GENERATING SPATIALLY CORRELATED RANDOM FIELDS WITH R

October 31, 2010 – 20:32

By *sbegueria*

Posted in *Geocomputation and Numerical Modelling*, *Geographic Information Science*

Tagged *code*, *fields*, *geographic variables*, *gstat*, *kriging*, *R*, *spatial correlation*, *spatial data*, *spatial fields*, *spatial variables*, *unconditional gaussian simulation*

Comments (34)

In several occasions I needed to generate synthetic data with a desired level of spatial autocorrelation. Here I will show how to generate as many such fields as we need by using R, an open-source port of the S language for statical analysis. I will concentrate on two alternative ways of generating spatially correlated random fields (commonly known as unconditional Gaussian simulation), using the libraries `gstat` and `fields`.

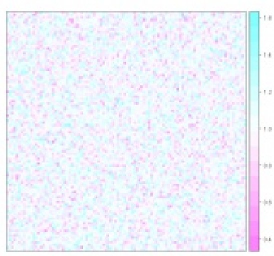


Figure 0. A purely random field. (Clic to enlarge)

1. Unconditional Gaussian simulation using gstat

Spatial modellers commonly use the term unconditional Gaussian simulation to refer to the process of generating spatially correlated random fields. For comparison purposes, the Figure 0 shows an example of a random field with no spatial correlation. The value of the measured property at one given cell is completely independent of the values of that property at neighbouring cells. This situation is very seldom (or I perhaps should better say never) found in nature, since spatially distributed variables always have a certain level of autocorrelation, i.e. co-variance between neighbours. This is often mentioned as the 'geographical law', stating that closer locations tend to have similar properties.

Generating spatially correlated random fields is interesting because it makes it possible testing different issues related to the statistical analysis of spatial data.

1.1. Generating the spatial field

We are going to use the `gstat` library, so we start by loading it:

```
library(gstat)
```

We create a 100 x 100 grid, and we convert it into a data frame (xyz structure) by taking all possible combinations of the x and y coordinates:

```
xy <- expand.grid(1:100, 1:100)
```

We give names to the variables:

```
names(xy) <- c('x','y')
```

Defining the spatial model and performing the simulations.

Second, we define the spatial model as a gstat object:

```
ormula=z~1, locations=~x+y, dummy=T, beta=1, model=vgm(psill=(
```

where `formula` defines the dependent variable (z) as a linear model of independent variables. For ordinary and simple kriging we can use the formula `z~1`; for simple kriging it is necessary to define a beta parameter too (see below); for universal kriging, if `z` is linearly dependent on `x` and `y` use the formula `z~x+y`. We are using simple kriging here. `locations` define the data coordinates, e.g. `~x+y` in our case here. `dummy` is a logical value, and it needs to be TRUE for unconditional simulation. `beta` is used only for simple kriging, and is a vector with the trend coefficients (including an intercept); if no independent variables are defined the model only contains an intercept, i.e. the simple kriging mean. `model` defines the variogram model, as defined by a call to `vgm`. `vgm` allows defining the (partial) sill, range and nugget paramaters, as well as the variogram model type (e.g. exponential, gaussian, spherical, etc). Anisotropy can also be used. `nmax` defines the number of nearest observations that should be used for a kriging prediction or simulation.

Now we are ready to make as many simulations as we like based on the gstat object (four simulations in this example):

```
yy <- predict(g.dummy, newdata=xy, nsim=4)
```

1.2. Displaying the simulations

To see one realisation of the simulations:

```
gridded(yy) = ~x+y
spplot(obj=yy[1])
```

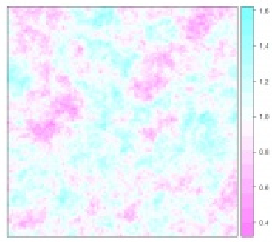


Figure 1. A spatially correlated random spatial field with mean=1, sill=0.025, range=5, exponential variogram model.

`spplot`, from the library `sp`, provides lattice (trellis) plot methods for spatial data with attributes. It's only compulsory parameter is `obj`, which must point to an object of class Spatial; gstat objects belong to this class, so there's no need to do anything extra. It is possible to show all four simulations in a single trellis plot:

```
spplot(yy)
```

1.3. Complete code

I include the complete code for convenience:

```
# unconditional simulations on a 100 x 100 grid using g
library(gstat)

# create structure
xy <- expand.grid(1:100, 1:100)
names(xy) <- c("x","y")

# define the gstat object (spatial model)
g.dummy <- gstat(formula=z~1, locations=~x+y, dummy=T,

# make four simulations based on the stat object
yy <- predict(g.dummy, newdata=xy, nsim=4)

# show one realization
gridded(yy) = ~x+y
spplot(yy[1])

# show all four simulations:
spplot(yy)
```

1.4. Variations

By modifying the range parameter in the variogram model it is possible to control the degree of spatial correlation. For example, by setting it at 15 instead of 5 we get a random field with a 'coarser' autocorrelation (Figure 2).

```
g.dummy <- gstat(formula=z~1, locations=~x+y, dummy=T, b
```

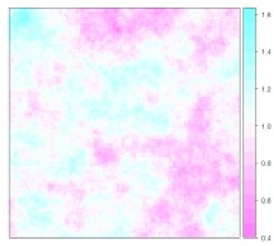


Figure 2. A spatially correlated random spatial field with mean=1, sill=0.025, range=15, exponential variogram model.

