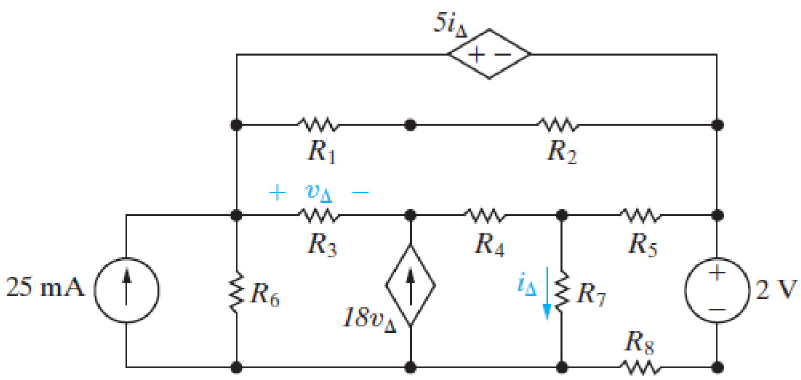
4.1 For the circuit shown in Fig. P4.1, state the numerical value of the number of (a) branches, (b) branches where the current is unknown, (c) essential branches, (d) essential branches where the current is unknown, (e) nodes, (f) essential nodes, and (g) meshes.

Figure P4.1



4.1

[a] 12 branches, 8 branches with resistors, 2 branches with independent sources, 2 branches with dependent sources.

[b] The current is unknown in every branch except the one containing the 25 mA current source, so the current is unknown in 11 branches.

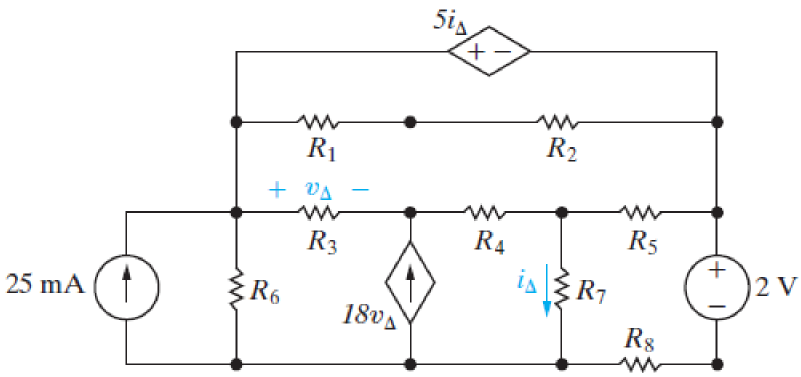
[c] 10 essential branches – $R_1 - R_2$ forms an essential branch as does R_8 - 2 V. The remaining eight branches are essential branches that contain a single element.

[d] The current is unknown only in the essential branch containing the current source, and is unknown in the remaining 9 essential branches.

[e] From the figure there are 7 nodes – three identified by rectangular boxes two identified by triangles, and two identified by diamonds.

[f] There are 5 essential nodes, three identified with rectangual boxes and two identified with triangles.

[g] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.



4.2 a) If only the essential nodes and branches are identified in the circuit in Fig. P4.1, how many simultaneous equations are needed to describe the circuit?

b) How many of these equations can be derived using Kirchhoff's current law?

c) How many must be derived using Kirchhoff's voltage law?

d) What two meshes should be avoided in applying the voltage law?

4.2

[a] 9 essential branches where the current is unknown, so we need 9 simultaneous equations to describe the circuit.

[b] 5 essential nodes, so we can apply KCL at (5-1) = 4 of these essential nodes. There would be also be a dependent source constraint equation.

[c] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

[d] We must avoid using the bottom left-most mesh, since it contains a current source, and we have no way of determining the voltage drop accross a current source. The two meshes on the bottom that share the dependent source must be handled in a special way.

4.3 Assume the voltage v_s in the circuit in Fig. P4.3 is known. The resistors $R_1 - R_7$ are also known.

a) How many unknown currents are there?

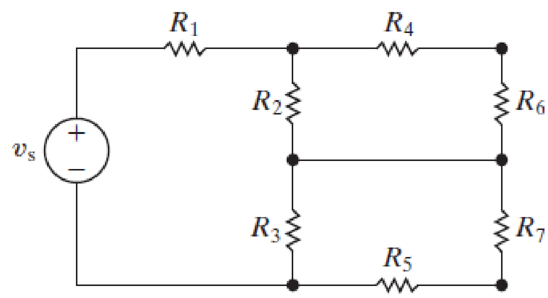
b) How many independent equations can be written using Kirchhoff's current law (KCL)?

c) Write an independent set of KCL equations.

d) How many independent equations can be derived from Kirchhoff's voltage law (KVL)?

e) Write a set of independent KVL equations.

Figure P4.3



4.3

[a] There are 8 circuit components, seven resistors and the voltage source. Because of this, there are 8 unknown currents. Nevertheless, the voltage source and the R_1 resistor are in series, so have the same current. The R_4 and R_6 resistors are also in series, so have the same current. So, we only need 5 equations to find the 5 distinct currents in this circuit.

[b] There are three essential nodes in this circuit, identified by the boxes. At two of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the third node would be dependent on the first two. So, there are two independent KCL equations.

[c] Sum the currents at any two of the three essential nodes a, b, and c. Using nodes a and c we get:

$$-i_1 + i_2 + i_4 = 0$$

$$i_1 + i_3 + i_5 = 0$$

[d] There are three meshes in this circuit: one on the left with the components v_s, R_1, R_2 and R_3; one on the top right with components R_2, R_4 and R_6; and one on the bottom right with components R_3, R_5 and R_7. We can write KVL equations for all three meshes, giving a total of three independent KVL equations.

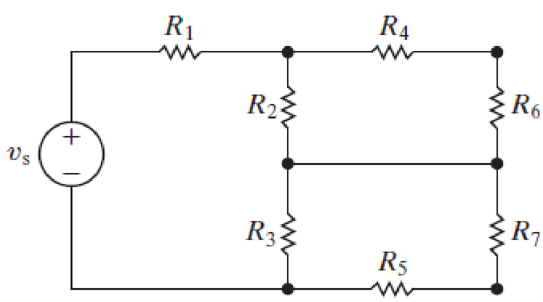
[e]
$$-v_s + R_1 i_1 + R_2 i_2 + R_3 i_3 = 0$$
$$R_4 i_4 + R_6 i_4 + R_2 i_2 = 0$$
$$R_3 i_3 + R_5 i_5 + R_7 i_5 = 0$$

4.4 A current leaving a node is defined as positive.

a) Sum the currents at each node in the circuit shown in Fig. P4.3.

b) Show that any one of the equations in (a) can be derived from the remaining three equations.

