Name: Yunika Upadhayaya

ID: 1001631183

CSE 4309 – 001

Assignment – 3

Task 1:

Output:

For training file = pendigits_training.txt

For test file = pendigits_test.txt

a) Degree = 1 and lambda = 0

- Training Phase:

```
w0=-6.3872
w1=0.0276
w2=0.0432
w3=0.0126
w4=0.0176
w5=0.0080
w6=-0.0058
w7=-0.0081
w8=0.0714
w9=-0.0153
w10=-0.0190
w11=0.0117
w12=0.0222
w13=-0.0018
w14=-0.0013
w15=0.0091
w16=0.0382
```

- Test Phase:

```
ID= 3498, output=      3.8514, target value =     4.0000, squared error = 0.0221
```

b) Degree = 1 and lambda = 1

- Training Phase:

w0=-6.2611
w1=0.0275
w2=0.0428
w3=0.0126
w4=0.0172
w5=0.0078
w6=-0.0059
w7=-0.0081
w8=0.0713
w9=-0.0154
w10=-0.0191
w11=0.0116
w12=0.0221
w13=-0.0018
w14=-0.0017
w15=0.0090
w16=0.0383

- Test Phase:

ID= 3498, output= 3.8528, target value = 4.0000, squared error = 0.0217

c) Degree = 2 and lambda = 0

- Training Phase:

w0=-7.5608
w1=0.0223
w2=0.0001
w3=0.0352
w4=0.0000
w5=0.0049
w6=-0.0000
w7=-0.0299
w8=0.0002
w9=0.0327
w10=-0.0001
w11=0.0694
w12=-0.0004
w13=0.0079
w14=-0.0002

w15=0.0596
w16=-0.0003
w17=-0.0184
w18=-0.0000
w19=0.0093
w20=0.0002
w21=0.0162
w22=-0.0000
w23=0.0398
w24=-0.0002
w25=-0.0041
w26=0.0001
w27=0.0538
w28=-0.0007
w29=-0.0149
w30=0.0002
w31=0.1215
w32=-0.0007

- Test Phase:

ID= 3498, output= 3.6074, target value = 4.0000, squared error = 0.1542

d) Degree = 2 and lambda = 1

- Training Phase:

w0=-7.0384
w1=0.0219
w2=0.0001
w3=0.0310
w4=0.0001
w5=0.0043
w6=-0.0000
w7=-0.0345
w8=0.0002
w9=0.0315
w10=-0.0001
w11=0.0678
w12=-0.0004
w13=0.0077
w14=-0.0002
w15=0.0574
w16=-0.0003
w17=-0.0192
w18=-0.0000

w19=0.0091 w20=0.0002 w21=0.0156 w22=-0.0000 w23=0.0401 w24=-0.0002 w25=-0.0050 w26=0.0001 w27=0.0536 w28=-0.0007 w29=-0.0155 w30=0.0002 w31=0.1208 w32=-0.0007

- Test Phase:

ID= 3498, output= 3.6001, target value = 4.0000, squared error = 0.1599

Task 2:

Answer:

$X_1 = 5.3$, $t_1 = 9.6$

$X_2 = 7.1$, $t_2 = 4.2$

$X_3 = 6.4$, $t_3 = 2.2$

We know:

$$\tilde{E}_D(\boldsymbol{w}) = \left\{\frac{1}{2}\sum_{n=1}^{N}[(t_n - \boldsymbol{w}^T\varphi(x_n))^2]\right\} + \frac{\lambda}{2}\boldsymbol{w}^T\boldsymbol{w}$$

Now, the value of w that minimizes the above equation is:

$$\boldsymbol{w} = (\lambda \mathrm{I} + \Phi^T\Phi)^{-1}\Phi^T\mathbf{t}$$

The value of Φ is:

$$\Phi = \begin{bmatrix} 1 & 5.3 \\ 1 & 7.1 \\ 1 & 6.4 \end{bmatrix}$$

The transpose of Φ is:

$$\Phi^T = \begin{bmatrix} 1 & 1 & 1 \\ 5.3 & 7.1 & 6.4 \end{bmatrix}$$

t is given by:

$$t = \begin{bmatrix} 9.6 \\ 4.2 \\ 2.2 \end{bmatrix}$$

Now,

$$\Phi^T\Phi = \begin{bmatrix} 1 & 1 & 1 \\ 5.3 & 7.1 & 6.4 \end{bmatrix} \begin{bmatrix} 1 & 5.3 \\ 1 & 7.1 \\ 1 & 6.4 \end{bmatrix}$$

