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Midterm Copy 2 Fall 2022 Oct. 31, 2022

Suggested solutions Dr. Y. Chen Page 1.

Question 1: (1) A (2) D (3) A (4) A (5) C.

Question 2: (1) $\begin{bmatrix} 1 & -\frac{x}{2} & \frac{x^2-2y}{8} \\ 0 & \frac{1}{2} & -\frac{x}{8} \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ (2) 2. (3) $\begin{bmatrix} \frac{2}{15} \\ \frac{7}{15} \end{bmatrix}$

(4) -1 (5) $\begin{bmatrix} 0 & -A^2 \\ I_n & 0 \end{bmatrix}$

Question 3:

$$A = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & -24 \end{bmatrix}}_{U}$$

Question 4: (a) A basis for the column space is: $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(b) No. $C(A) \perp N(A^T)$.

(c) $C(A) = C(B)$ $B = \begin{bmatrix} -1 & -2 & 0 & 1 \end{bmatrix}$.

Question 5: (a) $f(A+B) = f(A) + f(B)$, for all $A, B \in \mathbb{R}^{2\times 2}$.

$f(\lambda A) = \lambda f(A)$, for all $A \in \mathbb{R}^{2\times 2}$, $\lambda \in \mathbb{R}$.

(b) Verify that $\ker(f)$ is closed under addition and scalar multiplication.

$A \in \ker(f) \Leftrightarrow A$ is symmetric.

A basis of $\ker(f)$ is: $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

(c) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 \\ -1 & 0 & -1 & 1 \end{bmatrix}$ (d) $\lambda = 2$, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Question 6: Case 1: Xiaomeng can get to SUSTech by hot air ballon.

Case 2: Xiaomeng can't get to SUSTech.

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Question 7.

(a) $R = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$, $T = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$S = RT$

(b) Yes, for example

$T' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $R' = \begin{bmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}$.

Question 8:

(a) $$\underbrace{\begin{bmatrix} I_n & -AB \\ 0 & I_n \end{bmatrix}}_{C} M = \begin{bmatrix} I_n & -AB \\ 0 & I_n \end{bmatrix} \begin{bmatrix} A & B \\ B^{-1} & A^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} A-A & B-ABA^{-1} \\ B^{-1} & A^{-1} \end{bmatrix}$$

$$\overset{AB=BA}{=} \begin{bmatrix} 0 & B-BAA^{-1} \\ B^{-1} & A^{-1} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ B^{-1} & A^{-1} \end{bmatrix}}_{D}$$

$CM = D$, C is invertible

D is NOT invertible

$\Rightarrow$ M is NOT invertible.

(b) $\text{rank}(CM) = \text{rank}(D) \leq \text{rank}\, M$

$\text{rank}(M) = \text{rank}(C^{-1}CM) \leq \text{rank}(CM) = \text{rank}\, D$

$\Rightarrow \text{rank}\, M = \text{rank}\, D$.