Beam Cooling

Beam Temperature

- We define the beam temperature as:
  - The longitudinal temperature:

$$\frac{1}{2}kT_{\parallel} = \frac{1}{2}mv_{\parallel}^2 = \frac{1}{2}m\beta^2c^2\left(\frac{dp}{p}\right)^2$$

  - The transverse temperature

$$\frac{1}{2}kT_{\perp} = \frac{1}{2}mv_{\perp}^2 = \frac{1}{2}m\beta^2c^2\gamma^2\theta_{\perp}^2$$

Why beam is hot

- The beam is generated at source. They have temperature from the distribution
  - Electron come from cathodes, with initial Fermi-Dirac distribution
  - Proton come from the plasma, with Maxwell-Boltzmann distribution
- Later on the beam can be 'heated' due to mismatch, intra-beam scattering, residue gas, etc.

What is beam cooling

- Cooling is to reduce the phase space area of the beam or the temperature of the beam, or maintain the emittance against the factors to make emittance growth.
- We already learn ways to reduce emittance which are not cooling methods:
  - Acceleration
  - Collimation

Way of Cooling

- Synchrotron Radiation
- Ionization Cooling
- Electron Cooling
- Stochastic Cooling
- Coherent electron cooling

Electron does not need cooling:

- Synchrotron Radiation is a natural way of cooling (damping).
- Revisit the energy deviation motion:

$$\frac{d^2\Delta E}{dt^2} = \frac{eV\omega_0^2 h\cos\phi_s\eta}{2\pi E_0}\Delta E - \frac{\omega_0}{2\pi}\frac{dU}{dE}\frac{\Delta E}{dt}$$

$$v \sim \frac{d\Delta E}{dt} \qquad\qquad F \sim -\alpha_E v$$

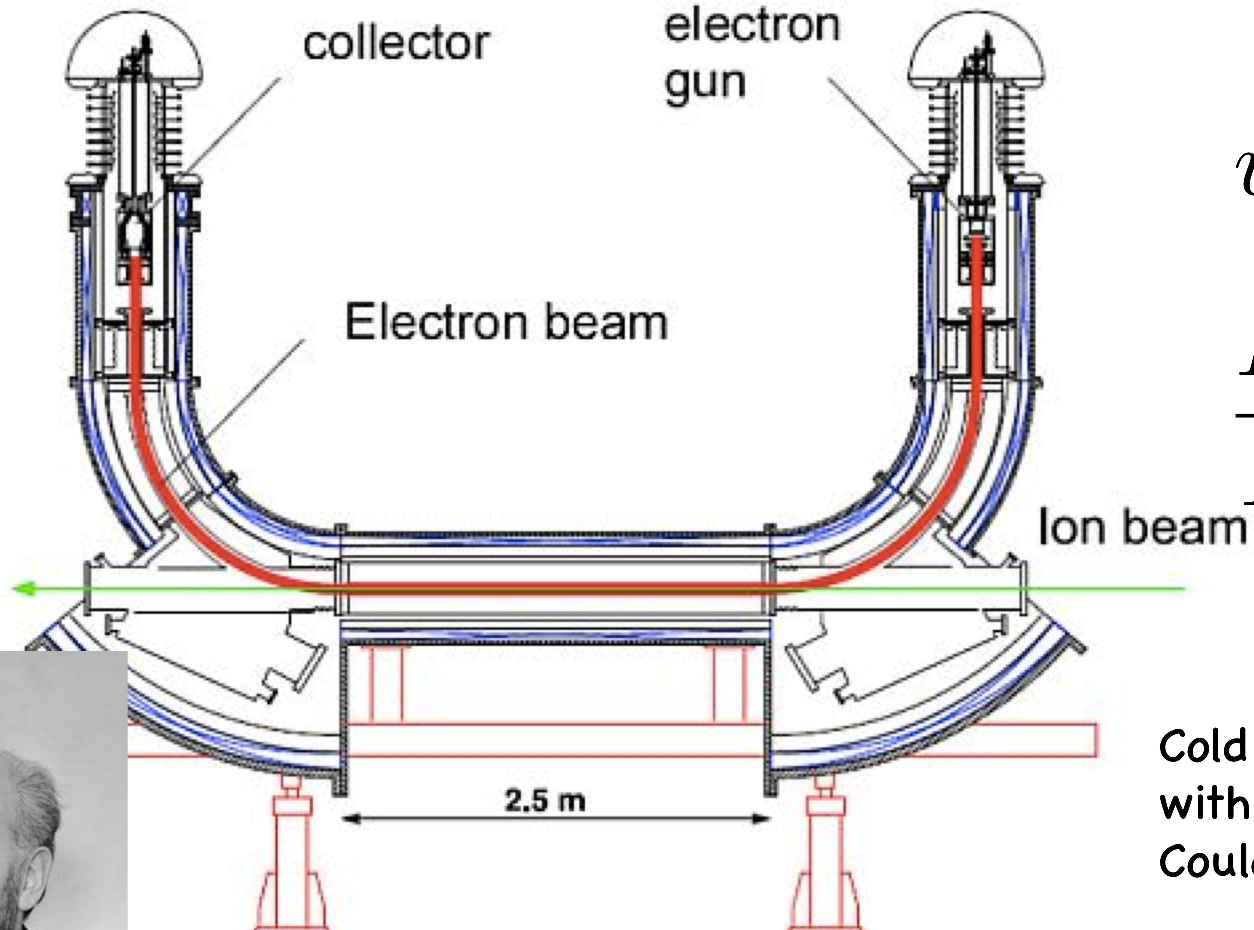
Synchrotron Radiation

- With the damping force that is proportional to the velocity, this is not a Hamiltonian system anymore.

$$\Delta E \sim \exp\left(\pm\sqrt{\alpha_E^2 - \omega^2} - \alpha_E\right) t$$

- SR is also bring large heating effect and balance out its damping effect, as we learned earlier.

