Network user equilibrium problems for the plug-in electric vehicles and gasoline vehicles subject to road length constraints

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ABSTRACT

This paper addresses a path-constrained user equilibrium (UE) traffic assignment problem (TAP) for transport networks with electric vehicles (EV) and gasoline vehicles (GV), where EV paths are restricted by the EV driving range limits. The suggested method, based on the classic Frank-Wolfe algorithm, incorporates an efficient constrained shortest-path algorithm as its subroutine. Two numerical examples are presented to verify the proposed model and solution algorithms. This paper focuses on the coexistence of various types of vehicles in the network, and evaluated the changes of network after the introduction of EVs which replace a certain proportion of GVs.

Keywords: electric vehicles, range anxiety, traffic assignment, user equilibrium, flow-dependent electricity consumption

1 INTRODUCTION

Carbon-based emissions and greenhouse gases (GHG) are critical global issues as addressed by the Kyoto Protocol in 1998 (1). The transport sector is a significant contributor to GHG emissions in most countries, comprising 23% (worldwide) of CO2 emissions from fossil fuel combustion in 2005 while automobile transport is the principal CO2 production source. From the energy safety point of view, the transport sector as a whole is 98% dependent on fossil oil which is also exceedingly affected by changes in energy resources (2). So changes to the current energy structure in transport sector are in urgent need.

Alternative fuels are addressed as a new fuel choice to reduce GHG emissions and electric vehicles (EV)are believed to be a sustainable solution(2). Governments and automotive companies have recognized the value of these vehicles in helping the environment and are encouraging the ownership of EV through economic incentives(3). It is mentioned that one million plug-in hybrid and electric vehicles will be on the road by 2015 in United States to reduce greenhouse gas emission and dependence on oil (4). According to Electric Drive Transportation Association, the plug-in electric vehicles (PEV) in US has exceeded 190,000 between January of 2011 and Mach of 2014 (5).

To eliminate the greenhouse gas emissions and to save the energy in automobile industry, Chinese government has promoted the commercialization of New Energy Vehicles (NEV) since 2009. Two rounds of NEV demonstration projects are launched. During the two rounds, the government promulgated a series of incentive policies to boom the development of EVs, in aspects of production, purchase, usage and charging infrastructures. The NEVs' sales have increased sharply since 2013.

Shanghai is one of the pilot cities in the first round promotion. EV consumers could have access to subsidies and free automobile licenses (the license price is over 89000RMB (10377dollars) for private cars in 2018). Meanwhile, EVs are not affected by the traffic restriction policy.

Although many cities are planning construction and expansion of charging infrastructures for EV, it is likely that in the foreseeable future EV commuters will need to charge their vehicles at home most of the time (6). Fast growth of the EV market faces two barriers. One is the high cost of battery packs. Another barrier is the lack of public charging facilities. EV users suffer from range anxiety inevitably for the limited battery range and the insufficient charging facilities. Given the long charging time, limited driving range and charging infrastructure, both EVs and GVs will coexist in the transportation networks for a long time.

It is obvious that the driving range limit inevitably adds a certain level of restrictions to EV drivers' travel behaviors, at least in a long future period prior to the coverage of recharging infrastructures reaching a sufficient level (7). EV companies are trying to overcome this limited range requirement with fast charging stations, where a vehicle can be charged in only a few minutes to near full capacity. Besides being much more costly to operate rapid recharge stations, the vehicles still take more time to recharge than a standard gasoline vehicle would take to refuel (8) .

However, the widespread adoption of EV calls for fundamental changes to the existing network flow modelling tools for properly capturing changed behaviors and induced constraints in forecasting travel demands and evaluating transportation development plans (7). In order to take into consideration of driving range limit and insufficient charging facility status in traffic assignment, Jiang et al. (9) proposed an approach to restrict flow of a path to zero if the path distance is greater than the driving range limit of EV. They employed a path travel time function that is the sum of the corresponding link cost such as the Bureau Public Road (BPR) function and showed the Lagrangian multiplier of its optimal solution stands for the unit out-of-range travel distance cost. Classic Frank-Wolfe algorithm with a constrained shortest path algorithm as its subroutine can be applied to solve this problem. The deterministic user equilibrium (DUE) condition characterizes route choice behavior where users have perfect traffic network information and always choose the shortest path accurately. A convex minimization model for DUE conditions can be built by adding path distance constraints into the Beckmann's conventional DUE model.

To sum up, the contributions of this study are multidimensional. First, a holistic methodology is proposed for DUE traffic assignment model with path distance constraints on EV scheme. Second, a Frank-Wolfe (F-W) algorithmic framework is proposed to solve the model, which is computationally efficient and have the potential to be applied in real networks. Thirdly, we attempt to investigate the traffic network's performance which contains various vehicles including EV and GV under different range limits, which is different from previous models with only one single type of EV. Therefore, we can evaluate the performance of network after the introduction of EVs which replace a certain proportion of GVs. The major part of this paper is a discussion of the modelling and solution methods for the DUE traffic assignment problem with path distance constraints.

