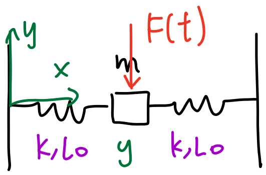
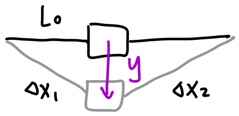


$\vec{r} = x\vec{e_x} + y\vec{e_y}$

$\dot{\vec{r}} = \dot{y}\vec{e_y}$

$\dot{x} = 0$

$F(t) = -F\sin(\omega t)$



$\Delta x_1 = \Delta x_2 = \sqrt{y^2 + L_0^2} - L_0$

$T = \frac{1}{2}m(\dot{y})^2$

$GPE = mgy$

$$
\begin{aligned}
EPE &= \frac{1}{2}k\left(\Delta x_1^2 + \Delta x_2^2\right) \\
&= k(\Delta x_1^2) \\
&= k\left(y^2 + L_0^2 - 2L_0\sqrt{y^2+L_0^2} + L_0^2\right) \\
&= k\left(2L_0^2 + y^2 - 2L_0\sqrt{y^2+L_0^2}\right)
\end{aligned}
$$

$L = T - V = \frac{1}{2}m\dot{y}^2 - mgy - k\left(2L_0^2 + y^2 - 2L_0\sqrt{y^2+L_0^2}\right)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = m\ddot{y}$

$$
\begin{aligned}
\frac{\partial L}{\partial y} &= -mg - k\left(2y - 2L_0\left(\tfrac{1}{\cancel{2}}(y^2+L_0^2)^{-\frac{1}{2}}(\cancel{2}y)\right)\right) \\
&= -mg - 2ky + \frac{2kL_0 y}{\sqrt{y^2+L_0^2}}
\end{aligned}
$$

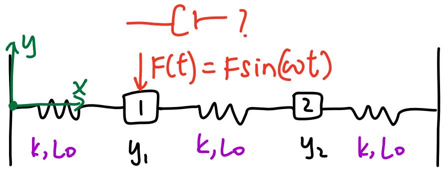
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = Q_y^{nc}$

$m\ddot{y} + mg + 2ky - \frac{2kL_0 y}{\sqrt{y^2+L_0^2}} = F(t)$

$\ddot{y} = \frac{1}{m}\left(F(t) - mg - 2ky + \frac{2kL_0 y}{\sqrt{y^2+L_0^2}}\right)$

★ no damping

Only ① being driven by F(t)



Assume $\Delta x = 0$, all springs the same, mass is the same

2dof → y_1, y_2

$y_1(0) = y_2(0) = 0$] initial condition

$\vec{r_1} = x_1\vec{e_x} + y_1\vec{e_y}$

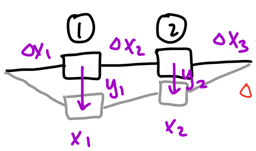
$\vec{r_2} = x_2\vec{e_x} + y_2\vec{e_y}$

$\dot{\vec{r_1}} = \dot{y_1}\vec{e_y}$

$\dot{\vec{r_2}} = \dot{y_2}\vec{e_y}$

$KE = \frac{1}{2}m(\dot{y_1})^2 + \frac{1}{2}m(\dot{y_2})^2 = T$

$GPE = mgy_1 + mgy_2 = mg(y_1+y_2)$



spring extension, $\Delta x_1 = \sqrt{L_o^2 + y_1^2} - L_o$, $(\Delta x_1)^2 = L_o^2 + y_1^2 - 2L_o\sqrt{L_o^2+y_1^2} + L_o^2$
$= 2L_o^2 + y_1^2 - 2L_o\sqrt{L_o^2+y_1^2}$

$\Delta x_2 = \sqrt{L_o^2+(y_2-y_1)^2} - L_o$ $\quad (\Delta x_2)^2 = 2L_o^2 + (y_2-y_1)^2 - 2L_o\sqrt{L_o^2+(y_2-y_1)^2}$

