BILKENT UNIVERSITY

ENGINEERING FACULTY

DEPARTMENT OF COMPUTER ENGINEERING

IE400
Principles of Engineering Management
Final Project Report

Aziz Utku Kağıtcı: 21602869

Furkan Kazım Akkurt: 21702900

Zeynep Cankara: 21703381

Fall 2020 - 2021

Contents

1 Introduction

In both Part (a) and Part (b) problems of the project, Santa should be deciding on choosing 4 village locations as gift centers among 30 villages to distribute the gifts which later can be picked up by parents. In these parts of the problem Santa aims to minimise the distance travelled by a parent to the gift center. This problem can be interpreted as center sub-problem in location problem in operations research. Part (c) and Part (d) of the project resembles the Travelling salesperson problem (TSP). In travelling salesman, salesperson problem (TSP), the main goal is to find the shortest possible route by minimizing the distance that the salesman travels. In a problem that is not optimized the distance of the route can be unnecessarily high. Since we want the salesman to pick the shortest route, we should be minimizing the total cost which is the total distance of the route. Then in part (c) he will be using his snowplow to visit all the villages. Finally, in part (d) all villages will be visited and returned back to the origin village by volunteers, within a time limit of 10 hours.

2 Project Description

We are provided with the necessary information in order to construct the models. The data given us contains the distance between all the village pairs in kilometers (d), and the probability that the road between any two village is out of use due to snow (P). This information is in the following format:

$$d = \begin{bmatrix} d_{11} & d_{12} & \dots \\ \vdots & \ddots & \\ d_{30,1} & & d_{30,30} \end{bmatrix}$$

$$P = \begin{bmatrix} P_{11} & P_{12} & \dots \\ \vdots & \ddots & \\ P_{30,1} & & P_{30,30} \end{bmatrix}$$

Both above matrices are 30x30, first representing the distance between the villages and the second representing the probabilities of the roads being out of use due to snow. The problem resembles the Travelling Salesman Problem (TSP), where a set of of cities and the distances between each pair of cities is given and the object is to find the shortest possible route which visits each village exactly once and returns to the origin village. In this problem the villages can be considered as the cities and the distances between them is the same as the distance between each village pair in TSP. In addition to TSP, this problem requires considering a few more constraints such as the probability of roads between villages being out of use, as well as the time constraint that each snowplow should turn back to the origin village after visiting the necessary villages no more than 10 hours.

3 Solution Procedure

In order to formulate the problem as similar to TSP, we need to define cost matrix $d_{i,j}$ which shows the distance between village i and village j.

$$d_{i,j} = \begin{cases} \text{distance between cities i and j} \end{cases}$$

Also the probability matrix $p_{i,j}$ shows the probability of the road between village i and village j being out of use due to snow.

$$P_{i,j} = \begin{cases} \text{probability of road between cities i and being closed} \end{cases}$$

3.1 Part (a)

Parameters

$d_{i,j}$: distance between each village

Decision variables:

$$y_{i,j} = \begin{cases} 1, & \text{if the village i is assigned to the gift center j} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$x_j = \begin{cases} 1, & \text{if the gift center located at village j} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$D = \text{maximum distance a parent should walk to the gift center} \tag{3}$$

IP Model:

Objective Function:

$$Minimise \text{ D} \tag{4}$$

Constraints:

$$\sum_{i=1}^{30} y_{i,j} = 1, \forall i = 1...30 \tag{5}$$

$$y_{i,j} \leq x_j, \forall i, j = 1...30 \tag{6}$$

$$\sum_{i=1}^{30} d_{i,j} \cdot y_{i,j} \leq \text{ D}, \forall i = 1...30 \tag{7}$$

$$\sum_{j=1}^{30} x_j = 4 \tag{8}$$

$$x_j \in \{0, 1\}, \forall j = 1...30 \tag{9}$$

$$y_{i,j} \in \{0, 1\}, \forall i, j = 1...30 \tag{10}$$

Constraint (5) implies that each village will be assigned to a gift center. Constraint (6) indicates that a village cannot be assigned to more than one gift centers. Constraint(7) implies that the travel time from a village to the assigned gift center cannot exceed D, decision variable that represents the maximum distance a parent should walk to the gift center. Constraint(8) implies there will be 4 gift centers chosen for the assignment. Constraint (9) and (10) are the integrity constraints. Objective function (4) enforces to minimise the distance between a village and it's assigned gift center which will be the distance taken by a parent to walk to the gift center. Decision variables defined by (1), (2) and (3) specifies how the village to gift center assignment will be done.

3.2 Part (b)

Part (b) can solved by introducing the additional variable $P_{i,j}$ which represents the probability that the road between the village i and village j will be blocked because of the snow. Thus, we can solve Part (b) by introducing the additional constraint to the IP Model of Part (a).

$$P_{i,j} \cdot y_{i,j} \leq 0.60, \forall i, j = 1...30 \tag{11}$$

Additionally, we are introducing the parameter $P_{i,j}$ to the part (a).