Figure P4.3



4.4

[a] At node a: $i_1 + i_2 + i_4 = 0$

At node b: $i_2 + i_3 + i_4 - i_5 = 0$

At node c: $i_1 + i_3 + i_5 = 0$

[b] There are many possible solutions. For example, adding the equations at nodes a and c gives the equation at node b:

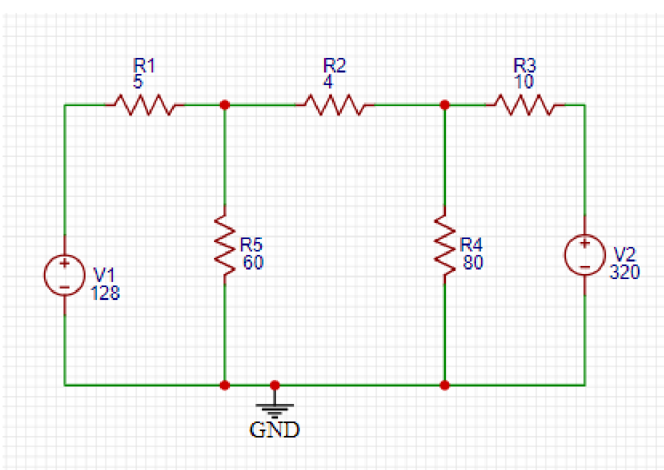
$(i_1 + i_2 + i_4) + (i_1 + i_3 + i_5) = 0$ so $i_2 - i_3 + i_4 + i_5 = 0$

This is the question at node b with both sides multiplied by -1.

4.11

a) Use the node-voltage method to find the branch currents $i_a - i_e$ in the circuit shown in Fig. P4.11.

b) Find the total power developed in the circuit.



4.11

[a] $\frac{V_1-128}{5} + \frac{V_1}{60} + \frac{V_1-V_2}{2} = 0$ $\Big\{$ $V_1\left(\frac{1}{5}+\frac{1}{60}+\frac{1}{4}\right) - \frac{V_2}{4} = \frac{128}{5}$

$\frac{V_2-V_1}{4} + \frac{V_2}{80} + \frac{V_2-320}{10} = 0$ $\Big|$ $-\frac{V_1}{4} + V_2\left(\frac{1}{4}+\frac{1}{80}+\frac{1}{10}\right) = 32$

$\underline{V_1 = 162V}$ $\underline{V_2 = 200V}$

$i_a = \frac{128-162}{5} = -6.8A$

$i_b = \frac{162}{60} = 2.7A$

$i_c = \frac{162-200}{4} = -9.5A$

$i_d = \frac{200}{80} = 2.5A$

$i_e = \frac{200-320}{10} = \boxed{-12A}$

[b]

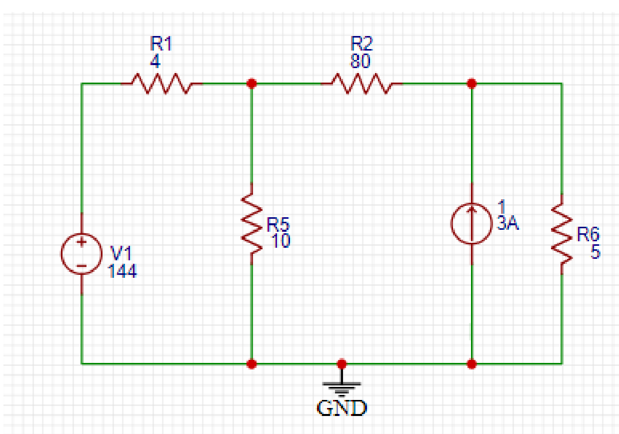
$P_{128V} = -128 \cdot -6.8 = 870.4W$

$P_{320} = 320 \cdot (-12) = -3840W$

$\boxed{\text{total Power} = 3840W}$

4.12

Use the node-voltage method to find v_1 and v_2 in the circuit in Fig. P4.12.



4.12

$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \Rightarrow 29v_1 - v_2 = 2880$$

$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \Rightarrow -v_1 + 17v_2 = 240$$

$$v_1 = 100\,V$$

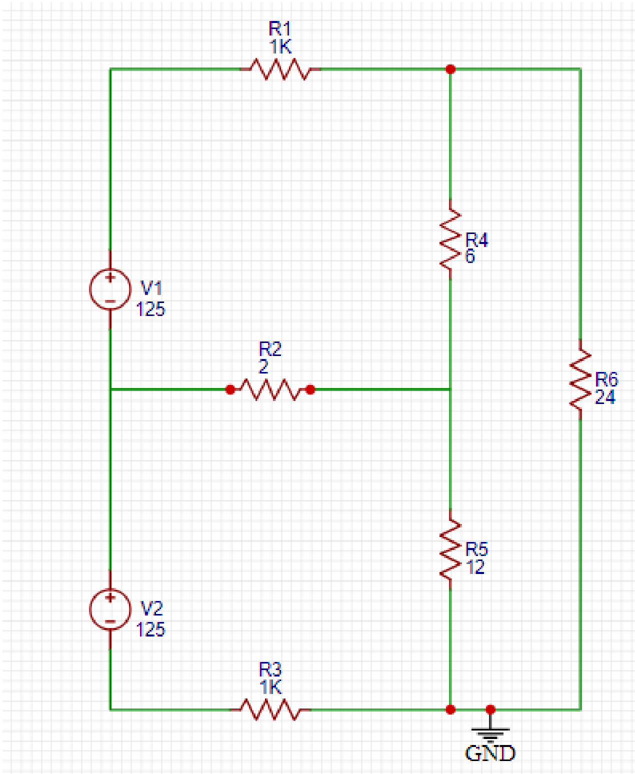
$$v_2 = 20\,V$$

4.15

The circuit shown in Fig. P4.15 is a dc model of a residential power distribution circuit.

a) Use the node-voltage method to find the branch currents $i_1 - i_6$.

b) Test your solution for the branch currents by showing that the total power dissipated equals the total power developed.



