Deep Learning for Handling Kernel/model Uncertainty in Image Deconvolution

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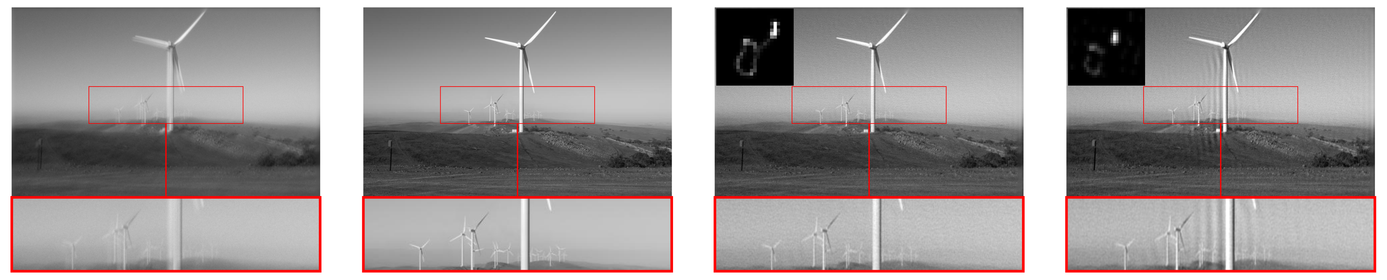
Background and Contribution

Background:

- Non-blind image deconvolution: observing blurry image $\boldsymbol{y}$ and kernel $\boldsymbol{k}$; aiming to recover sharp image $\boldsymbol{x}$ under the convolutional model

$$\boldsymbol{y} = \boldsymbol{k} \otimes \boldsymbol{x} + \boldsymbol{n}. \tag{1}$$

- Noise-free kernel is often unavailable/impractical due to
  1. Kernel error: observing $\widehat{\boldsymbol{k}}$ instead of $\boldsymbol{k}$;
  2. Model error: convolution is an approximation: $\boldsymbol{y} \approx \boldsymbol{k} \otimes \boldsymbol{x} + \boldsymbol{n}$.
- Image deconvolution is susceptible to kernel/model error.



Blurry input $\boldsymbol{y}$ Clear image $\boldsymbol{x}$ Result from $\boldsymbol{k}$ Result from $\widehat{\boldsymbol{k}}$

Key Contributions: Construct a *robust* image deblurring framework with

- A theoretical understanding by alleviating error-in-variable (EIV) model and total-least-squares (TLS) solver;
- A specially designed NN structure by unrolling TLS optimization;
- A sufficient training set by proposing kernel synthetic procedures.

Theoretical Formulation

Problem setting: Assume x, y is the colomn-wised vector of $\boldsymbol{x}, \boldsymbol{y}$, K and $\widehat{K}$ are the matrix form of the 2D convolution operator w.r.t. the unknown ground truth kernel $\boldsymbol{k}$ and estimated blur kernel $\widehat{\boldsymbol{k}}$,

$$y = Kx + n = (\widehat{K} - \Delta_K)x + n, \tag{2}$$

EIV perspective: Two error sources in EIV model: measurement noise n and model error Δ_K.

TLS estimator:

$$\min_{\Delta_K, n, x} \|\Delta_K\|_F^2 + \|n\|_2^2, \quad \text{s.t.} \quad (\widehat{K}x - \Delta_K x = y - n).$$

Certain prior $\phi(\cdot)$ is imposed to overcome the ill-posed kernel K

$$\min_{x,u} \|y - \widehat{K}x - u\|_2^2 + \phi(x) + \psi(u|x), \tag{3}$$

where

$$\psi(u|x) = \min_{\Delta_k \in \Omega} \|\Delta_k\|_F^2 + \lambda\|u - \Delta_K x\|_2^2.$$

The feasible set Ω for Δ_k denotes structure prior for the matrix Δ_K, e.g., the set of doubly Toeplitz matrices.

