VSRank: A Novel Framework for Ranking-Based Collaborative Filtering

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Collaborative filtering (CF) is an effective technique addressing the information overload problem. CF approaches generally fall into two categories: rating based and ranking based. The former makes recommendations based on historical rating scores of items and the latter based on their rankings. Ranking-based CF has demonstrated advantages in recommendation accuracy, being able to capture the preference similarity between users even if their rating scores differ significantly. In this study, we propose VSRank, a novel framework that seeks accuracy improvement of ranking-based CF through adaptation of the vector space model. In VSRank, we consider each user as a document and his or her pairwise relative preferences as terms. We then use a novel degree-specialty weighting scheme resembling TF-IDF to weight the terms. Extensive experiments on benchmarks in comparison with the state-of-the-art approaches demonstrate the promise of our approach.

Categories and Subject Descriptors: H.3.3 [**Information Storage and Retrieval**]: Information Search and Retrieval—*Information filtering*

General Terms: Algorithms, Performance, Experimentation

Additional Key Words and Phrases: Recommender systems, collaborative filtering, ranking-based collaborative filtering, vector space model, term weighting

ACM Reference Format:
Shuaiqiang Wang, Jiankai Sun, Byron J. Gao, and Jun Ma. 2014. VSRank: A novel framework for ranking-based collaborative filtering. ACM Trans. Intell. Syst. Technol. 5, 3, Article 51 (June 2014), 24 pages.
DOI: http://dx.doi.org./10.1145/2542048

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A preliminary version of this article was published in the *Proceedings of the 21st ACM International Conference on Information and Knowledge Management (CIKM)* [Wang et al. 2012].
This work was supported in part by the Natural Science Foundation of China (U1201258, 61272240, and 71171122), the National Science Foundation (OCI-1062439 and CNS-1058724), the Humanity and Social Science Foundation of Ministry of Education of China (12YJC630211), the Specialized Research Foundation of Ministry of Education of China for Returned Overseas Scholars, the Shandong Natural Science Funds for Distinguished Young Scholars (JQ201316), the Natural Science Foundation of Shandong Province of China (2012BSB01550), and the Specialized Research Foundation of Jinan for Returned Overseas Scholars (20120201).
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DOI: http://dx.doi.org./10.1145/2542048

1. INTRODUCTION

Ever since the web began to thrive, the world has been flooded with an overwhelming amount of information. Such a wealth of information has become increasingly unmanageable and created "a poverty of attention and a need to allocate that attention efficiently" [Simon 1971]. This so-called *information overload* problem represents one of today's major challenges on the web. As an effective technique addressing the problem, recommender systems generate item recommendations from a large collection in favor of user preferences. In recent years, recommender systems have become a de facto standard and must-own tool for e-commerce to promote business and help customers find products [Sarwar et al. 2000]. Prominent examples include eBay,[1] Amazon,[2] Last.fm,[3] Netflix,[4] Facebook,[5] and LinkedIn.[6]

1.1. Collaborative Filtering

The two main paradigms for recommender systems are content-based filtering and collaborative filtering (CF). Content-based filtering makes recommendations by finding regularities in the textual content information of users and items, such as user profiles and product descriptions [Belkin and Croft 1992]. CF is based on the assumption that if users X and Y rate n items similarly or have similar behaviors, they will rate or act on other items similarly [Goldberg et al. 1992; Resnick et al. 1994].

CF only utilizes the user–item rating matrix to make predictions and recommendations, avoiding the need for collecting extensive information about items and users. In addition, CF can be easily adopted in different recommender systems without requiring any domain knowledge [Liu and Yang 2008]. Given the effectiveness and convenience, many CF methods have been proposed, which fall into two categories: rating based and ranking based.

Rating-based CF. Rating-based CF methods recommend items for users based on their historical rating scores on items. As a classical CF paradigm, they have been extensively investigated, where most methods are either memory based [Resnick et al. 1994; Herlocker et al. 1999; Sarwar et al. 2001; Deshpande and Karypis 2004] or model based [Vucetic and Obradovic 2005; Shani et al. 2005; Si and Jin 2003; Tang et al. 2013; Jiang et al. 2012].

Rating-based CF utilizes similarity measures between two users based on their rating scores on the set of common items. A popular similarity measure is the Pearson correlation coefficient [Resnick et al. 1994; Herlocker et al. 2002]. However, the ultimate goal of a recommender system is to present a ranking or recommendation list to users rather than rating prediction [Shi et al. 2010; Herlocker et al. 2004]. In addition, as a common observation, such rating-based similarity measures would fail to capture preference similarity between users when their rating scores on items differ significantly [Adomavicius and Tuzhilin 2005; Gunawardana and Shani 2009], as illustrated in the following.

Example 1.1. Let $\{i_1, i_2, i_3\}$ be three items. Let $\{u, v\}$ be two users who have assigned ratings of $\{1, 2, 3\}$ and $\{3, 4, 5\}$ to the items, respectively. While u and v exhibit clear common relative preferences over the three items, their rating scores differ significantly, leading to small rating-based similarity between u and v.

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[1] http://www.ebay.com/.
[2] http://www.amazon.com/.
[3] http://www.last.fm/.
[4] http://www.netflix.com/.
[5] http://www.facebook.fm/.
[6] http://www.linkedin.com/.

A straightforward normalization based on average rating scores of users does not address this problem effectively. This is because when a given user only rates a very small set of items, his or her average rating may differ significantly from her "true" (but unknown) average rating, which can be defined as the average value of his or her ratings on all potential items in the item space. For example, suppose u is a very generous reviewer, and her "true" average rating is 4. Suppose u has reviewed five low-quality items, and her average rating is 2. Thus, an item with a rating of 3 will be interpreted as favorable by u after normalization based on the average rating of 2. However, the truth is that u dislikes the item because 3 is smaller than her "true" average rating of 4.

Ranking-based CF. Ranking-based CF methods recommend items for users based on their rankings of items. In particular, such methods utilize similarity measures between two users based on their rankings on the same set of items. A common similarity measure is the Kendall tau rank correlation coefficient [Kendall 1938; Marden 1995]. Recent efforts on ranking-based CF [Yang et al. 2011; Weimer et al. 2007; Liu and Yang 2008; Liu et al. 2009; Rendle et al. 2009; Cui et al. 2011; Shi et al. 2010; Kahng et al. 2011] have clearly demonstrated their advantages in recommendation accuracy.

However, conventional ranking-based CF algorithms treat pairwise relative preferences equally, without considering any weighting scheme for preferences in similarity measures. For example, relative preferences may have different *degrees* depending on how strong the preferences are. In addition, two users are considered similar if they share some *special* traits instead of common ones.

1.2. VSRank for Ranking-Based Collaborative Filtering

The vector space model [Baeza-Yates and Ribeiro-Neto 1999] is a standard and effective algebraic model widely used in information retrieval (IR). It treats a document or a query as a bag of terms and uses term weighting schemes such as TF-IDF to weight the terms. Then each document/query is represented as a vector of TF-IDF weights. In particular, term frequency (TF) measures the *degree* of the relevance between a given document d and a query term t, which is defined as the number of occurrences of t in d. Inverse document frequency (IDF) measures the *rarity* of a term t in the corpus. In information retrieval, document frequency (DF) for a term t is the number of documents in the corpus containing t. IDF, the inverse of DF, is the dampened (taking log value) ratio of $|D|$ (the total number of documents) over DF. Obviously, the weighting issues in ranking-based CF are very similar to those in the vector space model.

In this study, we propose VSRank, seeking recommendation accuracy improvement for ranking-based CF through adaptation of the vector space model. Similar (more straightforward) adaptation has been introduced for content-based filtering, demonstrating improvement in recommendation accuracy [Pazzani and Billsus 1997; Zhu et al. 2003; Debnath et al. 2008; Belkin and Croft 1992]. However, this technique has not been investigated in the context of CF.

To adapt the vector space model to ranking-based CF, we consider each user as a document and his or her pairwise relative preferences as terms. We then use a degree-specialty term weighting scheme resembling TF-IDF to weight the terms. After representing users as vectors of degree-specialty weights, we adopt ranking-based CF techniques to make recommendations for a given user.

Degree-specialty weighting. The key component in the adaptation is the degree-specialty weighting scheme. It is straightforward that relative preferences have different *degrees*. For example, while both users u and v rank item i_1 higher than item i_2, their actual rating scores for $\{i_1, i_2\}$ may be $\{5, 1\}$ and $\{2, 1\}$, respectively, reflecting the fact that user u prefers i_1 over i_2 much more strongly than user v. Obviously, stronger preferences with larger score differences are more important and should be given a larger degree. Degree resembles *term frequency* (TF), and a high degree for a preference term from a user can be interpreted in a way that the user repeatedly (frequently) confirms his or her preference.

