2016 Spring Notes

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Contents

1 各种相关背景知识

1.1 概率论

1.1.1 协方差矩阵

$$\Sigma = \mathrm{E}\Big[(\mathrm{X}-\mathrm{E}(\mathrm{X}))(\mathrm{X}-\mathrm{E}(\mathrm{X}))^T\Big] \tag{1}$$

$$\Sigma_{i,j} = Cov(X_i, X_j) \tag{2}$$

$$= \mathrm{E}\Big[(X_i-\mu_i)(X_j-\mu_j)\Big] \tag{3}$$

1.1.2 Multivariate normal distribution

- $$f_X(x_1,\ldots,x_n) = \frac{1}{\sqrt{(2\pi)^k|\Sigma|}}\exp(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu))$$

- 当为二维高斯分布时，条件概率服从（ ρ 为相关系数 ）：

$$X_1\Big|X_2 = x_2 \sim N(\mu_1+\frac{\sigma_1}{\sigma_2}\rho(x_2-\mu_2),(1-\rho^2)\sigma_1^2)$$

2 Information Theory

2.1 Differential Entropy

2.1.1 常见 Differential Entropy

- *Uniform distribution(from 0 to a)*

$$\log(a)$$

- *Normal Distribution*

$$X \sim N(0, \sigma^2)$$

$$h(X) = \frac{1}{2} \log 2\pi e \sigma^2$$

- *Multivariate Normal Distribution*

$$N_n \sim (\mu, K)$$

  μ is mean and K is covariance matrix

$$h(X_1, \ldots, X_n) = \frac{1}{2} \log(2\pi e)^n |K|$$

2.1.2 Properties

- $h(X + c) = h(X)$

- $h(aX) = h(X) + \log |a|$

- $h(AX) = h(X) + \log \left|\det(A)\right|$