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Assignment 2 (Written)

Task – 1:

Let Maine → M and Sahara → S, L → temp < 80, H → temp > 80

Given – P (M) = 5%, P (S) = 95%,

Conditions – Days are conditionally independent, and location of the sensor is also independent.

i.e., P (M) = 0.05 and P (S) = 0.95 (Prior probability of sensor placement)

For Maine:

P (temp >= 80) = 20 % = 0.2

P (temp < 80) = 80 % = 0.8 i.e., P (L | M) = 0.8

P (L | M^C) = 0.1 (Here, c is compliment)

For Sahara:

P (temp >= 80) = 90 % = 0.9

P (temp < 80) = 10 % = 0.1 i.e., P (L | S) = 0.1

a) P (M | L) = P (L | M) * P (M) / [P (M) P (L | M) + P (M^c) P (L | M^c)]
   = 0.8 * 0.05 / (0.05 * 0.8 + 0.95 * 0.1)
   = 0.2963
b) From (a), the posterior probability of sensor placement is 29.63% for Maine and 70.37% for Sahara.
   Using the law of total probability:
   P (L) = 0.2963 * 0.8 + 0.7037 * 0.1 = 0.23704 + 0.07037 = 0.30741

c) Probability of the first three emails that indicate daily highs under 80 degrees if the sensor is in Maine = P(M) * P (L | M) * P (L | M) * P (L | M) = 0.05 * 0.8 * 0.8 * 0.8 = 0.025
   Probability of the first three emails that indicate daily highs under 80 degrees if the sensor is in Sahara = P(S) * P (L | S) * P (L | S) * P (L | S) = 0.95 * 0.1 * 0.1 * 0.1 = 0.00095
   Thus, Probability (first 3 emails is under 80) = 0.025 + 0.00095 = 0.02655

Task – 2:

Given – Function P is defined on a sample S = {A, B, C, D}

P (A) = 0.3

P (B) = 0.6

P is a probability mass function (p.m.f) since A, B, C and D are discrete points.

P (X = x) = $f_x(x)$, $f_x(x) >= 0$, $\sum_x f_x(x) = 1$

Here, $f_A(A) = 0.3$

$F_B(B) = 0.6$

$F_C(C)$ and $F_D(D)$ are unknown. Thus,

P can be probability function if P(C) + P(D) = 1 – 0.3 – 0.6 = 0.1 or P (C) + P (D) <= 1 Otherwise, it cannot be a probability density function.

Task – 3:

Given – P(x) = 0.3 if x ∈ [0, 10]

For a function P(x) to be a probability density function, following conditions should be satisfied:

a) P(x) >= 0 for all x

b) $\int_{-\infty}^{\infty} P(x)dx = 1$

$$\int_{-\infty}^{\infty} P(x) = \overbrace{\int_{-\infty}^{0} P(x)dx}^{a} + \int_{0}^{10} P(x)dx + \overbrace{\int_{10}^{\infty} P(x)dx}^{b}$$

= a + $\int_0^{10} 0.3\ dx$ + b

= a + 3 + b

Since, we do not have the information about a and b, P(x) cannot be a valid probability density function.

P(x) can be a valid probability function if: P (- ∞ < x < 0) + P (10 < x < ∞) = 1 – P (0 < x < 10) = 1 – 0.3 = 0.7

Task – 4:

Given:

P (B = r) = 0.4

P (B = b) = 0.6

P (F = a | B = r) = 0.25

P (F = o | B = r) = 0.75

P (F = a | B = b) = 0.75

P (F = o | B = b) = 0.25

Here,

We know that if x = a:

$p(x = a) = \sum_{c_k \in C} p(x \mid c_k) * p(c_k)$ is the output given that (x = a) is picked.

Now,

p (F = a) = p (F = a | B = r) * p (B = r) + p (F = a + B = b) * p (B = b)

= 0.25 * 0.4 + 0.75 * 0.6

= 0.55

p (F = o) = p (F = o | B = r) * p (B = r) + p (F = o + B = b) * p (B = b)

= 0.75 * 0.4 + 0.25 * 0.6

= 0.45

Also,

p (B = r | F = a) = p (F = a | B = r) * p (B = r) / p (F = a) = 0.25 * 0.4 / 0.55 = 0.182

p (B = b | F = a) = p (F = a | B = b) * p (B = b) / p (F = a) = 0.75 * 0.6 / 0.55 = 0.82

Output – If the **box is blue**, then the **fruit is apple** 82% of the time.

p (B = r | F = o) = p (F = o | B = r) * p (B = r) / p (F = o) = 0.75 * 0.4 / 0.45 = 0.667%

p (B = b | F = o) = p (F = o | B = b) * p (B = b) / p (F = o) = 0.25 * 0.6 / 0.45 = 0.333%

Output – If the **box is red**, then **the fruit is orange** 67% of the time.

