

Problem Set 7 — Linear Algebra A (Fall 2021)

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Please hand in your assignment at the beginning of your Eighth tutorial session!

1. 证明：实上三角矩阵正交矩阵必为对角矩阵，且对角线上的元素为 ± 1 .
2. 在欧氏空间 \mathbb{R}^4 中确定一组基

$$\alpha_1 = (1, 1, 0, 0), \alpha_2 = (1, 0, 1, 0), \alpha_3 = (-1, 0, 0, 1), \alpha_4 = (1, -1, -1, 1).$$

通过施密特正交化过程, 把它们化为一组标准正交基.

3. 在欧氏空间 \mathbb{R}^{2n} 中求下列齐次线性方程组

$$x_1 - x_2 + x_3 - x_4 + \cdots + x_{2n-1} - x_{2n} = 0$$

的解空间的一组标准正交基.

4. 证明：任何一个 n 阶可逆实方阵 A 都可以表为一个实正交方阵 Q 和一个对角元全为正数的上三角方阵 T 的乘积, 即

$$A = QT.$$

而且这种表示法唯一.

5. If A is m by n with rank n , $\text{qr}(A)$ produces a square Q and zeros below R :

$$\text{The factors from MATLAB are } (m \times m)(m \times n) \quad A = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} R \\ 0 \end{bmatrix}$$

- (a) The n columns of Q_1 are an orthonormal basis for which fundamental subspace?
- (b) The $m - n$ columns of Q_2 are an orthonormal basis for which fundamental subspace?