



南方科技大学  
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目: 线 性 代 数

开课单位: 数 学 系

考试时长: 50 分钟

命题教师: Dr. Chen

题 号	1	2	3	4
分 值	15 分	15 分	10 分	10 分

本试卷共 (4) 大题, 满分 (50) 分. (考试结束后请将试卷、答题本、草稿纸一起交给监考老师)

This 50-minute long quiz includes 4 questions. Write **all your answers** on the examination book.

1. Let  $A$  be an  $n \times n$  real symmetric matrix, and  $A^2 = A$ ,  $\text{rank}(A) = r$ .

(a) Show that there exists an orthogonal matrix  $Q$ , such that  $Q^T A Q = \text{diag}(1, 1, \dots, 1, 0, \dots, 0)$ , and the number of 1's on the diagonal is  $r$ .

(b) Find  $|A - 2I|$ .

2. Let

$$A = \begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix}, i = \sqrt{-1}.$$

(a) Is  $A$  Hermitian?

(b) Find all the eigenvalues and eigenvectors of  $A$ .

3. (a) Please give the definition of positive definiteness of a matrix.

(b) Decide whether the following quadratic form is positive definite, positive semidefinite or indefinite.

$$f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + 5x_3^2 + 2x_1x_2 - 4x_1x_3.$$

4. There are six  $3 \times 3$  permutation matrices. Which are similar to each other?