Variational-EM-based Deep Learning for Noise-blind Image Deblurring

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Motivation and Contribution

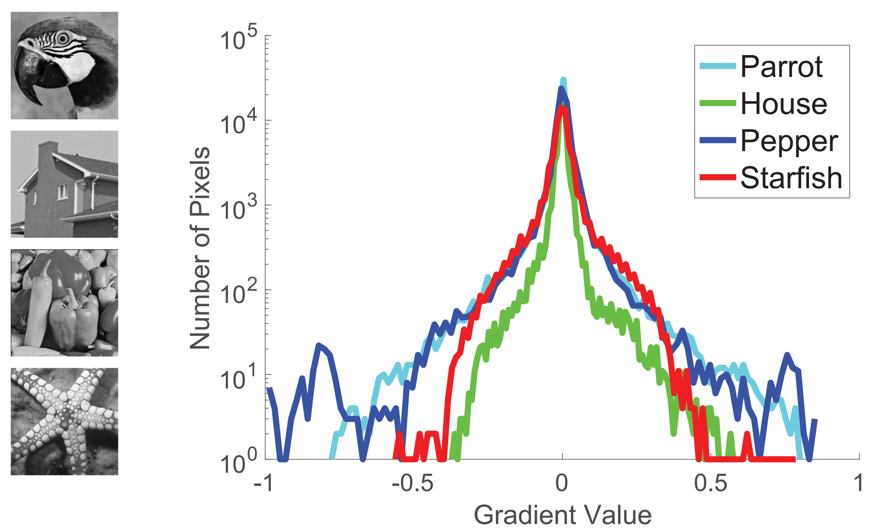
Background:

Non-blind image deconvolution: observing blurry image $\boldsymbol{y}$ and kernel $\boldsymbol{k}$; aiming to recover sharp image $\boldsymbol{x}$ under the convolutional model

$$\boldsymbol{y} = \boldsymbol{k} \otimes \boldsymbol{x} + \boldsymbol{n}. \tag{1}$$

Motivation:

- *Noise-blind* deblurring, training a universal model suitable for images with unknown noise level, is preferred.
- Imposed image prior $p(\boldsymbol{x})$ should be adaptive to each individual content.



Key Contributions: We construct a Variational-EM-based deblurring framework able to

- integrate the estimations of both noise level and image prior uncertainty;
- employ the NNs to model both image prior and model uncertainty;
- boost performance in comparison with both noise-blind and fixed noise level deblurring models.

Notation

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- The observed variable: blurry image $\boldsymbol{y} \in R^N$.
- The latent variables: sharp image $\boldsymbol{x} \in R^N$ and high frequency priors $\boldsymbol{z} = \{\boldsymbol{z}_i \in R^N | i = 1, \ldots, L\}$. The latent variable $\boldsymbol{x}$ follows the conditional distribution based on $\boldsymbol{z}$

$$p(\boldsymbol{x}|\boldsymbol{z}) \propto \Pi_{i=1}^{L} \mathcal{N}(\boldsymbol{f}_i \otimes \boldsymbol{x} | \boldsymbol{z}_i, \lambda_i^2 \mathbf{I}) \tag{2}$$

  The likelihood is $p(\boldsymbol{y}|\boldsymbol{x}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{k} \otimes \boldsymbol{x}, \sigma^2 \mathbf{I})$.

- The parameter set

$$\Theta = \{(\sigma, \lambda_i) | \sigma > 0, \lambda_i > 0, i = 1, \cdots, L\},$$

  where σ denotes the noise level, and the vector $\boldsymbol{\lambda} = [\lambda_i]_{i=1}^{L}$ denotes the standard deviation of the prior which follows a joint prior distribution $p(\boldsymbol{\lambda})$.

Variational-EM framework

E-step:

Provided an estimate $\theta := \theta^t \in \Theta$, the E-step estimates the $q(\boldsymbol{x}, \boldsymbol{z}) \in Q$ via minimization of KL divergence

$$q^{t+1}(\boldsymbol{x}, \boldsymbol{z}) = \underset{q \in Q}{\operatorname{argmin}} \, \mathrm{KL}(q(\boldsymbol{x}, \boldsymbol{z}) || p(\boldsymbol{x}, \boldsymbol{z} | \boldsymbol{y}, \theta^t)). \tag{3}$$

M-step:

Provided the variational distribution $q^{t+1}(\boldsymbol{x}, \boldsymbol{z})$, the parameters set $\theta = \{\sigma, \boldsymbol{\lambda}\}$ is updated by

$$\theta^{t+1} = \underset{\theta}{\operatorname{argmax}} \, \mathbb{E}_{q^{t+1}(\boldsymbol{x}, \boldsymbol{z})} \log(p(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{y}, \theta)), \tag{4}$$

