Decentralized Formation of Arbitrary Multi-Robot Lattices

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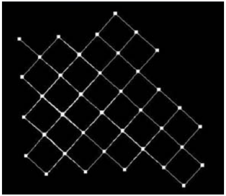
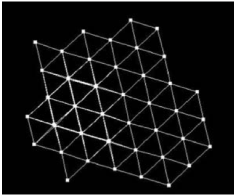
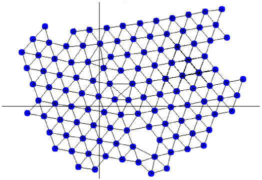
Related Work
Formation using virtual forces

W. Spears, D. Spears, J. Hamann and R. Heil, 2004

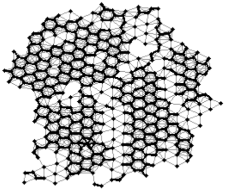




I. Navarro, J. Pugh, A. Martinoli, and F. Matia, 2008

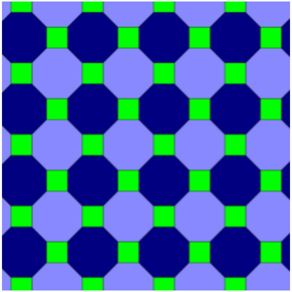


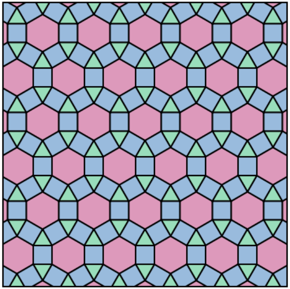
S. Prabhu, W. Li, J. McLurkin, 2012



Motivation

Question: How to use one algorithm to generate various (repeating) lattice pattern formations?





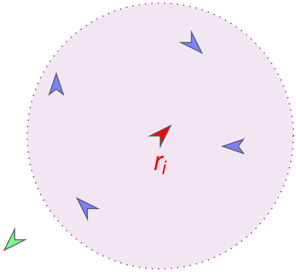
Objective

INPUT: A representation of a (repeating) lattice pattern.

OUTPUT: Move to this pattern using only local information.

Robot Model

- Differential Drive robots.
- Each robot has an unique **ID**.
- Use a vector $p = [x, y, \theta]^T$ to represent robot's **pose**.
- Each robot has a **range** within which it can sense and communicate with other robots.
- Each robot gets **observations** of its neighbors' IDs and relative poses in its body frame.

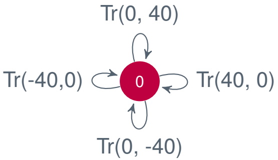


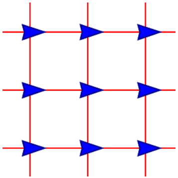
Robot r_i has 4 neighbors

Input: Lattice Graph

Definition

A **lattice graph** is a strongly connected directed multigraph in which each edge e is labeled with a rigid body transformation $T(e)$ and each $v \xrightarrow{T(e)} w$ has an inverse edge $w \xrightarrow{T(e)^{-1}} v$.



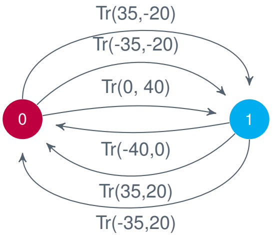


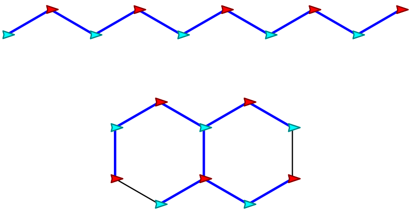
Role Function

Definition

Given a lattice graph $G = (V, E)$ and a set of robots $R = \{r_1, \ldots, r_n\}$, R **satisfies** G if there exists a role function $f : R \to V$ that preserves the neighborhood structure of G.

Specifically, for any i and j, if r_i and r_j are neighbors, there must exist an edge $e_{ij} : f(r_i) \longrightarrow f(r_j)$ in E, such that $T(r_j) = T(r_i)T(e_{ij})$.





