

Problem Set 6 — Linear Algebra A (Fall 2021)

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Please hand in your assignment at the beginning of your seventh tutorial session!

1. The points $P = (x, x, x)$ and $Q = (y, 3y, -1)$ are on two lines in space that don't meet. Choose x and y to minimize the squared distance

$$\|P - Q\|^2.$$

2. Three planes Π_1, Π_2, Π_3 in the space \mathbb{R}^3 are given by the equations

$$\Pi_1 : x + y + z = 0,$$

$$\Pi_2 : 2x - y + 4z = 0,$$

$$\Pi_3 : -x + 2y - z = 0.$$

Determine a matrix representative (in the standard basis of \mathbb{R}^3) of a linear transformation taking the xy plane to Π_1 , the yz plane to Π_2 and the zx plane to Π_3 .

3. Suppose P_1 and P_2 are projection matrices ($P_i^2 = P_i = P_i^T$). Prove: $P_1 P_2$ is a projection if and only if $P_1 P_2 = P_2 P_1$.
4. The space M of 2 by 2 matrices has the basis

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Suppose T multiplies each matrix by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find the matrix representing this linear transformation T with respect to the above mentioned basis.

5. 给定 \mathbb{R}^3 的两组基

$$u_1 = (1, 0, 1), u_2 = (2, 1, 0), u_3 = (1, 1, 1)$$

和

$$v_1 = (1, 2, -1), v_2 = (2, 2, -1), v_3 = (2, -1, -1).$$

定义线性变换: $T(u_i) = v_i, i = 1, 2, 3$.

- (a) 写出由基 u_1, u_2, u_3 到 v_1, v_2, v_3 的过渡矩阵;
- (b) 写出线性变换 T 在基 u_1, u_2, u_3 下的矩阵;
- (c) 写出线性变换 T 在基 v_1, v_2, v_3 下的矩阵.