3.3 Part (c)

Part (c) resembles the travelling salesman, salesperson problem (TSP) where we are trying to find shortest possible path in terms of time it takes to travel around the path to distribute the gifts to each village where we will avoid the roads which have the block by snow probability greater than 0.60. We accomplished these goals by setting the road distance to infinity whenever we encounter a road between village i and j is blocked by snow with probability greater than 0.60.

Parameters

$d_{i,j}$: distance between from village i to village j

$P_{i,j}$: Probability that a road from i to j will blocked by snow

Decision Variables:

$$x_{i,j} = \begin{cases} 1, & \text{if salesman goes from village i to village j} \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

IP Model:

Objective function:

$$Minimise \sum_{(i,j)} \frac{d_{i,j} \cdot x_{i,j}}{40} \tag{13}$$

Constraints:

$$\sum_{i=1}^{30} x_{i,j} = 1, \forall j = 1...30 \tag{14}$$

$$\sum_{j=1}^{30} x_{i,j} = 1, \forall i = 1...30 \tag{15}$$

$$P_{i,j} \cdot y_{i,j} \leq 0.60, \forall i, j = 1...30 \tag{16}$$

$$x_{i,i} = 0, \forall i = 1...30 \tag{17}$$

$$u_i - u_j + n \cdot x_{i,j} \leq n - 1, 2 \leq i \neq j \leq 30 \tag{18}$$

$$x_{i,j} \in \{0, 1\}, \forall i, j = 1...30 \tag{19}$$

Constraint (14), (15) implies that in the final path each village must connected to two other cities. Constraint (16) implies there can't exist a connection from village to itself. Constraint (17) implies that there should not be any sub-tours within the final path, we used Miller–Tucker–Zemlin formulation where u_i, u_j are the dummy variables. Constraint (18) is the integrity constraint. Objective function (13) enforces to minimise the time it takes for Santa to travel each village to distribute the gifts. Decision variable defined by (12), indicates the total distance covered by the final path which starts from village 1 and travels each village to distribute the gifts.

3.4 Part (d)

Parameters

$timeLimit$: 10 hours
$speed$: 40 km/hours
D : timeLimit * speed
A : An edge in the network of villages

Decision Variables

$$x_{i,j} = \begin{cases} 1, & \text{if volunteer goes from village i to village j} \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

$$y_{i,j} = \text{total distance from the origin to the village j traveled by a volunteer when it goes from i to j} \tag{21}$$

$$m = \text{Number of volunteers} \tag{22}$$

Objective function:

$$Minimise(m) \tag{23}$$

Constraints:

$$\sum_{i=2}^{30} x_{1,i} = m, \forall i = 1...30 \tag{24}$$

$$\sum_{i=2}^{30} x_{i,1} = m, \forall i = 1...30 \tag{25}$$

$$\sum_{i=1}^{30} x_{i,j} = 1, \forall j = 2...30 \tag{26}$$

$$\sum_{j=1}^{30} x_{i,j} = 1, \forall i = 2...30 \tag{27}$$

$$\sum_{j=1, j\neq i}^{30} y_{i,j} - \sum_{j=1, j\neq i}^{30} y_{j,i} - \sum_{j=1}^{30} d_{i,j} \cdot x_{i,j} = 0, \forall i = 2...30 \tag{28}$$

$$y_{1,i} = d_{1,i} \cdot x_{1,i}, \forall i = 2...30 \tag{29}$$

$$y_{i,j} \leq (D - d_{j,1}) \cdot x_{i,j}, j \neq 1, (i,j) \in A \tag{30}$$

$$y_{i,1} \leq D \cdot x_{i,1} \forall i = 2...30 \tag{31}$$

$$y_{i,j} \geq (d_{1,i} + d_{i,j}) \cdot x_{i,j}, i \neq 1, (i,j) \in A \tag{32}$$

$$x_{i,i} = 0, \forall i = 1...30 \tag{33}$$

$$x_{i,j} \in \{0,1\}, \forall i,j = 1...30 \tag{34}$$

Constraint (22), (23) implies that m vehicles must be used to travel villages. Constraint (24), (25) are degree constraints making sure that each village is visited just once. Constraint (26), (31) implies that the connectivity between villages established and makes sure it eliminates the sub-tours, and it guarantees that the solution do not have any illegal sub-tours. Constraints given in (27),(28) and (29) implies that 0 $y_{i,j}$ D and guarantees that the distance traveled do not exceed the total distance limit which is timeLimit * speed = D. Constraint (32) is the integrity constraint. Objective function (21) enforces to minimise the number of volunteers required to distribute all the gifts under 10 hours to each village. Decision variable defined by (19), (20) which volunteer goes from village i to j and the total distance covered by the volunteer by travelling from village i to j.

