CHAPTER

3

GENERATOR AND TRANSFORMER MODELS; THE PER-UNIT SYSTEM

3.1 INTRODUCTION

Before the power systems network can be solved, it must first be modeled. The three-phase balanced system is represented on a per-phase basis, which was described in Section 2.10. The single-phase representation is also used for unbalanced systems by means of symmetrical components which is treated in a later chapter. In this chapter we deal with the balanced system, where transmission lines are represented by the π model as described in Chapter 4. Other essential components of a power system are generators and transformers; their theory and construction are discussed in standard electric machine textbooks. In this chapter, we represent simple models of generators and transformers for steady-state balanced operation.

Next we review the one-line diagram of a power system showing generators, transformers, transmission lines, capacitors, reactors, and loads. The diagram is usually limited to major transmission systems. As a rule, distribution circuits and small loads are not shown in detail but are taken into account merely as lumped loads on substation busses.

In the analysis of power systems, it is frequently convenient to use the per-unit system. The advantage of this method is the elimination of transformers by simple impedances. The per-unit system is presented, followed by the impedance diagram of the network, expressed to a common MVA base.

3.2 SYNCHRONOUS GENERATORS

Large-scale power is generated by three-phase synchronous generators, known as *alternators*, driven either by steam turbines, hydroturbines, or gas turbines. The armature windings are placed on the stationary part called *stator*. The armature windings are designed for generation of balanced three-phase voltages and are arranged to develop the same number of magnetic poles as the field winding that is on the rotor. The field which requires a relatively small power (0.2–3 percent of the machine rating) for its excitation is placed on the rotor. The rotor is also equipped with one or more short-circuited windings known as *damper windings*. The rotor is driven by a prime mover at constant speed and its field circuit is excited by direct current. The excitation may be provided through slip rings and brushes by means of dc generators (referred to as *exciters*) mounted on the same shaft as the rotor of the synchronous machine. However, modern excitation systems usually use ac generators with rotating rectifiers, and are known as *brushless excitation*. The generator excitation system maintains generator voltage and controls the reactive power flow.

The rotor of the synchronous machine may be of cylindrical or salient construction. The cylindrical type of rotor, also called *round rotor*, has one distributed winding and a uniform air gap. These generators are driven by steam turbines and are designed for high speed 3600 or 1800 rpm (two- and four-pole machines, respectively) operation. The rotor of these generators has a relatively large axial length and small diameter to limit the centrifugal forces. Roughly 70 percent of large synchronous generators are cylindrical rotor type ranging from about 150 to 1500 MVA. The salient type of rotor has concentrated windings on the poles and nonuniform air gaps. It has a relatively large number of poles, short axial length, and large diameter. The generators in hydroelectric power stations are driven by hydraulic turbines, and they have salient-pole rotor construction.

3.2.1 GENERATOR MODEL

An elementary two-pole three-phase generator is illustrated in Figure 3.1. The stator contains three coils, aa', bb', and cc', displaced from each other by 120 electrical degrees. The concentrated full-pitch coils shown here may be considered to represent distributed windings producing sinusoidal mmf waves concentrated on the magnetic axes of the respective phases. When the rotor is excited to produce

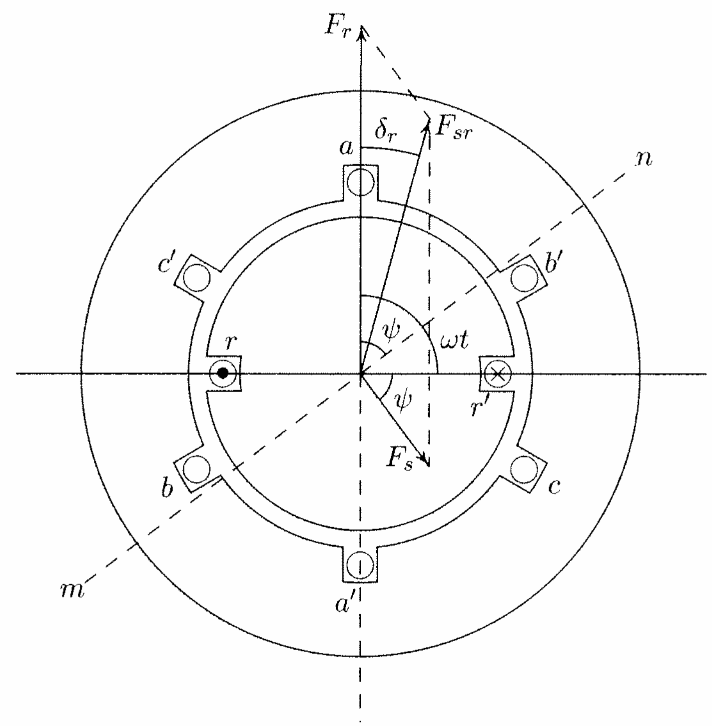


FIGURE 3.1
Elementary two-pole three-phase synchronous generator.

an air gap flux of ϕ per pole and is revolving at constant angular velocity ω, the flux linkage of the coil varies with the position of the rotor mmf axis ωt, where ωt is measured in electrical radians from coil aa' magnetic axis. The flux linkage for an N-turn concentrated coil aa' will be maximum ($N\phi$) at $\omega t = 0$ and zero at $\omega t = \pi/2$. Assuming distributed winding, the flux linkage λ_a will vary as the cosine of the angle ωt. Thus, the flux linkage with coil a is

$$\lambda_a = N\phi\cos\omega t \tag{3.1}$$

The voltage induced in coil aa' is obtained from Faraday's law as

$$\begin{aligned} e_a &= -\frac{d\lambda}{dt} = \omega N\phi\sin\omega t \\ &= E_{max}\sin\omega t \\ &= E_{max}\cos(\omega t - \frac{\pi}{2}) \end{aligned} \tag{3.2}$$

where

$$E_{max} = \omega N\phi = 2\pi f N\phi$$

Therefore, the rms value of the generated voltage is

$$E = 4.44 f N \phi \tag{3.3}$$

where f is the frequency in hertz. In actual ac machine windings, the armature coil of each phase is distributed in a number of slots. Since the emfs induced in different slots are not in phase, their phasor sum is less than their numerical sum. Thus, a reduction factor K_w, called the *winding factor*, must be applied. For most three-phase windings K_w is about 0.85 to 0.95. Therefore, for a distributed phase winding, the rms value of the generated voltage is

$$E = 4.44 K_w f N \phi \tag{3.4}$$

The magnetic field of the rotor revolving at constant speed induces three-phase sinusoidal voltages in the armature, displaced by $2\pi/3$ radians. The frequency of the induced armature voltages depends on the speed at which the rotor runs and on the number of poles for which the machine is wound. The frequency of the armature voltage is given by

$$f = \frac{P}{2}\frac{n}{60} \tag{3.5}$$

where n is the rotor speed in rpm, referred to as *synchronous speed*. During normal conditions, the generator operates synchronously with the power grid. This results in three-phase balanced currents in the armature. Assuming current in phase a is lagging the generated emf e_a by an angle ψ, which is indicated by line mn in Figure 3.1, the instantaneous armature currents are

$$\begin{aligned} i_a &= I_{max}\sin(\omega t - \psi) \\ i_b &= I_{max}\sin(\omega t - \psi - \frac{2\pi}{3}) \\ i_c &= I_{max}\sin(\omega t - \psi - \frac{4\pi}{3}) \end{aligned} \tag{3.6}$$

According to (3.2) the generated emf e_a is maximum when the rotor magnetic axis is under phase a. Since i_a is lagging e_a by an angle ψ, when line mn reaches the axis of coil aa', current in phase a reaches its maximum value. At any instant of time, each phase winding produces a sinusoidally distributed mmf wave with its peak along the axis of the phase winding. These sinusoidally distributed fields can be represented by vectors referred to as *space phasors*. The amplitude of the sinusoidally distributed mmf $f_a(\theta)$ is represented by the vector F_a along the axis of phase a. Similarly, the amplitude of the mmfs $f_b(\theta)$ and $f_c(\theta)$ are shown by vectors F_b and F_c along their respective axis. The mmf amplitudes are proportional to the instantaneous value of the phase current, i.e.,

$$\begin{aligned} F_a &= Ki_a = KI_{max}\sin(\omega t - \psi) = F_m \sin(\omega t - \psi) \\ F_b &= Ki_b = KI_{max}\sin(\omega t - \psi - \frac{2\pi}{3}) = F_m \sin(\omega t - \psi - \frac{2\pi}{3}) \\ F_c &= Ki_c = KI_{max}\sin(\omega t - \psi - \frac{4\pi}{3}) = F_m \sin(\omega t - \psi - \frac{4\pi}{3}) \end{aligned} \tag{3.7}$$

where K is proportional to the number of armature turns per phase and is a function of the winding type. The resultant armature mmf is the vector sum of the above mmfs. A suitable method for finding the resultant mmf is to project these mmfs on line mn and obtain the resultant in-phase and quadrature-phase components. The resultant in-phase components are

$$\begin{aligned} F_1 = {} & F_m \sin(\omega t - \psi)\cos(\omega t - \psi) + F_m \sin(\omega t - \psi - \frac{2\pi}{3}) \\ & \cos(\omega t - \psi - \frac{2\pi}{3}) + F_m \sin(\omega t - \psi - \frac{4\pi}{3})\cos(\omega t - \psi - \frac{4\pi}{3}) \end{aligned}$$

Using the trigonometric identity $\sin\alpha\cos\alpha = (1/2)\sin 2\alpha$, the above expression becomes

$$\begin{aligned} F_1 = {} & \frac{F_m}{2}[\sin 2(\omega t - \psi) + \sin 2(\omega t - \psi - \frac{2\pi}{3}) \\ & + \sin 2(\omega t - \psi - \frac{4\pi}{3})] \end{aligned}$$

The above expression is the sum of three sinusoidal functions displaced from each other by $2\pi/3$ radians, which adds up to zero, i.e., $F_1 = 0$.

The sum of quadrature components results in

$$\begin{aligned} F_2 = {} & F_m \sin(\omega t - \psi)\sin(\omega t - \psi) + F_m \sin(\omega t - \psi - \frac{2\pi}{3})\sin(\omega t - \psi - \frac{2\pi}{3}) \\ & + F_m \sin(\omega t - \psi - \frac{4\pi}{3})\sin(\omega t - \psi - \frac{4\pi}{3}) \end{aligned}$$

Using the trigonometric identity $\sin^2\alpha = (1/2)(1 - \cos 2\alpha)$, the above expression becomes

$$\begin{aligned} F_2 = {} & \frac{F_m}{2}[3 - \cos 2(\omega t - \psi) + \cos 2(\omega t - \psi - \frac{2\pi}{3}) \\ & + \cos 2(\omega t - \psi - \frac{4\pi}{3})] \end{aligned}$$

The sinusoidal terms of the above expression are displaced from each other by $2\pi/3$ radians and add up to zero, with $F_2 = 3/2F_m$. Thus, the amplitude of the resultant armature mmf or stator mmf becomes

$$F_s = \frac{3}{2}F_m \tag{3.8}$$

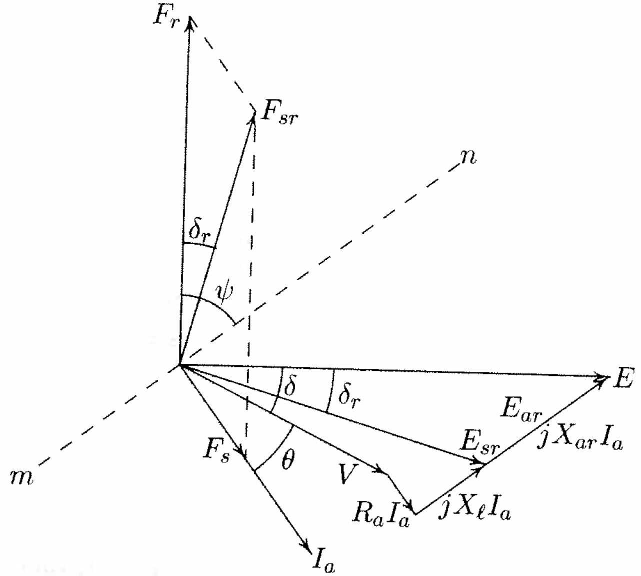


FIGURE 3.2
Combined phasor/vector diagram for one phase of a cylindrical rotor generator.

We thus conclude that the resultant armature mmf has a constant amplitude perpendicular to line mn, and rotates at a constant speed and in synchronism with the field mmf F_r. To see a demonstration of the rotating magnetic field, type **rotfield** at the *MATLAB* prompt.

