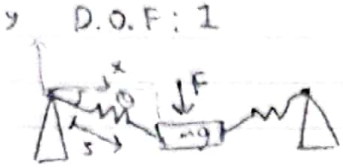
D.O.F : 1



Original length of spring

$L_0 = s\cos\theta$

$s = \frac{L}{\cos\theta}$

$\vec{r} = s\vec{e}_s$

$= -s\cos\theta\vec{e}_x - s\sin\theta\vec{e}_y$

$= -s(\cos\theta\vec{e}_x + \sin\theta\vec{e}_y)$

$= -L\vec{e}_x - L\tan\theta\vec{e}_y$

$\dot{\vec{r}} = -L\dot{\theta}\sec^2\theta\vec{e}_y$

$K.E. = \frac{1}{2}m\dot{\vec{r}}\cdot\dot{\vec{r}}$

$= \frac{1}{2}mL^2\dot{\theta}^2\sec^4\theta$

$y = \sec^4\theta$

$y' = 4(\sec^3\theta)(\sec\theta\tan\theta)$

$= 4\sec^4\theta\tan\theta$

$G.P.E = -mgs\sin\theta$

$= -mgL\tan\theta$

$\text{Spring } P.E = \frac{1}{2}k(s-L)^2 \quad \times 2$

$= k\left(\frac{L}{\cos\theta} - L\right)^2$

$= kL^2\left(\frac{1}{\cos\theta} - 1\right)^2$

$= kL^2(\sec\theta - 1)^2$

$\therefore \quad L = T - V$

$= \frac{1}{2}mL^2\dot{\theta}^2\sec^4\theta + mgL\tan\theta - kL^2(\sec\theta - 1)^2$

$\frac{\partial L}{\partial\dot{\theta}} = mL^2\dot{\theta}\sec^4\theta$

$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} = mL^2\ddot{\theta}\sec^4\theta + 4mL^2\dot{\theta}\sec^4\theta\tan\theta$

$\frac{\partial L}{\partial\theta} = 2mL^2\dot{\theta}^2\sec^4\theta\tan\theta + mgL\sec^2\theta - 2kL^2(\sec\theta - 1)\sec\theta\tan\theta$

$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} - \frac{\partial L}{\partial\theta} = 0$

$mL^2\ddot{\theta}\sec^4\theta + 4mL^2\dot{\theta}\sec^4\theta\tan\theta - 2mL^2\dot{\theta}^2\sec^4\theta\tan\theta - mgL\sec^2\theta - 2kL^2(\sec\theta - 1)\sec\theta\tan\theta = 0$