EEE319 Optimisation
Lecture 3 Graphical Optimisation

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Outline

- Constraints
- Steps of graphical optimisation
- Examples
- Limitations

Constraints

- An optimisation problem could be no constraints on the variables. This is called unconstrained optimisation. If there are constraints on the variables, this is called constrained optimisation.
- Types of constraints
  - Equality constraints
  - Inequality constraints

$$\min f(x)$$

Subject to $g_i(x) = c_i$ *i*=1, 2, …, *n* *Equality constraints*

$h_j(x) \geq d_j$ *j*=1, 2, 3, …, *n* Inequality constraints

Graphical Optimisation

- Used to solve 2 variable optimisation problems
- Objective function's complexity is not too high

Graphical Optimisation

- 6 steps
  I. Formulate the problem (objective function and constraints functions)
  II. Frame the graph (one variable on horizontal and the other on vertical axes)
  III. Plot the constraints (inequality to equality)
  IV. Outline the feasible regions (satisfy the constraints)
  V. Find the optimal solutions

Graphical Optimisation

- Example (Step 1)

Given an optimisation problem as follows

$$\max Z = 4x + 3y$$ Objective function

$$\text{Subject to } x + 2y \leq 30$$

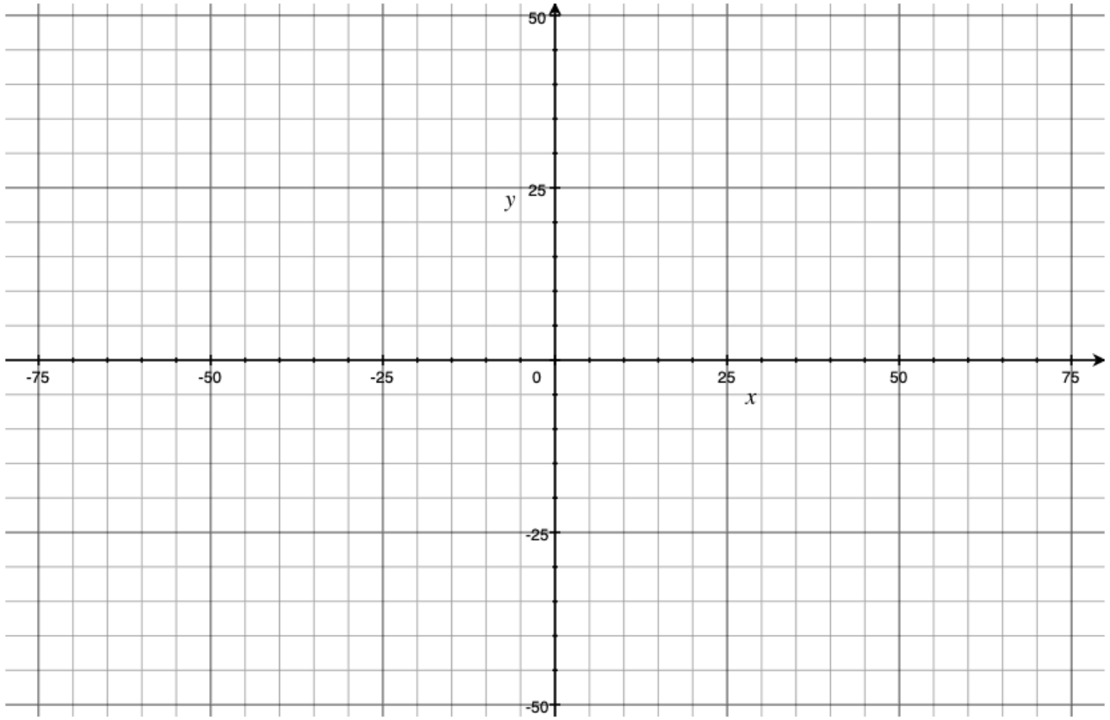
$$3x + y \leq 45 \text{ and}$$ Constraints function

$$x \geq 0, y \geq 0$$

From the sketch, find the optimal solution of the problem.