Now we explain *specialty*. A straightforward way of adapting IDF to ranking-based CF is to use the original definition of IDF, which is the dampened ratio of $|U|$ (the total number of users) over the number of users holding the preference (DF). However, our experiments have returned unsatisfactory results for this method.

A deeper analysis shows that although we conceptually treat preferences as textual terms, they have fundamental differences. While a textual term is undirectional involving only one entity, a preference is directional involving two entities. A preference always has an "enemy," which is its opposite preference. In light of this, instead of a literal word for word translation, our specialty is essentially a phrasal translation (conveying the sense of the original) of IDF, which measures the rarity of the preference in the users who hold the same or the opposite preferences on the same items.

However, this ratio statistic would suffer from a subtle "small sample" problem because the confidence information (indicated by the number of users) would be cancelled out. We solve this problem by introducing a novel *confidence calibration* technique that adjusts specialty toward its true value. While the formal definition will be introduced later, the following example illustrates how specialty truly captures the rarity of preferences.

The effectiveness of VSRankcan be understood from another perspective. Terms are features. Feature selection and weighting has been one of the most frequently used techniques in pattern recognition, machine learning, and data mining for data analysis, in particular, classification tasks [Han et al. 2011; Bishop 2006]. It eliminates irrelevant, redundant, and noisy data. Although numerous classification frameworks and algorithms have been proposed, predicting accuracy is upper bounded by the amount of noise in historical data. Reducing noise has the most direct and immediate effects on predicting accuracy, as it would for recommendation accuracy.

In implementing VSRank, we use two similarity measures, cosine similarity and weighted Kendall tau correlation coefficient, for the discovery of neighborhood users. We also use two preference aggregation methods, an order-based and a score-based method, for the prediction of item ranking, resulting in $2 \times 2 = 4$ ranking-based CF algorithms. We conduct extensive experiments on benchmarks in comparison with the state-of-the-art approaches to validate the effectiveness of these algorithms.

Contribution. We make the following contributions.

(1) We propose VSRank, a framework for adapting the vector space model to ranking-based collaborative filtering for improved recommendation accuracy.
(2) We present the novel degree-specialty weighting scheme resembling TF-IDF. We also reveal insightful connections between cosine similarity and the Kendall tau rank correlation coefficient.
(3) We implement four recommender systems based on VSRank. Extensive experiments on benchmarks demonstrate the promise of our framework.

Organization. The rest of the article is organized as follows. Section 2 reviews the related work. Section 3 presents the preliminaries. Section 4 proposes VSRank, the framework for adapting the vector space model to ranking-based CF. Section 5 implements four recommender systems based on VSRank. Section 6 reports the experimental results. Section 7 concludes the article.

2. RELATED WORK

2.1. Recommender Systems

Most existing techniques for recommender systems fall into two categories: *collaborative filtering* and *content-based filtering*. Content-based techniques make recommendations based on regularities in the content information of users and items, where users and items are represented by explicit features [Belkin and Croft 1992; Basu et al. 1998]. CF only utilizes the user–item rating matrix to make predictions and recommendations, avoiding the need for collecting extensive information about items and users. In addition, CF can be easily adopted in different recommender systems without requiring any domain knowledge [Liu and Yang 2008]. Given the effectiveness and convenience, many CF approaches have been proposed, which are either rating based or ranking based. Hybrid recommender systems have also been proposed [Burke 2002].

Rating-based CF. Rating-based CF techniques can be memory based or model based. Memory-based methods make predictions based on similarities between users or items. The user-based paradigm [Resnick et al. 1994; Herlocker et al. 1999] is more common, which estimates the unknown ratings of a target user based on the ratings by a set of neighboring users that tend to rate similarly to the target user. In the item-based paradigm [Deshpande and Karypis 2004; Sarwar et al. 2001], item–item similarity is used to select a set of neighboring items that have been rated by the target user and the ratings on the unrated items are predicted based on his or her ratings on the neighboring items. Since the number of items is usually much less than the number of users in most applications, item–item similarities are less sensitive to the data sparsity problem. Many commercial systems such as Amazon.com are memory based since they are relatively easy to implement [Hofmann 2004].

Model-based methods estimate or learn a model to make predictions. For example, Vucetic and Obradovic [2005] proposed a regression-based approach to collaborative filtering tasks, which built a collection of simple linear models and then combined them efficiently to provide rating predictions for an active user. Shani et al. [2005] used a Markov decision process (MDP) model for recommender systems, which viewed the recommendation process as a sequential optimization problem. Si and Jin [2003] presented a flexible mixture model (FMM) for collaborative filtering. FMM is an extension of partitioning/clustering algorithms, which cluster both users and items together simultaneously without assuming that each user and item should only belong to a single cluster. Tang et al. [2013] proposed a matrix factorization-based framework LOCABAL, taking advantage of both local and global social context for recommendation. Jiang et al. [2012] incorporated social recommendation on the basis of psychology and sociology studies into a probabilistic matrix factorization-based CF algorithm. Comprehensive surveys of rating-based CF can be found in Adomavicius and Tuzhilin [2005], Gunawardana and Shani [2009], Herlocker et al. [2004], and Su and Khoshgoftaar [2009].

Ranking-based CF. Ranking-based CF is able to capture the preference similarity between users even if their rating scores differ significantly. Recently, the formulation of the recommendation problem is shifting away from rating based to ranking based [McNee et al. 2006]. CCF [Yang et al. 2011] learned user preferences using the context of the user behavior of choices in recommender systems and employed a ranking-oriented latent factor model to characterize the dyadic utility function. CoFiRank [Weimer et al. 2007] used maximum margin matrix factorization to optimize ranking of items for collaborative filtering. EigenRank [Liu and Yang 2008] measured the similarity between users with the Kendall tau rank correlation coefficient for neighborhood selection, predicted the relative preferences of items with the preference function, and aggregated these preferences into a total ranking. Liu et al. [2009] adopted a probabilistic latent preference analysis model (pLPA) that made ranking predictions by directly modeling user preferences with respect to a set of items rather than the rating scores on individual items. Rendle et al. [2009] proposed a Bayesian probabilistic model for personalized ranking from implicit feedback. Cui et al. [2011] proposed HF-NMF, a hybrid factor nonnegative matrix factorization approach for item-level social influence modeling.

It is natural to apply learning to rank to ranking-based recommender systems. In Shi et al. [2010], ListRank-MF was proposed, which combined a list-wise learning-to-rank algorithm with probabilistic matrix factorization. In Kahng et al. [2011], a context-aware learning-to-rank method was proposed that incorporated several context features, such as time and location of users, into the ranking model.

Existing ranking-based CF methods treat relative preferences equally. We adapt the vector space model and weight preferences according to their importance for improved recommendation accuracy. Similar adaptation has been introduced in content-based filtering approaches, demonstrating improvement in recommendation performance [Pazzani and Billsus 1997; Zhu et al. 2003; Debnath et al. 2008; Belkin and Croft 1992]. However, this technique has not been investigated in the context of CF.

2.2. Vector Space Model

The vector space model [Baeza-Yates and Ribeiro-Neto 1999] is a standard algebraic model commonly used in information retrieval (IR). It also has other interesting applications. For example, in content-based filtering, the descriptive user profiles can be considered as documents [Pazzani and Billsus 2007] and the vector space model can be applied to make recommendations based on user similarity. In rating-based collaborative filtering, the generalized vector space model can be used to transform vectors of users from the user space into the item space, and then similarity between users and items can be easily measured with cosine similarity [Soboroff and Nicholas 2000]. In image processing, local interest points of images can be clustered, and each cluster can be considered as a visual word [Yang et al. 2007; Kesorn and Poslad 2012], based on which the vector space model can be applied for classification and recognition. In spam detection, features representing web documents can be partially generated from the vector space model [Niu et al. 2010]. In song sentiment classification, the sentiment vector space model has been proposed to categorize songs into light-hearted and heavy-hearted [Xia et al. 2008], where the song lyrics are regarded as documents and the sentiment words are used to construct the model. In spoken language identification, spoken utterances can be used as term features to build a vector space classifier [Li et al. 2007].

3. PRELIMINARIES

3.1. Collaborative Filtering

The following notations will be used throughout the article. Let U be a set of users and I be a set of items. In a recommender system, for each user $u \in U$, a set of items $I_u \subseteq I$ is rated by u. Let R be a rating matrix, where each element $r_{u,m} \in \mathbb{N}$ is the rating score of the m^{th} item i_m with respect to u, and $\mathbb{N}$ is the natural number set indicating different relevance scores.

CF recommends items to users based on the rating scores predicted by neighborhood users (similar users). In particular, for user u, the similarity between u and each user in U is computed from the rating matrix R. Then a set of neighborhood users $U_u \subset U$ is selected, based on which recommendations are made.