4 Solving Models with Gams Software

4.1 Data Pre-processing

The project assignment already gave the distance and probability between each village in a spreadsheet format which made for us easy to feed it into the Gams software for both Part (a) and Part (b). In order to feed the distance and probability tables into the Gams software we introduced additional row and column indicating the village number.

4.2 Part (a) Results

In part (a) of the project we were asked to solve a problem that asks which villages to be chosen as gift centers in order to minimize the maximum distance that a parent should walk. We came up with a model which we have introduced in subsection 3.1. This model has two decision variables which are named $y_{i,j}$ and x_j. The aim is to minimize D, which is the maximum distance a parent should walk in order to receive a gift. Then we have additional constraints, such as one that is implying each village will be assigned to a gift center. Feeding this model into Gams software, we obtained some results.

The results we have gotten after solving the model using Gams software tells us about which villages to allocate as gift center in order to minimize the maximum distance that a parent walks in order to receive a gift, as well as the minimized maximum distance. The villages that should be allocated as gift center are as follows:

15, 19, 24, 30

Also, the maximum distance that a parent should walk is:

50.50 kilometers

4.3 Part (b) Results

This part of the problem can be seen as some kind of extension to the part (a). Here we can simply say that the roads whose probability of being out of use is less than .6 are chosen by Santa. Here we have added a new decision variable and a constraint to our model. Decision variable $P_{i,j}$ represents the mentioned probability, and new constraint, which is,

$$P_{i,j} \cdot y_{i,j} \leq 0.60, \forall i, j = 1...30 \tag{35}$$

guarantees that roads having probability greater than .6 are not to be used.

The results we have gotten after solving the model using Gams software tells us about which villages to allocate as gift center in order to minimize the maximum distance that a parent walks in order to receive a gift, as well as the minimized maximum distance. Here, in this problem, Santa also considers that parent from a village can only walk to a gift center only if the probability of road between those two villages being out of use is less than .6. The villages that should be allocated as gift center are as follows:

9, 13, 20, 24

Also, the maximum distance that a parent should walk is:

54.50 kilometers

4.4 Part (c) Results

In this part of the problem Santa is said to be visiting all the villages. He has a snowplow on which he travels with an average speed of 40km/h. We are asked to minimize the time it takes for Santa to visit all the villages using his snowplow. Additionally, in this part of the problem, Santa again cannot use the roads that have probability of being out of use greater than .6. We are minimizing the total time it takes for Santa to distribute visit all villages by dividing the total distance he travels by 40, which is the average speed of his snowplow. The model can be found in the subsection 3.3, and the results we have gotten after solving the model using Gams software can be found below [2].

Time spent to distribute all the gifts following the shortest path:

27.875 hours

TSP Path starting from Village 1:

1–>30–>17–>14–>25–>27–>9–>13–>18–>22–>
16–>12–>28–>6–>5–>20–>23–>4–>3–>15–>
7–>8–>29–>24–>2–>10–>26–>19–>11–>21–>1

4.5 Part (d) Results

This part of the problem resembles the distance constrained vehicle routing problem which can be found in the paper "Arc based integer programming formulations for the Distance Constrained Vehicle Routing Problem" [1]. We have used a similar model to one that was presented in the paper. We set our time limit to 10 hours and shown the number of volunteers with letter m. We are minimizing m, which represents the number of volunteers helping Santa, as it was asked in the question. Results from the Gams software demonstrated that at least 3 volunteers should help Santa in order to visit all the villages within 10 hours. The path every volunteer follows and the required time can be found by looking at the below results.

Volunteer 1

385.5 kilometers

1 –> 9 –> 29 –> 24 –> 4 –> 5 –> 3 –> 30 –> 2 –> 1

Volunteer 2

393.5 kilometers

1 –> 10 –> 26 –> 19 –> 27 –> 6 –> 14 –> 17 –> 23 –> 20 –> 11 –> 21 –> 1

Volunteer 3

394.5 kilometers

1 –> 15 –> 7 –> 8 –> 25 –> 28 –> 12 –> 13 –> 18 –> 22 –> 16 –> 1

We know that the bottleneck for the time value will be the longest distance travelled by a volunteer. In our case the longest distance travelled by a volunteer is 394.5 km. Dividing this value with the speed of 40 km/h we found out that the time required to travel all villages under 10 hours with the help of minimum number of volunteers is 9,8625 hours.

394.5 km / 40 km/h = 9,8625h

5 References

[1] Kara, I. (2011). Arc based integer programming formulations for the Distance Constrained Vehicle Routing problem. 3rd IEEE International Symposium on Logistics and Industrial Informatics, 34-36. doi:10.1109/lindi.2011.6031159

[2] Kalvelagen, E, Model Building with GAMS. forthcoming [Source code]. $www.gams.com/latest/gamslib_ml/libhtml/gamslib_tsp2.html$