Task – 5:

Output produced by the training stage when given **yeast_training.txt** as the input file:

```
Class 1, attribute 1, mean = 0.52, std = 0.10
Class 1, attribute 2, mean = 0.54, std = 0.10
Class 1, attribute 3, mean = 0.52, std = 0.07
Class 1, attribute 4, mean = 0.41, std = 0.17
Class 1, attribute 5, mean = 0.50, std = 0.01
Class 1, attribute 6, mean = 0.00, std = 0.01
Class 1, attribute 7, mean = 0.50, std = 0.05
Class 1, attribute 8, mean = 0.24, std = 0.05
Class 2, attribute 1, mean = 0.45, std = 0.11
Class 2, attribute 2, mean = 0.45, std = 0.10
Class 2, attribute 3, mean = 0.53, std = 0.06
Class 2, attribute 4, mean = 0.23, std = 0.11
Class 2, attribute 5, mean = 0.50, std = 0.04
Class 2, attribute 6, mean = 0.00, std = 0.01
Class 2, attribute 7, mean = 0.49, std = 0.06
Class 2, attribute 8, mean = 0.33, std = 0.14
Class 3, attribute 1, mean = 0.43, std = 0.10
Class 3, attribute 2, mean = 0.48, std = 0.11
Class 3, attribute 3, mean = 0.36, std = 0.06
Class 3, attribute 4, mean = 0.22, std = 0.08
Class 3, attribute 5, mean = 0.51, std = 0.05
Class 3, attribute 6, mean = 0.00, std = 0.01
Class 3, attribute 7, mean = 0.51, std = 0.04
Class 3, attribute 8, mean = 0.27, std = 0.09
Class 4, attribute 1, mean = 0.79, std = 0.07
Class 4, attribute 2, mean = 0.76, std = 0.07
Class 4, attribute 3, mean = 0.38, std = 0.06
Class 4, attribute 4, mean = 0.32, std = 0.11
Class 4, attribute 5, mean = 0.50, std = 0.01
Class 4, attribute 6, mean = 0.00, std = 0.01
Class 4, attribute 7, mean = 0.51, std = 0.07
Class 4, attribute 8, mean = 0.27, std = 0.09
Class 5, attribute 1, mean = 0.74, std = 0.16
Class 5, attribute 2, mean = 0.62, std = 0.13
Class 5, attribute 3, mean = 0.42, std = 0.08
Class 5, attribute 4, mean = 0.30, std = 0.12
Class 5, attribute 5, mean = 0.50, std = 0.01
Class 5, attribute 6, mean = 0.00, std = 0.01
Class 5, attribute 7, mean = 0.51, std = 0.06
Class 5, attribute 8, mean = 0.24, std = 0.04
Class 6, attribute 1, mean = 0.54, std = 0.14
Class 6, attribute 2, mean = 0.50, std = 0.12
Class 6, attribute 3, mean = 0.51, std = 0.05
```

```
Class 6, attribute 4, mean = 0.24, std = 0.10
Class 6, attribute 5, mean = 0.50, std = 0.01
Class 6, attribute 6, mean = 0.49, std = 0.39
Class 6, attribute 7, mean = 0.51, std = 0.03
Class 6, attribute 8, mean = 0.24, std = 0.05
Class 7, attribute 1, mean = 0.48, std = 0.11
Class 7, attribute 2, mean = 0.47, std = 0.09
Class 7, attribute 3, mean = 0.54, std = 0.06
Class 7, attribute 4, mean = 0.22, std = 0.12
Class 7, attribute 5, mean = 0.50, std = 0.04
Class 7, attribute 6, mean = 0.00, std = 0.03
Class 7, attribute 7, mean = 0.50, std = 0.06
Class 7, attribute 8, mean = 0.26, std = 0.09
Class 8, attribute 1, mean = 0.74, std = 0.11
Class 8, attribute 2, mean = 0.73, std = 0.11
Class 8, attribute 3, mean = 0.49, std = 0.05
Class 8, attribute 4, mean = 0.29, std = 0.07
Class 8, attribute 5, mean = 0.50, std = 0.01
Class 8, attribute 6, mean = 0.00, std = 0.01
Class 8, attribute 7, mean = 0.46, std = 0.08
Class 8, attribute 8, mean = 0.23, std = 0.02
Class 9, attribute 1, mean = 0.55, std = 0.14
Class 9, attribute 2, mean = 0.56, std = 0.16
Class 9, attribute 3, mean = 0.51, std = 0.07
Class 9, attribute 4, mean = 0.20, std = 0.07
Class 9, attribute 5, mean = 0.50, std = 0.01
Class 9, attribute 6, mean = 0.00, std = 0.01
Class 9, attribute 7, mean = 0.53, std = 0.05
Class 9, attribute 8, mean = 0.24, std = 0.05
Class 10, attribute 1, mean = 0.78, std = 0.06
Class 10, attribute 2, mean = 0.73, std = 0.12
Class 10, attribute 3, mean = 0.48, std = 0.11
Class 10, attribute 4, mean = 0.33, std = 0.07
Class 10, attribute 5, mean = 1.00, std = 0.01
Class 10, attribute 6, mean = 0.00, std = 0.01
Class 10, attribute 7, mean = 0.55, std = 0.02
Class 10, attribute 8, mean = 0.23, std = 0.01
```

Output produced by the last line of the test stage of the program given **yeast_test.txt** as the input file is:

```
Classification accuracy = 0.4483
```