Ranking-based CF. Ranking-based CF recommends items based on their rankings derived from the rating matrix R. The similarity between two users u and v, $\tau_{u,v}$, can be computed by the standard Kendall tau rank correlation coefficient [Marden 1995] based on the two rankings from u and v on their common item set:

$$\tau_{u,v} = \frac{N_c - N_d}{\frac{1}{2}N(N-1)}, \tag{1}$$

where N_c and N_d are the numbers of the concordant pairs and discordant pairs, respectively.

Let $sgn_{u,v}(m,n)$ be an indicator function such that $sgn_{u,v}(m,n) = 1$ if items i_m and i_n are concordant in u and v, and $sgn_{u,v}(m,n) = -1$ if items i_m and i_n are discordant, formally:

$$sgn_{u,v}(m,n) = \begin{cases} 1, & \text{if } (r_{u,m} - r_{u,n})(r_{v,m} - r_{v,n}) > 0 \\ -1, & \text{if } (r_{u,m} - r_{u,n})(r_{v,m} - r_{v,n}) < 0. \end{cases} \tag{2}$$

The sum of $sgn_{u,v}(m,n)$ for all item pairs is $N_c - N_d$, that is, the number of concordant pairs minus the number of discordant pairs. Thus, $\tau_{u,v}$ can be represented as follows:

$$\tau_{u,v} = \frac{\sum_{m=1}^{N} \sum_{n=m+1}^{N} sgn_{u,v}(m,n)}{\frac{1}{2}N(N-1)}. \tag{3}$$

For user u, the preference on a pair of items (i_m, i_n) can be predicted with a preference function $\Psi_u(m,n)$ as follows:

$$\Psi_u(m,n) = \frac{\sum_{v \in U_u^{m,n}} \tau_{u,v}(r_{v,m} - r_{v,n})}{\sum_{v \in U_u^{m,n}} \tau_{u,v}}, \tag{4}$$

where $U_u^{m,n}$ is the set of similar users of u who have rated both items i_m and i_n.

Based on the predicted pairwise preferences, a total ranking of items for user u can be obtained by applying a preference aggregation algorithm.

3.2. Vector Space Model

The vector space model [Baeza-Yates and Ribeiro-Neto 1999] is a standard algebraic model commonly used in IR. It treats a textual document as a bag of words, disregarding grammar and even word order. It typically uses TF-IDF (or a variant weighting scheme) to weight the terms. Then each document is represented as a vector of TF-IDF weights. Queries are also considered as documents. Cosine similarity is used to compute similarity between document vectors and the query vector. Large similarity indicates high relevancy of documents with respect to the query.

TF-IDF. The term frequency $TF_{t,d}$ of term t in document d is defined as the number of times that t occurs in d. It positively contributes to the relevance of d to t.

The inverse document frequency IDF_t of term t measures the rarity of t in a given corpus. If t is rare, then the documents containing t are more relevant to t. IDF_t is obtained by dividing N by DF_t and then taking the logarithm of that quotient, where N is the total number of documents and DF_t is the document frequency of t, that is, the number of documents containing t. Formally, $IDF_t = \log_{10}(\frac{N}{DF_t})$. The TF-IDF value of a term is commonly defined as the product of its TF and IDF values. $TF\text{-}IDF_{t,\boldsymbol{d}} = TF_{t,\boldsymbol{d}} \times IDF_t$.

Example 3.1. Let d be a document containing the term "recommendation" three times; then the TF value of "recommendation" for d is 3. Suppose there are 10 out of

$N = 100$ documents containing "recommendation"; then the IDF value of "recommendation" is $\log_{10}(\frac{100}{10}) = 1$. Then, the TF-IDF value of "recommendation" in d is $3 \times 1 = 3$.

Cosine similarity. Cosine similarity is a standard measure estimating pairwise document similarity in the vector space model. It corresponds to the cosine of the angle between two vectors, and it has the effect of normalizing the length of documents. Let $q = \langle w_{q,1}, w_{q,2}, \ldots, w_{q,N} \rangle$ and $d = \langle w_{d,1}, w_{d,2}, \ldots, w_{d,N} \rangle$ be two N-dimensional vectors corresponding to a query and a document, and their cosine similarity $s_{q,d} = \frac{q \cdot d}{||q|| \times ||d||}$.

4. THE VSRANK FRAMEWORK

In this section, we present VSRank, a novel framework for adapting the vector space model to ranking-based CF. In VSRank, users are considered as documents and relative preferences are considered as terms. The terms are weighted by a degree-specialty weighting scheme resembling TF-IDF. The target user u is considered as a query, which is also a document. Then recommendations can be made according to principles of ranking-based CF.

Generally, ranking-based CF works in the following two phases:

—**Phase I: Discovery of neighborhood users**. For each user, Phase I discovers a set of the most similar users as the neighborhood users.
—**Phase II: Prediction of item ranking**. Based on the neighborhood users, phase II predicts a ranking list of items by aggregating preferences of neighborhood users for recommendation purposes.

4.1. Representation of Users

We consider users as documents and pairwise relative preferences of items as terms. We adopt a bag-of-words model, where each user is represented as a *bag* of relative preferences, instead of a *set* as in other ranking-based CF methods.

In particular, for a user u, from the set I of items rated by u, we can derive a set of relative preferences $\{i_m \succ i_n | i_m, i_n \in I \wedge r_{u,m} > r_{u,n}\}$. Each preference $i_m \succ i_n$ is considered as a term, and the score difference $|r_{u,m} - r_{u,n}|$ indicates the number of "occurrences" of the preference in u.

Example 4.1. Suppose user u has assigned 4, 3, and 2 to items i_1, i_2, and i_3, respectively. The user u contains three preference terms and can be represented as "$i_1 \succ i_2, i_1 \succ i_3, i_1 \succ i_3, i_2 \succ i_3$."

4.2. Term Weighting

Degree. Similar to TF, the degree of preference $i_m \succ i_n$ in user u can be defined as the number of occurrences of $i_m \succ i_n$ in u. In this article, we use a logarithm variant of TF. Formally, let $r_{u,m}$ be the rating score of item i_m by user u; then the degree of term $i_m \succ i_n$ is defined as

$$w^{(D)}_{u,i_m \succ i_n} = \log_2 \left(1 + |r_{u,m} - r_{u,n}|\right). \tag{5}$$

Specialty. Similar to IDF, we want to use specialty to measure the rarity of preference terms in the set of users. Let us consider preference $i_m \succ i_n$. A straightforward method would be using IDF literally, which is the log value of $\frac{|U|}{N_{i_m \succ i_n}}$, where $|U|$ is the total number of users and $N_{i_m \succ i_n}$ is the DF, that is, the number of users holding the preference $i_m \succ i_n$.

However, we observe that textual terms and preference terms are fundamentally different. While a textual term is undirectional involving only one entity, a preference term is directional involving two entities. A preference term always has an "enemy," which is its opposite preference term. Also, a textual term t is either "contained" or "not contained" in a document d. However, a preference term $i_m \succ i_n$ can be "contained," "not contained," or "unknown" with respect to a user document u.

What exactly is *rarity* for preference terms? We say that *a preference term is rare if there are more opposite preference terms*. With the same interpretation, *a textual term is rare if there are more documents not containing the term*.

The original IDF captures this interpretation of rarity for textual terms, but not for preferences. The nominator of IDF is the total number of documents, which is the number of documents containing the term + the number of documents not containing the term. However, the total number of users is the number of users holding the preference + the number of users holding the opposite preference + the number of users who have not rated both items. Due to the typical sparsity of the rating matrix, most users have not rated both items.

In light of this, instead of using $|U|$ as the nominator, we use "$N_{i_m \succ i_n} + N_{i_m \prec i_n}$" as the nominator. This can be considered as a phrasal translation (conveying the sense of the original) of IDF, instead of a literal word-for-word one. Example 4.2 provides a clear illustration of this idea.

For each pair of items (i_m, i_n), the relative preferences can be either $i_m \succ i_n$ or $i_m \prec i_n$. For simplicity, we combine the two opposite preference terms into one notation of $i_m \Theta i_n$, where $\Theta \in \{\succ, \prec\}$. Based on the previous analysis, a possible specialty would be defined as follows:

$$\lambda_{i_m \Theta i_n} = \log_2 \left(\frac{N_{i_m \succ i_n} + N_{i_m \prec i_n}}{N_{i_m \Theta i_n}} \right). \tag{6}$$

However, this definition would suffer from a subtle "small sample" problem because the ratio in the formula cancels out the confidence information indicated by the number of users. To illustrate the small sample problem, suppose we want to estimate the ratio of the number of males over the number of females in a population. If the sample is too small, the ratio estimate would not be reliable and has a large degree of uncertainly. Note that IDF does not suffer much from this problem because it uses a fixed nominator (total number of documents).

