Xi'an Jiaotong-Liverpool University
西交利物浦大學

EEE109: Electronic Circuits

Organizational Information & Chapter 1

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Class Rules

- Attendance of the classes is IMPORTANT.

Reason 1: If you miss the class once for a good reason, your chance to miss more classes increases “exponentially”

Reason 2: Important announcements are usually given in the beginning and the end of the class.

Reason 3: There will be random class tests/assignments during the semester.

Reason 4: Enjoy the company of your friends in the battle field of classroom fighting for knowledge that will enrich your life.

Mandatory Textbook

- Mandatory Book:

  Microelectronic Circuit Analysis and Design, *4rd Ed*, *McGraw Hill*, by Donald Neaman

- Additional References Books:

1. A. S. Sedra and K. C. Smith, Microelectronic Circuits, 5th Ed, Oxford University Press, 2004.
2. David Comer and Donald Comer, Fundermentals of Electronic Circuit, John Wiley
3. A. R. Hambley, Electrical Engineering Principles and Applications, McGraw-Hill

Learning Outcomes

On successful completion of this module the student should:

- understand the behaviour, important properties, equivalent circuit representations and applications of diodes and transistors;
- understand circuit biasing, the role of decoupling capacitors and the performance of some commonly used circuit configurations and their practical significance;
- understand amplifier circuit design and circuit analysis;
- understand frequency response of amplifiers via simulation.
- Understand output stage and power amplifier.

Intellectual Abilities

On successful completion of this module the student should be able to

- analyse simple transistor circuits;
- determine components to meet a specification;
- design various types of amplifiers.

Practical Skills

On successful completion of this module the student should be able to:

- determine device properties from characteristics;
- calculate the output voltage and regulation of simple rectifier and stabilizer circuits;
- perform simple analysis of circuits containing bipolar and MOS transistors;
- construct and test simple transistor circuits and amplifiers;
- simulate frequency response of amplifiers using PSpice.

Module Assessment

Assessment Components:

1. Final Exam, 60 %
2. Midterm Exam, 15%
3. Assignments/HWs, 10 %
4. Lab/Lab reports 15%

Note:

a) You will not pass if you don't take the Midterm/HW/Lab seriously

b) Resit is in August 2016 and applies students who fail the class.

Module Assessment

Homework and Report Submission Requirements:

- Do not be late; No late HW will be accepted
- Do not do them in the class, *turn in your work electronically only (no hardcopy submissions)*
- Submit electronic copy in PDF format **only**
- Due in preannounced time and date via ICE

Lab Arrangement

Lab Schedule:

Week 8: Lab 1 (class 1) - Diodes Characteristics and Their Applications

Week 9: Lab 1 (class 2) - Diodes Characteristics and Their Applications

Week 10: Lab 2 (class 1) - Transistors and Their Applications

Week 11: Lab 2 (class 2) - Transistors and Their Applications

Week 12: Lab 3 PSpice lab (Class 1&2) – Frequency Response of BJT Amplifiers

Lab Reports:

3 Lab reports (15 % of module credit)

TAs:

5 TAs available

Background Knowledge

What is Electronics Anyway?

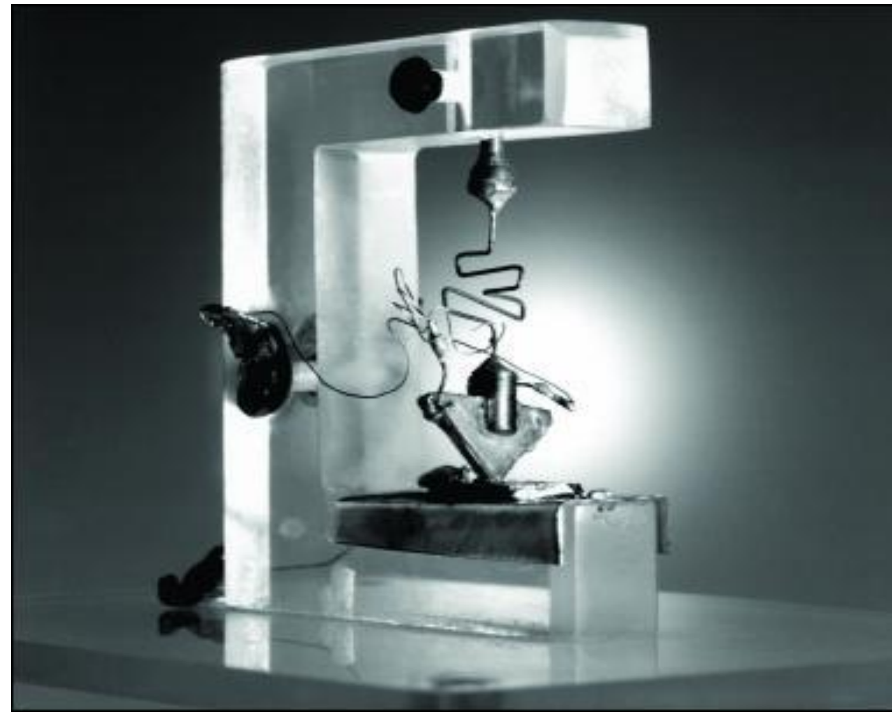
Microelectronics, Nanoelectronics, Optoelectronics, Terahertz Electronics, Power Electronics, Digital and Analog Electronics…

Def 1: **Electronics** is the branch of science that deals with the study of flow and control of electrons (electricity) and the study of their behavior and effects in vacuums, gases, and semiconductors, and with devices using such electrons. This control of electrons is accomplished by devices that resist, carry, select, steer, switch, store, manipulate, and exploit the electron.

Def 2: The institute of Radio Engineers has given a definition of electronics as "the field of science and engineering, which deals with electron devices and their utilization."

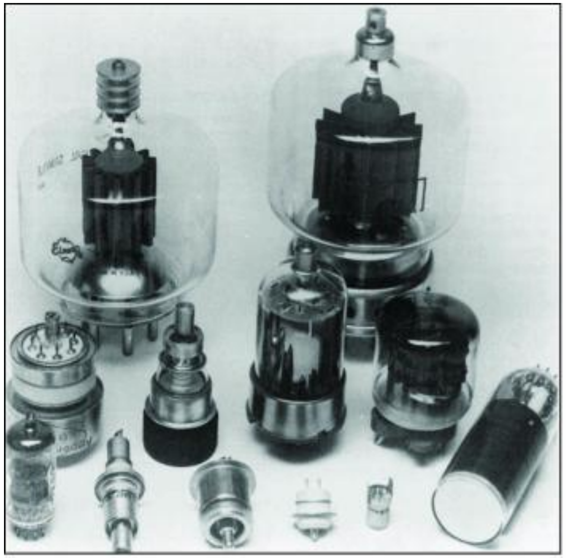
The Start of the Modern Electronics Era

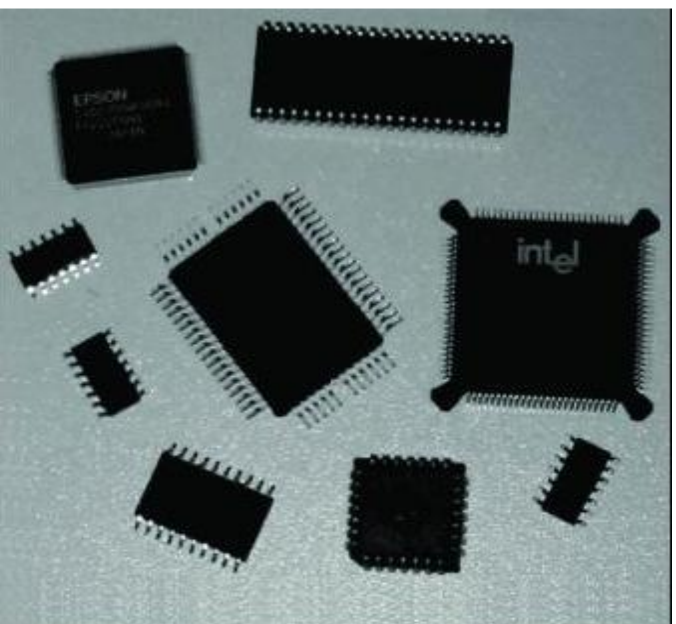
Bardeen, Shockley, and Brattain at Bell Labs - Brattain and Bardeen invented the bipolar transistor in 1947.



The first germanium bipolar transistor. Roughly 60 years later, electronics account for >10% (>8 trillion dollars) of the world GDP.
Note: World GDP ~78.9 Trillions in 2011

Evolution of Electronic Devices

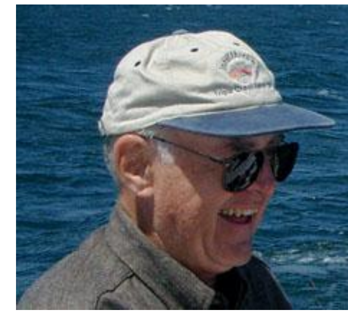




References by Jaeger/Blalock 7/1/03

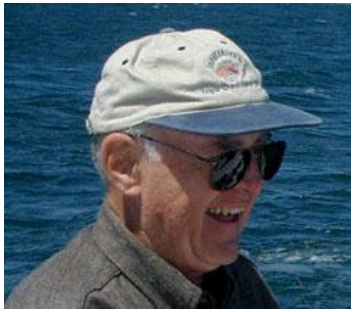
Microelectronics Proliferation

- The integrated circuit was invented in 1958.
- Moore's Law (predicted in 1966).



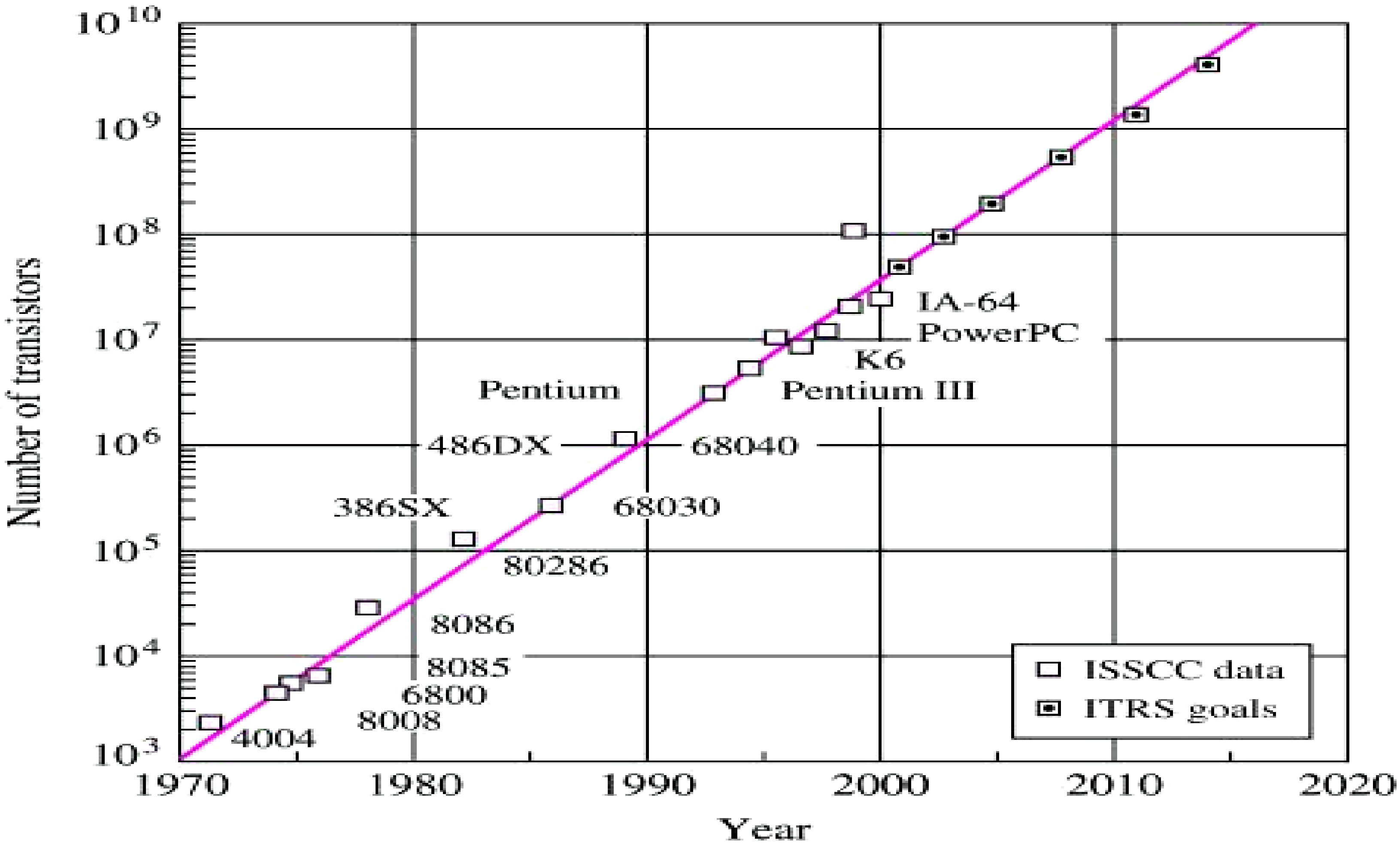
- World transistor production has more than doubled every year for the past twenty years.
- Every year, more transistors are produced than in all previous years combined.
- Nanoelectronics is the new kid in the block!

Microelectronics Proliferation



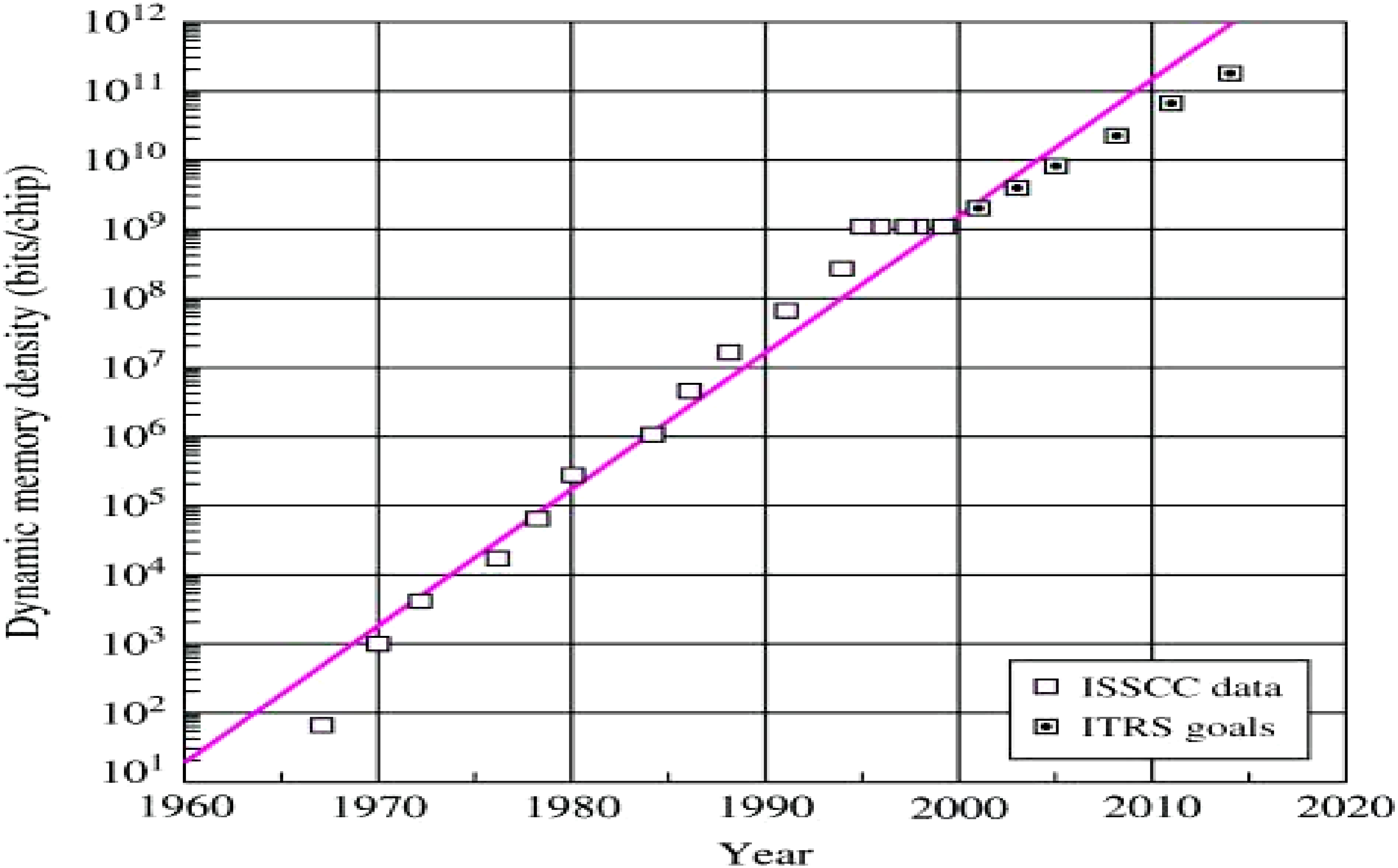
- On 13 April 2005, Gordon Moore stated in an interview that the law cannot be sustained indefinitely: "***It can't continue forever. The nature of exponentials is that you push them out and eventually disaster happens***". He also noted transistors would eventually reach the **limits of miniaturization** at **atomic** levels.

Microprocessor complexity versus time



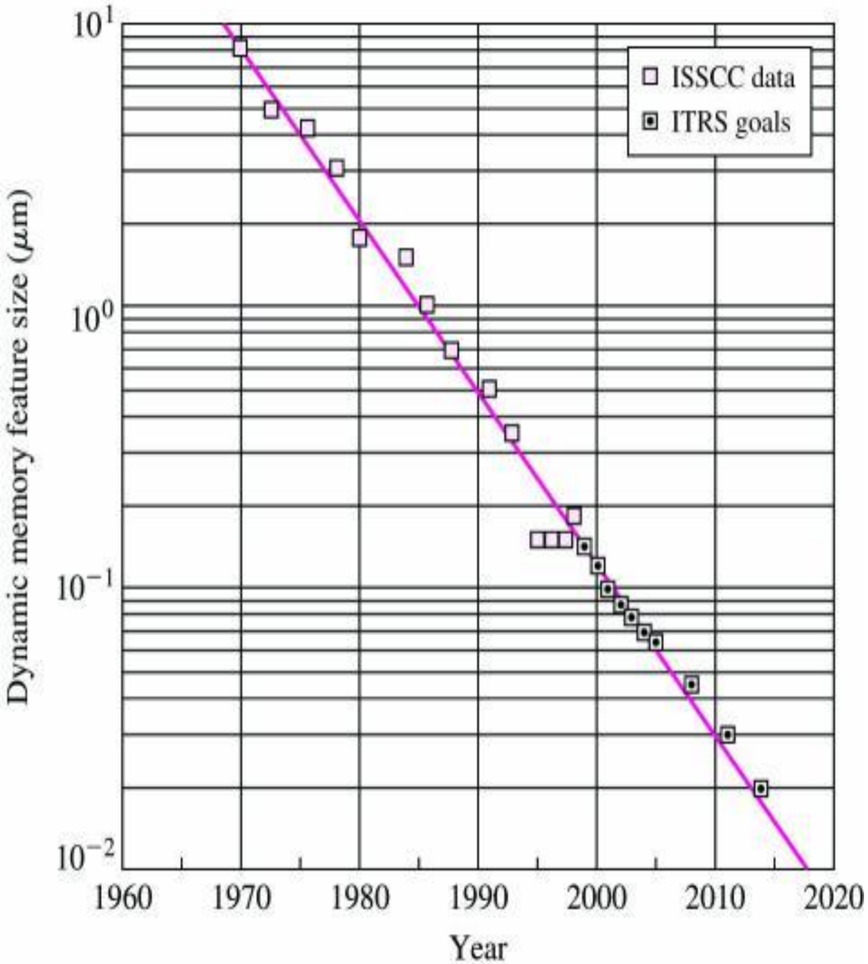
From Jaeger/Blalock 7/1/03

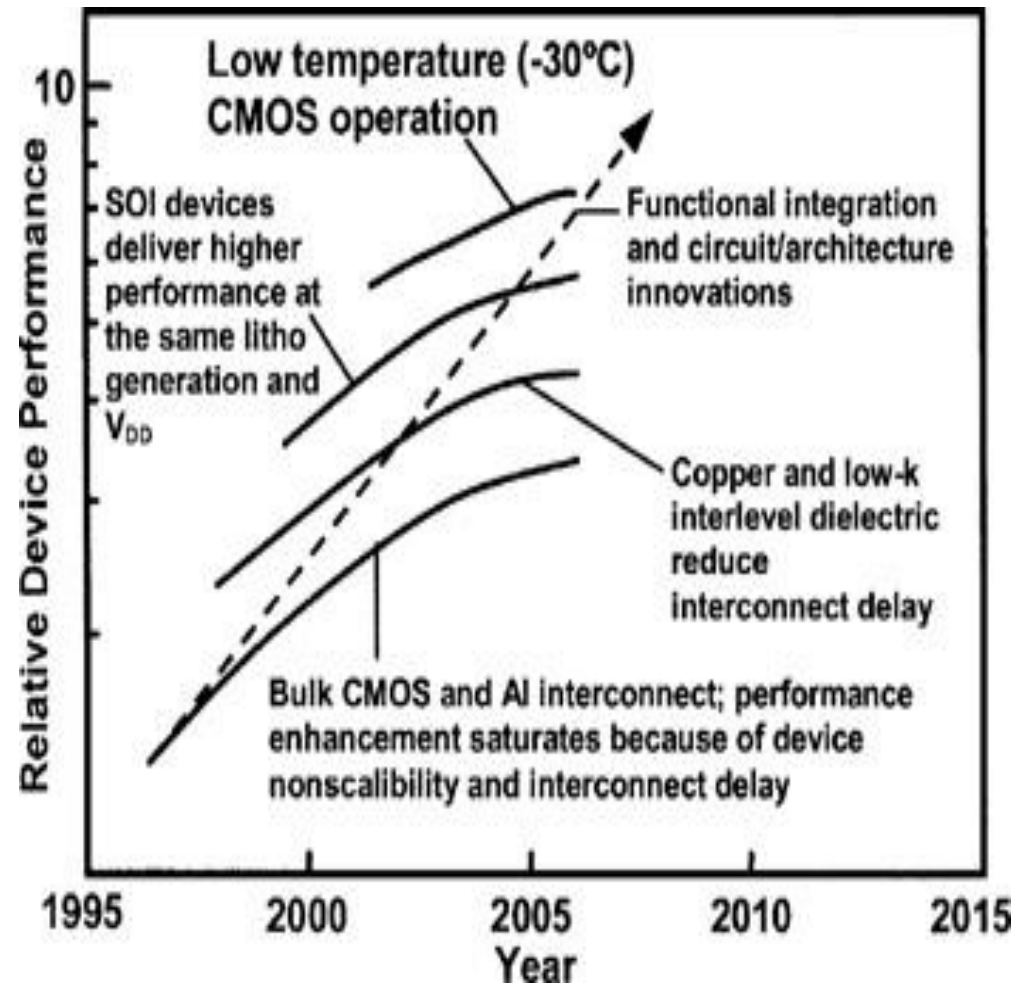
Memory chip density versus time



Taken from Jaeger/Blalock 7/1/03

Device Feature Size

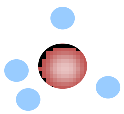




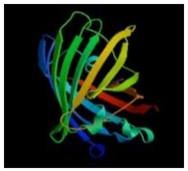
Take by Permission from Jaeger/Blalock 7/1/03

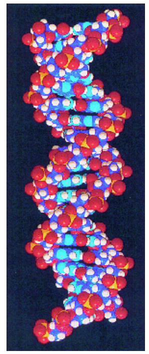
Length Scales

Simple molecules <1nm



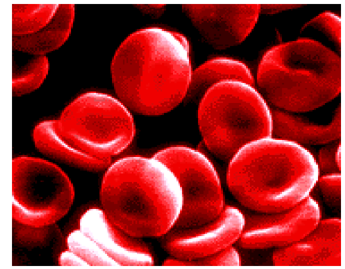
DNA proteins nm



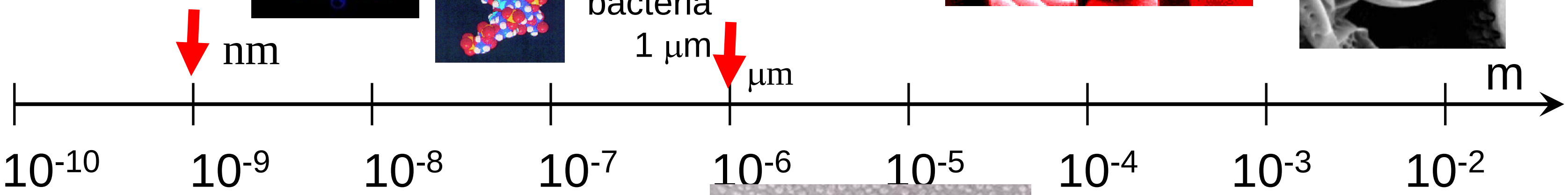


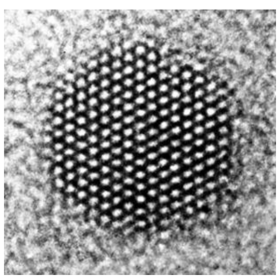
bacteria 1 μm

red blood cell ~5 μm (SEM)

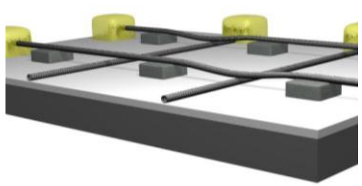


Diatom 30 μm

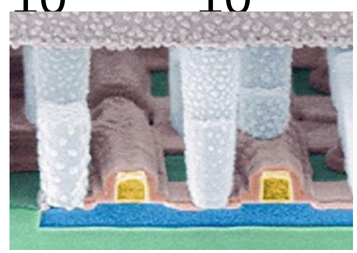




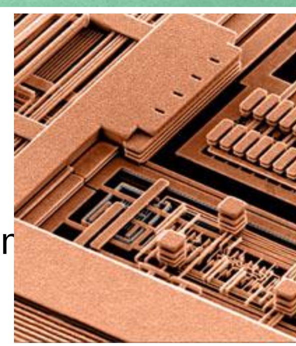
semiconductor nanocrystal (CdSe) 5nm



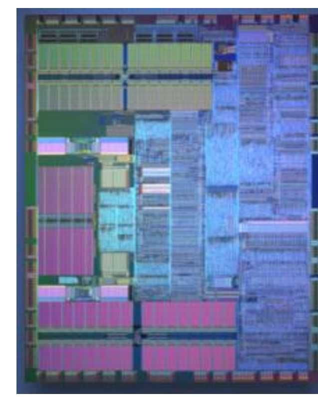
Nanometer memory element (Lieber) 10^{12} bits/cm^2 (1Tbit/cr



SOI transistor width 0.12μm



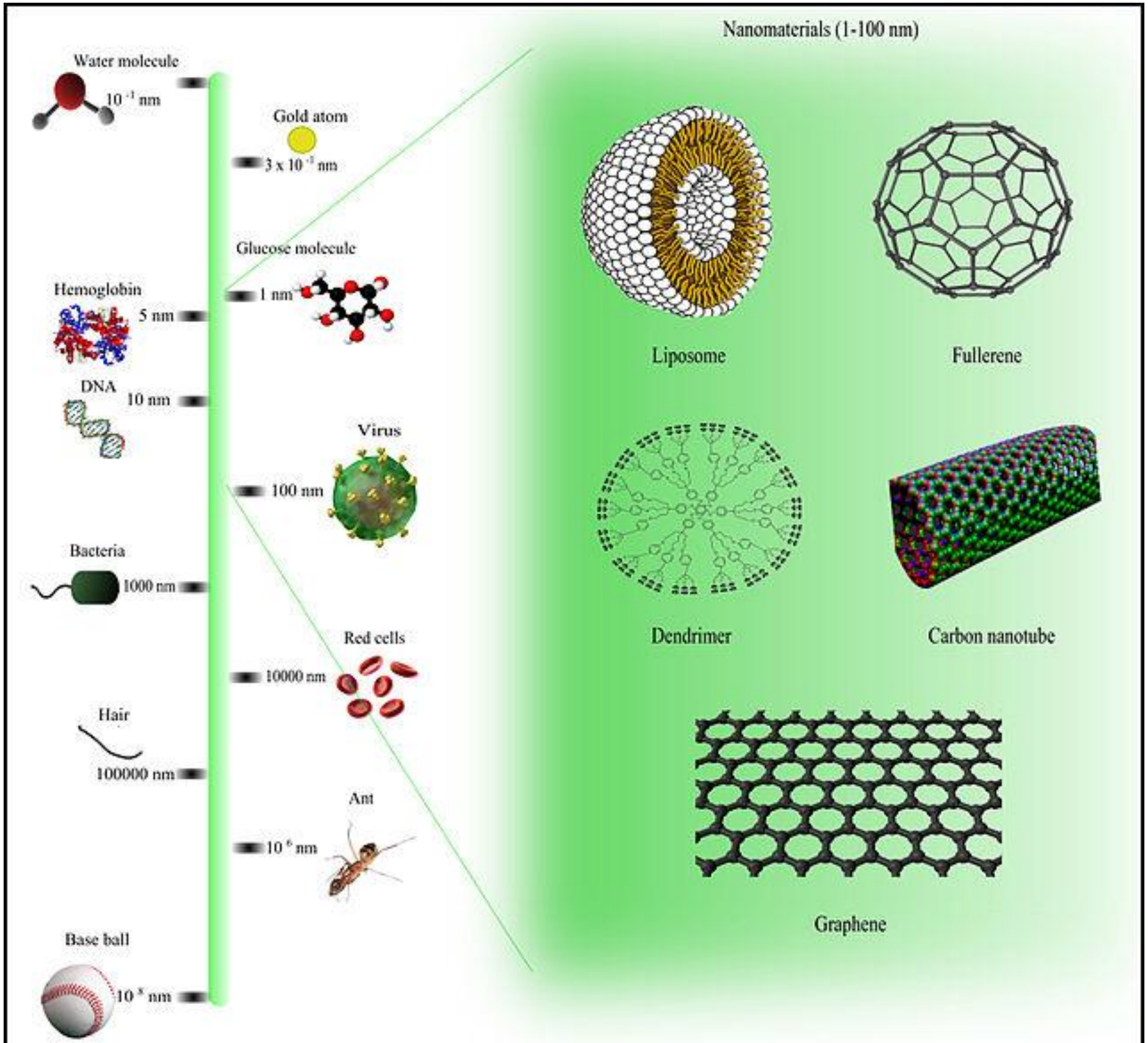
Circuit design Copper wiring width 0.2μm



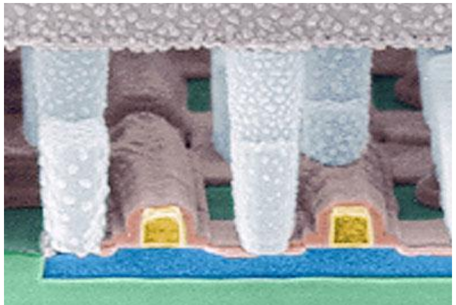
IBMPowerPC750™ Microprocessor7.56 mm × 8.799mm 6.35×10^6 transistors

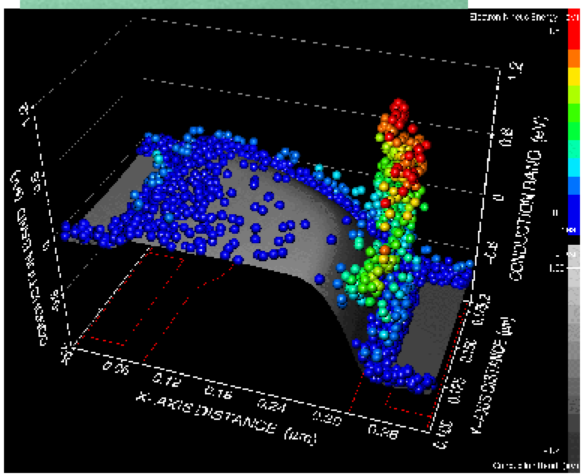
Photo credits, bio, L-R GFP: RCSB Protein Data Bank http://www.rcsb.org/pdb/ E.Coli: Dennis Kunkel http://www.pbrc.hawaii.edu/kunkel/catalog/by_category/ Red Blood Cells: James A. Sullivan, www.cellsalive.com Diatom: Dept of Biology, Indiana University Silicon, L-R CdSe nanocrystal: Andreas Kadavanich, Alivisatos Group, Dept of Chemistry, UC Berkeley Nanotube memory device: Lieber Group, Dept of Chemistry, Harvard University SOI transistor/Cu wiring/PowerPC Microprocessor chip: IBM

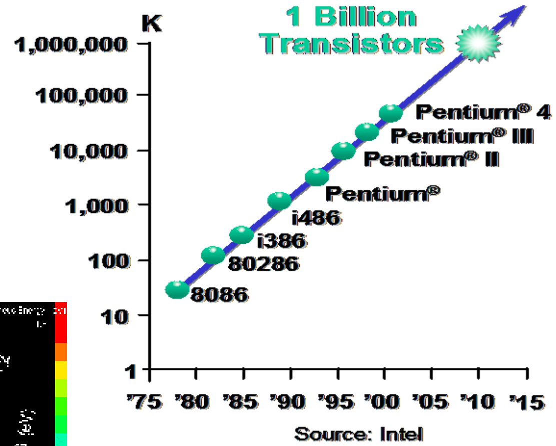
Comparison of Nanomaterial Sizes

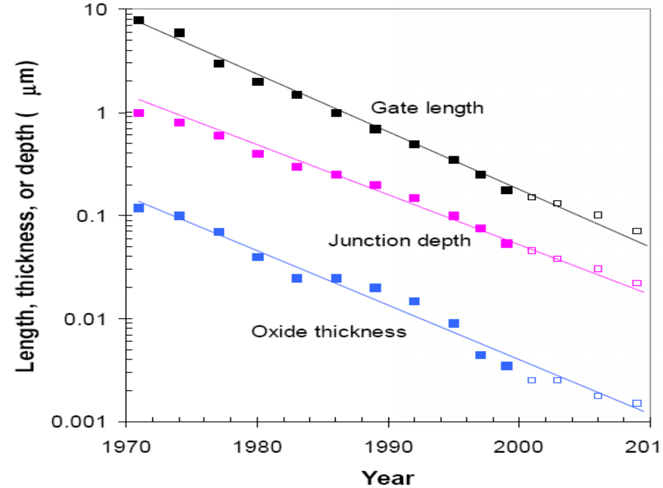


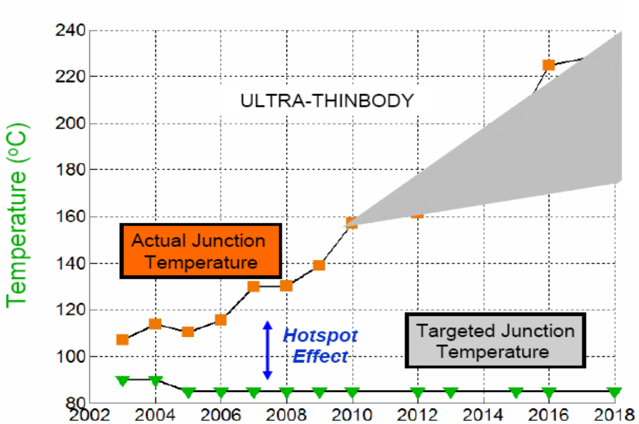
Revolution and Evolution in Electronics











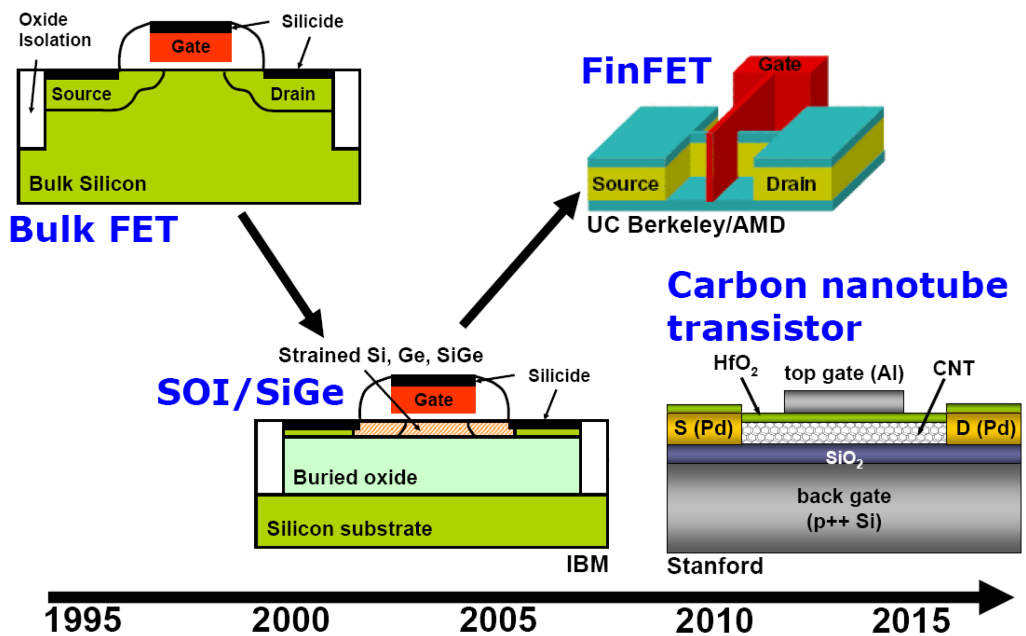


Photo credits, bio, L-R GFP: RCSB Protein Data Bank http://www.rcsb.org/pdb/ E.Coli: Dennis Kunkel http://www.pbrc.hawaii.edu/kunkel/catalog/by_category/ Red Blood Cells: James A. Sullivan, www.cellsalive.com Diatom: Dept of Biology, Indiana University Silicon, L-R CdSe nanocrystal: Andreas Kadavanich, Alivisatos Group, Dept of Chemistry, UC Berkeley Nanotube memory device: Lieber Group, Dept of Chemistry, Harvard University SOI transistor/Cu wiring/PowerPC Microprocessor chip: IBM

Goals for Chapter 1

- Gain a basic understanding of semiconductor material properties
  - Two types of charged carriers that exist in a semiconductor
  - Two mechanisms that generate currents in a semiconductor

- Determine the properties of a pn junction
  - Ideal current–voltage characteristics of a pn junction diode

- Examine dc analysis techniques for diode circuits using various models to describe the nonlinear diode characteristics

- Develop an equivalent circuit for a diode that is used when a small, time-varying signal is applied to a diode circuit

- Gain an understanding of the properties and characteristics of a few specialized diodes

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Semiconductor Material Properties

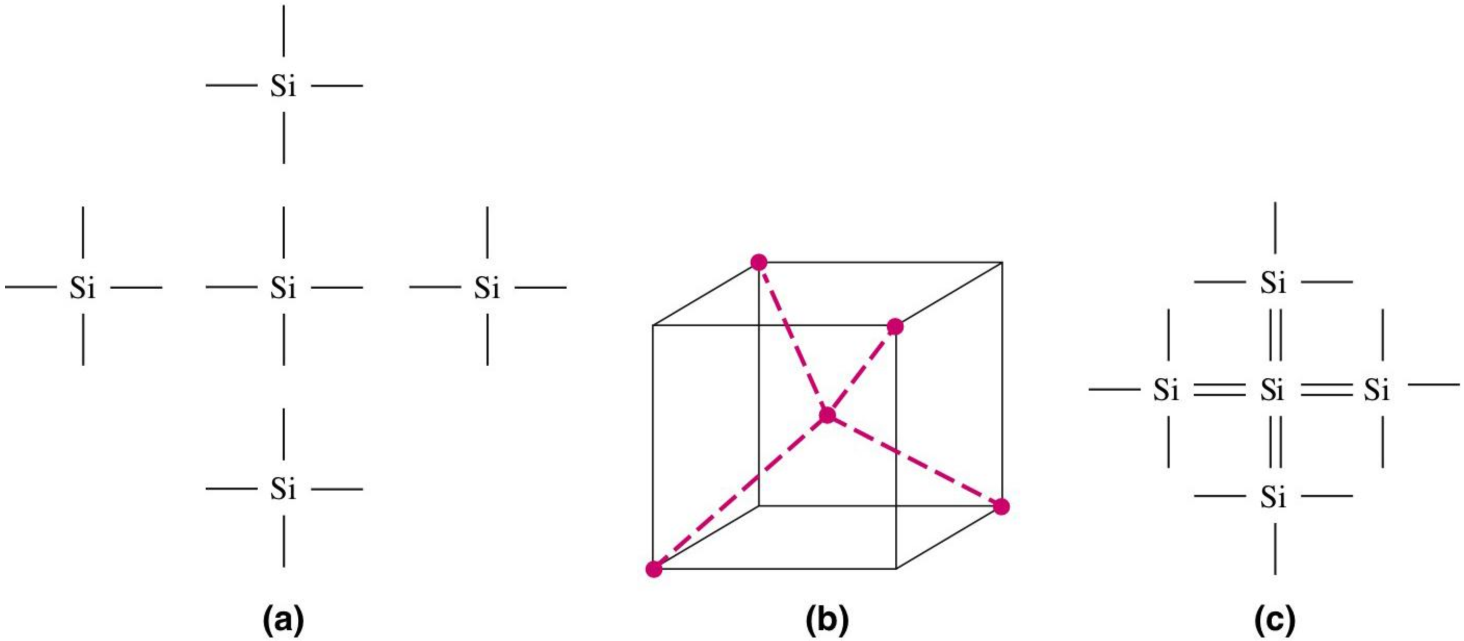
- Two types of charged carriers that exist in a semiconductor

- Two mechanisms that generate currents in a semiconductor

Intrinsic Semiconductors

- Ideally 100% pure material
  - Elemental semiconductors
    - Silicon (Si)
      - Most common semiconductor used today
    - Germanium (Ge)
      - First semiconductor used in p-n diodes
  - Compound semiconductors
    - Gallium Arsenide (GaAs)

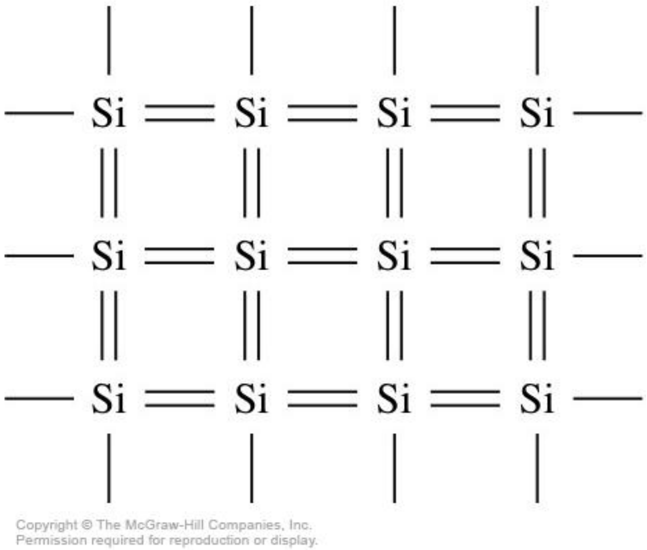
Silicon (Si)

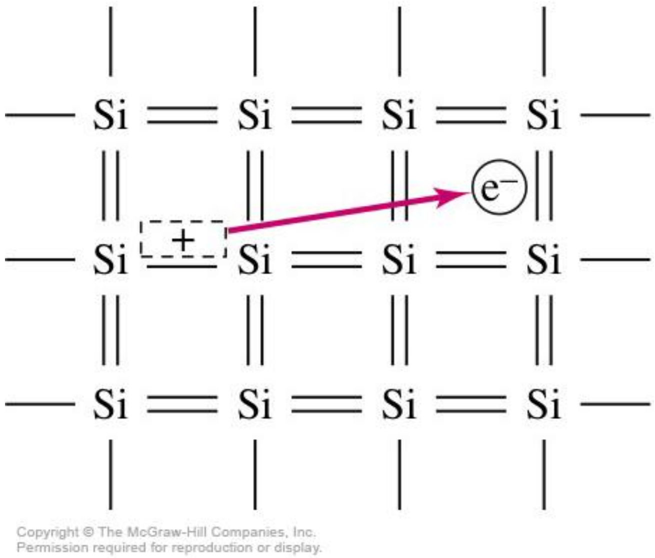


Covalent bonding of one Si atom with four other Si atoms to form tetrahedral unit cell.

Valence electrons available at edge of crystal to bond to additional Si atoms.

Effect of Temperature





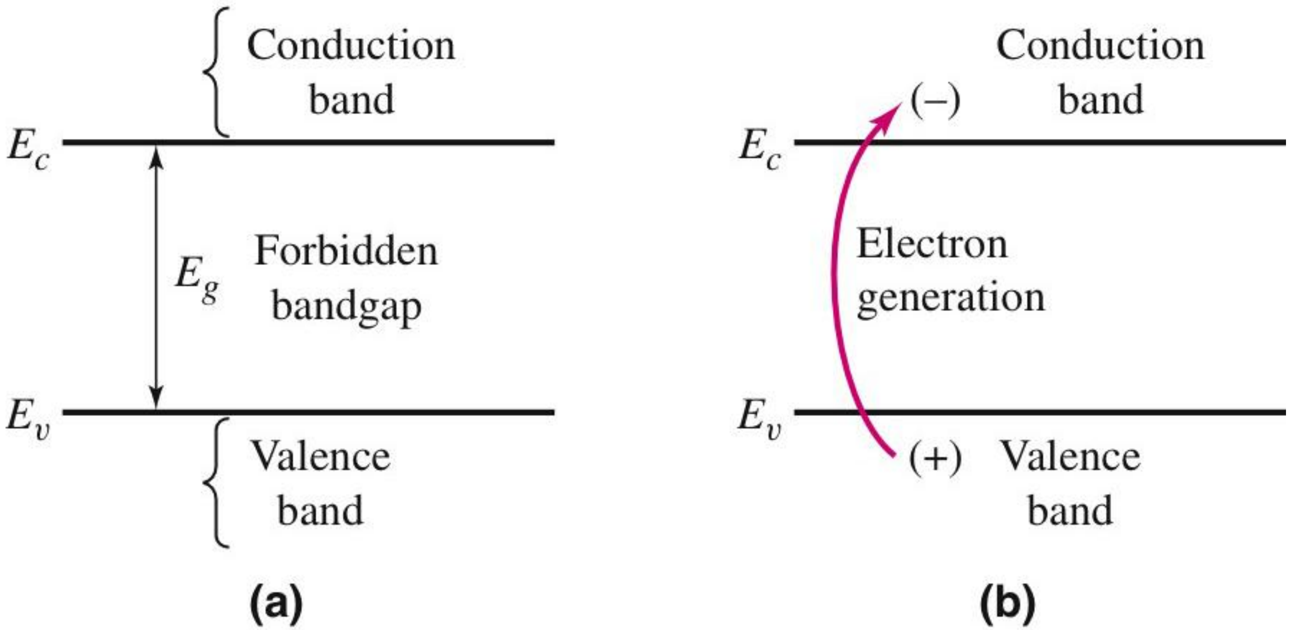
At 0K, no bonds are broken.

Si is an insulator.

As temperature increases, a bond can break, releasing a valence electron and leaving a broken bond (hole).

Current can flow.

Energy Band Diagram

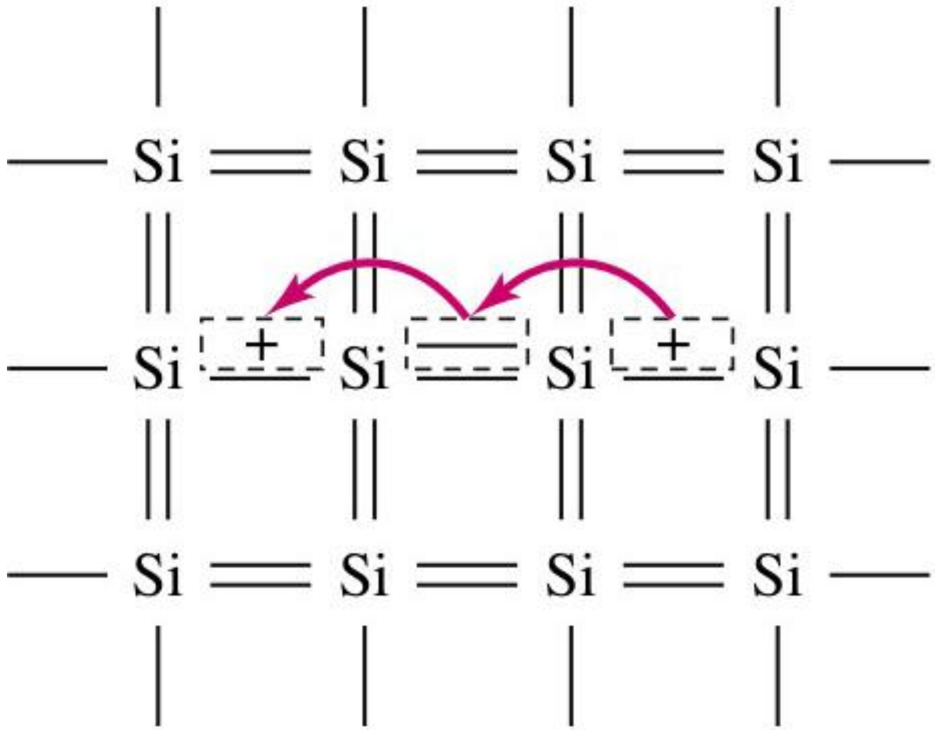


E_v – Maximum energy of a valence electron or hole

E_c – Minimum energy of a free electron

E_g – Energy required to break the covalent bond

Movement of Holes



A valence electron in a nearby bond can move to fill the broken bond, making it appear as if the 'hole' shifted locations.

Intrinsic Carrier Concentration

$$n_i = BT^{3/2} e^{\frac{-E_g}{2kT}}$$

B – coefficient related to specific semiconductor

T – temperature in Kelvin **0 °C = 273.15 K**

E_g – semiconductor bandgap energy

k – Boltzmann's constant

$$n_i(Si, 300K) = 1.5x10^{10}\,cm^{-3}$$

Extrinsic Semiconductors

- Impurity atoms replace some of the atoms in crystal
  - Column V atoms in Si are called donor impurities.
  - Column III in Si atoms are called acceptor impurities.

V·T·E

Periodic table

[hide]

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	Alkali metals	Alkaline earth metals													Pnictogens	Chalcogens	Halogens	Noble gases
Period 1	Hydrogen 1 H																	Helium 2 He
2	Lithium 3 Li	Beryllium 4 Be											Boron 5 B	Carbon 6 C	Nitrogen 7 N	Oxygen 8 O	Fluorine 9 F	Neon 10 Ne
3	Sodium 11 Na	Magnesium 12 Mg											Aluminium 13 Al	Silicon 14 Si	Phos-phorus 15 P	Sulfur 16 S	Chlorine 17 Cl	Argon 18 Ar
4	Potassium 19 K	Calcium 20 Ca	Scandium 21 Sc	Titanium 22 Ti	Vanadium 23 V	Chromium 24 Cr	Manganese 25 Mn	Iron 26 Fe	Cobalt 27 Co	Nickel 28 Ni	Copper 29 Cu	Zinc 30 Zn	Gallium 31 Ga	Germanium 32 Ge	Arsenic 33 As	Selenium 34 Se	Bromine 35 Br	Krypton 36 Kr
5	Rubidium 37 Rb	Strontium 38 Sr	Yttrium 39 Y	Zirconium 40 Zr	Niobium 41 Nb	Molyb-denum 42 Mo	Technetium 43 Tc	Ruthenium 44 Ru	Rhodium 45 Rh	Palladium 46 Pd	Silver 47 Ag	Cadmium 48 Cd	Indium 49 In	Tin 50 Sn	Antimony 51 Sb	Tellurium 52 Te	Iodine 53 I	Xenon 54 Xe
6	Caesium 55 Cs	Barium 56 Ba	*	Hafnium 72 Hf	Tantalum 73 Ta	Tungsten 74 W	Rhenium 75 Re	Osmium 76 Os	Iridium 77 Ir	Platinum 78 Pt	Gold 79 Au	Mercury 80 Hg	Thallium 81 Tl	Lead 82 Pb	Bismuth 83 Bi	Polonium 84 Po	Astatine 85 At	Radon 86 Rn
7	Francium 87 Fr	Radium 88 Ra	**	Ruther-fordium 104 Rf	Dubnium 105 Db	Sea-borgium 106 Sg	Bohrium 107 Bh	Hassium 108 Hs	Meitnerium 109 Mt	Darm-stadtium 110 Ds	Roent-genium 111 Rg	Coper-nicium 112 Cn	Ununtrium 113 Uut	Flerovium 114 Fl	Unun-pentium 115 Uup	Liver-morium 116 Lv	Unun-septium 117 Uus	Unun-octium 118 Uuo
		*	Lanthanum 57 La	Cerium 58 Ce	Praseo-dymium 59 Pr	Neo-dymium 60 Nd	Prome-thium 61 Pm	Samarium 62 Sm	Europium 63 Eu	Gadolinium 64 Gd	Terbium 65 Tb	Dysprosium 66 Dy	Holmium 67 Ho	Erbium 68 Er	Thulium 69 Tm	Ytterbium 70 Yb	Lutetium 71 Lu	
		**	Actinium 89 Ac	Thorium 90 Th	Protac-tinium 91 Pa	Uranium 92 U	Neptunium 93 Np	Plutonium 94 Pu	Americium 95 Am	Curium 96 Cm	Berkelium 97 Bk	Californium 98 Cf	Einsteinium 99 Es	Fermium 100 Fm	Mende-levium 101 Md	Nobelium 102 No	Lawren-cium 103 Lr	

Copper: transition metal; primordial; solid

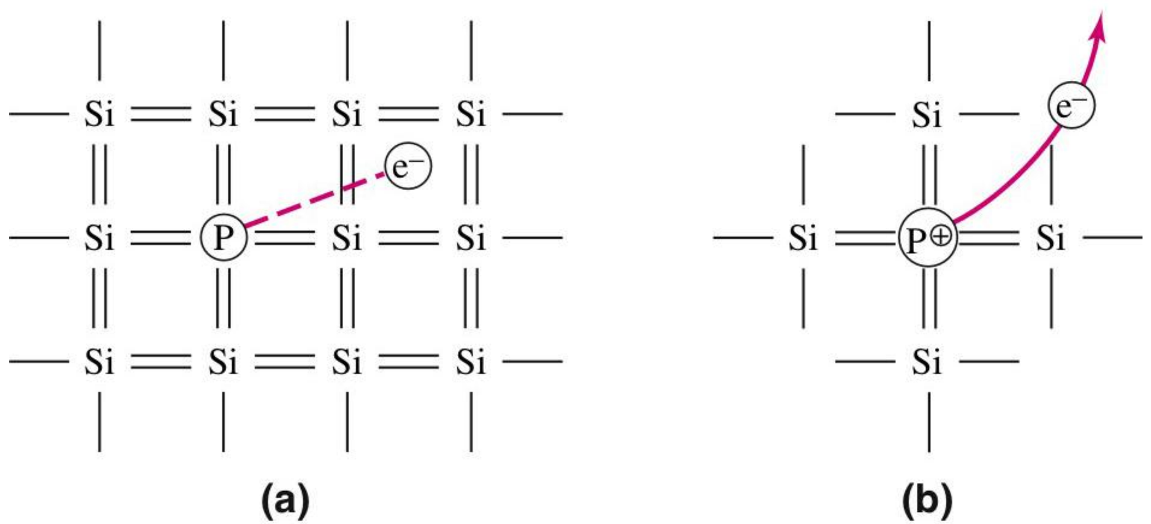
black=Solid | green=Liquid | red=Gas | grey=Unknown | **Color of the atomic number** shows state of matter (at 0 °C and 1 atm)

Primordial | From decay | Synthetic | **Border** shows natural occurrence of the element

Background color shows subcategory in the metal–nonmetal range:

Metal						Metalloid	Nonmetal			Unknown chemical properties
Alkali metal	Alkaline earth metal	Lanthanide	Actinide	Transition metal	Post-transition metal		Polyatomic nonmetal	Diatomic nonmetal	Noble gas	

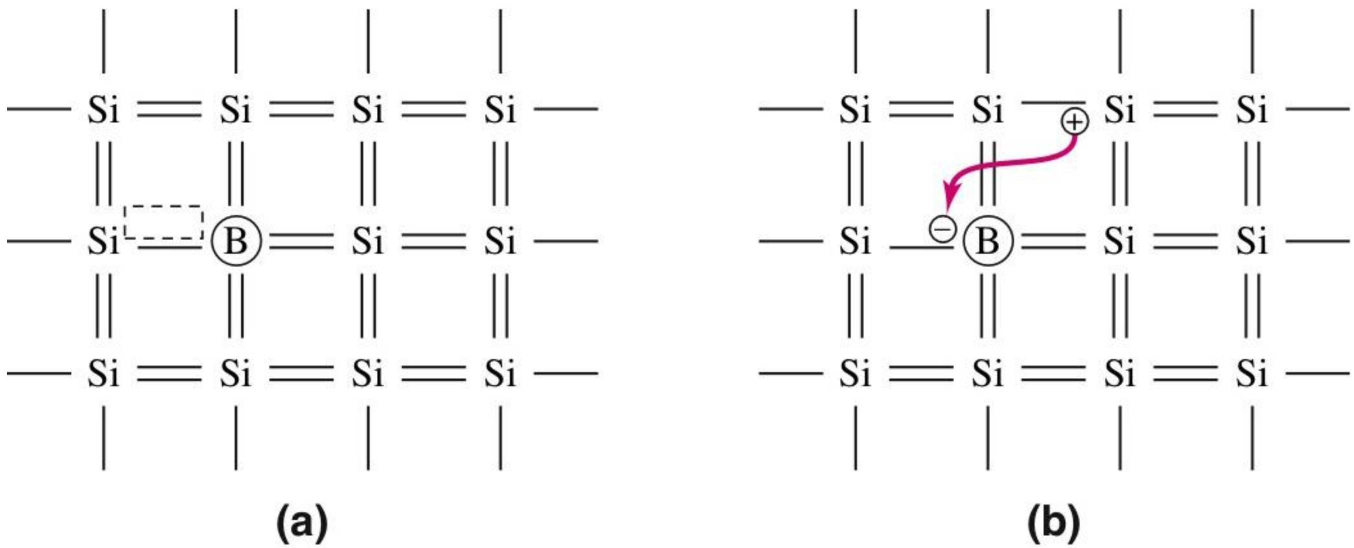
Phosphorous – Donor Impurity in Si



Phosphorous (P) replaces a Si atom and forms four covalent bonds with other Si atoms.

The **fifth** outer shell electron of P is easily freed to become a conduction band electron, adding to the number of electrons available to conduct current.

Boron – Acceptor Impurity in Si



Boron (B) replaces a Si atom and forms only **three** covalent bonds with other Si atoms.

The missing covalent bond is a hole, which can begin to move through the crystal when a valence electron from another Si atom is taken to form the fourth B-Si bond.

Electron and Hole Concentrations

n = electron concentration

p = hole concentration

$$n_i^2 = n \cdot p$$

n-type:

n = N_D, the donor concentration

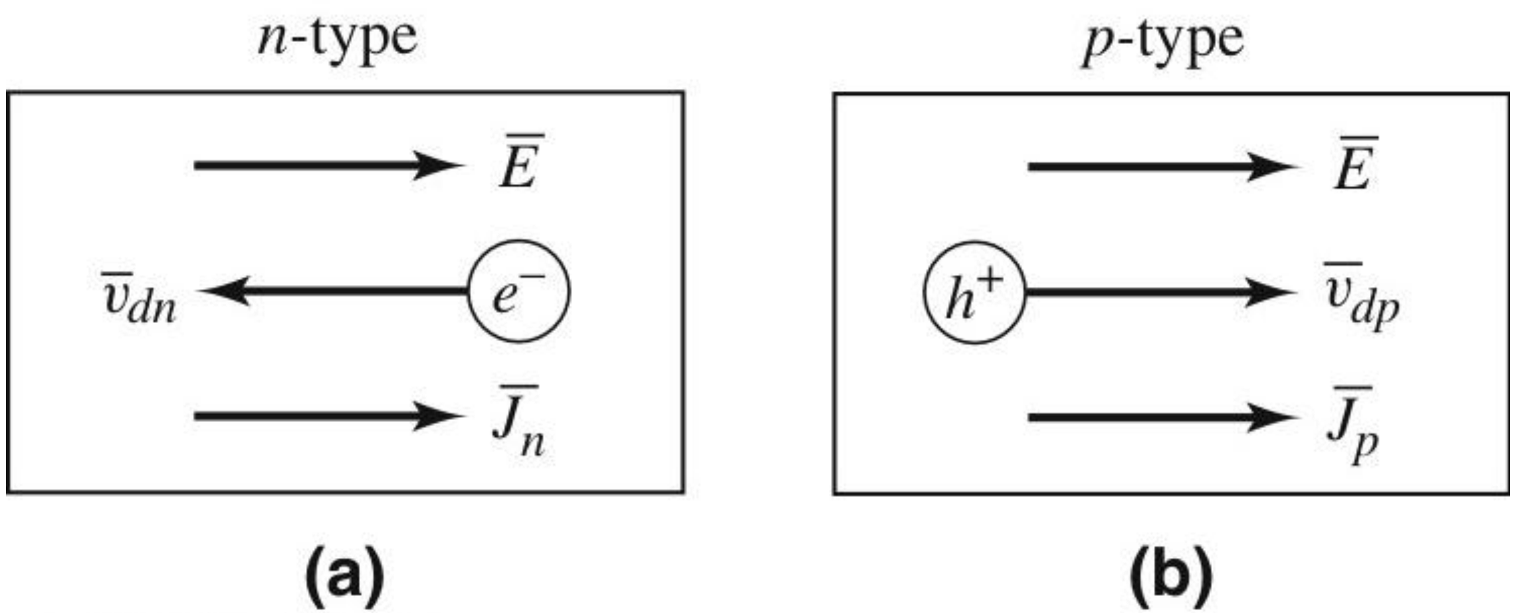
$$p = n_i^2 / N_D$$

p-type:

p = N_A, the acceptor concentration

$$n = n_i^2 / N_A$$

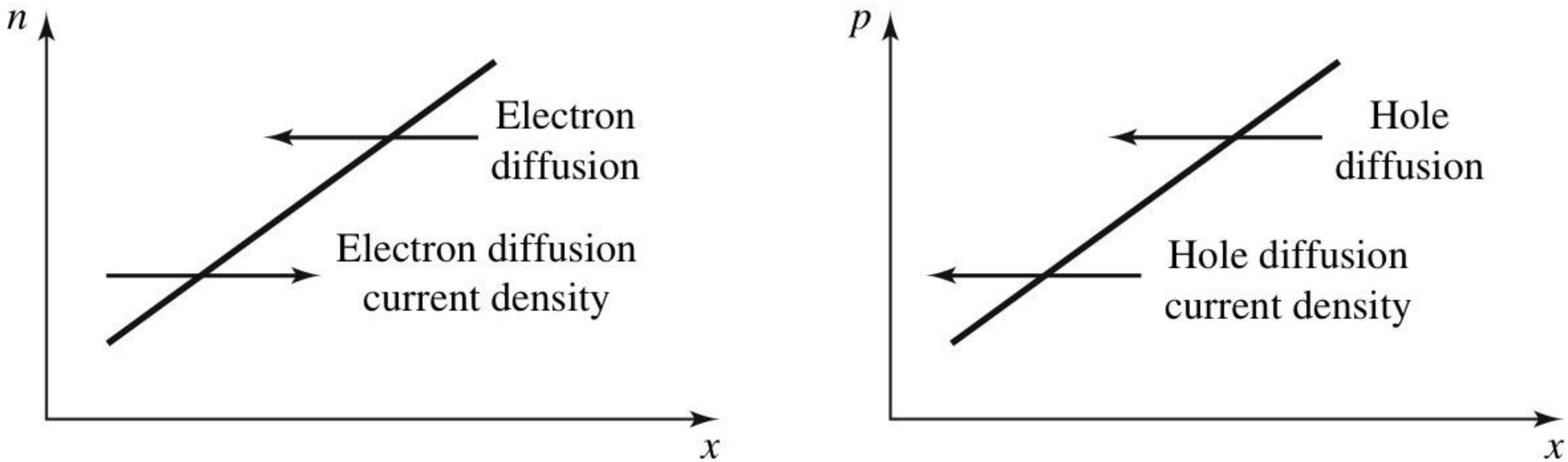
Drift Currents



Electrons and hole flow in opposite directions when under the influence of an electric field at different velocities.

The drift currents associated with the electrons and holes are in the same direction.

Diffusion Currents



(a) (b)

Both electrons and holes flow from high concentration to low.

The diffusion current associated with the electrons flows in the opposite direction when compared to that of the holes.

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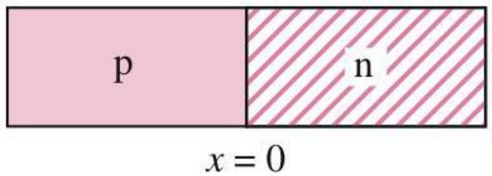
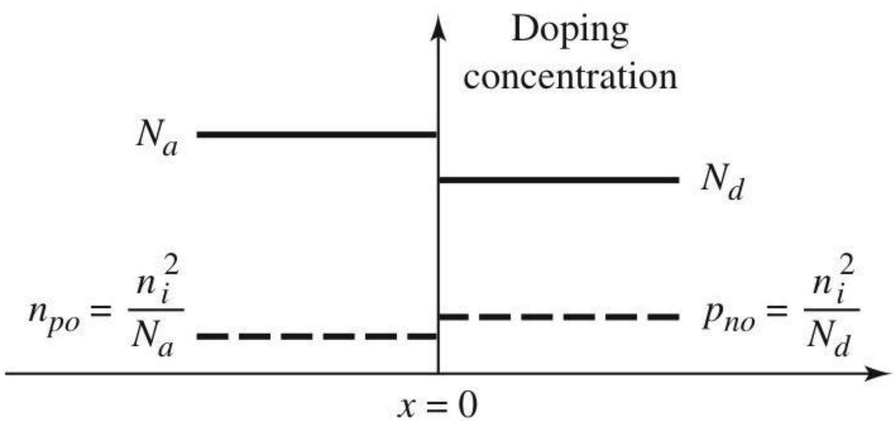
p-n Junctions

A simplified 1-D sketch of a p-n junction (a) has a doping profile (b).

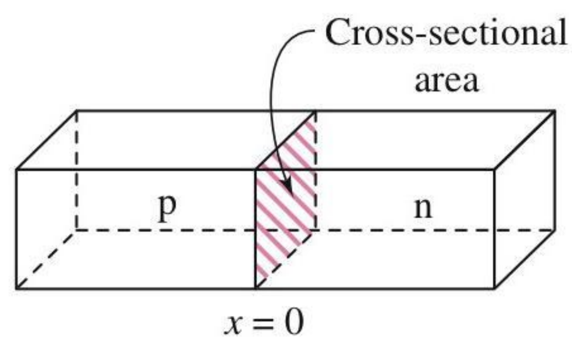
The 3-D representation (c) shows the cross sectional area of the junction.



(a)

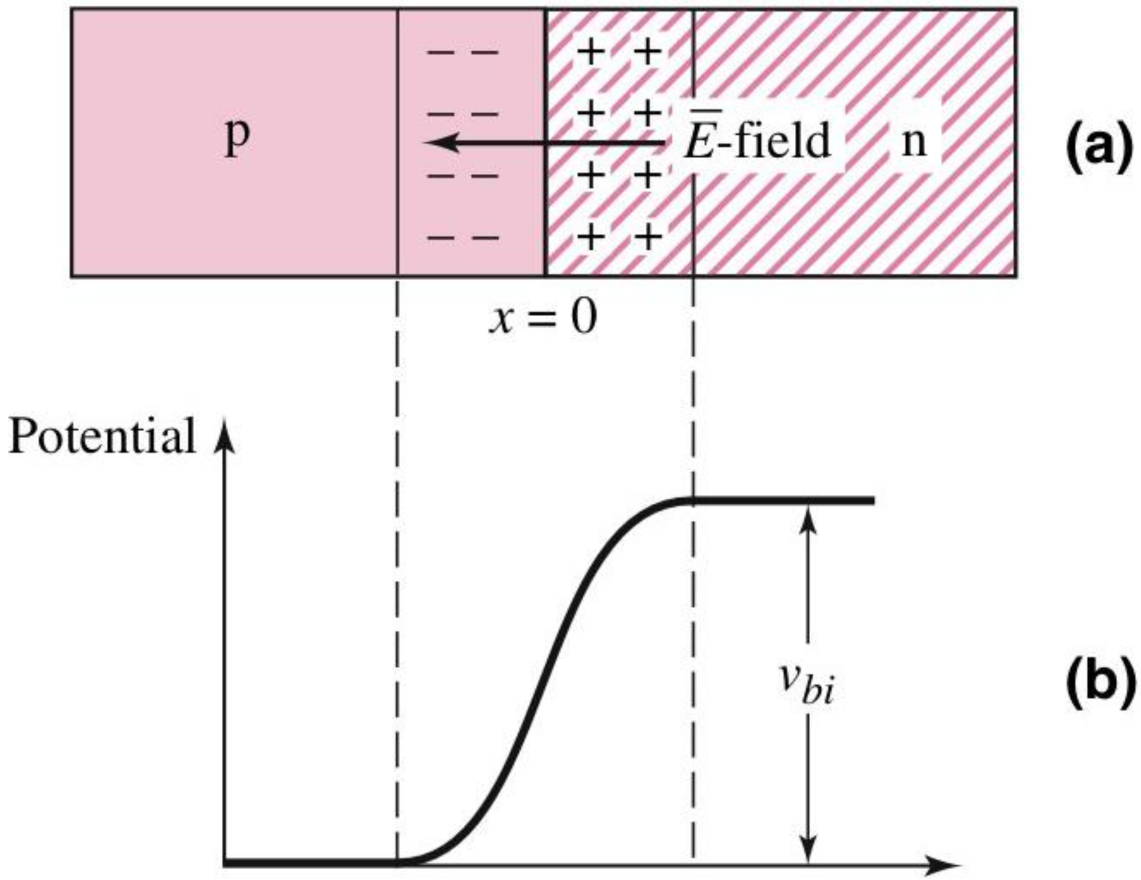


(b)



(c)

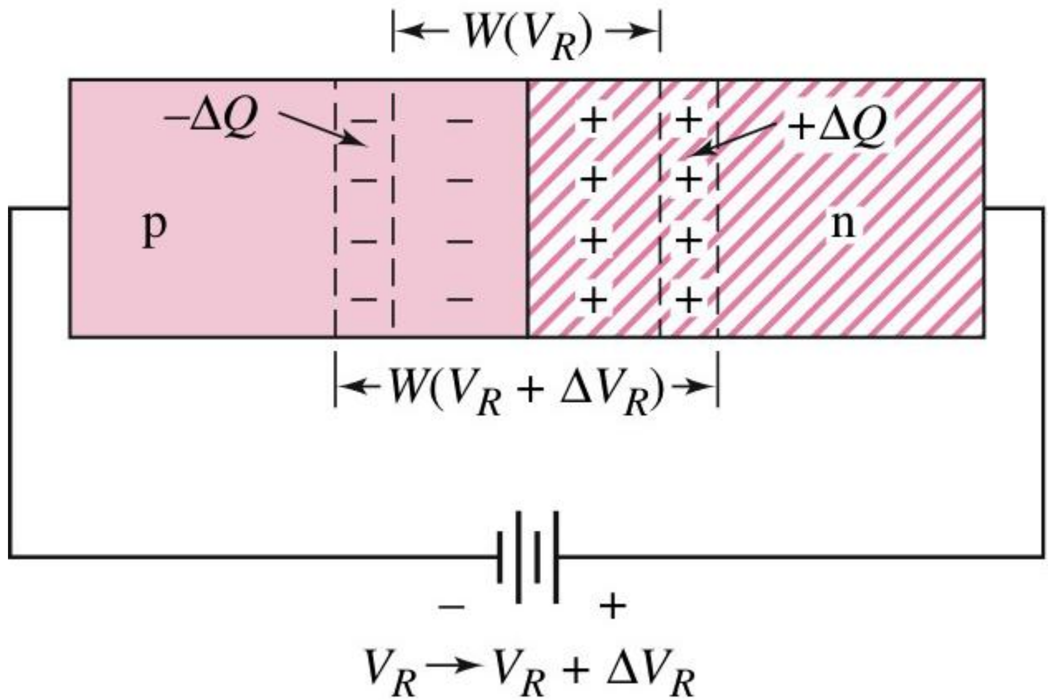
Built-in Potential



This movement of carriers creates a space charge or depletion region with an induced electric field near x = 0.

A potential voltage, v_{bi}, is developed across the junction.

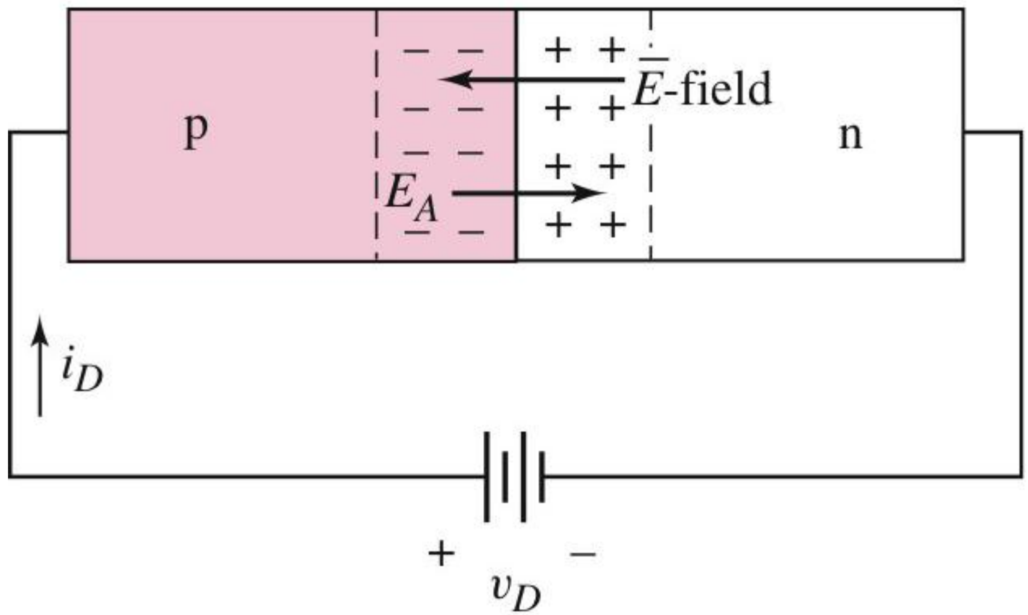
Reverse Bias



Increase in space-charge width, W, as V_R increases to $V_R+\Delta V_R$.

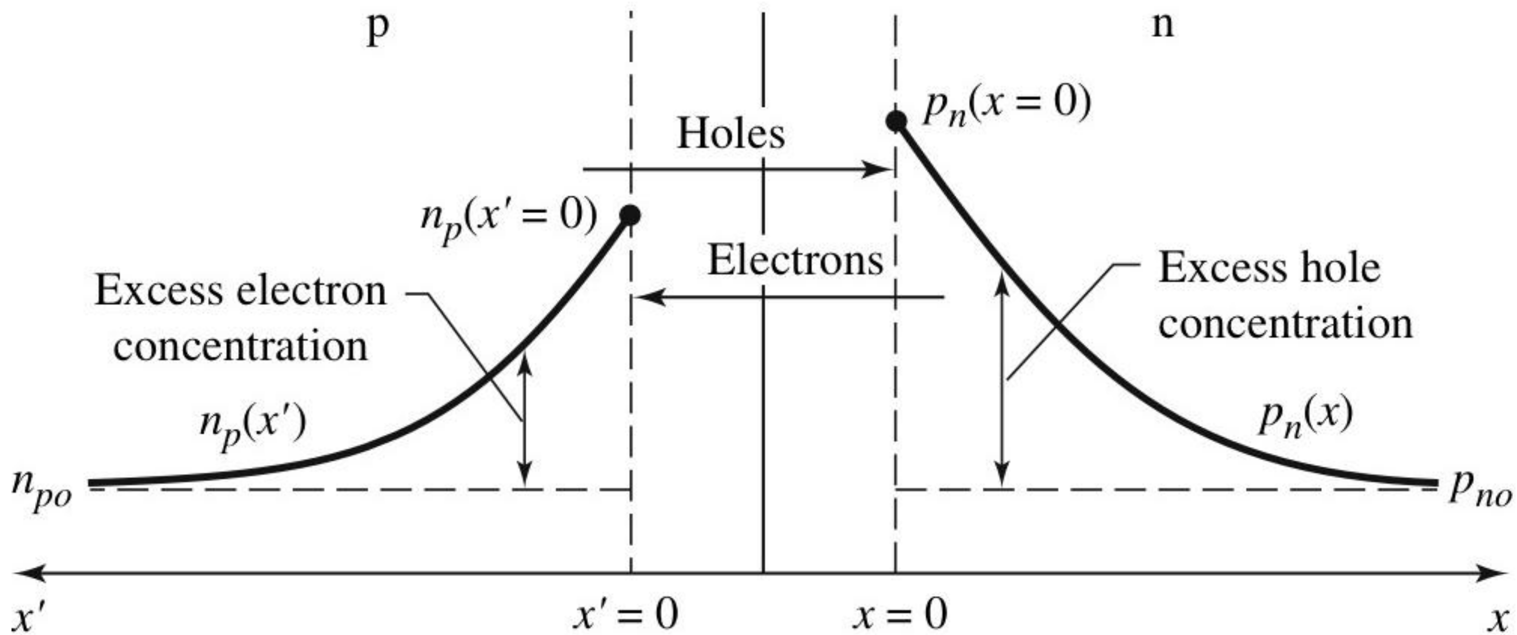
Creation of more fixed charges ($-\Delta Q$ and $+\Delta Q$) leads to junction capacitance.

Forward Biased p-n Junction



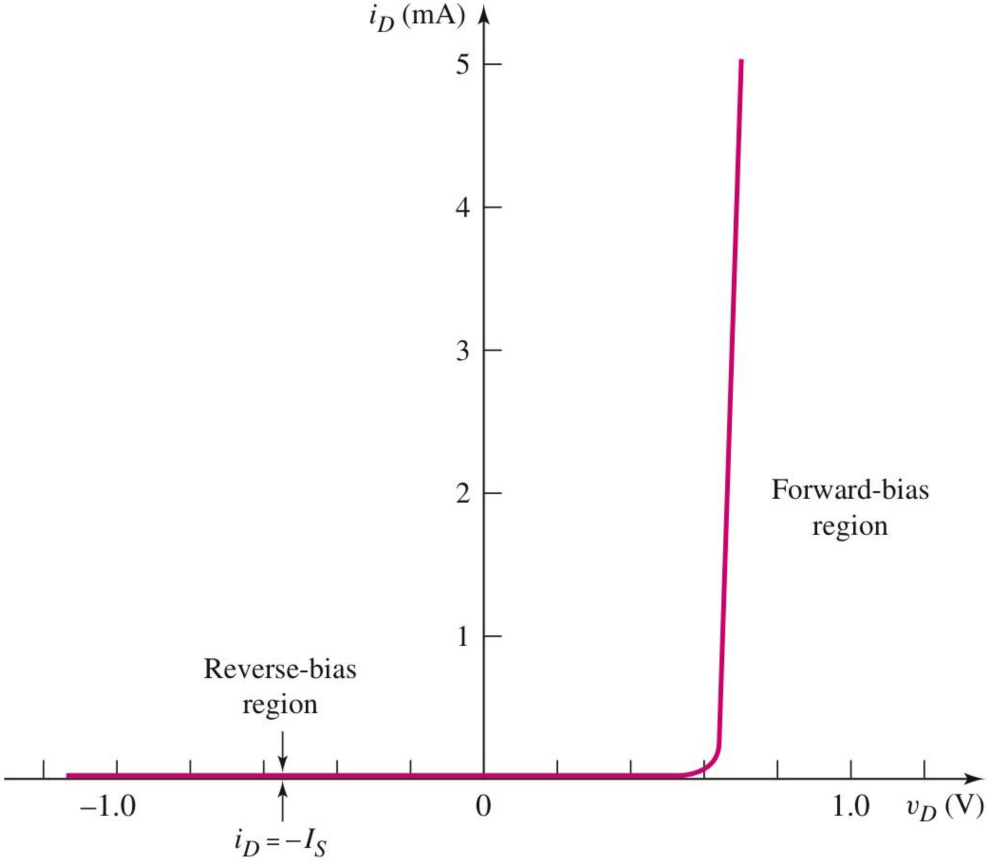
Applied voltage, v_D, induces an electric field, E_A, in the opposite direction as the original space-charge electric field, resulting in a smaller net electric field and smaller barrier between n and p regions.

Minority Carrier Concentrations



Gradients in minority carrier concentration generates diffusion currents in diode when forward biased.

Ideal Current-Voltage (I-V) Characteristics



The p-n junction only conducts significant current in the forward-bias region.

i_D is an exponential function in this region.

Essentially no current flows in reverse bias.

Ideal Diode Equation

A fit to the I-V characteristics of a diode yields the following equation, known as the ideal diode equation:

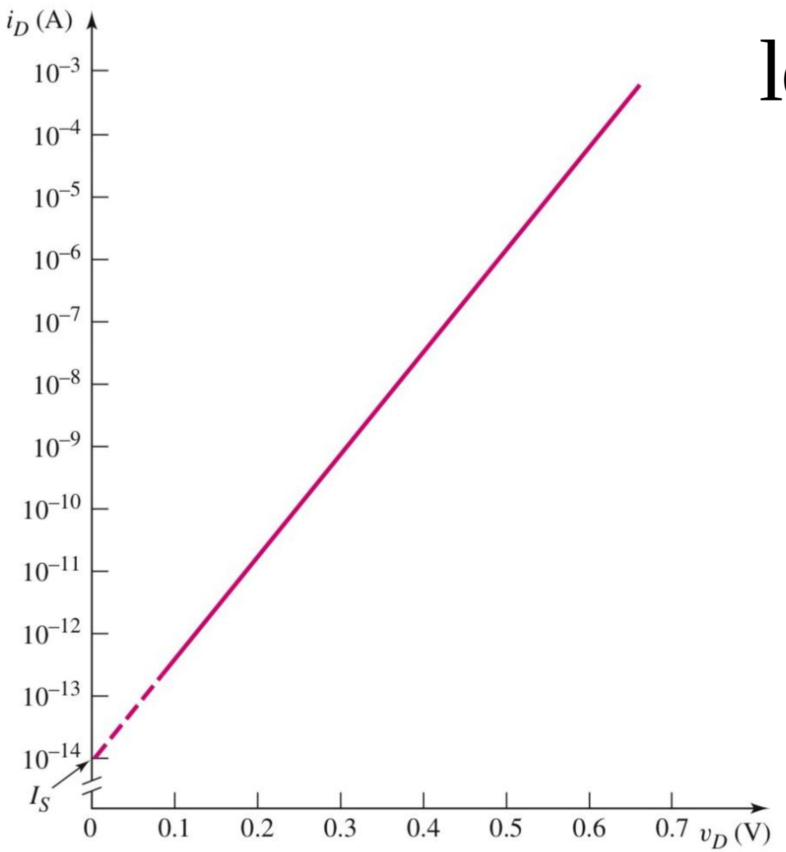
$$I_D = I_s\left(e^{\frac{qv_D}{nkT}} - 1\right)$$

kT/q is also known as the thermal voltage, V_T.

V_T = 25.9 mV when T = 300K, room temperature.

$$I_D = I_s\left(e^{\frac{v_D}{V_T}} - 1\right)$$

Ideal Diode Equation



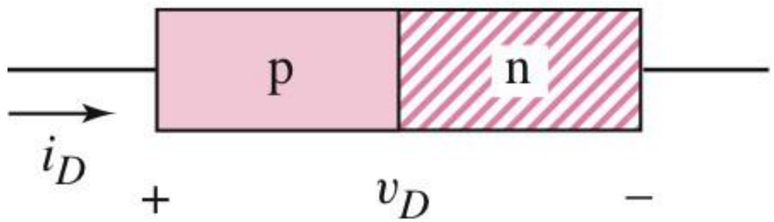
$$\log(i_D) \approx \frac{\log e}{nV_T} v_D + \log(I_s)$$

The y intercept is equal to I_S.

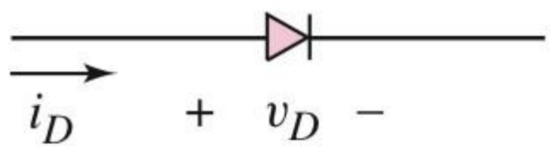
The slope is proportional to 1/n.

When n = 1, i_D increased by ~ one order of magnitude for every 60-mV increase in v_D.

Circuit Symbol



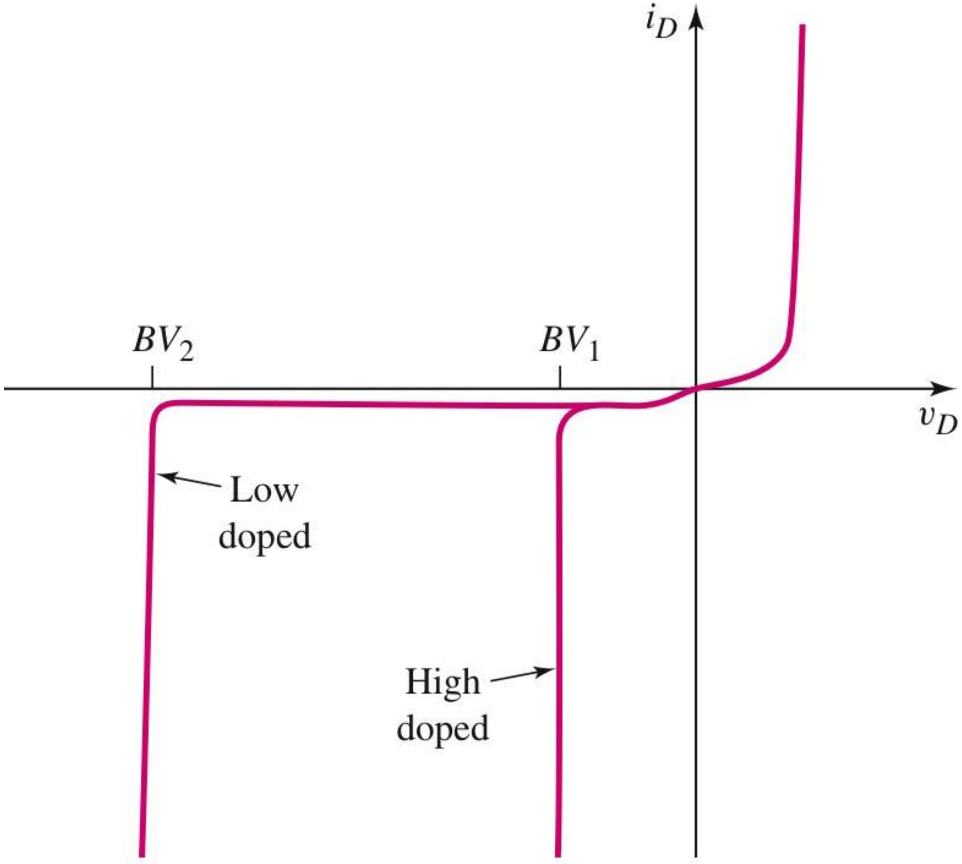
(a)



(b)

Conventional current direction and polarity of voltage drop is shown

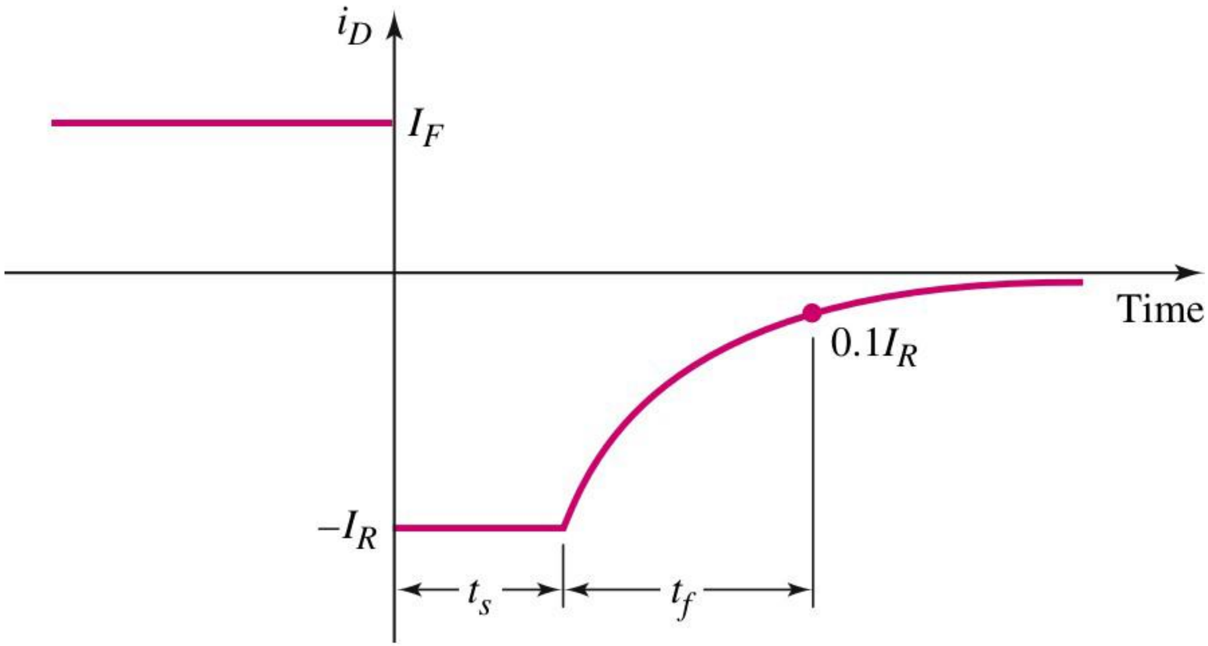
Breakdown Voltage



The magnitude of the breakdown voltage (BV) is smaller for heavily doped diodes as compared to more lightly doped diodes.

Current through a diode increases rapidly once breakdown has occurred.

Transient Response



Short reverse-going current pulse flows when the diode is switched from forward bias to zero or reverse bias as the excess minority carriers are removed.

It is composed of a storage time, t_s, and a fall time, t_f.

Goals for Chapter 1

- Gain a basic understanding of semiconductor material properties
  - Two types of charged carriers that exist in a semiconductor
  - Two mechanisms that generate currents in a semiconductor

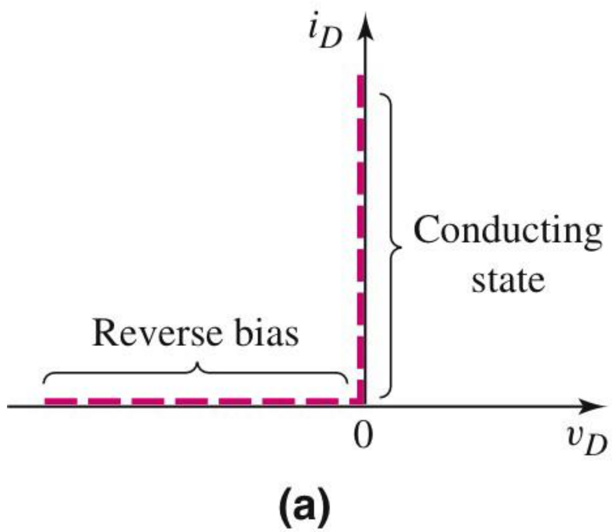
- Determine the properties of a pn junction
  - Ideal current–voltage characteristics of a pn junction diode

- Examine dc analysis techniques for diode circuits using various models to describe the nonlinear diode characteristics

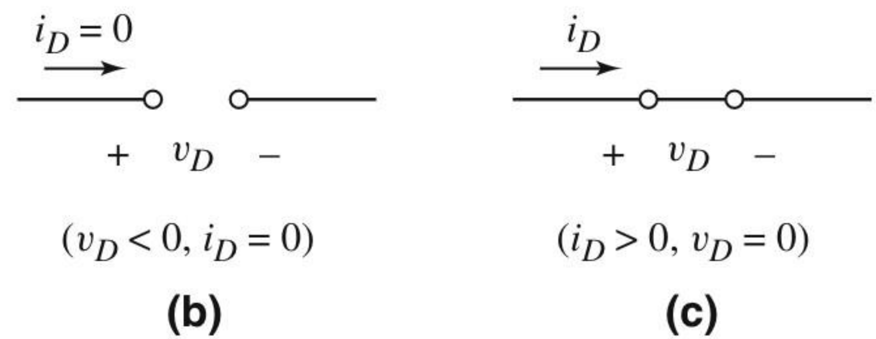
- Develop an equivalent circuit for a diode that is used when a small, time-varying signal is applied to a diode circuit

- Gain an understanding of the properties and characteristics of a few specialized diodes

dc Model of Ideal Diode



Equivalent Circuits

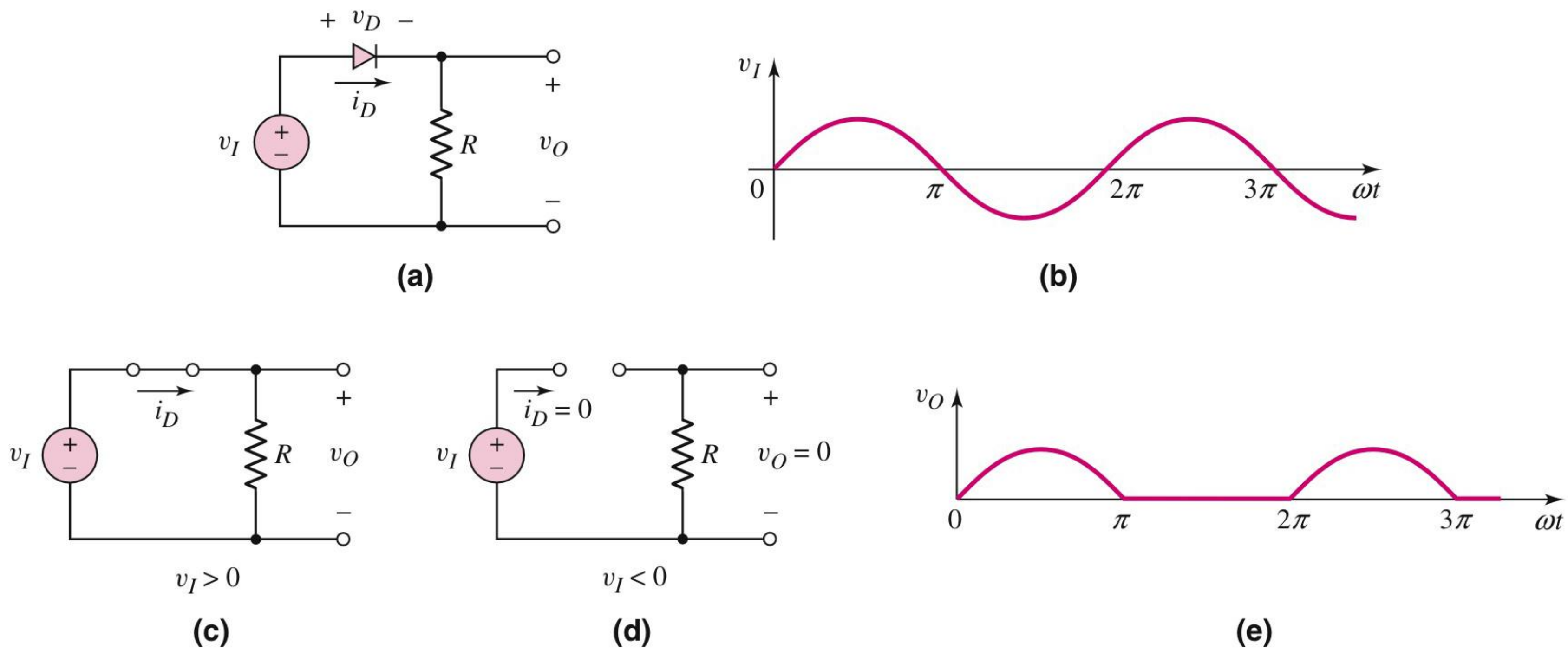


Assumes v_{bi} = 0.

No current flows when reverse biased (b).

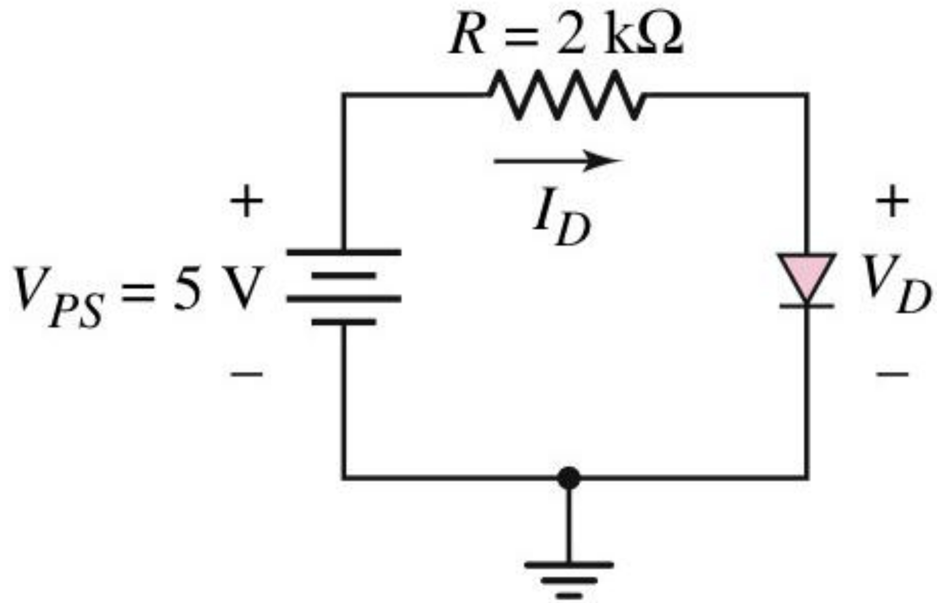
No internal resistance to limit current when forward biased (c).

Half-Wave Diode Rectifier



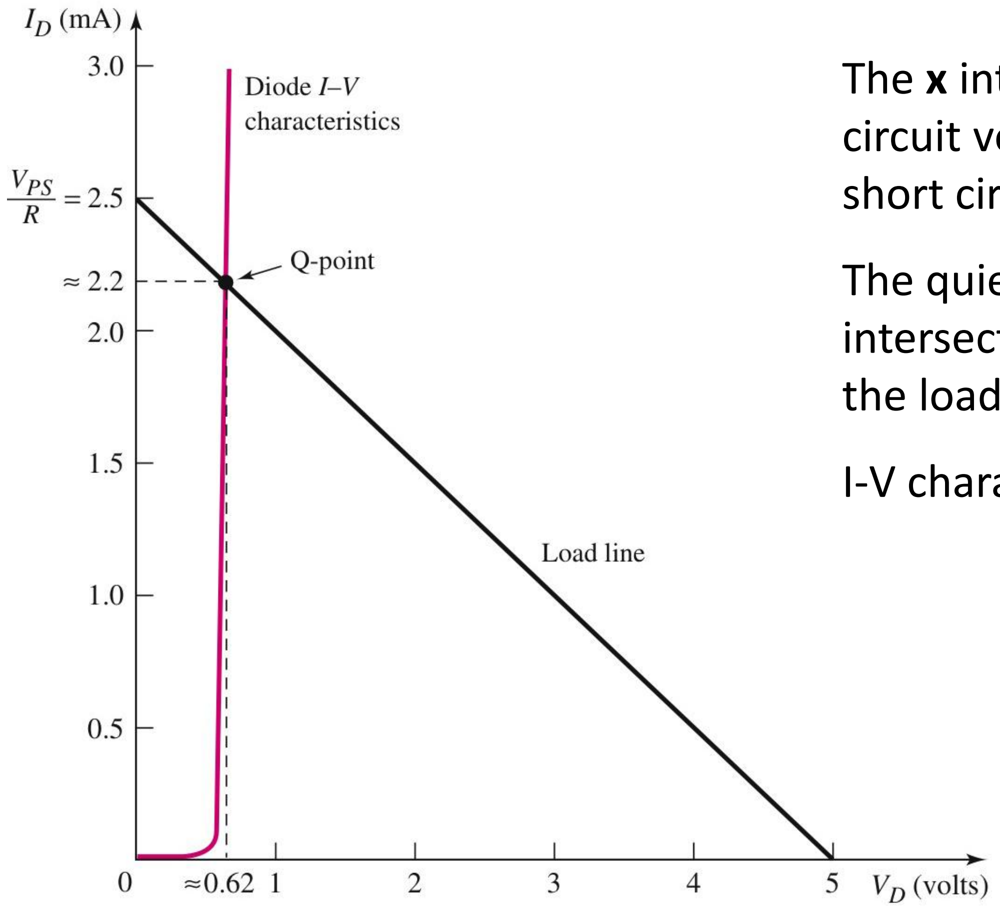
Diode only allows current to flow through the resistor when $v_I \geq 0V$. Forward-bias equivalent circuit is used to determine v_O under this condition.

Graphical Analysis Technique



Simple diode circuit where I_D and V_D are not known.

Load Line Analysis

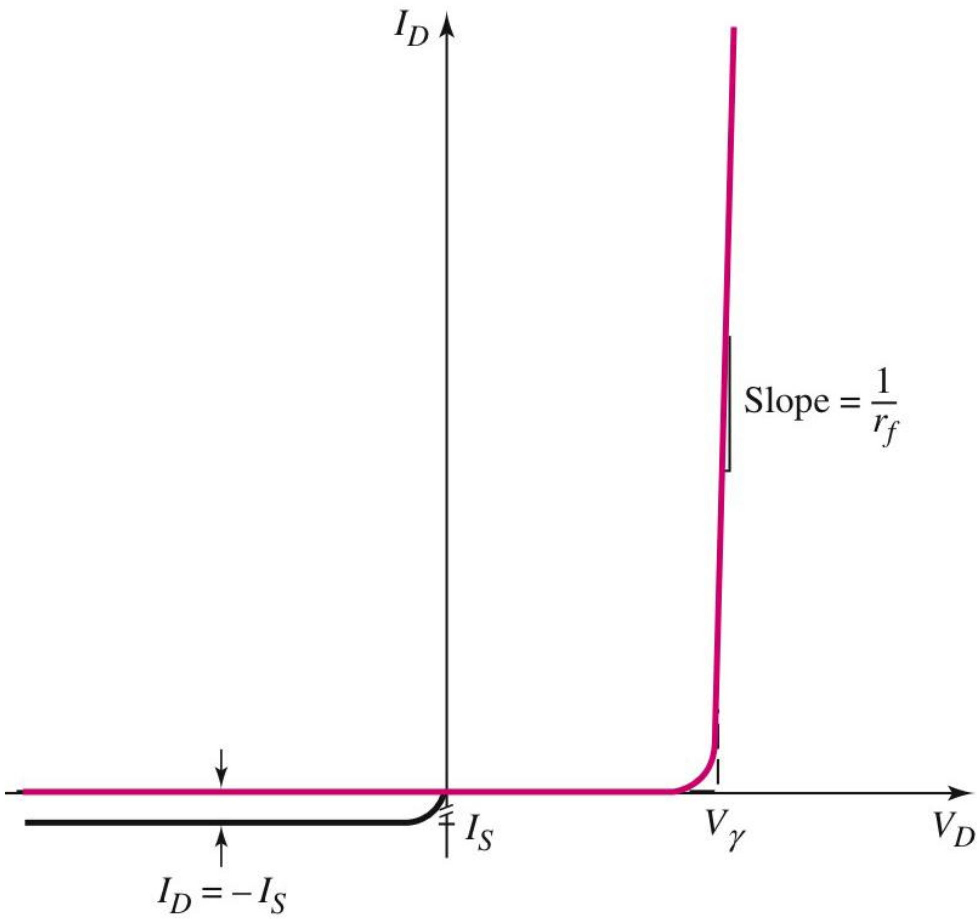


The **x** intercept of the load line is the open circuit voltage and the **y** intercept is the short circuit current.

The quiescent point or Q-point is the intersection of diode I-V characteristic with the load line.

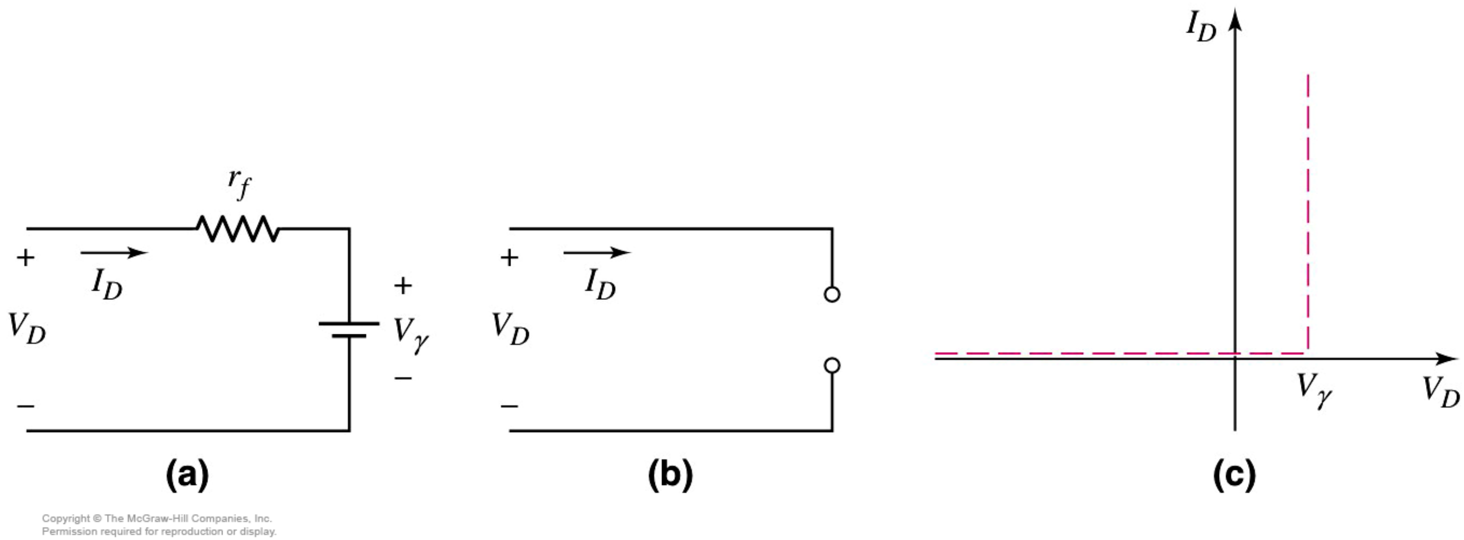
I-V characteristics of diode must be known.

Piecewise Linear Model



Two linear approximations are used to form piecewise linear model of diode.

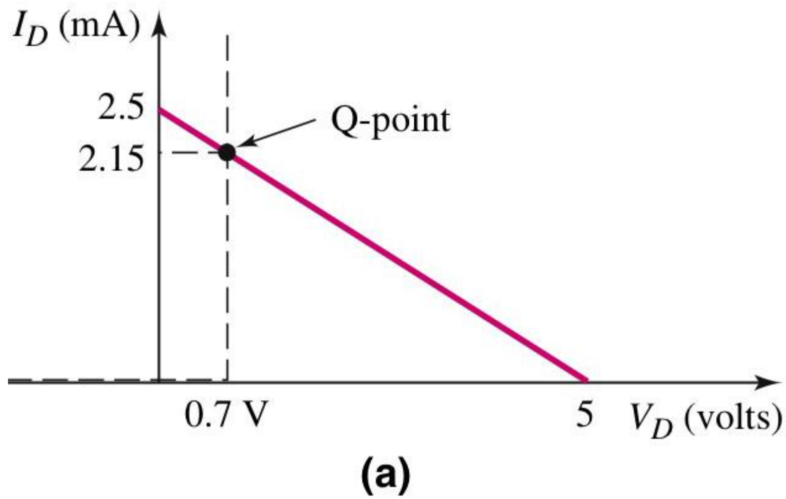
Diode Piecewise Equivalent Circuit

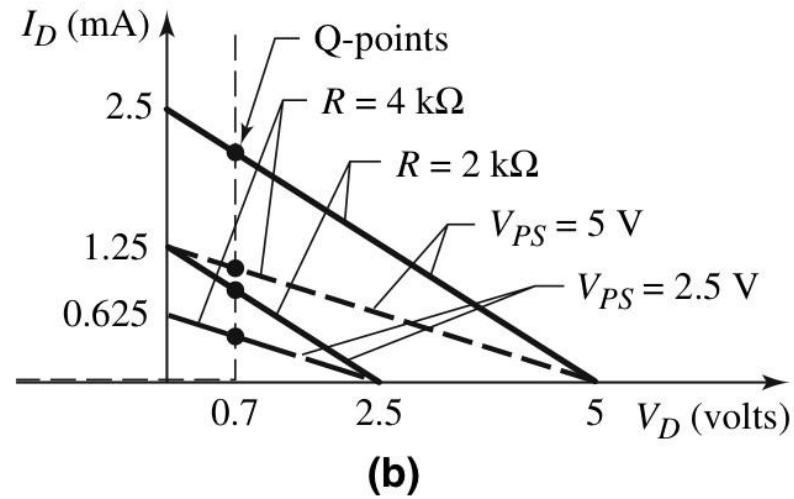


The diode is replaced by a battery with voltage, V_γ, with a a resistor, r_f, in series when in the 'on' condition (a) and is replaced by an open when in the 'off' condition, $V_D < V_\gamma$.

If $r_f = 0$, $V_D = V_\gamma$ when the diode is conducting.

Q-point

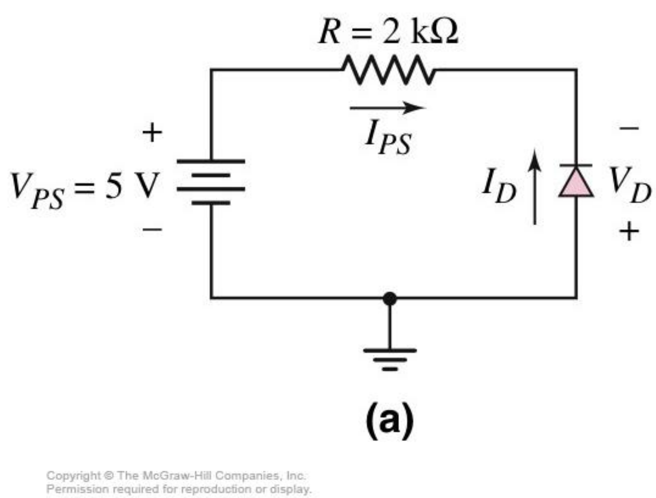


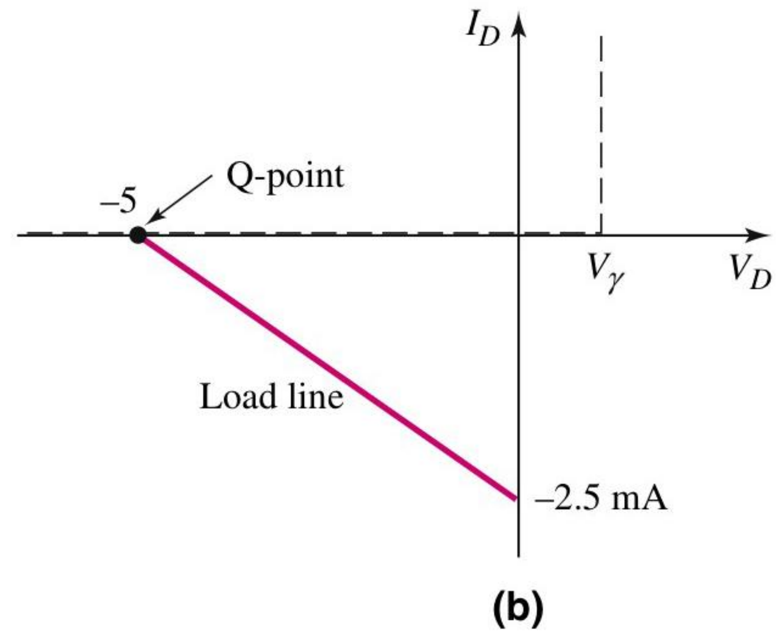


The **x** intercept of the load line is the open circuit voltage and the **y** intercept is the short circuit current.

The Q-point is dependent on the power supply voltage and the resistance of the rest of the circuit as well as on the diode I-V characteristics.

Load Line: Reverse Biased Diode





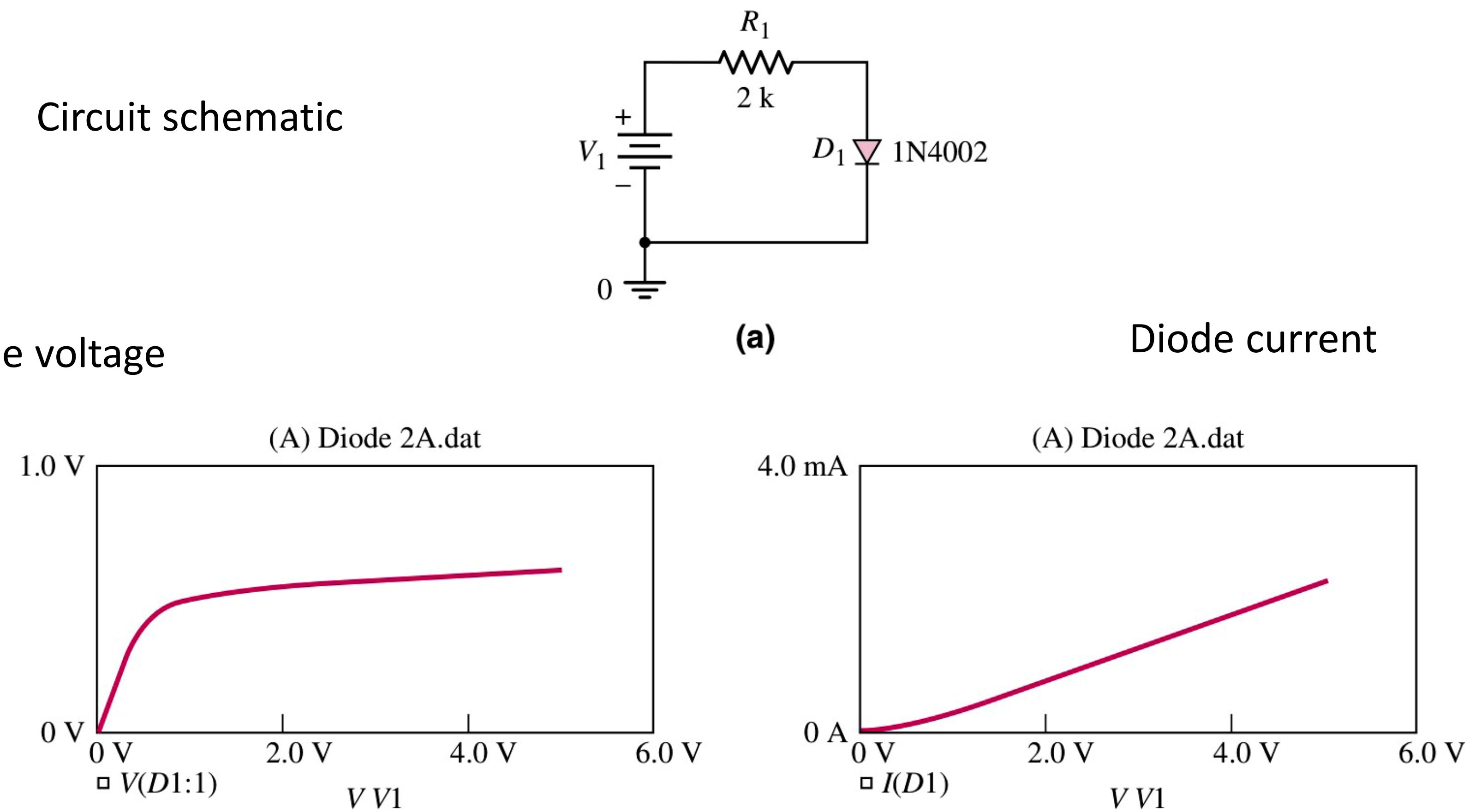
The Q-point is always $I_D = 0$ and V_D = the open circuit voltage when using the piecewise linear equivalent circuit.

PSpice Analysis

Circuit schematic

Diode voltage

Diode current



(a)

(b)

(c)

Goals for Chapter 1

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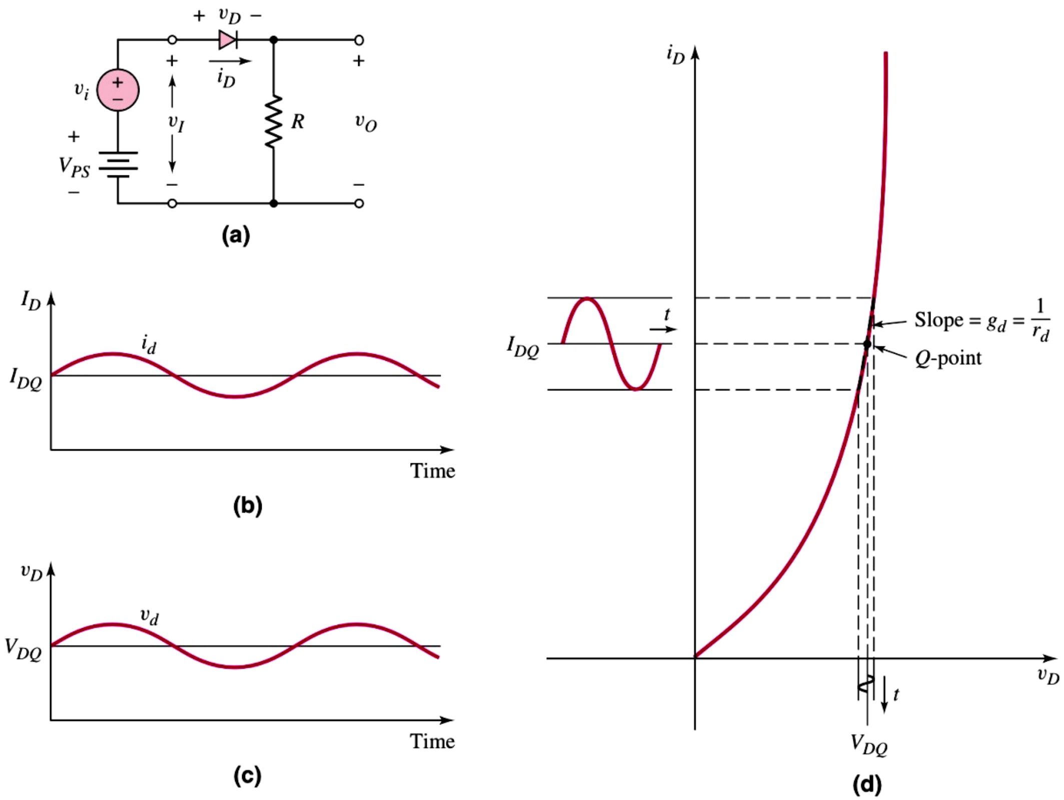
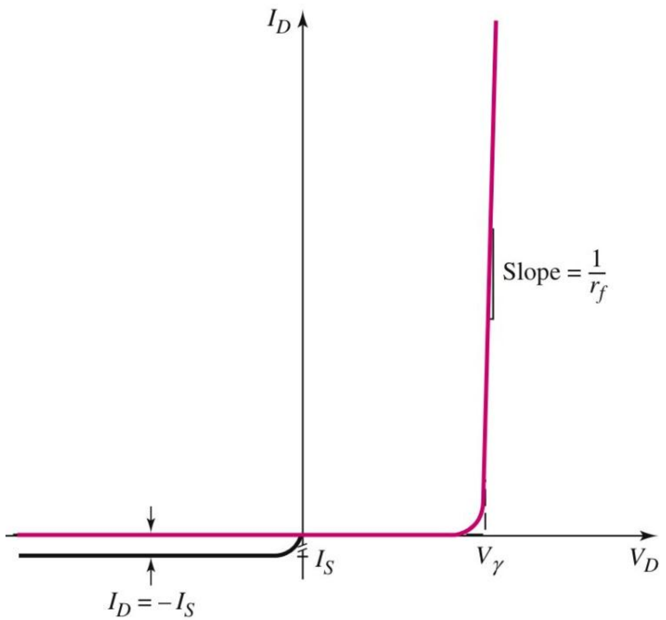
ac Circuit Analysis

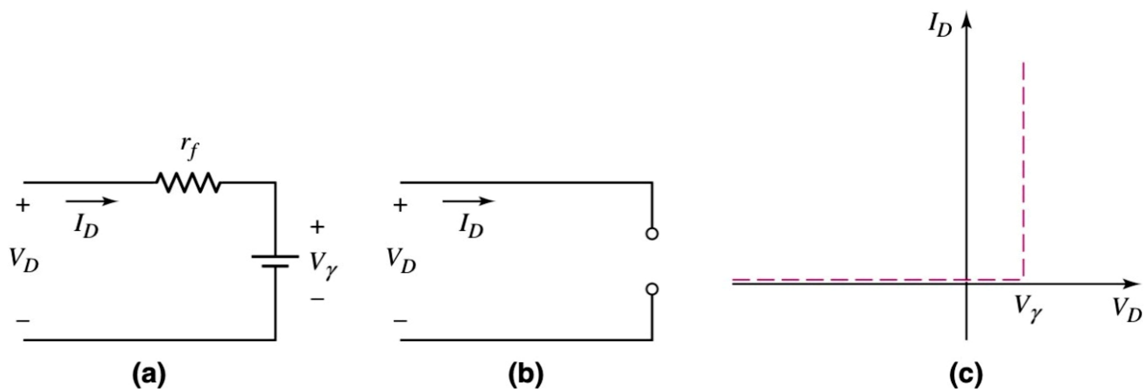


Combination of dc and sinusoidal input voltages modulate the operation of the diode about the Q-point.

Comparison

- Diode Piecewise Equivalent Circuit

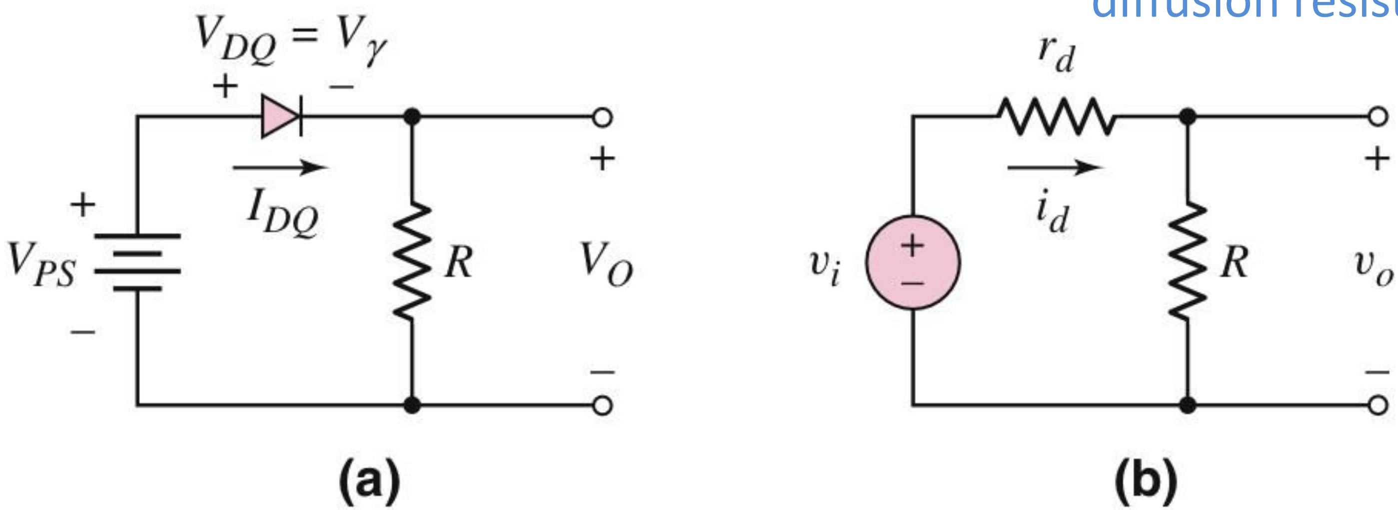




r_f is the forward diode resistance.
V_γ is the turn on voltage.

Equivalent Circuits

r_d is the small signal incremental resistance, or diffusion resistance.

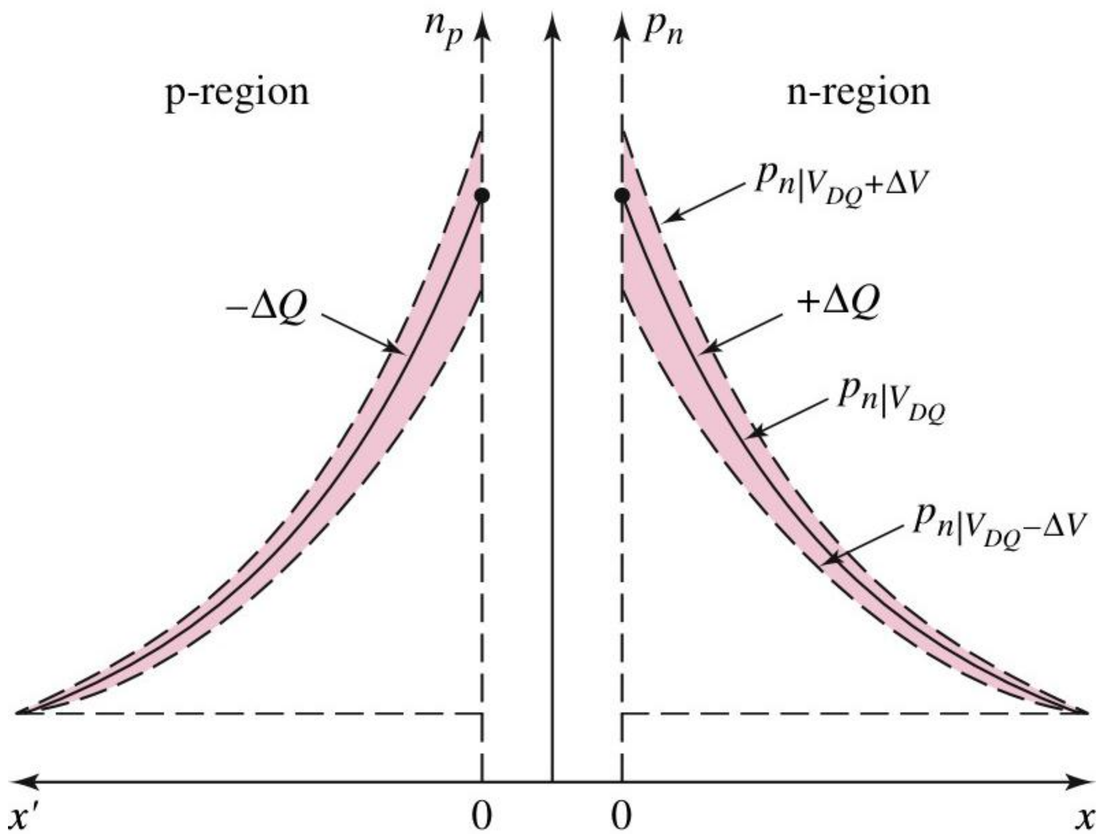


When ac signal is small, the dc operation can be decoupled from the ac operation.

First perform dc analysis using the dc equivalent circuit (a).

Then perform the ac analysis using the ac equivalent circuit (b).

Minority Carrier Concentration

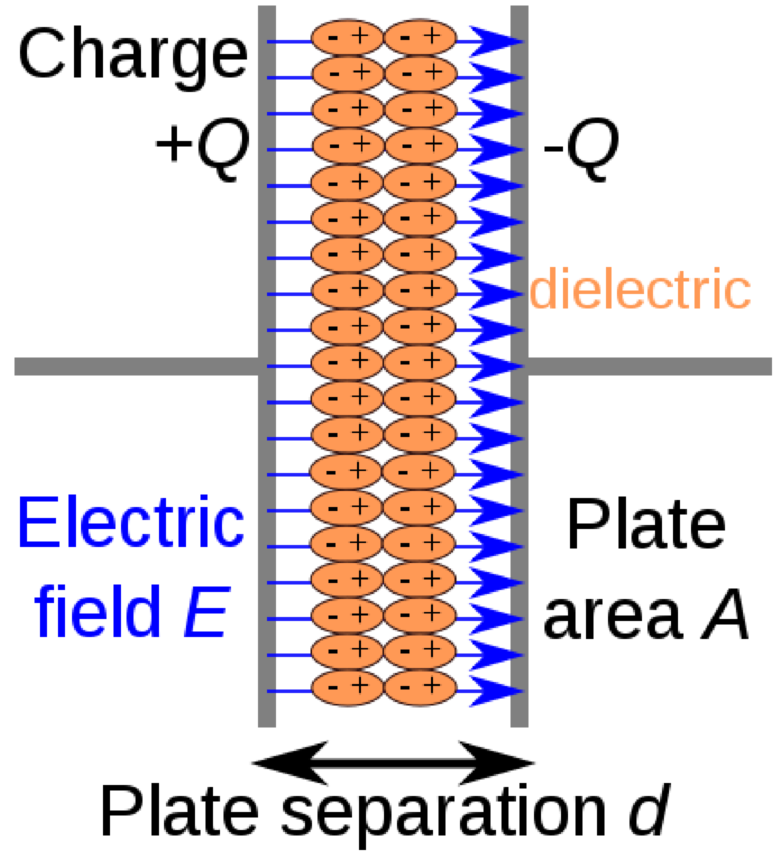


Time-varying excess charge leads to diffusion capacitance.

Capacitance

- An ideal capacitor is wholly characterized by a constant capacitance C, defined as the ratio of charge $\pm Q$ on each conductor to the voltage V between them:

$$C = \frac{Q}{V}$$

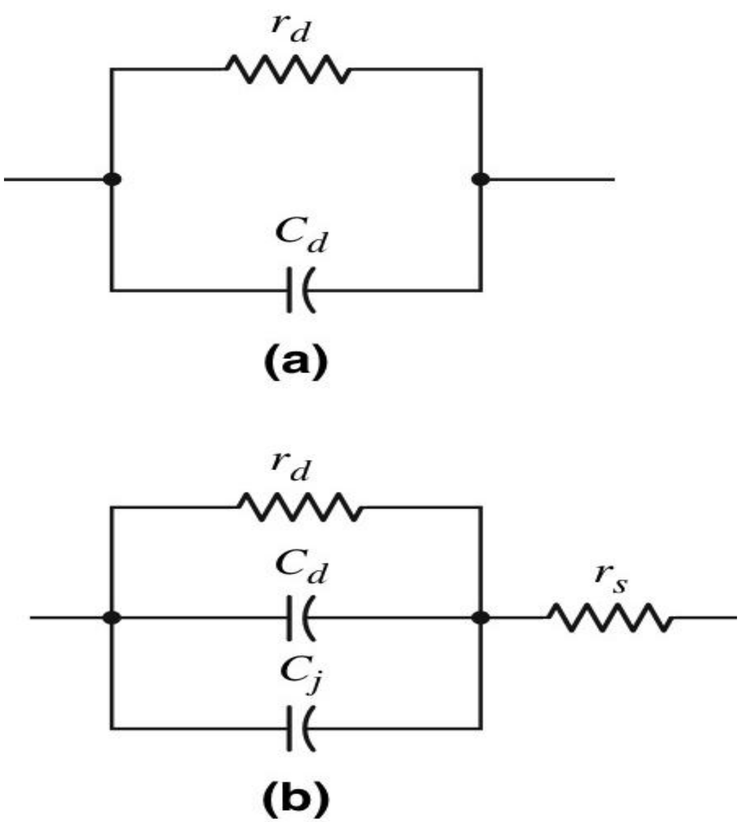


Capacitance

- Sometimes charge build-up affects the capacitor mechanically, causing its capacitance to vary. In this case, capacitance is defined in terms of incremental changes

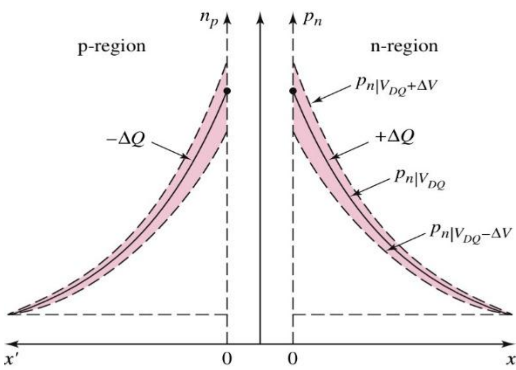
$$C = \frac{\mathrm{d}Q}{\mathrm{d}V}$$

Small Signal Equivalent Model



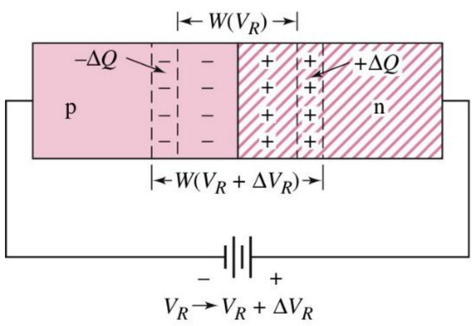
Simplified model, which can only be used when the diode is forward biased.

$$C_d = \frac{dQ}{dV_D}$$



Complete model

$$C_j = C_{j0}\left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$$



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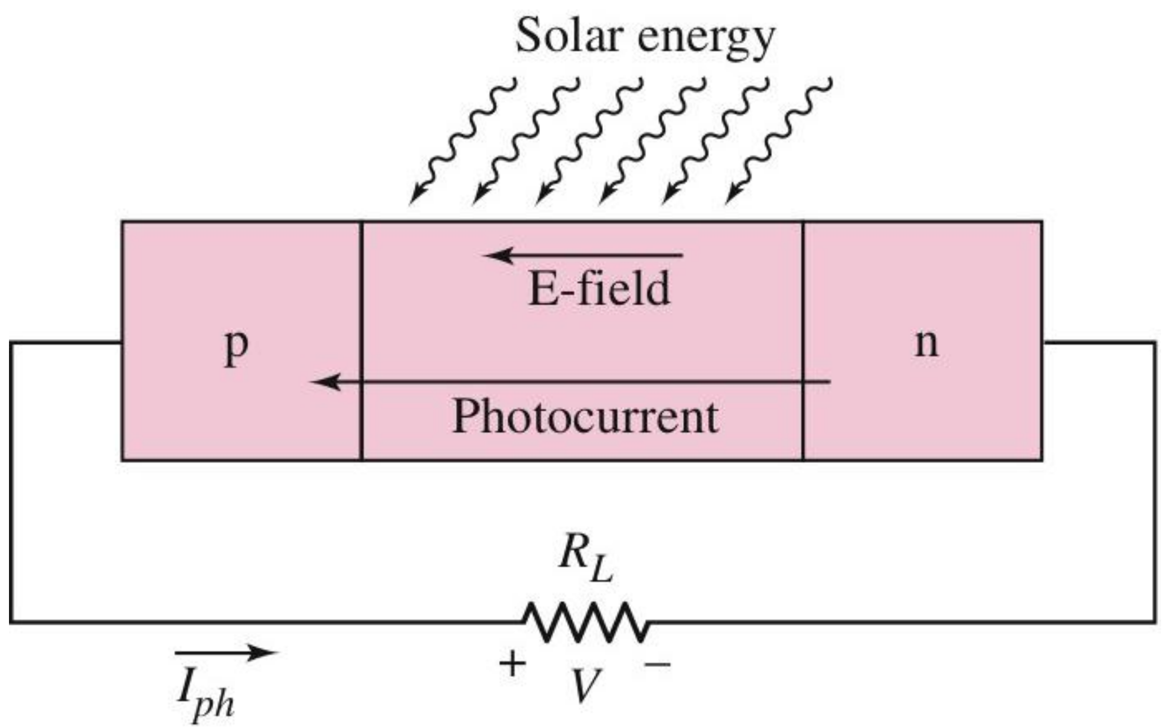
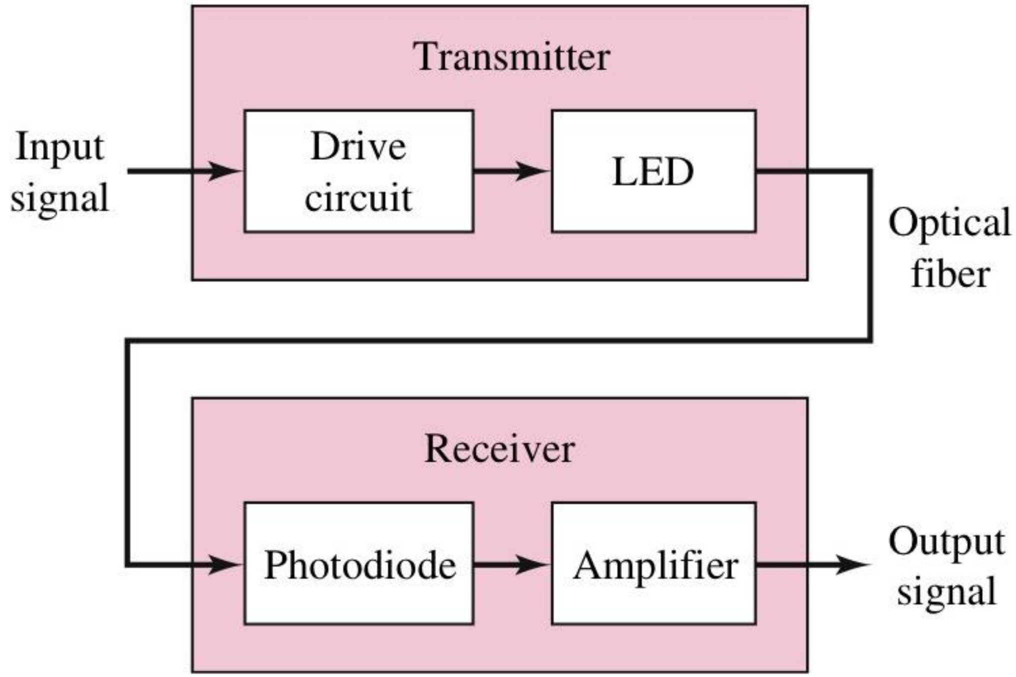
Photogenerated Current



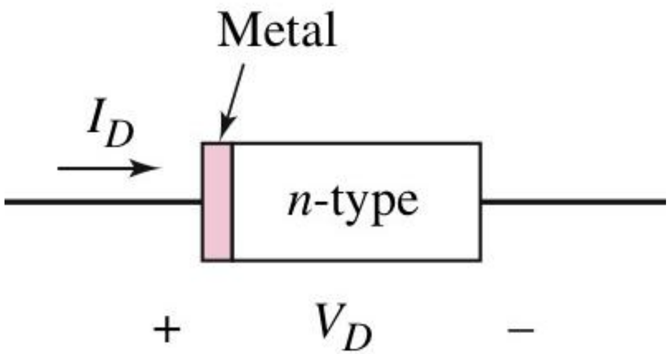
When the energy of the photons is greater than E_g, the photon's energy can be used to break covalent bonds and generate an equal number of electrons and holes to the number of photons absorbed.