A typical synchronous machine field alignment for operation as a generator is shown in Figure 3.2, using space vectors to represent the various fields. When the rotor is revolving at synchronous speed and the armature current is zero, the field mmf F_r produces the no-load generated emf E in each phase. The no-load generated voltage which is proportional to the field current is known as the *excitation voltage*. The phasor voltage for phase a, which is lagging F_r by $90°$, is combined with the mmf vector diagram as shown in Figure 3.2. This combined phasor/vector diagram leads to a circuit model for the synchronous machine. It must be emphasized that in Figure 3.2 mmfs are space vectors, whereas the emfs are time phasors. When the armature is carrying balanced three-phase currents, F_s is produced perpendicular to line mn. The interaction of armature mmf and the field mmf, known as *armature reaction*, gives rise to the resultant air gap mmf F_{sr}. The resultant mmf F_{sr} is the vector sum of the field mmf F_r and the armature mmf F_s. The resultant mmf is responsible for the resultant air gap flux ϕ_{sr} that induces the generated emf on-load, shown by E_{sr}. The armature mmf F_s induces the emf E_{ar}, known as the *armature reaction voltage*, which is perpendicular to F_s. The voltage E_{ar} leads I_a by $90°$ and thus can be represented by a voltage drop across a reactance X_{ar} due to the current I_a. X_{ar} is known as the *reactance of the armature reaction*. The phasor sum of E and E_{ar} is shown by E_{sr} perpendicular to F_{sr}, which represents the on-load generated emf.

$$E = E_{sr} + jX_{ar}I_a \tag{3.9}$$

The terminal voltage V is less than E_{sr} by the amount of resistive voltage drop R_aI_a and leakage reactance voltage drop $X_\ell I_a$. Thus

$$E = V + [R_a + j(X_\ell + X_{ar})]I_a \tag{3.10}$$

or

$$E = V + [R_a + jX_s]I_a \tag{3.11}$$

where $X_s = (X_\ell + X_{ar})$ is known as the *synchronous reactance*. The cosine of the angle between I and V, i.e., $\cos\theta$ represents the power factor at the generator terminals. The angle between E and E_{sr} is equal to the angle between the rotor mmf F_r and the air gap mmf F_{sr}, shown by δ_r. The power developed by the machine is proportional to the product of F_r, F_{sr} and $\sin\delta_r$. The relative positions of these mmfs dictates the action of the synchronous machine. When F_r is ahead of F_{sr} by an angle δ_r, the machine is operating as a generator and when F_r falls behind F_{sr}, the machine will act as a motor. Since E and E_{sr} are proportional to F_r and F_{sr}, respectively, the power developed by the machine is proportional to the products of E, E_{sr}, and $\sin\delta_r$. The angle δ_r is thus known as the *power angle*. This is a very important result because it relates the time angle between the phasor emfs with the space angle between the magnetic fields in the machine. Usually the developed power is expressed in terms of the excitation voltage E, the terminal voltage V, and $\sin\delta$. The angle δ is approximately equal to δ_r because the leakage impedance is very small compared to the magnetization reactance.

Due to the nonlinearity of the machine magnetization curve, the synchronous reactance is not constant. The unsaturated synchronous reactance can be found from the open- and short-circuit data. For operation at or near rated terminal voltage, it is usually assumed that the machine is equivalent to an unsaturated one whose magnetization curve is a straight line through the origin and the rated voltage point on the open-circuit characteristic. For steady-state analysis, a constant value known as the *saturated value of the synchronous reactance* corresponding to the rated voltage is used. A simple per-phase model for a cylindrical rotor generator based on (3.11) is obtained as shown in Figure 3.3. The armature resistance is generally much smaller than the synchronous reactance and is often neglected. The equivalent circuit connected to an infinite bus becomes that shown in Figure 3.4, and (3.11) reduces to

$$E = V + jX_sI_a \tag{3.12}$$

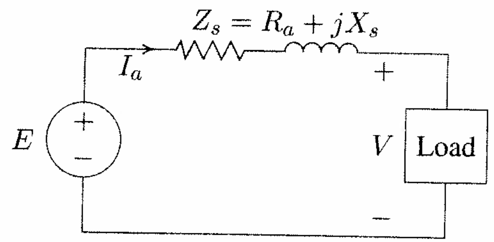


FIGURE 3.3
Synchronous machine equivalent circuit.

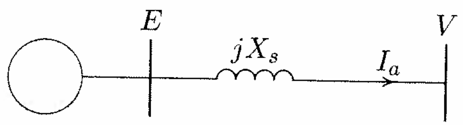
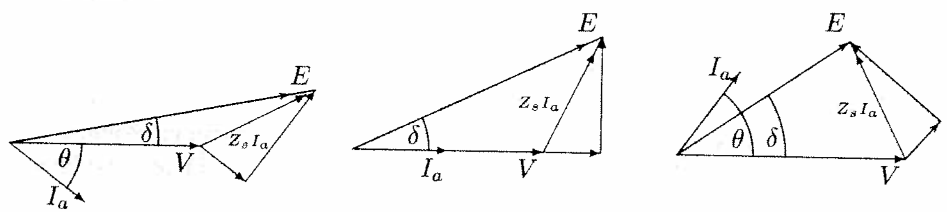


FIGURE 3.4
Synchronous machine connected to an infinite bus.

Figure 3.5 shows the phasor diagram of the generator with terminal voltage as reference for excitations corresponding to lagging, unity, and leading power factors. The voltage regulation of an alternator is a figure of merit used for comparison with other machines. It is defined as the percentage change in terminal voltage from no-load to rated load. This gives an indication of the change in field current required to maintain system voltage when going from no-load to rated load at some specific power factor.



(a) Lagging pf load (b) Upf load (c) Leading pf load

FIGURE 3.5
Synchronous generator phasor diagram.

$$\text{VR} = \frac{|V_{nl}| - |V_{rated}|}{|V_{rated}|} \times 100 = \frac{|E| - |V_{rated}|}{|V_{rated}|} \times 100 \tag{3.13}$$

The no-load voltage for a specific power factor may be determined by operating the machine at rated load conditions and then removing the load and observing the no-load voltage. Since this is not a practical method for very large machines, an accurate analytical method recommended by IEEE as given in reference [43] may be used. An approximate method that provides reasonable results is to consider a hypothetical linearized magnetization curve drawn to intersect the actual magnetization curve at rated voltage. The value of E calculated from (3.12) is then used to find the field current from the linearized curve. Finally, the no-load voltage corresponding to this field current is found from the actual magnetization curve.

3.3 STEADY-STATE CHARACTERISTICS—CYLINDRICAL ROTOR

3.3.1 POWER FACTOR CONTROL

Most synchronous machines are connected to large interconnected electric power networks. These networks have the important characteristic that the system voltage at the point of connection is constant in magnitude, phase angle, and frequency. Such a point in a power system is referred to as an *infinite bus*. That is, the voltage at the generator bus will not be altered by changes in the generator's operating condition.

The ability to vary the rotor excitation is an important feature of the synchronous machine, and we now consider the effect of such a variation when the machine operates as a generator with constant mechanical input power. The per-phase equivalent circuit of a synchronous generator connected to an infinite bus is shown in Figure 3.4. Neglecting the armature resistance, the output power is equal to the power developed, which is assumed to remain constant given by

$$P_{3\phi} = \Re[3VI_a^*] = 3|V||I_a|\cos\theta \tag{3.14}$$

where V is the phase-to-neutral terminal voltage assumed to remain constant. From (3.14) we see that for constant developed power at a fixed terminal voltage V, $I_a \cos\theta$ must be constant. Thus, the tip of the armature current phasor must fall on a vertical line as the power factor is varied by varying the field current as shown in Figure 3.6. From this diagram we have

$$cd = E_1 \sin\delta_1 = X_s I_{a1} \cos\theta_1 \tag{3.15}$$

Thus $E_1 \sin\delta_1$ is a constant, and the locus of E_1 is on the line ef. In Figure 3.6, phasor diagrams are drawn for three armature currents. Application of (3.12) for a lagging power factor armature current I_{a1} results in E_1. If θ is zero, the generator operates at unity power factor and armature current has a minimum value, shown by I_{a2}, which results in E_2. Similarly, E_3 is obtained corresponding to I_{a3} at a leading power factor. Figure 3.6 shows that the generation of reactive power can

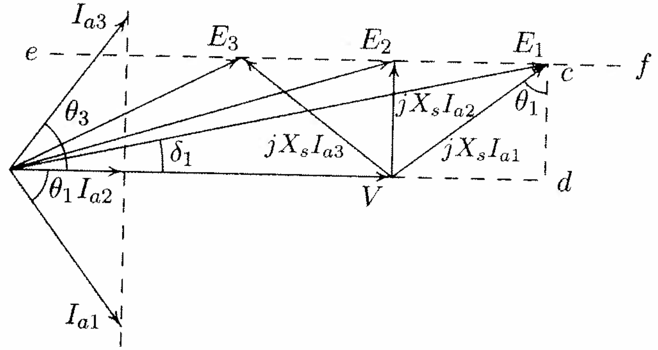


FIGURE 3.6
Variation of field current at constant power.

be controlled by means of the rotor excitation while maintaining a constant real power output. The variation in the magnitude of armature current as the excitation voltage is varied is best shown by a curve. Usually the field current is used as the abscissa instead of excitation voltage because the field current is readily measured. The curve of the armature current as the function of the field current resembles the letter V and is often referred to as the *V curve* of synchronous machines. These curves constitute one of the generator's most important characteristics. There is, of course, a limit beyond which the excitation cannot be reduced. This limit is reached when $\delta = 90°$. Any reduction in excitation below the stability limit for a particular load will cause the rotor to pull out of synchronism. The V curve is illustrated in Figure 3.7 (page 62) for the machine in Example 3.3.

3.3.2 POWER ANGLE CHARACTERISTICS

Consider the per-phase equivalent circuit shown in Figure 3.4. The three-phase complex power at the generator terminal is

$$S_{3\phi} = 3VI_a^* \tag{3.16}$$

Expressing the phasor voltages in polar form, the armature current is

$$I_a = \frac{|E|\angle\delta \; - |V|\angle 0}{|Z_s|\angle\gamma} \tag{3.17}$$

Substituting for I_a^* in (3.16) results in

$$S_{3\phi} = 3\frac{|E||V|}{|Z_s|}\angle\gamma - \delta \; - 3\frac{|V|^2}{|Z_s|}\angle\gamma \tag{3.18}$$

Thus, the real power $P_{3\phi}$ and reactive power $Q_{3\phi}$ are

$$P_{3\phi} = 3\frac{|E||V|}{|Z_s|}\cos(\gamma - \delta) - 3\frac{|V|^2}{|Z_s|}\cos\gamma \tag{3.19}$$

$$Q_{3\phi} = 3\frac{|E||V|}{|Z_s|}\sin(\gamma - \delta) - 3\frac{|V|^2}{|Z_s|}\sin\gamma \tag{3.20}$$

If R_a is neglected, then $Z_s = jX_s$ and $\gamma = 90°$. Equations (3.19) and (3.20) reduce to

$$P_{3\phi} = 3\frac{|E||V|}{X_s}\sin\delta \tag{3.21}$$

$$Q_{3\phi} = 3\frac{|V|}{X_s}(|E|\cos\delta - |V|) \tag{3.22}$$

Equation (3.21) shows that if $|E|$ and $|V|$ are held fixed and the power angle δ is changed by varying the mechanical driving torque, the power transfer varies sinusoidally with the angle δ. From (3.21), the theoretical maximum power occurs when $\delta = 90°$

$$P_{max(3\phi)} = 3\frac{|E||V|}{X_s} \tag{3.23}$$

The behavior of the synchronous machine can be described as follows. If we start with $\delta = 0°$ and increase the driving torque, the machine accelerates, and the rotor mmf F_r advances with respect to the resultant mmf F_{sr}. This results in an increase in δ, causing the machine to deliver electric power. At some value of δ the machine reaches equilibrium where the electric power output balances the increased mechanical power owing to the increased driving torque. It is clear that if an attempt were made to advance δ further than 90° by increasing the driving torque, the electric power output would decrease from the P_{max} point. Therefore, the excess driving torque continues to accelerate the machine, and the mmfs will no longer be magnetically coupled. The machine loses synchronism and automatic equipment disconnects it from the system. The value P_{max} is called the *steady-state stability limit* or *static stability limit*. In general, stability considerations dictate that a synchronous machine achieve steady-state operation for a power angle at considerably less than 90°. The control of real power flow is maintained by the generator governor through the frequency-power control channel.