4.15

[a]

$$v_1 - 125 + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} = 0$$

$$\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{24} = 0$$

$$\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} = 0$$

$$\Rightarrow \boxed{\begin{aligned} v_1 &= 104.24\,V \\ v_2 &= 10.66\,V \\ v_3 &= -106.57\,V \end{aligned}}$$

$i_1 = 125 - v_1 = 23.76\,A$ $\quad i_2 = v_2/2 = 5.33\,A$

$i_3 = v_3 + 125 = 18.43\,A$ $\quad i_4 = (v_1 - v_2)/6 = 15.10\,A$

$i_5 = (v_2 - v_3)/12 = 9.77\,A$ $\quad i_6 = (v_1 - v_3)/24 = 8.66\,A$

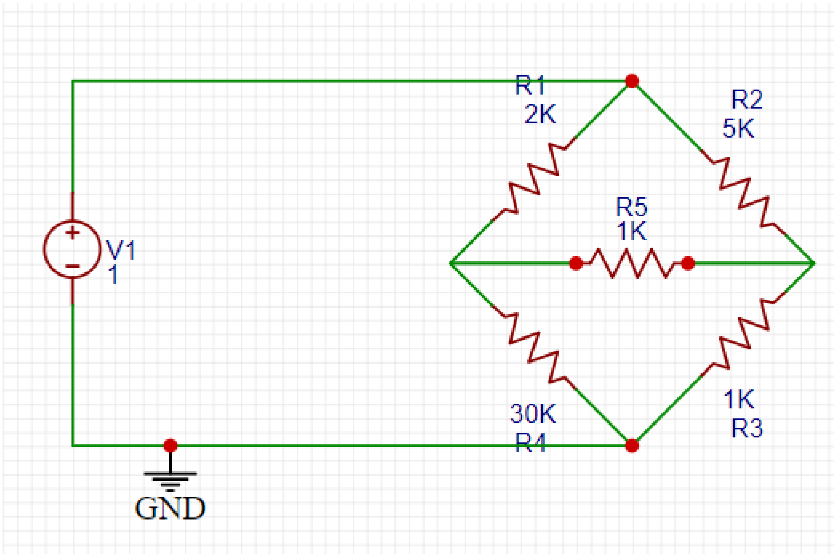
[b]

$$\sum P_{dev} = 125 i_1 + 125 i_3 = 5273.09\,W$$

$$\sum P_{dis} = i_1^2(1) + i_2^2(2) + i_3^2 1 + i_4^2 6 + i_5^2 12 + i_6^2 24 = 5273.09\,W$$

4.24

Use the node-voltage method to find i_o in the circuit in Fig. P4.24.



4.24.

$$\frac{V1}{30{,}000} + \frac{V1 - V2}{5000} + \frac{V1 - 20}{2000} = 0$$

So $22V1 - 6V2 = 300$

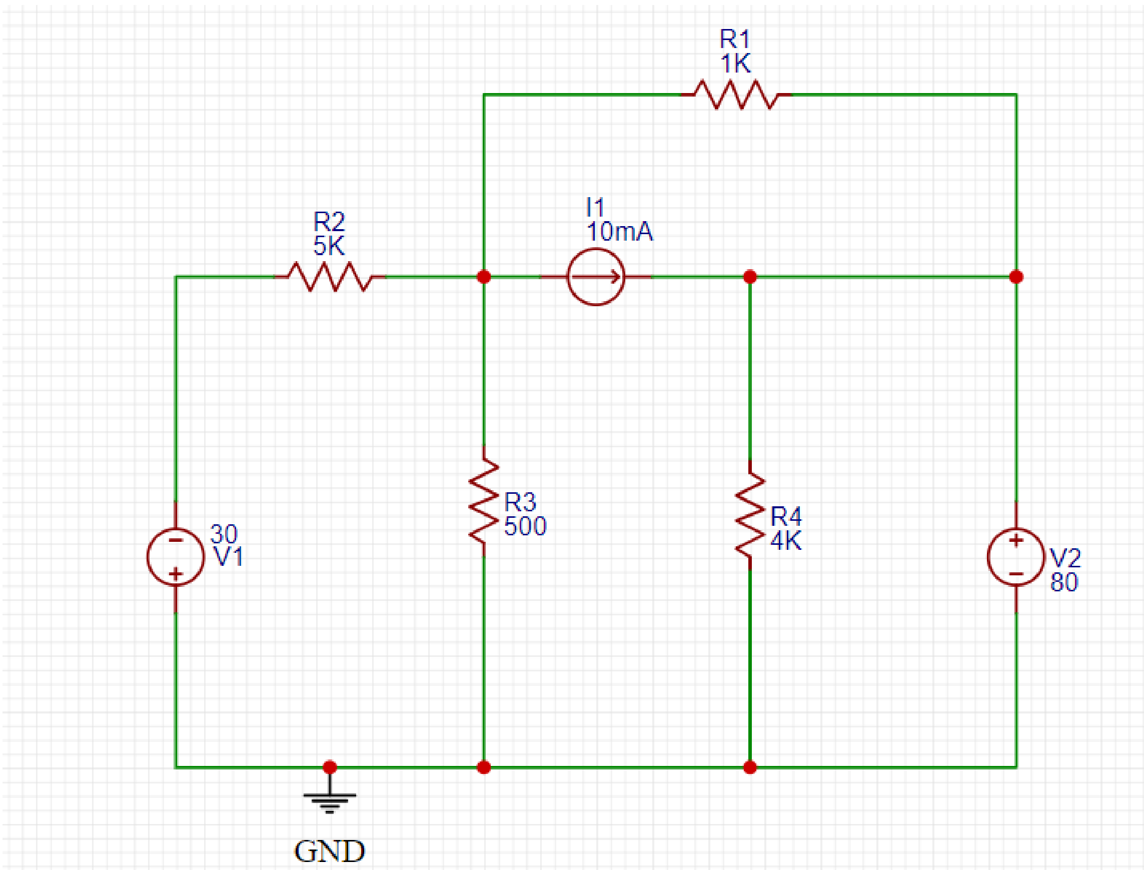
$$\frac{V2}{1000} + \frac{V2 - V1}{5000} + \frac{V2 - 20}{5000} = 0$$

So $-V1 + 7V2 = 20$

4.27

a) Use the node-voltage method to find the branch currents i_1, i_2, and i_3 in the circuit in Fig. P4.27.

b) Check your solution for i_1, i_2, and i_3 by showing that the power dissipated in the circuit equals the power developed.