The remainder of this paper is organized as follows. First, literature reviews are elaborated in Section 2. In Sections 3, we elaborate the problem formulation, and analyse its solution properties. Section 4 develops a UE model for the mixed EVs and GVs with path distance constraints. To solve the developed UE model, a Frank-Wolfe algorithmic framework is proposed. While Section 5 presents the numerical results from applying the algorithm procedure for a small network and Sioux Falls network with path distance constraints. We focus on the coexistence of various types of vehicles in the network, and evaluated the changes of network after the introduction of EVs which replace a certain proportion of GVs. In the end, conclusions and future research are presented in Section 6.

2 LITERATURE REVIEW

It is well known that the standard TAP under DUE can be solved efficiently with a Frank-Wolfe type algorithm whose linearized sub-problem finds shortest paths for each OD pair at each iteration. Nevertheless, insufficient charging stations and limited driving range for BEVs make traffic assignment problem (TAP) more challenging due to the incorporation of path distance constraints and battery capacity constraints. The existing TAP models should be modified to better describe commuters' behavior with the prevalence of BEVs. There have been many endeavors to address this problem.

The problem of finding the shortest path for an EV was originally discussed by Ichimori et al. (10), where a vehicle has a limited battery and is allowed to stop and recharge at certain locations. Lawler (11) developed a polynomial algorithm for its solution. Adler et al. (12) proposed an EV shortest-walk problem to determine the shortest travel distance route which may include cycles for detouring to recharging batteries from origins to destinations with minimum detouring. Kobayashi et al. (13) and Siddiqi et al. (14) included battery recharging stations in their shortest weight-constrained path problem models, which is known to be NP-Complete (15), and proposed heuristic techniques as solution methodologies. There has been some recent consideration of the effect of EV on traffic assignment and DUE. Jiang et al. (9) studied the effect of restricted the EV path distances and assumes charging events only occur at OD nodes, which corresponds to the real circumstance of insufficient charging facilities. It enforces flow of a path to be zero if the path distance is greater than the driving range limit of BEVs. The classic Frank- Wolfe method with a constrained shortest path algorithm can be applied to solve this problem under deterministic user equilibrium (DUE) (9). As an extension of static path distance constraint, stochastic range anxiety resulting in stochastic path distance constraint has been considered in networks (9, 16, 17).

Network equilibrium problem was further addressed when modeling transportation networks that accommodated both gasoline vehicles (GVs) and BEVs (7, 18, 19). A multi-class dynamic user equilibrium model was proposed to evaluate the performance of the mixed traffic flow network, where GVs chose paths with minimum travel time and BEVs selected paths to minimize the generalized costs including travel time, energy cost and range anxiety cost. It was also pointed out that the BEV energy consumption rate per unit distance traveled is lower at moderate speed than at higher speed resulting in an equilibrium that BEVs choose paths with lower speed to conserve battery energy (20). Relay/charging requirement has been taken into account in network equilibrium problems and was formulated as a nonlinear integer programming (21). It was found that traffic congestion would affect fuel economy of BEVs and BEVs might become more fuel-efficient as the average speed increases, particularly at local arterials (22). Hence, another work considered recharging time based on flow-independent energy consumption in the base network equilibrium model and further extended the proposed DUE model with flow-dependent energy consumption assumption (23).

3 METHODOLOGY

3.1 Notation, assumptions and problem description

Table 1 Notation

Sets	
N	Set of nodes, where N={n}
A	Set of links, where A={a}
R	Set of origin nodes, where R={r}
S	Set of destination nodes, where S={s}
K	Set of paths, where K ={k}
Parameters	
$q_m^{r,s}$	travel demand rate of *m*th class of vehicles from origin *r* to destination *s*
D_m	distance limit of *m*th class of vehicles
$\delta_{a,k}^{rs}$	Link-path incidence parameter, $\delta_{a,k}^{rs}$ is equal to 1 if link *a* is contained in *k*th path from origin *r* to destination *s*; otherwise, $\delta_{a,k}^{rs}$ is equal to 0.
d_a	distance of link *a*
Variables	
t_a	Travel cost on traffic link a
x_a	traffic flow rate on link *a*
l_k^{rs}	the length of path *k* from origin *r* to destination *s*
$f_{k,m}^{rs}$	traffic flow rate of *m*th class of vehicles on path *k* from origin *r* to destination *s*

Let us assume the transport network is modeled as a connected graph, denoted by G = (N, A). The path flows and link flows should comply with fundamental flow conservation equations:

$$\sum_k f_{k,m}^{rs} = q_m^{r,s}, \qquad \forall r, s, m \tag{1}$$

$$x_a = \sum_{rs}\sum_k\sum_m f_{k,m}^{rs}\delta_{a,k}^{rs} \quad \forall k, r, s, m, a \tag{2}$$

$$f_{k,m}^{rs} \geq 0 \quad \forall k, r, s, m \tag{3}$$

Based on EV's market potential, it is expected that in the future GV and EV will coexist in the automobile market. For this reason, the proposed model includes multiple classes of vehicles, namely GV and EV, which distinguish from each other in terms of driving distance range and travel cost composition. To derive the theoretical properties of the problem, we consider a set of assumptions regarding demand heterogeneity and travel behavior.