Equation (3.22) shows that for small δ, $\cos\delta$ is nearly unity and the reactive power can be approximated to

$$Q_{3\phi} \simeq 3\frac{|V|}{X_s}(|E| - |V|) \tag{3.24}$$

From (3.24) we see that when $|E| > |V|$ the generator delivers reactive power to the bus, and the generator is said to be overexcited. If $|E| < |V|$, the reactive power delivered to the bus is negative; that is, the bus is supplying positive reactive power to the generator. Generators are normally operated in the overexcited mode since the generators are the main source of reactive power for inductive load throughout the system. Therefore, we conclude that the flow of reactive power is governed mainly by the difference in the excitation voltage $|E|$ and the bus bar voltage $|V|$. The adjustment in the excitation voltage for the control of reactive power is achieved by the generator excitation system.

Example 3.1 (chp3ex1)

A 50-MVA, 30-kV, three-phase, 60-Hz synchronous generator has a synchronous reactance of 9 Ω per phase and a negligible resistance. The generator is delivering rated power at a 0.8 power factor lagging at the rated terminal voltage to an infinite bus.
(a) Determine the excitation voltage per phase E and the power angle δ.
(b) With the excitation held constant at the value found in (a), the driving torque is reduced until the generator is delivering 25 MW. Determine the armature current and the power factor.
(c) If the generator is operating at the excitation voltage of part (a), what is the steady-state maximum power the machine can deliver before losing synchronism? Also, find the armature current corresponding to this maximum power.

(a) The three-phase apparent power is

$$\begin{aligned} S_{3\phi} = 50\angle\cos^{-1}0.8 &= 50\angle 36.87^\circ \quad \text{MVA} \\ &= 40 \quad \text{MW} + j30 \quad \text{Mvar} \end{aligned}$$

The rated voltage per phase is

$$V = \frac{30}{\sqrt{3}} = 17.32\angle 0^\circ \quad \text{kV}$$

The rated current is

$$I_a = \frac{S^*_{3\phi}}{3V^*} = \frac{(50\angle -36.87)10^3}{3(17.32\angle 0^\circ)} = 962.25\angle -36.87^\circ \quad \text{A}$$

The excitation voltage per phase from (3.12) is

$$E = 17320.5 + (j9)(962.25\angle -36.87) = 23558\angle 17.1^\circ \quad \text{V}$$

The excitation voltage per phase (line to neutral) is 23.56 kV and the power angle is 17.1°.

(b) When the generator is delivering 25 MW from (3.21) the power angle is

$$\delta = \sin^{-1}\left[\frac{(25)(9)}{(3)(23.56)(17.32)}\right] = 10.591^\circ$$

The armature current is

$$I_a = \frac{(23,558\angle 10.591^\circ - 17,320\angle 0^\circ)}{j9} = 807.485\angle -53.43^\circ \quad \text{A}$$

The power factor is given by $\cos(53.43) = 0.596$ lagging.

(c) The maximum power occurs at $\delta = 90^\circ$

$$P_{max(3\phi)} = 3\frac{|E||V|}{X_s} = 3\frac{(23.56)(17.32)}{9} = 136 \quad \text{MW}$$

The armature current is

$$I_a = \frac{(23,558\angle 90^\circ - 17,320\angle 0^\circ)}{j9} = 3248.85\angle 36.32^\circ \quad \text{A}$$

The power factor is given by $\cos(36.32) = 0.8057$ leading.

Example 3.2 (chp3ex2)

The generator of Example 3.1 is delivering 40 MW at a terminal voltage of 30 kV. Compute the power angle, armature current, and power factor when the field current is adjusted for the following excitations.
(a) The excitation voltage is decreased to 79.2 percent of the value found in Example 3.1.
(b) The excitation voltage is decreased to 59.27 percent of the value found in Example 3.1.
(c) Find the minimum excitation below which the generator will lose synchronism.

(a) The new excitation voltage is

$$E = 0.792 \times 23,558 = 18,657 \quad \text{V}$$

From (3.21) the power angle is

$$\delta = \sin^{-1}\left[\frac{(40)(9)}{(3)(18.657)(17.32)}\right] = 21.8^\circ$$

The armature current is

$$I_a = \frac{(18657\angle 21.8^\circ - 17320\angle 0^\circ)}{j9} = 769.8\angle 0^\circ \quad \text{A}$$

The power factor is given by $\cos(0) = 1$.

(b) The new excitation voltage is

$$E = 0.5927 \times 23,558 = 13,963 \quad \text{V}$$

From (3.21) the power angle is

$$\delta = \sin^{-1}\left[\frac{(40)(9)}{(3)(13.963)(17.32)}\right] = 29.748^\circ$$

The armature current is

$$I_a = \frac{(13,963\angle 29.748^\circ - 17,320\angle 0^\circ)}{j9} = 962.3\angle 36.87^\circ \quad \text{A}$$

From current phase angle, the power factor is $\cos 36.87 = 0.8$ leading. The generator is underexcited and is actually receiving reactive power.

(c) From (3.23), the minimum excitation corresponding to $\delta = 90^\circ$ is

$$E = \frac{(40)(9)}{(3)(17.32)(1)} = 6.928 \quad \text{kV}$$

The armature current is

$$I_a = \frac{(6,928\angle 90^\circ - 17,320\angle 0^\circ)}{j9} = 2073\angle 68.2^\circ \quad \text{A}$$

The current phase angle shows that the power factor is $\cos 68.2 = 0.37$ leading. The generator is underexcited and is receiving reactive power.

Example 3.3 (chp3ex3)

For the generator of Example 3.1, construct the V curve for the rated power of 40 MW with varying field excitation from 0.4 power factor leading to 0.4 power factor lagging. Assume the open-circuit characteristic in the operating region is given by $E = 2000 I_f$ V.

The following *MATLAB* command results in the V curve shown in Figure 3.7.

```
P = 40;                                          % real power, MW
V = 30/sqrt(3)+ j*0;                             % phase voltage, kV
Zs = j*9;                                        % synchronous impedance
ang = acos(0.4);
theta=ang:-0.01:-ang;%Angle 0.4 leading to 0.4 lagging pf
P = P*ones(1,length(theta));%generates array of same size
Iam = P./(3*abs(V)*cos(theta));     % current magnitude kA
Ia = Iam.*(cos(theta) + j*sin(theta));    % current phasor
E = V + Zs.*Ia;                       % excitation voltage phasor
Em = abs(E);                  % excitation voltage magnitude, kV
If = Em*1000/2000;                            % field current, A
plot(If, Iam), grid, xlabel('If - A')
ylabel('Ia - kA'), text(3.4, 1, 'Leading pf')
text(13, 1, 'Lagging pf'), text(9, .71, 'Upf')
```

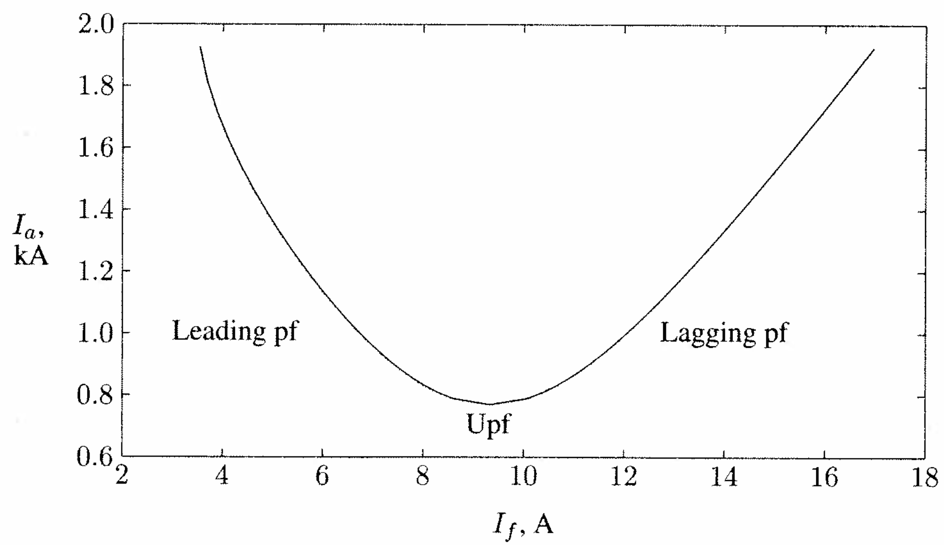


FIGURE 3.7
V curve for generator of Example 3.3.

3.4 SALIENT-POLE SYNCHRONOUS GENERATORS

The model developed in Section 3.2 is only valid for cylindrical rotor generators with uniform air gaps. The salient-pole rotor results in nonuniformity of the magnetic reluctance of the air gap. The reluctance along the polar axis, commonly referred to as the rotor *direct axis*, is appreciably less than that along the interpolar axis, commonly referred to as the *quadrature axis*. Therefore, the reactance has a high value X_d along the direct axis, and a low value X_q along the quadrature axis. These reactances produce voltage drop in the armature and can be taken into account by resolving the armature current I_a into two components I_q, in phase, and I_d in time quadrature, with the excitation voltage. The phasor diagram with the armature resistance neglected is shown in Figure 3.8. It is no longer possible to represent the machine by a simple equivalent circuit. The excitation voltage magnitude is

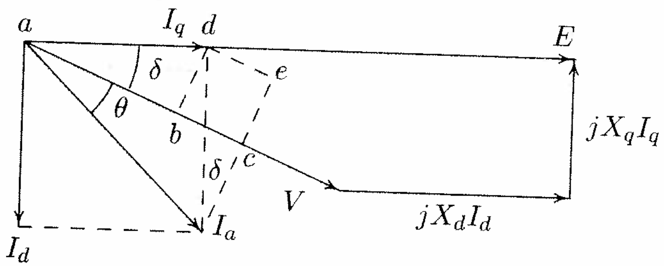


FIGURE 3.8
Phasor diagram for a salient-pole generator.

$$|E| = |V|\cos\delta + X_d I_d \tag{3.25}$$

The three-phase real power at the generator terminal is

$$P = 3|V||I_a|\cos\theta \tag{3.26}$$

The power component of the armature current can be expressed in terms of I_d and I_q as follows.

$$\begin{aligned}|I_a|\cos\theta &= ab + de\\ &= I_q\cos\delta + I_d\sin\delta\end{aligned} \tag{3.27}$$

Substituting from (3.27) into (3.26), we have

$$P = 3|V|(I_q\cos\delta + I_d\sin\delta) \tag{3.28}$$

Now from the phasor diagram given in Figure 3.8,

$$|V|\sin\delta = X_q I_q \tag{3.29}$$

or

$$I_q = \frac{|V|\sin\delta}{X_q} \tag{3.30}$$

Also, from (3.25), I_d is given by

$$I_d = \frac{|E| - |V|\cos\delta}{X_d} \tag{3.31}$$

Substituting for I_d and I_q from (3.31) and (3.30) into (3.28), the real power with armature resistance neglected becomes

$$P_{3\phi} = 3\frac{|E||V|}{X_d}\sin\delta + 3|V|^2\frac{X_d - X_q}{2X_dX_q}\sin 2\delta \tag{3.32}$$

The power equation contains an additional term known as the *reluctance power*. Equations (3.25) and (3.32) can be utilized for steady-state analysis. For short-circuit analysis, assuming a high X/R ratio, the power factor approaches zero and the quadrature component of current can often be neglected. In such a case, X_d merely replaces the X_s used for the cylindrical rotor machine. Generators are thus modeled by their direct axis reactance in series with a constant-voltage power source. Later in the text it will be shown that X_d takes on different values, depending upon the transient time following the short circuit. These reactances are usually expressed in per-unit and are available from the manufacturer's data.

3.5 POWER TRANSFORMER

Transformers are essential elements in any power system. They allow the relatively low voltages from generators to be raised to a very high level for efficient power transmission. At the user end of the system, transformers reduce the voltage to values most suitable for utilization. In modern utility systems, the energy may undergo four or five transformations between generator and ultimate user. As a result, a given system is likely to have about five times more kVA of installed capacity of transformers than of generators.