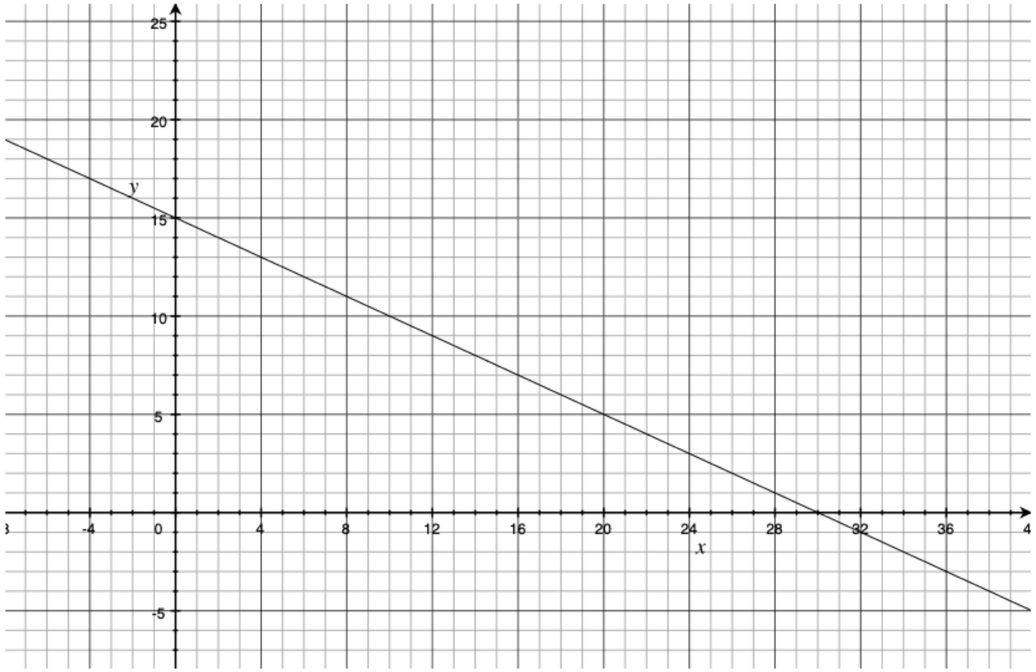
Graphical Optimisation

- Step 2 Frame the graph



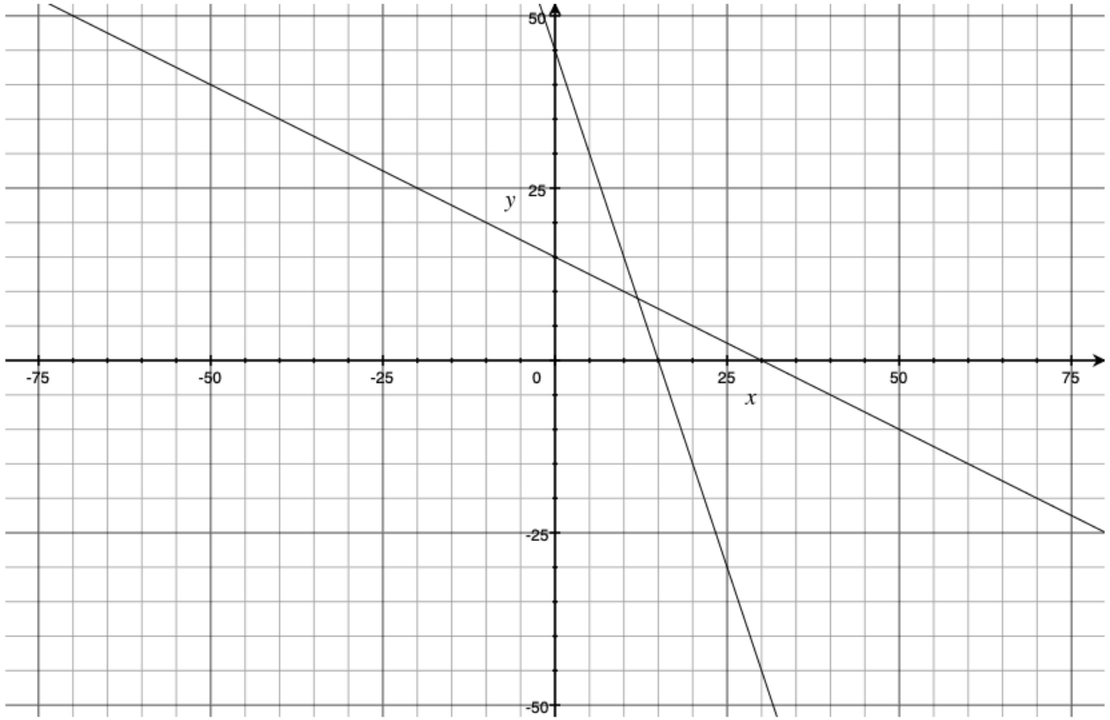
Graphical Optimisation

- Step 3 Plot the constraints
- First constraint $x + 2y \leq 30$
- (1) Inequality to equality