Electron Cooling



$$v_e = \bar{v}_i$$

$$\frac{E_e}{E_i} = \frac{m_e}{m_i}$$

Cold electron interact with hot ion beam via Coulomb interaction.

G. Budker

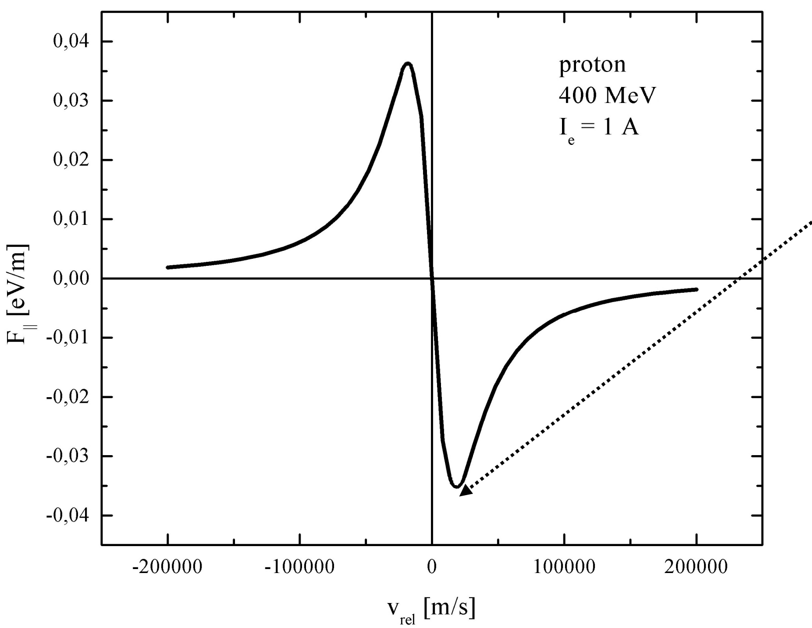
Longitudinal temperature: ~1e-4eV
Transverse temperature:~0.1eV

Electron Cooling II

In the electron beam velocity reference frame:

Faster ions

Slower ions



Maximum cooling force

For small relative velocity

$$F \sim v_r$$

For large relative velocity

$$F \sim v_r^{-2}$$

Electron Cooling III

- The cooling time:

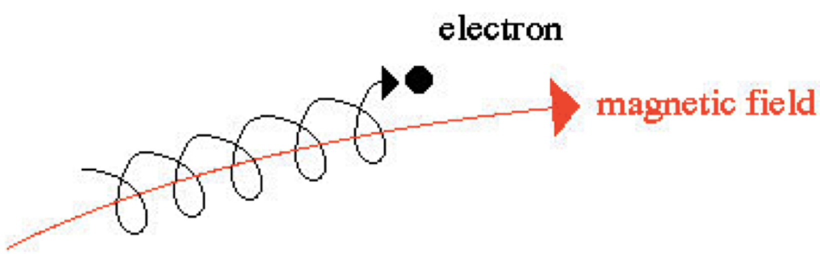
$$\tau_z \propto \frac{A}{Q^2}\frac{1}{n_e \eta}\beta^3\gamma^5\theta_z^3$$

- Therefore the electron cooling is good for:
  - Cold beam
  - Low energy beam
  - Dense electron beam
  - Highly charge beam
- Does not matter on the intensity of the ion beam.

Electron Cooling IV

- Electron is usually confined in longitudinal magnetic field. (Magnetized cooling)
- All electrons experience cyclotron motion.

