Supplementary material of the article entitled "Evaluating generation of chaotic time series by convolutional generative adversarial networks."

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Here we present supplementary results that we could not include in the main text.

## A. Samples of generated time series

Ten time-series samples, each generated using a GAN trained to generate logistic map time series, are shown, along with ten numerically computed time series (training time series) samples in Fig A.1.

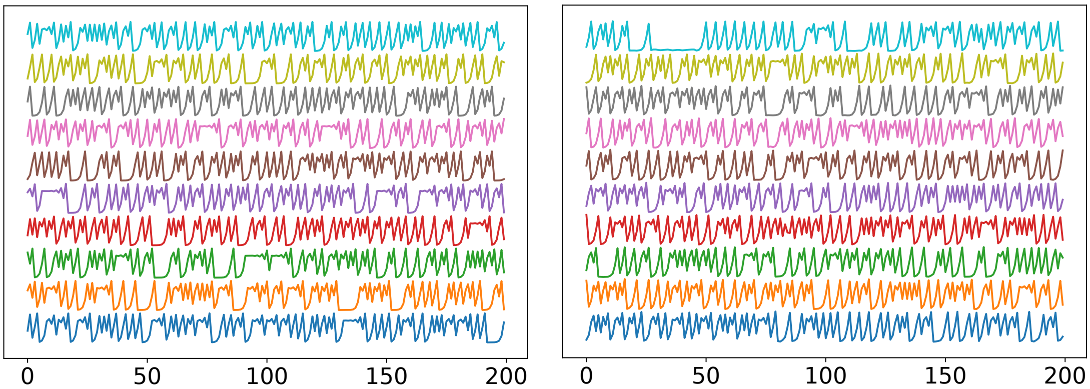


**Fig. A.1** Samples of numerically computed time series (left) and GAN-generated time series (right).

## B. Effect of Embedding Dimension on Largest Lyapunov Exponent Estimation

The largest Lyapunov exponent was estimated by changing the embedding dimension $m$ from 1 to 5. The radius $\varepsilon$ of the neighborhood search was scaled depending on $m$ so that the number of neighborhoods is sufficient, as described in the main text. Figure B.1 shows the slopes of $S(k)$ versus $k$ for each $m$. When $m = 1$, a jump of $S(k)$ was observed between k=0 and 1 in the GAN-generated time series. Such jumps are often observed when the embedding dimension is insufficient or when noise is added to the deterministic time series [1,14], both of which were likely to contribute to the jumps in this case. The jump was not prominent for $m = 2$ and 3, and well-defined linear slope intervals were obtained. For $m = 4$ and 5, because we used larger $\varepsilon$, the steps until the error increased to saturation became shorter, resulting in a looser estimate of the slope of $S(k)$ and thus a lower Lyapunov exponent than the theoretical value (Fig 3(right) in the main text). For all $m$, the GAN approximated the error expansion exhibited by the training time series better than any other conditions.