VEM-based iterative scheme:

$$\boldsymbol{x}^{t+1} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \, \|\boldsymbol{y} - \boldsymbol{k} \otimes \boldsymbol{x}\|_2^2 + \sum_{i=1}^{L} (\frac{\sigma^t}{\lambda_i^t})^2 \|\boldsymbol{f}_i \otimes \boldsymbol{x} - \boldsymbol{z}_i^t\|_2^2,$$

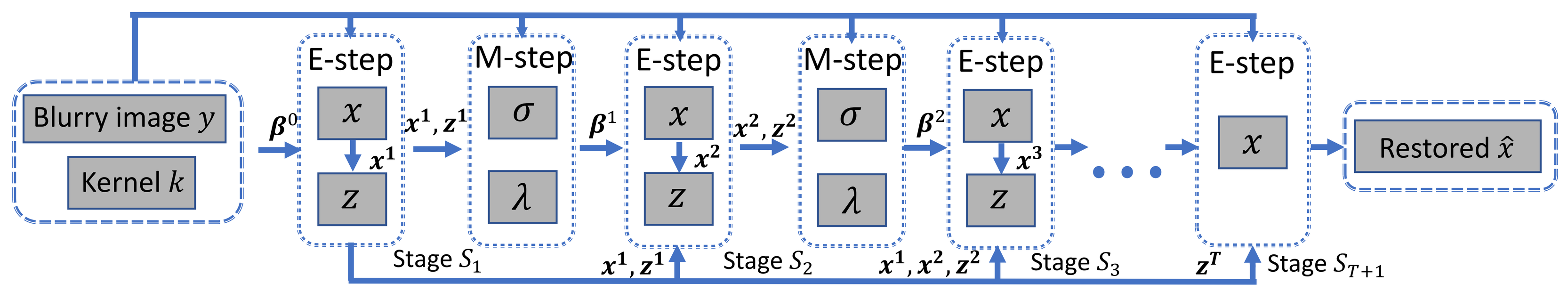
$$\boldsymbol{z}^{t+1} = \underset{\boldsymbol{z}}{\operatorname{argmin}} \sum_{i=1}^{L} \frac{1}{(\lambda_i^t)^2} \|\boldsymbol{f}_i \otimes \boldsymbol{x}^{t+1} - \boldsymbol{z}_i\|_2^2 + \log p(\boldsymbol{z}),$$

$$\sigma^{t+1} = \{\frac{1}{N}(\|\boldsymbol{y} - \boldsymbol{k} \otimes \boldsymbol{x}^{t+1}\|^2 + \gamma^2 \|\boldsymbol{k}\|^2)\}^{\frac{1}{2}},$$

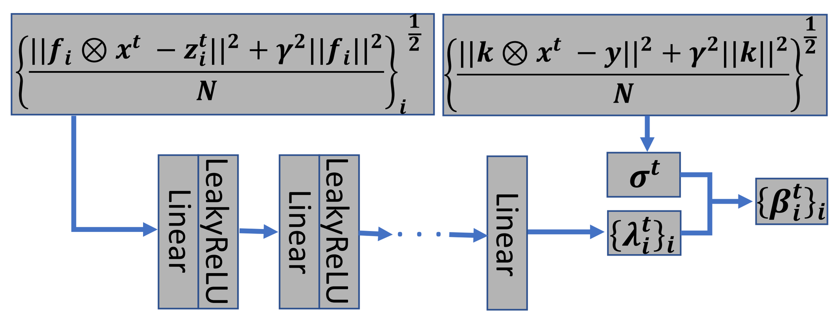
$$\boldsymbol{\lambda}^{t+1} = \underset{\boldsymbol{\lambda}}{\operatorname{argmin}} \sum_{i=1}^{L} \{\frac{1}{2\lambda_i^2} \|\boldsymbol{f}_i \otimes \boldsymbol{x}^{t+1} - \boldsymbol{z}_i^{t+1}\|_2^2 + \frac{\gamma^2}{2\lambda_i^2} \|\boldsymbol{f}_i\|^2 + N \log(\lambda_i)\} + \log p(\boldsymbol{\lambda}).$$

Method and Architecture

Computational Flow



Architecture of M-step



Loss function

$$\mathcal{L} := \|\boldsymbol{x}^{T+1} - \boldsymbol{x}\|_2^2 + \sum_{i=2}^{T} \mu_i \|\boldsymbol{x}^i - \boldsymbol{x}\|_2^2, \tag{5}$$