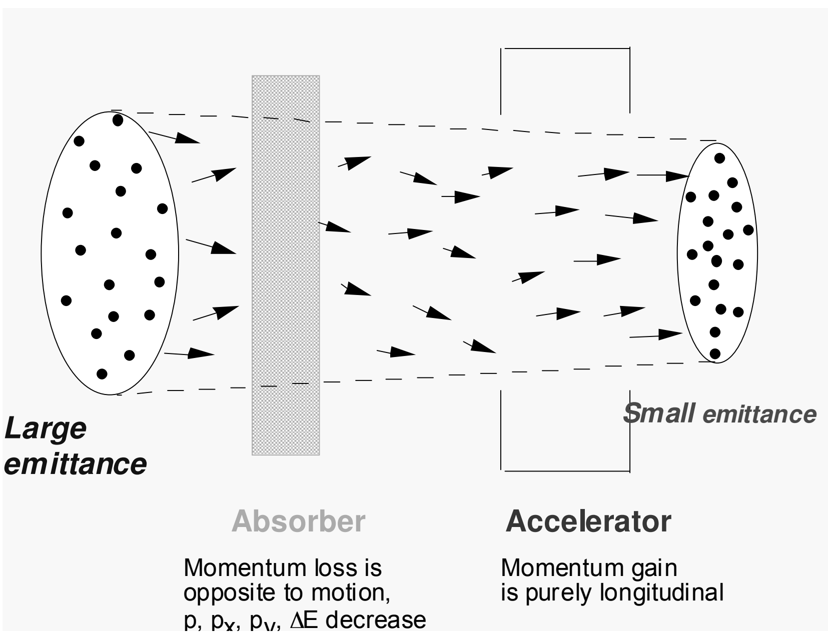
$$\omega_c = \frac{eB}{\gamma m_e}$$



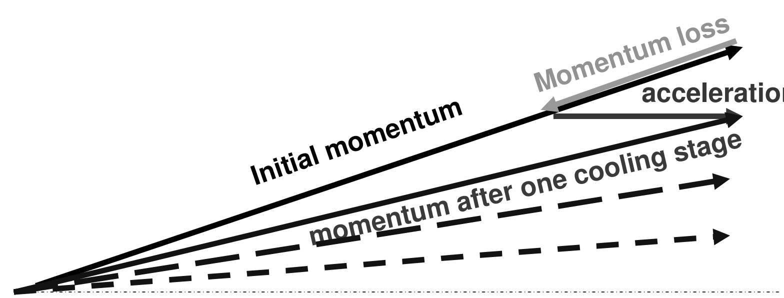
- Interaction time >> cyclotron period, the transverse temperature is not important, only depend on longitudinal temperature.

Ionization Cooling

- Muon collider needs much shorter cooing time, since they decay in 2.2 micro seconds



proposed for muon cooling



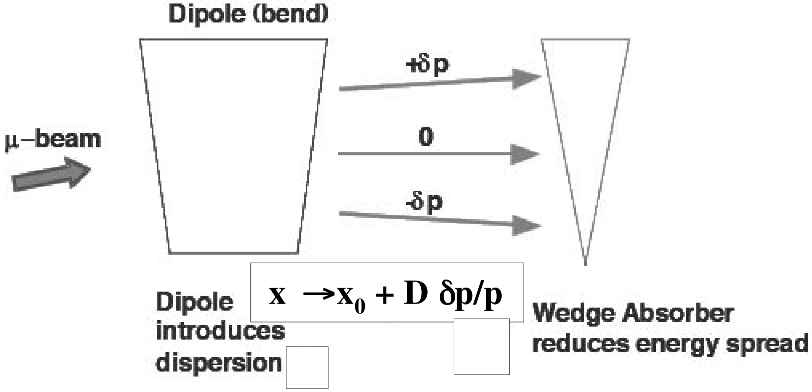
$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta\gamma\beta_\perp}{2}\frac{\langle\theta_{rms}^2\rangle}{ds}$$

$$= -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E}$$

Ionization Cooling II

- As we noticed, this is naturally a transverse process, we can use dispersive region to make it also work in longitudinal plane.

$$\frac{d\sigma_E^2}{ds} = -2\frac{\partial(dE/ds)}{\partial E}\sigma_E^2 + \frac{d\langle \Delta E_{rms}^2 \rangle}{ds}$$