Example 4.2. Let $\{i_1, i_2\}$ be two items. Suppose among the total number of 10,000 users, 1,000 have rated both i_1 and i_2, where 800 prefer i_1 to i_2 ($i_1 \succ i_2$) and 200 prefer i_2 to i_1 ($i_2 \succ i_1$). In this case, $i_2 \succ i_1$ is a rare preference because there are four times more users holding the opposite preference. The specialty for preference $i_1 \succ i_2$ is based on $\frac{1000}{800}$ instead of $\frac{10000}{800}$, and the specialty for preference $i_2 \succ i_1$ is based on $\frac{1000}{200}$ instead of $\frac{10000}{200}$.

Why can't we use the original IDF? Let $\{i_3, i_4\}$ be two items. Suppose among the total number of 10,000 users, 100 have rated both i_3 and i_4 with 80 holding preference $i_3 \succ i_4$ and 20 holding preference $i_4 \succ i_3$. In this case, $i_4 \succ i_3$ is a rare preference and $i_3 \succ i_4$ is a popular one. If IDF is used, then a popular preference $i_3 \succ i_4$ would have a much bigger IDF than a rare preference $i_2 \succ i_1$ because $\frac{10000}{80} > \frac{10000}{200}$.

Specialty solves the problem nicely. Without confidence calibration, the two rare preferences would have the same bigger specialty since $\frac{1000}{200} = \frac{100}{20}$, and the two popular preferences would have the same smaller specialty since $\frac{1000}{800} = \frac{100}{80}$. With confidence calibration, $i_2 \succ i_1$ would have a slightly bigger specialty than $i_4 \succ i_3$ and $i_1 \succ i_2$ would have a slightly smaller specialty than $i_3 \succ i_4$, both due to higher confidence.

To solve this problem, we propose a novel *confidence calibration* technique. After calibration, the statistic should be brought closer to its true value in the population. The idea is that we define a "prior ratio," which is the prior knowledge for the ratio. We make an adjustment of the computed ratio toward the prior ratio. When the sample is small, that is, $N_{i_m \succ i_n}$ and $N_{i_m \prec i_n}$ are small, there is more uncertainty and we make a bigger adjustment. Otherwise, we make a small adjustment because we have high confidence in the computed ratio.

Let $|U|$ be the total number of users. We use the following formula to estimate the "sample size" index for $i_m \Theta i_n$, which can be used to indicate the confidence level of $\lambda_{i_m \Theta i_n}$:

$$\alpha_{i_m,i_n} = \log_a \left(1 + (a-1)\frac{N_{i_m \succ i_n} + N_{i_m \prec i_n}}{|U|}\right).$$

Finally, the specialty for preference term $i_m \Theta i_n$ can be defined based on $\lambda_{i_m \Theta i_n}$ and α_{i_m,i_n} as follows:

$$w^{(S)}_{i_m \Theta i_n} = \alpha_{i_m,i_n} \times \lambda_{i_m \Theta i_n} + \left(1 - \alpha_{i_m,i_n}\right) \times 1. \tag{7}$$

In the formula, the default prior specialty is 1, which is the case when $N_{i_m \succ i_n} = N_{i_m \prec i_n}$ and $\log_2(\frac{2}{1}) = 1$. $w^{(S)}_{i_m \Theta i_n}$ drives $\lambda_{i_m \Theta i_n}$ toward the prior specialty of 1. It makes a bigger adjustment when α_{i_m,i_n} is small (small confidence) and a smaller adjustment otherwise (large confidence).

Degree-specialty. Resembling TF-IDF, degree-specialty is the product of degree and specialty. Specifically, for a user u, the degree-specialty weight of preference term $i_m \Theta i_n$ is defined using Equations (5) and (7) as follows:

$$w_{u,i_m \Theta i_n} = w^{(D)}_{u,i_m \Theta i_n} \times w^{(S)}_{i_m \Theta i_n}. \tag{8}$$

Example 4.3. Let $\{i_1, i_2\}$ be two items. Suppose among the total number of 10,000 users, 1,000 have rated both i_1 and i_2, where 800 prefer i_1 to i_2 ($i_1 \succ i_2$) and 200 prefer i_2 to i_1 ($i_2 \succ i_1$). Suppose user u has assigned scores 2 and 5 and user v has assigned scores 4 and 3 to items i_1 and i_2, respectively.

Then for user u, the degree-specialty for preference term $i_1 \prec i_2$ can be computed as follows:

$$\begin{aligned}
w^{(D)}_{u,i_1 \prec i_2} &= \log_2(1 + |2-5|) = 2, \\
w^{(S)}_{i_1 \prec i_2} &= \alpha_{i_1,i_2} \times \lambda_{i_1 \prec i_2} + \left(1 - \alpha_{i_1,i_2}\right) \times 1 \\
&= \log_{10} 1.9 \times \log_2\left(\frac{1000}{200}\right) + (1 - \log_{10} 1.9) \times 1 \\
&= 1.37, \\
w_{u,i_1 \prec i_2} &= 2 \times 1.37 = 2.74.
\end{aligned}$$

Similarly, for user v, the degree-specialty for preference term $i_1 \succ i_2$ can be computed as follows:

$$\begin{aligned}
w^{(D)}_{v,i_1 \succ i_2} &= \log_2(1 + |4-3|) = 1, \\
w^{(S)}_{i_1 \succ i_2} &= \alpha_{i_1,i_2} \times \lambda_{i_1 \succ i_2} + \left(1 - \alpha_{i_1,i_2}\right) \times 1 \\
&= \log_{10} 1.9 \times \log_2\left(\frac{1000}{800}\right) + (1 - \log_{10} 1.9) \times 1 \\
&= 0.81, \\
w_{v,i_1 \succ i_2} &= 1 \times 0.81 = 0.81.
\end{aligned}$$

Now, let $\{i_3, i_4\}$ be two items. Suppose among the total number of 10,000 users, 100 have rated both i_3 and i_4 with 80 holding preference $i_3 \succ i_4$ and 20 holding preference

$i_4 \succ i_3$. Suppose user u has assigned scores 2 and 5 and user v has assigned scores 4 and 3 to items i_3 and i_4, respectively.

Then, for user u, the degree-specialty for preference term $i_3 \prec i_4$ can be computed as follows:

$$\begin{aligned}
w^{(D)}_{u,i_3 \prec i_4} &= \log_2(1 + |2 - 5|) = 2, \\
w^{(S)}_{i_3 \prec i_4} &= \alpha_{i_3,i_4} \times \lambda_{i_3 \prec i_4} + \left(1 - \alpha_{i_3,i_4}\right) \times 1 \\
&= \log_{10} 1.09 \times \log_2\left(\frac{100}{20}\right) + (1 - \log_{10} 1.09) \times 1 \\
&= 1.05, \\
w_{u,i_3 \prec i_4} &= 2 \times 1.05 = 2.10.
\end{aligned}$$

Similarly, for user v, the degree-specialty for preference term $i_3 \succ i_4$ can be computed as follows:

$$\begin{aligned}
w^{(D)}_{v,i_3 \succ i_4} &= \log_2(1 + |4 - 3|) = 1, \\
w^{(S)}_{i_3 \succ i_4} &= \alpha_{i_3,i_4} \times \lambda_{i_3 \succ i_4} + \left(1 - \alpha_{i_3,i_4}\right) \times 1 \\
&= \log_{10} 1.09 \times \log_2\left(\frac{100}{80}\right) + (1 - \log_{10} 1.09) \times 1 \\
&= 0.97, \\
w_{v,i_3 \succ i_4} &= 1 \times 0.97 = 0.97.
\end{aligned}$$

From Example 4.3, we can see that strong (high degree) and rare (high specialty) preferences are given higher weights ($2.74 > 0.81$ and $2.10 > 0.97$). We can also see that confidence calibration makes smaller adjustments for specialty with more confidence toward the prior value of 1 ($2.74 > 2.10$ and $0.81 < 0.97$).

4.3. The VSRank Framework

Pseudocode. The pseudocode of the VSRank framework is shown in Algorithm 1. Lines 1–14 represent each user as a vector of relative preference terms based on the vector space model. In particular, lines 1–5 extract a bag of relative preference terms T_u for each user u, forming a set of preference terms T. Lines 6–8 compute the specialty weight for each term $t \in T$. Lines 9–14 compute the degree weights and then obtain a vector of degree-specialty weights for each user.

Lines 15–23 follow a ranking-based CF procedure to make recommendations for each user. In particular, for each user u, lines 16–18 compute similarity between u and the rest of users, based on which line 19 selects a set of neighborhood users U_u for u. Then lines 21–23 aggregate the preferences of the neighborhood users into a total ranking of items τ_u for recommendation.