4.27 There is only one node voltage equation:

$$\frac{Va+30}{5000}+\frac{Va}{500}+\frac{Va-80}{1000}+0.01=0$$

$Va+30+10va+5va-400+50=0$ so $16va=320$

$Va=20\,V$

Calculate the currents:

$i_1=(-30-20)/5000=-10\,mA$

$i_2=20/500=40\,mA$

$i_4=80/4000=20\,mA$

$i_5=(80-20)/1000=60\,mA$

$i_3+i_4+i_5-10\,mA=0$ so $i_3=0.01-0.02-0.06=-0.07=-70m$

$\boxed{b}$ $P_{30V}=(30)\cdot(-0.01)=-0.3\,W$

$P_{10mA}=(20-80)(0.01)=-0.6\,W$

$P_{80V}=(80).(-0.07)=-5.6\,W$

$P_{5k}=(-0.01)^2(5000)=0.5\,W$

$P_{500\Omega}=(0.04)^2(500)=0.8\,W$

$P_{1k}=(80-20)^2/(1000)=3.6\,W$

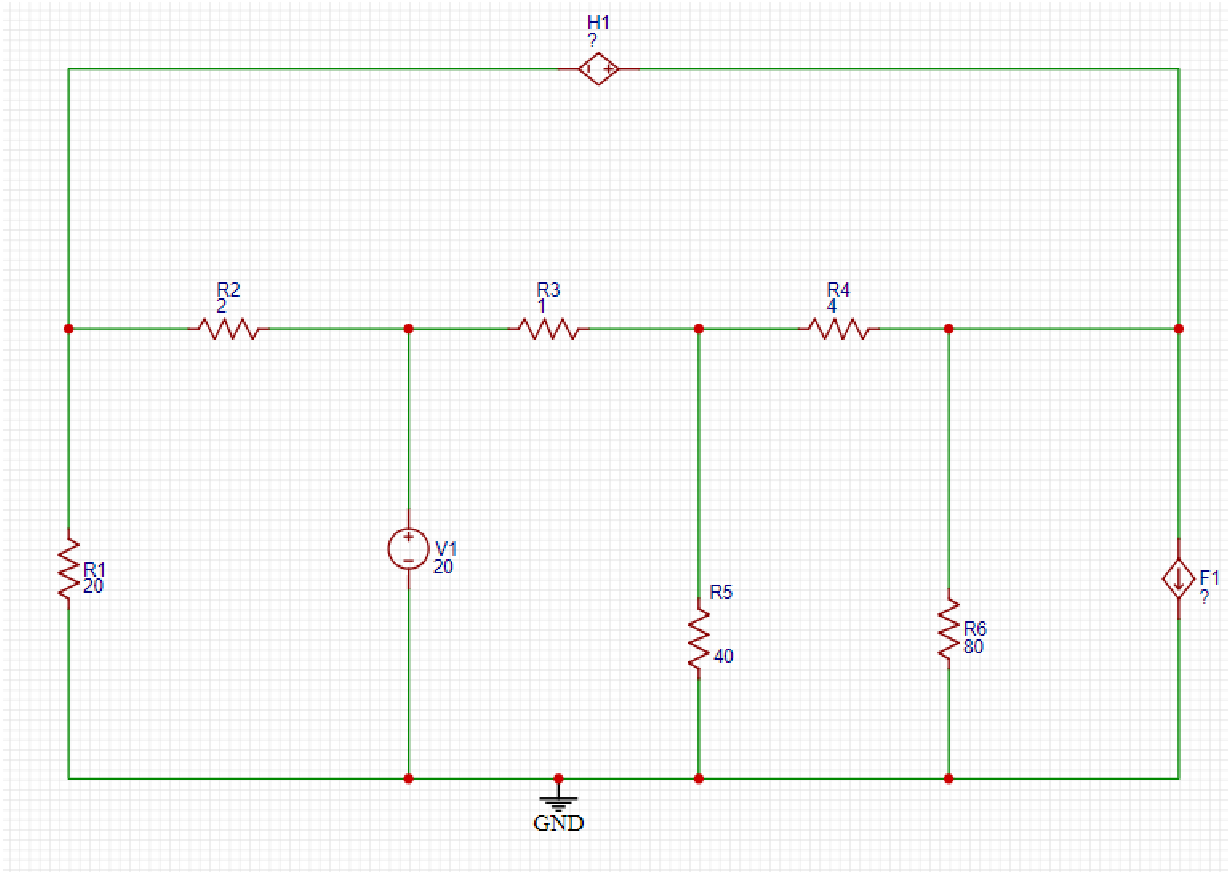
$P_{4k}=(80)^2/(4000)=1.6\,W$

$\sum P_{abs}=0.5+0.8+3.6+1.6=6.5\,W$

$\sum P_{del}=0.3+0.6+5.6=6.5\,W$

4.30

Use the node-voltage method to find the power developed by the 20 V source in the circuit in Fig. P4.30.



H1
?
R2
2
R3
1
R4
4
R1
20
V1
20
R5
40
R6
80
F1
?
GND

4.30 $\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125 v_\Delta = 0$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constrait equations:

$v_\Delta = 20 - v_2$

$v_1 = 35 i_\phi = v_3$

$i_\phi = v_2/40$

$v_1 = -20.25V$; $v_2 = 10V$; $v_3 = -29V$

Let i_g be the current delivered by the 20 V source, then,

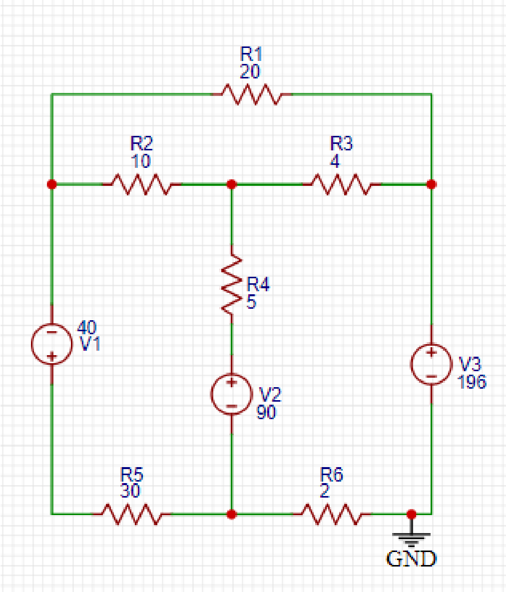
$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125 A$$

$P_g = 20(30.125) = 602.5 W$

4.36 a) Use the mesh-current method to find the total power developed in the circuit in Fig. P4.36.

PSPICE
MULTISIM

b) Check your answer by showing that the total power developed equals the total power dissipated.



