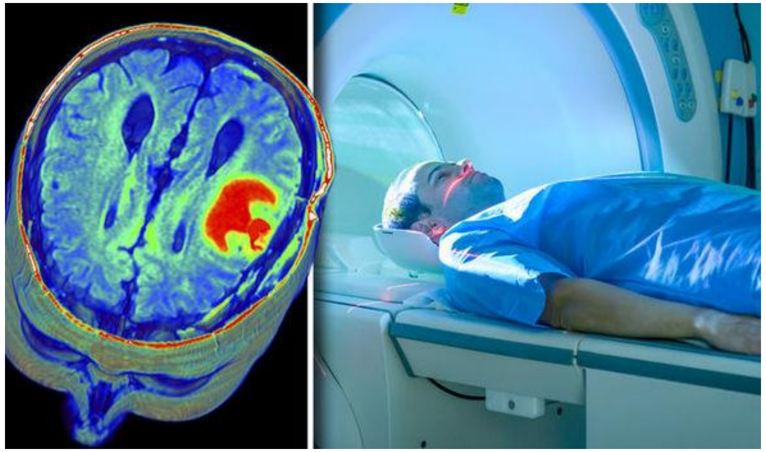
Qiegen Liu

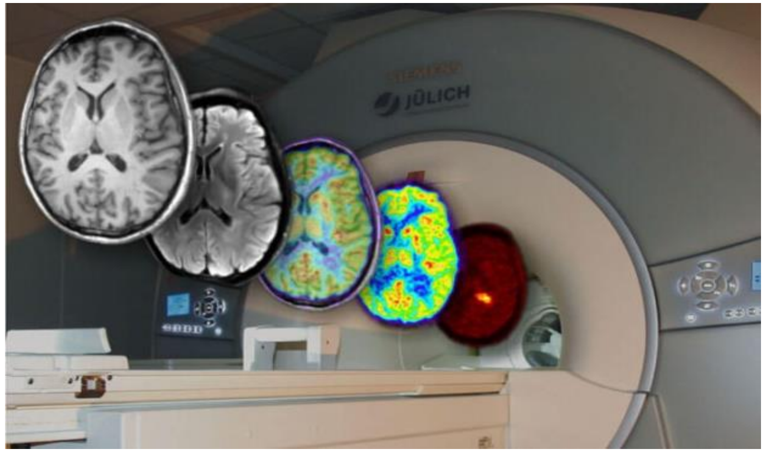
Nanchang University, China

Unsupervised Deep Learning for Fast Imaging: From DAE to Generative Model

Outline：

Part 1-From CS to AI





Fast MRI Techniques:

- ✓ **MR physics (1970’s)**

  Pulse sequence design

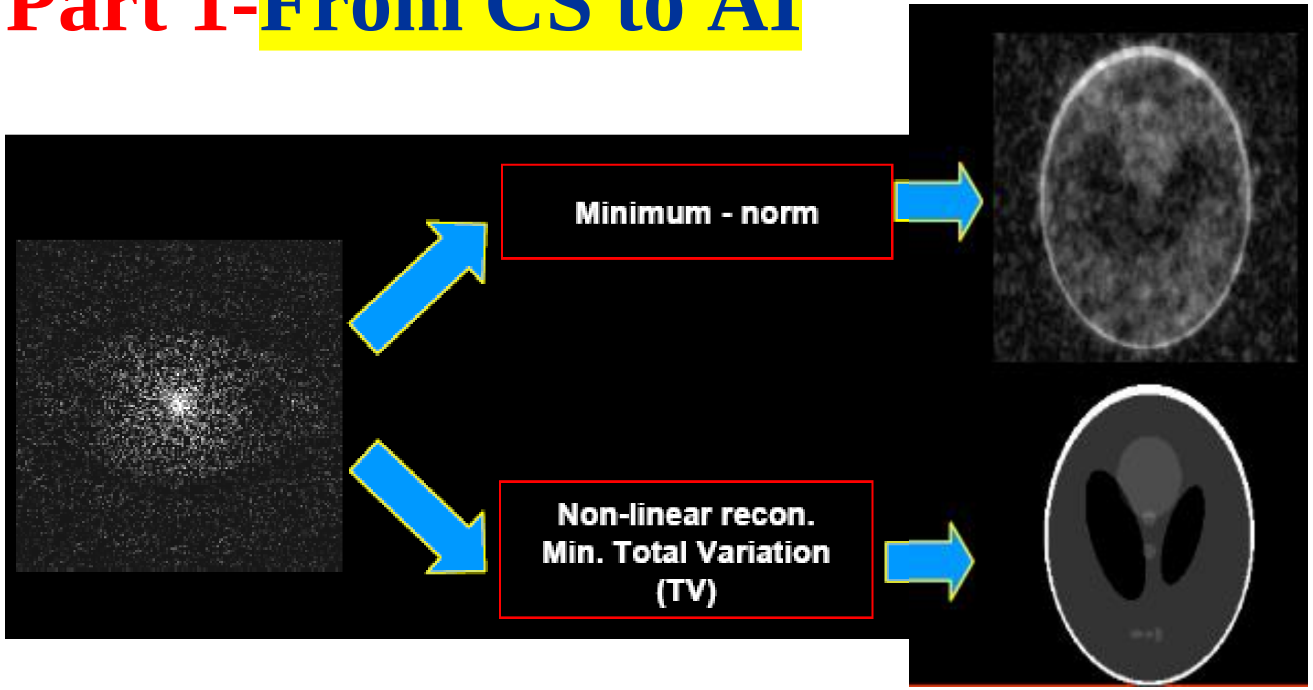
- ✓ **Hardware (2000’s)**

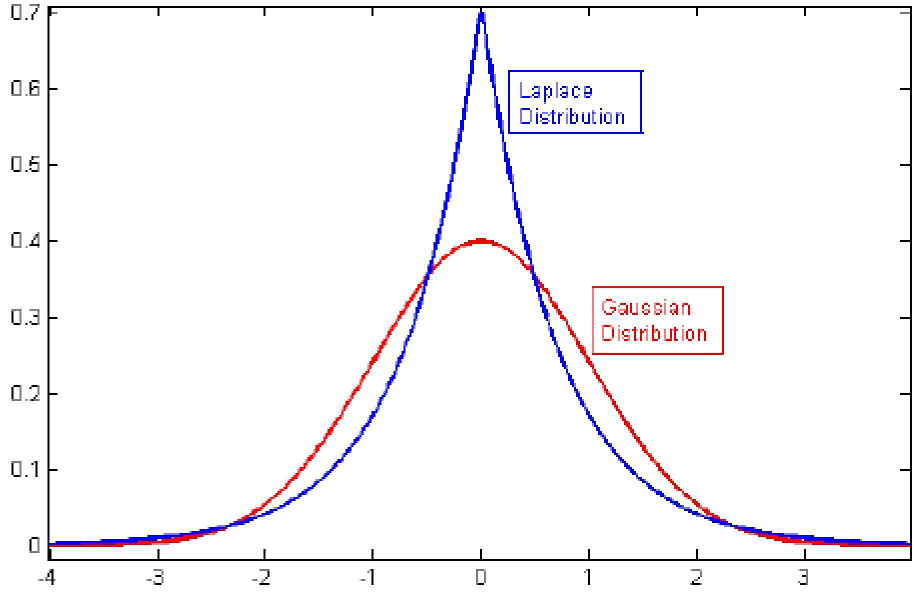
  Parallel imaging with phased array coils

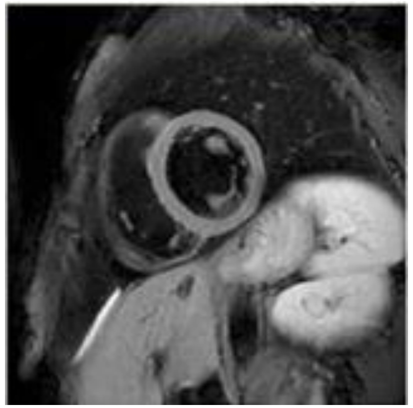
- ✓ **Partial K-space reconstruction** (past two decades)

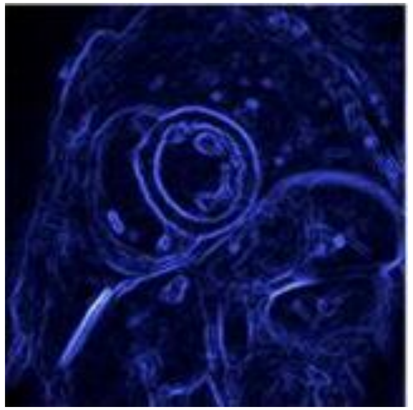
  Modeling using priori knowledge, etc.

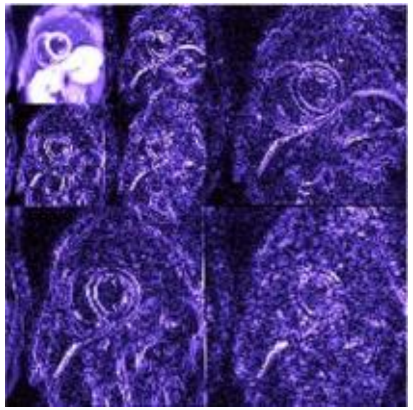
Part 1-From CS to AI





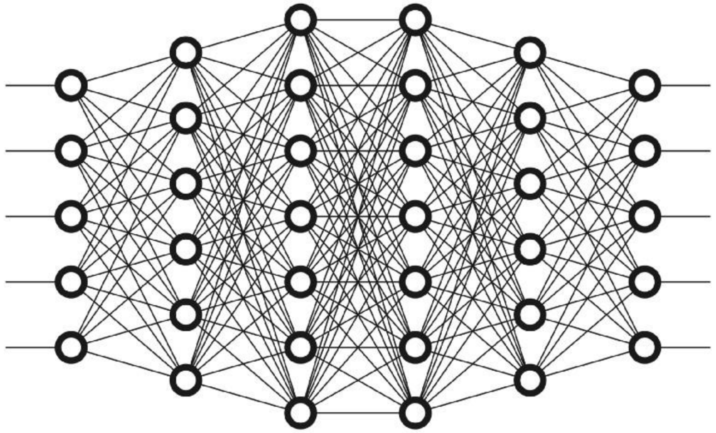






Sparse in Gradient

Sparse in Wavelet



From compressed sensing (CS) to Artificial intelligence (AI)