Experiments and Results

Quantitative comparison with noise-blind / fixed noise level methods

	Set12			Sun			Levin		
	2.55	7.65	12.75	2.55	7.65	12.75	2.55	7.65	12.75
IDD-BM3D	31.43	27.56	25.95	32.24	28.74	27.30	33.75	29.26	27.33
FDN	31.43	27.89	26.28	32.30	28.97	27.62	33.65	29.79	28.02
EPLL-NA	- / -	- / -	- / -	32.18	28.77	- / -	32.16	28.77	- / -
GradNet7S	- / -	- / -	- / -	31.75	28.04	- / -	31.43	27.55	- / -
DMSP	31.06	27.87	26.39	31.76	28.68	27.53	32.61	29.31	27.79
EPLL	27.61	25.24	23.87	30.53	27.46	26.08	32.03	28.31	27.15
CSF	29.37	26.41	25.10	31.04	27.84	26.53	29.85	27.28	26.25
FCNN	30.68	27.40	25.84	32.19	28.93	27.55	33.10	29.50	27.81
IRCNN	30.53	27.09	- / -	30.91	27.93	- / -	32.66	29.15	- / -
Ours	**31.93**	**28.47**	**26.77**	**32.73**	**29.41**	**28.04**	**34.31**	**30.50**	**28.52**

Visualization of intermediate results

$\boldsymbol{y}$: 14.54 $\boldsymbol{x}^{(0)}$: 16.02 $\boldsymbol{x}^{(1)}$: 26.29 $\boldsymbol{x}^{(2)}$:27.14 $\boldsymbol{x}^{(3)}$: 27.71 $\boldsymbol{x}^{(4)}$: 27.84 Truth image

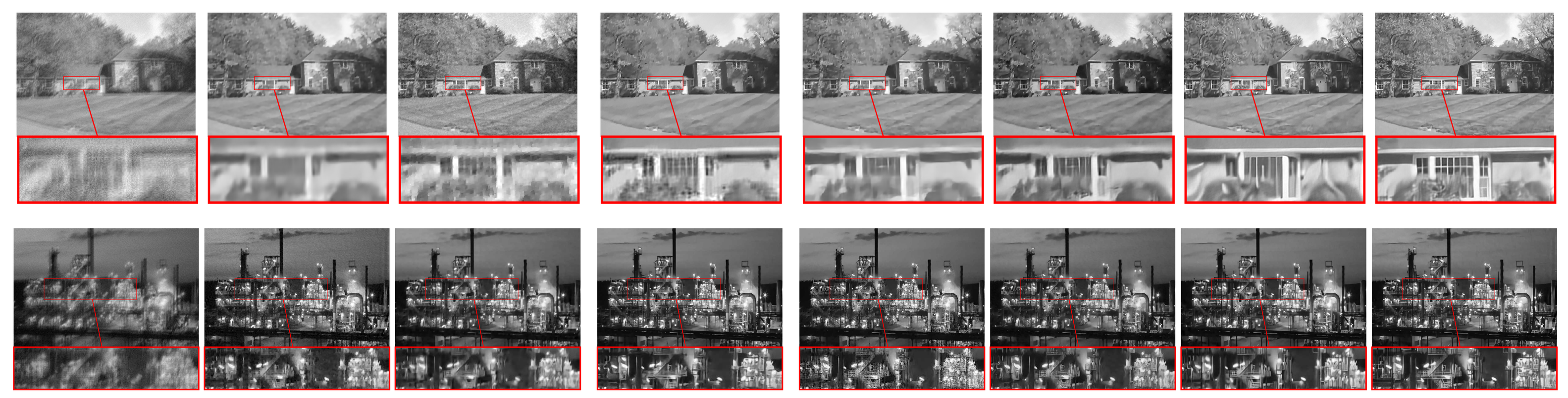
The noise level prediction

True σ	1%	2%	3%	4%	5%
Predicted $\hat{\sigma}$	1.3%	2.0%	2.8%	3.7%	4.6%

Evaluation on Poisson noise

Peaks	VST-BM3D	RWL2	FCNN	Ours
128	24.39	24.72	25.09	**25.69**
256	24.98	25.52	26.09	**26.69**
512	25.50	25.81	27.27	**27.93**
1024	-	26.30	27.95	**29.15**

Visualization of recovered results



Input EPLL CSF IDDBM3D FCNN FDN DMSP Ours

Further Thoughts and Discussion

- VEM-based parameter approximation is a powerful tool to model uncertainty for both noise level and prior uncertainty;
- Fourier-based architectures including FDN, FCNN, and ours all suffer from erroneous boundary condition for certain example. Future research can work on such issue.