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# INTRODUCTION TO GRAPH THEORY: EXERCISES

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# *Graphs*

## *Exercises*

1. $\mathcal{P}(\{1,2,3\}) \coloneqq \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

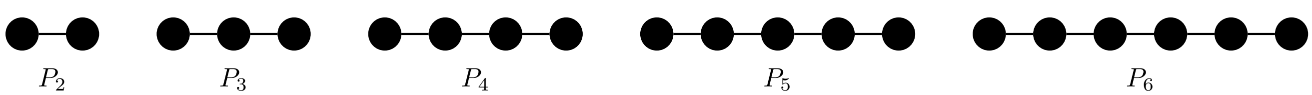
2.
$$\dfrac{\dfrac{\dfrac{\dfrac{\emptyset \not\subseteq A}{\exists x \in \emptyset : x \notin A} \qquad \dfrac{}{\neg\exists x \in \emptyset}}{\bot}}{\emptyset \subseteq A}}{}$$

$\square$

3.
$$\dfrac{\dfrac{}{S(b) \coloneqq \{ m \in V : m \notin S(m) \}} \qquad \dfrac{}{b \in V}}{b \in S(b) \iff b \notin S(b)} \equiv \dfrac{\dfrac{}{R = \{ x \notin x \}}}{R \in R \iff R \notin R}$$

4. Let $S$ be the collection of all sets that can be described in an English sentence of twenty-five words or less. $S$ is not a set, because $S$ can be described in fewer than twenty-five words, and if it were a set, then $S$ would have to be a member of itself, which violates the axiomatic definition of a set.

5. The first five *path graphs*.

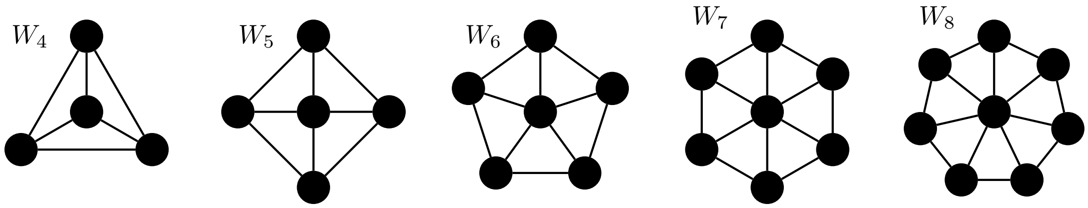



$$\begin{aligned} V(P_v) &= \{1, 2, ..., v\} \\ E(P_v) &= \{\{n-1, n\} : n \in \{2, ..., v\}\} \\ |E(P_v)| &= v - 1 \\ e &= v - 1 \end{aligned}$$

The number of edges in a *path graph* $P_v$, where $v \geq 2$, is given by the formula $e = v - 1$.

$\square$

6. The first five *wheel graphs*.




$$
\begin{aligned}
V(W_v) &= \{\, 1, 2, 3, ..., v \,\} \\
E(W_v) &= \{\, \{\, 1, 2 \,\}, \{\, 1, 3 \,\}, ..., \{\, 1, v \,\}, \\
&\quad\ \{\, 2, 3 \,\}, \{\, 3, 4 \,\}, ..., \{\, v-1, v \,\}, \\
&\quad\ \{\, v, 2 \,\} \,\} \\
&= \{\, \{\, \{\, 1, n \,\} : n \in \{\, 2, ..., v \,\} \,\}, \\
&\quad\ \{\, \{\, n-1, n \,\} : n \in \{\, 3, ..., v \,\} \,\}, \\
&\quad\ \{\, v, 2 \,\} \,\} \\
|E(W_v)| &= (v-1) + (v-2) + 1 \\
&= (v-1) + (v-1) \\
e &= 2(v-1)
\end{aligned}
$$

The number of edges in the *wheel graph* on $v$ vertices $W_v$, where $v \geq 4$, is given by the formula $e = 2(v-1)$.

□

7.

$$\begin{aligned} 1 + 2 + ... + (v-1) &= (1/2)v(v-1) \\ &= E(K_v) \qquad \text{(T2)} \\ &= (v-1) + (v-2) + ... + (v-(v-1)) \\ &= 1 + 2 + ...(v-1) \end{aligned}$$

Imagine drawing $K_v$ by joining vertex 1 to vertices 2 through $v$, creating $v-1$ edges; then joining vertex 2 to vertices 3 through $v$, creating $v-2$ edges; and so on, i.e. $(v-1)+(v-2)+...+(v-(v-1))$, or equivalently, $1+2+...+(v-1)$.

□

8.

p57

# *Glossary*

*path graph* If $v$ is a an integer greater than or equal to 2, the *path graph* on $v$ vertices, denoted "$P_v$", is the graph having the vertex set $\{1, 2, 3, ..., v\}$ and edge set $\{\{1, 2\}, \{2, 3\}, \{3, 4\}, ..., \{v - 1, v\}\}$. 5

*wheel graph* . 6

define