3.6 EQUIVALENT CIRCUIT OF A TRANSFORMER

The equivalent circuit model of a single-phase transformer is shown in Figure 3.9. The equivalent circuit consists of an ideal transformer of ratio $N_1{:}N_2$ together with elements which represent the imperfections of the real transformer. An ideal transformer would have windings with zero resistance and a lossless, infinite permeability core. The voltage E_1 across the primary of the ideal transformer represents the rms voltage induced in the primary winding by the mutual flux ϕ. This is the portion of the core flux which links both primary and secondary coils. Assuming

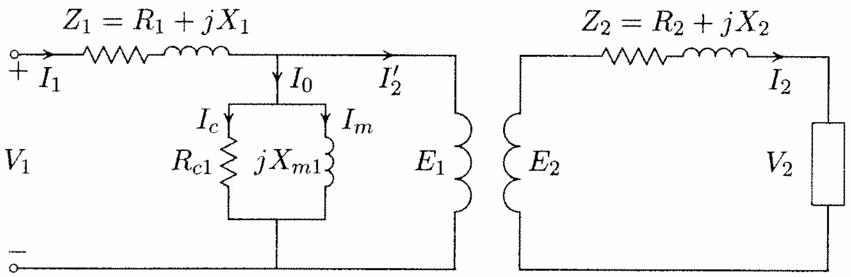


FIGURE 3.9
Equivalent circuit of a transformer.

sinusoidal flux $\phi = \Phi_{max} \cos \omega t$, the instantaneous voltage e_1 is

$$\begin{aligned} e_1 &= N_1 \frac{d\phi}{dt} \\ &= -\omega N_1 \Phi_{max} \sin \omega t \\ &= E_{1max} \cos(\omega t + 90^\circ) \end{aligned} \tag{3.33}$$

where

$$E_{1max} = 2\pi f N_1 \Phi_{max} \tag{3.34}$$

or the rms voltage magnitude E_1 is

$$E_1 = 4.44 f N_1 \Phi_{max} \tag{3.35}$$

It is important to note that the phasor flux is lagging the induced voltage E_1 by 90°. Similarly the rms voltage E_2 across the secondary of the ideal transformer represents the voltage induced in the secondary winding by the mutual flux ϕ, given by

$$E_2 = 4.44 f N_2 \Phi_{max} \tag{3.36}$$

In the ideal transformer, the core is assumed to have a zero reluctance and there is an exact mmf balanced between the primary and secondary. If I'_2 represents the component of current to neutralize the secondary mmf, then

$$I'_2 N_1 = I_2 N_2 \tag{3.37}$$

Therefore, for an ideal transformer, from (3.35) through (3.37) we have

$$\frac{E_1}{E_2} = \frac{I_2}{I'_2} = \frac{N_1}{N_2} \tag{3.38}$$

In a real transformer, the reluctance of the core is finite, and when the secondary current I_2 is zero, the primary current has a finite value. Since at no-load, induced voltage E_1 is almost equal to the supply voltage V_1, the induced voltage and the flux are sinusoidal. However, because of the nonlinear characteristics of the ferromagnetic core, the no-load current is not sinusoidal and contains odd harmonics. The third harmonic is particularly troublesome in certain three-phase connections of transformers. For the purpose of modeling, we assume a sinusoidal no-load current with the rms value of I_0, known as the *no-load current*. This current has a component I_m, in phase with flux, known as the *magnetizing current*, to set up the core flux. Since flux is lagging the induced voltage E_1 by 90°, I_m is also lagging the induced voltage E_1 by 90°. Thus, this component can be represented in the circuit by the magnetizing reactance jX_{m1}. The other component of I_0 is I_c, which supplies the eddy-current and hysteresis losses in the core. Since this is a power component, it is in phase with E_1 and is represented by the resistance R_{c1} as shown in Figure 3.9.

In a real transformer with finite reluctance, all of the flux is not common to both primary and secondary windings. The flux has three components: mutual flux, primary leakage flux, and secondary leakage flux. The leakage flux associated with one winding does not link the other, and the voltage drops caused by the leakage flux are expressed in terms of leakage reactances X_1 and X_2. Finally, R_1 and R_2 are included to represent the primary and secondary winding resistances.

To obtain the performance characteristics of a transformer, it is convenient to use an equivalent circuit model referred to one side of the transformer. From Kirchhoff's voltage law (KVL), the voltage equation of the secondary side is

$$E_2 = V_2 + Z_2 I_2 \tag{3.39}$$

From the relationship (3.38) developed for the ideal transformer, the secondary induced voltage and current are $E_2 = (N_2/N_1)E_1$ and $I_2 = (N_1/N_2)I'_2$, respectively. Upon substitution, (3.39) reduces to

$$\begin{aligned} E_1 &= \frac{N_1}{N_2} V_2 + \left(\frac{N_1}{N_2}\right)^2 Z_2 I'_2 \\ &= V'_2 + Z'_2 I'_2 \end{aligned} \tag{3.40}$$

where

$$Z'_2 = R'_2 + jX'_2 = \left(\frac{N_1}{N_2}\right)^2 R_2 + j\left(\frac{N_1}{N_2}\right)^2 X_2$$

Relation (3.40) is the KVL equation of the secondary side referred to the primary, and the equivalent circuit of Figure 3.9 can be redrawn as shown in Figure 3.10, so the same effects are produced in the primary as would be produced in the secondary.

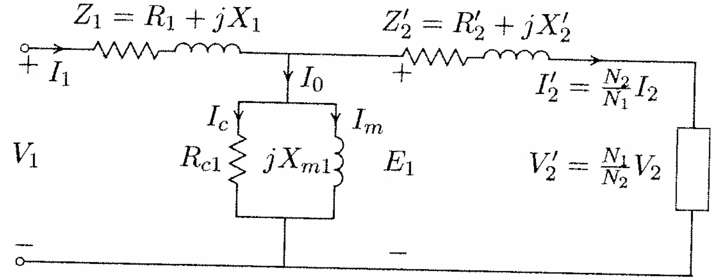


FIGURE 3.10
Exact equivalent circuit referred to the primary side.

On no-load, the primary voltage drop is very small, and V_1 can be used in place of E_1 for computing the no-load current I_0. Thus, the shunt branch can be moved to the left of the primary series impedance with very little loss of accuracy. In this manner, the primary quantities R_1 and X_1 can be combined with the referred secondary quantities R'_2 and X'_2 to obtain the equivalent primary quantities R_{e1} and X_{e1}. The equivalent circuit is shown in Figure 3.11 where we have dispensed with the coils of the ideal transformer. From Figure 3.11

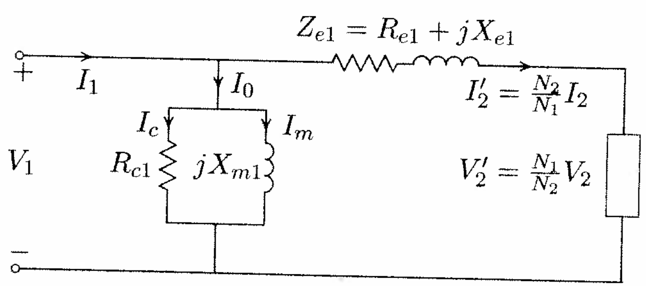


FIGURE 3.11
Approximate equivalent circuit referred to the primary.

$$V_1 = V'_2 + (R_{e1} + jX_{e1})I'_2 \tag{3.41}$$

where

$$R_{e1} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 \quad X_{e1} = X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2 \quad \text{and} \quad I'_2 = \frac{S^*_L}{3V'^*_2}$$

The equivalent circuit referred to the secondary is also shown in Figure 3.12. From Figure 3.12 the referred primary voltage V'_1 is given by

$$V'_1 = V_2 + (R_{e2} + jX_{e2})I_2 \tag{3.42}$$

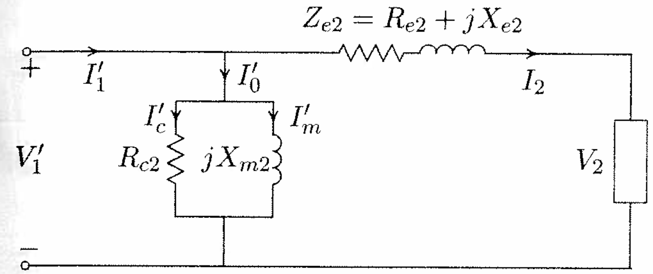


FIGURE 3.12
Approximate equivalent circuit referred to the secondary.

Power transformers are generally designed with very high permeability core and very small core loss. Consequently, a further approximation of the equivalent circuit can be made by omitting the shunt branch, as shown in Figure 3.13. The equivalent circuit referred to the secondary is also shown in Figure 3.13.

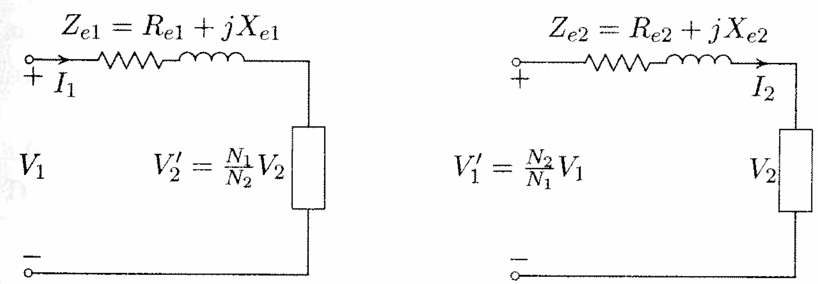


FIGURE 3.13
Simplified circuits referred to one side.

3.7 DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS

The parameters of the approximate equivalent circuit are readily obtained from open-circuit and short-circuit tests. In the open-circuit test, rated voltage is applied at the terminals of one winding while the other winding terminals are open-circuited. Instruments are connected to measure the input voltage V_1, the no-load input current I_0, and the input power P_0. If the secondary is open-circuited, the referred secondary current I'_2 will be zero, and only a small no-load current will be drawn from the supply. Also, the primary voltage drop $(R_1 + jX_1)I_0$ can be neglected, and the equivalent circuit reduces to the form shown in Figure 3.14.

Since the secondary winding copper loss (resistive power loss) is zero and the

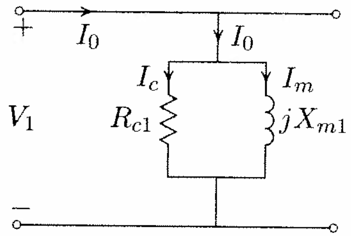


FIGURE 3.14
Equivalent circuit for the open-circuit test.

primary copper loss $R_1 {I_0}^2$ is negligible, the no-load input power P_0 represents the transformer core loss commonly referred to as *iron loss*. The shunt elements R_c and X_m may then be determined from the relations

$$R_{c1} = \frac{{V_1}^2}{P_0} \tag{3.43}$$

The two components of the no-load current are

$$I_c = \frac{V_1}{R_{c1}} \tag{3.44}$$

and

$$I_m = \sqrt{{I_0}^2 - {I_c}^2} \tag{3.45}$$

Therefore, the magnetizing reactance is

$$X_{m1} = \frac{V_1}{I_m} \tag{3.46}$$

In the short-circuit test, a reduced voltage V_{sc} is applied at the terminals of one winding while the other winding terminals are short-circuited. Instruments are connected to measure the input voltage V_{sc}, the input current I_{sc}, and the input power P_{sc}. The applied voltage is adjusted until rated currents are flowing in the windings. The primary voltage required to produce rated current is only a few percent of the rated voltage. At the correspondingly low value of core flux, the exciting current and core losses are entirely negligible, and the shunt branch can be omitted. Thus, the power input can be taken to represent the winding copper loss. The transformer appears as a short when viewed from the primary with the equivalent leakage impedance Z_{e1} consisting of the primary leakage impedance and the referred secondary leakage impedance as shown in Figure 3.15. The series elements

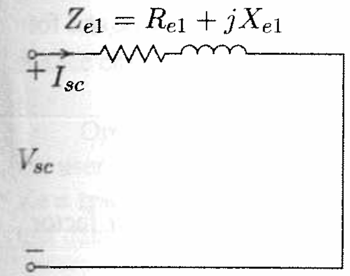


FIGURE 3.15
Equivalent circuit for the short-circuit test.