First without loss of generality, it is assumed that the demand population is only comprised of GV and EV. Plug-in hybrid electric vehicle (PHEV) are not explicitly considered since they can be simply treated as an in-between class of GV and EV in terms of the technological and economic features (i.e., driving range limit and travel cost composition), or a special type of GV with lower operating costs. Readily multiple types of EV with different driving range limits and operating costs can be incorporated into the model.

Second, we assume the total travel demand between each O-D pair for every vehicle type m is deterministically known a-priori. DUE concept is devised for route choice procedure, in which each traveler chooses a route that minimizes his/her travel cost while no one can reduce his/her travel cost by unilaterally switching to an alternative route. For an individual GV traveler, user equilibrium simply implies a conventional traffic assignment problem of searching for minimum cost (travel time); whereas for an EV traveler, it poses a path distance-constrained minimum cost problem. In this paper, we scrutinize the integrated effect of different vehicle types with various path constraints.

Third, without loss of generality, we assume that both GV and EV travelers use a common form of systematic travel cost function for determining their travel choices. The link travel time functions are assumed to be separable between different network links and identical for different vehicle classes, implying the travel time on a particular link only depends on its own traffic flow. These functions are assumed to be positive, monotonically increasing, and strictly convex.

In our network equilibrium analysis, it is implicitly assumed that all EV are fully charged at their origins. The possible availability of commercial battery-charging or battery-swapping stations emerging in urban areas can be considered in the future when charging infrastructures achieve a certain level of coverage. EV users would choose a path whose distance l_k^rs is less than or equal to the driving range limit of the vehicle type m, denoted by D_m. Hence, any feasible path flow pattern should satisfy the path distance constraints:

$$f_{k,m}^{rs}(D_m - l_k^{rs}) \geq 0 \ \ \forall k, r, s, m, a \tag{4}$$

which means that if the flow of that class of EV users going through this path is positive, the path distance is smaller than or equal to the driving range of a given class of EV; otherwise, the trip flow should equal to zero.

3.2 Mathematical model

As noted above, the classic TAP can be described by an equivalent mathematical program that is known as Beckmann's transformation. We can add the path distance constraint into this minimization model developed by Sheffi (1985) as follows, only if predetermining path set to ensure distances of all the used paths are less than the range limit for each O-D pair.

Let $t_a(x_a)$ denote the separable travel time function of link a that is assumed to be a positive, strictly increasing, convex and continuously differentiable function of the traffic flow on the link.

$$\min Z\big(x(f)\big) = \sum_a \int_0^{x_a} t_a(\omega)\,d\omega \tag{5}$$

Subject to :(1)(2)(3)(4)

Compared to Sheffi's model which can be solved as an unconstrained minimization problem and still yield a solution that satisfies the flow conservation constraints (1)(2)(3), the extra path distance constraints(4) are the constraints that needs a careful consideration. Then we prove the equivalency of the solution of the proposed optimization problem (1-5) and the equilibrium conditions defined in (1-4). Let $u_m^{r,s}$ and $\lambda_{k,m}^{rs}$ denote the dual variables associated with Equations 1 and 4, respectively. If $u_m^{r,s}$ is unrestricted in sign, and $\lambda_{k,m}^{rs}$ is restricted to be nonnegative, the relevant Lagrangian problem to the optimization problem is, if we relax constraints (1) and (4),

$$L(f,u,\lambda) = Z\big(x(f)\big) + \sum_{rs}\sum_{m} u_m^{r,s}\left(q_m^{r,s} - \sum_k f_{k,m}^{rs}\right) - \sum_{rs}\sum_{k}\sum_{m} \lambda_{k,m}^{rs}(D_m - l_k^{rs})f_{k,m}^{rs} \tag{6}$$

$$f_{k,m}^{rs} \geq 0 \;\; \forall k,r,s,m \tag{7}$$

$$\lambda_{k,m}^{rs} \geq 0 \; \forall k,r,s,m \tag{8}$$

By making use of the optimality conditions of the Lagrangian problem, we obtain the following system of equations and inequalities:

$$f_{k,m}^{rs*}\left[\sum_a t_a\delta_{a,k}^{rs} - \lambda_{k,m}^{rs*}(D_m - l_k^{rs}) - u_m^{r,s*}\right] = 0 \;\; \forall k,r,s,m \tag{9}$$

$$\sum_a t_a\delta_{a,k}^{rs} - \lambda_{k,m}^{rs*}(D_m - l_k^{rs}) - u_m^{r,s*} \geq 0 \;\; \forall k,r,s,m \tag{10}$$

$$f_{k,m}^{rs*} \geq 0 \;\; \forall k,r,s,m \tag{11}$$

$$\lambda_{k,m}^{rs*}(D_m - l_k^{rs})f_{k,m}^{rs} = 0 \;\; \forall k,r,s,m \tag{12}$$

$$(D_m - l_k^{rs})f_{k,m}^{rs} \geq 0 \;\; \forall k,r,s,m \tag{13}$$

$$\lambda_{k,m}^{rs*} \geq 0 \;\; \forall k,r,s,m \tag{14}$$

$$\sum_k f_{k,m}^{rs} = q_m^{r,s}, \qquad \forall r,s,m \tag{15}$$

It is readily proved that Objective Function 5 is convex and that the feasible region defined by Constraints 2 to 4 is convex. Therefore, this DCTAP model has a unique solution.