Unsupervised Deep Learning for Fast Imaging: From DAE to Generative Model

Outline：

1. Fast imaging: From CS to AI

2. A review of USL for MRI Rec

2.1 Five representatives of USL

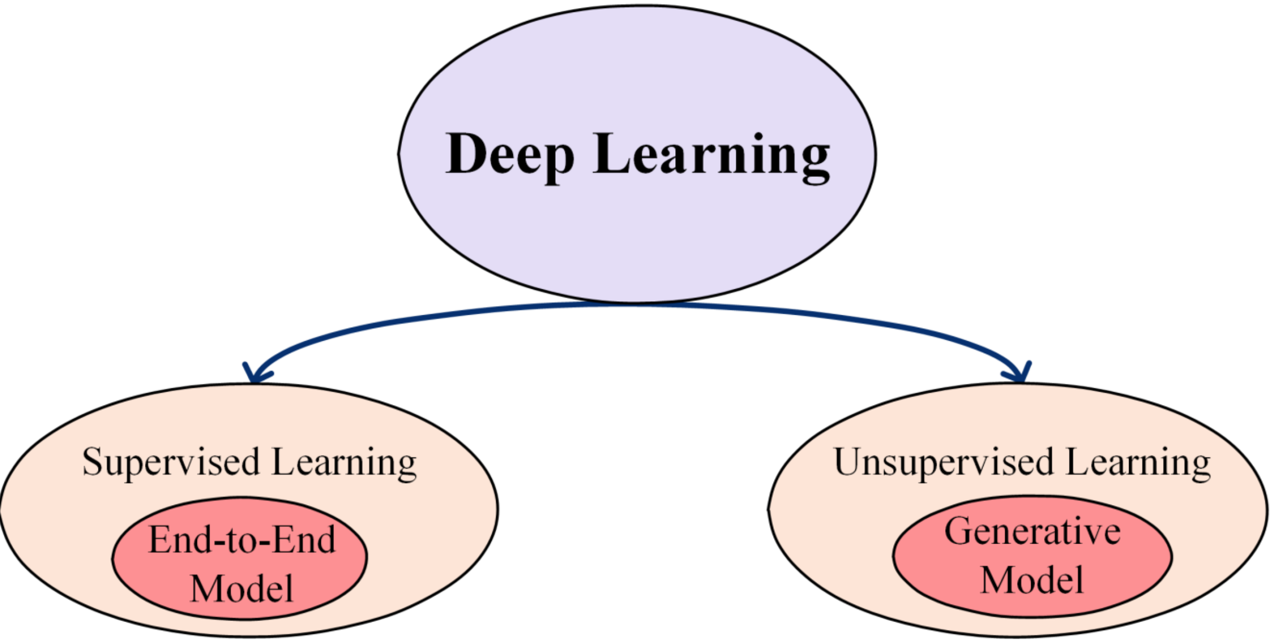
2.2 Underlying ideas for improvements

3. Our work of USL in fast imaging

3.1 Examples of USL from DAE to DSM

3.2 Examples of DSM from image domain to k-space domain

Part 2.1-Five representatives of USL



Generative model

01 Denoising autoencoding(DAE)

02 Variational Autoencoders (VAE)

03 Generative Adversarial Network (GAN)

04 PixelCNN

05 Generative Flow (Glow)

Z. He, Q. Liu, et al., A comparative study of unsupervised deep learning methods in MRI reconstruction, *iMRI*, 2020.

Part 2.1-Five representatives of USL

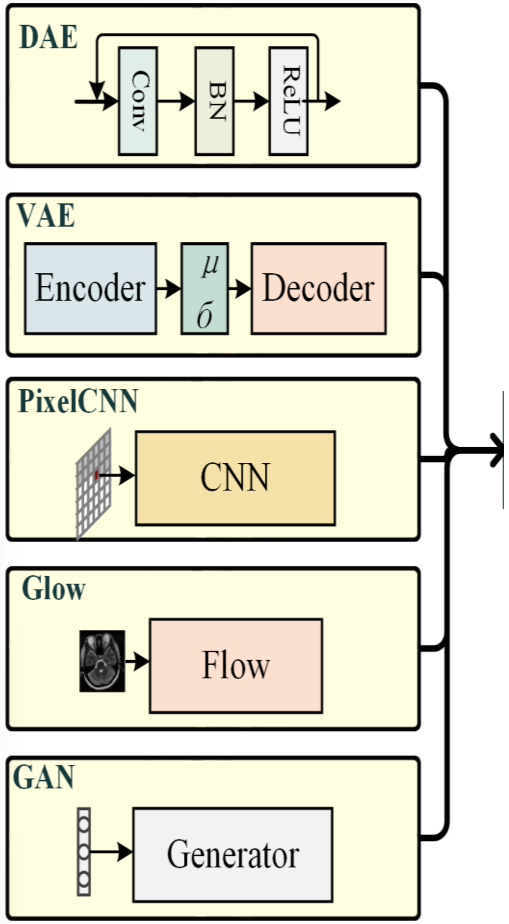
01 Denoising autoencoding(DAE)

02 Variational Autoencoders (VAE)

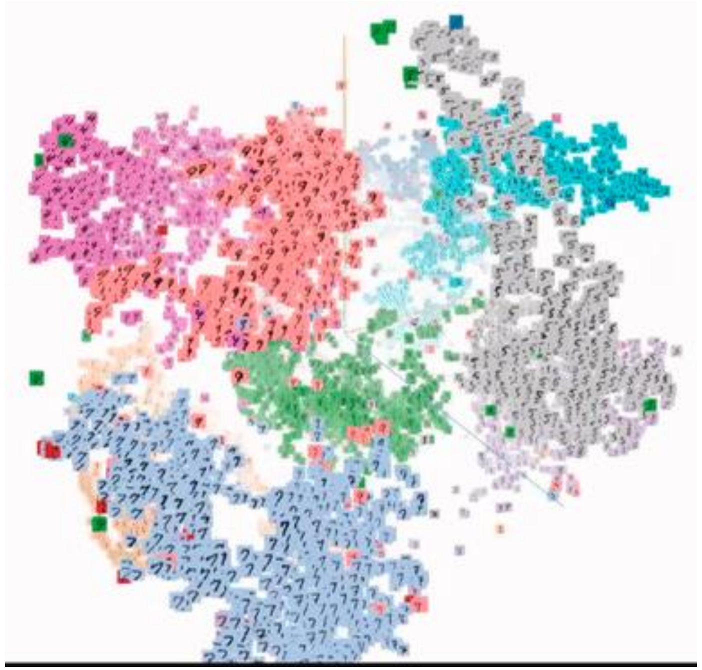
03 Generative Adversarial Network (GAN)

04 PixelCNN

05 Generative Flow (Glow)



Data distribution $\log p(x)$



Z. He, Q. Liu, et al., A comparative study of unsupervised deep learning methods in MRI reconstruction, *iMRI*, 2020.

Part 2.1-Five representatives of USL

POCS-like scheme for MRI Rec

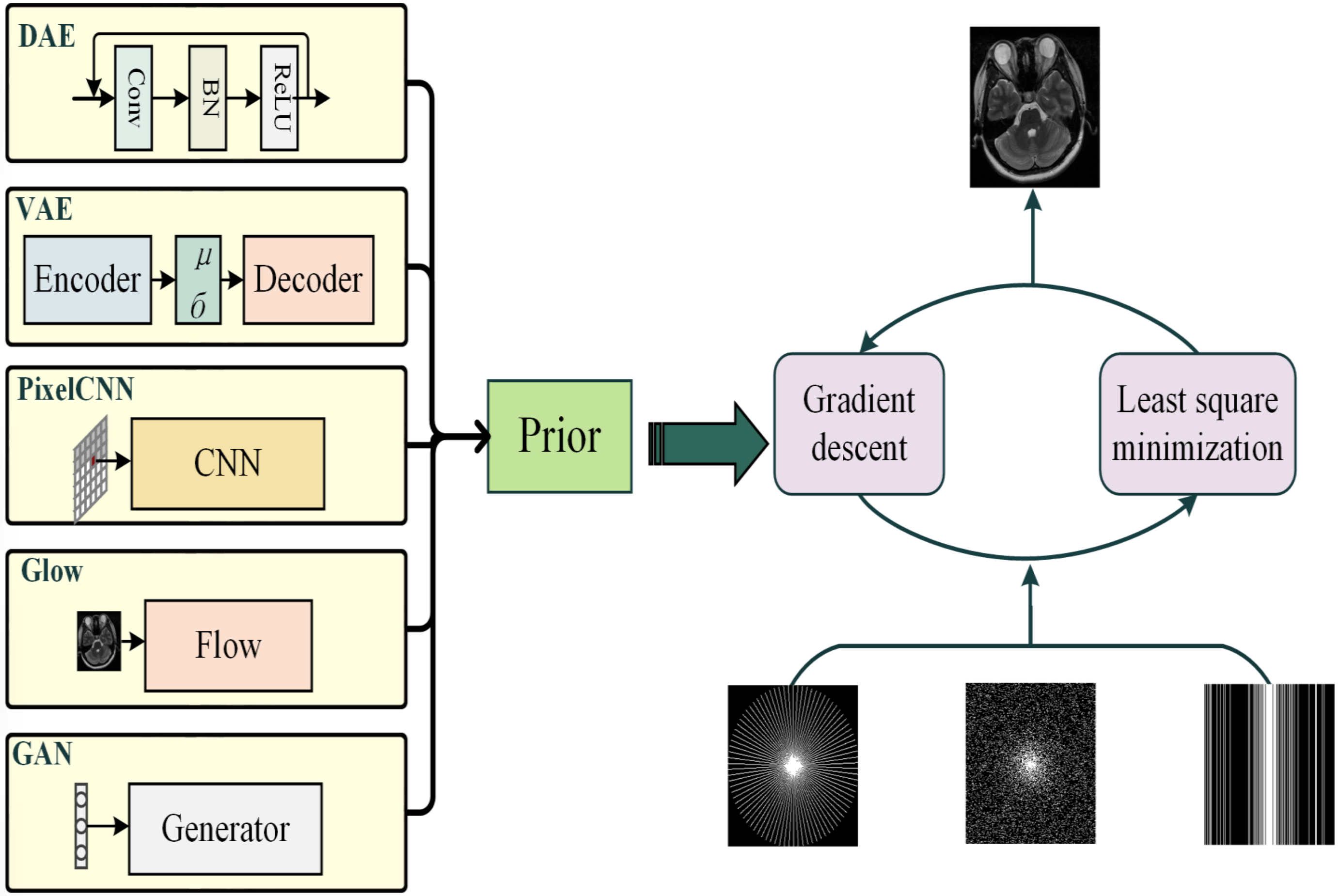
01 Denoising autoencoding(DAE)

02 Variational Autoencoders (VAE)

03 Generative Adversarial Network (GAN)

04 PixelCNN

05 Generative Flow (Glow)



Z. He, Q. Liu, et al., A comparative study of unsupervised deep learning methods in MRI reconstruction, *iMRI*, 2020.

Part 2.1-Five representatives of USL

- 01 Denoising autoencoding(DAE)
- 02 Variational Autoencoders (VAE)
- 03 Generative Adversarial Network (GAN)
- 04 PixelCNN
- 05 Generative Flow (Glow)

POCS-like scheme for MRI Rec

$$\begin{cases} x^{k+1/2} = x^k - \alpha \nabla prior(x^k) & \text{Update on prior term} \\ x^{k+1} = \arg\min_x \|F_p x - y\|_2^2 + \lambda \|x - x^{k+1/2}\|_2^2 & \text{Update on data-consistency} \end{cases}$$

Algorithm 1: Unsupervised Learning for Reconstruction (USLearn)

Prior learning stage

Input: MR dataset: $x \in C^{n \times n}$

Output: Trained network model by learning $p(x)$

Iterative reconstruction stage

1: Initialization: $x^0 = F_p^H y$
2: For $k = 0, 1, 2, \cdots, K$ do
3: Pre-process to get the corresponding network input $m^k = pre(x^k)$
4: Get gradient $\nabla_m prior(m)$ at m^k
5: Update $m^{k+1} = m^k + \alpha \nabla_m prior(m^k)$
6: Post-process $x^{k+1} = post(m^{k+1})$ for projection
7: Projection x^{k+1} in Eq. (8)
8: Return x^K

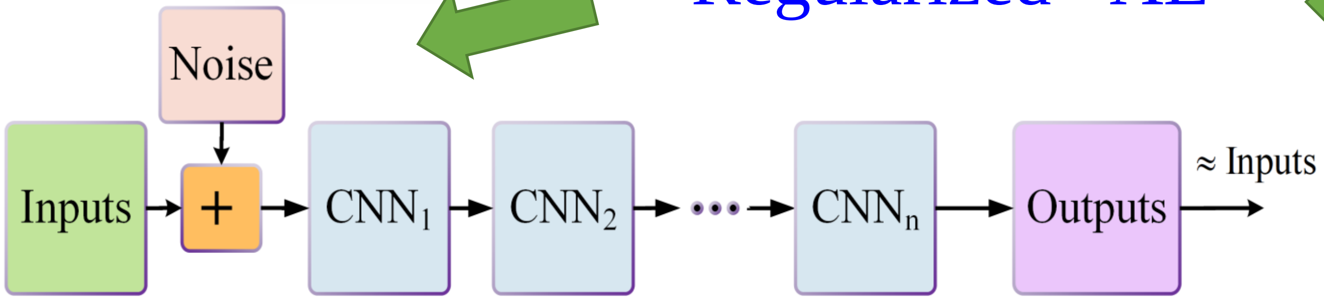
Z. He, Q. Liu, et al., A comparative study of unsupervised deep learning methods in MRI reconstruction, *iMRI*, 2020.