Optical Transmission System



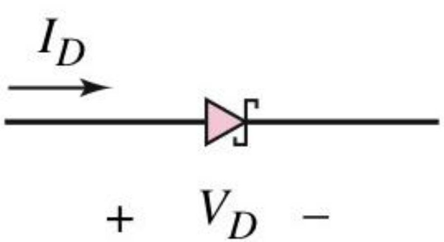
LED (**L**ight **E**mitting **Diode**) and photodiode are p-n junctions.

Schottky Barrier Diode



(a)

A metal layer replaces the p region of the diode.

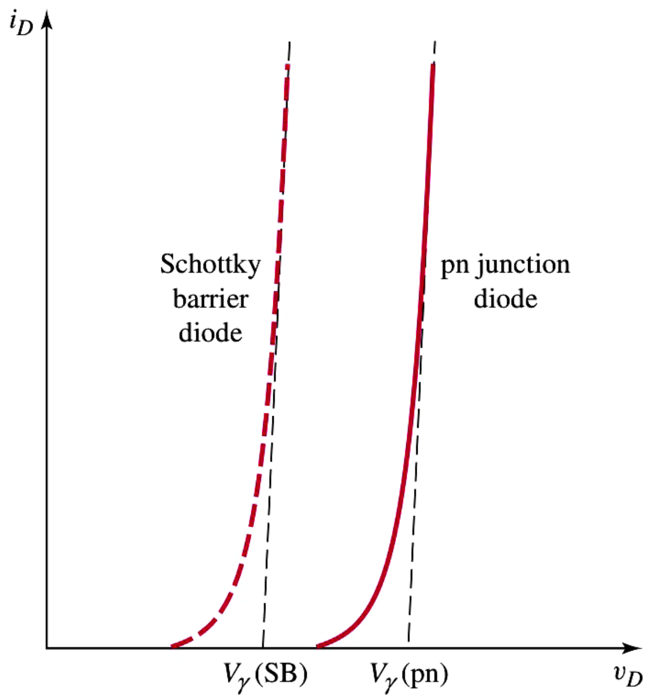


(b)

Circuit symbol showing conventional current direction of current and polarity of voltage drop.

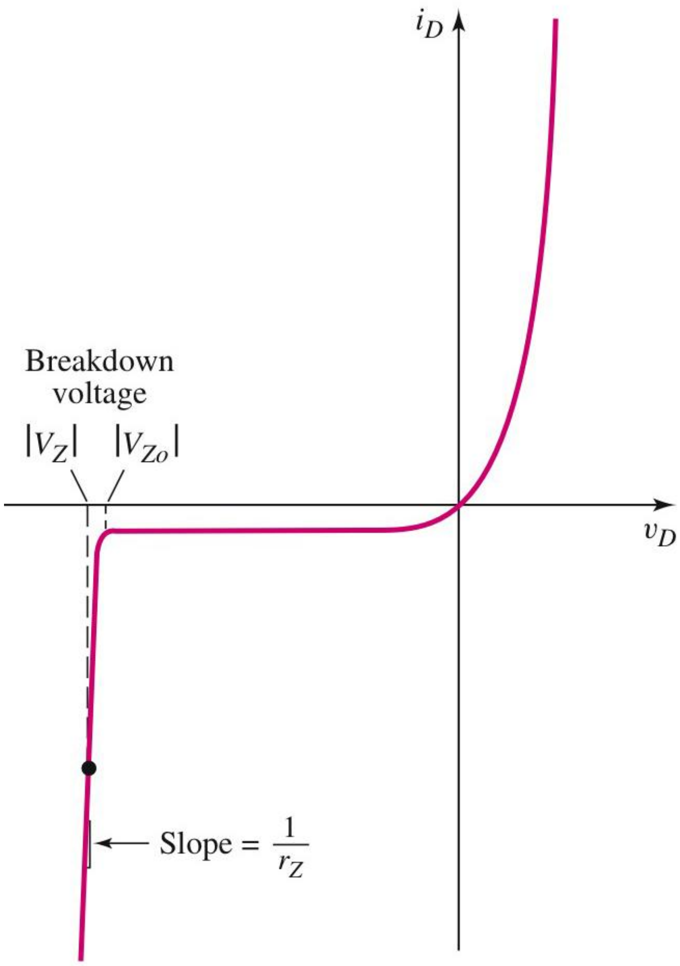
Comparison of I-V Characteristics:

Forward Bias

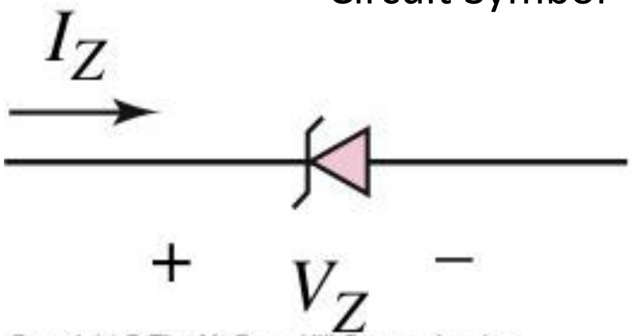


The built-in voltage of the Schottky barrier diode, V_γ(SB), is about ½ as large as the built-in voltage of the p-n junction diode, V_γ(pn),.

Zener Diode
I-V Characteristics



Circuit Symbol



Usually operated in reverse bias region near the breakdown or Zener voltage, V_Z.

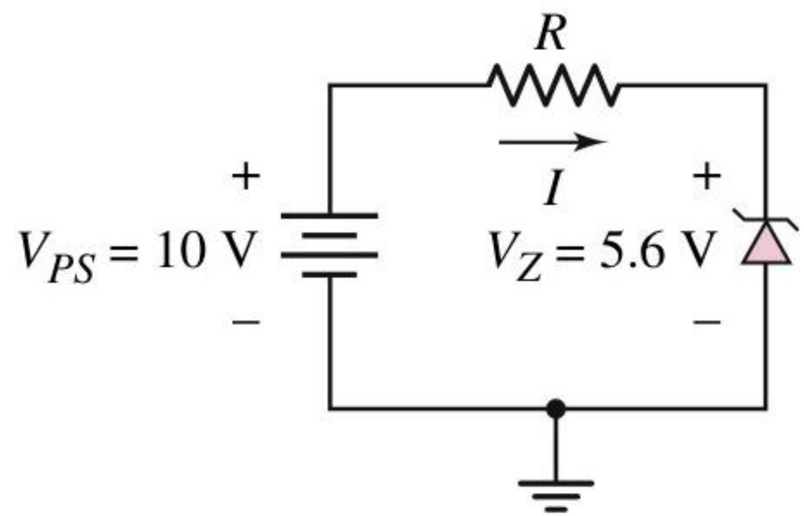
Note the convention for current and polarity of voltage drop.

Example 1

Given $V_Z = 5.6V$

$r_Z = 0\Omega$

Find a value for R such that the current through the diode is limited to 3mA



$$I = \frac{V_{PS} - V_Z}{R}$$

$$R = \frac{V_{PS} - V_Z}{I} = \frac{10V - 5.6V}{3mA} = 1.47k\Omega$$

$$P_Z = I_Z V_Z = 3mA \cdot 5.6V = 1.68mW$$

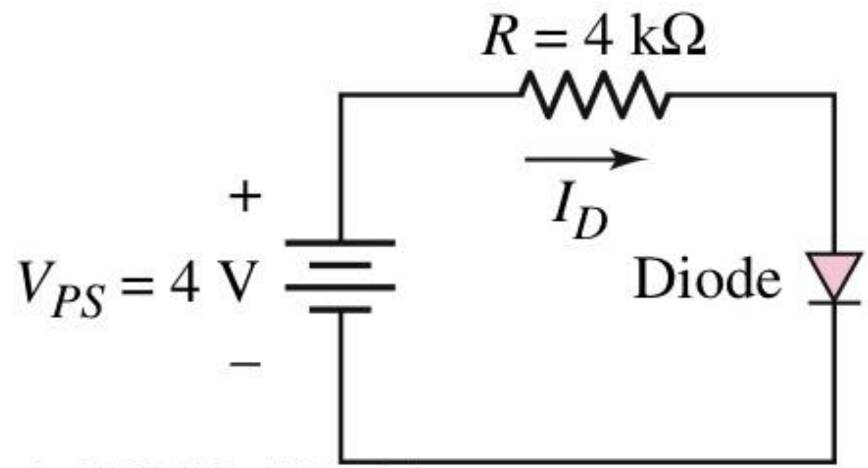
Example 2

Given V_γ (pn) = 0.7V

$V\gamma$ (SB) = 0.3V

r_f = 0Ω for both diodes

Calculate I_D in each diode.



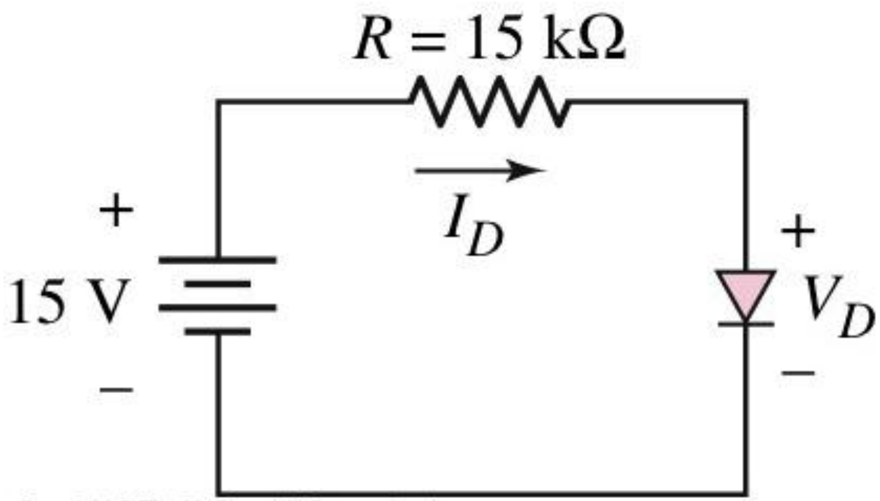
$$I = \frac{V_{PS} - V_\gamma}{R}$$

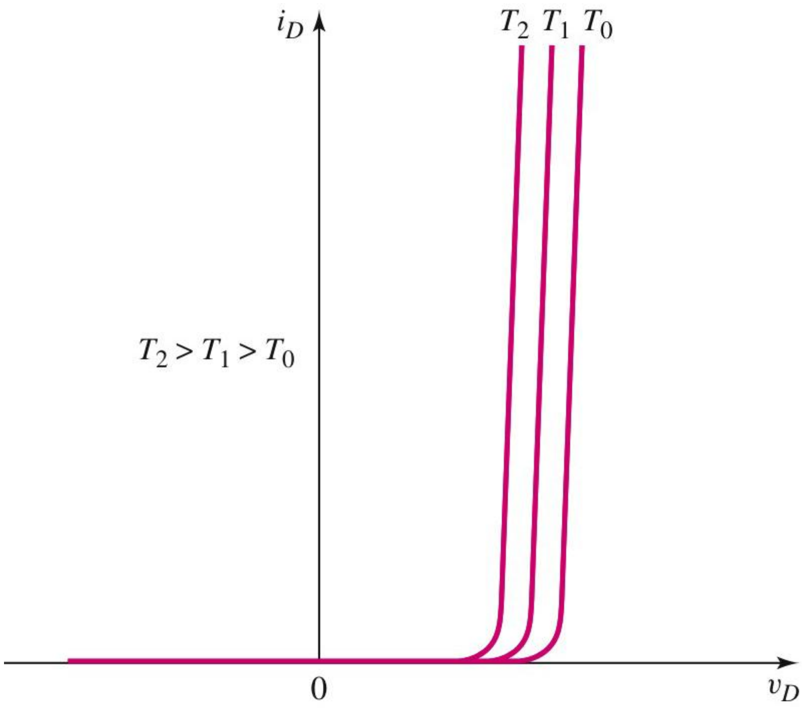
$$I = \frac{4V - 0.7V}{4k\Omega} = 0.825mA \text{ for the p-n junction diode}$$

$$I = \frac{4V - 0.3V}{4k\Omega} = 0.925mA \text{ for the Schottky diode}$$

Example 3: Digital Thermometer

Use the temperature dependence of the forward-bias characteristics to design a simple electronic thermometer.





Solution

Given: I_S = 10^{-13} A at T = 300K $\quad E_g / e = 1.12V$

Assume: Ideal diode equation can be simplified.

$$I_D \approx I_S e^{\frac{V_D}{V_T}} \propto n_i^2 e^{\frac{-E_g}{kT}} e^{\frac{V_D}{V_T}}$$

$$\frac{I_{D1}}{I_{D2}} = \frac{e^{\frac{-E_g}{kT_1}} e^{\frac{eV_{D1}}{kT_1}}}{e^{\frac{-E_g}{kT_2}} e^{\frac{eV_{D2}}{kT_2}}}$$

$$V_{D2} = -\frac{E_g}{e}\left(\frac{T_2}{T_1}\right) + \frac{E_g}{e} + V_{D1}\left(\frac{T_2}{T_1}\right) = 1.12\left(1 - \frac{T_2}{T_1}\right) + V_{D1}\left(\frac{T_2}{T_1}\right)$$

$$I_D = \frac{15V - V_D}{R} = I_S e^{\frac{V_D}{V_T}}$$

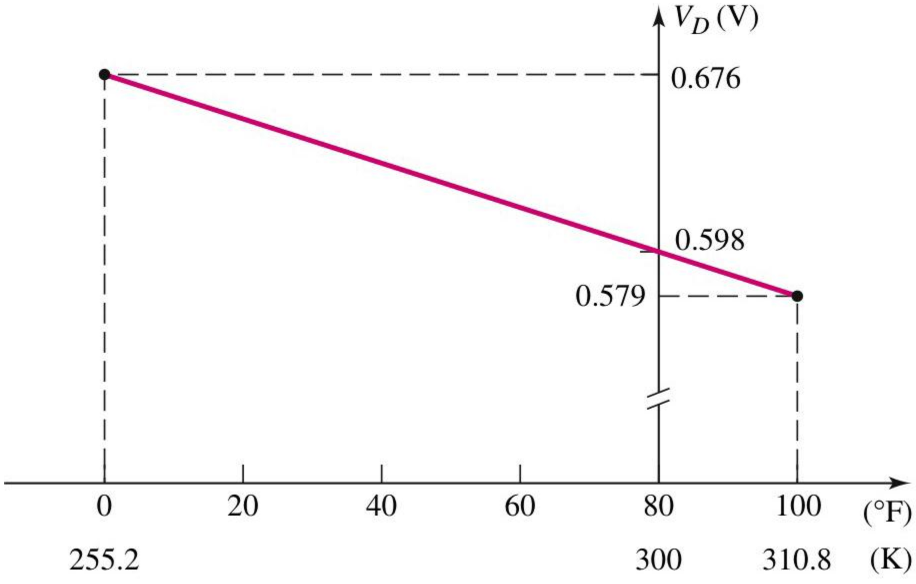
Thermometer con't

$$I_D = \frac{15V - V_D}{15x10^3\Omega} = 10^{-13}A \cdot e^{\frac{V_D}{V_T}} \quad \text{at T} = 300\text{K}$$

Through trial and error: $V_D = 0.5976V$ and $I_D = 0.960mA$

To find temperature dependence, let $\text{T}_1 = 300\text{K}$.

$$\text{V}_\text{D} = 1.12 - 0.522(\frac{T}{300})\ \text{V}$$



Chapter 1 Summary

- First class introduction to Microelectronics
- Overview of Microelectronics, its significance, current state of the art and future prospects
- Intro to semiconductor material properties mainly Si and GaAs, discuss the two types of charged carriers that exist in a semiconductor and the two mechanisms that generate currents in a semiconductor.
- Covered the properties of a pn junction including the ideal current–voltage characteristics of the pn junction diode.
- Examined the DC analysis techniques for diode circuits using various models to describe the nonlinear diode characteristics.
- Develop an equivalent circuit for a diode that is used when a small, time varying signal is applied to a diode circuit.
- Go over the properties and characteristics of a few specialized diodes.

Note: 1. Please Read Chapter 1 and do all even problems

2. Start Reading Chapter 2

3. Do all even problems in Chapter 1

4. I will assign HW next week to be due the week after

EEE109: Electronic Circuits

Diode Circuits

Contents of Chapter 2

- Determine the operation and characteristics of diode rectifier circuits, which is the first stage of the process of converting an ac signal into a dc signal in the electronic power supply.

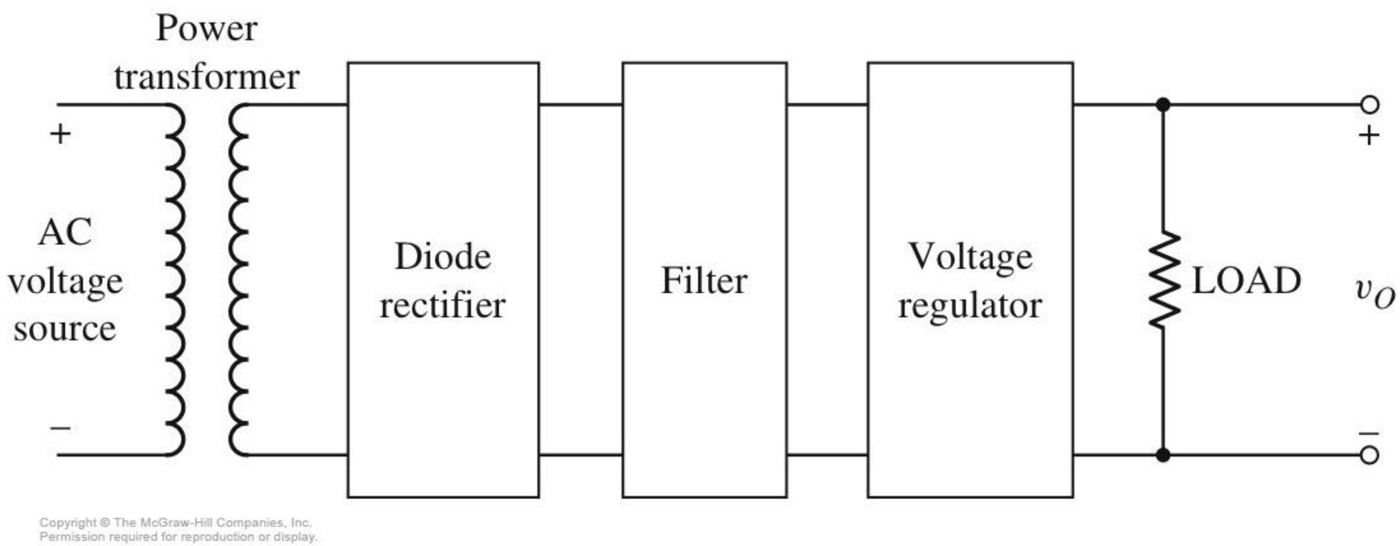
- Apply the characteristics of the Zener diode to a Zener diode voltage regulator circuit.

- Apply the nonlinear characteristics of diodes to create waveshaping circuits known as clippers and clampers.

- Examine the techniques used to analyze circuits that contain more than one diode.

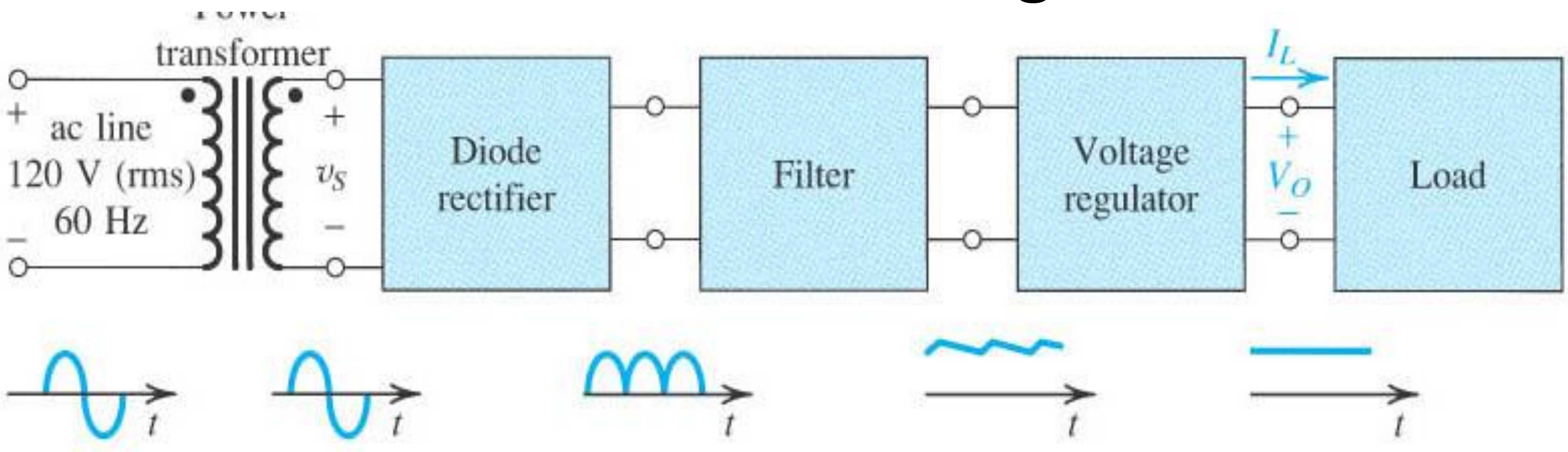
- Understand the operation and characteristics of specialized photodiode and light-emitting diode circuits.

Block Diagram for ac to dc Converter



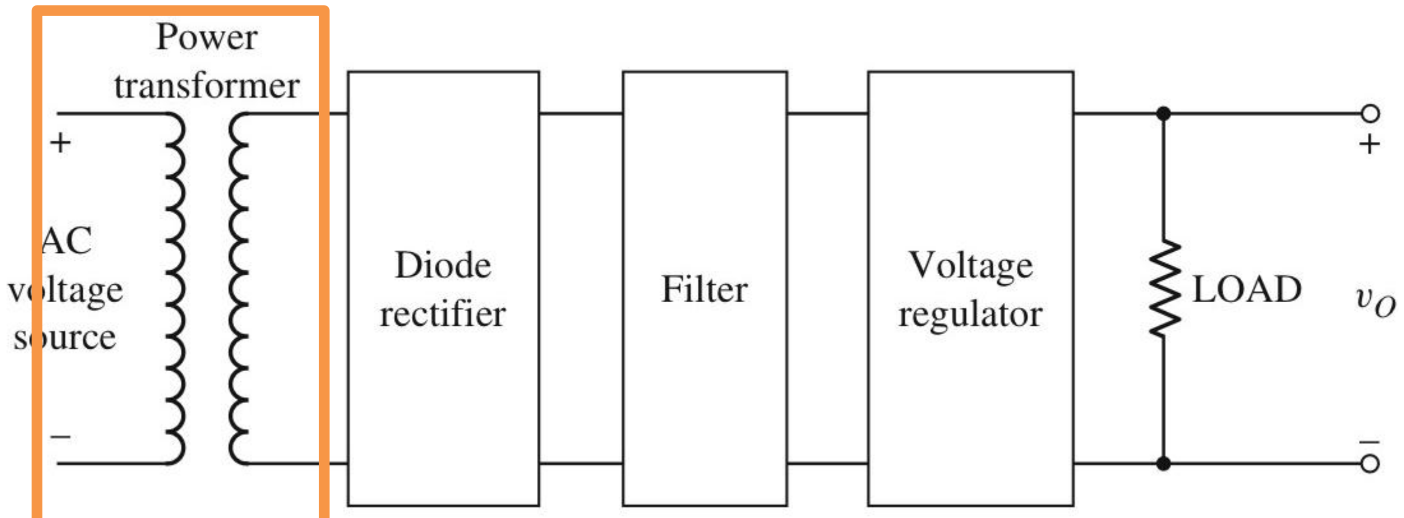
The diode rectifier, filter, and voltage regulator are diode circuits.

Rectifier Circuits Using Diodes



- Basic rectifier converts an ac voltage to a pulsating dc voltage.
- A filter then eliminates ac components of the waveform to produce a nearly constant dc voltage output.
- Rectifier circuits are used in virtually all electronic devices to convert the 120 V-60 Hz ac power line source to the dc voltages required for operation of the electronic device.
- In rectifier circuits, the diode state changes with time and a given piecewise linear model is valid only for a certain time interval.

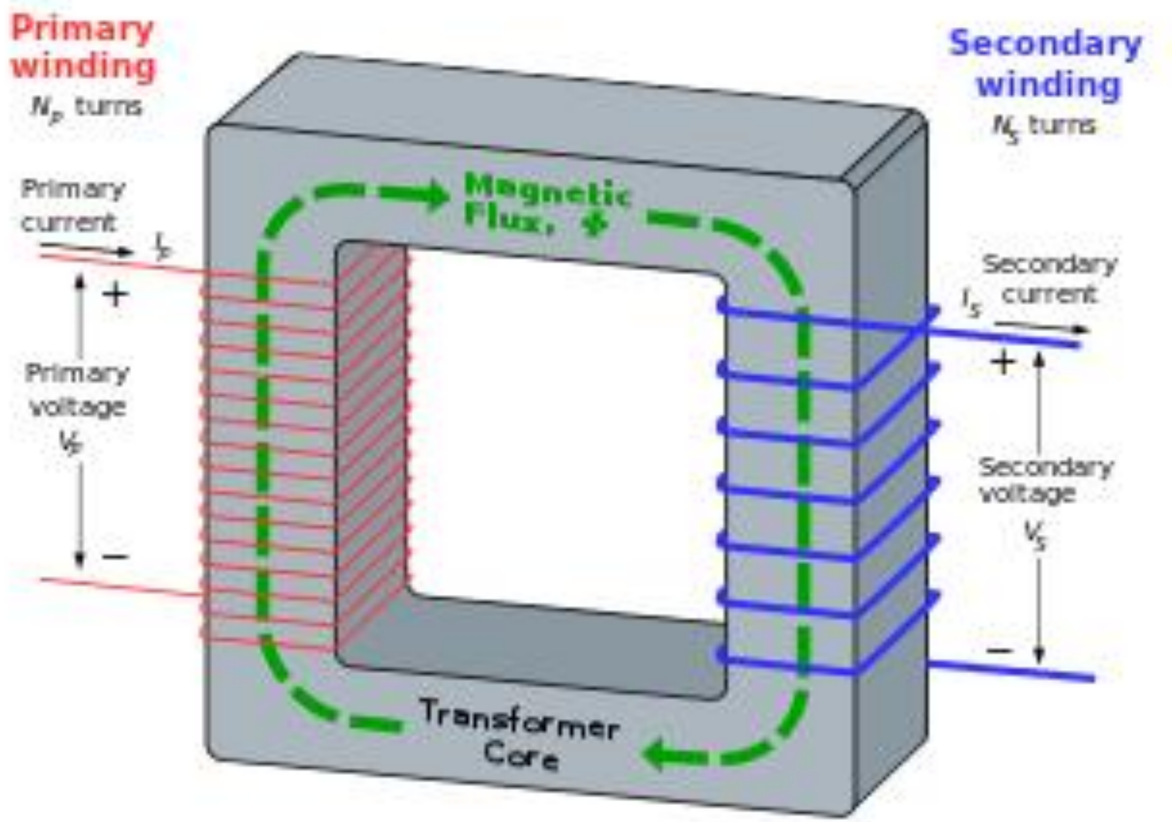
Block Diagram
for ac to dc Converter



The diode rectifier, filter, and voltage regulator are diode circuits.

Transformer

- A **transformer** is an electrical device that transfers energy between two or more circuits through electromagnetic induction.



Ideal transformer equations

- By **Faraday's law of induction**

  $$V_S = -N_S \frac{d\Phi}{dt} \quad (1)$$

  $$V_P = -N_P \frac{d\Phi}{dt} \quad (2)$$

- Combining ratio of (1) & (2)

  Turns ratio $= \frac{V_P}{V_S} = \frac{-N_P}{-N_S} = a$ (3)

  Where for step-down transformers, a > 1; for step-up transformers, a < 1.

- By **law of Conservation of Energy**, apparent, real and reactive power are each conserved in the input and output

  $$S = I_P V_P = I_S V_S \quad (4)$$

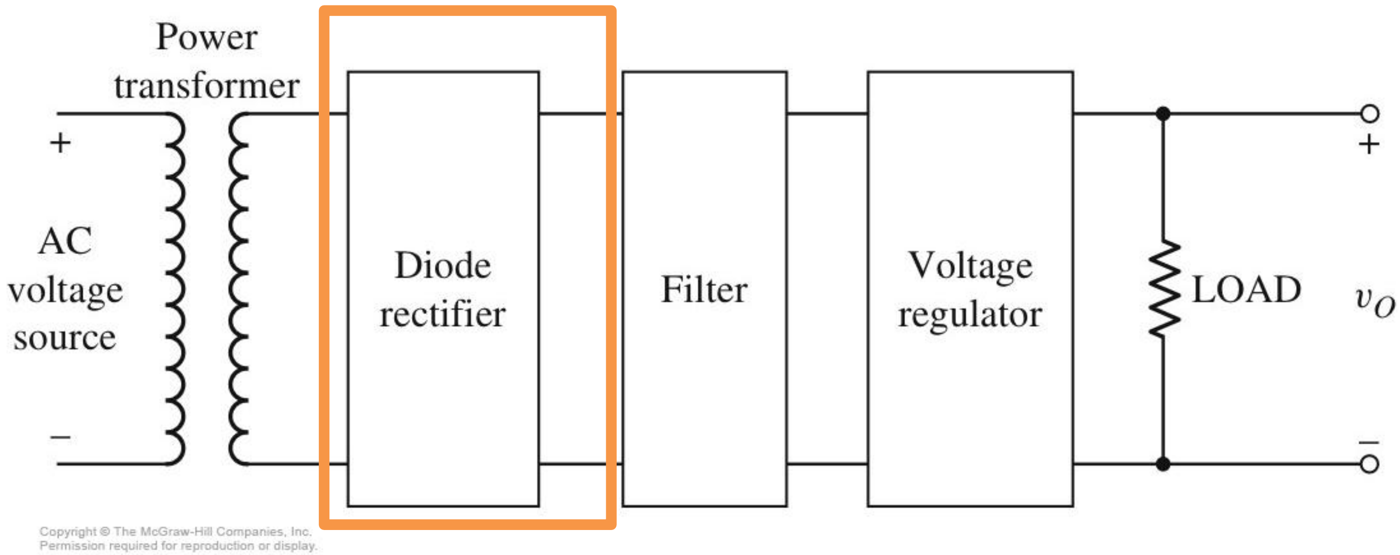
- Combining (3) & (4) with this endnote yields the **ideal transformer identity**:

  $$\frac{V_P}{V_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S} = \sqrt{\frac{L_P}{L_S}} = a \quad (5)$$

Problem-Solving Technique: Diode Circuits

1. Determine the input voltage condition such that the diode is conducting (on).
   a. Find the output signal for this condition.
2. Determine the input voltage such that the diode is not conducting (off).
   a. Find the output signal for this condition.

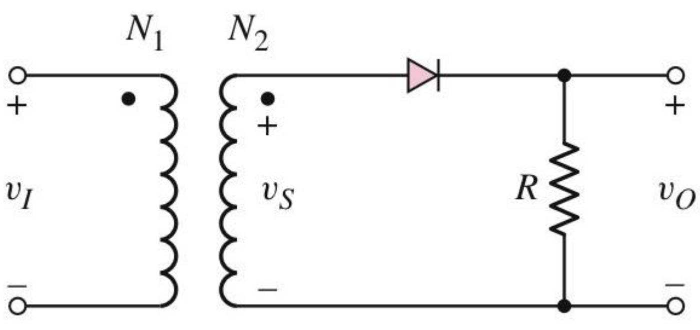
Block Diagram for ac to dc Converter



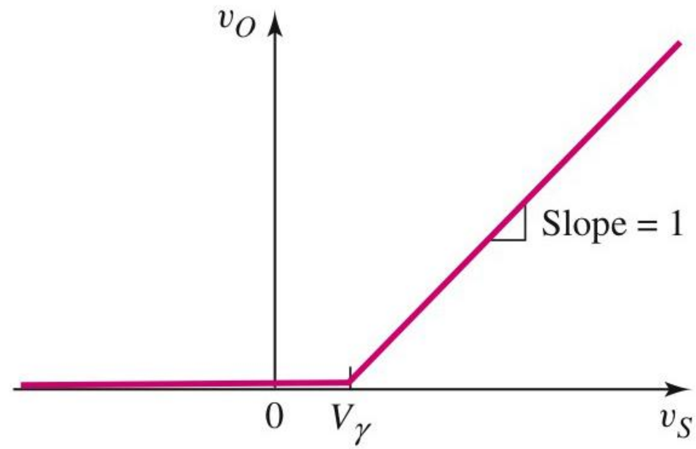
The diode rectifier, filter, and voltage regulator are diode circuits.

Half-wave Rectification

Half-Wave Rectifier



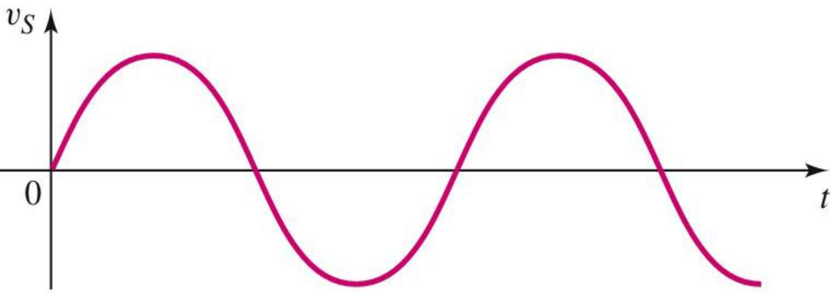
(a)



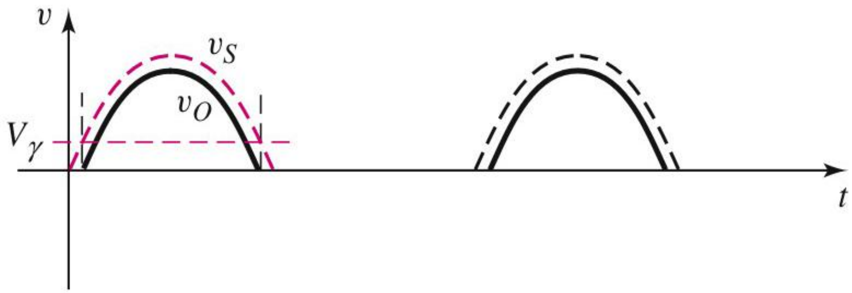
(b)

Voltage Transfer Characteristics

Signals of Half Wave Rectifier



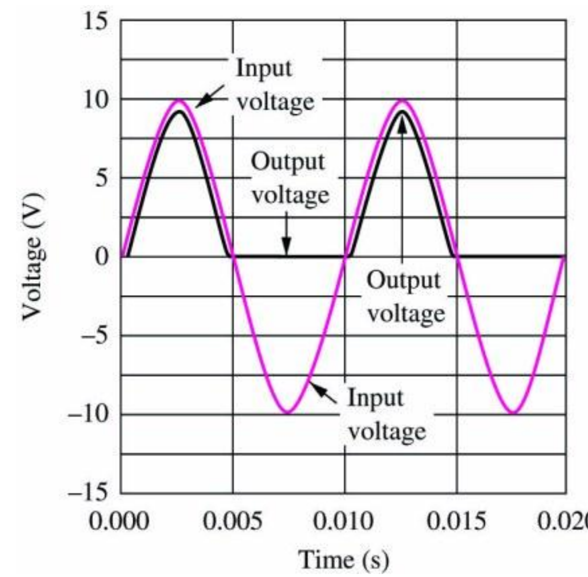
(a)

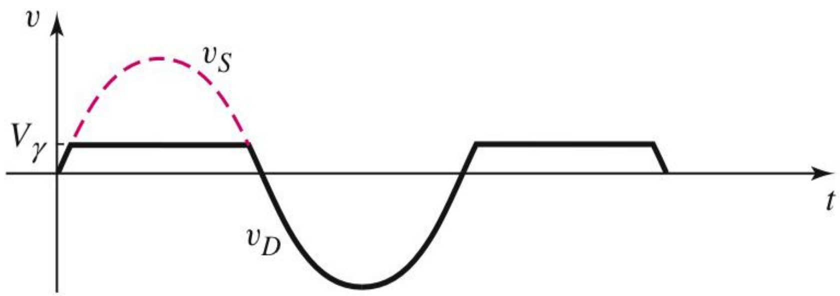


(b)

Output voltage

Input voltage



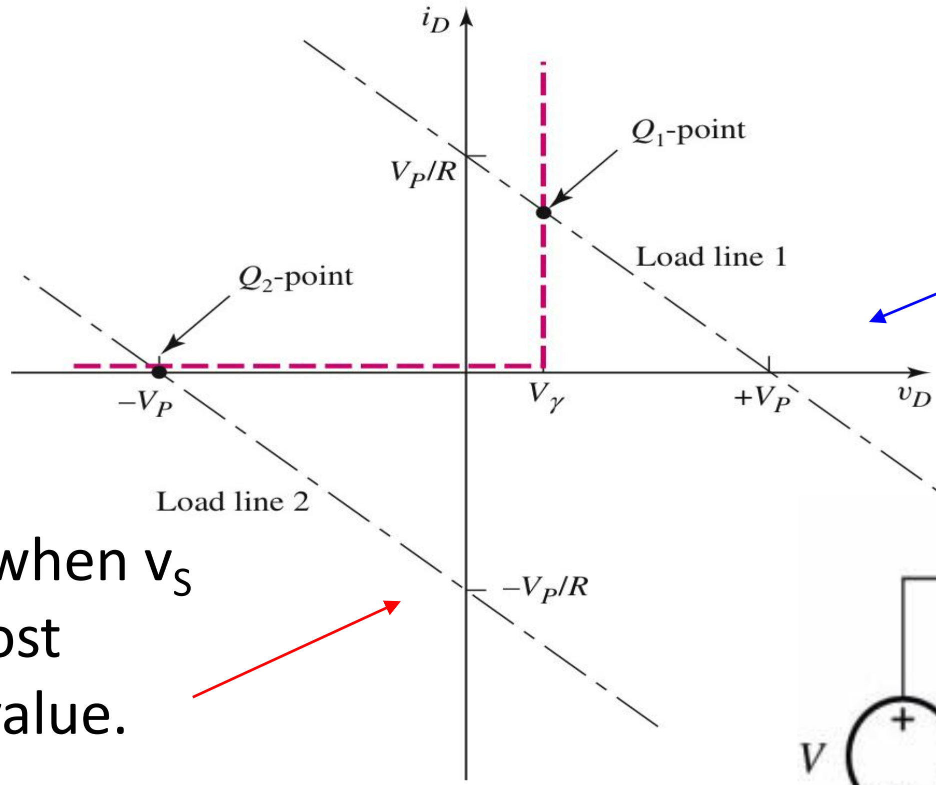


(c)

Diode voltage

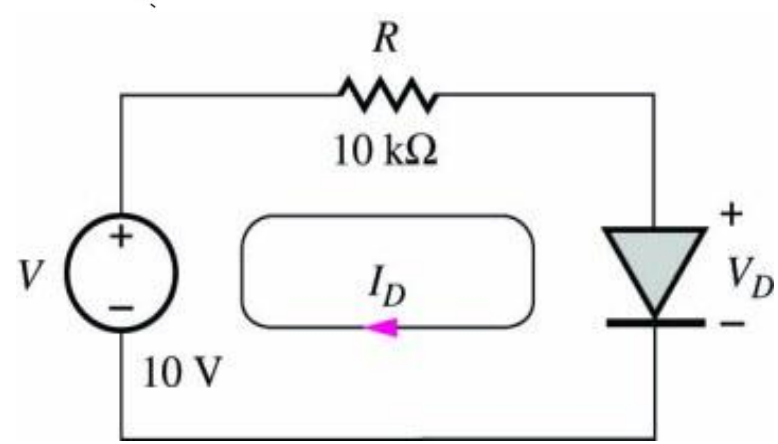
1. Peak current in the **forward direction**;
2. Largest peak inverse voltage (PIV) at **reverse direction**.

Load Line Analysis



Load line when v_S is at its maximum forward voltage.

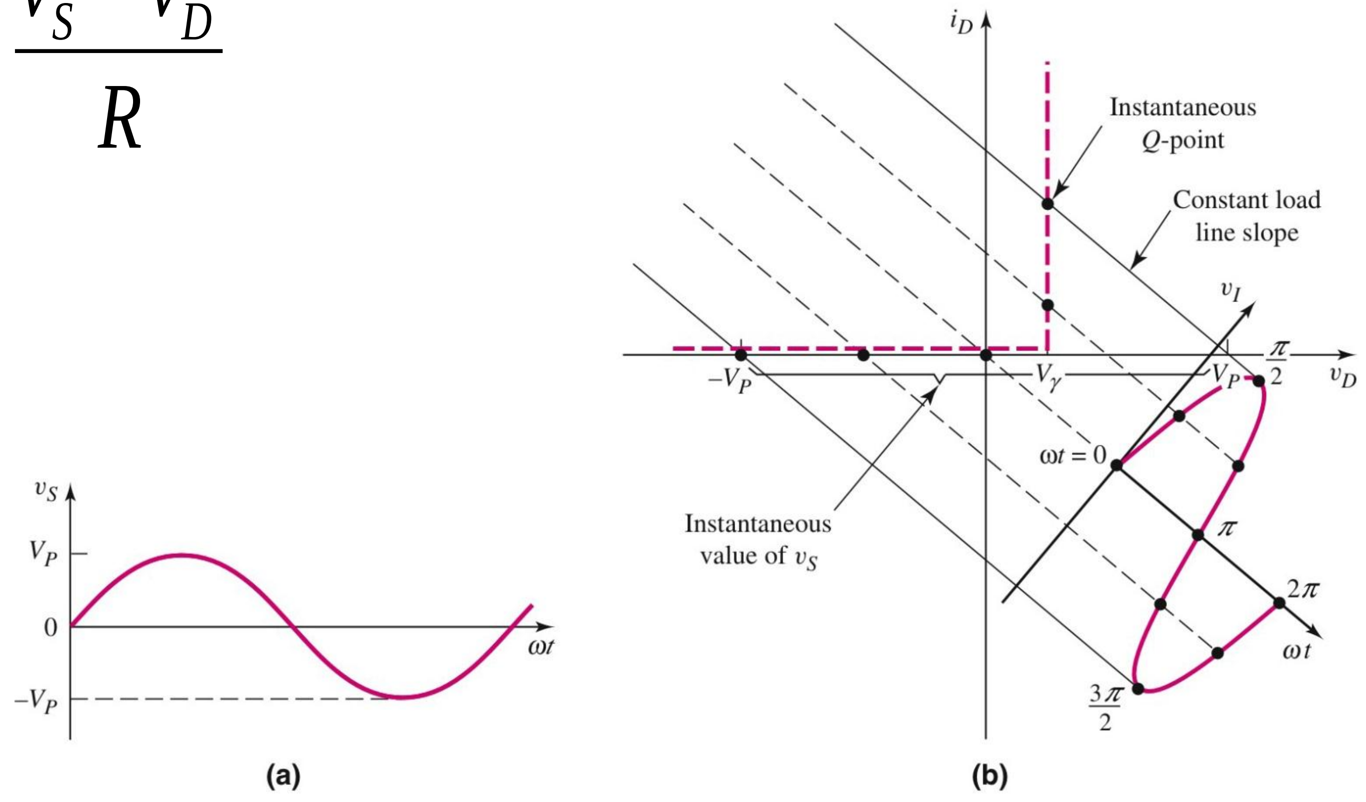
Load line when v_S is at its most **neg**ative value.



$$i_D = \frac{V_P - v_D}{R}$$

Load Line (con't)

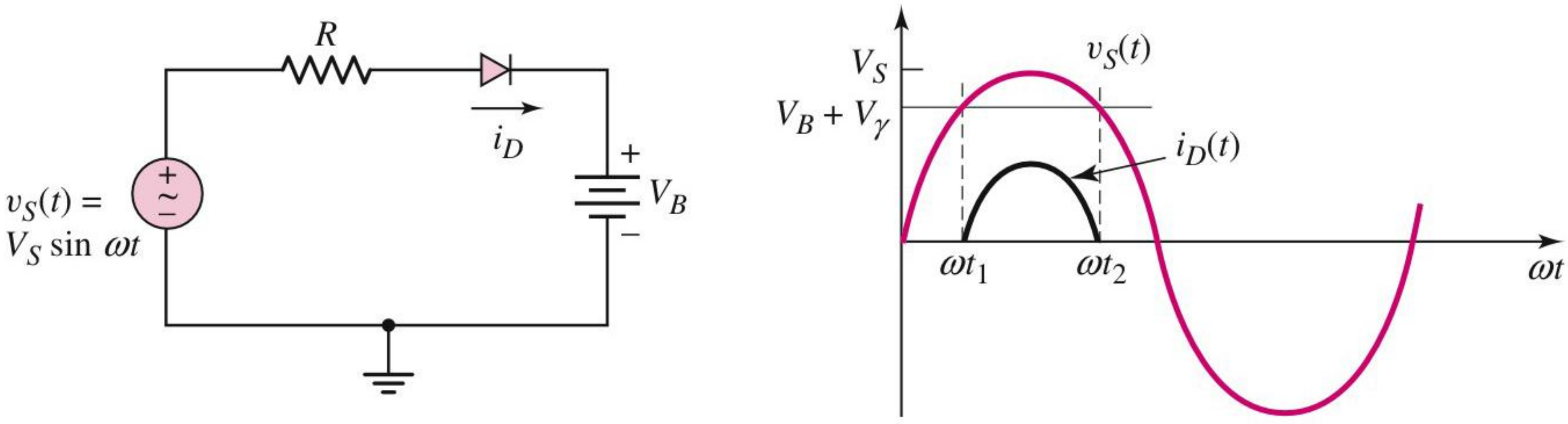
$$i_D = \frac{v_S - v_D}{R}$$



As v_S varies with time, the load line also changes, which changes the Q-point (v_D and i_D) of the diode.

Half-wave Rectifier As Battery Charger

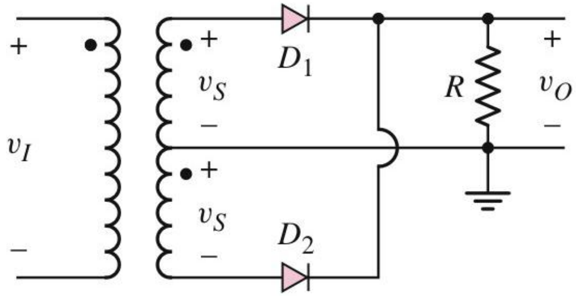
Half-Wave Rectifier as Battery Charger



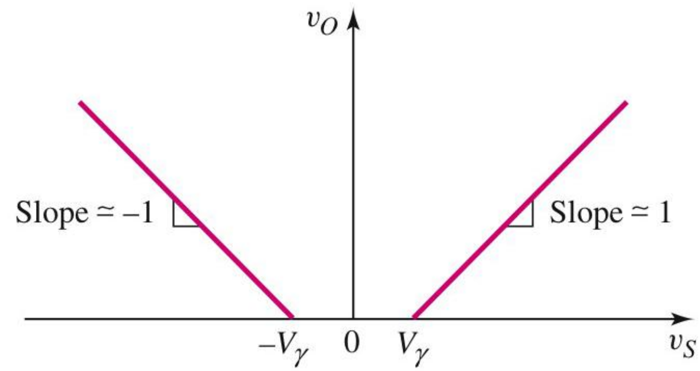
(a) (b)

Full-wave Rectification

Full-Wave Rectifier

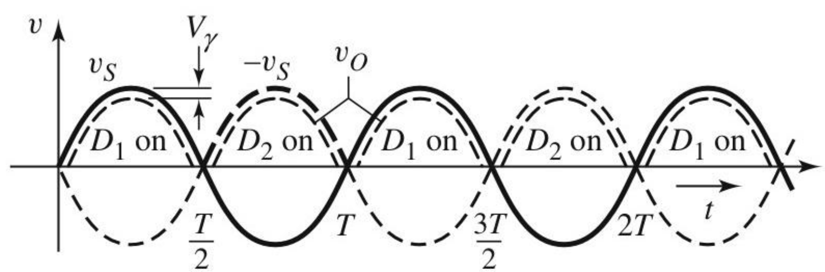


(a)



(b)

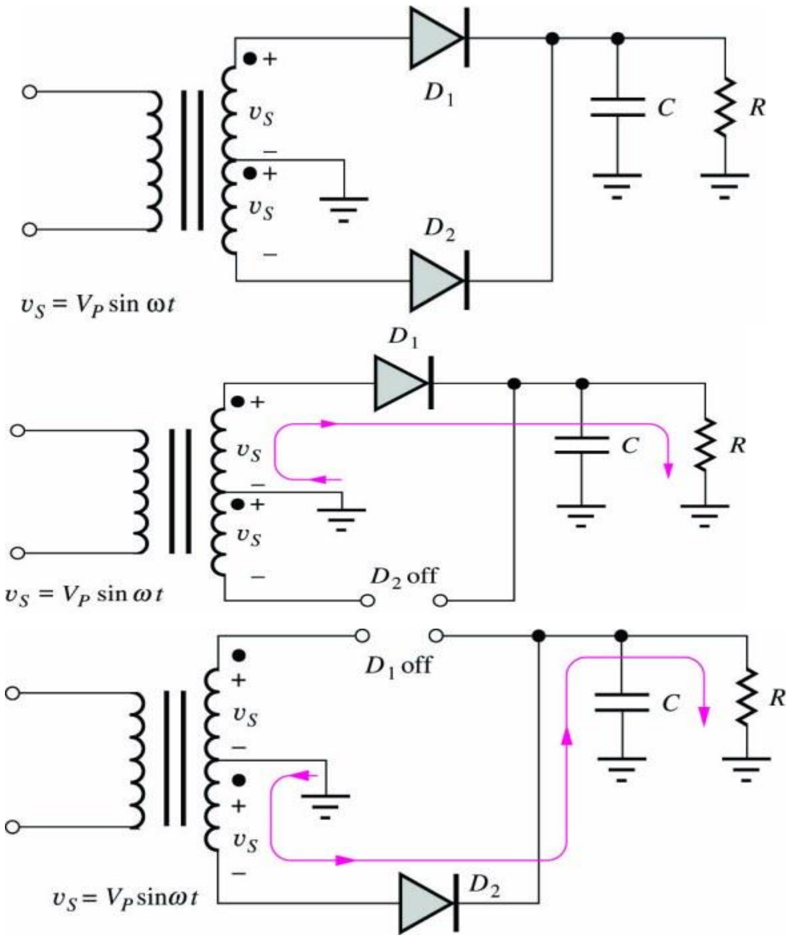
Voltage transfer characteristics

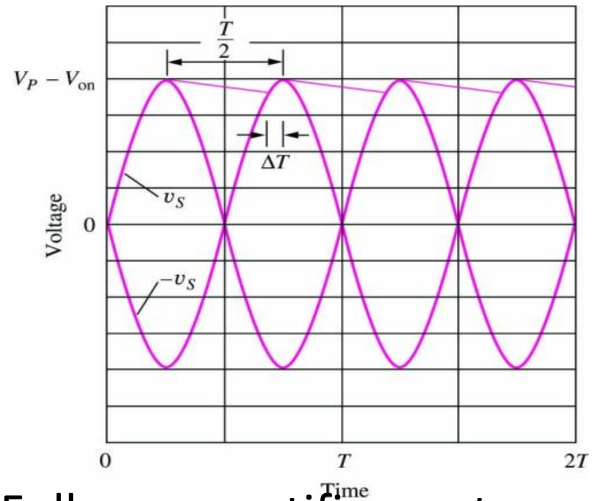


(c) Input and output waveforms

Two outputs from a center-tapped secondary winding that provides equal voltages with opposite polarities.

Full-Wave Rectifiers



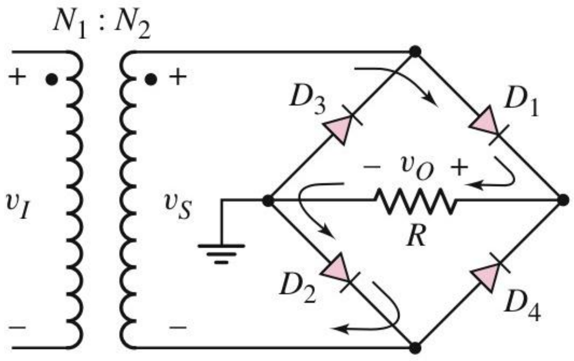


Full-wave rectifiers cut capacitor discharge time in half and require half the filter capacitance to achieve given ripple voltage. All specifications are same as for half-wave rectifiers.

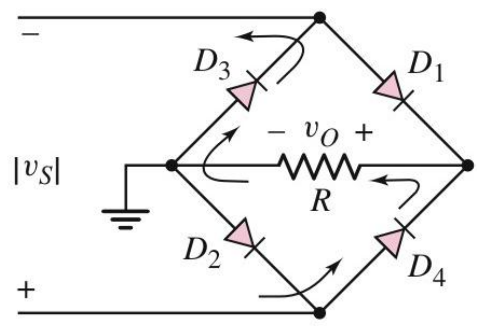
Reversing polarity of diodes gives a full-wave rectifier with negative output voltage.

Full-Wave **Bridge** Rectifier

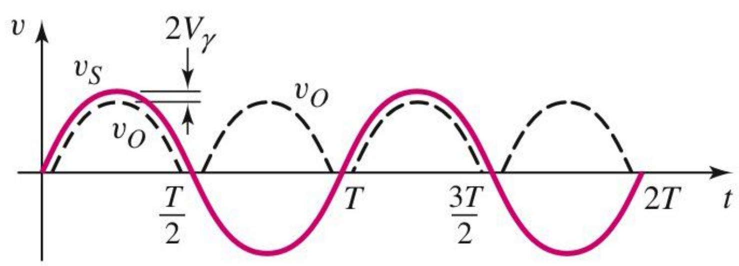
Full-Wave Bridge Rectifier



(a)



(b)

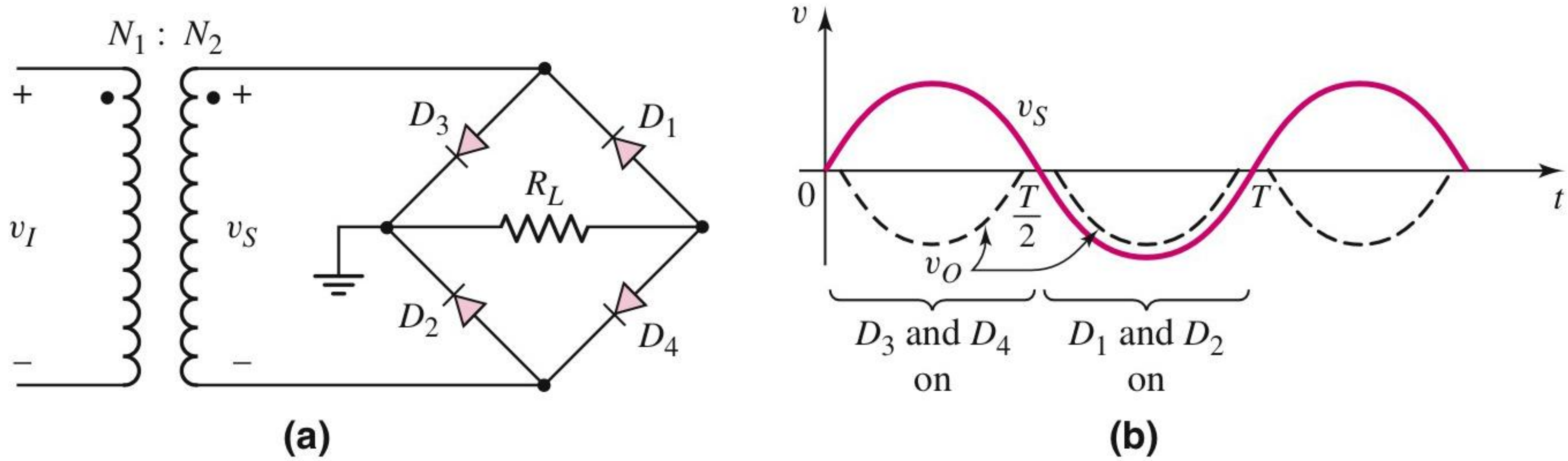


(c)

When v_S is positive, D_1 and D_2 are turned on (a). When v_S is negative, D_3 and D_4 are turned on (b). In either case, current flows through R in the same direction, resulting in an output voltage, v_O, shown in (c).

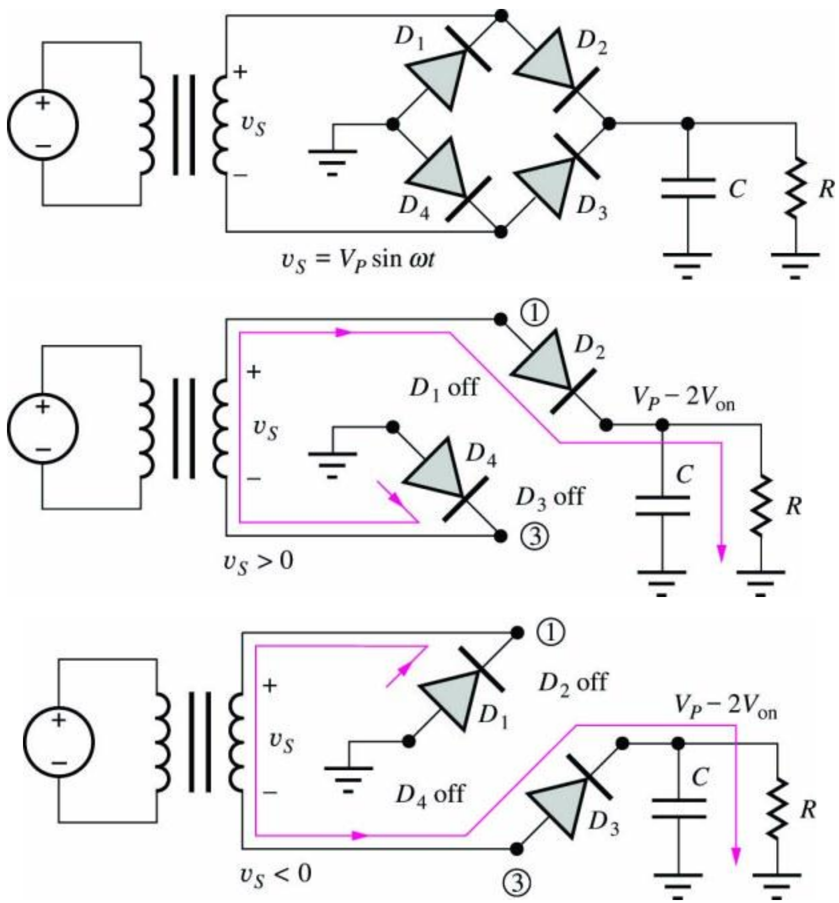
Do not require a center-tapped secondary winding!

Full-Wave Bridge Rectifier



The output voltage is now negative!

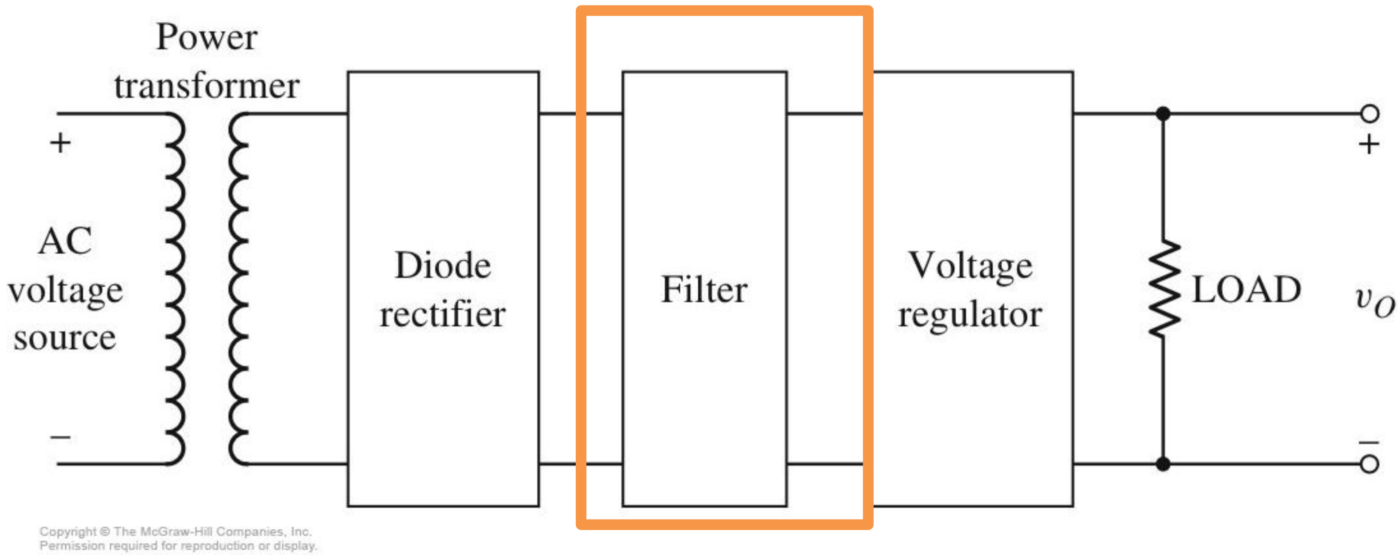
Full-Wave Bridge Rectification



Requirement for a center-tapped transformer in the full-wave rectifier is eliminated through the use of 2 extra diodes. All other specifications are same as for a half-wave rectifier except PIV=V_P.

Filters

Block Diagram for ac to dc Converter

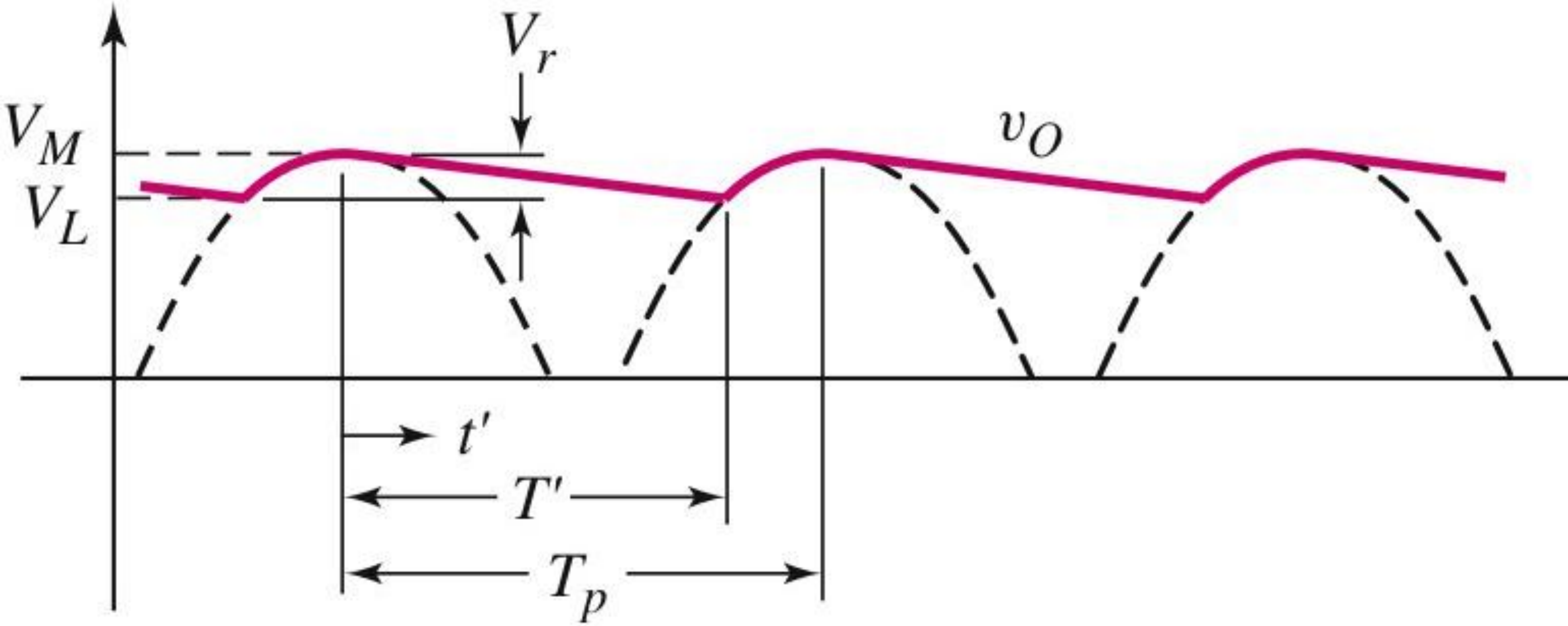


The diode rectifier, filter, and voltage regulator are diode circuits.

Rectifier Topology Comparison

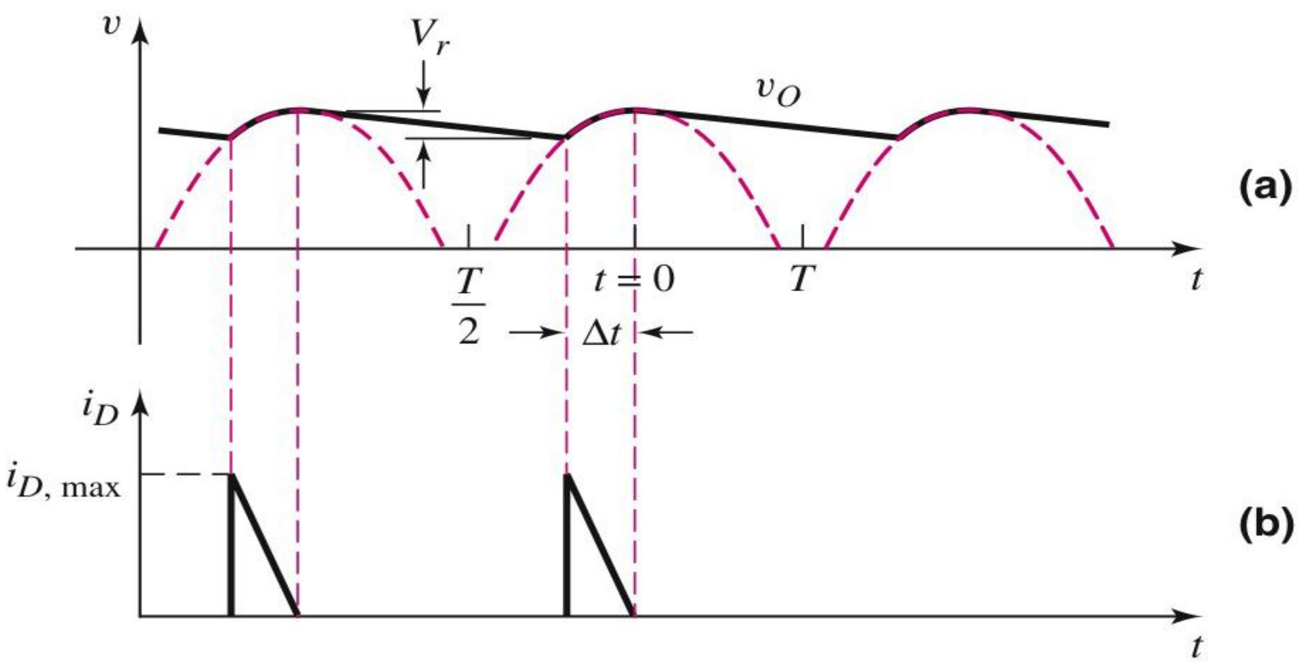
- Filter capacitor is a major factor in determining cost, size and weight in design of rectifiers.
- For given ripple voltage, full-wave rectifier requires half the filter capacitance as that in half-wave rectifier. Reduced peak current can reduce heat dissipation in diodes. Benefits of full-wave rectification outweigh increased expenses and circuit complexity (a extra diode and center-tapped transformer).
- Bridge rectifier eliminates center-tapped transformer, PIV rating of diodes is reduced. Cost of extra diodes is negligible.

Output Voltage of Full-Wave Rectifier with RC Filter



The ripple on the 'dc' output is $V_r = \dfrac{V_M}{2fRC}$ where $f = \dfrac{1}{2T_P}$

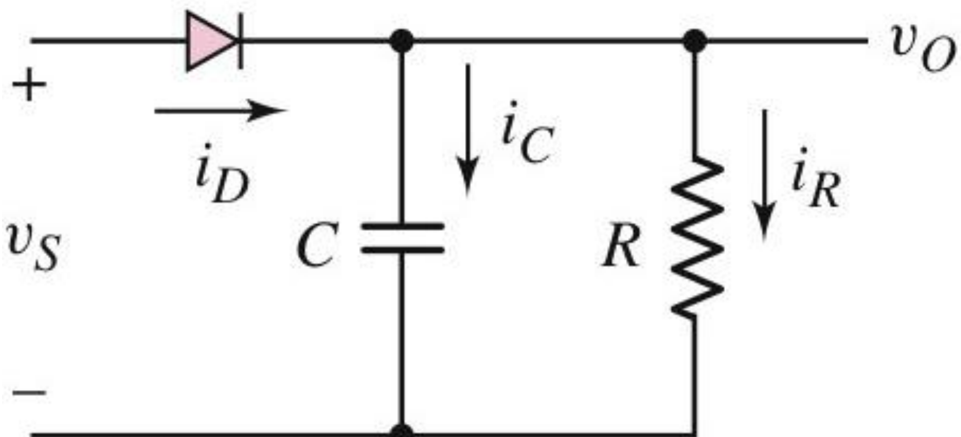
Output Voltage of Full-Wave Rectifier with RC Filter



$$\frac{\Delta t}{T} = \frac{1}{\pi}\sqrt{\frac{2V_r}{V_M}}$$

Diode conducts current for only a small portion of the period.

Equivalent Circuit During Capacitance Charging Cycle

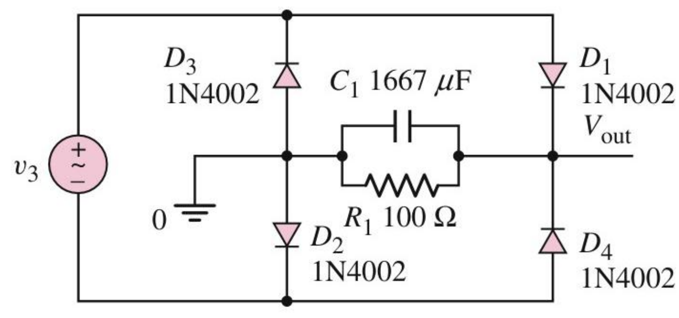


$$i_C = -\omega C V_M \omega t$$

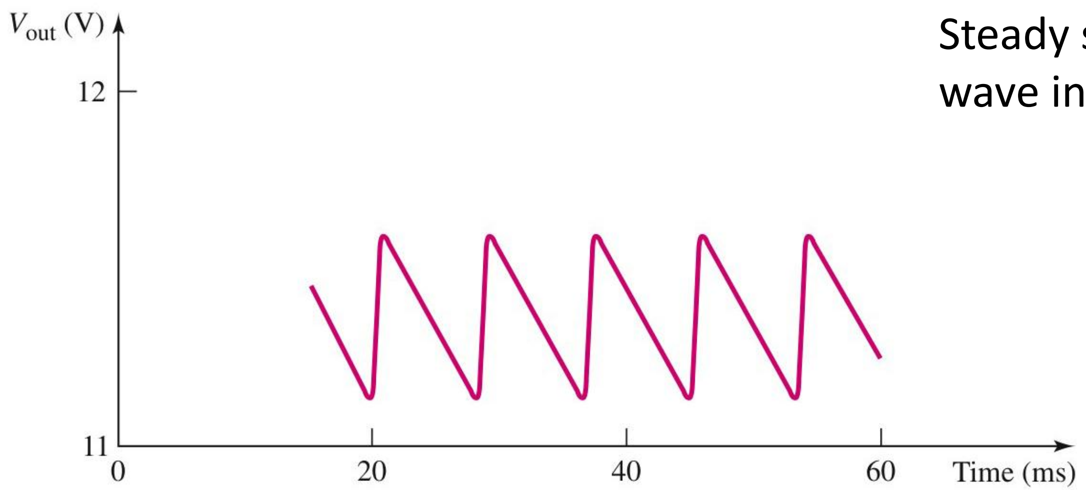
$$i_{C,peak} = +\omega C V_M \omega \Delta t$$

$$\omega \Delta t = \sqrt{\frac{2V_r}{V_M}}$$

PSpice Schematic of Diode Bridge Circuit



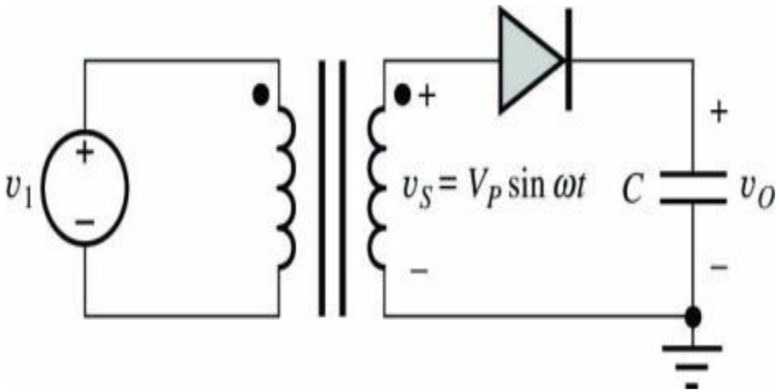
(a)

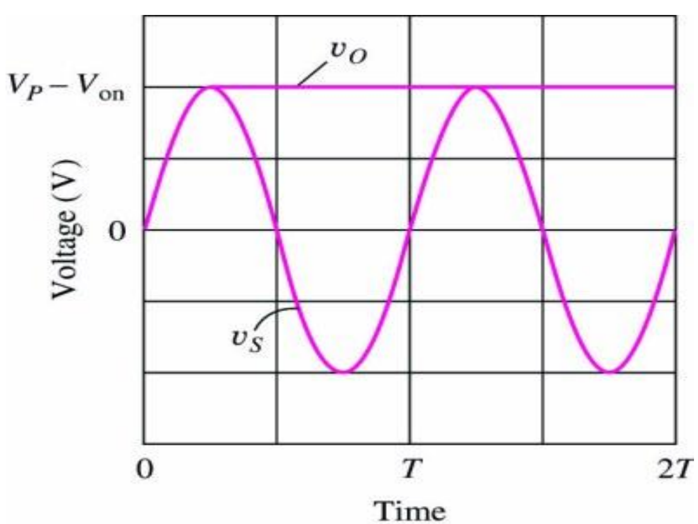


(b)

Steady state output voltage for a 60Hz sine wave input with peak value of 13.4V.

Rectifiers Applications: Peak Detector Circuit



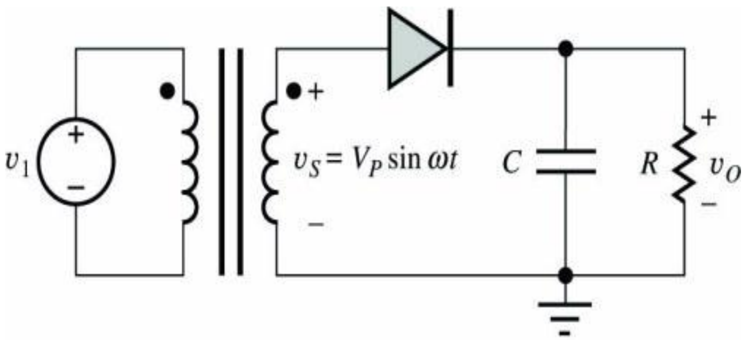


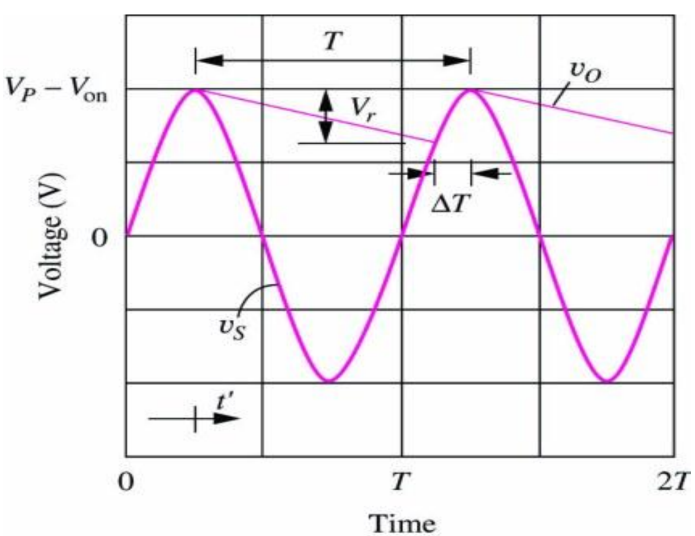
As input voltage rises, diode is on and capacitor (initially discharged) charges up to input voltage minus the diode voltage drop.

At peak of input, diode current tries to reverse, diode cuts off, capacitor has no discharge path and retains constant voltage providing constant output voltage

$V_{dc} = V_P - V_{on}$.

Half-Wave Rectifier Circuit with *RC* Load





As input voltage rises during first quarter cycle, diode is on and capacitor (initially discharged) charges up to peak value of input voltage.

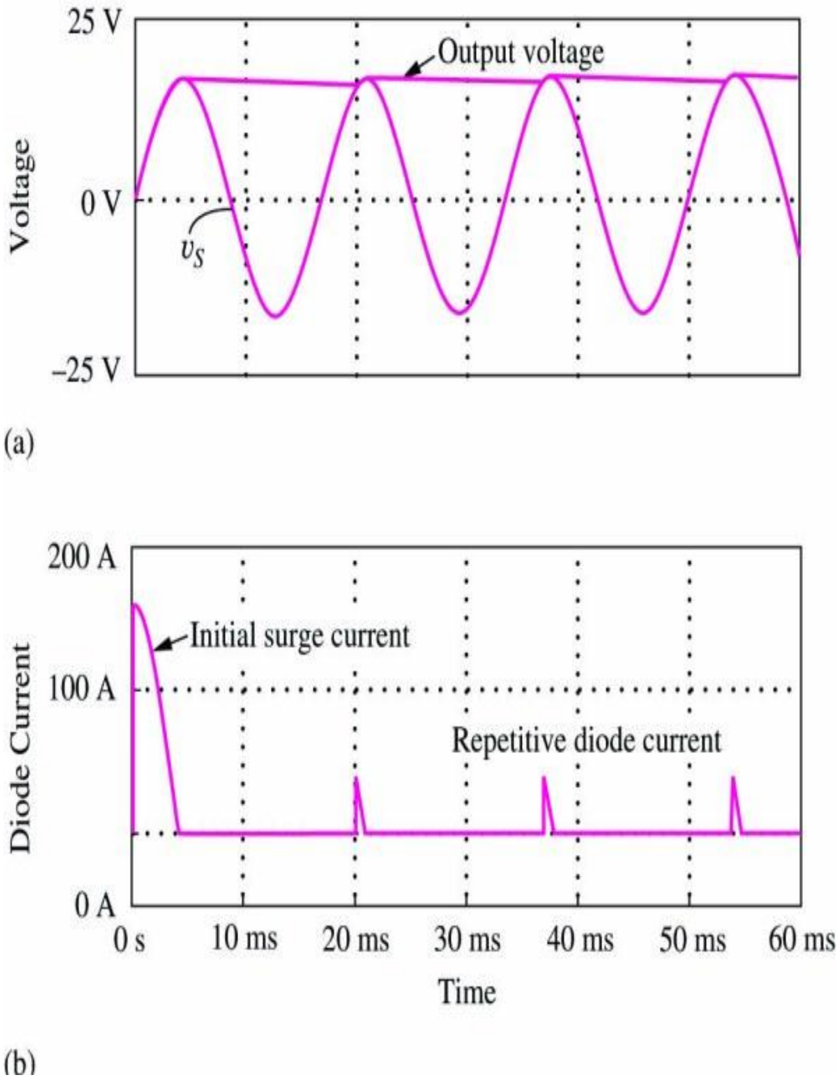
At peak of input, diode current tries to reverse, diode cuts off, capacitor discharges exponentially through *R*. Discharge continues till input voltage exceeds output voltage which occurs near peak of next cycle. Process then repeats once every cycle.

This circuit can be used to generate negative output voltage if the top plate of capacitor is grounded instead of bottom plate. In this case, $V_{dc} = -(V_P - V_{on})$

Half-Wave Rectifier Analysis: Example

Problem: Find dc output voltage, output current, ripple voltage, conduction interval, conduction angle.
Given data: secondary voltage V_{rms} 12.6 (60 Hz), R= 15 Ω, C= 25,000 μF, V_{on} = 1 V
Analysis:

Peak Diode Current



In rectifiers, nonzero current exists in diode for only a very small fraction of period *T*, yet an almost constant dc current flows out of filter capacitor to load.

Total charge lost from capacitor in each cycle is replenished by diode during short conduction interval causing high peak diode currents. If repetitive current pulse is modeled as triangle of height I_P and width ΔT,

$$I_P = I_{dc}\frac{2T}{\Delta T} = 48.6\text{A}$$

using values from previous example.

Surge Current

Besides peak diode currents, when power supply is turned on, there is an even larger current through diode called **surge current.**

During first quarter cycle, current through diode is approximately

$$i_d(t) = i_C(t) \cong C\left(\frac{d}{dt}V_P \sin\omega t\right) = \omega C V_P \cos\omega t$$

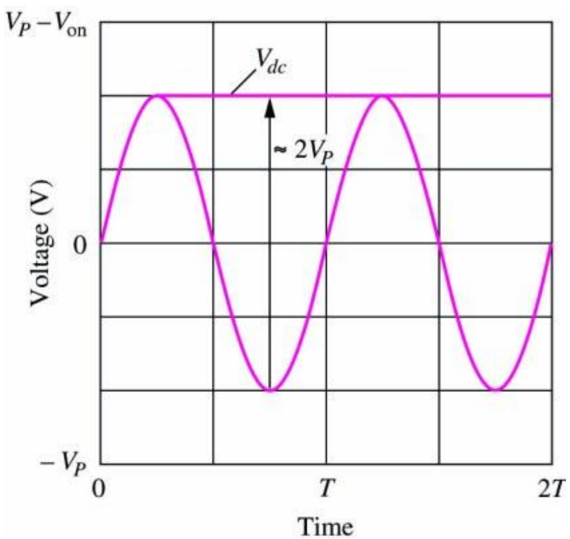
Peak values of this initial surge current occurs at $t = 0^+$:

$$I_{SC} = \omega C V_P = 168\text{A}$$

using values from previous example.

Actual values of surge current won't be as large as predicted because of neglected series resistance associated with rectifier diode as well as transformer.

Peak Inverse Voltage Rating



Peak inverse voltage (PIV) rating of the rectifier diode gives the breakdown voltage.

When diode is off, reverse-bias across diode is V_{dc} - v_s. When v_s reaches negative peak,

$$\text{PIV} \geq V_{dc} - v_s^{\min} = V_P - V_{on} - (-V_P) \cong 2V_P$$

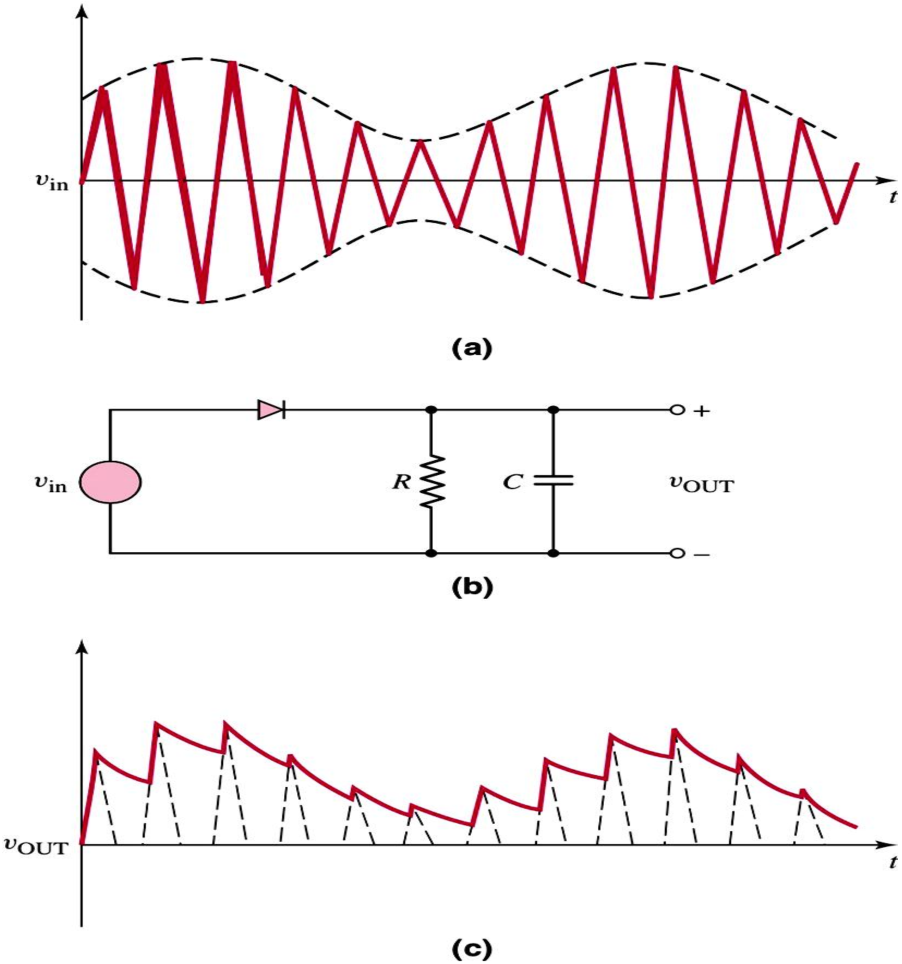
PIV value corresponds to minimum value of Zener breakdown voltage for rectifier diode.

Rectifier Design Analysis

Problem: Design rectifier with given specifications.
Given data: V_{dc} =15 V, V_r < 0.15 V, I_{dc} = 2 A
Analysis: Use full-wave bridge rectifier that needs smaller value of filter capacitance, smaller diode PIV rating and no center-tapped transformer.

AM Signal Detector

Demodulation of Amplitude-Modulated Signal

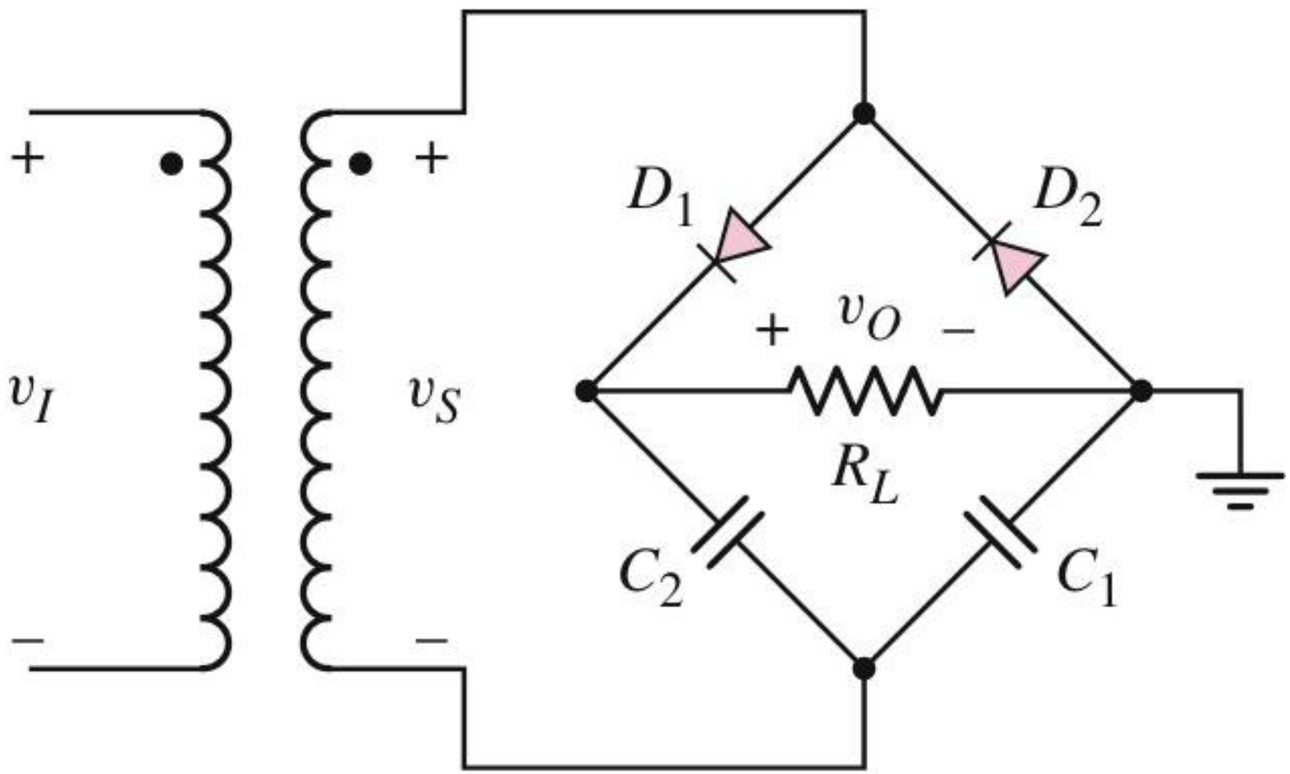


Modulated input signal

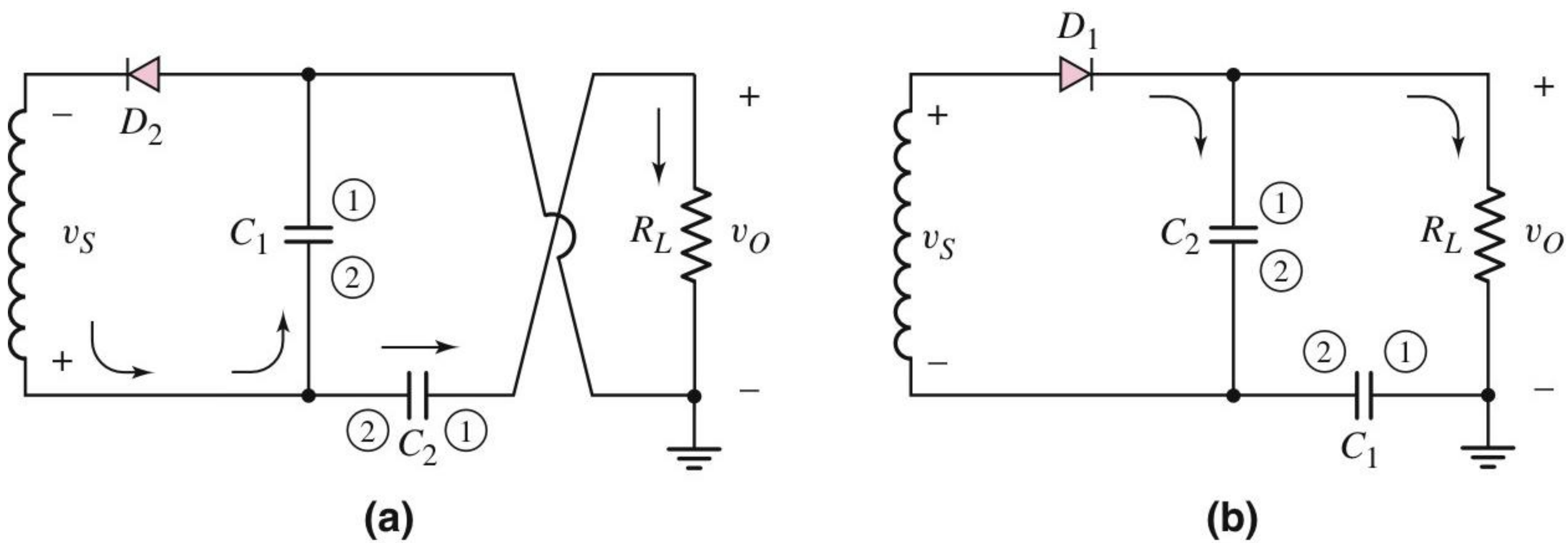
Detector circuit

Demodulated output signal

Voltage Doubler Circuit



Equivalent Circuits for Input Cycles

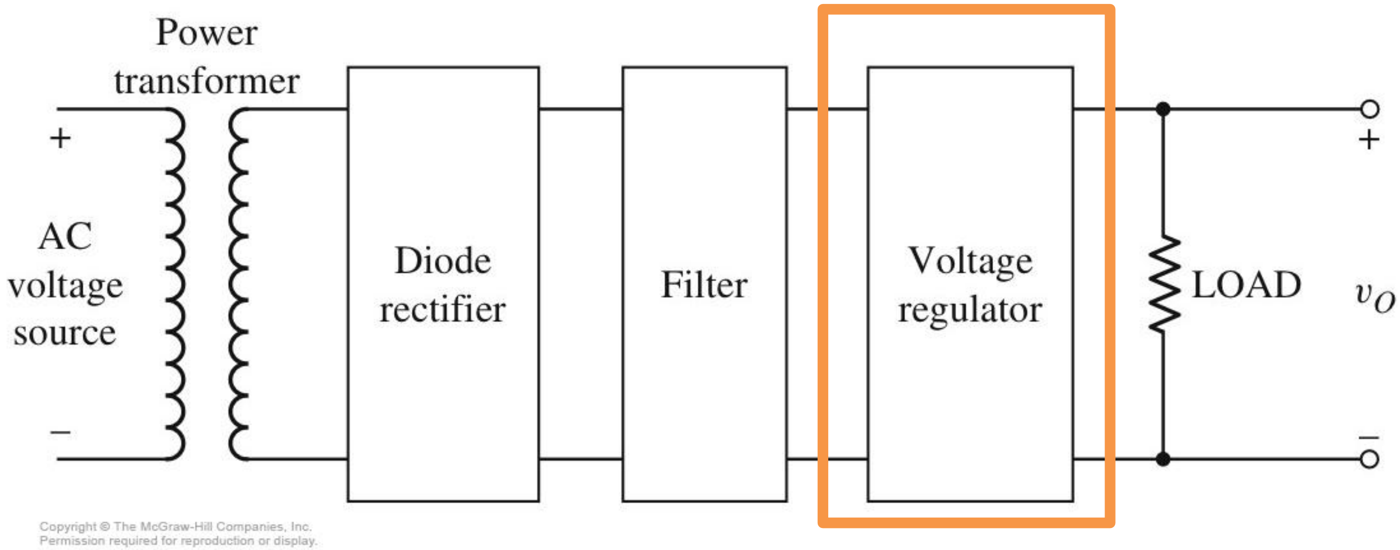


Negative input cycle

Positive input cycle

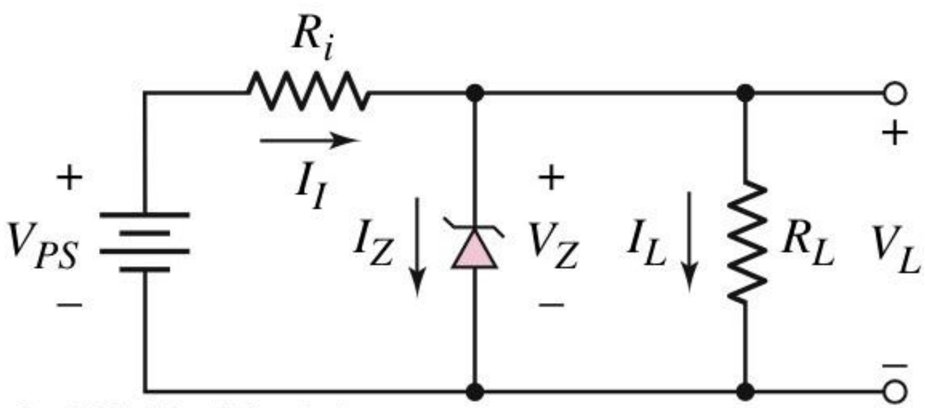
Zener Diode Circuits

Block Diagram for ac to dc Converter



The diode rectifier, filter, and voltage regulator are diode circuits.

Voltage Regulator



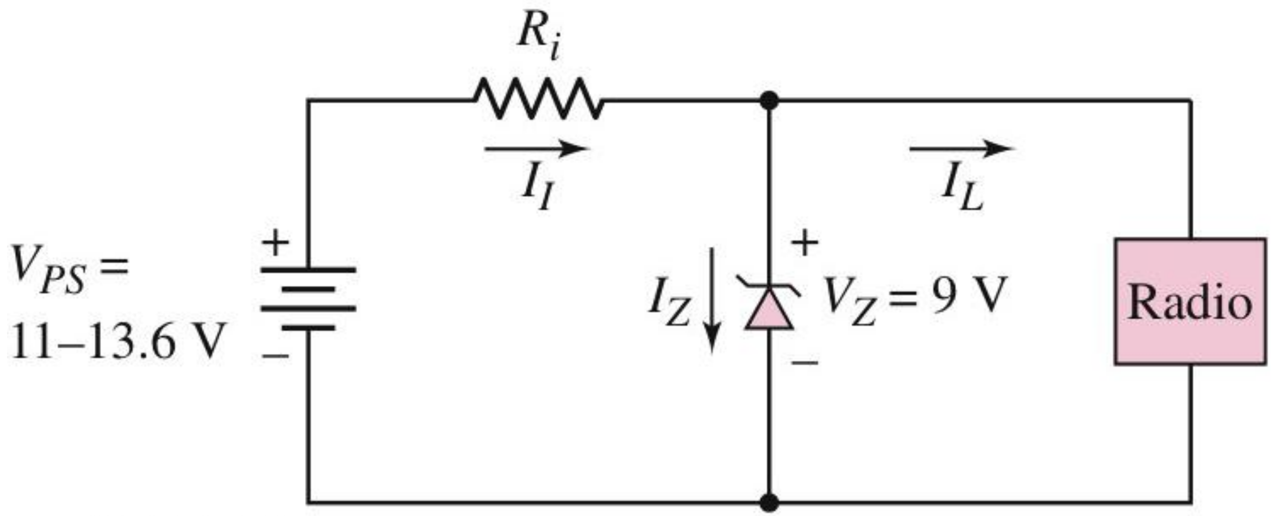
$$I_L = \frac{V_Z}{R_L}$$

$$I_I = \frac{V_{PS} - V_Z}{R_i}$$

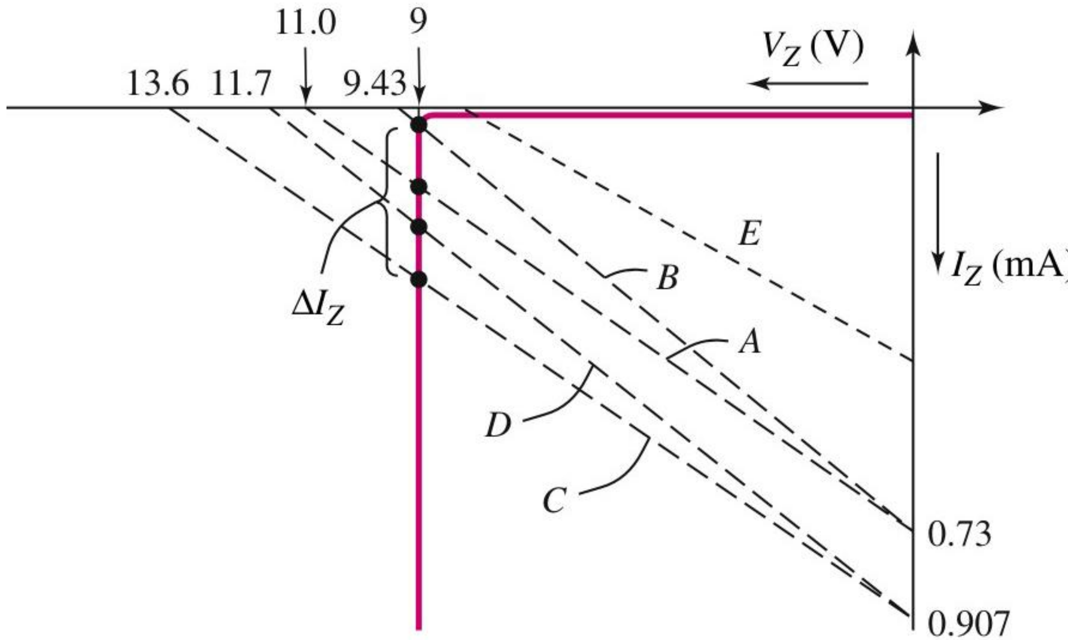
$$I_Z = I_I - I_L$$

The characteristics of the Zener diode determines V_L.

Design Example 2.5

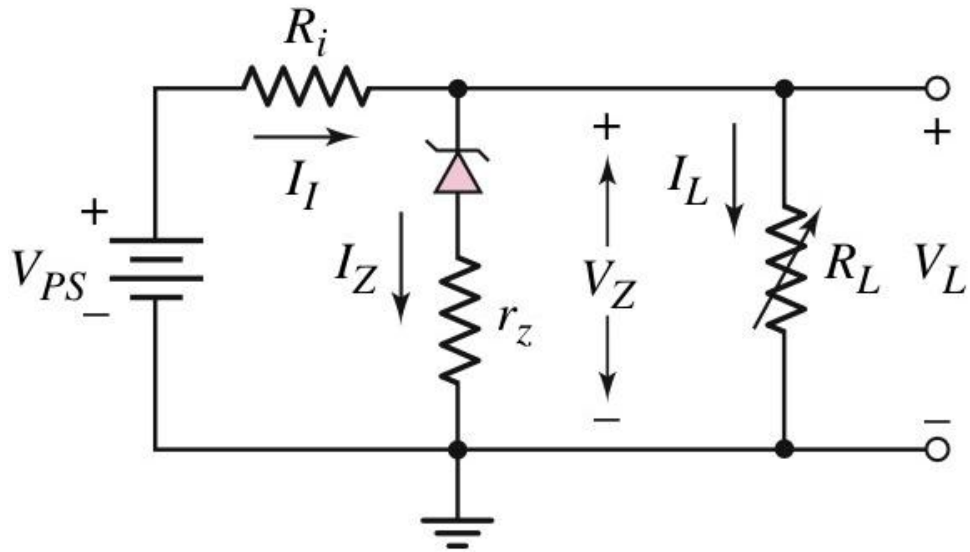


Load Line Analysis



The reverse bias I-V is important for Zener diodes.

Voltage Rectifier with nonzero Zener resistance



The Zener diode begins to conduct when $V_{PS} = V_Z$.

When $V_{PS} \geq V_Z$: $V_L = V_Z$

$I_L = V_Z/R_L$, but $V_Z \neq$ constant

$I_1 = (V_{PS} - V_Z)/R_i$

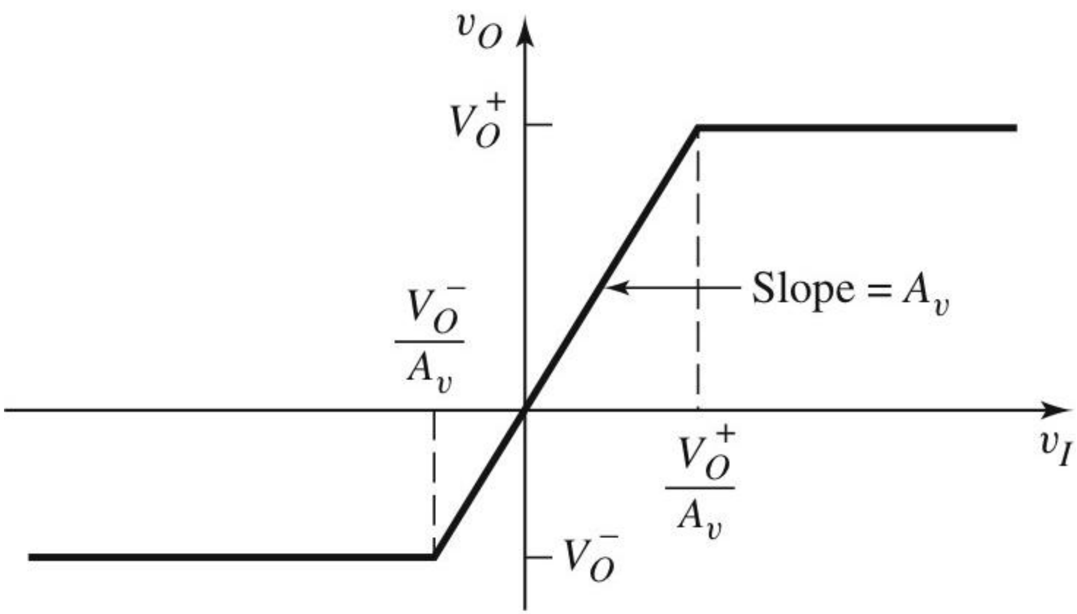
$I_Z = I_1 - I_L$

Clipper and Clamper Circuits

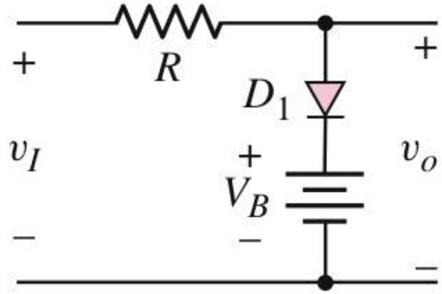
- More Applications of Diodes

Clipper Circuits

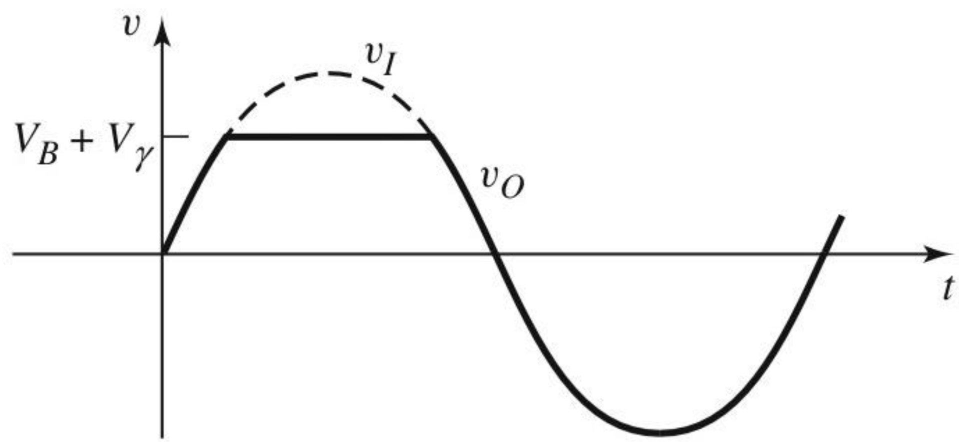
Voltage Transfer Characteristics of Limiter Circuit



Single Diode Clipper

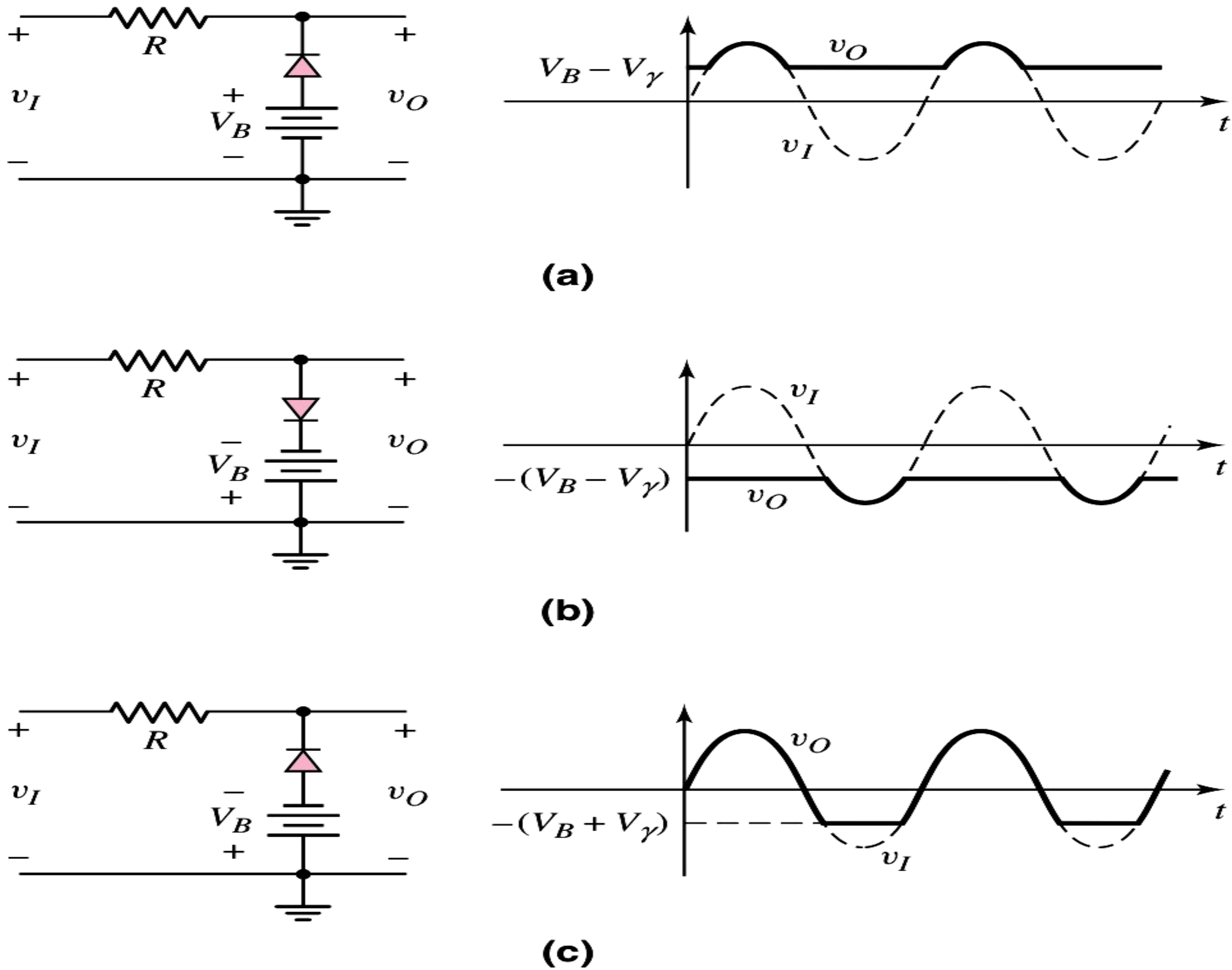


(a)

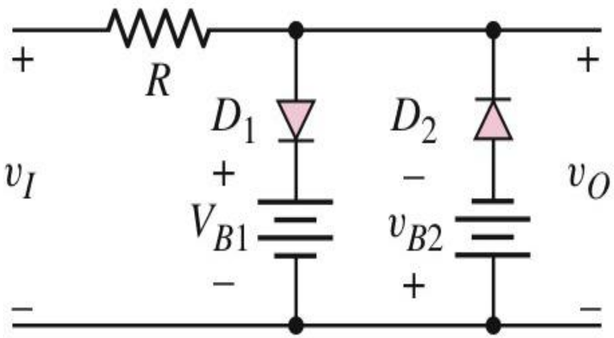


(b)

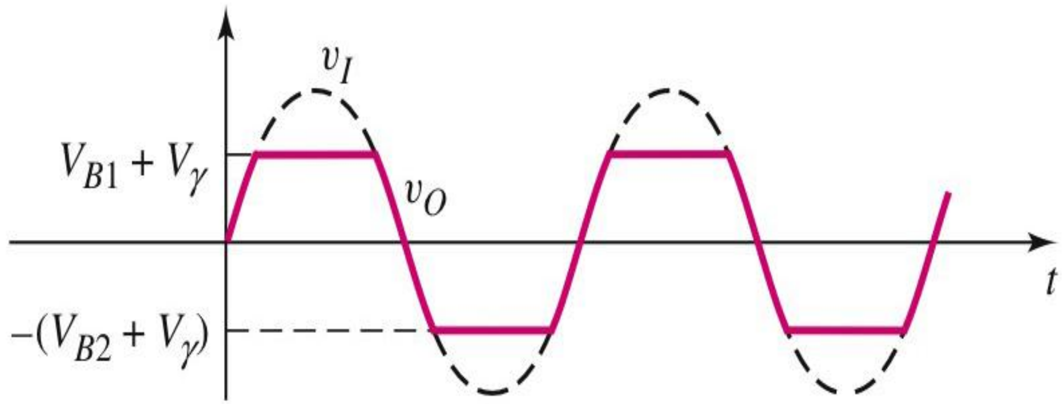
Additional Diode Clipper Circuits



Parallel-Based Diode Clipper Circuit

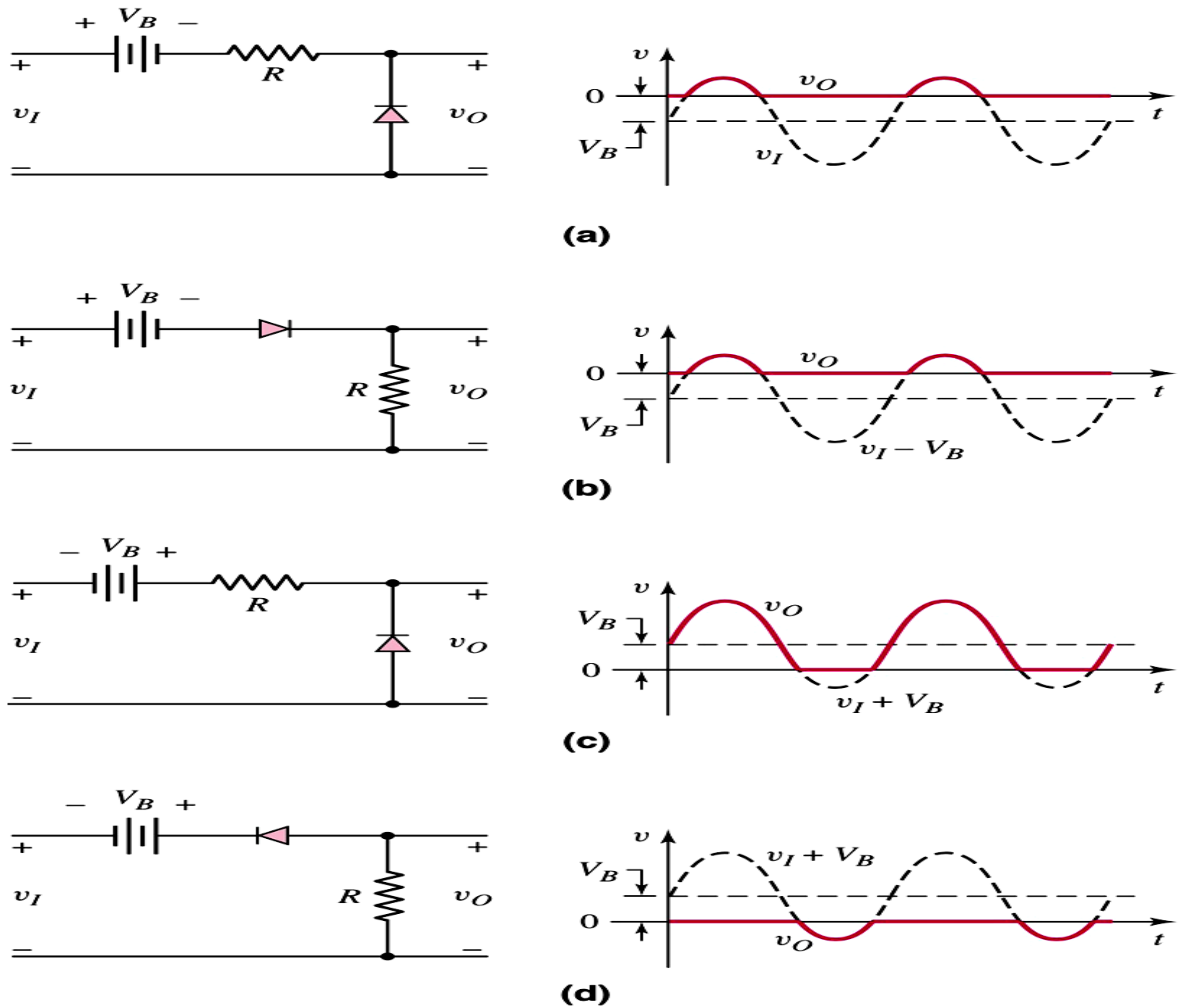


(a)

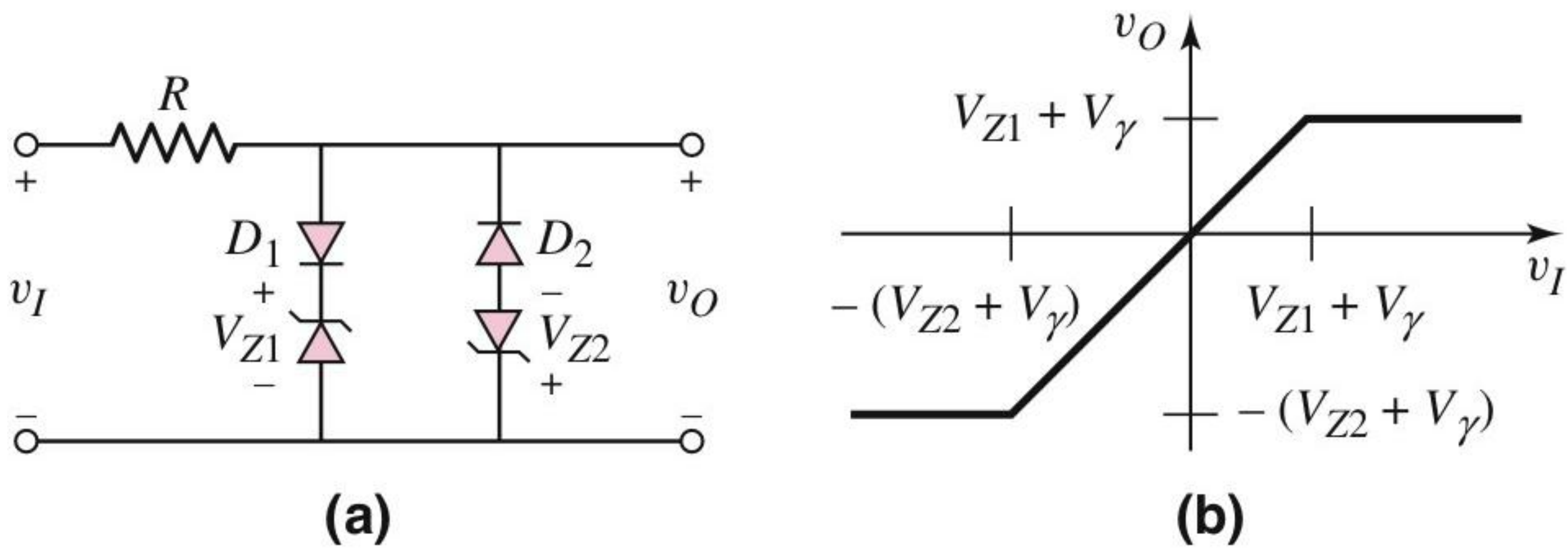


(b)

Series-Based Diode Clipper Circuits

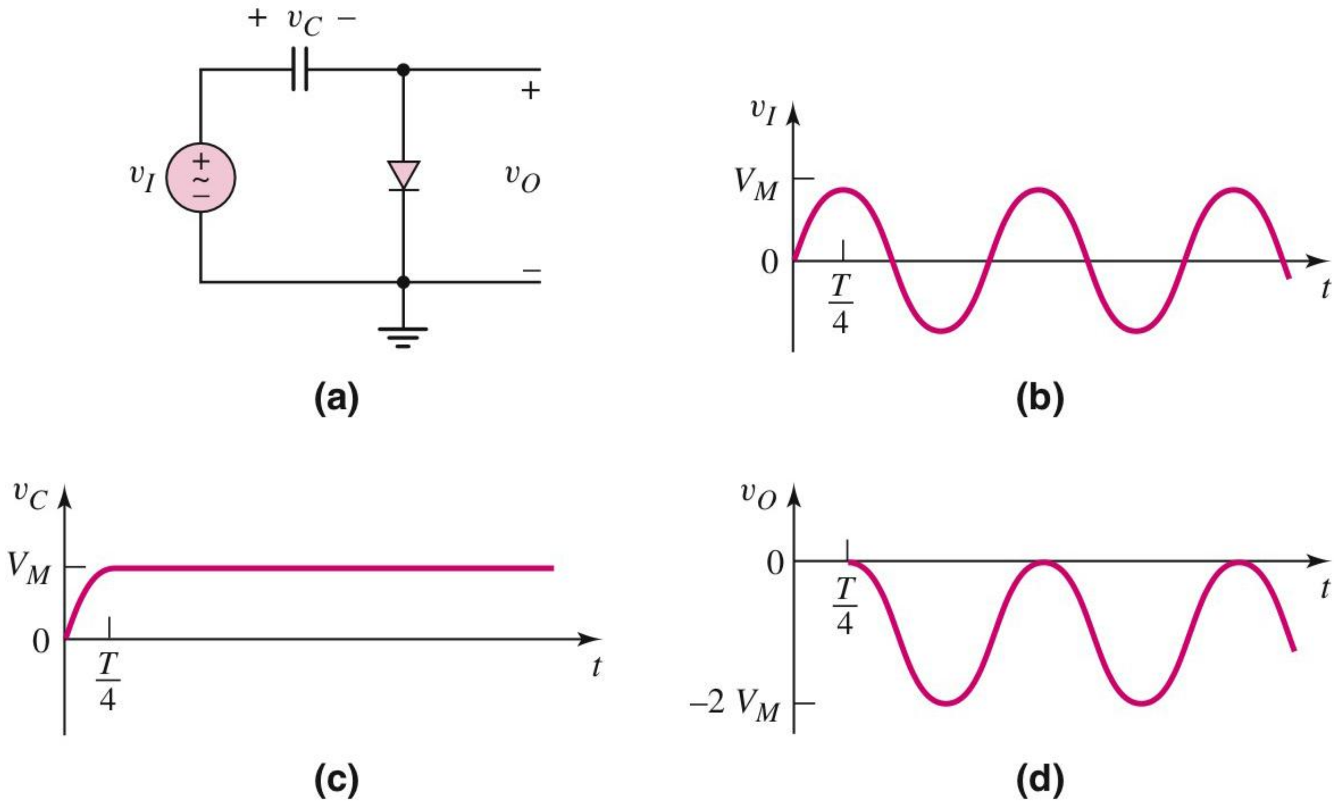


Parallel-Based Clipper Circuit Using Zener Diodes



Clamper Circuits

Diode Clamper Circuit



It adjusts the dc level without needing to know the exact waveform.

Impedance

- **Electrical impedance** is the measure of the opposition that a circuit presents to a current when a voltage is applied.

- Impedance extends the concept of resistance to AC circuits, and possesses both magnitude and phase, unlike resistance, which has only magnitude.

Impedance (Cont')

- In Cartesian form,

$$Z = R + jX$$

where the real part of impedance is the resistance R, and the imaginary part is the reactance X. j is the imaginary unit and there is $j^2 = -1$.

What is reactance?

Reactance

- The concept of impedance in AC circuits involves **TWO** additional **impeding mechanisms** compared to the normal *resistance* of DC circuits:

1. the induction of voltages in conductors self-induced by the magnetic fields of currents – *inductance, and*
2. the electrostatic storage of charge induced by voltages between conductors – *capacitance.*

- The impedance caused by these two effects is **collectively** referred to as reactance.

Reactance (Cont')

- The impedance of inductors increases as frequency increases

$$Z_L = j\omega L$$

- The impedance of capacitors decreases as frequency increases

$$Z_C = \frac{1}{j\omega C}$$

Capacitance

$$i_{\mathrm{C}}(t) = C\frac{\mathrm{d}\,v_{\mathrm{C}}(t)}{\mathrm{d}\,t}$$

$$v_{\mathrm{C}}(t) = V_p \sin(\omega t)$$

$$\frac{\mathrm{d}\,v_{\mathrm{C}}(t)}{\mathrm{d}\,t} = \omega V_p \cos(\omega t)$$

$$\frac{v_{\mathrm{C}}\,(t)}{i_{\mathrm{C}}\,(t)} = \frac{V_p \sin(\omega t)}{\omega V_p C \cos(\omega t)} = \frac{\sin(\omega t)}{\omega C \sin\left(\omega t + \frac{\pi}{2}\right)}$$

$$Z_{\text{capacitor}} = \frac{1}{\omega C} e^{-j\frac{\pi}{2}}$$

$$Z_{\text{capacitor}} = -j\frac{1}{\omega C} = \frac{1}{j\omega C}$$

Inductance

$$v_{\mathrm{L}}(t) = L\frac{\mathrm{d}\, i_{\mathrm{L}}(t)}{\mathrm{d}\, t}$$

$$i_{\mathrm{L}}(t) = I_p \sin(\omega t)$$

$$\frac{\mathrm{d}\, i_{\mathrm{L}}(t)}{\mathrm{d}\, t} = \omega I_p \cos(\omega t)$$

$$\frac{v_{\mathrm{L}}(t)}{i_{\mathrm{L}}(t)} = \frac{\omega I_p L \cos(\omega t)}{I_p \sin(\omega t)} = \frac{\omega L \sin\left(\omega t + \frac{\pi}{2}\right)}{\sin(\omega t)}$$

$$Z_{\text{inductor}} = \omega L e^{j\frac{\pi}{2}}$$

$$Z_{\text{inductor}} = j\omega L$$

Inductance

$$v_{\mathrm{L}}(t) = L\frac{\mathrm{d}\,i_{\mathrm{L}}(t)}{\mathrm{d}\,t}$$

$$i_{\mathrm{L}}(t) = I_p \sin(\omega t)$$

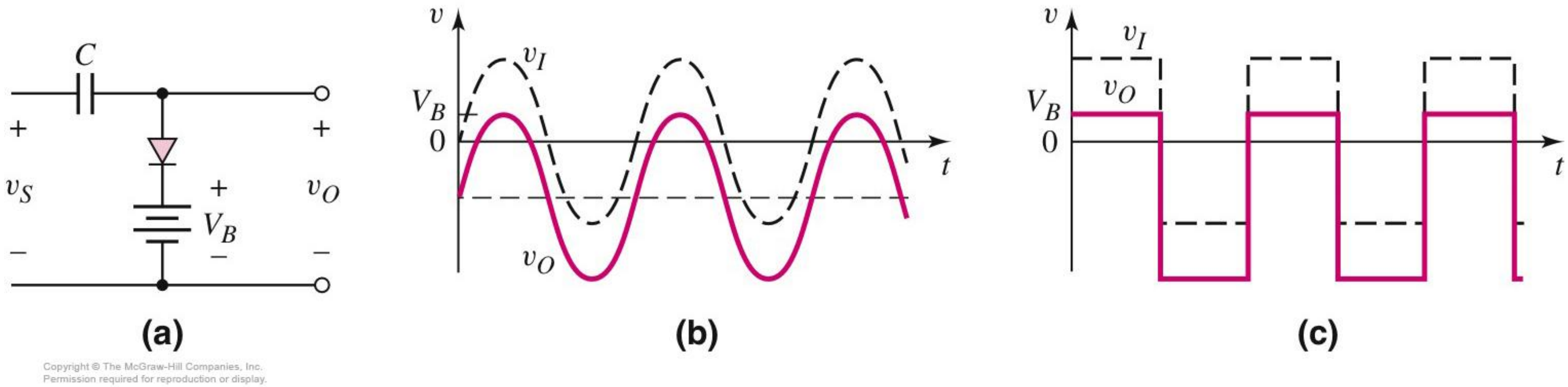
$$\frac{\mathrm{d}\,i_{\mathrm{L}}(t)}{\mathrm{d}\,t} = \omega I_p \cos(\omega t)$$

$$\frac{v_{\mathrm{L}}(t)}{i_{\mathrm{L}}(t)} = \frac{\omega I_p L \cos(\omega t)}{I_p \sin(\omega t)} = \frac{\omega L \sin\left(\omega t + \frac{\pi}{2}\right)}{\sin(\omega t)}$$

$$Z_{\text{inductor}} = \omega L e^{j\frac{\pi}{2}}$$

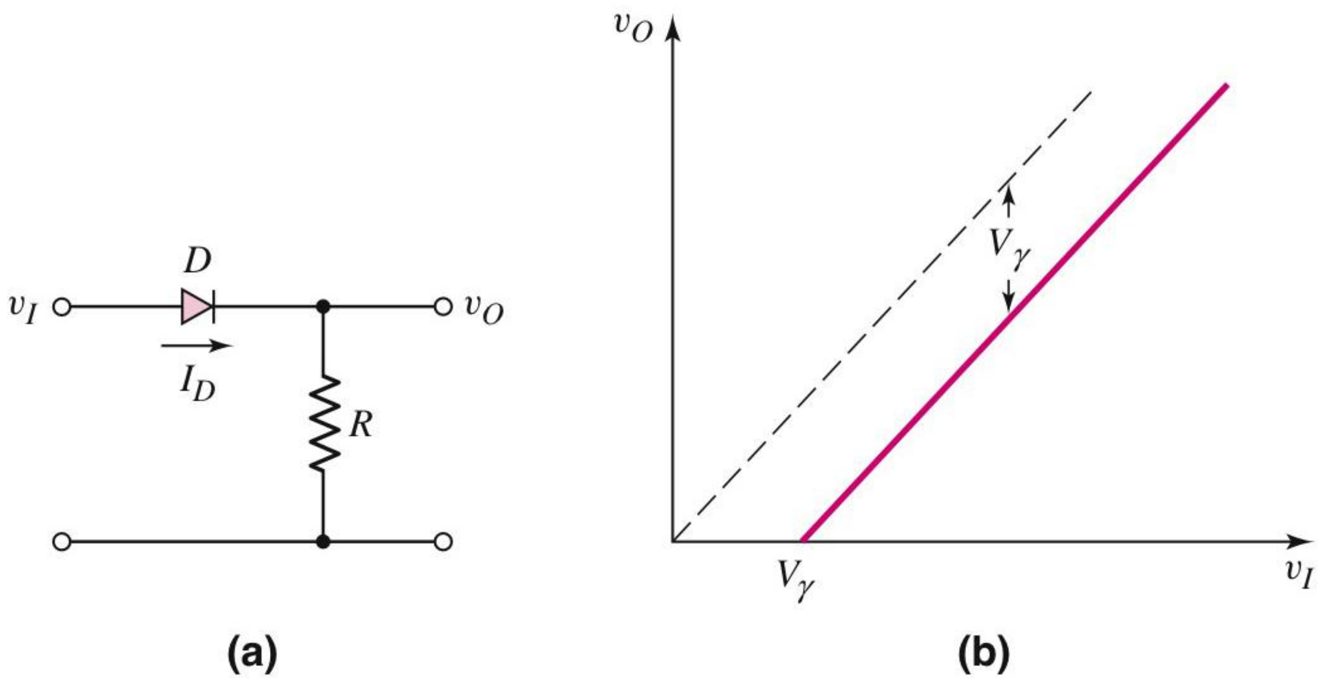
$$Z_{\text{inductor}} = j\omega L$$

Diode Clamper Circuit with Voltage Source



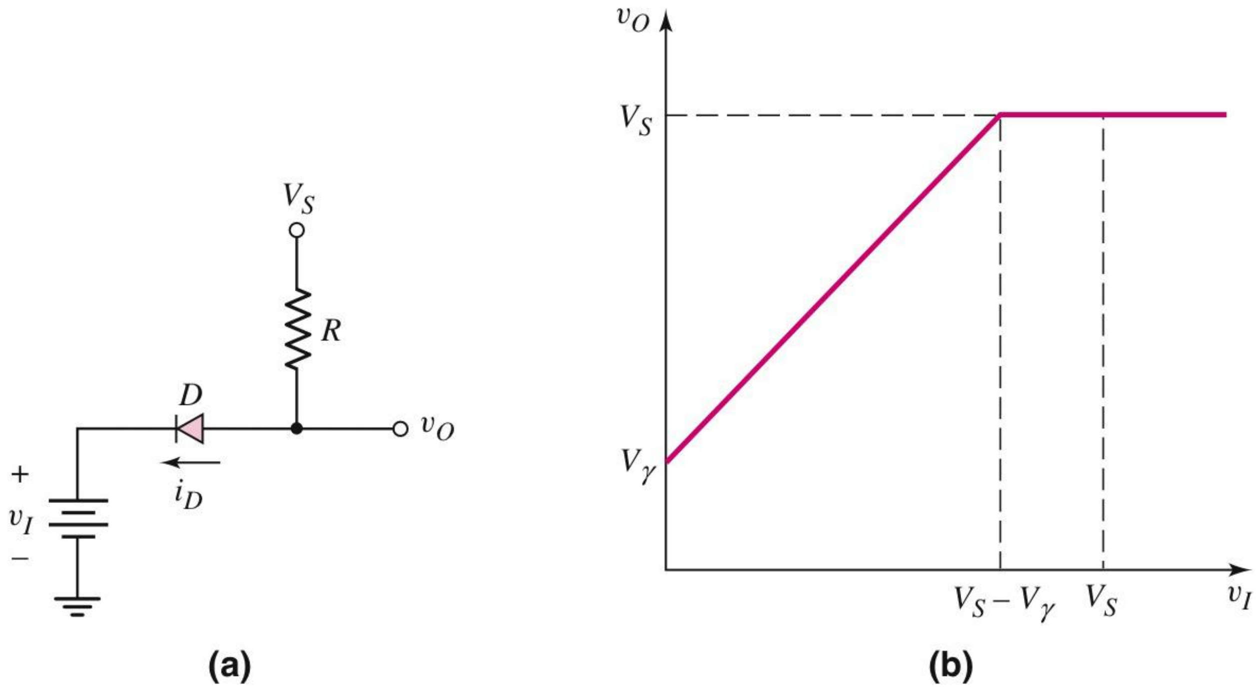
Example Diode Circuits

Diode and Resistor In Series



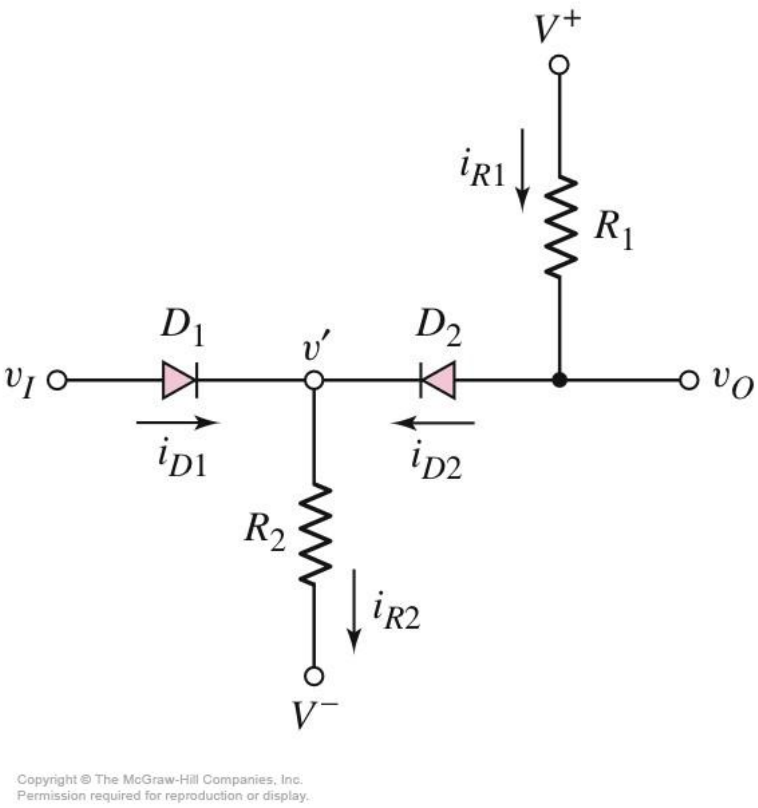
Voltage shift between input and output voltages in transfer characteristics is because the diode only conducts when $v_1 \geq V_\gamma$.

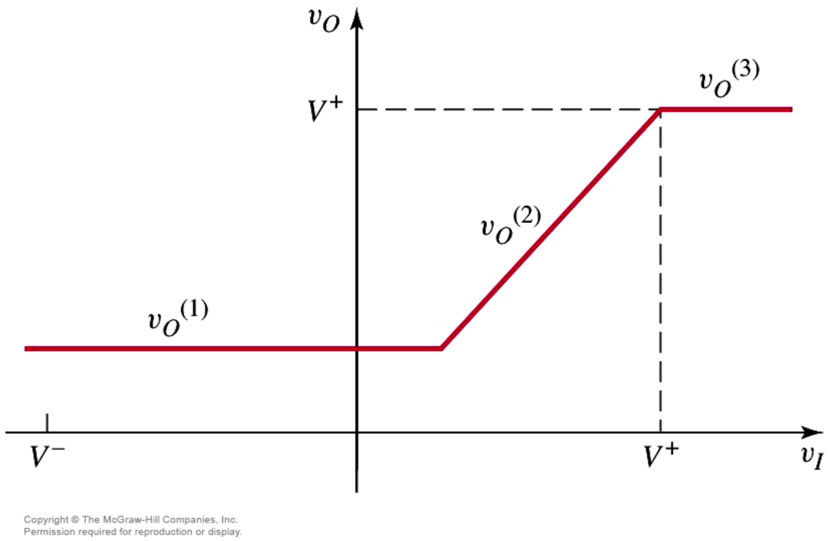
Diode with Input Voltage Source



Output voltage is a constant when the diode is not conducting, when $v_1 \geq V_s - V_\gamma$.

