7.3.5. Do two arbitrary processes have the same central tendency?

The nonparametric equivalent of the t test is due to Mann and Whitney, called the U test

By "arbitrary" we mean that we make no underlying assumptions about normality or any other distribution. The test is called the **Mann-Whitney U Test**, which is the nonparametric equivalent of the t test for means.

The U-test (as the majority of nonparametric tests) uses the rank sums of the two samples.

Procedure

The test is implemented as follows.

1. Rank all $(n_1 + n_2)$ observations in ascending order. Ties receive the average of their observations.
2. Calculate the sum of the ranks, call these T_a and T_b.
3. Calculate the U statistic,

$$U_a = n_1\, n_2 + 0.5\, n_1(n_1 + 1) - T_a$$

or

$$U_b = n_1\, n_2 + 0.5\, n_2(N_2 + 1) - T_b$$

where $U_a + U_b = n_1\, n_2$.

Null Hypothesis

The null hypothesis is: the two populations have the same central tendency. The alternative hypothesis is: The central tendencies are **NOT** the same.

Test statistic

The test statistic, U, is the smaller of U_a and U_b. For sample sizes larger than 20, we can use the normal z as follows:

$$z = \frac{U - \mathrm{E}\,(U)}{\sigma}\,,$$

where

$$\mathrm{E}\,(U) = 0.5\, n_1\, n_2 \quad \text{and} \quad \sigma^2 = \frac{n_1\, n_2\,(n_1 + n_2 + 1)}{12}\,.$$

The critical value is the normal tabled z for $\alpha/2$ for a two-tailed test or z at α level, for a one-tail test.

For small samples, tables are readily available in most textbooks on nonparametri

Example

An illustrative example of the U test

Two processing systems were used to clean wafers. The following data represent the (coded) particle counts. The null hypothesis is that there is no difference between the central tendencies of the particle counts; the alternative hypothesis is that there is a difference. The solution shows the typical kind of output software for this procedure would generate, based on the large sample approximation.

Group A	Rank	Group B	Rank
0.55	8	0.49	5
0.67	15.5	0.68	17
0.43	1	0.59	9.5
0.51	6	0.72	19
0.48	3.5	0.67	15.5
0.60	11	0.75	20.5
0.71	18	0.65	13.5
0.53	7	0.77	22
0.44	2	0.62	12
0.65	13.5	0.48	3.5
0.75	20.5	0.59	9.5

	N	Sum of Ranks	U	Std. Dev of U	Median
A	11	106.000	81.000	15.229	0.540
B	11	147.000	40.000	15.229	0.635

For $U = 40.0$ and $\mathrm{E}\,(U) = 0.5\, n_1\, n_2 = 60.5$, the test statistic is

$$z = \frac{U - \mathrm{E}\,(U)}{\sigma} = \frac{40.0 - 60.5}{15.23} = -1.346\,,$$

where

$$\sigma = \sqrt{\frac{n_1\, n_2\, (n_1 + n_2 + 1)}{12}} = \sqrt{\frac{11(11)(11 + 11 + 1)}{12}} = 15.23\,.$$

For a two-sided test with significance level α = 0.05, the critical value is $z_{1-\alpha/2}$ = 1.96. Since $|z|$ is less than the critical value, we do not reject the null hypothesis and conclude that there is not enough evidence to claim that two groups have different central tendencies.