Method and Architecture

Deduced iterative NN and corresponding modules

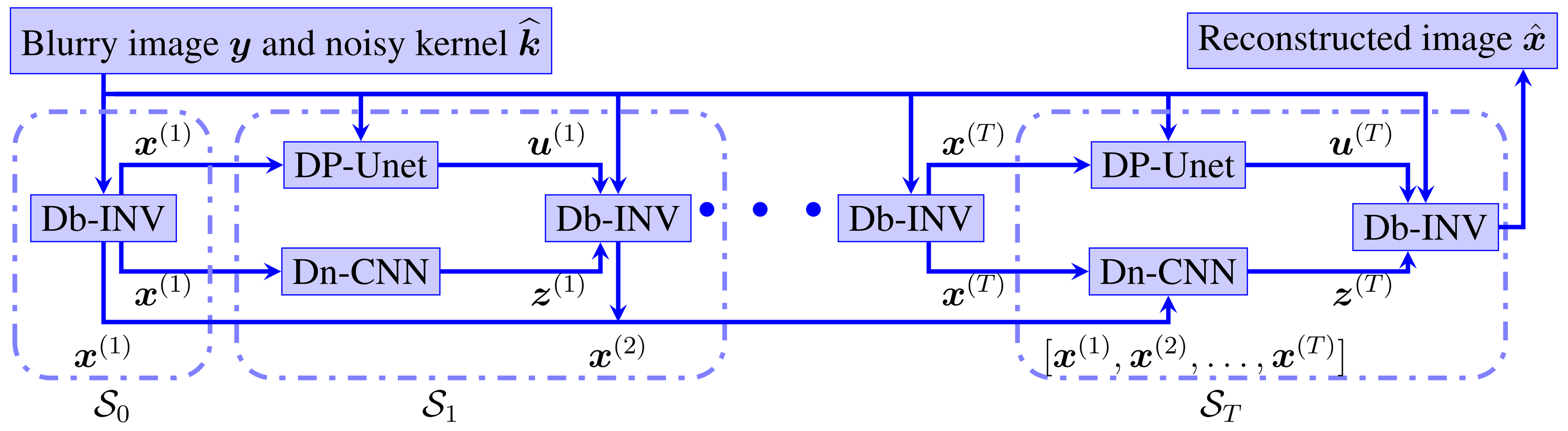
$$\boldsymbol{x}^{(t)} = \operatorname*{argmin}_{\boldsymbol{x}} \|\boldsymbol{y} - \widehat{\boldsymbol{k}} \otimes \boldsymbol{x} - \boldsymbol{u}^{(t-1)}\|_2^2 + \|\mathrm{diag}(\lambda)(\Gamma\boldsymbol{x} - \boldsymbol{z}^{(t-1)})\|_2^2; \quad \text{(Db-INV)}$$

$$\boldsymbol{z}^{(t)} = \operatorname*{argmin}_{\boldsymbol{z}} \|\mathrm{diag}(\lambda)(\Gamma\boldsymbol{x}^{(t)} - \boldsymbol{z})\|_2^2 + \rho(\boldsymbol{z}); \quad \text{(Dn-CNN)}$$

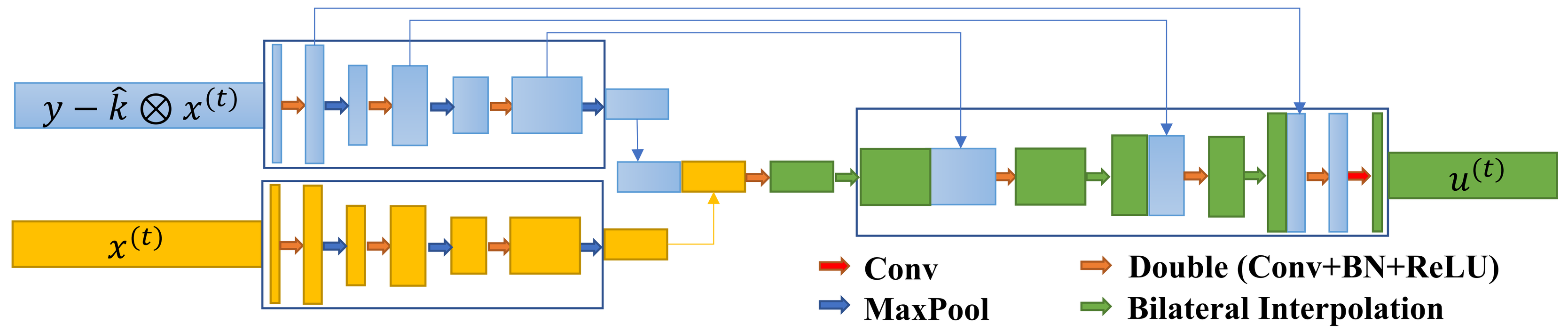
$$\boldsymbol{u}^{(t)} = \operatorname*{argmin}_{\boldsymbol{u}} \|\boldsymbol{y} - \widehat{\boldsymbol{k}} \otimes \boldsymbol{x}^{(t)} - \boldsymbol{u}\|_2^2 + \psi(\boldsymbol{u}|\boldsymbol{x}^{(t)}). \quad \text{(DP-Unet)}$$