R_{e1} and X_{e1} may then be determined from the relations

$$Z_{e1} = \frac{V_{sc}}{I_{sc}}$$

and

$$R_{e1} = \frac{P_{sc}}{(I_{sc})^2} \tag{3.47}$$

Therefore, the equivalent leakage reactance is

$$X_{e1} = \sqrt{{Z_{e1}}^2 - {R_{e1}}^2} \tag{3.48}$$

3.8 TRANSFORMER PERFORMANCE

The equivalent circuit can now be used to predict the performance characteristics of the transformer. An important aspect is the transformer efficiency. Power transformer efficiencies very from 95 percent to 99 percent, the higher efficiencies being obtained from transformers with the greater ratings. The actual efficiency of a transformer in percent is given by

$$\eta = \frac{\text{output power}}{\text{input power}} \tag{3.49}$$

and the conventional efficiency of a transformer at n fraction of the full-load power is given by

$$\eta = \frac{n \times S \times PF}{(n \times S \times PF) + n^2 \times P_{cu} + P_c} \tag{3.50}$$

where S is the full-load rated volt-ampere, P_{cu} is the full-load copper loss, and for a three-phase transformer, they are given by

$$\begin{aligned} S &= 3|V_2||I_2| \\ P_{cu} &= 3R_{e2}|I_2|^2 \end{aligned}$$

and P_c is the iron loss at rated voltage. For varying I_2 at constant power factor, maximum efficiency occurs when

$$\frac{d\eta}{d|I_2|} = 0$$

For the above condition, it can be easily shown that maximum efficiency occurs when copper loss equals core loss at n per-unit loading given by

$$n = \sqrt{\frac{P_c}{P_{cu}}} \tag{3.51}$$

Another important performance characteristic of a transformer is change in the secondary voltage from no-load to full-load. A figure of merit used to compare the relative performance of different transformers is the voltage regulation. Voltage regulation is defined as the change in the magnitude of the secondary terminal voltage from no-load to full-load expressed as a percentage of the full-load value.

$$\text{Regulation} = \frac{|V_{2nl}| - |V_2|}{|V_2|} \times 100 \tag{3.52}$$

where V_2 is the full-load rated voltage. V_{2nl} in (3.52) can be calculated by using equivalent circuits referred to either primary or secondary. When the equivalent circuit is referred to the primary side, the primary no-load voltage is found from (3.41), and the voltage regulation becomes

$$\text{Regulation} = \frac{|V_1| - |V_2'|}{|V_2'|} \times 100 \tag{3.53}$$

When the equivalent circuit is referred to the secondary side, the secondary no-load voltage is found from (3.42), and the voltage regulation becomes

$$\text{Regulation} = \frac{|V_1'| - |V_2|}{|V_2|} \times 100 \tag{3.54}$$

An interesting feature arises with a capacitive load. Because partial resonance is set up between the capacitance and the reactance, the secondary voltage may actually tend to rise as the capacitive load value increases.

A program called **trans** is developed for obtaining the transformer performance characteristics. The command **trans** displays a menu with three options:

Option 1 calls upon the function **[Rc, Xm] = troct(Vo, Io, Po)** which prompts the user to enter the no-load test data and returns the shunt branch parameters. Then **Ze = trsct(Vsc, Isc, Psc)** is loaded which prompts the user to enter the short-circuit test data and returns the equivalent leakage impedance.

Option 2 calls upon the function **[Zelv, Zehv] = wz2eqz(Elv, Ehv, Zlv, Zhv)** which prompts the user to enter the individual winding impedances and the shunt branch. This function returns the referred equivalent circuit for both sides.

Option 3 prompts the user to enter the parameters of the equivalent circuit.

The above functions can be used independently when the arguments of the functions are defined in the *MATLAB* environment. If the above functions are typed without the parenthesis and the arguments, the user will be prompted to enter the required data. After the selection of any of the above options, the program prompts the user to enter the load specifications and proceeds to obtain the transformer performance characteristics including an efficiency curve from 25 to 125 percent of full-load.

A new GUI program named **transformer** is developed for the transformer tests and analysis. This program obtains the transformer equivalent circuit from open-circuit and short-circuit tests. It also finds the transformer performance characteristics using the transformer parameters.

Example 3.4 (chp3ex4)

Data obtained from short-circuit and open-circuit tests of a 240-kVA, 4800/240-V, 60-Hz transformer are:

Open-circuit test, low-side data	Short-circuit test, high-side data
$V_1 = 240$ V	$V_{sc} = 187.5$ V
$I_0 = 10$ A	$I_{sc} = 50$ A
$P_0 = 1440$ W	$P_{sc} = 2625$ W

Determine the parameters of the equivalent circuit

The command

```
trans
```

display the following menu

Type of parameters for input	Select
To obtain equivalent circuit from tests	1
To input individual winding impedances	2
To input transformer equivalent impedance	3
To quit	0

```
Select number of menu → 1
Enter Transformer rated power in kVA, S = 240
Enter rated low voltage in volts = 240
Enter rated high voltage in volts = 4800

Open circuit test data
Enter 'lv' within quotes for data ref. to low side or
enter 'hv' within quotes for data ref. to high side →'lv'
Enter input voltage, in volts, Vo = 240
Enter no-load current in Amp, Io = 10
Enter no-load input power in Watt, Po = 1440

Short circuit test data
Enter 'lv' within quotes for data ref. to low side or
enter 'hv' within quotes for data ref. to high side →'hv'
Enter reduced input voltage in volts, Vsc = 187.5
Enter input current in Amp, Isc = 50
Enter input power in Watt, Psc = 2625

Shunt branch ref. to LV side      Shunt branch ref. to HV side
Rc = 40.000 ohm                   Rc = 16000.000 ohm
Xm = 30.000 ohm                   Xm = 12000.000 ohm

Series branch ref. to LV side     Series branch ref. to HV side
Ze = 0.002625 + j 0.0090 ohm      Ze = 1.0500 + j 3.6000 ohm

Hit return to continue
```

At this point the user is prompted to enter the load apparent power, power factor, and voltage. The program then obtains the performance characteristics of the transformer including the efficiency curve from 25 to 125 percent of full load as shown in Figure 3.16.

```
Enter load kVA, S2 = 240
Enter load power factor, pf = 0.8
Enter 'lg' within quotes for lagging pf
or 'ld' within quotes for leading pf -> 'lg'
Enter load terminal voltage in volt, V2 = 240
```

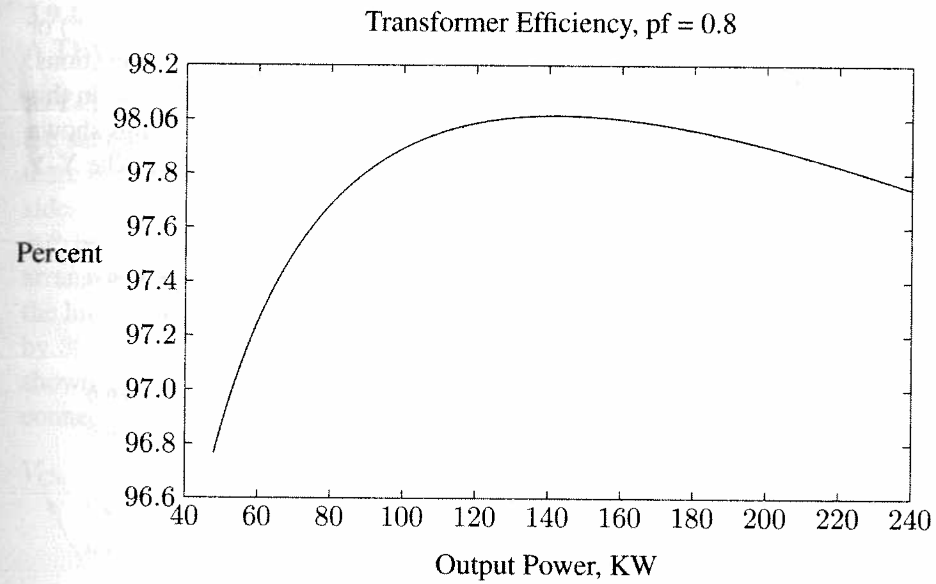


FIGURE 3.16
Efficiency curve of Example 3.4.

```
Secondary load voltage    =    240.000 V
Secondary load current    =   1000.000 A  at  -36.87 degrees
Current ref. to primary   =     50.000 A  at  -36.87 degrees
Primary no-load current   =      0.516 A  at  -53.13 degrees
Primary input current     =     50.495 A  at  -37.03 degrees
Primary input voltage     =   4951.278 V  at    1.30 degrees
Voltage regulation        =      3.152 %
Transformer efficiency    =     97.927 %

Maximum efficiency is 98.015 percent, occurs at 177.757 kVA
with 0.80 pf.
```

At the end of this analysis the program menu is displayed.

3.9 THREE-PHASE TRANSFORMER CONNECTIONS

Three-phase power is transformed by use of three-phase units. However, in large extra high voltage (EHV) units, the insulation clearances and shipping limitations may require a bank of three single-phase transformers connected in three-phase arrangements.

The primary and secondary windings can be connected in either wye (Y) or delta (Δ) configurations. This results in four possible combinations of connections: Y–Y, Δ–Δ, Y–Δ and Δ–Y shown by the simple schematic in Figure 3.17. In this diagram, transformer windings are indicated by heavy lines. The windings shown in parallel are located on the same core and their voltages are in phase. The Y–Y

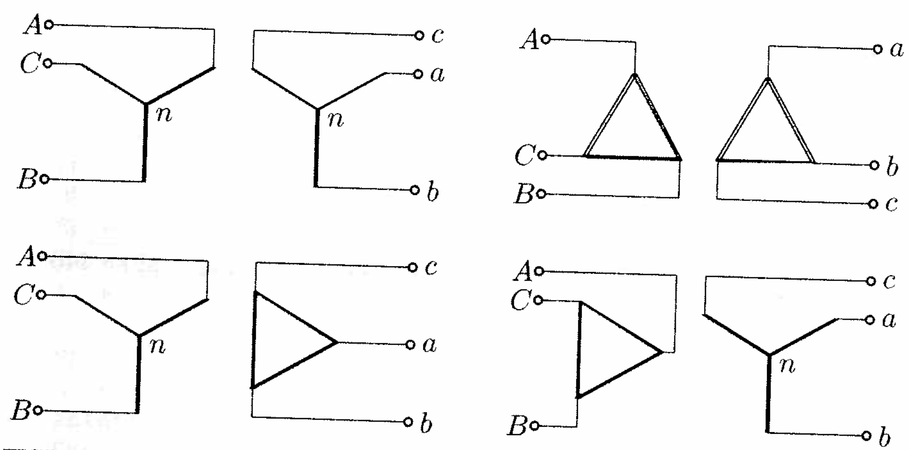


FIGURE 3.17
Three-phase transformer connections.

connection offers advantages of decreased insulation costs and the availability of the neutral for grounding purposes. However, because of problems associated with third harmonics and unbalanced operation, this connection is rarely used. To eliminate the harmonics, a third set of windings, called a *tertiary* winding, connected in Δ is normally fitted on the core to provide a path for the third harmonic currents. This is known as the *three-winding* transformer. The tertiary winding can be loaded with switched reactors or capacitors for reactive power compensation. The Δ–Δ provides no neutral connection and each transformer must withstand full line-to-line voltage. The Δ connection does, however, provide a path for third harmonic currents to flow. This connection has the advantage that one transformer can be removed for repair and the remaining two can continue to deliver three-phase power at a reduced rating of 58 percent of the original bank. This is known as the V connection. The most common connection is the Y–Δ or Δ–Y. This connection is more stable with respect to unbalanced loads, and if the Y connection is used on the high voltage side, insulation costs are reduced. The Y–Δ connection is commonly used to step down a high voltage to a lower voltage. The neutral point on the high voltage side can be grounded. This is desirable in most cases. The Δ–Y connection is commonly used for stepping up to a high voltage.

3.9.1 THE PER-PHASE MODEL OF A THREE-PHASE TRANSFORMER

In Y–Y and Δ–Δ connections, the ratio of the line voltages on HV and LV sides are the same as the ratio of the phase voltages on the HV and LV sides. Furthermore, there is no phase shift between the corresponding line voltages on the HV and LV sides. However, the Y–Δ and the Δ–Y connections will result in a phase shift of 30° between the primary and secondary line-to-line voltages. The windings are arranged in accordance to the ASA (American Standards Association) such that the line voltage on the HV side leads the corresponding line voltage on the LV side by 30° regardless of which side is Y or Δ. Consider the Y–Δ schematic diagram shown in Figure 3.17. The positive phase sequence voltage phasor diagram for this connection is shown in Figure 3.18, where V_{An} is taken as reference. Let the Y

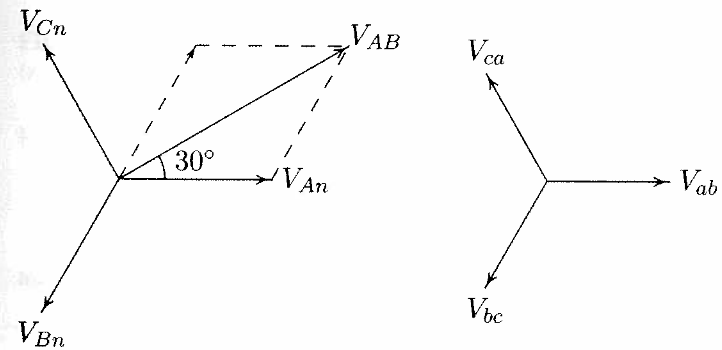


FIGURE 3.18
30° phase shift in line-to-line voltages of Y–Δ connection.

connection be the high voltage side shown by letter H and the Δ connection the low voltage side shown by X. We consider phase a only and use subscript L for line and P for phase quantities. If N_H is the number of turns on one phase of the high voltage winding and N_X is the number of turns on one phase of the low voltage winding, the transformer turns ratio is $a = N_H/N_X = V_{HP}/V_{XP}$. The relationship between the line voltage and phase voltage magnitudes is

$$V_{HL} = \sqrt{3}\, V_{HP}$$
$$V_{XL} = V_{XP}$$

Therefore, the ratio of the line voltage magnitudes for Y–Δ transformer is

$$\frac{V_{HL}}{V_{XL}} = \sqrt{3}\, a \tag{3.55}$$

Because the core losses and magnetization current for power transformers are on the order of 1 percent of the maximum ratings, the shunt impedance is neglected and only the winding resistance and leakage reactance are used to model the transformer. In dealing with Y–Δ or Δ–Y banks, it is convenient to replace the Δ connection by an equivalent Y connection and then work with only one phase. Since for balanced operations, the Y neutral and the neutral of the equivalent Y of the Δ connection are at the same potential, they can be connected together and represented by a neutral conductor. When the equivalent series impedance of one transformer is referred to the delta side, the Δ connected impedances of the transformer are replaced by equivalent Y-connected impedances, given by $Z_Y = Z_\Delta/3$. The per phase equivalent model with the shunt branch neglected is shown in Figure 3.19. Z_{e1} and Z_{e2} are the equivalent impedances based on the line-to-neutral connections, and the voltages are the line-to-neutral values.