Discussion. Let m and n be the numbers of users and items. In VSRank, each user maximally holds $\frac{1}{2}n(n-1)$ preferences. In the worst case, computing degree-specialty weights has a time complexity of $O(mn^2)$. Evaluating similarity between pairs of users has a time complexity of $O(m^2n^2)$. Predicting rankings of items has a time complexity of $O(mn^2)$ (see Algorithm 3). In total, VSRank has a time complexity of $O(m^2n^2)$, which is n times higher than that of rating-based CF $O(m^2n)$.

Note that the complexity analysis is based on the worst case. First of all, in the real-world cases, the rating matrix is very sparse, and each user only rates a very small portion of items [Su and Khoshgoftaar 2009]. Second, as Equations (2) and (3) have shown, in ranking-based CF, only pairs of items with different rating scores are considered as preference terms, which further reduces the number of preference terms.

Table I. The Four Recommender Systems in VSRank

	Order-based aggregation	Score-based aggregation
$\cos^w_{u,v}$	wVCOrder	wVCScore
$\tau^w_{u,v}$	wTauOrder	wTauScore

ALGORITHM 1: The VSRank Framework

Input: An item set I, a user set U, and a rating matrix R
Output: A set of rankings $\{\tau_u\}_{u\in U}$ of items for each user $u \in U$

$T \leftarrow \emptyset$;
for each $u \in U$ **do**
  $T_u \leftarrow$ `ExtractTerms` (u, I, R);
  $T \leftarrow T \cup T_u$;
end
for each $t \in T$ **do**
  $w^{(S)}_t \leftarrow$ `ComputeSpecialty` (T); `// Eq. 7`
end
for each $u \in U$ **do**
  for each $t \in T_u$ **do**
    $w^{(D)}_{u,t} \leftarrow$ `ComputeDegree` (T_u); `// Eq. 5`
    $w_{u,t} \leftarrow w^{(S)}_t \times w^{(D)}_{u,t}$; `// Eq. 8`
  end
end
for each $u \in U$ **do**
  for each $v \in U$ **and** $u \neq v$ **do**
    $s_{u,v} \leftarrow$`ComputeSimilarity` (w_u, w_v)
  end
  $U_u \leftarrow$`SelectNeighbors` $(\{s_{u,v}\}_{v\in U})$
end
for each $u \in U$ **do**
  $\tau_u \leftarrow$`Aggregate` $(\{T_v\}_{v\in U_u})$
end

For example, let user u rate l items, where $l \ll n$. Each user can maximally hold $\frac{1}{2}l(l-1)$ preferences, and the time complexity of similarity evaluation in ranking-based CF should be $O(m^2l^2)$ instead. Let the rating scale be from 1 to s, and the numbers of items with rating scores of $1, 2, \ldots, s$ be $l_1, l_2, \ldots, l_s$, respectively. The number of preference terms in u is $\frac{1}{2}l(l-1) - \frac{1}{2}\sum_i l_i(l_i - 1)$.

Furthermore, the recommendation algorithms are performed offline and can be significantly accelerated via parallel or distributed computing.

5. IMPLEMENTATION OF VSRANK

In implementing VSRank, we use two similarity measures, cosine similarity $\cos^w_{u,v}$ and Kendall tau correlation coefficient $\tau^w_{u,v}$, for similarity computation. For preference aggregation, we also use two methods, an order based and a score based, to predict item ranking.

Depending on the user similarity measure used in Phase I, $\cos^w_{u,v}$ or $\tau^w_{u,v}$, and the preference aggregation method used in Phase II, order based or score based, we name the resulting algorithms as wVCScore, wVCOrder, wTauScore, and wTauOrder, respectively. Table I shows the four algorithms.

5.1. Similarity Computation

Cosine similarity. The indicator p of a preference on a pair of items (i_m, i_n) can be defined as a number in {–1, 1}, where $p_{u,(m,n)} = -1$ for $i_m \prec i_n$ and $p_{u,(m,n)} = 1$ for $i_m \succ i_n$. Let $r_{u,m}$ and $r_{u,n}$ be the rating scores that have been assigned to items i_m and i_n, respectively, by user u. The value for the preference can be written as

$$p_{u,(m,n)} = \begin{cases} -1, & \text{if } r_{u,m} < r_{u,n} \\ 1, & \text{if } r_{u,m} > r_{u,n}. \end{cases} \tag{9}$$

The indicator $p_{u,(m,n)}$ indicates whether user u prefers item i_m to i_n or vice versa. Remember that we have defined an indicator function $sgn_{u,v}(m, n)$ in Equation (2), indicating whether two users u and v have a concordant/discordant preference on the pair of items (i_m, i_n). According to Equations (2) and (9), it is easy to prove that

$$p_{u,(m,n)} p_{v,(m,n)} = sgn_{u,v}(m, n). \tag{10}$$

With degree-specialty weighting, user u is represented as a vector of degree-specialty weights $\hat{w}_u$, where each element is represented as $\hat{w}_{u,(m,n)} = w_{u,i_m\Theta i_n} p_{u,(m,n)}$. Then, the similarity between two users u and v can be computed by the standard cosine similarity:

$$\cos_{u,v} = \frac{\hat{w}_u \cdot \hat{w}_v}{||\hat{w}_u|| \times ||\hat{w}_v||} = \frac{\sum_{m=1}^{N} \sum_{n=1}^{N} \hat{w}_{u,(m,n)} \times \hat{w}_{v,(m,n)}}{\sqrt{\sum_{m=1}^{N} \sum_{n=1}^{N} \hat{w}_{u,(m,n)}^2} \times \sqrt{\sum_{m=1}^{N} \sum_{n=1}^{N} \hat{w}_{v,(m,n)}^2}}. \tag{11}$$

Weighted Kendall tau. Shieh [1998] proposed τ^w, a class of weighted variants of Kendall tau rank correlation coefficient that can be used to compute similarity between users u and v, where each pair of ranks can be weighted separately. Formally,

$$\tau_{u,v}^w = \frac{\sum_{m=1}^{N} \sum_{n=m+1}^{N} w_{m,n} \times sgn_{u,v}(m, n)}{\sum_{m=1}^{N} \sum_{n=m+1}^{N} w_{m,n}}, \tag{12}$$

where $w_{m,n}$ is the weight for the pair of items (i_m, i_n), and $sgn_{u,v}(m, n)$ is an indicator function, as defined in Equation (2).

The weighted Kendall τ_w generalizes the Kendall τ rank correlation coefficient, and the latter is a special case when $w_{m,n} \equiv 1$, where $1 \leq m < n \leq N$.

In estimating similarity between users u and v, the degree-specialty weight of the item pair (i_m, i_n) in $\tau_{u,v}^w$ is represented as the product of the weights of u and v, formally,

$$w_{m,n} = w_{u,i_m\Theta i_n} w_{v,i_m\Theta i_n}. \tag{13}$$

Thus, the weighted Kendall tau correlation coefficient can be rewritten as follows:

$$\tau_{u,v}^w = \frac{\sum_{m=1}^{N} \sum_{n=m+1}^{N} w_{u,i_m\Theta i_n} w_{v,i_m\Theta i_n} sgn_{u,v}(m, n)}{\sum_{m=1}^{N} \sum_{n=m+1}^{N} w_{u,i_m\Theta i_n} w_{v,i_m\Theta i_n}}. \tag{14}$$

Relationship between $\cos_{u,v}$ and weighted $\tau_{u,v}^w$. Now we reveal interesting connections between cosine similarity $\cos_{u,v}$ and weighted Kendall tau rank correlation coefficient $\tau_{u,v}^w$.

THEOREM 5.1. $\cos_{u,v} = \tau^{w}_{u,v} \cos(w_u, w_v)$.