where Γ denotes the set of high-pass filters such that gradient operator ∇ or wavelet filter bank $\{\boldsymbol{f}_i \otimes\}$.

Computational Flow



Architecture of DP-Unet

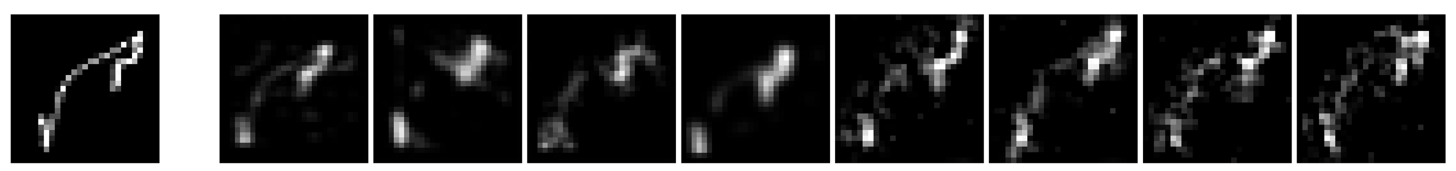


Data Preparation

Observation

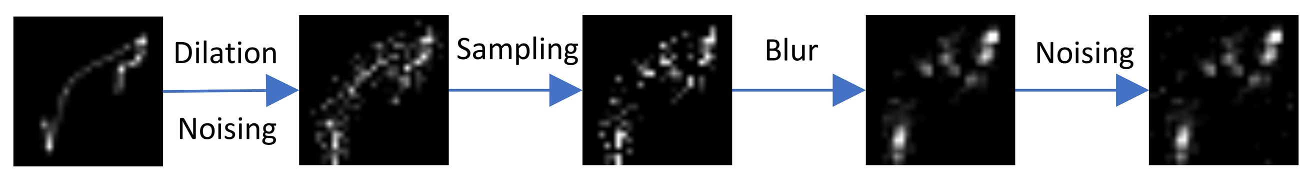
kernel degradation patterns including

- kernel diffusion (overly smoothed),
- missing pieces of kernels,
- random spike-like noise.