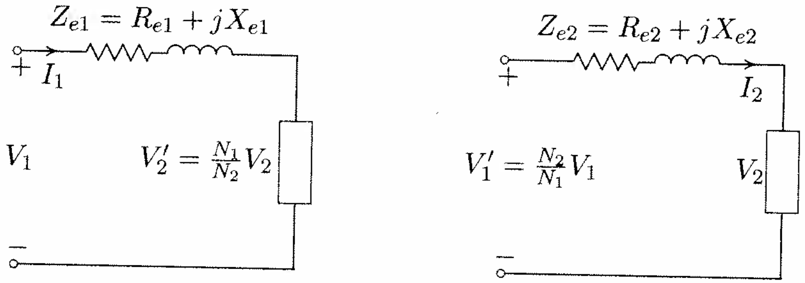


FIGURE 3.19
The per phase equivalent circuit.

3.10 AUTOTRANSFORMERS

Transformers can be constructed so that the primary and secondary coils are electrically connected. This type of transformer is called an autotransformer. A conventional two-winding transformer can be changed into an autotransformer by connecting the primary and secondary windings in series. Consider the two-winding transformer shown in Figure 3.20(a). The two-winding transformer is converted to an autotransformer arrangement as shown in Figure 3.20(b) by connecting the two windings electrically in series so that the polarities are additive. The winding from X_1 to X_2 is called the series winding, and the winding from H_1 to H_2 is called the common winding. From an inspection of this figure it follows that an autotransformer can operate as a step-up as well as a step-down transformer. In both cases, winding part H_1H_2 is common to the primary as well as the secondary side of the transformer. The performance of an autotransformer is governed by the fundamental considerations already discussed for transformers having two separate windings. For determining the power rating as an autotransformer, the ideal transformer relations are ordinarily used, which provides an adequate approximation to the actual transformer values.

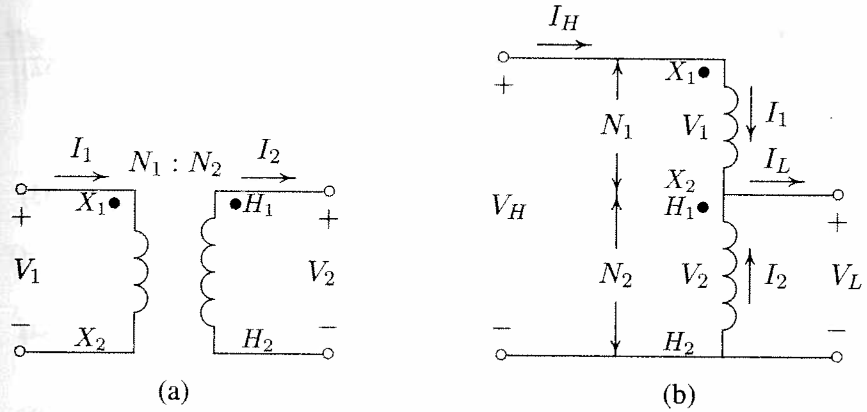


FIGURE 3.20
(a) Two-winding transformer, (b) reconnected as an autotransformer.

From Figure 3.20(a), the two-winding voltages and currents are related by

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a \tag{3.56}$$

and

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = a \tag{3.57}$$

where a is the turns ratio of the two-winding transformer. From Figure 3.20(b), we have

$$V_H = V_2 + V_1 \tag{3.58}$$

Substituting for V_1 from (3.56) into (3.58) yields

$$V_H = V_2 + \frac{N_1}{N_2}V_2 \tag{3.59}$$

Since $V_2 = V_L$, the voltage relationship between the two sides of an autotransformer becomes

$$\begin{aligned} V_H &= V_L + \frac{N_1}{N_2}V_L \\ &= (1+a)V_L \end{aligned} \tag{3.60}$$

or

$$\frac{V_H}{V_L} = 1 + a \tag{3.61}$$

Since the transformer is ideal, the mmf due to I_1 must be equal and opposite to the mmf produced by I_2. As a result, we have

$$N_2 I_2 = N_1 I_1 \tag{3.62}$$

From Kirchhoff's law, $I_2 = I_L - I_1$, and the above equation becomes

$$N_2(I_L - I_1) = N_1 I_1 \tag{3.63}$$

or

$$I_L = \frac{N_1 + N_2}{N_2} I_1 \tag{3.64}$$

Since $I_1 = I_H$, the current relationship between the two sides of an autotransformer becomes

$$\frac{I_L}{I_H} = 1 + a \tag{3.65}$$

The ratio of the apparent power rating of an autotransformer to a two-winding transformer, known as the *power rating advantage*, is found from

$$\frac{S_{auto}}{S_{2-w}} = \frac{(V_1 + V_2)I_1}{V_1 I_1} = 1 + \frac{N_2}{N_1} = 1 + \frac{1}{a} \tag{3.66}$$

From (3.66), we can see that a higher rating is obtained as an autotransformer with a higher number of turns of the common winding (N_2). The higher rating as an autotransformer is a consequence of the fact that only S_{2-w} is transformed by the electromagnetic induction. The rest passes from the primary to secondary without being coupled through the transformer's windings. This is known as the *conducted power*. Compared with a two-winding transformer of the same rating, autotransformers are smaller, more efficient, and have lower internal impedance. Three-phase autotransformers are used extensively in power systems where the voltages of the two systems coupled by the transformers do not differ by a factor greater than about three.

Example 3.5 (chp3ex5)

A two-winding transformer is rated at 60 kVA, 240/1200 V, 60 Hz. When operated as a conventional two-winding transformer at rated load, 0.8 power factor, its efficiency is 0.96. This transformer is to be used as a 1440/1200-V step-down autotransformer in a power distribution system.
(a) Assuming ideal transformer, find the transformer kVA rating when used as an autotransformer.
(b) Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

The two-winding transformer rated currents are:

$$I_1 = \frac{60,000}{240} = 250 \text{ A}$$
$$I_2 = \frac{60,000}{1200} = 50 \text{ A}$$

The autotransformer connection is as shown in Figure 3.21.

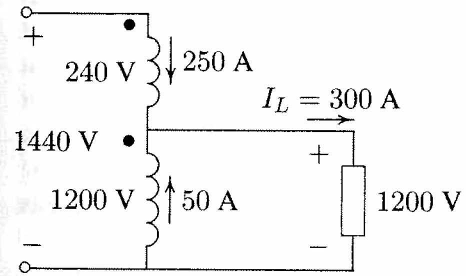


FIGURE 3.21
Auto transformer connection for Example 3.5.

(a) The autotransformer secondary current is

$$I_L = 250 + 50 = 300 \text{ A}$$

With windings carrying rated currents, the autotransformer rating is

$$S = (1200)(300)(10^{-3}) = 360 \text{ kVA}$$

Therefore, the power advantage of the autotransformer is

$$\frac{S_{auto}}{S_{2-w}} = \frac{360}{60} = 6$$

(b) When operated as a two-winding transformer at full-load, 0.8 power factor, the losses are found from the efficiency formula

$$\frac{(60)(0.8)}{(60)(0.8) + P_{loss}} = 0.96$$

Solving the above equation, the total transformer loss is

$$P_{loss} = \frac{48(1 - 0.96)}{0.96} = 2.0 \text{ kW}$$

Since the windings are subjected to the same rated voltages and currents as the two-winding transformer, the autotransformer copper loss and the core loss at the rated values are the same as the two-winding transformer. Therefore, the autotransformer efficiency at rated load, 0.8 power factor, is

$$\eta = \frac{(360)(0.8)}{(360)(0.8) + 2} \times 100 = 99.31 \text{ percent}$$

3.10.1 AUTOTRANSFORMER MODEL

When a two-winding transformer is connected as an autotransformer, its equivalent impedance expressed in per-unit is much smaller compared to the equivalent value of the two-winding connection. It can be shown that the effective per-unit impedance of an autotransformer is smaller by a factor equal to the reciprocal of the power advantage of the autotransformer connection. It is common practice to consider an autotransformer as a two-winding transformer with its two winding connected in series as shown in Figure 3.22, where the equivalent impedance is referred to the N_1-turn side.

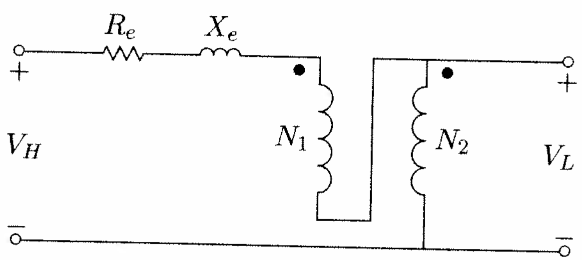


FIGURE 3.22
Autotransformer equivalent circuit.

3.11 THREE-WINDING TRANSFORMERS

Transformers having three windings are often used to interconnect three circuits which may have different voltages. These windings are called primary, secondary, and tertiary windings. Typical applications of three-winding transformers in power systems are for the supply of two independent loads at different voltages from the same source and interconnection of two transmission systems of different voltages. Usually the tertiary windings are used to provide voltage for auxiliary power purposes in the substation or to supply a local distribution system. In addition, the switched reactor or capacitors are connected to the tertiary bus for the purpose of reactive power compensation. Sometimes three-phase Y-Y transformers and Y–connected autotransformers are provided with Δ–connected tertiary windings for harmonic suppression.

3.11.1 THREE-WINDING TRANSFORMER MODEL

If the exciting current of a three-winding transformer is neglected, it is possible to draw a simple single-phase equivalent T-circuit as shown in Figure 3.23.

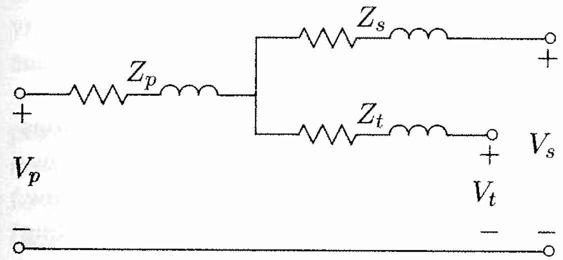


FIGURE 3.23
Equivalent circuit of three-winding transformer.

Three short-circuit tests are carried out on a three-winding transformer with N_p, N_s, and N_t turns per phase on the three windings, respectively. The three tests are similar in that in each case one winding is open, one shorted, and reduced voltage is applied to the remaining winding. The following impedances are measured on the side to which the voltage is applied.

Z_{ps} = impedance measured in the primary circuit with the secondary short-circuited and the tertiary open.
Z_{pt} = impedance measured in the primary circuit with the tertiary short-circuited and the secondary open.
Z'_{st} = impedance measured in the secondary circuit with the tertiary short-circuited and the primary open.

