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EMBRACING UNCERTAINTY: A PRIMER

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Introduction

WILLIAM Gibson's masterful short story *The Gernsback Continuum*[1] follows a man haunted by an idea. His protagonist is pursued by *semiotic ghosts*, lingering packets of meaning, there in the form of ray-gun spacemen and ten-engine airships sprung from the mind of legendary science fiction author Hugo Gernsback. This character's dilemma is Gibson's reaction to the starry-eyed view of science fiction in the 1970's – a view stuck in the chrome-plated, streamlined future of the 1930's which could never be.

[1] William Gibson. *Burning Chrome*. Orion Publishing Group, 1986

We as engineers in the early 2000's are haunted by a different idea, that of an engineering certainty which never existed. Our tools are predicated on a deterministic, omniscient view of the universe which is woefully at odds with the reality of our occupation. Our traditional approach is to make uncertainty irrelevant – to pile on margin and arbitrary safety factors until we creep away from failure. This safety comes at the expense of inefficient, expensive designs, and a lack of understanding of the relevant uncertainties, precluding improvement. Underlying this deterministic framework is a sentiment of revulsion, perhaps even fear.

This book is an attempt at exorcism. Rather than making it irrelevant, or becoming paralyzed by anxiety, we aim to *Embrace Uncertainty*.

Purpose

This book is foremost a *primer*. We shall not attempt to cover an exhaustive treatment of all relevant material, but rather shall seek to understand important concepts, and to grasp some useful tools.

The topic is uncertainty, interpreted broadly. Humans, as a species, are generally quite bad at managing uncertainty. At best it tends to make us nervous: Think of the anxiety that strikes when moving to a new city, or asking a stranger on a date. If the outcomes were certain – if finding new friends were guaranteed or our desired would accept – there would be no anxiety. At worst, we tend to ignore uncertainty: Think of politicians who make self-assured[2] assertions.

[2] Often self-contradictory...

Anxiety prevents us from taking action. Ignorance prevents us from learning. The foremost objective of this primer is to catalyze a *change in perspective*; to convince you, dear reader, that confronting uncertainty is useful, possible, and necessary.

Furthermore, uncertainty is *exciting*, and helps contribute to a rich, well-lived life. Think of how boring a drama would be if all the plot-points were spelled out before the first establishing shot – amateur critics of films often lament when a movie is predictable. *Reframing* the examples from before, socializing is exciting *because* it is uncertain. As Richard Powers is fond of saying, we should welcome chance encounters. Of course, chance encounters in engineering design are quite unlike those in social dance – a misstep in engineering can cost lives.

With the above waxing philosophic complete, we can fully understand our mission of *embracing* uncertainty as akin to living in the ideal[3] relationship. Just as a successful relationship must be built on mutual respect and continual growth, we as engineers must embrace the uncertainty endemic to our chosen profession, respect and appreciate the unknowns that face us, and continually grow in our understanding. Of course, we ought not take this metaphor too far...

[3] non-Hollywood

This book is intended for students of engineering, either early graduate or advanced undergraduate. The content will make modest demands on your background; we will assume fluency with differential and integral calculus. Regretfully, many engineering curricula shortchange probability and (especially) statistics. Both are powerful tools for handling uncertainty, and we will make thorough use of these disciplines. Two appendices are provided to cover the bare essentials of probability and statistics.

How to use this book

Acknowledgements

A great many people have influenced my thoughts on uncertainty, and their influence is reflected in this book. Mike Baiocchi first introduced me to the framing of *embracing* uncertainty, within the context of the reproducibility crisis. It is a testament to the power of language that a simple little phrase can spur a fundamental change in one's philosophy, and I have Mike to thank for this particular thought-germ; one that is still mutating my understanding of the subject.

A Demonstration

(Here we give a tour of UQ through an example; a la Saltelli's Primer.)

Formulate

Data

Data are our quantitative anchor to reality.[4] They allow us to make numerical statements about the physical world, and thus are invaluable to solving problems. However, data are not infallible – one must have both skepticism and appreciation of variability in order to use data effectively. We will consider two kinds of data in this primer:

[4] One could make a case for data in the plural (data are facts) or in the singular (data is information). We will use the former.

Physical Data are comprised of measurements of the physical world. The vast majority of physical measurements involve the comparison of some physical quantity of interest against a defined standard; this is most obviously seen in simple measurements of length, where one can compare a meter stick against a physical object. More complicated measurements involve some quantity which cannot be directly measured, but can instead be obtained through a transformation based on a physical model. An example: It would be challenging to measure pressure directly. However, we can build a barometer out of a sealed body with fluid, which essentially converts pressure changes into length changes. Carrying out the transformation from measured lengths to pressures requires an appeal to hydrostatics, which carries with it some particular assumptions.

Variation in physical data arises from *unknown variables*, what we sometimes call *lurking variables*.[5] In the barometer example above, suppose that our instrument experienced both temperature and pressure changes. If our barometer used a fluid whose density changed with the pressure, *and if we did not account for the temperature changes*, then even at a constant external pressure, we might measure fluctuating values. In this case, we would call temperature a lurking variable. Detecting and controlling lurking variables is challenging, but necessary to improve physical measurements.[6]

[5] George E. P. Box. Use and abuse of regression. *Technometrics*, 8(4):625–629, 1966

[6] Brian L. Joiner. Lurking variables: Some examples. *The American Statistician*, 35(4):227–233, 1981; and Z. del Rosario, M. Lee, and G. Iaccarino. Lurking Variable Detection via Dimensional Analysis. *ArXiv e-prints*, November 2017

Luckily, the variations in physical data tend to exhibit 'nice' properties, and so often lend themselves to statistical characterization. For this reason, the tools of probability and statistics are very well-suited to tackling physical data.

