The **yaev** package

Yves Deville deville.yves@alpestat.com

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Contents

1 Extreme-Value distributions

The probability functions for the GPD and GEV distributions depend smoothly on the parameters: they are infinitely differentiable functions of the parameters. However these functions are not analytic functions of the parameters and a singularity exists for all the functions when the shape parameter say ξ is zero. In practice, the functions are given with different formulas depending on whether ξ is zero or not; the formulas for $\xi = 0$ relate to the exponential and Gumbel distributions and correspond to the limit for $\xi \to 0$ of the functions given by the formulas for $\xi \neq 0$.

As an example consider the quantile function of the Generalized Pareto distribution with shape ξ and unit scale

$$q(p) = \begin{cases} [(1-p)^{-\xi} - 1]/\xi & \xi \neq 0 \\ -\log(1-p) & \xi = 0. \end{cases}$$

It can be shown that for $\xi \approx 0$

$$q \approx -\log(1-p), \qquad \frac{\partial q}{\partial \xi} \approx \frac{1}{2}\log^2(1-p), \qquad \frac{\partial^2 q}{\partial \xi^2} \approx -\frac{1}{3}\log^2(1-p).$$

It is quite easy to obtain expressions for the the derivatives. We can even rely on the derivation method `D` available in R

```
qEx <- function(p, xi) ((1 - p)^(-xi) - 1) / xi
dqEx <- D(expression(((1 - p)^(-xi) - 1) / xi), name = "xi")
d2qEx <- D(dqEx, name = "xi")
p <- 0.9; q <- 1 - p
for (xi in c(1e-4, 1e-7, 1e-9)) {
    r <- rbind("ord 0" = c("lim" = -log(q), "der" = qEx(p = p, xi = xi)),
               "ord 1" = c("lim" = log(q)^2 / 2, "der" = eval(dqEx, list(p = p, xi = xi))),
               "ord 2" = c("lim" = -log(q)^3 / 3, "der" = eval(d2qEx, list(x = p, xi = xi))))
    cat("xi = ", xi, "\n")
    print(r)
}
```

```
## xi =  1e-04
##            lim      der
## ord 0 2.302585 2.302850
## ord 1 2.650949 2.651356
## ord 2 4.069357 4.069899
## xi =  1e-07
##            lim           der
## ord 0 2.302585      2.302585
## ord 1 2.650949      2.652455
## ord 2 4.069357 -30116.312500
## xi =  1e-09
##            lim           der
## ord 0 2.302585  2.302585e+00
## ord 1 2.650949  8.674716e+01
## ord 2 4.069357 -1.681924e+11
```

The limit can be used with a quite high precision for the functions. However it turns out that the precision is not as good for the derivatives. The formulas for the functions can be wrong when ξ is as small as 1e-16, and this may be considered as good enough in practice. For the derivatives, the formulas can be wrong when ξ is about 1e-8 for the first-order derivatives and when ξ is about 1e-4 for the second-order derivatives. The reason is that the formulas for the derivatives involve fractions or small quantities since ξ or ξ^2 comes at the denominator. Although not yet widespread, the use of the derivatives w.r.t. the parameters is of great help in the optimization tasks required in EVA. These tasks of course involve the maximisation of the likelihood, but the profile-likelihood inference for models with covariates may require constrained optimization, and differential equations can be also used to derive confidence intervals. Although automatic differentiation is nowdays widely available, it is unclear that the derivatives are evaluated with a good precision even if the exact formulas are used.

This report gives the exact expressions for the first-order and the second-order derivatives of the probability functions w.r.t. the parameters and also provides workable approximations for the case $\xi \approx 0$.

- The **raw expressions given by Maxima** are reported in green. The expressions can be regarded as exact, not being influenced by manual computations. However these formulas are usually difficult to use in a compiled code.
- The **simplified expressions** are derived by us from the raw expressions. They are reported in red. The expressions are influenced by manual computations hence could in principle contain errors although they have been carefully checked. These formulas are used to write the compiled code.