ECE 365 **Fall 2019**

September 5, 2019

Name:

NetID:

1. **[Bayes Classifier (10 points)]**
Consider the binary hypothesis testing problem:

$$H_{-1}: \ X \sim p(x|-1)$$
$$H_1: \ X \sim p(x|1)$$

where

$$p(x|-1) = \begin{cases} 0.1 & x=0 \\ 0.2 & x=1 \\ 0.3 & x=2 \\ 0.4 & x=3 \\ 0 & \text{otherwise} \end{cases} \qquad p(x|1) = \begin{cases} 0.25 & x=0 \\ 0.25 & x=1 \\ 0.2 & x=2 \\ 0.3 & x=3 \\ 0 & \text{otherwise} \end{cases}$$

with priors $\pi_{-1} = 0.5, \pi_1 = 0.5$.

(a) Find the Bayes classifier (MAP decision rule).
Hint: It may be easier to use the likelihood ratio form of the MAP test, rather than the joint probability matrix, to solve this problem.
Solution: The Bayes classifier chooses 1 when

$$\frac{p(x|1)}{p(x|-1)} > \frac{\pi_{-1}}{\pi_1} = 1$$

i.e., when $x = 0$ or $x = 1$, and chooses -1 when $x = 2$ or $x = 3$.

(b) Calculate the probability of error for the Bayes classifier.
Solution: The probability of error is given by:

$$\begin{aligned} \mathrm{P_e} &= \pi_{-1}\mathrm{P}\left\{\text{say } H_1 \mid H_{-1} \text{ true}\right\} + \pi_1 \mathrm{P}\left\{\text{say } H_{-1} \mid H_1 \text{ true}\right\} \\ &= 0.5(0.1+0.2) + 0.5(0.2+0.3) = 0.15 + 0.25 = 0.4 \end{aligned}$$

2. **[Classifier performance (10 points)]**

(a) Consider the *0-1 loss* function. Explain clearly how you can design a classifier to have a training error of 0.
Solution: Set

$$f(\underline{x}_i) = \begin{cases} y_i & \text{if } (\underline{x}_i, y_i) \in \mathcal{T} \\ 1 & \text{otherwise.} \end{cases}$$

(b) Explain the difference between training error and prediction (generalization) error.
Solution: Training error is the average error that the classifier incurs over the training samples, whereas the prediction error is the average error that classifier incurs over the entire population.

3. **[Linear Classifier (10 points)]**
Consider a 3-ary linear classifier, with classes 1, 2, and 3, for which the three linear discriminant functions are:

$$g_{12}(\underline{x}) = x_1 + 2x_2 - 3$$
$$g_{13}(\underline{x}) = 2x_1 + x_2 - 1$$
$$g_{23}(\underline{x}) = x_1 - x_2 + 2$$

Classify the input $\underline{x}$ with $x_1 = 1, x_2 = 2$.

Solution: Note that $g_{12}(\underline{x}) = 2$, which favors 1 over 2, $g_{13}(\underline{x}) = 3$, which favors 1 over 3, and $g_{23}(\underline{x}) = 1$, which favors 2 over 3. Thus Class 1 dominates both the other classes, which means that the point is classified as 1.