2 Diode Circuit



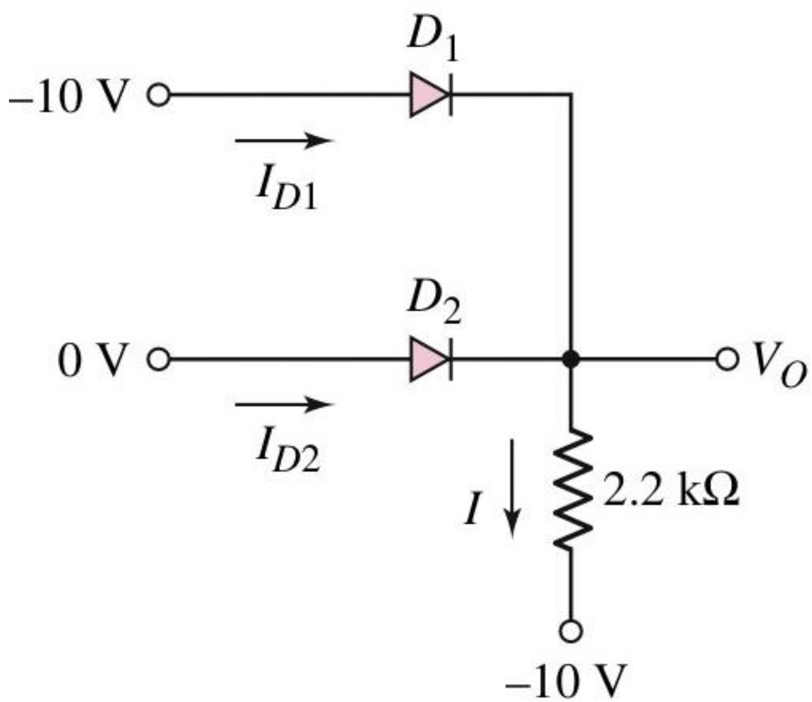


Voltage transfer characteristics

Problem-Solving Technique: Multiple Diode Circuits

1. Assume the state of the diode.
   a. If assumed on, $V_D = V_\gamma$
   b. If assumed off, $I_D = 0$.
2. Analyze the 'linear' circuit with assumed diode states.
3. Evaluate the resulting state of each diode.
4. If any initial assumptions are proven incorrect, make new assumption and return to Step 2.

Exercise problem

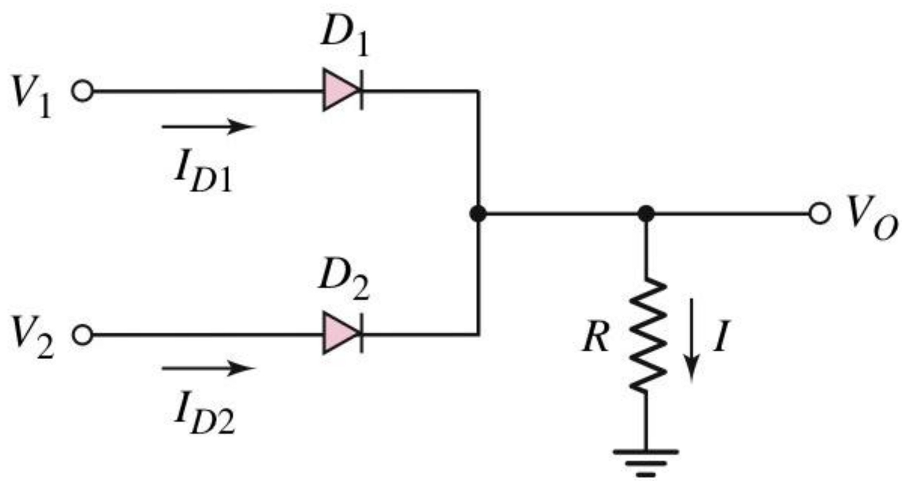


D_1 is not on.

D_2 is on.
This pins V_O to -0.6V

Diode Logic Circuits

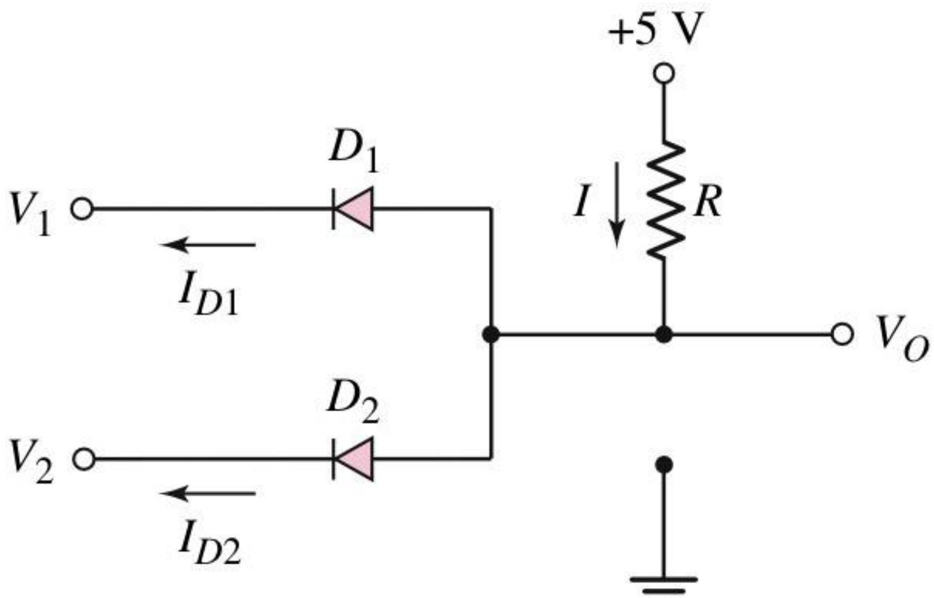
Diode Logic Circuits: 2-Input OR Gate



V_1 (V)	V_2 (V)	V_O (V)
0	0	0
5	0	4.3
0	5	4.3
5	5	4.3

$V_\gamma = 0.7V$

Diode Logic Circuits: 2-Input AND Gate

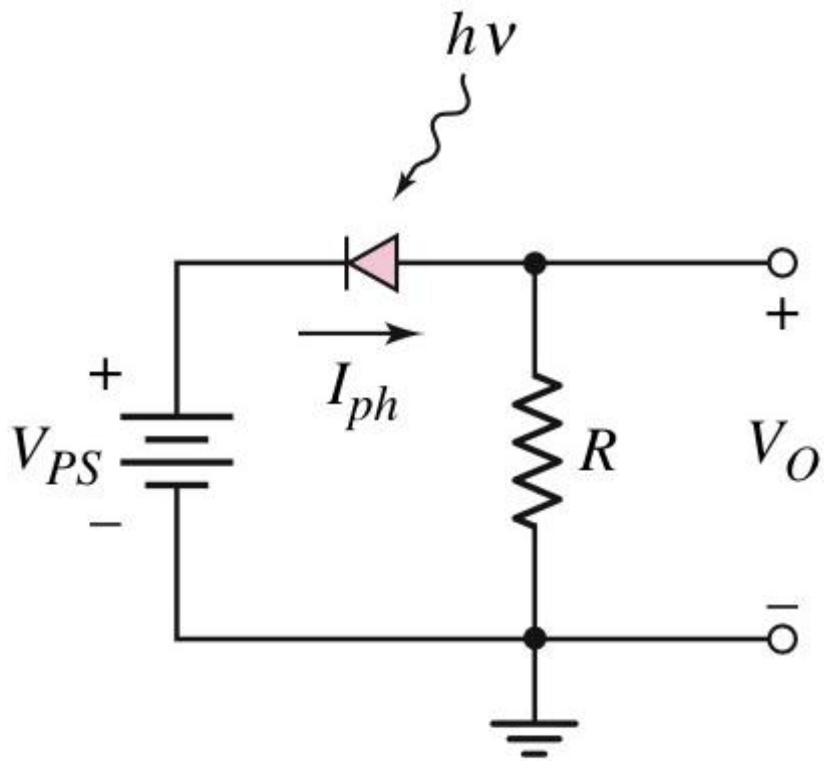


V_1 (V)	V_2 (V)	V_O (V)
0	0	0
5	0	0
0	5	0
5	5	4.3

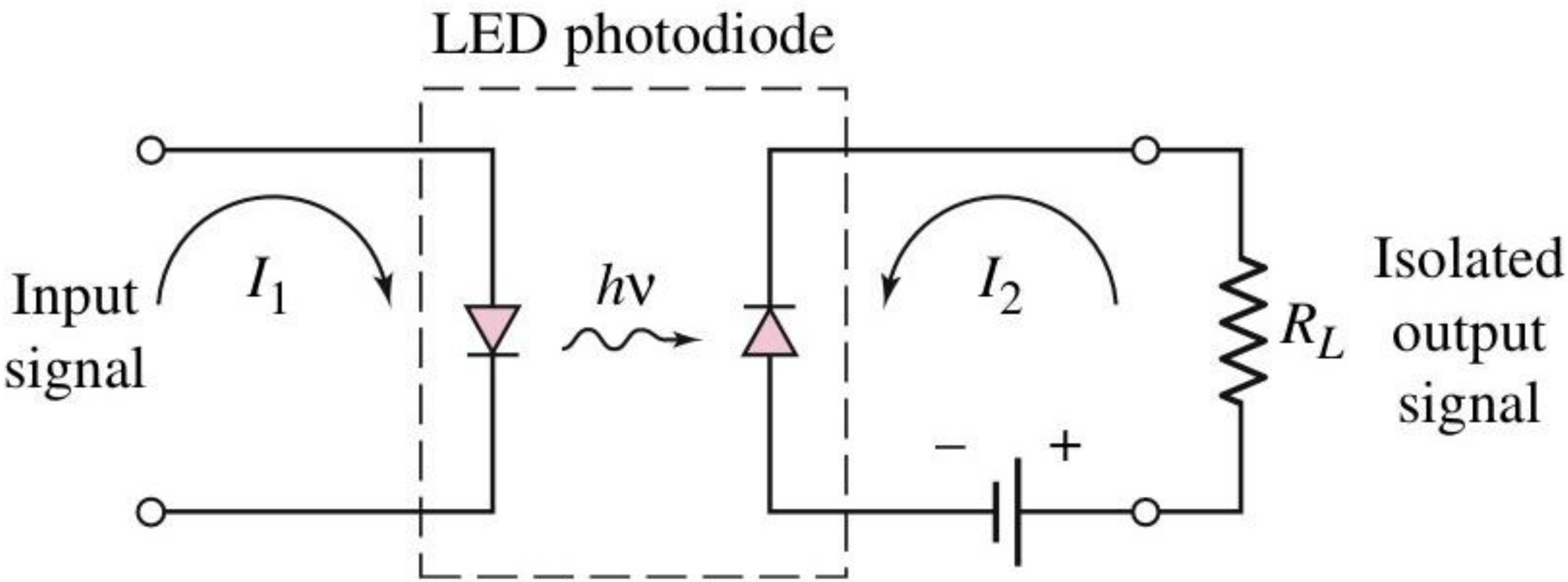
$V_\gamma = 0.7V$

Photodiode and LED Circuits

Photodiode Circuit

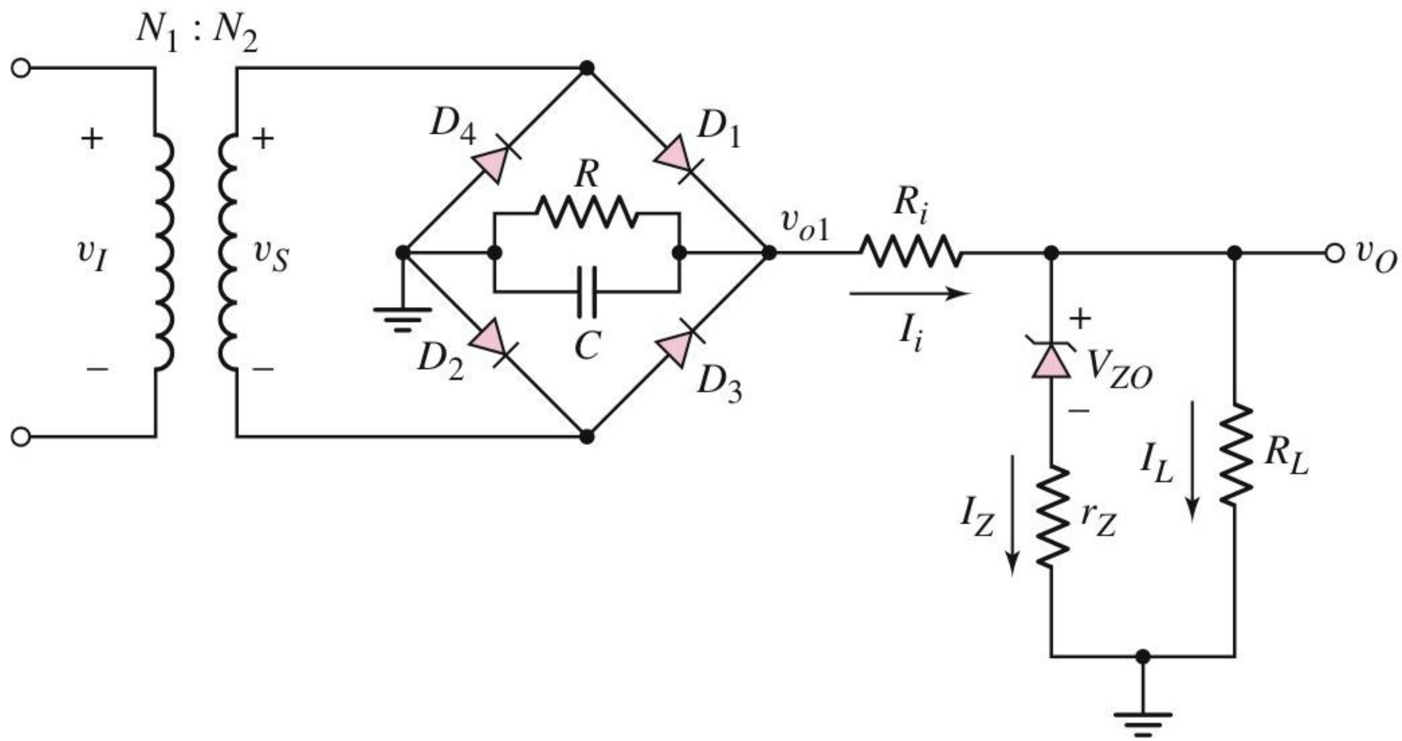


Optoisolator



Design Application
- DC Power Supply

Design DC Power Supply Circuit



Chapter 2 Summary

1. Determine the operation and characteristics of diode rectifier circuits, which is the first stage of the process of converting an ac signal into a dc signal in the electronic power supply.

2. Apply the characteristics of the Zener diode to a Zener diode voltage regulator circuit.

3. Apply the nonlinear characteristics of diodes to create waveshaping circuits known as clippers and clampers.

4. Examine the techniques used to analyze circuits that contain more than one diode.

5. Understand the operation and characteristics of specialized photodiode and light-emitting diode circuits.

EEE109: Electronic Circuits

The Bipolar Junction Transistor

Contents

- Discuss the physical structure and operation of the bipolar junction transistor.
- Understand the dc analysis and design techniques of bipolar transistor circuits.
- Examine three basic applications of bipolar transistor circuits.
- Investigate various dc biasing schemes of bipolar transistor circuits, including integrated circuit biasing.
- Consider the dc biasing of multistage or multi-transistor circuits.

MOFET VS BIPOLAR Comparison of Characteristics

- For MOSFET, the important feature is how V_{GS} controls I_D, shown as the **transfer characteristic**, I_D against V_{GS} .

- For a BJT the transfer characteristic is almost a straight line with slope β (transfer characteristics don't vary) so it is not necessary to examine it.

- For the BJT the **input characteristic** is examined (<u>I_B as a function of V_{BE}</u>) whereas no current flows into a MOSFET so it has the input characteristic is $I_G = 0$.

An enhancement MOSFET is "*on*" – "*active*" - if $V_{GS} > V_T$

, where V_T is the **threshold voltage**.

Comparison of Characteristics

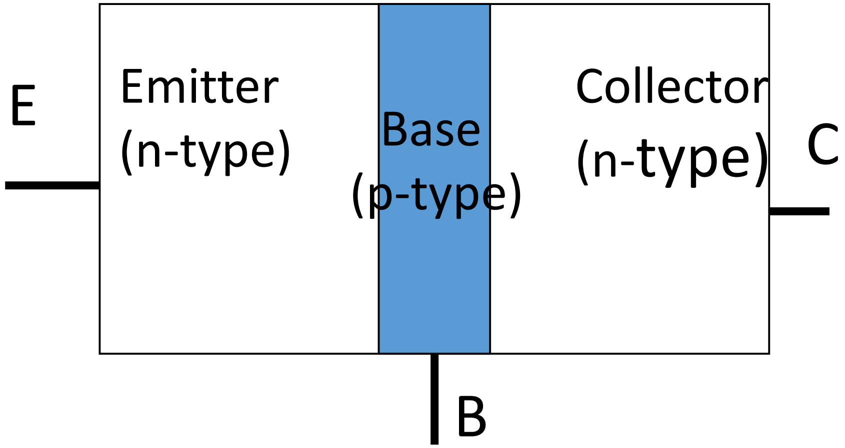
Bipolar

- I_B controls the collector current
- Control varies greatly from transistor to transistor as β varies by a large amount
- Input voltage threshold is $V_{BE} > 0.7$ volts.
- When conducting V_{BE} is almost constant at about 0.7V

MOSFET

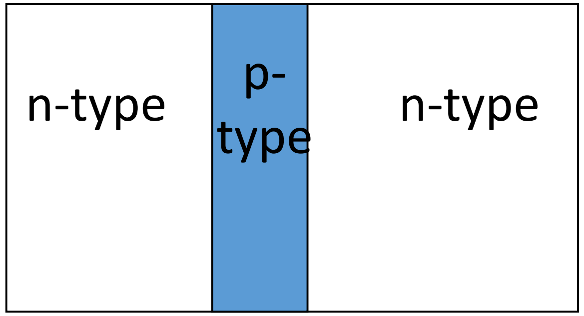
- V_{GS} controls the drain current (strictly V_{GS} - V_{T})
- Control varies greatly from transistor to transistor as V_T varies by a large amount
- Input voltage threshold is V_T - varies with transistor.
- I_G = 0 for all situations so is constant

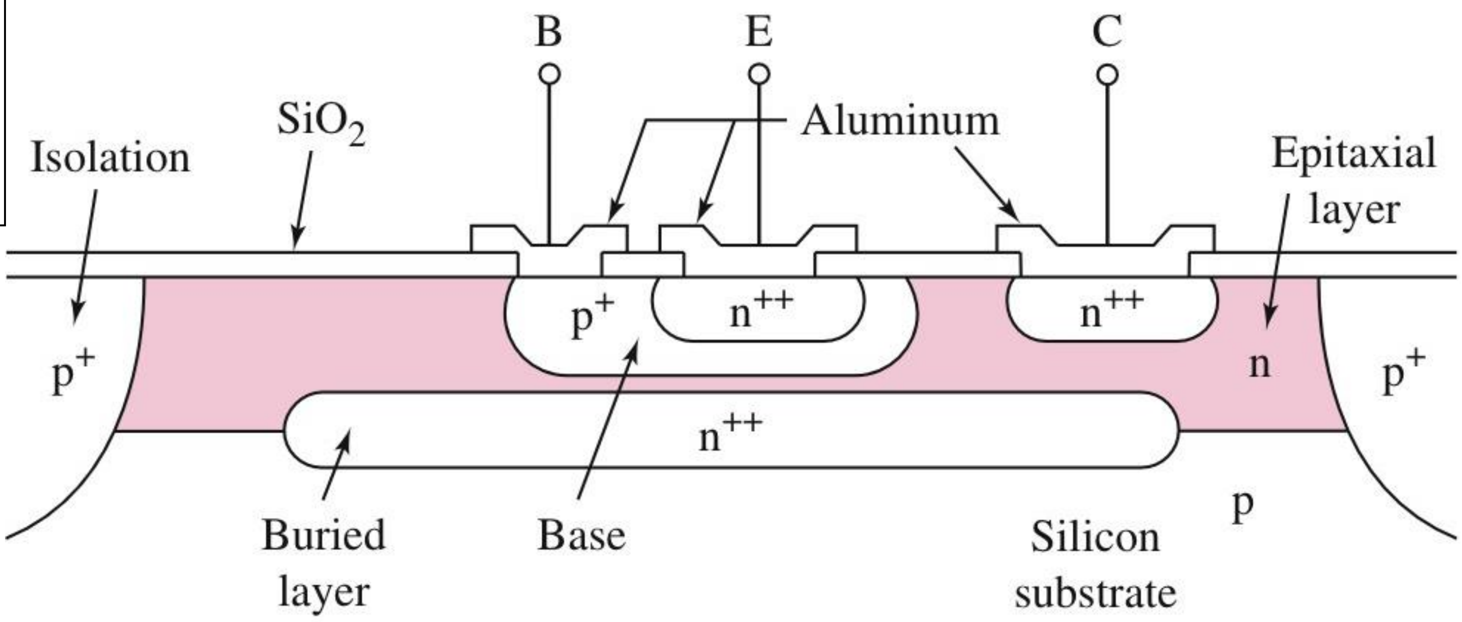
Bipolar Junction Transistors (BJTs)



- The bipolar junction transistor is a semiconductor device constructed with three doped regions.
- These regions essentially form two 'back-to-back' p-n junctions in the same block of semiconductor material (silicon).
- The most common use of the BJT is in linear amplifier circuits (linear means that the output is proportional to input). It can also be used as a switch (in, for example, logic circuits).

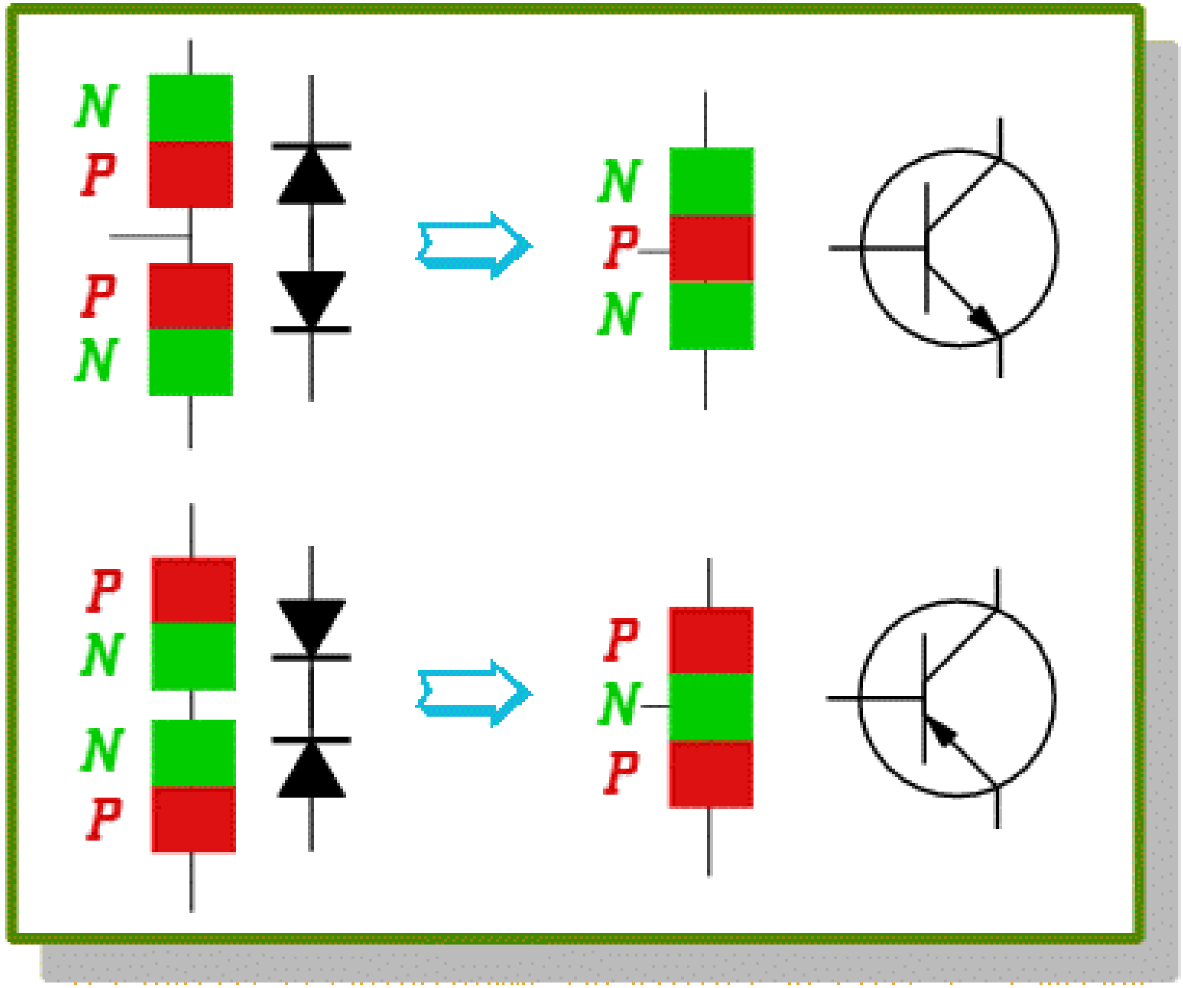
Cross Section of Integrated Circuit npn Transistor

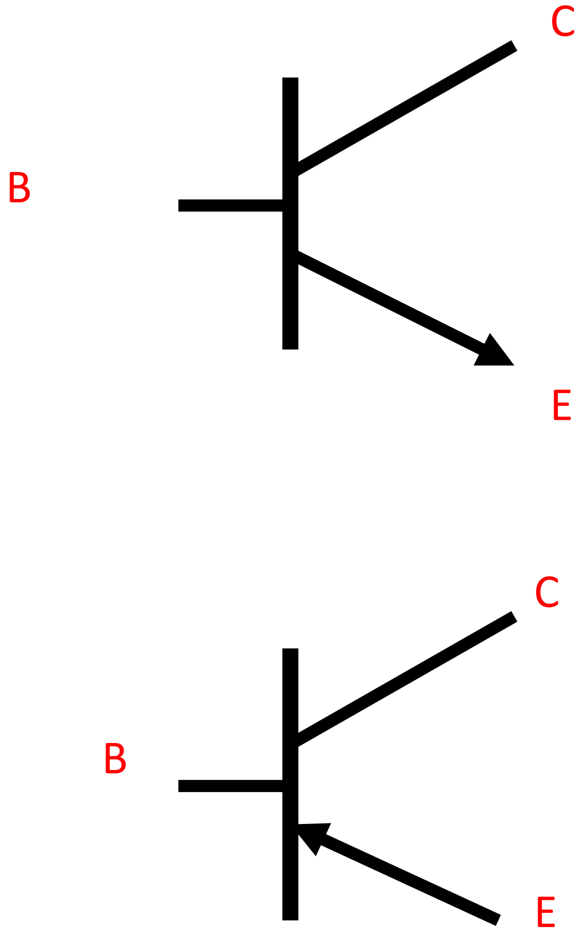




Impurity doping concentrations in the three regions are substantially different.

npn BJT Symbol





The direction of the arrow on the emitter is reversed

Common Configuration

- NPN Transistor Most Common Configuration

- Base, Collector, and Emitter
  - Base is a very thin region with **less dopants**
  - Base collector junction reversed biased
  - Base emitter junction forward biased

  Current flow analogy:
  - If current flows into the base, a **much larger current** can flow from the collector to the emitter
  - If a signal to be amplified is applied as a current to the base, a valve between the collector and emitter opens and closes in response to signal fluctuations

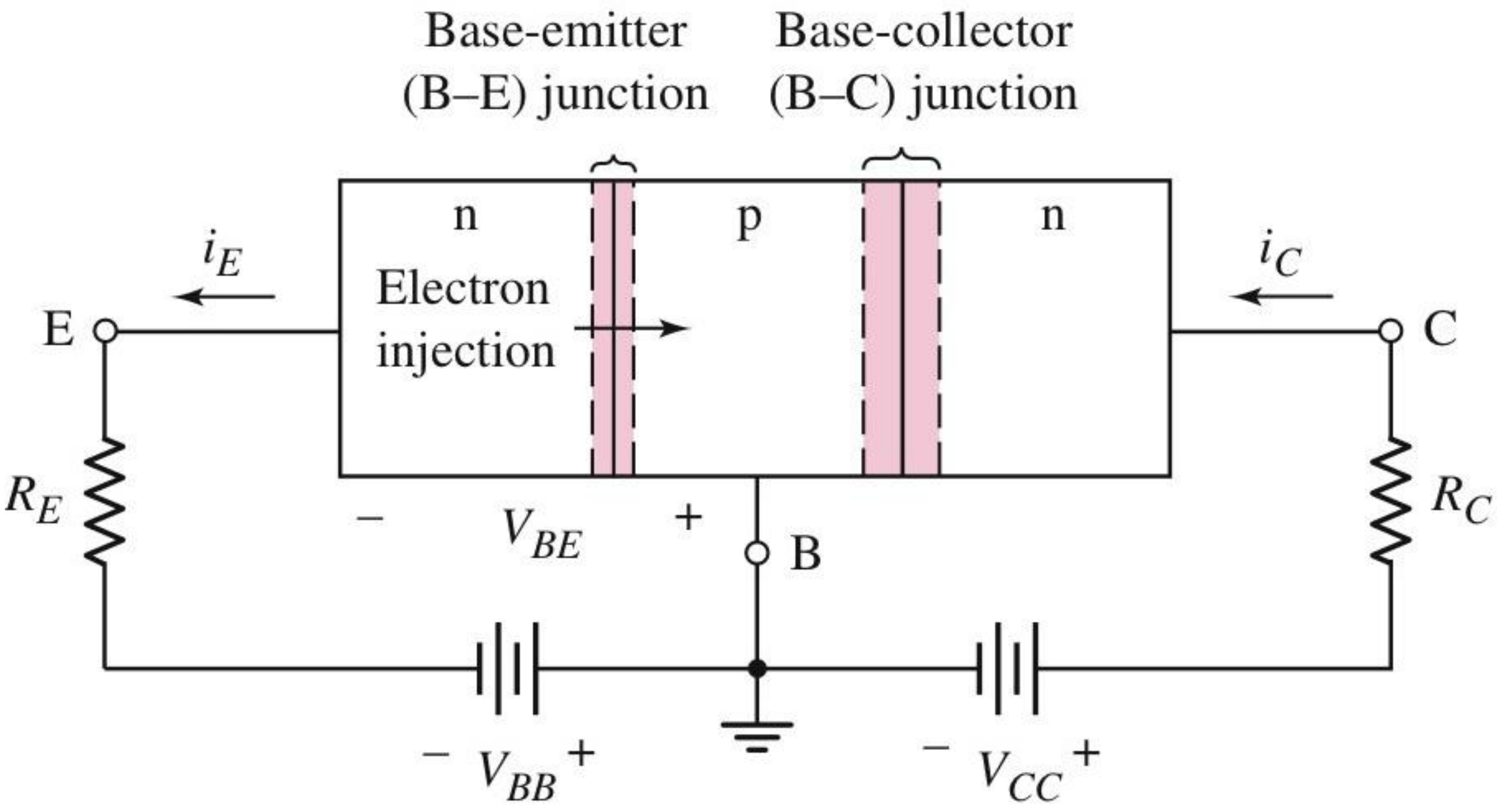
- PNP Transistor essentially the same except for directionality

Modes of Operation

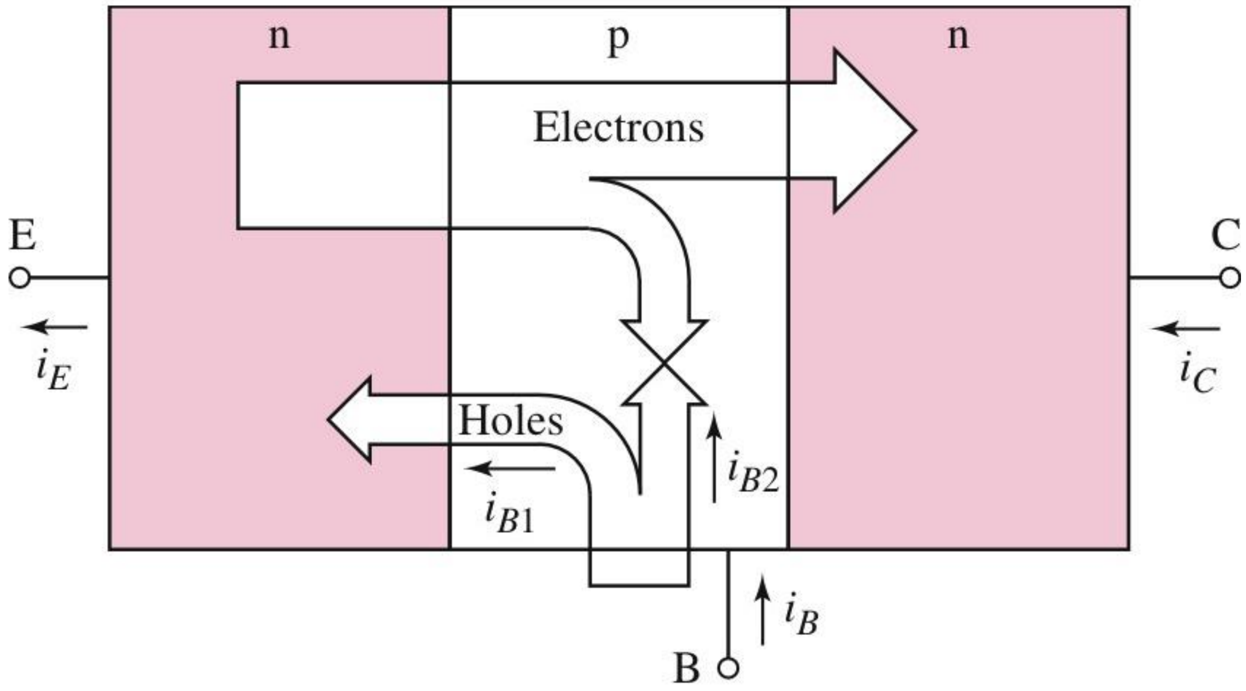
Modes of Operation

- Forward-Active
  - B-E junction is forward biased
  - B-C junction is reverse biased
- Saturation
  - B-E and B-C junctions are forward biased
- Cut-Off
  - B-E and B-C junctions are reverse biased
- Inverse-Active (or Reverse-Active)
  - B-E junction is reverse biased
  - B-C junction is forward biased

npn BJT in Forward-Active



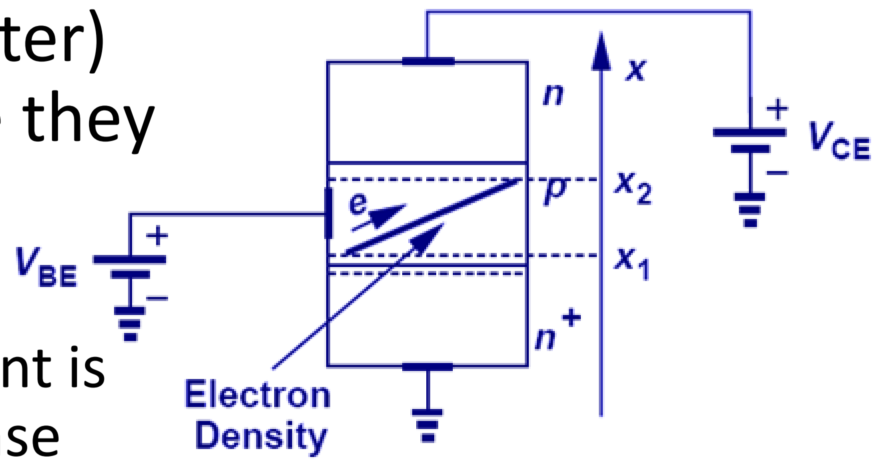
Electrons and Holes in npn BJT



Carrier Transport in the Base Region

- Since the width of the quasi-neutral base region ($W_B = x_2 - x_1$) is much smaller than the minority-carrier diffusion length, very few of the carriers injected (from the emitter) into the base recombine before they reach the collector-junction depletion region.

  → Minority-carrier diffusion current is ~constant in the quasi-neutral base

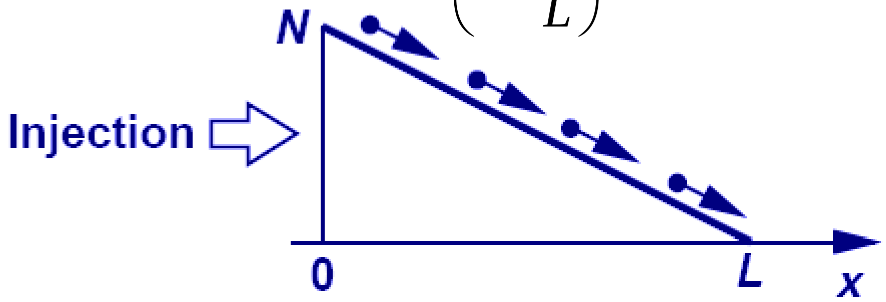


- The minority-carrier concentration at the edges of the collector-junction depletion region are ~0.

Diffusion Example Redux

- Linear concentration profile
  → constant diffusion current

$$p = N\left(1 - \frac{x}{L}\right)$$

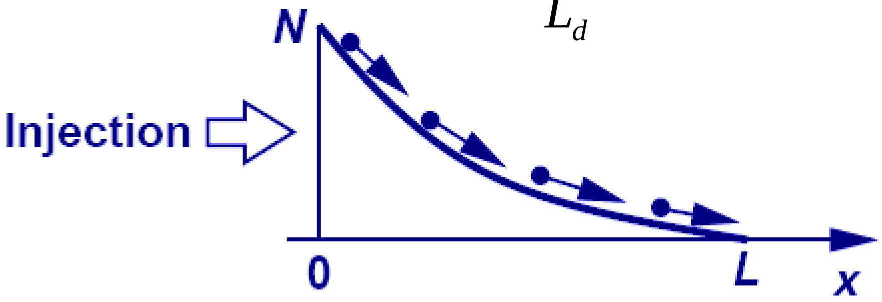


$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= qD_p \frac{N}{L}$$

- Non-linear concentration profile
  → varying diffusion current

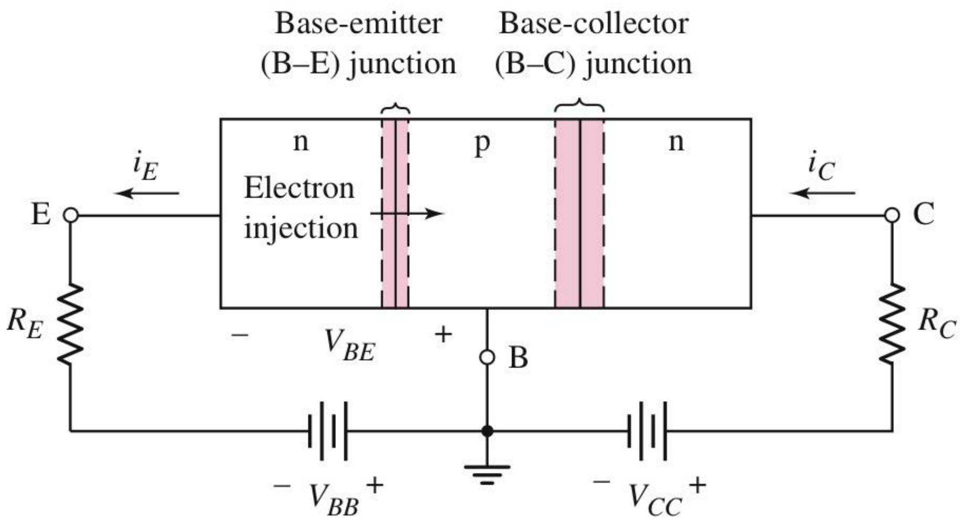
$$p = N \exp \frac{-x}{L_d}$$

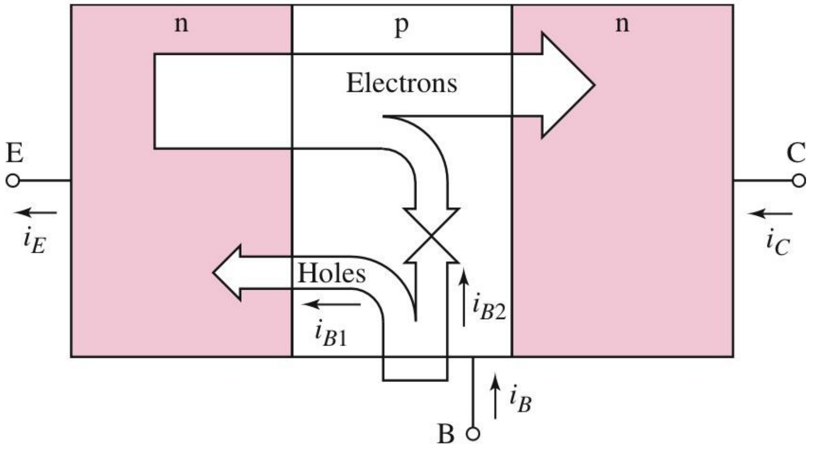


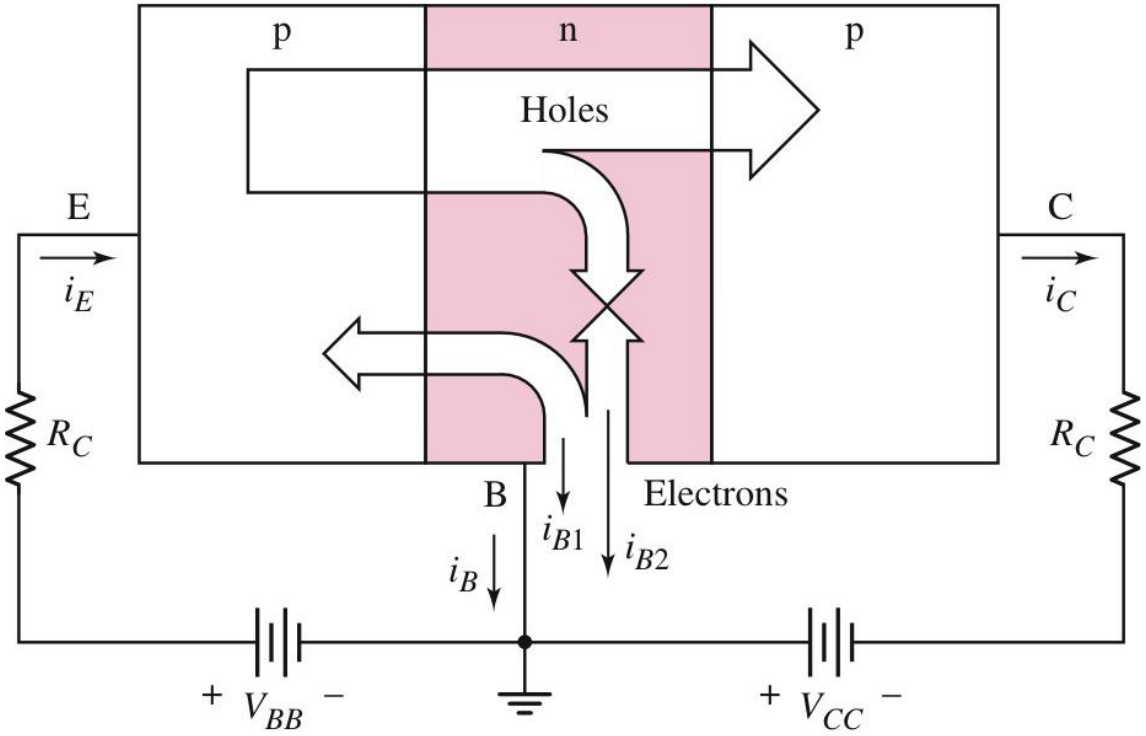
$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= \frac{qD_p N}{L_d} \exp \frac{-x}{L_d}$$

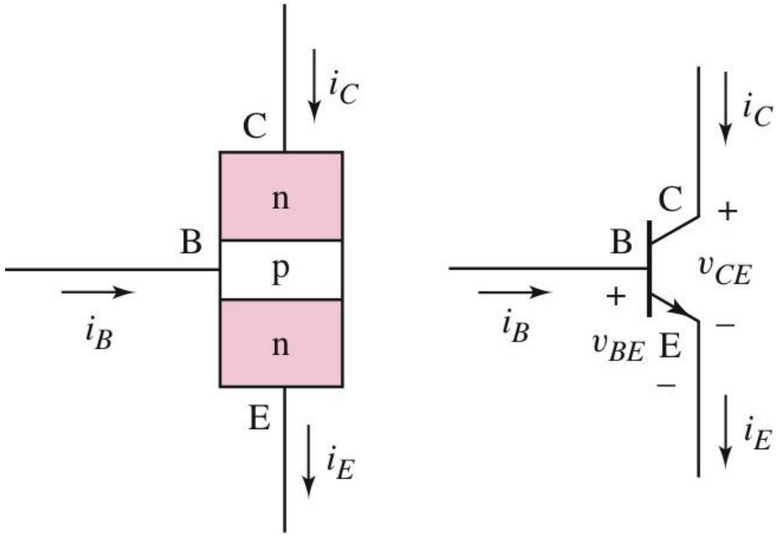
Electrons and Holes in pnp BJT



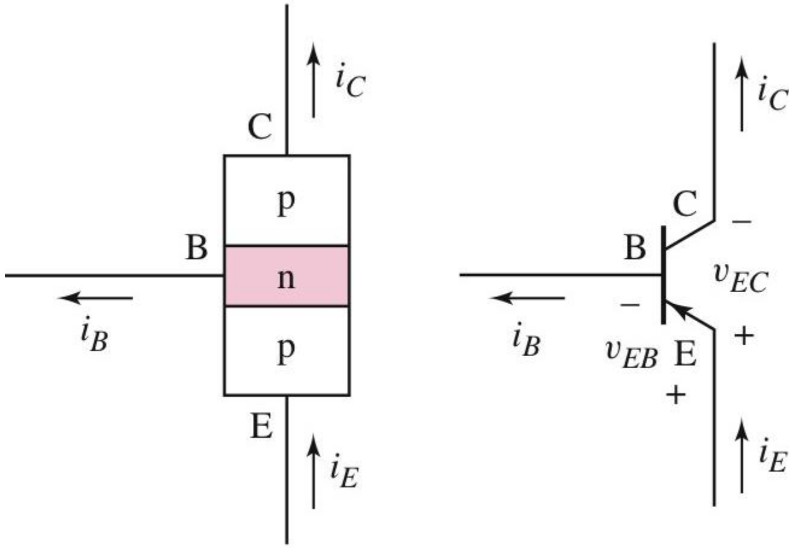




Circuit Symbols and Current Conventions

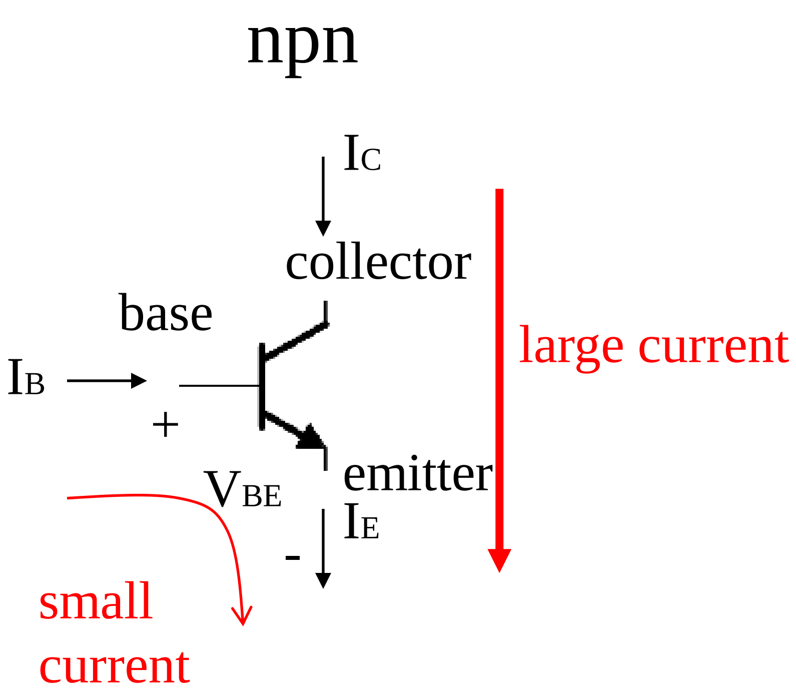


(a) (b)

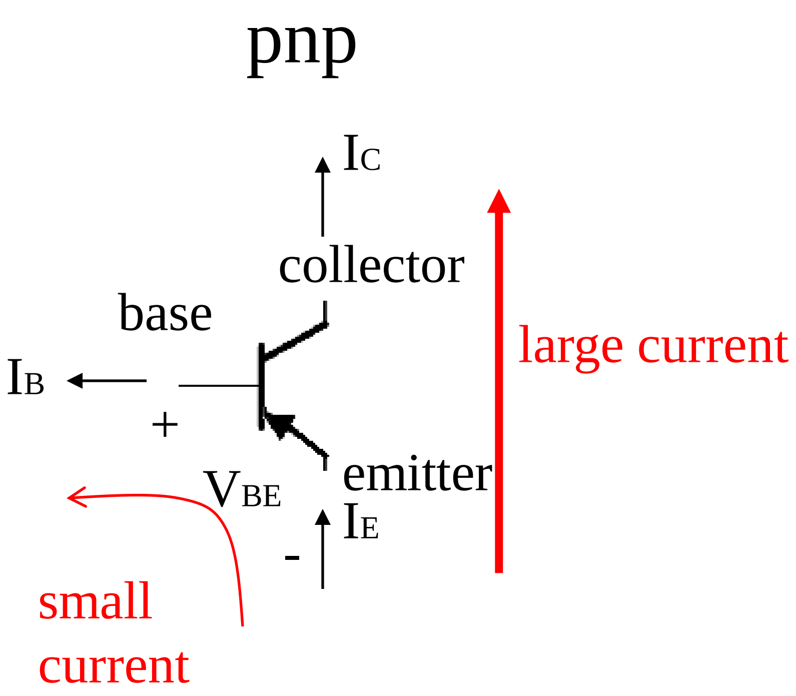


(a) (b)

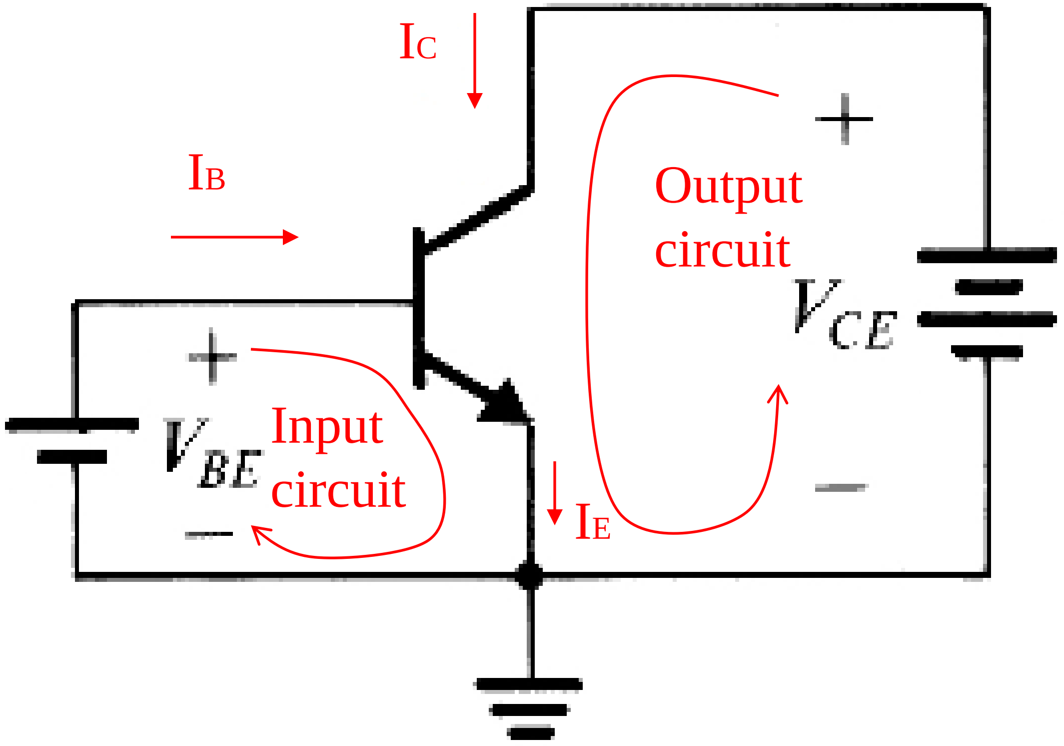
Summary of npn Transistor Behavior



Summary of pnp Transistor Behavior

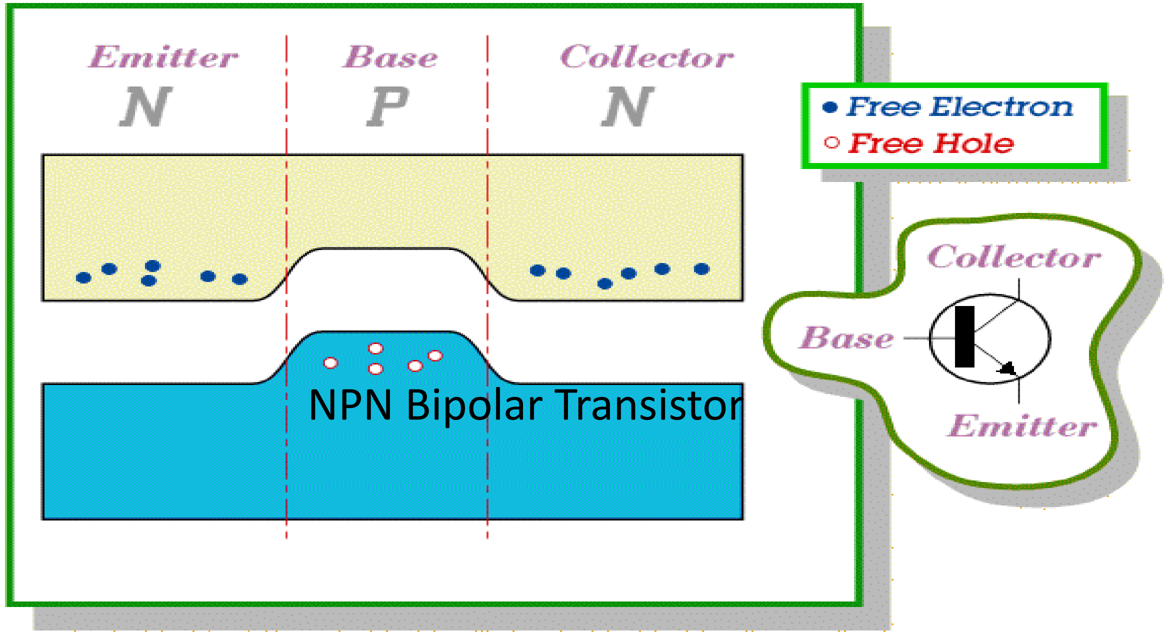


Graphical Representation of Transistor Characteristics



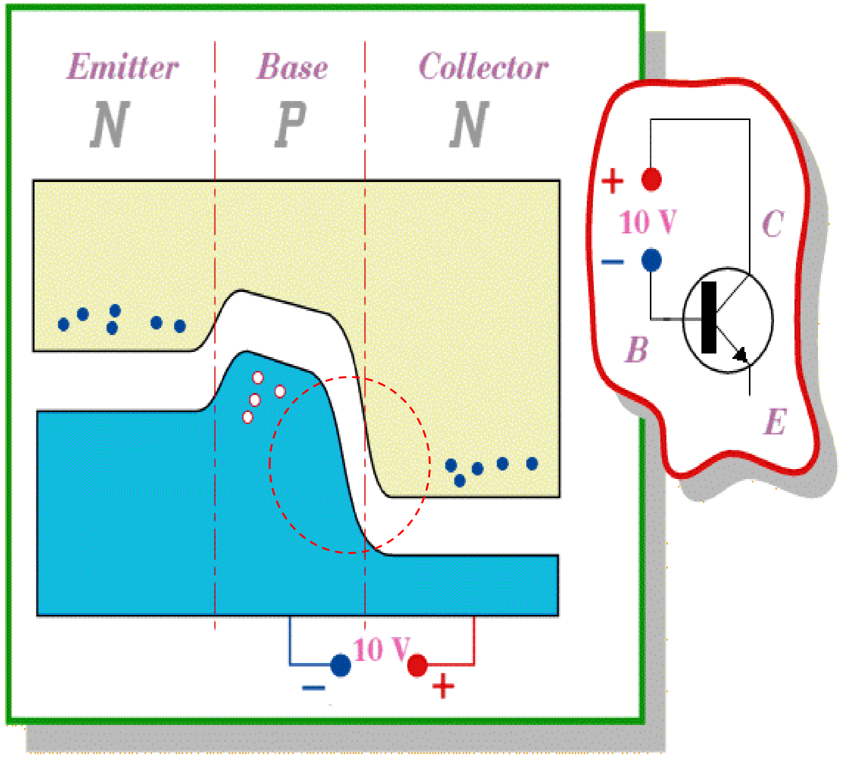
Understand DC Analysis from Micro-level

How the BJT works



- Figure shows the energy levels in an NPN transistor under **no externally applying voltages**.
- In each of the N-type layers conduction can take place by the free movement of electrons in the conduction band.
- In the P-type (filling) layer conduction can take place by the movement of the free holes in the valence band.
- However, in the absence of any externally applied electric field, we find that depletion zones form at both PN-Junctions, so no charge wants to move from one layer to another.

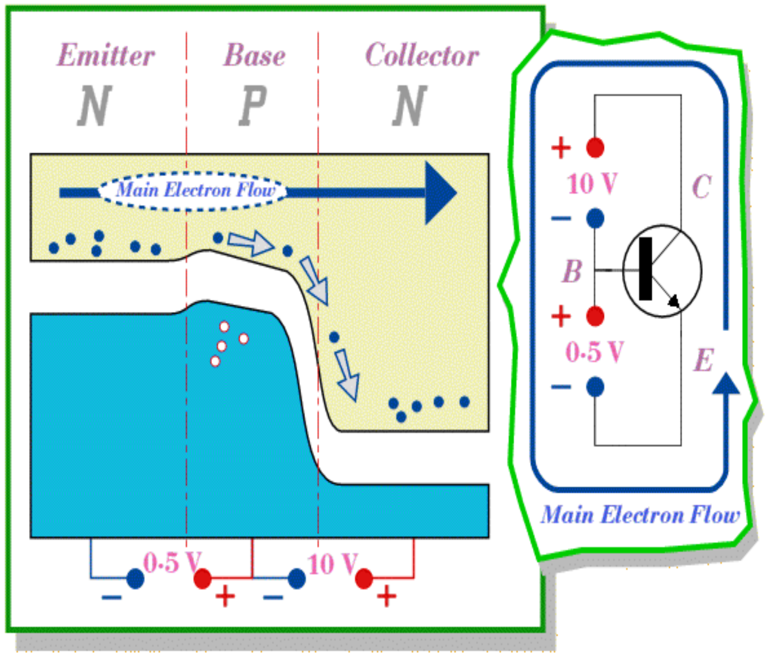
How the BJT works



Apply a Collector-Base voltage

- What happens when we apply a moderate voltage between the collector and base parts.
- The polarity of the applied voltage is chosen to increase the force pulling the N-type electrons and P-type holes apart.
- This widens the depletion zone between the collector and base and so no current will flow.
- In effect we have reverse-biassed the Base-Collector diode junction.

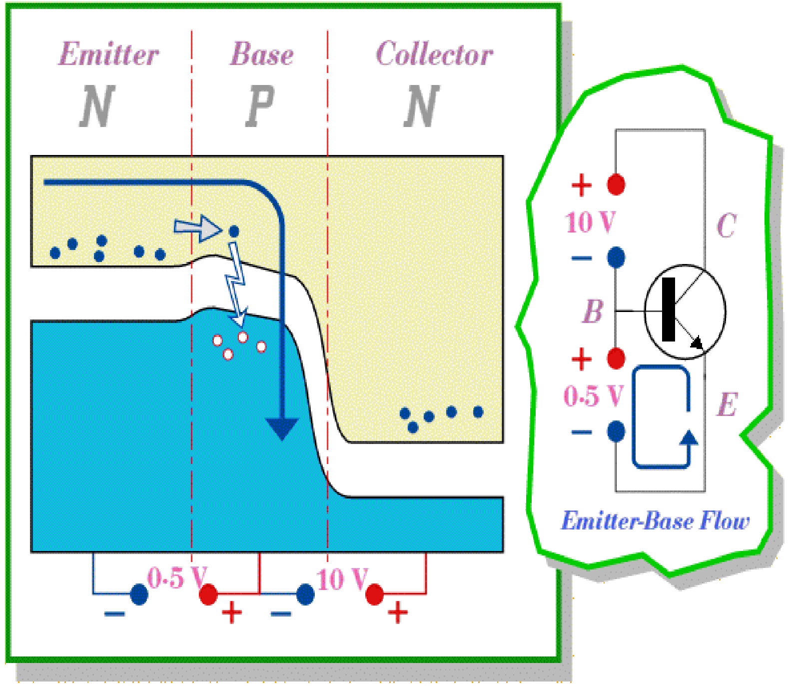
Charge Flow



Apply an Emitter-Base voltage

- What happens when we apply a relatively small Emitter-Base voltage whose polarity is designed to forward-bias the Emitter-Base junction.
- This 'pushes' electrons from the Emitter into the Base region and sets up a current flow across the Emitter-Base boundary.
- Once the electrons have managed to get into the Base region they can respond to the attractive force from the positively-biassed Collector region.
- As a result the electrons which get into the Base move swiftly towards the Collector and cross into the Collector region.
- Hence a Emitter-Collector current magnitude is set by the chosen Emitter-Base voltage applied.
- Hence an external current flowing in the circuit.

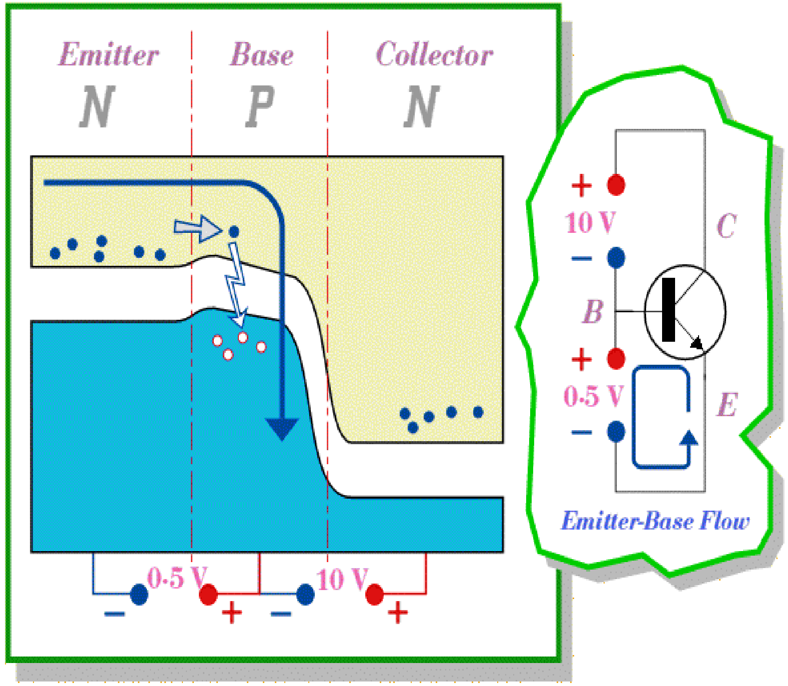
Charge Flow



Some electron fall into a hole

- Some of free electrons crossing the Base encounter a hole and 'drop into it'.
- As a result, the Base region loses one of its positive charges (holes).
- The Base potential would become more negative (because of the removal of the holes) until it was negative enough to repel any more electrons from crossing the Emitter-Base junction.
- The current flow would then stop.

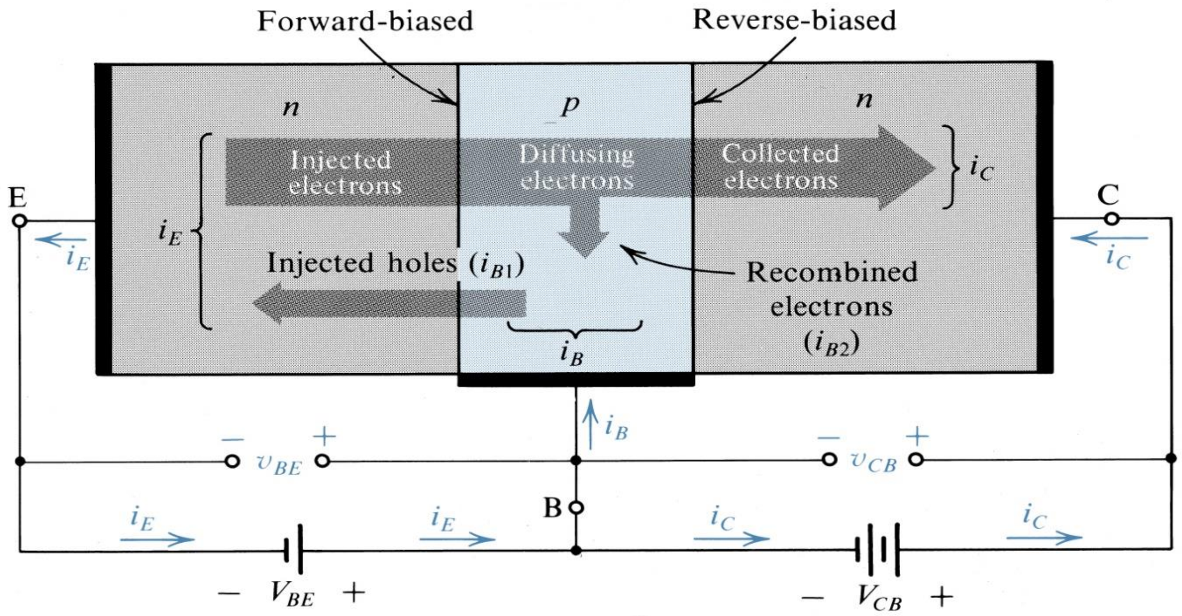
Charge Flow



Some electron fall into a hole

- To prevent this happening we use the applied E-B voltage to remove the captured electrons from the base and maintain the number of holes.
- The effect, some of the electrons which enter the transistor via the Emitter emerging again from the Base rather than the Collector.
- For most practical BJT only about 1% of the free electrons which try to cross Base region get caught in this way.
- Hence a Base current, I_B, which is typically around one hundred times smaller than the Emitter current, I_E.

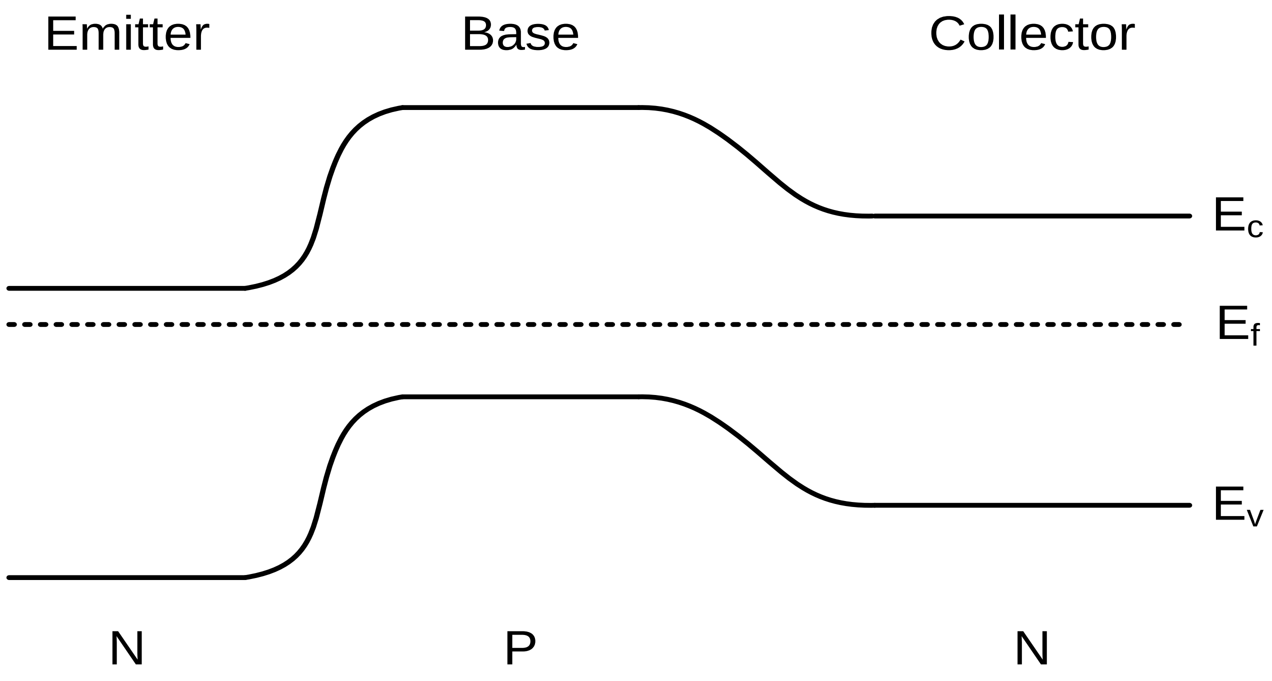
BJT in Active Mode



- Operation
  - Forward bias of EBJ injects electrons from emitter into base (small number of holes injected from base into emitter)
  - Most electrons shoot through the base into the collector across the reverse bias junction (think about band diagram)
  - Some electrons recombine with majority carrier in (P-type) base region

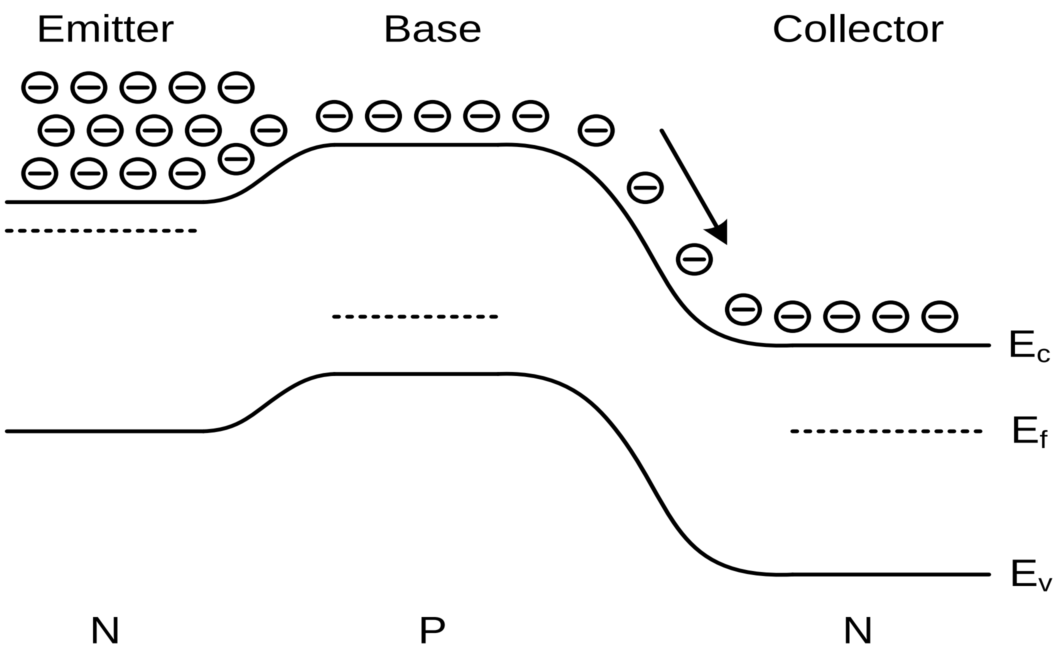
Band Diagrams (In equilibrium)

- No current flow
- Back-to-back PN diodes



Band Diagrams (Active Mode)

- EBJ forward biased
  - Barrier reduced and so electrons diffuse into the base
  - Electrons get **swept across** the base into the collector
- CBJ reverse biased
  - **Electrons roll down the hill (high E-field)**



Current Relationships and IV Characteristics

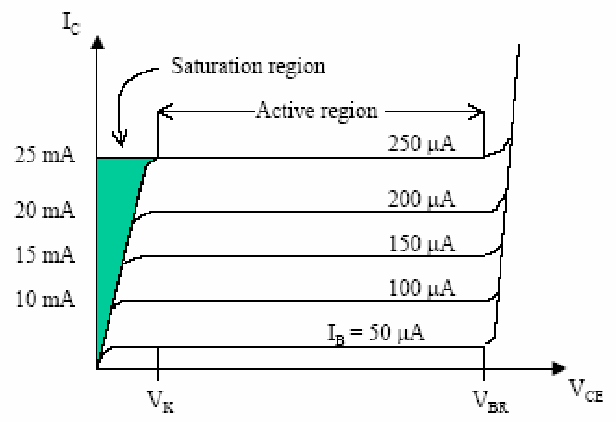
Collector Current

- Electrons that diffuse across the base to the CBJ junction are swept across the CBJ depletion region to the collector b/c of the higher potential applied to the collector.

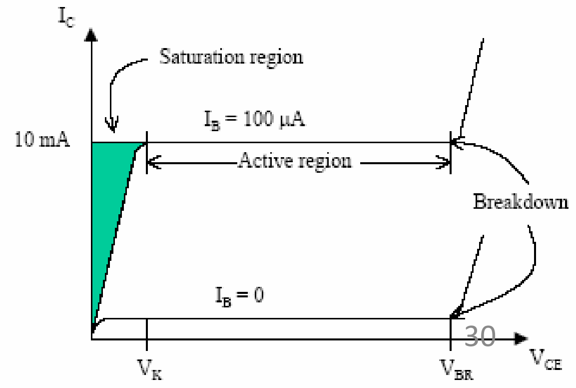
$$i_C = I_s e^{v_{BE}/V_T} \text{ where the saturation current is } I_S = qA_E D_n n_{p0}/W$$

and we can rewrite the saturation current as:

$$\boxed{I_S = \frac{qA_E D_n n_i^2}{N_A W}}$$



- Note that i_C is independent of v_{CB} (potential bias across CBJ) ideally
- Saturation current is
  - inversely proportional to W and directly proportional to A_E
    - Want short base and large emitter area for high currents
  - dependent on temperature due to n_i^2 term



Base Current

- Base current i_B composed of two components:
  - holes injected from the base region into the emitter region

$$i_{B1} = \frac{qA_E D_p n_i^2}{N_D L_P} e^{v_{BE}/V_T}$$

  - holes supplied due to recombination in the base with diffusing electrons and depends on minority carrier lifetime τ_b in the base

$$i_{B2} = \frac{Q_n}{\tau_b}$$

And the Q in the base is

$$Q_n = \frac{qA_E W n_i^2}{N_A} e^{v_{BE}/V_T}$$

So, current is

$$i_{B2} = \frac{qA_E W n_i^2}{N_A \tau_b} e^{v_{BE}/V_T}$$

- Total base current is

$$i_B = \left(\frac{qA_E D_p n_i^2}{N_D L_P} + \frac{qA_E W n_i^2}{N_A \tau_b} \right) e^{v_{BE}/V_T}$$

Beta

- Can relate i_B and i_C by the following equation

$$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T}$$

and β is

$$\beta = \frac{1}{\frac{D_p}{D_n}\frac{N_A}{N_D}\frac{W}{L_p} + \frac{1}{2}\frac{W^2}{D_n \tau_b}}$$

  - Beta is constant for a particular transistor
  - On the order of 100-200 in modern devices (but can be higher)
  - Called the common-emitter current gain
- For high current gain, want small W, low N_A, high N_D

Current Relationships

Common-emitter current gain: β

Common-base current gain: α

$$i_E = i_C + i_B$$

$$i_C = \beta i_B$$

$$i_E = (1+\beta i_B)$$

$$i_C = \alpha i_E$$

$$\beta = \frac{\alpha}{1-\alpha}$$

Current Relationships

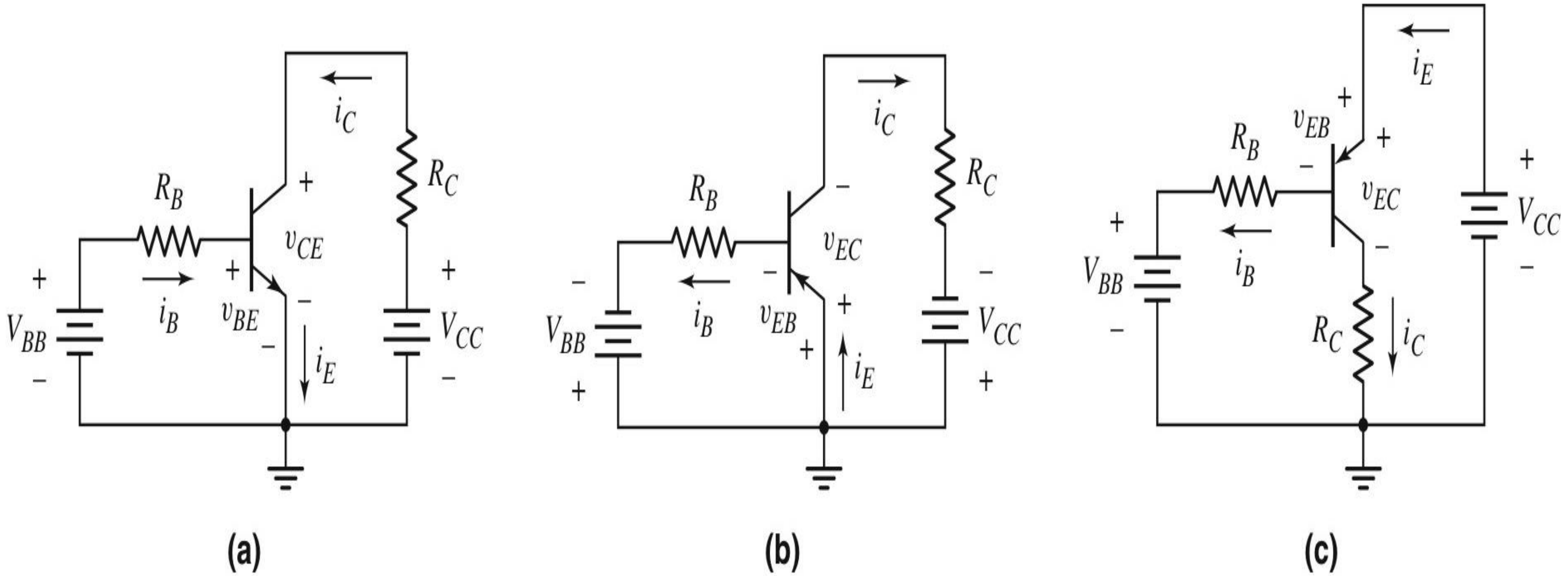
- Common-base current Gain **α** :
  - α is the fraction of electrons that diffuse across the narrow Base region
  - 1- α is the fraction of electrons that recombine with holes in the Base region to create base current
- The common-emitter current Gain **β** is expressed in terms of the **β** (beta) of the transistor (often called h_{fe} by manufacturers).
- β is Temperature and Voltage dependent.
- β can vary a lot among transistors (common values for signal BJT: 20 - 200).

$$I_C = \alpha I_E$$

$$I_B = (1-\alpha) I_E$$

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha}$$

Common-Emitter Configurations

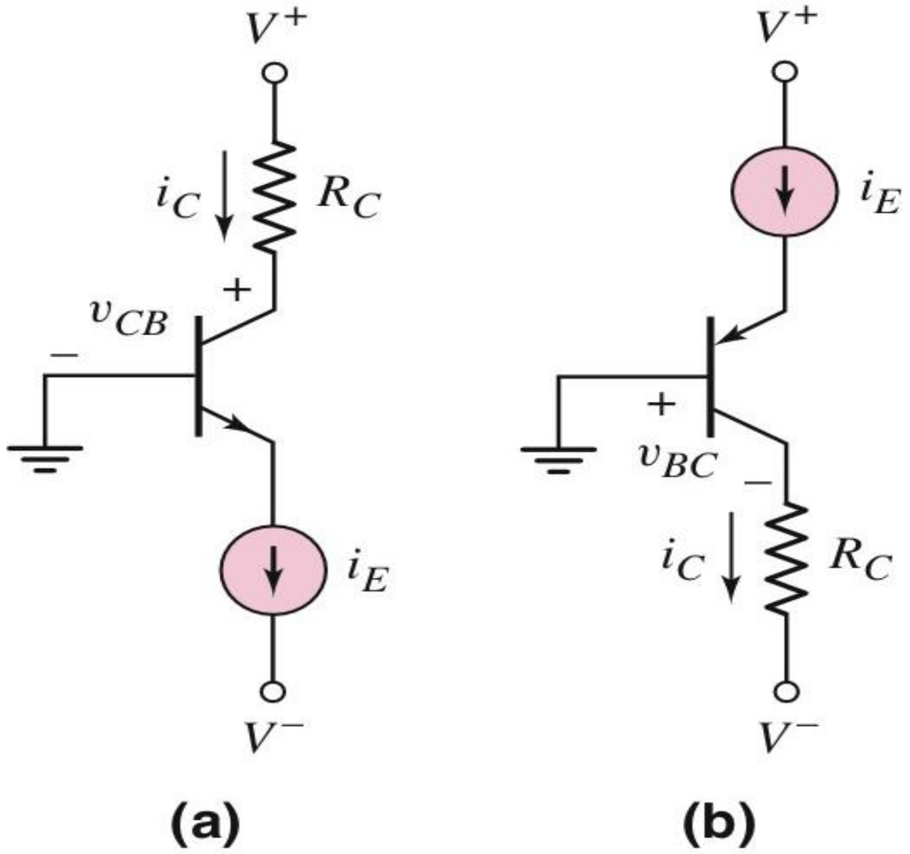


(a) CE circuit with npn transistor

(b) CE circuit with pnp transistor

(c) CE circuit with pnp transistor with a positive voltage source

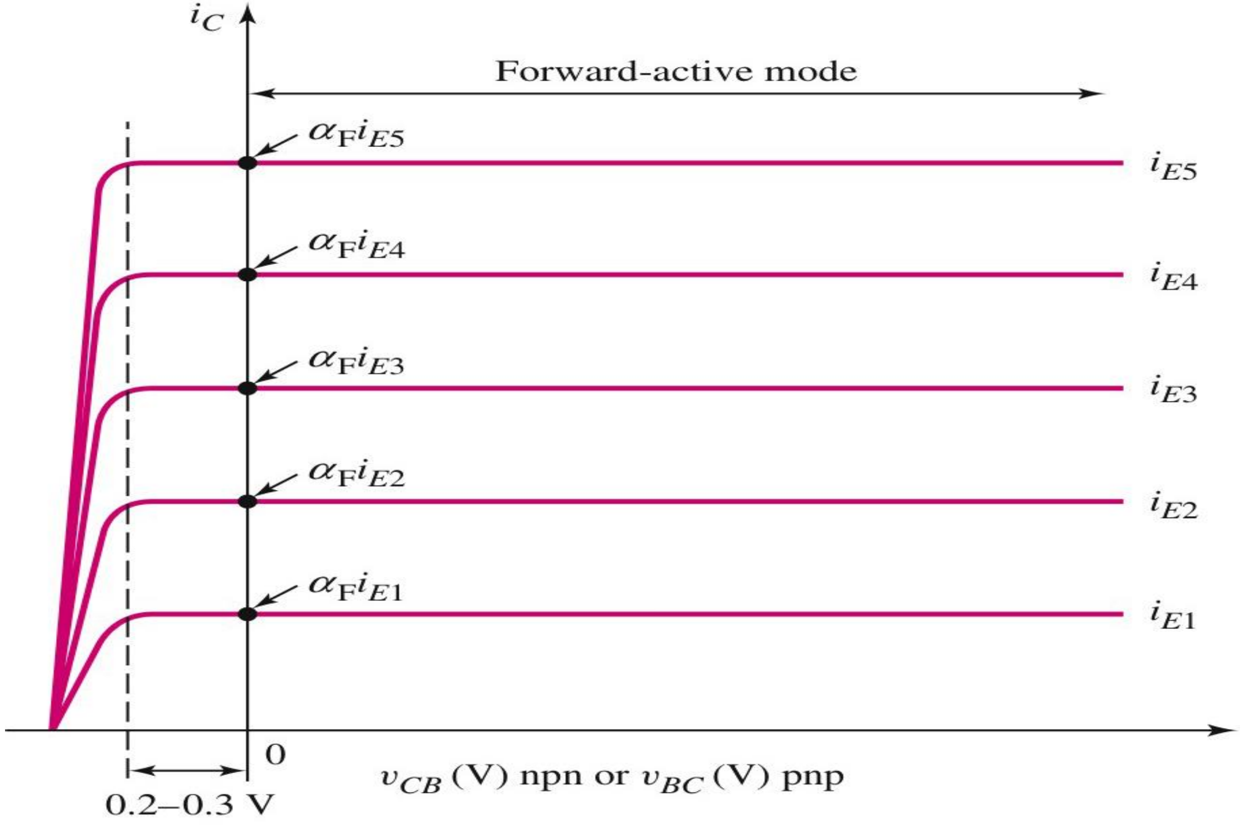
Common-Base Configuration



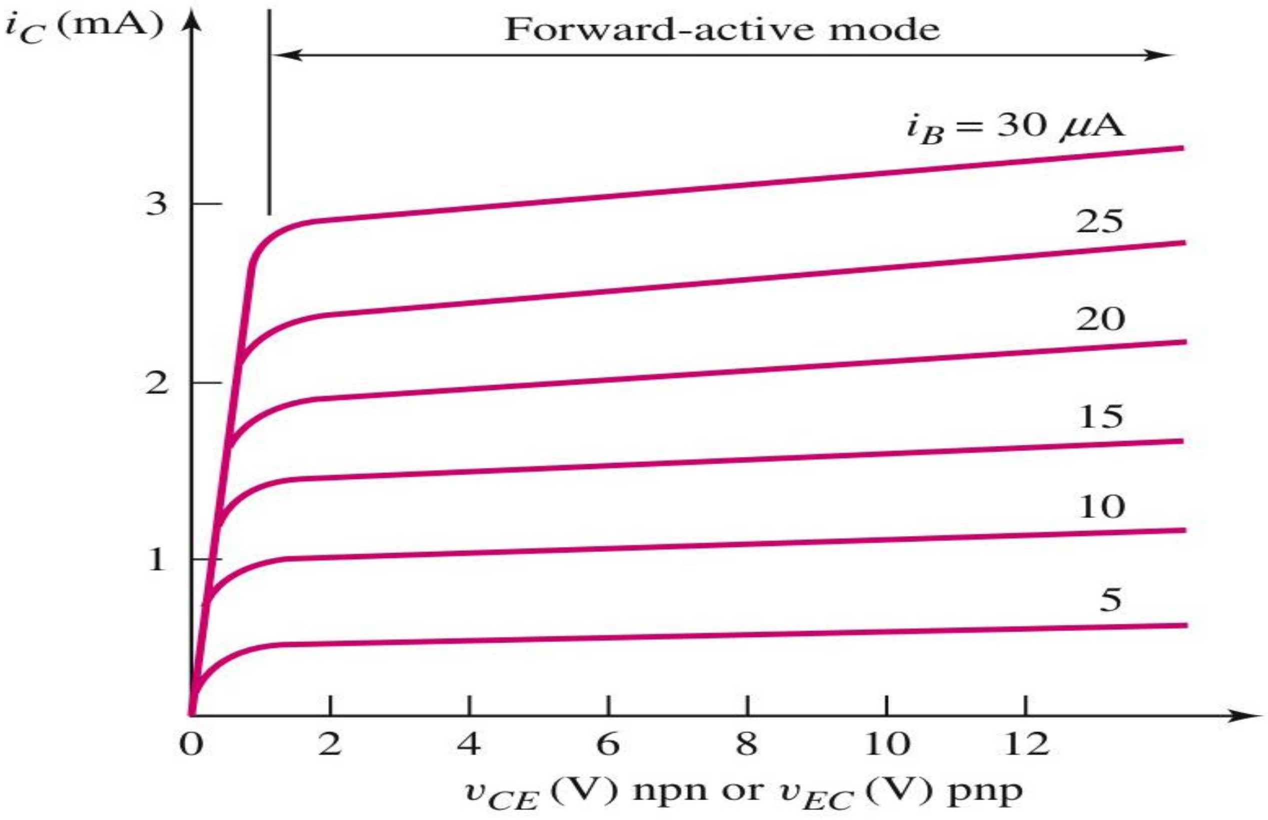
The common-base device is nearly an ideal constant current source.

Common-base current gain: α

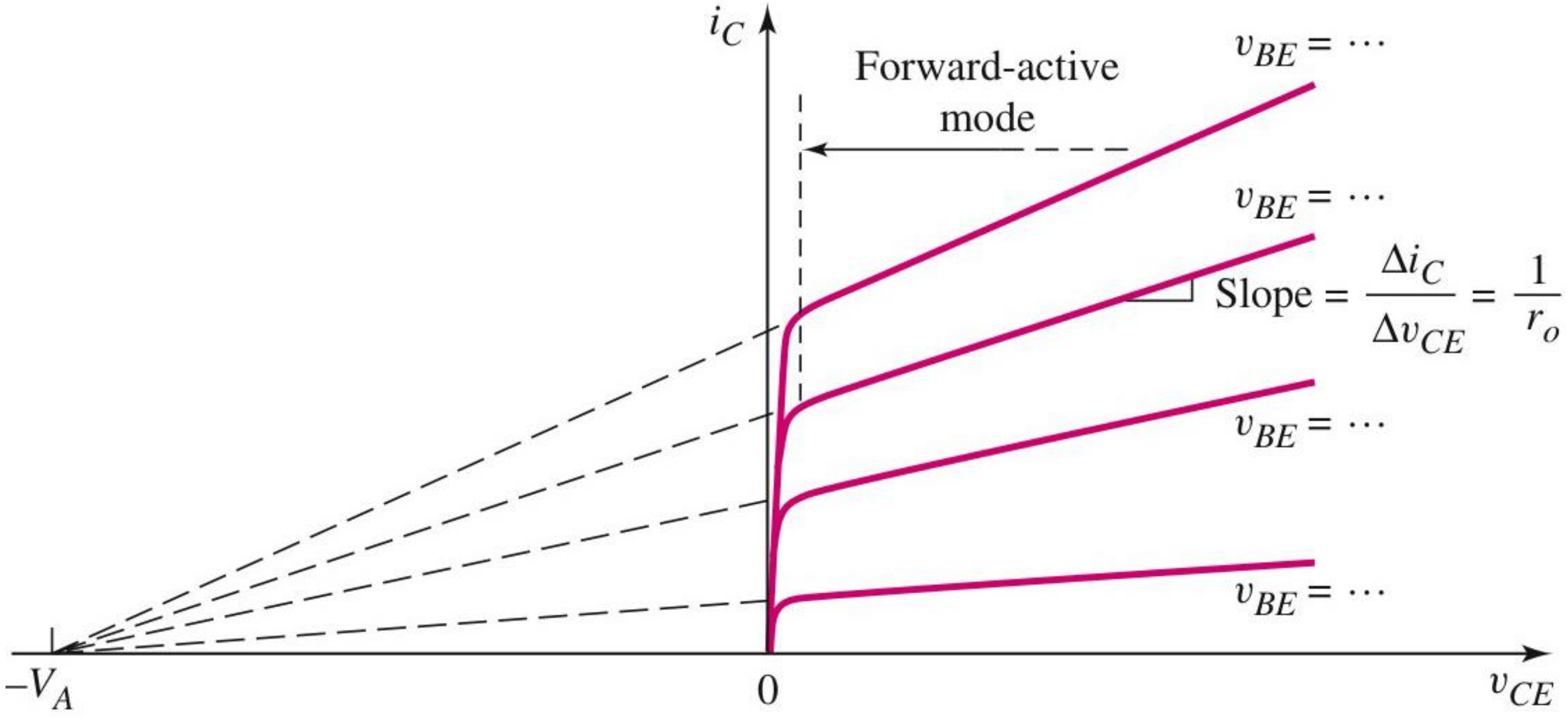
Current-Voltage Characteristics of a Common-Base Circuit



Current-Voltage Characteristics of a Common-Emitter Circuit



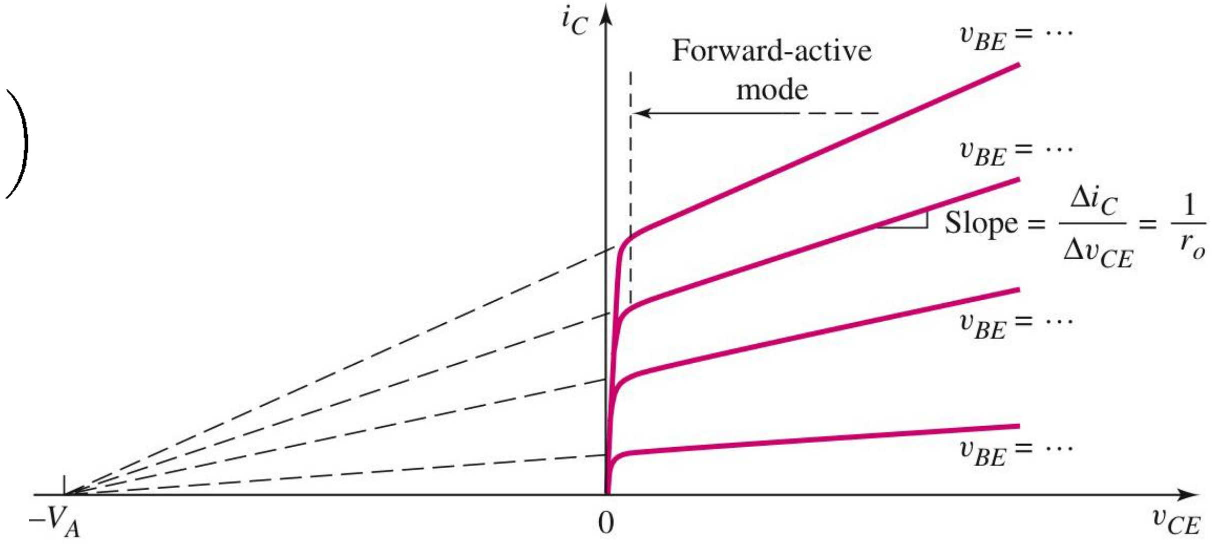
Early Voltage/Finite Output Resistance



Base-width modulation – Early Effect!

Early Effect

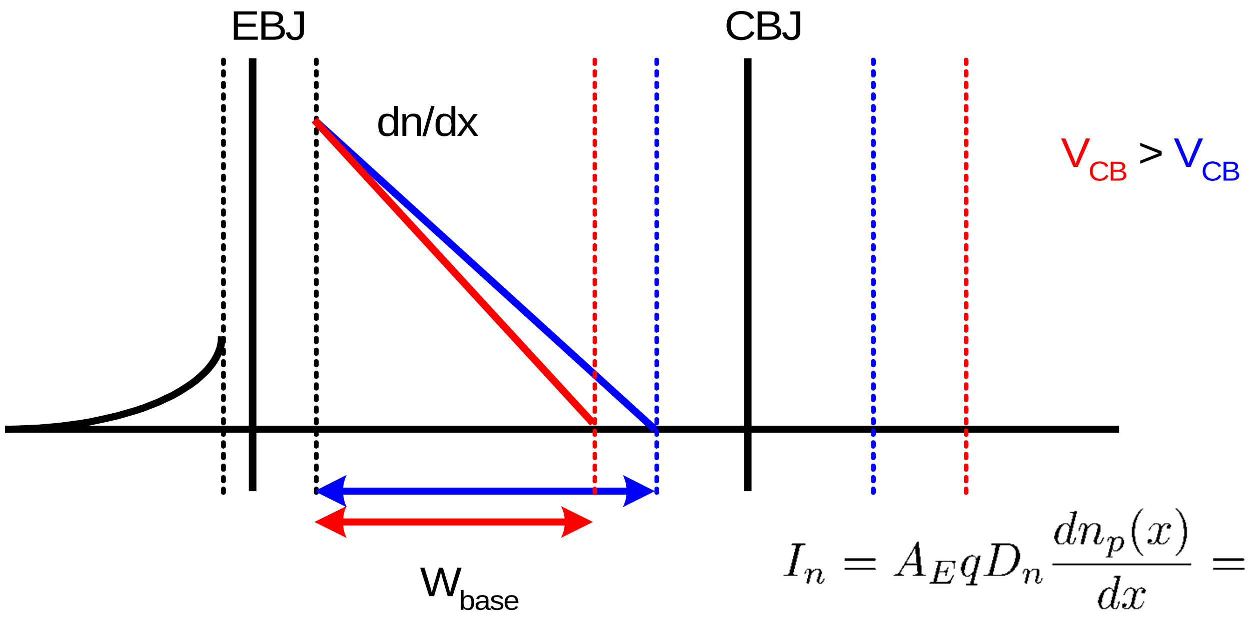
$$I_C = I_S e^{V_{BE}/V_T}\left(1 + \frac{V_{CE}}{V_A}\right)$$



- Early Effect
    - Current in active region depends (slightly) on v_{CE}
    - V_A is a parameter for the BJT (50 to 100) and called the Early voltage
    - Due to a decrease in effective base width W as reverse bias increases
    - Account for Early effect with additional term in collector current equation
    - Nonzero slope means the output resistance is NOT infinite, but…
        - I_C is collector current at the boundary of active region

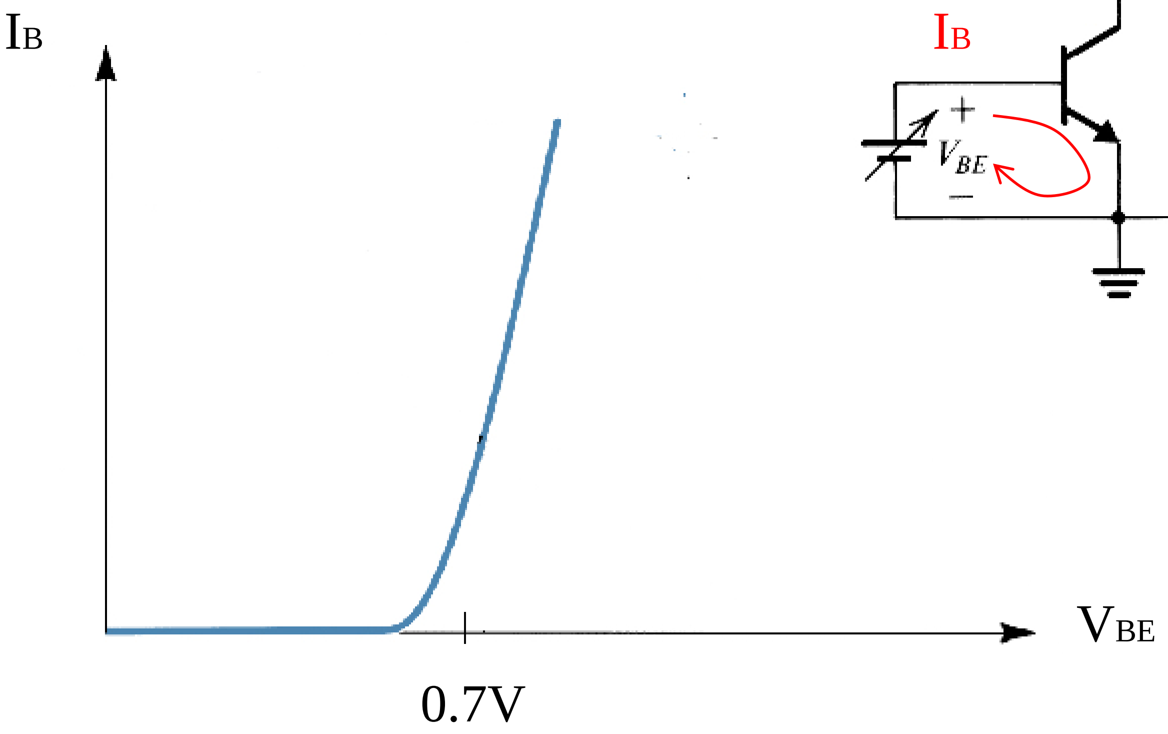
Early Effect

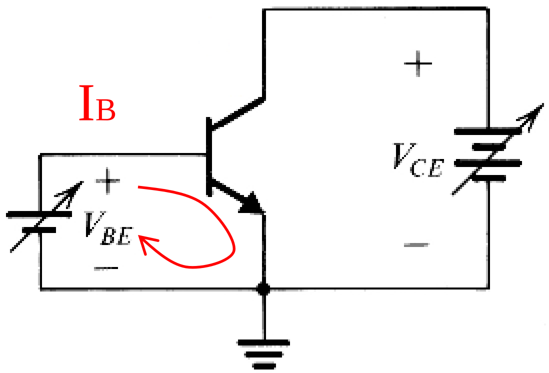
- What causes the Early Effect?
  - Increasing V_{CB} causes depletion region of CBJ to grow and so the effective base width decreases (base-width modulation)
  - Shorter effective base width → higher dn/dx



$$I_n = A_E q D_n \frac{dn_p(x)}{dx} = A_E q D_n \left(-\frac{n_p(0)}{W}\right)$$

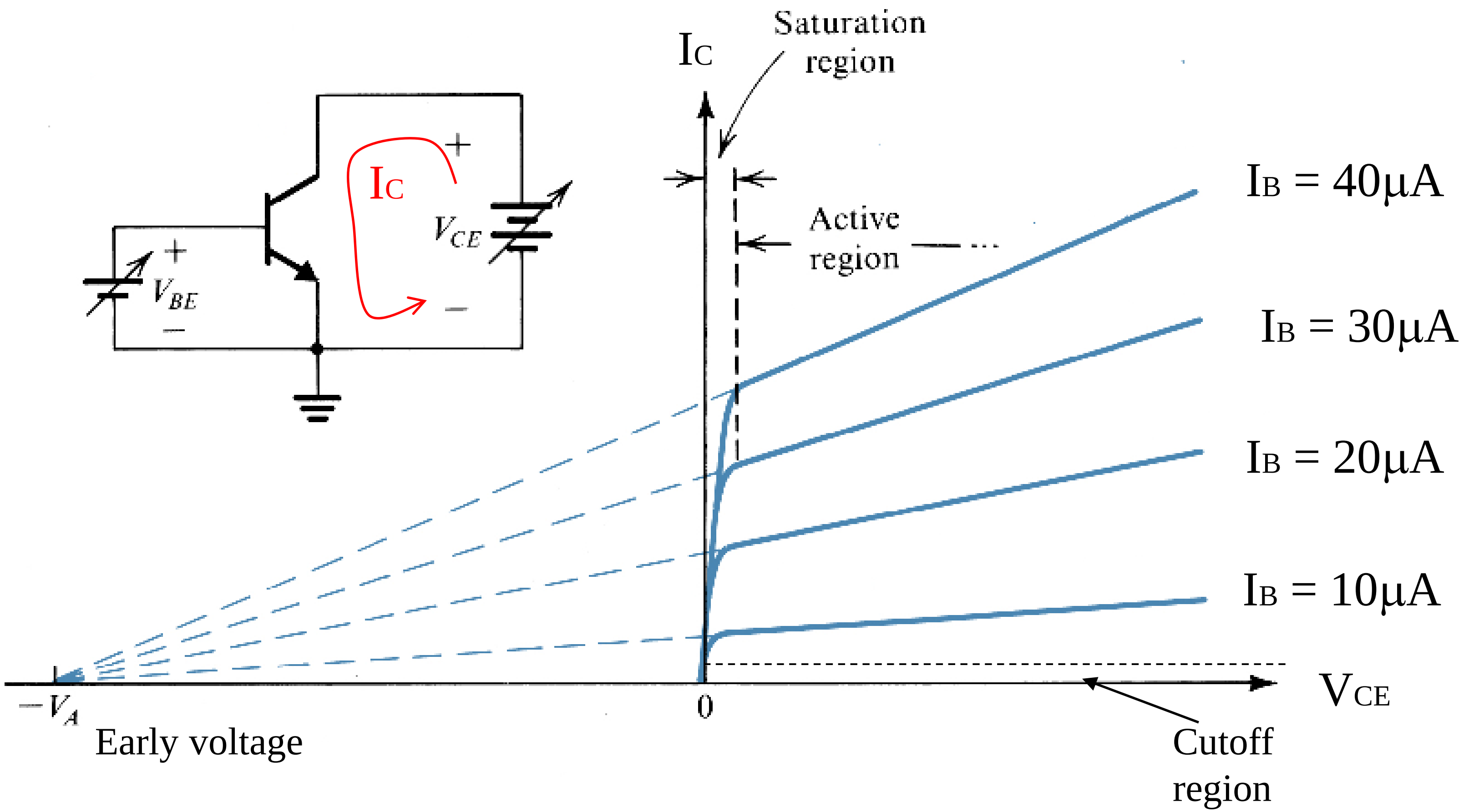
Summary: Input characteristics





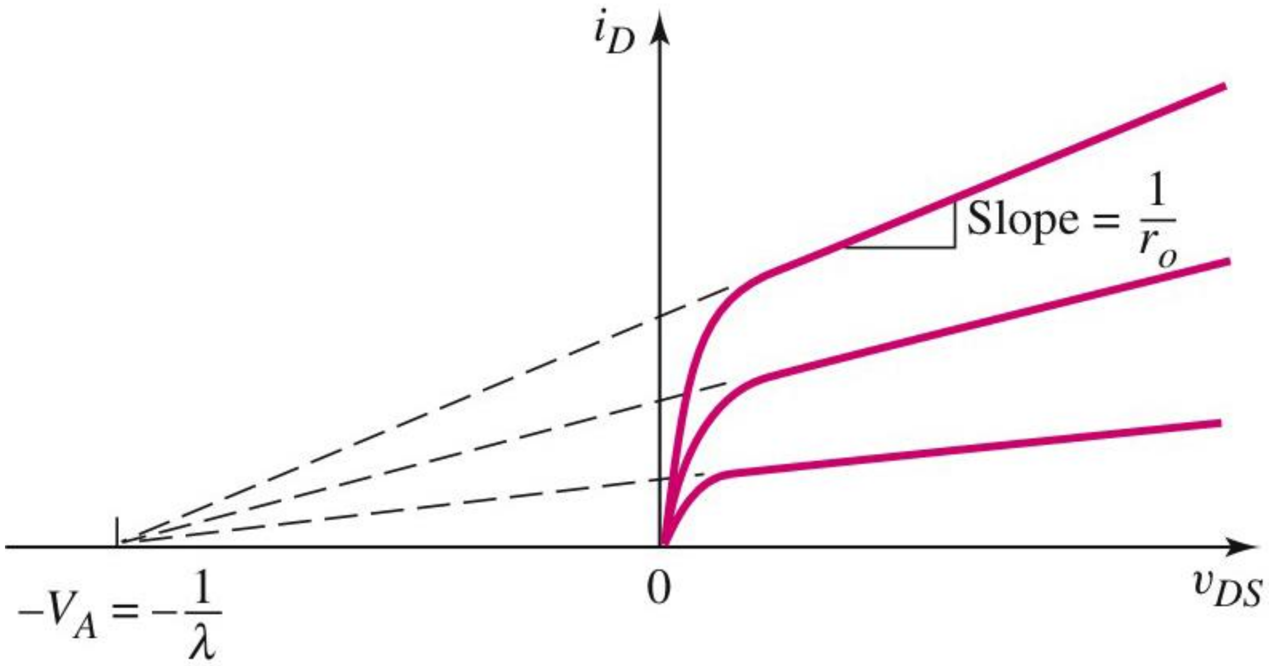
- Acts as a diode
- $V_{BE} \approx 0.7V$

Summary: Output characteristics

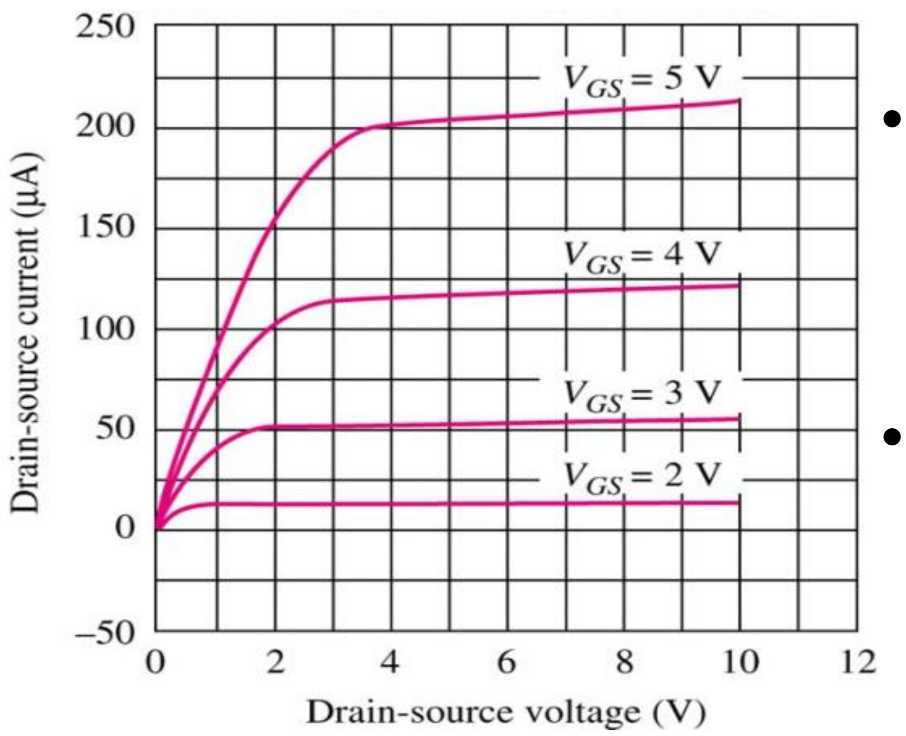


- At a fixed I_B, I_C is not dependent on V_{CE}
- Slope of output characteristics in linear region is near 0 (scale exaggerated)

Recall: Channel Length Modulation: Early Voltage



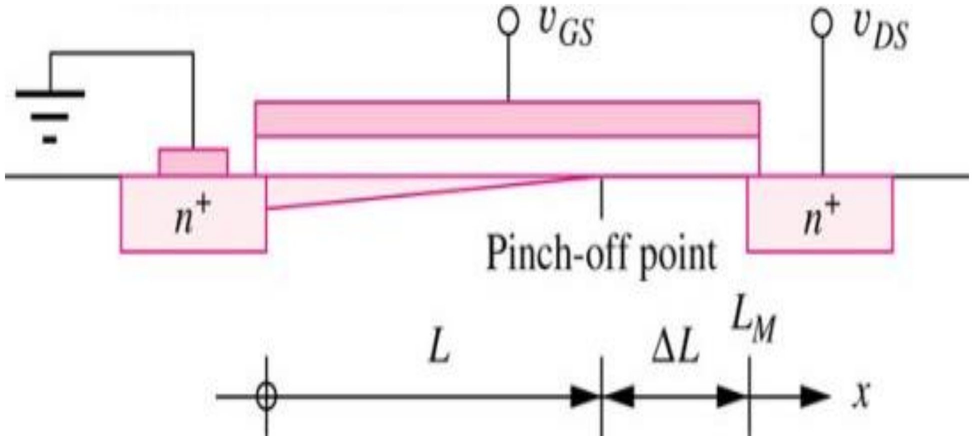
Recall: Channel-Length Modulation



- As v_{DS} increases above v_{DSAT}, length of depleted channel beyond **pinch-off point**, ΔL, increases and actual L decreases.
- i_D increases slightly with v_{DS} instead of being constant.

$$i_D = \frac{K_n}{2}\frac{W}{L}\left(v_{GS} - V_{TN}\right)^2\left(1 + \lambda v_{DS}\right)$$

λ= channel length modulation parameter



Recall: Channel-Length Modulation

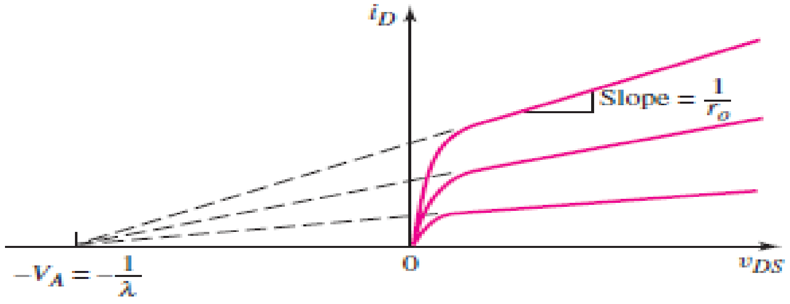


Figure 3.20 Family of i_D versus v_{DS} curves showing the effect of channel length modulation producing a finite output resistance

The parameters λ and V_A are related. From Equation (3.7), we have $(1 + \lambda v_{DS}) = 0$ at the extrapolated point where $i_D = 0$. At this point, $v_{DS} = -V_A$, which means that $V_A = 1/\lambda$.

The output resistance due to the channel length modulation is defined as

$$r_o = \left(\frac{\partial i_D}{\partial v_{DS}}\right)^{-1}\Bigg|_{v_{GS}=\text{const.}} \tag{3.8}$$

From Equation (3.7), the output resistance, evaluated at the Q-point, is

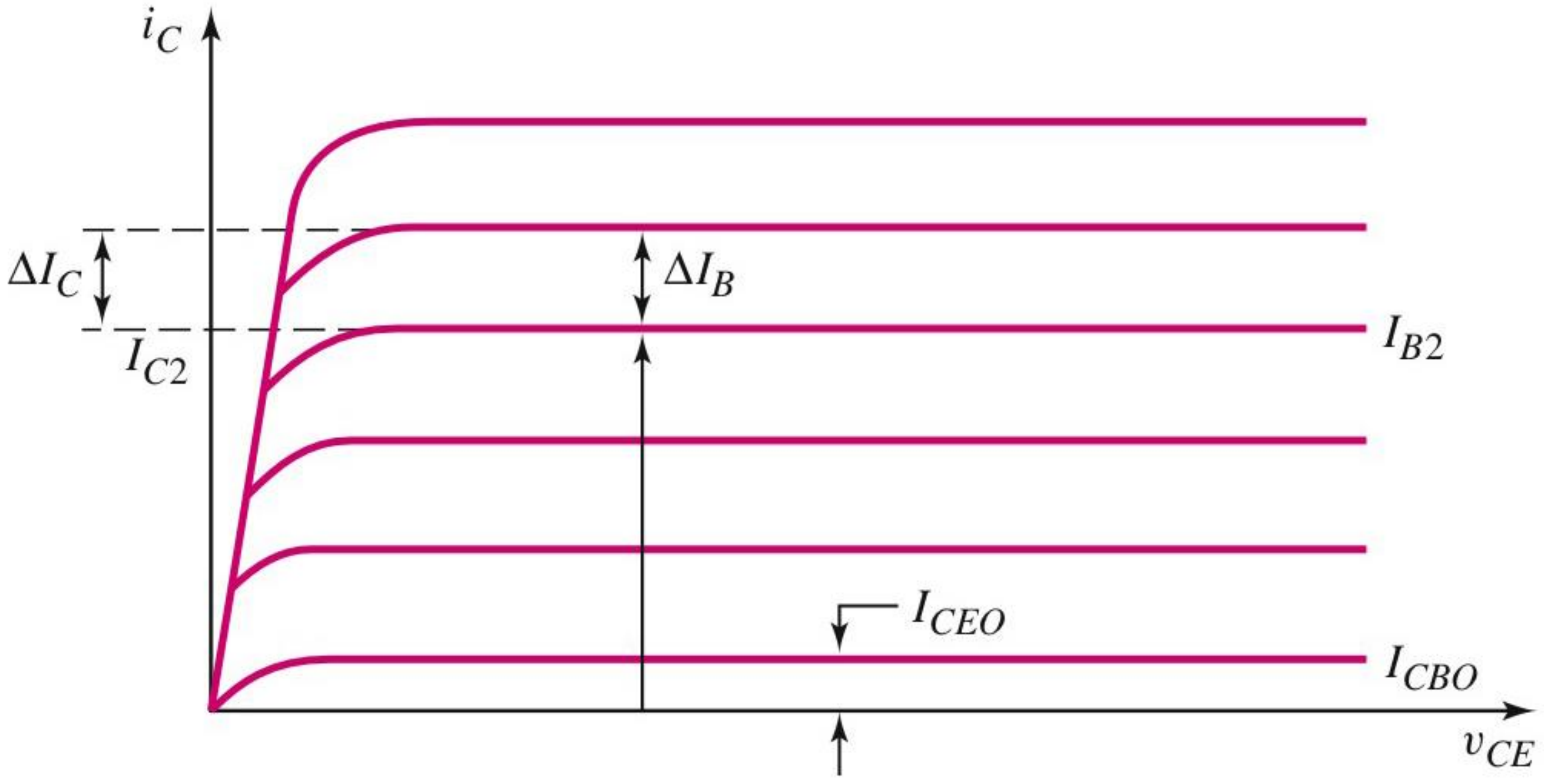
$$r_o = [\lambda K_n (V_{GSQ} - V_{TN})^2]^{-1} \tag{3.9(a)}$$

or

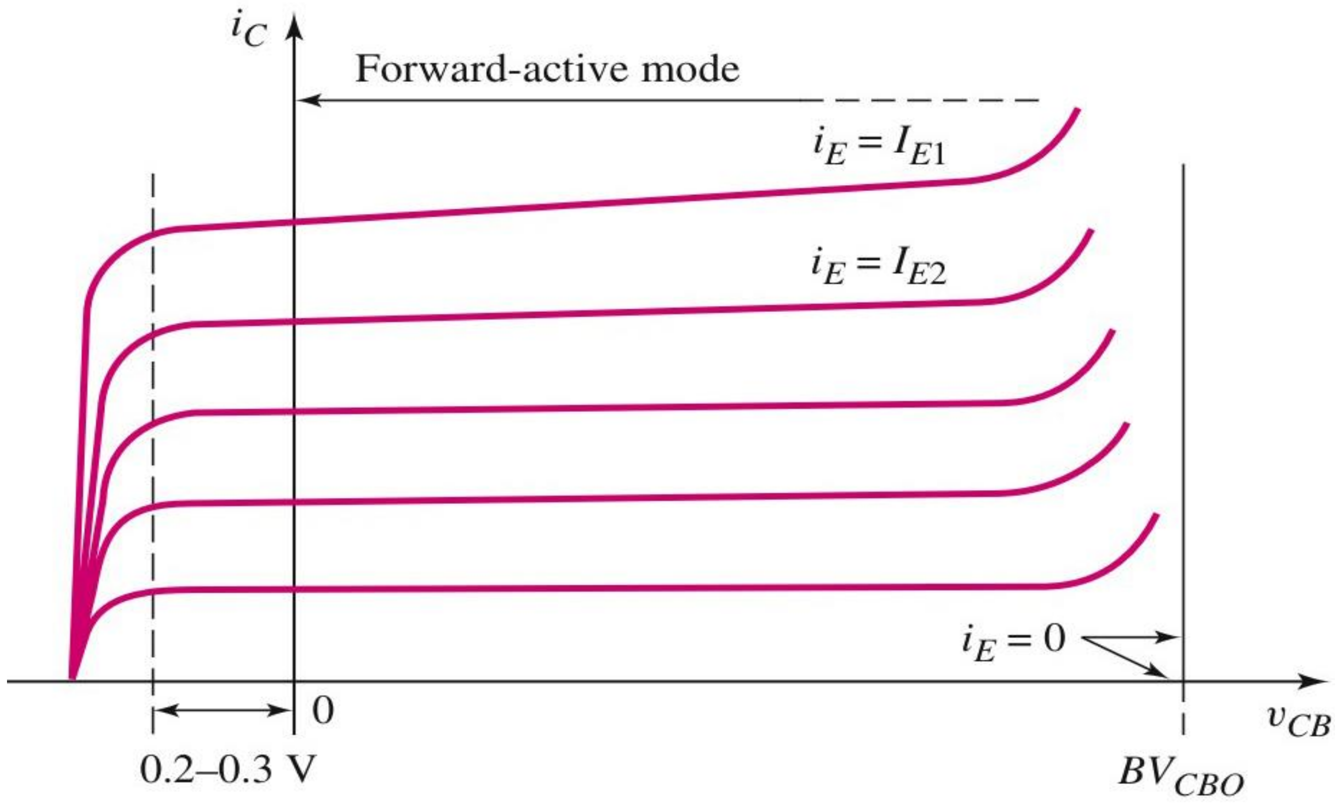
$$r_o \cong [\lambda I_{DQ}]^{-1} = \frac{1}{\lambda I_{DQ}} = \frac{V_A}{I_{DQ}} \tag{3.9(b)}$$

The output resistance r_o is also a factor in the small-signal equivalent circuit of the MOSFET, which is discussed in the next chapter.

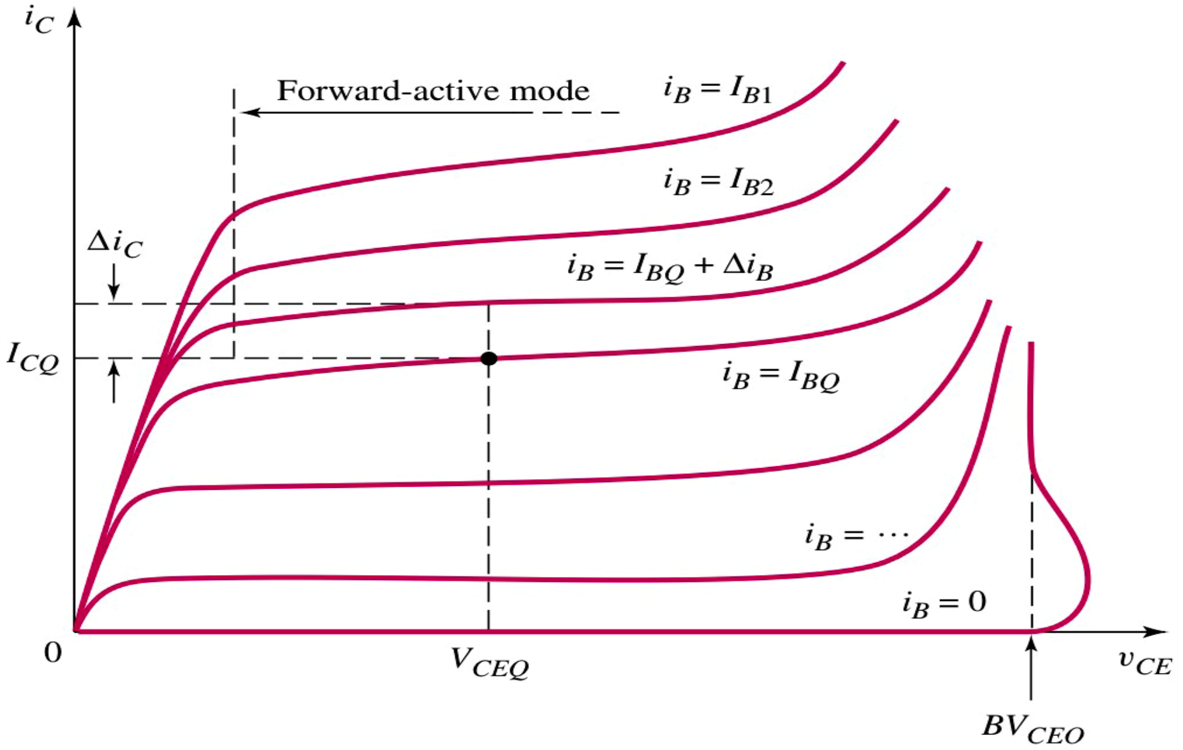
Effects of Leakage Currents on I-V Characteristics



Effect of Collector-Base Breakdown on Common Base I-V Characteristics



Effect of Collector-Base Breakdown on Common Emitter I-V Characteristics



Breakdown Voltages

- The basic limitation of the max. voltage in a transistor is the same as that in a *pn* junction diode.
- However, the voltage breakdown depends not only on the nature of the junction involved but also on the external circuit arrangement.
- In **Common Base** configuration, the maximum voltage between the collector and base with the emitter open, BV_{CBO} is determined by the avalanche breakdown voltage of the CBJ.
- In **Common Emitter** configuration, the maximum voltage between the collect and emitter with the base open, BV_{CEO} can be much smaller than BV_{CBO}.

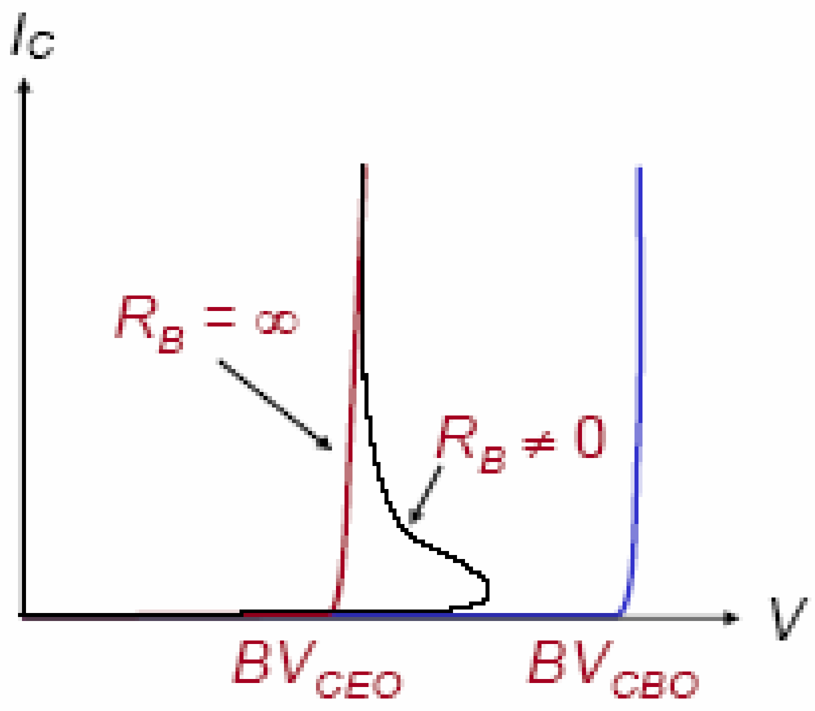
Breakdown Voltages

In general, BV_{CEO} is related to BV_{CBO} by the following expression,

$$BV_{CEO} = BV_{CBO}\sqrt[n]{1-\alpha} \approx BV_{CBO}/\sqrt[n]{\beta}$$

The typical value of n is between 2 to 4 in silicon.

In general, the actual breakdown voltage is between BV_{CEO} and BV_{CBO}, depending on the external resistance seen by the base, R_B.



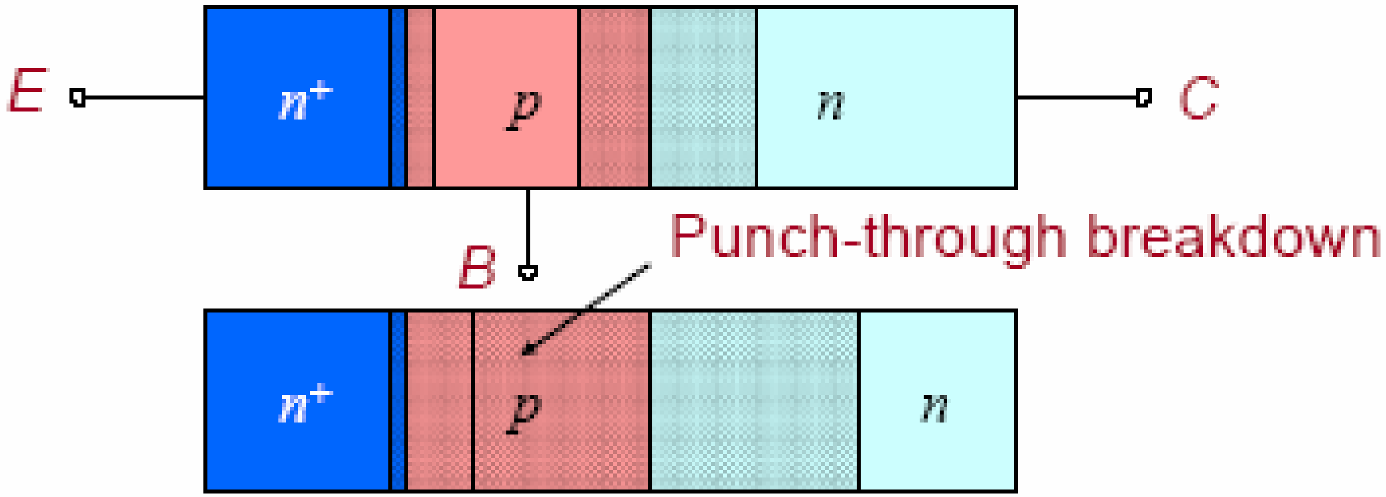
Breakdown Voltages

For a planar abrupt p^+n junction, the avalanche breakdown voltage is given by

$$BV = \frac{\varepsilon_s \varepsilon_0 E_{crit}^2}{2qN_D}$$

where E_{crit} is the critical electric field, and N_D is the doping concentration for the low doping region.

Breakdown Voltages



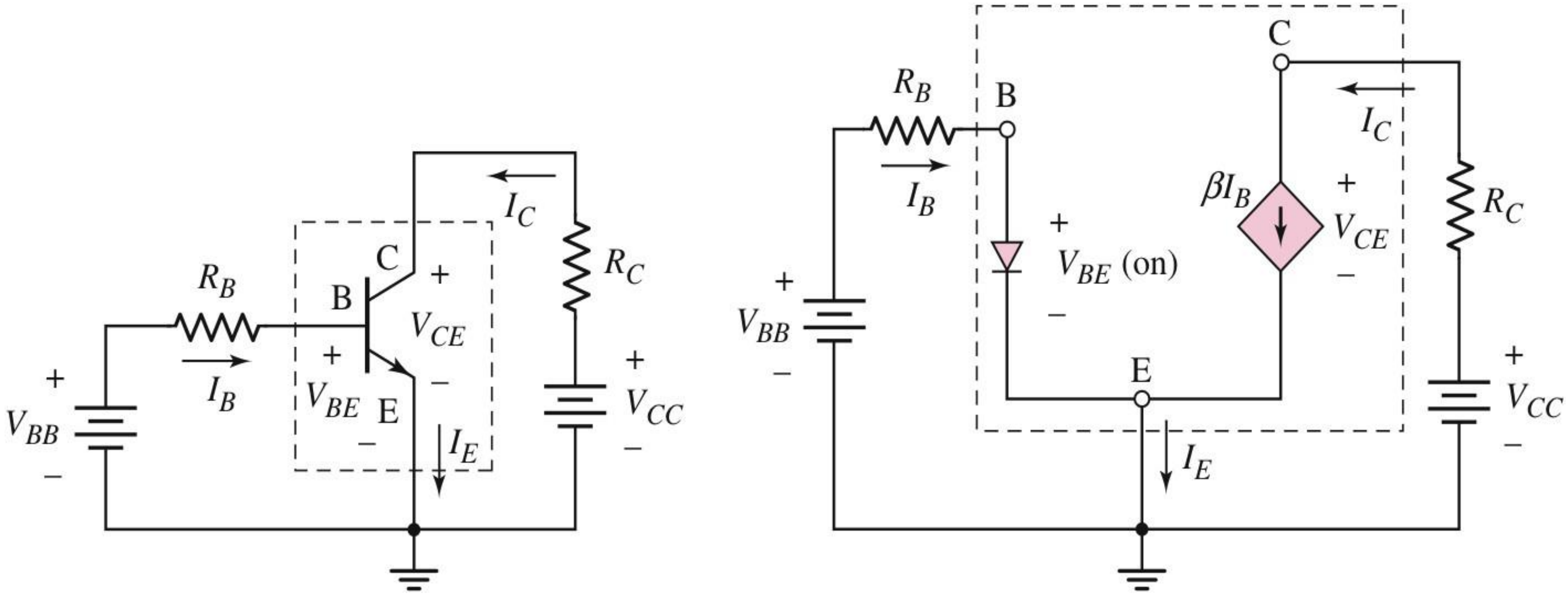
- As V_{CB} (or V_{CE}) increases, the depletion region will continue to spread into the base region.
- If the base become completely depleted, the depletion region from the collector and emitter touch each other, resulting in a short between the *n*+ and *n* regions.

Operation region summary

Operation Region	***I_B or V_{CE} Char.***	***BC and BE Junctions***	***Mode***
Cutoff	I_B = Very small	Reverse & Reverse	Open Switch
Saturation	V_{CE} = Small	Forward & Forward	Closed Switch
Active Linear	V_{CE} = Moderate	Reverse & Forward	Linear Amplifier
Break-down	V_{CE} = Large	Beyond Limits	Overload

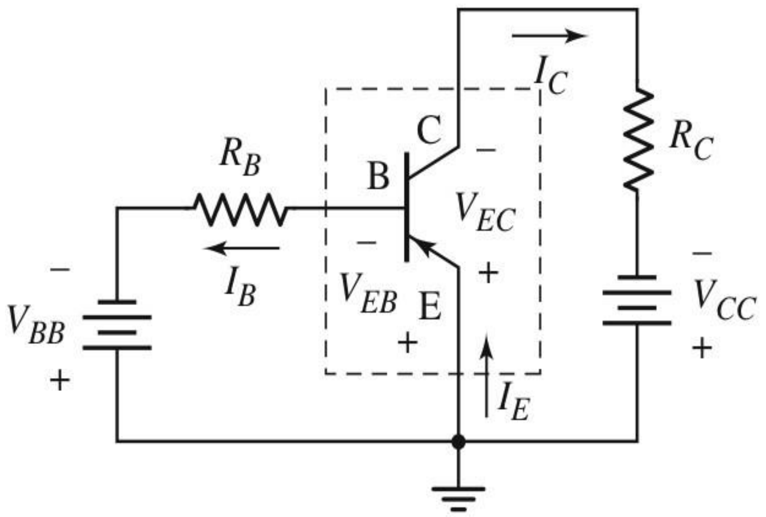
DC Equivalent Circuit

DC Equivalent Circuit for npn Common Emitter

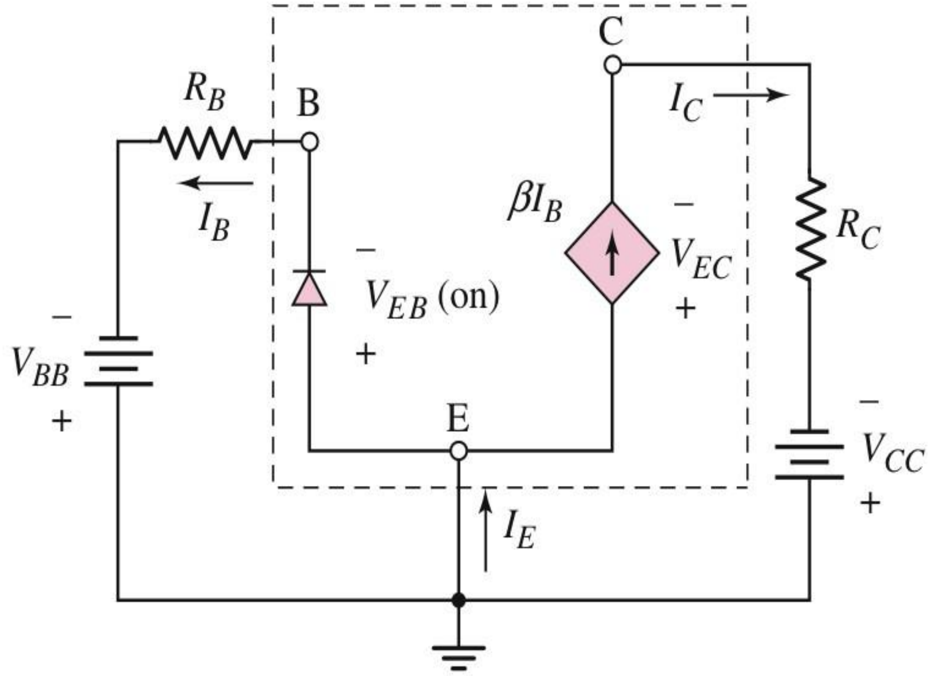


(a) (b)

DC Equivalent Circuit for pnp Common Emitter

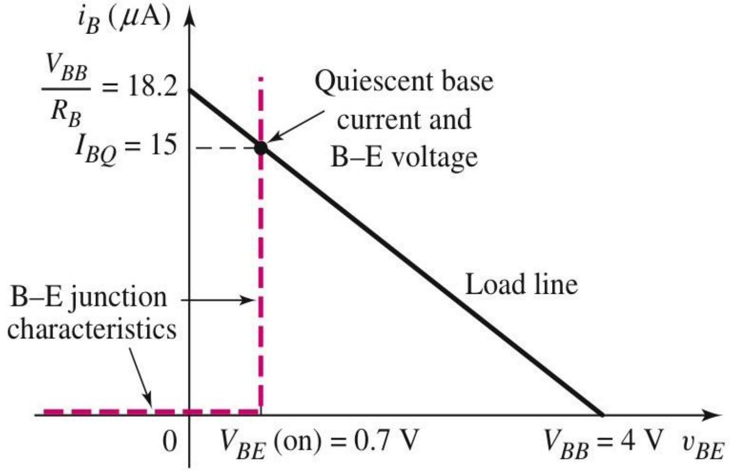


(a)



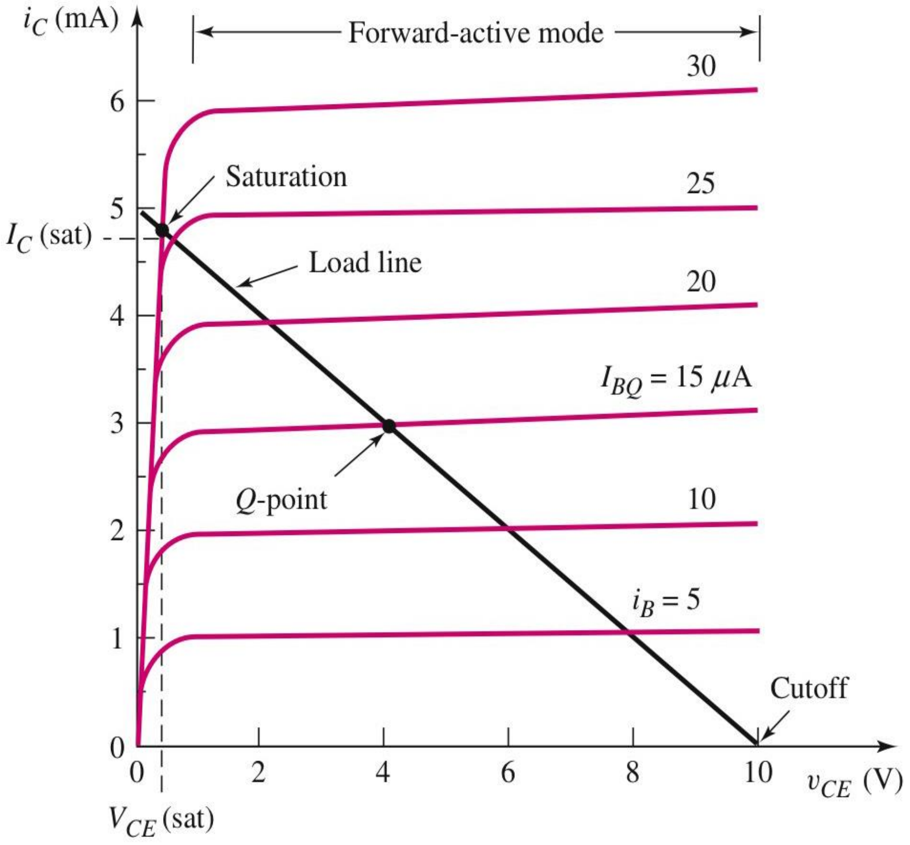
(b)

Load Line



i_B (μA)
$\frac{V_{BB}}{R_B} = 18.2$
$I_{BQ} = 15$
Quiescent base current and B–E voltage
Load line
B–E junction characteristics
0
V_{BE} (on) = 0.7 V
$V_{BB} = 4$ V
v_{BE}

(a)



i_C (mA)
Forward-active mode
30
25
20
$I_{BQ} = 15\ \mu$A
10
$i_B = 5$
Saturation
Load line
Q-point
Cutoff
I_C (sat)
6
5
4
3
2
1
0
0
2
4
6
8
10
v_{CE} (V)
V_{CE} (sat)

(b)

Problem-Solving Technique: Bipolar DC Analysis

1. Assume that the transistor is biased in forward active mode
   a. $V_{BE} = V_{BE}(on)$, $I_B > 0$, & $I_C = \beta I_B$
2. Analyze 'linear' circuit.
3. Evaluate the resulting state of transistor.
   a. If $V_{CE} > V_{CE}(sat)$, assumption is correct
   b. If $I_B < 0$, transistor likely in cutoff
   c. If $V_{CE} < 0$, transistor likely in saturation
4. If initial assumption is incorrect, make new assumption and return to Step 2.