4.36 $40+10(i_1+i_2)+5(i_1+i_3)-90+30i_1=0$

$20i_2+4(i_2-i_3)+10(i_2-i_1)=0$

$196+2i_3-90+5(i_3-i_1)+4(i_3-i_2)=0$

$i_1=-5A$; $i_2=-3A$; $i_3=-13A$

$P_{40}= 40i_1=-200\ W$ (del)

$P_{90}= 90(i_1-i_3)=720\ W$ (abs)

$P_{196}=196i_3=-2548\ W$ (del)

$\sum P_{dev}=2748\ W$

[5] $P_{20\Omega} = (-3)^2(20) = 180\,W$

$P_{10\Omega} = (2)^2(10) = 40\,W$

$P_{4\Omega} = (10)^2(4) = 400\,W$

$P_{5\Omega} = (8)^2(5) = 320\,W$

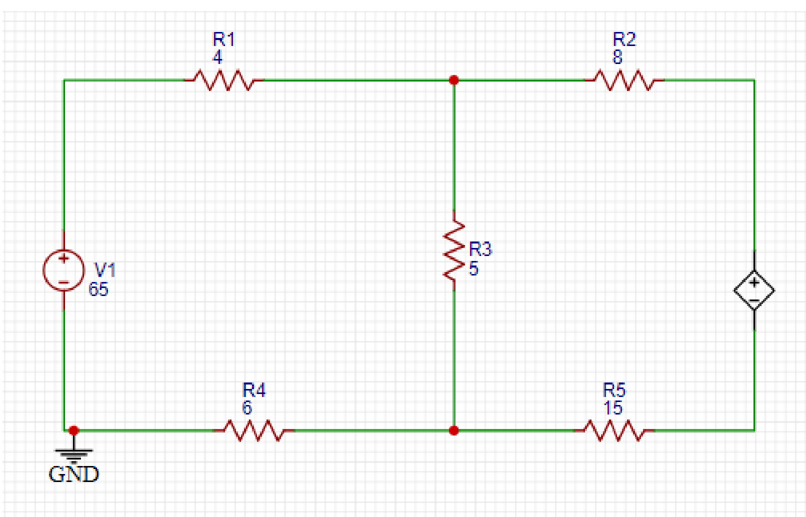
$P_{30\Omega} = (-5)^2(30) = 750\,W$

$P_{2\Omega} = (-13)^2(2) = 338\,W$

$\sum P_{abs} = 770 + 180 + 40 + 400 + 320 + 750 + 338 = 2798\,W$

4.39 Use the mesh-current method to find the power dissipated in the $15\,\Omega$ resistor in the circuit in Fig. P4.39.

PSPICE
MULTISIM



4.39

$-65 + 4i_1 + 5(i_1 - i_2) + 6i_1 = 0$

$8i_2 + 3v_\Delta + 15i_2 + 5(i_2 - i_1) = 0$

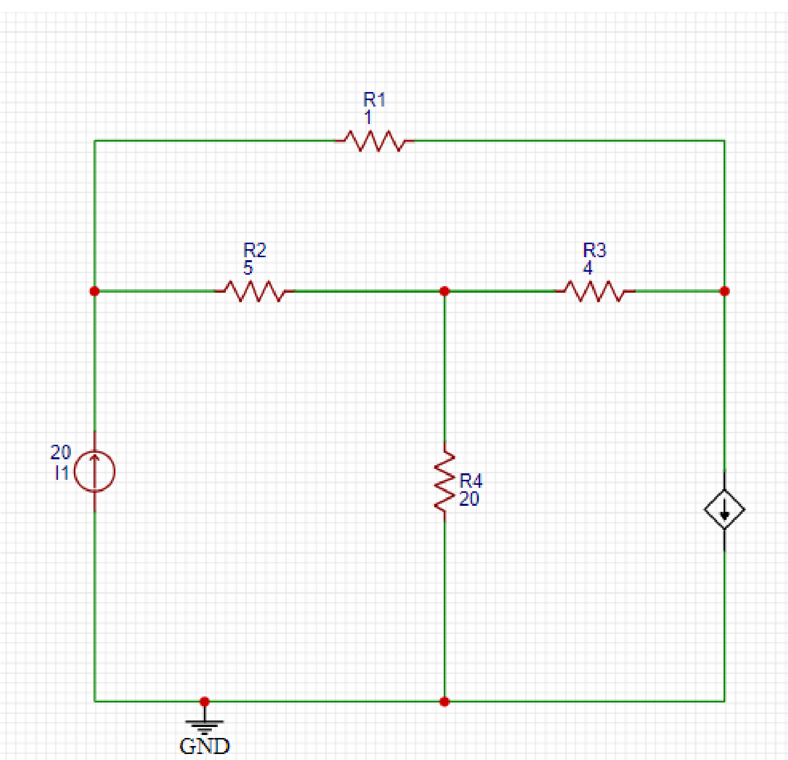
$v_\Delta = 4i_1$

$i_1 = 4\,A$ $\quad i_2 = -1\,A$ $\quad v_\Delta = 16\,V$

$P_{15\Omega} = (-1)^2(15) = 15\,W$

4.46 Use the mesh-current method to find the total power developed in the circuit in Fig. P4.46.

PSPICE
MULTISIM



4.46

$$10i_\Delta - 4i_1 = 0$$

$$-4i_\Delta + 24i_1 + 6.5i_\Delta = 400$$

$i_1 = 15$ A; $i_\Delta = 16$ A

$$v_{20A} = 1i_\Delta + 6.5i_\Delta = 7.5(16) = 120 \text{ V}$$

$$p_{20A} = -20v_{20A} = -(20)(120) = -2400 \text{ W (del)}$$

$$p_{6.5i_\Delta} = 6.5i_\Delta i_1 = (6.5)(16)(15) = 1560 \text{ W (abs)}$$

So, the independent source is developing 2400 W, all other elements are absorbing power, and the total power developed is thus 2400 W.

