Vector space:

A vector space is a set V along with an addition on V and a multiplication on V such that the following properties hold:

commutativity: ~~uv~~ $u+v=v+u$ for all $u,v \in V$;

associativity: $(u+v)+w=u+(v+w)$ and $(ab)v=a(bv)$ for all $u,v,w \in V$ and $a,b \in \mathbb{F}$;

additive identity: there exists an element $0 \in V$ such that $v+0=v$ for all $v \in V$;

additive inverse: for every $v \in V$, there exists $w \in V$ such that $v+w=0$;

multiplicative identity: $1v=v$ for all $v \in V$;

distributive properties: $a(u+v)=au+av$ and $(a+b)v=av+bv$ for all $a,b \in \mathbb{F}$ and all $u,v \in V$.

这一定义是最严谨的定义，其中加、乘大家直接理解作向量的加法与数乘即可，$\mathbb{F}$指数域，若不了解认为成$\mathbb{R}$（实数域）或$\mathbb{C}$（复数域）即可，下面附上陈老师Lecture上给的定义

A real vector space is a set of vectors together with rules for vector addition and multiplication by real numbers. There are eight fundamental properties:

1. $x+y=y+x$ (commutativity of addition)
2. $x+(y+z)=(x+y)+z$ (associativity of addition)
3. There is a unique "zero vector" such that $x+0=x$ for all x
4. For each x there is a unique vector $-x$ such that $x+(-x)=0$
5. $1x=x$
6. $(c_1c_2)x=c_1(c_2x)$
7. $c(x+y)=cx+cy$
8. $(c_1+c_2)x=c_1x+c_2x$

The elements $x+y$ and cx are called the sum of x and y and the product of c and x, respectively.

注意到陈老师定义中强调了3,4条中的unique，两者是一致的课上已给出证明，此处不作赘述

Subspace: 判定更为重要，因为~~其~~比上者简洁，更可能会考

i. additive identity: $0 \in U$.

ii. closed under addition: $\forall u,v \in U$, we have $u+v \in U$.

iii. closed under multiplication: $\forall u \in V$ and $a \in \mathbb{R}$, we have $au \in U$.

（先给大家限定至实数空间，上面$\mathbb{R}$也可换为$\mathbb{F}$或$\mathbb{C}$，第i条因第iii条a可取零可以不写）