Since we introduced path distance constraints, the DCTAP problem may be unfeasible since the shortest path length is longer that the vehicle's range limit. Therefore, before assigning demands, we should compare every OD pair's shortest path length with vehicles' range limit.

4 SOLUTION METHOD

Given the convex objective function and linear constraint sets, it was shown that the Frank-Wolfe algorithm can be applied to the path distance constrained traffic assignment problem (DCTAP) with a direction-finding step different from that of classic TAP and feasibility check step. To find a descent direction to the optimization problem in Equations 1 to 4, the Frank-Wolfe algorithm searches the entire feasible region for an auxiliary feasible solution, y_n, such that the direction from x_n (the current solution at the nth iteration) to y_n provides a maximum drop in the objective function value. This direction can be constructed by solution of the following linearization problem:

$$\min Z^n(y) = \nabla Z(x^n)\cdot y^T = \sum_a t_a(x_a)y_a \tag{18}$$

Subject to

$$\sum_k g_{k,m}^{rs} = q_m^{r,s}, \qquad \forall r,s,m \tag{19}$$

$$g_{k,m}^{rs} \geq 0 \;\; \forall k,r,s,m \tag{20}$$

$$(D_m - l_k^{rs})g_{k,m}^{rs} \geq 0 \;\; \forall k,r,s,m \tag{21}$$

$g_{k,m}^{rs}$ is the auxiliary flow rate of the mth class of vehicles on path k connecting O-D pair r-s and $y_a = \sum_{rs}\sum_k\sum_m g_{k,m}^{rs}\delta_{a,k}^{rs}$, and is the auxiliary link flow rate. Given that $\mathrm{x_a^n}$ is the current link flow rate obtained from the last iteration, link travel cost $t_a(\mathrm{x_a^n})$ is constant here.

Obviously, the direction-finding step of the Frank-Wolfe method can be derived with the gradient of Objective Function 1 for path flows. The program then becomes

$$\min Z^n(g) = \nabla_f Z[x(f^n)] \cdot g^T = \sum_{rs}\sum_k\sum_m c_k^{rs,n} g_{k,m}^{rs} \tag{22}$$

$$c_k^{rs,n} = \sum_a t_a \delta_{a,k}^{rs} \tag{23}$$

Subject to :(19)(20)(21)

Where $c_k^{rs,n}$ is the travel cost on path k connecting O-D pair r and s at the nth iteration of the algorithm. In a UE problem, $g_{k,m}^{rs}$ is equal to $q_m^{r,s}$ if l_k^{rs} is $\leq D_m$ and $c_h^{rs,n}$ is $\leq c_k^{rs,n}$ for all k satisfying $l_k^{rs} \leq D_m$, and $g_{k,m}^{rs}$ is equal to 0 for all other paths.

Once the path flow pattern $g_{k,m}^{rs}$ is found, the auxiliary link flow pattern can be calculated; that is, y_a^n is equal to $\sum_{rs}\sum_k\sum_m g_{k,m}^{rs,n}\delta_{a,k}^{rs}$, The descent direction (d^n) can then be obtained as $y_a^n - x_a^n$. Once the descent direction is determined, any line search method can be applied to obtain the move size so that the maximum drop of the objective function value is achieved.

This program is a convex program, we can use Frank-Wolfe Algorithm to solve it, the steps of the algorithm are displayed as follows:

Step 0: Feasibility check. For each OD pair, find the shortest path according to physical distance. If the distance of this path is longer than the range limit of a certain type of vehicle and the corresponding travel demand is positive, then there is no feasible path for this type of vehicle between this OD pair. Record this OD pair and infeasible vehicle type to Set A.

Step 1: Initialization. Set $\mathrm{x_a^0} = 0$, $\mathrm{t_a^0} = ta\ [\mathrm{x_a^0}]$. For each OD pair, find the shortest path for each class of vehicles in terms of free flow travel time. If the path distance is greater than the range limit of this class of vehicles, set the path travel time to infinite. Calculate the probability of choose each path, record them as initial path set and perform stochastic network loading to assign all the demand of each class of vehicles between this OD pair to the corresponding shortest paths. This yields x_a^1. Set iteration counter n = 1 .

Step 2: Update. Calculate a new link cost in terms of $\mathrm{t_a^n} = t_a(\mathrm{x_a^n})$, ∀a.

Step 3: Direction finding. For each O-D pair and vehicle class, find the distance-constrained least-cost path on the basis of new link travel cost $\mathrm{t_a^n}$. Perform the all-or-nothing assignment of each class of vehicles between this O-D pair based on the tna. This yields auxiliary flow $\{y_a^n\}$.

Step 4: Line search. Apply any of the interval reduction line search methods, such as the bisection method, the golden section method(this paper uses this), or other applicable method, to find the optimal value of θ-the optimal move size-by determination of the solution to

$$\min_{0\leq\theta\leq1} \sum_a \int_0^{x_a^n+\theta(y_a^n-x_a^n)} t_a(\omega)d\omega \tag{24}$$

Step 5: Move. Find the new flow pattern by setting $x_a^{n+1} = x_a^n + \theta(y_a^n - x_a^n)$.

