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INTRODUCTION TO GRAPH THEORY: EXERCISES

Contents

Graphs

Exercises

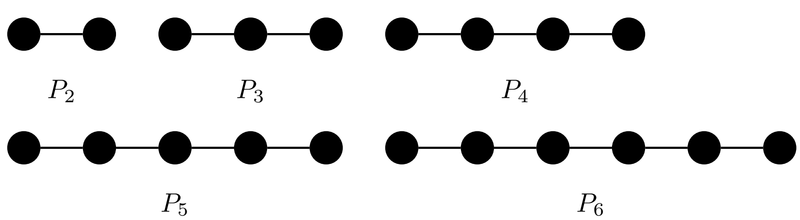
1. $\mathcal{P}(\{1,2,3\}) \coloneqq \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

2.
$$\dfrac{\dfrac{\dfrac{\emptyset \nsubseteq A}{\exists x \in \emptyset : x \notin A} \qquad \dfrac{}{\neg \exists x \in \emptyset}}{\bot}}{\emptyset \subseteq A}$$

$\square$

3.
$$\dfrac{\dfrac{}{S(b) \coloneqq \{ m \in V : m \notin S(m) \}} \qquad \dfrac{}{b \in V}}{b \in S(b) \iff b \notin S(b)} \equiv \dfrac{\dfrac{}{R = \{ x \notin x \}}}{R \in R \iff R \notin R}$$

4. Let S be the collection of all sets that can be described in an English sentence of twenty-five words or less. S is not a set, because S can be described in fewer than twenty-five words, and if it were a set, then S would have to be a member of itself, which violates the axiomatic definition of a set.

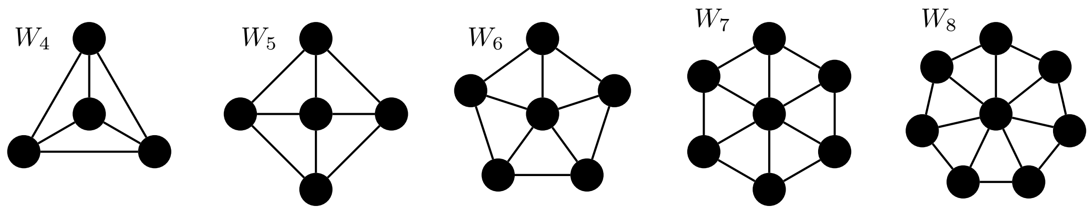
5.


$$\begin{aligned} V(P_v) &= \{ 1, 2, ..., v \} \\ E(P_v) &= \{ \{ n-1, n \} : n \in \{ 2, ..., v \} \} \\ |E(P_v)| &= v - 1 \\ e &= v - 1 \end{aligned}$$

The number of edges in a *path graph* P_v, where $v \geq 2$, is given by the formula $e = v - 1$.

$\square$

6.



$$\begin{aligned}
V(W_v) &= \{\, 1, 2, 3, ..., v \,\} \\
E(W_v) &= \{\, \{\, 1, 2 \,\}, \{\, 1, 3 \,\}, ..., \{\, 1, v \,\}, \\
&\qquad \{\, 2, 3 \,\}, \{\, 3, 4 \,\}, ..., \{\, v-1, v \,\}, \\
&\qquad \{\, v, 2 \,\} \,\} \\
&= \{\, \{\, \{\, 1, n \,\} : n \in \{\, 2, ..., v \,\} \,\}, \\
&\qquad \{\, \{\, n-1, n \,\} : n \in \{\, 3, ..., v \,\} \,\}, \\
&\qquad \{\, v, 2 \,\} \,\} \\
|E(W_v)| &= (v-1) + (v-2) + 1 \\
&= (v-1) + (v-1) \\
e &= 2(v-1)
\end{aligned}$$

The number of edges in *the wheel graph on v vertices* W_v, where $v \geq 4$, is given by the formula $e = 2(v-1)$.

$\square$

7.

$$\begin{aligned} 1 + 2 + ... + (v - 1) &= (1/2)v(v - 1) \\ &= E(K_v) \qquad\qquad\qquad\qquad \text{(T2)} \\ &= (v - 1) + (v - 2) + ... + (v - (v - 1)) \\ &= 1 + 2 + ...(v - 1) \end{aligned}$$

Imagine drawing K_v by joining vertex 1 to vertices 2 through v, creating $v - 1$ edges; then joining vertex 2 to vertices 3 through v, creating $v-2$ edges; and so on, i.e. $(v-1)+(v-2)+...+(v-(v-1))$, or equivalently, $1 + 2 + ... + (v - 1)$. □

8. p57