$$\Phi^T\Phi = \begin{bmatrix} 3 & 18.8 \\ 18.8 & 119.46 \end{bmatrix}$$

Then,

$$\Phi^T\mathrm{t} = \begin{bmatrix} 1 & 1 & 1 \\ 5.3 & 7.1 & 6.4 \end{bmatrix} \begin{bmatrix} 9.6 \\ 4.2 \\ 2.2 \end{bmatrix}$$

$$\Phi^T\mathrm{t} = \begin{bmatrix} 16 \\ 94.78 \end{bmatrix}$$

So,

$$\boldsymbol{w} = (\lambda \mathrm{I} + \Phi^T\Phi)^{-1}\Phi^T\mathbf{t}$$

$$\boldsymbol{w} = \left[(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 18.8 \\ 18.8 & 119.46 \end{bmatrix})^{-1} \begin{bmatrix} 16 \\ 94.78 \end{bmatrix} \right]$$

$$\boldsymbol{w} = \left[(\begin{bmatrix} 3+\lambda & 18.8 \\ 18.8 & 119.46+\lambda \end{bmatrix})^{-1} \begin{bmatrix} 16 \\ 94.78 \end{bmatrix} \right]$$

$$\boldsymbol{w} = \begin{bmatrix} \frac{119.46 + x}{x^2 + 112.46x + 4.94} & \frac{-18.8}{x^2 + 112.46x + 4.94} \\ \frac{-18.8}{x^2 + 112.46x + 4.94} & \frac{3 + x}{x^2 + 112.46x + 4.94} \end{bmatrix} \begin{bmatrix} \frac{16}{94.78} \end{bmatrix}$$

$$\boldsymbol{w} = \begin{bmatrix} \frac{16x + 129.496}{x^2 + 122.46x + 4.94} \\ \frac{94.78x - 16.46}{x^2 + 122.46x + 4.94} \end{bmatrix}$$

$$\boldsymbol{w} = \lim_{x \to \infty} \begin{bmatrix} \frac{16x + 129.496}{x^2 + 122.46x + 4.94} \\ \frac{94.78x - 16.46}{x^2 + 122.46x + 4.94} \end{bmatrix}$$

$$\boldsymbol{w} = \lim_{x\to\infty} \begin{bmatrix} \dfrac{\frac{16}{x} + \frac{129.496}{x^2}}{1 + \frac{122.46x}{x^2} + \frac{4.94}{x^2}} \\ \dfrac{\frac{94.78}{x} - \frac{16.46}{x^2}}{1 + \frac{122.46}{x} + \frac{4.94}{x^2}} \end{bmatrix}$$

$$\boldsymbol{w} = \begin{bmatrix} \frac{0}{1} \\ \frac{0}{1} \end{bmatrix}$$

Thus, $\boldsymbol{w} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Task 3:

Answer:

a) $X_1 = 5.3$, $t_1 = 9.6$

- ➔ f (x) = 3.1 x + 4.2 = 3.1 * 5.3 + 4.2 = 20.63 ---- eq (I)
- ➔ f (x) = 2.4 x – 1.5 = 2.4 * 5.3 – 1.5 = 11.2 --- eq (II)

b) $X_2 = 7.1$, $t_2 = 4.2$

- ➔ f (x) = 3.1 x + 4.2 = 3.1 * 7.1 + 4.2 = 26.21--- eq (I)
- ➔ f (x) = 2.4 x – 1.5 = 2.4 * 7.1 – 1.5 = 15.54 --- eq (II)

c) $X_3 = 6.4$, $t_3 = 2.2$

- ➔ f (x) = 3.1 x + 4.2 = 3.1 * 6.4 + 4.2 = 24.04 --- eq (I)
- ➔ f (x) = 2.4 x – 1.5 = 2.4 * 6.4 – 1.5 = 13.86 --- eq (II)

From eq (I)s:

$(20.63 – 9.6)^2 + (26.21 – 4.2)^2 + (24.04 – 2.2)^2 = 1083.087$

From eq (II)s:

$(11.2 – 9.6)^2 + (15.54 – 4.2)^2 + (13.86 – 2.2)^2 = 267.11$

Thus, f (x) = 2.4 x – 1.5 is a better solution out of the above two solutions because it produces least value when using sum-of-squares criterion.