Simulation Data are the result of models. They are connected to reality only insofar as their generating model is connected to reality. However, simulation data has a huge advantage over physical data; one can use simulation data to make quantitative statements about reality *without physical testing*. This can be useful for overcoming constraints of cost (as with aircraft testing) or legality (as with nuclear weapons). Perhaps one of the greatest triumphs of simulation is human spaceflight. Using little more than Newton's laws, early pioneers of spaceflight managed to compute trajectories that sent astronauts to the moon and back – physical data alone could not have stood up this effort.

Of course, we should not oversell the value of simulation. The Apollo program was built on top of the experience and physical data gathered from Project Gemini, and before that Mercury. Models themselves are built from studying physical data, and always carry some assumptions – assumptions which may be disconnected from reality. While physical data are subject to variability, models are often subject to *discrepancy*, often modeled as a difference between a model prediction and the 'true' value that would arise from a perfect measurement in physical reality.[7]

Despite the fundamental difference between variability and discrepancy, discrepancy is often modeled using probability and statistics as well.[8] As I am writing this sentence, there is an open debate about the suitability of this approach. Rather than wade into this argument, we will accept the use of probability for discrepancy as normative, and proceed to treat both physical and simulation data with the same toolkit.

For what follows, we will consider a set of data $X_i \in \mathbb{R}$ for $i = 1, \ldots, n$. These data are measurements of some true underlying value $X_i = Y + \epsilon_i$, where the ϵ_i are errors arising either from lurking variables or discrepancies.

[7] Dave Higdon, Marc Kennedy, James C Cavendish, John A Cafeo, and Robert D Ryne. Combining field data and computer simulations for calibration and prediction. *SIAM Journal on Scientific Computing*, 26(2):448–466, 2004

[8] Marc C. Kennedy and Anthony O'Hagan. Bayesian calibration of computer models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 63(3):425–464, 2001; and Dave Higdon, Marc Kennedy, James C Cavendish, John A Cafeo, and Robert D Ryne. Combining field data and computer simulations for calibration and prediction. *SIAM Journal on Scientific Computing*, 26(2):448–466, 2004

Summaries

Central tendency is a key concept with a rather descriptive name. A measure of central tendency is a single number which describes the 'location' or 'typical value' of data. There are two important measures of central tendency: the *mean* and the *median*.

The *mean* (average) is defined via

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i. \tag{1}$$

In terms of the true value, the mean is given by $\overline{X} = Y + \frac{1}{n} \sum_{i=1}^{n} \epsilon_i$. While the mean has a number of useful statistical properties, it has an intuitive justification: In the case where the errors ϵ_i are 'evenly' distributed about zero, they will cancel to yield the true value Y.

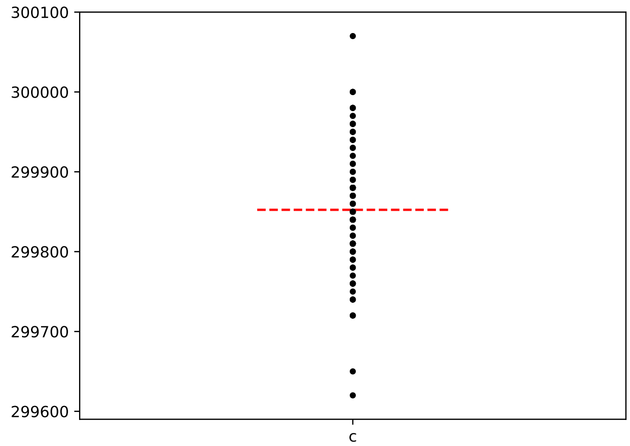

Figure 1: Albert Michelson's 1880 speed-of-light measurements. The mean (pictured as a red line) is appropriate as a nominal value.

While the mean is a sensible measure of central tendency, it does have some weaknesses. We will illustrate these by contrasting against another measure of central tendency.

The *median*

Variability

Skepticism

Distributions

Models

Questions

Bibliography

[1] George E. P. Box. Use and abuse of regression. *Technometrics*, 8(4):625–629, 1966.

[2] Z. del Rosario, M. Lee, and G. Iaccarino. Lurking Variable Detection via Dimensional Analysis. *ArXiv e-prints*, November 2017.

[3] William Gibson. *Burning Chrome*. Orion Publishing Group, 1986.

[4] Dave Higdon, Marc Kennedy, James C Cavendish, John A Cafeo, and Robert D Ryne. Combining field data and computer simulations for calibration and prediction. *SIAM Journal on Scientific Computing*, 26(2):448–466, 2004.

[5] Brian L. Joiner. Lurking variables: Some examples. *The American Statistician*, 35(4):227–233, 1981.

[6] Marc C. Kennedy and Anthony O'Hagan. Bayesian calibration of computer models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 63(3):425–464, 2001.