$\Delta x_3 = \sqrt{L_o^2+y_2^2} - L_o$ $\quad (\Delta x_3)^2 = 2L_o^2 + y_2^2 - 2L_o\sqrt{L_o^2+y_2^2}$

$$(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 = 6L_o^2 + y_1^2 + y_2^2 + y_2^2 - 2y_2y_1 + y_1^2 - 2L_o\left(\sqrt{L_o^2+y_1^2} + \sqrt{L_o^2+(y_2-y_1)^2} + \sqrt{L_o^2+y_2^2}\right)$$
$$= 6L_o^2 + 2y_1^2 + 2y_2^2 - 2y_2y_1 - 2L_o\left(\sqrt{L_o^2+y_1^2} + \sqrt{L_o^2+(y_2-y_1)^2} + \sqrt{L_o^2+y_2^2}\right)$$

$$EPE_{spring} = \frac{1}{2}k(\Delta x_1)^2 + \frac{1}{2}k(\Delta x_2)^2 + \frac{1}{2}k(\Delta x_3)^2$$
$$= \frac{1}{2}k\left[6L_o^2 + 2y_1^2 + 2y_2^2 - 2y_2y_1 - 2L_o\left(\sqrt{L_o^2+y_1^2} + \sqrt{L_o^2+(y_2-y_1)^2} + \sqrt{L_o^2+y_2^2}\right)\right]$$

$$V = mg(y_1+y_2) + \frac{1}{2}k\left[6L_o^2 + 2y_1^2 + 2y_2^2 - 2y_2y_1 - 2L_o\left(\sqrt{L_o^2+y_1^2} + \sqrt{L_o^2+(y_2-y_1)^2} + \sqrt{L_o^2+y_2^2}\right)\right]$$

$$L = T - V = \frac{1}{2}m(\dot{y_1}^2 + \dot{y_2}^2) - \frac{1}{2}k\left[6L_o^2 + 2y_1^2 + 2y_2^2 - 2y_1y_2 - 2L_o\left(\sqrt{L_o^2+y_1^2} + \sqrt{L_o^2+(y_2-y_1)^2} + \sqrt{L_o^2+y_2^2}\right)\right] - mg(y_1+y_2)$$

y_1

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1}\right) = m\ddot{y}_1$$

$$\left(\frac{\partial L}{\partial y_1}\right) = -\frac{1}{2}k\left[4y_1 - 2y_2 - 2L_0\left(\cancel{\tfrac{1}{2}}(L_0^2+y_1^2)^{-\frac{1}{2}}(\cancel{2}y_1) + \cancel{\tfrac{1}{2}}(L_0^2+(y_2-y_1)^2)^{-\frac{1}{2}}(\cancel{2}(y_2-y_1)(-1)\right)\right] - mg$$

$$= -\frac{1}{2}k\left[4y_1 - 2y_2 - 2L_0\left(\frac{y_1}{\sqrt{L_0^2+y_1^2}} - \frac{(y_2-y_1)}{\sqrt{L_0^2+(y_2-y_1)^2}}\right)\right] - mg$$

$$\delta W = \vec{F}\cdot\delta r$$

$$= -F\sin(\omega t)\cdot dy_1$$

$$Q^{nc}_{y_1} = -F\sin(\omega t)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1}\right) - \left(\frac{\partial L}{\partial y_1}\right) = Q^{nc}_{y_1}$$

$$m\ddot{y}_1 + \frac{k}{2}\left[4y_1 - 2y_2 - 2L_0\left(\frac{y_1}{\sqrt{L_0^2+y_1^2}} - \frac{(y_2-y_1)}{\sqrt{L_0^2+(y_2-y_1)^2}}\right)\right] + mg = -F\sin(\omega t)$$

$$L = T - V = \frac{1}{2}m(\dot{y}_1^2 + \dot{y}_2^2) - \frac{1}{2}k\left[6L_0^2 + 2y_1^2 + 2y_2^2 - 2y_1y_2 - 2L_0\left(\sqrt{L_0^2+y_1^2} + \sqrt{L_0^2+(y_2-y_1)^2} + \sqrt{L_0^2+y_2^2}\right)\right] - mg(y_1+y_2)$$

y_2

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_2}\right) = m\ddot{y}_2$$

$$\frac{\partial L}{\partial y_2} = -\frac{1}{2}k\left[4y_2 - 2y_1 - 2L_0\left(\cancel{\tfrac{1}{2}}(L_0^2+(y_2-y_1)^2)^{-\frac{1}{2}}(\cancel{2}(y_2-y_1)(1)) + \cancel{\tfrac{1}{2}}(L_0^2+y_2^2)^{-\frac{1}{2}}(\cancel{2}y_2)\right)\right] - mg$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_2}\right) - \frac{\partial L}{\partial y_2} = Q^{nc}_{y_2}, \quad Q^{nc}_{y_2} = 0$$

$$m\ddot{y}_2 + \frac{1}{2}k\left[4y_2 - 2y_1 - 2L_0\left(\frac{y_2-y_1}{\sqrt{L_0^2+(y_2-y_1)^2}} + \frac{y_2}{\sqrt{L_0^2+y_2^2}}\right)\right] + mg = 0$$