Referring Z'_{st} to the primary side, we obtain

$$Z_{st} = \left(\frac{N_p}{N_s}\right)^2 Z'_{st} \tag{3.67}$$

If Z_p, Z_s, and Z_t are the impedances of the three separate windings referred to the primary side, then

$$\begin{aligned} Z_{ps} &= Z_p + Z_s \\ Z_{pt} &= Z_p + Z_t \\ Z_{st} &= Z_s + Z_t \end{aligned} \tag{3.68}$$

Solving the above equations, we have

$$\begin{aligned} Z_p &= \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st}) \\ Z_s &= \frac{1}{2}(Z_{ps} + Z_{st} - Z_{pt}) \\ Z_t &= \frac{1}{2}(Z_{pt} + Z_{st} - Z_{ps}) \end{aligned} \tag{3.69}$$

3.12 VOLTAGE CONTROL OF TRANSFORMERS

Voltage control in transformers is required to compensate for varying voltage drops in the system and to control reactive power flow over transmission lines. Transformers may also be used to control phase angle and, therefore, active power flow. The two commonly used methods are tap changing transformers and regulating transformers.

3.12.1 TAP CHANGING TRANSFORMERS

Practically all power transformers and many distribution transformers have taps in one or more windings for changing the turns ratio. This method is the most popular since it can be used for controlling voltages at all levels. Tap changing, by altering the voltage magnitude, affects the distribution of vars and may therefore be used to control the flow of reactive power. There are two types of tap changing transformers

(i) Off-load tap changing transformers.
(ii) Tap changing under load (TCUL) transformers.

The off-load tap changing transformer requires the disconnection of the transformer when the tap setting is to be changed. Off-load tap changers are used when it is expected that the ratio will need to be changed only infrequently, because of load growth or some seasonal change. A typical transformer might have four taps in addition to the nominal setting, with spacing of 2.5 percent of full-load voltage between them. Such an arrangement provides for adjustments of up to 5 percent above or below the nominal voltage rating of the transformer.

Tap changing under load (TCUL) is used when changes in ratio may be frequent or when it is undesirable to de-energize the transformer to change a tap. A large number of units are now being built with load tap changing equipment. It is used on transformers and autotransformers for transmission tie, for bulk distribution units, and at other points of load service. Basically, a TCUL transformer is a transformer with the ability to change taps while power is connected. A TCUL transformer may have built-in voltage sensing circuitry that automatically changes taps to keep the system voltage constant. Such special transformers are very common in modern power systems. Special tap changing gear are required for TCUL transformers, and the position of taps depends on a number of factors and requires special consideration to arrive at an optimum location for the TCUL equipment. Step-down units usually have TCUL in the low voltage winding and de-energized taps in the high voltage winding. For example, the high voltage winding might be equipped with a nominal voltage turns ratio plus four 2.5 percent fixed tap settings to yield ± 5 percent buck or boost voltage. In addition to this, there could be provision, on the low voltage windings, for 32 incremental steps of $\frac{5}{8}$ each, giving an automatic range of ± 10 percent.

Tapping on both ends of a radial transmission line can be adjusted to compensate for the voltage drop in the line. Consider one phase of a three-phase transmission line with a step-up transformer at the sending end and a step-down transformer at the receiving end of the line. A single-line representation is shown in Figure 3.24, where t_S and t_R are the tap setting in per-unit. In this diagram, V_1' is the supply phase voltage referred to the high voltage side, and V_2' is the load phase voltage, also referred to the high voltage side. The impedance shown includes the

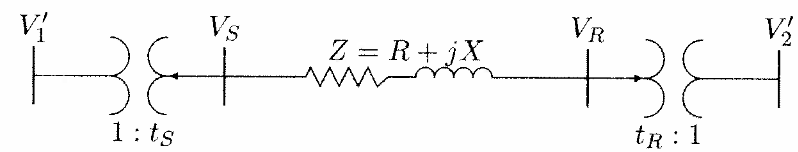


FIGURE 3.24
A radial line with tap changing transformers at both ends.

line impedance plus the referred impedances of the sending end and the receiving end transformers to the high voltage side. If V_S and V_R are the phase voltages at both ends of the line, we have

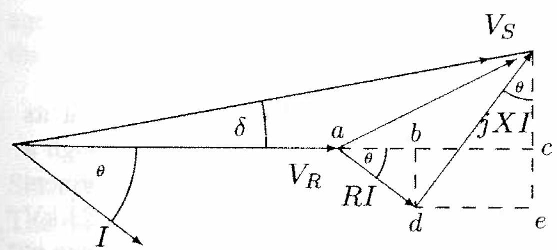


FIGURE 3.25
Voltage phasor diagram.

$$V_R = V_S + (R + jX)I \tag{3.70}$$

The phasor diagram for the above equation is shown in Figure 3.25.

The phase shift δ between the two ends of the line is usually small, and we can neglect the vertical component of V_S. Approximating V_S by its horizontal component results in

$$\begin{aligned} |V_S| &= |V_R| + ab + de \\ &= |V_R| + |I|R\cos\theta + |I|X\sin\theta \end{aligned} \tag{3.71}$$

Substituting for $|I|$ from $P_\phi = |V_R||I|\cos\theta$ and $Q_\phi = |V_R||I|\sin\theta$ will result in

$$|V_S| = |V_R| + \frac{RP_\phi + XQ_\phi}{|V_R|} \tag{3.72}$$

Since $V_S = t_S V_1'$ and $V_R = t_R V_2'$, the above relation in terms of V_1', and V_2' becomes

$$t_S|V_1'| = t_R|V_2'| + \frac{RP_\phi + XQ_\phi}{t_R|V_2'|} \tag{3.73}$$

or

$$t_S = \frac{1}{|V_1'|}\left(t_R|V_2'| + \frac{RP_\phi + XQ_\phi}{t_R|V_2'|}\right) \tag{3.74}$$

Assuming the product of t_S and t_R is unity, i.e., $t_S t_R = 1$, and substituting for t_R in (3.74), the following expression is found for t_S.

$$t_S = \sqrt{\frac{\frac{|V_2'|}{|V_1'|}}{1 - \frac{RP_\phi + XQ_\phi}{|V_1'||V_2'|}}} \tag{3.75}$$

Example 3.6 (chp3ex6)

A three-phase transmission line is feeding from a 23/230-kV transformer at its sending end. The line is supplying a 150-MVA, 0.8 power factor load through a step-down transformer of 230/23 kV. The impedance of the line and transformers at 230 kV is $18 + j60\ \Omega$. The sending end transformer is energized from a 23-kV supply. Determine the tap setting for each transformer to maintain the voltage at the load at 23 kV.

The load real and reactive power per phase are

$$P_\phi = \frac{1}{3}(150)(0.8) = 40 \text{ MW}$$
$$Q_\phi = \frac{1}{3}(150)(0.6) = 30 \text{ Mvar}$$

The source and the load phase voltages referred to the high voltage side are

$$|V_1'| = |V_2'| = \left(\frac{230}{23}\right)\left(\frac{23}{\sqrt{3}}\right) = \frac{230}{\sqrt{3}}$$

From (3.75), we have

$$t_S = \sqrt{\frac{1}{1 - \frac{(18)(40)+(60)(30)}{(230/\sqrt{3})^2}}} = 1.08 \text{ pu}$$

and

$$t_R = \frac{1}{1.08} = 0.926 \text{ pu}$$

3.12.2 REGULATING TRANSFORMERS OR BOOSTERS

Regulating transformers, also known as *boosters*, are used to change the voltage magnitude and phase angle at a certain point in the system by a small amount. A booster consists of an exciting transformer and a series transformer.

VOLTAGE MAGNITUDE CONTROL

Figure 3.26 shows the connection of a regulating transformer for phase a of a three-phase system for voltage magnitude control. Other phases have identical arrangement. The secondary of the exciting transformer is tapped, and the voltage obtained from it is applied to the primary of the series transformer. The corresponding voltage on the secondary of the series transformer is added to the input voltage. Thus, the output voltage is

$$V'_{an} = V_{an} + \Delta V_{an} \tag{3.76}$$

Since the voltages are in phase, a booster of this type is called an *in-phase booster*. The output voltage can be adjusted by changing the excitation transformer taps. By changing the switch from position 1 to 2, the polarity of the voltage across the series transformer is reversed, so that the output voltage is now less than the input voltage.

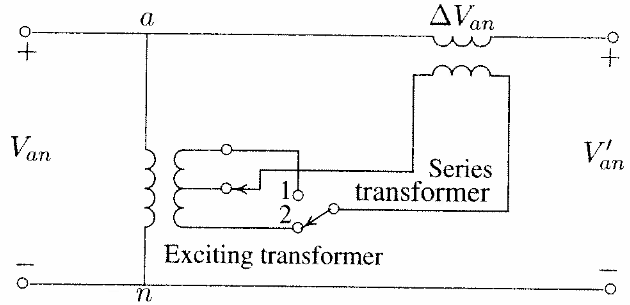


FIGURE 3.26
Regulating transformer for voltage magnitude control.

PHASE ANGLE CONTROL

Regulating transformers are also used to control the voltage phase angle. If the injected voltage is out of phase with the input voltage, the resultant voltage will have a phase shift with respect to the input voltage. Phase shifting is used to control active power flow at major intertie buses. A typical arrangement for phase a of a three-phase system is shown in Figure 3.27.

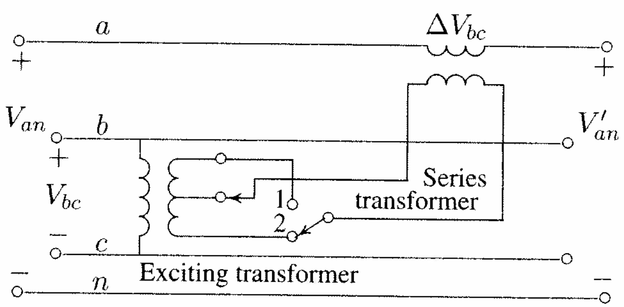


FIGURE 3.27
Regulating transformer for voltage phase angle control.

The series transformer of phase a is supplied from the secondary of the exciting transformer bc. The injected voltage ΔV_{bc} is in quadrature with the voltage V_{an}, thus the resultant voltage V'_{an} goes through a phase shift α, as shown in Figure 3.28. The output voltage is

$$V'_{an} = V_{an} + \Delta V_{bc} \tag{3.77}$$

Similar connections are made for the remaining phases, resulting in a balanced three phase output voltage. The amount of phase shift can be adjusted by changing the excitation transformer taps. By changing the switch from position 1 to 2, the

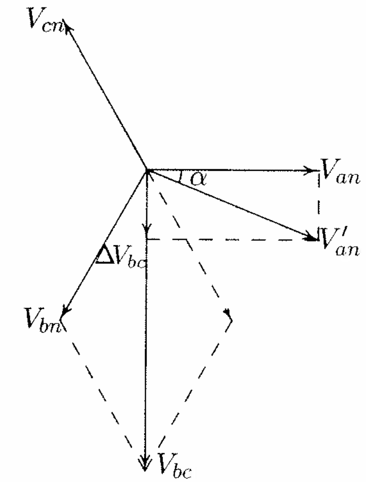


FIGURE 3.28
Voltage phasor diagram showing phase shifting effect for phase a.

output voltage can be made to lag or lead the input voltage. The advantages of the regulating transformers are

1. The main transformers are free from tappings.
2. The regulating transformers can be used at any intermediate point in the system.
3. The regulating transformers and the tap changing gears can be taken out of service for maintenance without affecting the system.