Step 6: Convergence test. If a convergence criterion is met, stop, the current solution $\{x_a^{n+1}\}$ is the set of equilibrium link flows; otherwise, set n = n +1 and go to step 1. In our program, the convergence test is as follows ($\epsilon = 1e-3$):

$$\frac{|x_a^{n+1} - x_a^n|}{|x_a^n|} < \epsilon \tag{25}$$

5 NUMERICAL ANALYSIS

The purpose of this numerical analysis is fourfold: (a) to justify the validity of the model and algorithm, (b) to examine the impact of the distance constraint on network flow patterns, (c) to evaluate the change of computational costs caused by the distance constraint, and (d) to evaluate the change of computational costs caused by the improvement of the rate of EV.

The solution procedure is first applied to a small and simple network with eight nodes and 10 links, as shown in **Figure 1**. The number beside each link is the link length. Nodes 1 and 2 are origins, and Nodes 3 and 4 are destinations. In this small example, the focus is on analysis of how different distance limits affect the routing behavior of vehicles and how the proportion between GV and EV influence the whole network assignment. The travel demand between each O-D pair (O-D Pairs 1–3, 1–4, 2–3, and 2–4) for this class of vehicles is 20 flow units. Link costs and distances of the connectors (i.e., those links connecting origins or destinations with internal nodes) are assumed to be 0, and all other links have the same cost function, namely, $t_a = 1 + x_a^2$.

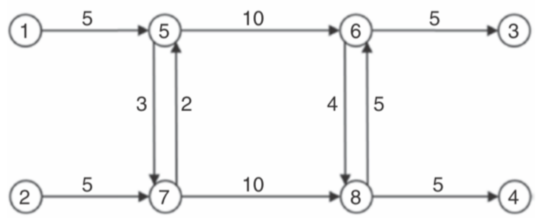

FIGURE 1 Test Networks: A Small Network

Table 2 Link Flow Pattern Under Different Constraint Conditions and Different Vehicle Composition

GV Rate	Link	Link Flow Distance Limit:23	24	25	27	———
GV	5-6			40		
	5-7			10		
	6-8			10		
	7-5			10		
	7-8			40		
	8-6			10		
75%	5-6	40	40	40	40	40
	5-7	10	10	10	10	10
	6-8	10	10	10	10	10
	7-5	10	10	10	10	10
	7-8	40	40	40	40	40
	8-6	10	10	10	10	10
50%	5-6	40	40	40	40	40
	5-7	10	10	10	10	10
	6-8	10	10	10	10	10
	7-5	10	10	10	10	10
	7-8	40	40	40	40	40
	8-6	10	10	10	10	10

25%	5-6	**40**	**41**	40	40	40
	5-7	**15**	**14**	10	10	10
	6-8	**5**	**6**	10	10	10
	7-5	**15**	**15**	10	10	10
	7-8	**40**	**39**	40	40	40
	8-6	**5**	**5**	10	10	10
EV	5-6	**40**	**42**	40	40	40
	5-7	**20**	**18**	10	10	10
	6-8	**0**	**2**	10	10	10
	7-5	**20**	**20**	10	10	10
	7-8	**40**	**38**	40	40	40
	8-6	**0**	**0**	10	10	10

Table 3 System Total Travel Cost of Different Link Flow Pattern

	Link Flow						
Flow Pattern	Link 5-6	Link 5-7	Link 6-8	Link 7-5	Link 7-8	Link 8-6	System Total Travel Cost
1	40	10	10	10	40	10	3606
2	41	14	6	15	39	5	3690
3	40	15	5	15	40	5	3706
4	42	18	2	20	38	0	3941
5	40	20	0	20	40	0	4005

It is observed that the DUE assignment with different path distance limit resulted in different equilibrium link flows. For some distance limits, e.g. link 25 & 27, under any GV rate, the network obtaines same assignment results, while for the other distance limits, it can be seen that the more EV use a certain link, the less GV choose that link, because the EV's path choices are restricted by its driving range and EV user prefer paths of short distance. When EV users crowded into those links with short distance, they became over-saturated, thus increasing corresponding link travel time, and GV user would rather use those unsaturated links to reduce their travel time to obtain equilibrium.

We find that the results of traffic assignment are same under the circumstance that the proportion of GV is more that 50%. The bold face in the **Table 2** shows the different traffic assignment outcomes when the distance limit is strict and the rate of EV is high.

When distance limit is equal to 24, to EV, only O-D Pair 1–4 has two available paths, and both of them carry flows. Because of this tighter limit, the second path between O-D Pair 2–3 (2–7–8–6–3) is no longer feasible. Traffic flows switch from this path to the first path, which causes the cost of those paths through Link 5–6 to increase, whereas the cost of those through Link 7–8 decreases. The links which are abandoned by EV will be available and used by GV.

As the limit becomes tighter, the number of paths used decreases. When distance limit is equal to 23, only one path can be used for each O-D pair for EV. As a result, some links, such as Link 6–8 and Link 8–6, will not be used by EV at all. When there are 25% GV(5 vehicles), they will use Link 6–8 and Link 8–6 since the cost of these paths are minimum.

Table 3 shows that the total network travel cost increases as the distance limit gets tighter. This is not surprising because when the distance constraint is set to be tighter, the number of feasible paths in the network typically decreases and those remaining feasible paths become more congested.

