MIT 18.06 Exam 2, Fall 2017
Johnson

Your name: ______________________________

Recitation: ______________________________

problem	**score**
1	/40
2	/30
3	/30
total	/100

Problem 1 (40 points):

The complete solution to $Ax = b$ is $x = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ for all possible scalars α_1 and α_2.

(a) A is an $m \times n$ matrix of rank r. Describe all possible values of m, n, and r.

(b) If $b = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, give a possible matrix A. (Look carefuly at x: can you identify likely free and pivot columns of A from how we usually construct the particular and special solutions?)

(c) Look carefully at x, and write down the matrix P that performs orthogonal projection onto $N(A)$. (Not much calculation should be needed!)

(blank page for your work if you need it)

Problem 2 (30 points):

(a) Give a possible 4×3 matrix A with three *different, nonzero* columns such that blindly applying Gram–Schmidt to the columns of A will lead you to **divide by zero** at some point.

(b) The reason Gram–Schmidt didn't work is that your A does not have ____.

(c) To find an orthonormal basis for $C(A)$, you should instead apply Gram–Schmidt to what matrix (for your A)?

(blank page for your work if you need it)

Problem 3 (30 points):

Given two $m \times n$ matrices A and B, and two right-hand sides $b, c \in \mathbb{R}^m$, suppose that we want to minimize

$$f(x) = \|b - Ax\|^2 + \|c - Bx\|^2$$

over all $x \in \mathbb{R}^n$, i.e. we want to minimize the *sum of two least-squares fitting errors.*

(a) $\|b\|^2 + \|c\|^2$ can be written as the length squared $\|w\|^2$ of a single vector w. What is w?

(b) Write down a matrix equation $C\hat{x} = d$ whose solution $\hat{x}$ is the minimum of $f(x)$. (Give explicit formulas for C and d in terms of A, B, b, c.) Hint: your answer from the previous part should give you an idea to convert this into a "normal" least-squares problem.

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