$$p_{1\Omega} = (16)^2(1) = 256 \text{ W}$$

$$p_{5\Omega} = (20-16)^2(5) = 80 \text{ W}$$

$$p_{4\Omega} = (1)^2(4) = 4 \text{ W}$$

$$p_{20} = (20-15)^2(20) = 500 \text{ W}$$

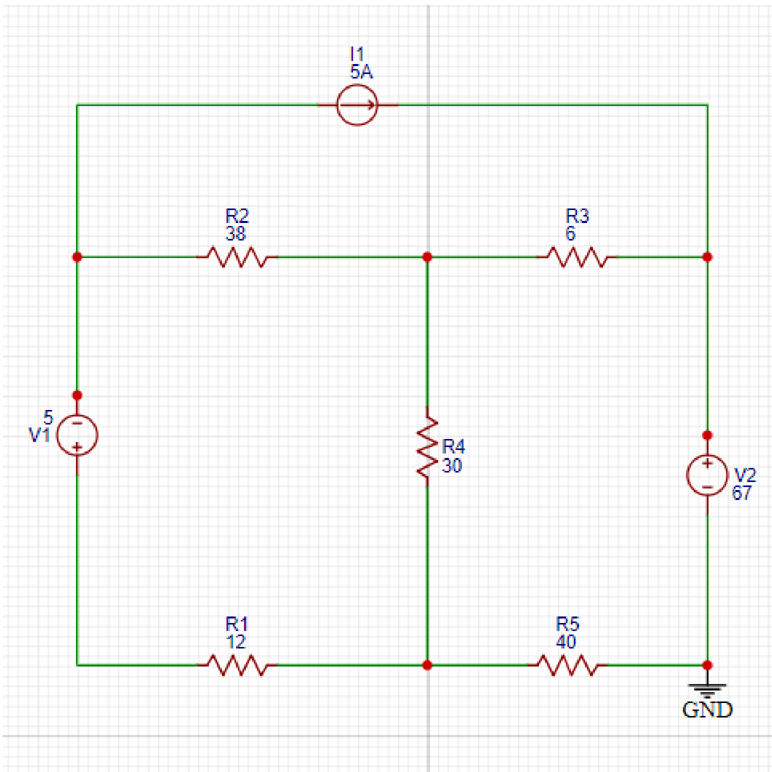
$$\sum P_{abs} = 1560 + 256 + 80 + 4 + 500 = 2400 \text{ W}$$

4.47

a) Use the mesh-current method to find how much power the 5 A current source delivers to the circuit in Fig. P4.47.

b) Find the total power delivered to the circuit.

c) Check your calculations by showing that the total power developed in the circuit equals the total power dissipated



4.47

$5 + 38i_1 - 190 + 30i_1 - 30i_2 + 12i_1 = 0$ $\qquad i_1 = 2.5A$

$67 + 40i_2 + 30i_2 - 30i_1 + 6i_2 - 30 = 0$ $\qquad i_2 = 0.5A$

[a] $V_{5A} = 38 \cdot 2 + 6(-4.5) = -122V$

$P_{5A} = 5 \cdot -122 = \boxed{-610W}$

[b] $P_{5V} = 12.5W$ $\quad P_{67} = 67/2 = \underline{33.5W}$

[c] $\sum P_{resistors} = 564W$

$\sum P_{abs} = 564 + 12.5 + 33.5 = 610W = \sum P_{del}$

4.54

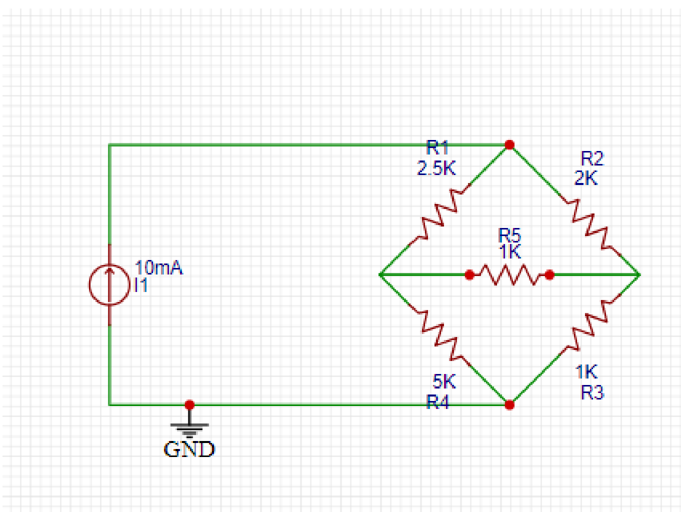
Assume you have been asked to find the power dissipated in the horizontal 1 kΩ resistor in the circuit in Fig. P4.54.

a) Which method of circuit analysis would you recommend? Explain why.

b) Use your recommended method of analysis to find the power dissipated in the horizontal 1 kΩ resistor.

c) Would you change your recommendation if the problem had been to find the power developed by the 10 mA current source? Explain.

d) Find the power delivered by the 10 mA current source.



4.54 [a] There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b] $2500(i_1 - 0.01) + 2000 i_1 + 1000(i_1 - i_2) = 0$

$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000 i_2 = 0$

Place the equations in standard form

$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$

$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$

$i_1 = 6\,mA; \quad i_2 = 8\,mA$

$i_{1k} = i_1 - i_2 = -2\,mA$

$P_{1k} = (-0.002)^2(1000) = 4\,mW$

[c] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equation is less work than solving three node voltage equations.

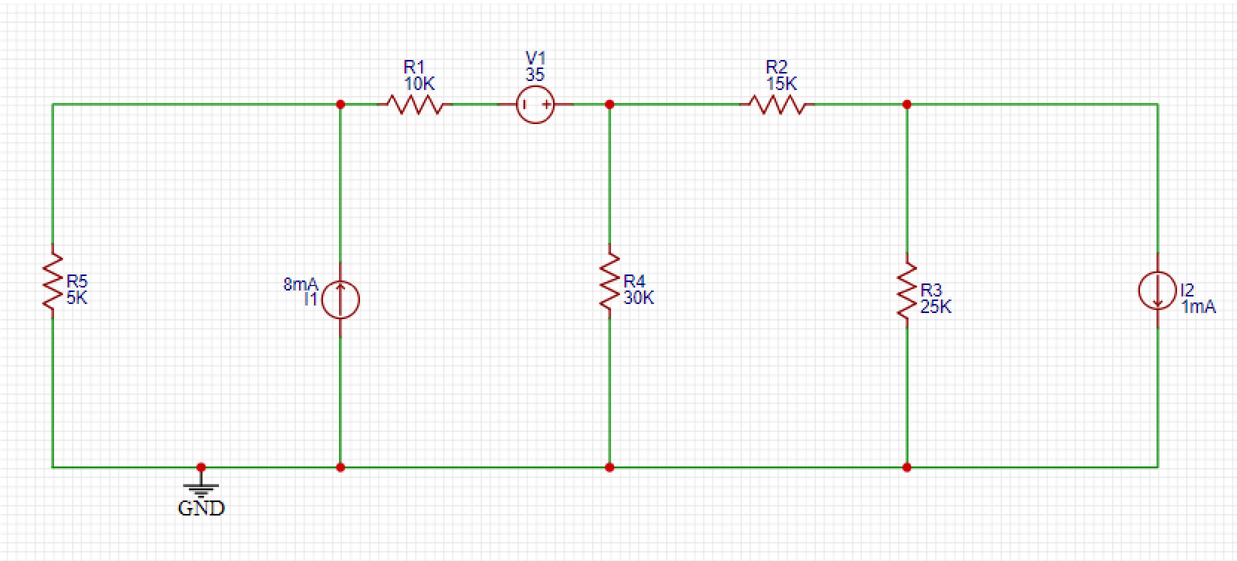
[d] $v_g = 2000 i_1 + 1000 i_2 = 12 + 8 = 20\,V$

$P_{10mA} = -(20)(0.01) = -200\,mW$

Thus the 10mA source develops 200 mW

4.59 a) Make a series of source transformations to find the voltage v_0 in the circuit in Fig. P4.59.

b) Verify your solution using the mesh-current method.