- $x + 2y = 30$
- (2) two points (0, 15), (30, 0)



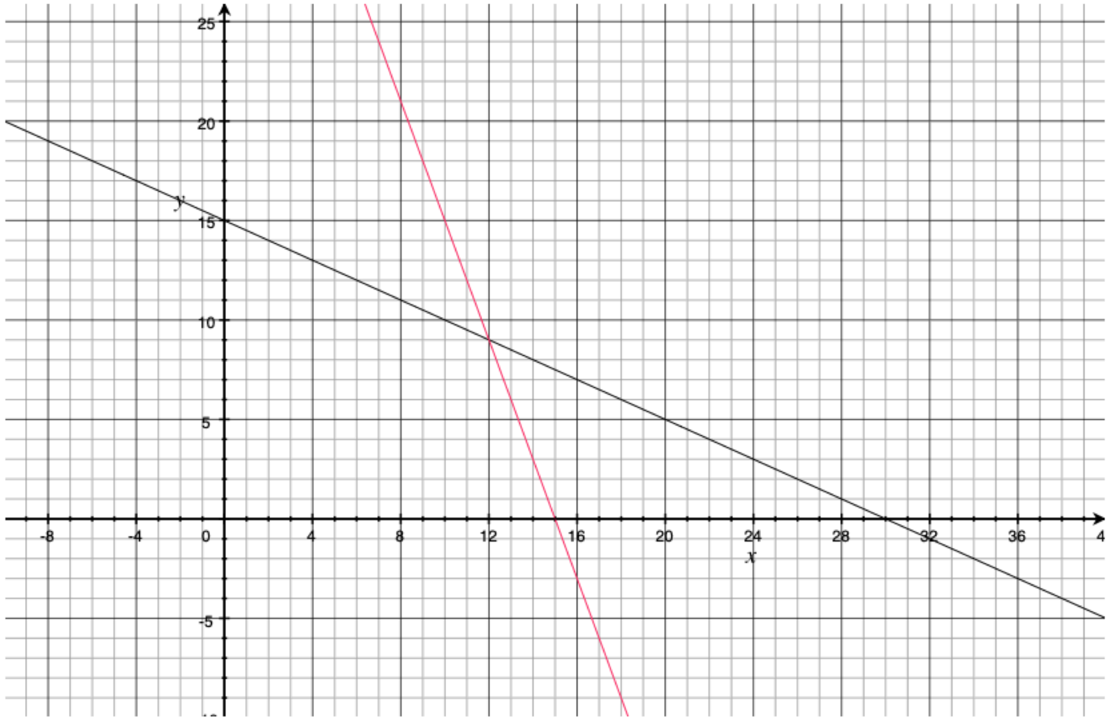
Graphical Optimisation

- Step 3 Plot the constraints
- 2nd constraint $3x + y \leq 45$
- (1) Inequality to equality
- $3x + y = 45$
- (2) two points (0, 45), (15, 0)



