Name:

NetID:

ECE398BD - Quiz 4
February 17, 2016
Time: 20 minutes
Please print your answers neatly (especially if you have poor handwriting). Circle your answers as well.

Problem 1 *Eigenvalues, Eigenvectors and SVD* [20 points]

1. Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. You do not need to normalize the eigenvectors. [5 points for eigenvalues, 5 points for eigenvectors]

2. Let $A = \frac{1}{\sqrt{10}}\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \end{bmatrix} + \frac{2}{\sqrt{10}}\begin{bmatrix} -1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \end{bmatrix}$.

   What are the left singular vectors of A? [3 points] What are the right singular vectors of A? [3 points] What are the singular values of A? [4 points]

   To recieve full credit, make sure to number your singular vectors and singular values such that they are appropriately matched to each other. Clearly mark which vectors are left singular vectors and which are right singular vectors.

1. $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$\det(\lambda I - A) = \det\begin{pmatrix} \lambda-3 & -1 \\ -1 & \lambda-3 \end{pmatrix}$.

$= (\lambda-3)^2 - 1 = 0$.

$\lambda = 4$ or 2.

For $\lambda = 4$. $\vec{u_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

$\lambda = 2$ $\vec{u_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$\lambda = 4$. or 2.

$\vec{u} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ or $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

2. left singular vectors.

$U = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$.

right singular vectors

$V = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$.

$S = \begin{bmatrix} \frac{1}{\sqrt{10}} & 0 \\ 0 & \frac{2}{\sqrt{10}} \end{bmatrix}$.

$A = U \cdot S V^T$.

Problem 2 *Principal Component analysis* [10 points]

1. Fill in the blank: The i-th principal component explains the most variance in the data in a direction orthogonal to the prior principal components $(1, \ldots, i-1)$. [5 points]

2. Draw the principal components of the data below. Clearly mark which principal component is the first principal component and which is the second principal component [5 points].