Part 2.1-Five representatives of USL

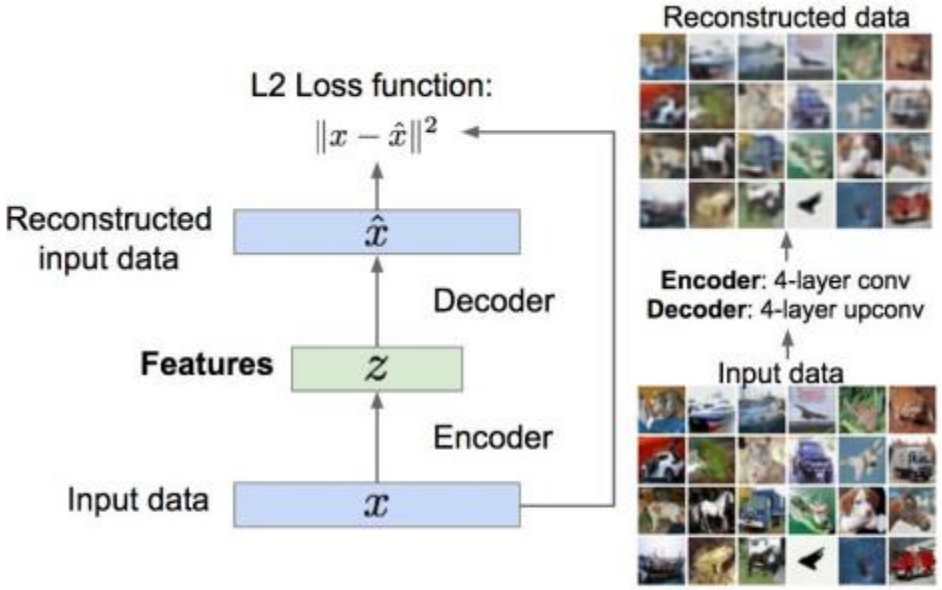
01

DAE

Regularized - AE



$$L_{DAE}(A) = E_{x,\ \mu}[\|A_{\sigma}(x+\mu)-x\|^{2}]$$



Autoencoders (Feature learning)

The relationship between DAE and generative model are not fully revealed by researchers in early period. DAE can be regarded as a generative model until Song et al. used denoising sco re matching and Langevin dynamics method to generate images

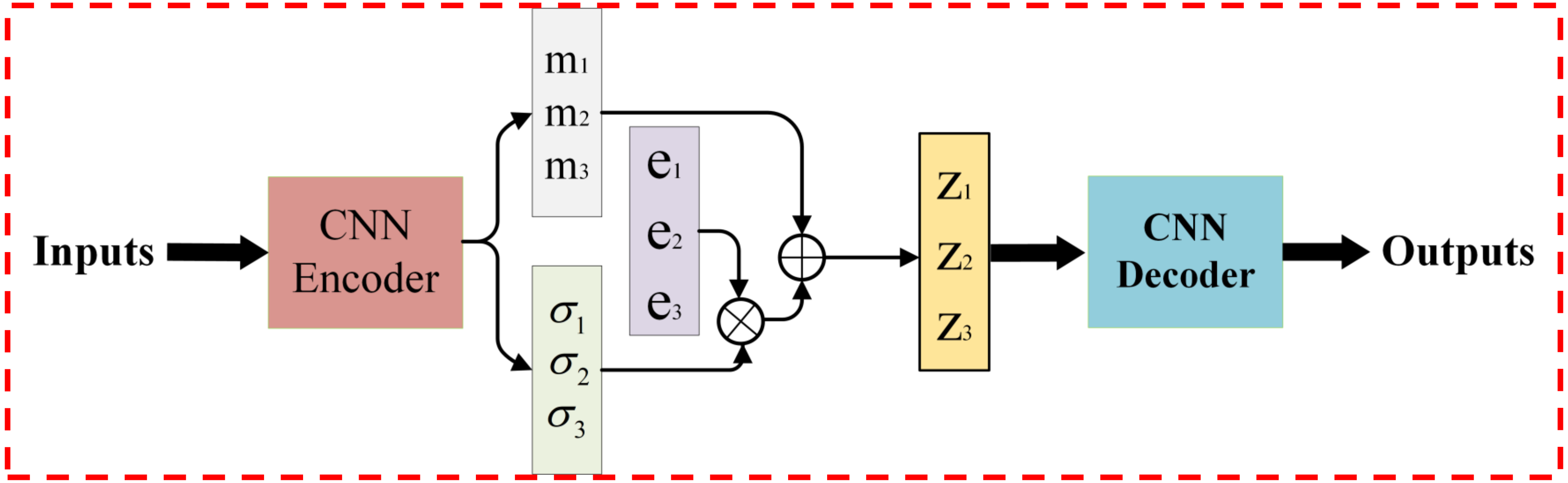
Y. Song, and S. Ermon, Generative modeling by estimating gradients of the data distribution, *Proc. Adv. Neural Inf. Process. Syst.*, pp. 11895-11907, 2019.

Part 2.1-Five representatives of USL

02

VAE

VAE realizes image generation by constraining the latent coding vectors to be Gaussian distribution.



$$p(x) = \int_Z p(x,z)dz = \int_Z p(x|z)p(z)dz$$

$$\log p(x) = E_{q(z|x)}\left[\log \frac{p(x,z)}{q(z|x)}\right] + D_{KL}[q(z|x) \,||\, p(z|x)]$$

VAE is an approximate generative model, as it maximizes the lower bound of the data likelihood function.

Tezcan et al. used it to estimate the distribution of MR image patch as prior.

Tezcan KC, Baumgartner CF, Luechinger R, Pruessmann KP, Konukoglu E. MR image reconstruction using deep density priors. *IEEE Trans. Med. Imag.*, 38:1633-1642, 2018.

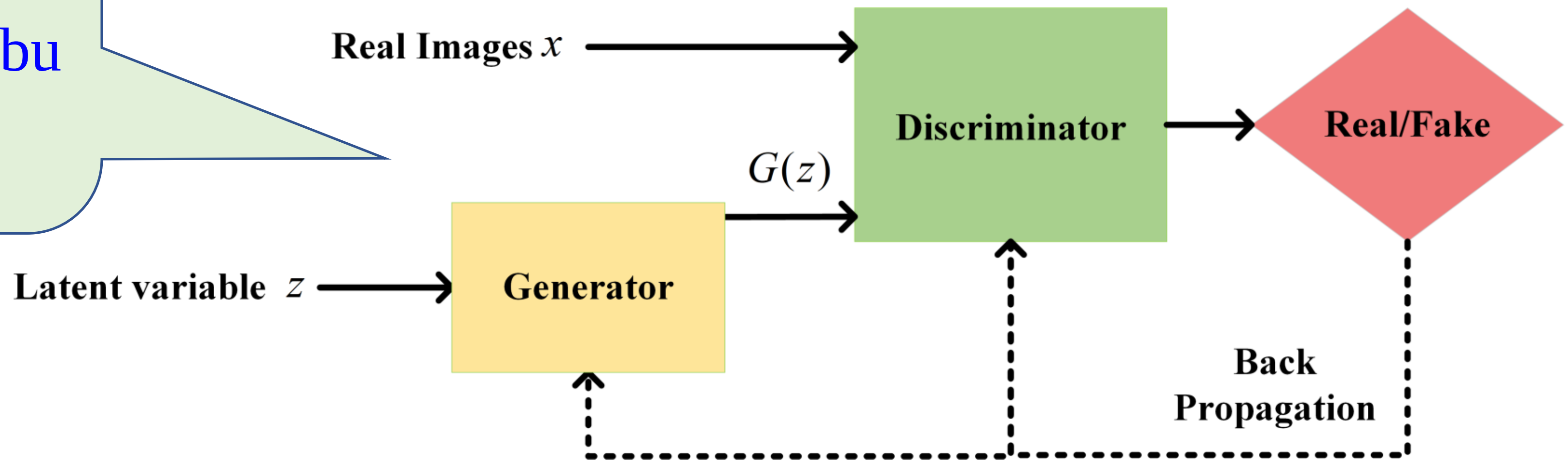
Part 2.1-Five representatives of USL

03 GAN

GAN gradually optimizes the generator and discriminator to generate image which has the same distribution as the real image

$$\min_{G}\max_{D} L(D,G)=E_{x\sim p_{data}}[\log D(x)]+E_{z\sim p_z(z)}[\log(1-D(G(z)))]$$

$$y=Ax,\quad x=G(z)$$



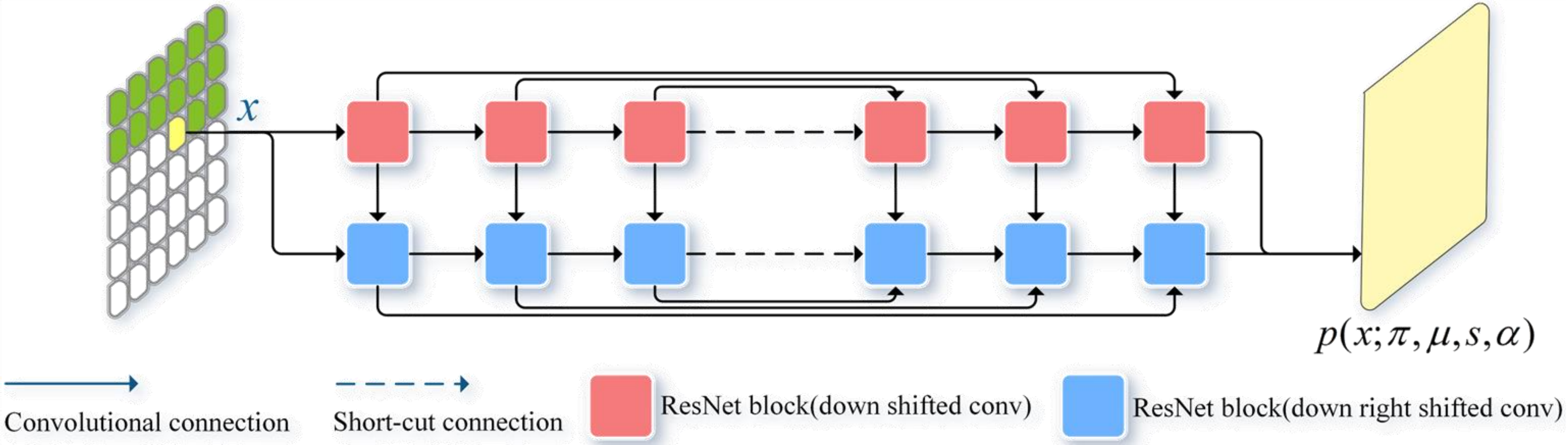
A. Bora, A. Jalal, E. Price, A. Dimakis, Compressed sensing using generative models, *ICML* 2017.

V. Shah, C. Hegde, Solving linear inverse problems using GAN priors: An algorithm with provable guarantees, 2018.

Part 2.1-Five representatives of USL

04 PixelCNN

The autoregressive model optimizes the explicit likelihood function through chain rule



For real-valued image: $$p(x)=\prod_{i=1}^{n^2} p\left(x_i \middle| x_1,\dots,x_{i-1}\right) \quad v \,\square \sum_{i=1}^{K} \pi_i \mathrm{logistic}\left(\mu_i, s_i\right)$$

For complex-valued image: $$p(x_{ij}|C_{ij}) = p(\mathrm{Re}(x_{ij}),\mathrm{Im}(x_{ij})\,|\,C_{ij}) = P(\mathrm{Re}(x_{ij})\,|\,\mu_{\mathrm{Re}}(C_{ij}), s_{\mathrm{Re}}(C_{ij}))\times P(\mathrm{Im}(x_{ij})\,|\,\mu_{\mathrm{Im}}(C_{ij},\mathrm{Re}(x_{ij}))s_{\mathrm{Im}}(C_{ij}))$$

G. Luo, N. Zhao, W. Jiang, E.S. Hui, C. Peng. MRI reconstruction using deep Bayesian estimation, *Magn. Reason. Med.*, 2020. https://doi.org/10.1002/mrm.28274