True Kernel Real kernel errors Synthetic kernel errors

Kernel synthesis pipeline

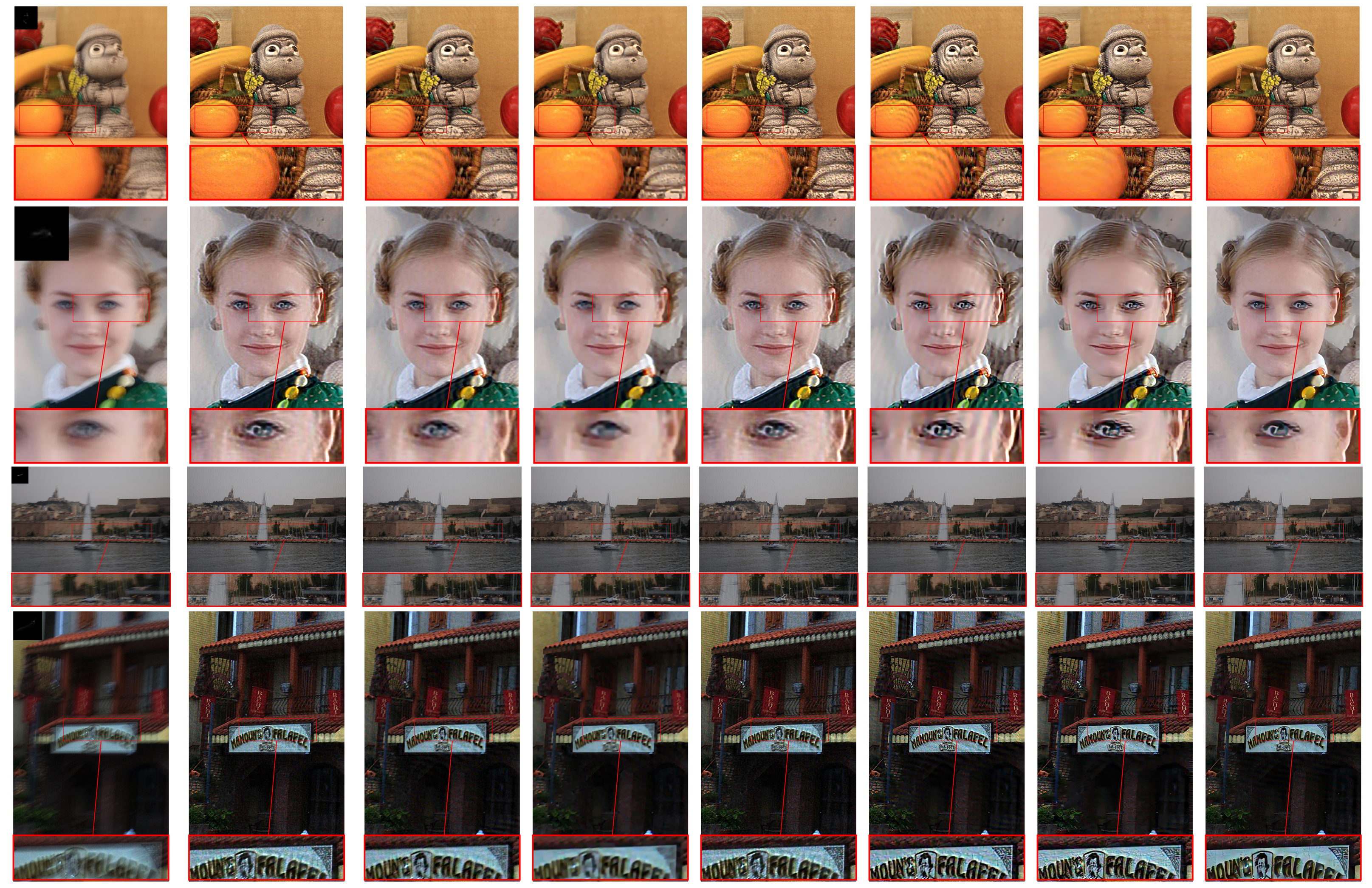


Experiments and Results

Quantitative Results on Synthetic Dataset

$\sigma = 1\%$	Levin *et al.*	Sun *et al.*	Lai *et al.*
Krishnan-09	28.85/0.812	28.90/0.792	19.30/0.661
Ji-11	28.45/0.843	27.72/0.755	19.48/0.673
Zoran-11	28.31/0.830	28.84/0.805	20.08/0.723
Whyte-14	28.65/0.818	28.79/0.783	19.84/0.703
Kruse-17	29.89/0.887	30.01/0.869	- / -
Zhang-17	29.70/0.872	30.03/0.855	19.68/0.708
Ours	**30.68/0.900**	**30.52/0.867**	**22.15/0.722**
$\sigma = 0\%$	Levin *et al.*	Sun *et al.*	Lai *et al.*
Vasu-18	32.27/0.922	30.94/0.880	22.44/0.758
Ours	**32.49/0.930**	**32.16/0.917**	**24.37/0.792**

Visual Results on Real Dataset



Input Krishnan-09 Zoran-11 Ji-11 Whyte-14 Zhang-17 Vasu-18 Ours

Visualization of learned DP-Unet

Sharp image $\boldsymbol{x}$ Blurry image $\boldsymbol{y}$ Recovered image $\hat{\boldsymbol{x}}$ $\Delta\boldsymbol{k} \otimes \boldsymbol{x}$ $\boldsymbol{u}$

Further Thoughts and Discussion

- Kernel/model uncertainty modeling is a step towards bridgeing the gap between blind and non-blind deconvlution.
- A trade-off in non-blind deblurring: believing in kernel information recovers more details but undermines robustness.
- Modeling error-in-variable improves the robustness. Except from learned method and ℓ_1 norm, a concise but powerful prior is still uncovered.