3.13 THE PER-UNIT SYSTEM

The solution of an interconnected power system having several different voltage levels requires the cumbersome transformation of all impedances to a single voltage level. However, power system engineers have devised the *per-unit system* such that the various physical quantities such as power, voltage, current and impedance are expressed as a decimal fraction or multiples of base quantities. In this system, the different voltage levels disappear, and a power network involving generators, transformers, and lines (of different voltage levels) reduces to a system of simple impedances. The per-unit value of any quantity is defined as

$$\text{Quantity in per-unit} = \frac{\text{actual quantity}}{\text{base value of quantity}} \tag{3.78}$$

For example,

$$S_{pu} = \frac{S}{S_B} \quad V_{pu} = \frac{V}{V_B} \quad I_{pu} = \frac{I}{I_B} \quad \text{and} \quad Z_{pu} = \frac{Z}{Z_B}$$

where the numerators (actual values) are phasor quantities or complex values and the denominators (base values) are always real numbers. A minimum of four base quantities are required to completely define a per-unit system: volt-ampere, voltage, current, and impedance. Usually, the three-phase base volt-ampere S_B or MVA_B and the line-to-line base voltage V_B or kV_B are selected. Base current and base impedance are then dependent on S_B and V_B and must obey the circuit laws. These are given by

$$I_B = \frac{S_B}{\sqrt{3}\,V_B} \tag{3.79}$$

and

$$Z_B = \frac{V_B/\sqrt{3}}{I_B} \tag{3.80}$$

Substituting for I_B from (3.79), the base impedance becomes

$$Z_B = \frac{(V_B)^2}{S_B}$$
$$Z_B = \frac{(kV_B)^2}{MVA_B} \tag{3.81}$$

The phase and line quantities expressed in per-unit are the same, and the circuit laws are valid, i.e.,

$$S_{pu} = V_{pu} I_{pu}^* \tag{3.82}$$

and

$$V_{pu} = Z_{pu} I_{pu} \tag{3.83}$$

The load power at its rated voltage can also be expressed by a per-unit impedance. If $S_{L(3\phi)}$ is the complex load power, the load current per phase at the phase voltage V_P is given by

$$S_{L(3\phi)} = 3V_P I_P^* \tag{3.84}$$

The phase current in terms of the ohmic load impedance is

$$I_P = \frac{V_P}{Z_P} \tag{3.85}$$

Substituting for I_P from (3.85) into (3.84) results in the ohmic value of the load impedance

$$Z_P = \frac{3|V_P|^2}{S_{L(3\phi)}^*}$$
$$= \frac{|V_{L-L}|^2}{S_{L(3\phi)}^*} \tag{3.86}$$

From (3.81) the load impedance in per-unit is

$$Z_{pu} = \frac{Z_P}{Z_B} = \left|\frac{V_{L-L}}{V_B}\right|^2 \frac{S_B}{S_{L(3\phi)}^*} \tag{3.87}$$

or

$$Z_{pu} = \frac{|V_{pu}|^2}{S_{L(pu)}^*} \tag{3.88}$$

3.14 CHANGE OF BASE

The impedance of individual generators and transformers, as supplied by the manufacturer, are generally in terms of percent or per-unit quantities based on their own ratings. The impedance of transmission lines are usually expressed by their ohmic values. For power system analysis, all impedances must be expressed in per unit on a common system base. To accomplish this, an arbitrary base for apparent power is selected; for example, 100 MVA. Then, the voltage bases must be selected. Once a voltage base has been selected for a point in a system, the remaining voltage bases are no longer independent; they are determined by the various transformer turns ratios. For example, if on a low-voltage side of a 34.5/115-kV transformer the base voltage of 36 kV is selected, the base voltage on the high-voltage side must be $36(115/34.5) = 120$ kV. Normally, we try to select the voltage bases that are the same as the nominal values.

Let Z_{pu}^{old} be the per-unit impedance on the power base S_B^{old} and the voltage base V_B^{old}, which is expressed by

$$Z_{pu}^{old} = \frac{Z_\Omega}{Z_B^{old}} = Z_\Omega \frac{S_B^{old}}{(V_B^{old})^2} \tag{3.89}$$

Expressing Z_Ω to a new power base and a new voltage base, results in the new per-unit impedance

$$Z_{pu}^{new} = \frac{Z_\Omega}{Z_B^{new}} = Z_\Omega \frac{S_B^{new}}{(V_B^{new})^2} \tag{3.90}$$

From (3.89) and (3.90), the relationship between the old and the new per-unit values is

$$Z_{pu}^{new} = Z_{pu}^{old} \frac{S_B^{new}}{S_B^{old}} \left(\frac{V_B^{old}}{V_B^{new}} \right)^2 \tag{3.91}$$

If the voltage bases are the same, (3.91) reduces to

$$Z_{pu}^{new} = Z_{pu}^{old} \frac{S_B^{new}}{S_B^{old}} \tag{3.92}$$

The advantages of the per-unit system for analysis are described below.

- The per-unit system gives us a clear idea of relative magnitudes of various quantities, such as voltage, current, power and impedance.
- The per-unit impedance of equipment of the same general type based on their own ratings fall in a narrow range regardless of the rating of the equipment. Whereas their impedance in ohms vary greatly with the rating.
- The per-unit values of impedance, voltage and current of a transformer are the same regardless of whether they are referred to the primary or the secondary side. This is a great advantage since the different voltage levels disappear and the entire system reduces to a system of simple impedance.
- The per-unit systems are ideal for the computerized analysis and simulation of complex power system problems.
- The circuit laws are valid in per-unit systems, and the power and voltage equations as given by (3.82) and (3.83) are simplified since the factors of $\sqrt{3}$ and 3 are eliminated in the per-unit system.

Example 3.7 demonstrates how a per-unit impedance diagram is obtained for a simple power system network.

Example 3.7 (chp3ex7)

The one-line diagram of a three-phase power system is shown in Figure 3.29. Select a common base of 100 MVA and 22 kV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in per-unit. The manufacturer's data for each device is given as follow:

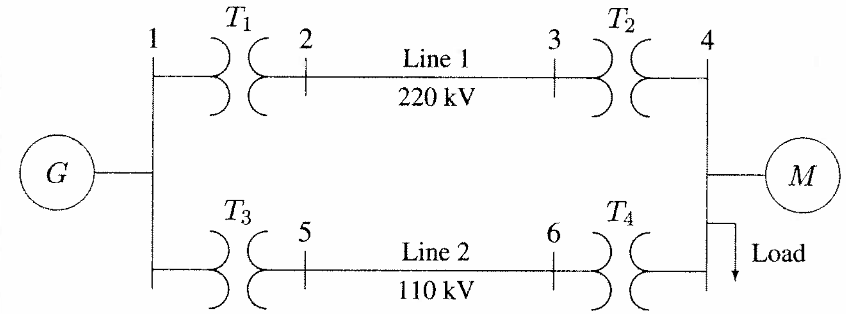


FIGURE 3.29
One-line diagram for Example 3.7.

G:	90 MVA	22 kV	$X = 18\ \%$
T_1:	50 MVA	22/220 kV	$X = 10\ \%$
T_2:	40 MVA	220/11 kV	$X = 6.0\ \%$
T_3:	40 MVA	22/110 kV	$X = 6.4\ \%$
T_4:	40 MVA	110/11 kV	$X = 8.0\ \%$
M:	66.5 MVA	10.45 kV	$X = 18.5\ \%$

The three-phase load at bus 4 absorbs 57 MVA, 0.6 power factor lagging at 10.45 kV. Line 1 and line 2 have reactances of 48.4 and 65.43 Ω, respectively.

First, the voltage bases must be determined for all sections of the network. The generator rated voltage is given as the base voltage at bus 1. This fixes the voltage bases for the remaining buses in accordance to the transformer turns ratios. The base voltage V_{B1} on the LV side of T_1 is 22 kV. Hence the base on its HV side is

$$V_{B2} = 22(\frac{220}{22}) = 220 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B3} = 220$ kV, and on its LV side at

$$V_{B4} = 220(\frac{11}{220}) = 11 \quad \text{kV}$$

Similarly, the voltage base at buses 5 and 6 are

$$V_{B5} = V_{B6} = 22(\frac{110}{22}) = 110 \quad \text{kV}$$

Since generator and transformer voltage bases are the same as their rated values, their per-unit reactances on a 100 MVA base, from (3.92) are

$$G:\ X = 0.18\left(\frac{100}{90}\right) = 0.20 \quad \text{pu}$$

$$T_1:\ X = 0.10\left(\frac{100}{50}\right) = 0.20 \quad \text{pu}$$

$$T_2:\ X = 0.06\left(\frac{100}{40}\right) = 0.15 \quad \text{pu}$$

$$T_3:\ X = 0.064\left(\frac{100}{40}\right) = 0.16 \quad \text{pu}$$

$$T_4:\ X = 0.08\left(\frac{100}{40}\right) = 0.2 \quad \text{pu}$$

The motor reactance is expressed on its nameplate rating of 66.5 MVA and 10.45 kV. However, the base voltage at bus 4 for the motor is 11 kV. From (3.91) the motor reactance on a 100 MVA, 11-kV base is

$$M:\ X = 0.185\left(\frac{100}{66.5}\right)\left(\frac{10.45}{11}\right)^2 = 0.25 \quad \text{pu}$$

Impedance bases for lines 1 and 2, from (3.81) are

$$Z_{B2} = \frac{(220)^2}{100} = 484 \quad \Omega$$

$$Z_{B5} = \frac{(110)^2}{100} = 121 \quad \Omega$$

Line 1 and 2 per-unit reactances are

$$\text{Line 1:}\ X = \left(\frac{48.4}{484}\right) = 0.10 \quad \text{pu}$$

$$\text{Line 2:}\ X = \left(\frac{65.43}{121}\right) = 0.54 \quad \text{pu}$$

The load apparent power at 0.6 power factor lagging is given by

$$S_{L(3\phi)} = 57\angle 53.13^\circ \quad \text{MVA}$$

Hence, the load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(10.45)^2}{57\angle -53.13^\circ} = 1.1495 + j1.53267 \quad \Omega$$

The base impedance for the load is

$$Z_{B4} = \frac{(11)^2}{100} = 1.21 \quad \Omega$$

Therefore, the load impedance in per-unit is

$$Z_{L(pu)} = \frac{1.1495 + j1.53267}{1.21} = 0.95 + j1.2667 \quad \text{pu}$$

The per-unit equivalent circuit is shown in Figure 3.30.

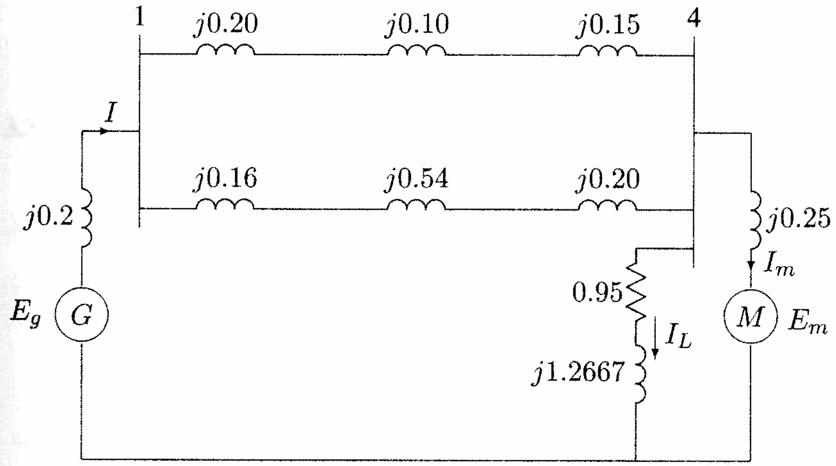


FIGURE 3.30
Per-unit impedance diagram for Example 3.7.

Example 3.8 (chp3ex8)

The motor of Example 3.7 operates at full-load 0.8 power factor leading at a terminal voltage of 10.45 kV.

(a) Determine the voltage at the generator bus bar (bus 1).
(b) Determine the generator and the motor internal emfs.

(a) The per-unit voltage at bus 4, taken as reference is

$$V_4 = \frac{10.45}{11} = 0.95\angle 0^\circ \quad \text{pu}$$

The motor apparent power at 0.8 power factor leading is given by

$$S_m = \frac{66.5}{100}\angle -36.87^\circ \quad \text{pu}$$

Therefore, current drawn by the motor is

$$I_m = \frac{S_m^*}{V_4^*} = \frac{0.665\angle 36.87}{0.95\angle 0^\circ} = 0.56 + j0.42 \quad \text{pu}$$

and current drawn by the load is

$$I_L = \frac{V_4}{Z_L} = \frac{0.95\angle 0^\circ}{0.95 + j1.2667} = 0.36 - j0.48 \quad \text{pu}$$

Total current drawn from bus 4 is

$$I = I_m + I_L = (0.56 + j0.42) + (0.36 - j0.48) = 0.92 - j0.06 \quad \text{pu}$$

The equivalent reactance of the parallel branches is

$$X_{\|} = \frac{0.45 \times 0.9}{0.45 + 0.9} = 0.3 \quad \text{pu}$$

The generator terminal voltage is

$$\begin{aligned} V_1 = V_4 + Z_{\|} I = 0.95\angle 0^\circ + j0.3(0.92 - j0.06) &= 0.968 + j0.276 \\ &= 1.0\angle 15.91^\circ \quad \text{pu} \\ &= 22\angle 15.91^\circ \quad \text{kV} \end{aligned}$$

(b) The generator internal emf is

$$\begin{aligned} E_g = V_1 + Z_g I = 0.968 + j0.276 + j0.20(0.92 - j0.06) &= 1.0826\angle 25.14^\circ \quad \text{pu} \\ &= 23.82\angle 25.14^\circ \quad \text{kV} \end{aligned}$$

and the motor internal emf is

$$\begin{aligned} E_m = V_4 - Z_m I_m = 0.95 + j0 - j0.25(0.56 + j0.42) &= 1.064\angle -7.56^\circ \quad \text{pu} \\ &= 11.71\angle -7.56^\circ \quad \text{kV} \end{aligned}$$