Part 2.1-Five representatives of USL

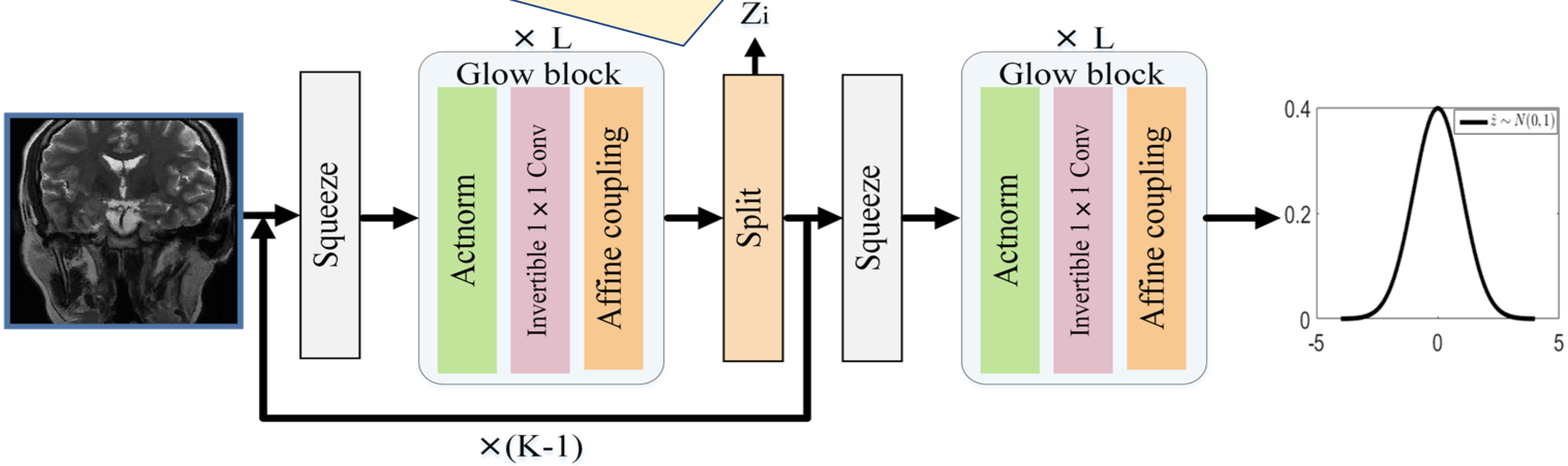
05 Generative Flow (Glow)

By designing a chain of invertible transformations with Jacobian determinants, flow-based generative models map points from the simple distribution to a complex one.

$$p_\theta(x) = p_z(z)\left|\partial z/\partial x\right|$$

$$\log p_\theta(x) = \log p_Z(z) + \sum_{i=1}^{L} \log\left|\partial h_i/\partial h_{i-1}\right|$$

$$x \overset{F_1}{\leftrightarrow} h_1 \overset{F_2}{\leftrightarrow} h_2 \overset{F_3}{\leftrightarrow} \cdots \overset{F_L}{\leftrightarrow} z$$



V.A. Kelkar, S. Bhadra, M.A. Anastasio. Compressible latent-space invertible networks for generative model-constrained image reconstruction. https://arxiv.org/pdf/2007.02462.pdf

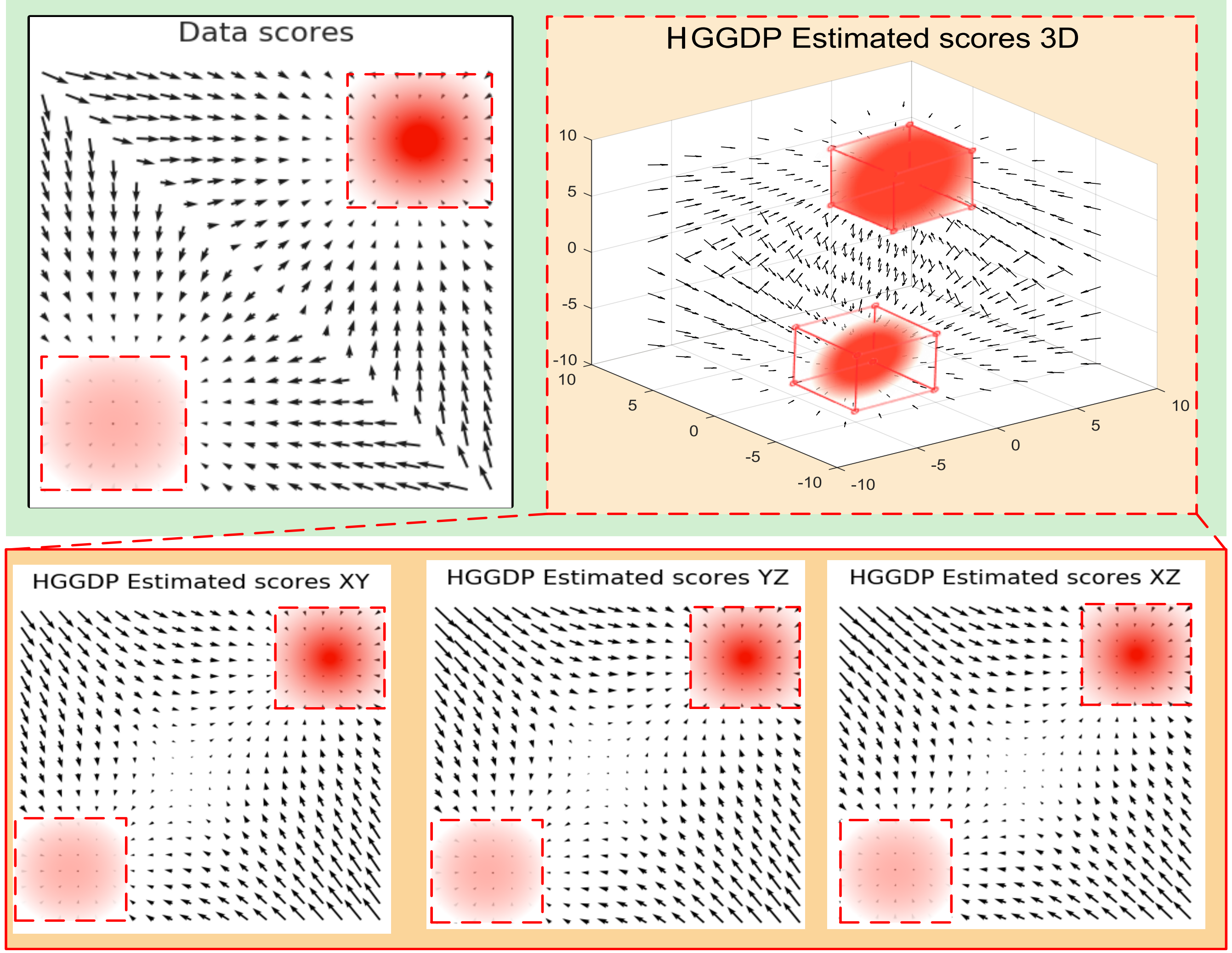
Part 2.2-Underlying ideas for improvements

01

How to estimate $\nabla_x \log p_{data}(x)$?

Learning prior density in higher-dimensional space

With variable *X*, not with *x* itself



C. Quan, J. Zhou, Y. Zhu, Y. Chen, S. Wang, D. Liang, Q. Liu, Homotopic gradients of generative density prior s for MR image reconstruction, *IEEE Trans. Med. Imag.*, 2021.

Part 2.2-Underlying ideas for improvements

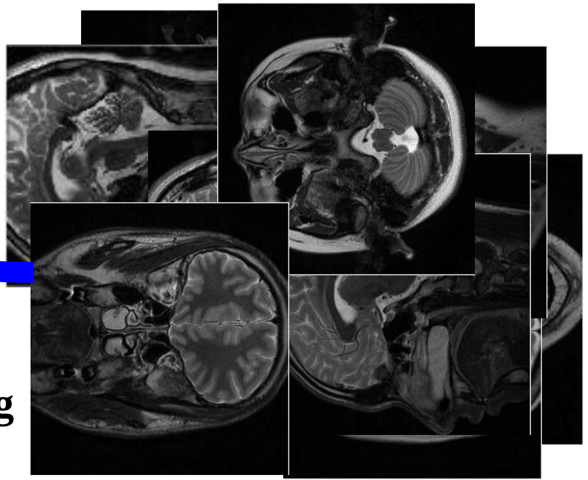
Learning prior density in different modality

02 Prior knowledge across modality?

With variable z , not with x itself

Prior learned from different modalities for CT recon

CT dataset	$p = 0.8$	47.47/0.9909
	$p = 1$	47.44/0.9908
	$p = 1.5$	47.18/0.9902
	$p = 2$	46.69/0.9888
DIV2K dataset	$p = 0.8$	47.54/0.9908
	$p = 1$	47.52/0.9907
	$p = 1.5$	47.24/0.9901
	$p = 2$	46.71/0.9887
MRI dataset	$p = 0.8$	46.69/0.9890
	$p = 1$	46.65/0.9889
	$p = 1.5$	46.41/0.9882
	$p = 2$	45.82/0.9863



F. Zhang, M. Zhang, B. Qin, Y. Zhang, Z. Xu, D. Liang, Q. Liu, REDAEP: Robust and enhanced denoising autoencoding prior for sparse-view CT reconstruction, *IEEE Trans. Radiat. Plasma Med. Sci.*, 2020.

Unsupervised Deep Learning for Fast Imaging: From DAE to Generative Model

Outline：

1. Fast imaging: From CS to AI

2. A review of USL

2.1 From DAE to generative model

2.2 Underlying ideas for improvements

3. Our work of USL in fast imaging

3.1 Examples of USL from DAE to DSM

3.2 Examples of DSM from image domain to k-space domain

Part 3.1-USL from DAE to DSM

Q. Liu, Q. Yang, H. Cheng, S. Wang, M. Zhang, D. Liang, Highly undersampled magnetic resonance imaging reconstruction using autoencoding priors, ***Magn. Reson. Med.***, vol. 83, no. 1, pp. 322-336, 2020.

DAEP

DAE

S. Wang, J. Lv, Z. He, D. Liang, Y. Chen, M. Zhang, Q. Liu, Denoising auto-encoding priors in undecimated wavelet domain for MR image reconstruction, ***Neurocomputing***, vol.437, pp.325-338, 2021.

DMSP

DSM

Generative model

M. Zhang, M. Li, J. Zhou, Y, Zhu, S. Wang, D. Liang, Y. Chen, Q. Liu. High-dimensional embedding network derived prior for compressive sensing MRI reconstruction, ***Med. Image Anal.***, vol. 64, 101717, 2020.

DMSP

C. Quan, J. Zhou, Y. Zhu, Y. Chen, S. Wang, D. Liang, Q. Liu, Homotopic gradients of generative density priors for MR image reconstruction, ***IEEE Trans. Med. Imag.***, 2021.

DSM

DAE Prior

DAE is trained by minimizing the following objective function:

$$L_{DAE}(A) = E_{x,\,\mu}[\|A_\sigma(x+\mu) - x\|^2]$$

The autoencoder error is proportional to the gradient of the log-likelihood of the smoothed density

$$A_\sigma(x) = x + \sigma^2 \nabla \log[g_\sigma * p](x)$$

where *p*(x) denotes the data density. * represents the convolutional operator.

$$R(x) = \|A_\sigma(x) - x\|^2 = \|\sigma^2 \nabla \log[g_\sigma * p](x)\|^2$$

Regularization term

G. Alain, Y. Bengio, What regularized auto-encoders learn from the data-generating distribution, *Journal of machine learning research*, 15, 3743-3773, 2014.

