

MIT 18.06 Exam 3, Fall 2017
Johnson

Your name: _____

Recitation: _____

problem	score
1	/30
2	/30
3	/40
total	/100

Problem 1 (30 points):

- (a) **Give a matrix** A where $\det(A - \lambda I) = 0$ has exactly two roots $\lambda = 1$ and $\lambda = 3$, but the trace of A does *not* equal 4.
- (b) The eigenvalues of $(A + A^T)^{-1}$ for any real, square matrix A (assuming $A + A^T$ is invertible) must be _____.
- (c) If $A = Q^T \Lambda Q$ for a diagonal matrix Λ and a real orthogonal matrix Q , then the eigenvectors of A are the _____ of Q .
- (d) If A is real, and $e^{At} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = e^{(3+4i)t} \begin{pmatrix} 1+i \\ 2-2i \end{pmatrix} + e^{\alpha t} \begin{pmatrix} \beta \\ \gamma \end{pmatrix}$, then the (t -independent) scalars α, β, γ are _____?
- (e) If A is a 4×4 matrix with $\det A = 5$, then $\frac{d}{dt} \det(A^T A t) =$ _____.

(blank page for your work if you need it)

Problem 2 (30 points):

You are given a matrix $A = e^{-B^T B}$ for some real 3×3 matrix B . The nullspace $N(B)$ is spanned by $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

- (a) **Circle** any of the following vectors that *cannot possibly* be eigenvectors of A , and put a **rectangle** around any vectors that *must be* eigenvectors of A :

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

- (b) $A^n x$ for some $x \neq 0$ may do what for large n (circle *all possibilities*)?
Oscillate / **decay** / **diverge** / **go to a nonzero constant vector**.
- (c) For $x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, give a **good approximation** for $A^n x$ for a very large n .

(blank page for your work if you need it)

Problem 3 (40 points):

The vector $x(t)$ satisfies the ODE

$$(I + A) \frac{dx}{dt} = (A^2 - I)x$$

for the **diagonalizable** matrix $A = \begin{pmatrix} 0.9 & 0.0 & 0.3 \\ 0.0 & 0.8 & 0.4 \\ 0.1 & 0.2 & 0.3 \end{pmatrix}$. If we square this, we

get $A^2 = \begin{pmatrix} 0.84 & 0.06 & 0.36 \\ 0.04 & 0.72 & 0.44 \\ 0.12 & 0.22 & 0.2 \end{pmatrix}$.

- (a) If A has an eigenvalue λ and an eigenvector v , give a **nonzero solution** $x(t)$ satisfying the ODE above, in terms of λ , v , and t .
- (b) Both A and A^2 are _____ matrices. By inspection of A^2 , what can you say (with no arithmetic! don't calculate λ !) **about the magnitudes** $|\lambda|$ of the three eigenvalues of A^2 ? What does this tell you about the magnitudes $|\lambda|$ of the eigenvalues of A ?
- (c) Give the **eigenvalue** λ of A with the biggest magnitude. A corresponding **eigenvector** is $\begin{pmatrix} \alpha \\ 2 \\ 1 \end{pmatrix}$ for what α ?
- (d) For an initial conditions $x(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, circle what would you expect the solutions $x(t)$ to do for large t : **Oscillate** / **decay** / **diverge** / **go to a nonzero constant vector**? Give a good **approximation for** $x(t)$ for a large t — if you can't figure it out exactly, at least give a vector that $x(t)$ is nearly parallel to.

(blank page for your work if you need it)