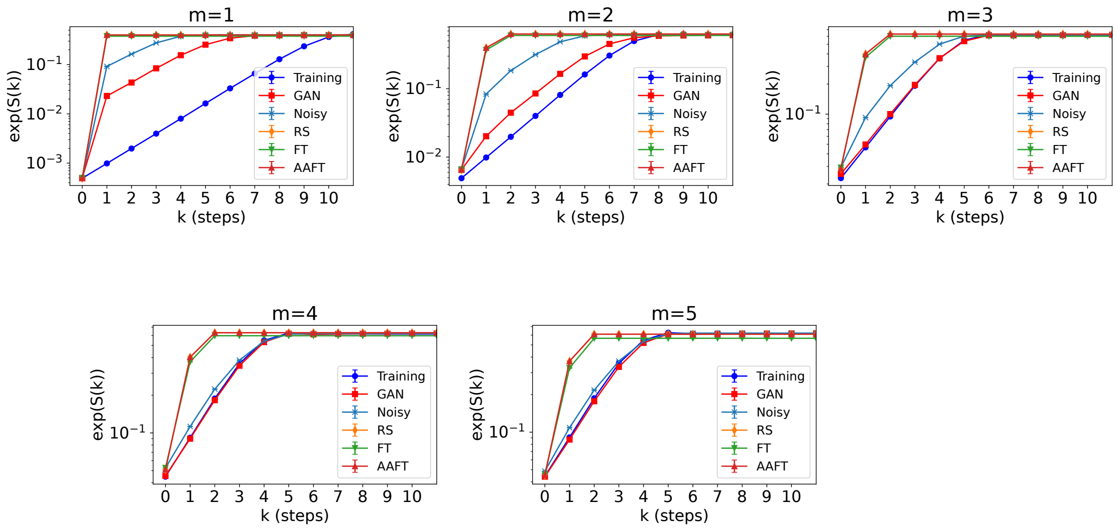


**Fig. B.1** The slopes of S(k) for different embedding dimensions m.

## C. $n > 1$ step errors

In the main text, only errors in transitions one step ahead were analyzed for GAN-generated time series $X = \{\ldots, x_{t-1}, x_t, x_{t+1}, \ldots x_N\}$. Here we deal with errors for $n > 1$ steps. Let $\{y_t^0, y_t^1, \ldots, y_t^n, \ldots\}$ be the time series such that $y_t^0 = x_t$ and $y_t^n = f(y_t^{n-1})$ $(t = 1,2, \ldots,$ and $n = 1, 2, \ldots)$. Then the $n$-step mean absolute error can be defined by $\frac{1}{N-n}\sum_{t=1}^{N-n}|x_{t+n} - y_t^n|$. However, there is a subtle difficulty here that needs to be considered. When a trajectory starts from an initial value outside the unit interval $I = [0,1]$, it diverges to negative infinity in the case of the logistic map. However, in the GAN-generated time series, even if the trajectory temporarily leaves $I$, it soon returns to $I$ and never diverges. Therefore, if the $n$-step error of the trajectory from a point outside $I$ is evaluated as is, the error will be quite large due to the divergence of $y_t^n$. Thus, we evaluated the $n$-step error only for initial values starting from within $I$. Let $X_I = \{x_t | x_t \in I \cap X\}$ be a set of GAN-generated values that are in $I$. The $n$-step mean absolute error was calculated by

$$E(n) = \frac{1}{|X_I|}\sum_{x_t \in X_I} |x_{t+n} - y_t^n|.$$

Figure C.1 shows the expansion of the mean error as a function of the number of steps $n$. It shows a similar exponential growth trend to that seen in the analysis of Lyapunov exponents

( Figure 3 (left) in the main text). The dashed line indicates the slope of the theoretical error expansion (the Lyapunov exponent equals $\ln 2$.)

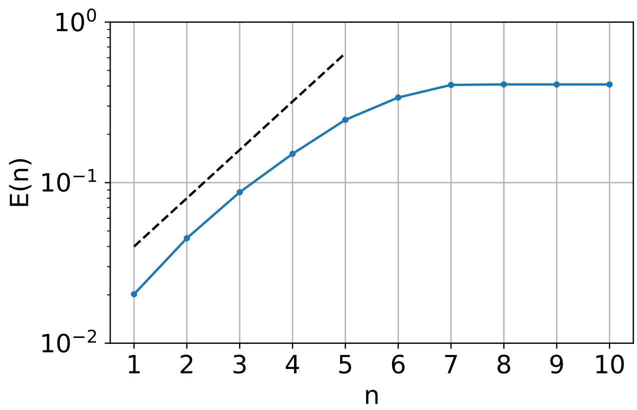


**Figure C.1** Mean absolute n-step error of GAN-generated time series.

## D. Model architecture

Figure D.1 shows the full architectures of the generator and the discriminator used in this study.