Summary of DC Problem

• Bias transistors so that they operate in the linear region B-E junction forward biased, C-E junction reversed biased

• Use V_{BE} = 0.7 (npn), $I_C \approx I_E$, $I_C = I_B$

• Represent base portion of circuit by the Thevenin circuit

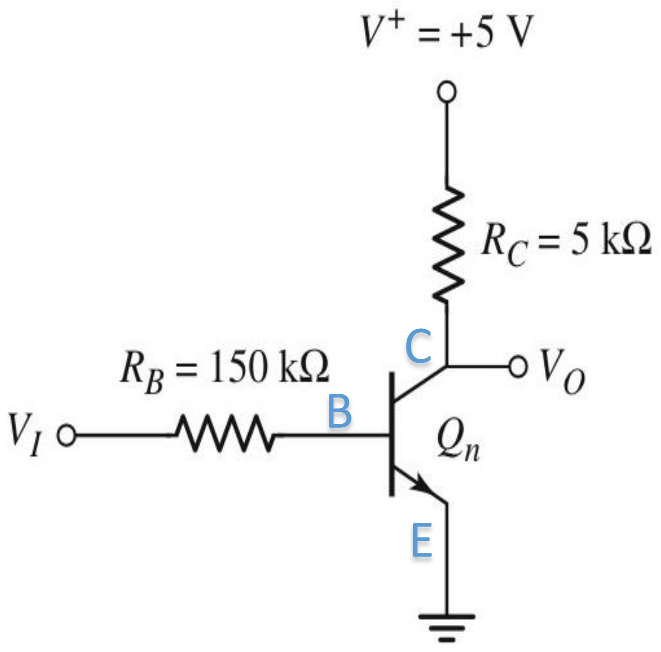
• Write B-E, and C-E voltage loops.

• For analysis, solve for I_C, and V_{CE}.

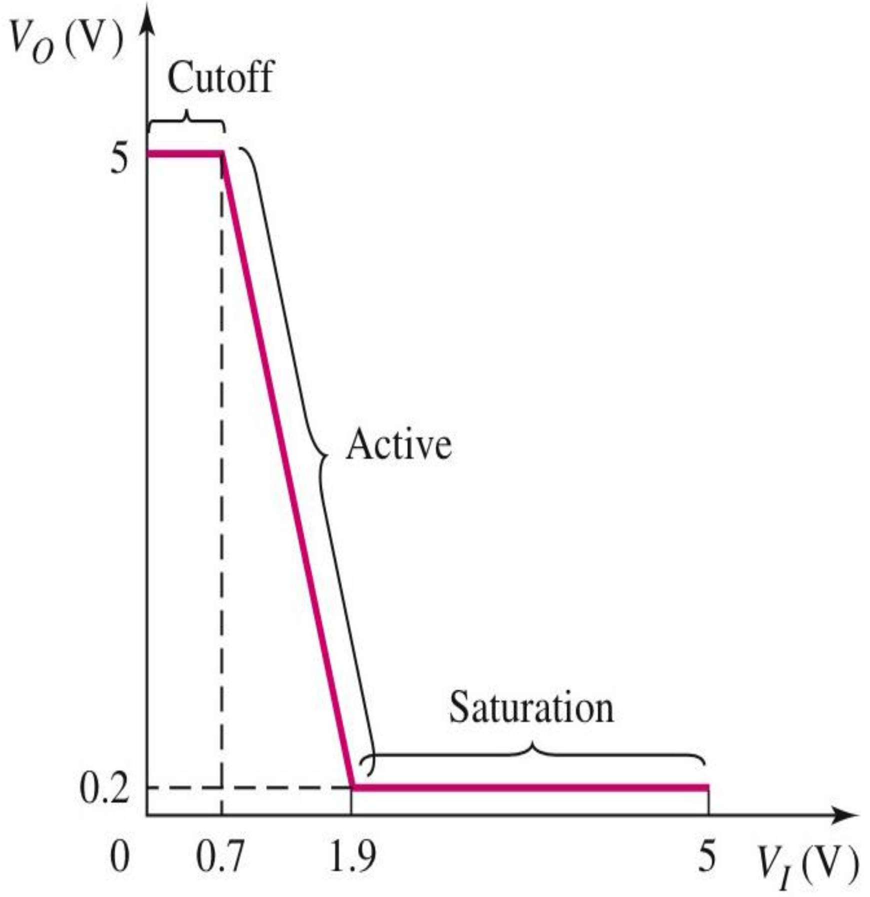
• For design, solve for resistor values (I_C and V_{CE} specified).

Voltage Transfer Characteristic

Voltage Transfer Characteristic for npn Circuit



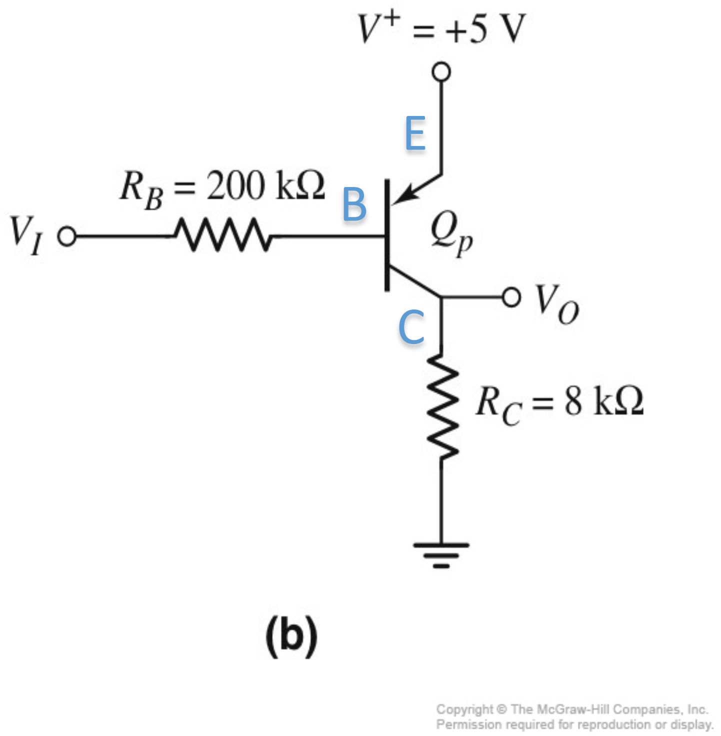
(a)

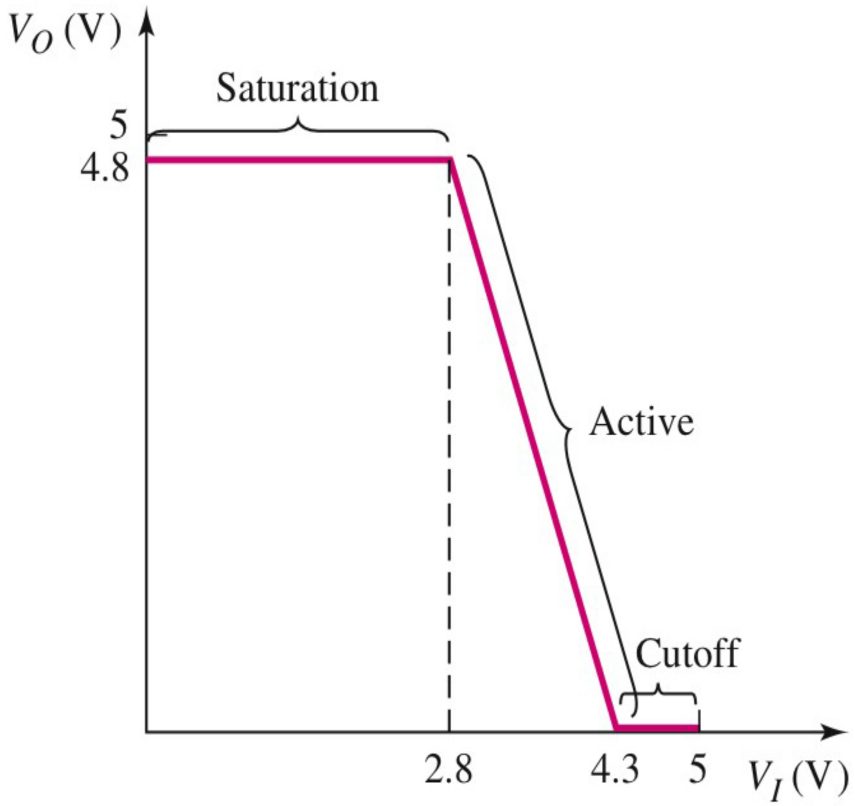


V_{CE}(sat) = 0.2 V

β = 120

Voltage Transfer Characteristic for pnp Circuit





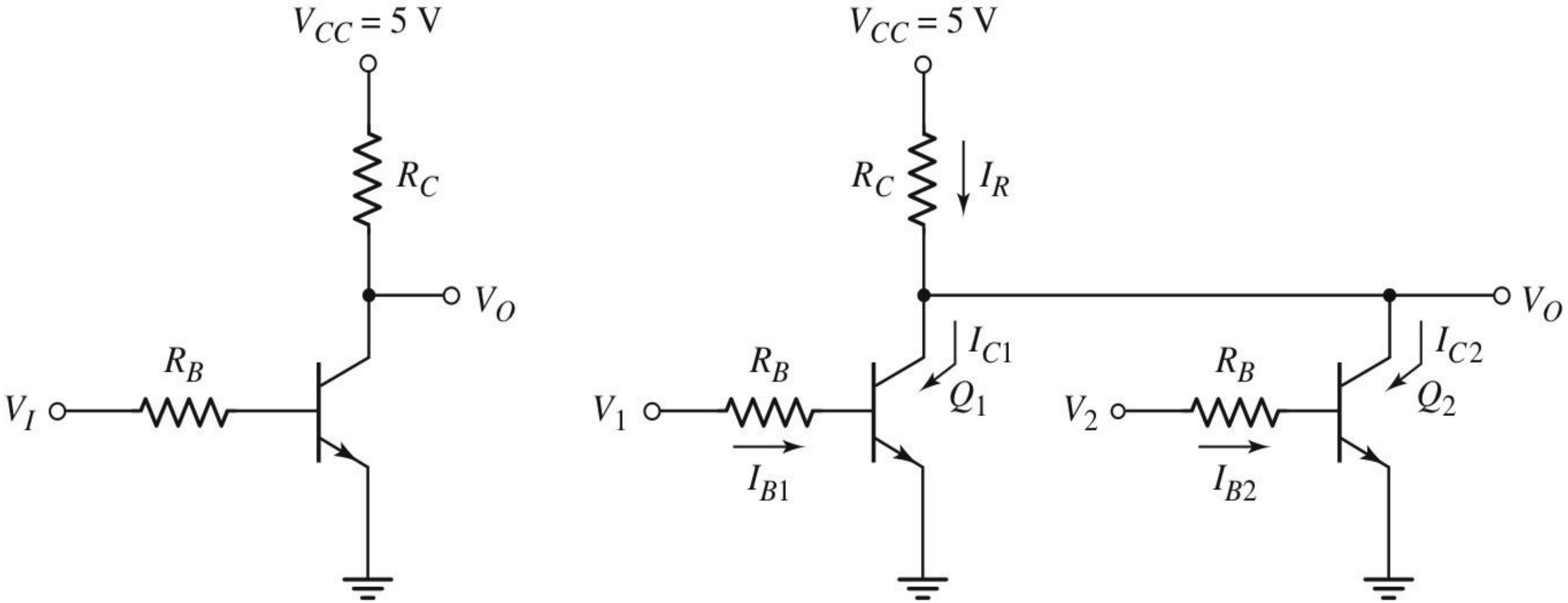
V_{EC}(sat) = 0.2 V

$V_{EB} < 0.7$V

β = 80

Digital Logic Circuit

Digital Logic

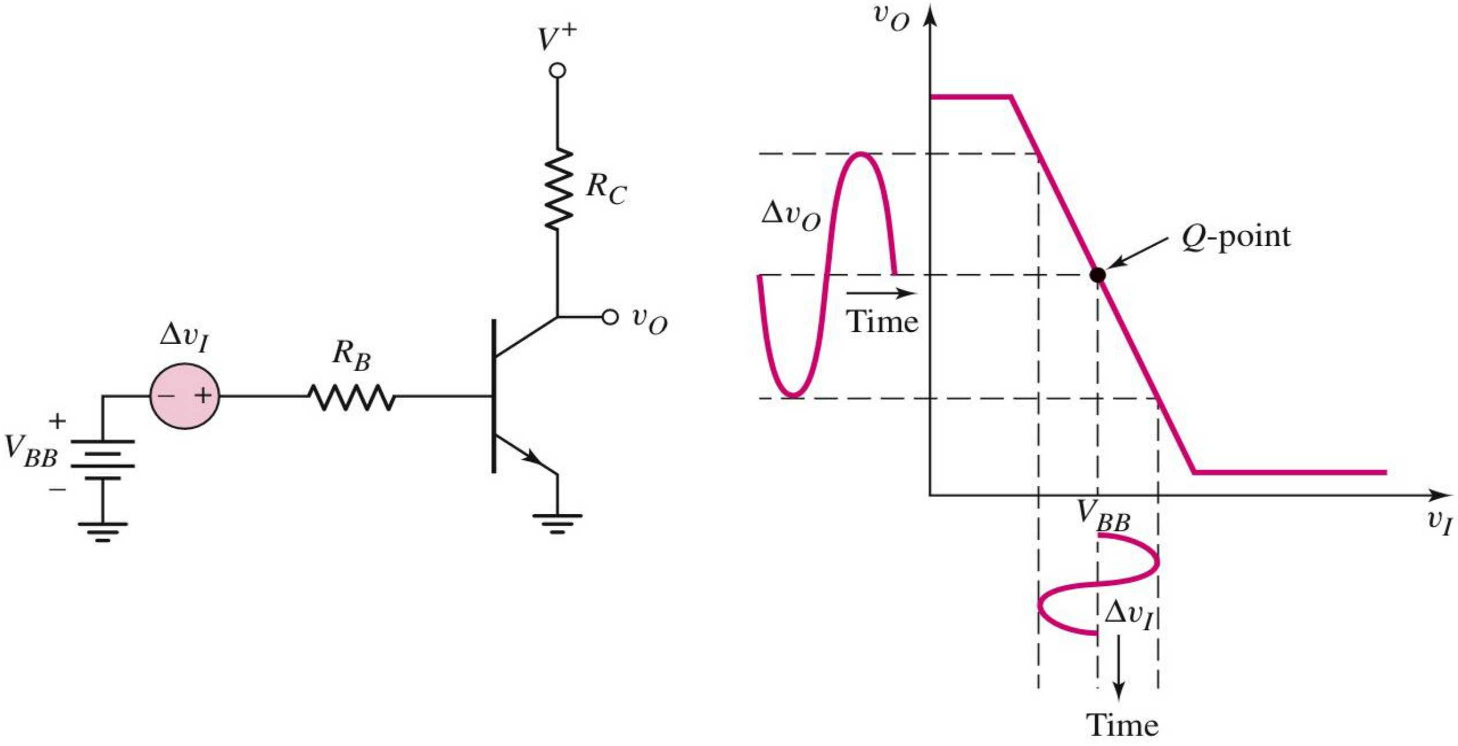


(a) Inverter **(b)** NOR gate

INPUT		OUTPUT
A	B	A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

V1 (V)	*V2* (V)	*Vo* (V)
0	0	5
5	0	0.2
0	5	0.2
5	5	0.2

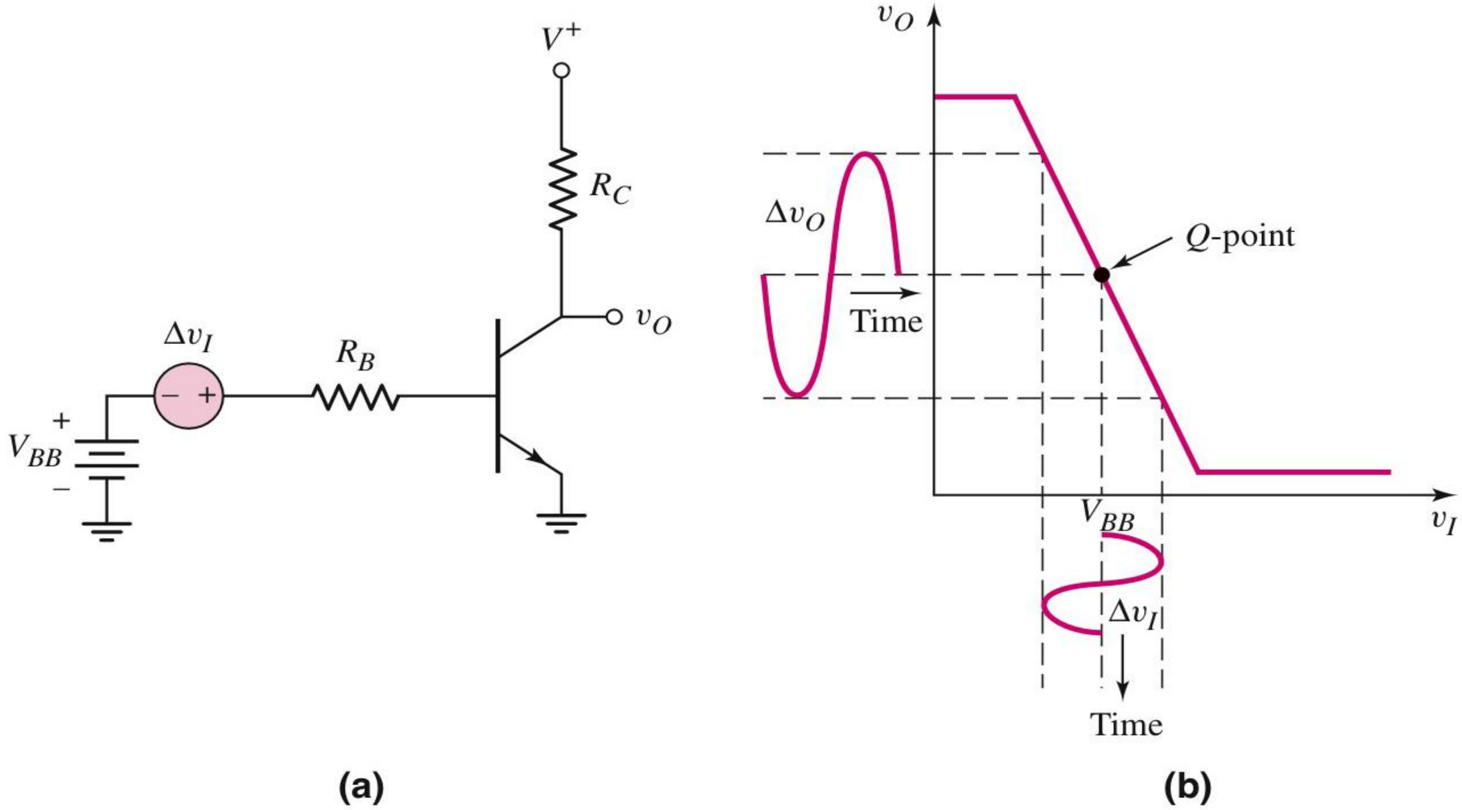
Bipolar Inverter as Amplifier



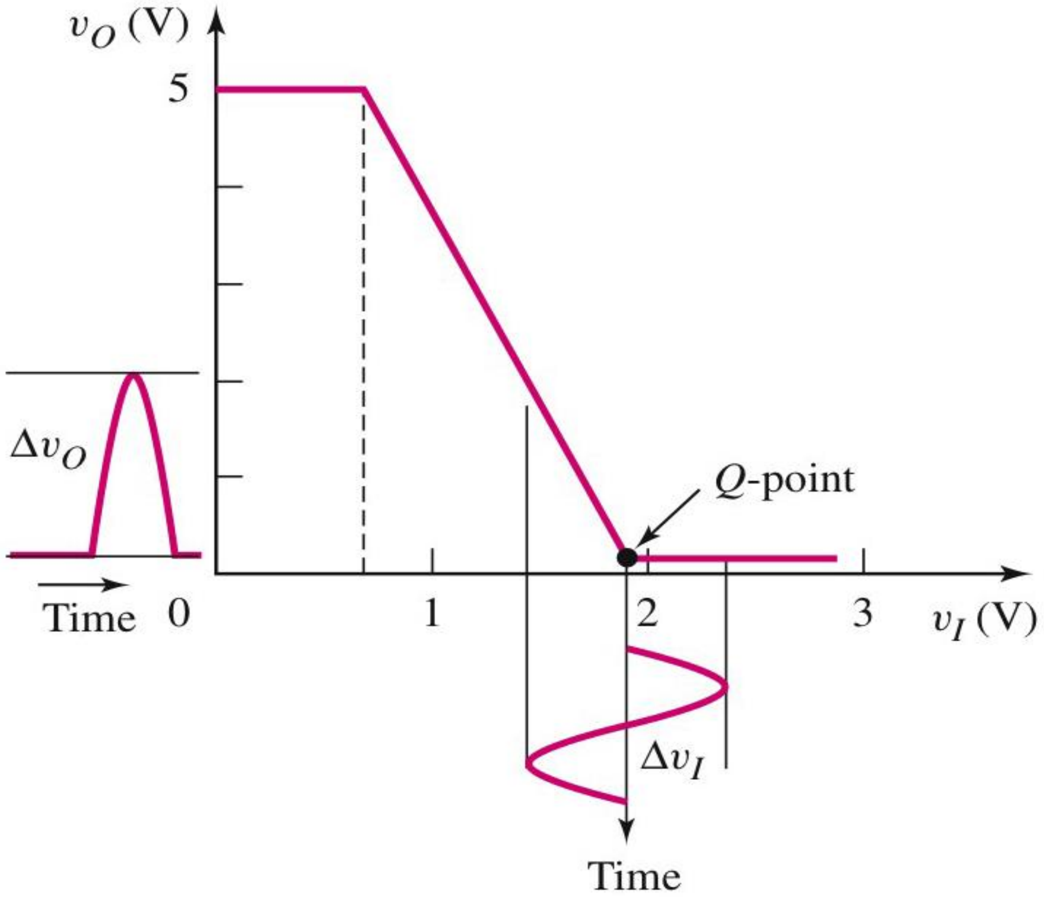
(a) (b)

Bipolar Transistor Biasing

Bipolar Inverter as Amplifier



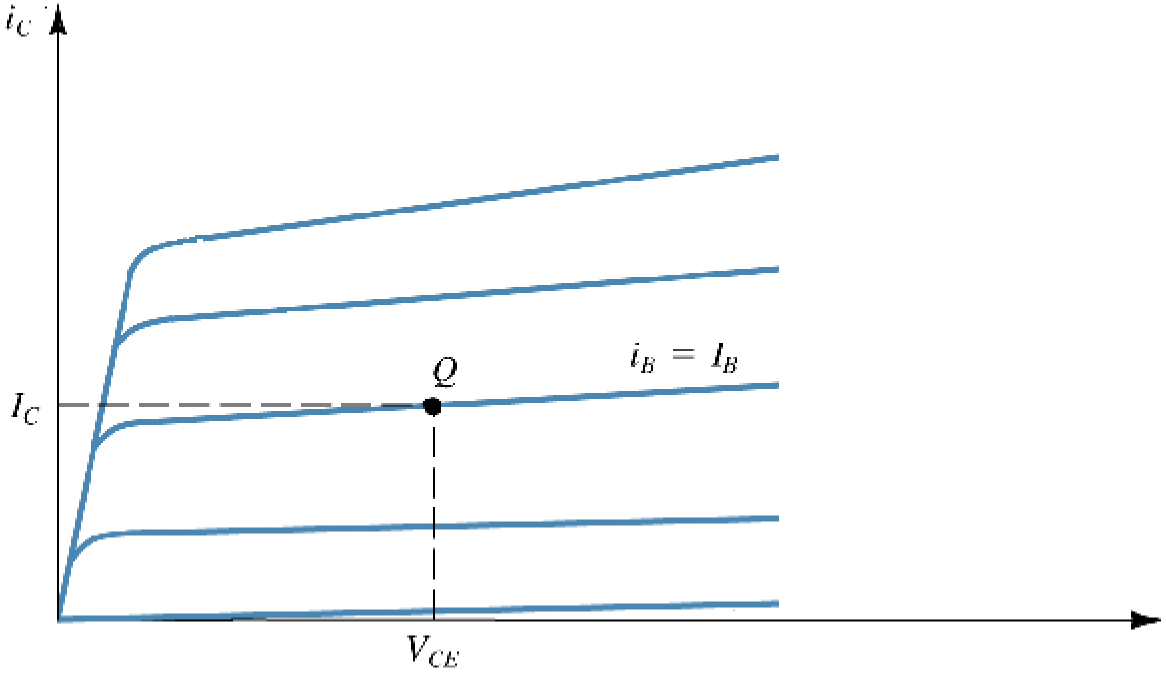
Effect of Improper Biasing on Amplified Signal Waveform



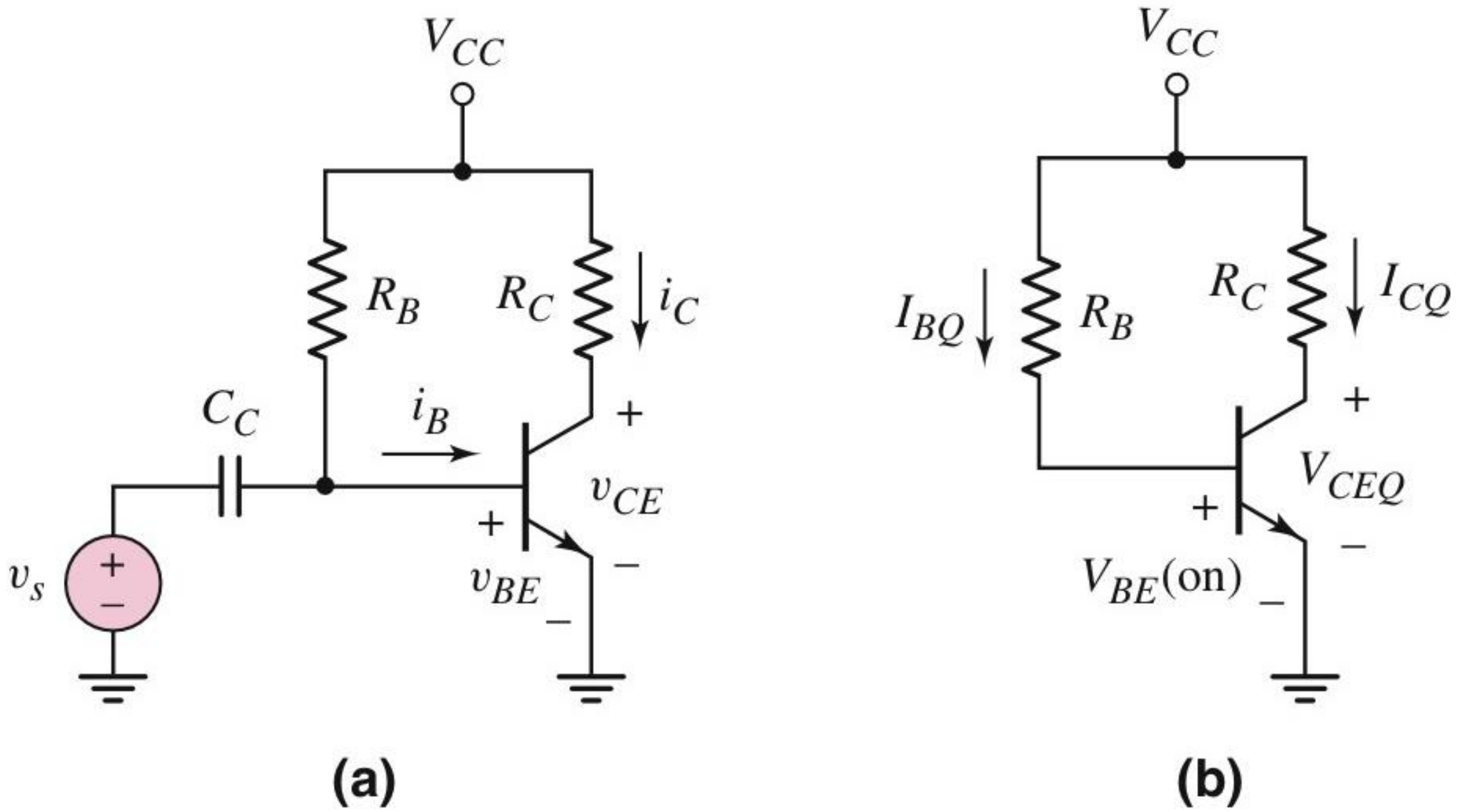
Biasing a Transistor

- We must operate the transistor in the linear region.
- A transistor's operating point (Q-point) is defined by

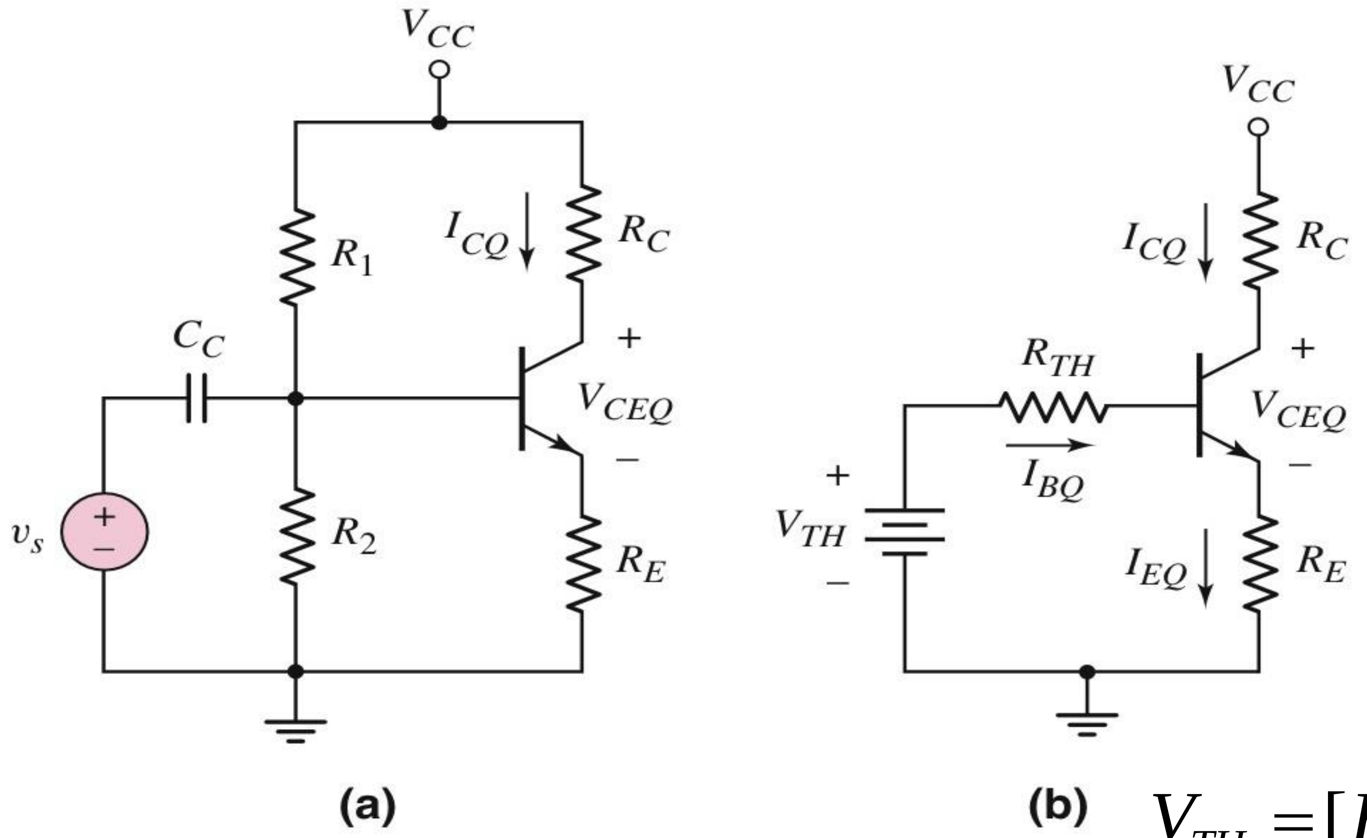
I_C, V_{CE}, and I_B.



Single Base Resistor Biasing

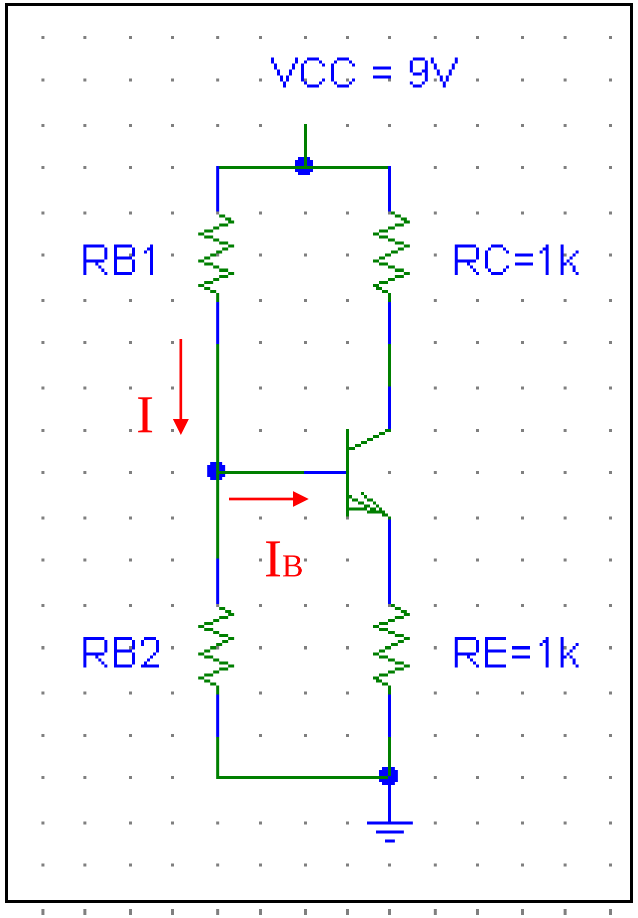


Common Emitter with Voltage Divider Biasing and Emitter Resistor



$$V_{TH} = [R_2 / (R_1 + R_2) V_{CC}$$

Example:



•Use a voltage divider, R_{B1} and R_{B2} to bias V_B to avoid two power supplies.

•Make the current in the voltage divider about 10 times I_B to simplify the analysis. Use $V_B = 3V$ and $I = 0.2mA$.

(a) R_{B1} and R_{B2} form a voltage divider.

Assume $I >> I_B$ $I = V_{CC}/(R_{B1} + R_{B2})$

$.2mA = 9 /(R_{B1} + R_{B2})$

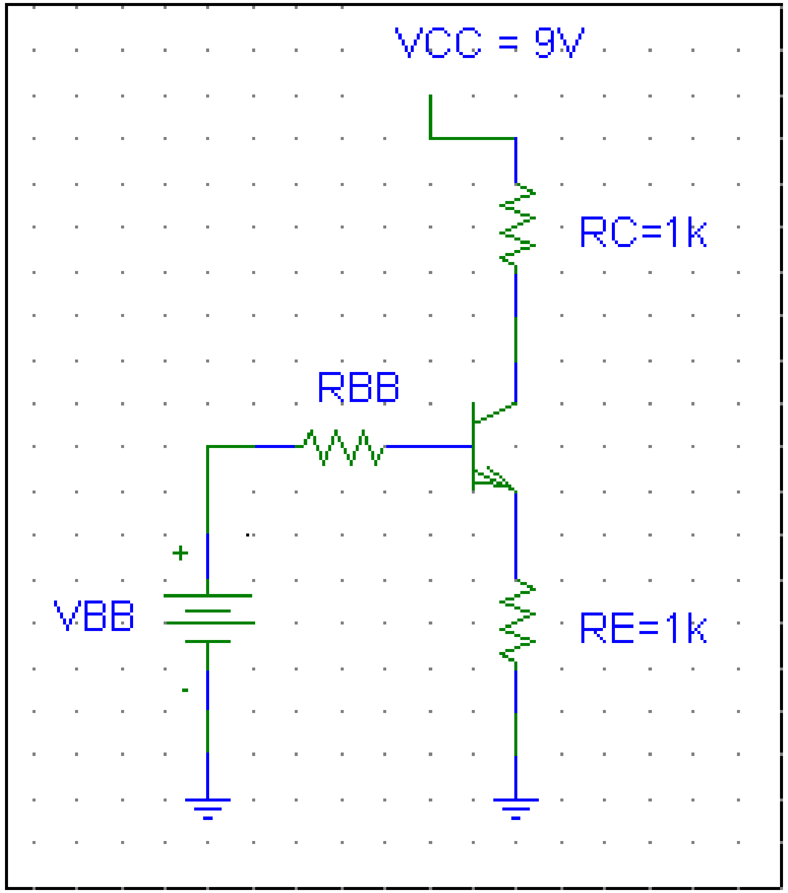
AND

$V_B = V_{CC}[R_{B2}/(R_{B1} + R_{B2})]$

$3 = 9 [R_{B2}/(R_{B1} + R_{B2})]$, Solve for R_{B1} and R_{B2}.

$R_{B1} = 30K\Omega$, and $R_{B2} = 15K\Omega$.

Example (Cont')



Find the operating point

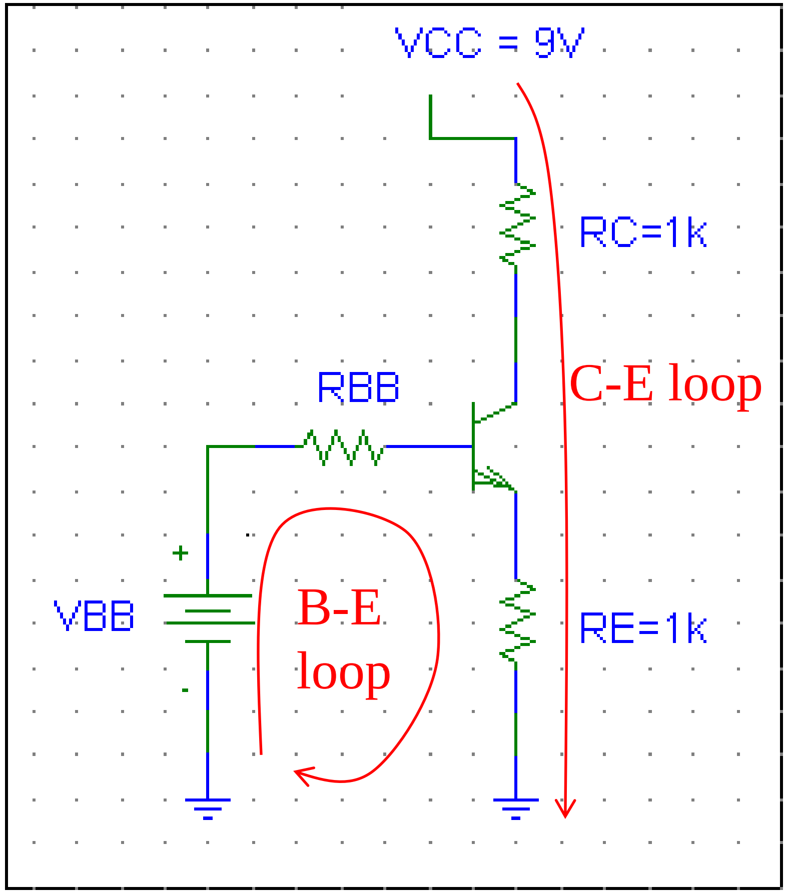
- Use the Thevenin equivalent circuit for the base

- Makes the circuit simpler

- $V_{BB} = V_B = 3V$

- R_{BB} is measured with voltage sources grounded
- $R_{BB} = R_{B1} || R_{B2} = 30K\Omega \; || \; 15K\Omega = 10K\Omega$

Example (Cont')



Write B-E loop and C-E loop

B-E loop

$V_{BB} = I_B R_{BB} + V_{BE} + I_E R_E$

$I_E = 2.09$ mA

C-E loop

$V_{CC} = I_C R_C + V_{CE} + I_E R_E$

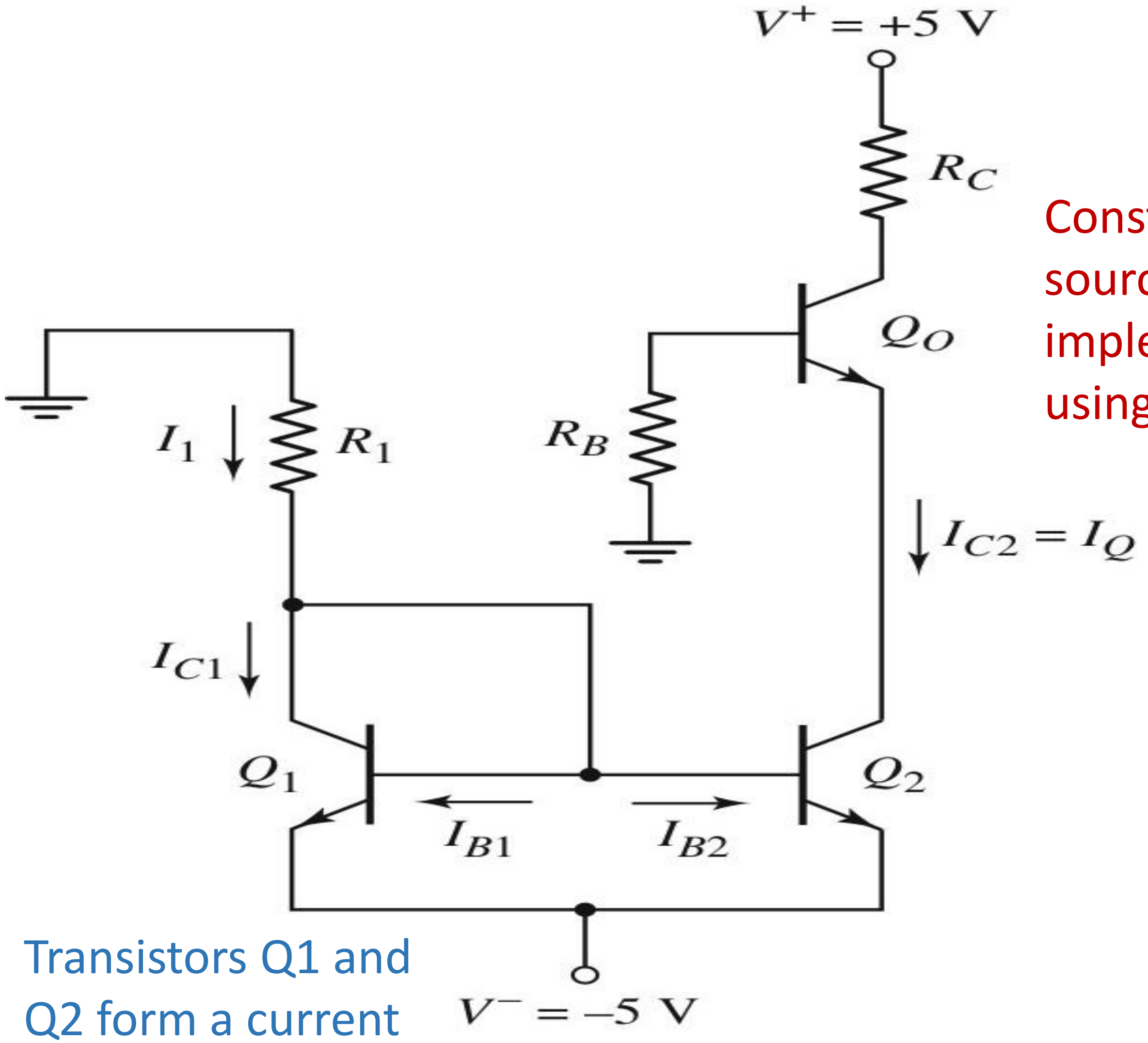
$V_{CE} = 4.8$ V

This is how all DC circuits are analyzed

and designed!

Integrated Circuit Biasing

$$I_C = I_Q = \frac{I_1}{1 + \frac{2}{\beta}}$$

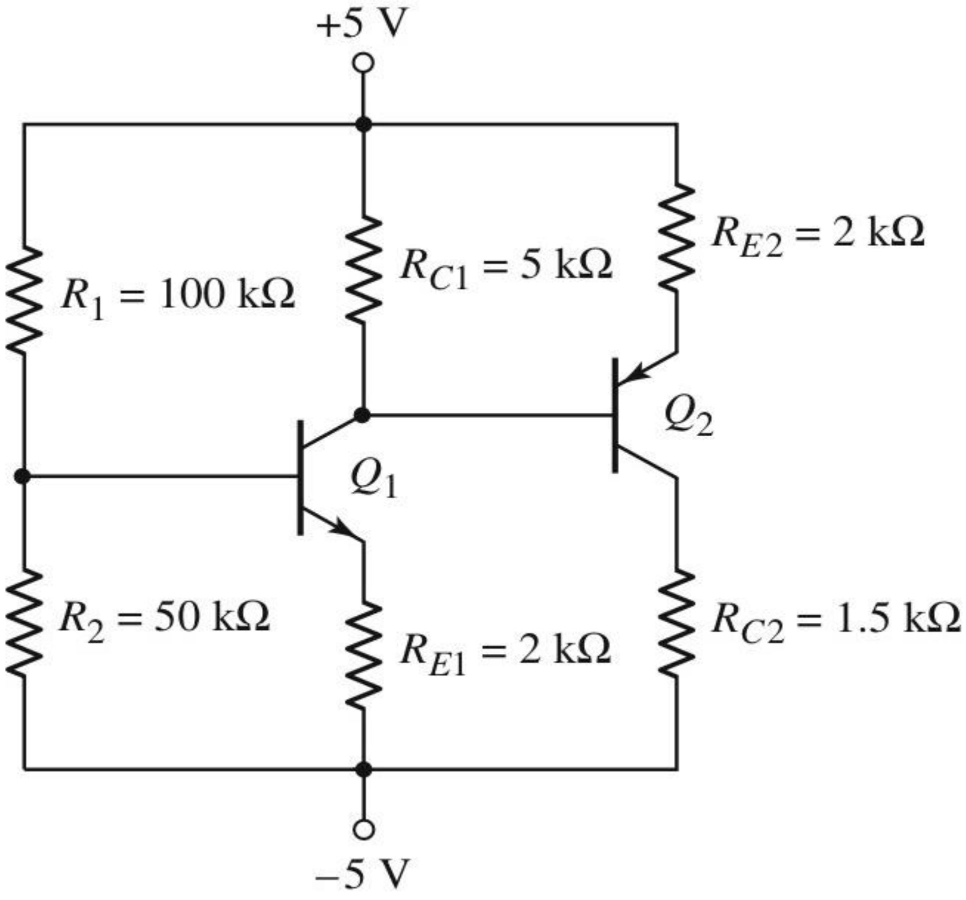


Constant current source implemented by using transistors.

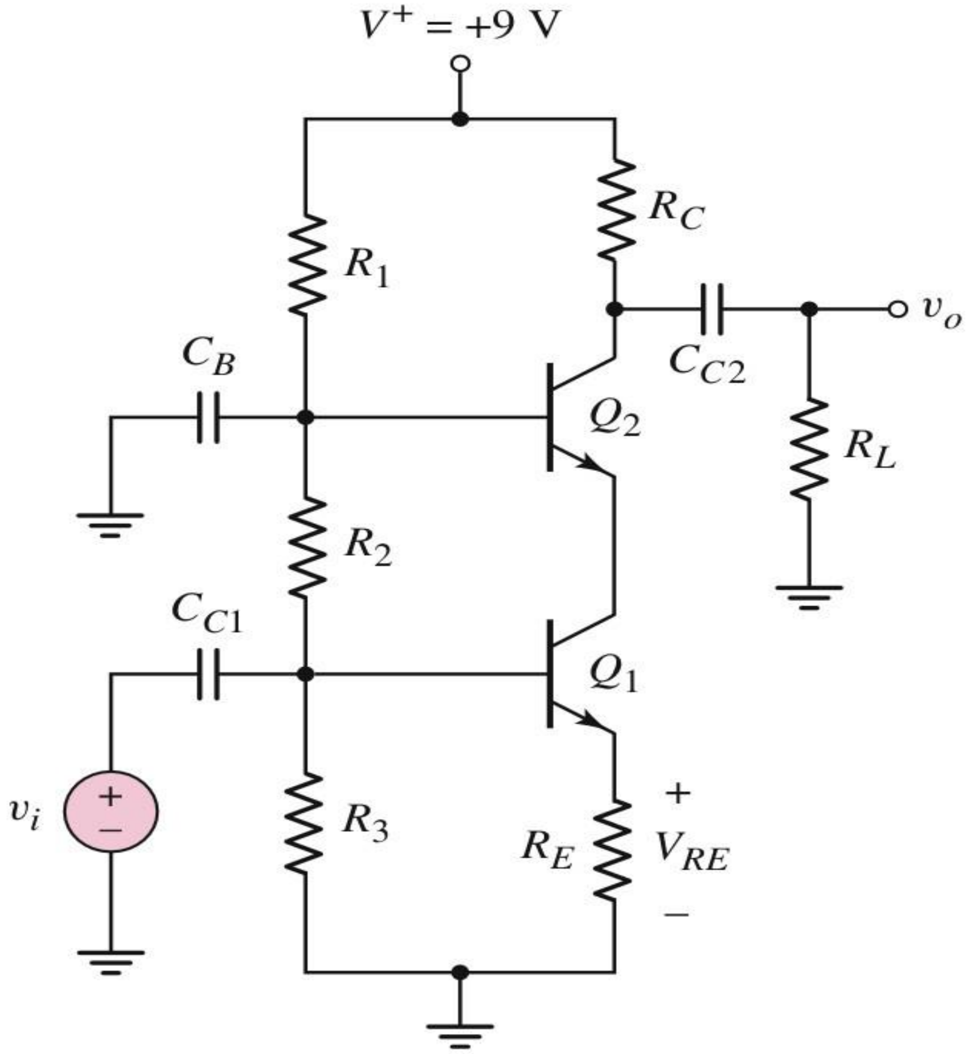
Transistors Q1 and Q2 form a current mirror.

Multistage Circuits

Multistage Cascade Transistor Circuit



Multistage Cascode Transistor Circuit



Contents of Chapter

- Discuss the physical structure and operation of the bipolar junction transistor.
- Understand the dc analysis and design techniques of bipolar transistor circuits.
- Examine three basic applications of bipolar transistor circuits.
- Investigate various dc biasing schemes of bipolar transistor circuits, including integrated circuit biasing.
- Consider the dc biasing of multistage or multi-transistor circuits.

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EEE109: Electronic Circuits

Basic BJT Amplifiers – Part 1

Contents

- Understand the concept of an analog signal and the principle of a linear amplifier.
  - Investigate how a transistor circuit can amplify a small, time-varying input signal.

- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-emitter amplifier.
  - Analyze the emitter-follower amplifier.
  - Analyze the common-base amplifier.

- Analyze multitransistor or multistage amplifiers.
  circuit.

- Understand the concept of signal power gain in an amplifier

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- Analyze multitransistor or multistage amplifiers. circuit.

- Understand the concept of signal power gain in an amplifier

General Definition of Amplifiers

Amplifiers - Definition

- An ideal amplifier is a unit with two input terminals and two output terminals. A signal (a voltage or current that varies with time) is applied to the input terminals and an exact copy of the signal but of larger magnitude is produced at the output terminals. That is if the input is S(t) then the output is $A \times S(t)$ where A is a constant numeric value that is usually greater than one.
- There are **four possibilities** because we can consider the input signal source to be a **current** or a **voltage** source. Similarly the output may act as a current or a voltage source. This gives four cases if we define

$i_{in}(t)$ is the input current. $v_{in}(t)$ is the input voltage

$i_{out}(t)$ is the output current. $v_{out}(t)$ is the output voltage

Amplifier - Classification

- Possible amplifiers are

$$v_{out}(t) = A \times v_{in}(t)$$ voltage amplifier

$$i_{out}(t) = A \times i_{in}(t)$$ current amplifier

$$v_{out}(t) = A \times i_{in}(t)$$ transimpedance amplifier

$$i_{out}(t) = A \times v_{in}(t)$$ transconductance amplifier

Although all have applications by far the largest number of cases examined are **voltage amplifiers** (and Thévénin and Nortons Theorems allow many of the others to be manipulated to this form).

Amplifier – Circuit Representation (1)

- Any amplifier can be considered to behave as the generic amplifier although it may not do so in an exact manner. The generic four terminal voltage amplifier is shown in Figure 2.1.

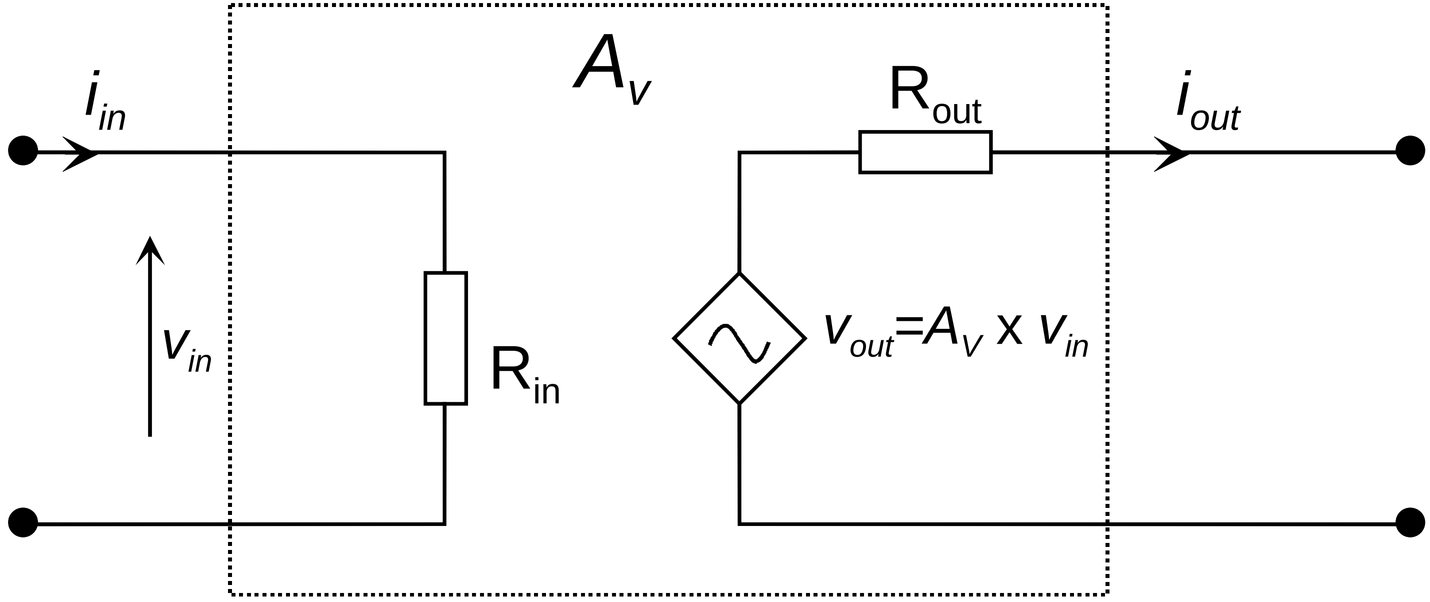


Figure 2.1

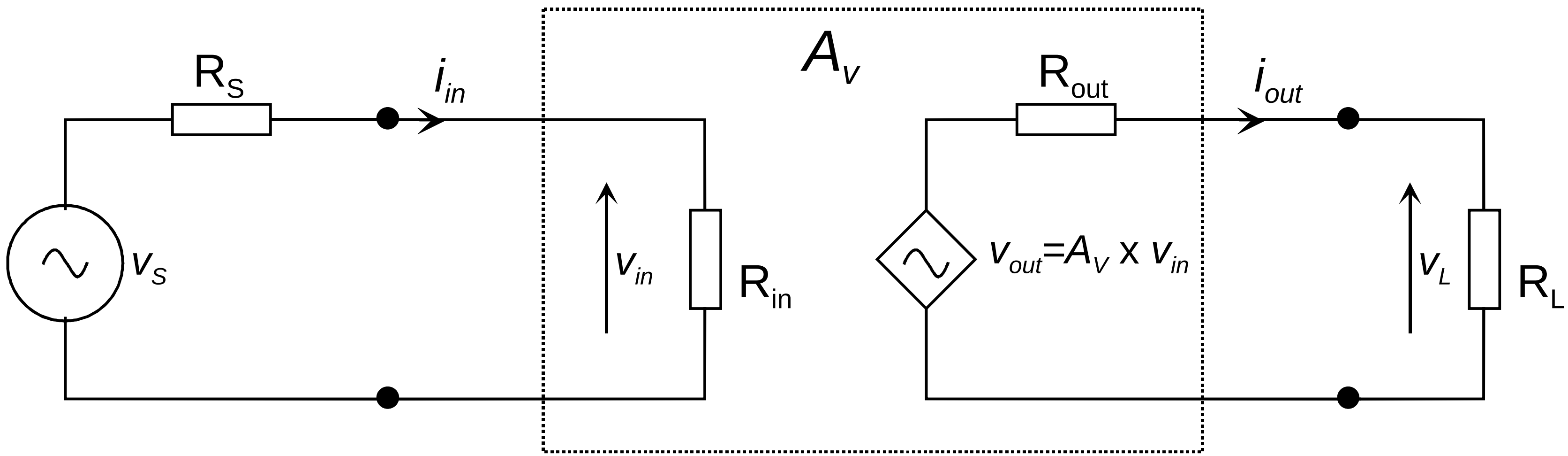
Amplifier - Circuit Representation (2)

- **Notes:**
  Usually the input signal is supplied from a source which can be regarded as a Thévénin circuit as in Figure 2.2. Therefore **is not the same as the source voltage**

- Usually the output signal is applied to a resistive load as in Figure 2.2. Therefore **is not the same as the voltage across the load**.

- In real circuits it is unusual for the input and output circuits to be totally isolated from each other. One of the most common arrangements – BUT NOT THE ONLY ONE – is for one input terminal and one output terminal to be joined.

Amplifier - Circuit Representation (3)



<u>**Figure 2.2**</u>

Amplifier – Input (1)

- R_S and R_{in} form a potential divider, therefore $v_{in} = v_S \dfrac{R_{in}}{R_S + R_{in}}$

- For the largest possible output (largest amplification of the signal, v_S) v_{in} must be as large as possible. Therefore **R_{in} should be much larger than R_S** - for general purpose voltage amplification a large input resistance is required. **However** you will learn in later years that in some situations other problems can arise and for these **matching is necessary -** the output resistance of the signal source must equal, match, the input resistance of the circuit to which it is connected requiring that $R_S = R_{in}$.

Amplifier – Input (2)

- In voltage amplifier design usually one of two cases arises

  either maximum voltage output is required so

  $R_{in} >> R_s$ – very high input resistance

  or it is required that

  $R_{in} = R_s$ – source resistance and input resistance are matched

Amplifier – Output

- R_{out} and R_L form a potential divider, therefore

$$v_L = A_v v_{in} \frac{R_L}{R_{out} + R_L}$$

- If R_L cannot be varied because the amplifier is required to drive a specified load then to get the largest possible output requires that R_L should be much larger than R_{out}. For general purpose voltage amplification **a small output resistance is required**. Again in some situations **matching** is necessary and the output resistance must match the load.

Amplifier – Matching (1)

- For simple voltage amplification often the output must be as large as possible. However often R_{out} is fixed and it is necessary to get maximum **power** possible in the load by choosing R_L. How does R_L affect the power in the load?

$$P_L = v_L \times i_{out} = \frac{v_L^2}{R_L} = A_v^2 \times v_{in}^2 \times \left(\frac{R_L}{R_{out} + R_L}\right)^2 \times \frac{1}{R_L}$$

- If the gain, input signal and output resistance are all fixed then only R_L affects the power output. Therefore differentiate P_L with respect to R_L to find how power varies with value of R_L

Amplifier – Matching (2)

$$P_L = \frac{K \times R_L}{(R_{out} + R_L)^2} \quad \text{so} \quad \frac{dP_L}{dR_L} = K\left(\frac{1}{(R_{out} + R_L)^2} - 2\frac{R_L}{(R_{out} + R_L)^3}\right)$$

A function is a maximum (or minimum) when the first differential is ze

$$\frac{dP_L}{dR_L} = 0$$

which is
when

$$\frac{1}{(R_{out} + R_L)^2} = 2\frac{R_L}{(R_{out} + R_L)^3}$$

Re-arranging reduces this to $R_{out} = R_L$ and checking the second differential shows that this is the maximum case.

Amplifier – Matching (3)

- Hence for **maximum power output** from a circuit **the load should equal the output resistance**. **Note** that this is maximum power, not maximum voltage or maximum current.

maximum power output

$$R_{out} = R_L$$

Amplifier – Output Resistance (1)

- Usually voltage amplifier design requirements will lead one of two cases

either R_L is a fixed requirement and the maximum output voltage is required so

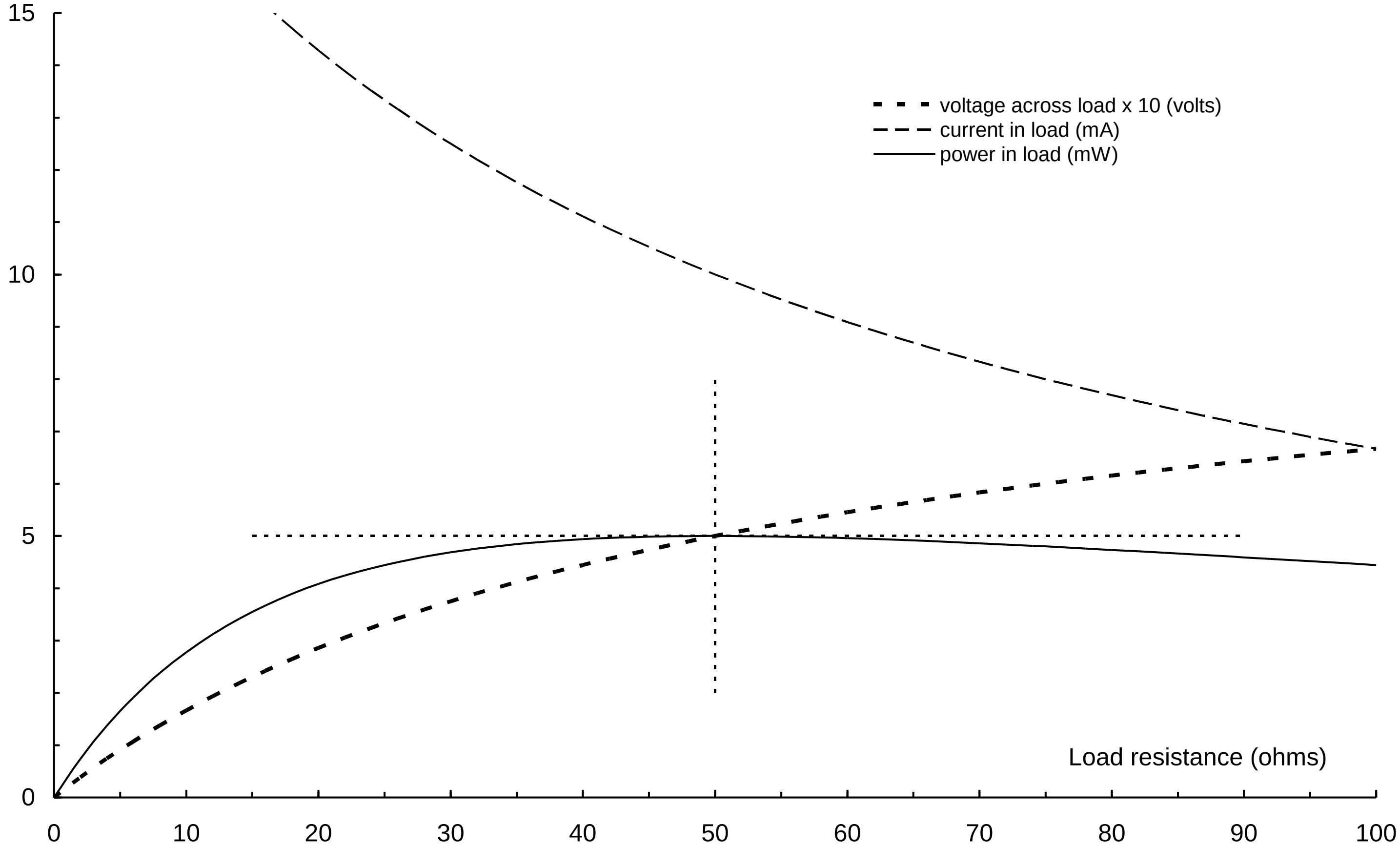
$R_{out} << R_L$ – very low output resistance

or

R_{out} is fixed and R_L can be selected; maximum power is required so

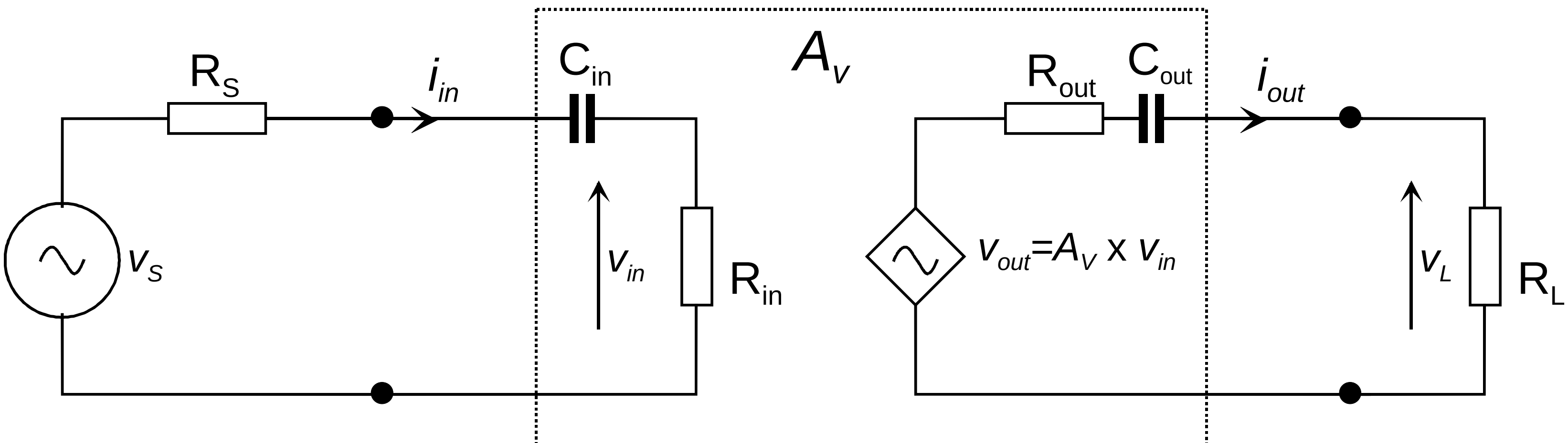
$R_{out} = R_L$ – output and load resistances are matched

Amplifier – Output Resistance (2)



Amplifier – AC Version

Later in the course the a.c coupled version of the amplifier will be considered.



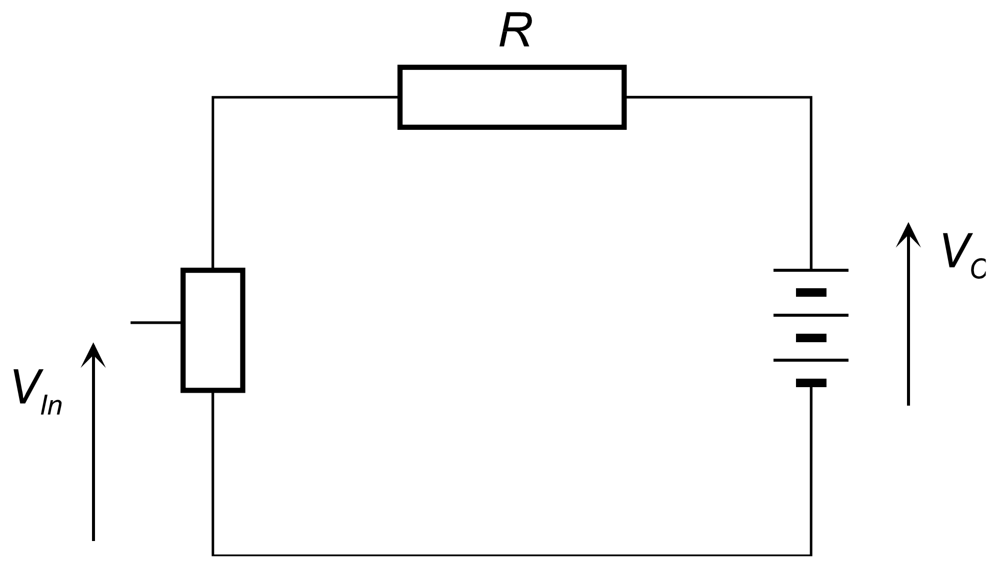
Bipolar Transistor Revisited

- Transistor in a circuit
- Bipolar transistor
- Transistor equivalent circuit

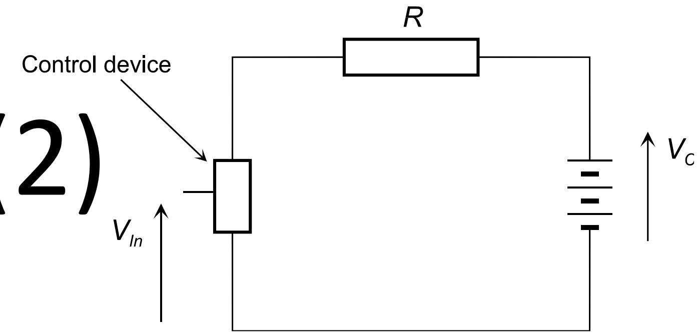
Transistor in a Circuit (1)

- Transistor behaviour is introduced in the 1st semester of the course. This is a very brief outline from the viewpoint of the transistor as an electronic circuit element – ***not construction and physics of operation***

- Basic circuit with a transistor



Transistor in a Circuit (2)



- The **battery** is a store of energy (potential energy), the resistor absorbs energy.

- The box *'Control device'* with input signal V_{in} is some form of ***electronic control valve,*** usually a transistor.

- A change in the input, ΔV_{In} and ΔI_{In}, causes a change in the energy dissipated in resistor R by controlling the energy flow from the battery.

- If the control device requires **very little energy change** at its **input** to produce a **large change** in the **current through the resistor** there will be a gain – **amplification**.

Transistor in a Circuit (3)

If ΔI_R is the **change in current** through the resistor and ΔV_R is the **change in voltage** across the resistor then

$$A_i = \frac{\Delta I_R}{\Delta I_{In}}$$ and is the current gain

$$A_V = \frac{\Delta V_R}{\Delta V_{In}}$$ and is the voltage gain

$$A_P = A_V \times A_I = \frac{\Delta I_R}{\Delta I_{In}} \times \frac{\Delta V_R}{\Delta V_{In}}$$ and is the power gain

The transistor is the solid state electronic analogue of a control valve. *It is used to control (or switch) energy flow from an electrical energy supply into a* <u>*load*</u> *in which the* <u>*energy is dissipated or stored*</u>*.*

Bipolar Transistor (1)

They have **three terminals**; <u>base</u>, <u>collector</u> and <u>emitter</u> for **bipolar transistors.**

***Non-linear Transistor Characteristics*:**

- All transistors in all forms and materials have non-linear characteristics. Initially we examine bipolar transistors as these are the most suitable for laboratory work. In a **simplistic** form non-linear means the transistor's behaviour **cannot** be represented by **<u>straight line equations</u>** such as

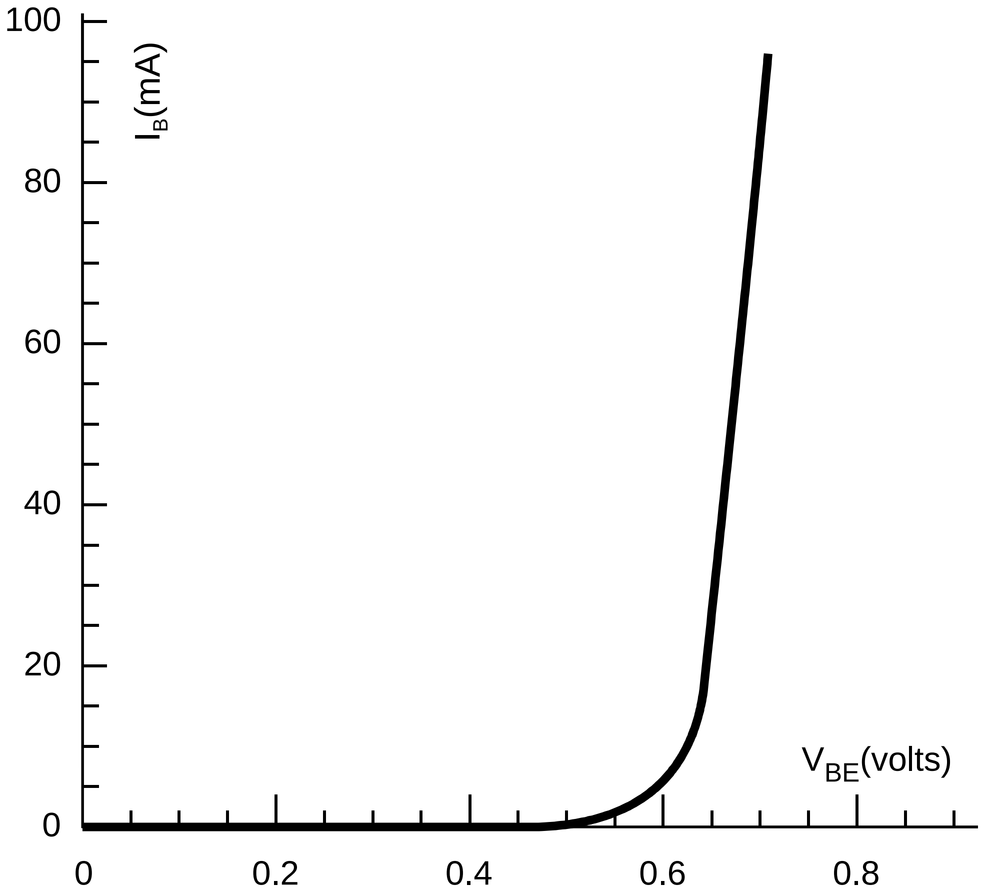
$$y = m \times x \quad \textbf{or} \quad y = m \times x + c$$

Bipolar Transistor (2)

Circuit analysis in other courses used **linear components** that **obey** **Ohm's Law**. The current through a resistor is related to the voltage across it by $V = I \times R$. Ohm's Law is a **linear equation.**

Capacitors and inductors also obey **Ohm's Law** ***in a time varying form***. For transistors the various current and voltages cannot be related by simple straight line equations.

Bipolar Transistor (3)

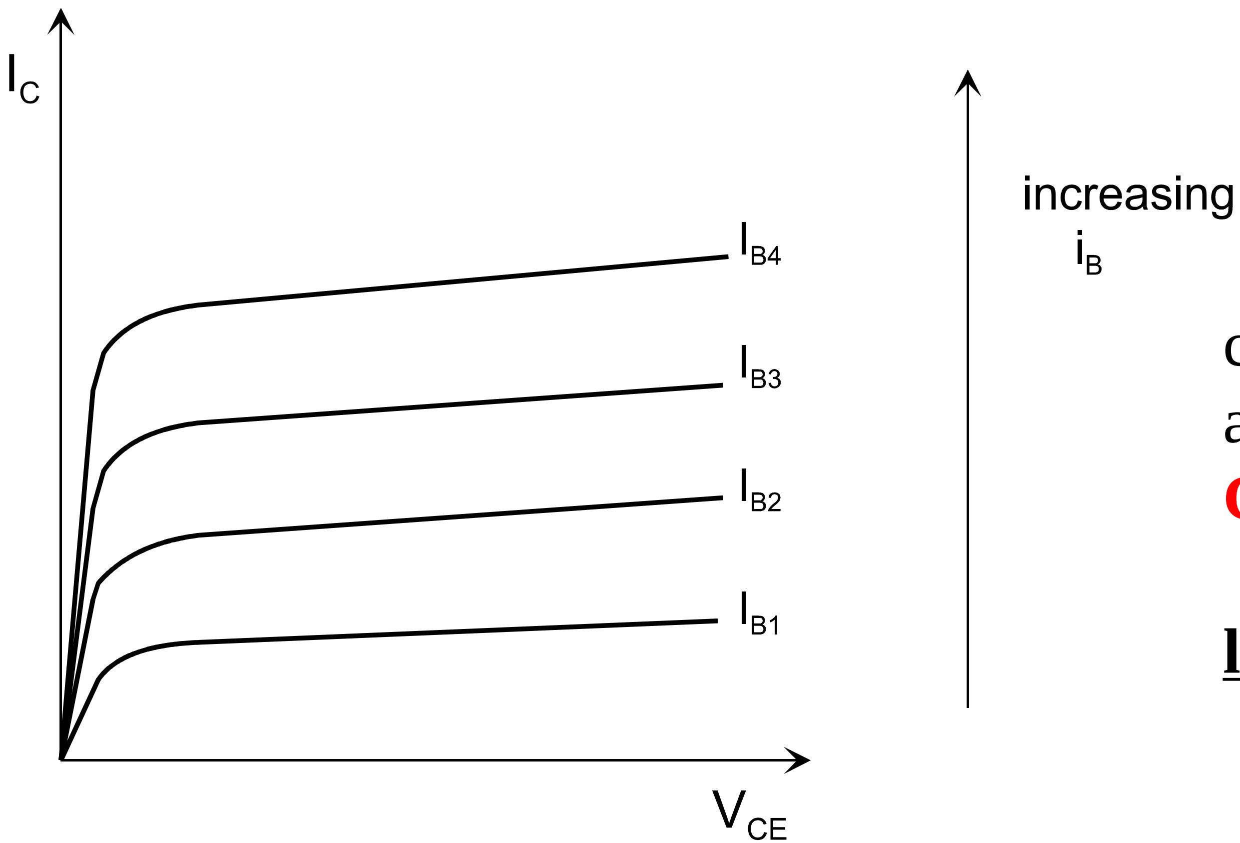


$$I_B = I_S . \exp\left(\frac{qV_{BE}}{kT} + 1\right)$$

the variation of I_B with V_{BE} is the same as the variation of a diode current with voltage – **not a straight line, not linear**

Typical input characteristic, base current as a function of base-emitter voltage

Bipolar Transistor (4)



Output characteristics are also called **transfer Characteristics**

– they are **non-linear**

Typical output characteristics, collector current as a function of collector-emitter voltage

Bipolar Transistor (5)

Why is non-linearity a problem?

• Many applications require the circuit output to be proportional to the input.

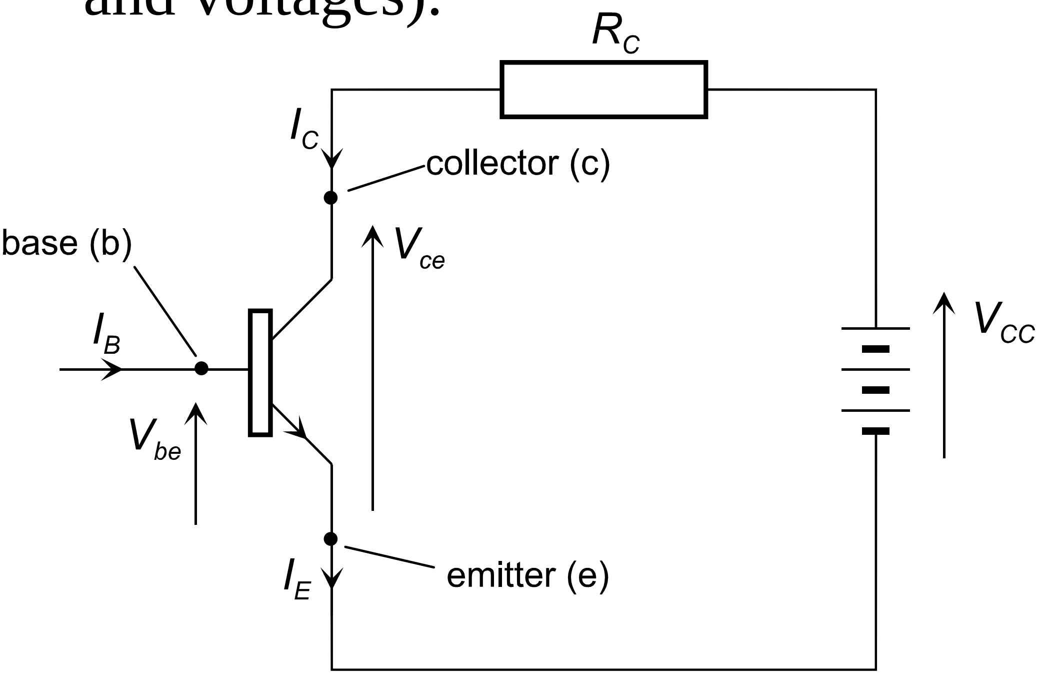
e.g. $V_{out} = k \times V_{in}$ to give **faithful reproduction** of the signal

• In design synthesis and in analysis, we require equations that have analytical solutions.

• For analysis super-position is used, it only works for linear systems

Bipolar Transistor (6)

An npn transistor in a simple circuit (for pnp reverse all currents and voltages).



Rc is in series with the collector.
The current Ic flows through Rc and the collector lead.

The transistor, Rc and the supply form a loop; using Kirchoff's voltage law

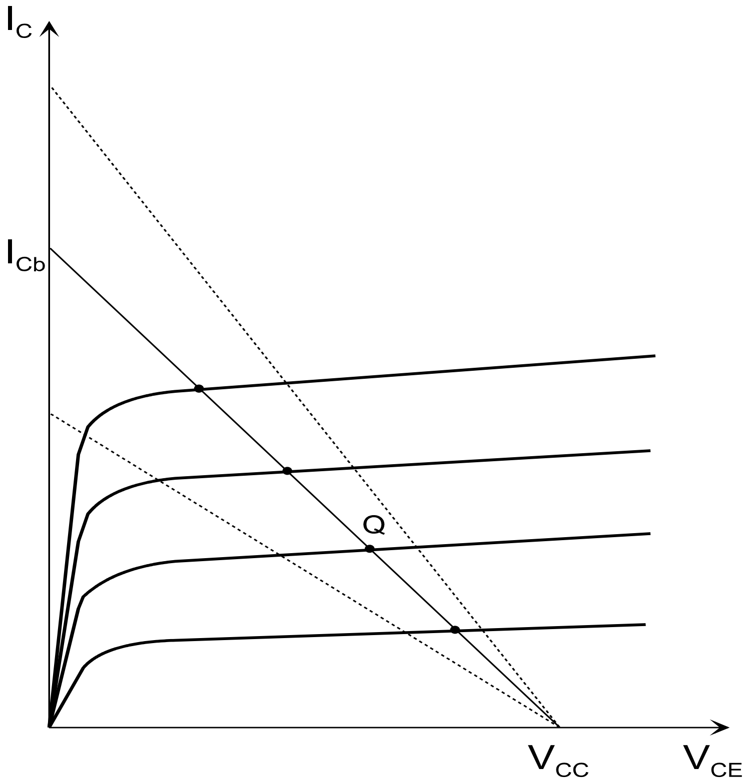
$$V_{CE} = V_{CC} - I_C \times R_C$$

Bipolar Transistor (7)

- I_C and V_{CE} must satisfy the linear relationship of this equation. V_{CC} and R_C are constants – values of circuit components.
- **BUT** I_C and V_{CE} **must also satisfy** the equation (curve, characteristic) which describes the behaviour of the transistor.
- That is the equation of the output characteristic for the particular value of current flowing into the base. There are now **two equations**, the straight line and the transistor output characteristic (which is complicated), these are **simultaneous equations**.

Bipolar Transistor (8) – Solving simultaneous equation (a)

One method of solution to find **the operating point** is **graphical**.
Draw line $V_{CE} = V_{CC} - (I_C \times R_C)$, called a **load line**, and the characteristic on the same axes
(line $I_C = (V_{CC} - V_{CE}) / R_C$ for the axes are shown).



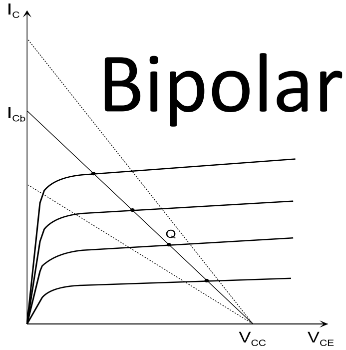
The straight line is easily determined. When

$$I_C = 0 \quad V_{CC} = V_{CE}$$

and if $V_{CE} = 0 \quad I_C = I_{Cb} = \dfrac{V_{CC}}{R_C}$

These two points are on the required straight line – **it must go through both**!

*The broken lines are for different values of R_C



Bipolar Transistor (9) – Solving simultaneous equation (b)

For a specified value of base current, I_B, the values of I_C and V_{CE} are related by the characteristic curve for that value of base current. I_C and V_{CE} are <u>also related</u> by the straight line. The **only** values of I_C and V_{CE} which satisfy both of these are **where the characteristic and the line cross.**

If the transistor **transfer characteristics**, $I_C = f\{I_B, V_{CE}\}$, are known it is possible to determine the values of I_C and V_{CE}. **Note that the transfer characteristic** used is set by I_B which itself is set by the way additional circuits cause the transistor to behave – **to be examined later** – and I_B is related to V_{BE} by the **<u>input (diode) characteristic</u>** (next page)

Bipolar Transistor (10) – Solving simultaneous equation (c)

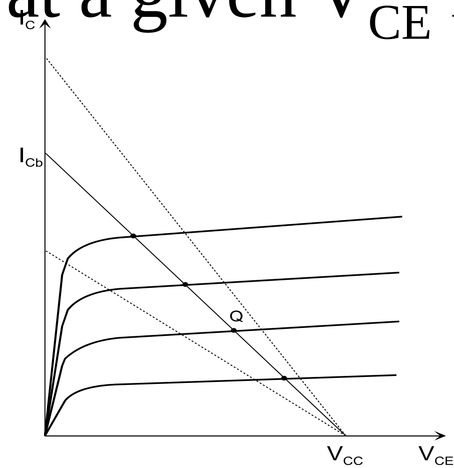
As I_B is changed the output characteristic to be used changes but the load line remains the same, the operating point moves along the line and the value of I_C changes.

Conversely if we force I_C to a value as I_C changes I_B will change. **A change in I_B results in a change in I_C (and vice versa)**

The ratio of I_B to I_C for the transistor at a given V_{CE} is defined as β,

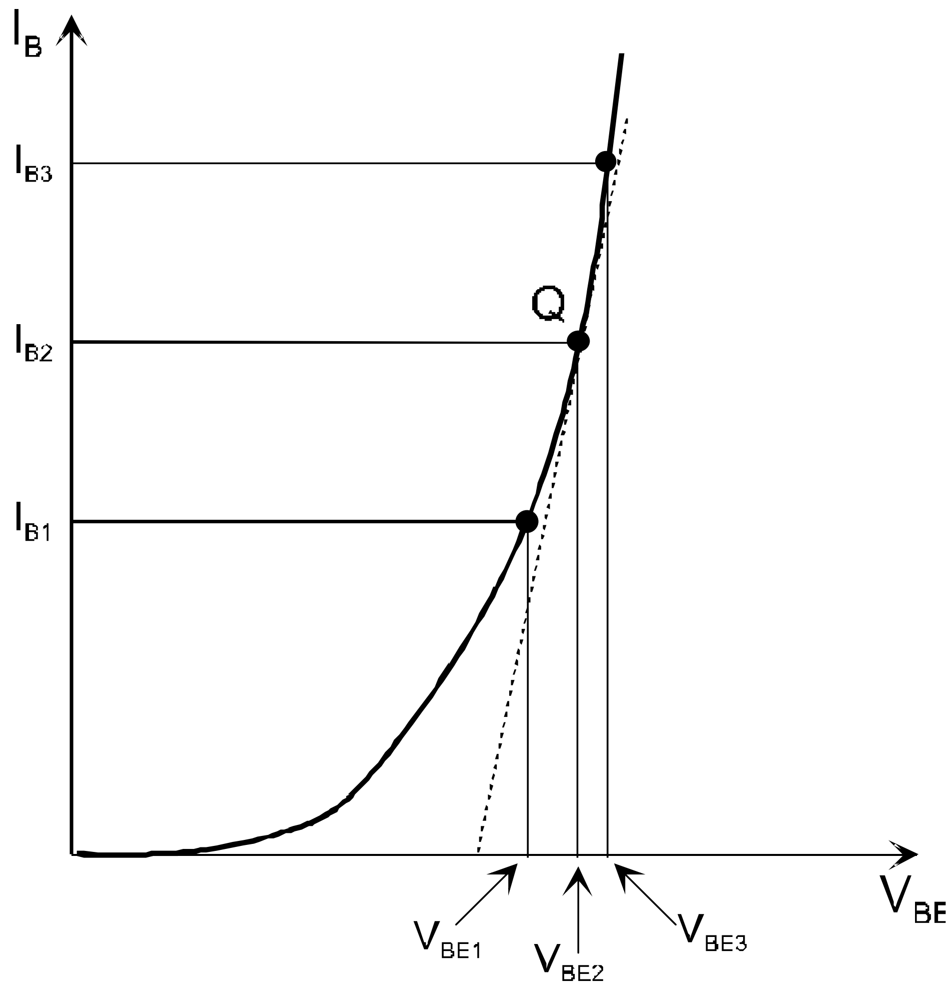
where

$$\beta = \frac{I_C}{I_B}$$



Bipolar Transistor (11) – Solving simultaneous equation (d)

β is known as the <u>static current gain</u> of the transistor, it is also called the DC gain denoted by h_{FE} or β_{DC}.



Bipolar Transistor (12)

- **Two common electronic engineering tasks**

Circuit Design - so that the transistor is at a required operating point and amplifies an input signal.

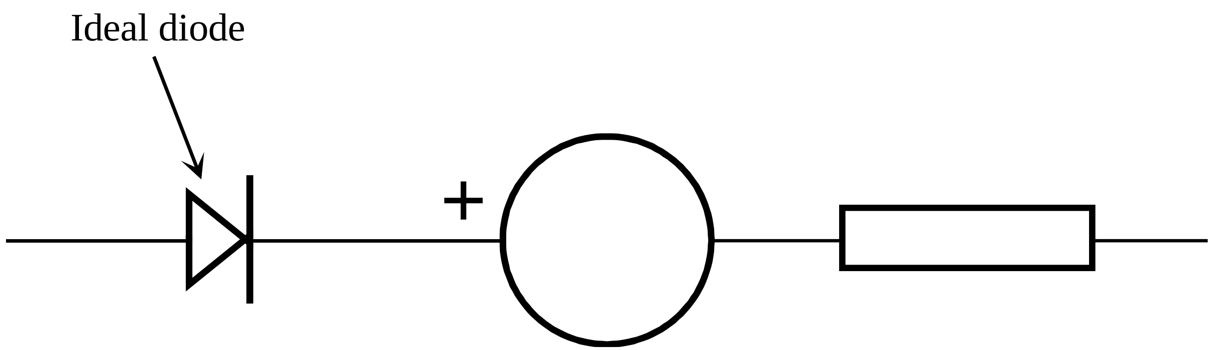
Circuit Analysis – determine the operating point and amplification factor from the circuit.
It often assists in design and analysis if the transistor is represented by an equivalent circuit.

Bipolar Transistor Equivalent Circuit

- h-parameter model
- Hybrid Pi model

Transistor Equivalent Circuit (1)

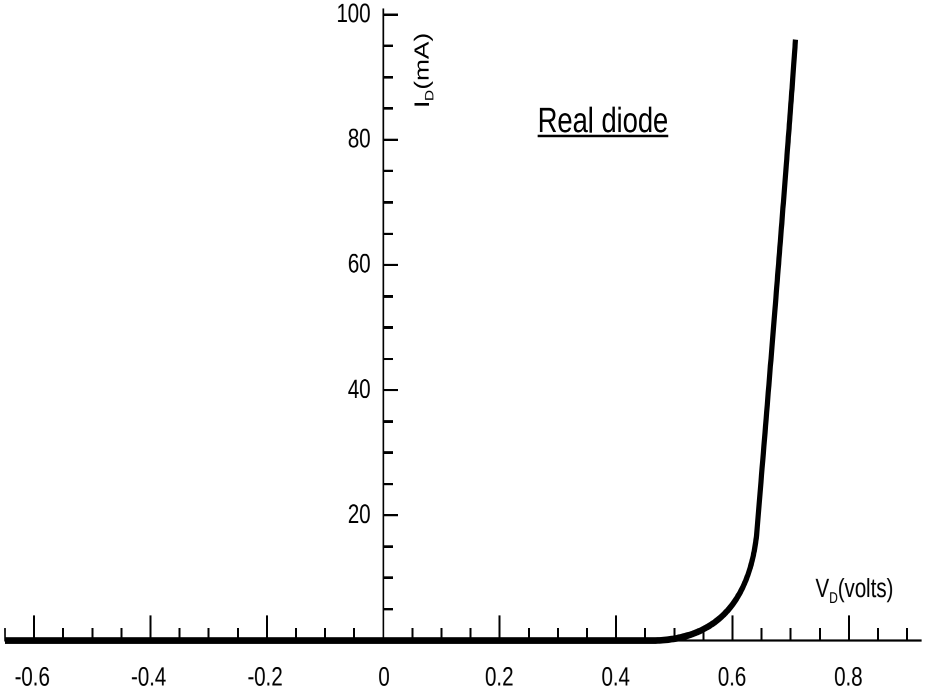
It was possible to replace the diode by an **equivalent circuit** using linear circuit components, this corresponds to **straight line approximations** to the characteristic curves.

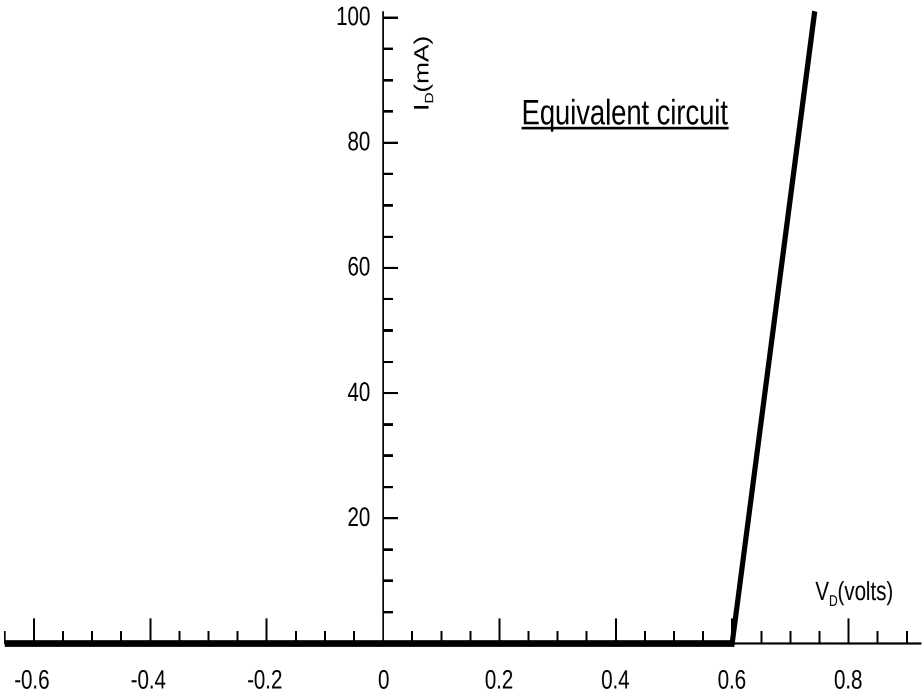


An equivalent circuit for a diode

Ideal diode means no resistance, no voltage drop across it when forward biased; no current flow when reverse biased (a perfect switch operated by the voltage across it).

Transistor Equivalent Circuit (2)





Characteristics for real diode and for the equivalent circuit

Transistor Equivalent Circuit (3)

Perform a similar task for the transistor

– the more complicated curves mean that we divide the transistor operation into two parts**.**

Large signal or steady state or d.c.

Sets operation in a small selected area of the characteristics. This year (and often in practice) this will be done without an equivalent circuit – work with the characteristics.

Small signal or dynamic system or a.c.

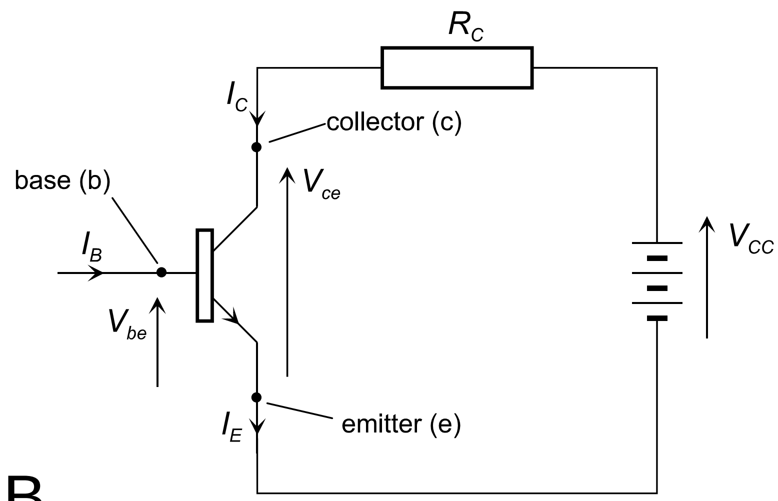
Consideration of the circuit behaviour when small changes in conditions are made at relatively high speed. The transistor's dynamic characteristics – effects of rapid changes.

Transistor Equivalent Circuit (4)

We will return to this division into two parts later, first obtain the small signal equivalent circuit. It should help your understanding if you can follow the development of the equivalent circuit but if you find it difficult jump straight to the actual equivalent circuit.

Kirchoff's current law (if I_B, I_C and I_E are in the directions in the figure on slide 27)

$$I_B + I_C = I_E$$

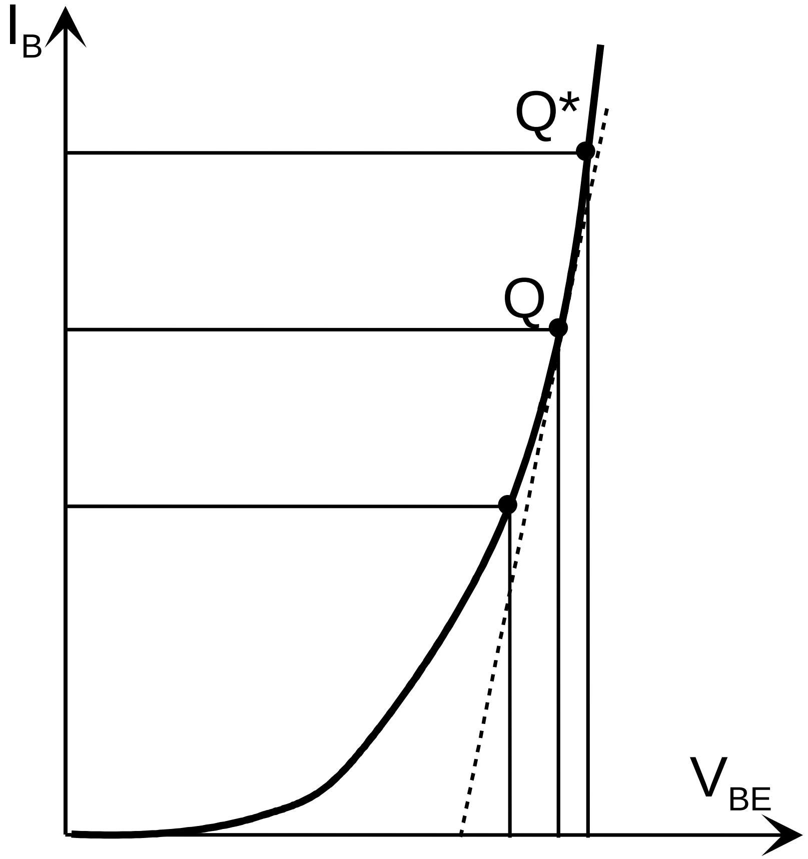


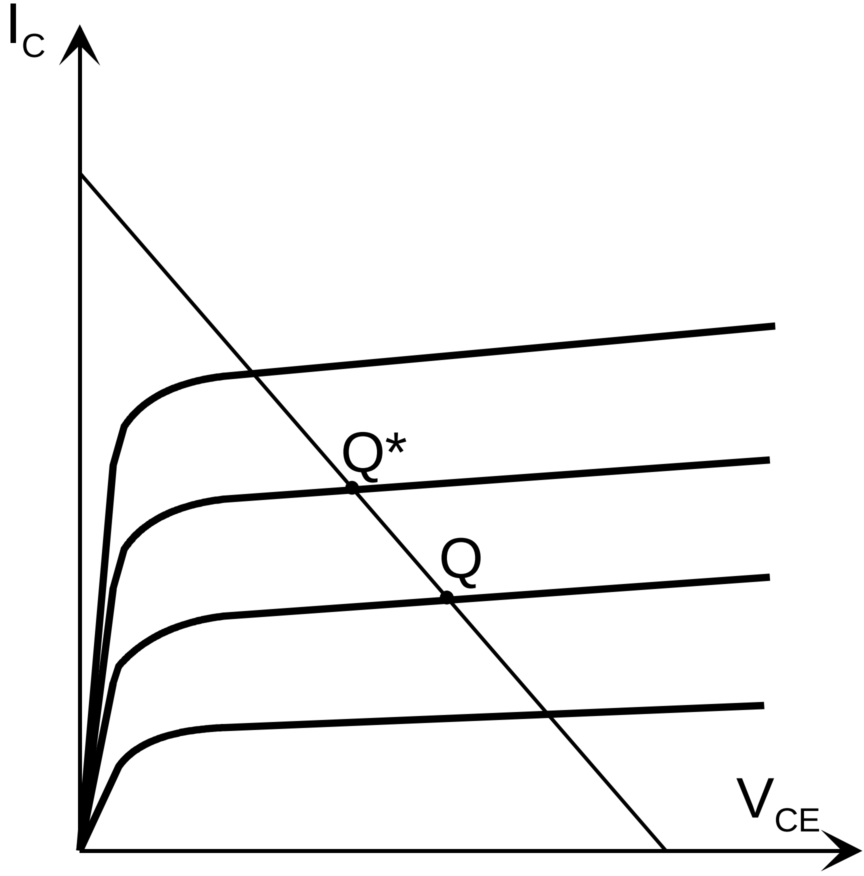
Using the expression for β

$$I_E = (1 + \beta) \times I_B$$

Transistor Equivalent Circuit (5)

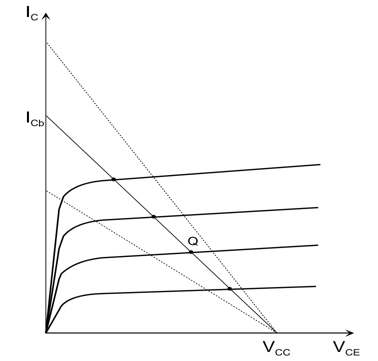
Consider the transistor set at the operating point Q on the input and output characteristics. A small change ΔV_{BE} causes a change in the base current of ΔI_B determined from the input characteristic. *The change in I_B* moves the operating point to new positions, Q*, on both characteristics.





Transistor Equivalent Circuit (6)

On the output characteristics the new point, Q*, is on the load line where it intersects the characteristic for the **<u>new base current</u>**. i.e. the change ΔI_B moves the operating point changing ΔI_C and ΔV_{CE}.



If ΔI_B is small compared to I_B the consequent changes ΔI_C and ΔV_{CE} will be small compared to I_C and V_{BE} respectively and we can approximate the transistor behaviour using linear equations. This **linear behaviour** (model) is **<u>only valid for small changes</u>** in the transistor currents and voltages.

Transistor Equivalent Circuit (7)

Writing $I_C = f_1\{I_B, V_{CE}\}$ and $V_{BE} = f_2\{I_B, V_{CE}\}$ then

$$\Delta I_C = \left.\frac{\partial f_1}{\partial I_B}\right|_{V_{CE}} \times \Delta I_B + \left.\frac{\partial f_1}{\partial V_{CE}}\right|_{I_B} \times \Delta V_{CE} = \frac{\partial I_C}{\partial I_B} \times \Delta I_B + \frac{\partial I_C}{\partial V_{CE}} \times \Delta V_{CE}$$

$$\Delta V_{BE} = \left.\frac{\partial f_2}{\partial I_B}\right|_{V_{CE}} \times \Delta I_C + \left.\frac{\partial f_2}{\partial V_{CE}}\right|_{I_B} \times \Delta V_{CE} = \frac{\partial V_{BE}}{\partial I_B} \times \Delta I_B + \frac{\partial V_{BE}}{\partial V_{CE}} \times \Delta V_{CE}$$

Transistor Equivalent Circuit (8)

This *mathematical linearisation* of transistor behaviour leads to an equivalent circuit that describes the transistor behaviour for small changes. *'Change'* implies variation in time so changes correspond to a.c. signals and this leads to the **small signal a.c. equivalent circuit**.

One convention

• use capital (upper case) letters and subscripts for d.c. conditions

• use small (lower case) letters and subscripts for a.c. conditions

Transistor Equivalent Circuit (9)

Hence I_C is the steady d.c. collector current and i_c is the varying a.c. collector current. Drop the Δ notation as ΔI_C is the change (variation) in I_C so ΔI_C is i_c . In this notation the equations become

$$i_c = \frac{\partial i_c}{\partial i_b} \times i_b + \frac{\partial i_c}{\partial \nu_{ce}} \times \nu_{ce} \quad \text{and} \quad \nu_{be} = \frac{\partial \nu_{be}}{\partial i_b} \times i_b + \frac{\partial \nu_{be}}{\partial \nu_{ce}} \times \nu_{ce}$$

$\frac{\partial i_c}{\partial i_b}$ defines change in collector current due to change in base current. The symbol h_{fe} is used for this and is the **small signal current gain**.

Transistor Equivalent Circuit (10)

Define

$$\frac{\partial i_c}{\partial i_b} = h_{fe}$$ = small signal current gain (a number, no units)

$$\frac{\partial i_c}{\partial \nu_{ce}} = h_{oe}$$ = output admittance (units of Ω^{-1})

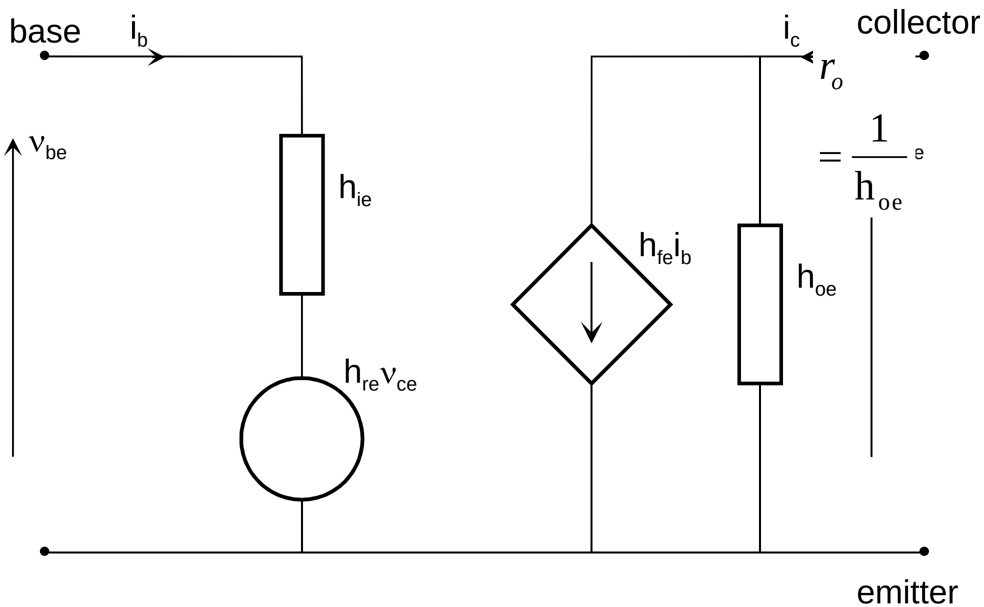
$$\frac{\partial \nu_{be}}{\partial i_b} = h_{ie}$$ = input resistance (units of Ω)

$$\frac{\partial \nu_{be}}{\partial \nu_{ce}} = h_{re}$$ = voltage source amplitude factor (a number, no units)

Transistor Equivalent Circuit (11)

These are the "h" parameters for a linearised transistor model, **"h" = hybrid** because the parameters have mixed (different) units.

After setting the operating point the transistor can be represented by the **<u>small signal equivalent circuit</u>** for examination of the circuit behaviour for small changes in currents and voltages. The h parameters are the circuit elements of this model, the model is a circuit which behaves as the linearised characteristics.



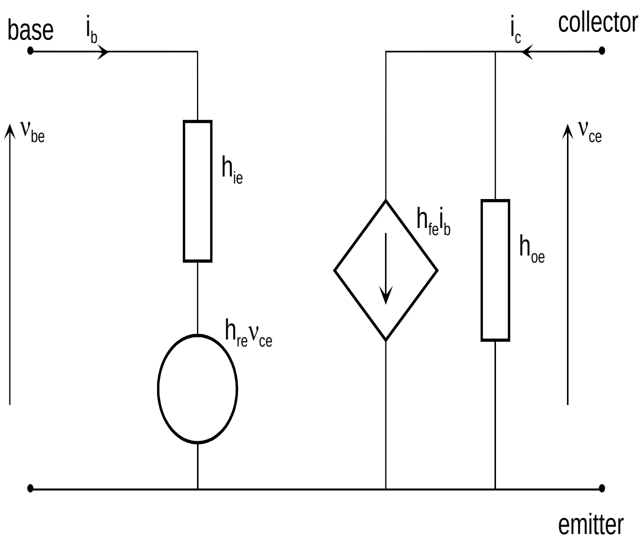
Transistor Equivalent Circuit (12)

The emitter is the reference level for base ***and*** collector voltages. The emitter is common to the input and output parts of the circuit, this is a **<u>common emitter</u>** equivalent circuit.

For most purposes it is reasonable to assume that $\frac{\partial \nu_{be}}{\partial \nu_{ce}} = h_{re} \cong 0$

Reason:
As the base current changes by a small amount (and *νce* changes) the exponential input characteristic is so steep that *νbe* is very small, VBE changes very little.

Remember if any change in VBE is so small it can be ignored then for most purposes VBE can be regarded as a constant value of about 0.7 volts



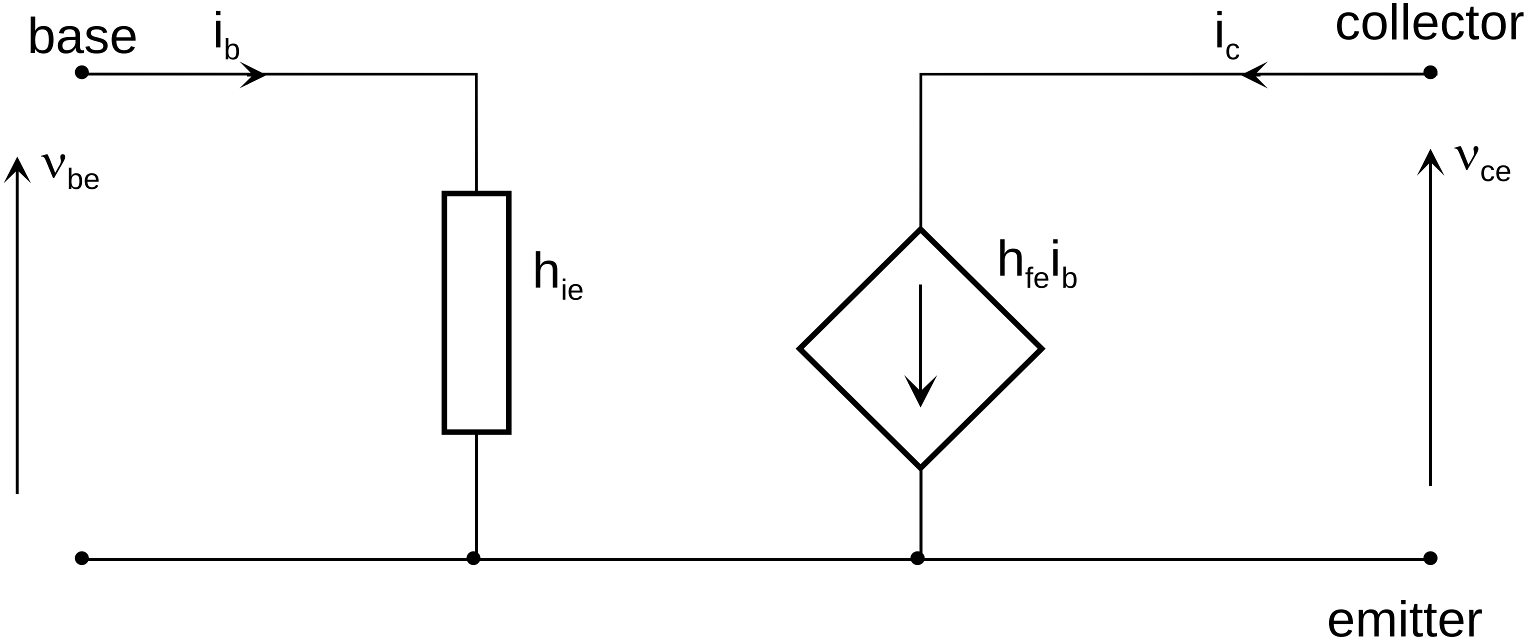
For a transistor with ideal characteristics output admittance is approx 0

$$\frac{\partial i_c}{\partial \nu_{ce}} = h_{oe} \cong 0$$

Transistor Equivalent Circuit (13)

In this semester, it will always be assumed that h_{re} is zero.
In almost all cases h_{oe} will be assumed to be zero.

In the most simple case the small signal a.c. equivalent circuit is



Note the value of the current generator in the collector side is set by the base current

Hybrid Pi Model

The Hybrid Pi model – An Introduction (1)

- We now appreciate the application of the ***h* parameter** transistor model to the study of AC transistor operation.

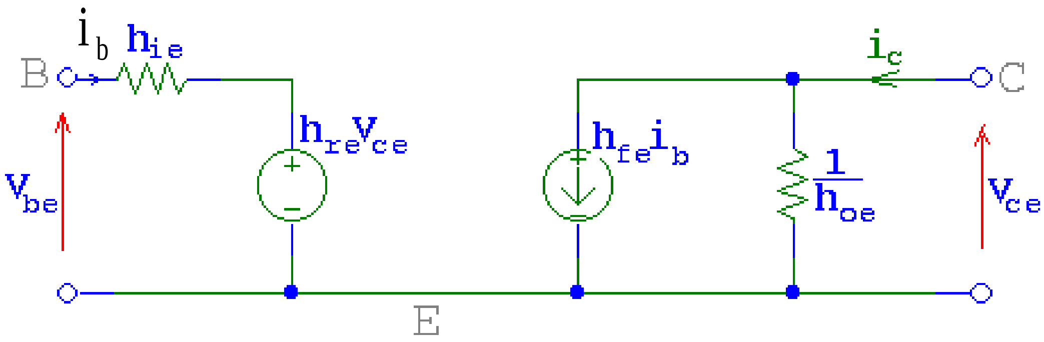
- The **hybrid pi model** of the transistor, which is more versatile than the *h parameter model* (as will be shown later), can be related to the *h* parameters.

- This is particularly important, as manufacturers typically supply values for the *h parameters* in their transistor **specification sheets**, and NOT the hybrid pi model parameters.

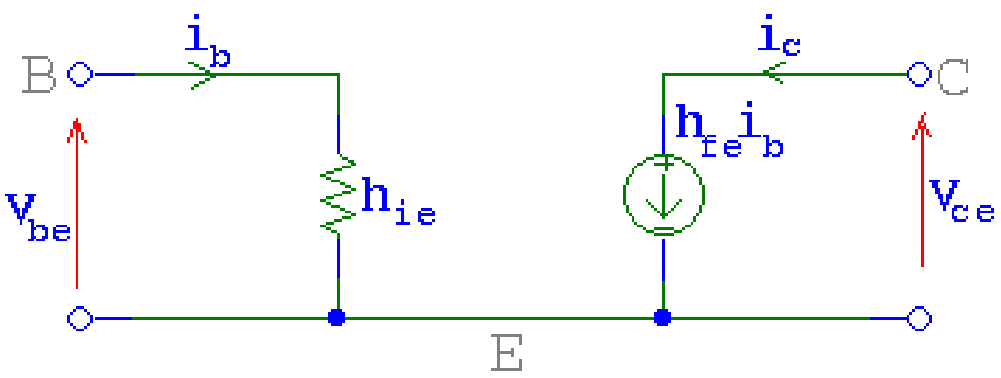
- Therefore one needs to be able to **derive** the **hybrid pi model** parameters **from** the ***h* parameters**.

The Hybrid Pi model – An Introduction (2)

For the CE BJT amplifier, the *h* parameter model is shown below

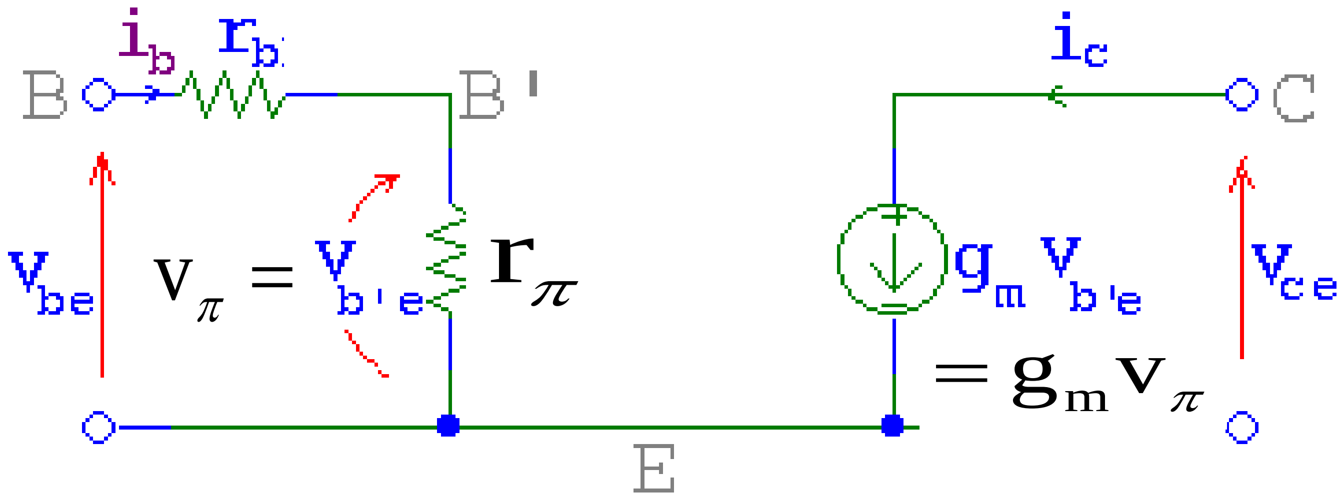


As h_{re} is typically a relatively small quantity and $1/h_{oe}$ is very large, the above model can be simplified, with only a minor effect on any calculations made.



The Hybrid Pi model – An Introduction (3)

The corresponding simplified hybrid pi model is



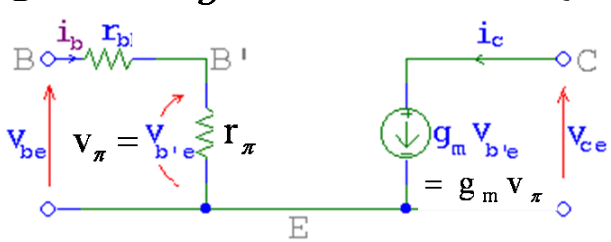
B' is NOT physically accessible BUT represents the internal base node at the junction, separated from the external node B by r_b .

g_m = **transconductance** of the BJT transistor.

The Hybrid Pi model – An Introduction (4)

The parameter r_b is the series resistance of the semiconductor material between the external base terminal B and an idealized internal base region B'. Typically, r_b is **a few tens of ohms** and is usually **much smaller** than r_π ; therefore ,r_b is normally negligible (a short circuit) at low frequencies. However, at high frequencies, r_b may not be negligible, since the input impedance becomes capacitive, as we will see in later on.

In the lecture notes, when we use the hybrid-π equivalent circuit model, we will **neglect** r_b unless they are specifically included.



The Hybrid Pi model – An Introduction (5)

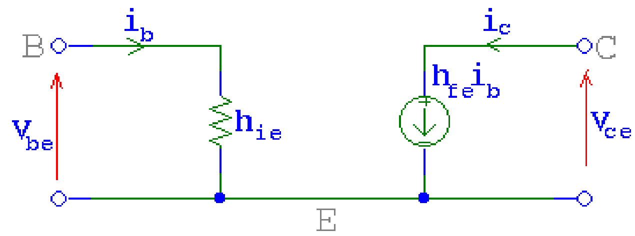
Comparing the 2 models, it can be seen that

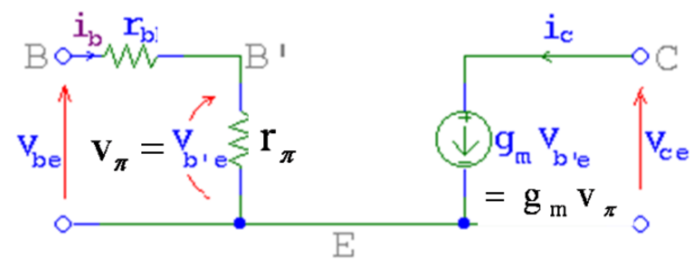
- $h_{ie} = r_b + r_\pi$ .....(1.1)
- $h_{fe}\, i_b = g_m v_{be} = g_m\, r_\pi\, i_b$

where $g_m = |I_c| / V_T$

OR

- $h_{fe} = g_m\, r_\pi$ ....(1.2)





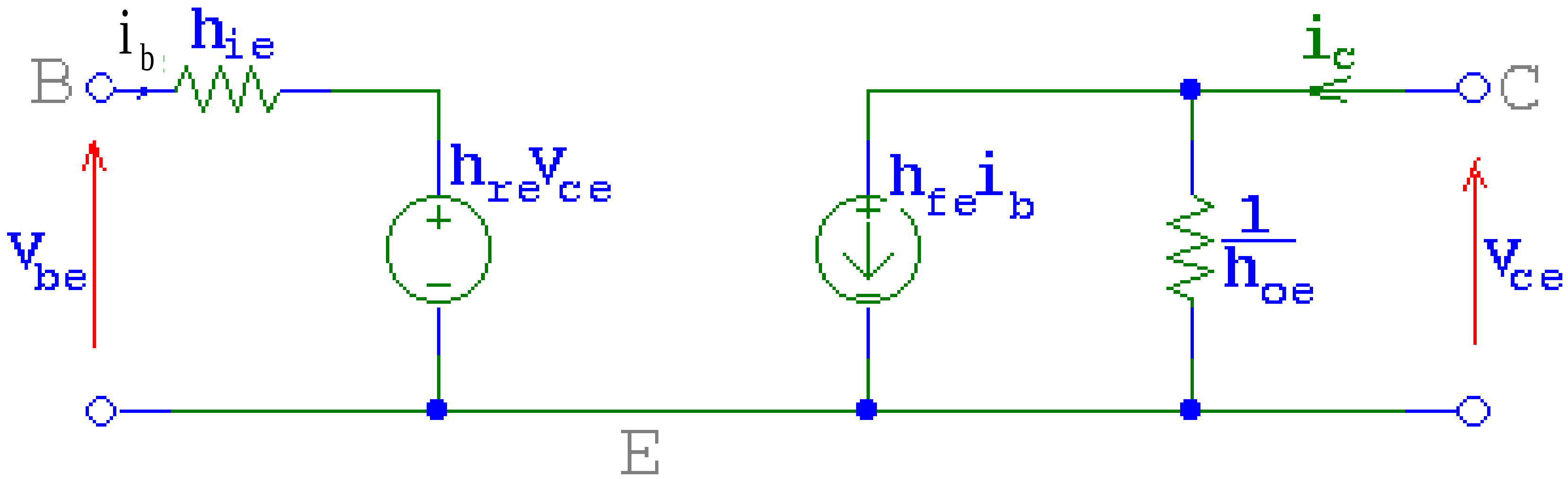
For a BJT

- $g_m = |Ic|(mA) / 0.026V$ at room temperature ..(1.3)

NB: The above simplified models are ONLY applicable at LOW frequencies (up to midband range of frequencies) !!

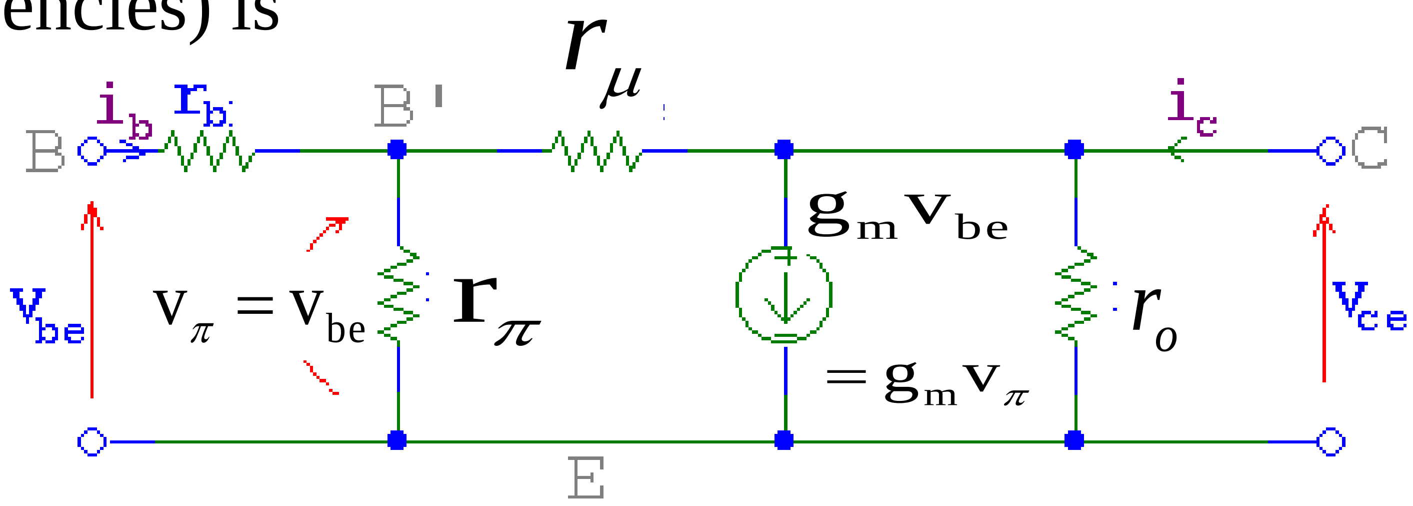
The Hybrid Pi model – An Introduction (6)

What about h_{oe} and h_{re} ?? How do they relate to the hybrid pi model ? To answer this question, we must return to the complete h-parameter model.

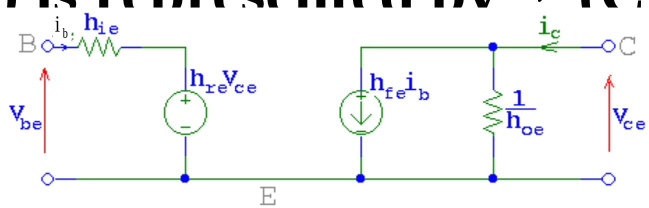


The Hybrid Pi model – An Introduction (7)

The corresponding complete hybrid pi model (at LOW frequencies) is



Note the effect of h_{oe} in the h parameter model is represented partly by r_o (i.e. the finite output resistance of the transistor). The effect of h_{re} in the h parameter model (showing some feedback from the output back to the input) is represented by r_μ (C = output, B' = input)

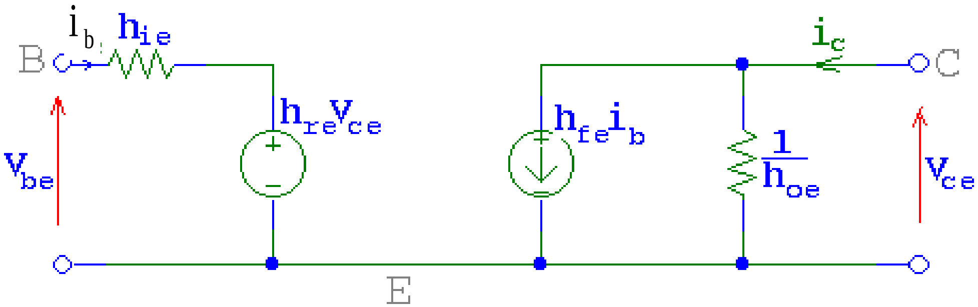


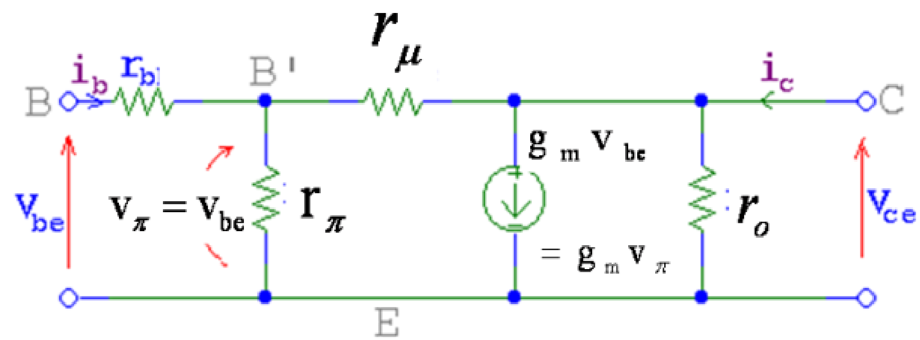
The Hybrid Pi model – An Introduction (8)

The parameter r_μ is the **reverse-biased diffusion resistance** of the base-collector junction. This resistance is typically on the **order of megohms** and can normally be neglected (an open circuit). However, the resistance does provide some feedback between the output and input, meaning that the base current is a slight function of the collector-emitter voltage.

In the lecture notes, when we use the hybrid-π equivalent circuit model, we will **neglect** r_μ unless they are specifically included.

The Hybrid Pi model – An Introduction (9)





$$h_{re} = V_{be}/V_{ce}\big|_{i_b=0} = V_\pi/V_{ce} = r_\pi/(r_\pi + r_\mu) \approx r_\pi/r_\mu \qquad (1.4)$$

$$h_{oe} = i_c/V_{ce}\big|_{i_b=0} \qquad (\text{assuming } r_\pi << r_\mu)$$

$$\text{Under these conditions } i_c = (V_{ce}/r_o) + V_{ce}/(r_\pi + r_\mu) + g_m V_\pi$$

$$\text{But from (1.4) for } i_b = 0,\ V_\pi = h_{re}V_{ce}$$

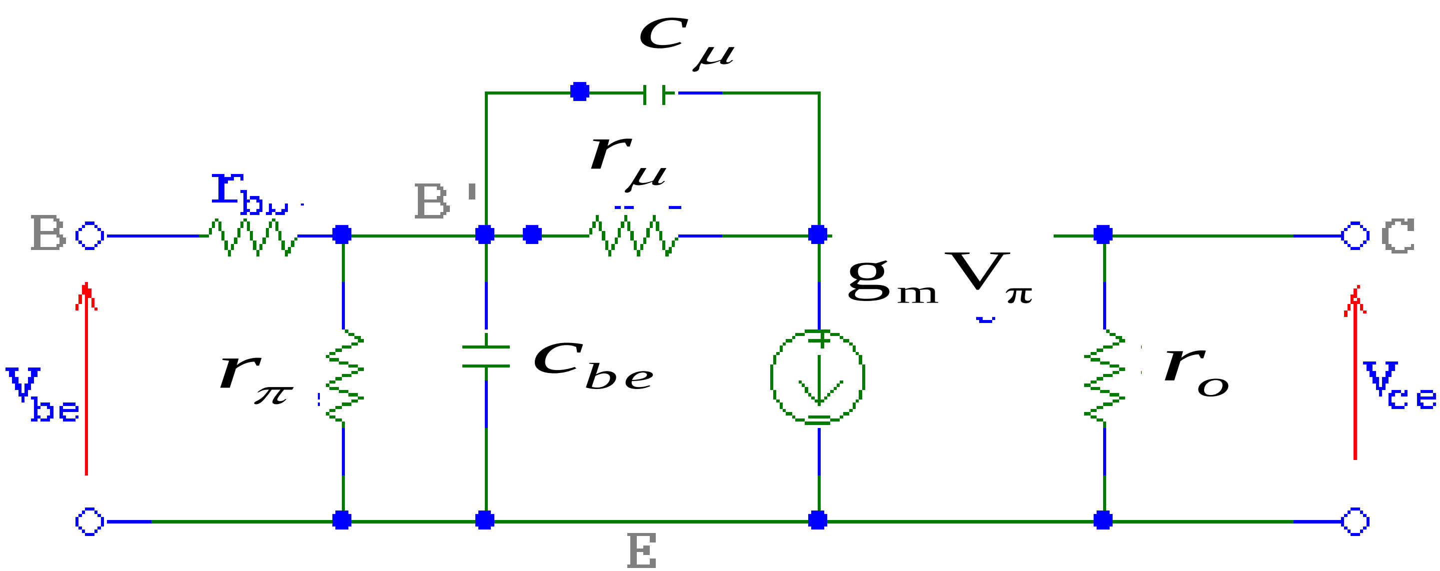
$$\text{Therefore, } h_{oe} = i_c/V_{ce} = 1/r_o + 1/r_\mu + g_m h_{re} \quad (\text{assuming } r_\pi << r_\mu)$$

$$\text{As } g_m = h_{fe}/r_\pi,\quad h_{re} \approx r_\pi/r_\mu \qquad (\text{assuming } r_\pi << r_\mu)$$

$$\text{Therefore, } h_{oe} = 1/r_o + (1/r_\mu)(1 + h_{fe}) \qquad (1.5)$$

The Hybrid Pi model –
High Frequency Analysis of Transistor Operation

The h parameter model is only suitable at **low frequencies**. The high frequency behaviors of a transistor can easily be taken into consideration in the hybrid pi model if the following addition are made:
The **emitter diffusion capacitance** is added between terminals E and B' and the **collector transition capacitance** is placed between C & B'
The final hybrid pi model becomes



Summary

If the CE h paramaters at **LOW frequencies** are known at the collector current I_c, the hybrid pi model circuit parameters can then be calculated from the following equations, in the order given (derived from equations (1.1) - (1.5) above)

$$g_m = |I_c|(mA)/0.026 \quad \text{at room temperature} \tag{1.6}$$

$$r_\pi = h_{fe}/g_m \tag{1.7}$$

$$r_b = h_{ie} - r_\pi \tag{1.8}$$

$$r_\mu \approx r_\pi / h_{re} \tag{1.9}$$

$$1/r_o = h_{oe} - (1/r_\mu)(1 + h_{fe}) \tag{1.10}$$

Example

Typical values of h parameters for a BJT transistor at room temperature and I_C = 1.3mA are

- $h_{ie} = 2K1$
- $h_{re} = 10^{-4}$
- $h_{oe} = 10^{-5}$ A/V
- $h_{fe} = 100$

$$g_m = |I_c|(\text{mA})/0.026$$

$$r_\pi = h_{fe}/g_m \qquad r_\mu \approx r_\pi / h_{re}$$

$$r_b = h_{ie} - r_\pi$$

$$1/r_o = h_{oe} - (1/r_\mu)(1 + h_{fe})$$

The corresponding hybrid pi model circuit parameters are $g_m =$

- $\mathbf{r_\pi} =$
- $\mathbf{r_b} =$
- $\mathbf{r_\mu} =$
- $\mathbf{r_o} =$

Coming Up

- Understand the concept of an analog signal and the principle of a linear amplifier.
  – Investigate how a transistor circuit can amplify a small, time-varying input signal.
- Discuss and compare the three basic transistor amplifier configurations.
  – Analyze the common-emitter amplifier.
  – Analyze the emitter-follower amplifier.
  – Analyze the common-base amplifier.
- Analyze multitransistor or multistage amplifiers.
  circuit.
- Understand the concept of signal power gain in an amplifier

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EEE109: Electronic Circuits

Basic BJT Amplifiers – Part 2

Contents

- Understand the concept of an analog signal and the principle of a linear amplifier.
  - Investigate how a transistor circuit can amplify a small, time-varying input signal.
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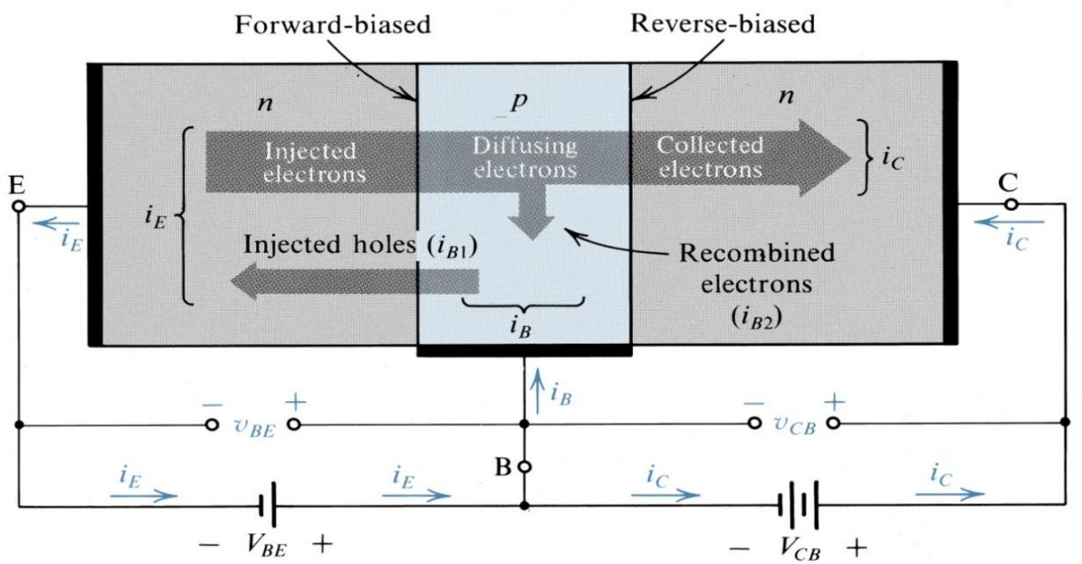
Analyse the Common-Emitter Amplifier

Physical Mechanism: BJT in Active Mode



- Operation
  - Forward bias of EBJ injects electrons from emitter into base (small number of holes injected from base into emitter)
  - Most electrons shoot through the base into the collector across the reverse bias junction (think about band diagram)
  - Some electrons recombine with majority carrier in (P-type) base region

Physical Mechanism: Collector Current

- Electrons that diffuse across the base to the CBJ junction are swept across the CBJ depletion region to the collector b/c of the higher potential applied to the collector.