PROOF.

$$\begin{aligned}
&\hat{w}_u \cdot \hat{w}_v \\
&= \sum_{m=1}^{N} \sum_{n=m+1}^{N} (w_{u,i_m \Theta i_n} p_{u,(m,n)})(w_{v,i_m \Theta i_n} p_{v,(m,n)}) \\
&= \sum_{m=1}^{N} \sum_{n=m+1}^{N} (w_{u,i_m \Theta i_n} w_{v,i_m \Theta i_n})(p_{u,(m,n)} p_{v,(m,n)})
\end{aligned}$$

According to Equation (10):

$$\hat{w}_u \cdot \hat{w}_v = \sum_{m=1}^{N} \sum_{n=m+1}^{N} (w_{u,i_m \Theta i_n} w_{v,i_m \Theta i_n} sgn_{u,v}(m, n)),$$

since

$$\begin{aligned}
&||\hat{w}_u|| \times ||\hat{w}_v|| \\
&= \sqrt{\sum_{m=1}^{N} \sum_{n=m+1}^{N} \hat{w}^2_{u,i_m \Theta i_n}} \sqrt{\sum_{m=1}^{N} \sum_{n=m+1}^{N} \hat{w}^2_{v,i_m \Theta i_n}} \\
&= \sqrt{\sum_{m=1}^{N} \sum_{n=m+1}^{N} w^2_{u,i_m \Theta i_n}} \sqrt{\sum_{m=1}^{N} \sum_{n=m+1}^{N} w^2_{v,i_m \Theta i_n}} \\
&= ||w_u|| \times ||w_v|| \\
&= \frac{w_u \cdot w_v}{\cos(w_u, w_v)} \\
&= \frac{\sum_{m=1}^{N} \sum_{n=m+1}^{N} w_{u,i_m \Theta i_n} w_{v,i_m \Theta i_n}}{\cos(w_u, w_v)}.
\end{aligned}$$

Hence:

$$\begin{aligned}
&\cos_{u,v} \\
&= \frac{\hat{w}_u \cdot \hat{w}_v}{||\hat{w}_u|| \times ||\hat{w}_v||} \\
&= \frac{\left(\sum_{m=1}^{N} \sum_{n=m+1}^{N} (w_{u,i_m \Theta i_n} w_{v,i_m \Theta i_n} sgn_{u,v}(m, n))\right) \cos(w_u, w_v)}{\sum_{m=1}^{N} \sum_{n=m+1}^{N} w_{u,i_m \Theta i_n} w_{v,i_m \Theta i_n}} \\
&= \tau^{w}_{u,v} \cos(w_u, w_v). \quad \square
\end{aligned}$$

COROLLARY 5.2. *Without weighting, cosine similarity* $\cos_{u,v}$ *is equivalent to the Kendall tau rank correlation coefficient* $\tau_{u,v}$.

The corollary can be easily derived from Theorem 5.1 for the special unweighted case of $w_u = w_v = \langle 1, 1, \ldots, 1 \rangle$.

Discussion. The previous theoretical results reveal valuable insights. Existing ranking-based CF methods do not weight preferences, and Kendall tau rank correlation coefficient $\tau^{w}_{u,v}$ is the standard similarity measure. We have shown that it is equivalent to cosine similarity for the unweighted case after adapting the vector space model. However, with preference weighting, $\cos_{u,v} = \tau^w_{u,v} \cos(w_u, w_v)$. Then comparing $\cos_{u,v}$ with $\tau^w_{u,v}(w_u, w_v)$, the former incorporates length normalization whereas the latter does not. This explains our experimental results that the former performed better than the latter.

5.2. Ranking Prediction

As introduced in Section 4, ranking-based CF discovers a set of the most similar users in Phase I and predicts a ranking list of items for recommendation in Phase II.

We have discussed Phase I in Section 5.1. In this section, we discuss Phase II, ranking prediction, where we aggregate the partial preference rankings of the neighborhood users into a total ranking of items that can be used for recommendation. Cohen et al. [1999] proved that it is an NP-hard problem.

Aggregation algorithms can be classified into two categories: order based and score based [Aslam and Montague 2001; Gleich and Lim 2011]. The former only uses relative preference orders of items to generate a total ranking of items with an order-based aggregation function. The latter predicts a score for each pair of items to indicate the preference degree of items, based on which a total ranking of items are generated with a score-based aggregation function. Generally speaking, order-based algorithms are less biased, whereas score-based algorithms are more effective for sparse data [Baskin and Krishnamurthi 2009].

In this study, we provide two algorithms for preference aggregation. First of all, we adopt Schulze's method for order-based aggregation. The Schulze method [Schulze 2003] is a voting system to create a sorted list of winners with votes iteratively, which satisfies the properties of pareto, monotonicity, resolvability, independence of clones, and reversal symmetry. For score-based aggregation, we use a greedy method, which is straightforward, easy to implement, yet highly effective.

Order-based preference aggregation. For a given user u, order-based preference aggregation attempts to maximize the number of consistences between the aggregated ranking τ_u and preference rankings from the neighborhood users, and simultaneously minimize the number of inconsistences. Let $U_u^{m,n}$ be the set of neighborhood users of u who have rated both items i_m and i_n; the objective function for optimization is given as follows:

$$\arg\max \sum_{\forall (i_m, i_n): \tau_u(i_m) < \tau_u(i_n)} \sum_{v \in U_u^{m,n}} p_{v,(m,n)}, \tag{15}$$

where $p_{v,(m,n)}$ indicates whether user v prefers i_m to i_n or vice versa, outputting 1 or -1 respectively (see Equation (9)), and $\tau_u(i_m) < \tau_u(i_n)$ indicates that i_m is prior to i_n in the aggregated ranking τ_u.

In this study, we adopt the Schulze method to implement order-based aggregation as shown in Algorithm 2. The algorithm has a time complexity of $O(n^3)$, where n is the number of items.

In the algorithm, line 1 introduces an $|I| \times |I|$ matrix M, where each element $M_{m,n}$ indicates the relative preference degree of the pair of items (i_m, i_n) for the neighborhood users of u. Then lines 2–5 initialize each element $M_{m,n}$ of the matrix as the number of preferences $i_m \succ i_n$ voted by the neighborhood users.

Lines 7–12 estimate the highest preference degree for each element $M_{m,n}$ of the matrix iteratively and update it with $\max(M_{n,k}, \min(M_{n,m}, M_{m,k}))$. For example, suppose that currently there are 10 neighborhood users who prefer i_n to i_k, 20 neighbors who prefer i_n to i_m, and 15 neighbors who prefer i_m to i_k. There are two preference paths from i_n to i_k: one from i_n to i_k directly with the preference degree of 10, and the

ALGORITHM 2: The Schulze method for order-based preference aggregation

Input: An item set I, a user u, and a preference prediction function Ψ_u
Output: A ranking τ_u of items for user u

1 $M_{|I|\times|I|} \leftarrow 0$;
2 **for** $m \leftarrow 1$ **to** $|I|$ **do**
3   **for** $n \leftarrow 1$ **to** $|I|$ **and** $m \neq n$ **do**
4         $M_{m,n} \leftarrow N_{i_m \succ i_n}$;
5   **end**
6 **end**
7 **for** $m \leftarrow 1$ **to** $|I|$ **do**
8   **for** $n \leftarrow 1$ **to** $|I|$ **and** $m \neq n$ **do**
9   **for** $k \leftarrow 1$ **to** $|I|$ **and** $k \neq m$ **and** $k \neq n$ **do**
10             $M_{n,k} \leftarrow \max(M_{n,k}, \min(M_{n,m}, M_{m,k}))$;
11   **end**
12   **end**
13 **end**
14 **for each** $i_m \in I$ **do**
15     $\tau_u(i_m) \leftarrow \sum_{\forall i_n \in I \setminus \{i_m\}} 1(M_{m,n} > M_{n,m})$;
16 **end**

other from i_n through i_m to i_k with the preference degree of $\min(20, 15) = 15$. Thus, the preference path with the highest preference degree is the latter with a value of $\max(10, \min(20, 15)) = 15$.

Lines 14–16 iteratively produce a total ranking of items. In particular, the item i_m is prior to i_n if $M_{m,n} > M_{n,m}$, and the rank position of i_m is equal to the number of occurrences of $M_{m,n} > M_{n,m}$ for any other item i_n.

Score-based preference aggregation. Order-based algorithms only use the relative preference orders of items to generate a total ranking. On the other hand, score-based algorithms consider the similarity scores between the given user and his or her neighborhood users as weights of their preferences. Let $s_{u,v}$ be the similarity between two users u and v. For a given user u, score-based preference aggregation attempts to maximize the *weighted* consistences between the aggregated ranking τ_u and the preferences of the neighborhood users of u:

$$\arg\max \sum_{\forall (i_m,i_n):\tau_u(i_m)<\tau_u(i_n)} \frac{\sum_{v\in U_u^{m,n}} s_{u,v}\, p_{v,(m,n)}}{\sum_{v\in U_u^{m,n}} s_{u,v}}. \tag{16}$$

Similar to EigenRank [Liu and Yang 2008], we define a preference prediction function $\Psi_u(i_m, i_n)$ as follows:

$$\Psi_u(i_m, i_n) = \frac{\sum_{v\in U_u^{m,n}} s_{u,v}\, p_{v,(m,n)}}{\sum_{v\in U_u^{m,n}} s_{u,v}}. \tag{17}$$

For a given user u, the preference prediction function $\Psi_u(i_m, i_n) : I \times I \to \mathbb{R}$ assigns real number confidence scores to preferences, where I is the item set and $\mathbb{R}$ is the real number set. $\Psi_u(i_m, i_n) > 0$ indicates that item m is more preferable to n by user u and vice versa. The magnitude of the preference function $|\Psi_u(i_m, i_n)|$ implies the evidence of the preference, and a value of zero means that there is no preference between the two items. Thus, the objective function for optimization in a score-based approach can be

Table II. Statistics About MovieLens and EachMovie

	EachMovie	MovieLens
Number of users	74,418	6,040
Number of movies	1,648	3,952
Number of ratings	2,811,983	1,000,209
Rating scales	0–5	1–5

rewritten as follows:

$$\arg\max \sum_{\forall (i_m, i_n): \tau_u(i_m) < \tau_u(i_n)} \Psi_u(i_m, i_n). \tag{18}$$

In this study, we provide a greedy method for score-based preference aggregation as shown in Algorithm 3.