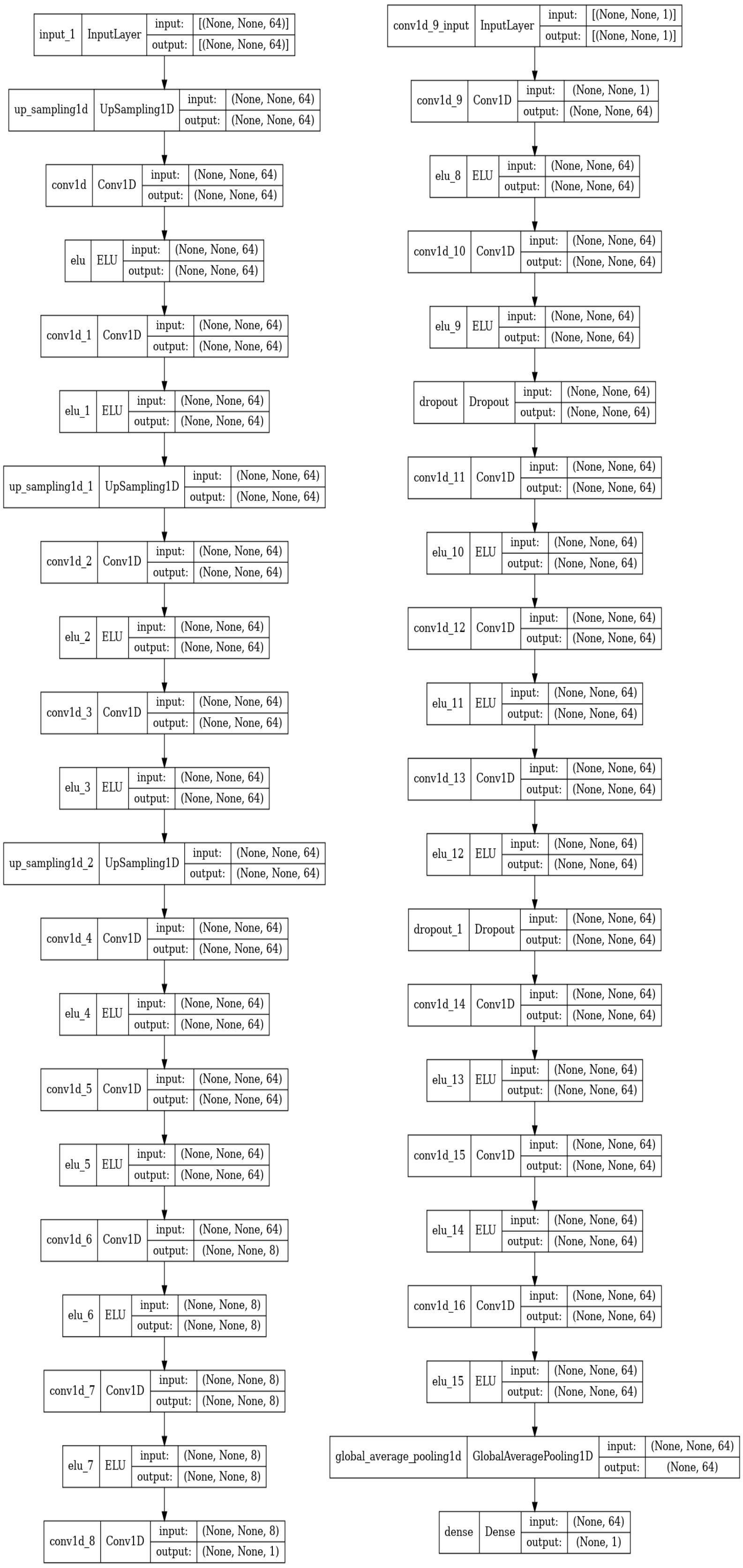


**Figure D.1** Architectures of the generator (left) and the discriminator (right).