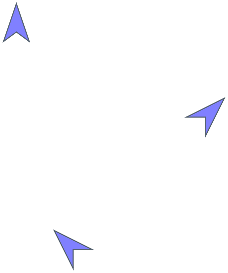
Algorithm

General Description

Robots broadcast messages containing their:

- authority
- matching.

1. Form tree structure.
2. Use tree structure to compute local task assignment.
3. Make movement decision.



Authority

Definition

An **authority** is an ordered list of robot IDs

$$(\mathrm{id}_1, \ldots, \mathrm{id}_k)$$

The first ID in the list, id_1 is called the **root** ID. The final ID in the list, id_k is called the **sender** ID.

Definition

Authority A_2 is **higher than** A_1 if:

- root ID of $A_2 >$ root ID of A_1, or
- length of $A_2 <$ length of A_1 if they have the same root, or
- sender ID of $A_2 >$ sender ID of A_1 if they have the same root and length.

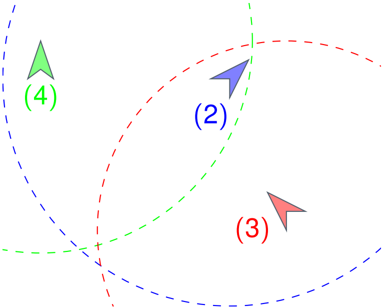
1.Construct Authority Tree

Decide to be root or descendant

The robots use these authorities to establish a collection of authority trees

1. Discards any message in which the authority contains its own ID.
2. Forms an authority containing only its own ID, compares it with the authorities of remaining messages and selects the highest authority.

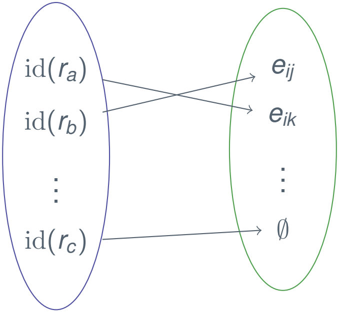
- If its authority is the highest, then it is a root;
- Otherwise, it selects the one who sends the highest authority as its parent. Append its own ID to the highest authority to create its own authority.



Matching

Definition

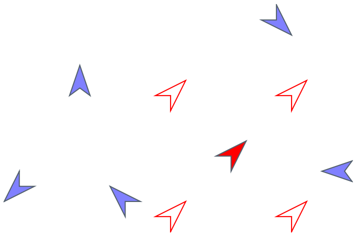
A **matching** for a robot is a function $\eta : \{\mathrm{id}(r_a), \mathrm{id}(r_b), ...\} \rightarrow \{\emptyset, e_{ij}, e_{ik}, ...\}$ that associates each neighbor ID with either a lattice graph edge from its role vertex or with the null value $\emptyset$.



2. Local Task Assignment
Hungarian Algorithm

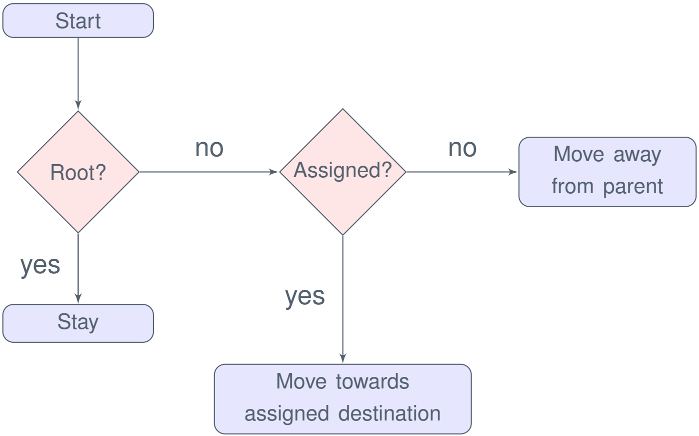
To compute an optimal matching of a robot with N neighbors and E out-going edges of its role in the lattice graph, define a weight matrix of size $N \times \max(N, E)$ and apply Hungarian Algorithm (H. W. Kuhn, 1955).

