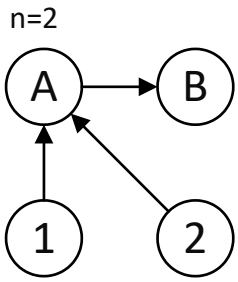
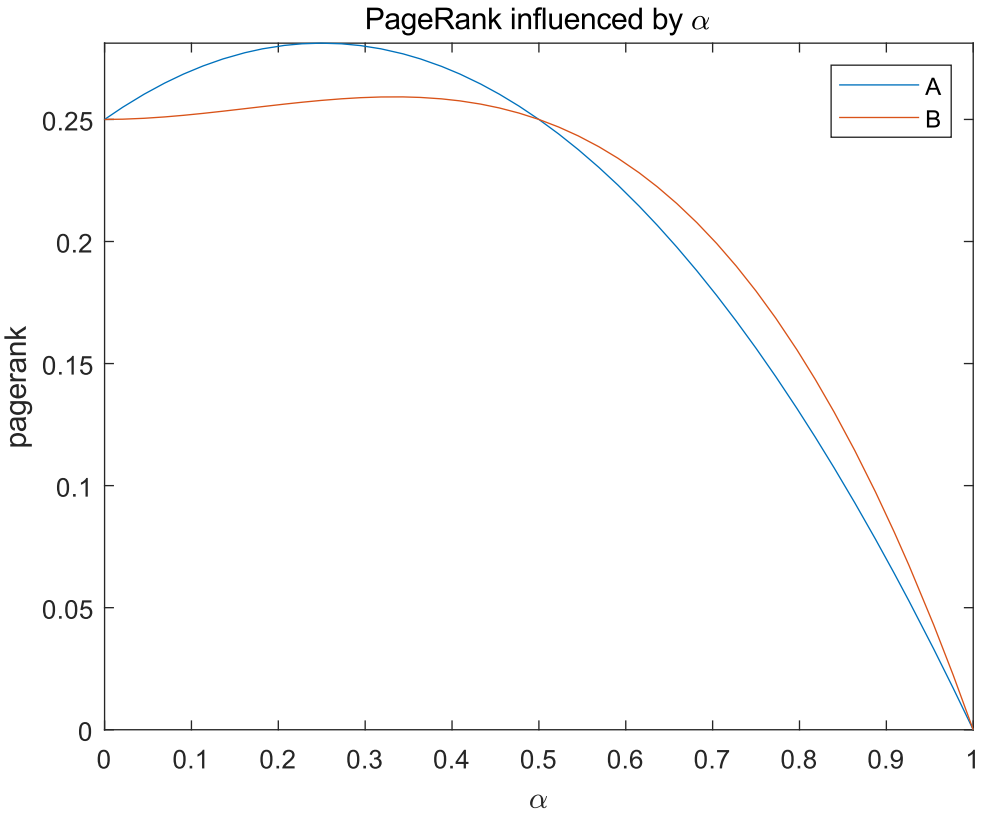
Pagerank most challenging problem

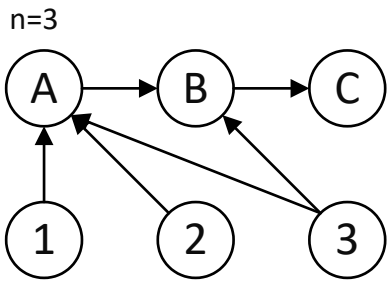
1. A problem from [Exercise 5.14, Mining of Massive Datasets]

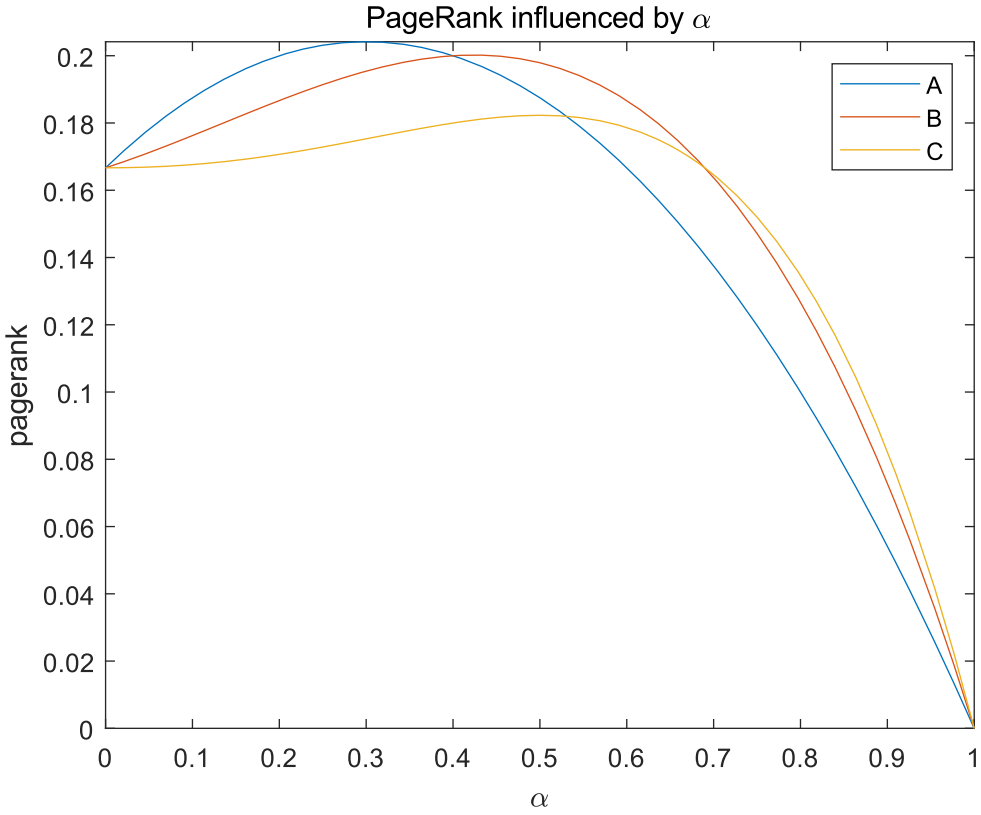
> Construct, for any integer n, a Web such that, depending on α, any of the n nodes can have the highest PageRank among those n. It is allowed for there to be other nodes in the Web besides these n.