Algorithm overview

❑ Multi-channel learning scheme (self-copy)

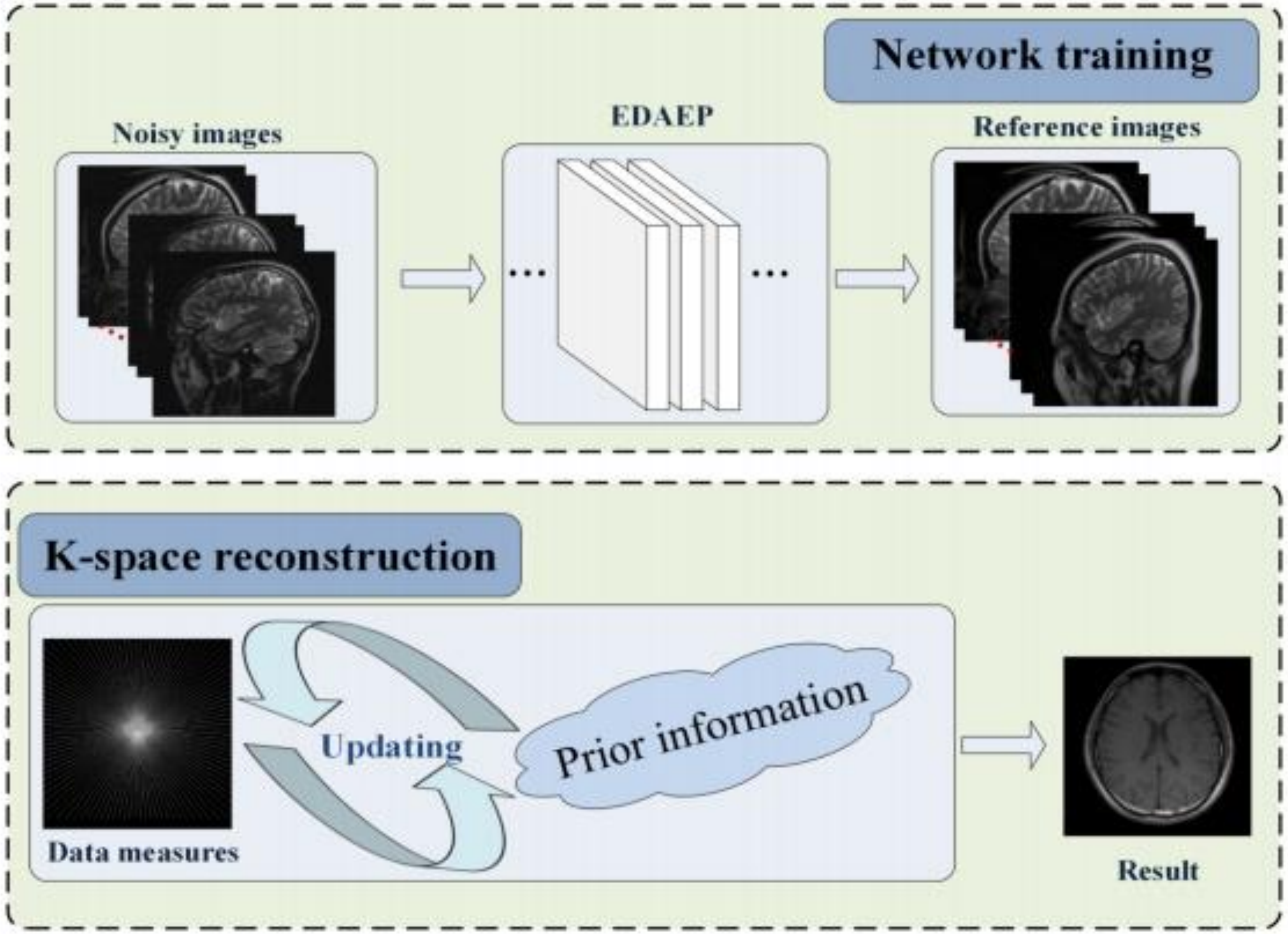
EDAEPRec

Training stage:

$$\{X = [x, x, x]\}$$

Testing stage:

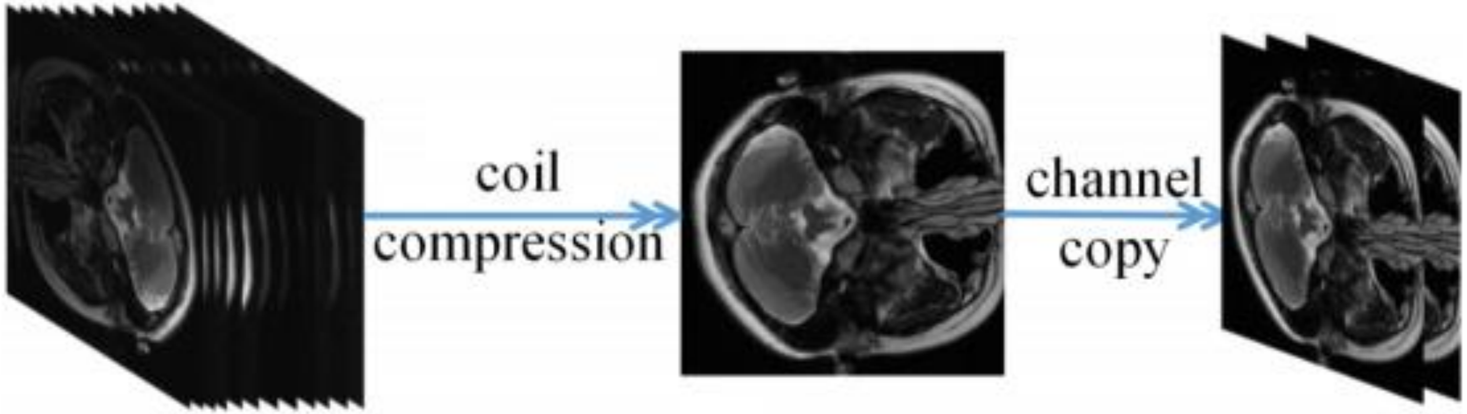
$$X^k = [x^k, x^k, x^k]$$



Q. Liu, Q. Yang, H. Cheng, S. Wang, M. Zhang, D. Liang, Highly undersampled magnetic resonance imaging reconstruction using autoencoding priors, *Magn. Reson. Med.*, vol. 83, no. 1, pp. 322-336, 2020.

Algorithm overview

- Multi-channel learning scheme (self-copy)



Schematic illustration of generating the three-channel training data.

Mathematical model for MRI Recon is:

EDAEPRec

$$\operatorname*{Min}_{x} \frac{1}{2}\left\|X - A_{\sigma_{\eta 1}}(X)\right\|^2 + \frac{1}{2}\left\|X - A_{\sigma_{\eta 2}}(X)\right\|^2 + \nu\left\|F_p x - f\right\|^2$$

We convert the input of EDAEP from $C^{m\times n\times 3}$ space into $R^{m\times n\times 6}$ space.

Q. Liu, Q. Yang, H. Cheng, S. Wang, M. Zhang, D. Liang, Highly undersampled magnetic resonance imaging reconstruction using autoencoding priors, *Magn. Reson. Med.*, vol. 83, no. 1, pp. 322-336, 2020.

From DAEP to Score-based generative model

Deep mean-shift prior (DMSP):

$$\nabla prior(x) = \nabla \log \int g_\sigma(\eta) p(x+\eta) d\eta$$
$$= [(A_\sigma(x) - x)] / \sigma^2$$

Denoising score matching (DSM):

$$S(x) \to \nabla \log p_\sigma(x)$$

Theorem 1. The DAE loss $L_{DAE}(A) = E_{x\sim p,\ \eta\sim g_\sigma}[\|A(x+\eta) - x\|^2]$

and the DSM loss $L_{DSM}(S) = E_{p_\sigma}[\|S(x) - \nabla \log p_\sigma(x)\|^2]$

with $S(x) = \dfrac{A(x) - x}{\sigma^2}$

are equivalent up to a term that does not depend on A or S.

Score-based generative model

Prior learning stage:

$$p_\sigma(\tilde{x}\,|\,x) = N(\tilde{x}\,|\,x, \sigma^2 \mathrm{I})$$

$$\nabla_{\tilde{x}} \log p_\sigma(\tilde{x}\,|\,x) = -(\tilde{x} - x)/\sigma^2$$

$$\begin{aligned}\ell(\theta;\sigma) &\triangleq \frac{1}{2} E_{p_\sigma(x)}[\|S_\theta(x,\sigma) - \nabla_x \log p_\sigma(x)\|_2^2] \\ &= \frac{1}{2} E_{p_\sigma(\tilde{x},x)}[\|S_\theta(\tilde{x},\sigma) - \nabla_{\tilde{x}} \log p_\sigma(\tilde{x}\,|\,x)\|_2^2] + C \\ &= \frac{1}{2} E_{p_\sigma(\tilde{x},x)}[\|S_\theta(\tilde{x},\sigma) + (\tilde{x} - x)/\sigma^2\|_2^2] + C\end{aligned}$$

$$L(\theta;\{\sigma_i\}_{i=1}^I) \stackrel{\Delta}{=} \frac{1}{I}\sum_{i=1}^{I} \lambda(\sigma_i)\ell(\theta;\sigma_i)$$

Training DSM network S_θ for all $\{\sigma_i\}_{i=1}^I$

Beside of the statistical derivation to Eq. (8) [33], [34], we provide a new intuitive derivation for it. In fact, as stated in **Theorem 1**, we get $S_\theta(x,\sigma) = [A(x+\eta) - (x+\eta)]/\sigma^2$. On the other hand, as described in Fig. 3, if we denote $D_\theta(x,\sigma) = (x+\eta) - A(x+\eta)$ and $A(x+\eta) \to x$, then we get $S_\theta(x,\sigma) = -D_\theta(x,\sigma)/\sigma^2 \to -\eta/\sigma^2 = -(\tilde{x} - x)/\sigma^2$.

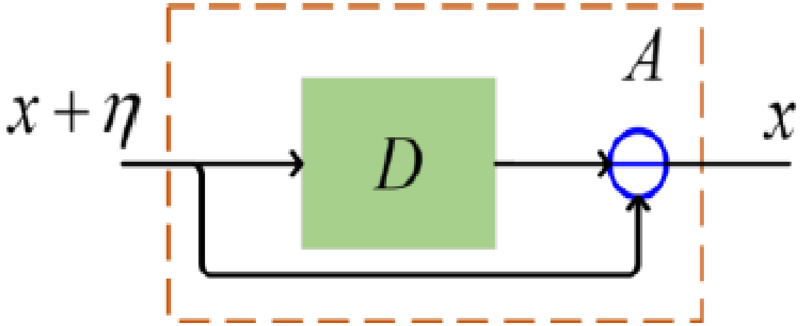


Fig. 3. Visual exhibition of the network A and D.

C. Quan, J. Zhou, Y. Zhu, Y. Chen, S. Wang, D. Liang, Q. Liu, Homotopic gradients of generative density prior s for MR image reconstruction, *IEEE Trans. Med. Imag.*, 2021.

Score-based generative model

Iterative reconstruction stage： Annealed Langevin dynamics

Langevin dynamics

$$
\begin{aligned}
\tilde{x}_t &= \tilde{x}_{t-1} + \frac{\alpha_i}{2} \nabla_x \log p_{\sigma_i}(\tilde{x}_{t-1}) + \sqrt{\alpha_i}\, z_t \\
&= \tilde{x}_{t-1} + \frac{\alpha_i}{2} S_\theta(\tilde{x}_{t-1}, \sigma_i) + \sqrt{\alpha_i}\, z_t
\end{aligned}
$$

Algorithm 1 Annealed Langevin dynamics.

Require: $\{\sigma_i\}_{i=1}^{L}, \epsilon, T$.

Initialize $\tilde{\mathbf{x}}_0$

for $i \leftarrow 1$ to L **do**

  $\alpha_i \leftarrow \epsilon \cdot \sigma_i^2 / \sigma_L^2$ ▷ α_i is the step size.