1. Each row corresponds to a neighbor;
2. Each column corresponds to an out-going edge of robot's role or a null value $\emptyset$.
3. The entries of the matrix are the Euclidean distance between current position of each neighbor and the desired position if matched with a lattice graph edge.



5 neighbors, 4 out-going edges.

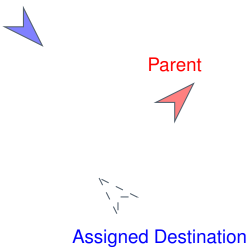
3. Robot Movement Strategy



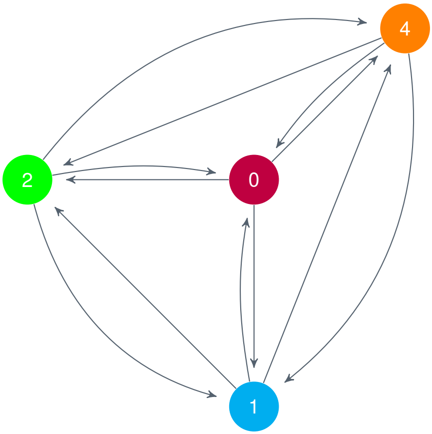
Bounded Movement

Goal: stay within the range of its parent

- Within the set O (Red circle), the parent is guaranteed to get observation at next stage.
- Set P (blue circle) denotes reachable point of the descendant in one step.
- The real destination for descendant is the closest point in the intersection ($O \cap P$) to the assigned destination.



Lattice graph for the octagon-square lattice



Simulation

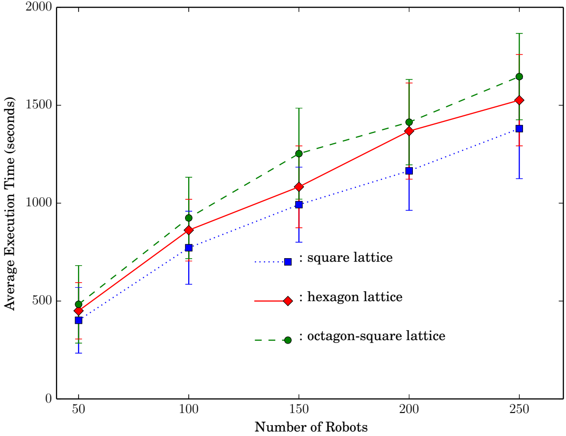
Octagon-square formation with 100 robots

Simulation

Hexagon formation with 100 robots

Experiment Results

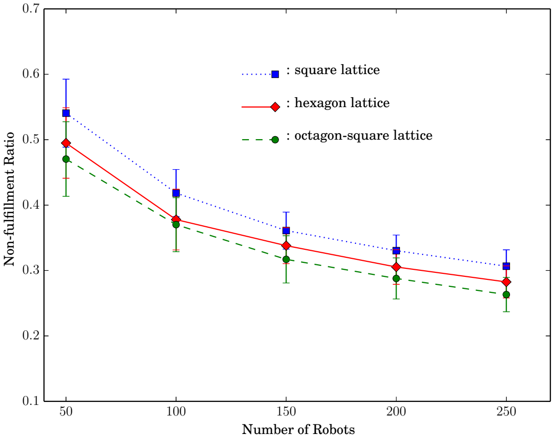
on three kinds of repeated lattice patterns



Average time to the static position with 50 trials, uniform distribution.

Experiment Results

on three kinds of repeated lattice patterns



Average non-fulfillment ratio $\Gamma = \frac{1}{n}\sum_{i=1}^{n}\frac{E_i - N_i}{E_i}$ with 50 trials, uniform distribution.

Conclusions

Summary

- Robots can form different types of geometric formations, including repeated lattice patterns.
- Algorithm scales reasonably well with increasing numbers of robot.
- Algorithm is robust to the situation when some robots are removed from or with more robots added to the system.

Future Work

- Improve the motion strategy
- Prove convergence
- Nonholonomic constraints

Questions

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