ALGORITHM 3: The greedy method for score-based preference aggregation

Input: An item set I, a user u, and a preference prediction function Ψ_u
Output: A ranking τ_u of items for user u

1 $N \leftarrow |I|$;
2 **for each** $i \in I$ **do**
3     $\pi_u(i) \leftarrow \sum_{\forall j \in I} \Psi_u(i, j) - \sum_{\forall j \in I} \Psi_u(j, i)$
4 **end**
5 **while** $I \neq \varnothing$ **do**
6     $t \leftarrow \arg\max_i \pi_u(i)$;
7     $\tau_u(t) \leftarrow N - |I|$;
8     $I \leftarrow I \backslash \{t\}$;
9   **for each** $i \in I$ **do**
10         $\pi_u(i) \leftarrow \pi_u(i) + \Psi_u(t, i) - \Psi_u(i, t)$;
11   **end**
12 **end**

For a given user u, Algorithm 3 assigns to each item $i \in I$ a *potential* value $\pi_u(i)$, which is the sum of the evidence scores of the preferences starting with i minus the sum of the evidence scores of the preferences ending with i (lines 2–4). Then the algorithm iteratively produces the rank of each item in a greedy strategy until I is empty (lines 5–12). In particular, the algorithm first picks some item i with maximum potential value and assigns it a rank $\tau_u(i) = N - |I|$ (lines 6–7). Then it deletes i from I (line 8) and updates the potential values of the remaining items (lines 9–11).

The algorithm has a time complexity of $O(n^2)$, where n is the number of the items. It can be proved to have an approximation ratio of 2, that is, $\Lambda_S(\tau_u) \geq \frac{1}{2}\Lambda_S(\tau_u^*)$ [Cohen et al. 1999].

6. EXPERIMENTS

6.1. Methodology

Datasets. We used two real movie rating datasets in our experiments, EachMovie and MovieLens.[7] The EachMovie dataset contains about 2.8 million ratings, which are made by 74,418 users on 1,648 movies. The MovieLens dataset consists of 1 million ratings assigned by 6,040 users to a collection of 3,952 movies. The EachMovie rating scale is from 0 to 5, while the MovieLens rating scale is from 1 to 5. Table II lists the detailed statistics about the two datasets.

[7]http://www.grouplens.org/node/12.

Evaluation measures. For rating-based collaborative filtering, the standard evaluation criterion is the rating prediction accuracy. Commonly used accuracy measures include the Mean Absolute Error (MAE) and the Root Mean Square Error (RMSE). Both measures depend on differences between true rating and predicted rating. Since our study focuses on improving item rankings instead of rating prediction, we employ two ranking-oriented evaluation measures: Normalized Discounted Cumulative Gain (NDCG) [Järvelin and Kekäläinen 2002] and Mean Average Precision (MAP). They are popular in information retrieval for evaluating ranked results, where documents are assigned graded relevance judgments in NDCG and binary relevance judgments in MAP.

In the context of collaborative filtering, item ratings assigned by users can naturally serve as relevance judgments. The NDCG metric is evaluated over some number n of the top items on the ranked item list. Let U be the set of users and $r_{u,p}$ be the rating score assigned by user u to the item at the pth position of the ranked list from u. The NDCG at the nth position with respect to the given user u is defined as follows:

$$NDCG_u@n = Z_u \sum_{p=1}^{n} \frac{2^{r_{u,p}} - 1}{\log(1+p)}. \tag{19}$$

NDCG at the nth position takes the mean of the NDCG values at the same position over the set of users U.

P@n represents the precision within the top n results of the ranked list of items for a user. Average precision (AP) for user u is defined as the average of the P@n values for all relevant items:

$$AP_u = \frac{\sum_{p=1}^{N} \left(P@n \times rel_{u,p}\right)}{\text{\# relevant items for user u}}, \tag{20}$$

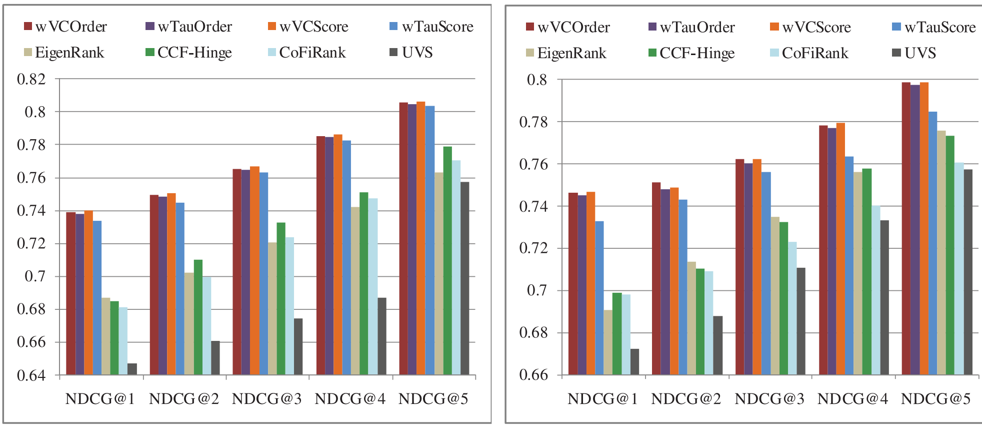
where $rel(n)$ is a binary function mapping a document to either 1 (relevant) or 0 (irrelevant). In this experiment, we regarded the rating scores of 5 as relevant and scores less than 5 as irrelevant. MAP takes the mean of the AP values over the set of users U.

Comparison partners. We used three state-of-the-art ranking-based collaborative filtering algorithms, EigenRank [Liu and Yang 2008], CCF [Yang et al. 2011], and CoFiRank [Weimer et al. 2007], as our main comparison partners. In Yang et al. [2011], two CCF algorithms of CCF-Softmax and CCF-Hinge were provided with softmax and hinge loss functions, respectively, achieving similar recommendation performances. In our experiments, we used CCF-Hinge for comparison. In addition, we also included comparisons with UVS [Breese 1998], a conventional user-based collaborative filtering method. UVS measured similarity between users using the vector cosine similarity and then ranked the items for each user according to their predicted rating scores for the purpose of obtaining a ranking of items.

Experimental setup. In our experiments, we randomly selected 80% rated items for training and used the remaining 20% for testing. In order to guarantee that there are an adequate number of common rating items between each neighborhood user and the target user, we filtered those users who have rated fewer than 50 items in MovieLens and 100 items in EachMovie. We ran each algorithm five times and reported the average performance.

6.2. Accuracy

In the first series of experiments, we evaluated the accuracy performance of wVCOrder, wTauOrder, wVCScore, and wTauScorein comparison with EigenRank, CoFiRank, CCF-Hinge, and UVS on EachMovie and MovieLens.



(a) EachMovie (b) MovieLens

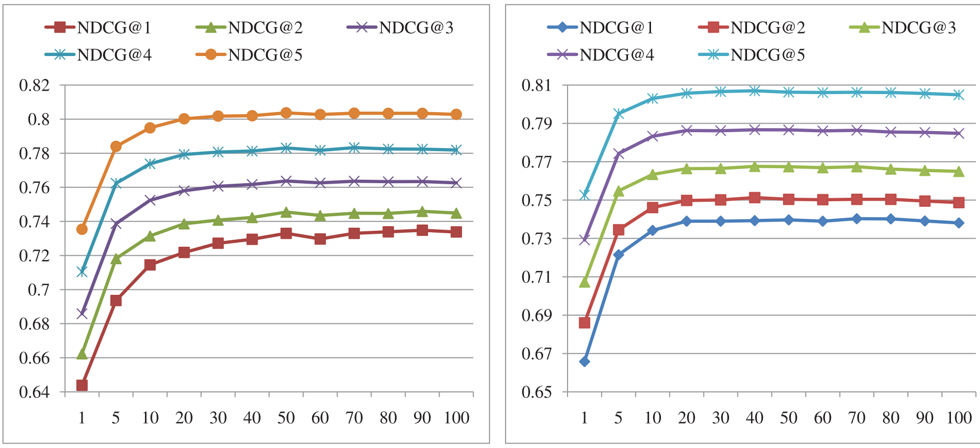
Fig. 1. Accuracy in NDCG@n.