  for $t \leftarrow 1$ to T **do**

    Draw $\mathbf{z}_t \sim \mathcal{N}(0, I)$

    $\tilde{\mathbf{x}}_t \leftarrow \tilde{\mathbf{x}}_{t-1} + \frac{\alpha_i}{2} \mathbf{s}_{\boldsymbol{\theta}}(\tilde{\mathbf{x}}_{t-1}, \sigma_i) + \sqrt{\alpha_i}\, \mathbf{z}_t$

  end for

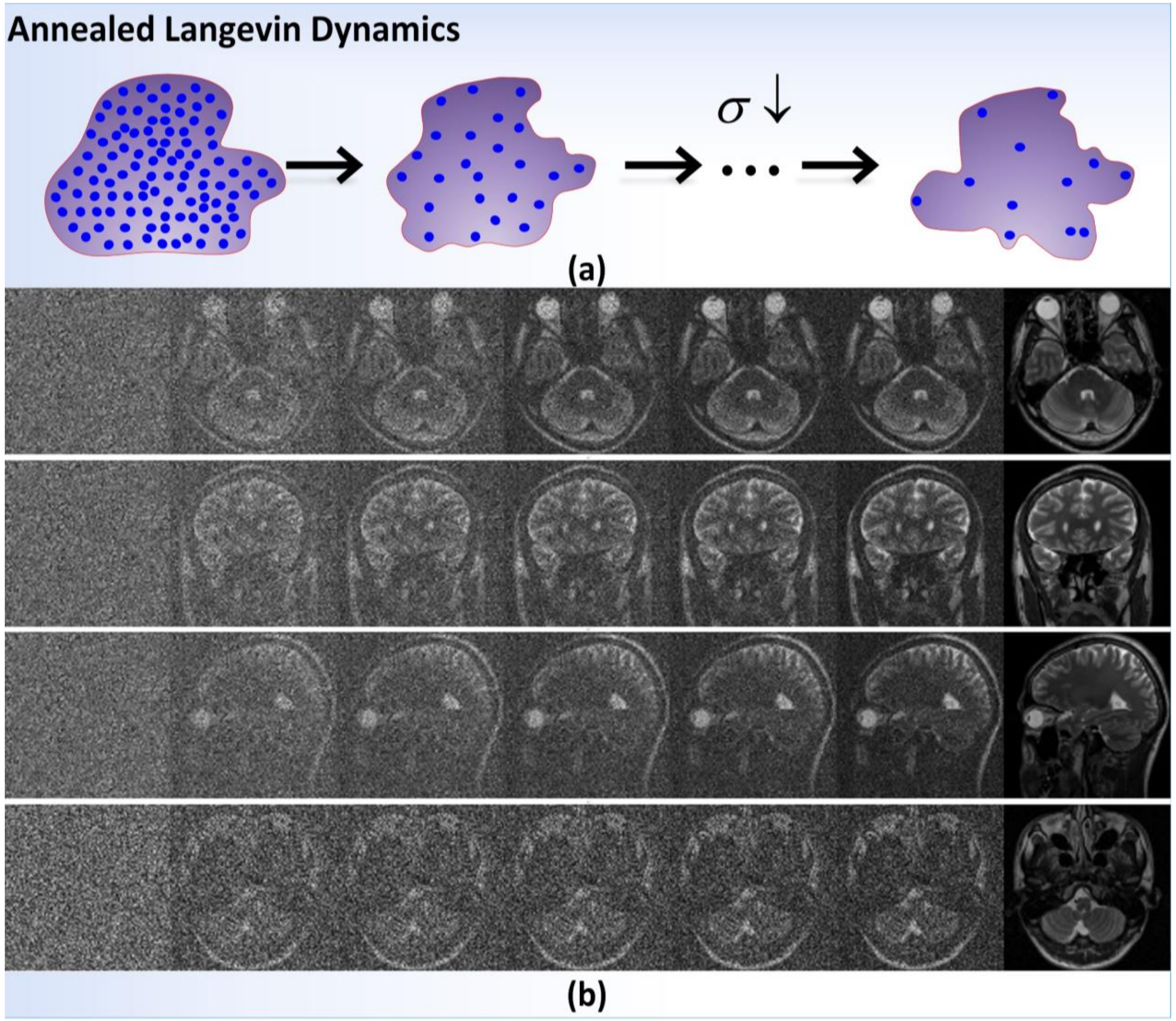
  $\tilde{\mathbf{x}}_0 \leftarrow \tilde{\mathbf{x}}_T$

end for

return $\tilde{\mathbf{x}}_T$

C. Quan, J. Zhou, Y. Zhu, Y. Chen, S. Wang, D. Liang, Q. Liu, Homotopic gradients of generative density priors for MR image reconstruction, *IEEE Trans. Med. Imag.*, 2021.

Iterative image generation



(a) Conceptual diagram of the sampling on high-dimensional noisy data distribution with multi-view noise.

(b) Intermediate samples of annealed Langevin dynamics.

HGGDPRec

Iterative reconstruction stage

Algorithm 1 HGGDPRec

Training stage

Dataset: Multi- channel dataset: $X=\{X_1, X_2, X_3, \cdots X_N\}$

Outputs: Trained HGGDP $S_\theta(X,\sigma)$

Reconstruction stage

Setting: $\sigma \in \{\sigma_i\}_{i=1}^{I}, \in, T$, and x^0

for $i \leftarrow 1$ to I **do (Outer loop)**

$\quad \alpha_i = \in \cdot \sigma_i^2 / \sigma_I^2$

$\quad$ **for** $t \leftarrow 1$ to T **do (Inner loop)**

$\quad\quad$ Draw $z_t \sim N(0,1)$ and $X^{t-1}=\{x^{t-1}, x^{t-1}, \cdots, x^{t-1}\}$

$$X^t = X^{t-1} + \frac{\alpha_i}{2} S_\theta(X_i^{t-1}, \sigma_i) + \sqrt{\alpha_i} z_t$$

$\quad\quad$ Update $x^t = Mean(X^t)$ and Eq. (17)

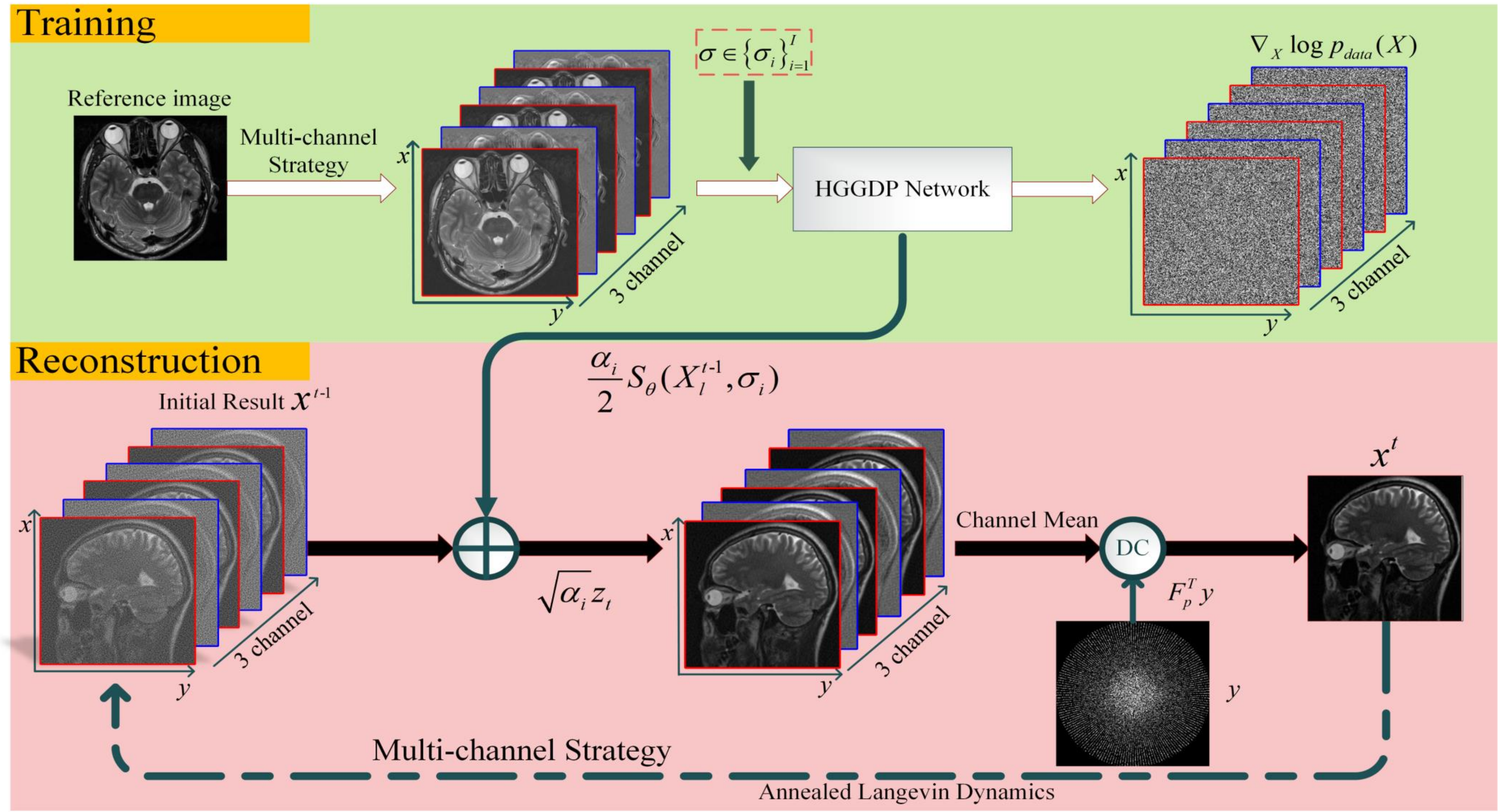
$\quad$ **end for**

$\quad x^0 \leftarrow x^T$

end for

Return x^T

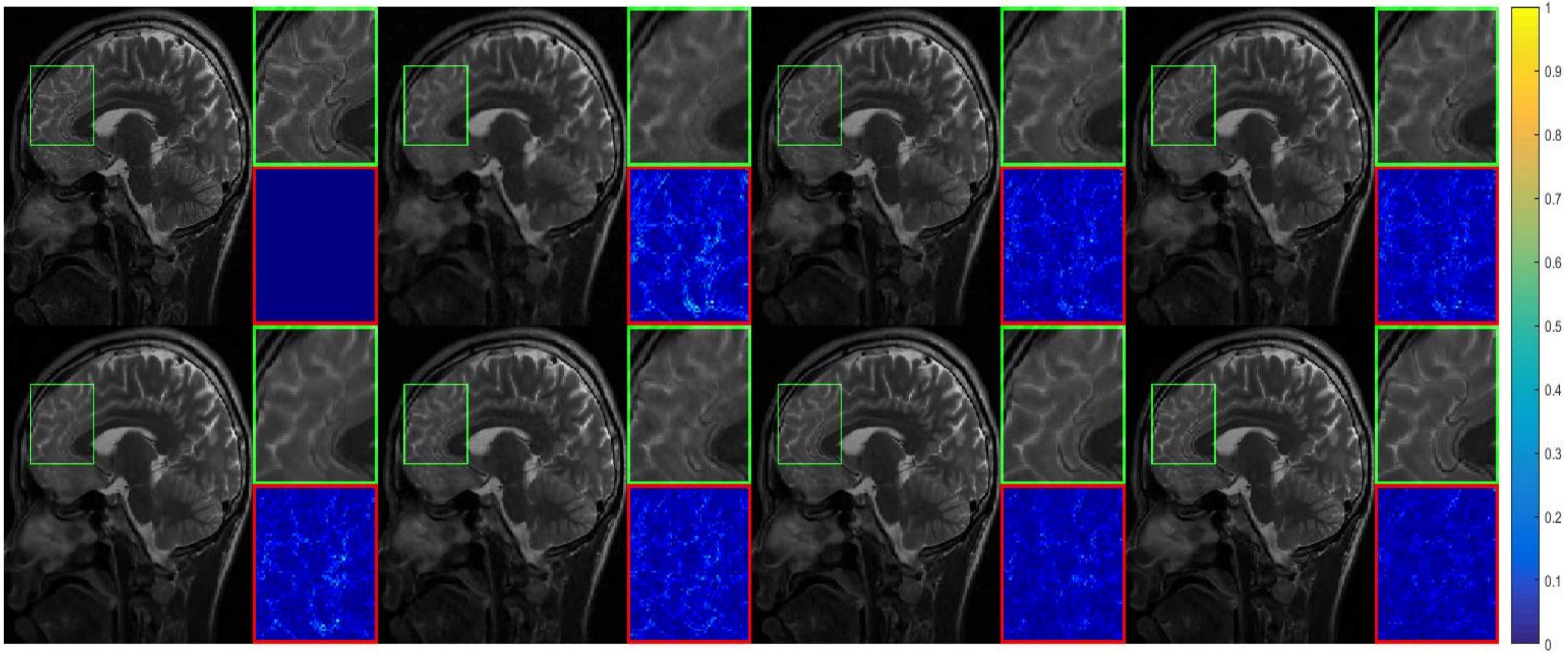
HGGDPRec



Pipeline of HGGDP training process for prior learning and HGGDPRec procedure for MRI reconstruction.

C. Quan, J. Zhou, Y. Zhu, Y. Chen, S. Wang, D. Liang, Q. Liu, Homotopic gradients of generative density priors for MR image reconstruction, *IEEE Trans. Med. Imag.*, 2021.