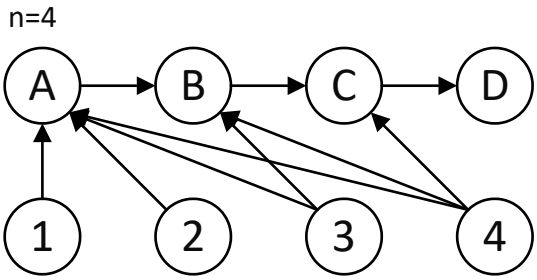
The solution:

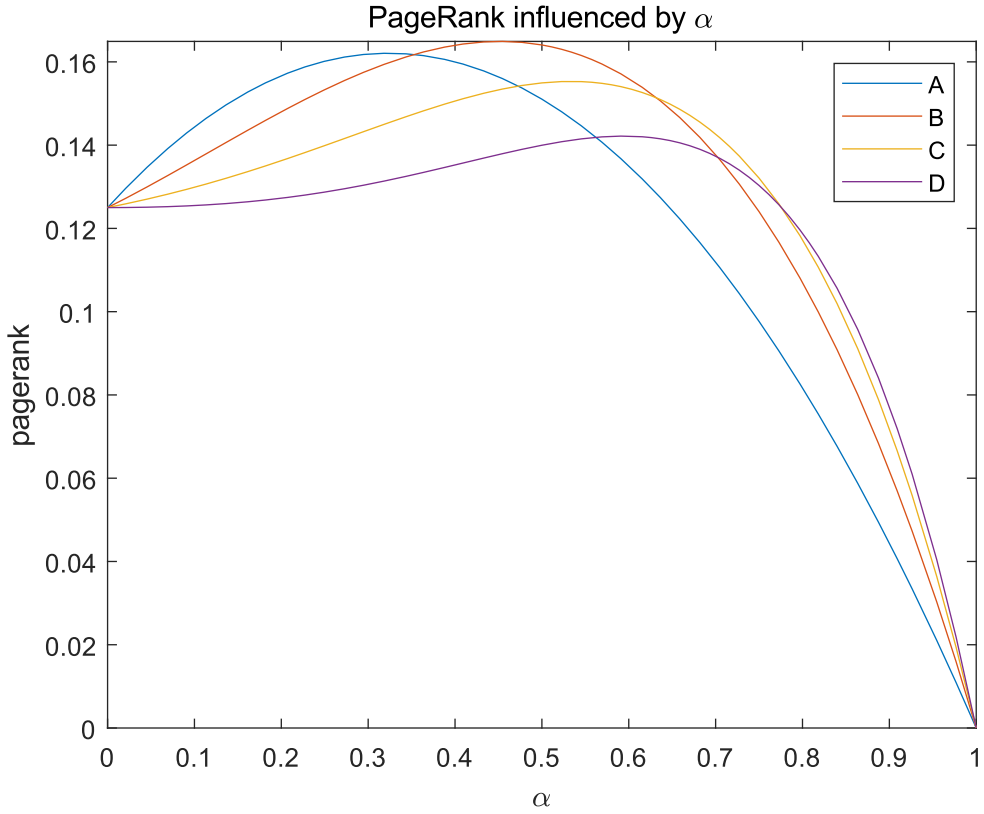





n=3
A
B
C
1
2
3


PageRank influenced by α
pagerank
α
A
B
C
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


n=4
A
B
C
D
1
2
3
4



Deep thinking:

The main idea is how we donate the influence of α in the probability of π

we have

$$\pi = \alpha P\pi + (1-\alpha)v \tag{1}$$
$$1^T\pi = 1$$

so we can get

$$\pi = (1-\alpha)(I-\alpha P)^{-1}v \tag{2}$$
$$\pi = \frac{1-\alpha}{\det(I-\alpha P)}(I-\alpha P)^* v$$

here, $P \in R^{2n\times 2n}$ and we only look at $\pi[1:n]$ (nodes $A, B, \ldots$). $\frac{1-\alpha}{\det(I-\alpha P)}$ and v are same roles in each entry of π

let $A = (I-\alpha P)^*$ and we only need to see the sum of each row $\sum_{j=1}^{n} A_{ij}$

Code:

```
syms alpha

n = 2;
P = zeros(2 * n);
v = ones(2 * n, 1) / (2 * n);

for i = n:-1:2
    P(i, i - 1) = 1;
end

for i = 2*n:-1:n+1
```

```
    for j = 1:i-n-1
        P(j, i) = 1 / (i - n - 1);
    end
end

P(1, n+1) = 1;

pi = (1 - alpha) * ((eye(2 * n) - alpha * P) \ v);

fplot(pi(1:n),[0 1])
title('PageRank influenced by \alpha')
xlabel('\alpha')
ylabel('pagerank')
legend('A','B')
```