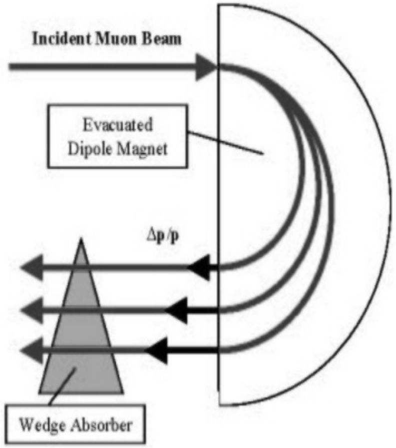


Figure 1. Use of a Wedge Absorber for Emittance Exchange

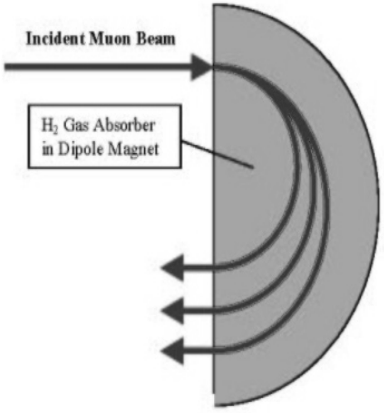


Figure 2. Use of Continuous Gaseous Absorber for Emittance Exchange

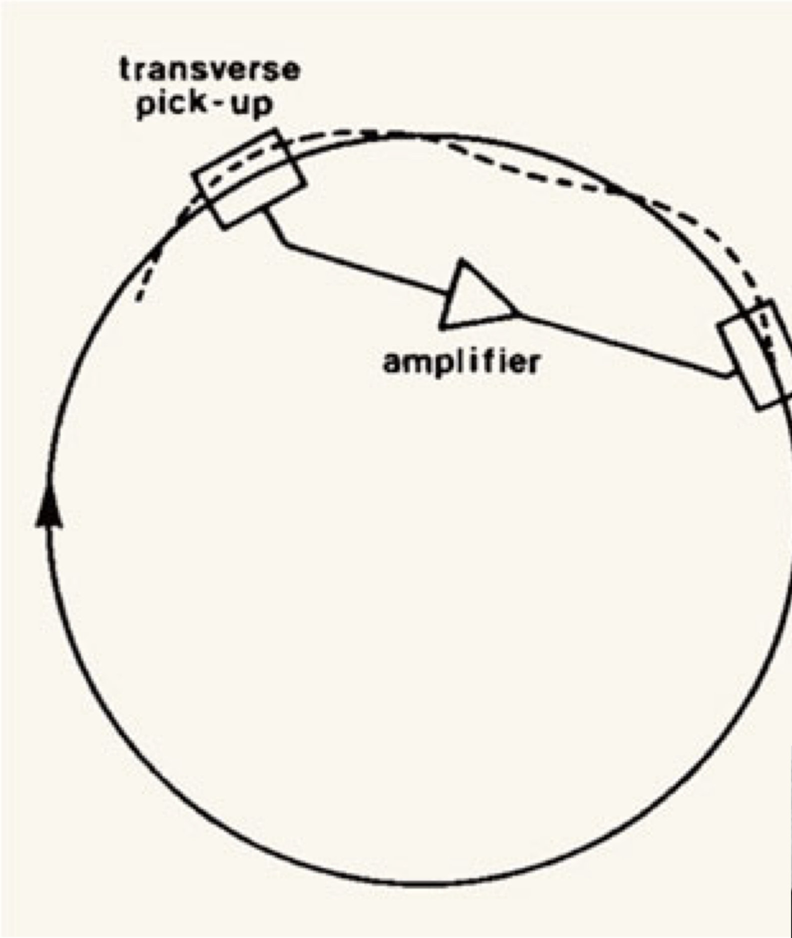
Stochastic cooling

First brought up and realized in CERN by Simon van der Veer.

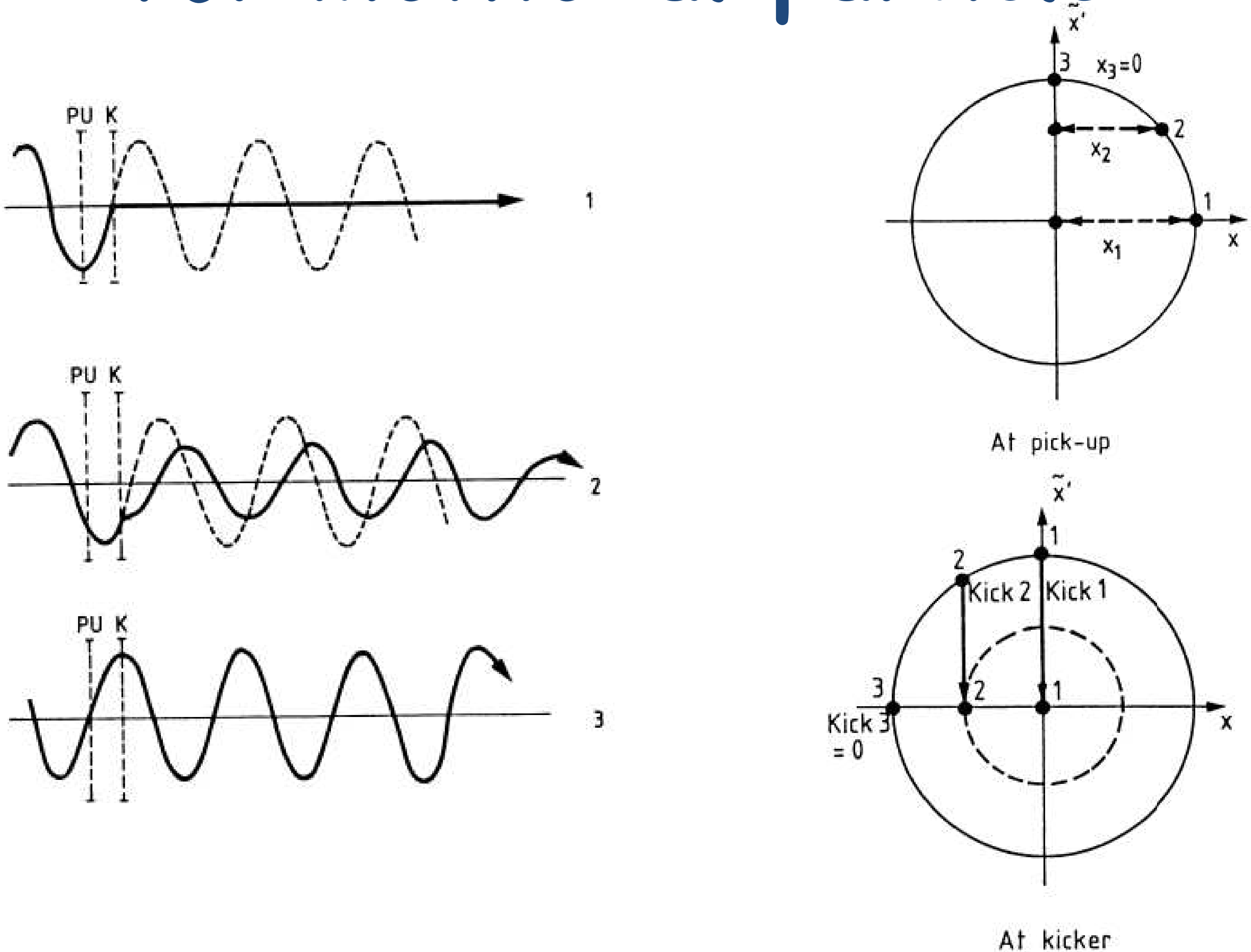
Nobel Laureate.