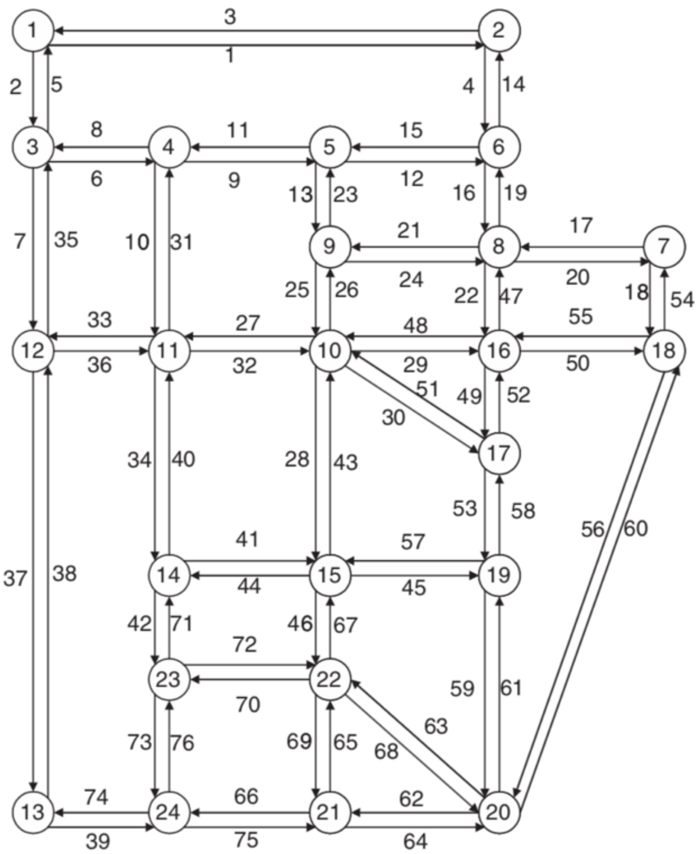

FIGURE 2 Test Networks: Sioux Falls Network

The solution procedure is applied to a famous network, Sioux Falls, as shown in **Figure 2**. The number beside each link is the link length. Nodes 1 and 2 are origins, and Nodes 10 and 20 are destinations. In this example, the focus is on analysis of how different distance limits and rate of EV affect the routing behavior of vehicles; for simplicity, only two classes of vehicles are considered, and the free-flow travel time is used as a proxy for the link length for each link. The travel demand between each O-D pair (O-D Pairs 1-10,1-20,2-10,2-20), the total demand on the network is 135(divided by rate). Link costs and distances of the connectors (i.e., those links connecting origins or destinations with internal nodes) are assumed to be 0, travel time on each other link is defined by the following BPR (Bureau of Public Road) type function ,

$$t_a(v_a) = t_a^0(1 + \alpha \times (\frac{v_a}{H_a})^{\beta})$$

The solution procedure described above was run with different values of the distance constraint and different rate of GV. The same equilibrium flow patterns can be explained by **Table 4**, which shows that, when all vehicles on the network are GV with no distance limit, the path cost of the four O-D pairs are 6634(O-D: 1-10), 7035(O-D: 2-10), 7121(O-D: 1-20), 7522(O-D: 2-20).

When 25% GV transfer into EV with distance limit parameter $\beta = 1$(distance limit is 115), the GV and EV flow show different characteristics (there is 75% GVs on the network). It shows the EV flow has no choice but choosing the only path to get the destination that makes the cost very high. When limit parameter relaxes to $\beta = 2$(distance limit is 230), the different between the cost of EVs and GVs get much smaller. Due to the relaxation, EVs have more links to choose, the distribution of EVs will be more separately, so the cost of GVs becomes more. The node 1 and node 2 can get to each other easily, and origin node 2 has less path to get to the destination than origin node 1, the swing in the cost of O-D pairs whose origin is node 1 can be explained. When there is no GV on the network, path cost of the EVs is decreasing with the relaxation of distance limit obviously.

According to the outcome, it can be easily seen that, the bigger the parameter β is, the more paths EVs can choose, and the greater cost of GVs in network will be.

Table 4 Link Flow Pattern Under Different Constraint Conditions and Different Vehicle Composition for Sioux Falls network

		Path cost				
GV Rate	O-D	β=1	β=1.2	β=1.5	β=1.8	β=2
100%	1-10	6633.63				
	2-10	7034.651				
	1-20	7121.102				
	2-20	7522.268				
75%	1-10	6308.745	6315.204	6450.269	6446.48	6433.872
	2-10	6673.558	6674.339	6815.361	6809.461	6799.276
	1-20	7006.596	7008.007	6912.37	6923.687	6929.054
	2-20	7373.963	7377.775	7290.397	7292.292	7298.038
50%	1-10	6111.77	6107.501	6511.675	6542.25	6520.216
	2-10	6240.45	6243.952	6641.458	6910.449	6906.621
	1-20	7079.10	7080.982	6978.655	7031.776	7035.913
	2-20	7227.38	7228.686	7110.91	7409.221	7422.164
25%	1-10	5193.694	5337.304	6226.56	6473.19	6431.715
	2-10	5183.352	5288.954	6188.438	6645.23	6797.472
	1-20	6354.553	6498.843	6774.984	6917.916	6925.342
	2-20	6348.806	6455.592	6742	7109.358	7296.461
0%	1-10	3820.102	3880.772	4817.465	5092.57	6345.198
	2-10	3818.102	3877.772	4814.465	5089.412	6542.812
	1-20	5168.883	5270.082	5350.737	5557.635	6875.961
	2-20	5166.883	5267.082	5347.737	5554.477	7073.574

Some results are summarized in **Table 4** (there is only a few EVs with different distance limit, the complete table is in appendix). It shows out the flow on alternative links will change when the distance limit becomes bigger.