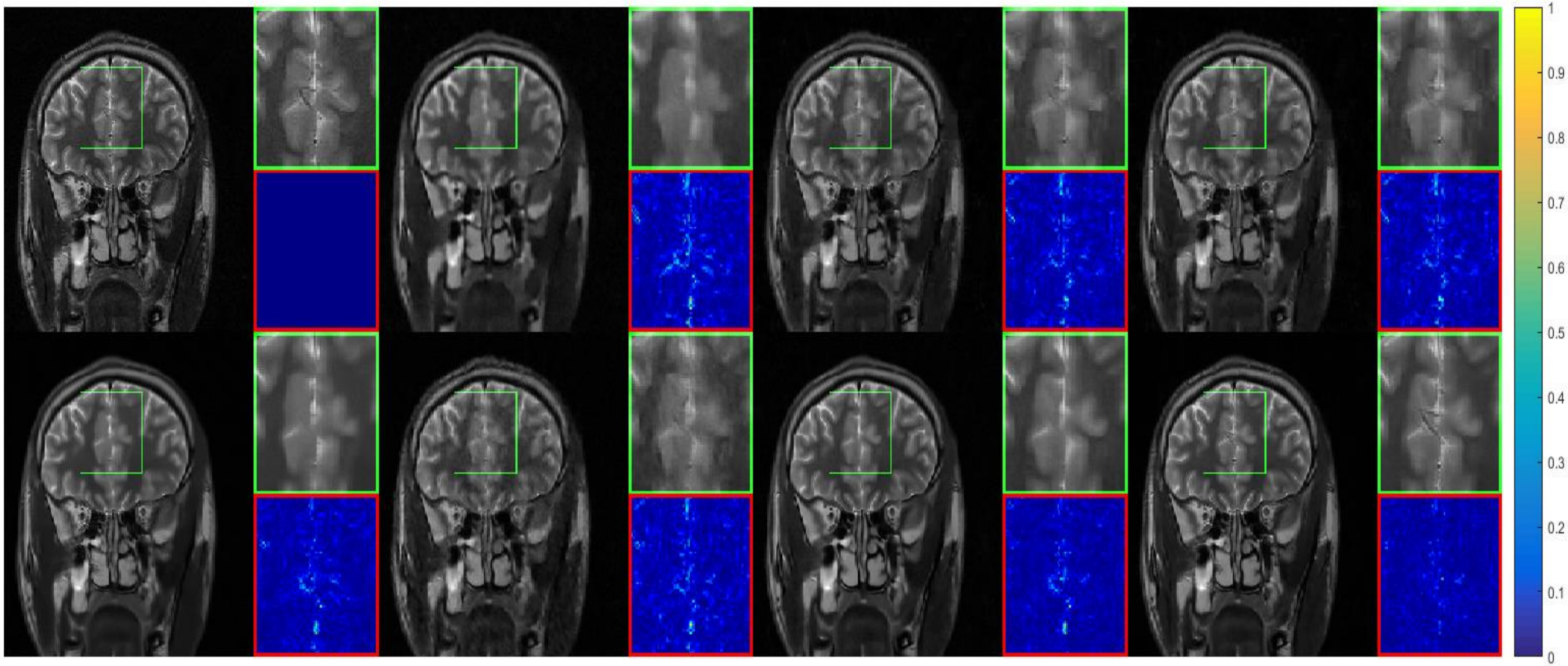
Experimental results



Reconstruction comparison on pseudo radial sampling at acceleration factor R=6.7. Top: Reference, reconstruction by DLMRI, PANO, FDLCP; Bottom: Reconstruction by NLR-CS, DC-CNN, EDAEPRec, HGGDPRec. Green and red boxes illustrate the zoom in results and error maps, respectively.

C. Quan, J. Zhou, Y. Zhu, Y. Chen, S. Wang, D. Liang, Q. Liu, Homotopic gradients of generative density prior s for MR image reconstruction, *IEEE Trans. Med. Imag.*, 2021.

Experimental results

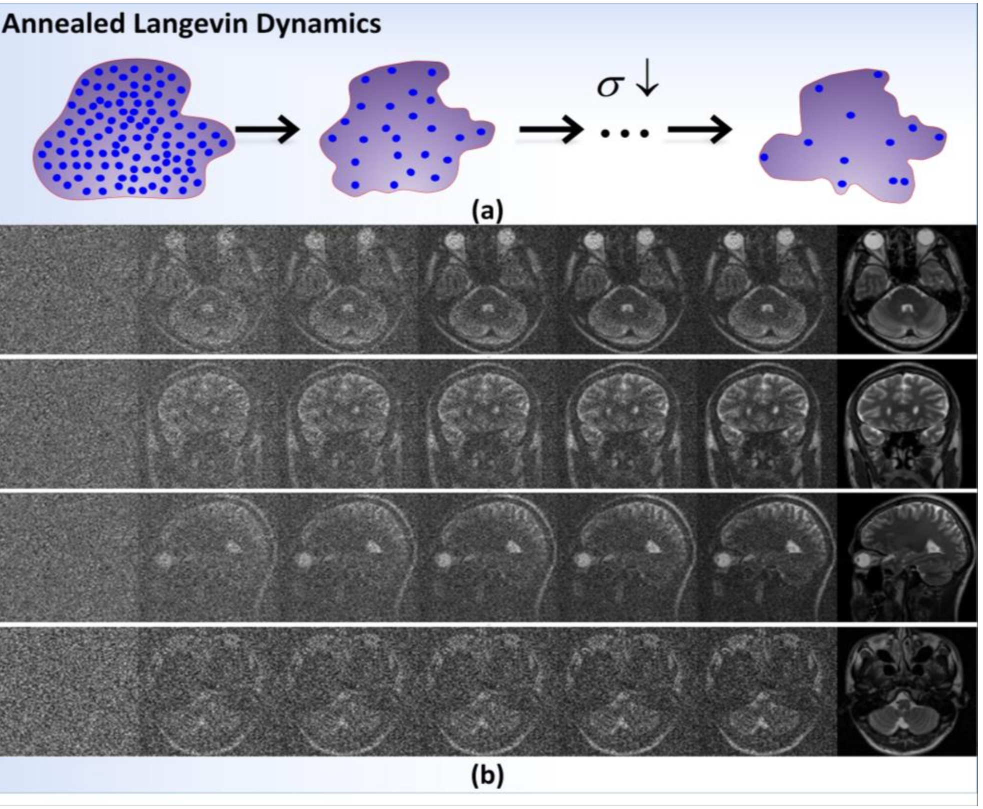


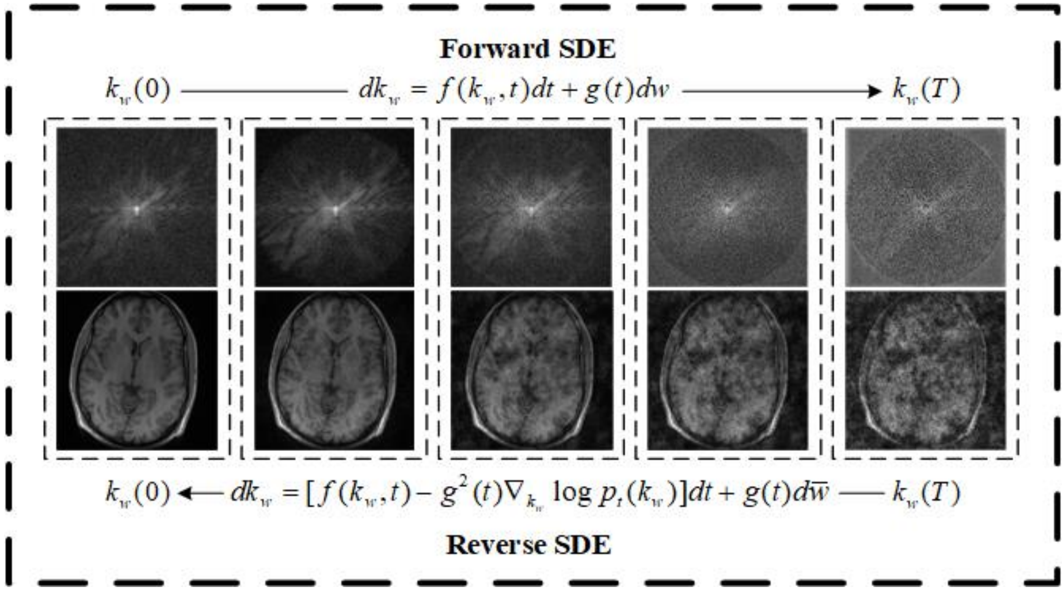
Reconstruction comparison on 2D Random sampling at acceleration factor R=6.7. Top: Reference, reconstruction by DLMRI, PANO, FDLCP; Bottom: Reconstruction by NLR-CS, DC-CNN, EDAEPRec, HGGDPRec. Green and red boxes illustrate the zoom in results and error maps, respectively.

C. Quan, J. Zhou, Y. Zhu, Y. Chen, S. Wang, D. Liang, Q. Liu, Homotopic gradients of generative density priors for MR image reconstruction, *IEEE Trans. Med. Imag.*, 2021.

Part 3.2-DSM from image domain to k-space domain

Generative model in image domain



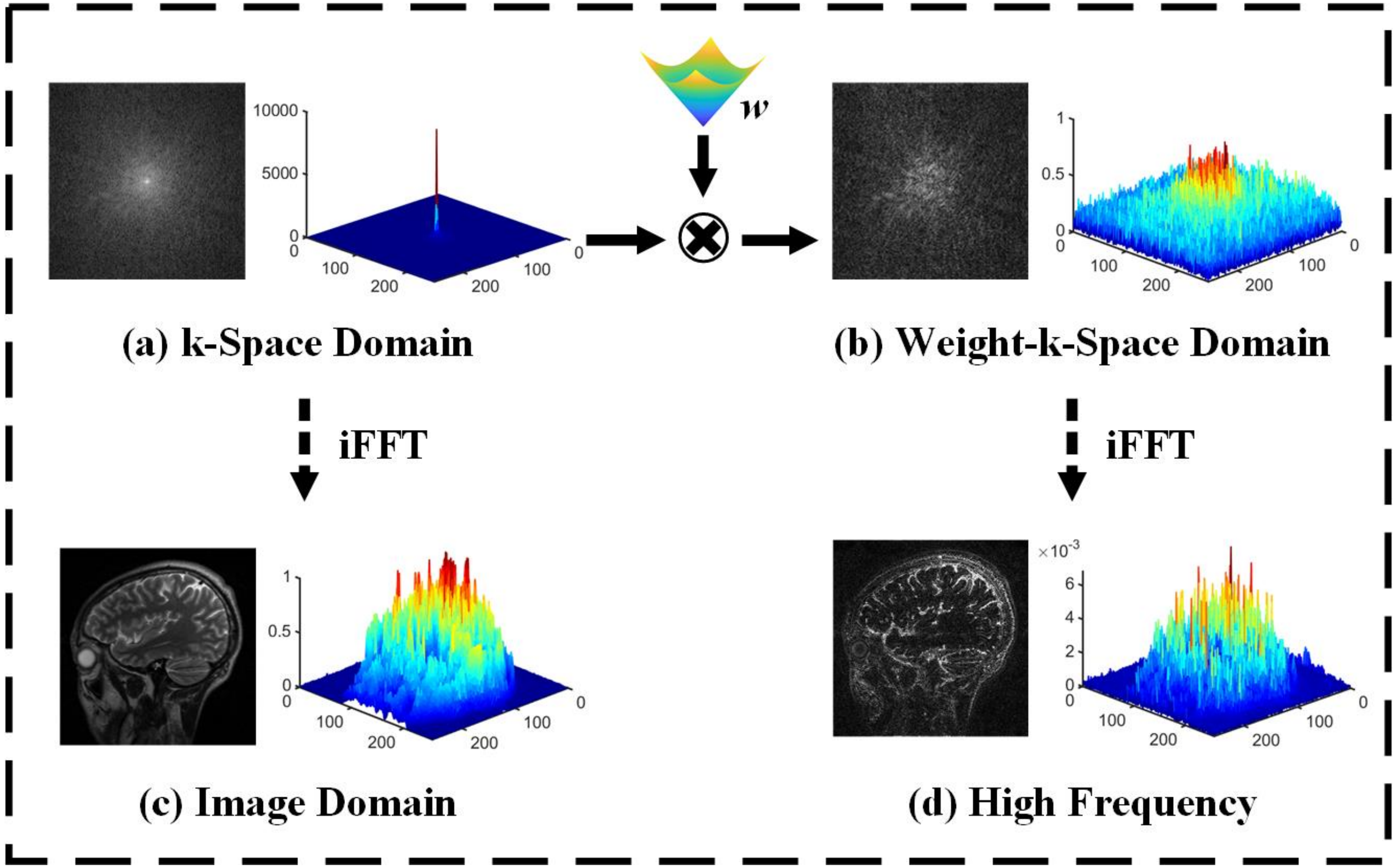


Generative model in k-space domain

Z. Tu, D. Liu, X. Wang, C. Jiang, M. Zhang, Q. Liu, D. Liang, WKGM: Weight-K-space generative model for parallel imaging reconstruction, https://arxiv.org/abs/2205.03883, 2022.