A negative feedback system for individual particles' signal.

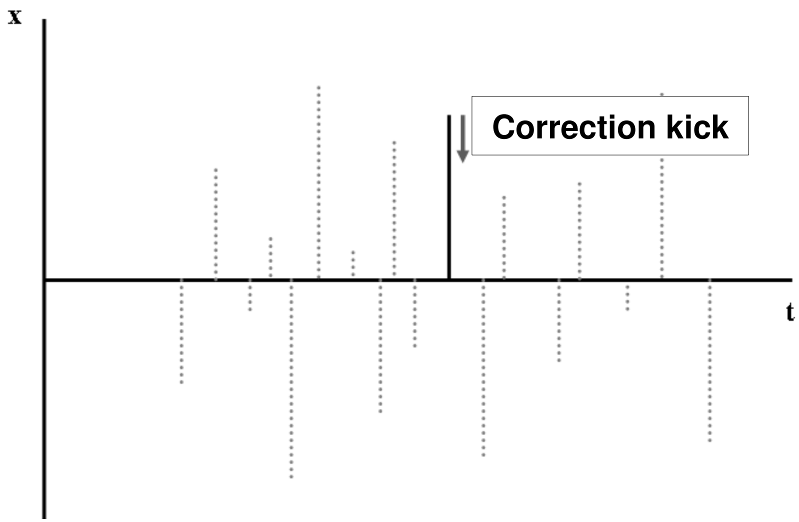
(N+0.5) pi phase advance between pickup and kicker



For individual particle



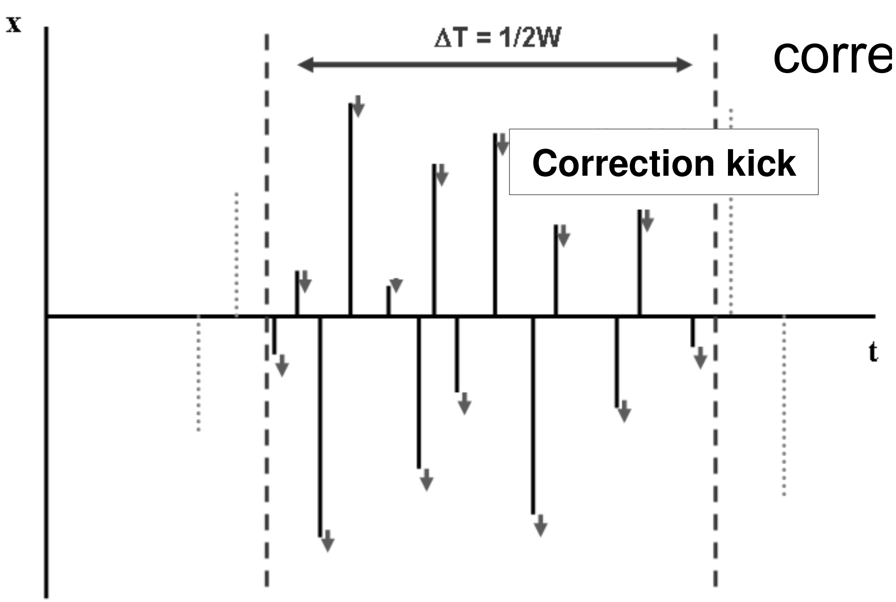
Bandwidth Rules



$$\Delta x = g \times x$$

Requires infinite bandwidth

For a finite bandwidth W, the signal is averaged in time window of 1/2W.