Table 5 Link Flow Under Different Constraint Conditions and Different Vehicle Composition for Sioux Falls Network

Link length	O	D	β=1	β=1.1	β=1.2	β=1.5	β=1.8	β=2	No Limit
1	1	2	25	25	25	4.861	2.237	2.454	1.69
2	1	3	56.712	56.795	57.26	63.21	64.305	65.565	65.746
3	2	1	36.712	36.795	37.26	23.071	21.542	23.019	22.437
4	2	6	78.288	78.205	77.74	71.79	70.695	69.435	69.254
5	3	1	0	0	0	0	0	0	0
6	3	4	32.302	32.183	32.907	34.697	33.511	28.785	28.54
7	3	12	24.41	24.613	24.353	28.513	30.794	36.78	37.206

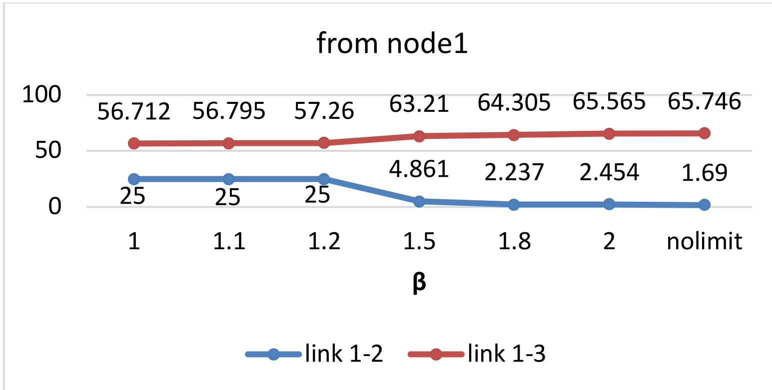

FIGURE 3 Link Flow from Node 1 Under Different β

From node 1, the EVs have to choose 1-3 when $\beta \leq 1.2$, they change into link 1-2 when $\beta \geq 1.5$. It reveals that the link 1-3 is very congested, but it do have short length, so when the limit is tight, the EVs have no choice but to choose the congested link 1-3. As the limit get looser, the EVs turn to the other link which is not congested quickly.

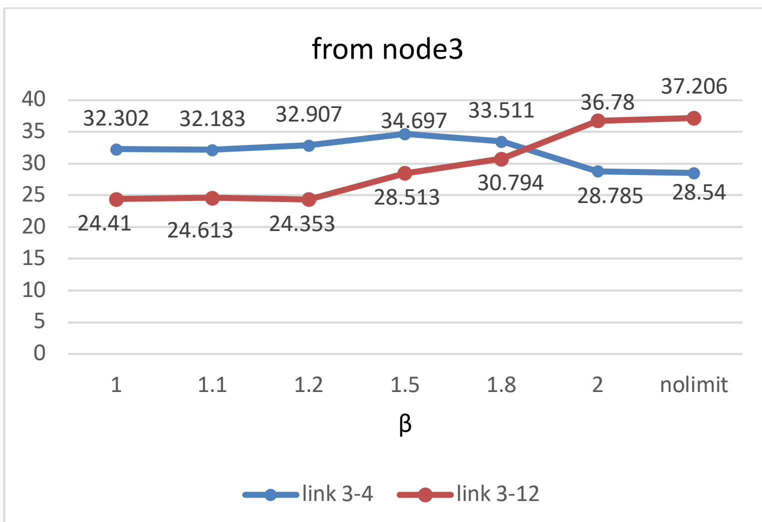

FIGURE 4 Link Flow from Node 3 Under Different β

Link 3-4 and link 3-12 show that, if the congestion of two link are similar, the translation of flow is not so obviously. But all the users are seeking for the best path for themselves.

6 CONCLUSIONS AND FUTURE WORK

The traffic flow under different distance limits is loaded onto the road network at the same time, and the path cost of different classes of cars under the UE condition is shown. The famous Frank-Wolfe algorithm is used here. The DCTAP model is validated and expanded to show the effect of cars with distance limit (EV) on the path selection of unrestricted cars (GV). We use simple parameter β to limit the travel distance of EV. In terms of model expansion, more realistic and specific parameter system can be used to compute the distance limit, and even consider the effect of charging station on distance limit and choice of paths.

Although the model runs well on the famous network, Sioux Falls, it may be necessary to make the algorithm more efficiency.In addition, when analyzing the phenomenon of the outcome, it is found difficult to locate which path the traffic flow on each link comes from. Therefore, there is space for expansion in this aspect. If the flow on each link from which path can be located, we can better classify the impact of different vehicles on the path selection, and analyze it from the micro levels.