4.59

[a] $V_o = 12500/1000 = 12.5V$

[b] $5000 i_1 + 40000 i_2 - 30000 i_3 = 35$

$i_2 - i_1 = 0.008$

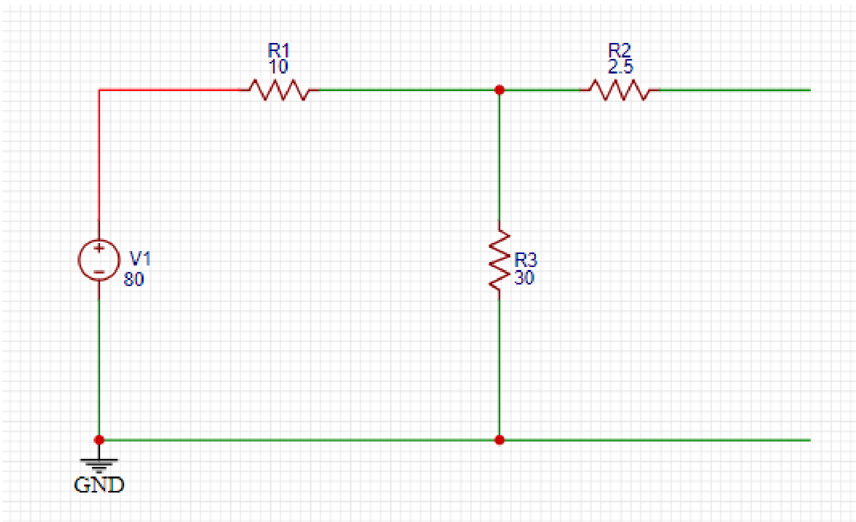
$-30000 i_2 + 70000 i_3 = 25$

$i_1 = -5.33mA$ $\quad i_2 = 2.667mA$ $\quad i_3 = 1.5mA$

$V_o = 25000/5000 = 12.5V$

4.64

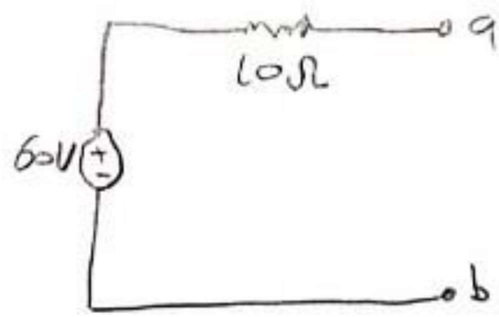
Find the Thévenin equivalent with respect to the terminals a,b for the circuit in Fig. P4.64.



4.64

$$V_{Th} = \frac{30}{40} 80 = \underline{60V}$$

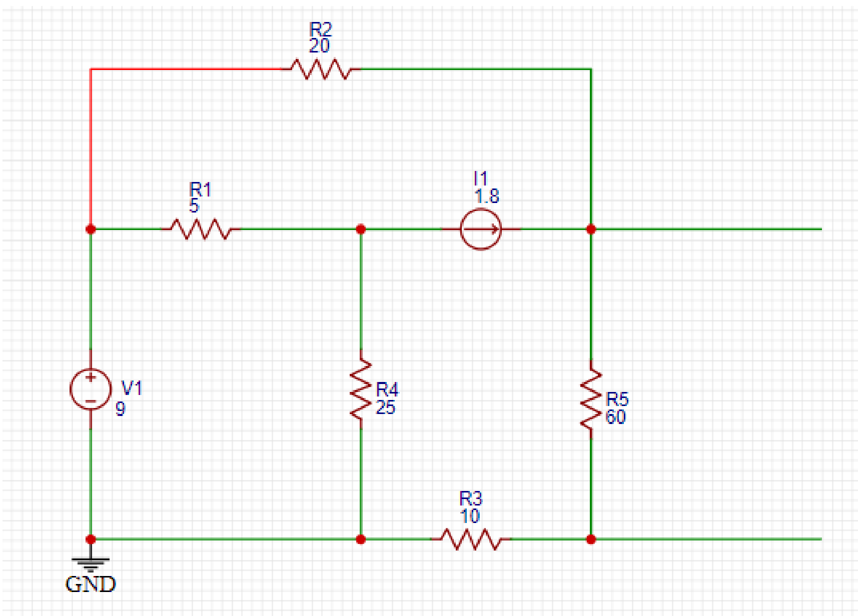
$$R_{Th} = 2.5 + \frac{300}{40} = 10\,\Omega$$



4.78 a) Find the Thévenin equivalent with respect to the terminals a,b for the circuit in Fig. P4.78 by finding the open-circuit voltage and the short-circuit current.

PSPICE
MULTISIM

b) Solve for the Thévenin resistance by removing the independent sources. Compare your result to the Thévenin resistance found in (a).



4.78

[a] $\frac{v_2-9}{20}+\frac{v_2}{10}-1.8=0 \quad v_2=35\,V$

Open circ:

$V_{Th}=\frac{6}{7}v_2=30\,V$

Short circ:

$\frac{v_2-9}{20}+\frac{v_2}{10}-1.8=0 \quad v_2=15\,V$

$i_a=\frac{9-15}{20}=-0.3\,A$

$i_{sc}=1.8-0.3=1.5\,A$

$R_{Th}=\frac{30}{1.5}=20\,\Omega$

[b]