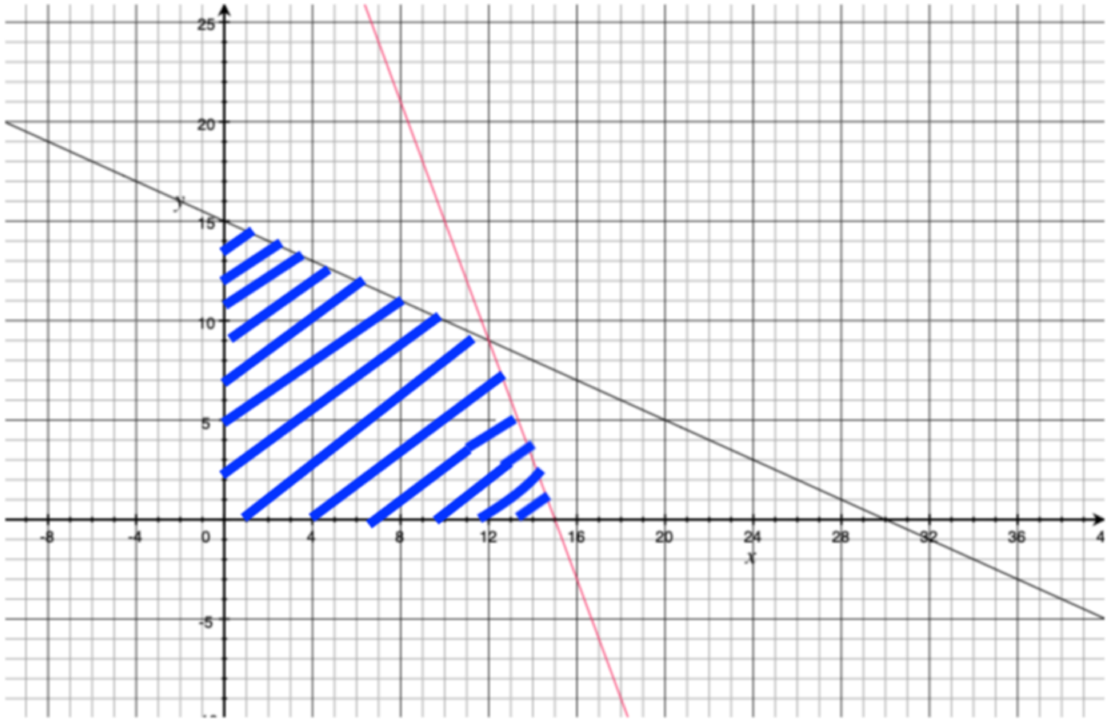
Graphical Optimisation

- Step 3 Plot the constraints
- Similar to first two constraints
- $x \geq 0;\ y \geq 0$