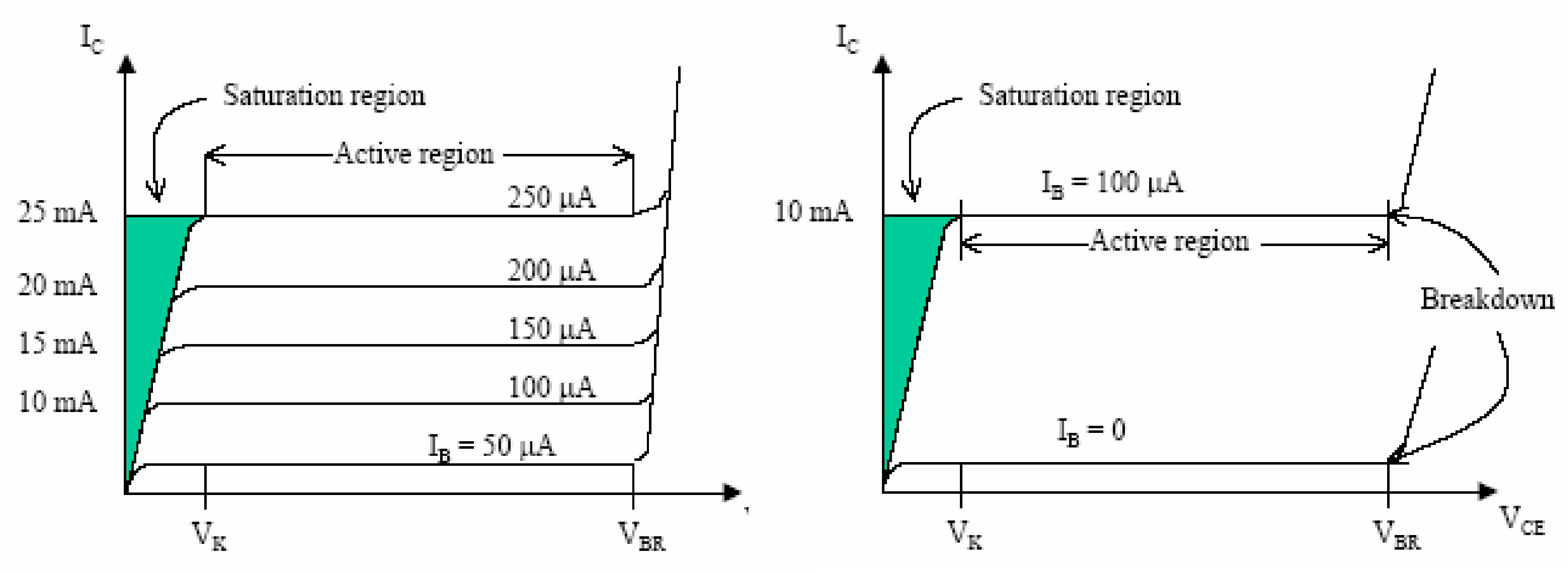
$\boxed{i_C = I_s e^{v_{BE}/V_T}}$ where the saturation current is $I_S = qA_E D_n n_{p0}/W$

and we can rewrite the saturation current as:

$$\boxed{I_S = \frac{qA_E D_n n_i^2}{N_A W}}$$

- Note that i_C is independent of v_{CB} (potential bias across CBJ) ideally
- Saturation current is
  - inversely proportional to W and directly proportional to A_E
    - Want short base and large emitter area for high currents
  - dependent on temperature due to n_i^2 term

Physical Mechanism: Collector Current



Physical Mechanism: Base Current

- Base current i_B composed of two components:
  - holes injected from the base region into the emitter region

$$i_{B1} = \frac{qA_E D_p n_i^2}{N_D L_P} e^{v_{BE}/V_T}$$

  - holes supplied due to recombination in the base with diffusing electrons and depends on minority carrier lifetime t_b in the base

$$i_{B2} = \frac{Q_n}{\tau_b}$$

And the Q in the base is $$Q_n = \frac{qA_E W n_i^2}{N_A} e^{v_{BE}/V_T}$$

So, current is $$i_{B2} = \frac{qA_E W n_i^2}{N_A \tau_b} e^{v_{BE}/V_T}$$

- Total base current is $$\boxed{i_B = \left(\frac{qA_E D_p n_i^2}{N_D L_P} + \frac{qA_E W n_i^2}{N_A \tau_b}\right) e^{v_{BE}/V_T}}$$

Beta

- Can relate i_B and i_C by the following equation

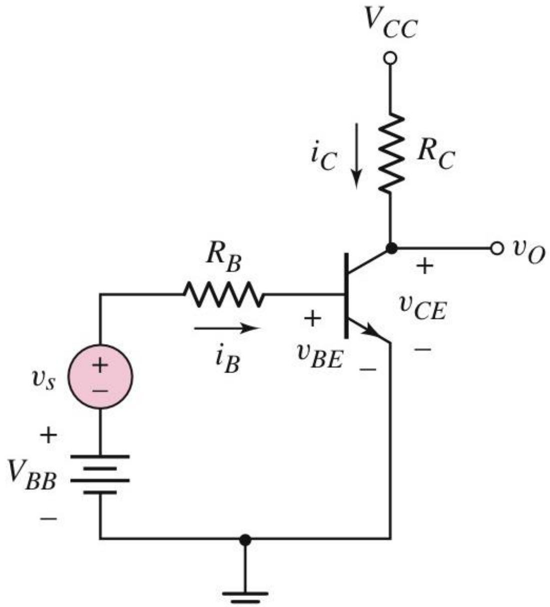
$$\boxed{i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T}}$$

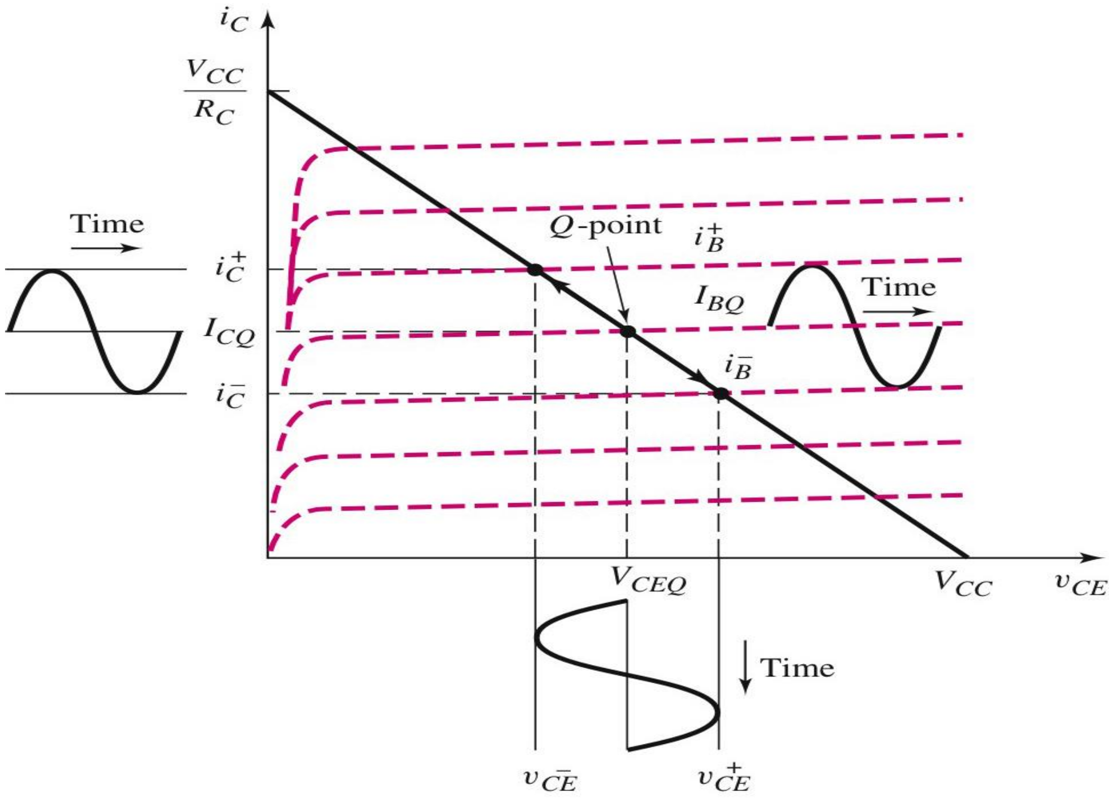
and *beta* is

$$\beta = \frac{1}{\frac{D_p}{D_n}\frac{N_A}{N_D}\frac{W}{L_p} + \frac{1}{2}\frac{W^2}{D_n \tau_b}}$$

  - Beta is constant for a particular transistor
  - On the order of 100-200 in modern devices (but can be higher)
  - Called the common-emitter current gain
- For high current gain, want small W, low N_A, high N_D

Common Emitter with Time-Varying Input





I_B Versus V_{BE} Characteristic

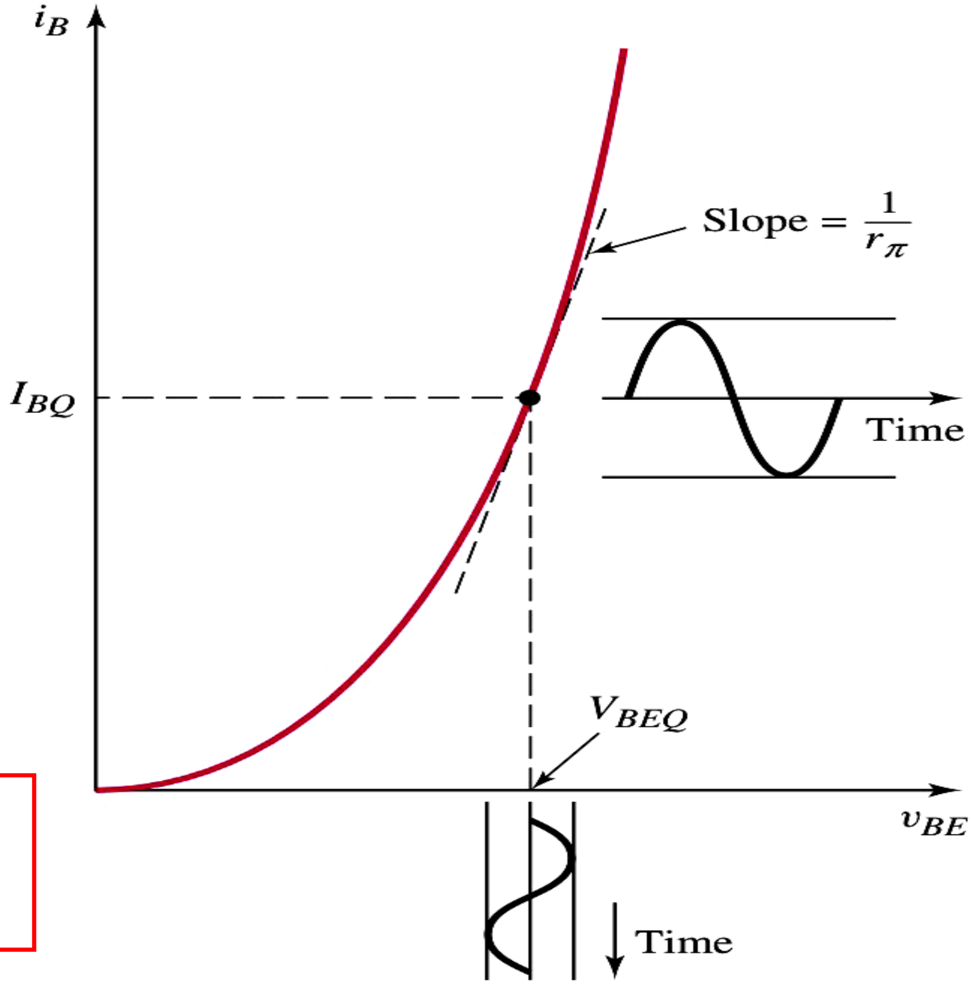
$$\boxed{i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T}}$$

If $v_{be} \ll V_T$,

We can expand the exponential term in a Talor series, keeping only the **linear term**.

$$\boxed{i_B \cong I_{BQ}(1+\frac{v_{be}}{V_T}) = I_B + i_b}$$

The approximation is what is meat by small signal!



Small Signal Implications

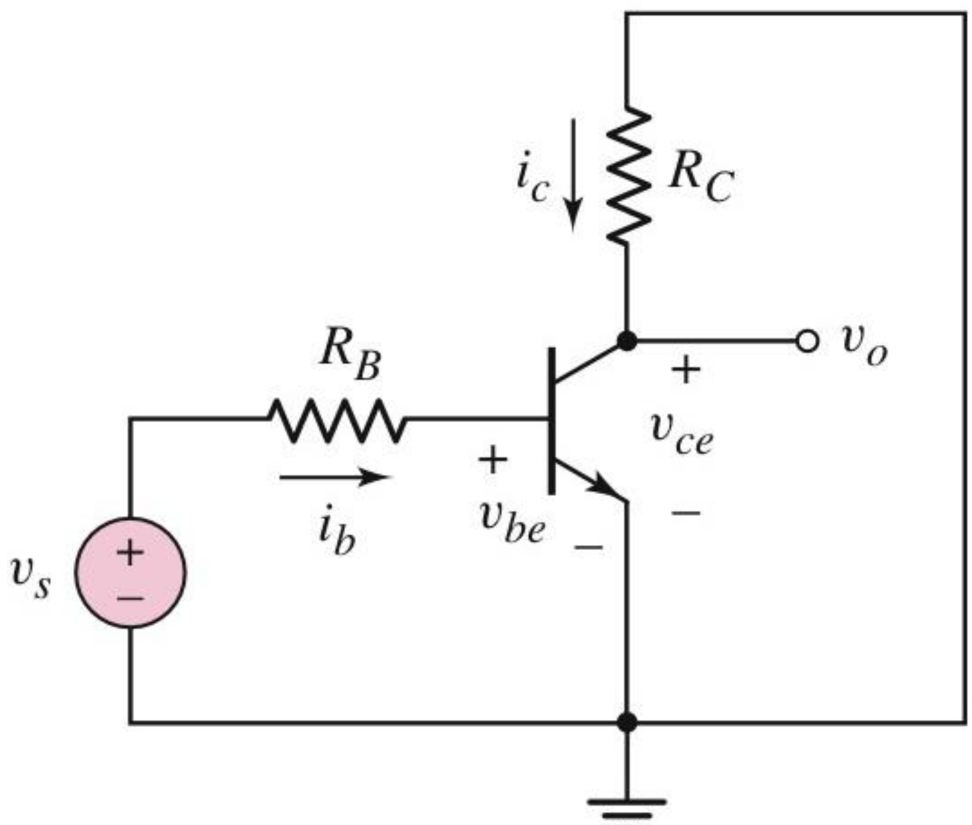
$$i_B \cong I_{BQ}(1+\frac{v_{be}}{V_T}) = I_B + i_b$$

$$i_b = \left(\frac{I_{BQ}}{V_T}\right) v_{be}$$

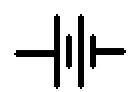
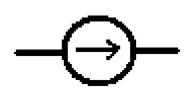
The **two** linear equations have two interpretations:

1. The *total instantaneous values* of current i_B can be written as an ac current **<u>superimposed</u>** on a dc quiescent value.
2. If v_{be} is sufficiently small, i_b and v_{be} have linear relationship.

ac Equivalent Circuit for Common Emitter



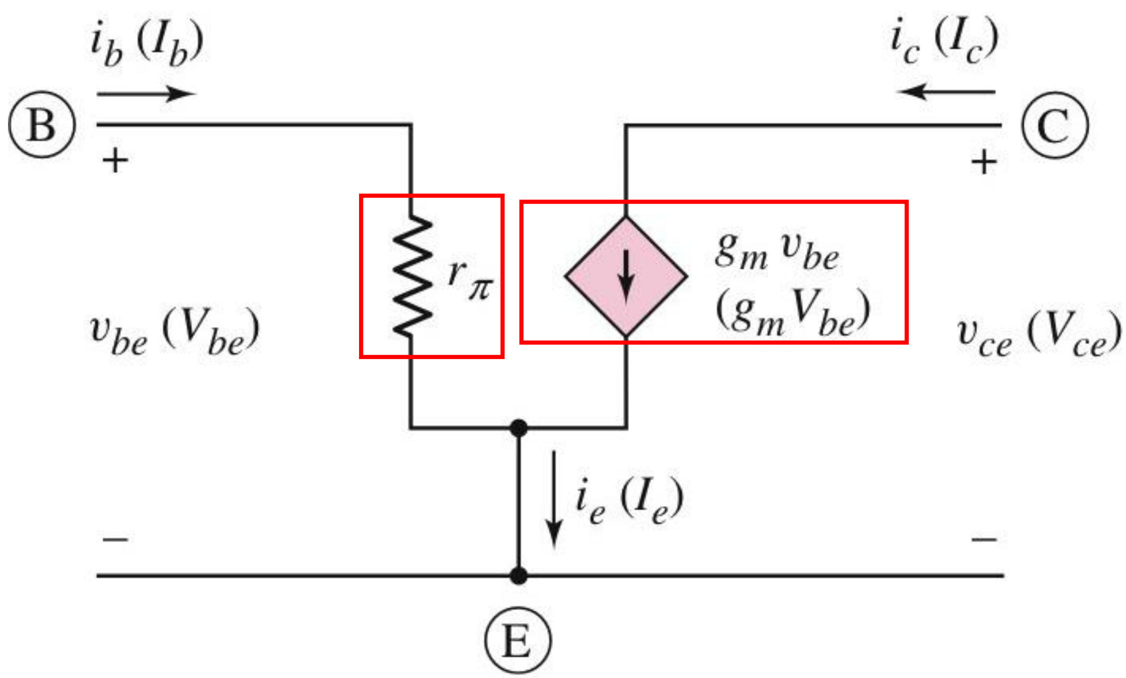
Transformation of Elements

Element	DC Model	AC Model
Resistor	R	R
Capacitor	Open	C
Inductor	Short	L
Diode	$+V_\gamma, r_f -$ 	$r_d = V_T/I_D$
Independent Constant Voltage Source	$+ V_S -$ 	Short
Independent Constant Current Source	I_S 	Open

Small-Signal Hybrid π Model for npn BJT

$$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T}$$

$$i_C = I_s e^{v_{BE}/V_T}$$



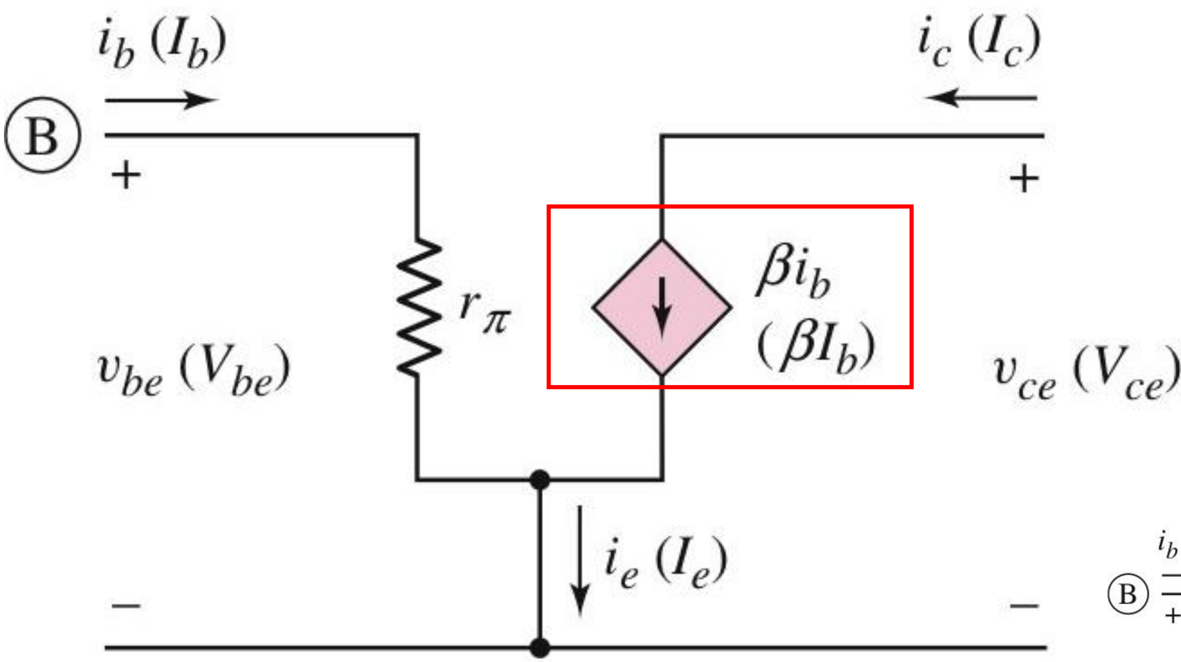
$$g_m = \frac{I_{CQ}}{V_T}$$

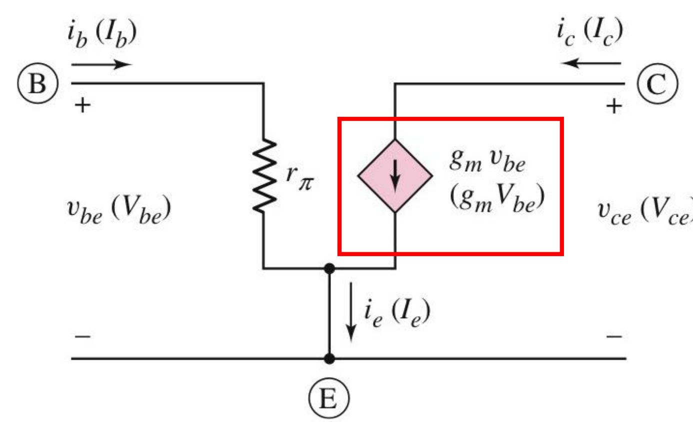
$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$

$$\boxed{g_m r_\pi = \beta}$$

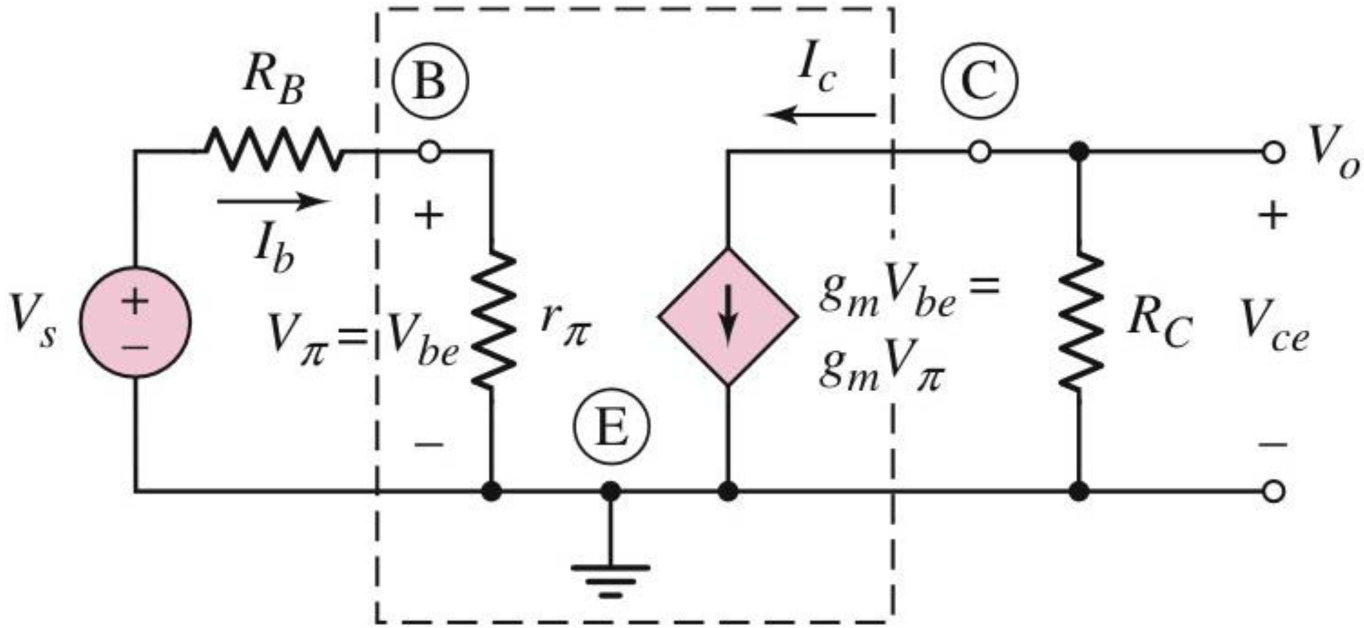
Phasor signals are shown in parentheses.

Small-Signal Equivalent Circuit Using Common-Emitter Current Gain





Small-Signal Equivalent Circuit for npn Common Emitter circuit

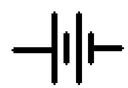
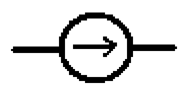


$$A_v = -(g_m R_C)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

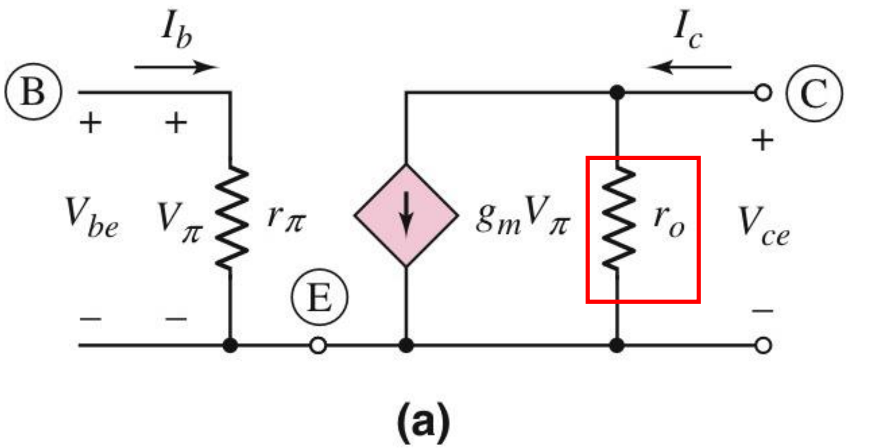
Problem-Solving Technique: BJT AC Analysis

1. Analyze circuit with only dc sources to find Q point.
2. Replace each element in circuit with small-signal model, including the hybrid π model for the transistor.
3. Analyze the small-signal equivalent circuit after setting dc source components to zero.

Transformation of Elements

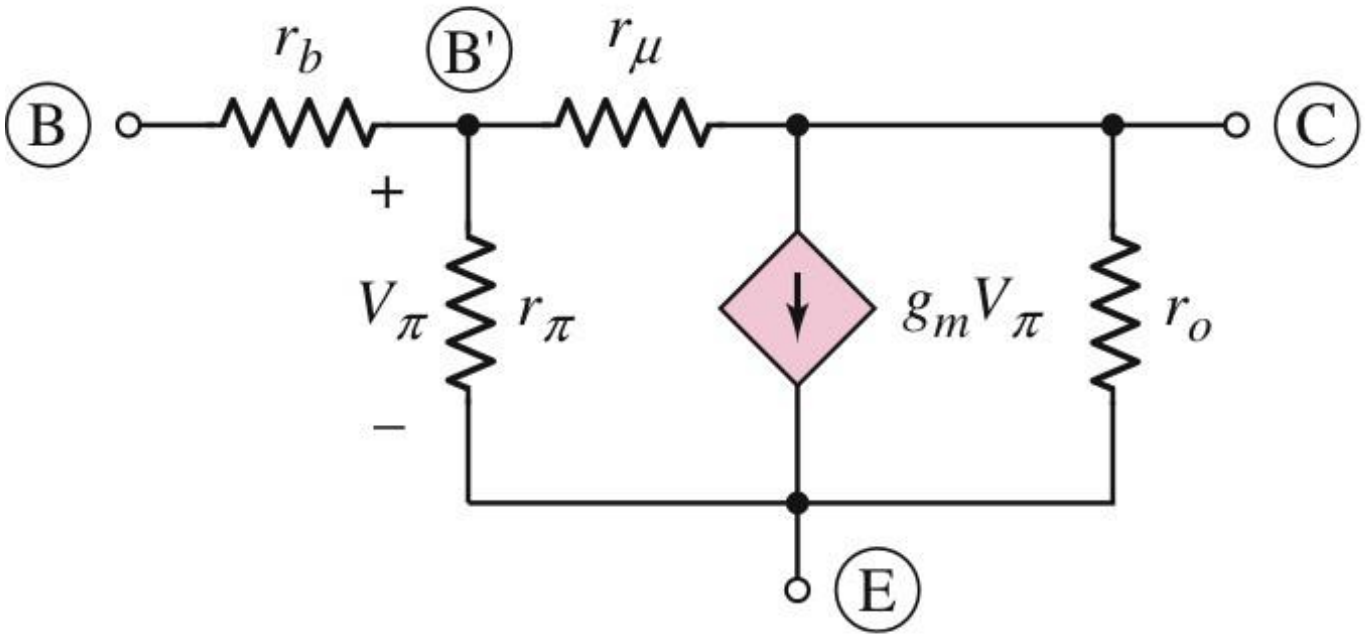
Element	DC Model	AC Model
Resistor	R	R
Capacitor	Open	C
Inductor	Short	L
Diode	$+V_{\gamma}$, r_f – 	$r_d = V_T/I_D$
Independent Constant Voltage Source	+ V_S - 	Short
Independent Constant Current Source	I_S 	Open

Hybrid π Model for npn with Early Effect



$$\boxed{r_o = \frac{V_A}{I_{CQ}}}$$

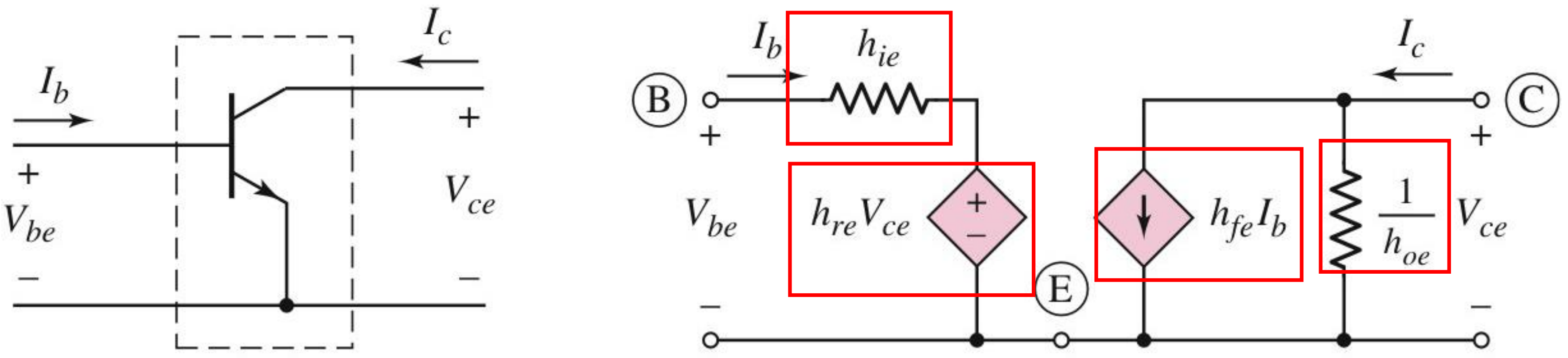
Expanded Hybrid π Model for npn



r_b is the series resistance of the semiconductor material between the external base terminal B and an idealised internal base region B'.

r_μ is the reverse-biased diffusion resistance of the base-collector junction.

h-Parameter Model for npn



(a) **(b)**

$$h_{ie} = r_b + r_\pi \| r_\mu$$

$$h_{re} \cong \frac{r_\pi}{r_\mu}$$

$$h_{fe} = \beta$$

$$h_{oe} = \frac{1+\beta}{r_\mu} + \frac{1}{r_o}$$

C-E Amplifier Properties and Examples

- Common-Emitter (C-E) Amplifier Properties and Example

  - H-parameter Model

- Common-Emitter (C-E) Amplifier Properties and Example

  - Hybrid pi Model

Common-Emitter (C-E) Amplifier Properties and Example (H-parameter Model)

- Common Emitter Transistor Amplifier Properties

✓ Equivalent circuit

✓ Input circuit

✓ Output circuit

✓ Approximation and simplification

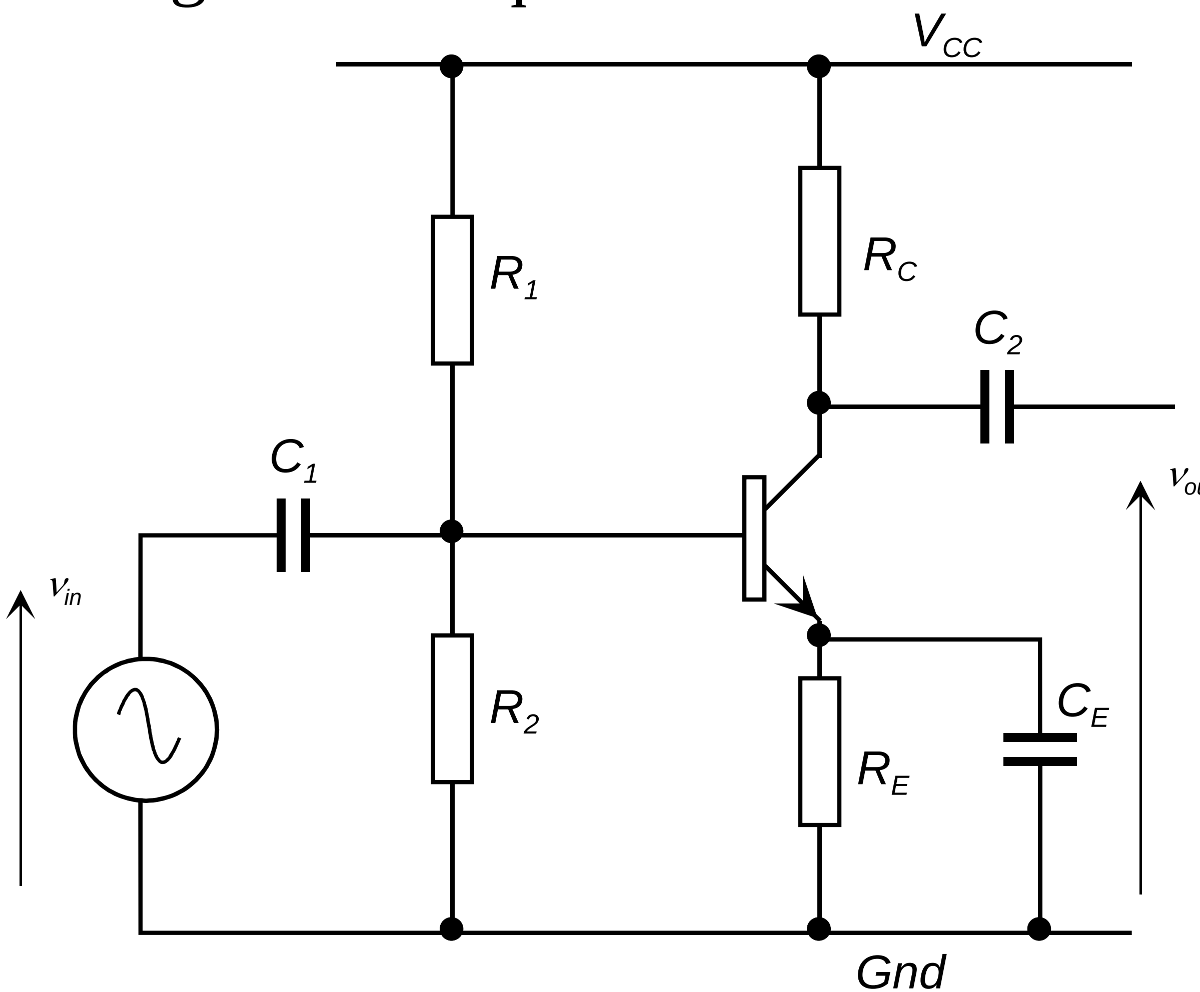
✓ Maximum power gain

✓ Gain in decibel

■ Appendix: Decibels and gain

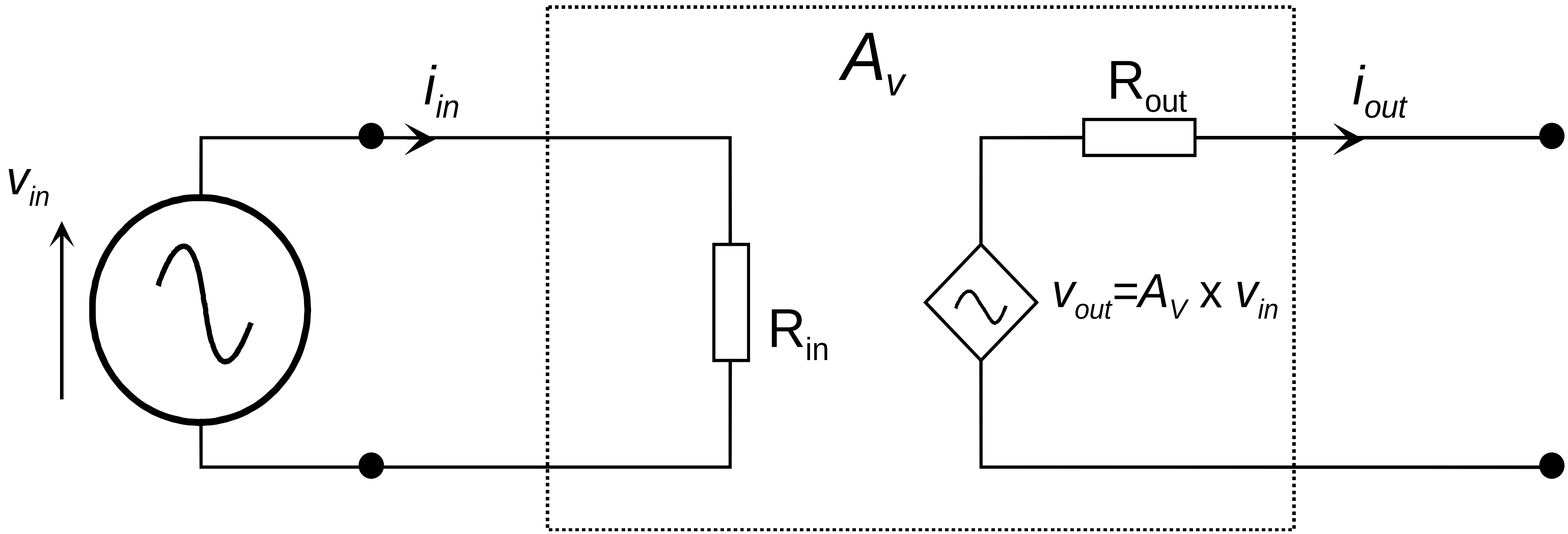
Common Emitter Transistor – Equivalent Circuit (1)

Determination of the a.c. behaviour of the common emitter amplifier using the a.c. equivalent circuit.



Common Emitter Transistor – Equivalent Circuit (2)

Any amplifier can be considered to behave as the generic amplifier although it may not do so in an exact manner.

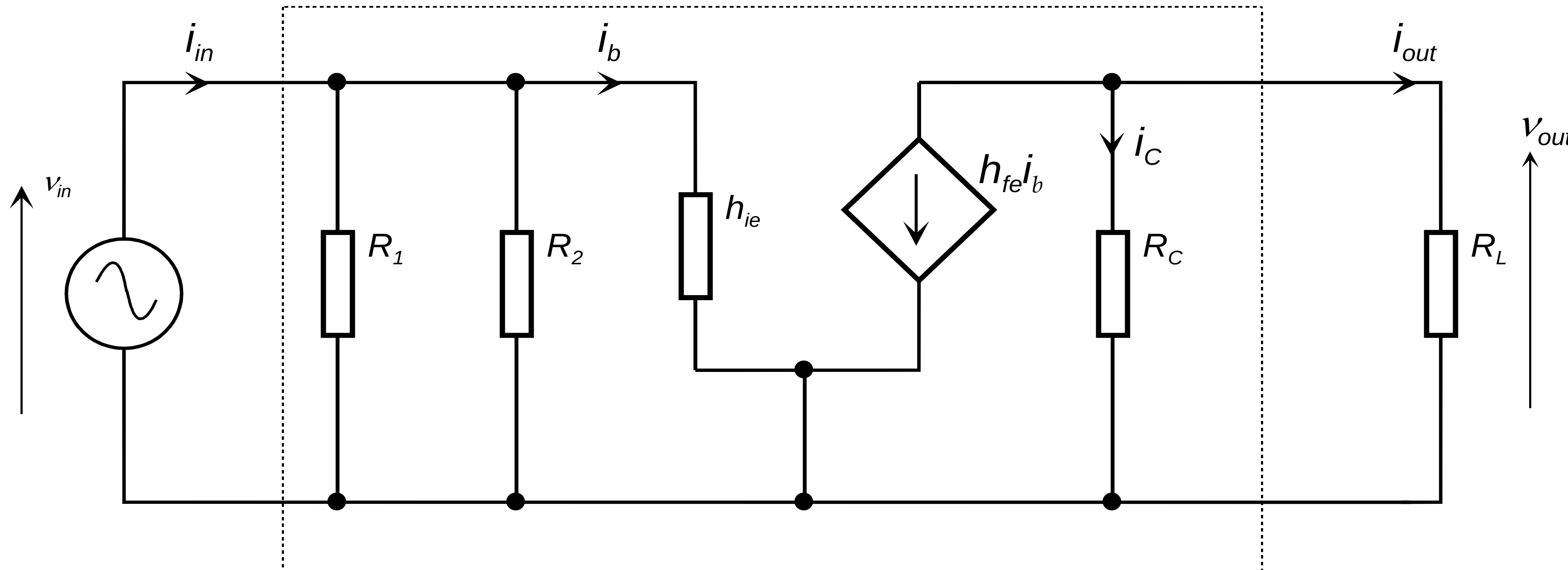


For a common emitter amplifier the primary concerns are

- Voltage gain (open circuit)
- Power gain
- Input Resistance
- Output Resistance

Common Emitter Transistor – Equivalent Circuit (3)

The common-emitter amplifier will usually have a load at the output, a resistor between the output and the common supply. At mid-frequency the capacitors are short circuits so the equivalent circuit becomes:



The voltage gain A_V (open circuit) was determined as

$$A_v = \frac{v_{out}}{v_{in}} = -h_{fe}\frac{R_C}{h_{ie}} \qquad 1.1$$

Common Emitter Transistor – Input Circuit

From equivalent circuit for an input signal ν_{in} , $i_b = \dfrac{\nu_{in}}{h_{ie}}$

R_{in} is the **input resistance** of the generic amplifier and for this circuit is R_1, R_2 and h_{ie} in parallel

$$R_{in} = \frac{R_1 R_2 h_{ie}}{\left[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2\right]} \qquad 1.2$$

The **input power** is

$$P_{in} = \frac{\nu_{in}^2}{R_{in}} \qquad 1.3$$

giving

$$P_{in} = \frac{\nu_{in}^2 \left[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2\right]}{R_1 R_2 h_{ie}} \qquad 1.4$$

As $\nu_{in} = i_b h_{ie}$

$$P_{in} = \frac{i_b^2 h_{ie} \left[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2\right]}{R_1 R_2} \qquad 1.5$$

Common Emitter Transistor – Output Circuit

The **output voltage** is $v_{out} = i_{out} R_L$ 1.6

Use Kirchoff's current law at the collector node (with directions shown) and Ohm's Law to obtain

$$\nu_{out} = \frac{-h_{fe} i_b R_c R_L}{R_C + R_L} \tag{1.7}$$

Output power is $P_{out} = \frac{\nu_{out}{}^2}{R_L} = \frac{h_{fe}^2 i_b{}^2 R_C{}^2 R_L}{(R_C + R_L)^2}$ 1.8

Power gain is $A_p = \frac{P_{out}}{P_{in}} = \frac{h_{fe}^2 R_C^2 R_L}{(R_C + R_L)^2} \frac{R_1 R_2}{h_{ie}[R_1 R_2 + R_1 h_{ie} + R_2 h_{ie}]}$ 1.9

Common Emitter Transistor – Approximation and Simplification (1)

This is complicated but allows consideration of choices necessary to design an amplifier for a specified purpose. Except for choice of transistor the values of h_{ie} and h_{fe} are fixed. R_C and R_L are usually set by the application requirements. So for high power gain it is necessary to have the term

$$\frac{R_1 R_2}{\left[R_1 R_2 + R_1 h_{ie} + R_2 h_{ie}\right]}$$

as large as possible. It is always less than 1 but can be made close to 1 if

$R_1 R_2 >> R_1 h_{ie}$ and $R_1 R_2 >> R_2 h_{ie}$, that is $R_2 >> h_{ie}$ and $R_1 >> h_{ie}$

(Earlier R_1 and R_2 were required to be small enough for the current through them to be much greater than I_B – an upper limit. This new requirement gives a lower limit for *'small'*).

Common Emitter Transistor – Approximation and Simplification (2)

If the additional inequalities are met then $\frac{R_1 R_2}{h_{ie}\left[R_1 R_2 + R_1 h_{ie} + R_2 h_{ie}\right]} \approx \frac{1}{h_{ie}}$

h_{ie} is usually of order 1k for a bipolar transistor and it is usual to choose $R_1 + R_2$ in the 10k to 500k range.
With the approximations

$$A_p = \frac{P_{out}}{P_{in}} = \frac{h_{fe}^2 R_C^2 R_L}{\left(R_C + R_L\right)^2 h_{ie}} \qquad 1.10$$

and the input resistance reduces to $R_{in} \approx h_{ie}$

Common Emitter Transistor – Output Resistance (1)

A general approach to determining the output resistance R_o of the equivalent generic amplifier commonly uses one of three methods:

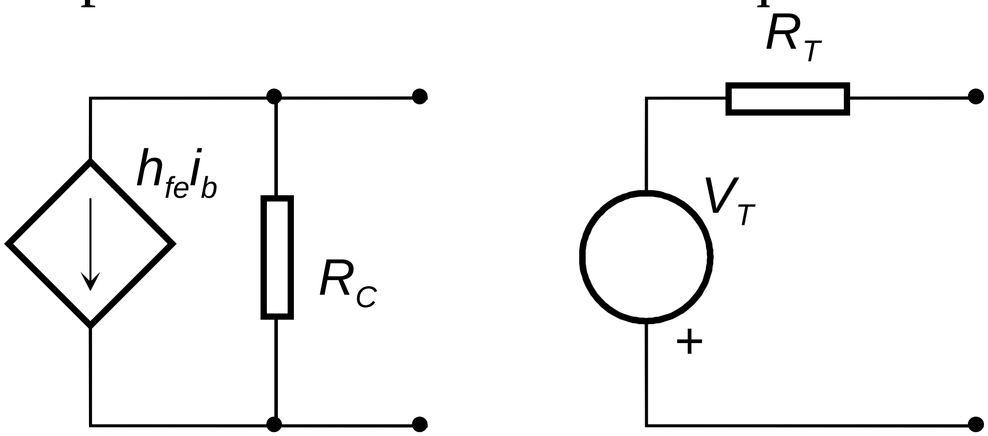
1. $$\frac{\textit{open circuit output voltage}}{\textit{short circuit output current}} = \frac{v_{oc}}{i_{sc}}$$

2. short circuit the input, connect a supply at the output and measure the current flowing into the amplifier at the output.

3. derive the ratio of a change in output voltage to the change in output current for a small change in applied load

Common Emitter Transistor – Output Resistance (2)

However for the common emitter amplifier with capacitor C_E present the equivalent circuit leads to a very simple evaluation. The output circuit of the amplifier and the Thévenin equivalent are



(a) (b)

R_T is the output resistance of the circuit. Remembering Thévenin's and Norton's Theorems then Thévenin and Norton resistances have the same value. Therefore

$$R_{out} = R_T = R_C \qquad 1.11$$

Also

$$V_T = h_{fe} i_b R_C \qquad 1.12$$

Common Emitter Transistor – Summary of Results

Initially it was stated that for a common emitter amplifier the primary concerns are

Voltage gain (open circuit)
Power gain
Input Resistance
Output Resistance

The results are

$$A_v = \frac{\nu_{out}}{\nu_{in}} = -h_{fe}\frac{R_C}{h_{ie}} \qquad 1.13$$

$$A_p = \frac{P_{out}}{P_{in}} \approx \frac{h_{fe}^2 R_C^2 R_L}{(R_C + R_L)^2 h_{ie}} \qquad 1.14$$

$$R_{in} = \frac{R_1 R_2 h_{ie}}{[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2]} \approx h_{ie} \qquad 1.15$$

$$R_{out} = R_C \qquad 1.16$$

Common Emitter Transistor – Maximum Power Gain

Maximum power transfer to a load (examined in circuit theory) is when the Thévenin (or Norton) resistance equals the load resistance. Hence maximum power in the load requires $R_L = R_C$ It also requires $R_1 >> h_{ie}$ and $R_2 >> h_{ie}$ as these give high power gain (the approximations made are better and less signal power is lost in these resistors).

If these inequalities hold $R_{in} \cong h_{ie}$ and if also $R_L = R_C$

$$A_{pmax} = \frac{h_{fe}^2 R_C}{4h_{ie}} = \frac{h_{fe}^2 R_L}{4h_{ie}} \qquad 1.17$$

If R_L is similar to h_{ie} (around 1k), power gain is about $\frac{h_{fe}^2}{4}$ typically around 2000.

C-E Amplifier Properties - H-parameter Model

Voltage gain:

$$A_v = -h_{fe}\frac{R_C}{h_{ie}}$$

Output Resistance:

$$R_{out} = R_C$$

Input Resistance:

$$R_{in} = \frac{R_1 R_2 h_{ie}}{\left[R_1 R_2 + h_{ie} R_1 + h_{ie} R_2\right]}$$

Approximation:

$$R_{in} \approx h_{ie}$$

Power Gain:

$$A_p = \frac{h_{fe}^2 R_C^2 R_L}{\left(R_C + R_L\right)^2} \frac{R_1 R_2}{h_{ie}\left[R_1 R_2 + R_1 h_{ie} + R_2 h_{ie}\right]}$$

Approximation:

$$A_p \approx \frac{h_{fe}^2 R_C^2 R_L}{\left(R_C + R_L\right)^2 h_{ie}}$$

* Assume $R_2 >> h_{ie}$ and $R_1 >> h_{ie}$, $R_1 + R_2$ in the 10k to 500k range. h_{ie} is typically 1k.

Common Emitter Transistor – Gain in Decibel (1)

Decibel Gain **-** Commonly gain is expressed in decibels (**a note on dBs is in Appendix**)

$$G_{dB} = 10\,log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10log_{10}\left(A_p\right) \qquad 1.18$$

If the gain is 2000 then $G_{dB} = 10\,log_{10}\left(2000\right) \cong 33dB$

Often voltage gain is expressed in dBs – strictly this is wrong as dBs are a measure of a ratio of two powers. The power gain may be written

$$A_P = \frac{P_{out}}{P_{in}} = \frac{\nu_{out}^2}{R_{out}}\frac{R_{in}}{\nu_{in}^2} = \left(\frac{\nu_{out}}{\nu_{in}}\right)^2 \frac{R_{in}}{R_{out}}$$

Common Emitter Transistor – Gain in Decibel (2)

If it is assumed that

$$R_{in} = R_{out} \quad \text{(\underline{\textit{not true in most cases}} – see 1.15 and 1.16)}$$

then

$$G_{dB} = 10\,log_{10}\left(\left(\frac{\nu_{out}}{\nu_{in}}\right)^2\right) = 20 log_{10}\left(\frac{\nu_{out}}{\nu_{in}}\right)$$

$$G_{dB} = 20\,log_{10}\left(\left|A_V\right|\right) \qquad 1.19$$

where

$$A_V = \frac{\nu_{out}}{\nu_{in}} = \text{ the voltage gain}$$

Common Emitter Transistor – Gain in Decibel (3)

It is common to express amplifier voltage gain in decibels as in 1.19 even although **<u>this is not strictly correct</u>**. For example for the amplifier being examined in 1.1 gave

$$A_v = \frac{\nu_{out}}{\nu_{in}} = -h_{fe}\frac{R_C}{h_{ie}} \qquad 1.1$$

If $R_C = h_{ie}$ and $h_{fe} = 100$ then $A_V = h_{fe}$ and

$$G_{dB} = 20\,log_{10}(100) = 20 \times 2 = 40dB$$

Reminder: previous value for power gain was 33dB

Calculating voltage gain this way gives a value in dB, which is useful but is not the power gain.

Common-Emitter (C-E) Amplifier Properties and Example - Hybrid pi Model

- Common Emitter Amplifier Circuit Hybrid Pi Model

✓ Basic common emitter amplifier circuit
✓ Small signal equivalent circuit
✓ Example – 1.1
✓ Example – 1.2
✓ Circuit with emitter resistor
✓ Example – 1.3
✓ Example – 1.4

- Basic Common-Emitter Circuit

Figure below shows the basic common-emitter circuit with voltage-divider biasing. The signal from the signal source is coupled into the base of the transistor through the **coupling capacitor Cc**, which provides **dc isolation** between the amplifier and the signal source. The **dc transistor biasing** is establishing by **R1** and **R2**, and is not disturbed when the signal source is capacitively coupled to the amplifier.

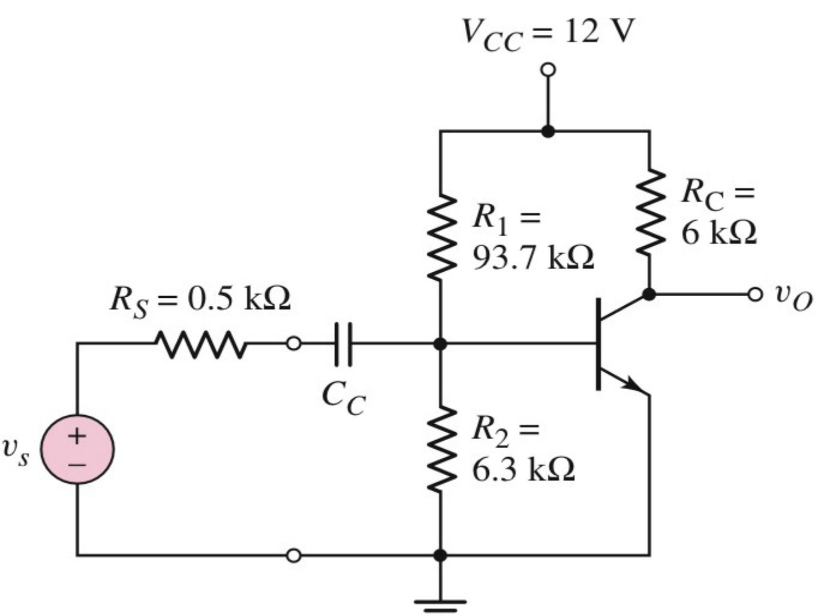


Figure 1.1: A common-emitter circuit with a voltage-divider biasing circuit and a coupling capacitor

- Small-signal Equivalent Circuit

If the signal source is a sinusoidal voltage at frequency f , then the magnitude of the capacitor impedance is

$$|Z_c| = \frac{1}{2\pi f Cc} \quad (1.1)$$

The small-signal equivalent circuit in which the coupling capacitor is assumed to be a **short circuit** is shown in Figure 1.2.

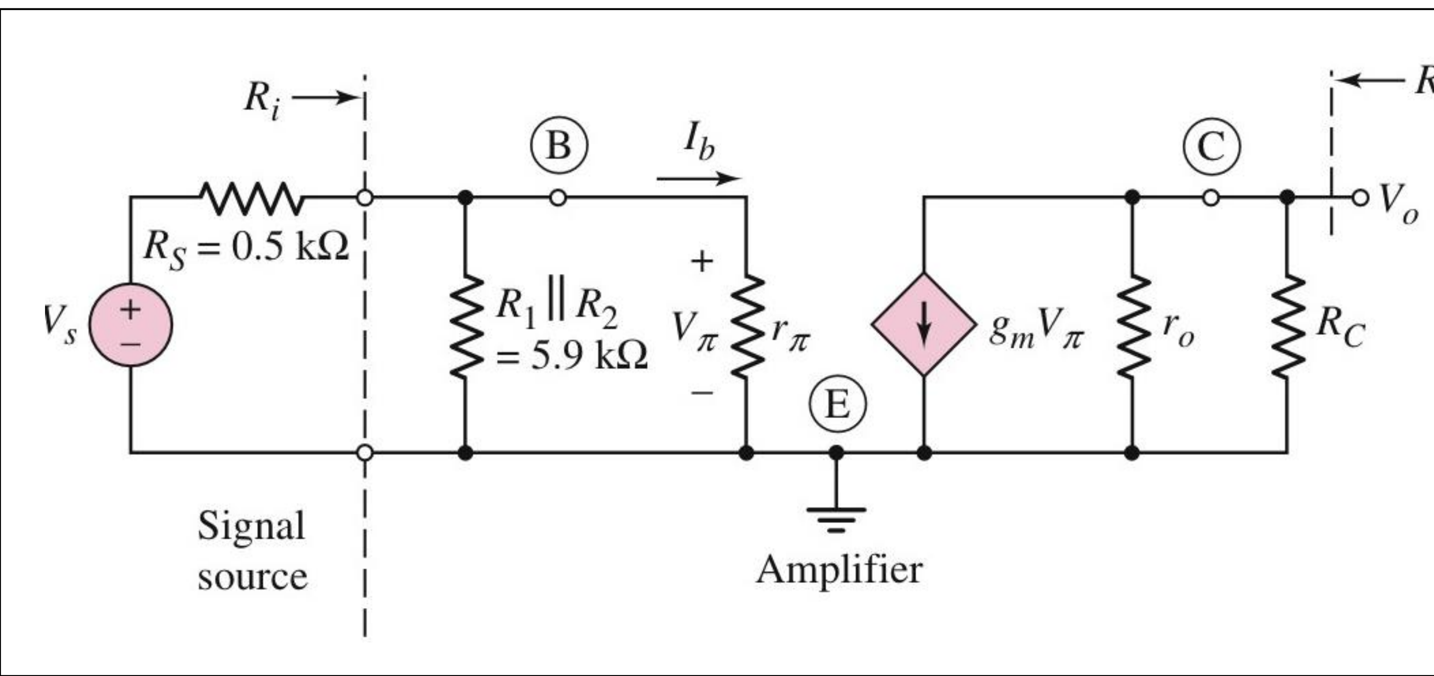
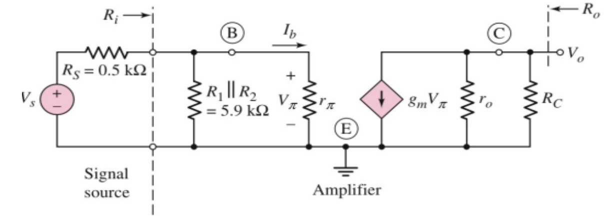


Figure 1.2

Example - 1.1



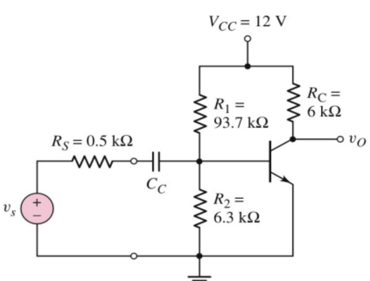
Determine the **small-signal voltage gain**, **input resistance**, and **output resistance** of the circuit shown in figure 1.1.

Assume the transistor parameters are : $\beta = 100, V_{BE}(on) = 0.7V$, and $V_A = 100V$

DC solution is given for this example: $I_{CQ} = 0.95mA$ and $V_{CEQ} = 6.31V$

AC solution:

The small-signal hybrid-π parameters for the equivalent circuit are



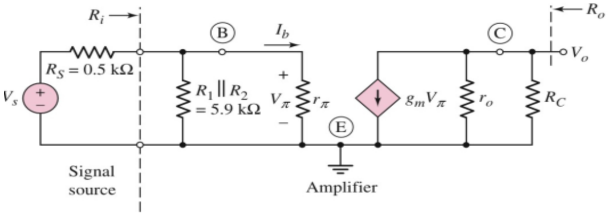
$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{(0.95)} = 2.74K\Omega$$

$$g_m = |I_c|(mA)/0.026$$

$$r_\pi = h_{fe}/g_m \qquad r_\mu \approx r_\pi / h_{re}$$

$$r_b = h_{ie} - r_\pi$$

$$1/r_o = h_{oe} - (1/r_\mu)(1+h_{fe})$$



$$g_m = \frac{I_{CQ}}{V_T} = \frac{(0.95)}{(0.026)} = 36.5\mathrm{mA/V}$$

and

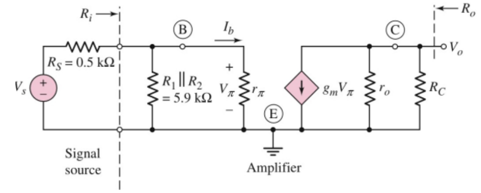
$$r_o = \frac{V_A}{I_{CQ}} = \frac{(100)}{(0.95)} = 105K\Omega$$

Assuming that Cc acts as short circuit, figure 1.2 shows the small-signal equivalent circuit. The small-signal output voltage is

$$V_o = -(g_m V_\pi)(r_o // R_c)$$

The dependent current $g_m V_\pi$ flows through the parallel combination of r_o and R_C , but in a direction that produces a negative output voltage. We can relate the control voltage V_π to the input voltage V_S by a voltage divider, we have

$$V_\pi = \left(\frac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S} \right) . V_S$$



We can then write the **small-signal voltage gain** as

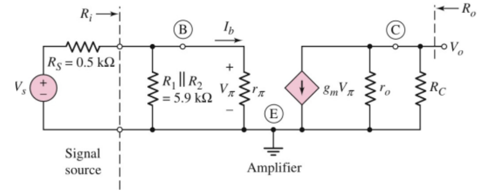
$$A_v = \frac{V_o}{V_S} = -g_m\left(\frac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S}\right)(r_o // R_C)$$

or

$$A_v = -(36.5)\left(\frac{5.9 // 2.74}{5.9 // 2.74 + 0.5}\right)(105 // 6) = -163$$

We can also calculate R_i , which is the **resistance to the amplifier**. From figure 2.2, we see that

$$R_i = R_1 // R_2 // r_\pi = 5.9 // 2.74 = 1.87K\Omega$$



The output resistance R_o is found by setting the independent source V_S equal to zero. In this case, there is no excitation to the input portion of the circuit so $V_\pi = 0$, which implies that $g_m V_\pi = 0$ (am open circuit). The **output resistance** looking back into the output terminals is then

$$R_o = r_o // R_C = 105 // 6 = 5.68K\Omega$$

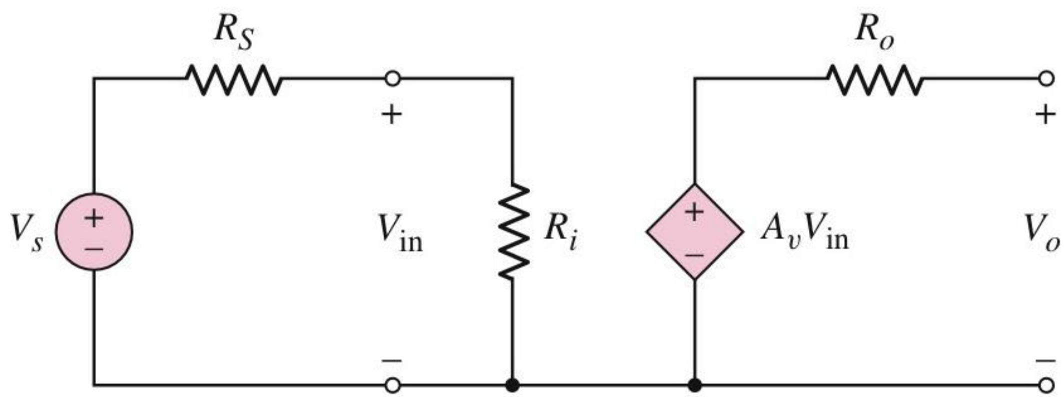
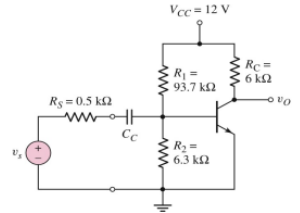


Figure 1.3: two-port equivalent circuit for the amplifier

Example - 1.2



The circuit parameters in figure 2.1 are changed to V_{CC}=5V, R_1= 35.2 Kohms, R_2= 5.83 Kohms, R_C= 10 Kohms, R_S=0. assume the transistor parameter are same as listed in example 1.1. Determine the **quiescent collector current** and **collector-emitter voltage**, and find the **small-signal voltage gain**.

$$R_{TH} = R_1 \,||\, R_2 = 35.2 \,||\, 5.83 = 5\ k\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{5.83}{5.83 + 35.2}\right)(5)$$

or

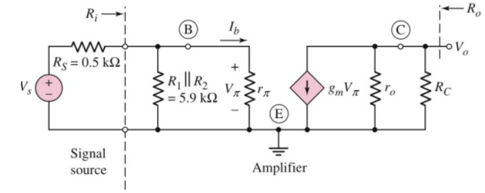
$$V_{TH} = 0.7105\ V$$

Then

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH}} = \frac{0.7105 - 0.7}{5}$$

or

$$I_{BQ} = 2.1\ \mu A$$



and

$$I_{CQ} = \beta I_{BQ} = (100)(2.1\ \mu A) = 0.21\ mA$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C = 5 - (0.21)(10)$$

and

$$V_{CEQ} = 2.9\ V$$

Now

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.21}{0.026} = 8.08\ mA$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.21} = 476\ k\Omega$$

And

$$A_v = -g_m (r_o \| R_C) = -(8.08)(476 \| 10)$$

so

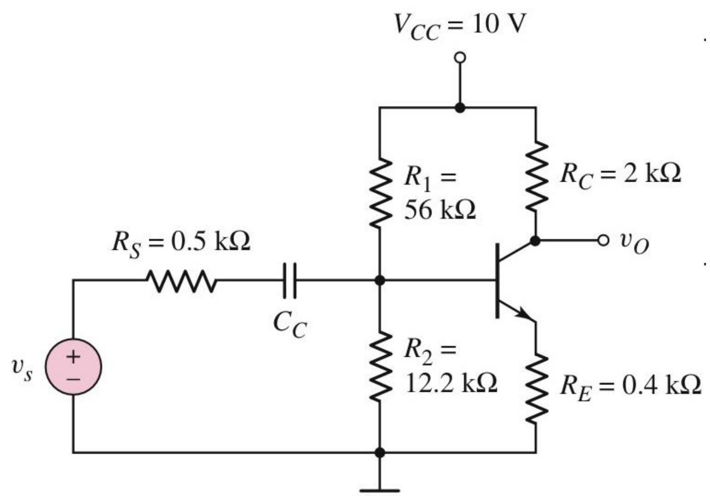
$$A_v = -79.1$$

Basic Common-Emitter Amplifier Circuit Pi Model

- Circuit with Emitter Resistor (1)

For the circuit in figure 1.1, the bias resistors R1 and R2 in conjunction with if $V_{BE} = 0.7V$, then $i_B = 9.5\mu A$ and $i_C = 0.95mA$.

But if changed to $V_{BE} = 0.6V$, then $i_B = 26\mu A$, which is sufficient to drive the transistor into **saturation**. Therefore, the circuit shown in figure 1.1 is not practical. An improved **dc biasing design** includes an emitter resistor.



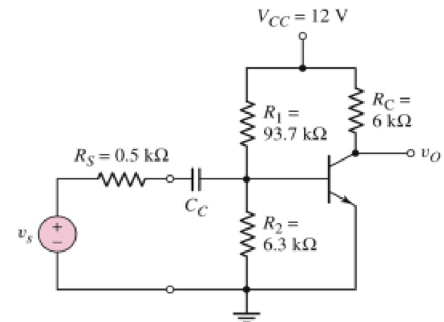


Figure 1.4: An npn common-emitter circuit with an emitter transistor

Basic Common-Emitter Amplifier Circuit Pi Model

- Circuit with Emitter Resistor (2)

Figure below shows the **small-signal hybrid-pi equivalent circuit** (three terminals of the transistor).

Sketch the hybrid-pi equivalent circuit between the three terminals and then sketch in the remaining circuit elements around these terminals.

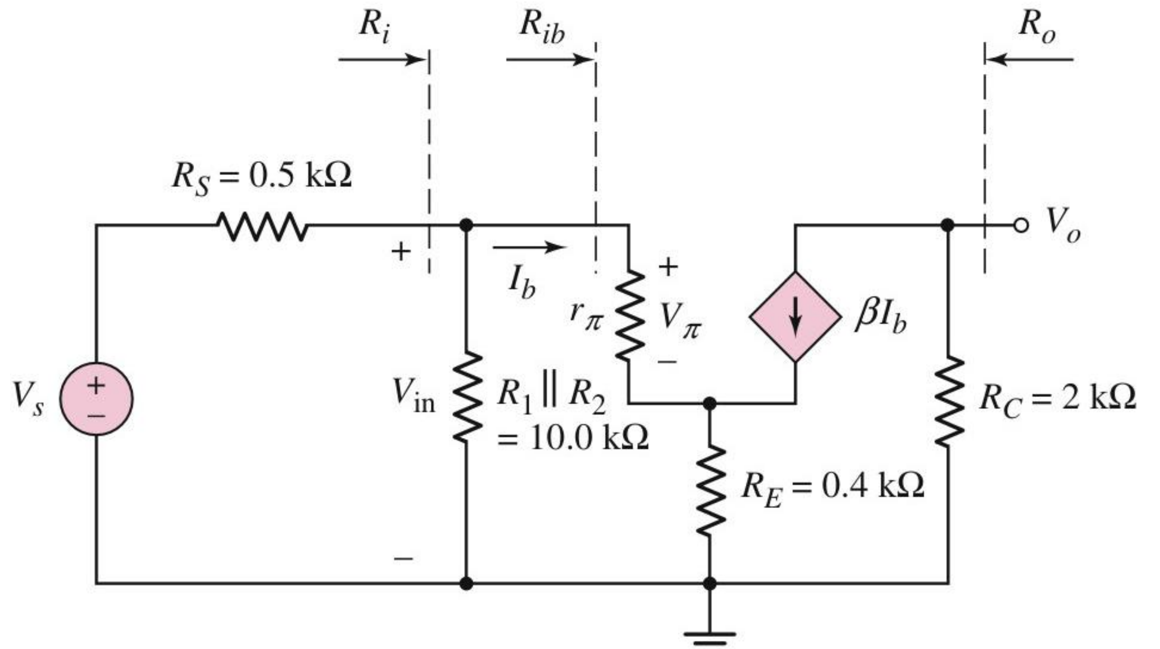


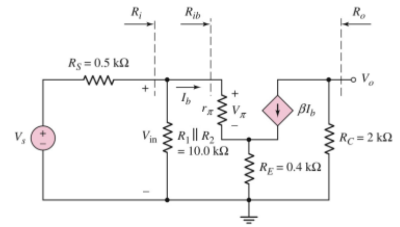
Figure 1.5: The small-signal equivalent circuit of the circuit shown in figure 1.4

Basic Common-Emitter Amplifier Circuit Pi Model

- Circuit with Emitter Resistor (3)

In this case, we are using the equivalent circuit with the **current gain parameter** β , and we are assuming that the **Early voltage is infinite** so the transistor **output resistance** r_o can be neglected (an open circuit).

The a.c. outpt voltage is



$$V_o = -(\beta I_b)R_C \tag{1.2}$$

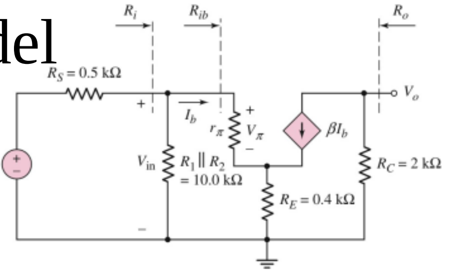
To find the **small-signal voltage gain**, it is worthwhile finding the input resistance first (R_{ib}) . We can write the following loop equation

$$V_{in} = I_b r_\pi + (I_b + \beta I_b)R_E \tag{1.3}$$

The **input resistance** R_{ib} is then defined as, and found to be,

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta)R_E \tag{1.4}$$

Basic Common-Emitter Amplifier Circuit Pi Model



- Circuit with Emitter Resistor (4)

The **input resistance to the amplifier** is now

$$R_i = R_1 \,//\, R_2 \,//\, R_{ib} \qquad (1.5)$$

We can again relate V_{in} to V_s through a **voltage-divider** equation as

$$V_{in} = \left(\frac{R_i}{R_i + R_S} \right) . V_S \qquad (1.6)$$

Combining Equations (1.2), (1.4),and (1.6), we find the **small-signal voltage gain** is

$$A_v = \frac{V_o}{V_S} = -\frac{(\beta I_b) R_C}{V_S} = -\beta R_C \left(\frac{V_{in}}{R_{ib}} \right) . \left(\frac{1}{V_S} \right) \qquad (1.7a)$$

or

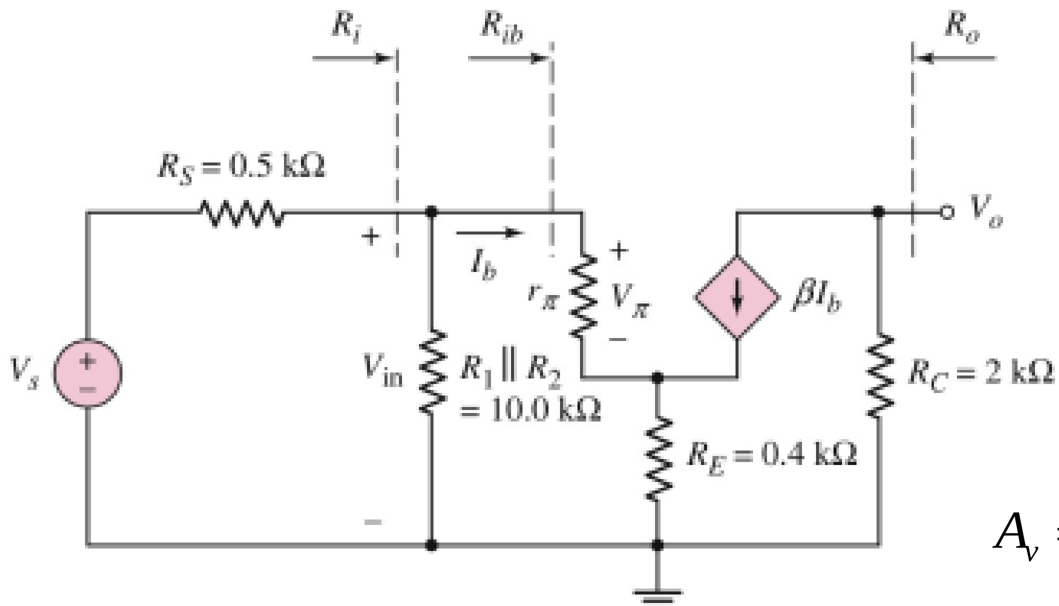
$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E} \left(\frac{R_i}{R_i + R_s} \right) \qquad (1.7b)$$

Basic Common-Emitter Amplifier Circuit Pi Model

- Circuit with Emitter Resistor (5)

From this equation, we see that if Ri >>Rs and if $(1+\beta)R_E >> r_\pi$, then the small-signal voltage gain is approximately

$$A_v \cong \frac{-\beta R_C}{(1+\beta)R_E} \cong \frac{-R_C}{R_E} \qquad (1.8)$$



$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E}\left(\frac{R_i}{R_i + R_s}\right)$$

Example - 1.3

Determine the **small-signal voltage gain** and **input resistance** of a common-emitter circuit with an emitter resistor.

Assume the transistor parameters are : $\beta = 100, \mathrm{V_{BE}}(\mathrm{on}) = 0.7\mathrm{V}$, and $\mathrm{V_A} = \infty$

DC solution:

From a dc analysis of the circuit, we determine that $I_{CQ} = 2.16\mathrm{mA}$ and $V_{CEQ} = 4.81\mathrm{V}$, which shows that the transistor is biased in the forward-active mode

AC solution:

The small-signal hybrid-π parameters for the equivalent circuit are

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{(2.16)} = 1.2K\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.16}{(0.026)} = 83.1\text{mA/V}$$

and

$$r_o = \frac{V_A}{I_{CQ}} = \infty$$

The input resistance to be base can be determined as

$$R_{ib} = r_\pi + (1+\beta)R_E = 1.20 + (101)(0.4) = 41.6K\Omega$$

And the input resistance to be amplifier is now found to be

$$R_i = R_1 \,//\, R_2 \,//\, R_{ib} = 10 \,//\, 41.6 = 8.06K\Omega$$

Using the exact expression for the voltage gain, we find

$$A_v = \frac{-(100)(2)}{1.20 + (101)(0.4)}\left(\frac{8.06}{8.06 + 0.5}\right) = -4.53$$

If we use the approximation given by equation (1.8), we obtain

$$A_v = \frac{-R_C}{R_E} = \frac{-2}{0.4} = -5$$

Example - 1.4

For the circuit in figure 1.6 , let R_E=0.6Kohms, R_C=5.6Kohms, R_1= 250 Kohms, R_2= 75 Kohms, V_{BE}(on)= 0.7 V, and $\beta = 120$.

(a)For V_A = ∞, determine the input resistance looking into the base of the transistor and determine the small-signal voltage gain.

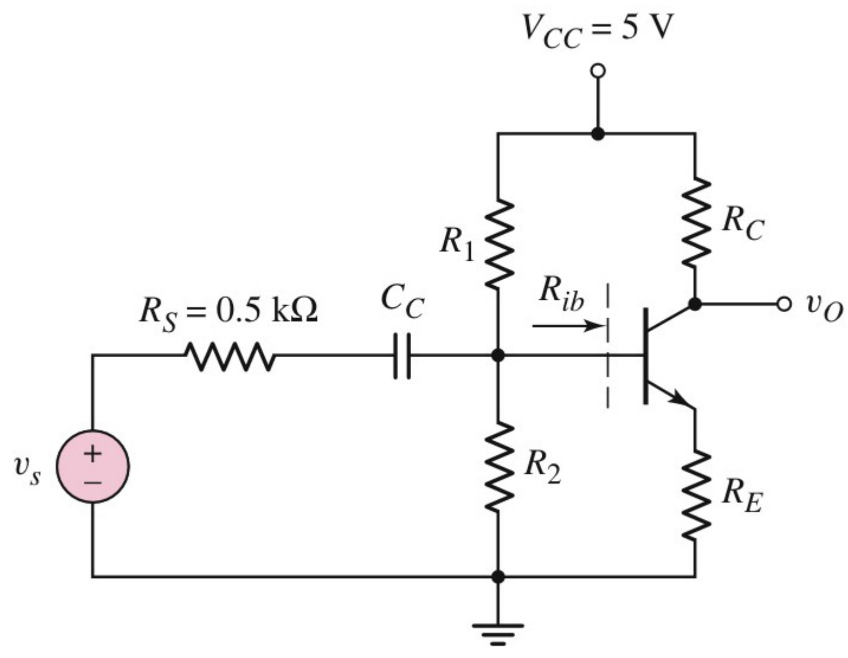


Figure 1.6

$$R_{TH} = R_1 \| R_2 = 250 \| 75 = 57.7\ k\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \left(\frac{75}{75 + 250}\right)(5)$$

or

$$V_{TH} = 1.154\ V$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1 + \beta) R_E}$$

or

$$I_{BQ} = 3.48\ \mu A$$

$$I_{CQ} = \beta I_{BQ} = (120)(3.38\ \mu A) = 0.418\ mA$$

(a)

Now

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.418}{0.026} = 16.08\,mA/V$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.418} = 7.46\,k\Omega$$

We have

$$V_o = -g_m V_\pi R_C$$

We find

$$R_{ib} = r_\pi + (1+\beta) R_E = 7.46 + (121)(0.6)$$

or

$$R_{ib} = 80.1\ k\Omega$$

Also

$$R_1 \| R_2 = 250 \| 75 = 57.7\ k\Omega$$

$$R_1 \| R_2 \| R_{ib} = 57.7 \| 80.1 = 33.54\ k\Omega$$

We find

$$V_s' = \left(\frac{R_1 \,||\, R_2 \,||\, R_{ib}}{R_1 \,||\, R_2 \,||\, R_{ib} + R_S}\right) \cdot V_s = \left(\frac{33.54}{33.54 + 0.5}\right) \cdot V_s$$

or

$$V_s' = (0.985) V_s$$

Now

$$V_s' = V_\pi \left[1 + \left(\frac{1+\beta}{r_\pi}\right) R_E\right] = V_\pi \left[1 + \left(\frac{121}{7.46}\right)(0.6)\right]$$

or

$$V_\pi = (0.0932) V_s' = (0.0932)(0.985) V_s$$

So

$$A_v = \frac{V_o}{V_s} = -(16.08)(0.0932)(0.985)(5.6)$$

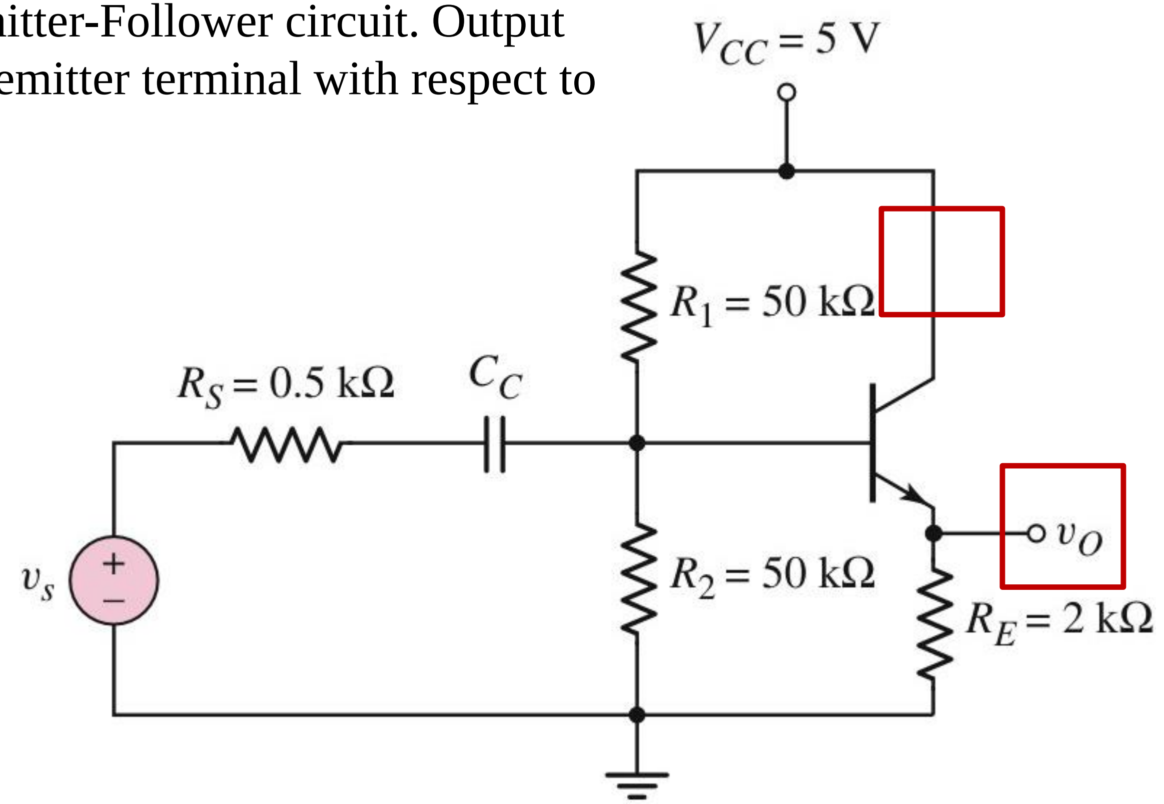
or

$$A_v = -8.27$$

Analyse the Common-Collector (C-C) Amplifier

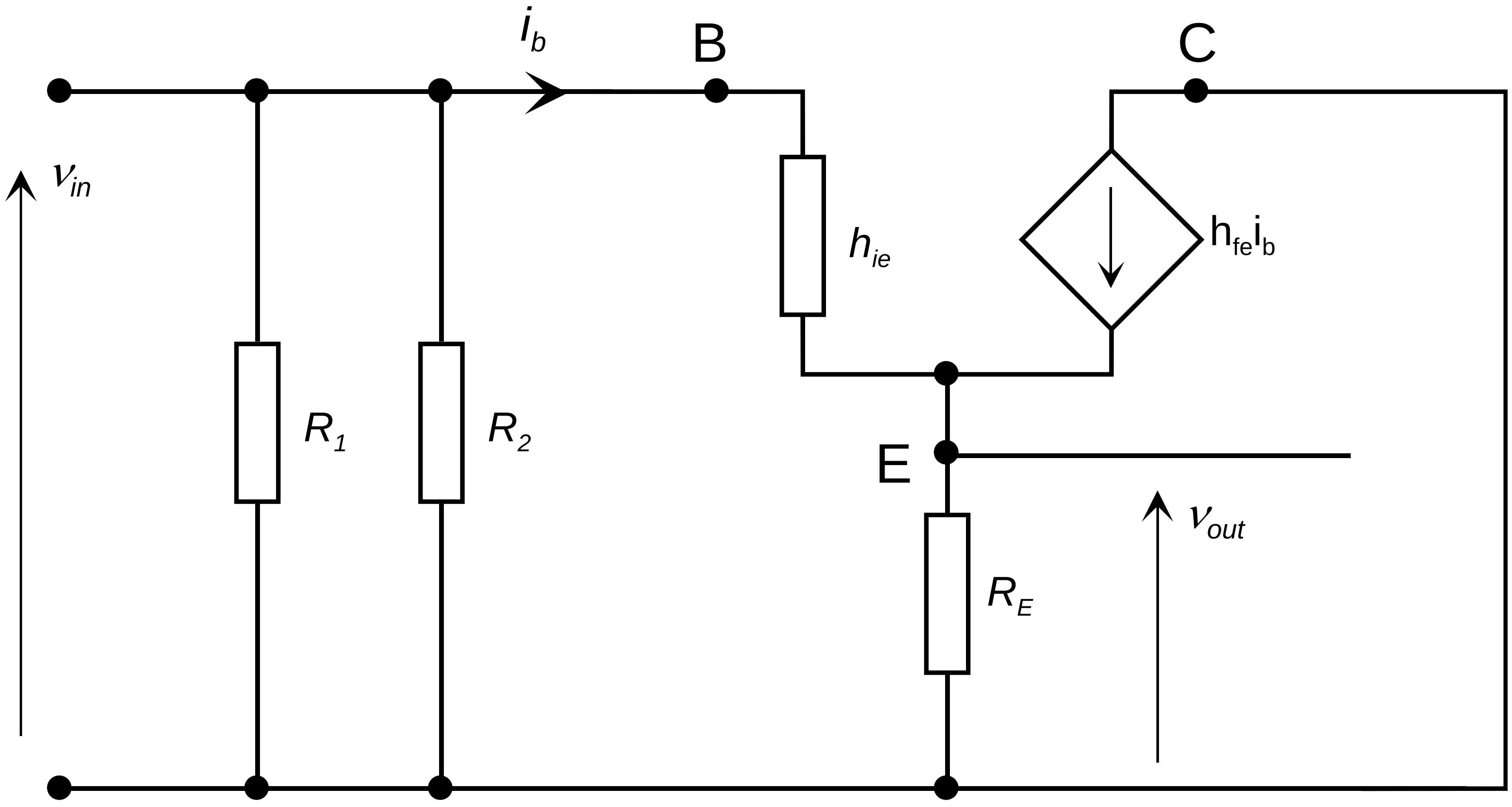
Common-Collector
or Emitter-Follower Amplifier

Figure 2.1 : Emitter-Follower circuit. Output signal is at the emitter terminal with respect to ground



The **output signal** is taken off of the **emitter** with respect to ground and the **collector** is connected directly to **Vcc**. Since Vcc is at signal ground in the ac equivalent circuit, we have the name ***common-collector*** *(emitter follower).*

C-C Amplifier Equivalent Circuit
- H-parameter Model



C-C Amplifier Properties
- H-parameter Model

Voltage Gain:

$$A_V = \frac{v_{out}}{v_{in}} = \frac{1}{\dfrac{h_{ie}}{(1+h_{fe})R_E} + 1}$$

Approximation:

$$\boxed{A_V \approx 1}$$

The output voltage at the ***emitter*** is ***in phase***, and essentially ***equal*** to the input signal voltage

Input Resistance:

$$R_{in} = \frac{R_b\left(h_{ie} + (1+h_{fe})R_E\right)}{R_b + h_{ie} + (1+h_{fe})R_E}$$

Approximation:

$$R_{in} \approx \frac{R_b(1+h_{fe})R_E}{R_b + (1+h_{fe})R_E}$$

*typically 10k or higher

C-C Amplifier Properties

- H-parameter Model (Cont')

Output Resistance:

$$R_{out} = \frac{R_E h_{ie}}{h_{ie} + \left(1 + h_{fe}\right) R_E}$$

Approximation:

$$R_{out} \approx \frac{h_{ie}}{\left(1 + h_{fe}\right)}$$

* h_{ie} is typically 1k and h_{fe} typically 100 or more. R_E is usually at least 10k.

* With h_{ie} ~ 1k and h_{fe} ~ 100 then R_{out} ~ 10 Ω.

Current Gain:

$$A_i = \frac{R_{in}}{R_L} = \frac{R_b\left(h_{ie} + (1 + h_{fe})R_E\right)}{R_L\left(h_{ie} + R_b + \left(1 + h_{fe}\right)R_E\right)}$$

Approximation:

$$A_i \approx \frac{R_b\left(1 + h_{fe}\right)R_E}{R_L\left(R_b + \left(1 + h_{fe}\right)R_E\right)}$$

$$\approx \left(1 + h_{fe}\right)\frac{R_E}{R_L}$$

* Assuming $h_{ie} << \left(1 + h_{fe}\right)R_E$ and $R_b > \left(1 + h_{fe}\right)R_E$

C-C Amplifier Hybrid-pi Parameter Properties and Examples

- Common-collector Amplifier

✓ Definition

✓ Small signal voltage gain

✓ Example 2.1

✓ Example 2.2

✓ Input resistance

✓ Output resistance

✓ Small signal current gain

✓ Example 2.3

Common-collector Amplifier – Small Signal Voltage Gain (1)

The hybrid-pi model of the bipolar transistor can also be used in the small-signal analysis of this circuit. Figure below shows the small-signal equivalent circuit of the circuit shown in figure 2.1. The collector terminal is at signal ground and the transistor output resistance r_o is in parallel with the dependent current source.

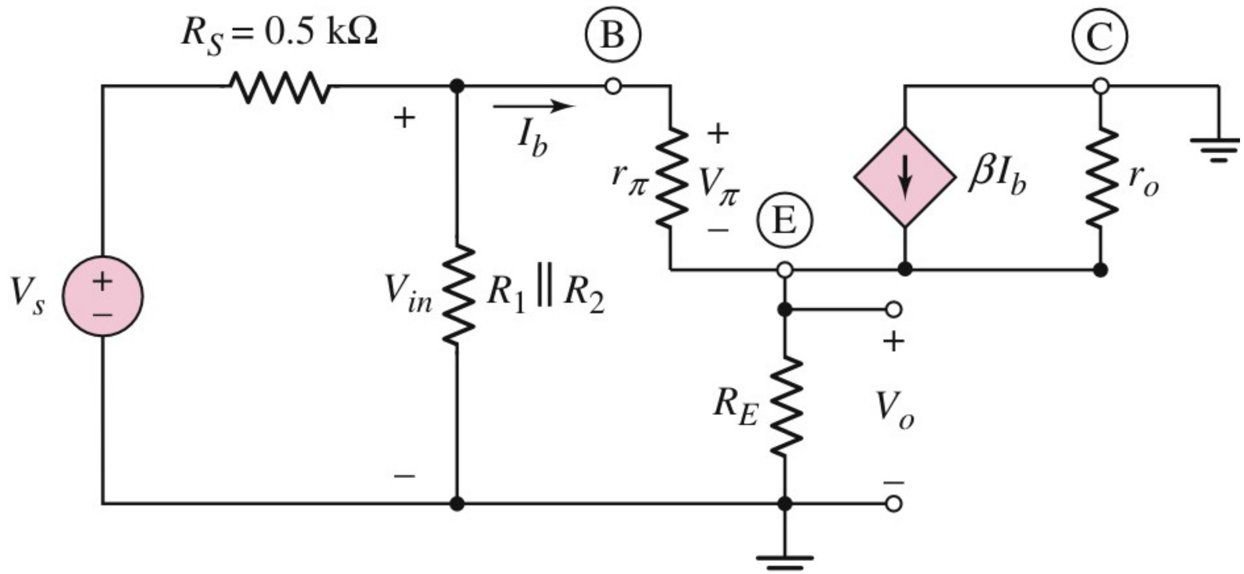


Figure 2.2: Small-signal equivalent circuit of the emitter-follower

Common-collector Amplifier – Small Signal Voltage Gain (2)

Figure below shows the equivalent circuit rearranged so that all signal grounds are at the same point

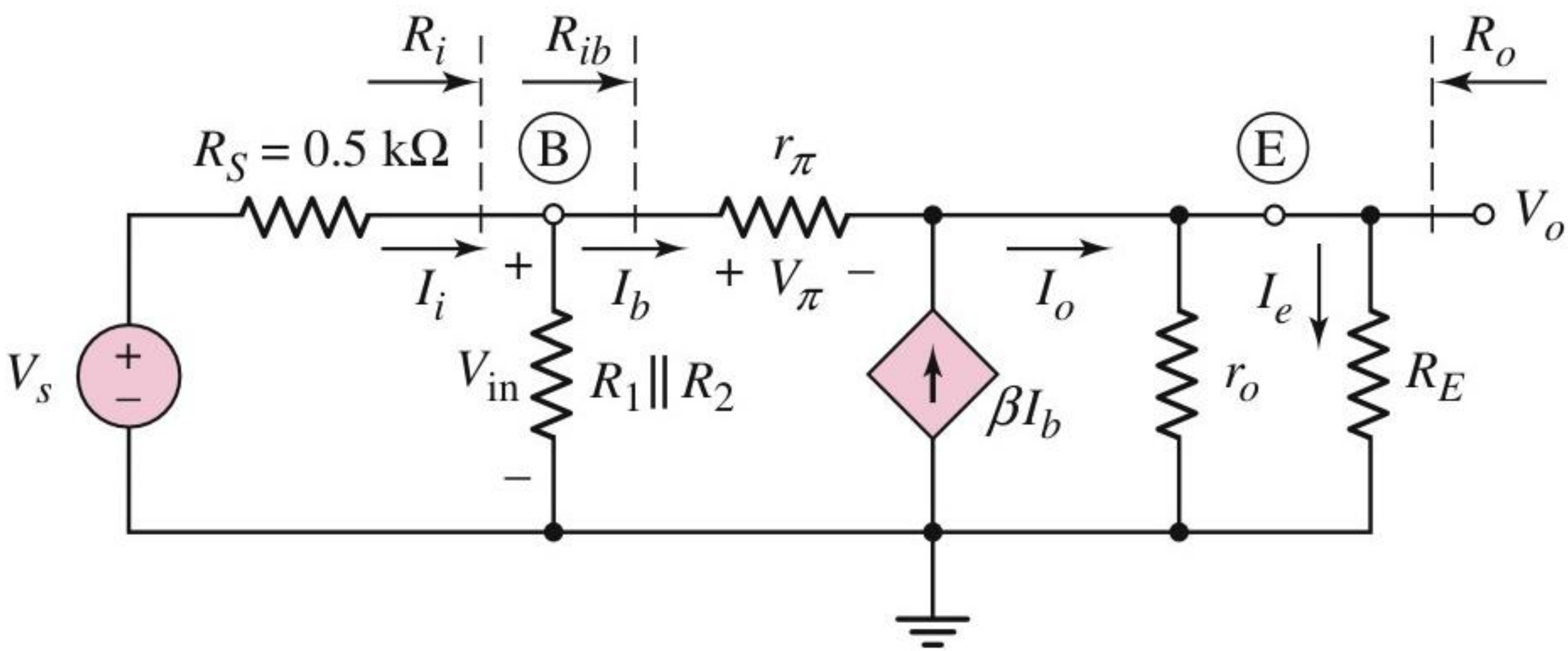
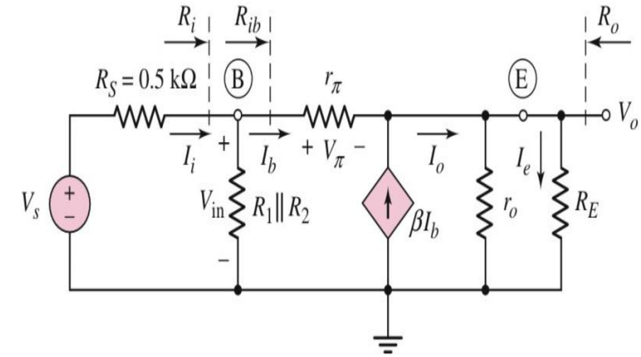


Figure 2.3: Small-signal equivalent circuit of the emitter-follower with all signal grounds are at the same point

Common-collector Amplifier – Small Signal Voltage Gain (3)

We see that

$$I_o = (1+\beta) I_b \qquad (2.1)$$



So the output voltage can be written as

$$V_o = I_b (1+\beta)(r_o \,//\, R_E) \qquad (2.2)$$

Writing a KVL equation around the base-emitter loop, we obtain

$$V_{in} = I_b \left[r_\pi + (1+\beta)(r_o \,//\, R_E) \right] \qquad (2.3a)$$

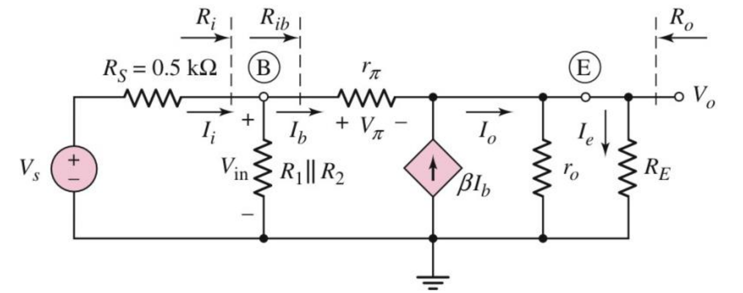
or

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta)(r_o \,//\, R_E) \qquad (2.3b)$$

Common-collector Amplifier – Small Signal Voltage Gain (4)

We can also write

$$V_{in} = \left(\frac{R_i}{R_i + R_s} \right) . V_S \qquad (2.4)$$



Where $R_i = R_1 // R_2 // R_{ib}$

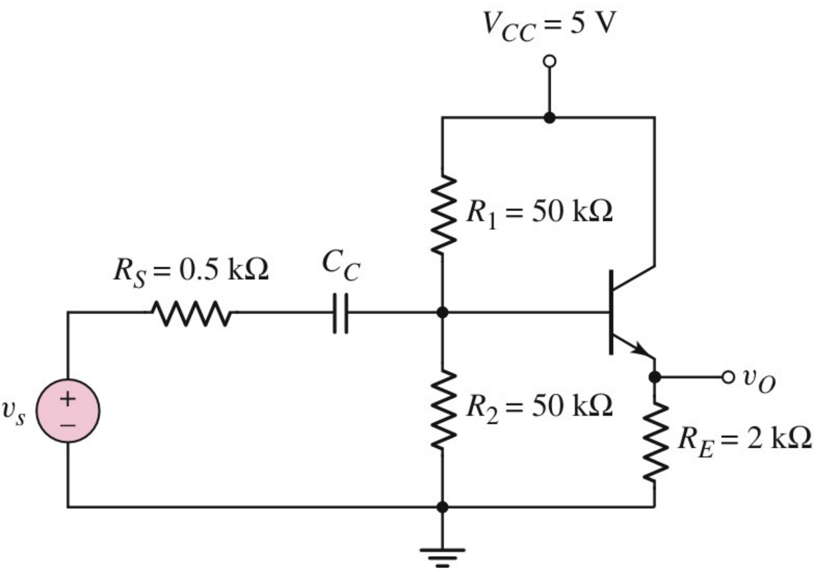
Combining Equations (2.2), (2.3b), and (2.4), the small-signal voltage gain is

$$A_v = \frac{V_o}{V_s} = \frac{(1+\beta)(r_o // R_E)}{r_\pi + (1+\beta)(r_o // R_E)} \left(\frac{R_i}{R_i + R_s} \right) \qquad (2.5)$$

Common-collector Amplifier – Example 2.1 (1)

Calculate the small-signal voltage gain of an emitter-follower circuit. For the circuit shown in figure 2.1, assume the transistor parameters are:

$\beta = 100$, $V_{BE}(on) = 0.7V$, and $V_A = 80V$



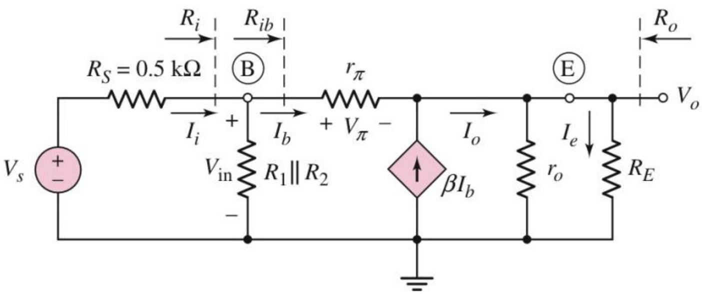
solution:

The dc analysis shows that $I_{CQ} = 0.793\text{mA}$ and $V_{CEQ} = 3.4\text{V}$. The small-signal hybrid-pi parameters are determined to be

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{0.793} = 3.28K\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.793}{0.026} = 30.5mA/V$$

Common-collector Amplifier – Example 2.1 (2)



The small-signal voltage gain is then

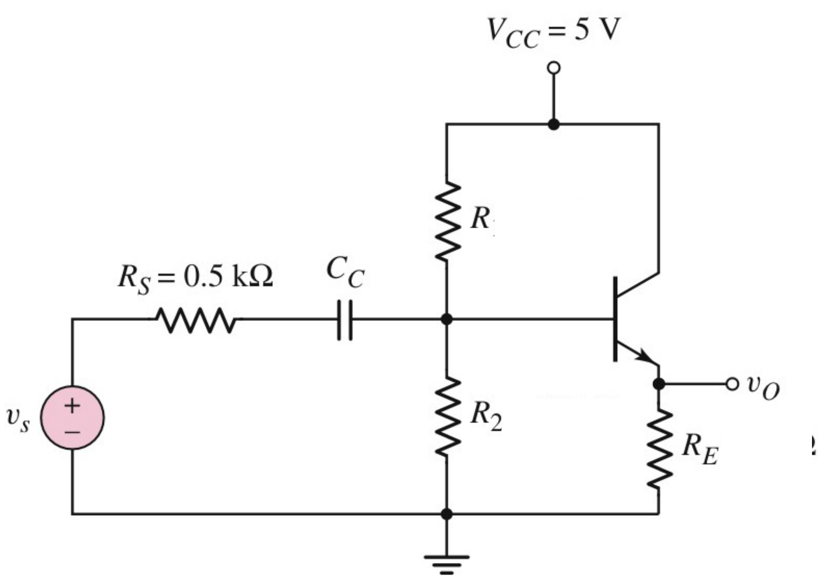
$$A_v =$$

or

Common-collector Amplifier – Example 2.2

For the circuit shown in figure 2.1, let $V_{cc} = 5V,\ \beta = 120,\ V_A = 100V,\ R_E = 1K\Omega$
$V_{BE}(on) = 0.7V,\ R_1 = 25K\Omega$, and $R_2 = 50K\Omega$,

a) Determine the small-signal voltage gain . b) Find the input resistance looking into the base of the transistor.



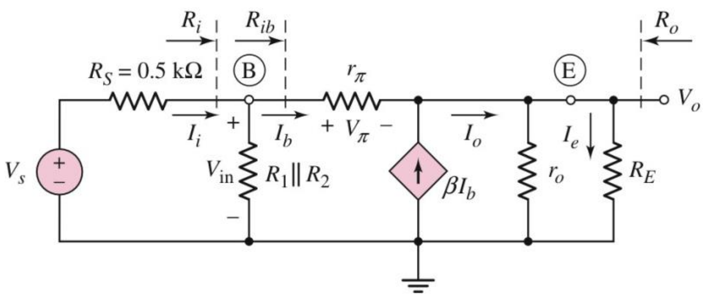
Common-collector Amplifier – Example 2.2 (2)

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.29}{0.026} = 88.1\ mA/V$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{2.29} = 1.36\ k\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{2.29} = 43.7\ k\Omega$$

$$V_s' = \left(\frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S}\right) \cdot V_s$$



$$R_{ib} = r_\pi + (1+\beta)(R_E \| r_o) = 1.36 + (121)(1 \| 43.7)$$

or

$$R_{ib} = 120\ k\Omega \text{ and } R_1 \| R_2 = 16.7\ k\Omega$$

Then

$$R_1 \| R_2 \| R_{ib} = 16.7 \| 120 = 14.7\ k\Omega$$

Now

$$V_s' = \left(\frac{14.7}{14.7 + 0.5}\right) \cdot V_s = (0.967) V_s$$

and

$$V_o = \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right)(R_E \| r_o) = V_\pi \left(\frac{1+\beta}{r_\pi}\right) R_E \| r_o$$

Example 2.2 (3)

We have

$$V_s' = V_\pi + V_o$$

then

$$V_\pi = \frac{V_s'}{1+\left(\frac{1+\beta}{r_\pi}\right)R_E \parallel r_o} = \frac{(0.967)V_s}{1+\left(\frac{1+\beta}{r_\pi}\right)R_E \parallel r_o}$$

We then obtain

$$A_v = \frac{V_o}{V_s} = \frac{(0.967)\left(\frac{1+\beta}{r_\pi}\right)R_E \parallel r_o}{1+\left(\frac{1+\beta}{r_\pi}\right)R_E \parallel r_o}$$

$$= \frac{(0.967)(1+\beta)R_E \parallel r_o}{r_\pi + (1+\beta)R_E \parallel r_o}$$

Now

$$R_E \parallel r_o = 1 \parallel 43.7 = 0.978\ k\Omega$$

Then

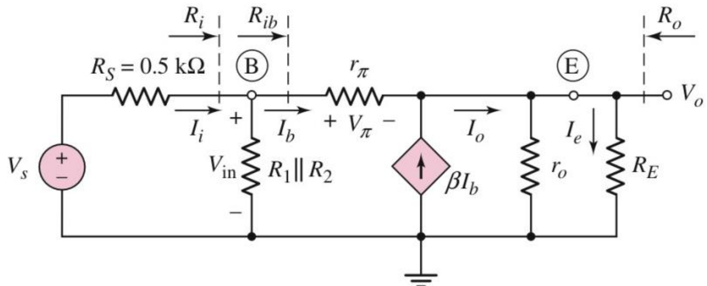
$$A_v = \frac{(0.967)(121)(0.978)}{1.36+(121)(0.978)} = 0.956$$

(b)

$$R_{ib} = r_\pi + (1+\beta)(R_E \parallel r_o)$$

or

$$R_{ib} = 1.36 + (121)(0.978) = 120\ k\Omega$$

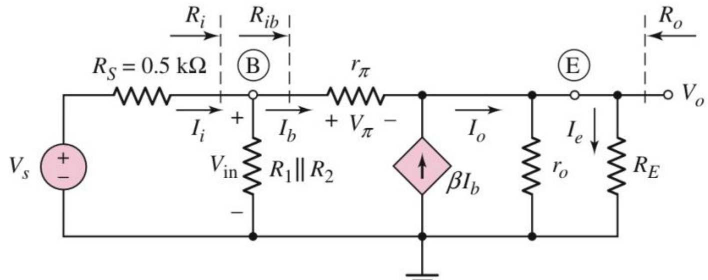


Common-collector Amplifier – Input Resistance

The input impedance, or small-signal input resistance for low-frequency signals, of the emitter-follower is determined in the same manner as for the common-emitter circuit. The input resistance R_{ib} was given by equation (2.3b)

$$R_{ib} = r_\pi + (1+\beta)(r_o \,//\, R_E)$$

Since the emitter current is $(1+\beta)$ times the base current, the effective impedance in the emitter is multiplied by $(1+\beta)$. We saw this same effect when an emitter resistor was included in a common-emitter circuit. This multiplication by $(1+\beta)$ is again called the resistance reflection rule.



Common-collector Amplifier – Output Resistance (1)

Initially, to find the output resistance of the emitter-follower circuit shown in figure 2.1, we will assume that the input signal source is ideal and that $R_s = 0$. The Figure below is derived from the small-signal equivalent circuit shown in figure 2.3 by setting the independent voltage source V_s equal to zero, which means that V_s acts as a short circuit.

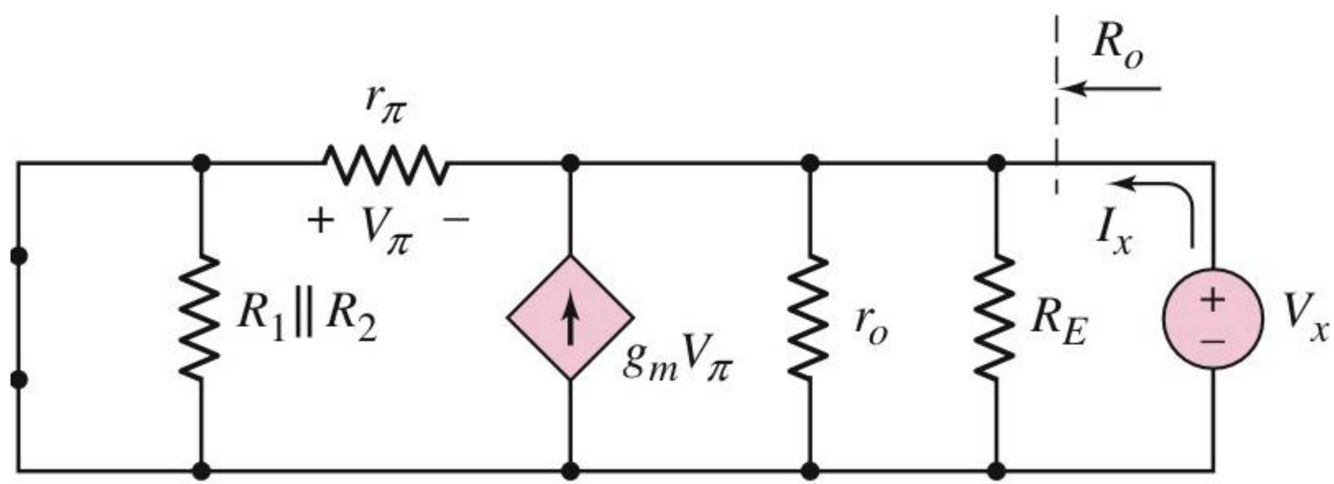


Figure 2.4: Small-signal equivalent circuit of the emitter-follower used to determine the output resistance.

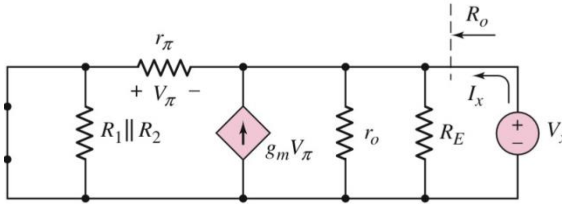
Common-collector Amplifier – Output Resistance (2)

A test voltage V_x is applied to the output terminal and the resulting test current is I_x . The output resistance, R_o , is given by

$$R_o = \frac{V_x}{I_x} \qquad (2.6)$$

In this case, the control voltage V_π is not zero, but is a function of the applied test voltage. From figure 2.4, we see that $V_\pi = -V_x$ .summing currents at the output node, we have

$$I_x + g_m V_\pi = \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi} \qquad (2.7)$$



Since $V_\pi = -V_x$, equation (2.7) can be written as

$$\frac{I_x}{V_x} = \frac{1}{R_o} = g_m + \frac{1}{R_E} + \frac{1}{r_o} + \frac{1}{r_\pi} \qquad (2.8)$$

Common-collector Amplifier – Output Resistance (3)

Or the output resistance is given by

$$R_o = \frac{1}{g_m} \,//\, R_E \,//\, r_o \,//\, r_\pi \qquad (2.9)$$

The output resistance may also be written in a slightly different form, Equation (2.8) can be written in the form

$$\frac{1}{R_o} = \left(g_m + \frac{1}{r_\pi} \right) + \frac{1}{R_E} + \frac{1}{r_o} = \left(\frac{1+\beta}{r_\pi} \right) + \frac{1}{R_E} + \frac{1}{r_o} \qquad (2.10)$$

Or the output resistance can be written in the form

$$R_o = \frac{r_\pi}{1+\beta} \,//\, R_E \,//\, r_o \qquad (2.11)$$

This is an important result and is called the inverse resistance reflection rule and is the inverse of the reflection rule looking to the base.

Common-collector Amplifier – Output Resistance (4)

We can determine the output resistance of the emitter-follower circuit taking into account a nonzero source resistance. The circuit in figure below is derived from the small-signal equivalent circuit shown in figure 2.3 and can be used to find R_o

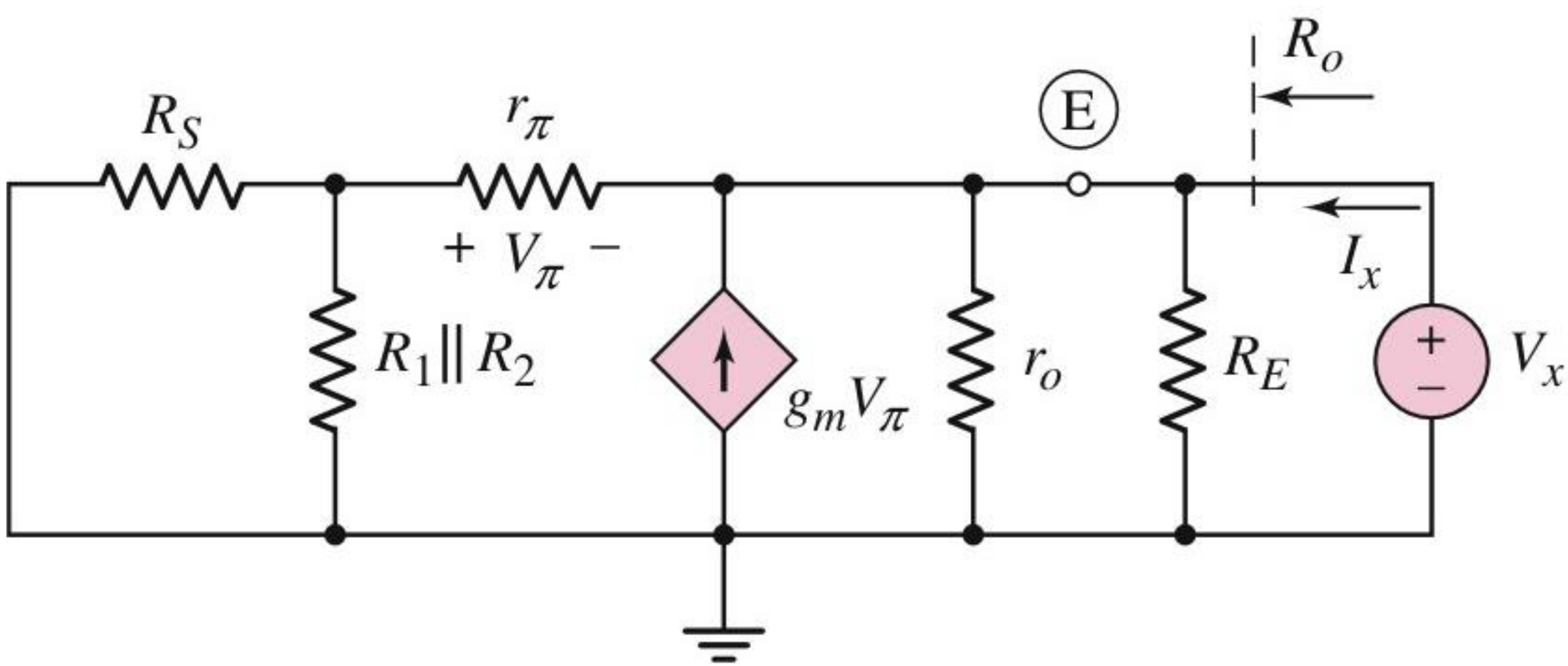
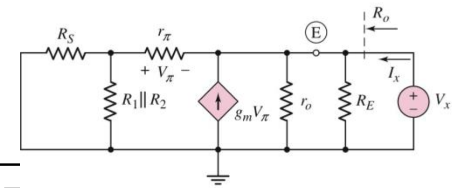


Figure 2.5: Small-signal equivalent circuit of the emitter-follower used to determine the output resistance including the effect of the source resistance R_s

Common-collector Amplifier – Output Resistance (5)

The independent source V_s is set equal to zero and test voltage V_x is applied to the output terminals. Again, the control voltage V_π is not zero, but is a function of the test voltage. Summing currents at the output node, we have



$$I_x + g_m V_\pi = \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi + R_1 // R_2 // R_s} \qquad (2.12)$$

The control voltage can be written in terms of the test voltage by a voltage divider equation as

$$V_\pi = -\left(\frac{r_\pi}{r_\pi + R_1 // R_2 // R_s}\right).V_x \qquad (2.13)$$

Equation (2.12) can then be written as

$$I_x = \left(\frac{g_m r_\pi}{r_\pi + R_1 // R_2 // R_s}\right).V_x + \frac{V_x}{R_E} + \frac{V_x}{r_o} + \frac{V_x}{r_\pi + R_1 // R_2 // R_s} \qquad (2.14)$$

Common-collector Amplifier – Output Resistance (6)

Noting that $g_m r_\pi = \beta$, we find

$$\frac{I_x}{V_x} = \frac{1}{R_o} = \left(\frac{1+\beta}{r_\pi + R_1 // R_2 // R_s} \right) + \frac{1}{R_E} + \frac{1}{r_o} \qquad (2.15)$$

or

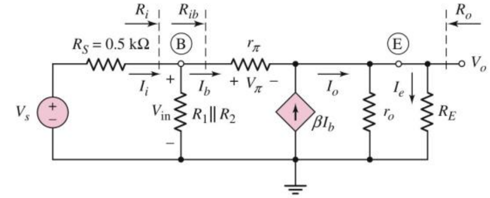
$$R_o = \left(\frac{r_\pi + R_1 // R_2 // R_s}{1+\beta} \right) // R_E // r_o \qquad (2.16)$$

In this case, the source resistance and bias resistances contribute to the output resistance

Common-collector Amplifier – Small Signal Current Gain (1)

We can determine the small-signal current gain of an emitter-follower by using the input resistance and the concept of current dividers. The small-signal current gain is defined as

$$A_i = \frac{I_e}{I_i} \qquad (2.16)$$



Where I_e and I_i are the output and input current phasors.

Using current divider equation, we can write the base current in terms of the input current, as follows:

$$I_b = \left(\frac{R_1 // R_2}{R_1 // R_2 + R_{ib}} \right) I_i \qquad (2.17)$$

Since $g_m V_\pi = \beta I_b$, then

$$I_0 = (1+\beta) I_b = (1+\beta) \left(\frac{R_1 // R_2}{R_1 // R_2 + R_{ib}} \right) I_i \qquad (2.18)$$

Common-collector Amplifier – Small Signal Current Gain (2)

Writing the load current in terms of I_o produces

$$I_e = \left(\frac{r_o}{r_o + R_E} \right) I_0 \qquad (2.19)$$

Combining equations (2.18) and (2.19) , we obtain the small-signal current gain, as follows:

$$A_i = \frac{I_e}{I_i} = (1+\beta)\left(\frac{R_1 \,//\, R_2}{R_1 \,//\, R_2 + R_{ib}} \right)\left(\frac{r_o}{r_o + R_E} \right) \qquad (2.20)$$

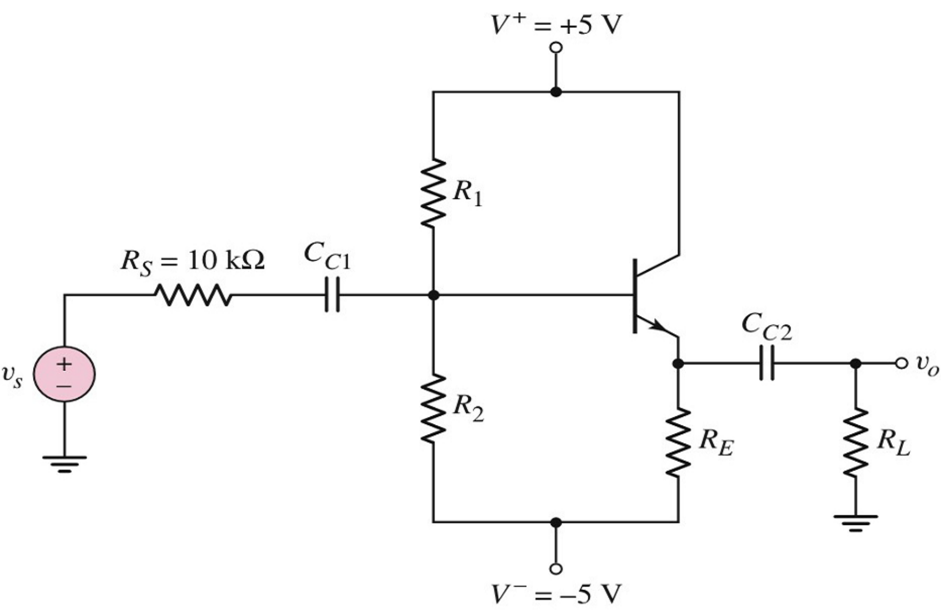
If we assume that $R_1 \,//\, R_2 >> R_{ib}$ and $r_o >> R_E$,then

$$A_i \cong (1+\beta) \qquad (2.21)$$

Which is current gain of the transistor.

Common-collector Amplifier – Example 2.3

For the circuit shown in figure below, let $\beta = 100, V_A = 125V$, and $V_{BE}(on) = 0.7V$ Assume $R_s = 0$, and $R_L = 1K\Omega$, a) Design a bias-stable circuit such that $I_{CQ} = 125mA$, and $V_{CEQ} = 4V$, b) What is small-signal Current gain $A_i = i_o / i_i$ c)What is output resistance looking back into the output terminal



We have

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(10) - 5$$

or

$$V_{TH} = \frac{1}{R_1}(481) - 5$$

We can write $I_{BQ} = \dfrac{V_{TH} - 0.7 - (-5)}{R_{TH} + (1+\beta)R_E}$

Or

$$0.0125 = \frac{\frac{1}{R_1}(481) - 5 - 0.7 + 5}{48.1 + (101)(4.76)}$$

which yields

$R_1 = 65.8\ k\Omega$

Since $R_1 \parallel R_2 = 48.1\ k\Omega$, we obtain

$R_2 = 178.8\ k\Omega$

(b)

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.25} = 2.08\ k\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{125}{1.25} = 100\ k\Omega$$

We may note that

$$g_m V_\pi = g_m\left(I_b r_\pi\right) = \beta I_b$$

Also

$$R_{ib} = r_\pi + (1+\beta)\left(R_E \parallel R_L \parallel r_o\right)$$
$$= 2.08 + (101)\left(4.76\|1\|100\right)$$

or

$$R_{ib} = 84.9\ k\Omega$$

Now

$$I_o = \left(\frac{R_E \| r_o}{R_E \| r_o + R_L}\right)(1+\beta) I_b$$

where

$$I_b = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}}\right) \cdot I_s$$

We can then write

$$A_I = \frac{I_o}{I_s} = \left(\frac{R_E \| r_o}{R_E \| r_o + R_L} \right)(1+\beta)\left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right)$$

We have

$$R_E \| r_o = 4.76 \| 100 = 4.54\ k\Omega$$

so

$$A_I = \left(\frac{4.54}{4.54+1} \right)(101)\left(\frac{48.1}{48.1+84.9} \right)$$

or

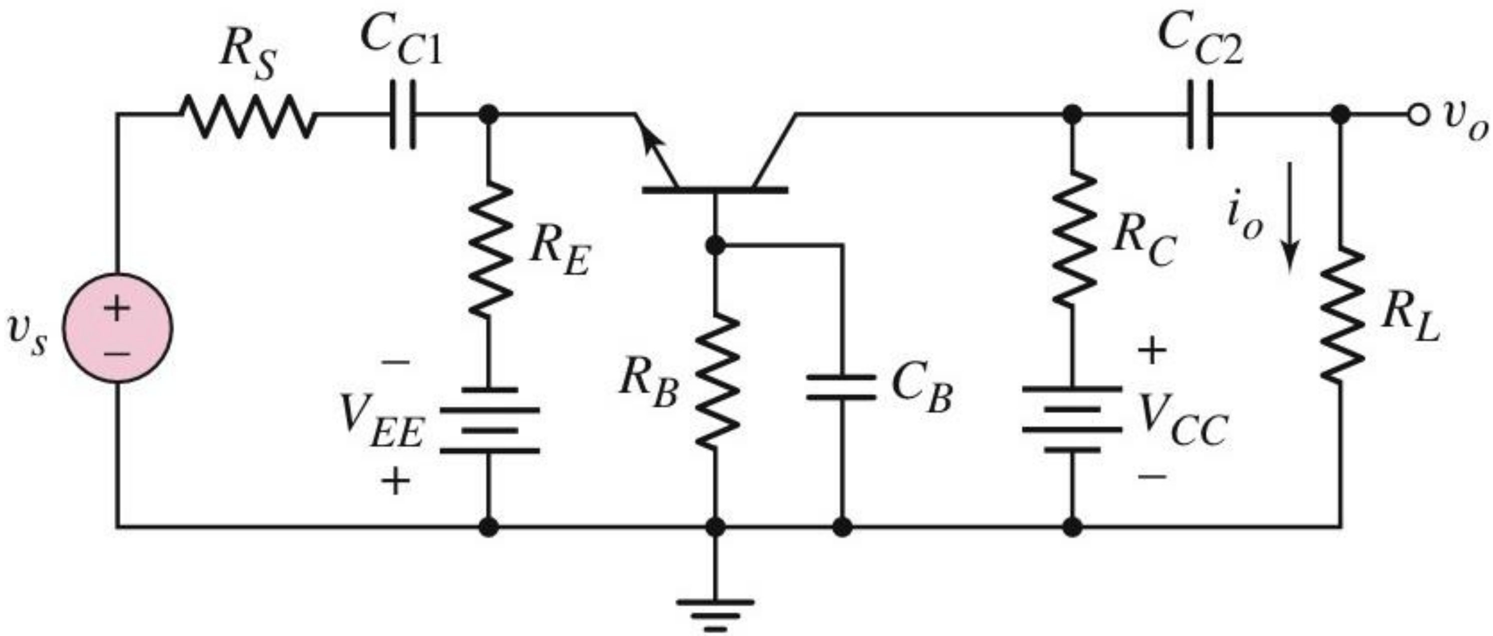
$A_I = 29.9$

(c) $$R_o = R_E \| r_o \left\| \frac{r_\pi}{1+\beta} = 4.76 \right\| 100 \left\| \frac{2.08}{101} \right.$$

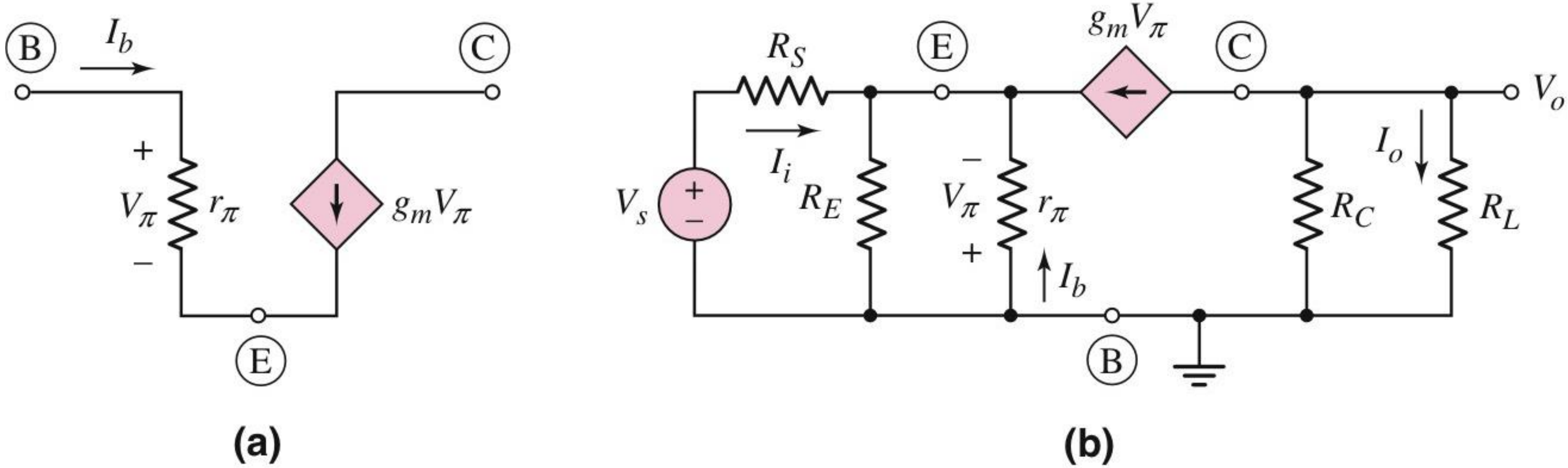
or $R_o = 20.5\ \Omega$

Analyse the Common-Base Amplifier

Common-Base Amplifier



Small-Signal Equivalent Circuit: Common Base

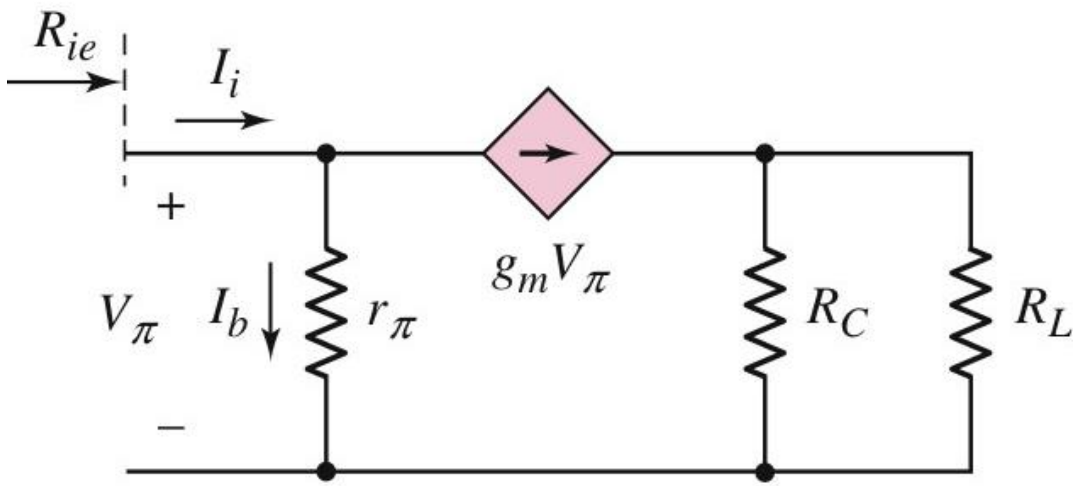


$$A_v = g_m (R_C \| R_L)$$

$$A_i = g_m \left(\frac{R_C}{R_C + R_L}\right)\left[\frac{r_\pi}{1+\beta} \| R_E\right]$$

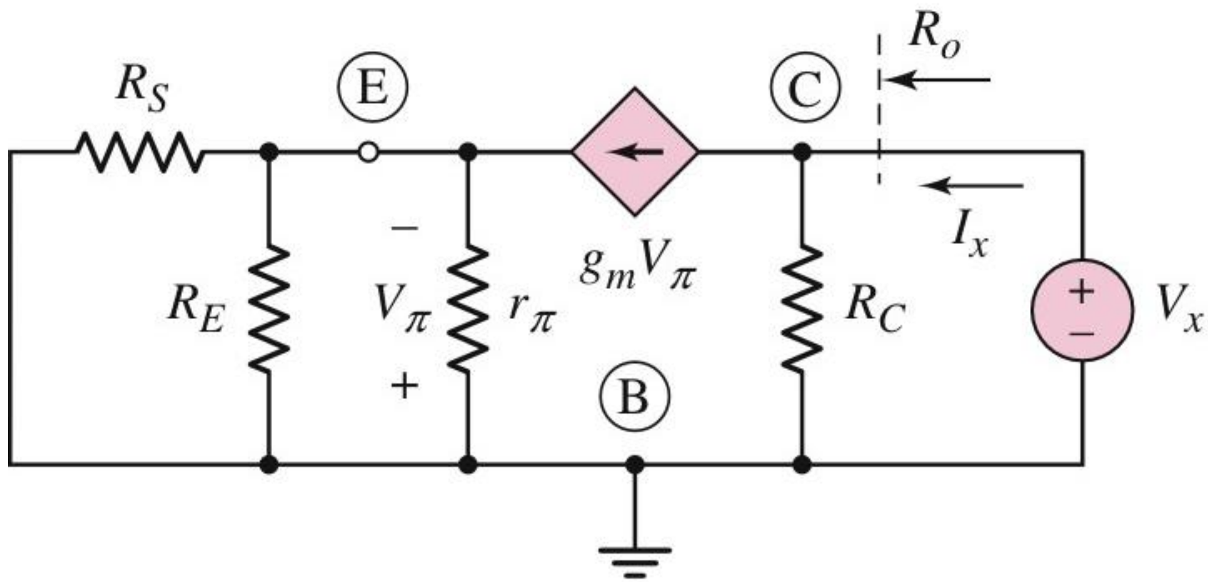
If R_E approaches infinity and R_L approaches zero, the current gain becomes the short-circuit current gain given by $A_{io} = \alpha$.

Input Resistance: Common Base



$R_{ie} = r_\pi/(1+\beta)$

Output Resistance: Common Base



$R_O = R_C$

Summary and Comparison

Configuration	Voltage gain	Current gain	Input resistance	Output Resistance
Common emitter	$A_V > 1$	$A_I > 1$	Moderate	Moderate to high
Emitter Follower	$A_V \cong 1$	$A_i > 1$	High	Low
Common base	$A_V > 1$	$A_i \cong 1$	Low	Moderate to high

Contents of Chapter

- Understand the concept of an analog signal and the principle of a linear amplifier.
  - Investigate how a transistor circuit can amplify a small, time-varying input signal.

- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-emitter amplifier.
  - Analyze the emitter-follower amplifier.
  - Analyze the common-base amplifier.

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EEE109: Electronic Circuits

Frequency Response

- Capacitor Effect and Examples

Contents

Input Coupling Capacitor Common-Emitter (1)

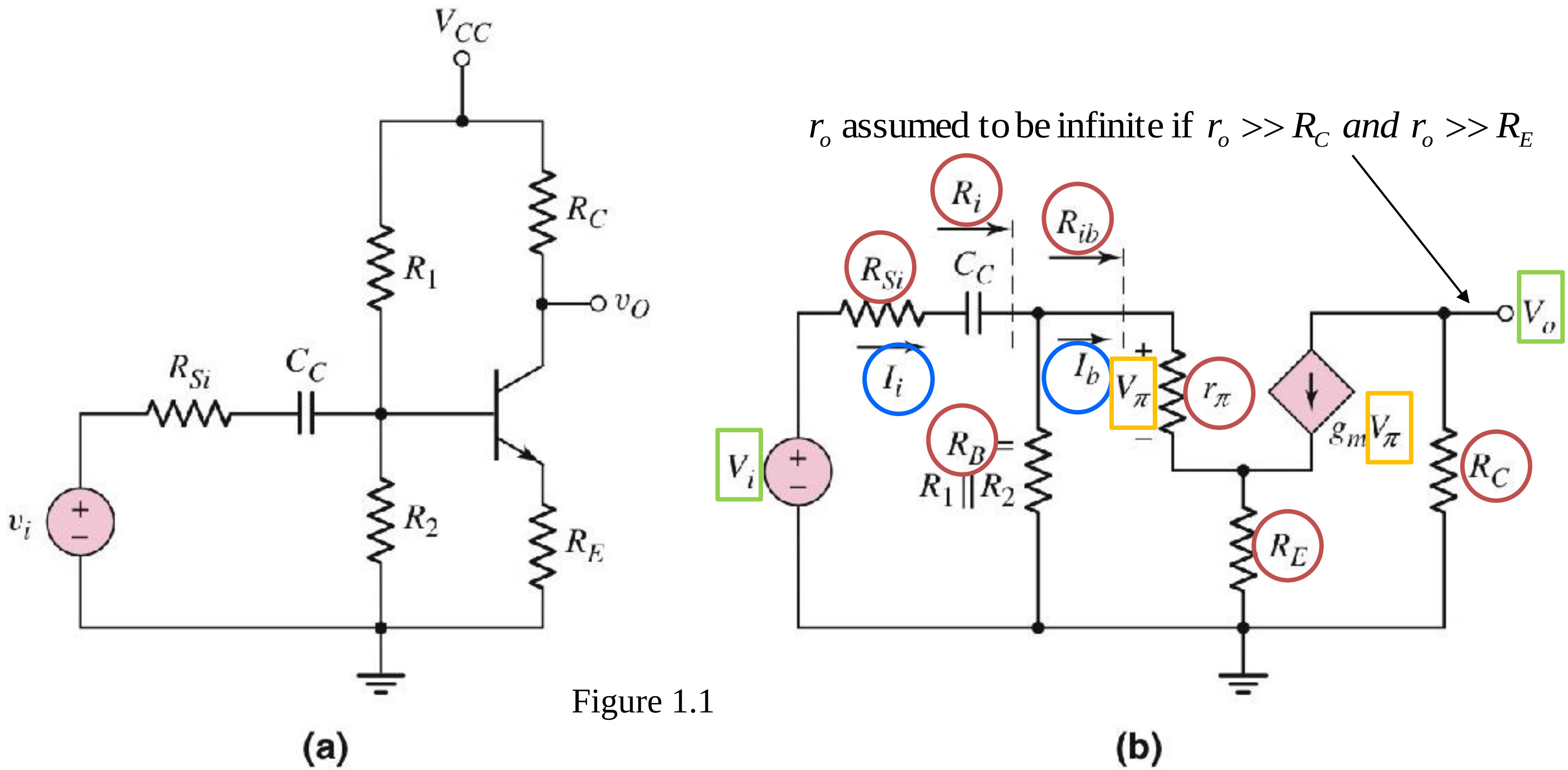
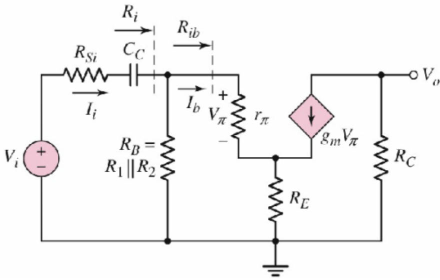


Figure 1.1

a) Common-emitter circuit with input coupling capacitor

b) Small-signal equivalent circuit

Input Coupling Capacitor Common-Emitter (2)



The input current can be written as

$$I_i = \frac{V_i}{R_{Si} + \dfrac{1}{s\,C_C} + R_i} \qquad (1.1)$$

Where the input resistance s and R_i is given by

$$s = j\omega$$

$$R_i = R_B // \left[r_\pi + (1+\beta) R_E \right] = R_B // R_{ib} \qquad (1.2)$$

Using a current divider, we determine the base current to be

$$I_b = \left(\frac{R_B}{R_B + R_{ib}} \right) I_i \qquad (1.3)$$

and then

$$V_\pi = I_b \; r_\pi \qquad (1.4)$$

Input Coupling Capacitor Common-Emitter (3)

The output voltage is given by

$$V_o = -g_m V_\pi R_C \qquad (1.5)$$

Combining Equation (1.1) through (1.5)

$$V_o = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}} \right) \left(\frac{V_i}{R_{si} + \dfrac{1}{s C_C} + R_i} \right) \qquad (1.6)$$

Therefore, the small-signal voltage gain is

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}} \right) \left(\frac{s C_C}{1 + s(R_{si} + R_i) C_C} \right) \qquad (1.7)$$

Input Coupling Capacitor Common-Emitter (4)

Which can be written in the form

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{-g_m\ r_\pi\ R_C}{(R_{si}+R_i)}\left(\frac{R_B}{R_B+R_{ib}}\right)\left(\frac{s\,\tau_s}{1+s\,\tau_s}\right) \qquad (1.8)$$

Where the time constant is

$$\tau_s = (R_{si}+R_i)\,C_C \qquad (1.9)$$

The corner frequency is

$$f_L = \frac{1}{2\pi\,\tau_s} = \frac{1}{2\pi(R_{si}+R_i)\,C_C} \qquad (1.10)$$

and the maximum magnitude, in decibels, is

$$\left|A_v(\max)\right|_{\mathrm{DB}} = 20\log_{10}\left(\frac{g_m\,r_\pi\,R_C}{R_{si}+R_i}\right)\left(\frac{R_B}{R_B+R_{ib}}\right) \qquad (1.11)$$

Example 1.1:

Calculate the corner frequency and maximum gain of a bipolar common-emitter circuit with a coupling capacitor.

For the circuit shown in figure 1.1, the parameters are: R_1=51.2 Kohms, R_2=9.6Kohms, R_C=2 Kohms, R_E=0.4Kohms, R_{si}=0.1 Kohms, C_C=1uF, and V_{CC}=10V. The transistor parameter are $V_{BE}(on)$ =0.7V, β =100, $V_A = \infty$,

$I_{CQ} = 1.81\text{mA}$

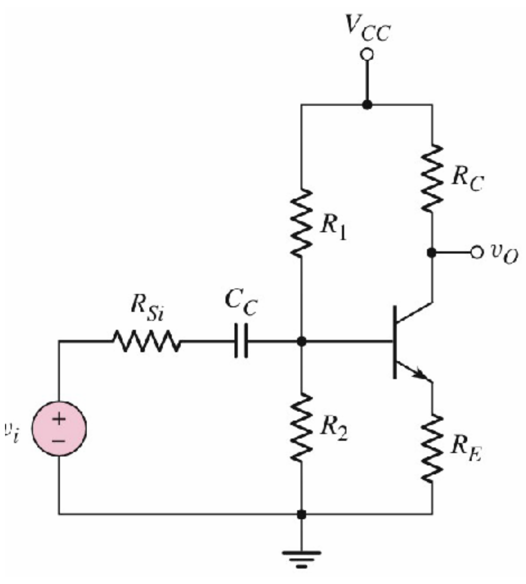


Figure 1.1

Example 1.1 :

Given :

$R_1 = 51.2\ K\Omega$ $\quad C_c = 1\ \mu F$ $\quad V_{cc} = 10\ V$

$R_2 = 9.6\ k\Omega$ $\quad V_{BE(on)} = 0.7\ V$

$R_C = 2\ k\Omega$ $\quad I_{CQ} = 1.81\ mA$ $\quad V_A = \infty$

$R_E = 0.4\ K\Omega$

$R_{S_i} = 0.1\ K\Omega$ $\quad \beta = 100$

Calculate :

① f_L ; ② Maximum gain $|A_v(max)|$

Solution :

step1 : Calcute π model parameter g_m & r_π .

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.81}{0.026} = 69.6\ mA/V$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100 \cdot 0.026}{1.81} = 1.44\ K\Omega$$

step2 : Calculate corner frequency f_L

① input resistance R_i

$$R_i = R_1 \| R_2 \| [r_\pi + (1+\beta)R_E]$$
$$= 51.2 \| 9.6 \| [1.44 + 101 \cdot 0.4] = 6.77\ K\Omega$$

② time constant τ_s

$$\tau_s = (R_{S_i} + R_i) C_c = (0.1 + 6.77) \cdot 10^3 \cdot 1 \cdot 10^{-6} = 6.87\ ms$$

③ corner frequency f_L

$$f_L = \frac{1}{2\pi \tau_s} = \frac{10^3}{2\pi \cdot 6.87} = 23.2\ HZ$$

No.

Date

[redacted]

Step 3: Calculate maximum gain (magnitude) $|A_v(max)|$

① Calculate input resistance at the base R_{ib}

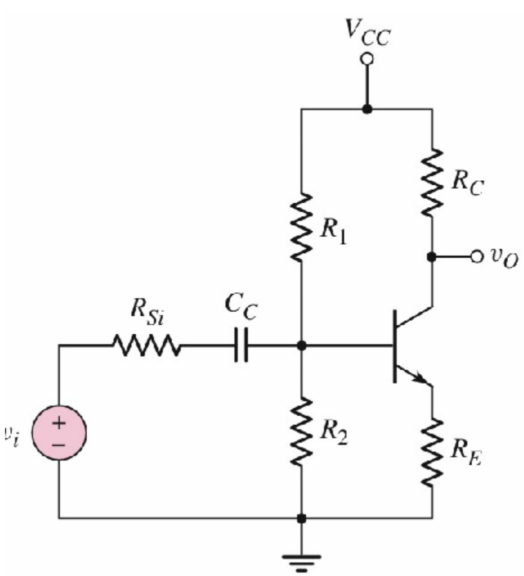
$$R_{ib} = r_\pi + (1+\beta) R_E = 41.8\ k\Omega$$

② $|A_v(max)|$

$$= \left(\frac{g_m r_\pi R_C}{R_{S_i} + R_i}\right)\left(\frac{R_B}{R_B + R_{ib}}\right) = 4.72$$

Example 1.2:

Calculate the τ_s , **corner frequency** and **maximum gain** of a bipolar common-emitter circuit with a coupling capacitor.
For the circuit shown in figure 1.1, the parameters are: R_1=20 Kohms, R_2=2.2Kohms, R_C=2 Kohms, R_E=0.1Kohms, R_{si}=0.1 Kohms, C_C=47uF, and V_{CC}=10V. The transistor parameter are V_{BE}(on) =0.7V, β=200, $V_A=\infty$



Example 1.2:

Given:

$R_1 = 20\ k\Omega$ $\quad C_c = 47\ \mu F$ $\quad V_{CC} = 10\ V$

$R_2 = 2.2\ k\Omega$ $\quad V_{BE(on)} = 0.7\ V$

$R_C = 2\ k\Omega$ $\quad \beta = 200$ $\quad V_A = \infty$

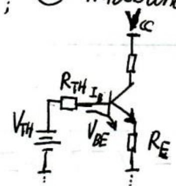
$R_E = 0.1\ k\Omega$

$R_{S_i} = 0.1\ k\Omega$

Calculate:

① τ_s ; ② f_L ; ③ midband voltage gain. A_v

Solution:



step1: DC analysis

① $R_{TH} = R_1 \| R_2 = 20 \| 2.2 = 1.98\ k\Omega$

$$V_{TH} = \frac{R_2}{R_1 + R_2} \cdot V_{CC} = \frac{2.2}{20+2.2} \cdot 10 = 0.991\ V$$

② $$I_{BQ} = \frac{V_{TH} - V_{BE(on)}}{R_{TH} + (1+\beta) R_E} = \frac{0.991 - 0.7}{1.98 + 201 \cdot 0.1} = 0.0132\ mA$$

$$I_{CQ} = \beta I_{BQ} = 200 \times 0.0132 = 2.64\ mA$$

step2: Calcute π-paramters.

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.64}{0.026} = 101.4\ mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{101.4} = 1.97\ k\Omega$$

step3: Calculate τ_s

① resistors R_B, R_i, R_{iB} involved

$$R_B = R_1 \| R_2 = R_{TH} = 1.98\ k\Omega$$

$$R_{ib} = r_\pi + (1+\beta) R_E = 1.97 + 201 \cdot 0.1 = 22.1\ k\Omega$$

$$\Rightarrow R_i = R_{ib} \parallel R_B$$

$$= 22.1 \parallel 1.98 = 1.817\ k\Omega$$

$$\Rightarrow \tau_S = (R_i + R_{Si}) \cdot C_C = (1.817 + 0.1) \cdot 10^3 \times 47 \times 10^{-6}$$

$$= 90.1\ ms$$

Step 4: Calculate corner frequency

$$f_L = \frac{1}{2\pi \tau_S} = \frac{10^3}{2\pi \cdot 901.1} = 1.77\ HZ$$

Step5 :

$$V_o = -g_m V_\pi \cdot R_C$$

$$KVL \Rightarrow \frac{R_i}{R_{Si} + R_i} \cdot V_i = I_b \cdot R_{ib} = I_b \cdot [r_\pi + (1+\beta) R_E]$$

$$= \frac{V_\pi}{r_\pi} \cdot [r_\pi + (1+\beta) R_E]$$

$$\Rightarrow V_i = \frac{R_{Si} + R_i}{R_i} \cdot \frac{V_\pi}{r_\pi} \cdot [r_\pi + (1+\beta) R_E]$$

$$\Rightarrow A_V = \frac{V_o}{V_i} = \frac{-g_m \cancel{V_\pi} \cdot R_C}{\frac{R_{Si}+R_i}{R_i} \cdot \cancel{V_\pi}/r_\pi \cdot [r_\pi + (1+\beta) R_E]} = \frac{-g_m R_C \cdot r_\pi}{r_\pi + (1+\beta) R_E} \cdot \frac{R_i}{R_i + R_{Si}}$$

Output Coupling Capacitor – Common-Source Circuit (1)

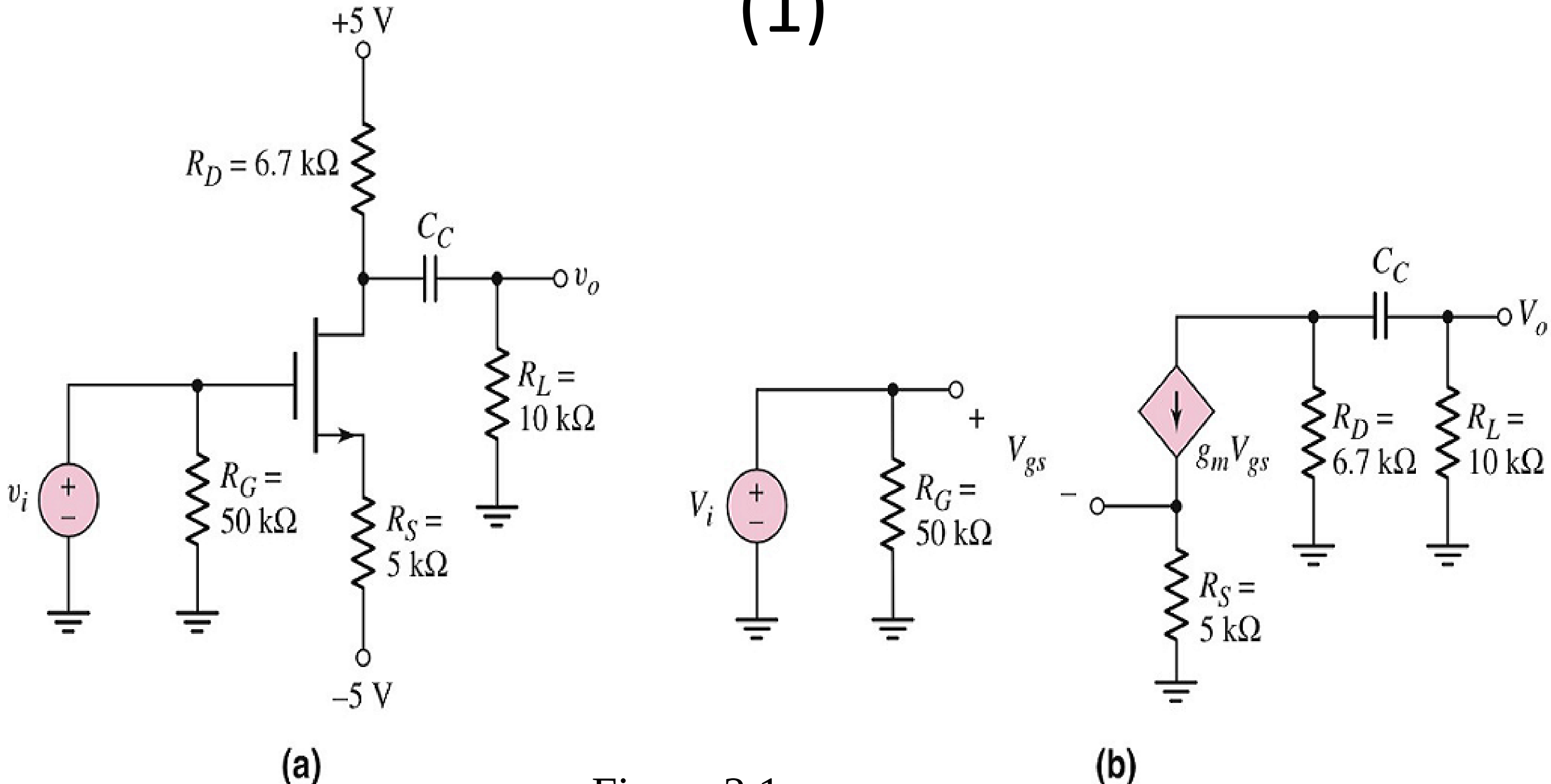


Figure 2.1

a) Common-source circuit with output coupling capacitor

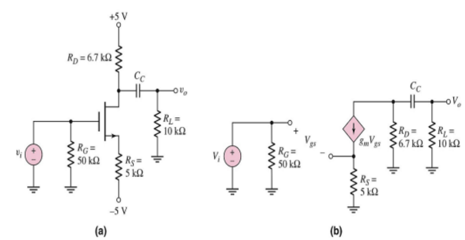
b) Small-signal equivalent circuit

Output Coupling Capacitor – Common-Source Circuit (2)

We assume that the resistance of the signal generator is much less than R_G and can therefore be neglected.

The small-signal equivalent circuit, assuming r_o is infinite, is shown in Figure 2.1(b). The maximum output voltage, assuming C_C is short circuit, is

$$|V_o|_{\max} = g_m V_{gs}(R_D \,//\, R_L) \qquad (2.1)$$



and the input voltage can be written as

$$V_i = V_{gs} + g_m R_S V_{gs} \qquad (2.2)$$

Therefore, the maximum small-signal gain is

$$|A_v|_{\max} = \frac{g_m(R_D \,//\, R_L)}{1 + g_m R_S} \qquad (2.3)$$

Output Coupling Capacitor – Common-Source Circuit (3)

The time constant is a function of the effective resistance seen by capacitor . C_C , which is determined by setting all independent sources equal to zero. Since $V_i = 0$, then $V_{gs} = 0$ and $g_m V_{gs} = 0$ and the effective resistance seen by C_C is $(R_D + R_L)$.

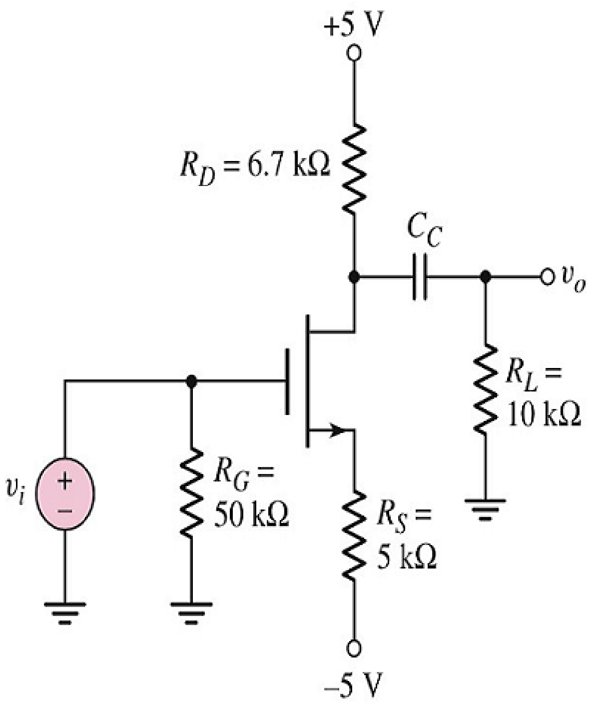
$$\tau_s = (R_D + R_L) C_C$$

and the corner frequency is

$$f_L = \frac{1}{2\pi \, \tau_s}$$

Example 2.1:

The circuit in figure 2.1(a) is to be used as a simple audio amplifier. Design the circuit such that the lower corner frequency is $f_L = 20Hz$



Example 2.1

Given: $R_G = 50\,K\Omega$ $\quad V_{cc} = 5\ V$

$R_D = 6.7\,K\Omega$

$R_S = 5\,K\Omega$ $\quad f_L = 20\ HZ$

$R_L = 10\,K\Omega$

Calculate: C_c

Solution:

step1: Calcute τ_s

$$\because f_L = \frac{1}{2\pi \tau_s}$$

$$\therefore \tau_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi \cdot 20} = 7.96\ ms$$

step2:

$$\because \tau_s = (R_D + R_L)\, C_c$$

$$\therefore C_c = \frac{\tau_s}{R_D + R_L} = \frac{7.96 \times 10^{-3}}{(6.7 + 10)10^3} = 4.77 \times 10^{-7}\ F = 47.7\ \mu F$$

Output Coupling Capacitor – Emitter-follower Circuit (1)

An emitter follower with a coupling capacitor in the output portion of the circuit is shown in figure 3.1(a). We assume that coupling capacitor C_{C1} . , which is part of original emitter follower, is very large and that it acts as a short circuit to input signal

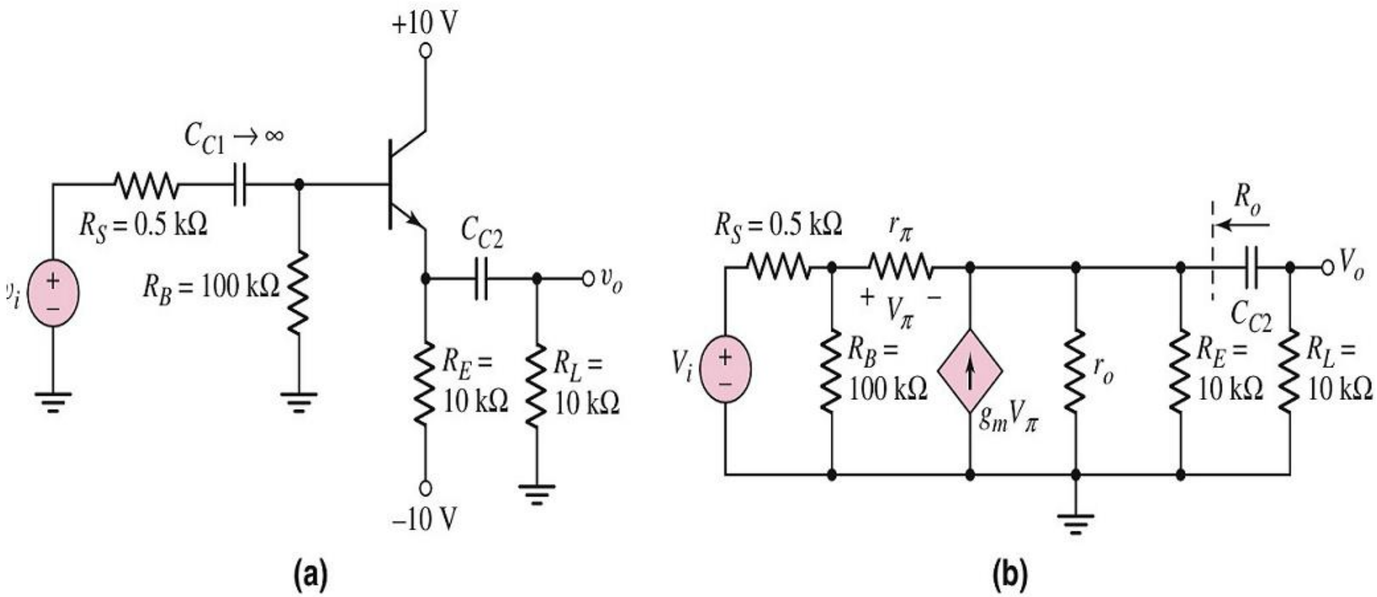


Figure 3.1: (a) Emitter-follower circuit with output coupling capacitor and (b) small-signal equivalent circuit.

Output Coupling Capacitor – Emitter-follower Circuit (2)

The equivalent resistance (r_o) seen by coupling capacitor C_{C2} is $\left[R_o + R_L\right]$, and the time constant is

$$\tau_S = \left[R_o + R_L\right] C_{C2} \qquad (3.1)$$

Where R_o is the output resistance as defined in figure 11pi-3(b) and

$$R_o = R_E \,//\, r_o \,//\left\{\frac{\left[r_\pi + \left(R_S \,//\, R_B\right)\right]}{1+\beta}\right\} \qquad (3.2)$$

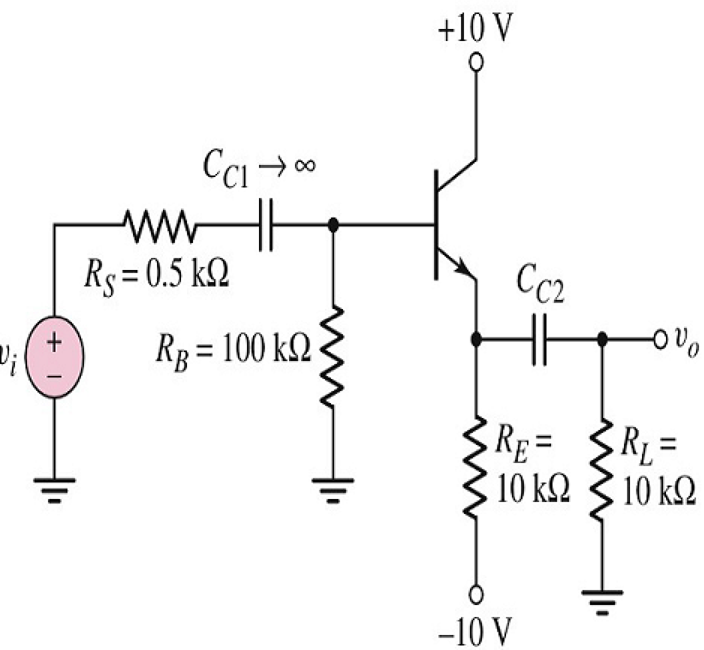
Example 3.1:

Determine the 3 dB frequency of an emitter-follower amplifier circuit with an output coupling capacitor.

Consider the circuit shown in figure below with transistor parameters .

$\beta = 100, V_{BE}(on) = 0.7, \text{and}, V_A = 120V$. The output coupling capacitance is

$C_{C2} = 1\mu F$



Solution:

A DC analysis shows that $I_{CQ} = 0.838\ \text{mA}$. Therefore the small signal parameters are:

The output resistance R_o of the emitter follower is

Emitter-follower Amplifier

Example 3.1

Given: $R_S = 0.5\ k\Omega$ $\quad V_+ = 10\ V,\ V_- = -10\ V$ $\quad C_{C2} = 1\ \mu F$

$R_B = 100\ k\Omega$ $\quad V_{BE(on)} = 0.7\ V$ $\quad C_{C1} \to \infty$

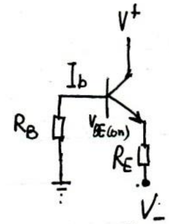
$R_E = 10\ k\Omega$ $\quad V_A = 120\ V$

$R_L = 10\ k\Omega$ $\quad \beta = 100$

Calculate: f_L

Solution:

step 1: DC analysis



KVL

$$\Rightarrow 0 - V_- = I_{BQ} \cdot (R_B + (1+\beta) R_E) + V_{BE(on)}$$

$$I_{BQ} = \frac{-V_{BE(on)} - V_-}{R_B + (1+\beta) R_E} = \frac{10 - 0.7}{100 + 101 \cdot 10} = 8.38 \times 10^{-6}\ A$$

$$I_{CQ} = \beta I_{BQ} = 0.838\ mA$$

step 2: Calculate π-model parameters

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.838}{0.026} = 32.2\ mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{32.2} = 3.1\ k\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{120}{0.838} = 143\ k\Omega$$

step 3: Calculate output resistance R_o

$$R_o = R_E \parallel r_o \parallel \frac{r_\pi + R_B \parallel R_S}{1+\beta} = 35.5\ \Omega$$

step4: Time constant τ_s

$$\tau_s = (R_o + R_L)\cdot C_{C2}$$
$$= (35.5 + 10^4)\cdot 1\times10^{-6} = 1\times10^{-2}\ s$$

step 5: Corner frequency

$$f_L = \frac{1}{2\pi\tau_s} = 15.9\ HZ$$

Bypass Capacitor Effects (1)

The bypass capacitors are assumed to act as short circuits at the signal frequency. However, to guide us in choosing a bypass capacitor, we must determine the circuit response in the frequency range where these capacitors are neither open or short circuits.

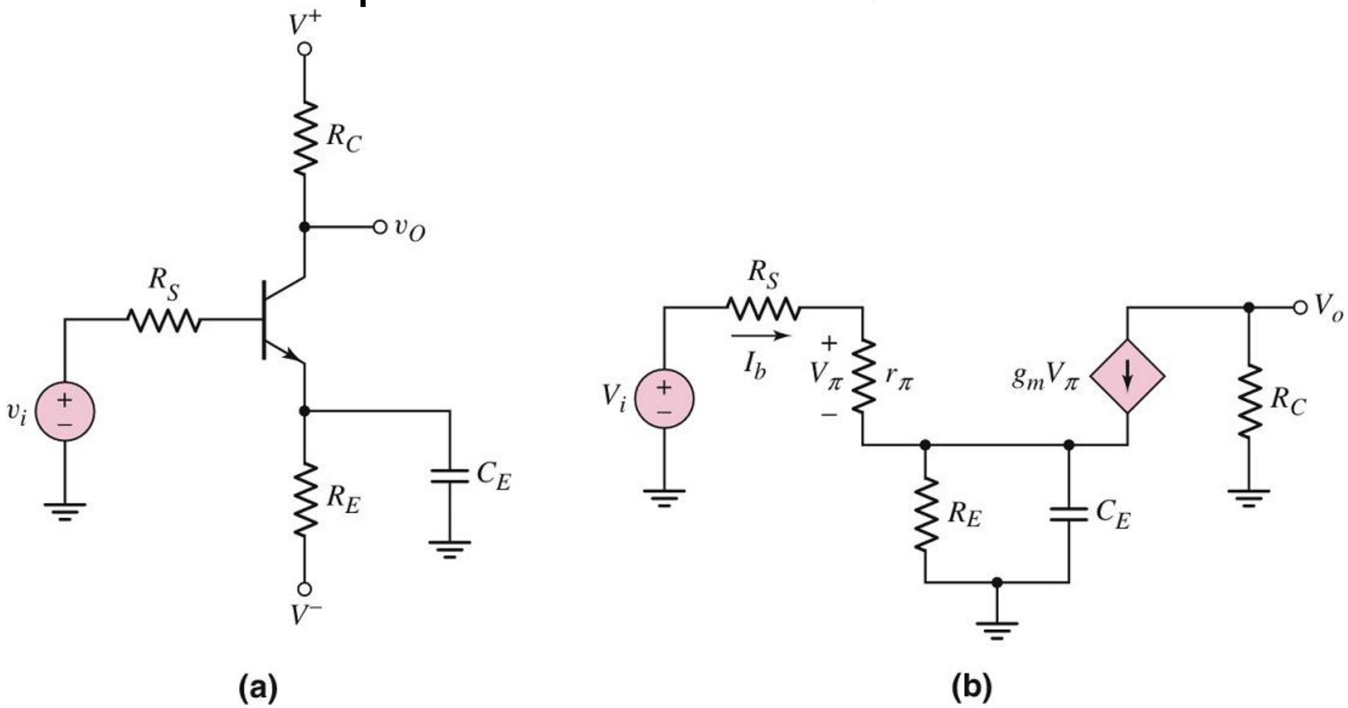


Figure 4.1: (a) Circuit with emitter bypass capacitor (b) small-signal equivalent circuit.

Bypass Capacitor Effects (2)

We can find the small-signal voltage gain as a function of frequency. Using the impedance reflection rule, the small-signal input current is

$$I_b = \frac{V_i}{R_S + r_\pi + (1+\beta)\left(R_E \,//\, \dfrac{1}{s\,C_E}\right)} \qquad (4.1)$$

The total impedance in the emitter is multiplied by the factor $(1+\beta)$. The control voltage is

$$V_\pi = I_b\, r_\pi \qquad (4.2)$$

and the output voltage is

$$V_o = -g_m\, V_\pi\, R_C \qquad (4.3)$$

Combining equations produces the small-signal voltage gain, as follows:

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{-g_m\, r_\pi\, R_C}{R_S + r_\pi + (1+\beta)\left(R_E \,//\, \dfrac{1}{s\,C_E}\right)} \qquad (4.4)$$

Bypass Capacitor Effects (3)

Expanding the parallel combination of R_E and $1/sC_E$ and rearranging terms, we find

$$A_v = \frac{-g_m\, r_\pi\, R_C}{\left[R_S + r_\pi + (1+\beta)R_E\right]} \text{ x } \frac{(1 + s\, R_E C_E)}{\left[1 + \frac{s\, R_E (R_S + r_\pi) C_E}{\left[R_S + r_\pi + (1+\beta)R_E\right]}\right]} \qquad (4.5)$$

Equation 11pi.31 can be written in terms of time constant as

$$A_v = \frac{-g_m\, r_\pi\, R_C}{\left[R_S + r_\pi + (1+\beta)R_E\right]} \text{ x } \frac{1 + s\tau_A}{1 + s\tau_B} \qquad (4.6)$$

The Bode plot of the voltage gain magnitude has two limiting horizontal asymptotes. If we set $s = j\omega$, we can then consider the limit as $\omega \to 0$ and the limit as $\omega \to \infty$. For $\omega \to 0$, C_E acts as an open circuit; for $\omega \to \infty$, C_E acts as a short circuit. From Equation (4.5), we have

$$\left|A_v\right|_{\omega \to 0} = \frac{g_m\, r_\pi\, R_C}{\left[R_S + r_\pi + (1+\beta)R_E\right]} \qquad (4.7)$$

Bypass Capacitor Effects (4)

and $$|A_v|_{\omega\to\infty} = \frac{g_m\, r_\pi\, R_C}{R_S + r_\pi} \qquad (4.8)$$

From these results, we see that for $\omega \to 0$, R_E is included in the gain expression , and for $\omega \to \infty$, R_E is not part of the gain expression, since it has been effectively shorted out by C_E.

If we assume that the time constants τ_A and τ_B in equation (4.6) differ substantially in magnitude, then the corner frequencies due to τ_A and are τ_B

$$f_B = 1/2\pi\tau_B \qquad (4.9(a))$$

$$f_A = 1/2\pi\tau_A \qquad (4.9(b))$$

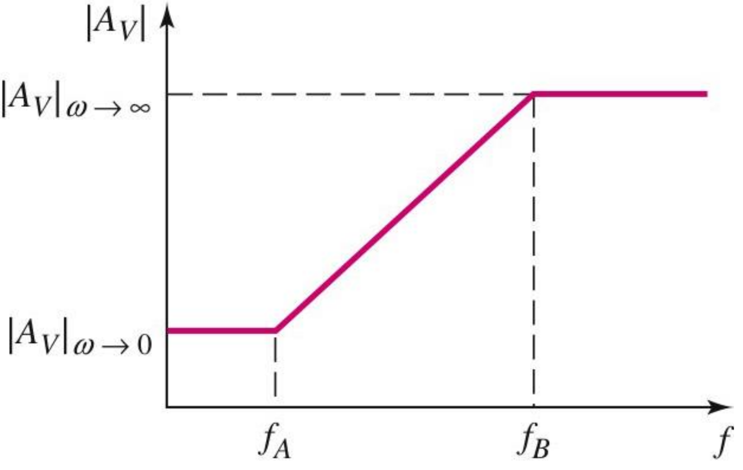


Figure 4.2: Bode plot of the voltage gain magnitude for the circuit with an emitter bypass capacitor.

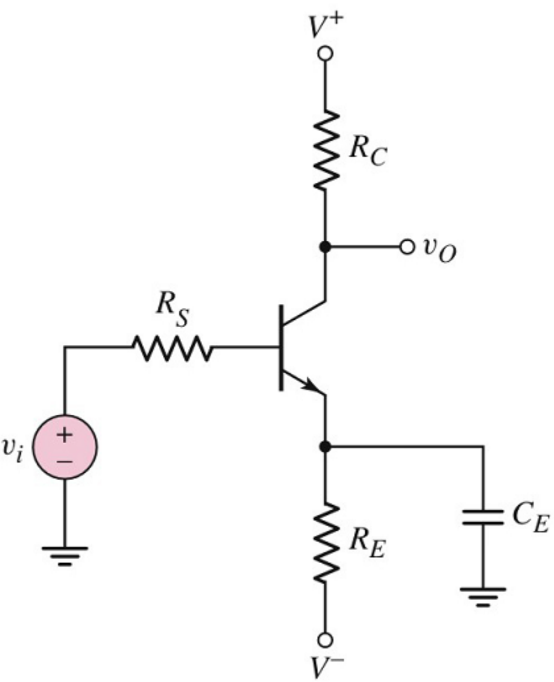
Example 4.1:

Determine the **corner frequencies** and **limiting horizontal asymptotes** of a common-emitter circuit with an emitter bypass capacitor.

Consider the circuit in figure below with parameters $R_E = 4K\Omega$, $R_C = 2K\Omega$

$R_S = 0.5K\Omega$, $C_E = 1\mu\text{F}$, $\text{V}^+ = 5\text{V}$, and $\text{V}^- = -5\text{V}$

The transistor parameters are $\beta = 100$, $V_{BE}(on) = 0.7$, and, $r_o = \infty$



Solution:

A DC analysis shows that $I_{CQ} = 1.06\text{ mA}$. Therefore the transconductance is :

Example 4.1 (Common - Emitter)

Given: $R_E = 4\ k\Omega$ $C_E = 1\ \mu F$ $V+ = 5V$

$R_C = 2\ k\Omega$ $V- = -5\ V$

$R_S = 0.5\ k\Omega$ $\beta = 100$ $V_{BE(on)} = 0.7$

$r_o = \infty$

Calculate: ① f_A . f_B ; ② $|A_v|_{\omega \to 0}$, $|A_v|_{\omega \to \infty}$

Solution:

Step 1: DC analysis

$$KVL \Rightarrow -V^- - V_{BE(on)} = I_{BQ}(R_s + (1+\beta)R_E)$$

$$\Rightarrow I_{BQ} = \frac{4.3}{0.5 + 101 \cdot 4} = 0.0106\ mA$$

$$\Rightarrow I_{CQ} = \beta I_{BQ} = 1.06\ mA$$

step 2: π-model

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.06}{0.026} = 40.77\ mA/V$$

$$r_\pi = \beta / g_m = 2.45\ k\Omega$$

Step 3: Calculate time constants

$$\tau_A = R_E \cdot C_E = 4 \times 1 = 4\ ms$$

$$\tau_B = \frac{R_E(R_S + r_\pi)}{R_S + r_\pi + (1+\beta)R_E} = 2.9 \times 10^{-5}\ S$$

Step4: Calculate corner frequency

$$f_A = \frac{1}{2\pi\tau_A} = 39.8\ HZ$$

$$f_B = \frac{1}{2\pi\tau_B} = 5.49\ KHZ$$

Step5: Voltage gains.

① $\omega \to 0 \Rightarrow \frac{1}{sC_E} \to \infty$

$$|A_v|_{\omega\to 0} = \frac{g_m r_\pi \cdot R_c}{R_s + r_\pi + (1+\beta)R_E} = 0.491$$

② $\omega \to \infty \Rightarrow \frac{1}{sC_E} = 0$

$$|A_v|_{\omega\to\infty} = \frac{g_m r_\pi R_c}{R_s + r_\pi} = 67.7$$

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EEE109: Electronic Circuits

Frequency Response

General Purposes

- Discuss the general frequency response characteristics of amplifiers.
- Derive the system transfer functions
  - Develop the Bode diagrams of the magnitude and phase of the transfer functions.
- Analyze the frequency response of transistor circuits with capacitors.

Contents

- Capacitor Involved in Previous Circuits
- RC Circuits as Lowpass and Highpass filters
- Frequency Response Characteristics of Amplifiers
- Frequency Response of the Common Emitter Amplifiers

Capacitors Involved in Precious Circuits

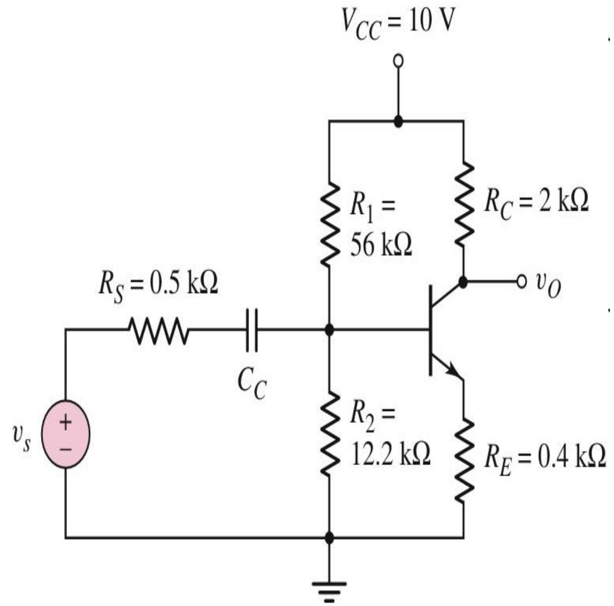
Previously it was assumed that the ***input coupling capacitor***, ***output coupling capacitor*** and ***emitter or source bypass capacitor*** are **short** circuits at **a.c.** signal frequencies of interest and **open** circuits for **d.c.**

At frequencies of interest the impedances of the capacitors are so small compared with the impedances of other circuit elements that they may be regarded as **zero**

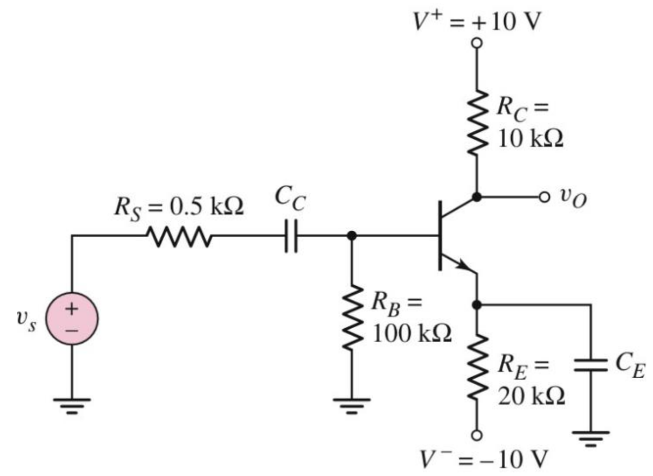
$$\frac{1}{j\omega C_1} \to 0 \qquad \frac{1}{j\omega C_2} \to 0$$

$$\frac{1}{j\omega C_X} \to 0$$

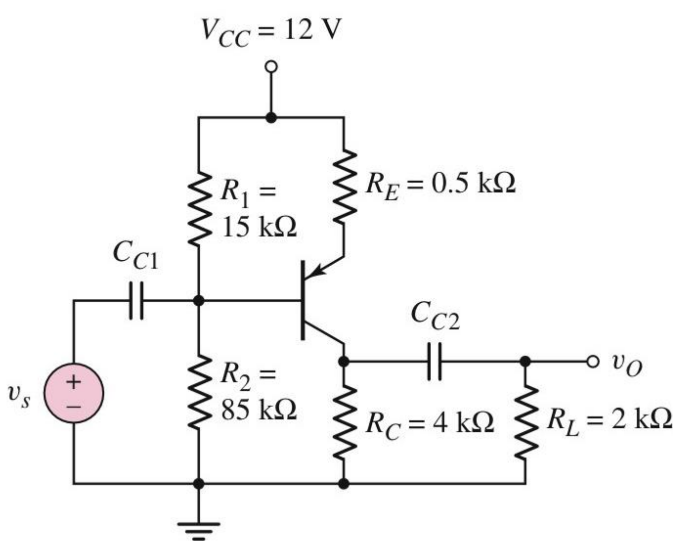
(X is E or S) at the operating frequency $f = \frac{\omega}{2\pi}$



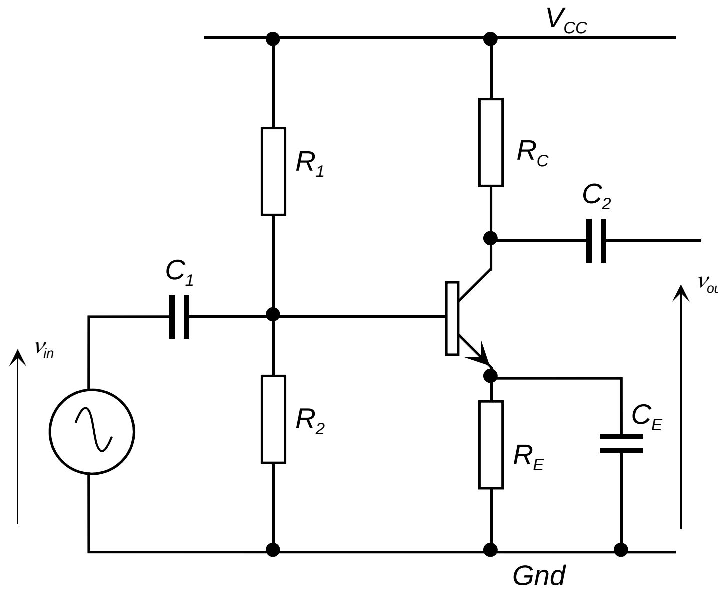
$V_{CC} = 10$ V
$R_1 =$ 56 kΩ
$R_C = 2$ kΩ
$R_S = 0.5$ kΩ
v_O
C_C
v_s
$R_2 =$ 12.2 kΩ
$R_E = 0.4$ kΩ



$V^+ = +10$ V
$R_C =$ 10 kΩ
$R_S = 0.5$ kΩ
C_C
v_O
v_s
$R_B =$ 100 kΩ
$R_E =$ 20 kΩ
C_E
$V^- = -10$ V



$V_{CC} = 12$ V
$R_1 =$ 15 kΩ
$R_E = 0.5$ kΩ
C_{C1}
C_{C2}
v_O
v_s
$R_2 =$ 85 kΩ
$R_C = 4$ kΩ
$R_L = 2$ kΩ



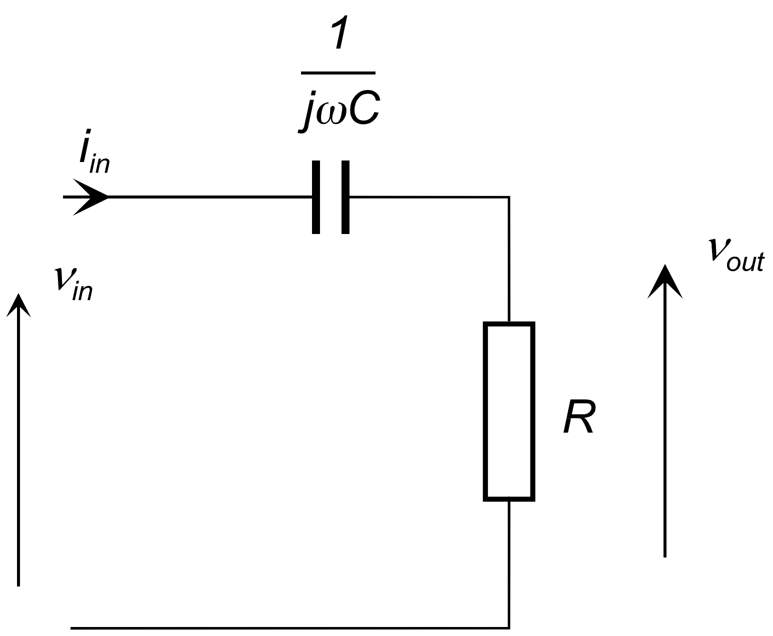
V_{CC}
R_1
R_C
C_2
C_1
v_{out}
v_{in}
R_2
C_E
R_E
Gnd

Contents (Cont')

- Capacitor Involved in Previous Circuits
- RC Circuits as Lowpass and Highpass filters
- Frequency Response Characteristics of Amplifiers
- Frequency Response of the Common Emitter Amplifiers

RC Circuit as a Highpass Filter (1)

Consider the simple *RC* circuit

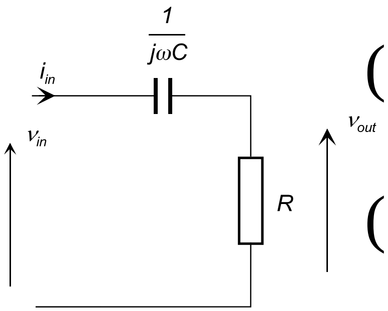


Resistor impedance is R, **capacitor** impedance is $\dfrac{1}{j\omega C}$

and **inductor** impedance is $j\omega L$.

RC Circuit as a Highpass Filter (2)

Kirchoff's Law gives $$v_{in} = i_{in}R - j\frac{i_{in}}{\omega C} \qquad (1.1)$$



Ohm's Law $$v_{out} = i_{in}R \qquad (1.2)$$

Hence $$v_{out} = \frac{\omega^2 R^2 C^2 + j\omega RC}{\omega^2 R^2 C^2 + 1} v_{in} \qquad (1.3)$$

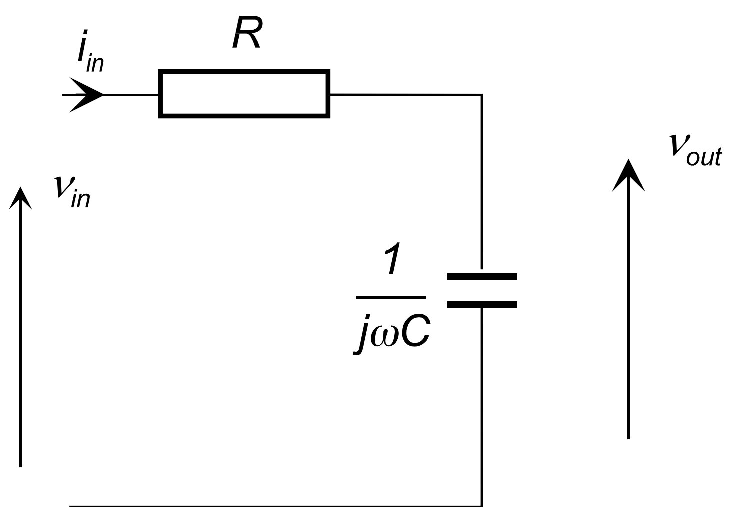
If $\omega \rightarrow 0$ then $v_{out} \rightarrow \frac{0}{1} \rightarrow 0$ and if $\omega \rightarrow \infty$ then $\omega^2 R^2 C^2 >> 1$

and $v_{out} \rightarrow v_{in}$

An output voltage appears across the resistor only if the **input frequency** is **sufficiently high**, this *RC* circuit configuration is a **high pass** filter (a high pass stage).

RC Circuit as a Lowpass Filter (1)

For the same *RC* circuit, but with the **voltage output** taken across the **capacitor**.



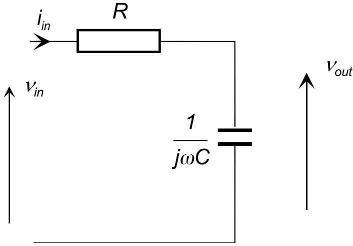
Kirchoff's Law still gives

$$v_{in} = i_{in}R - j\frac{i_{in}}{\omega C} \quad (1.1)$$

but now

$$v_{out} = -i_{in}\frac{j}{\omega C} \quad (1.4)$$

RC Circuit as a Lowpass Filter (2)



Hence $$v_{out} = \frac{1 - j\omega RC}{\omega^2 R^2 C^2 + 1} v_{in} \qquad (1.5)$$

If $\omega \to 0$ then $v_{out} \to \frac{1}{1} v_{in} \to v_{in}$ and if $\omega \to \infty$ then $\omega^2 R^2 C^2 >> 1$ and $v_{out} \to 0$

In this case, $v_{out} \to v_{in}$ if $\omega << \frac{1}{RC}$, i.e. frequency must be **below** some characteristic value for the **output to be close to the input**. This circuit is a **low pass filter**

At $\omega = \frac{1}{RC}$ (or $R = \frac{1}{\omega C}$) the impedances of R and C have **equal magnitude**

ω is the cut-off angular frequency, the cut-off point of an *RC* filter (often ω_0 is used for this value of ω).

RC Circuit as Lowpass/Highpass Filters

From 1.3 and 1.5

$$\left|\frac{\nu_{out}}{\nu_{in}}\right|_{\substack{high \\ pass}} = \frac{\omega RC}{\sqrt{\omega^2 R^2 C^2 + 1}} \quad (1.6)$$

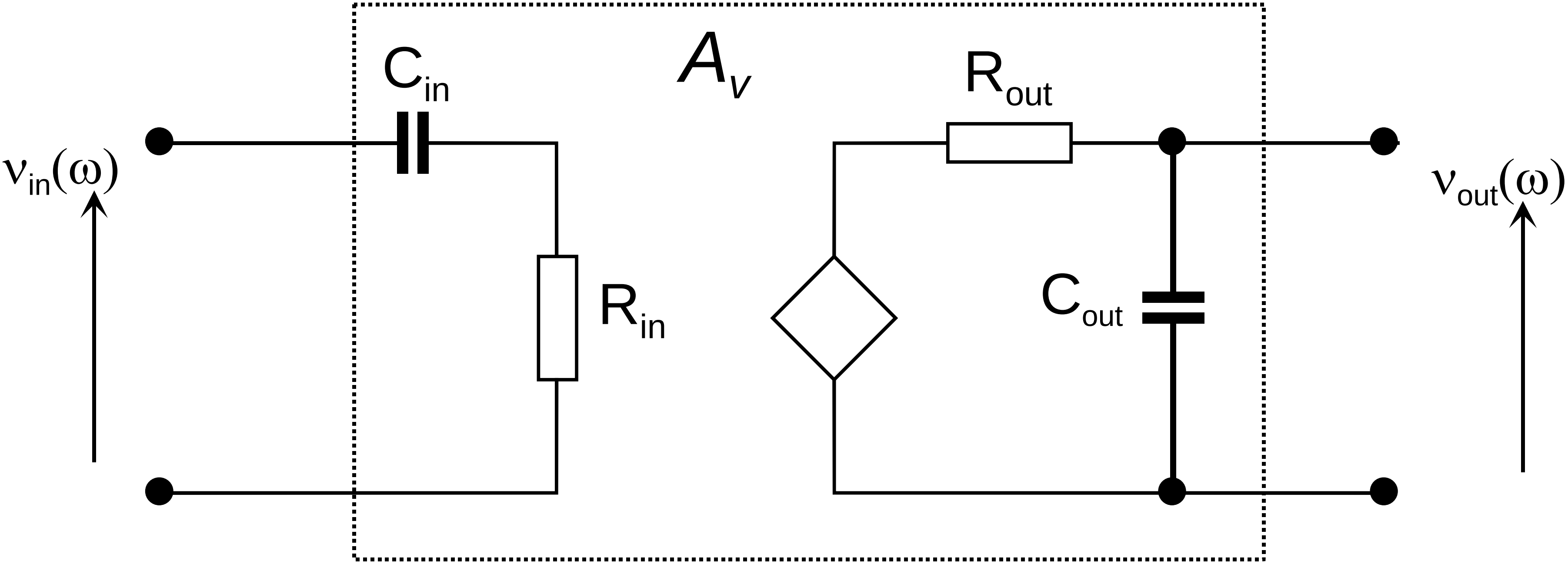
$$\left|\frac{v_{out}}{v_{in}}\right|_{\substack{low \\ pass}} = \frac{1}{\sqrt{\omega^2 R^2 C^2 + 1}} \quad (1.7)$$

Contents (Cont')

- Capacitor Involved in Previous Circuits
- RC Circuits as Lowpass and Highpass filters
- Frequency Response Characteristics of Amplifiers
- Frequency Response of the Common Emitter Amplifiers

Frequency Response Characteristics of Amplifiers (1)

Frequency response characteristics may be included in the **generic four terminal amplifier** as



Frequency Response Characteristics of Amplifiers (2)

The input is a **high pass stage**, C_{in} and R_{in}. The output is a **low pass stage**, C_{out} and R_{out}. The cut-off frequency of the **high pass filter** at a much lower frequency than that of the low pass filter

The input signal $v_{in}(\omega)$ only appears across the input resistance R_{in} when $\omega \gg \dfrac{1}{R_{in}C_{in}}$. For the output voltage to be $v_{out}(\omega) = A_V v_{in}(\omega)$ requires that in addition $\omega \ll \dfrac{1}{R_{out}C_{out}}$

The **voltage gain** of the amplifier is

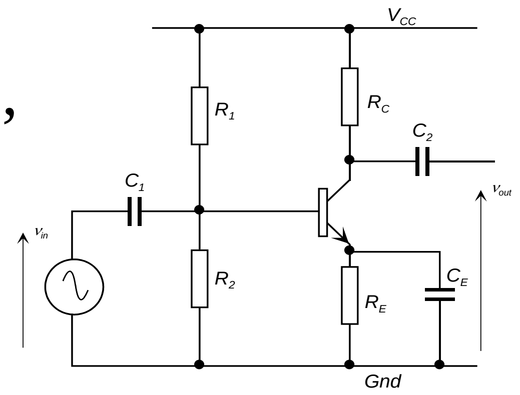
$$|A(\omega)| = \left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right|$$

or in decibels

$$|A(\omega)|_{dB} = 20\,log_{10}\left(\left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right|\right)$$

Frequency Response Characteristics of Amplifiers (3)

For an amplifier $R_{out} << R_{in}$ and $C_{out} << C_{in}$,



hence

$$\frac{1}{R_{out}C_{out}} >> \frac{1}{R_{in}C_{in}}$$

For the single transistor common emitter and common source amplifiers examined, C_{in} is an **equivalent capacitor** representing the effects of **coupling capacitors** C_1 and C_2 combined with the emitter/source **bypass capacitor**.

Important - C_1 is not the input capacitor C_{in} and C_2 is not the output capacitor C_{out}. C_{out} represents **residual capacitance** between the ***output terminal*** and ***earth*** plus effects of ***internal capacitances*** within the transistor. The internal capacitances of transistors are not examined in detail in year 2.

Frequency Response Characteristics of Amplifiers (4)

Five frequency ranges can be identified.

Very low frequency $\omega << \dfrac{1}{R_{in}C_{in}}$

Low frequency $\omega \sim \dfrac{1}{R_{in}C_{in}}$

Mid-range $\omega >> \dfrac{1}{R_{in}C_{in}}$ and $\omega << \dfrac{1}{R_{out}C_{out}}$

High frequency $\omega \sim \dfrac{1}{R_{out}C_{out}}$

Very high frequency $\omega >> \dfrac{1}{R_{out}C_{out}}$

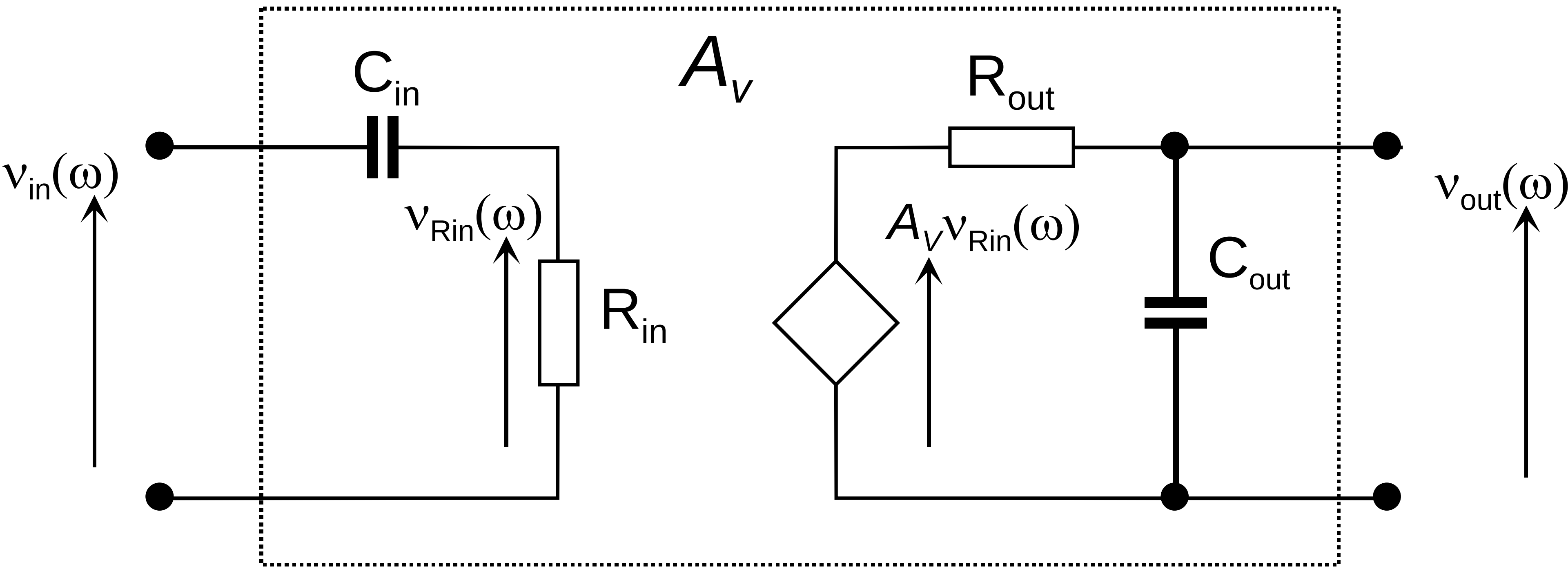
Frequency Response Characteristics of Amplifiers (5)

Mid-range –already examined, neither filter has any significant effect, the output voltage is A_V times the input voltage, that is

$$\left|A(\omega)\right| = \left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right| = A_V$$

Below mid-range the low pass filter in the output has no effect but the high pass filter in the input circuit does affect the circuit. To examine this it is convenient to consider the input circuit as a **voltage divider** with a reduced voltage defined as v_{Rin} developed across R_{in} and then amplified

Frequency Response Characteristics of Amplifiers (6)



Frequency Response Characteristics of Amplifiers (7)

$$\left|\frac{v_{out}}{v_{in}}\right|_{\substack{high\\pass}} = \frac{\omega RC}{\sqrt{\omega^2 R^2 C^2 + 1}}$$

Very low frequency - Equation 1.6 describes the input circuit, if

$\omega << \dfrac{1}{R_{in}C_{in}}$ then 1 is much larger than $\omega^2 R_{in}^2 C_{in}^2$ so the voltage

v_{Rin} across R_{in} is given by 1.6 simplified to $\left|\dfrac{v_{Rin}}{v_{in}}\right| = \omega R_{in} C_{in}$

As a result the **overall behaviour** in this frequency region is

$$|A(\omega)| = \left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right| = A_V \omega R_{in} C_{in}$$

or in dBs $$|A(\omega)|_{dB} = 20\log_{10}\left(\left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right|\right) = 20\log_{10}\left(A_V \omega R_{in} C_{in}\right)$$

$$= 20\,log_{10}\left(A_V R_{in} C_{in}\right) + 20\,log_{10}(\omega)$$

Frequency Response Characteristics of Amplifiers (8)

A plot of gain in dBs against log of frequency at very low frequency is a **straight line rising** as frequency increases. There is enough information to plot gain against frequency at very low frequency and mid frequency (next page).

Note gain at angular frequency 2ω is

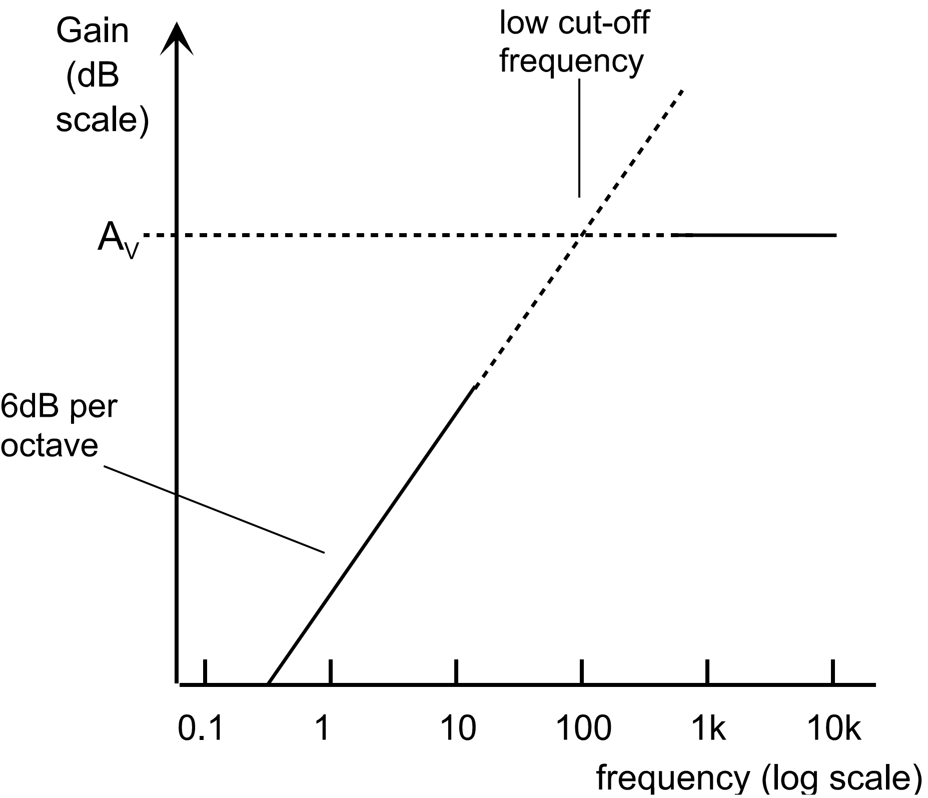
$$\left|A(\omega)\right|_{dB} = 20\,log_{10}\left(A_V R_{in} C_{in}\right) + 20\,log_{10}(2\omega)$$

and the **difference in gain** from ω to 2ω is

$$20\,log_{10}(2) = 6 \text{ dBs.}$$

Gain **increases by 6dBs** when the **frequency doubles** (6dBs **per octave**), calculation for frequency increase of **ten** gives **20dBs** increase, 20dBs **per decade**.

Frequency Response Characteristics of Amplifiers (9)



Increasing the straight line to meet the horizontal line of constant gain, A_V, shows that they cross when

$$0 = log_{10}(\omega R_{in} C_{in})$$

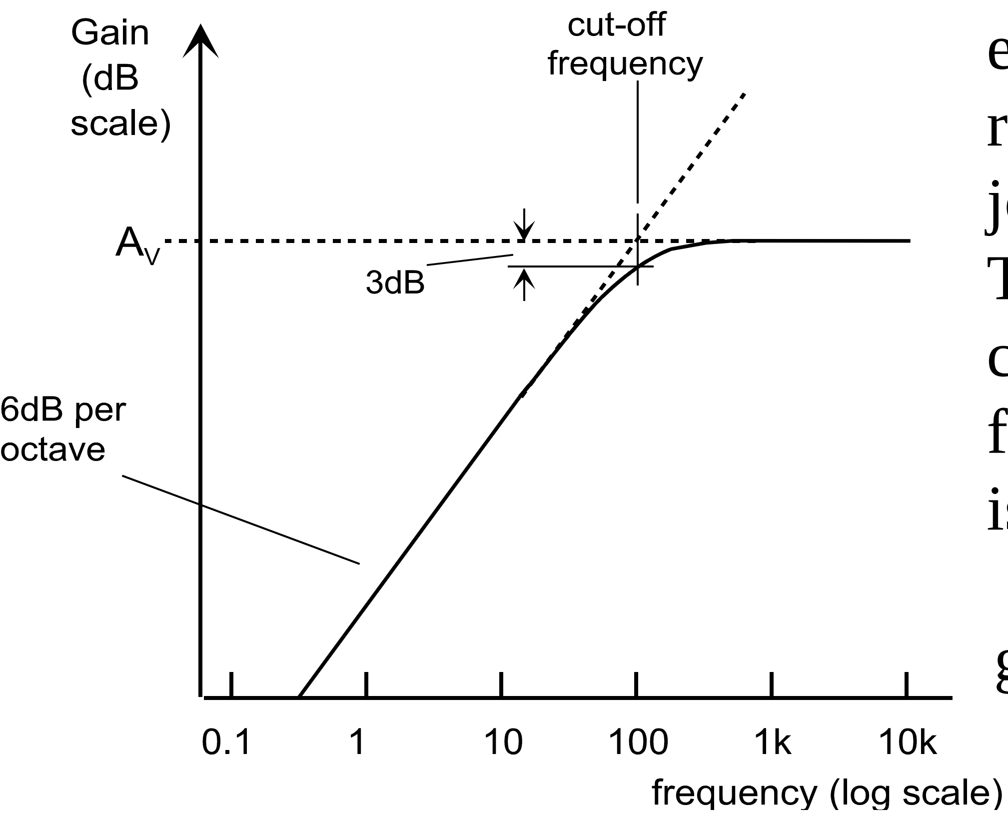
or $$1 = \omega R_{in} C_{in}$$,

i.e. at the low cut-off frequency

$$\omega = \frac{1}{R_{in} C_{in}}$$

Low frequency Say about **0.2 times to 5 times** the cut off.

Frequency Response Characteristics of Amplifiers (10)



In this region the exact form of equation 1.6 must be used – the results will form a smooth curve joining the two straight lines. The exact form can be used to calculate the gain at the cut-off frequency, the result is that the gain is $\frac{1}{\sqrt{2}}$ **lower** than the **mid-frequency** gain.

Therefore the cut-off is also known as the lower -3dB frequency or **3dB point**.

Frequency Response Characteristics of Amplifiers (11)

Above mid-range the situation is the **reverse** of the **low frequency one**. The high pass filter in the input has no effect but the low pass filter in the output does. The treatment is similar to the low pass case. Use the low pass result in equation 1.7 at the output of the amplifier. Now there is a **high frequency cut-off point** – also with gain 3dB below mid frequency - given by

$$\omega = \frac{1}{R_{out}C_{out}}$$

Very high frequency is when and $\omega \gg \dfrac{1}{R_{out}C_{out}}$ and the gain falls at 6dB per octave (20dB per decade). Equation 1.7 gives the approximate result

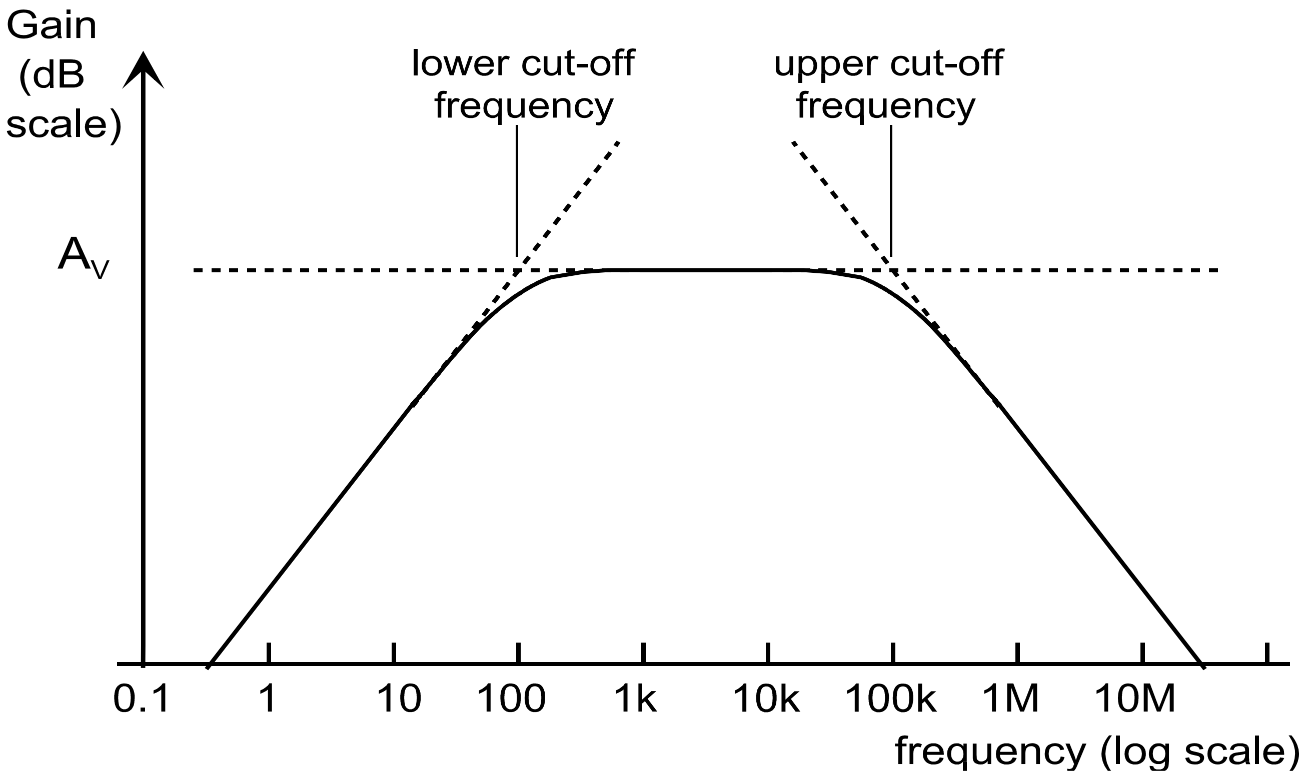
$$|A(\omega)| = \left|\frac{v_{out}(\omega)}{v_{in}(\omega)}\right| = \frac{A_V}{\omega R_{out}C_{out}}$$

or in dBs

$$|A(\omega)|_{dB} = 20\log_{10}\left(\frac{A_V}{R_{out}C_{out}}\right) - 20\log_{10}(\omega)$$

Frequency Response Characteristics of Amplifiers (12)

High frequency region - the exact form of equation 1.7 must be used.



The complete plot of gain in decibels against frequency on a log scale is known as a **Bode plot**.

Contents (Cont')

- Capacitor Involved in Previous Circuits
- RC Circuits as Lowpass and Highpass filters
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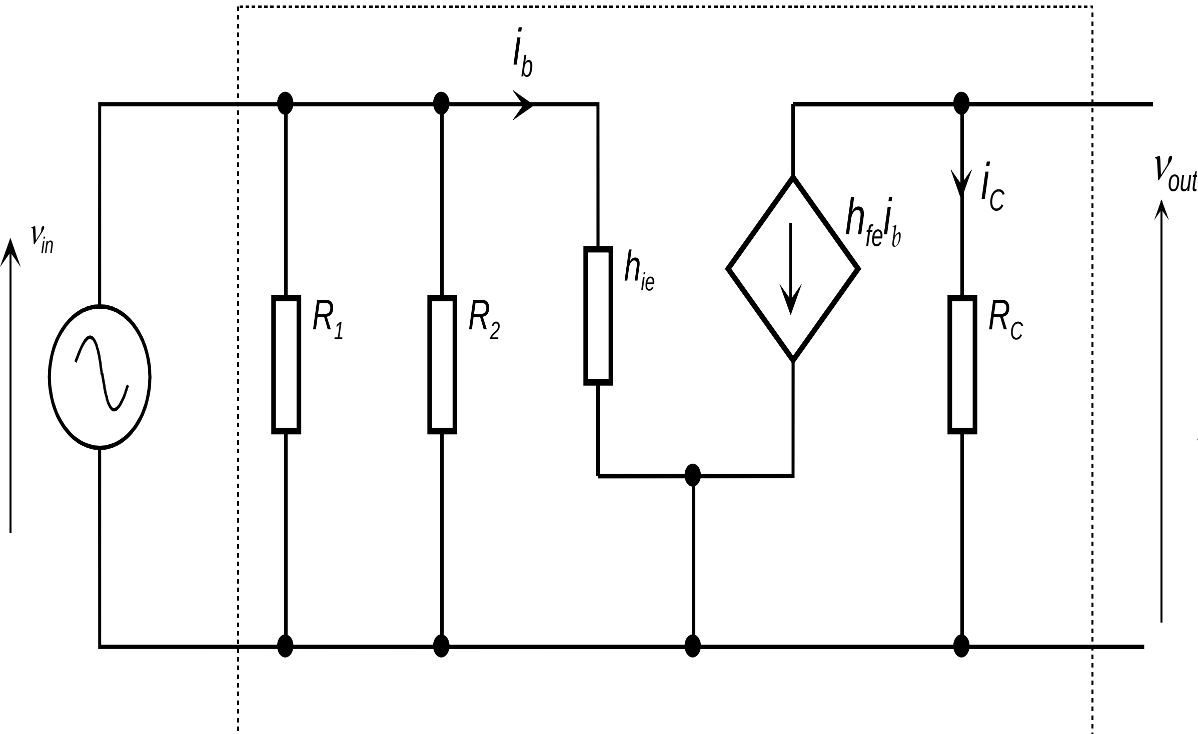
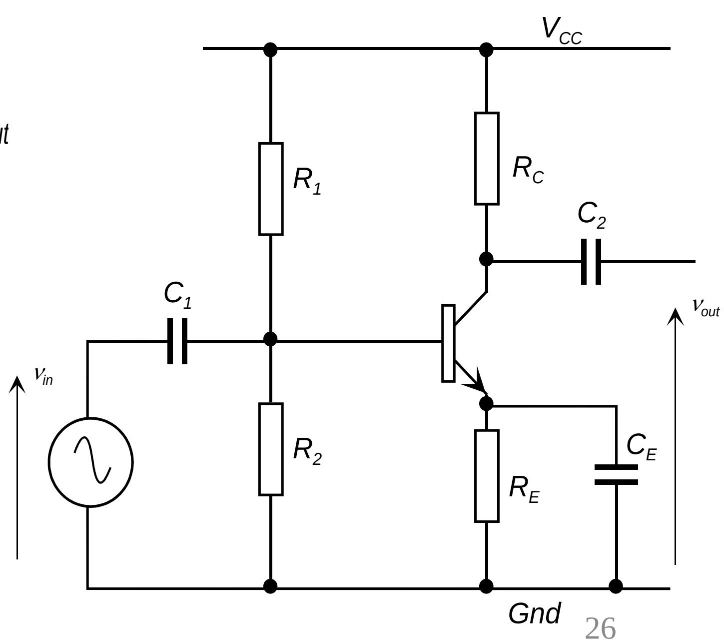
Frequency Response of the Common Emitter Amplifier (1)

The **common emitter amplifier** is examined - treatment of the common source amplifier is similar.

Mid Frequency

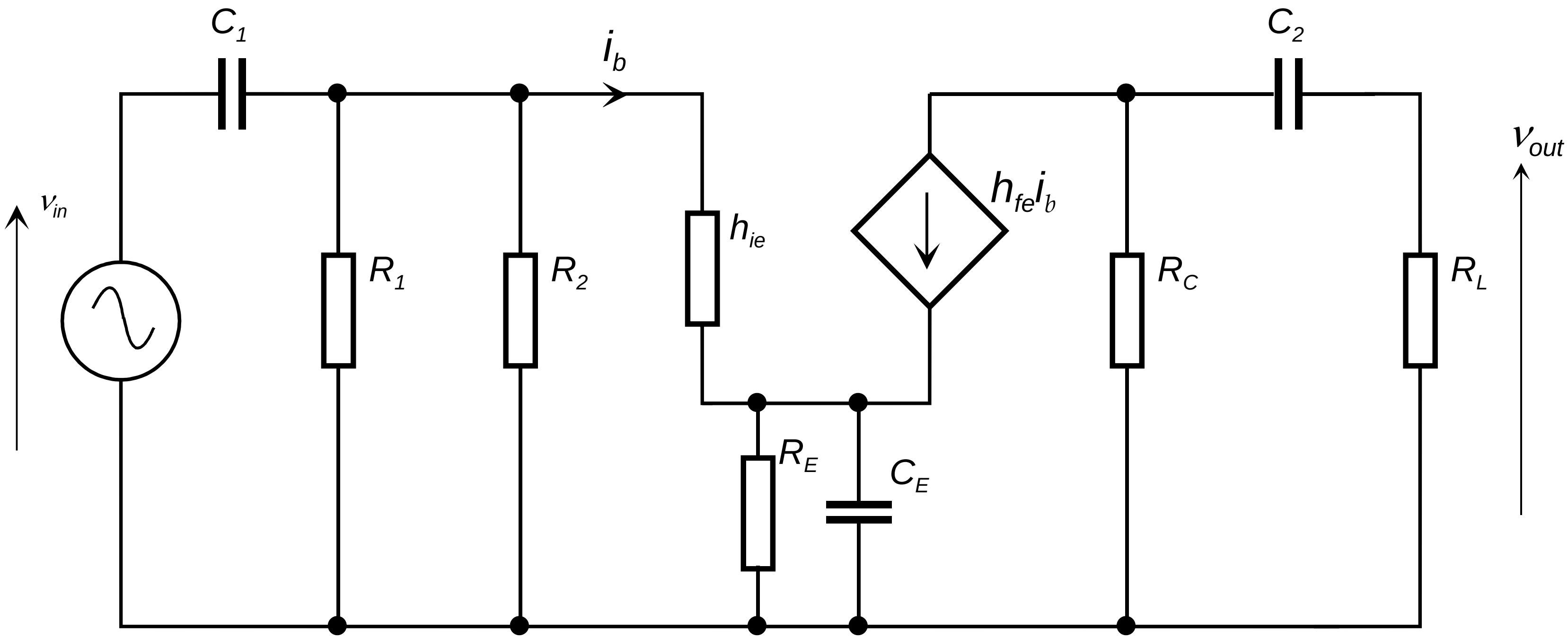
Small signal equivalent circuit for the CE amplifier developed earlier (assuming all capacitors are short circuits) is:





Frequency Response of the Common Emitter Amplifier (2)

Applies to the **mid-frequency region** ONLY, at other frequencies the capacitors must be included



Frequency Response of the Common Emitter Amplifier (3)

All the capacitors affect the **low frequency cut-off frequency** - all are parts of ***three high pass filters***. The high frequency cut-off is set by the transistor properties **and stray capacitances** in the circuit**.**

To find the lower 3dB frequency, $f_L = \dfrac{\omega_L}{2\pi}$

the effects of all three high pass filters must be *determined* and *combined*. **One approach** is the method of **short circuit time constants,** this is approximate but gives a result which is close to the exact one. Each capacitor circuit is examined with the other capacitors assumed so large at the frequency concerned that they **behave as short circuits**. That is it can be considered that the capacitor being examined sets the low frequency cut-off, the others would cause much lower values.

Frequency Response of the Common Emitter Amplifier (4)

- Approximate, but the result is **close** to the **exact** one.

- Useful because each capacitor is examined with the other capacitors assumed so large at the cut-off frequency that they are **short circuits** (a common approach by designers).

- It is as though the capacitor being examined sets the low frequency cut-off, the others would cause much lower values.

- Analyse each capacitor circuit as an **equivalent high-pass stage**.

- To calculate the **resistance** of the high-pass stage short circuit the source and make all the other **capacitors** have **infinite** value at the frequency considered ($\frac{1}{\omega C} \to 0$ as $\omega C \to \infty$).

Frequency Response of the Common Emitter Amplifier (5)

Take each possible case of two of the capacitors very large so they may be neglected.

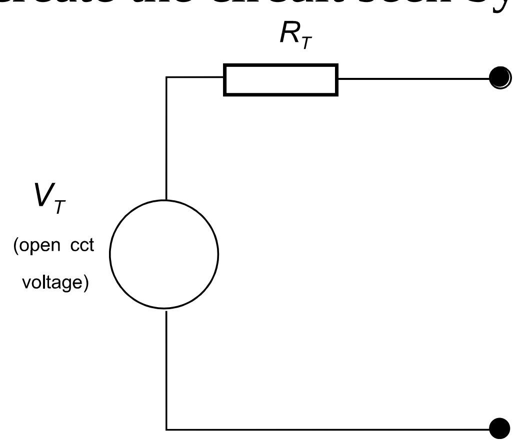
Case 1. If $C_E = C_2 \rightarrow \infty$ then $C1$ forms part of a low pass stage with associated resistance R_{C1} where

$$R_{C1} = R_B \| h_{ie} = \frac{R_B h_{ie}}{R_B + h_{ie}} \quad \text{and} \quad R_B = R_1 \| R_2 \qquad (2.1)$$

Case 2 At the output; the current generator has a source resistance (so must be open circuited) then

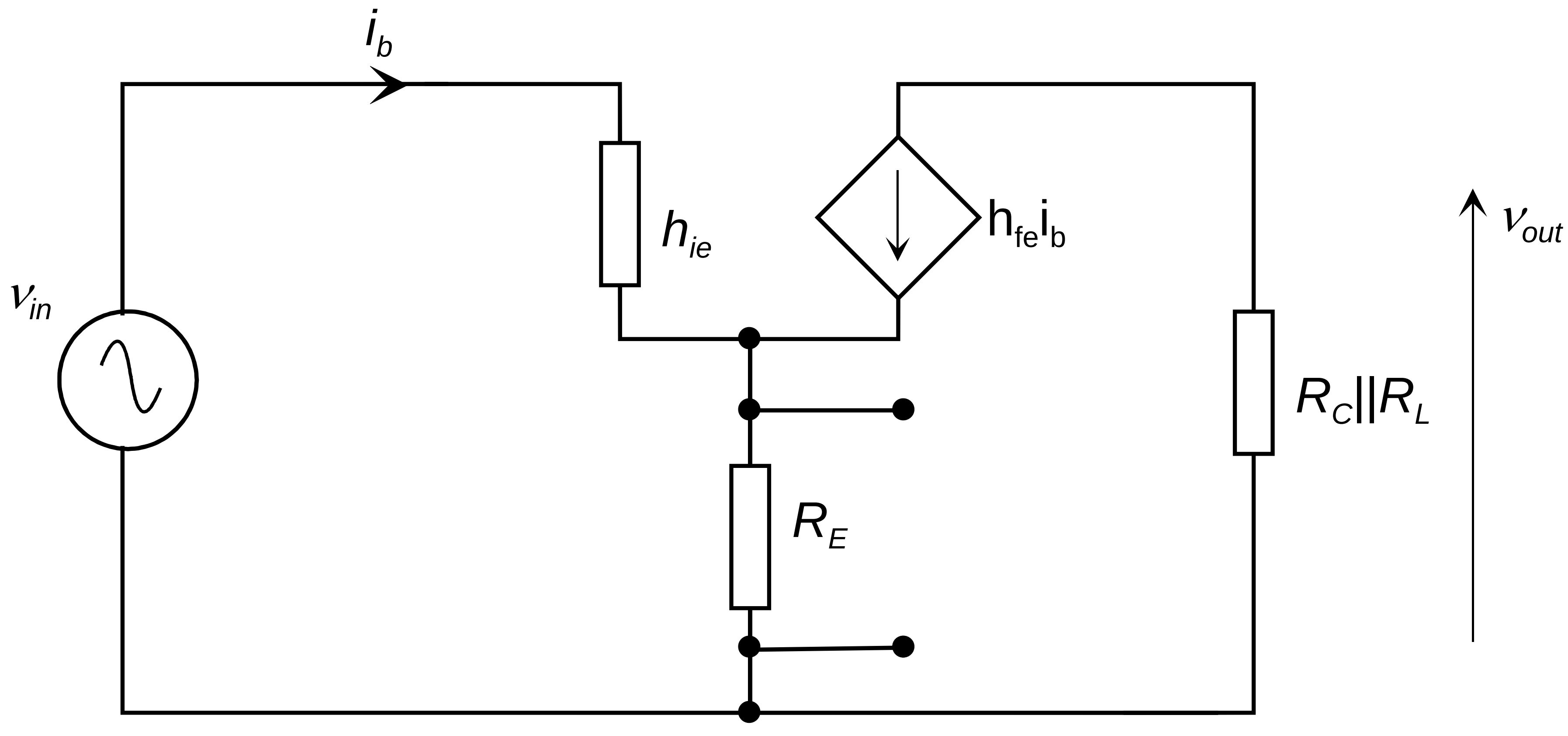
$$R_{C2} = R_C + R_L \qquad (2.2)$$

Case 3 Calculation of R_{CE} is a trickier because it is complicated by the coupling between input and output circuits via the current generator. Use **Thévenin's Theorem**to create the circuit seen by C_E.



Frequency Response of the Common Emitter Amplifier (6)

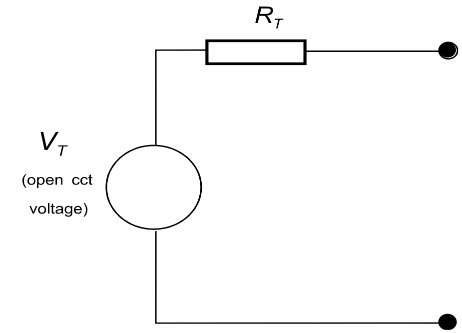
Take all the circuit *except* C_E and examine it with C_1 and C_2 set as short circuits. Call the R_C and R_L combination R_{out}. R_1 and R_2 are removed because they are across a **perfect voltage source**, the circuit is



Frequency Response of the Common Emitter Amplifier (7)

An alternative method of finding R_T for the Thévenin equivalent is

$$R_T = \frac{v_{oc}}{i_{sc}}$$

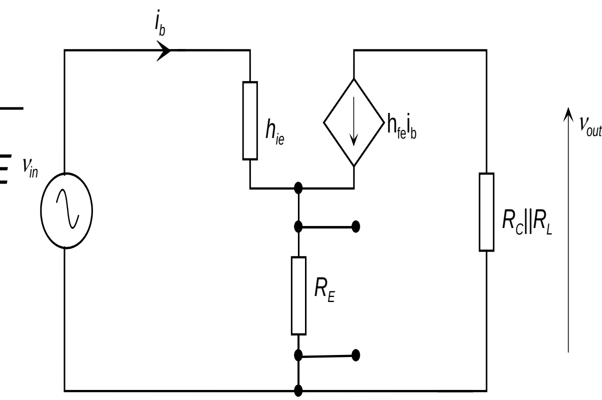


$$v_{oc} = V_T = i_{boc}(1 + h_{fe})R_E$$

When **open circuit** $\quad i_{boc} = \dfrac{v_{in}}{h_{ie} + (1 + h_{fe})R_E}$

Hence $\quad v_{oc} = V_T = v_{in}\dfrac{(1 + h_{fe})R_E}{h_{ie} + (1 + h_{fe})R_E}$

$$v_{oc} = V_T = i_{boc}(1 + \beta)R_E$$



When short circuit $\quad i_{sc} = i_{bsc}(1 + h_{fe})$

but as all input voltage is across h_{ie} in this case $\quad i_{bsc} = \dfrac{v_{in}}{h_{ie}}$

Frequency Response of the Common Emitter Amplifier (8)

Putting it all together

$$R_T = R_{CE} = \frac{\nu_{oc}}{i_{sc}} = \frac{h_{ie}R_E}{h_{ie} + (1 + h_{fe})R_E} \qquad (2.3)$$

The full form of 2.3 is usually required. **Often** h_{ie} and R_E have similar magnitude, if they are assumed equal then the effective resistance across C_E is approx. $\boldsymbol{R_E}$ divided by $(1 + h_{fe})$ and the required magnitude of C_E for a particular cut-off frequency is about $(1 + h_{fe})$ times larger than might be expected.

Once R_{C1}, R_{C2} and R_{CE} are known, the approximate low frequency **3dB point** occurs at

$$\omega_L \cong \frac{1}{C_1 R_{C1}} + \frac{1}{C_2 R_{C2}} + \frac{1}{C_E R_{CE}} \qquad (11.4)$$

Frequency Response of the Common Emitter Amplifier (9)

In practice a designer would usually attempt to make **one capacitor** determine the **low frequency response**.

That is set one at the value for $\omega_L = \dfrac{1}{RC}$ give cut-off and make all

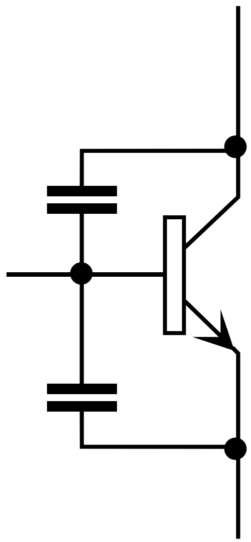
the others much larger so they frequencies well below ω_L

Either make C_E the **controlling capacitor** because it is going to be the largest and this avoids having to make it even larger

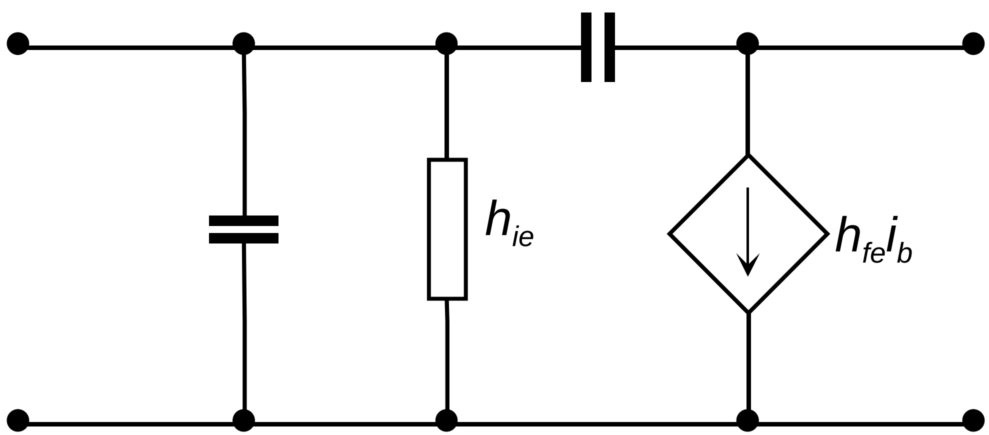
or make C_1 the **controlling capacitor** because it is good design to put filters removing unwanted signals as early as possible in the circuit.

Frequency Response of the Common Emitter Amplifier (10)

Detailed consideration of **high frequency** behaviour is outside the second year course as it depends on transistor properties not yet examined. This is a very brief introduction. At high frequencies **internal capacitances** of the transistor have a significant effect, the revised small signal equivalent circuit is



(a)



(b)

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EEE109: Electronic Circuits

Output Stages and Power Amplifiers

Contents

- Describe the concept of a power amplifier.
- Describe the characteristics of BJT and MOSFET power transistors
  - analyze the temperature and heat flow characteristics of devices using heat sinks.
- Define the various classes of power amplifiers and determine the maximum power efficiency of each class of amplifier.
  - class-A power amplifiers.
  - class-AB power amplifiers.
- Design an output stage using power MOSFETs as the output devices.

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Outlines

- Introduction of Power Amplifier
- Power Efficiency and Amplifier Classification
- Basic Class A Amplifier

Introduction (1)

- An electronic **power amplifier**, amplifier, or (informally) amp is an electronic device that **increases** the **power of a signal**.

- It does this by taking energy from a power supply and controlling the output to *match* the input signal shape but with a **large amplitude**. In this sense, an amplifier modulates the output of the power supply.

Introduction (2)

- Power amplifiers are used to deliver a relatively **high amount of power**, usually to a **low resistance load**.

- Typical load values range from 300W (for transmission antennas) to 8W (for audio speaker).

- Although these load values do not cover every possibility, they do illustrate the fact that power amplifiers **usually drive low-resistance loads**.

Introduction (3)

- Typical output power rating of a power amplifier will be **1W or higher**.

- **Ideal** power amplifier will deliver **100%** of the power it draws from the supply to load. In **practice**, this can never occur.

- The reason for this is the fact that the *components* in the amplifier will all **dissipate** some of the power that is being drawn form the *supply*.

Introduction (4)

- Numerous types of electronic amplifiers are specialized to various applications.

- An amplifier can refer to anything from an electrical circuit that uses a single active component, to a complete system such as a packaged audio hi-fi amplifier.

Introduction (5)

- In general a power amplifier is designated as the **last amplifier** in a transmission chain (the **output stage**) and is the amplifier stage that typically requires most attention to power efficiency.

- **Efficiency considerations** lead to various classes of power amplifier based on the **biasing** of the **output transistors.**

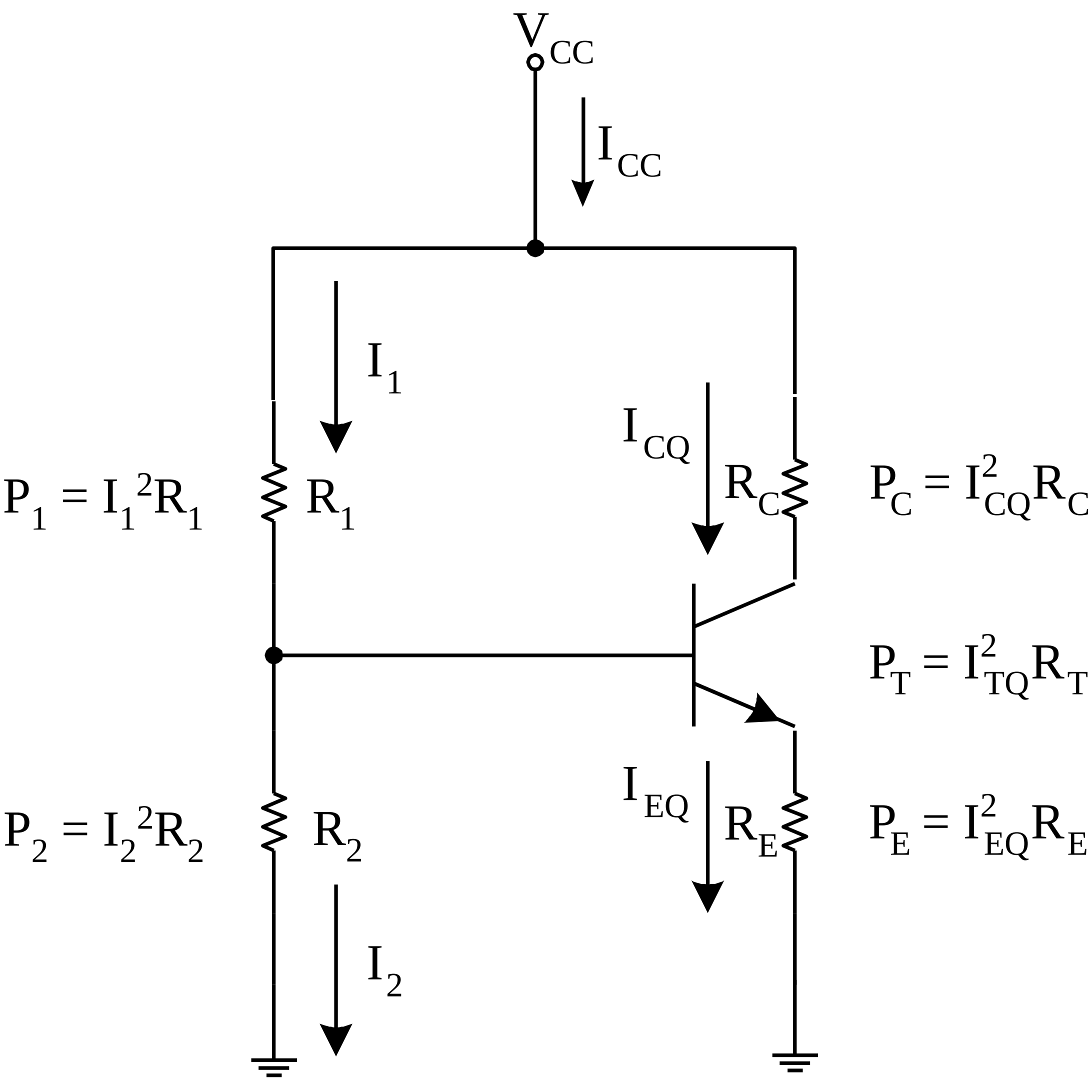
Amplifier Power Dissipation

The **total** amount of power being dissipated by the amplifier, $\boldsymbol{P_S}$, is

$$P_S = P_1 + P_2 + P_C + P_T + P_E$$

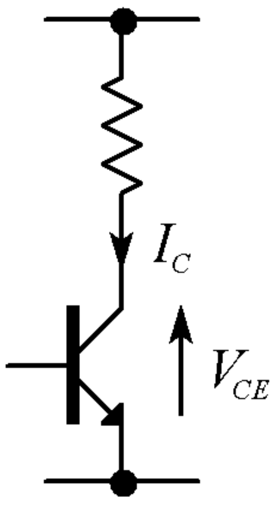
The difference between this total value and the total power being drawn from the supply is the power that actually goes to the **load – i.e. output power**.

$\Rightarrow$ **Amplifier Efficiency η**



Efficiency / Dissipation

The efficiency, η , of an amplifier is the ratio between the power delivered to the load and the total power supplied: $\eta = \frac{P_L}{P_S}$



Power supply requirements and transistor power dissipation ratings depend on the efficiency.

Power that isn't delivered to the load will be dissipated by the output device(s) in the form of heat.

$$P_D = P_S - P_L$$

$$= V_{CE} I_C$$

(for amplifier shown)

Amplifier Efficiency η

- A **figure of merit** for the power amplifier is its efficiency, η .
- **Efficiency (η)** of an amplifier is defined as the ratio of ac output power (power delivered to load) to dc input power .
- By formula :

$$\eta = \frac{ac \quad output \quad power}{dc \quad input \quad power} \times 100\% = \frac{P_o(ac)}{P_i(dc)} \times 100\%$$

- As we will see, certain amplifier configurations have much higher efficiency ratings than others.
- This is primary consideration when deciding which type of power amplifier to use for a specific application.

- $\Rightarrow$ **Amplifier Classifications**

Power amplifier classes

- Power amplifiers are classified according to the **percent of time** that **collector current** is **nonzero**.

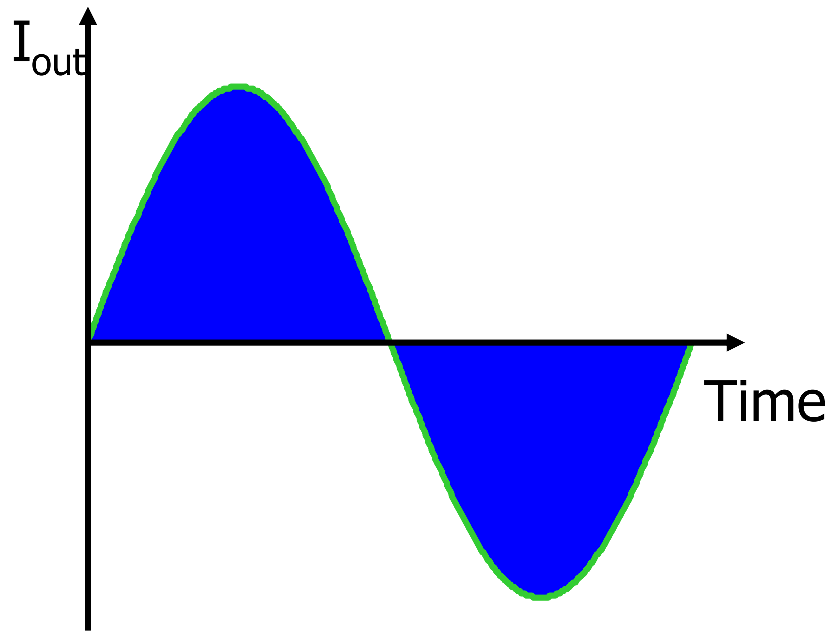
- The amount the ***output*** signal varies over **one cycle** of operation for a **full cycle** of ***input*** signal.

- Power amplifier circuits (output stages) are classified as **A**, **B**, **AB** and **C** for analog designs, and class **D** and **E** for switching designs based on the proportion of each input cycle (**conduction angle**), during which an amplifying device is *passing current*.

Conduction Angle

- The image of the conduction angle is derived from amplifying a sinusoidal signal.
- If the device is always on, the conducting angle is 360° .
- If it is on for only half of each cycle, the angle is 180° .
- The angle of flow is closely related to the amplifier **power efficiency**.

Class A Operating Mode

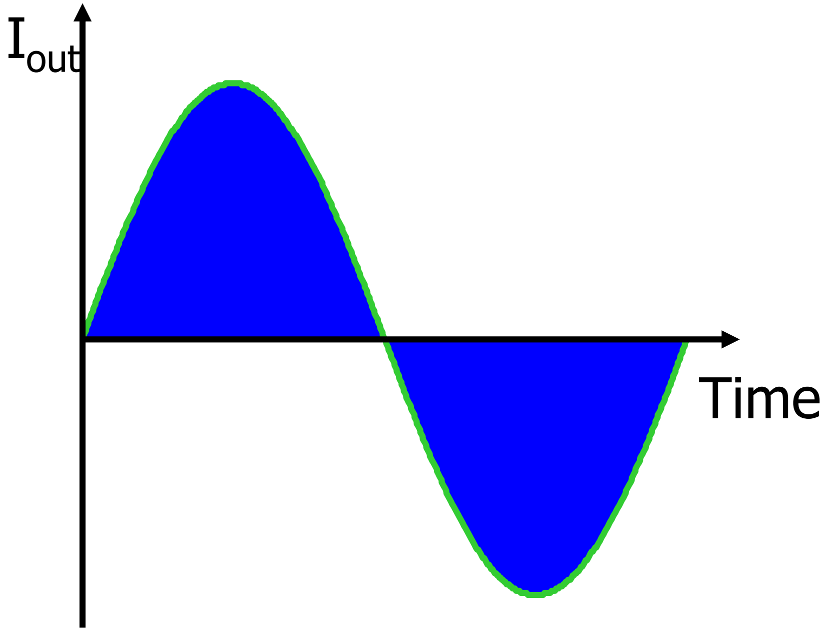


One device conducts for the whole of the a.c. cycle.

Conduction angle = 360°.

The Class A stage must be biased at a current greater than the amplitude of the signal current.

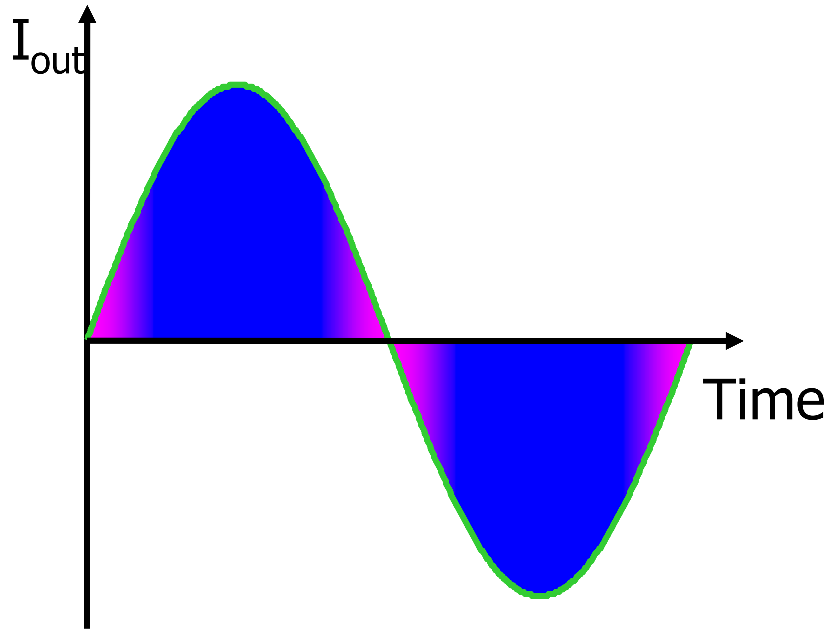
Class B Operating Mode



Two devices, each conducting for half of the a.c. cycle.

Conduction angle = 180°.

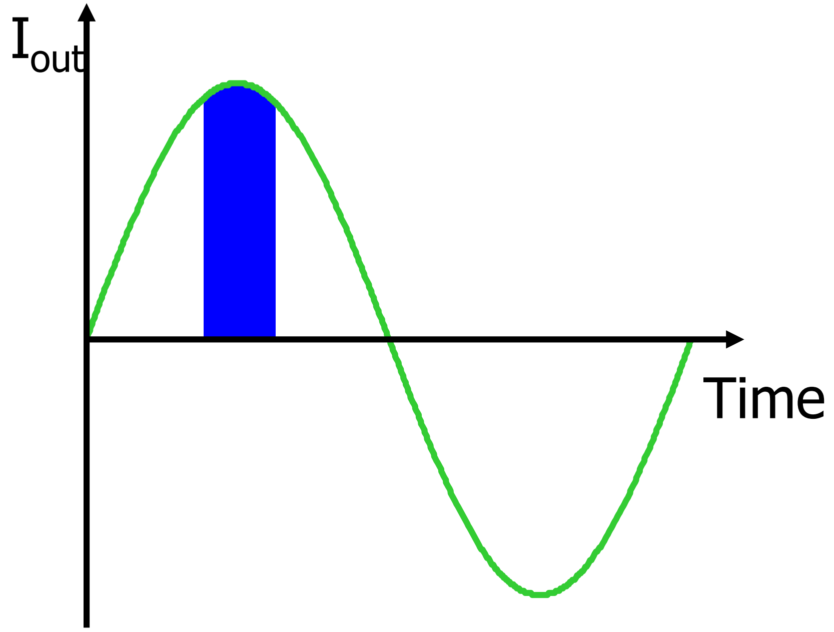
Class AB Operating Mode



Two devices, each conducting for just over half of the a.c. cycle.

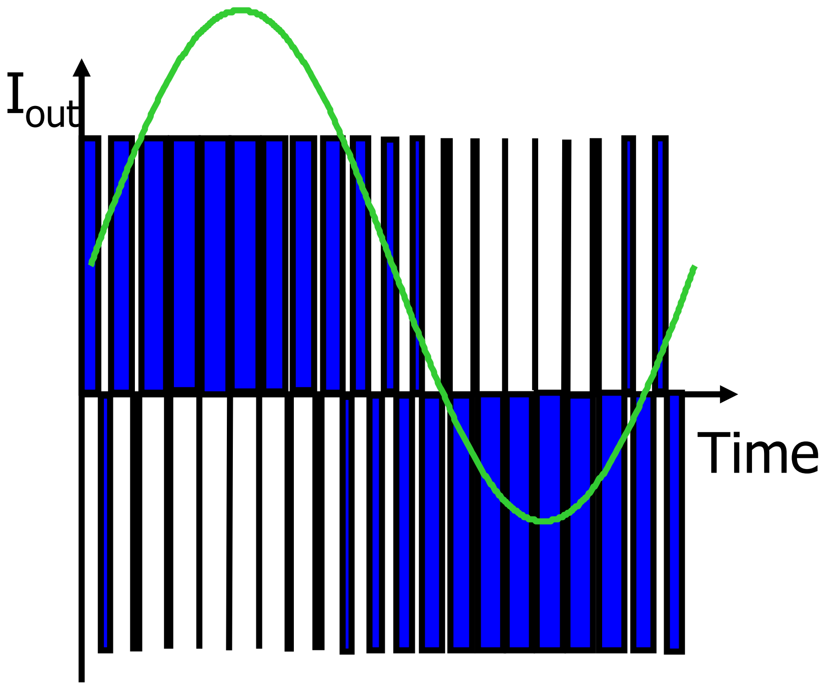
Conduction angle > 180 ° but << 360 ° .

Class C Operating Mode



One device conducts a small portion of the a.c. cycle.
Conduction angle << 180 ° .

Class D Operating Mode



Each output device always either fully on or off – theoretically zero power dissipation.
Example: The built-in speaker in a PC is driven by a Class D type “on/off’ circuit.

Efficiency Ratings

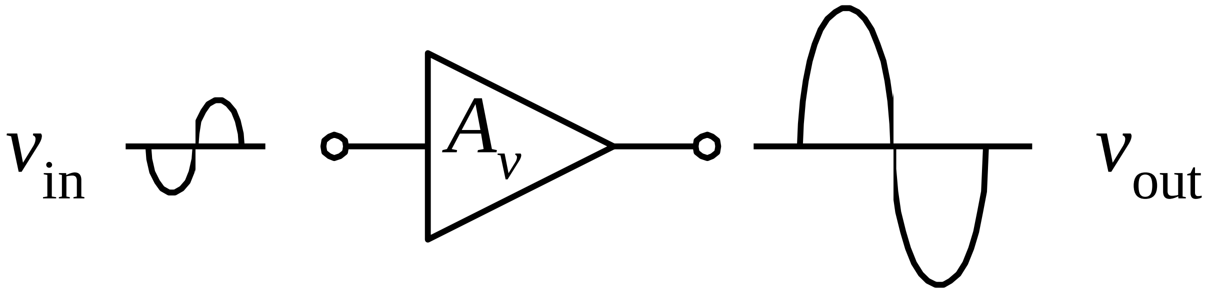
- The **maximum theoretical efficiency** ratings of class-A, B, and C amplifiers are:

Amplifier	Maximum Theoretical Efficiency, $\eta_{\mathbf{max}}$
Class A	25%
Class B	78.5%
Class C	99%

Differences Between Classes

- Class A : Linear operation, very inefficient.
- Class B : High efficiency, non-linear response.
- Class AB : Good efficiency and linearity, more complex than classes A or B though.
- Class C : Very high efficiency but requires narrow band load.
- Class D : Very high efficiency but requires low pass filter on load. Complex and expensive to get high quality results.

Class A Amplifier



- v_{output} waveform $\rightarrow$ **same shape** $\rightarrow$ v_{input} waveform **+ 180° phase shift.**

- The collector current is **nonzero** 100% of the time.

- The active element remains conducting all of the time.

Class A Amplifier

- 100% of the input signal is used (conduction angle $\Theta = 360^\circ$).

- Amplifying devices operating in class A conduct over the whole of the input cycle.

- A class-A amplifier is distinguished by the **output stage** being biased into class A.

$\rightarrow$ **inefficient**, since even with zero input signal, I_{CQ} is nonzero (i.e. transistor dissipates power in the rest, or quiescent, condition)

Advantages of class-A amplifiers

- Class-A designs are **simpler** than other classes; for example class-AB and –B designs require two devices (push–pull output) to handle both halves of the waveform; Class A can use a single device single-ended.

- Because the device is never shut off completely there is no "turn on" time, little problem with charge storage, and generally better high frequency performance and feedback loop stability (and usually **fewer high-order harmonics**).

Disadvantages of class-A amplifiers

- They are very **inefficient**.

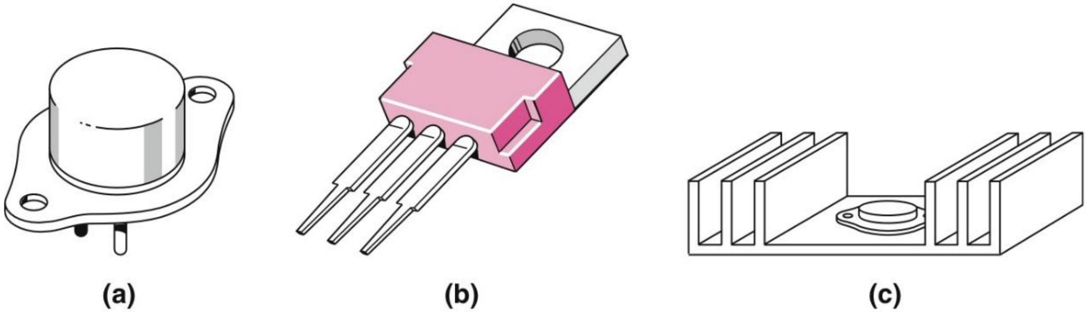
- A theoretical maximum of 50% is obtainable with **inductive** output coupling and only 25% with **capacitive** coupling.

- If **high output powers** are needed from a class-A circuit, the **power waste** (and the accompanying heat) becomes ***significant***.

Disadvantages of class-A amplifiers

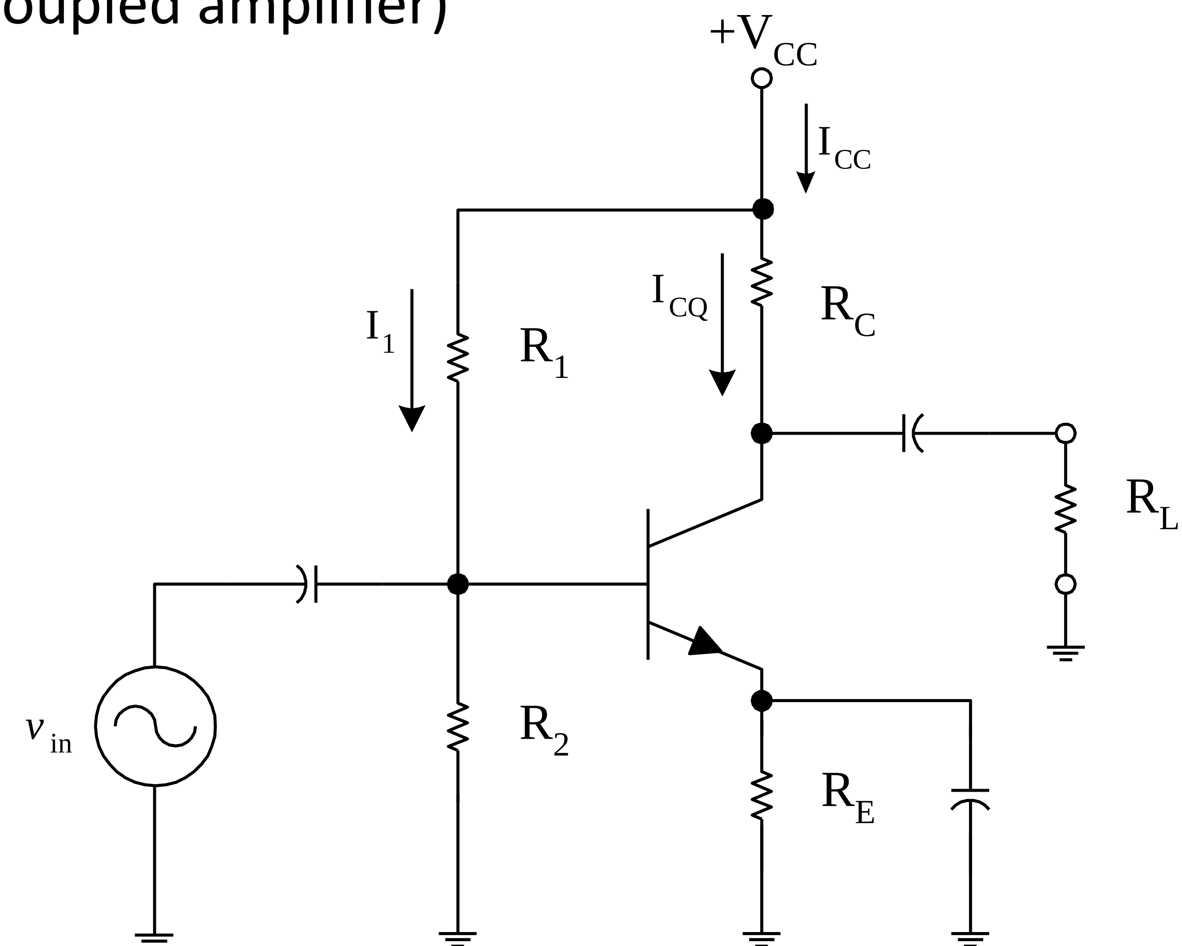
- For **every watt** delivered to the load, the amplifier itself, at best, dissipate **another watt**.

- For large powers this means very large and *expensive* **power supplies** and **heat sinking**.

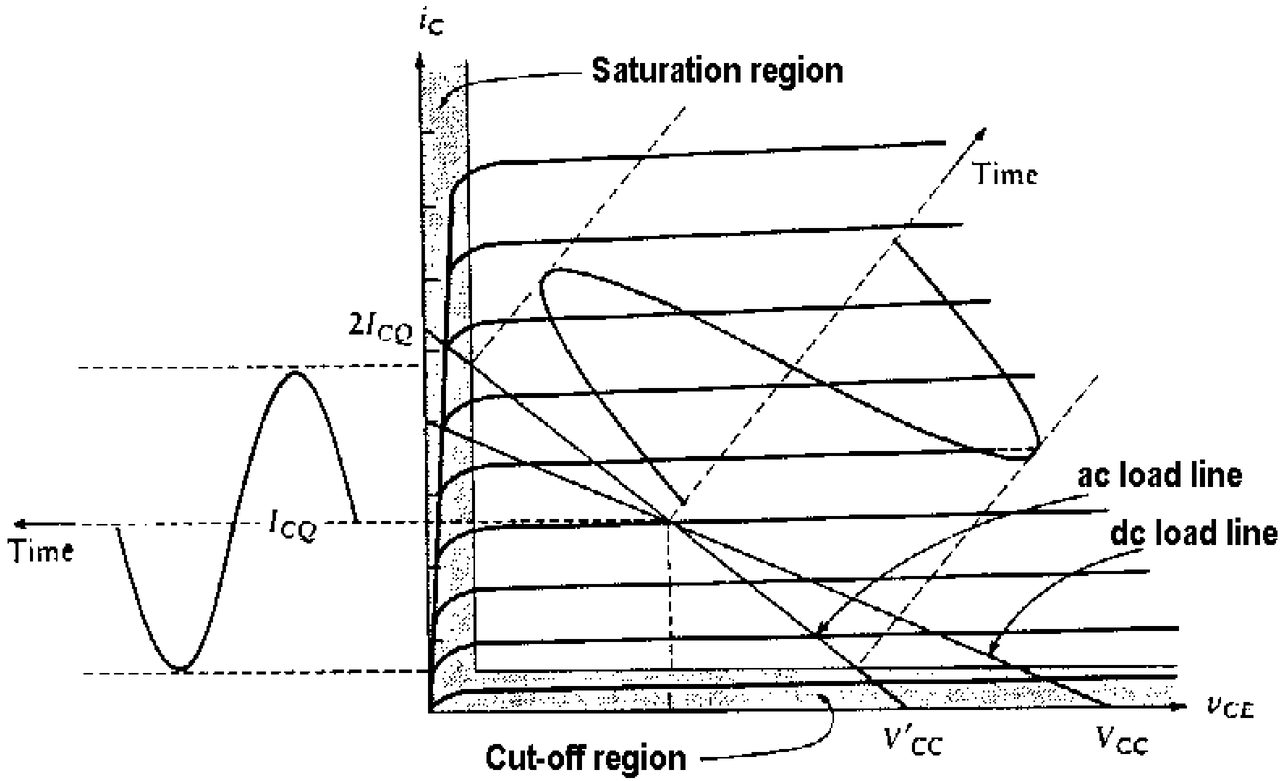


Basic Operation of Class-A Amplifier with CE Configuration

- Common-emitter (voltage-divider) configuration (RC-coupled amplifier)



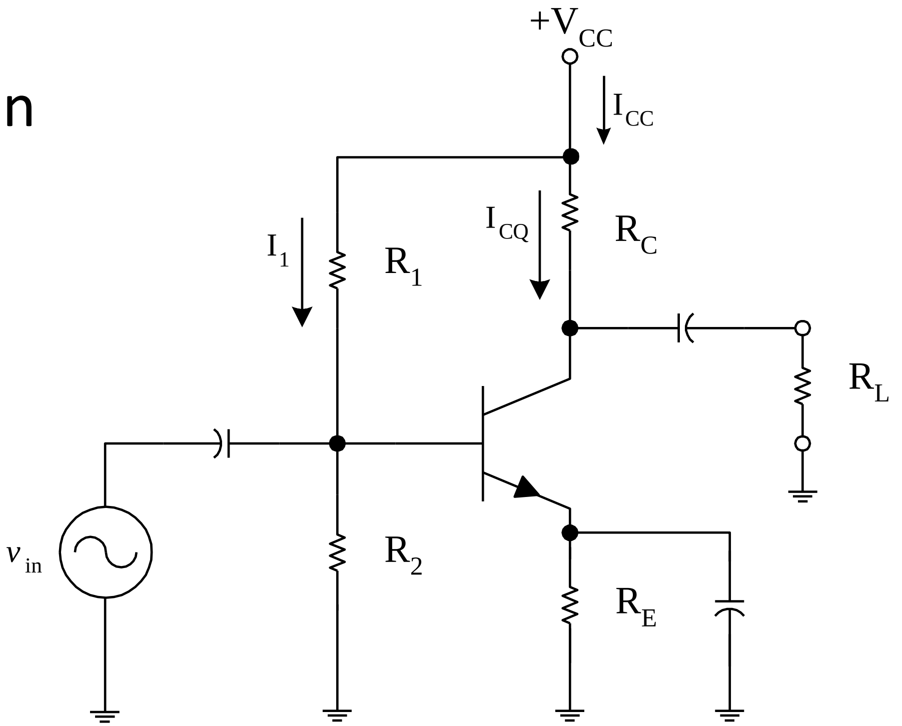
Typical Characteristic Curves for Class-A Operation



The current, I_{CQ} , is usually set to be in the **center** of the ac load line.

DC Input Power

The total dc power, $P_i(dc)$, that an amplifier draws from the power supply :



$$P_i(dc) = V_{CC} I_{CC}$$

$$I_{CC} = I_{CQ} + I_1$$

$$I_{CC} \approx I_{CQ} \qquad (I_{CQ} >> I_1)$$

$$P_i(dc) = V_{CC} I_{CQ}$$

Note that this equation is valid for most amplifier power analyses.

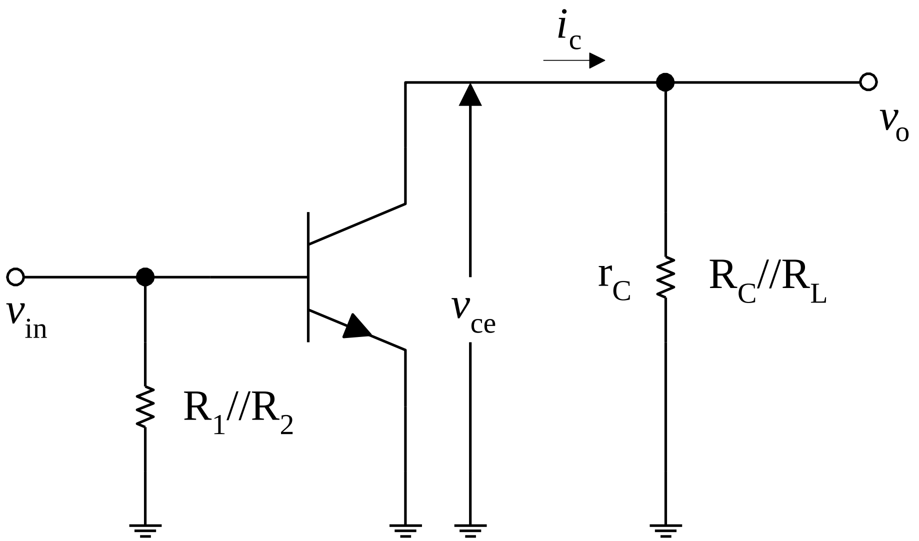
We can rewrite for the above equation for the **ideal** amplifier as

$$P_i(dc) = 2V_{CEQ} I_{CQ}$$

AC Output Power

AC output (or load) power, $P_o(ac)$

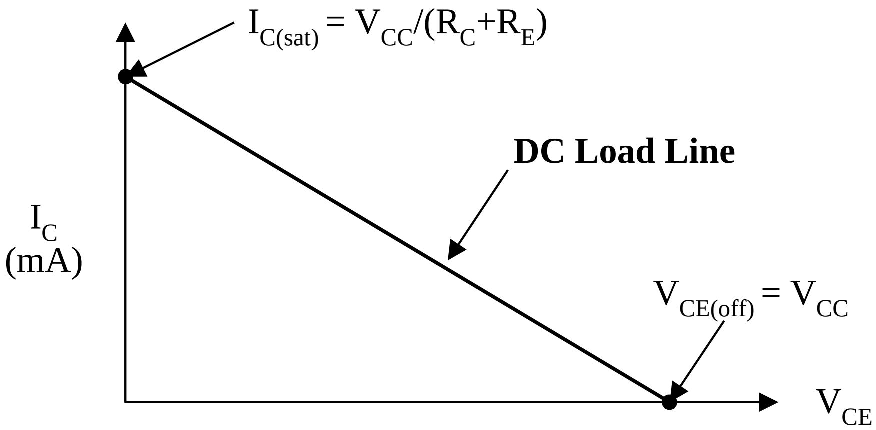
$$P_o(ac) = i_{c(rms)} v_{o(rms)} = \frac{v_{o(rms)}^2}{R_L}$$

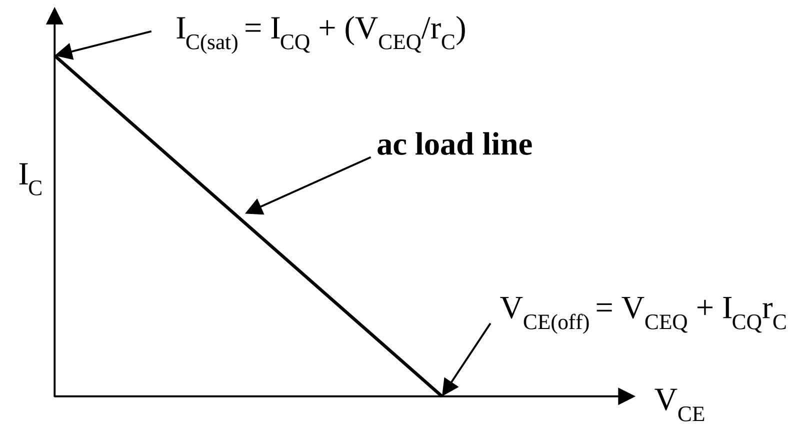


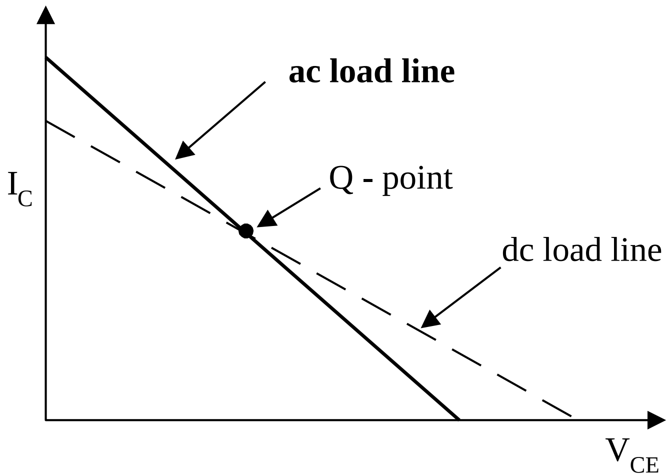
Above equations can be used to calculate the **maximum** possible value of ac load power.

Disadvantage of using class-A amplifiers is the fact that their efficiency ratings are so low, $\eta_{max} \approx 25\%$.

A majority of the power that is drawn from the supply by a class-A amplifier is used up by the amplifier itself.



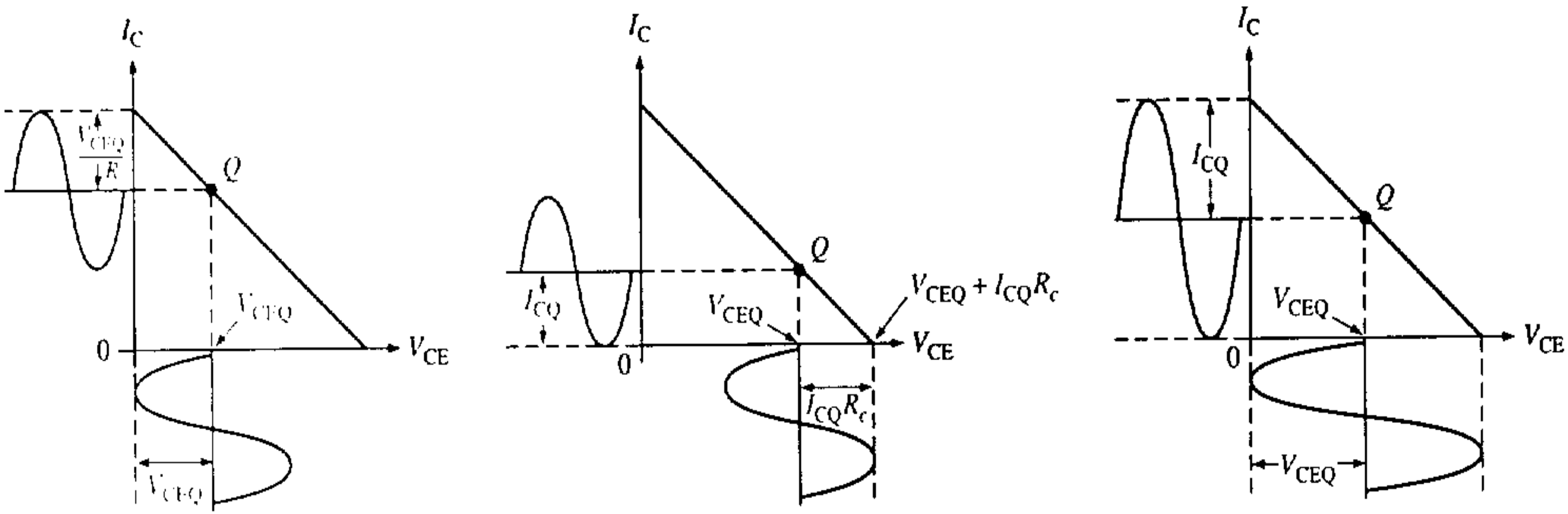




$$P_o(ac) = \left(\frac{V_{CEQ}}{\sqrt{2}}\right)\left(\frac{I_{CQ}}{\sqrt{2}}\right) = \frac{1}{2}V_{CEQ}I_{CQ} = \frac{V_{PP}^2}{8R_L}$$

$$\eta = \frac{P_{o(ac)}}{P_{i(dc)}} \times 100\% = \frac{\frac{1}{2}V_{CEQ}I_{CQ}}{2V_{CEQ}I_{CQ}} \times 100\% = 25\%$$

Limitation



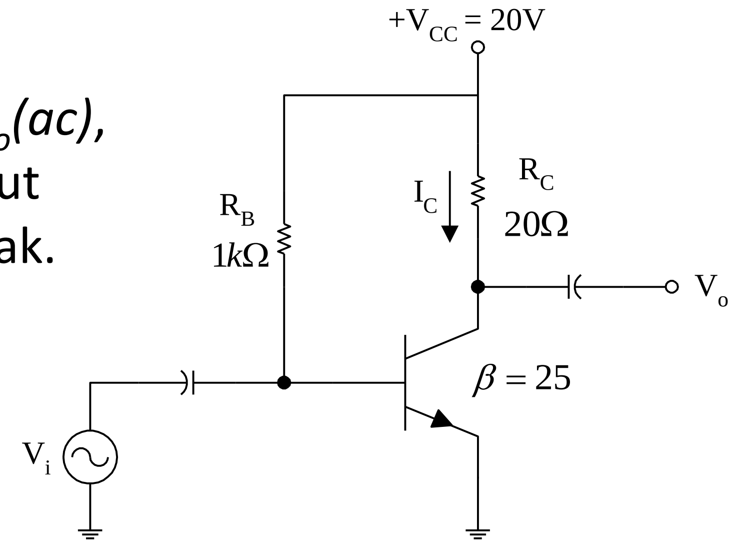
(a) Limited by saturation

(b) Limited by cutoff

(c) Centered Q-point

Example

Calculate the input power $P_i(dc)$, output power $P_o(ac)$, and efficiency ***h*** of the amplifier circuit for an input voltage that results in a base current of 10mA peak.



$$I_{BQ} = \frac{V_{CC} - V_{BE}}{R_B} = \frac{20V - 0.7V}{1k\Omega} = 19.3mA$$

$$I_{CQ} = \beta I_B = 25(19.3mA) = 482.5mA \cong 0.48A$$

$$V_{CEQ} = V_{CC} - I_C R_C = 20V - (0.48A)(20\Omega) = 10.4V$$

$$I_{c(sat)} = \frac{V_{CC}}{R_C} = \frac{20V}{20\Omega} = 1000mA = 1A$$

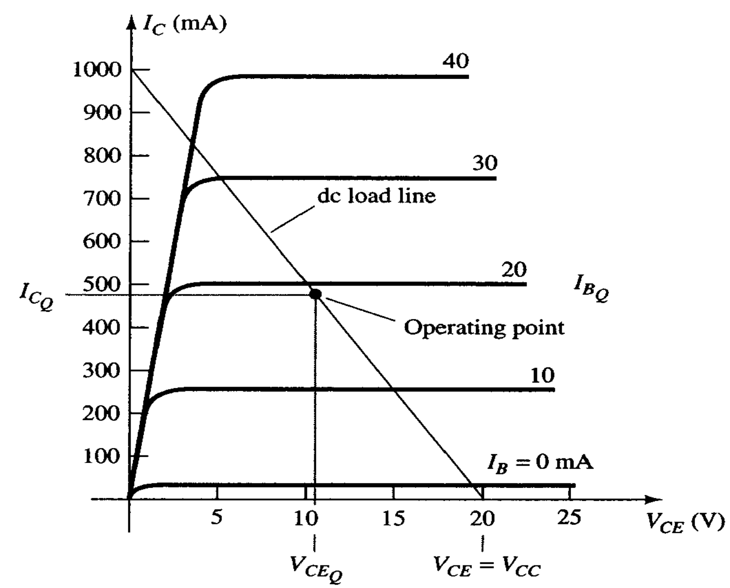
$$V_{CE(cutoff)} = V_{CC} = 20V$$

$$I_{C(peak)} = \beta I_{b(peak)} = 25(10mA\ peak) = 250mA\ peak$$

$$P_{o(ac)} = \frac{I^2_{C(peak)}}{2} R_C = \frac{\left(250\times10^{-3}A\right)^2}{2}(20\Omega) = 0.625W$$

$$P_{i(dc)} = V_{CC} I_{CQ} = (20V)(0.48A) = 9.6W$$

$$\eta = \frac{P_{o(ac)}}{P_{i(dc)}} \times 100\% = 6.5\%$$



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EEE109: Electronic Circuits

The Field Effect Transistor

Contents

- Study and understand the operation and characteristics of the various types of MOSFETs.
- Understand and become familiar with the dc analysis and design techniques of MOSFET circuits.
- Examine three applications of MOSFET circuits.
- Investigate current source biasing of MOSFET circuits, such as those used in integrated circuits.
- Analyze the dc biasing of multistage or multitransistor circuits.
- Understand the operation and characteristics of the junction field-effect transistor, and analyze the dc response of JFET circuits.