$R_{Th}=20+10\|60=20\,\Omega$

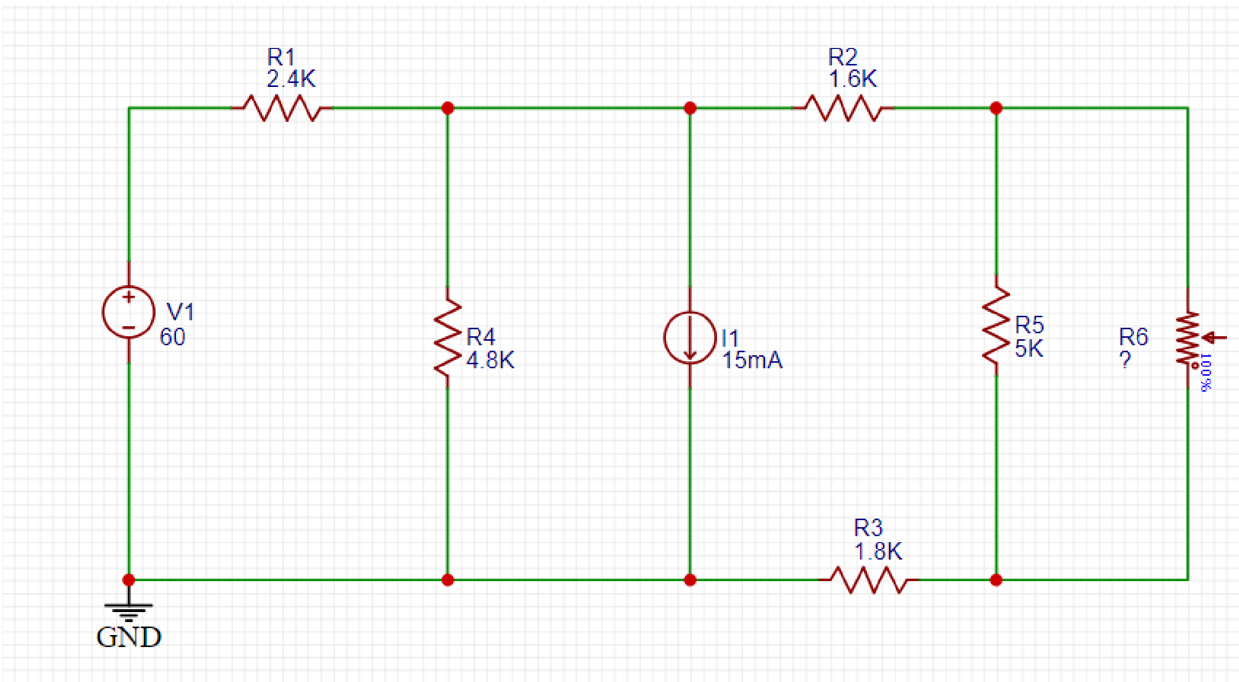
4.82 The variable resistor in the circuit in Fig. P4.82 is adjusted for maximum power transfer to R_o.

PSPICE
MULTISIM

a) Find the value of R_o.

b) Find the maximum power that can be delivered to R_o.

c) Find a resistor in Appendix H closest to the value in part (a). How much power is delivered to this resistor?



4.82

[a] $R_{Th} = 5000 \| (1600 + 2400 \| 4800 + 1800) = 2.5\ k\Omega$

$R_o = R_{Th} = 2.5\ k\Omega$

[b] $7200 i_1 - 4800 i_2 = 60$

$-4800 i_1 + 4800 i_2 + 8400 i_3 = 0$

$i_2 - i_3 = 0.015$

$i_1 = 19.4\,mA; \quad i_2 = 16.6\,mA; \quad i_3 = 1.6\,mA$

$P_{max} = (1.6 \times 10^{-3})^2 (2500) = 6.4\ mW$

[c] The resistor closest to 2.5 kΩ from Appendix H has a value of 2.7 kΩ. Use voltage division to find the voltage drop accross this load resistor, and use the voltage to find the power delivered to it;

$$V_{2.7k} = \frac{2700}{2700 + 2500}(8) = 4.154\,V$$

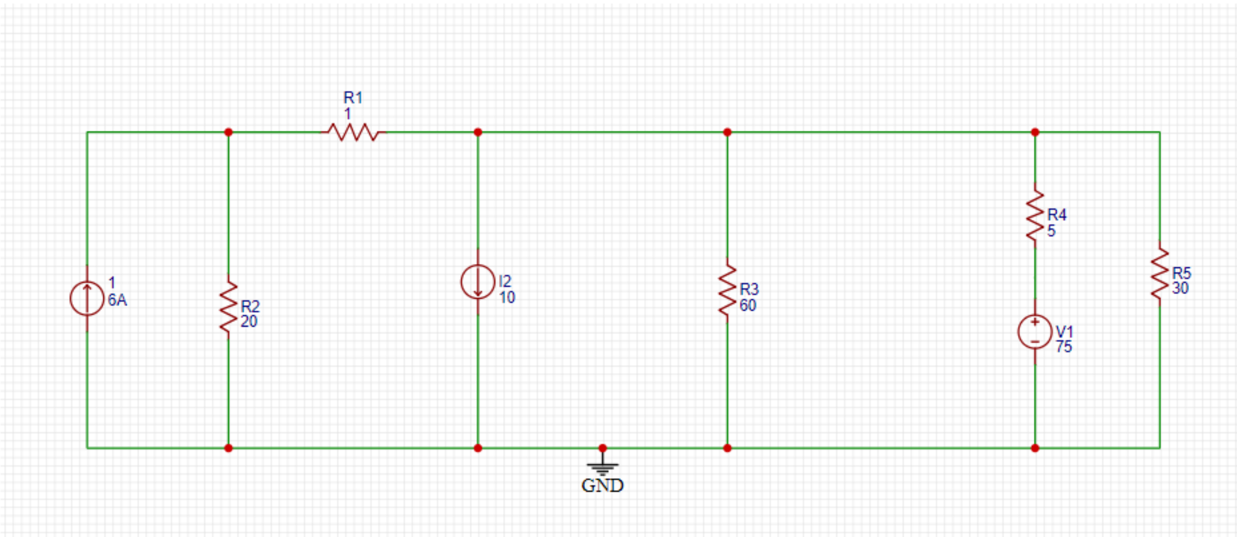
$$P_{2.7k} = \frac{(4.154)^2}{2700} = 6.391\ mW$$

The percent error between the maximum power and the power delivered to the best resistor from Appendix H is

$$\%\ error = \left(\frac{6.391}{6.4} - 1\right)(100) = -0.1\ \%$$

4.95 Use the principle of superposition to find the current i_o in the circuit shown in Fig. P4.95.

PSPICE
MULTISIM



4.95

$30 \| 5 \| 60 = 4\Omega$

$i_{o1} = \frac{20}{25} = 4.8\,A$

$i_{o2} = \frac{4}{25} \cdot 10 = 1.6A$

$i_{o3} = \frac{-4}{25} \cdot 15 = -2.4A$

$\Rightarrow i_o = 4.8 + 1.6 + (-2.4) = 4A$