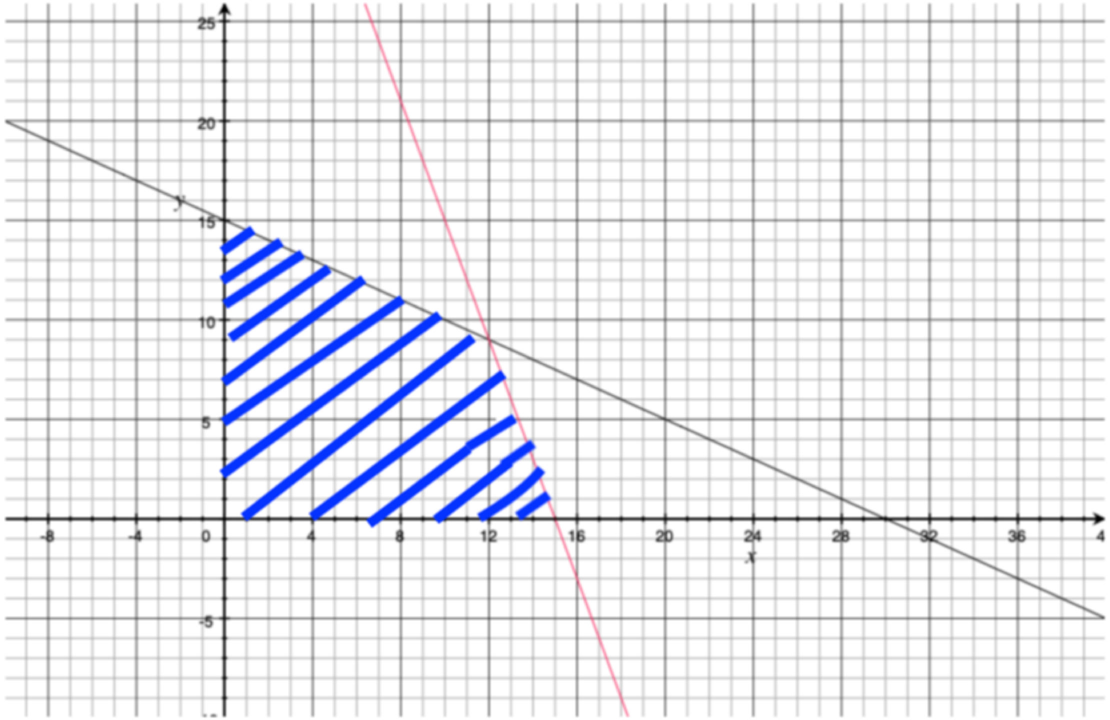
Graphical Optimisation

- Step 4 Feasible regions
  - The area with blue lines



Graphical Optimisation

- Step 5 Find optimal values
  - Two methods
    - Iso-profit line for maximisation/iso-cost line for minisation
    - Extreme point evaluation



Graphical Optimisation

- Step 5 Find optimal values
  - Iso-profit line for maximization

    (1) arbitrary line of objective function by finding any two points (or more points for a nonlinear objective function), starting from zero profit