General Overview

MOSFET

- Metal-Oxide-Semiconductor Field Effect Transistor - MOSFET

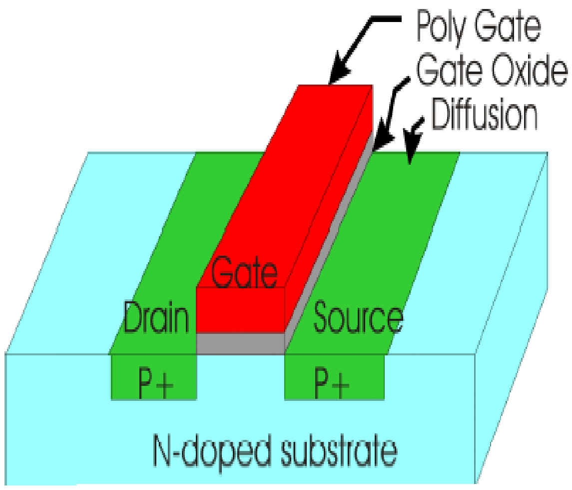
- **Current** is controlled by an electric field applied perpendicular to both semiconductor surface and to the direction of current.

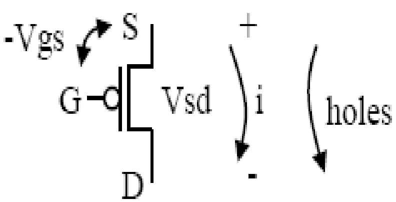
- **Field effect** – Phenomenon to control the current in the semiconductor by applying an electric field perpendicular to the surface.

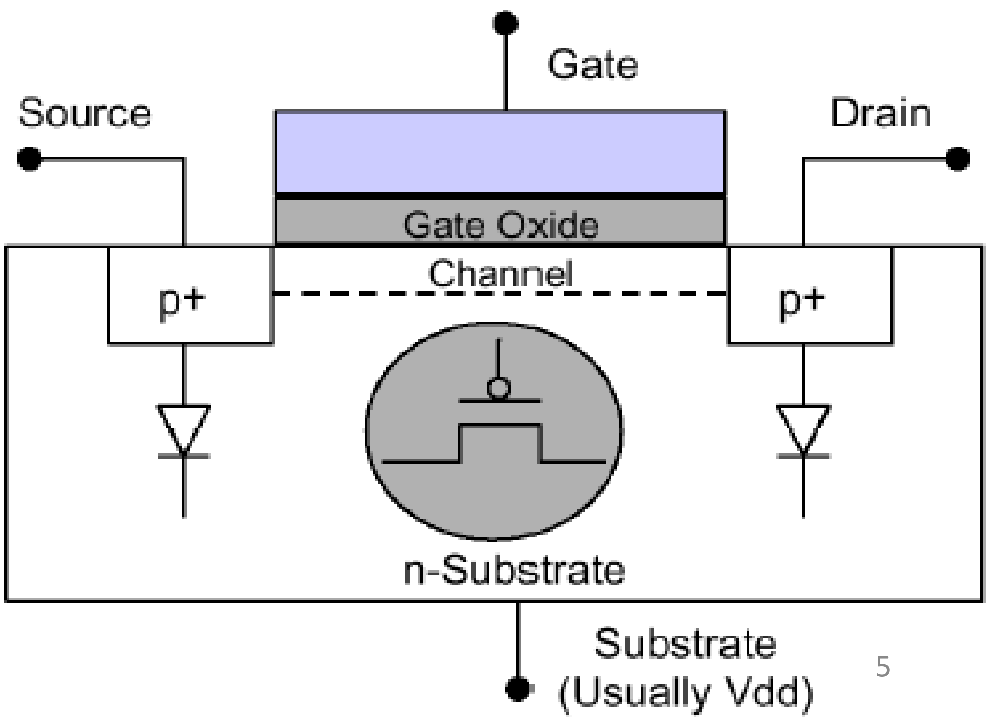
- **The basic transistor principle**: ***The voltage between two terminals controls the current through the third terminal.***

Types of Field-Effect Transistors

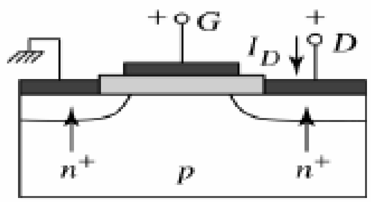
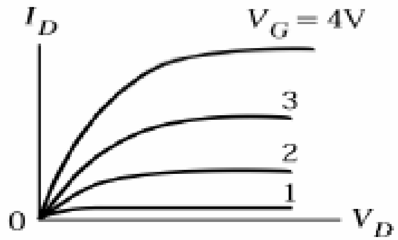
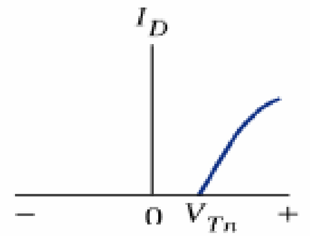
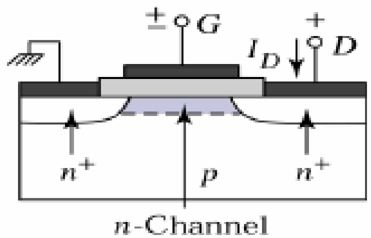
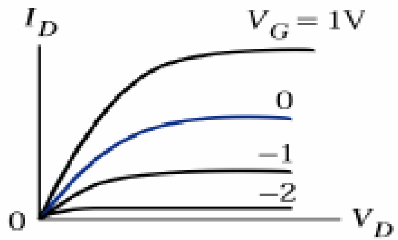
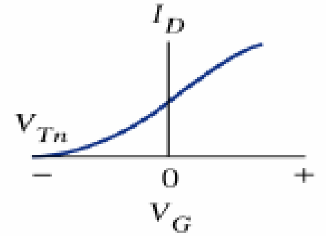
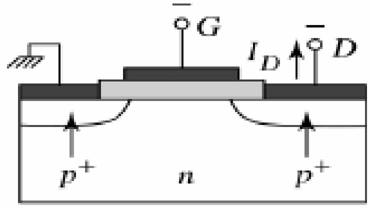
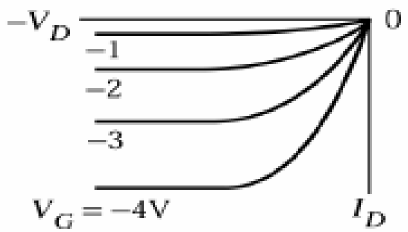
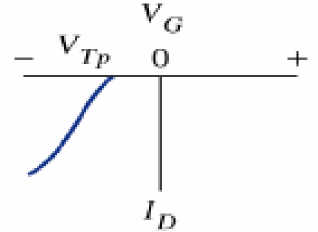
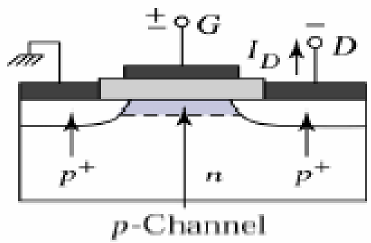
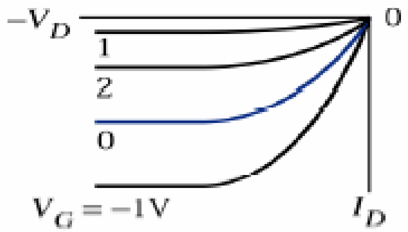
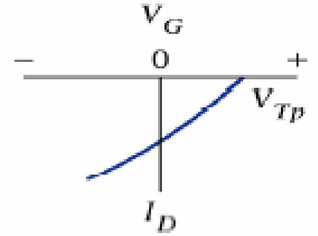
- MOSFET (Metal-Oxide Semiconductor Field-Effect Transistor)
- Depletion and enhancement mode (Primary component in high-density VLSI chips such as memory chips and microprocessors)
- JFET (Junction Field-Effect Transistor) finds application especially in analog and RF circuit design
- MESFET (a MOSFET with no oxide)
- All device and circuit analysis are the same!



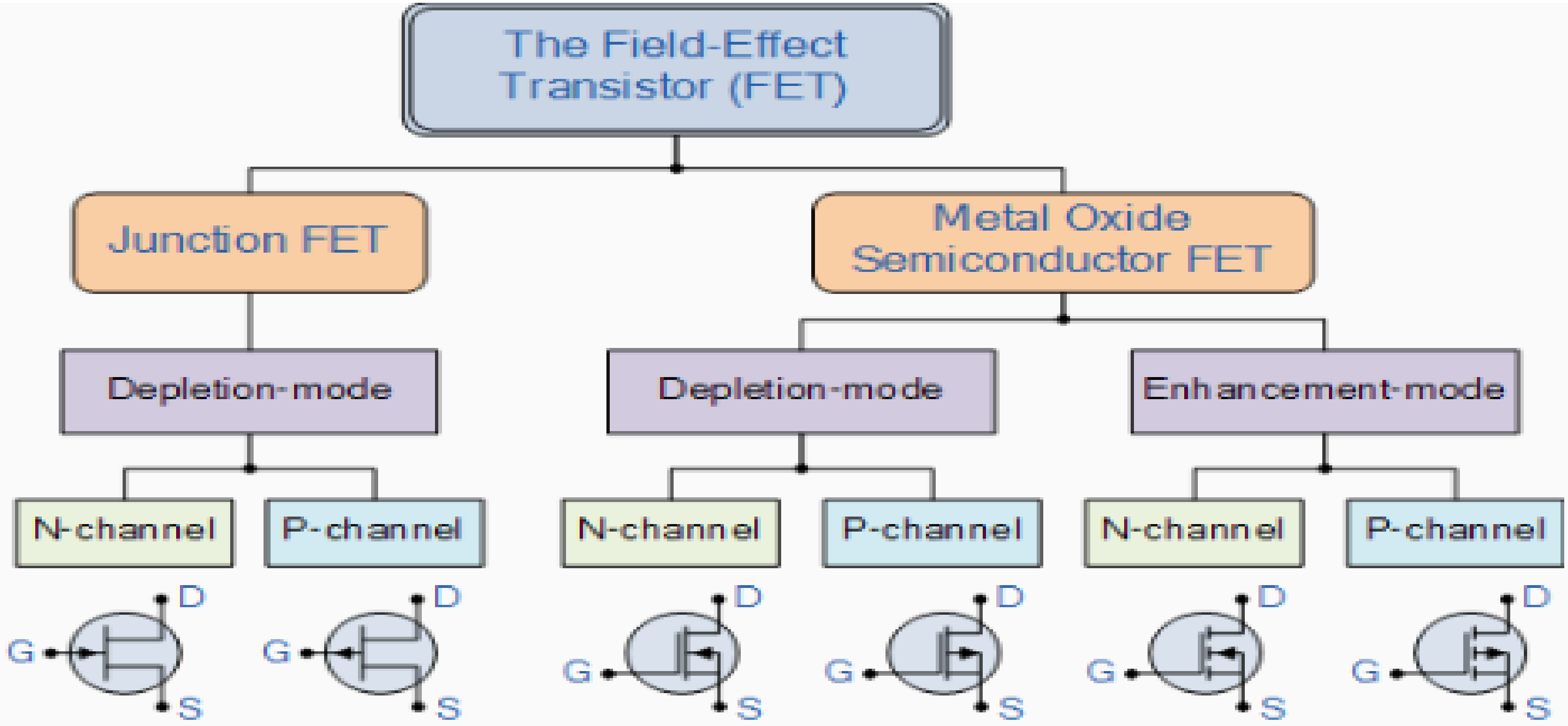




Types of MOSFETs

Type	Cross Section	Output Characteristics	Transfer Characteristics
n-Channel Enhancement (Normally Off)			
n-Channel Depletion (Normally On)			
p-Channel Enhancement (Normally Off)			
p-Channel Depletion (Normally On)			

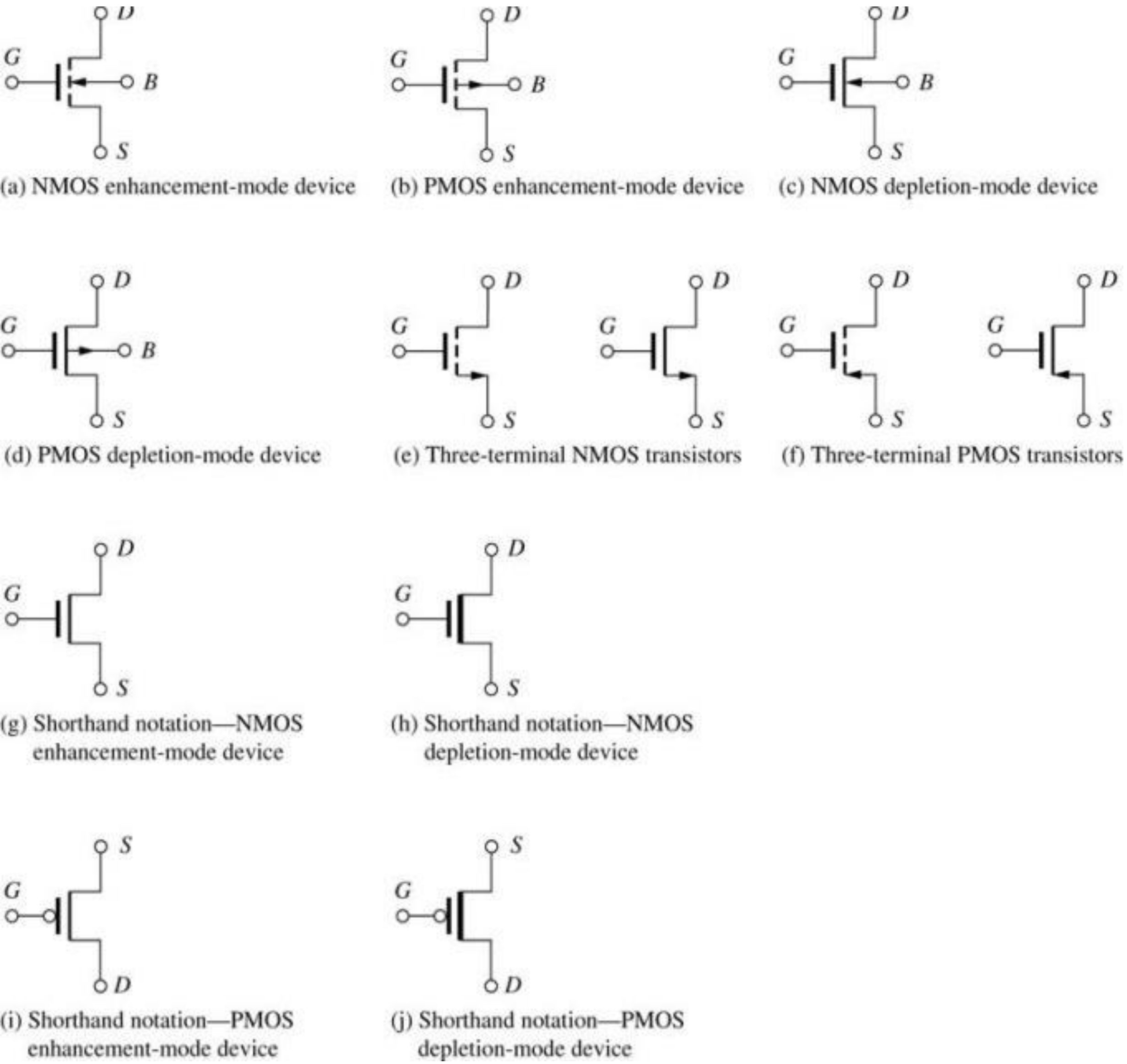
Field Effect Transistor Tree



Biasing of the Gate for both the junction field effect transistor, (JFET) and the metal oxide semiconductor field effect transistor, (MOSFET) configurations are given as:

Type	Junction FET		Metal Oxide Semiconductor FET			
	Depletion Mode		Depletion Mode		Enhancement Mode	
Bias	ON	OFF	ON	OFF	ON	OFF
N-channel	0v	-ve	0v	-ve	+ve	0v
P-channel	0v	+ve	0v	+ve	-ve	0v

MOSFET Circuit Symbols



(a) NMOS enhancement-mode device (b) PMOS enhancement-mode device (c) NMOS depletion-mode device

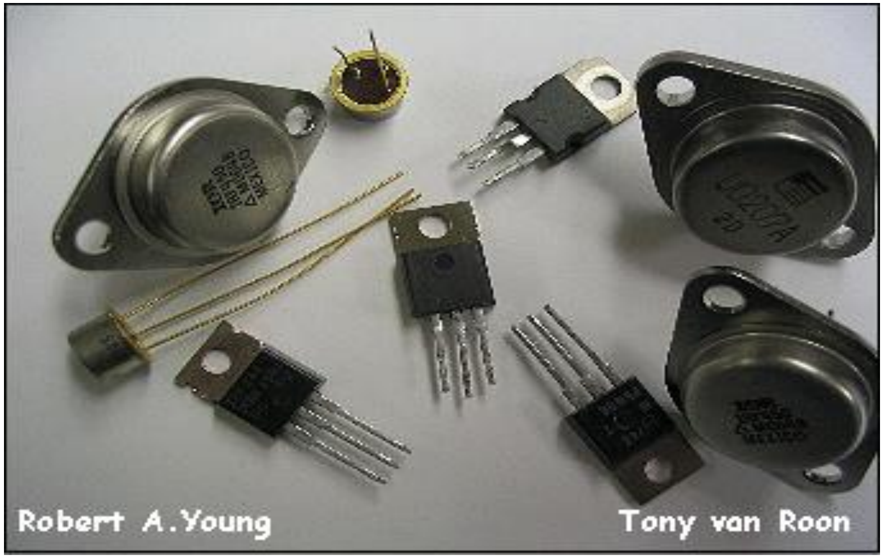
(d) PMOS depletion-mode device (e) Three-terminal NMOS transistors (f) Three-terminal PMOS transistors

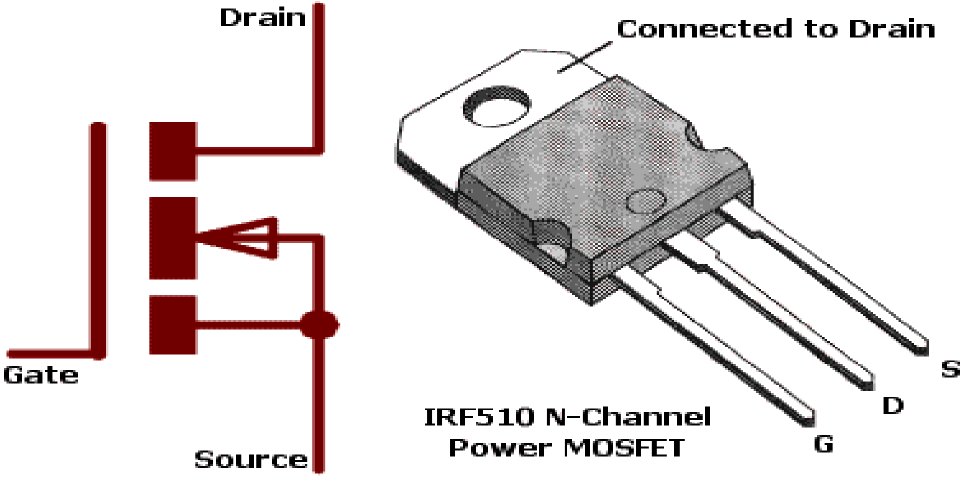
(g) Shorthand notation—NMOS enhancement-mode device (h) Shorthand notation—NMOS depletion-mode device

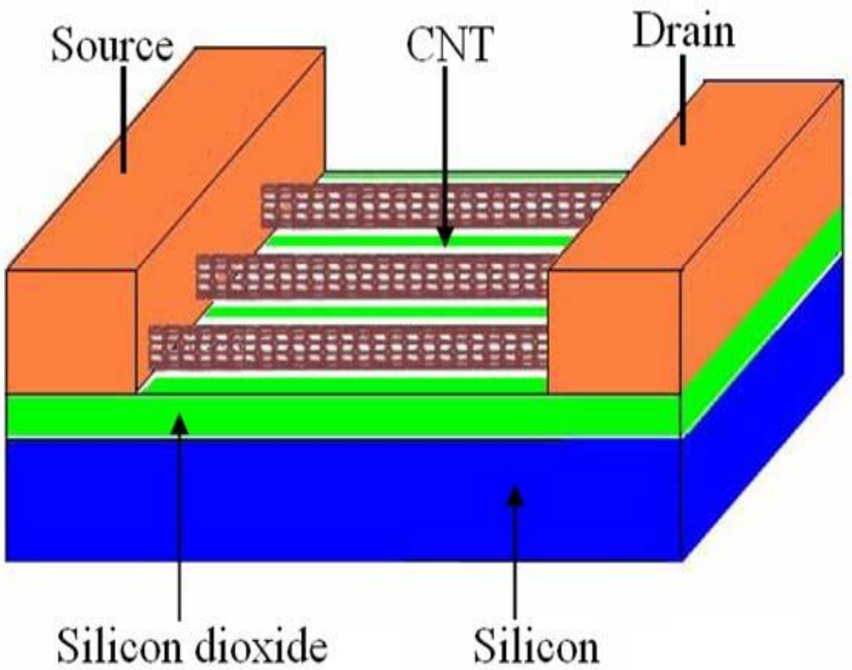
(i) Shorthand notation—PMOS enhancement-mode device (j) Shorthand notation—PMOS depletion-mode device

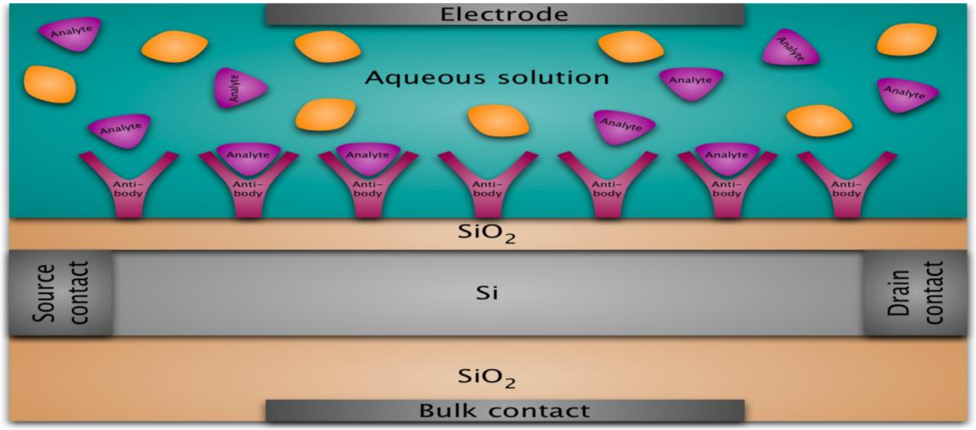
- (g) and(i) are the most commonly used symbols in VLSI logic design.
- MOS devices are symmetric.
- In NMOS, n^+ region at higher voltage is the drain.
- In PMOS p^+ region at lower voltage is the drain

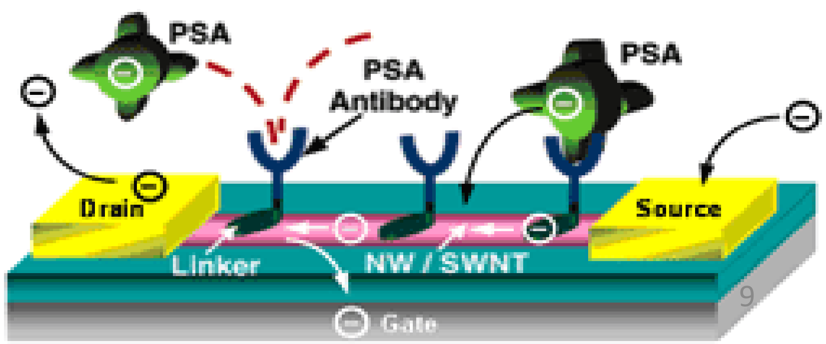
FETs


Robert A.Young
Tony van Roon


Drain
Gate
Source
Connected to Drain
IRF510 N-Channel
Power MOSFET
S
D
G


Source
CNT
Drain
Silicon dioxide
Silicon


Electrode
Aqueous solution
Analyte
Anti-body
SiO2
Source contact
Si
Drain contact
SiO2
Bulk contact


PSA
PSA
PSA Antibody
Drain
Source
Linker
NW / SWNT
Gate

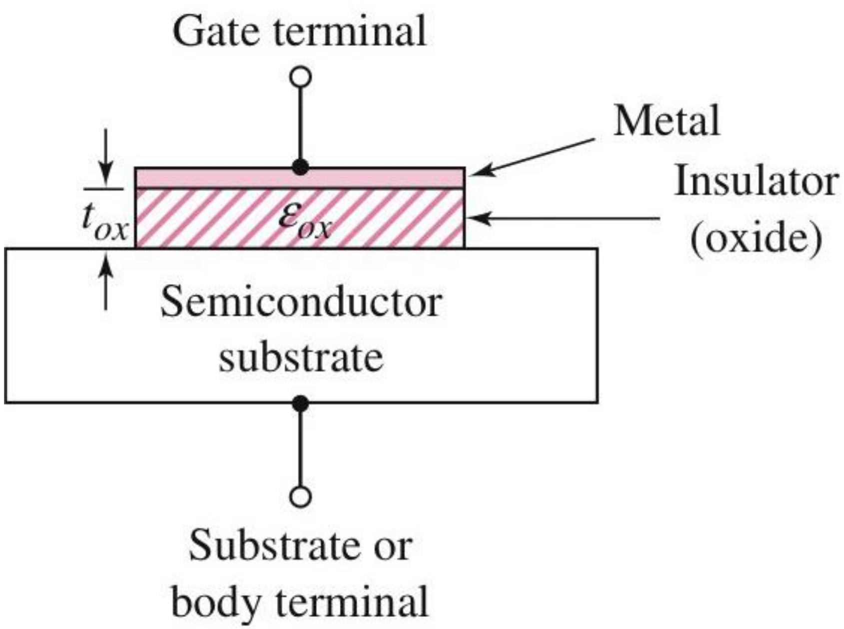
Two-Terminal MOS Structure

Basic Structure of MOS Capacitor

First electrode-Gate : Consists of low-resistivity material such as polycrystalline silicon

Second electrode- Substrate or Body: *n*- or *p*-type semiconductor

Dielectric-Silicon dioxide: stable high-quality electrical insulator between gate and substrate.



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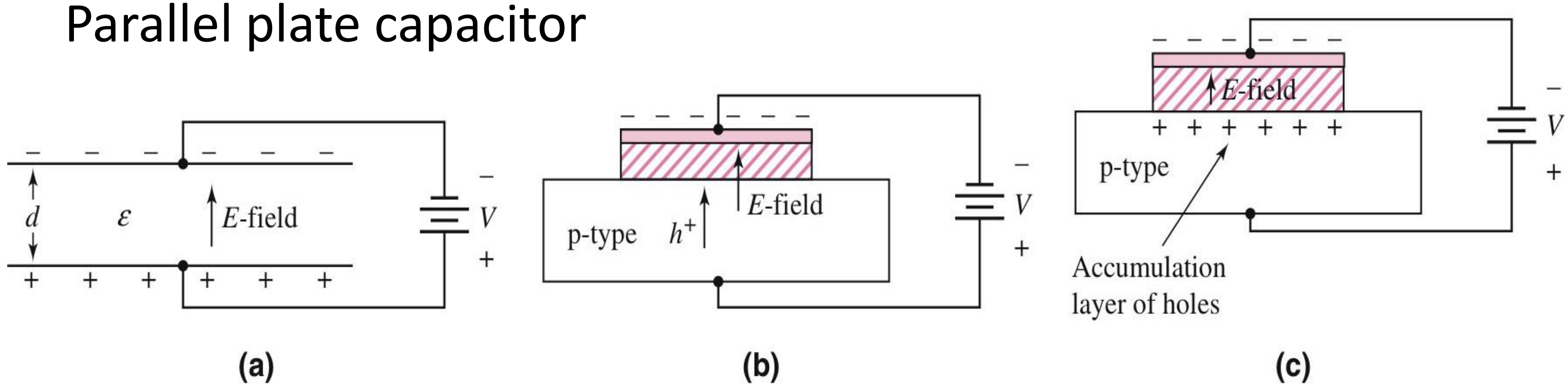
Heart of MOSFET!

MOS Capacitor Under Bias: Electric Field and Charge

Parallel plate capacitor



Negative gate bias:

Holes attracted to gate

Positive gate bias:

Electrons attracted to gate

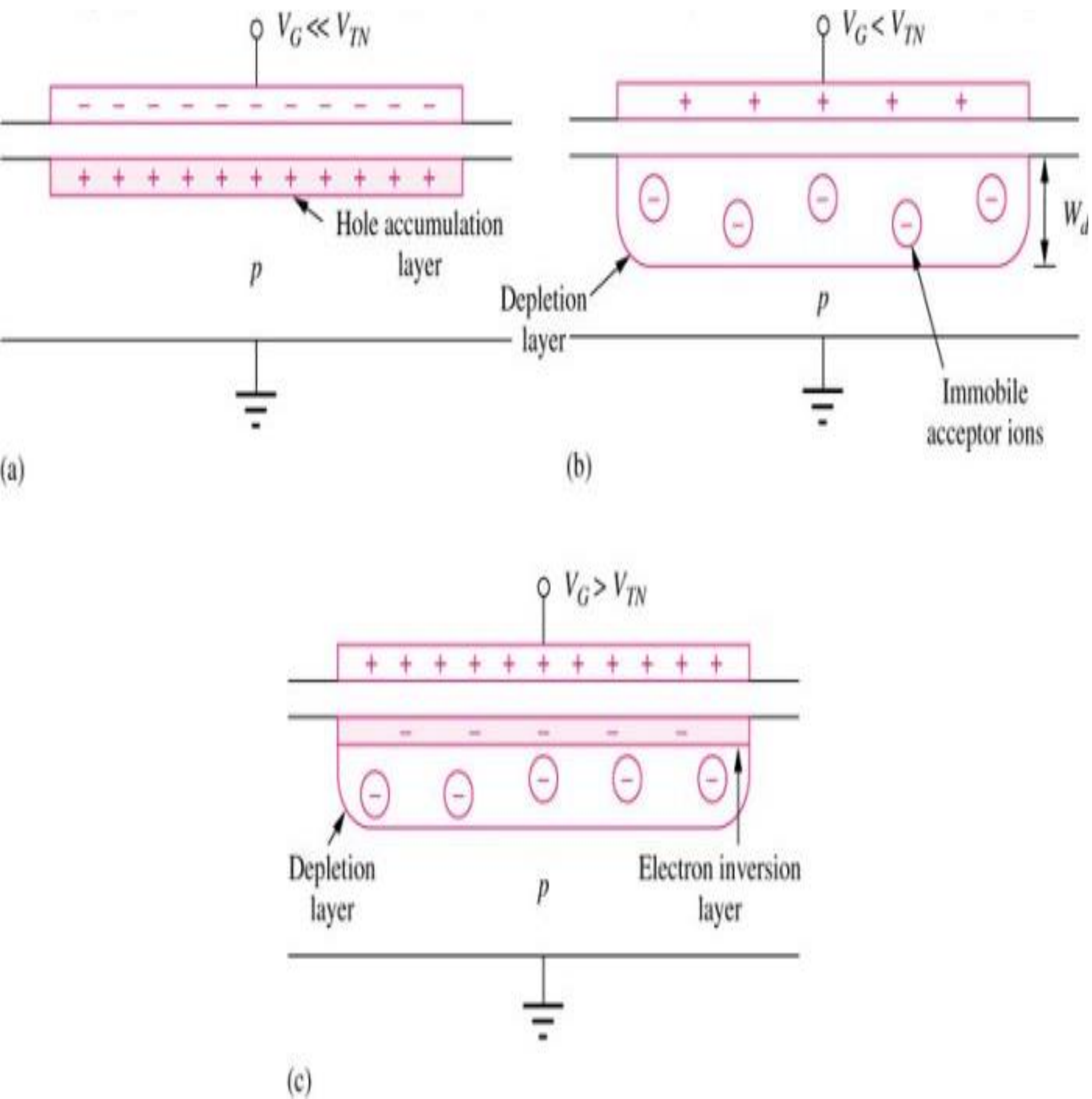
$C = \varepsilon A/d$

ε – permittivity

A – area of one plate

d – distance between plates

Substrate Conditions for Different Biases



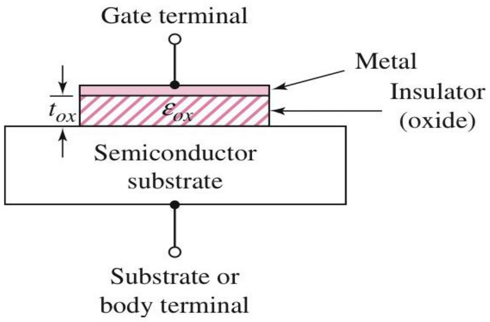
- Region of minority carrier electron inversion layer attracted to the oxide semiconductor surface!

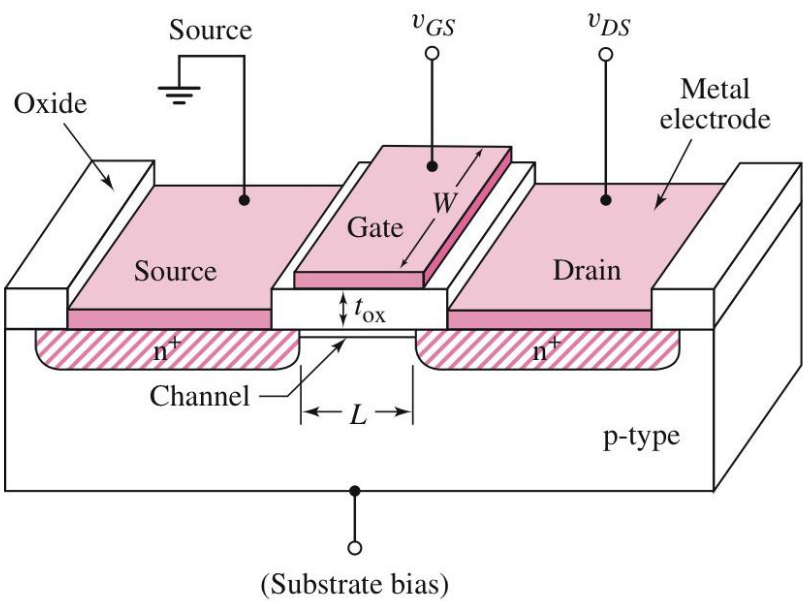
n-Channel Enhancement-Mode MOSFET

Enhancement mode & n-Channel

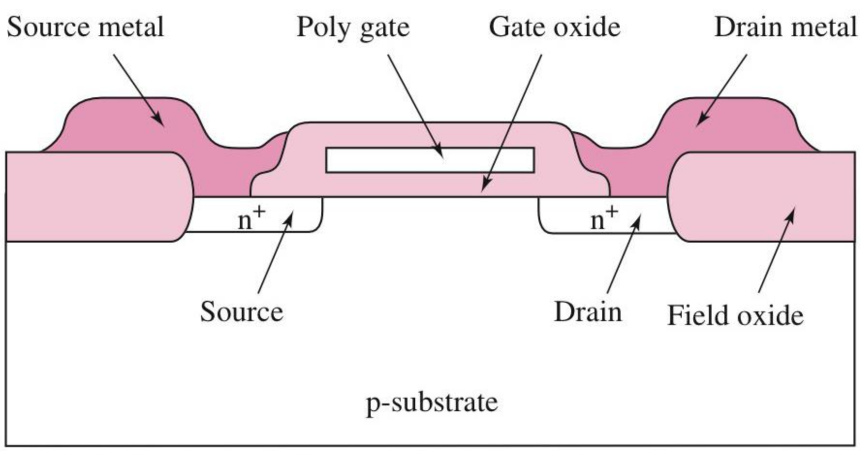
- Enhancement-mode -> a voltage must be applied to the gate to create an inversion layer
- n-type substrate -> positive gate voltage
- p-type substrate -> negative gate voltage

Schematic of n-Channel Enhancement Mode MOSFET





(a)



(b)

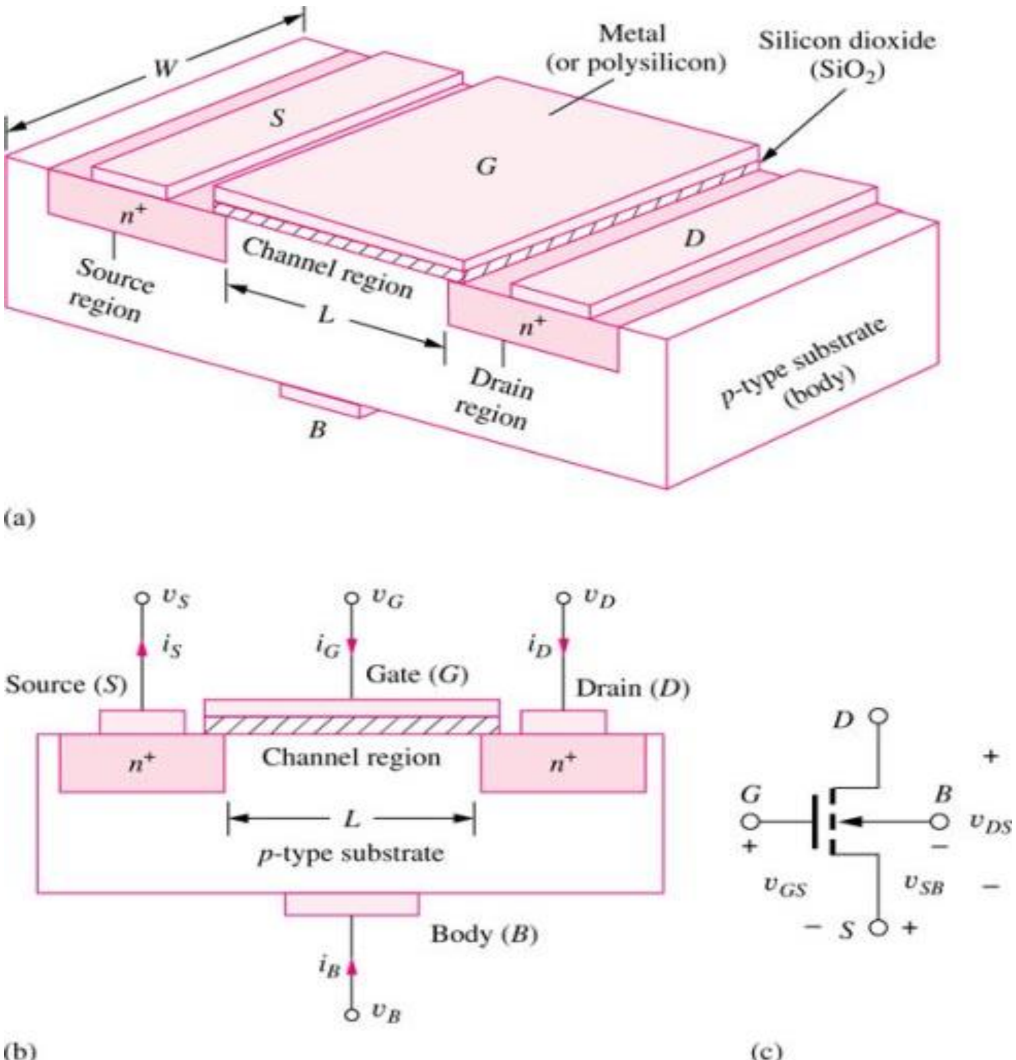
t_{ox} is typically on the order of 400 angstroms.

International System of Units

Standard prefixes for the SI units of measure

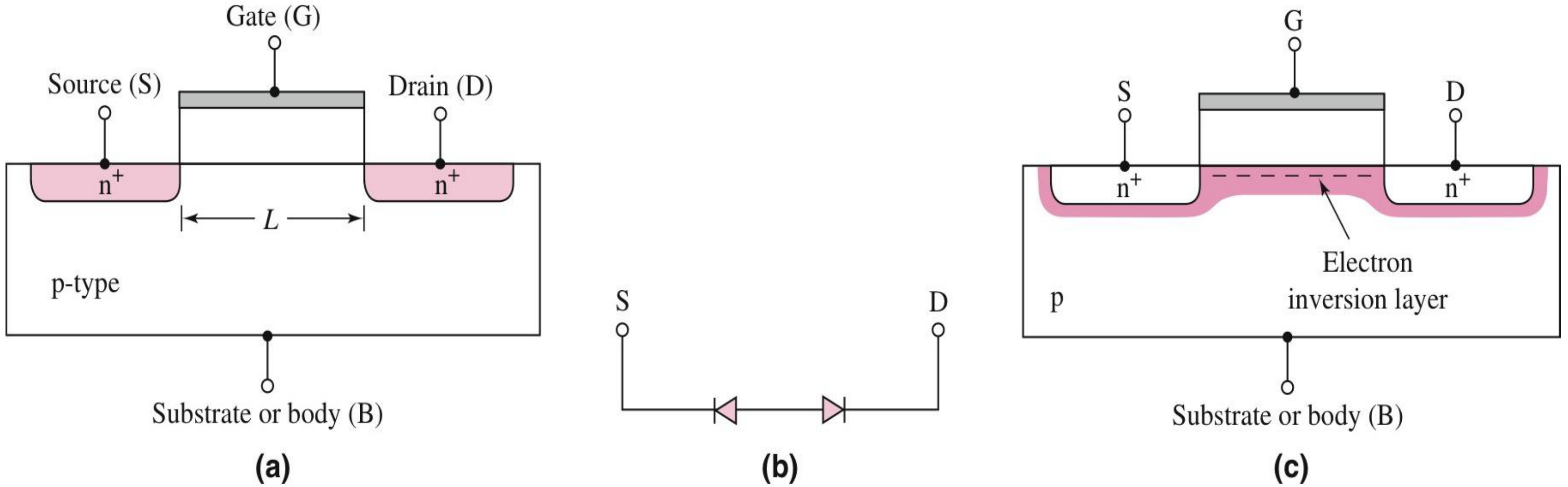
Multiples	**Name**		deca-	hecto-	kilo-	mega-	giga-	tera-	peta-	exa-	zetta-	yotta-
	Prefix		da	h	k	M	G	T	P	E	Z	Y
	Factor	10^0	10^1	10^2	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}	10^{21}	10^{24}
Fractions	**Name**		deci-	centi-	milli-	micro-	nano-	pico-	femto-	atto-	zepto-	yocto-
	Prefix		d	c	m	µ	n	p	f	a	z	y
	Factor	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}	10^{-18}	10^{-21}	10^{-24}

NMOS Transistor: Structure



- 4 device terminals: Gate(G), Drain(D), Source(S) and Body(B).
- Source and drain regions form *pn* junctions with substrate.
- v_{SB}, v_{DS} and v_{GS} always positive during normal operation.
- v_{SB} always < v_{DS} and v_{GS} to reverse bias *pn* junctions

Basic Transistor Operation



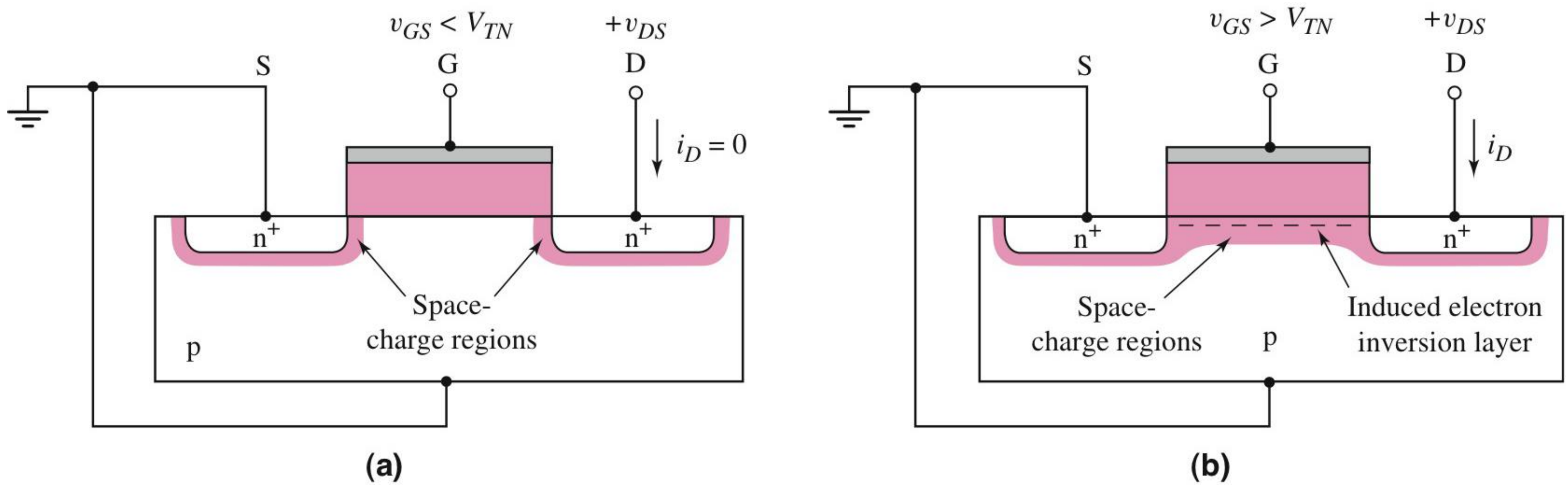
Before electron inversion layer is formed

After electron inversion layer is formed

The current in MOSFET is the result of the flow of charge in the inversion layer!

For NMOS, the carrier in the inversion layer is the electron!

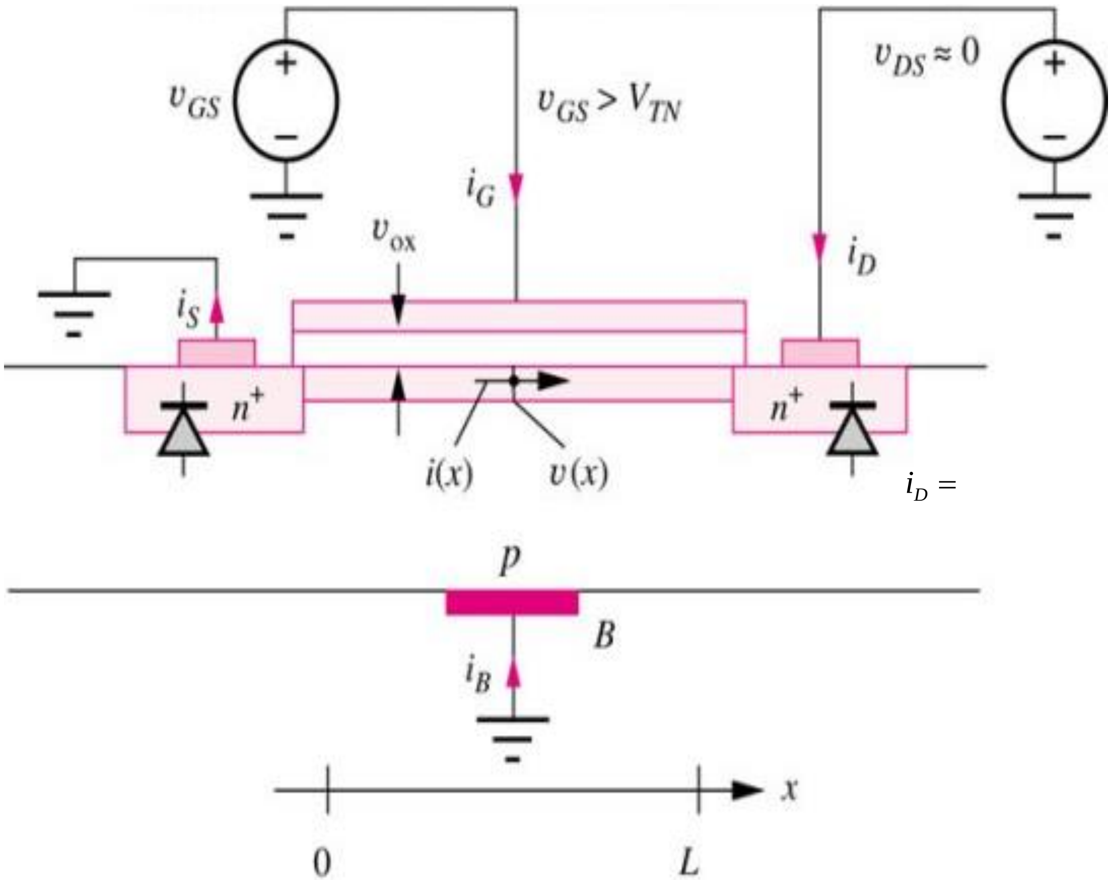
Basic Transistor Operation



Electrons flow from the source of the drain with an applied drain-to-source voltage!

The magnitude of current is a function of amount of charge in the inversion layer + a function of applied gate voltage.

NMOS Transistor: Triode Region Characteristics



$i_D =$

for $v_{GS} - V_{TN} \geq v_{DS} \geq 0$

where, $K_n = K_n' W/L$

$K_n' = \mu_n C_{ox}''$ (A/V^2)

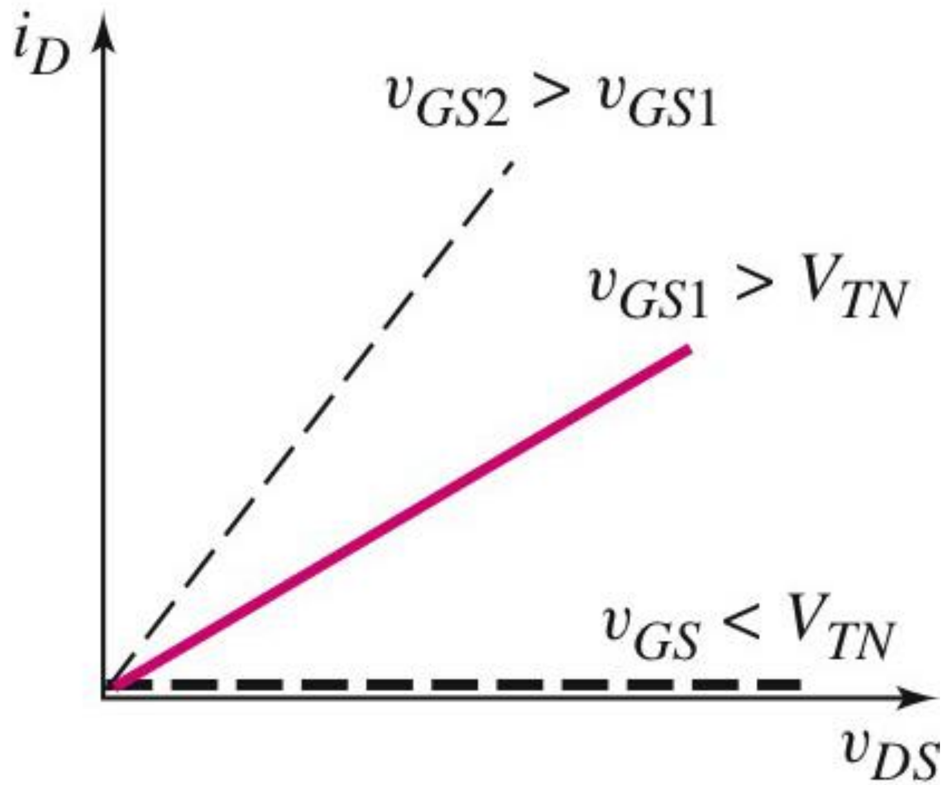
$C_{ox}'' = \varepsilon_{ox}/T_{ox}$

ε_{ox}=oxide permittivity (F/cm)

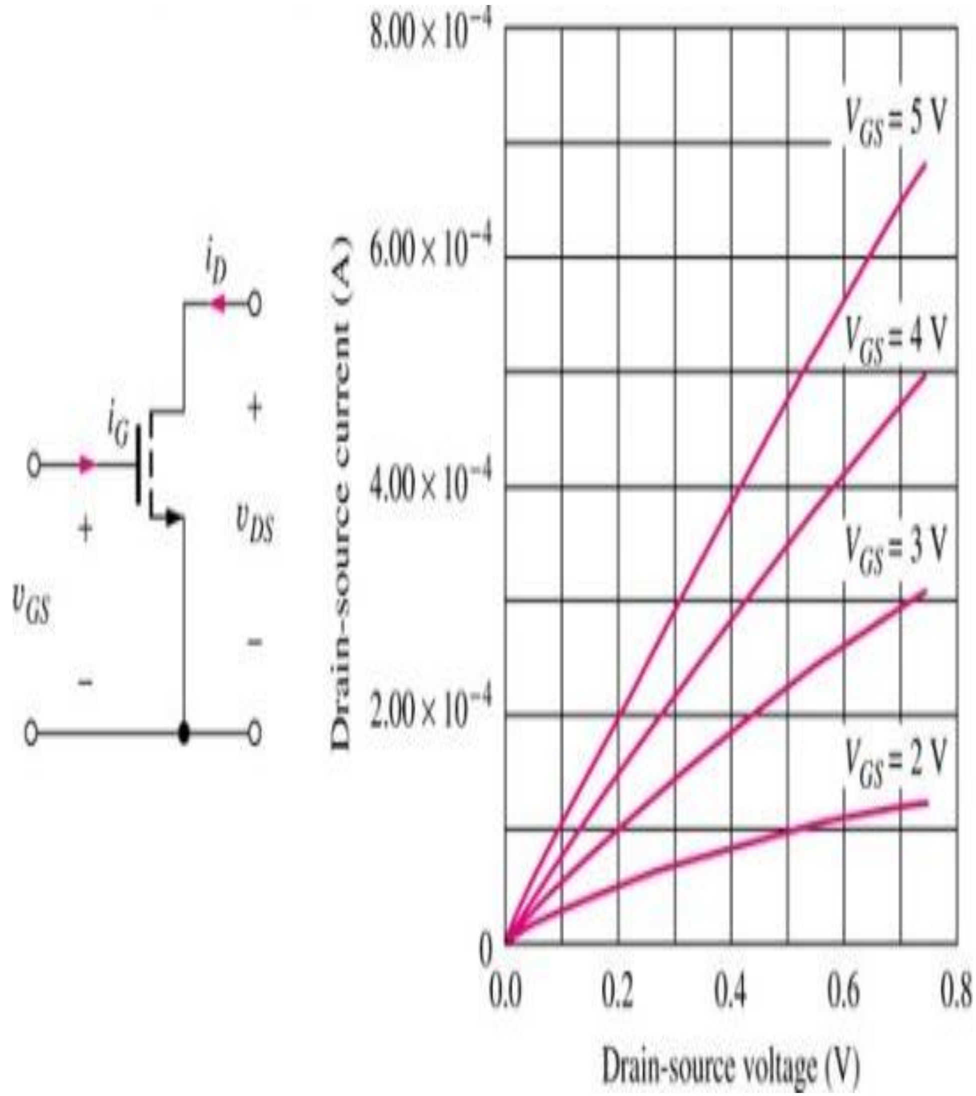
T_{ox}= oxide thickness (cm)

$$i_D = K_n \frac{W}{L}\left(v_{GS} - V_{TN} - \frac{v_{DS}}{2}\right) v_{DS}$$

Current Versus Voltage Characteristics: Enhancement-Mode nMOSFET



NMOS Transistor: Triode Region Characteristics (contd.)

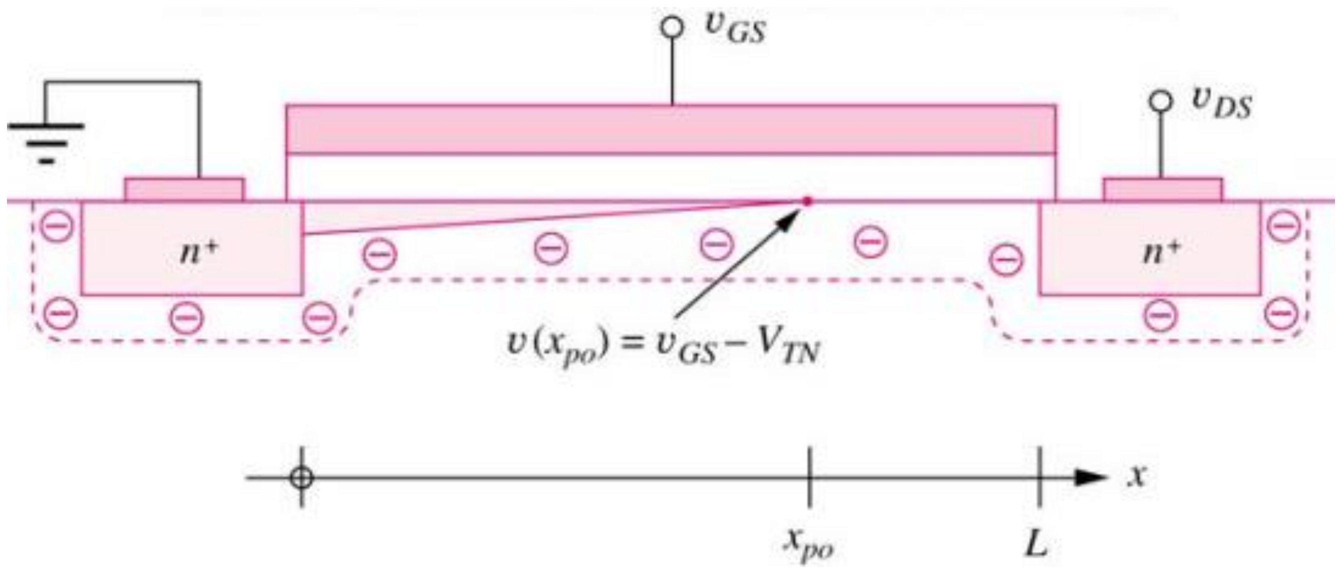


- Output characteristics appear to be linear.

- FET behaves like a gate-source voltage-controlled resistor between source and drain with

$$R_{on} = \frac{1}{K_n' \frac{W}{L}\left(V_{GS} - V_{TN}\right)}$$

NMOS Transistor: Saturation Region (contd.)

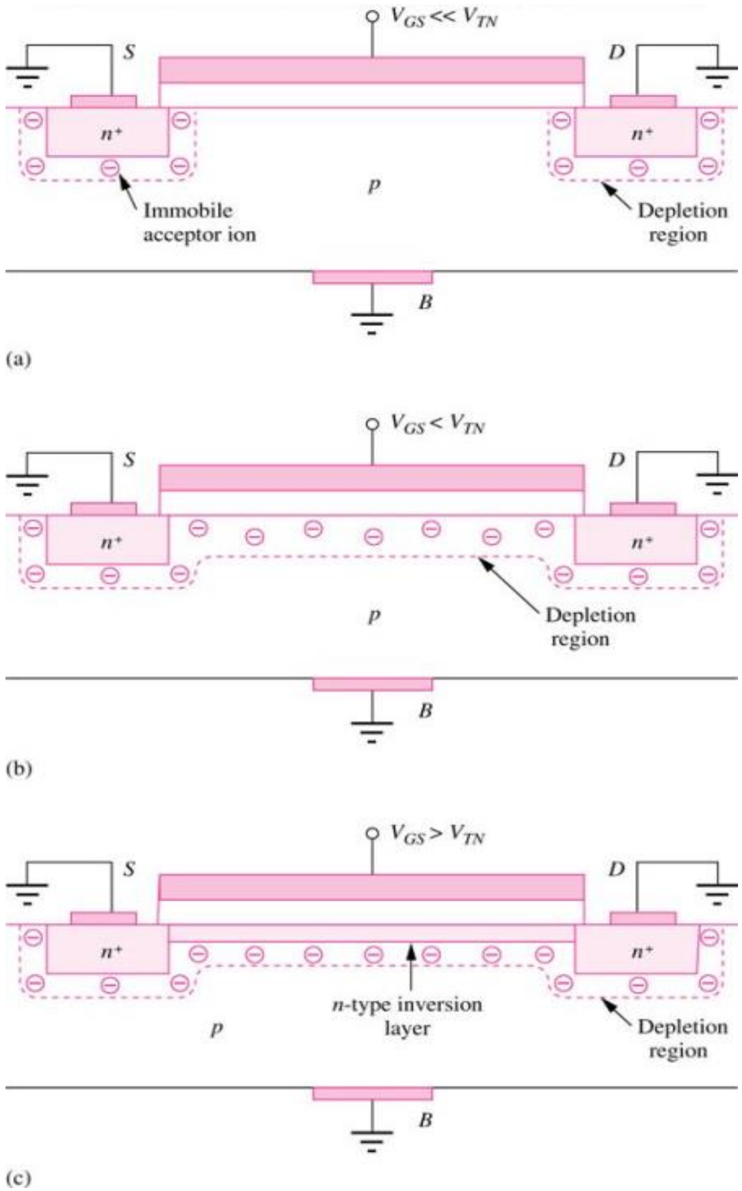


$$i_D = \frac{K_n}{2}\frac{W}{L}\left(v_{GS} - V_{TN}\right)^2$$ for $$v_{DS} \geq v_{GS} - V_{TN}$$

$$i_D \propto \left(v_{GS} - V_{TN}\right)^2$$ which quadratic function

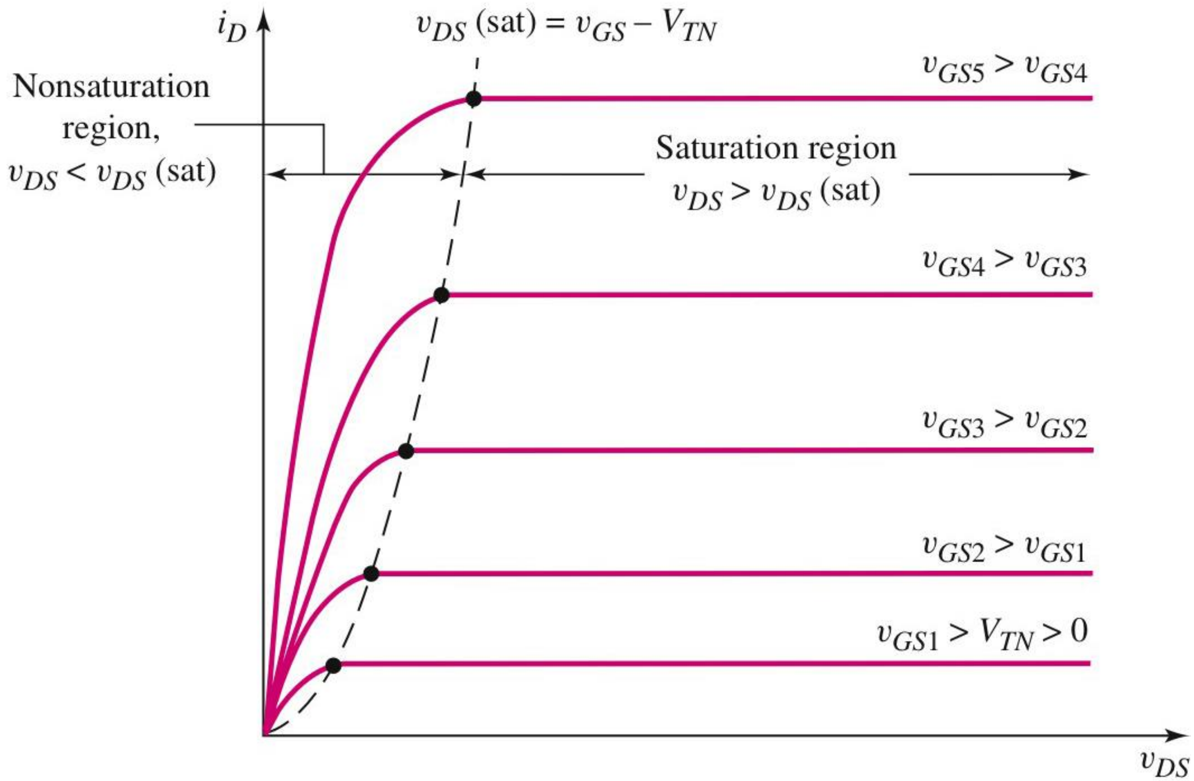
$v_{DSAT} = v_{GS} - V_{TN}$ is also called saturation or pinch-off voltage

NMOS Transistor: Qualitative I-V Behavior



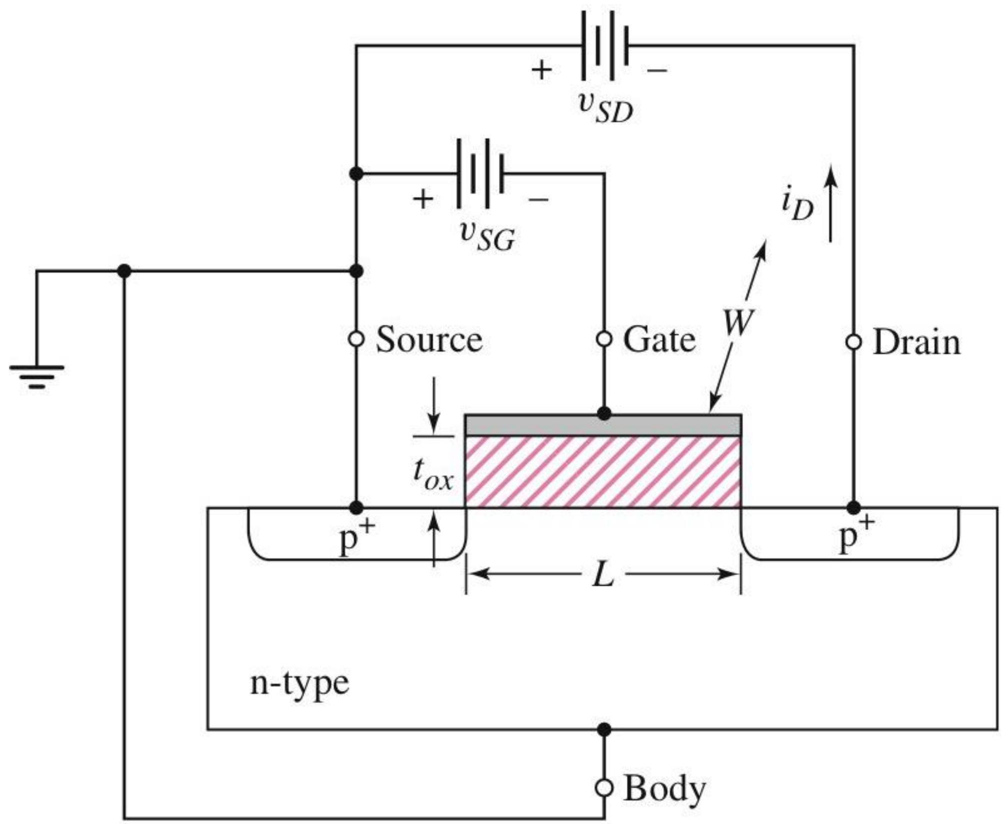
- V_{GS}<<V_{TN} : Only small leakage current flows.
- V_{GS}<V_{TN}: Depletion region formed under gate merges with source and drain depletion regions. No current flows between source and drain.
- V_{GS}>V_{TN}: Channel formed between source and drain. If v_{DS}>0,, finite i_D flows from drain to source.
- $i_B=0$ and $i_G=0$.

Family of i_D Versus v_{DS} Curves: Enhancement-Mode nMOSFET

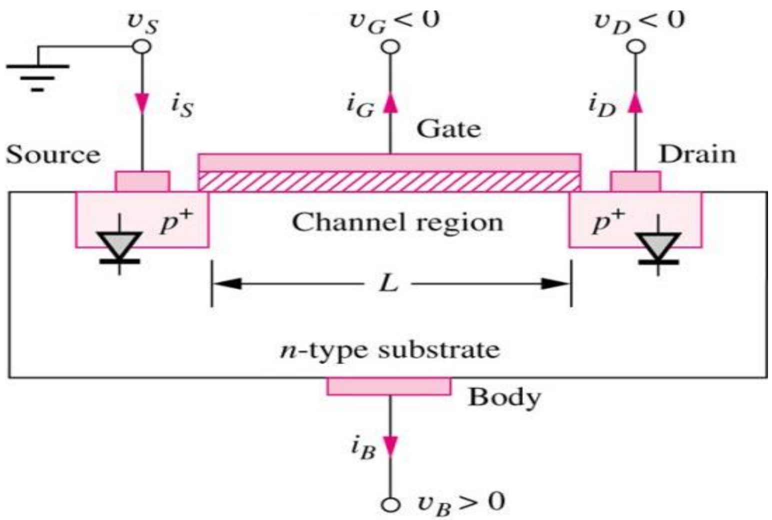


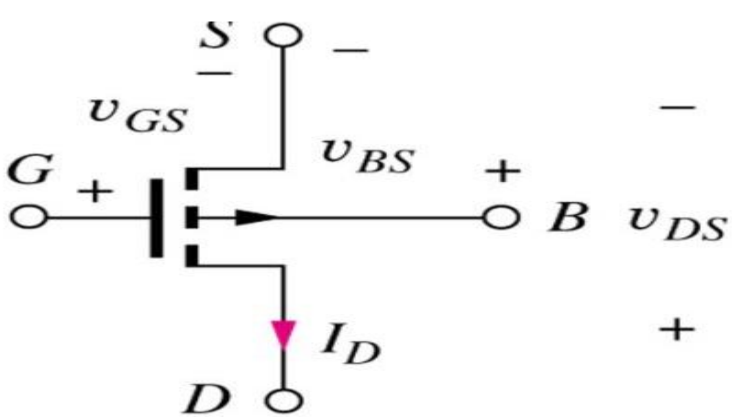
p-Channel Enhancement-Mode MOSFET

p-Channel Enhancement-Mode MOSFET



Enhancement-Mode PMOS Transistors: Structure

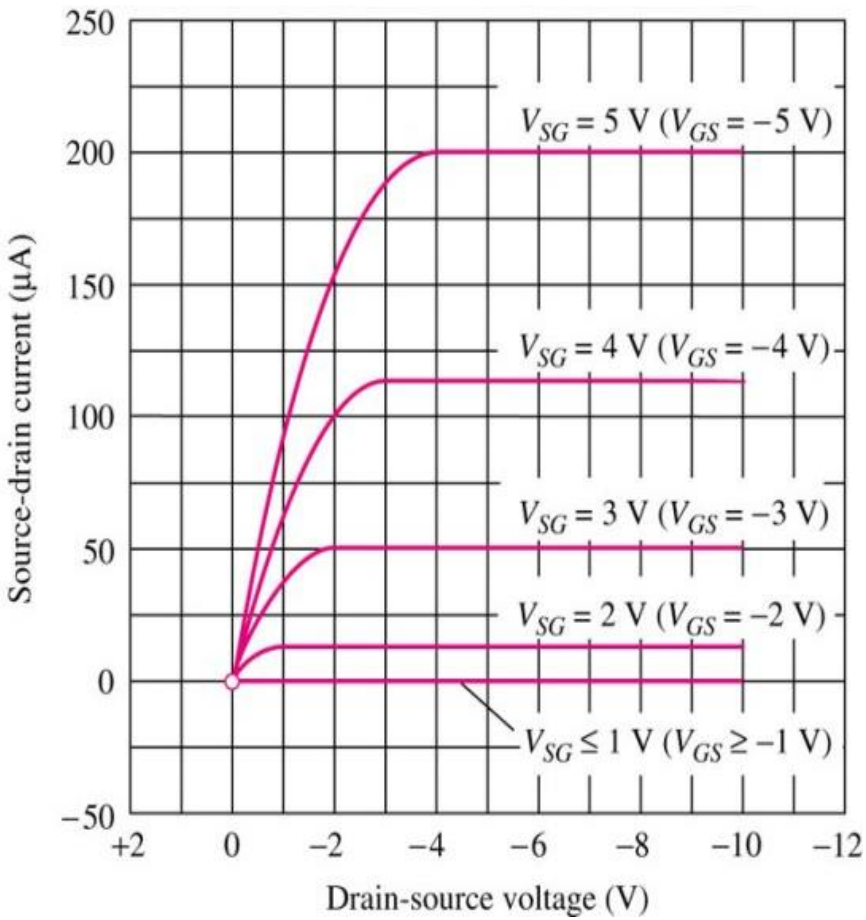




PMOS transistor

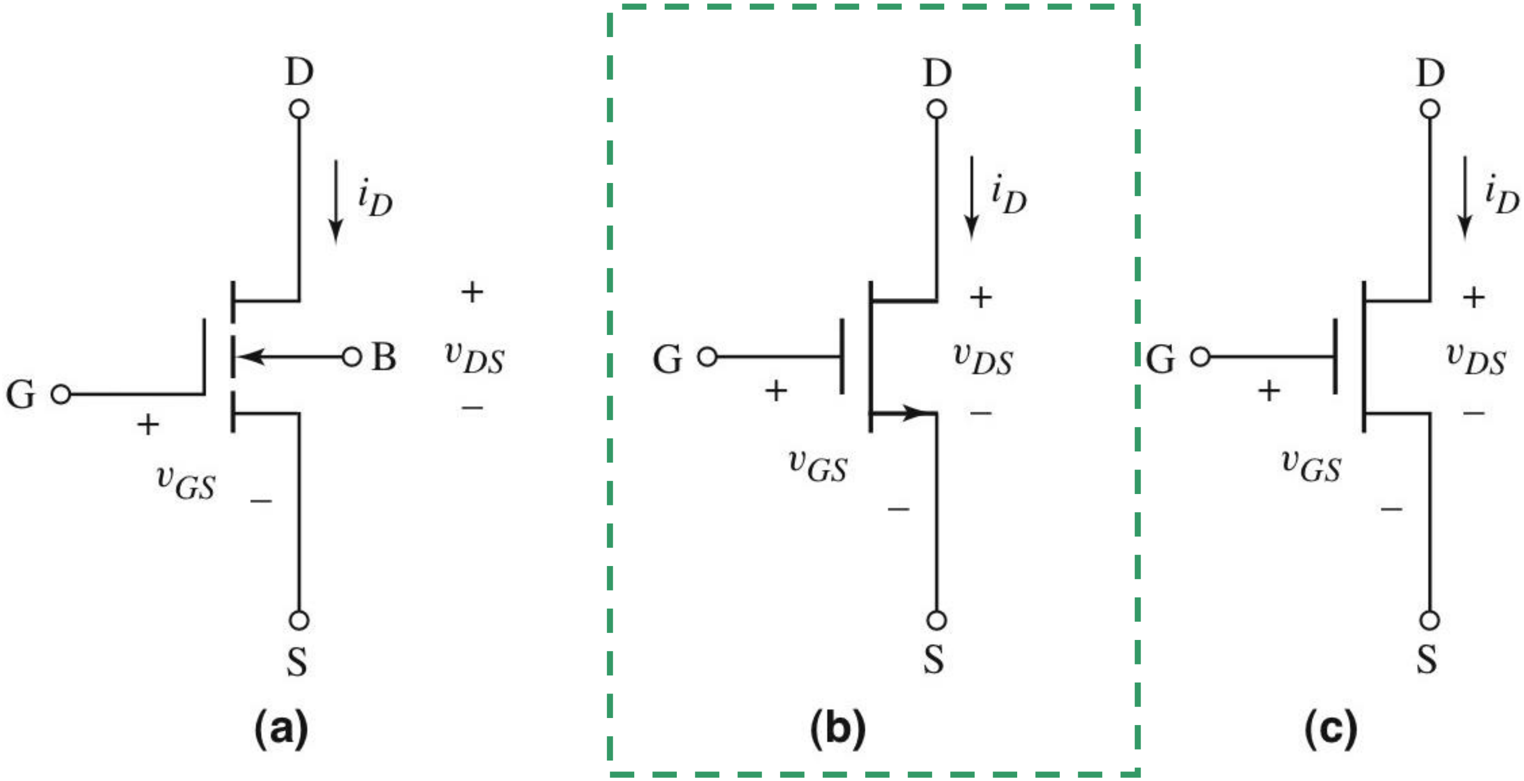
- *P*-type source and drain regions in *n*-type substrate.
- $v_{GS}<0$ required to create p-type inversion layer in channel region
- For current flow, $v_{GS} < v_{TP}$
- To maintain reverse bias on source-substrate and drain-substrate junctions, $v_{SB}<0$ and $v_{DB}<0$
- Positive bulk-source potential causes V_{TP} to become more negative

Enhancement-Mode PMOS Transistors: Output Characteristics

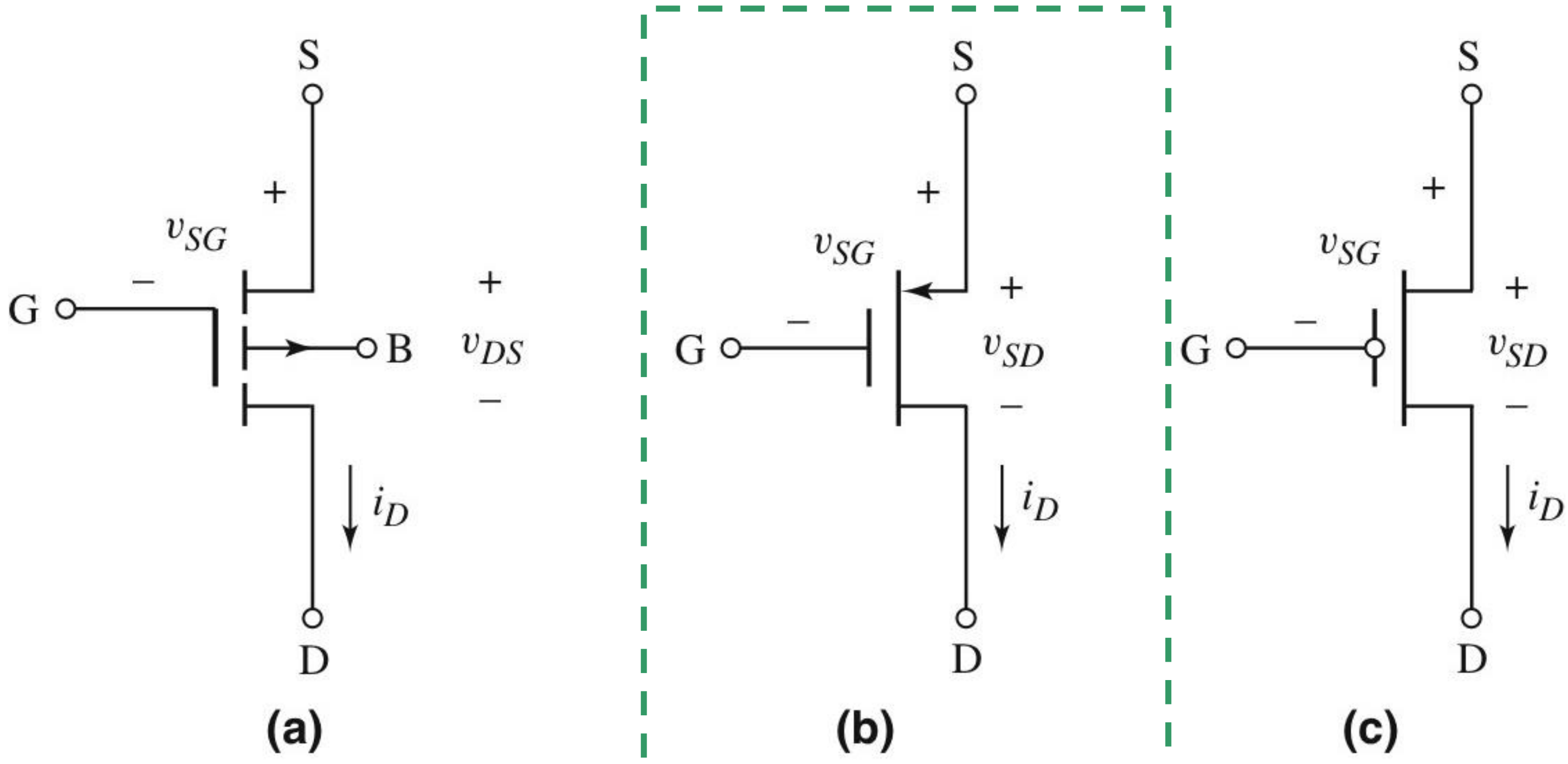


- For $V_{GS} \geq V_{TP}$, transistor is off.
- For more negative v_{GS}, drain current increases in magnitude
- PMOS is in triode region for small values of V_{DS} and in saturation for larger values.

Symbols for n-Channel Enhancement-Mode MOSFET

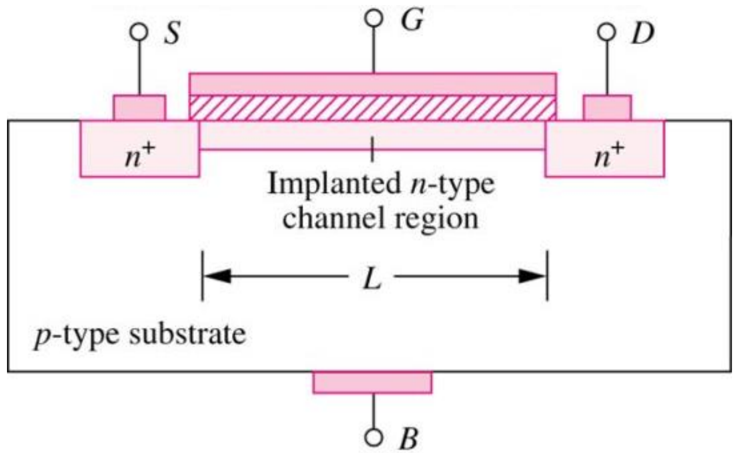


Symbols for p-Channel Enhancement-Mode MOSFET



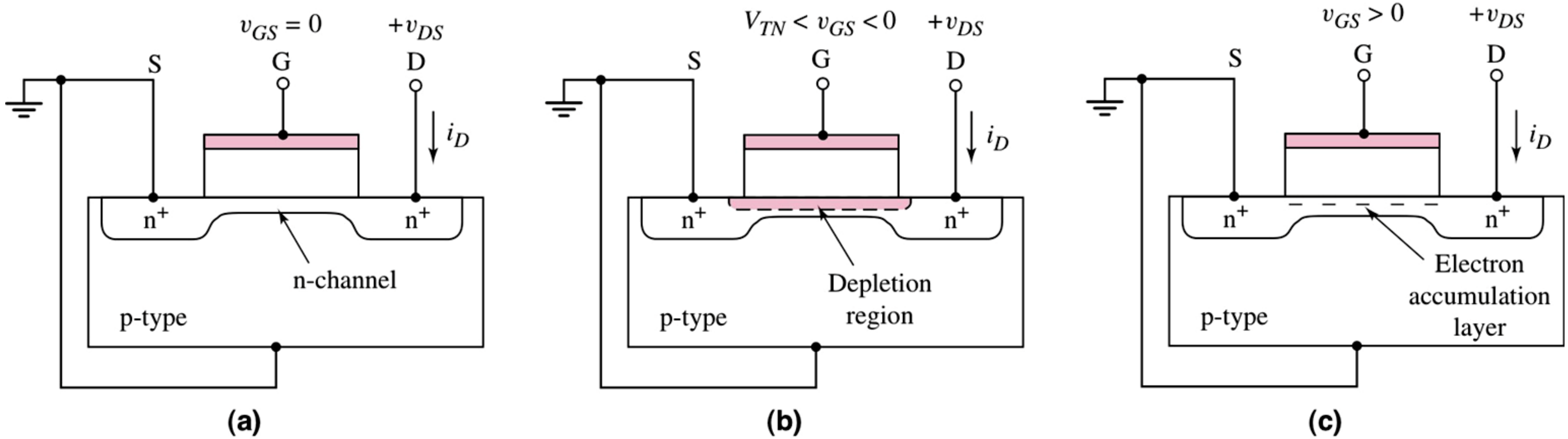
n-Channel Depletion-Mode MOSFET

Depletion-Mode MOSFETS

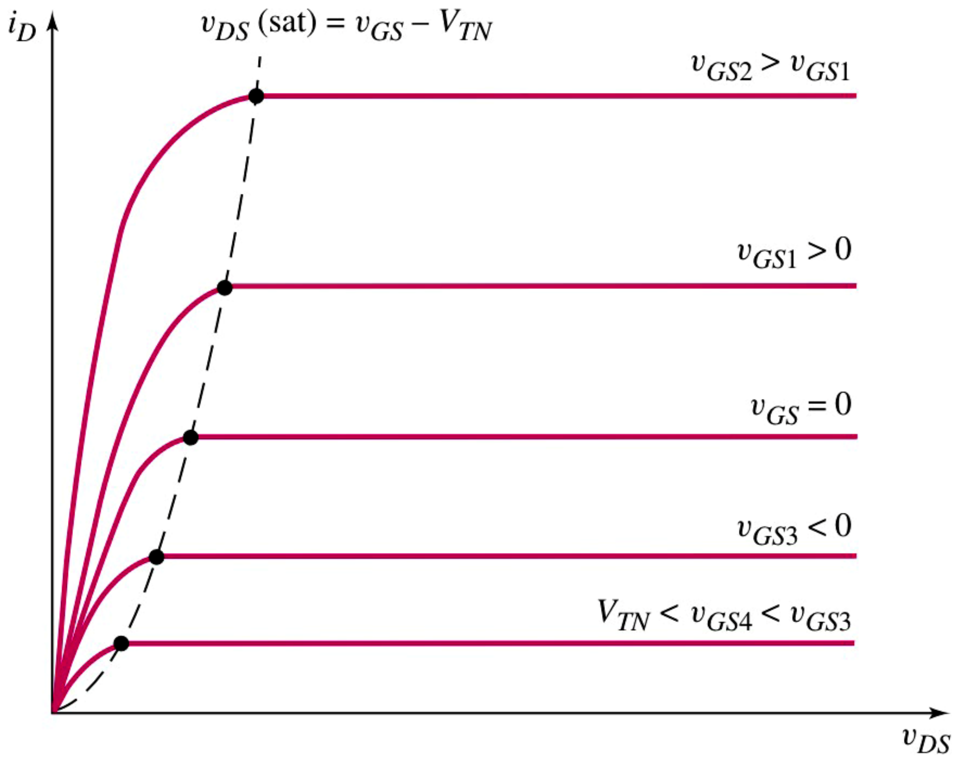


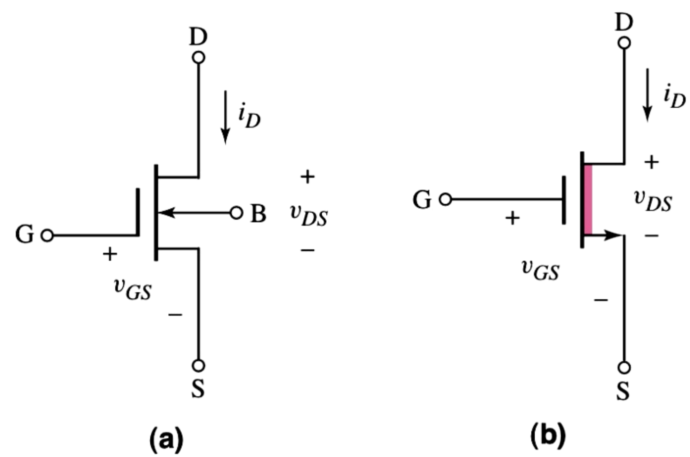
- NMOS transistors with $V_{TN} \leq 0$
- Ion implantation process used to form a built-in *n*-type channel in device to connect source and drain by a resistive channel
- Non-zero drain current for v_{GS}=0, negative v_{GS} required to turn device off.

n-Channel Depletion-Mode MOSFET



Family of i_D Versus v_{DS} Curves: Depletion-Mode nMOSFET

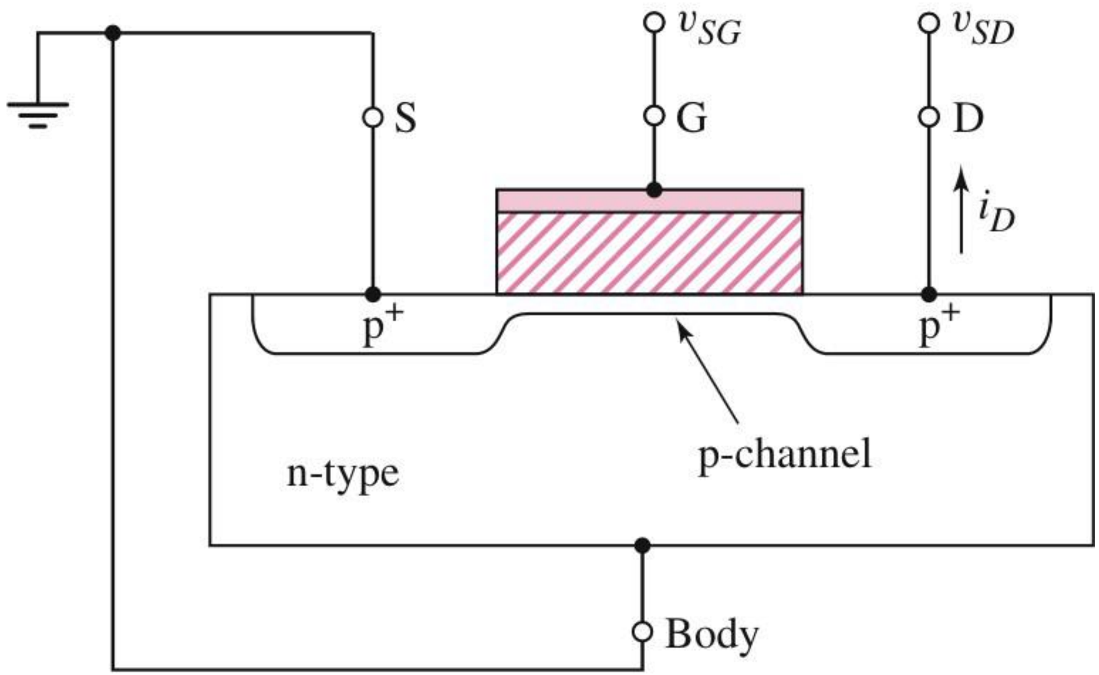


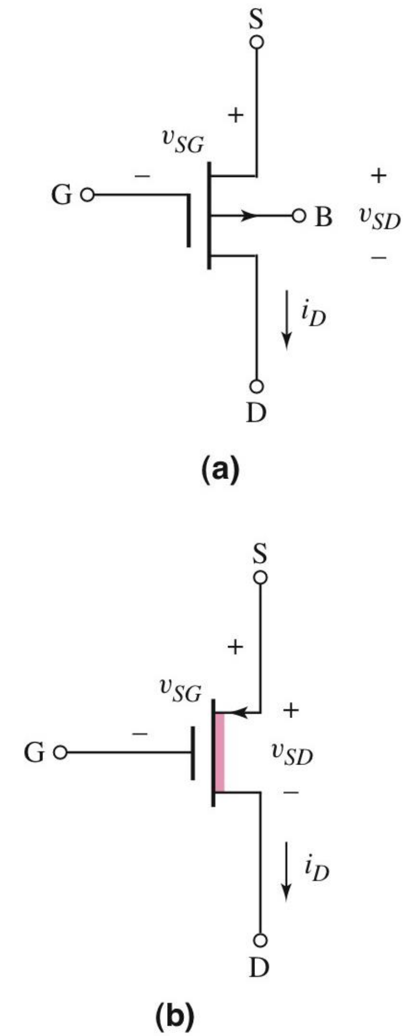


Symbols

p-Channel Depletion-Mode MOSFET

p-Channel Depletion-Mode MOSFET

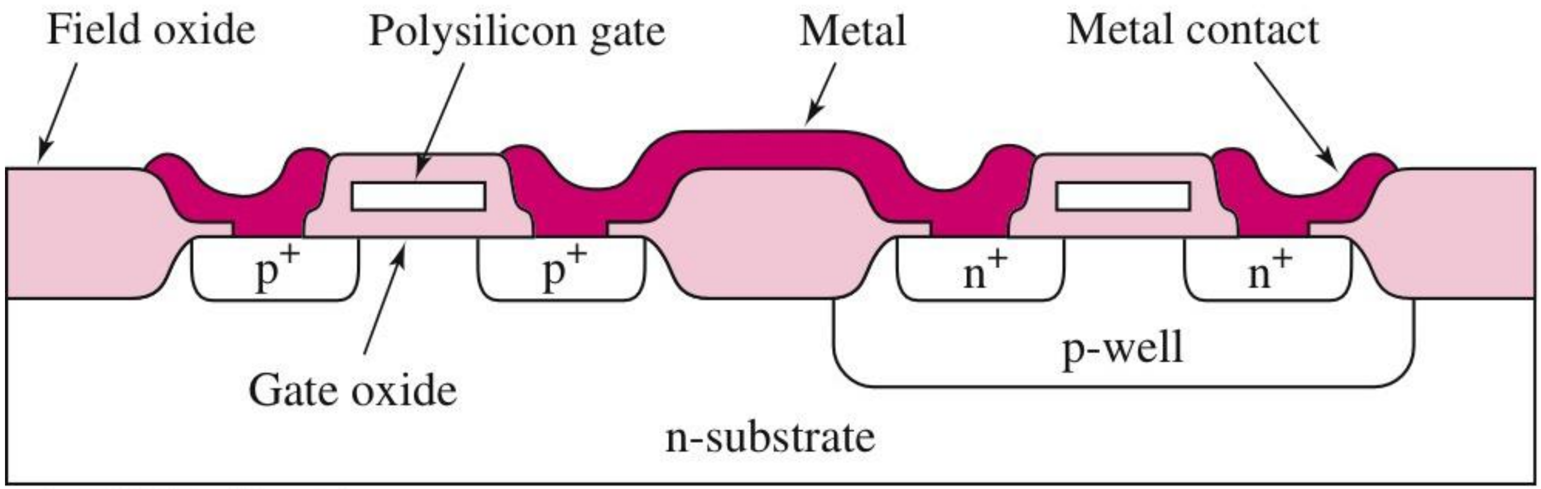




Symbols

Complementary MOSFETs (CMOS)

Cross-Section of nMOSFET and pMOSFET



Both transistors are used in the fabrication of CMOS circuitry.

Summary of I-V Relationships

Region	NMOS	PMOS
Nonsaturation	$v_{DS}<v_{DS}(sat)$ $i_D = K_n[2(v_{GS} - V_{TN})v_{DS} - v_{DS}^2]$	$v_{SD}<v_{SD}(sat)$ $i_D = K_p[2(v_{SG} + V_{TP})v_{SD} - v_{SD}^2]$
Saturation	$v_{DS}>v_{DS}(sat)$ $i_D = K_n[v_{GS} - V_{TN}]^2$	$v_{SD}>v_{SD}(sat)$ $i_D = K_p[v_{SG} + V_{TP}]^2$
Transition Pt.	$v_{DS}(sat) = v_{GS} - V_{TN}$	$v_{SD}(sat) = v_{SG} + V_{TN}$
Enhancement Mode	$V_{TN} > 0V$	$V_{TP} < 0V$
Depletion Mode	$V_{TN} < 0V$	$V_{TP} > 0V$

Conduction Parameters

- NMOSFET

$$K_n = \frac{W\mu_n C_{ox}}{L} = k_n^{'} \frac{W}{L}$$

- PMOSFET

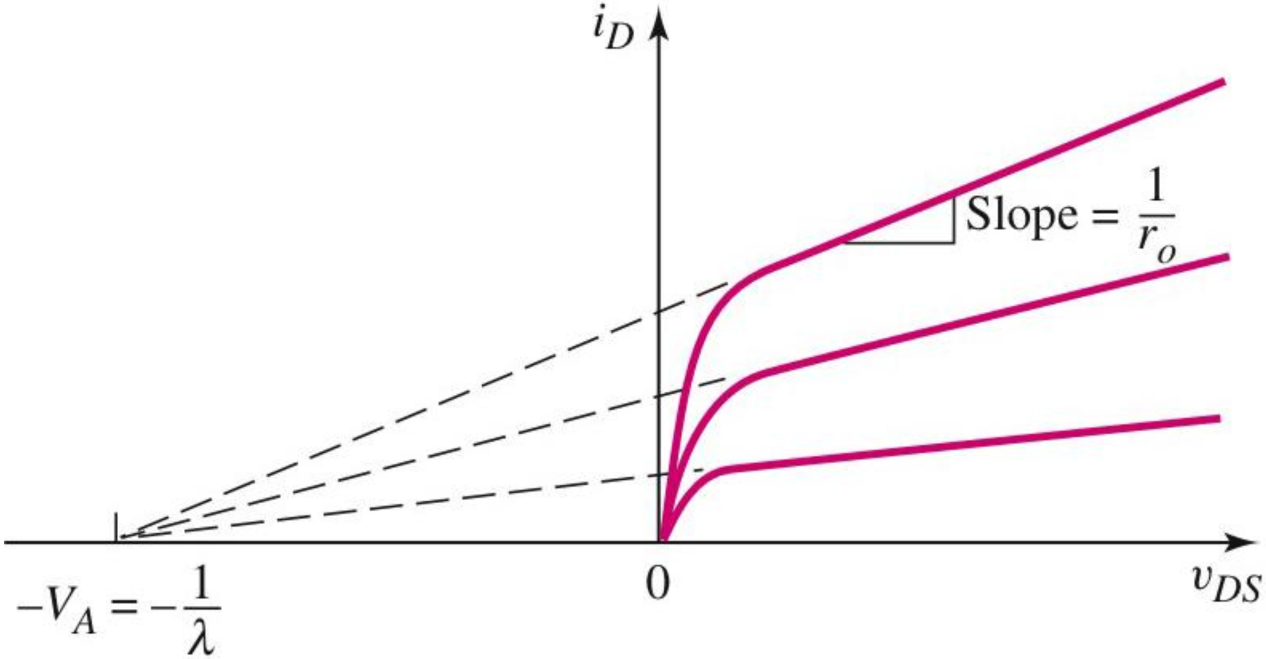
$$K_p = \frac{W\mu_p C_{ox}}{L} = k_p^{'} \frac{W}{L}$$

where:

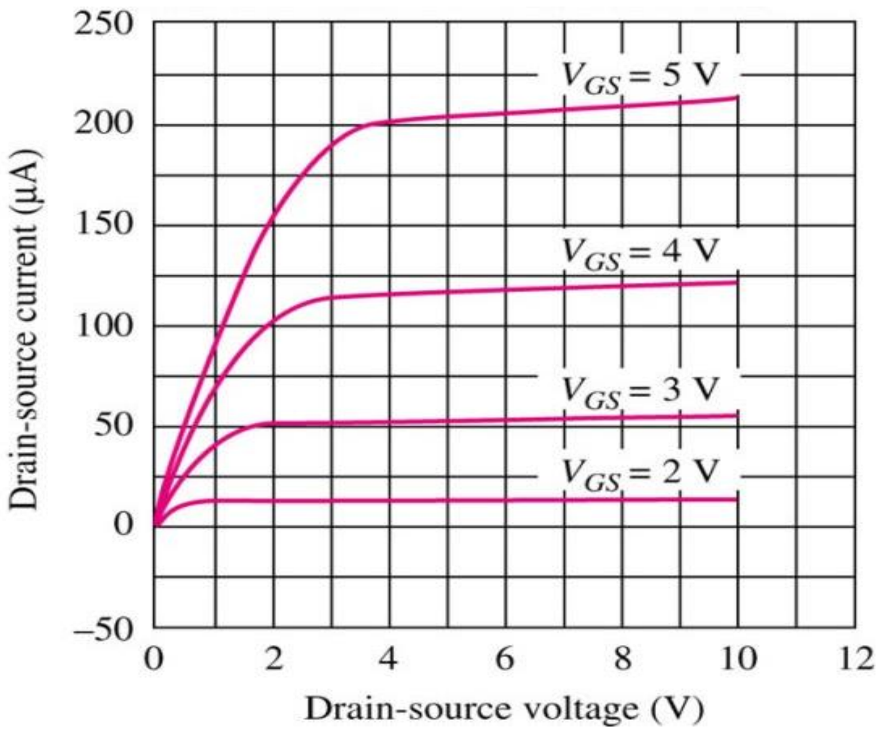
$$C_{ox} = \varepsilon_o / t_{ox}$$

Nonideal I-V Characteristics

Channel Length Modulation: Early Voltage



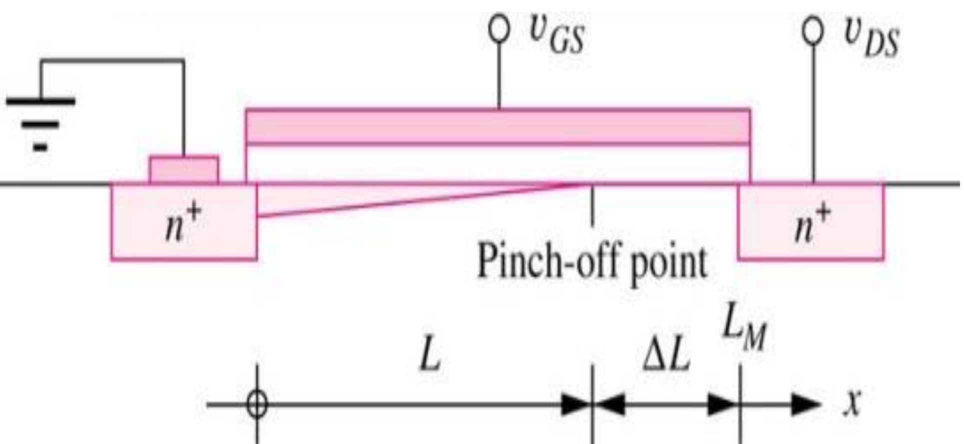
Channel-Length Modulation



- As v_{DS} increases above v_{DSAT}, length of depleted channel beyond **pinch-off point**, ΔL, increases and actual L decreases.
- i_D increases slightly with v_{DS} instead of being constant.

$$i_D = \frac{K_n}{2}\frac{W}{L}\left(v_{GS} - V_{TN}\right)^2\left(1 + \lambda v_{DS}\right)$$

λ= channel length modulation parameter



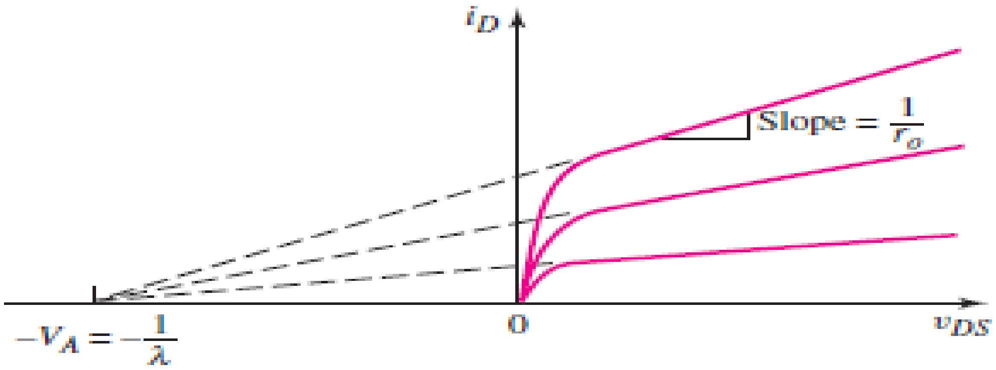


Figure 3.20 Family of i_D versus v_{DS} curves showing the effect of channel length modulation producing a finite output resistance

The parameters λ and V_A are related. From Equation (3.7), we have $(1 + \lambda v_{DS}) = 0$ at the extrapolated point where $i_D = 0$. At this point, $v_{DS} = -V_A$, which means that $V_A = 1/\lambda$.

The output resistance due to the channel length modulation is defined as

$$r_o = \left(\frac{\partial i_D}{\partial v_{DS}}\right)^{-1}\Bigg|_{v_{GS}=\text{const.}} \tag{3.8}$$

From Equation (3.7), the output resistance, evaluated at the Q-point, is

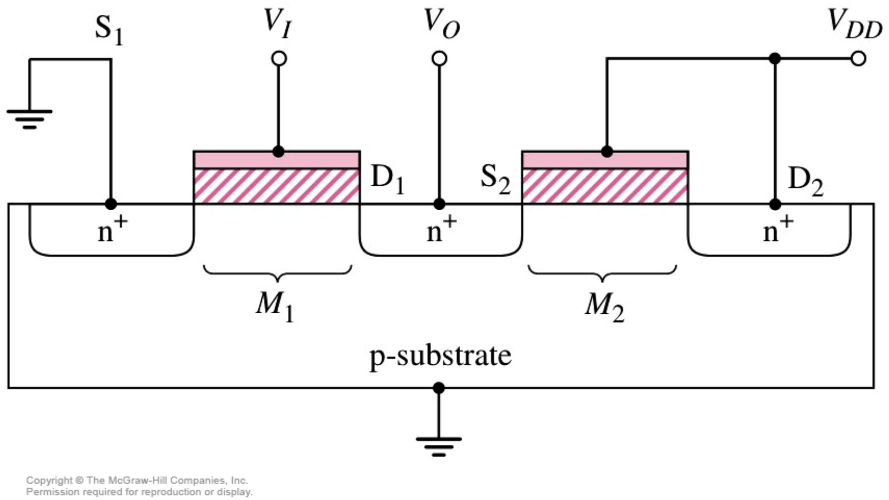
$$r_o = [\lambda K_n (V_{GSQ} - V_{TN})^2]^{-1} \tag{3.9(a)}$$

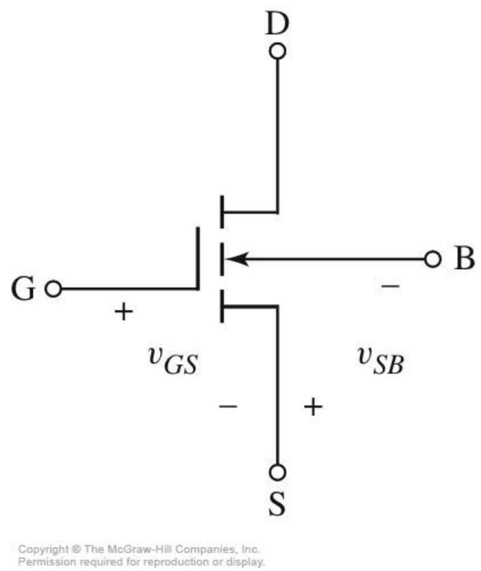
or

$$r_o \cong [\lambda I_{DQ}]^{-1} = \frac{1}{\lambda I_{DQ}} = \frac{V_A}{I_{DQ}} \tag{3.9(b)}$$

The output resistance r_o is also a factor in the small-signal equivalent circuit of the MOSFET, which is discussed in the next chapter.

Body Effect



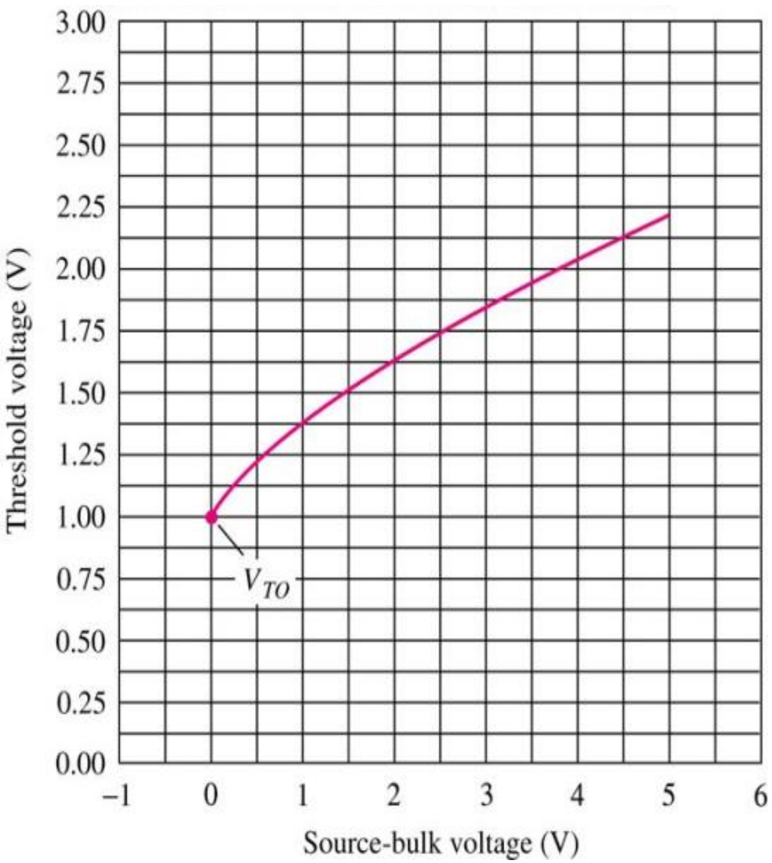


A zero or reverse-bias voltage exists across the source-substrate pn junction.

A change in the source-substrate junction voltage changes the threshold voltage.

Body Effect or Substrate Sensitivity

The assumption that the body lead is connected to the source lead. In integrated circuit design. In this case, any signal voltage on the source lead causes an a signal voltage between the body and source. The effect of this voltage is called the body effect.



This non-zero V_{SB} changes threshold voltage, causing substrate sensitivity to be modeled by:

$$V_{TN} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F}\right)$$

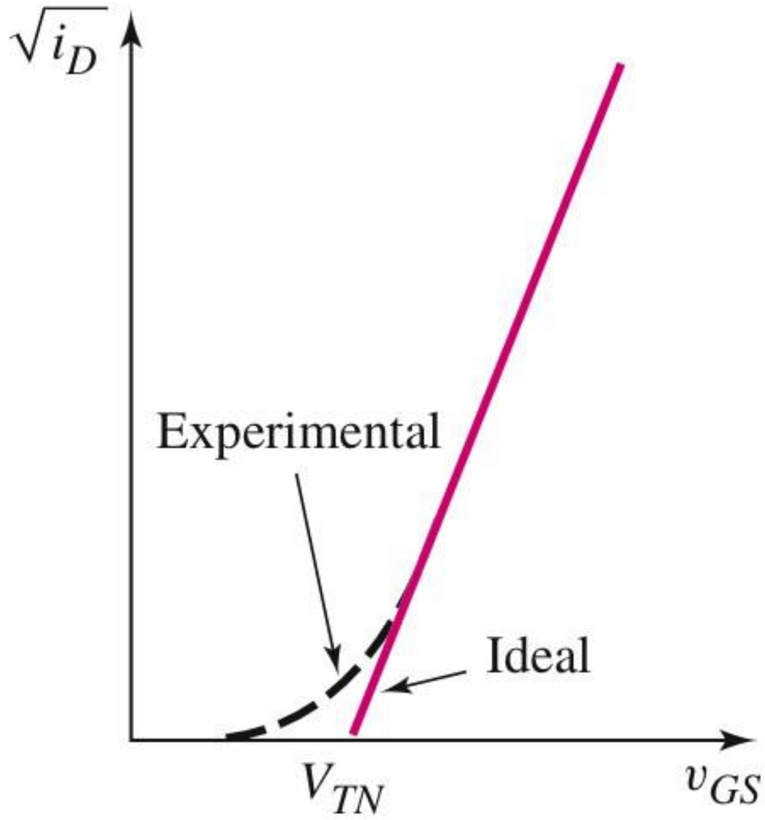
Where

V_{TO}= zero substrate bias for V_{TN} (V)

γ = body-effect parameter ($\sqrt{V}$)

$2F_F$= surface potential parameter (V)

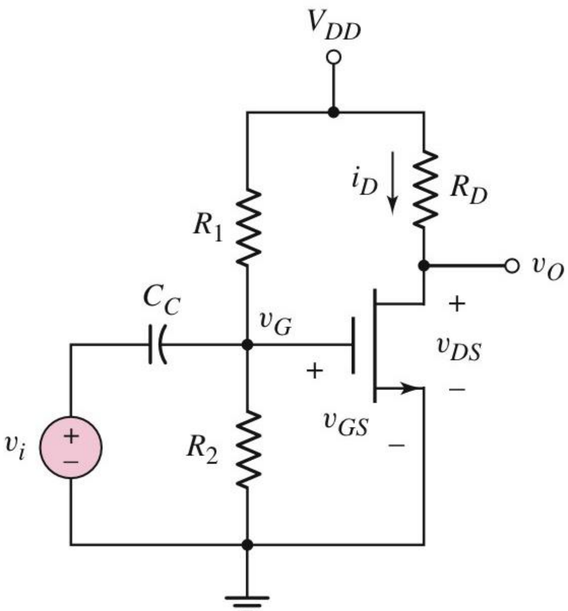
Subthreshold Condition

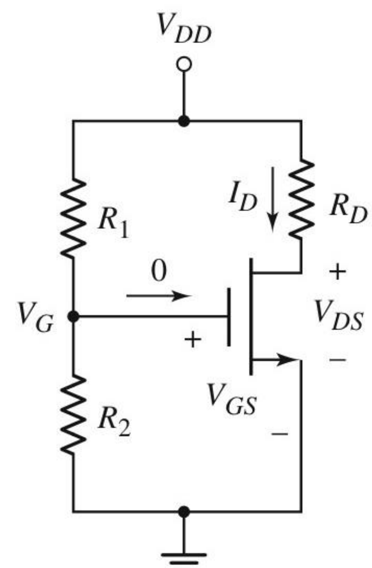


The drain current is non-zero; v_{gs} is slightly less than V_{TN}.

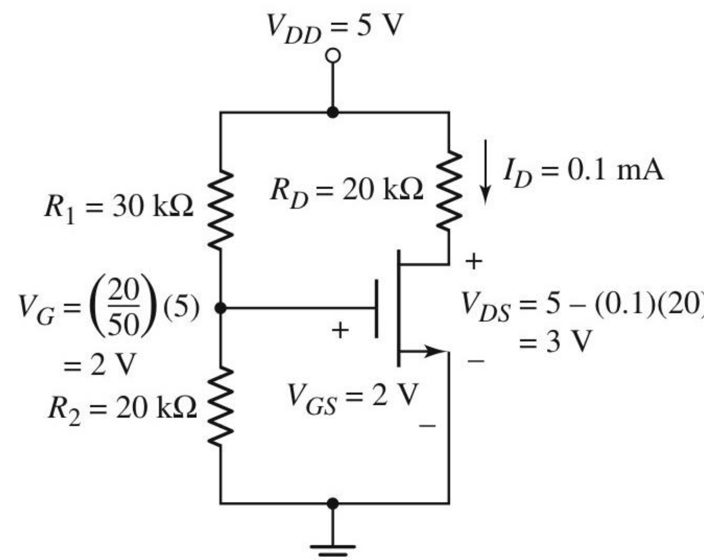
MOSFET DC Circuit Analysis

NMOS Common-Source Circuit



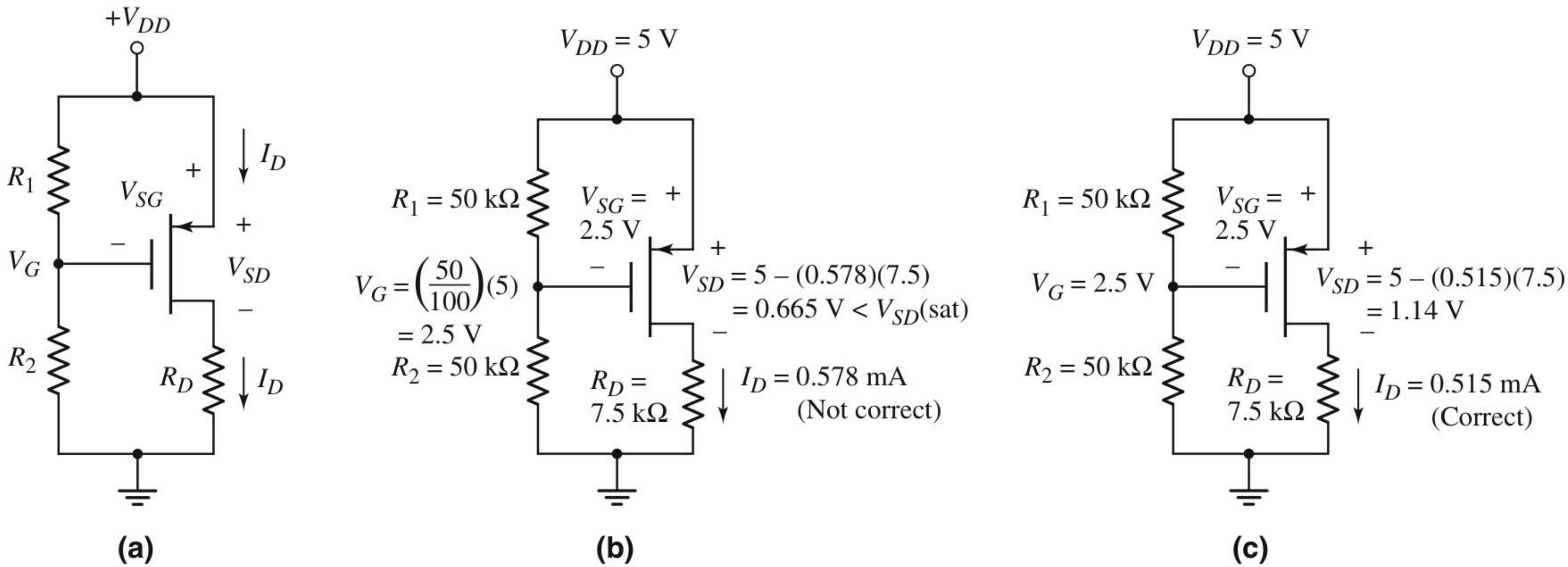


(a)

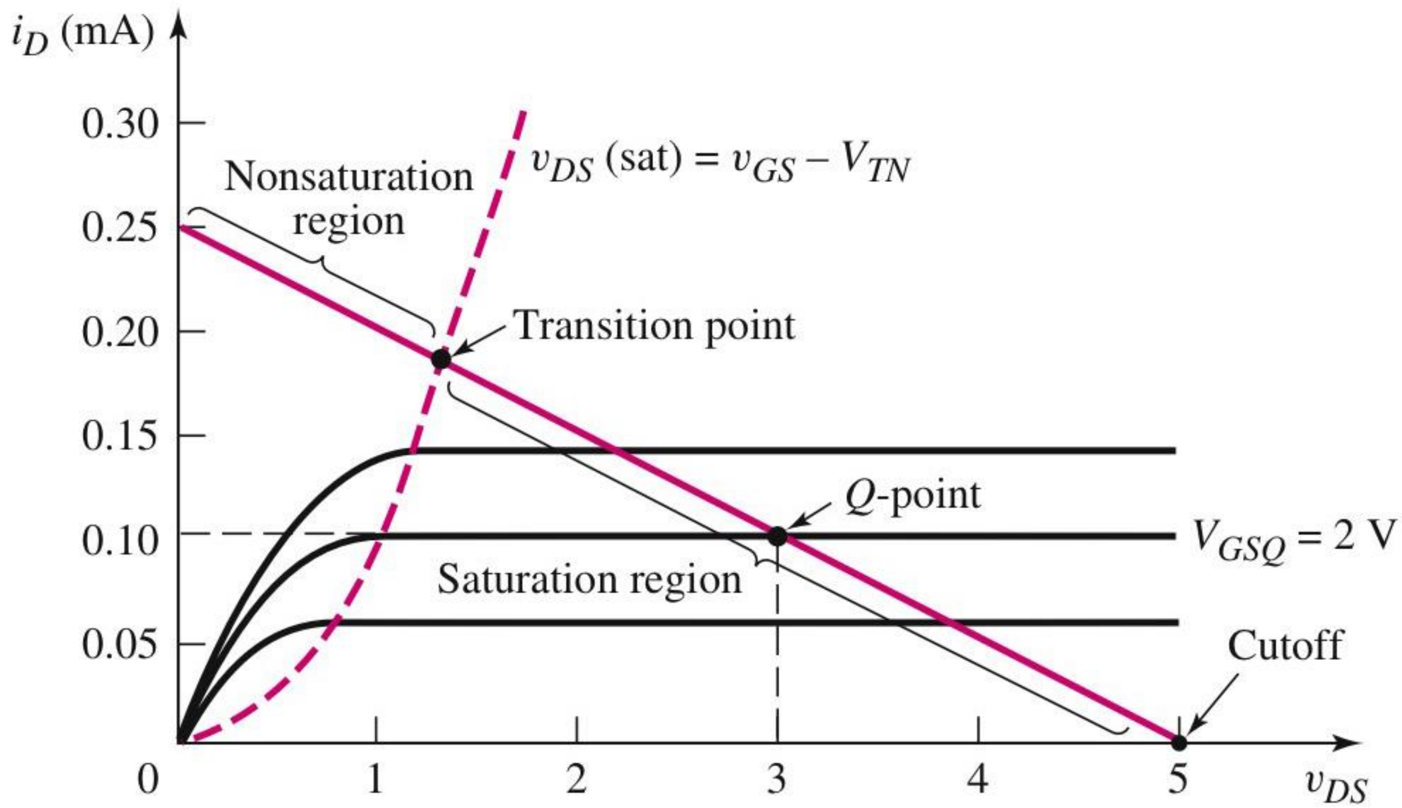


(b)

PMOS Common-Source Circuit



Load Line and Modes of Operation: NMOS Common-Source Circuit

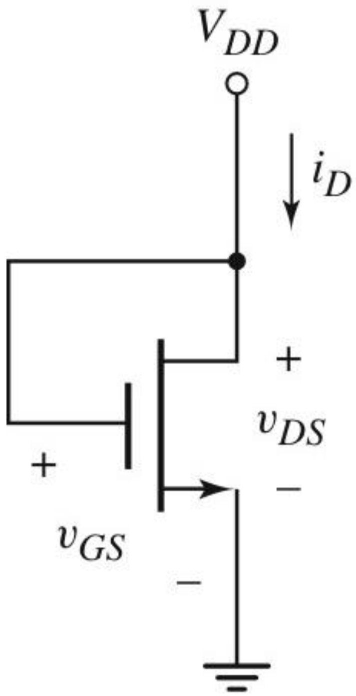


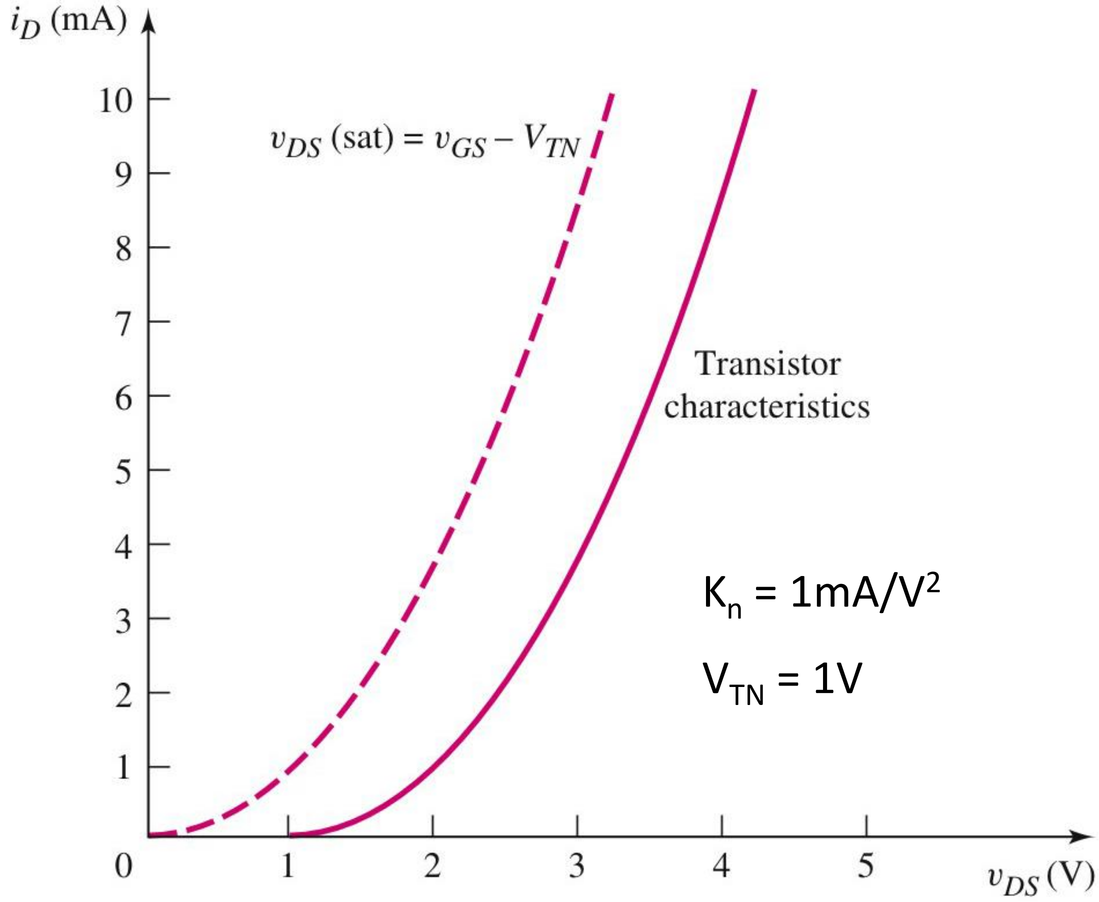
Problem-Solving Technique: NMOSFET DC Analysis

1. Assume the transistor is in saturation.
   a. $V_{GS} > V_{TN}$, $I_D > 0$, & $V_{DS} \geq V_{DS}(sat)$
2. Analyze circuit using saturation I-V relations.
3. Evaluate resulting bias condition of transistor.
   a. If $V_{GS} < V_{TN}$, transistor is likely in cutoff
   b. If $V_{DS} < V_{DS}(sat)$, transistor is likely in nonsaturation region
4. If initial assumption is proven incorrect, make new assumption and repeat Steps 2 and 3.

n-Channel Enhancement-Load Device

Enhancement Load Device



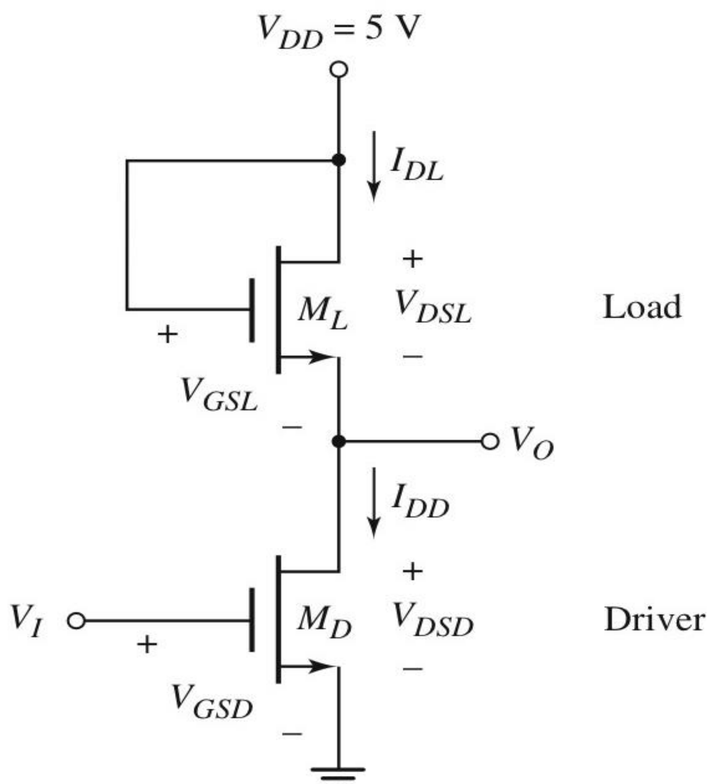


K_n = 1mA/V^2

V_{TN} = 1V

$$i_D = K_n(v_{GS} - V_{TN})^2 = K_n(v_{DS} - V_{TN})^2$$

Circuit with Enhancement Load Device and NMOS Driver

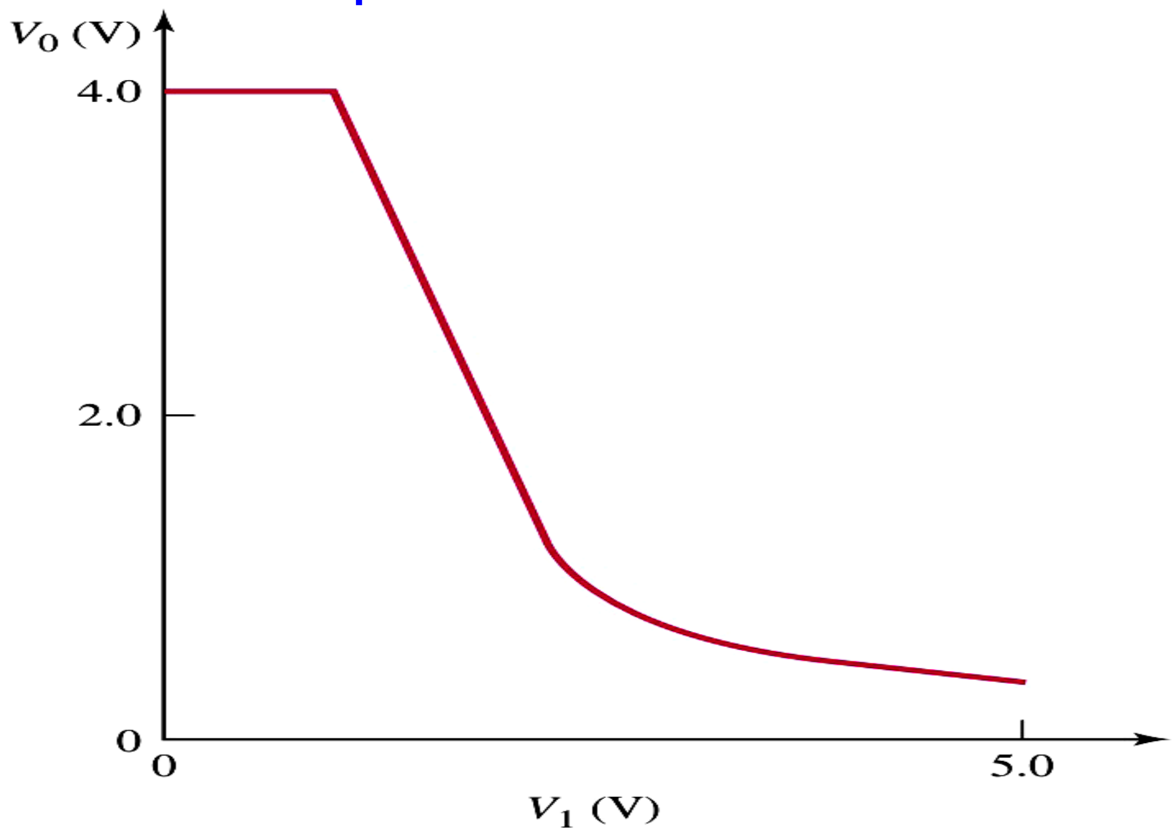


M_L is always in **saturation.**

M_D can be biased either in **saturation** or **nonsaturation** region.

