HOMEWORK 4

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Problem 1

a. $a_0 = -2^{(0+1)} = -2$ $\quad a_1 = -2^{(1+1)} = -4$ $\quad a_2 = -2^{(2+1)} = -8$

$a_3 = -2^{(3+1)} = -16$

if $n=1 \rightarrow a_1 = 3a_{(1-1)} + 2^1 = -2$

b. $a_0 = 1$ $\quad a_{n-1} = 1$

$a_n = r$ $\quad f^{(h)}(n) = c_1 . 3^n$

$r = 3$

$a_n^{(p)} = A . 2^n$

$a_{n-1}^{(p)} = A . 2^{n-1}$

$a_n = 3a_{n-1} + 2^n$

$A . 2^n = 3(A . 2^{n-1}) + 2^n \rightarrow A . 2^n = \frac{3}{2} A . 2^n + 2^n \rightarrow A = \frac{3}{2} A + 1$

$A = -2$ $\quad a_n = c_1 . (3)^n + (-2) . 2^n$

$a_0 = c_1 . (3)^0 + (-2) . 2^0 \rightarrow c_1 - 2 = 1 \rightarrow c_1 = 3$

$a_n = 3^{n+1} - 2^{n+1}$

Problem 2

$f(n) - 4f(n-1) + 4f(n-2) = n^2$

$P(r) = r^2 - 4r + 4 + 4 = (r-2)^2$

$r = 2$

$f^{(h)}(n) = c1 . 2^n + c2 . 2^n . n$

$f^{(p)}(n) = An^2 + Bn + C$

$f^{(p)}(n-1) = A(n-1)^2 + B(n-1) + C$

$f^{(p)}(n-2) = A(n-2)^2 + B(n-2) + C$

- $(An^2 + Bn + C) + (-4A(n-1)^2 - 4B(n-1) - 4C) + (4A(n-2)^2 + 4B(n-2) + 4C) = n^2$
- $(An^2 + Bn + C) - 4(A(n-1)^2 + B(n-1) + C) + 4(A(n-2)^2 + B(n-2) + C) = n^2$
- $(An^2 + Bn + C) + (-4A(n^2 - 2n + 1) - 4B(n-1) - 4C) + (4A(n^2 - 4n + 4) + 4B(n-2) + 4C) = n^2$

$\to An^2 + Bn + C - 4An^2 + 8An - 4A - 4Bn + 4B + 4C + 4An^2 - 16An + 16A + 4Bn - 8B + 4C = n^2$

$\to An^2 + (-8A + B)n + 12A - 4B = n^2$

$\to An^2 + Bn + C - 8An + 12A - 4B = n^2$

$\underline{A = 1}$ $\quad -8A + B = 0 \quad \underline{B = 8}$

$12A - 4B + C = 0 \quad C = 20$

$f^{(p)}(n) = n^2 + 8n + 20$

$f(n) = f^{(h)}(n) + f^{(p)}(n)$

$f(n) = C1 . 2^n + C2 . 2^n . n + n^2 + 8n + 20$

$f(0) = 2 \quad f(1) = 5$

$f(0) = C1 . 2^0 + C2 . 2^0 . 0 + 0^2 + 8 . 0 + 20 = 2$

$\hookrightarrow C1 + 20 = 2 \rightarrow C1 = -18$

$f(1) = -18 . 2^1 + C2 . 2^1 . 1 + 1^2 + 8 \times 1 + 20 = 5$

$\hookrightarrow -36 + 2 . C2 + 29 = 5 \rightarrow C2 = 6$

$f(n) = -18 . 2^n + 6 . 2^n . n + n^2 + 8n + 20$

$f(n) = (n-3) . (6 . 2^n) + n^2 + 8n + 20$

Problem 3

a.

$a_n = 2a_{n-1} - 2a_{n-2}$

$a_n = r^2 \quad a_{n-1} = r \quad a_{n-2} = 1$

$r^2 - 2r + 2 = 0 \; | \; r = \frac{2 + \sqrt{(-2)^2 - 4 . (1) . (2)}}{2}$

$r = \frac{2 + \sqrt{-4}}{2} \quad r = \frac{2 + 2\sqrt{-1}}{2} \quad r = \frac{2 + 2i}{2} \quad r = 1 + 1i$

$a_n^{(h)} = C1 . (1 - i)^n + C2 . (1 + i)^n$

b) $a_n^{(h)} = c_1\,(1-i)^n + c_2\,.(1+i)^n$

if $n=0 \rightarrow a_0^{(h)} = c_1.(1-i)^0 + c_2.(1+i)^0 = 1$

$\quad \hookrightarrow c_1 + c_2 = 1$

if $n=1 \rightarrow a_1^{(h)} = c_1.(1-i)^1 + c_2\,(1+i)^1 = 2$

$\quad \hookrightarrow c_1 - c_1 i + c_2 + c_2 i = 2 \rightarrow c_1 + c_2 - c_1 i + c_2 i = 2$

$c_2 i - c_2 i = 1$

$i\,(c_1 + c_2) = i(1)$

$c_1 i + c_2 i = i$

$c_2 i - c_1 i = 1$

$2c_2 = 1 + i \rightarrow 1 + i = 2ic_2$

$c_2 = \frac{1+i}{2i}$

$c = \frac{(1+i)\,i}{(2i)\,i} \rightarrow \frac{(i+i^2)}{(2i^2)} \rightarrow \frac{i-1}{-2}$

$c_2 = \frac{1}{2} - \frac{i}{2}$

$c_1 = 1 - c_2$

$c_1 = 1 - \left(\frac{1}{2} - \frac{i}{2}\right) \rightarrow 1 - \frac{1}{2} + \frac{i}{2}$

$c_1 = \frac{1}{2} + \frac{i}{2}$

$a_n^{(h)} = c_1.(1-i)^n + c_2.(1+i)^n$

$a_n = \left(\frac{1}{2} + \frac{i}{2}\right).(1-i)^n + \left(\frac{1}{2} - \frac{i}{2}\right).(1+i)^n$