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Appendix

Table Link Flow Under Different Constraint Conditions and Different Vehicle Composition for Sioux Falls Network

LinkLength	O-D	β=1	β=1.1	β=1.2	β=1.5	β=1.8	β=2	No Limit
1	1-2	25	25	25	4.861	2.237	2.454	1.69
2	1-3	56.712	56.795	57.26	63.21	64.305	65.565	65.746
3	2-1	36.712	36.795	37.26	23.071	21.542	23.019	22.437
4	2-6	78.288	78.205	77.74	71.79	70.695	69.435	69.254
5	3-1	0	0	0	0	0	0	0
6	3-4	32.302	32.183	32.907	34.697	33.511	28.785	28.54
7	3-12	24.41	24.613	24.353	28.513	30.794	36.78	37.206
8	4-3	0	0	0	0	0	0	0
9	4-5	20.106	21.971	16.806	21.559	6.299	3.063	2.445
10	4-11	12.197	12.766	16.102	20.414	28.091	28.719	29.359
11	5-4	0	0	0	7.277	0.879	2.996	3.264
12	5-6	0	0	0	0.661	0	0	0
13	5-9	23.394	25.176	19.546	36.369	32.894	28.823	28.069
14	6-2	0	0	0	0	0	0	0
15	6-5	3.288	3.205	2.74	22.747	27.475	28.757	28.888
16	6-8	75	75	75	49.704	43.22	40.678	40.365
17	7-8	0	0	0	0	0	0	0
18	7-18	75	75	75	37.552	20.242	17.915	17.526
19	8-6	0	0	0	0	0	0	0
20	8-7	75	75	75	37.552	20.242	17.915	17.526
21	8-9	0	0	0	0	1.06	1.692	1.528
22	8-16	0	9.277	7.265	37.448	27.129	24.758	24.477
23	9-5	0	0	0	0	0	0	0
24	9-8	0	9.277	7.265	25.296	5.21	3.687	3.165
25	9-10	23.394	15.899	12.281	11.073	28.743	26.828	26.432
26	10-9	0	0	0	0	0	0	0
27	10-11	0	0	0	0	0	0	0
28	10-15	0	0	0	0	8.942	6.289	4.105
29	10-16	0	0	0	0	0	0	0.272
30	10-17	0	0	0	0	4.964	2.676	1.692
31	11-4	0	2.554	0	0	0	0	0
32	11-10	36.606	34.824	26.941	31.007	34.093	26.869	25.111
33	11-12	0	0	0	0	0	0	0
34	11-14	0	0	13.514	17.92	24.793	19.407	21.512
35	12-3	0	0	0	0	0	0	0
36	12-11	24.41	24.613	24.353	28.513	30.794	17.558	17.263
37	12-13	0	0	0	0	0	19.223	19.943
38	13-12	0	0	0	0	0	0	0
39	13-24	0	0	0	0	0	19.223	19.943
40	14-11	0	0	0	0	0	0	0

41	14-15	0	0	13.514	17.92	24.793	19.407	13.394
42	14-23	0	0	0	0	0	0	8.118
43	15-10	0	0	13.514	17.92	1.893	2.901	1.868
44	15-14	0	0	0	0	0	0	0
45	15-19	0	0	0	0	11.83	8.407	8.988
46	15-22	0	0	0	0	20.012	14.388	8.061
47	16-8	0	0	0	0	0	0	0
48	16-10	0	9.277	7.265	0	8.78	9.996	10.355
49	16-17	0	0	0	0	9.376	9.827	10.067
50	16-18	0	0	0	37.448	9.222	4.935	4.703
51	17-10	0	0	0	0	0.398	2.371	2.304
52	17-16	0	0	0	0	0	0	0
53	17-19	0	0	0	0	13.942	10.132	9.813
54	18-7	0	0	0	0	0	0	0
55	18-16	0	0	0	0	0.248	0	0.376
56	18-20	75	75	75	75	29.216	22.85	21.853
57	19-15	0	0	0	0	0	0	0.901
58	19-17	0	0	0	0	0	0	0.357
59	19-20	0	0	0	0	25.772	18.539	17.542
60	20-18	0	0	0	0	0	0	0
61	20-19	0	0	0	0	0	0	0
62	20-21	0	0	0	0	0	0	0
63	20-22	0	0	0	0	0	0	0
64	21-20	0	0	0	0	0	19.223	17.66
65	21-22	0	0	0	0	0	0	0.365
66	21-24	0	0	0	0	0	0	0
67	22-15	0	0	0	0	0	0	0.517
68	22-20	0	0	0	0	20.012	14.388	17.945
69	22-21	0	0	0	0	0	0	3.302
70	22-23	0	0	0	0	0	0	0
71	23-14	0	0	0	0	0	0	0
72	23-22	0	0	0	0	0	0	13.337
73	23-24	0	0	0	0	0	0	0
74	24-13	0	0	0	0	0	0	0
75	24-21	0	0	0	0	0	19.223	14.724
76	24-23	0	0	0	0	0	0	5.219

Work Assignment

Content	Personal
Programming, introduction, literature review, methodology, solution, numerical analysis of small network, reference	Lu Yu
Programming, literature review, numerical analysis of big network, conclusion, appendix	Yuanmi Cao