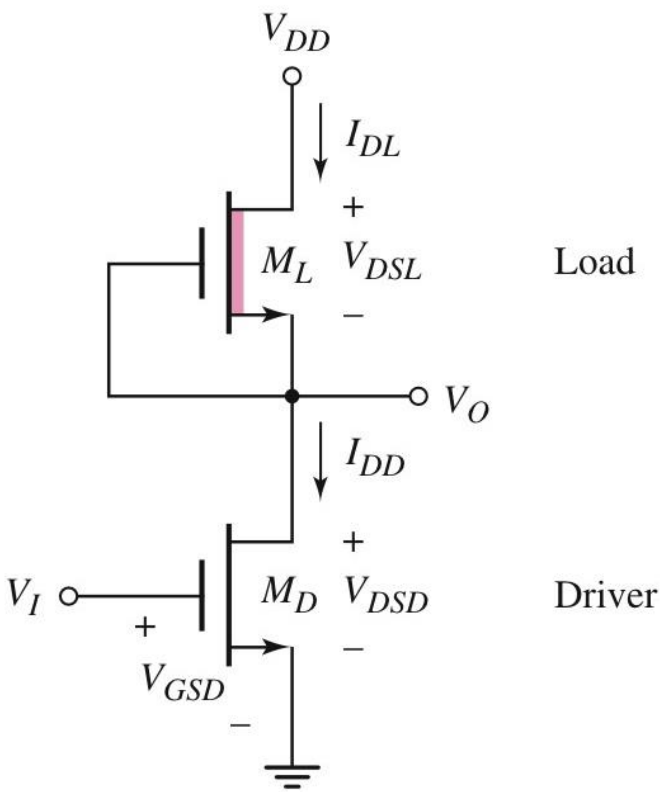
Voltage Transfer Characteristics:
NMOS Inverter with Enhancement Load Device

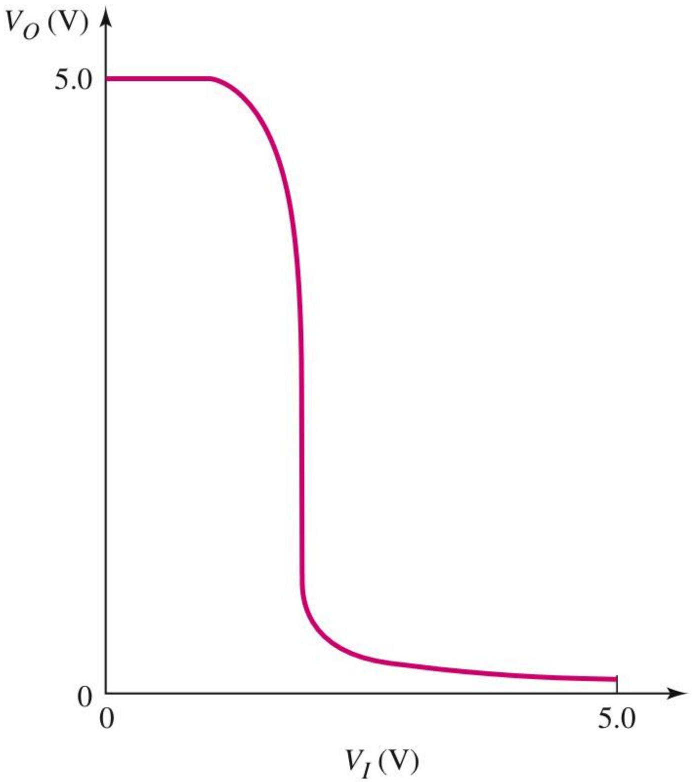
$v_I < V_{TN}$ | $v_I > V_{TN}$



n-Channel Depletion-Load Device

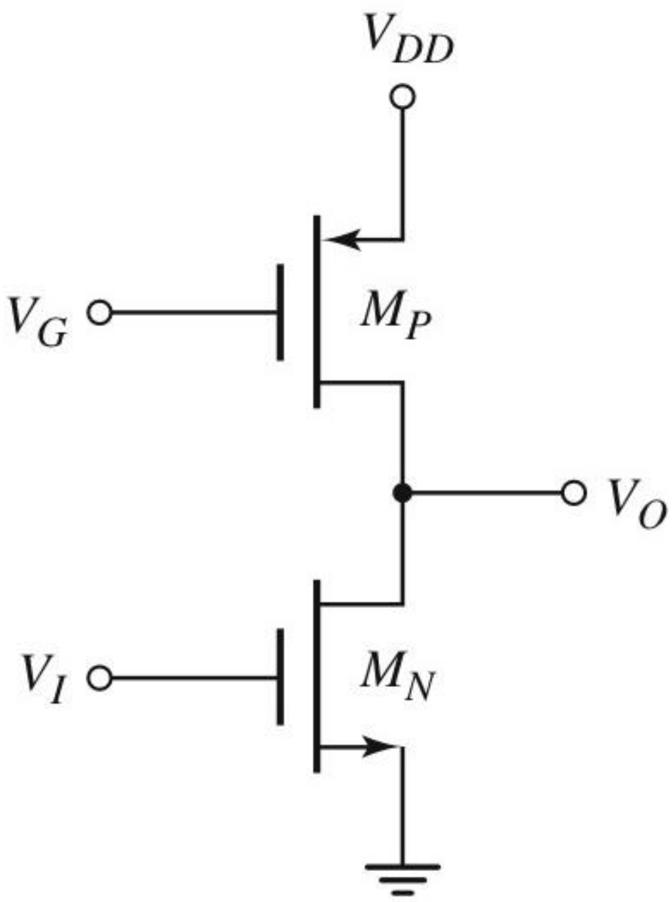
NMOS Inverter with Depletion Load Device

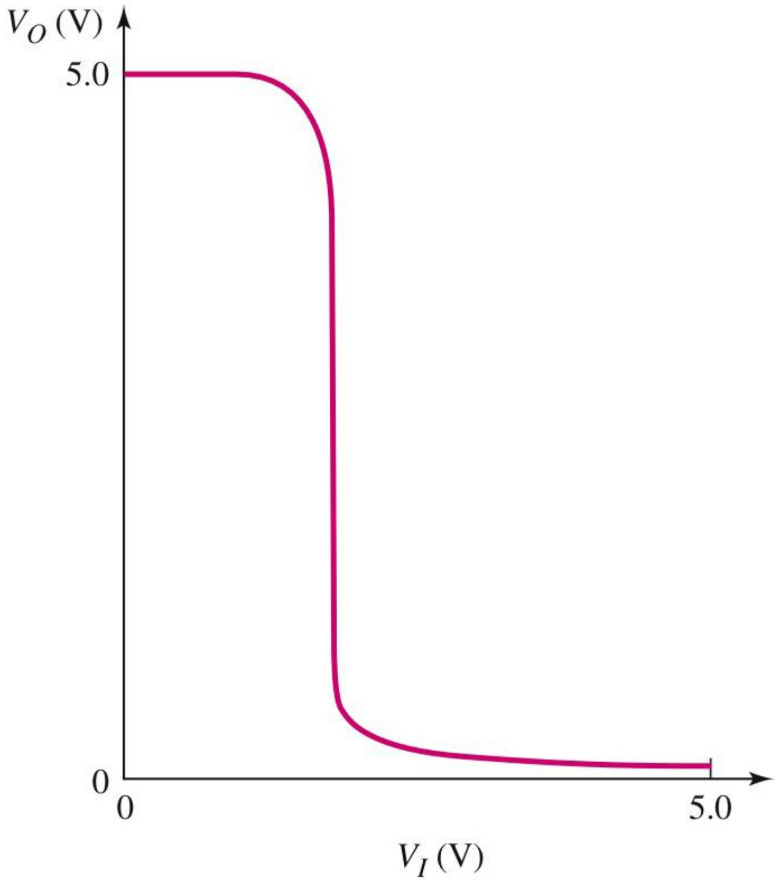




p-Channel Enhancement-Load Device

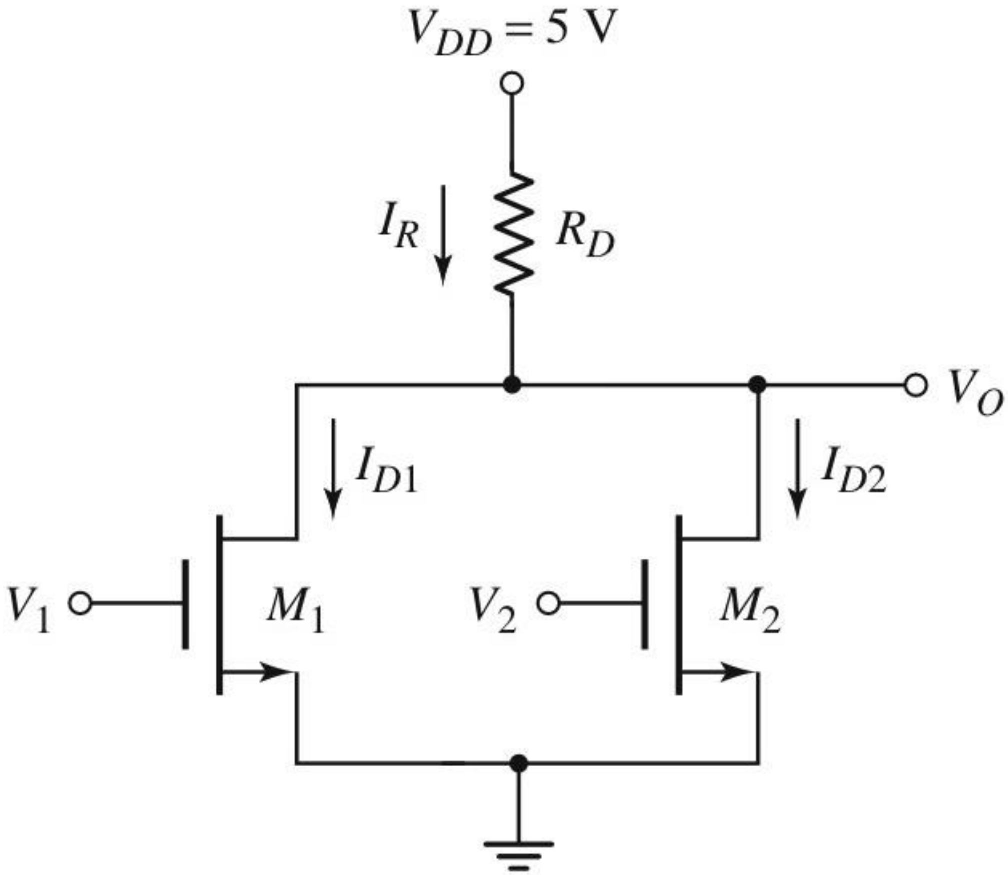
CMOS Inverter





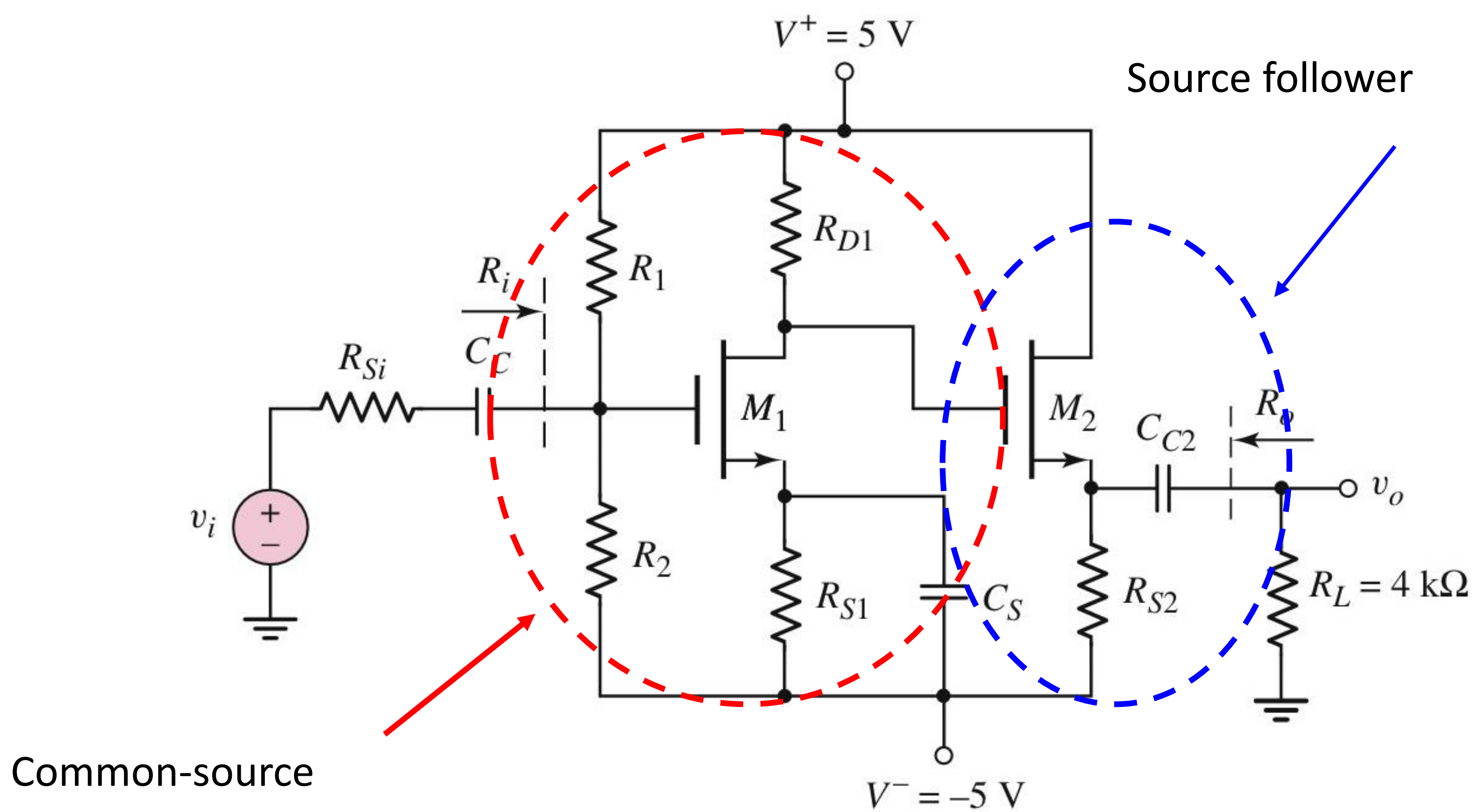
Basic MOSFET Applications

2-Input NMOS NOR Logic Gate



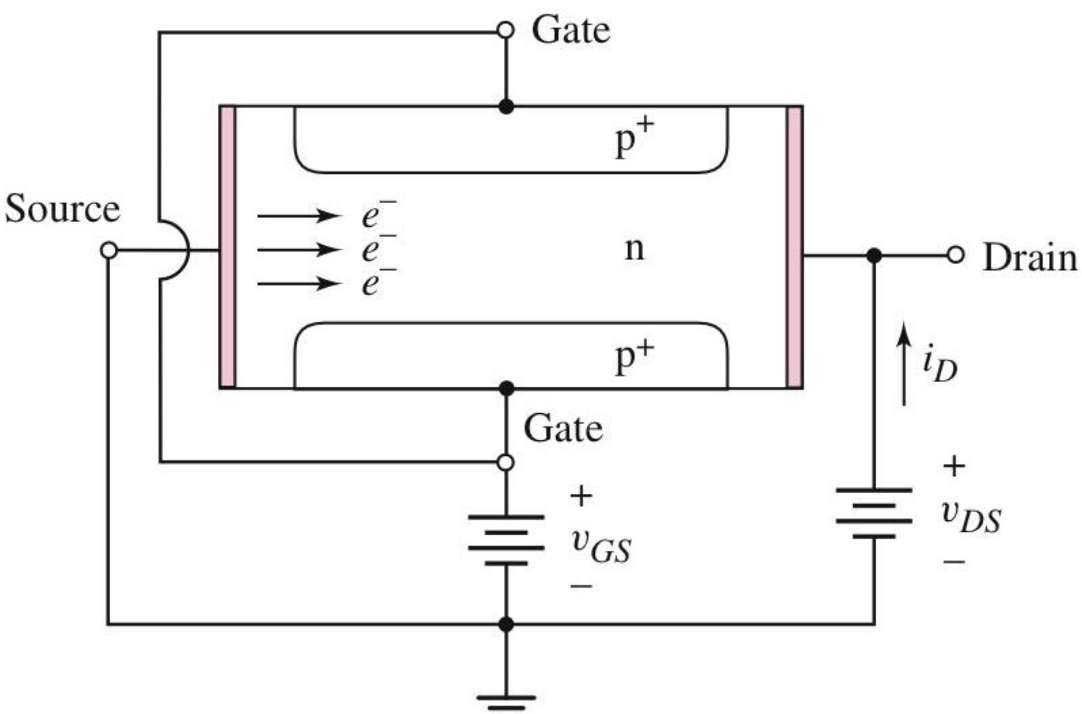
V_1 (V)	V_2 (V)	V_O (V)
0	0	High
5	0	Low
0	5	Low
5	5	Low

2-Stage Cascade Amplifier



Junction Field-Effect Transistor

Cross Section of n-Channel Junction Field Effect Transistor (JFET)

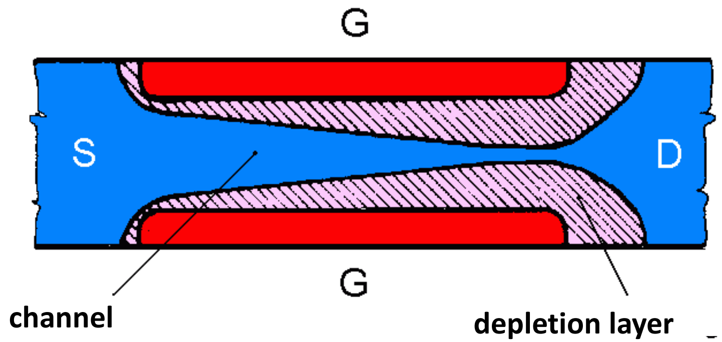


JUNCTION FET: depletion layers of pn-junctions close the channel

Most important parameter: pinch-off voltage

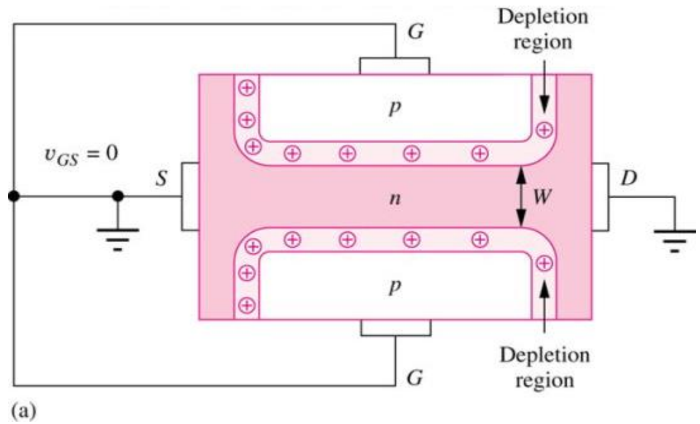
The JFET

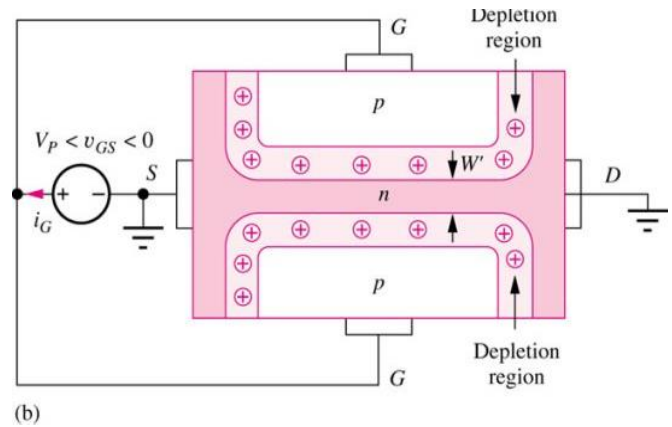
The width of the closed PN junction controls the conductivity of the channel

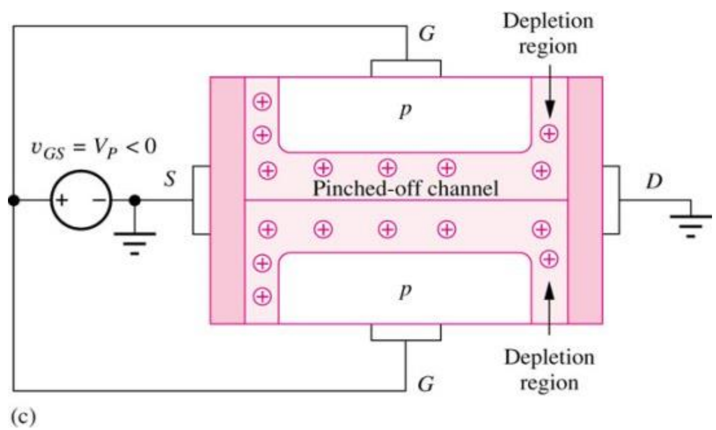


PN junction $\Rightarrow$ junction FET

JFET with Gate-Source Bias

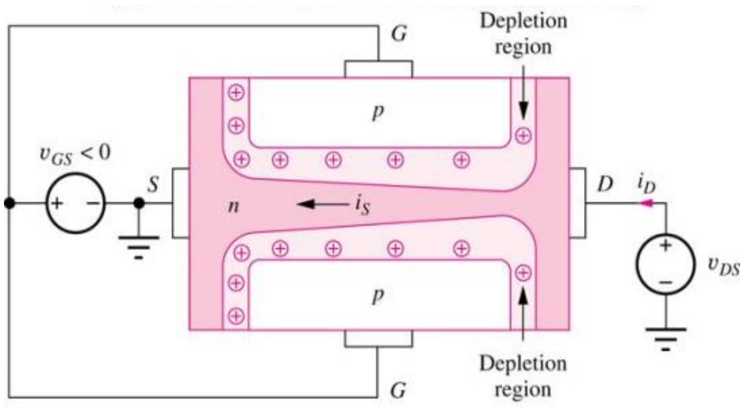




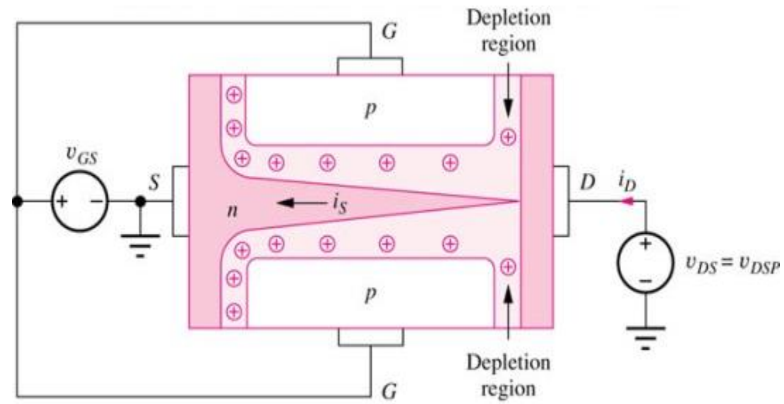


- v_{GS} =0, gate isolated from channel.
- $V_P < v_{GS}$ <0, W'<W, channel resistance increases, gate-source junction reverse-biased, i_G almost 0.
- $v_{GS} = V_P$ <0, channel region pinched-off, channel resistance infinite.

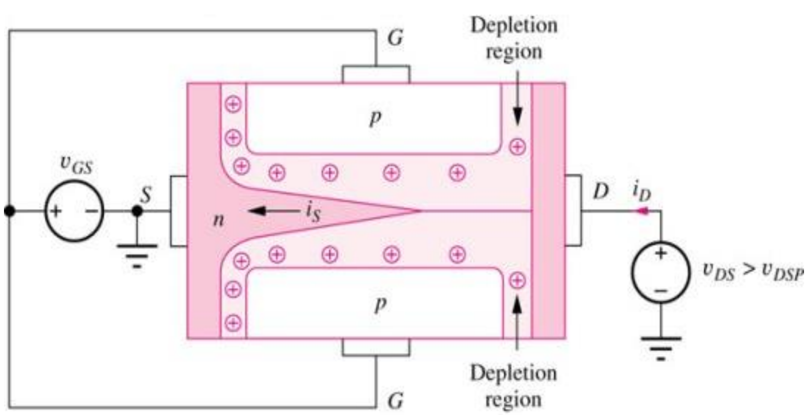
JFET Channel with Drain-Source Bias



(a)



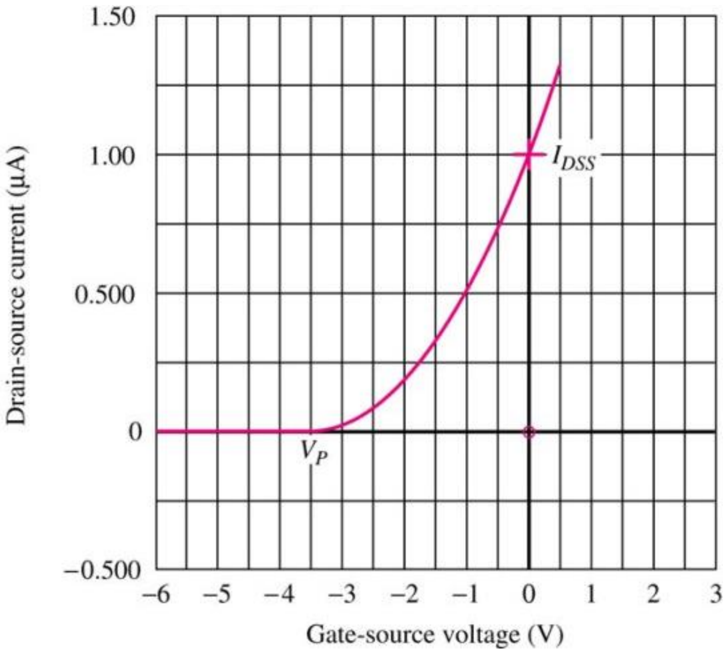
(b)



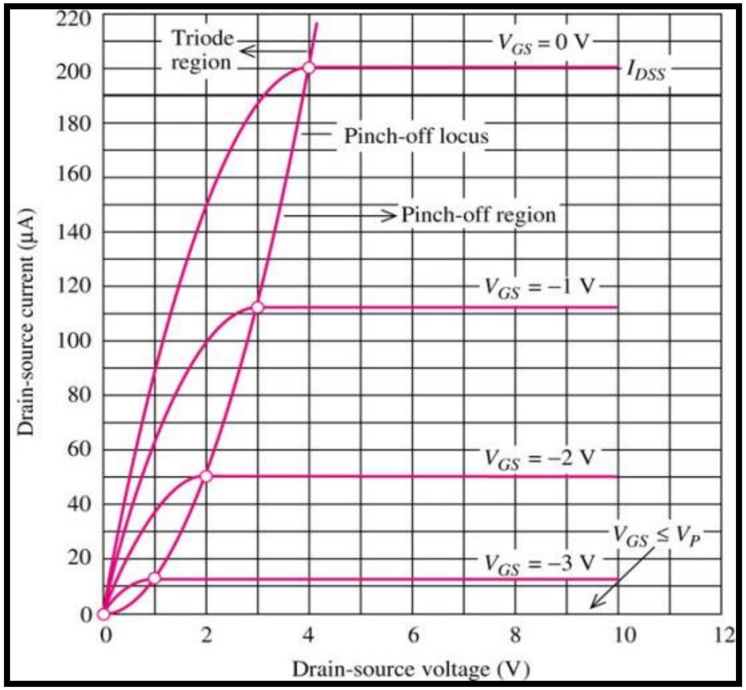
(c)

- With constant v_{GS}, depletion region near drain increases with v_{DS}.
- At $v_{DSP} = v_{GS} - V_P$,channel is totally pinched-off, i_D is saturated.
- JFET also suffers from channel length modulation like MOSFET at larger values of v_{DS}.

N-Channel JFET: i-v Characteristics



Transfer Characteristics



Output Characteristics

Chapter

•Study and understand the structure, operation and characteristics of MOSFETs.

•Understand and become familiar with the dc analysis and design techniques of MOSFET circuits.

•Examine three applications of MOSFET circuits.

•Investigate current source biasing of MOSFET circuits, such as those used in integrated circuits.

•Analyze the dc biasing of multistage or multi-transistor circuits.

•Understand the operation and characteristics of the junction field-effect transistor, and analyze the dc response of JFET circuits.

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西交利物浦大學

EEE109: Electronic Circuits

Basic FET Amplifiers

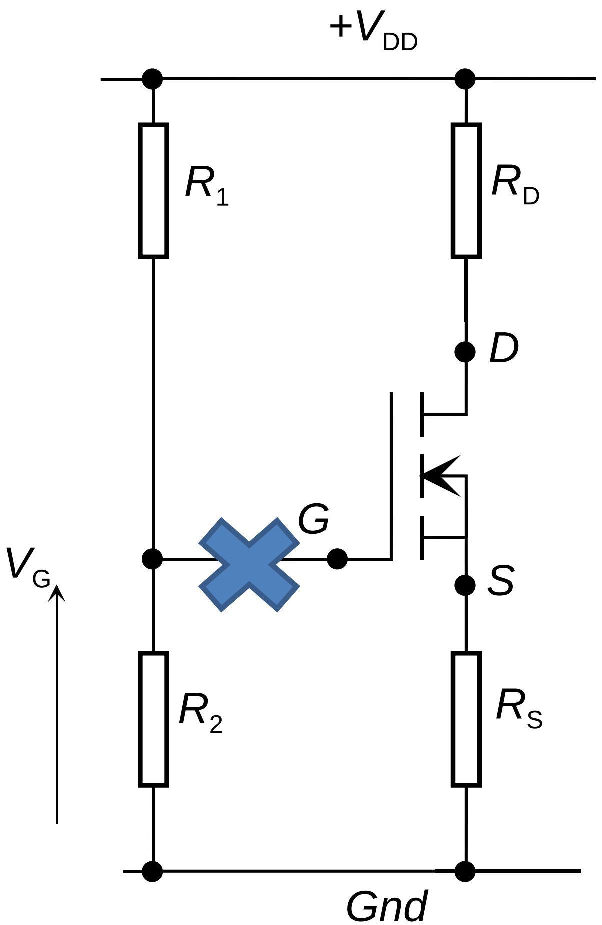
Contents of Chapter

- Investigate a single-transistor circuit that can amplify a small, time-varying input signal
  - Develop small-signal models that are used in the analysis of linear amplifiers.
- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-source amplifier.
  - Analyze the source-follower amplifier.
  - Analyze the common-gate amplifier.
- Analyze multitransistor or multistage amplifiers.
- Develop the small-signal model of JFET devices and analyze basic JFET amplifiers.
- Design a two-stage MOSFET amplifier circuit.

General Amplifier Characteristics

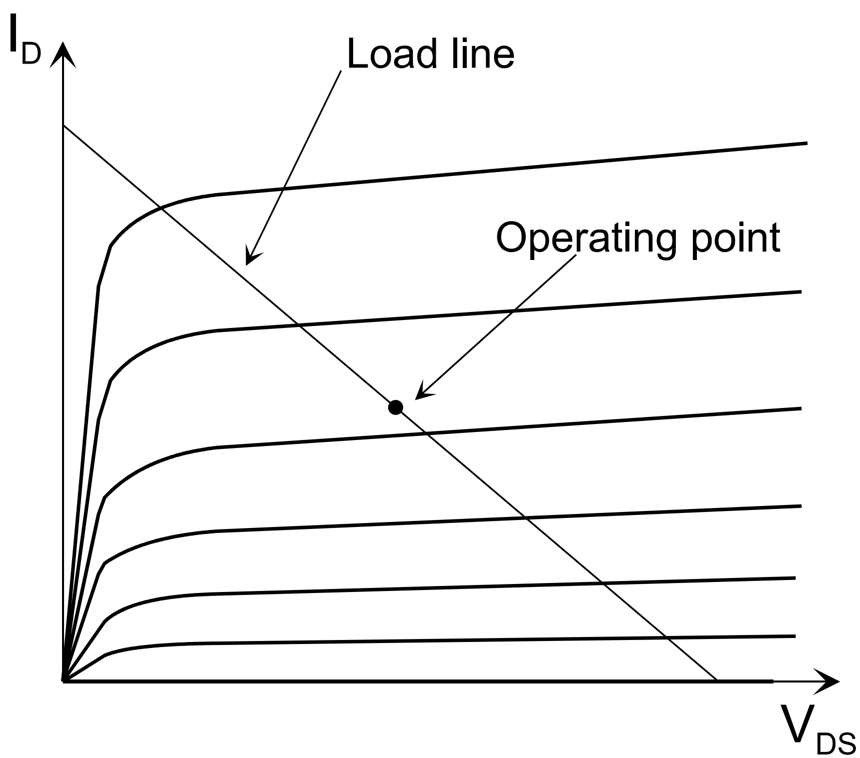
Amplifier Characteristics (1)

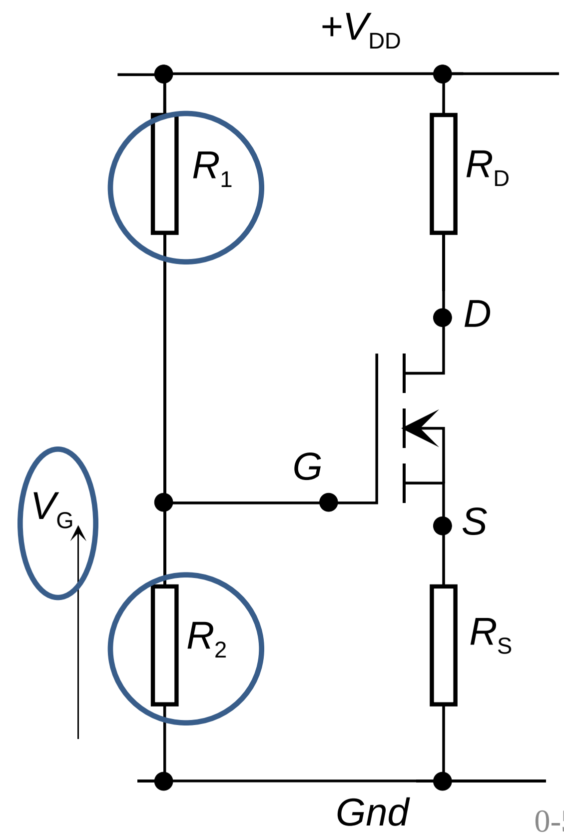
The MOSFET may be used as a **switch** or biased to operate as an **amplifier**; circuits are similar to those for the bipolar transistor **EXCEPT** **no current flows into the gate**. Using four resistor biasing a common source amplifier is formed.



Amplifier Characteristics (2)

The gate voltage V_G is set by the two resistors R_1 and R_2 and the voltage divider gives V_G. The MOSFET only conducts if $V_G > V_T$. The bias condition (operating point) gives the required values of V_{DS} and I_D.





Circuit Notation Conventions

Table 4.1 Summary of notation

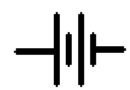
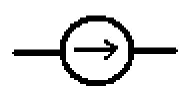
Variable	Meaning
i_D, v_{GS}	Total instantaneous values
I_D, V_{GS}	DC values
i_d, v_{gs}	Instantaneous ac values
I_d, V_{gs}	Phasor values

Small Signal Analysis Problem Solving Skills

Problem-Solving Technique: MOSFET AC Analysis

1. Analyze circuit with only the dc sources to find quiescent solution. Transistor must be biased in saturation region for linear amplifier.
2. Replace **elements** with small-signal model.
3. Analyze small-signal equivalent circuit, setting dc sources to zero, to produce the circuit to the time-varying input signals only.

Transformation of Elements

Element	DC Model	AC Model
Resistor	R	R
Capacitor	Open	C
Inductor	Short	L
Diode	$+V_\gamma$, r_f − 	$r_d = V_T/I_D$
Independent Constant Voltage Source	$+\ V_S\ -$ 	Short
Independent Constant Current Source	I_S 	Open

Practical Skill: General DC & AC Analysis (General Procedure)

- **DC analysis:**
  - Find DC equivalent circuit by replacing all capacitors by open circuits and inductors by short circuits.
  - Find Q-point from DC equivalent circuit by using appropriate large-signal transistor model.
- **AC analysis:**
  - Find AC equivalent circuit by replacing all capacitors by short circuits, inductors by open circuits, DC voltage sources by ground connections and DC current sources by open circuits.
  - Replace transistor by small-signal model
  - Use small-signal AC equivalent to analyze AC characteristics of amplifier.
  - Combine end results of DC and AC analysis to yield total voltages and currents in the network.

Note: Since we are dealing with linear amplifiers (saturation mode), the principle of superposition holds

Practical Skill: General DC & AC Analysis (Equivalent Circuits)

- **DC equivalent circuit:**

1) replacing all capacitances with open circuits,
2) replacing inductances with short circuits,
3) reducing AC sources to zero:
   - ✓ Replacing AC voltage sources by short circuits and
   - ✓ Replacing AC current sources by open circuits.

- **AC equivalent circuit:**

1) reducing all DC sources to zero:
   - ✓ Replacing DC voltage sources with short circuits and
   - ✓ Replacing DC current sources with open circuits.
2) replacing all capacitances with short circuits,
3) replacing inductances with open circuits,
4) replacing transistor with small signal equivalent circuit.

NMOS Transistor Small-Signal Parameters

- Values depends on Q-point

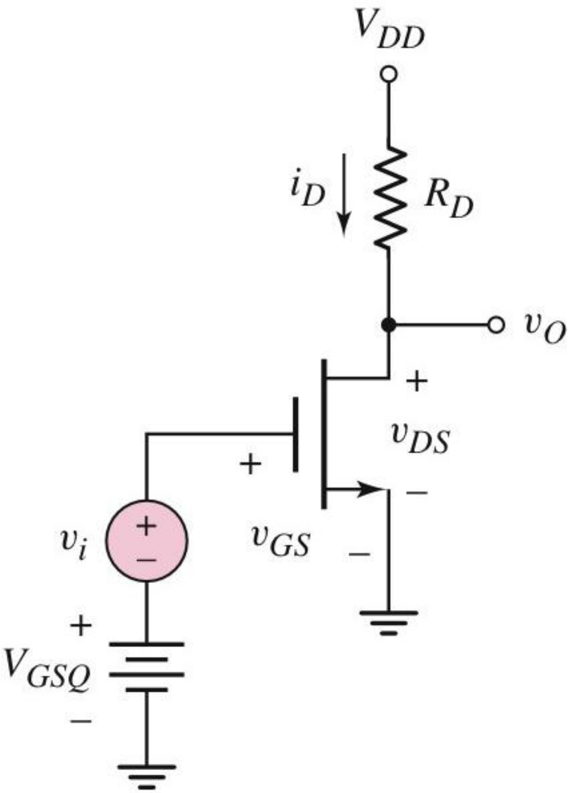
$$g_m = \frac{\partial i_D}{\partial v_{GS}} = \frac{i_d}{v_{gs}}$$

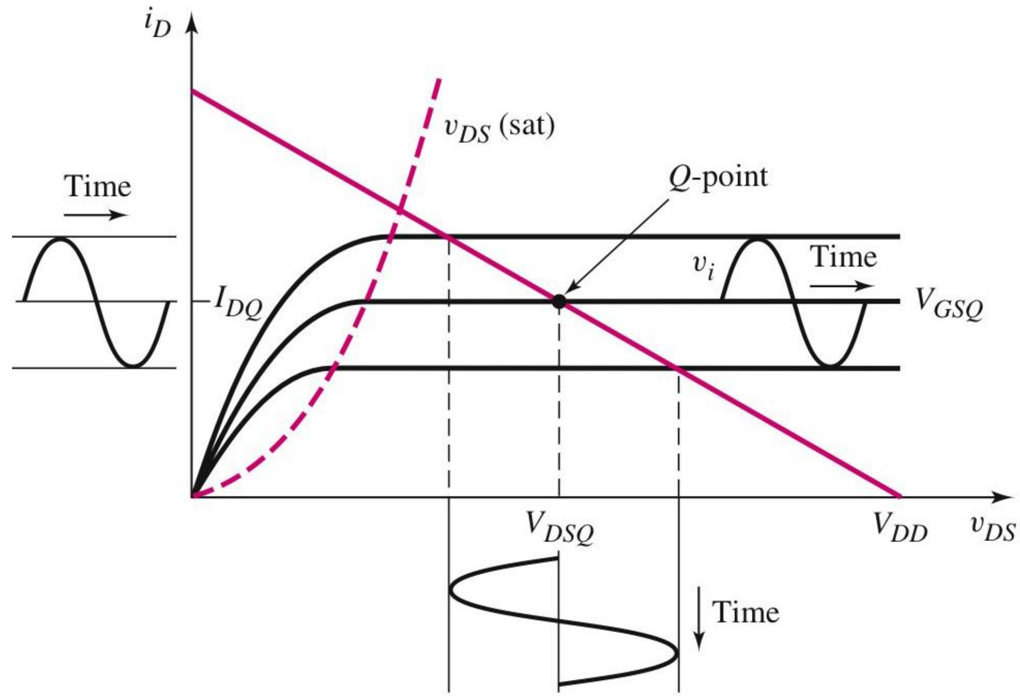
$$g_m = 2K_n(V_{GSQ} - V_{TN}) = \boxed{2\sqrt{K_n I_{DQ}}}$$

$$r_o = \left(\frac{\partial i_D}{\partial v_{DS}}\right)^{-1}$$

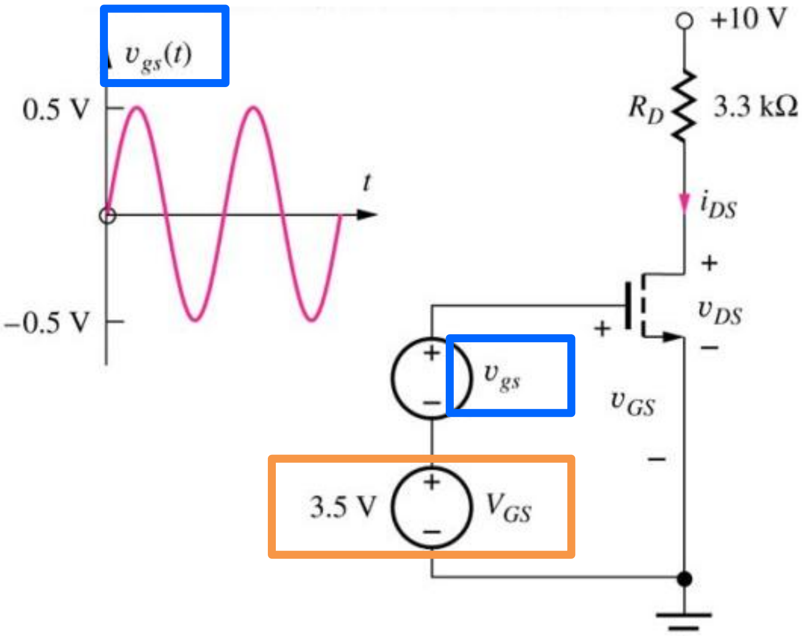
$$r_o = [\lambda K_n (V_{GSQ} - V_{TN})^2]^{-1} \cong \boxed{[\lambda I_{DQ}]^{-1}}$$

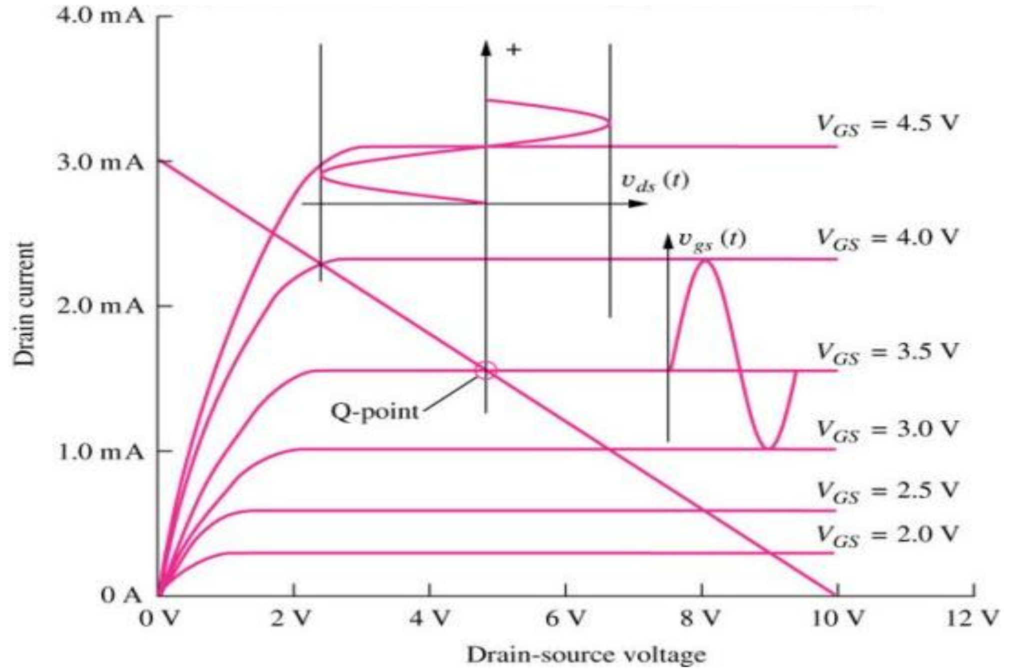
NMOS Common-Source Circuit





MOSFET Amplifier Example





MOSFET is biased in active region by DC voltage source V_{GS}.

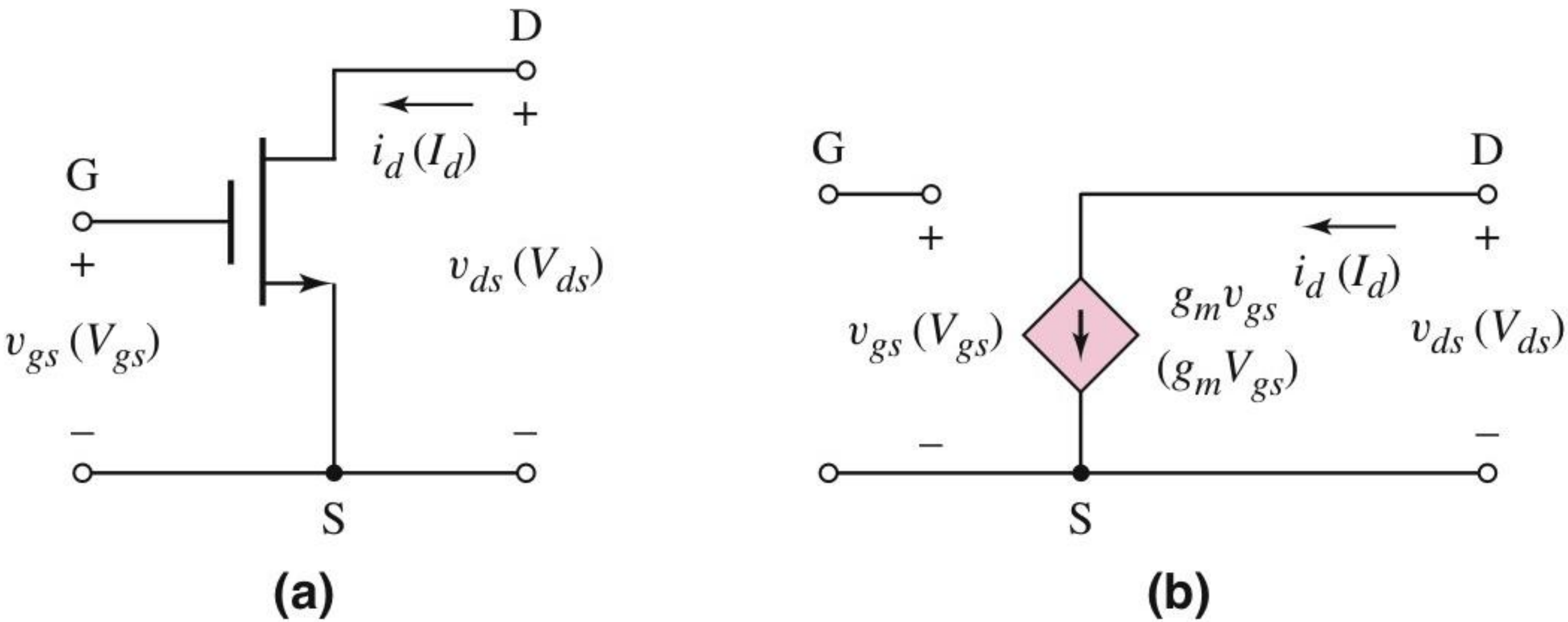
Q-point is set at (V_{DS}, I_D) = (4.8 V, 1.56 mA) @ V_{GS} = 3.5 V.

Total gate-source voltage is: $v_{GS}(t) = V_{GS} + v_{gs}(t)$

The input: 1 V *p-p* change in v_{GS}

The output: 1.25 mA *p-p* change in i_D and 4 V *p-p* change in v_{DS}.

Simple NMOS Small-Signal Transistor Equivalent Circuit (1)



Simple NMOS Small-Signal Transistor Equivalent Circuit (2)

$$v_{GS} = V_{GSQ} + v_i = V_{GSQ} + v_{gs} \tag{4.1}$$

where V_{GSQ} is the dc component and v_{gs} is the ac component. The instantaneous drain current is

$$i_D = K_n(v_{GS} - V_{TN})^2 \tag{4.2}$$

Substituting Equation (4.1) into (4.2) produces

$$i_D = K_n[V_{GSQ} + v_{gs} - V_{TN}]^2 = K_n[(V_{GSQ} - V_{TN}) + v_{gs}]^2 \tag{4.3(a)}$$

or

$$i_D = \boxed{K_n(V_{GSQ} - V_{TN})^2} + \boxed{2K_n(V_{GSQ} - V_{TN})v_{gs}} + \boxed{K_n v_{gs}^2} \tag{4.3(b)}$$

The first term in Equation (4.3(b)) is the dc or quiescent drain current I_{DQ}, the second term is the time-varying drain current component that is linearly related to the signal v_{gs}, and the third term is proportional to the square of the signal voltage. For a sinusoidal input signal, the squared term produces undesirable harmonics, or nonlinear distortion, in the output voltage. To minimize these harmonics, we require

$$v_{gs} \ll 2(V_{GSQ} - V_{TN}) \tag{4.4}$$

Simple NMOS Small-Signal Transistor Equivalent Circuit (3)

$$i_D = I_{DQ} + i_d \tag{4.5}$$

Again, small-signal implies linearity so that the total current can be separated into a dc component and an ac component. The ac component of the drain current is given by

$$i_d = 2K_n(V_{GSQ} - V_{TN})v_{gs} \tag{4.6}$$

The small-signal drain current is related to the small-signal gate-to-source voltage by the transconductance g_m. The relationship is

$$\boxed{g_m = \frac{i_d}{v_{gs}} = 2K_n(V_{GSQ} - V_{TN})} \tag{4.7}$$

The transconductance is a transfer coefficient relating output current to input voltage and can be thought of as representing the gain of the transistor.

The transconductance can also be obtained from the derivative

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS}=V_{GSQ}=\text{const.}} = 2K_n(V_{GSQ} - V_{TN}) \tag{4.8(a)}$$

Common Source with Channel Modulation

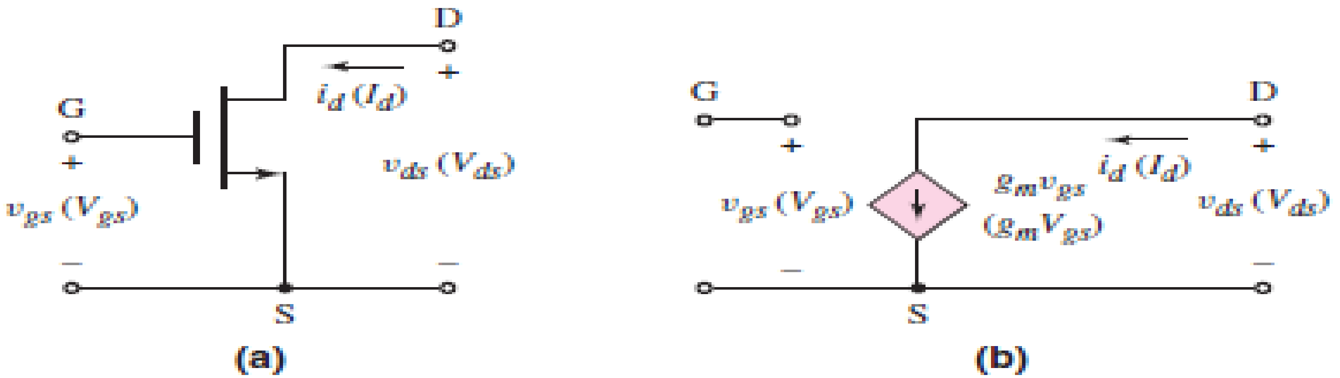


Figure 4.5 (a) Common-source NMOS transistor with small-signal parameters and (b) simplified small-signal equivalent circuit for NMOS transistor

$$i_D = K_n[(v_{GS} - V_{TN})^2(1 + \lambda v_{DS})] \tag{4.16}$$

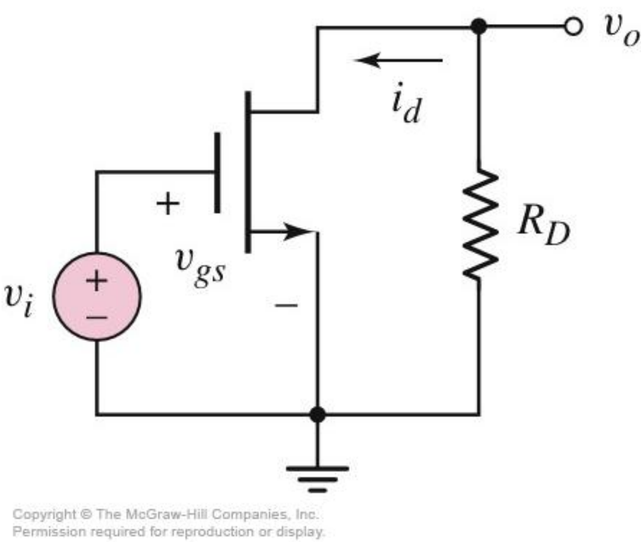
where λ is the channel-length modulation parameter and is a positive quantity. The small-signal output resistance, as previously defined, is

$$r_o = \left(\frac{\partial i_D}{\partial v_{DS}}\right)^{-1}\Bigg|_{v_{GS}=V_{GSQ}=\text{const.}} \tag{4.17}$$

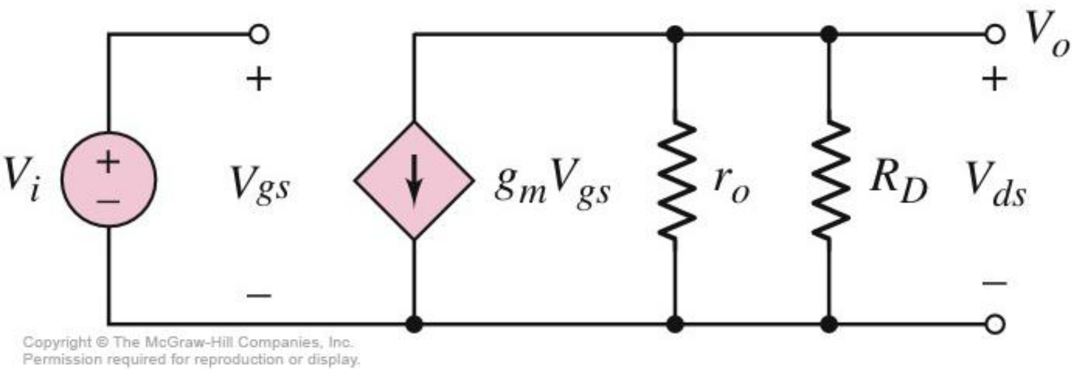
or

$$r_o = [\lambda K_n(V_{GSQ} - V_{TN})^2]^{-1} \cong [\lambda I_{DQ}]^{-1} \tag{4.18}$$

NMOS Common-Source Circuit



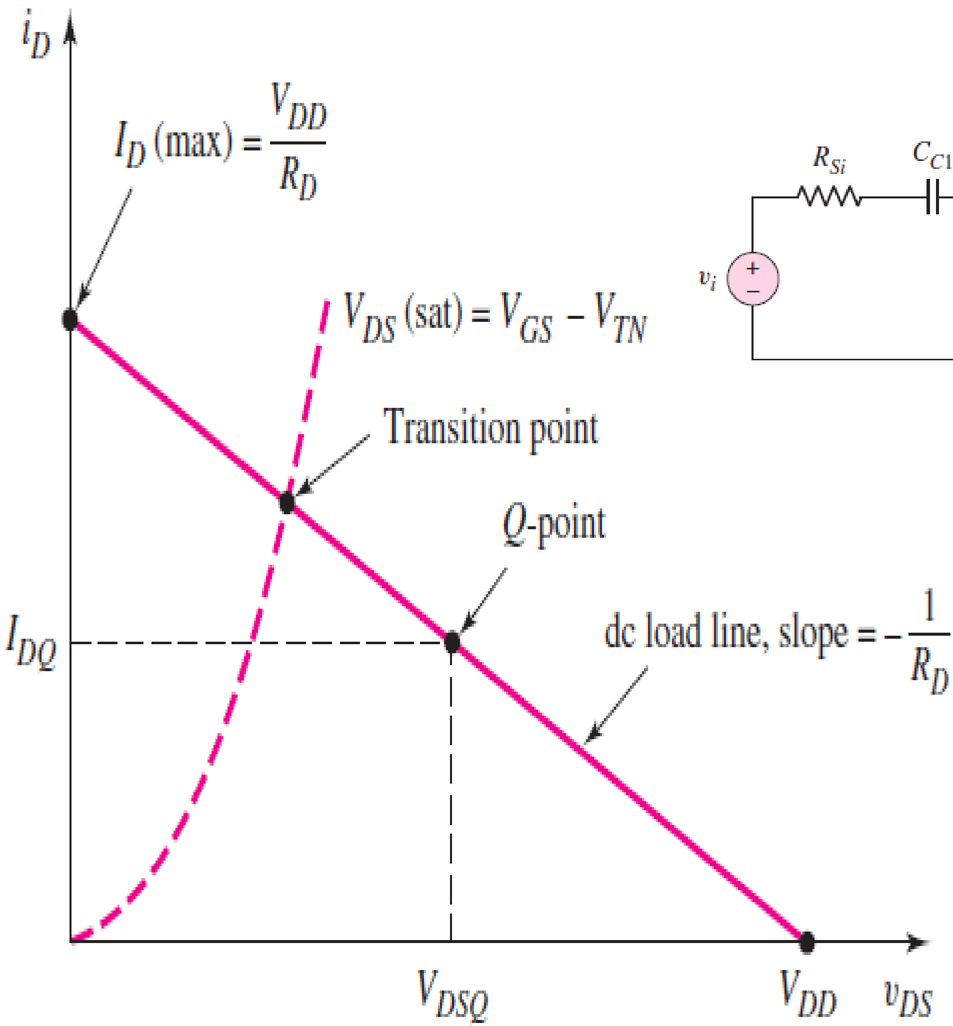
AC

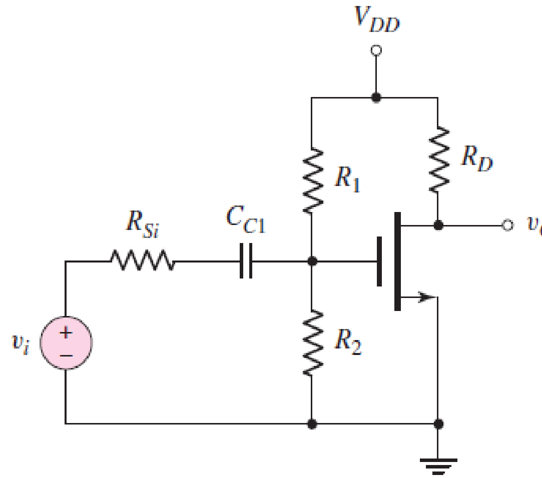


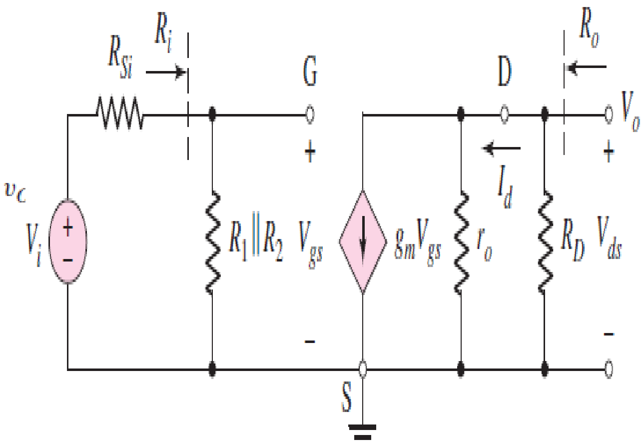
Small-signal

$$A_v = V_o / V_i = -g_m (r_o \| R_D)$$

NMOS Common-Source Circuit Analysis







$$V_o = -g_m V_{gs} (r_o \| R_D)$$

The input gate-to-source voltage is

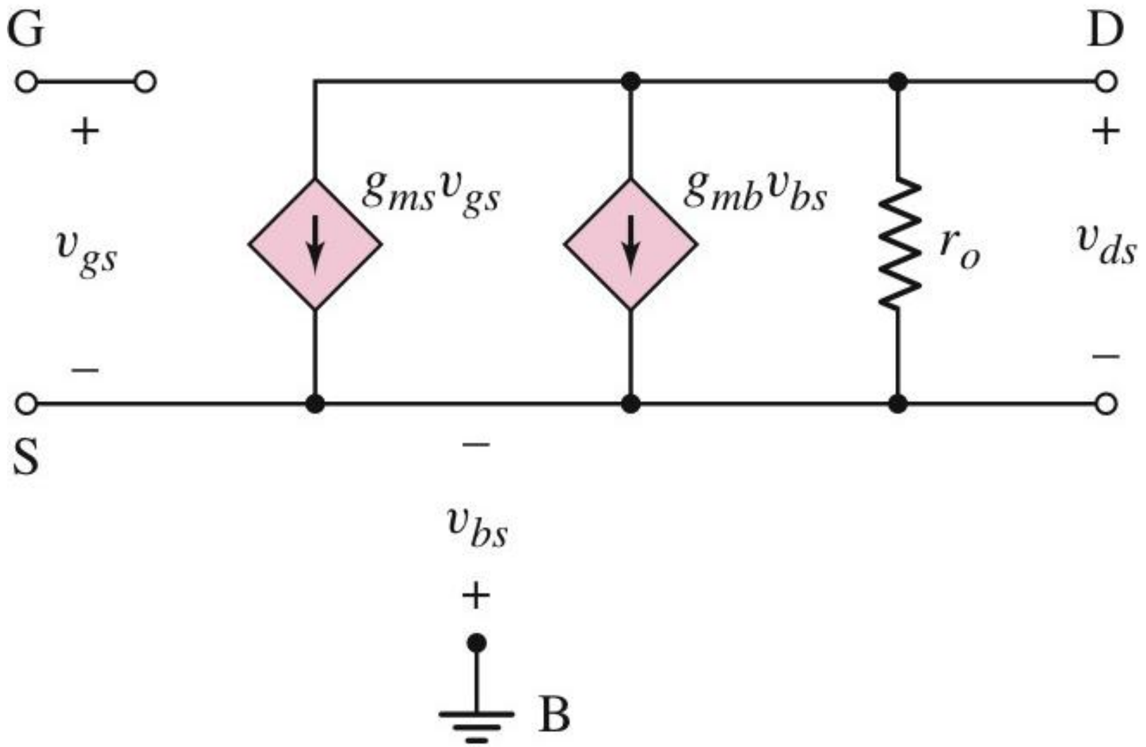
$$V_{gs} = \left(\frac{R_i}{R_i + R_{Si}} \right) \cdot V_i$$

so the small-signal voltage gain is

$$A_v = \frac{V_o}{V_i} = -g_m (r_o \| R_D) \cdot \left(\frac{R_i}{R_i + R_{Si}} \right)$$

Modeling the Body Effects

Modeling the Body Effects



Body Effect Consideration (1)

Modeling the Body Effect: As mentioned in Section 3.1.9, the body effect occurs in a MOSFET in which the **substrate**, or **body**, is **not** directly connected to the **source**. For an NMOS device, the body is connected to the most negative potential in the circuit and will be at signal ground.

$$i_D = K_n(v_{GS} - V_{TN})^2$$

and the threshold voltage is given by

$$V_{TN} = V_{TNO} + \gamma\left[\sqrt{2\phi_f + v_{SB}} - \sqrt{2\phi_f}\right]$$

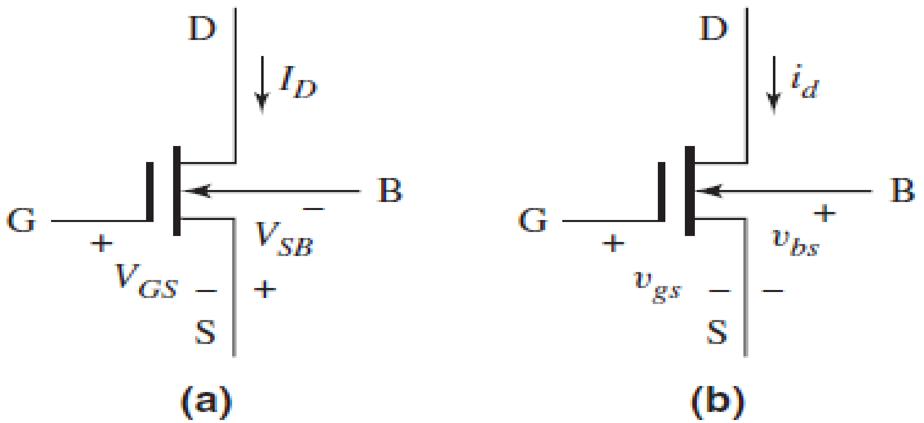


Figure 4.12 The four-terminal NMOS device with (a) dc voltages and (b) ac voltages

Body Effect Consideration (2)

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{BS}}\right|_{Q\text{-}pt} = \left.\frac{-\partial i_D}{\partial v_{SB}}\right|_{Q\text{-}pt} = -\left.\left(\frac{\partial i_D}{\partial V_{TN}}\right)\cdot\left(\frac{\partial V_{TN}}{\partial v_{SB}}\right)\right|_{Q\text{-}pt}$$

Using Equation (4.22), we find

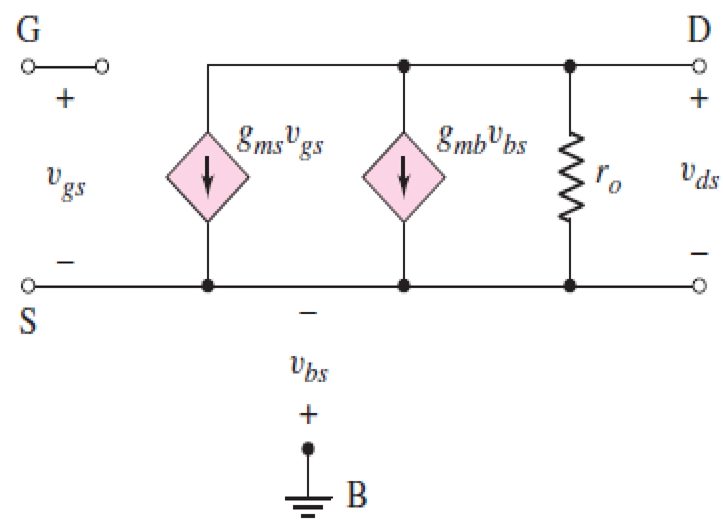
$$\frac{\partial i_D}{\partial V_{TN}} = -2K_n(v_{GS} - V_{TN}) = -g_m$$

and using Equation (4.23), we find

$$\frac{\partial V_{TN}}{\partial v_{SB}} = \frac{\gamma}{2\sqrt{2\phi_f + v_{SB}}} \equiv \eta$$

The back-gate transconductance is then

$$g_{mb} = -(-g_m)\cdot(\eta) = g_m\eta$$



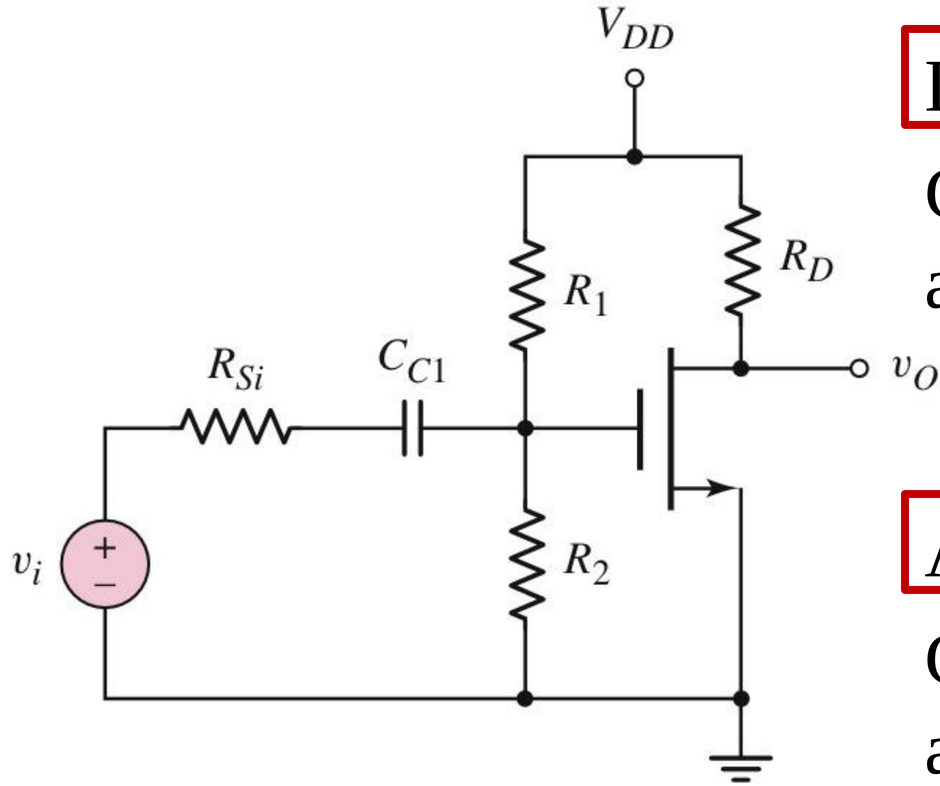
Small-signal equivalent circuit of NMOS device including body effect Test

Three Type of MOSFET Amplifier Circuit

- Common Source Amplifier
- Common Drain Amplifier
- Common Base Amplifier

Common Source Amplifier Circuit

Common-Source Configuration



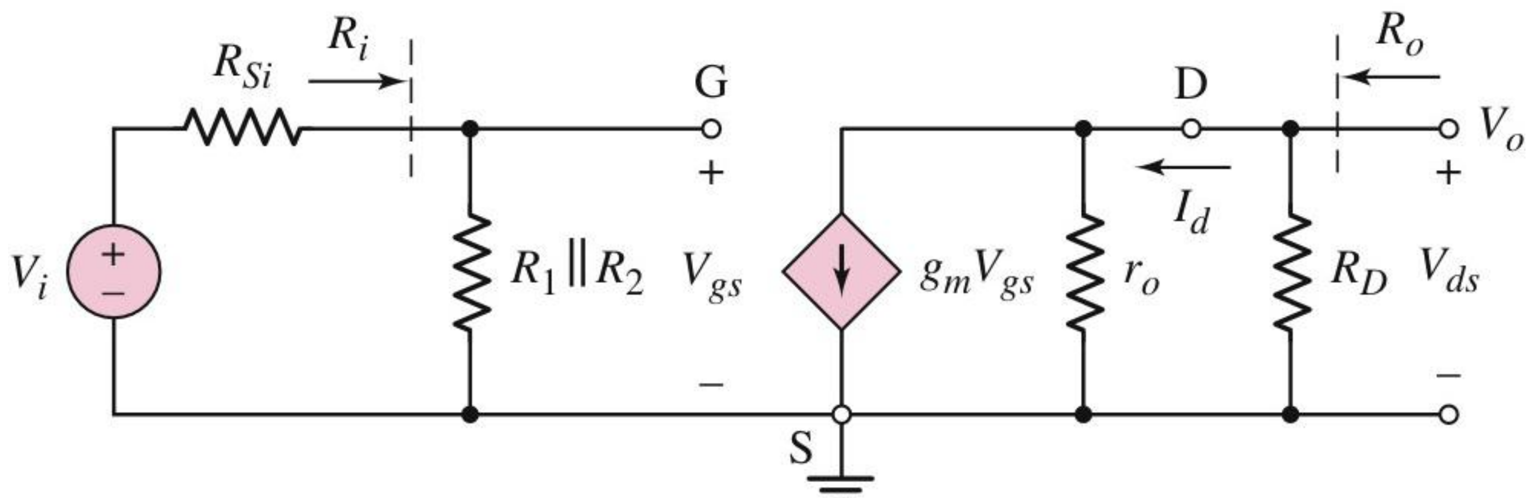
$\boxed{\text{DC}}$ analysis:

Coupling capacitor is assumed to be open.

$\boxed{\text{AC}}$ analysis:

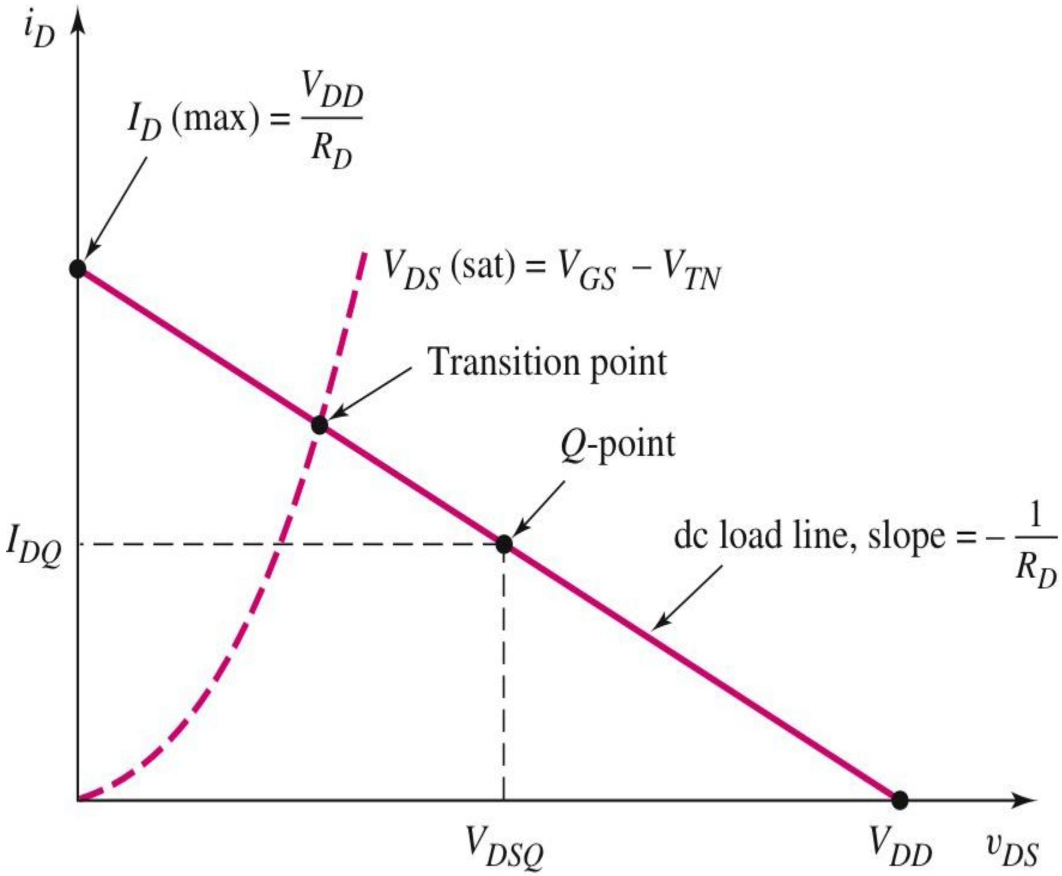
Coupling capacitor is assumed to be a short. DC voltage supply is set to zero volts.

Small-Signal Equivalent Circuit



$$A_v = V_o / V_i = -g_m (r_o \| R_D)\left(\frac{R_i}{R_i + R_{Si}}\right)$$

DC Load Line

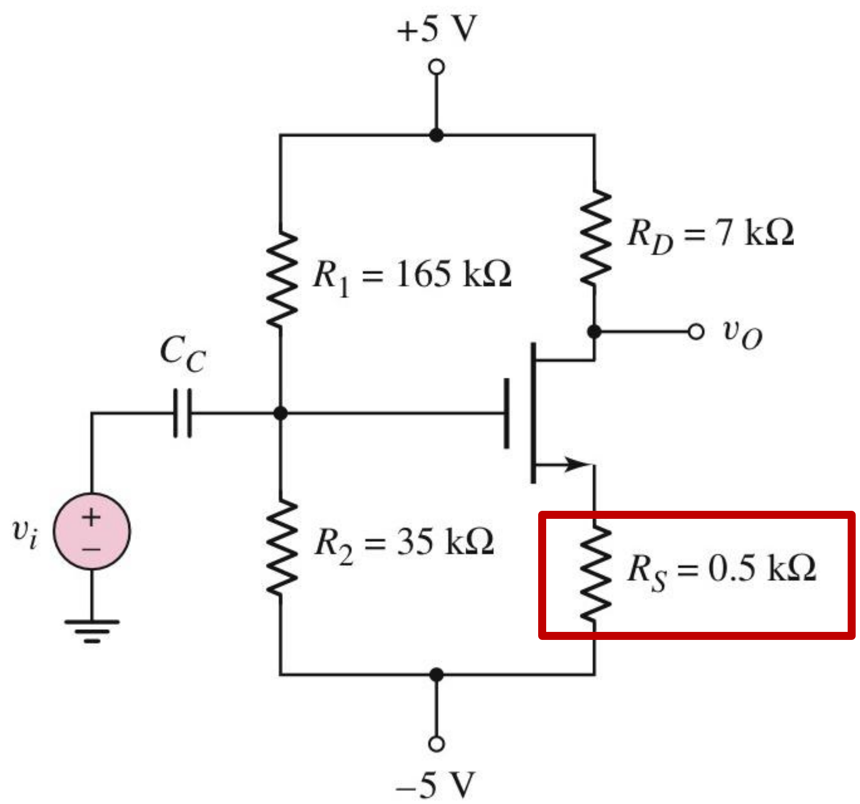


Q-point near the middle of the saturation region for maximum symmetrical output voltage swing,.

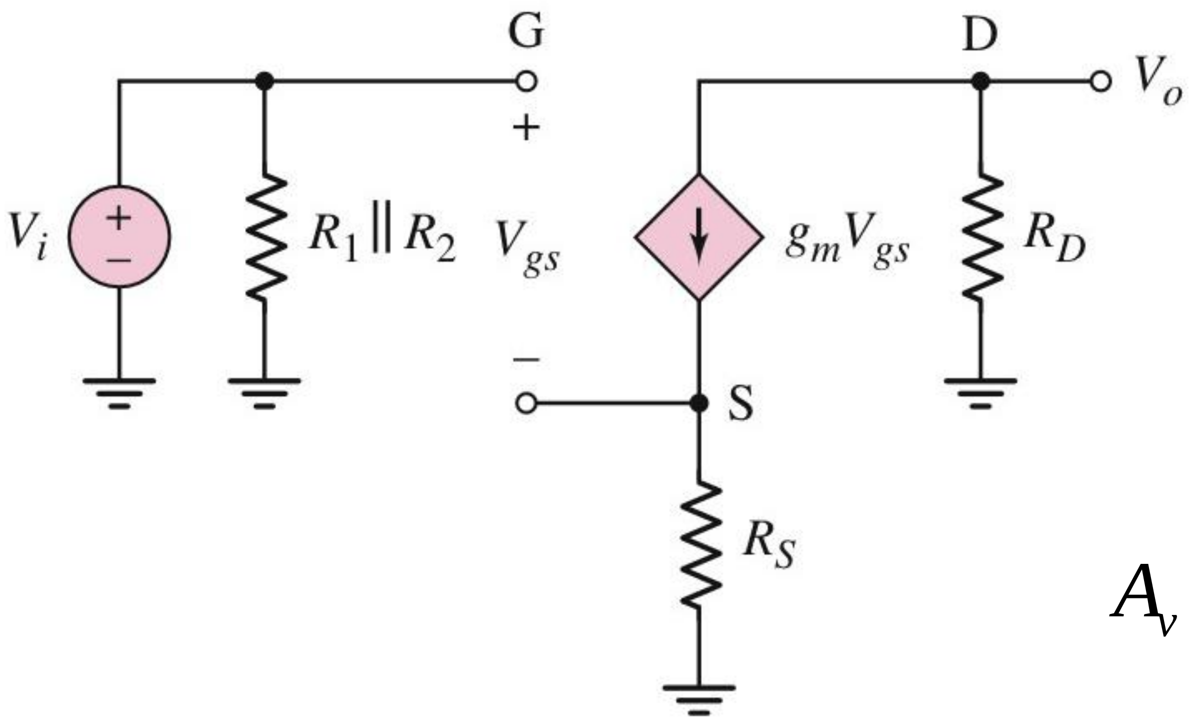
Small AC input signal for output response to be linear.

The Effect of Source Resistance

Common-Source Amplifier with Source Resistor



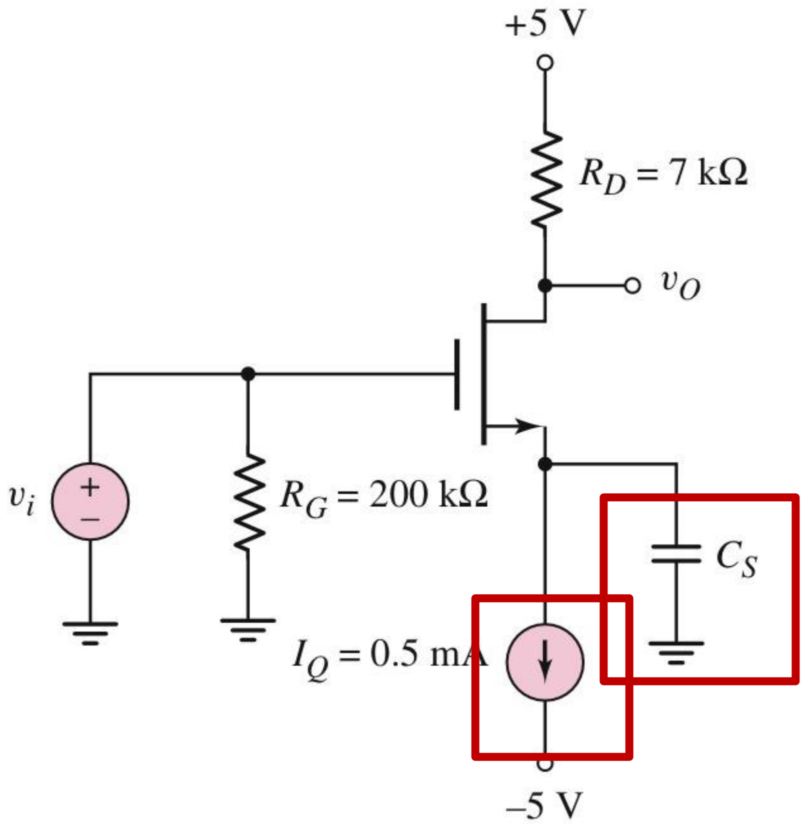
Small-Signal Equivalent Circuit for Common-Source with Source Resistor



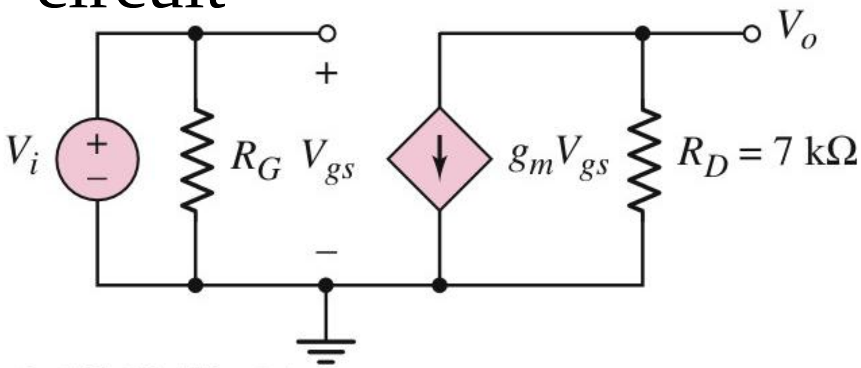
$$A_v = \frac{-\boxed{g_m} R_D}{1 + \boxed{g_m} \boxed{R_S}}$$

- The source resistor is added to stabilize the Q-point.
- Voltage gain is reduced by an increase in the denominator.

Common-Source Amplifier with Bypass Capacitor



Small-signal equivalent circuit

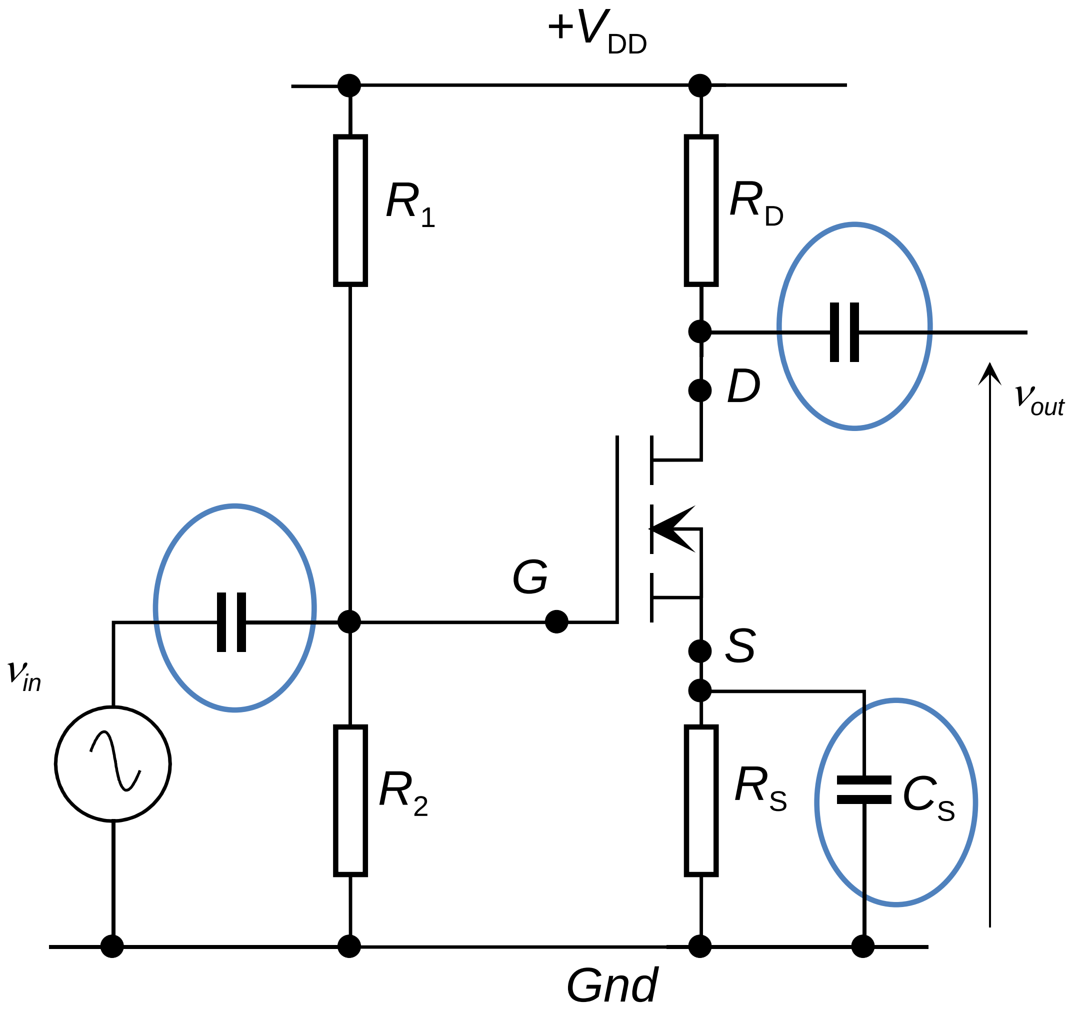


- Bypass capacitor is added to minimize the loss in the voltage gain.
- The source resistor is replaced by the current source to further stabilize the Q-poi

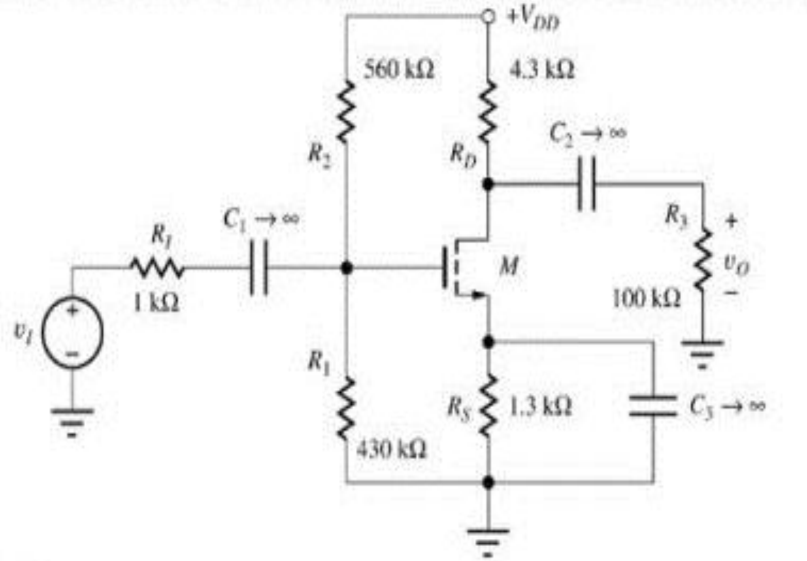
Common Source Small Signal Analysis Parameters

Small Signal Amplifier

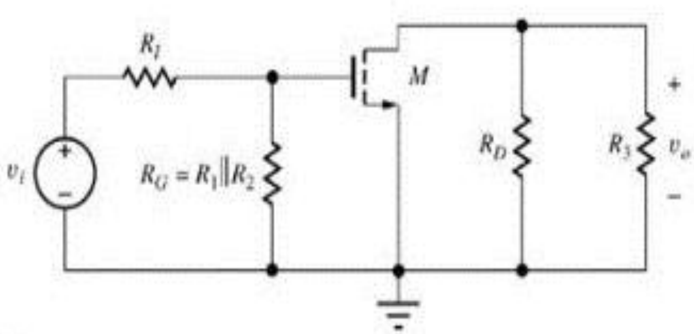
A small-signal amplifier is built by adding input and output coupling capacitors and a source bypass capacitor



Small-Signal Analysis of Complete C-S Amplifier: AC Equivalent



(a)



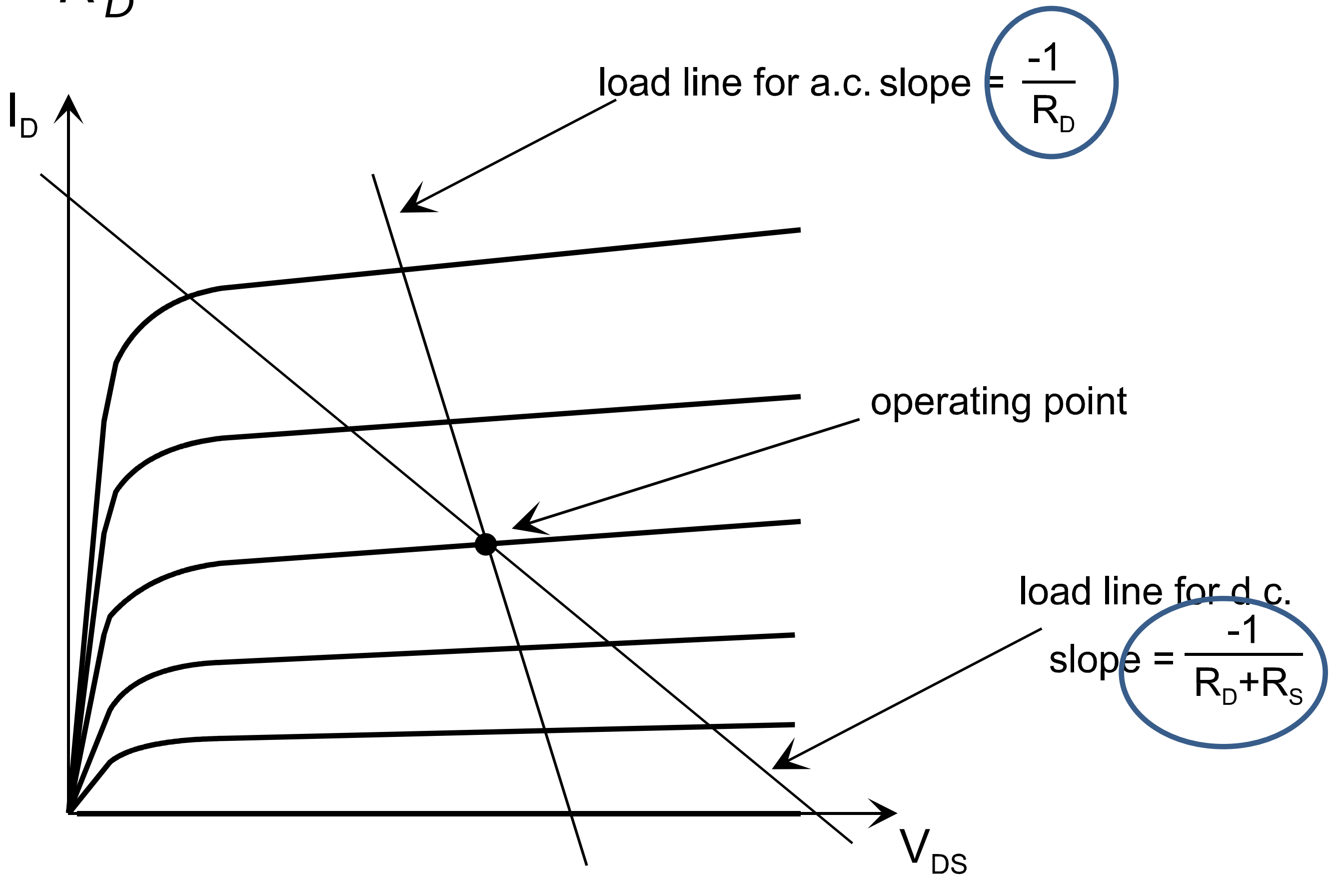
(b)

- AC equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources represent AC grounds.
- Assume that Q-point is already known.

$$R_G = R_1 \| R_2$$

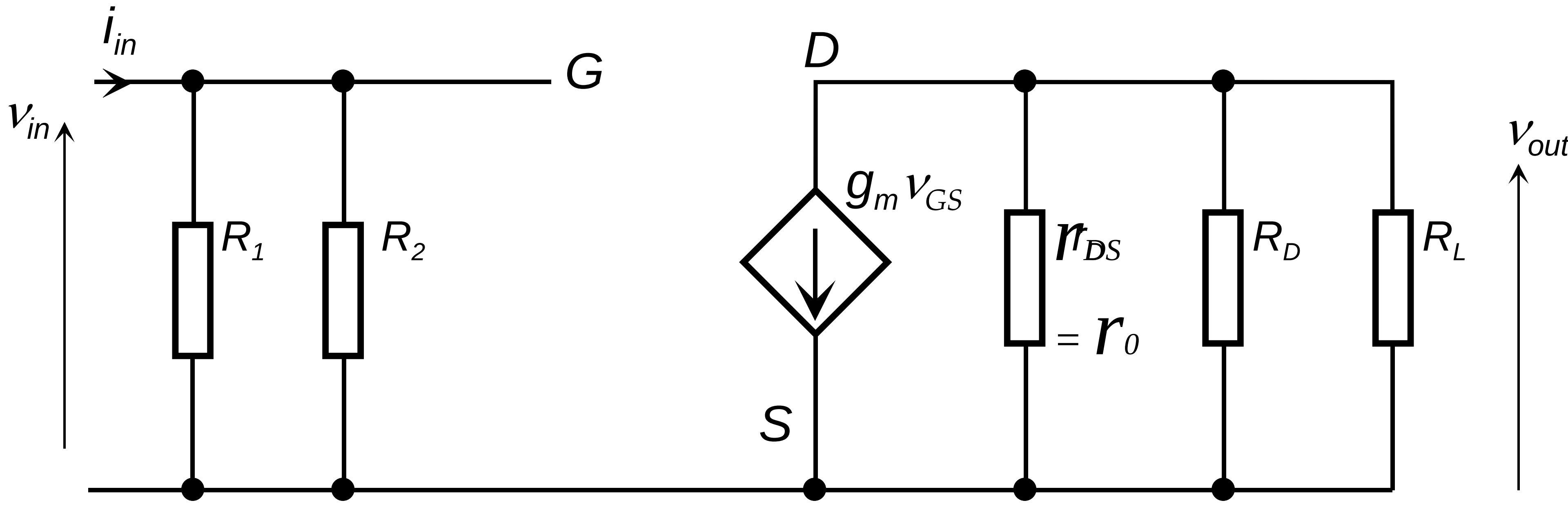
A.c. Load Line

Note that when the **source bypass capacitor** is used, only the **drain resist**
R_D determines the a.c. response at the operating point, the a.c. load line slope is $-\dfrac{1}{R_D}$ at the operating point.



Small Signal Equivalent Circuit

The small signal equivalent circuit of the common-source amplifier is



Input Resistance and Voltage Gain

The **input resistance** is $R_{in} = \frac{v_{in}}{i_{in}}$ and $i_{in} = \frac{v_{in}}{R_G}$ where $R_G = \frac{R_1 R_2}{R_1 + R_2}$

Hence $$R_{in} = R_G = \frac{R_1 R_2}{R_1 + R_2}$$

Large values may be selected for R_1 and R_2 – the upper limit is set by high frequency requirements

The **open circuit voltage gain** is

$$A_v = \frac{v_{out}}{v_{in}}$$

$$v_{out} = -g_m v_{gs} R_D$$

$$v_{in} = v_{gs}$$

Hence $$A_v = \frac{v_{out}}{v_{in}} = -g_m R_D$$

Output Resistance

Note g_m has units of Siemens, Ω^{-1}, so A_V is dimensionless (has no units) as it should.

$$v_{oc} = -g_m v_{gs} R_D$$

$$i_{sc} = -g_m v_{gs}$$

Hence $$r_{out} = R_D$$

Summary - the generic (black box) amplifier properties of the common source amplifier are

$$R_{in} = R_G = \frac{R_1 R_2}{R_1 + R_2} \qquad A_v = \frac{v_{out}}{v_{in}} = -g_m R_D \qquad R_{out} = R_D$$

Common Source Circuit Examples

Example 1

Consider an n-channel enhancement-mode MOSFET with the following parameter: $V_{TN} = 0.75\text{V}, W = 40\mu\text{m}, L = 4\mu\text{m}, \mu_n = 650c\,\text{m}^2/\text{V}-\text{s}$, $t_{\text{ox}} = 450\,\overset{0}{\text{A}}$, and $\varepsilon_{\text{ox}} = (3.9)(8.85\,\text{x}\,10^{-14})\text{F/cm}$.

Determine the current when $V_{\text{GS}} = 2V_{TN}$, for the transistor biased on the saturation region.

Solution:

The conduction parameter is determine by equation R4. First, consider the units involved in this equation, as follows:

$$K_n = \frac{W(\text{cm}).\,\mu_n\left(\dfrac{\text{cm}^2}{\text{V - s}}\right).\,\varepsilon_{\text{ox}}\left(\dfrac{\text{F}}{\text{cm}}\right)}{2\text{L}(\text{cm}).\,t_{\text{ox}}(\text{cm})} = \frac{\text{A}}{\text{V}^2}$$

Example 1 (Cont')

$$K_n = \frac{\left(40 \text{ x } 10^{-4}\right)\left(650\right)\left(3.9\right)\left(8.85 \text{ x } 10^{-14}\right)}{2\left(4 \text{ x } 10^{-4}\right)\left(450 \text{ x } 10^{-8}\right)} = 0.249 \text{ mA/V}^2$$

$$i_D = K_n\left(v_{GS} - V_{TN}\right)^2 = \left(0.249\right)\left(1.5 - 0.75\right)^2 = 0.140\text{mA}$$

Example 2

An n-channel enhancement-mode MOSFET with $V_{TN} = 1\text{V}$ has a drain Current $i_D = 0.8\text{mA}$ when $V_{\text{GS}} = 3\text{V}$ and $V_{\text{DS}} = 4.5\text{V}$.

Calculate the drain current when : .

a) $V_{\text{GS}} = 2\text{V}$; $V_{\text{DS}} = 4.5\text{V}$

b) $V_{\text{GS}} = 3\text{V}$; $V_{\text{DS}} = 1\text{V}$

Solution:

$$V_{TN} = 1\ V,\ V_{GS} = 3\ V,\ V_{DS} = 4.5\ V$$

$$V_{DS} = 4.5 > V_{DS}(sat) = V_{GS} - V_{TN} = 3 - 1 = 2\ V$$

Transistor biased in the saturation region

$$I_D = K_n\left(V_{GS} - V_{TN}\right)^2 \Rightarrow 0.8 = K_n\left(3-1\right)^2 \Rightarrow K_n = 0.2\ mA/V^2$$

Example 2 (Cont')

(a) $V_{GS} = 2\ V,\ V_{DS} = 4.5\ V$

Saturation region:

$$I_D = (0.2)(2-1)^2 \Rightarrow \underline{I_D = 0.2\ mA}$$

(b) $V_{GS} = 3\ V,\ V_{DS} = 1\ V$

Nonsaturation region:

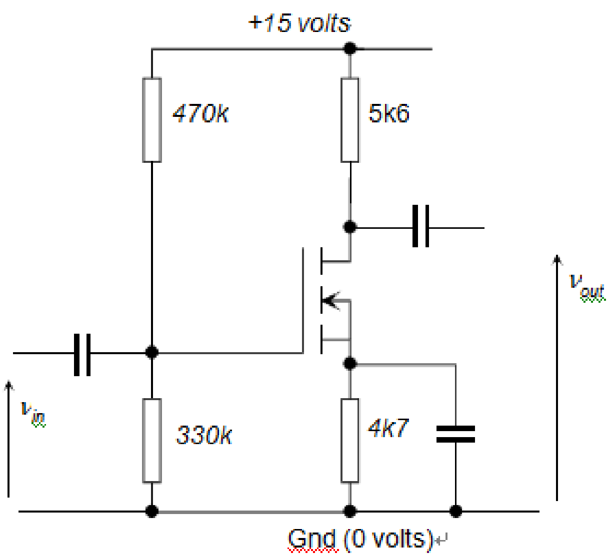
$$I_D = (0.2)\left[2(3-1)(1)-(1)^2\right] \Rightarrow \underline{I_D = 0.6\ mA}$$

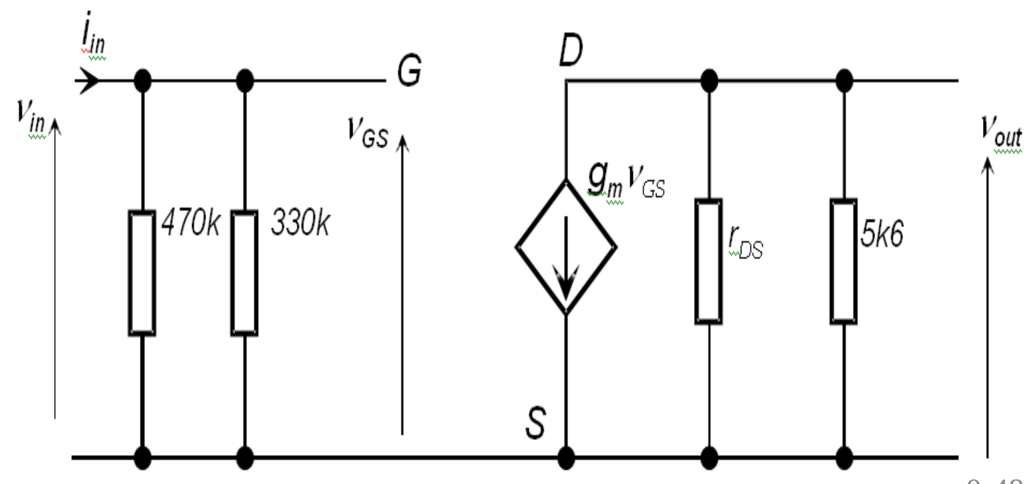
Example 3

A common source amplifier circuit based around a single n-channel MOSFET is shown in Figure Ex3.1. The equivalent hybrid pi model of the circuit is shown in the Figure Ex3.2.

a) Calculate the input resistance.
b) Calculate the output resistance.
c) Calculate the open circuit voltage gain .
d) Calculate the current gain when the amplifier has a load of 10k .

Assume the transconductance g_m = 30 mA/volts and assume that r_{DS} is so large it may be neglected.

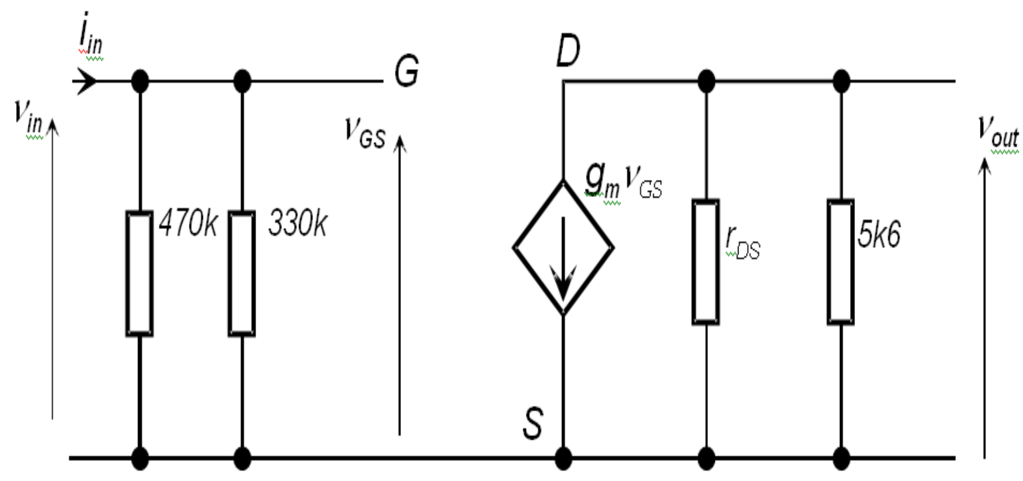




Example 3 (Cont')

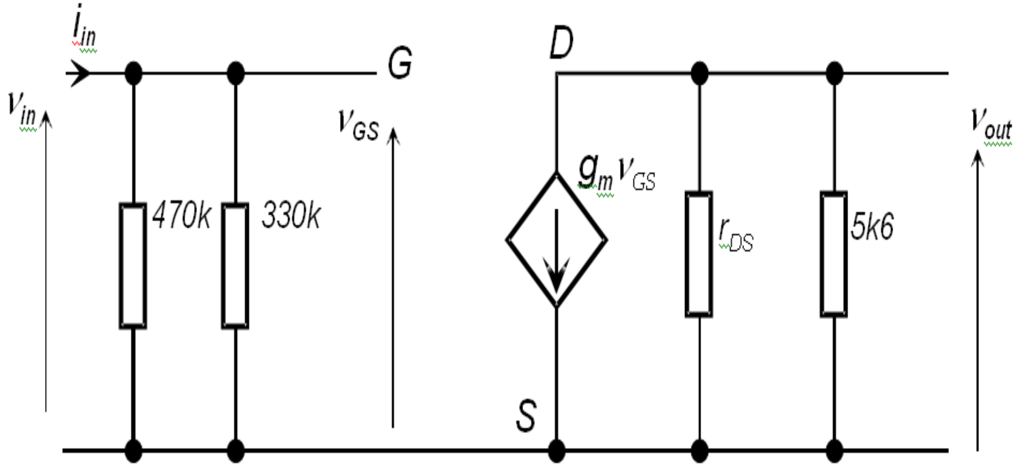
Solution:

a) Input resistance is 470k and 330k in parallel R_{in} = $\left(\frac{1}{470k}+\frac{1}{330k}\right)^{-1}$ = 194k

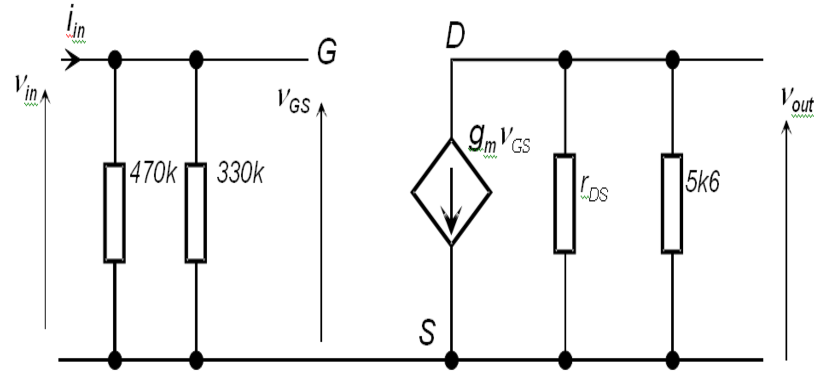


Example 3 (Cont')

b) Ignoring r_{DS}, output resistance is Rout = 5k6



Example 3 (Cont')



c) Open circuit voltage gain $A_v = \dfrac{v_{out}}{v_{in}}$

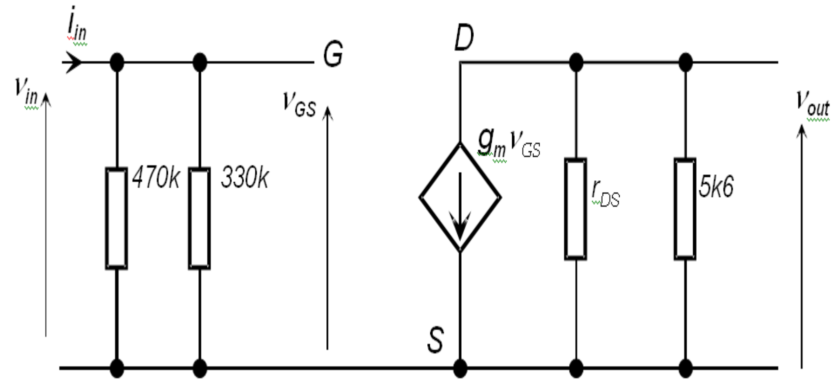
Since $v_{out} = -g_m v_{gs} R_D$ and $v_{in} = v_{gs}$

then

$$A_v = \frac{v_{out}}{v_{in}} = -g_m R_D$$

$$= -30 \times 10^{-3} \times 5.6 \times 10^{+3} = -168$$

Example 3 (Cont')



d) Input current is $i_{in} = v_{in} / 194k$ mA

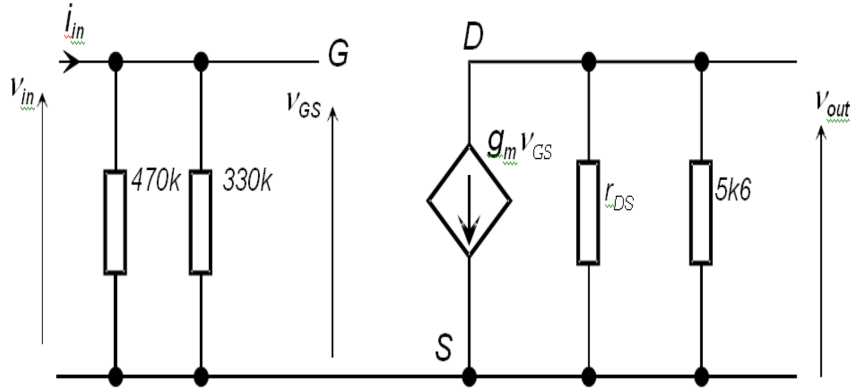
Combined output resistance with load is 5k6 and 10k in parallel is $R_C = \left(\frac{1}{5600} + \frac{1}{10000}\right)^{-1}$ = 3590Ω

Output voltage, $v_{out} = -g_m v_{gs} R_C$

$= - 30 \times 10^{-3} \times 3.59 \times 10^{+3} \times v_{in}$

$= - 107.7\, v_{in}$

Example 3 (Cont')



d) Output current

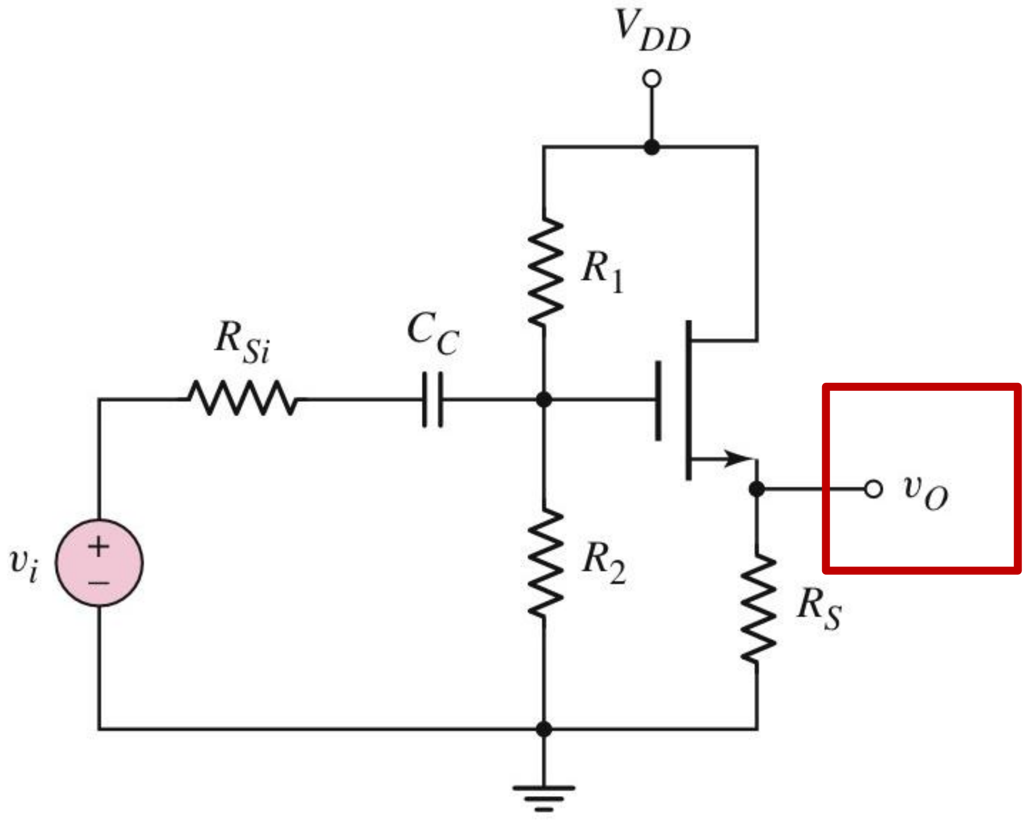
$$i_{out} = v_{out} / R_L = -107.7 v_{in} / 10^4 = \frac{-107.7 \times 194k \times i_{in}}{10k}$$

Hence current gain is

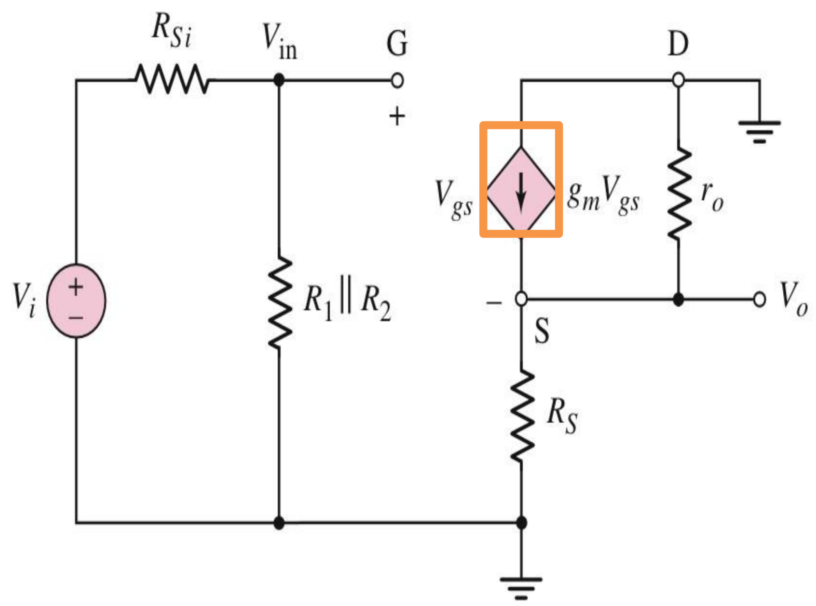
$$A_i = i_{out} / i_{in} = \text{-2089}$$

Common Drain or Source Follower Amplifier Circuit

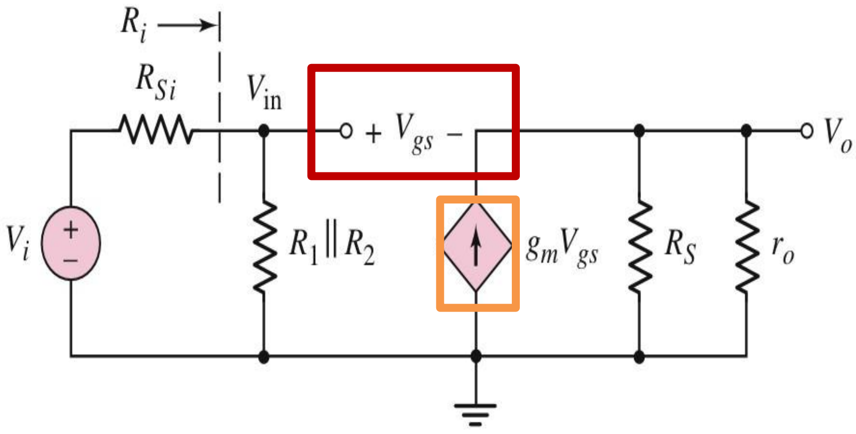
NMOS Source-Follower or Common Drain Amplifier



Small-Signal Equivalent Circuit for Source Follower



(a)



(b)

Common Drain Amplifier

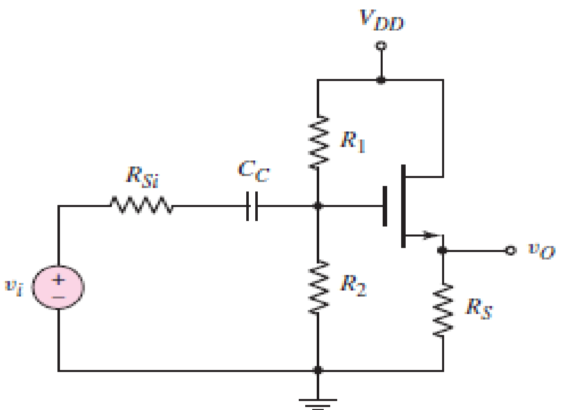


Figure 4.26 NMOS source-follower or common-drain amplifier

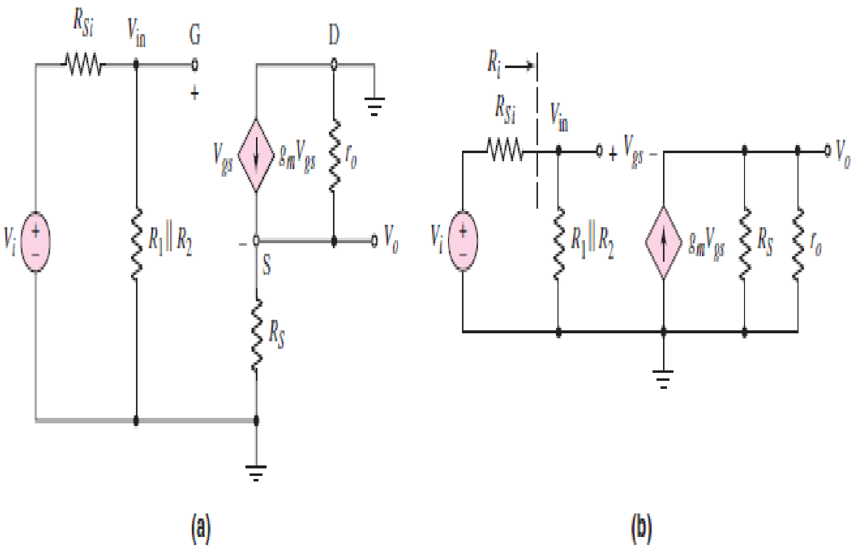


Figure 4.27 (a) Small-signal equivalent circuit of NMOS source follower and (b) small-signal equivalent circuit of NMOS source follower with all signal grounds at a common point

$$V_o = (g_m V_{gs})(R_S \| r_o) \tag{4.30}$$

Writing a KVL equation from input to output results in the following:

$$V_{in} = V_{gs} + V_o = V_{gs} + g_m V_{gs}(R_S \| r_o) \tag{4.31(a)}$$

Therefore, the gate-to-source voltage is

$$V_{gs} = \frac{V_{in}}{1 + g_m(R_S \| r_o)} = \left[\frac{\frac{1}{g_m}}{\frac{1}{g_m} + (R_S \| r_o)}\right] \cdot V_{in} \tag{4.31(b)}$$

Equation (4.31(b)) is written in the form of a voltage-divider equation, in which the gate-to-source of the NMOS device looks like a resistance with a value of $1/g_m$. More accurately, the effective resistance looking into the source terminal (ignoring r_o) is $1/g_m$. The voltage V_{in} is related to the source input voltage V_i by

$$V_{in} = \left(\frac{R_i}{R_i + R_{Si}}\right) \cdot V_i \tag{4.32}$$

where $R_i = R_1 \| R_2$ is the input resistance to the amplifier.

Substituting Equations (4.31(b)) and (4.32) into (4.30), we have the small-signal voltage gain:

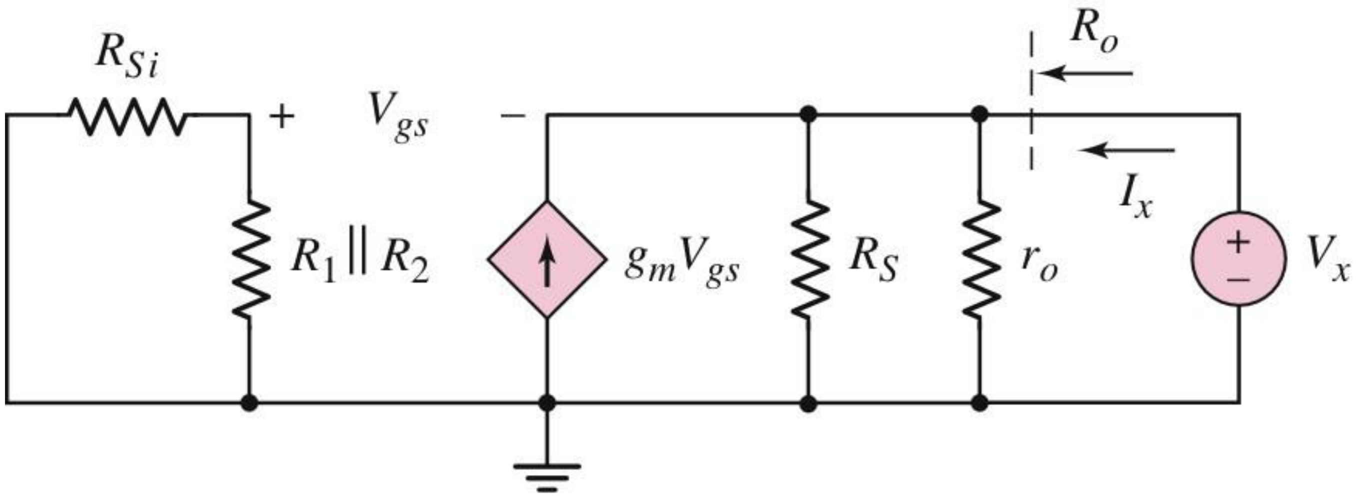
$$A_v = \frac{V_o}{V_i} = \frac{g_m(R_S \| r_o)}{1 + g_m(R_S \| r_o)} \cdot \left(\frac{R_i}{R_i + R_{Si}}\right) \tag{4.33(a)}$$

or

$$A_v = \frac{R_S \| r_o}{\frac{1}{g_m} + R_S \| r_o} \cdot \left(\frac{R_i}{R_i + R_{Si}}\right) \tag{4.33(b)}$$

which again is written in the form of a voltage-divider equation. An inspection of Equation 4.33(b) shows that the magnitude of the voltage gain is always less than unity.

Determining Output Impedance NMOS Source Follower



$$R_O = \frac{1}{g_m} \Big\| R_S \| r_o$$

Input and Output Impedances

$$R_o = \frac{V_x}{I_x} \quad (4.34)$$

Writing a KCL equation at the output source terminal produces

$$I_x + g_m V_{gs} = \frac{V_x}{R_S} + \frac{V_x}{r_o} \quad (4.35)$$

Since there is no current in the input portion of the circuit, we see that $\boxed{V_{gs} = -V_x}$. Therefore, Equation (4.35) becomes

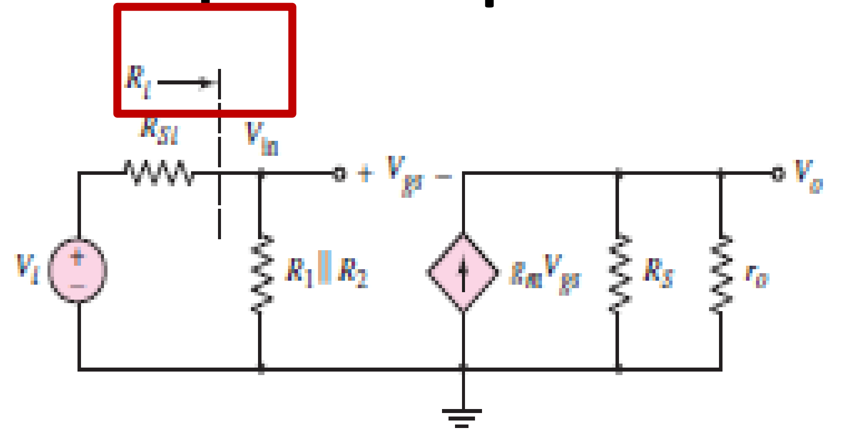
$$I_x = V_x\left(g_m + \frac{1}{R_S} + \frac{1}{r_o}\right) \quad (4.36(a))$$

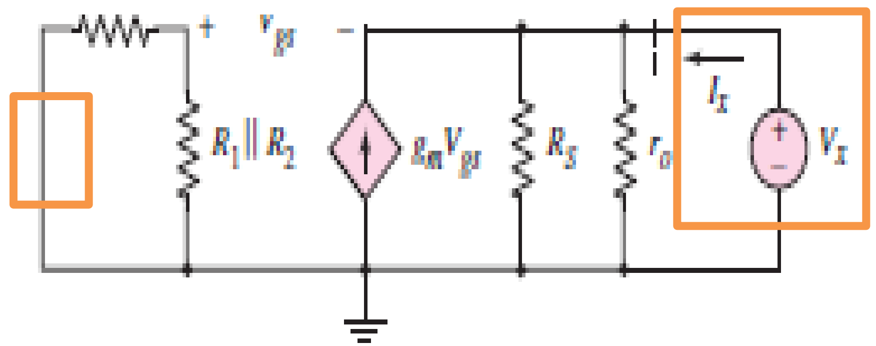
or

$$\frac{I_x}{V_x} = \frac{1}{R_o} = g_m + \frac{1}{R_S} + \frac{1}{r_o} \quad (4.36(b))$$

The output resistance is then

$$\boxed{R_o = \frac{1}{g_m} \| R_S \| r_o} \quad (4.37)$$

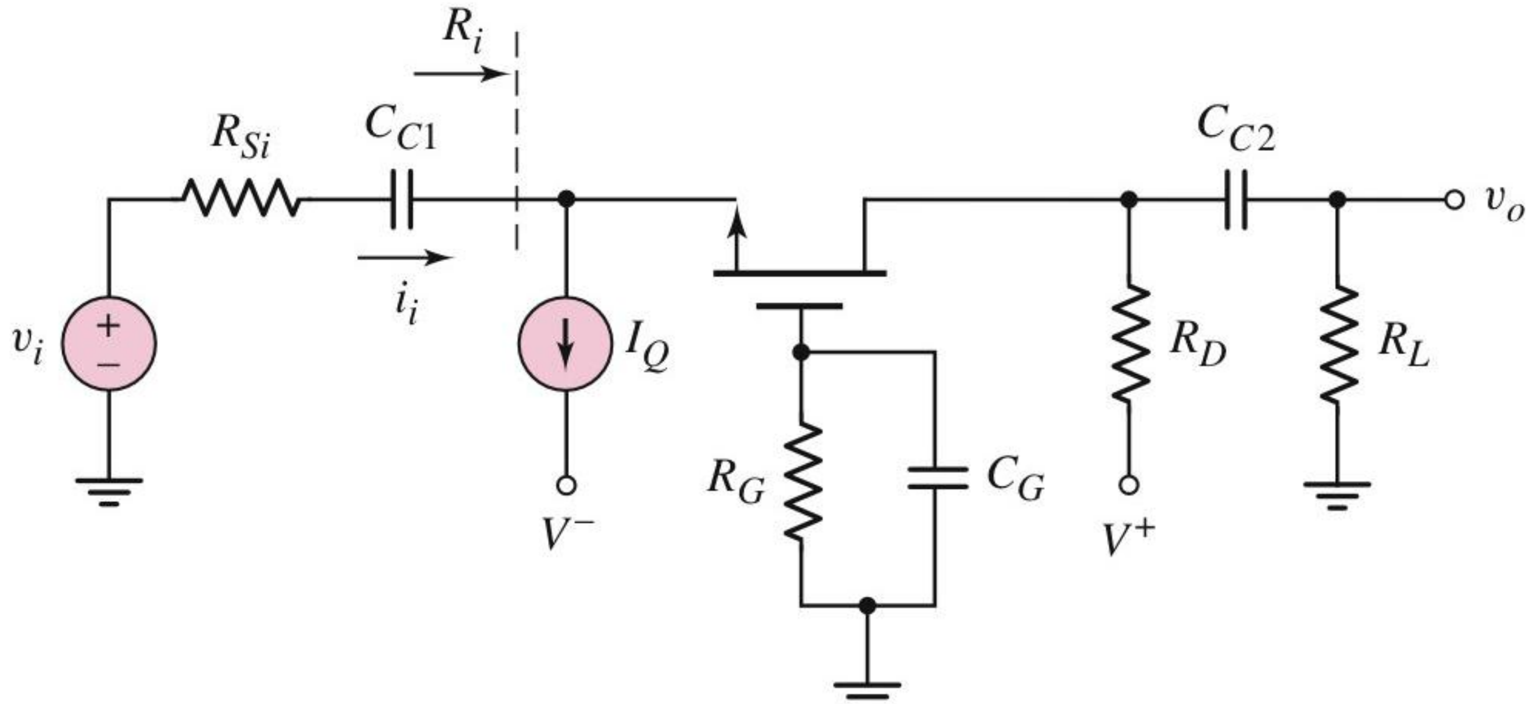




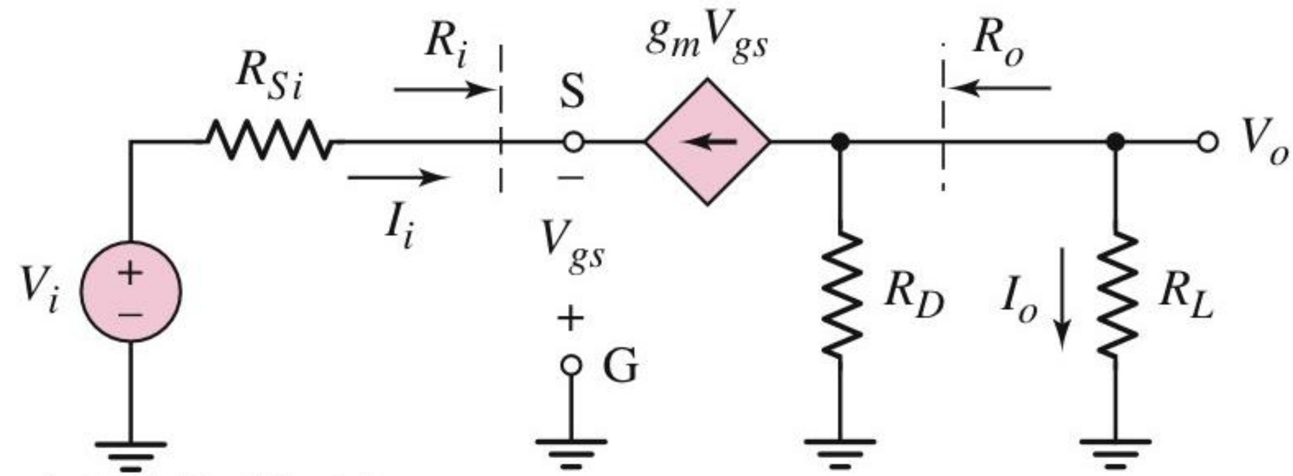
- The small-signal input resistance *Ri* as defined as the Thevenin equivalent resistance of the bias resistors (see figure above). So Ri = $R_1 \,||\, R_2$
- To calculate the small-signal output resistance, we set all independent small signal sources equal to zero, apply a test voltage to the output terminals, and measure a test current.

Common Gate Amplifier Circuit

Common-Gate Circuit



Small-Signal Equivalent Circuit for Common Gate



$$A_v = \frac{g_m (R_D \| R_L)}{1 + g_m R_{Si}}$$

$$A_i = \frac{I_O}{I_i} = \left(\frac{R_D}{R_D + R_L}\right)\left(\frac{g_m R_{Si}}{1 + g_m R_{Si}}\right)$$

Common Gate Amplifier

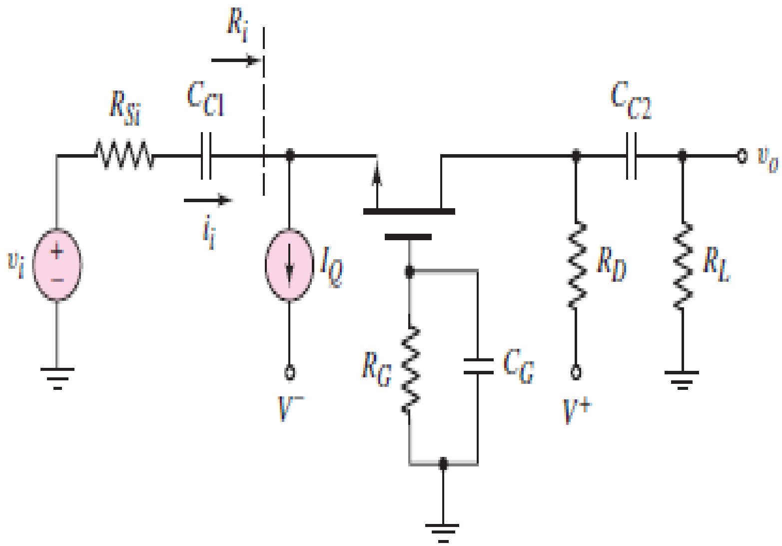


Figure 4.32 Common-gate circuit

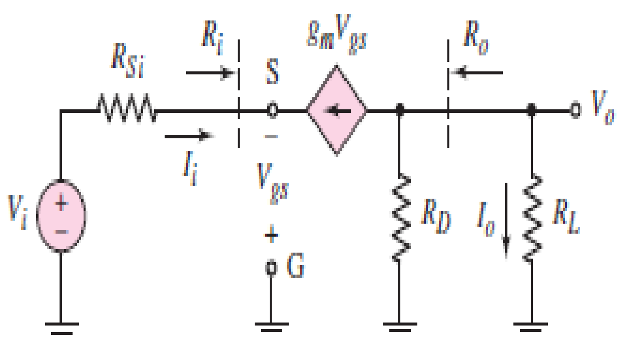


Figure 4.33 Small-signal equivalent circuit of common-gate amplifier

$$V_o = -(g_m V_{gs})(R_D \| R_L) \tag{4.38}$$

Writing the KVL equation around the input, we find

$$V_i = I_i R_{Si} - V_{gs} \tag{4.39}$$

where $I_i = -g_m V_{gs}$. The gate-to-source voltage can then be written as

$$V_{gs} = \frac{-V_i}{1 + g_m R_{Si}} \tag{4.40}$$

The small-signal voltage gain is found to be

$$\boxed{A_v = \frac{V_o}{V_i} = \frac{g_m (R_D \| R_L)}{1 + g_m R_{Si}}} \tag{4.41}$$

Also, since the voltage gain is positive, the output and input signals are in phase.

In many cases, the signal input to a common-gate circuit is a current. Figure 4.34 shows the small-signal equivalent common-gate circuit with a Norton equivalent circuit as the signal source. We can calculate a current gain. The output current I_o can be written

$$I_o = \left(\frac{R_D}{R_D + R_L}\right)(-g_m V_{gs}) \tag{4.42}$$

At the input we have

$$I_i + g_m V_{gs} + \frac{V_{gs}}{R_{Si}} = 0 \tag{4.43}$$

or

$$V_{gs} = -I_i \left(\frac{R_{Si}}{1 + g_m R_{Si}}\right) \tag{4.44}$$

The small-signal current gain is then

$$\boxed{A_i = \frac{I_o}{I_i} = \left(\frac{R_D}{R_D + R_L}\right) \cdot \left(\frac{g_m R_{Si}}{1 + g_m R_{Si}}\right)} \tag{4.45}$$

We may note that if $R_D \gg R_L$ and $g_m R_{Si} \gg 1$, then the current gain is essentially unity.

Input and Output Resistance

4.5.2 Input and Output Impedance

In contrast to the common-source and source-follower amplifiers, the common-gate circuit has a low input resistance because of the transistor. However, if the input signal is a current, a low input resistance is an advantage. The input resistance is defined, using Figure 4.33, as

$$R_i = \frac{-V_{gs}}{I_i} \tag{4.46}$$

Since $I_i = -g_m V_{gs}$, the input resistance is

$$\boxed{R_i = \frac{1}{g_m}} \tag{4.47}$$

This result has been obtained previously.

We can find the output resistance by setting the input signal voltage equal to zero. From Figure 4.33, we see that $V_{gs} = -g_m V_{gs} R_{Si}$, which means that $V_{gs} = 0$. Consequently, $g_m V_{gs} = 0$. The output resistance, looking back from the load resistance, is therefore

$$\boxed{R_o = R_D} \tag{4.48}$$

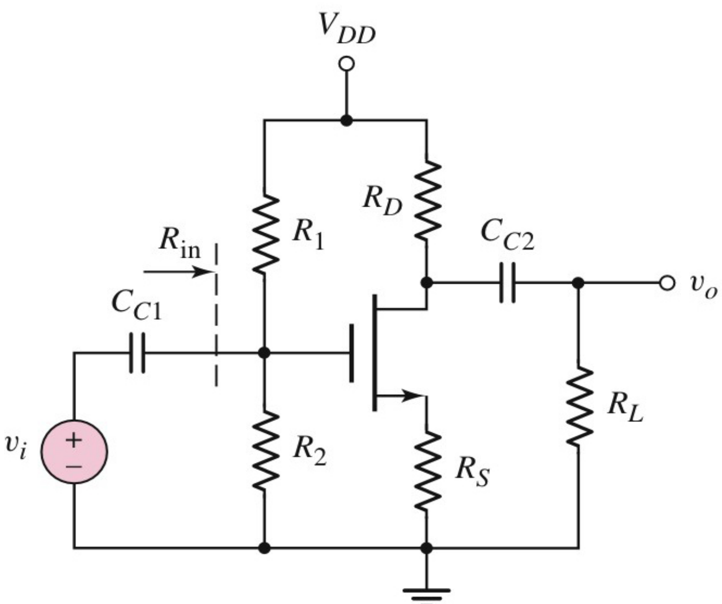
Comparison of Three Amplifier Circuits

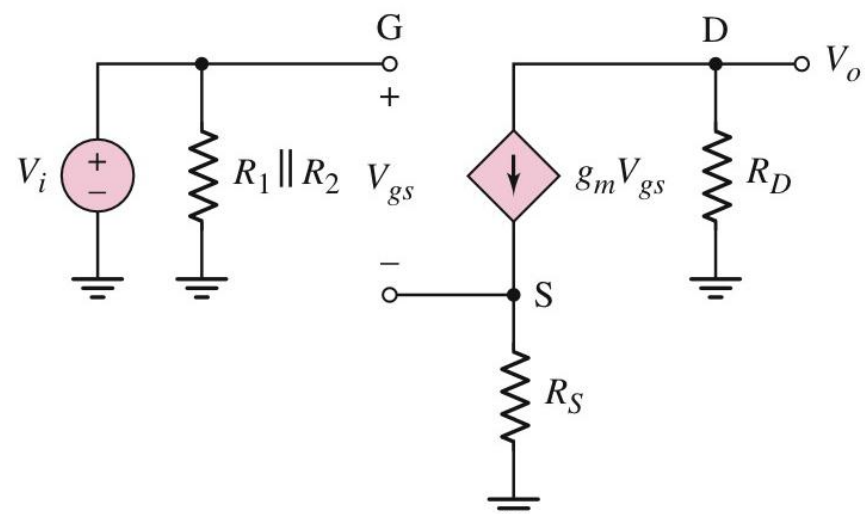
Comparison of 3 Basic Amplifiers

Configuration	Voltage Gain	Current Gain	Input Resistance	Output Resistance
Common Source	$A_v > 1$	—	$\boxed{R_{TH}}$	Moderate to high
Source Follower	$\boxed{A_v \approx 1}$	—	$\boxed{R_{TH}}$	$\boxed{\text{Low}}$
Common Gate	$A_v > 1$	$\boxed{A_i \approx 1}$	$\boxed{\text{Low}}$	Moderate to high

Example 4

The parameters of the circuit shown in Figure P4.15 are V_{DD} = 12 V, R_S = 0.5 kΩ, R_{in} = 250 kΩ, and R_L = 10 kΩ. The transistor parameters are V_{TN} = 1.2 V, K_n = 1.5 mA/V^2, are λ = 0. (a) Design the circuit such that I_{DQ} = 2 mA and V_{DSQ} = 5 V. (b) Determine the small signal voltage gain. (The small signal output resistance $r_O \cong \frac{1}{\lambda I_{DQ}}$. The transconductance $g_m = 2\sqrt{K_{n_{I_{DQ}}}}$.)





Solution

(a) $\boxed{V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)}$

$5 = 12 - (2)(R_S + R_D) \Rightarrow \boxed{R_S + R_D = 3.5\ \text{k}\Omega}$

$R_S = 0.5\ \text{k}\Omega$, then $\boxed{R_D = 3\ \text{k}\Omega}$

$\boxed{I_{DQ} = K_n (V_{GSQ} - V_{TN})^2}$

$2 = 1.5(V_{GSQ} - 1.2)^2 \Rightarrow \boxed{V_{GSQ} = 2.355\ \text{V}}$

$\boxed{V_G = V_{GSQ} + I_{DQ}R_S} = 2.355 + (2)(0.5) = 3.355\ \text{V}$

$$\boxed{V_G = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{DD}} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$3.355 = \frac{1}{R_1}(250)(12) \Rightarrow \boxed{R_1 = 894\ \text{k}\Omega}$$

$R_1 \| R_2 = 894 \| R_2 = 250 \Rightarrow \boxed{R_2 = 347\ \text{k}\Omega}$

(b) $g_m = 2\sqrt{(1.5)(2)} = 3.464\ \text{mA/V}$

$$A_v = \frac{-g_m(R_D \| R_L)}{1 + g_m R_S} = \frac{-(3.464)(3\|10)}{1 + (3.464)(0.5)} = -2.93$$

Chapter

- Investigate a single-transistor circuit that can amplify a small, time-varying input signal
  - Develop small-signal models that are used in the analysis of linear amplifiers.
- Discuss and compare the three basic transistor amplifier configurations.
  - Analyze the common-source amplifier.
  - Analyze the source-follower amplifier.
  - Analyze the common-gate amplifier.