Table III. Accuracy in MAP

Dataset	wVCOrder	wTauOrder	wVCScore	wTauScore	EigenRank	CCF-Hinge	CoFiRank	UVS
EachMovie	**0.5284**	0.5184	0.5267	0.5201	0.5036	0.5234	0.5032	0.4789
MovieLens	0.5806	0.5791	**0.5808**	**0.5808**	0.5731	0.5743	0.5759	0.5189

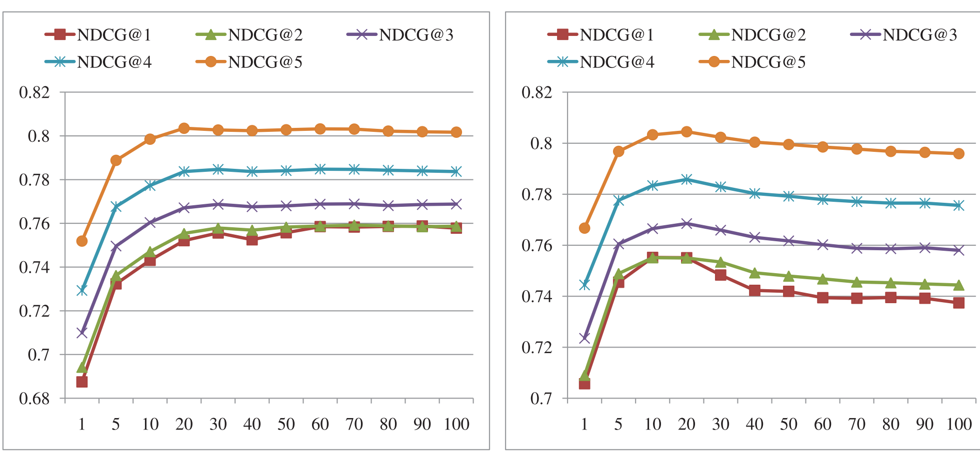
Figures 1(a) and 1(b) and Table III show the comparison of performance evaluated with NDCG and MAP measures. From the figures and the table, we can see the following:

(1) Our proposed degree-specialty weighting scheme can discover a more accurate set of neighborhood users, resulting in improved recommendation accuracy. For the two benchmark datasets, the four recommender systems wVCScore, wTauOrder, wVCScore, and wTauOrder outperformed all other comparison partners.
(2) Cosine similarity $\cos^w$ used in our vector space model is more effective than the weighted Kendall tau rank correlation coefficient τ^w, evidenced by the fact that wVCScore and wVCOrder outperformed wTauScore and wTauOrder, respectively.
(3) Order-based aggregation has less variance than score based, evidenced by the fact that the standard deviations of wVCOrder and wTauOrder are much smaller than those of wVCScore and wTauScore. For example, for EachMovie, the standard deviations of wVCOrder and wTauOrder are 0.0007 and 0.0006 on NDCG@1–2 compared to 0.0045 and 0.0040 for those of wVCScore and wTauScore. For MovieLens, the standard deviations of wVCOrder and wTauOrder are 0.0006 and 0.0021 on NDCG@1–2 compared to 0.0097 and 0.0040 for those of wVCScore and wTauScore.
(4) Ranking-based collaborative filtering can have advantages over rating-based methods on the NDCG evaluation measure. In our experiments, all the ranking-based methods outperformed the rating-based method UVS. For example, for EachMovie, wVCScore achieved 0.7402 and 0.7504 on NDCG@1–2 compared to 0.6471 and 0.6609 for UVS, gaining a 14.39% and 13.54% improvement, respectively. For MovieLens, wVCScore achieved 0.7467 and 0.7489 on NDCG@1–2 compared to 0.6723 and 0.688 for UVS, gaining an 11.07% and 8.85% improvement, respectively.
(5) From Table III we can see that, while in general our algorithms outperformed the comparison partners in MAP, their advantages were not as obvious as in NDCG. For example, CCF-Hinge outperformed two of our four algorithms wTauOrder and



(a) wVCOrder (b) wVCScore

Fig. 2. Sensitivity of neighborhood size on EachMovie.



(a) wTauOrder (b) wTauScore

Fig. 3. Sensitivity of neighborhood size on MovieLens.

wTauScore on EachMovie. This is mainly because unlike NDCG, MAP is not the most appropriate measure for multilevel ratings. MAP natively handles binary ratings, whereas EachMovie contains six-level ratings (0–5) and MovieLens contains five-level ratings (1–5).

6.3. Sensitivity of Neighborhood Size

The size of the neighborhood has a significant impact on the prediction quality for conventional collaborative filtering [Herlocker et al. 1999; Liu and Yang 2008]. For example, for EigenRank, the NDCG values gradually increase as the neighborhood size increases and reach the peaks at the neighborhood size of 100 [Liu and Yang 2008].

We studied the sensitivity of this parameter on our methods. We conducted a series of experiments on wVCOrder and wVCScore for the EachMovie and MovieLens datasets with the number of neighborhood users varying from 1 to 100. The experiment results are reported in Figures 2 and 3. From the results, we can see that the curves tend to be

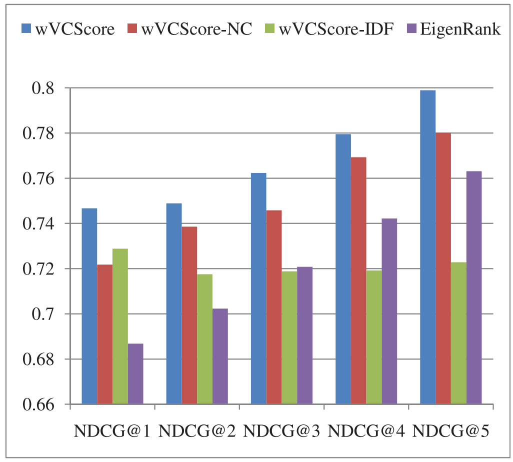


Fig. 4. Arguments on specialty.

flat for EachMovie and even decline for MovieLens after the neighborhood size exceeds 20. Based on the results, we have the following observations:

(1) The size of the neighborhood has an impact on the prediction quality of our methods. Prediction is accurate when the neighbors are very similar to the target user. When the neighborhood size exceeds 20, the performance starts to decrease because more dissimilar users are selected into the neighborhood, introducing noise to $\Psi(m, n)$.
(2) An effective term-weighting scheme can significantly benefit from the discovery of good neighborhood. wVCOrder and wVCScore are able to discover the most similar users with the neighborhood size of 20, compared to 100 for EigenRank, the conventional ranking-based collaborative filtering method.

6.4. Discussions on Specialty

Based on the experiments, we have the following observations: (1) specialty is more appropriate than the original IDF for measuring rarity of preference terms in the context of ranking-based collaborative filtering, and (2) confidence calibration can be used to adjust specialty toward its true value.

In particular, we applied wVCScore, which uses specialty, on the MovieLens dataset. We then repeated the experiments with two modified recommender systems of wVCScore-IDF and wVCScore-NC. The former replaces specialty with the original IDF, and the latter removes confidence calibration from specialty. The comparison results are reported in Figure 4, from which we can see the following:

(1) "Specialty" significantly outperformed "IDF." For example, wVCScore achieved 0.7795 and 0.7989 on NDCG@4–5 compared to 0.7192 and 0.7228 for wVCScore-IDF, gaining an 8.4% and 10.5% improvement, respectively. With "Degree-IDF," the performance of wVCScore-IDF is even slightly worse than that of EigenRank, the ranking-based CF without weighting.
(2) "Confidence calibration" can help improve accuracy. For example, wVCScore achieved 0.7467 and 0.7989 on NDCG@1 and 5 compared to 0.7218 and 0.7802 for wVCScore-NC, gaining a 3.4% and 2.4% improvement, respectively.

7. CONCLUSION

In this article, we have proposed VSRank, a framework for adapting the vector space model to ranking-based collaborative filtering for improved recommendation accuracy. Different from existing ranking-based CF methods that treat each user as a set of preferences, we adopt the bag-of-words model capturing the "frequency" of preferences. Different from existing ranking-based CF methods that treat preferences equally, we use a novel degree-specialty weighting scheme resembling TF-IDF. Users are represented as vectors of degree-specialty weights, and ranking-based CF techniques are used to predict a ranking of items for accurate recommendation to the target user. Comprehensive experiments have validated the effectiveness of our framework.

There are several interesting directions for future work. First, other ranking-based similarity measures can be experimented with for improving neighborhood quality. Second, knowing that there are many TF-IDF variants, we plan to investigate other possible variants of degree-specialty and study their performance in different applications. Last but not least, the proposed adaptation framework is not limited to ranking-based CF. We plan to explore a similar adaptation of the vector space model to rating-based CF and examine its effectiveness.

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Received May 2013; revised September 2013; accepted October 2013