    $$4x + 3y = 0$$

    Two points (0, 0) and (-3, 4), blue line

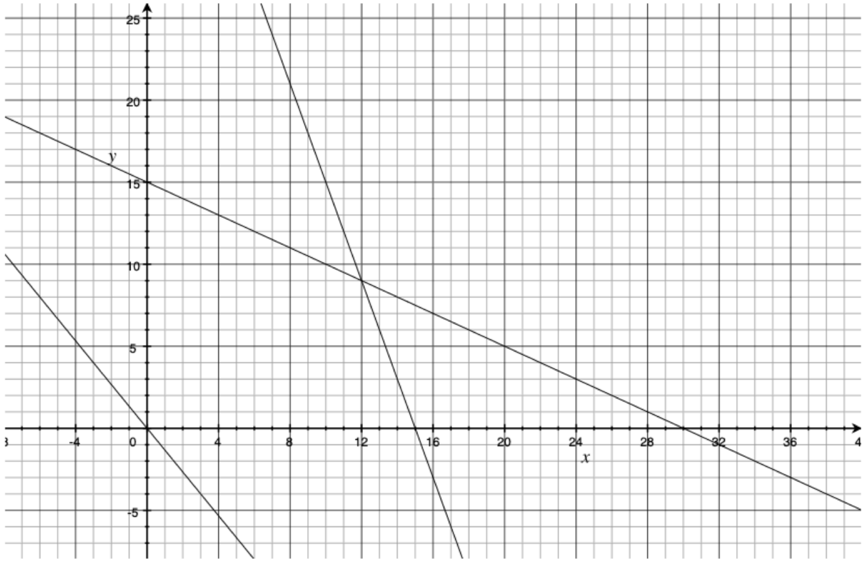
    (2) Change the profit value, i.e.,

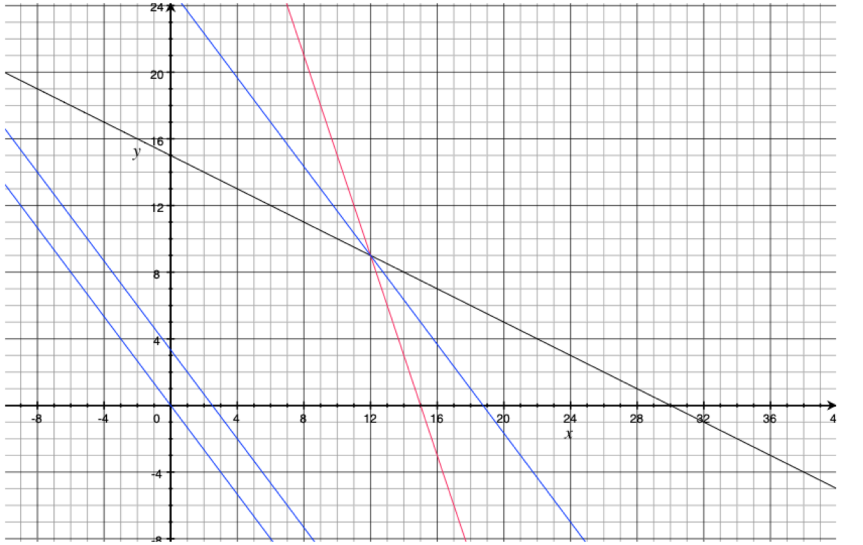
    $$4x + 3y = 10$$

    (3) Keep increasing the objective function value until the iso-profit line cannot increase further in the feasible region (12,9)

    (4) Optimal solution x=12, y=9

    $$Z = 4x + 3y = 4 * 12 + 3 * 9 = 75$$





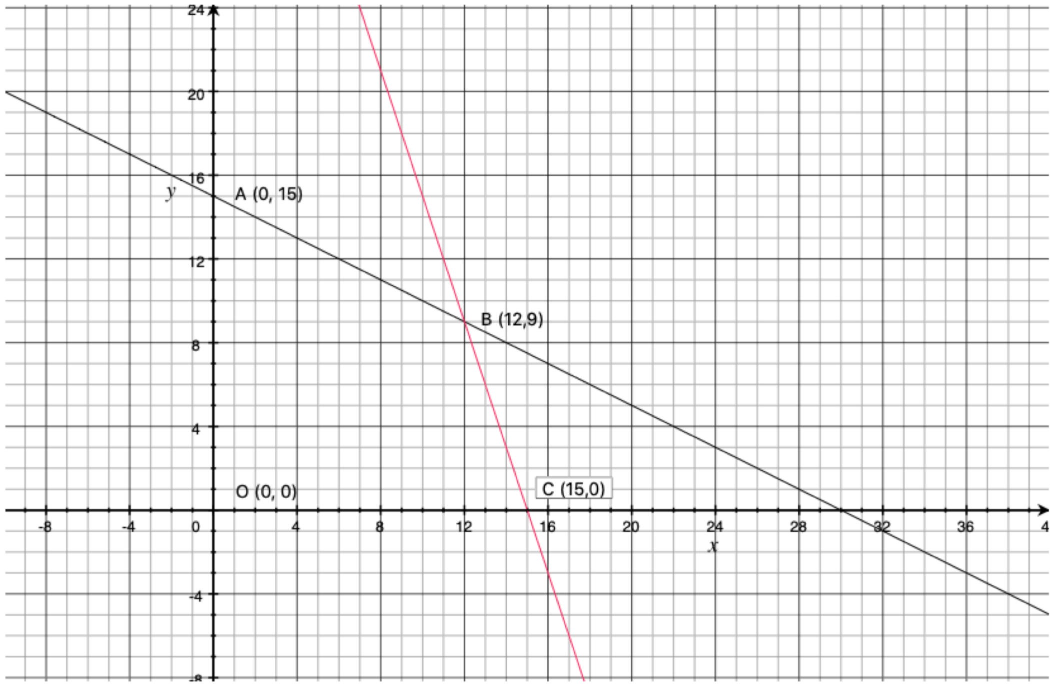
Graphical Optimisation

- Step 5 Find optimal values
    - Extreme points – corner points

      O(0,0) A(0,15), B(12,9), and C(15,0)

      Z=0, 45, 75, 60

      Optimal solution: x=12, y=9, Z=75



Graphical Optimisation

- Another example

$$\text{Min}\, z = 5x_1 + 2x_2$$

Subject to

$$2x_1 + 5x_2 \geq 10$$

$$4x_1 - x_2 \geq 12$$

$$x_1 + x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

Solve this example by yourself

Summary

- Easy to understand
- Limited to two variable
- It is difficult if the objective function is nonlinear, logarithmic, or exponential.

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