WKGM

Algorithm overview



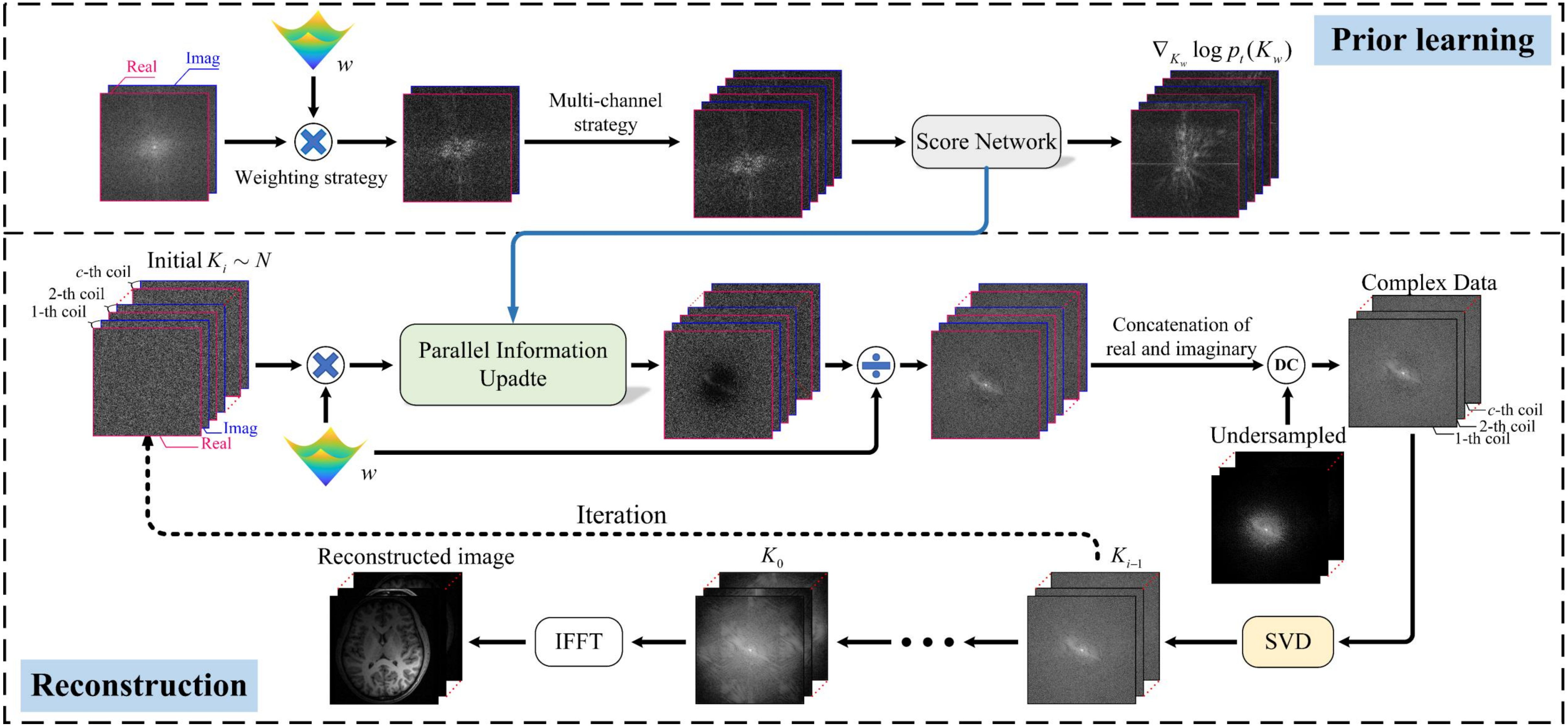
Visual comparison of the amplitude values in k-space domain and weight-k-space domain.

Prior learning in weighted k-space domain is more efficient !

Z. Tu, D. Liu, X. Wang, C. Jiang, M. Zhang, Q. Liu, D. Liang, WKGM: Weight-K-space generative model for parallel imaging reconstruction, https://arxiv.org/abs/2205.03883, 2022.

WKGM

Algorithm overview



Z. Tu, D. Liu, X. Wang, C. Jiang, M. Zhang, Q. Liu, D. Liang, WKGM: Weight-K-space generative model for parallel imaging reconstruction, https://arxiv.org/abs/2205.03883, 2022.

WKGM

Algorithm overview

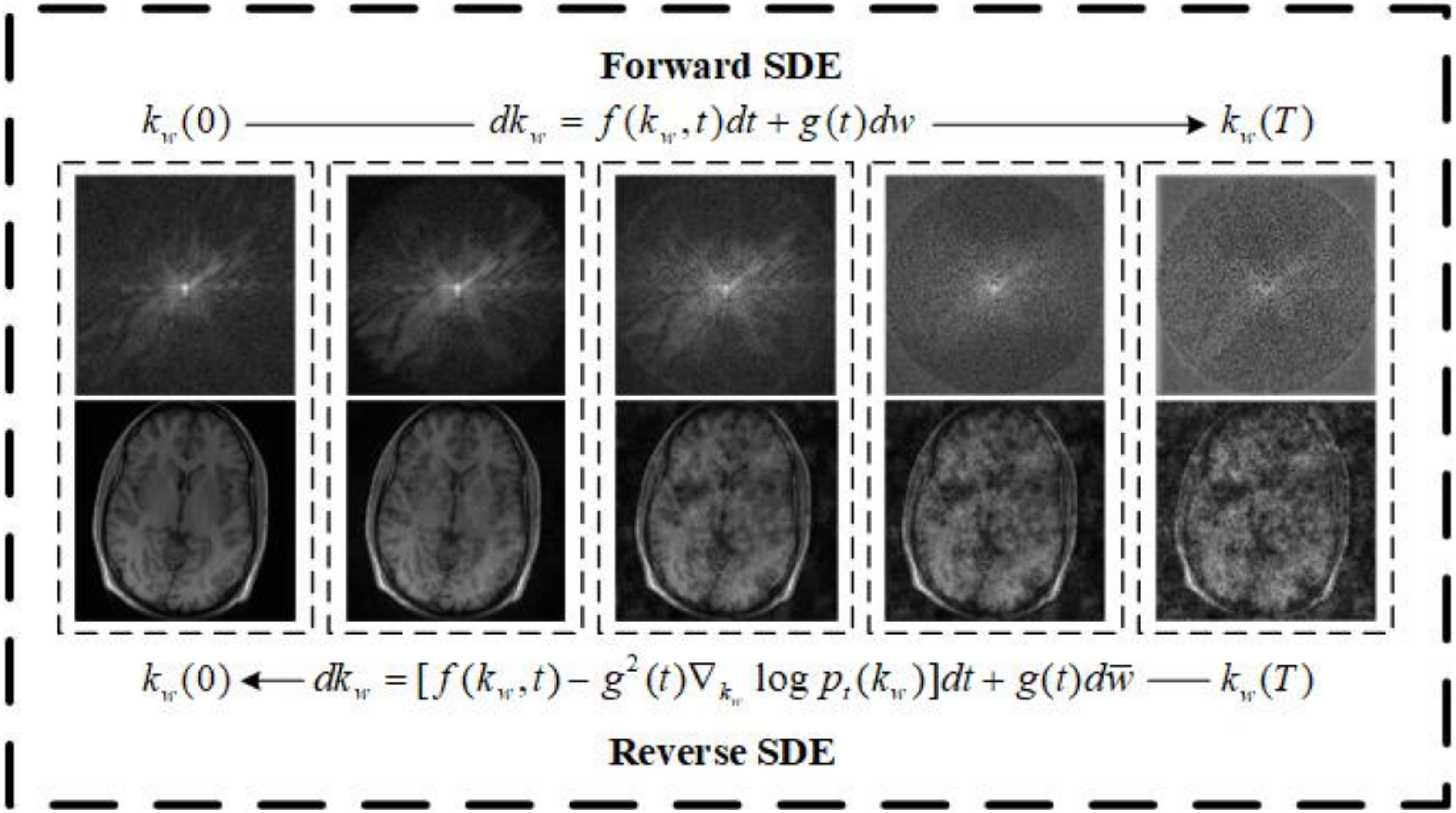
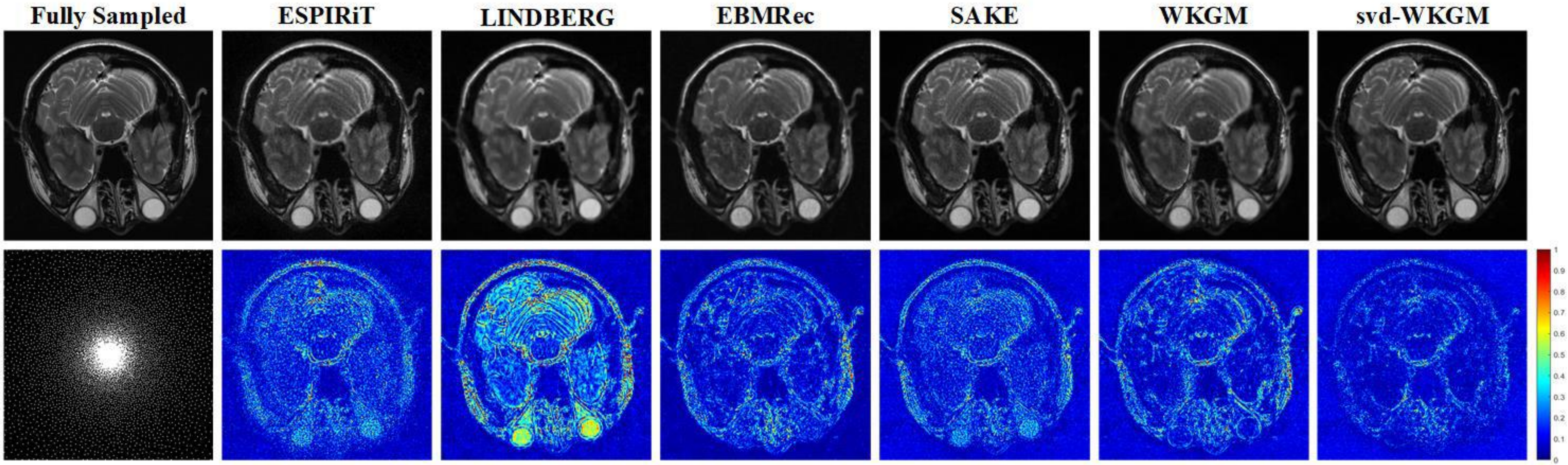


Illustration of the forward and reverse processes of k-space data and corresponding image. The forward process can be accomplished with a continuous-time SDE.

Z. Tu, D. Liu, X. Wang, C. Jiang, M. Zhang, Q. Liu, D. Liang, WKGM: Weight-K-space generative model for parallel imaging reconstruction, https://arxiv.org/abs/2205.03883, 2022.

WKGM

Experimental results



Parallel imaging reconstruction results by ESPIRiT, LINDBERG, EBMRec, SAKE, WKGM and svd-WKGM in *T 2 Transversal Brain* at *R*=10 2D Poisson disk under-sampling mask. The intensity of residual maps is five times magnify.

Z. Tu, D. Liu, X. Wang, C. Jiang, M. Zhang, Q. Liu, D. Liang, WKGM: Weight-K-space generative model for parallel imaging reconstruction, https://arxiv.org/abs/2205.03883, 2022.

Thanks all !

Thanks Zhipei Liang, Henry Leung, Dong Liang, Shanshan Wang, Yanjie Zhu
, Minghui Zhang, Yuhao Wang, Haifeng Wang, Sen Jia and Students Zhuonan
He, Cong Quan, Jinjie Zhou, Siyuan Wang, Tao Deng, Yu Guan, Yuanzhen Zhu, Jiaojiao X
iong, Sanqian Li, Fengqin Zhang, Qinxin Yang, Jin Li, Wanyun Li, Wenzhao Zhao, Wanqin
g Zhu, Kai Hong, MengtingLi, Zongjiang Tu, Xianghao Liang, Cailian Yang, ……