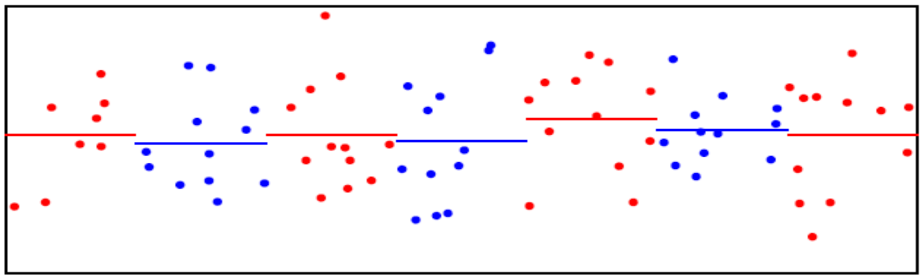
corre

$$\Delta x = \frac{g}{N_s} \times \sum_{i=1..N_s} x_i$$

$$N_s = N\frac{\Delta T}{T_0} = \frac{N}{2WT_0}$$

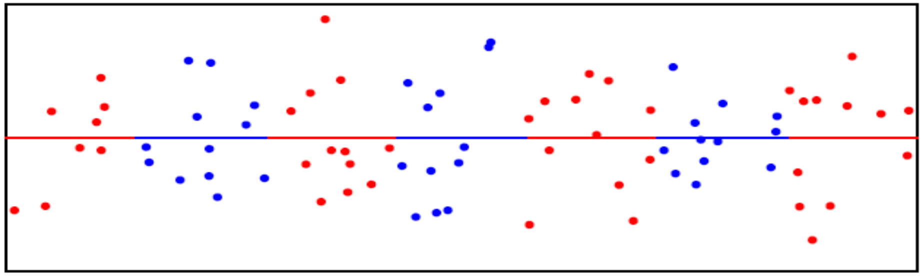
Mixing

before kicker

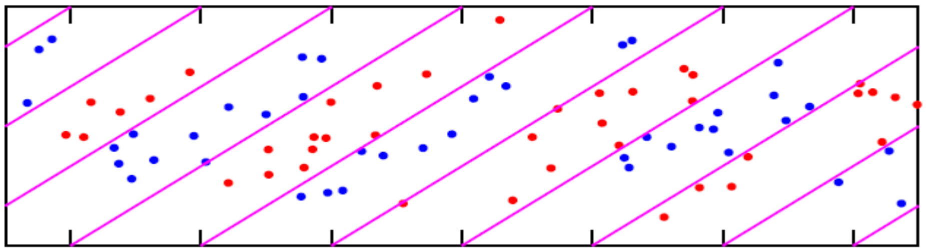


relative arrival time

after kicker



after mixing



Without cooling, the cooling process will stop after the first iteration.

$$\lambda = \tau^{-1} = \frac{2W}{N}\left(\overset{\text{cooling}}{\underline{2g}} - \overset{\text{heating}}{\underline{g^2(M+U)}}\right)$$

$$\boxed{\begin{array}{c}\textbf{maximum of cooling rate}\\ \lambda_{max} = \dfrac{2W}{N}\dfrac{1}{M+U}\end{array}}$$

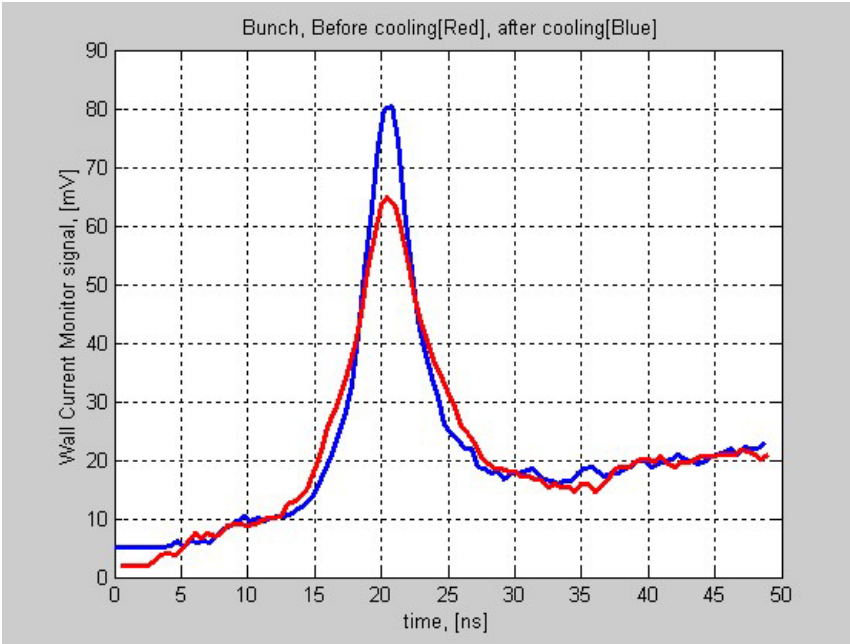
Good mixing:

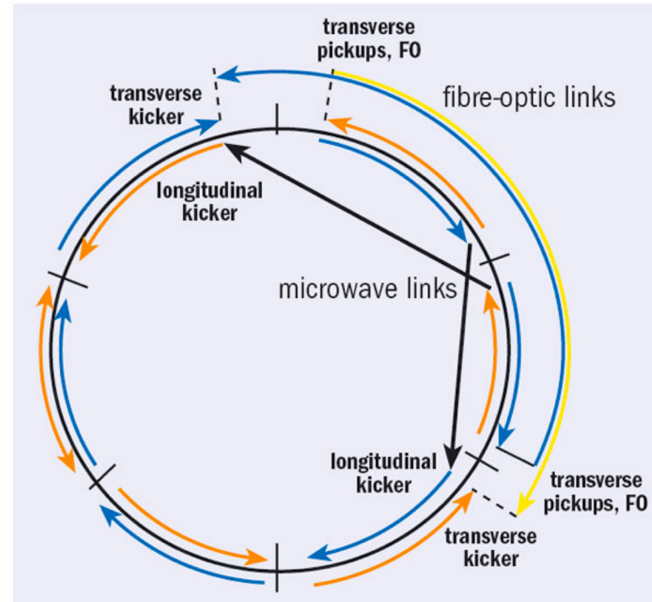
kicker to pickup

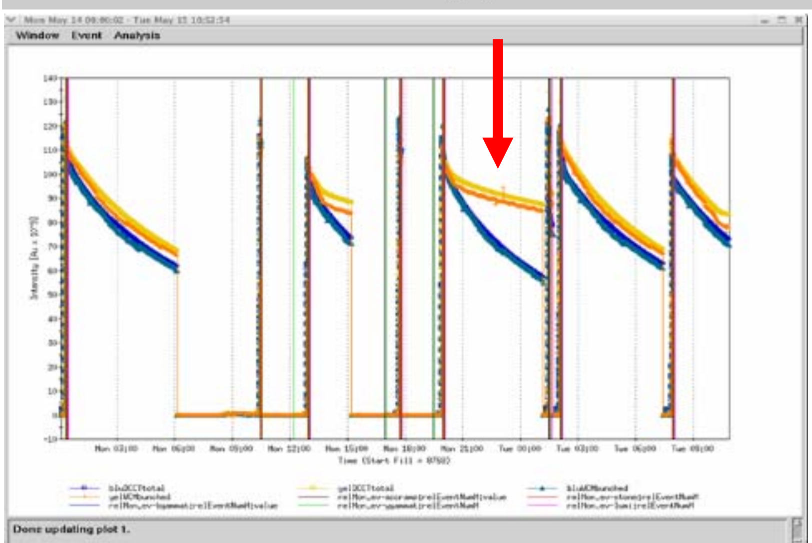
Bad mixing:

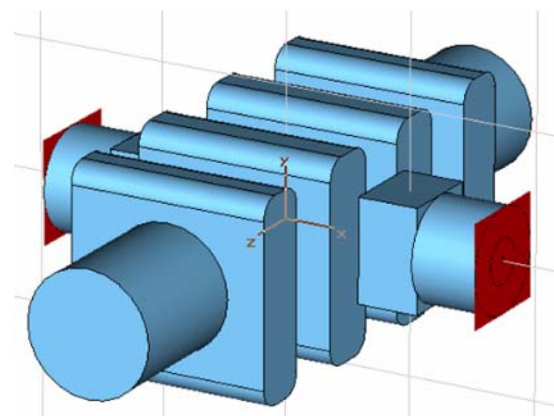
pickup to kicker

Stochastic Cooling at RHIC







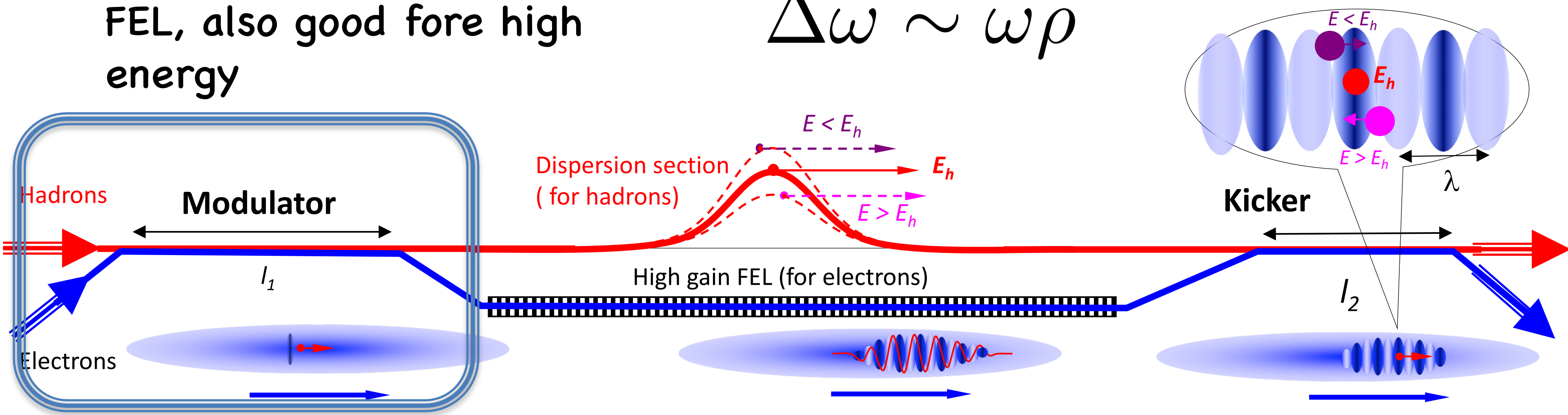


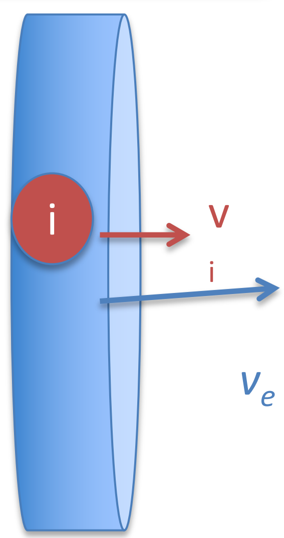
Kicker Cavity

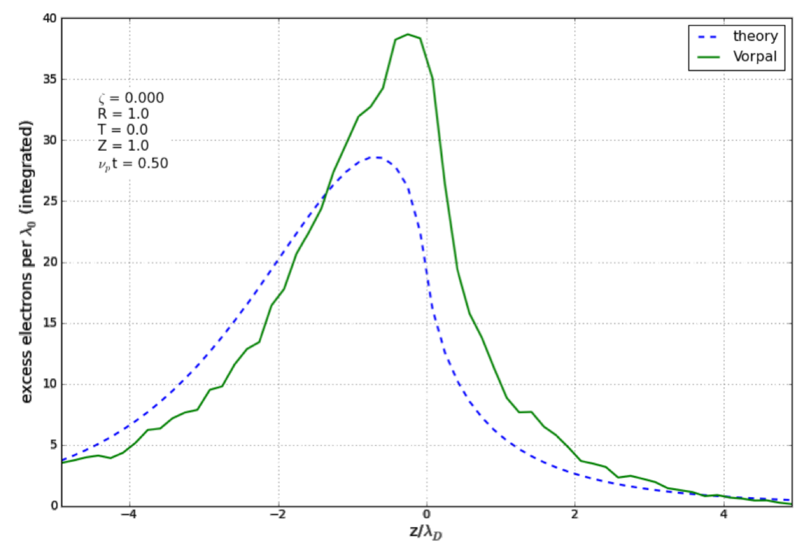
Bandwidth Improvement Coherent Electron Cooling

Bandwidth determined by FEL, also good fore high energy

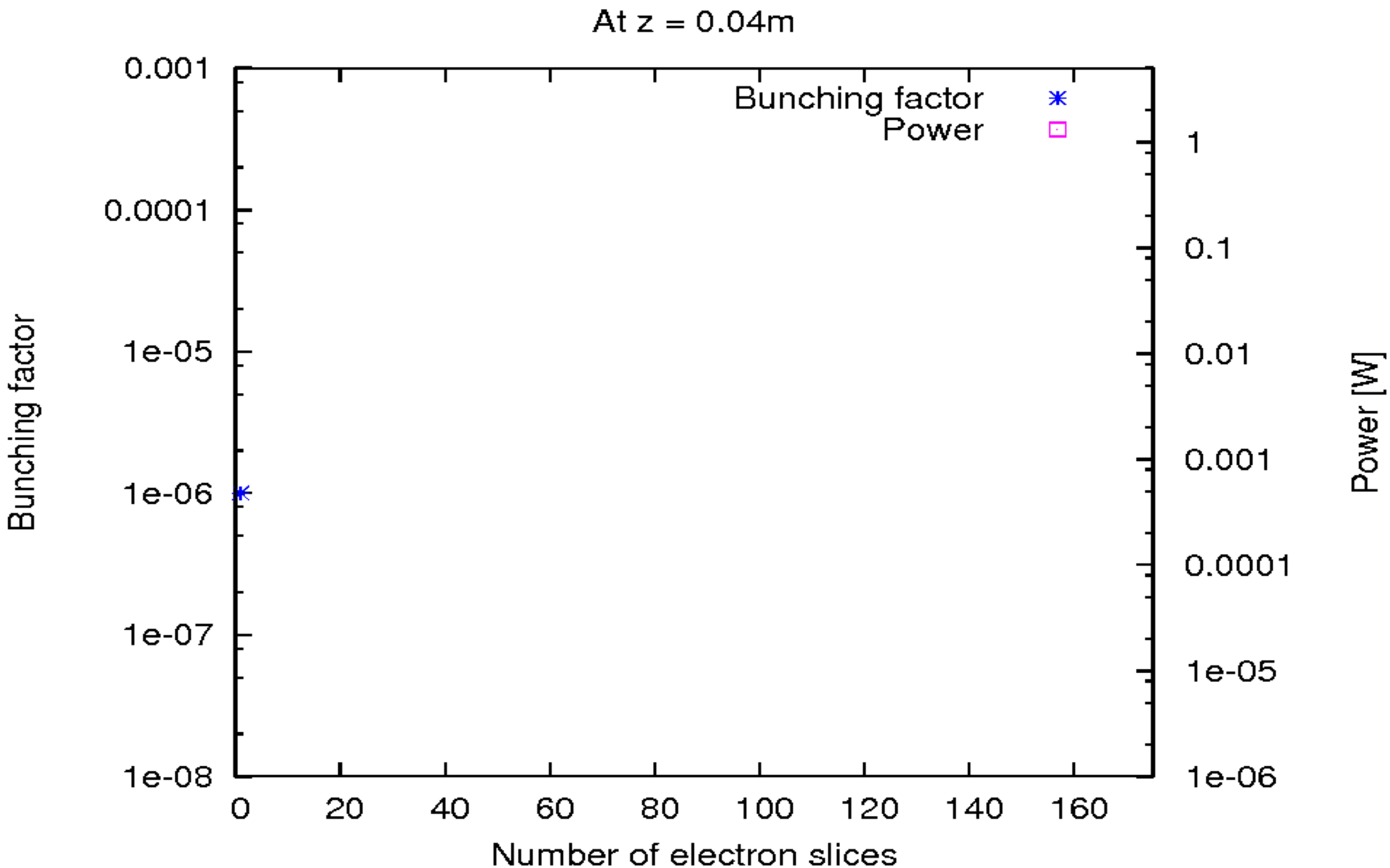
$$\Delta\omega \sim \omega\rho$$







CEC II


At z = 0.04m
Bunching factor
Power
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
0
20
40
60
80
100
120
140
160
Number of electron slices
Bunching factor
Power [W]