For including a linear trend surface in the simulation we can perform a universal kriging. For doing so it is necessary to specify it in the formula parameter as `~1+x+y`, and coefficients for the x and y components need to be specified in the beta parameter. For example, the following defines a model with a spatial trend in the y dimension (Figure 3):

```
g.dummy <- gstat(formula=z~1+y, locations=~y, dummy=T, b
```

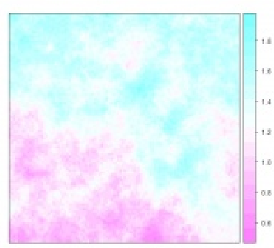


Figure 3. A spatially correlated random spatial field with mean=1, sill=0.025, range=15, y_trend=0.005, exponential variogram model.

The following code defines a model with a trend in both dimensions:

```
g.dummy <- gstat(formula=z~1+x+y, locations=~x+y, dummy=
```

2. Unconditional Gaussian simulation using fields

Spatially correlated random fields can also be generated using the `fields` library. I put an example code here, and leave the details to the reader to investigate.

```
library(fields)
# load Maunga Whau volcano (Mt Eden) elevation dataset
data(volcano)

# reduce size
volcano2 <- volcano[10:55, 14:51]
filled.contour(volcano2, color.palette=terrain.colors,
cols <- length(volcano2[1,])
rows <- length(volcano2[,1])
```

```
# create dataframe (xyz format)
X <- rep(1:cols, each=rows)
Y <- rep(1:rows, cols)
Z <- as.vector(volcano2)
volcano.df <- data.frame(X,Y,Z,cellid=1:cols*rows)
attach(volcano.df)
quilt.plot(Y,X,Z,nrow=rows,ncol=cols,add=F)

# create a spatial autocorrelation signature
# coordinate list
coords <- data.frame(X,Y)
# distance matrix
dist <- as.matrix(dist(coords))

# create a correlation structure (exponential)
str <- -0.1 # strength of autocorrelation, inv. proport
omega1 <- exp(str*dist)

# calculate correlation weights, and invert weights mat
weights <- chol(solve(omega1))
weights_inv <- solve(weights)

# create an autocorrelated random field
set.seed(1011)
error <- weights_inv %*% rnorm(dim(dist)[1])
quilt.plot(Y,X,error,nrow=rows,ncol=cols,add=F)

# create a variable as a linear function of the elevati
a <- 10
b <- 0.5
Z2 <- a + b*Z
quilt.plot(Y,X,Z2,nrow=rows,ncol=cols,add=F)

# add the autocorrelated error to the new variable
Z3 <- Z2 + error
quilt.plot(Y,X,Z3,nrow=rows,ncol=cols,add=F)

# export data (xyz format)
write.table(data.frame(X,Y,Z,Z2,Z3), file="data.txt", r
```

34 COMMENTS

James Hodden wrote:
August 30, 2012 at 22:17

Thank you for this clear explanation for how you generate random fields.

Felipe S. wrote:
September 28, 2012 at 19:18

Thank you very interesing post, by the way do you know how to make lognormal random fields simulation?

admin wrote:
November 11, 2013 at 15:36

Sequential Gaussian simulation would yield a normally distributed data at each point, i.e. with mean and standard deviation equal to the kriged mean and standard deviation. I am not sure about how to generate lognormal fields, but it is an interesting question and I will do a little research.

guru wrote:
October 3, 2013 at 09:50

Que pequeño es el mundo! Buscando como simular procesos gaussianos en R y aquí te encuentro. Muchas gracias por las instrucciones!

Laura wrote:

Hi, I am working on simulating a gaussian random field and I want to incorporate a y trend (as in your example). I want this

November 27, 2013 at 12:33

trend to be the same as in some data I have (higher values for larger y) – am I correct in thinking that you find the values for beta by looking at the coefficients of the linear model data~x+y? Doing this gives me beta=(0.73,0,-1.2) but I then get the trend in the wrong direction (higher values for smaller y). Where am I going wrong here?

admin wrote:
December 17, 2013 at 13:13

Hi Laura, it's difficult to say with so little information. The procedure you describe is basically right: you fit a linear model and look at the beta coefficients from that regression. I do not know why you get a negative beta for y, you should at your data and try to figure out…

mike give wrote:
December 17, 2013 at 12:15

Hi, this gives one a good starting point. in geoR, how would one differentiate a regular lattice from a fine grid?

admin wrote:
December 17, 2013 at 13:11

Hi Mike, I don't use geoR, I'm sorry.

pd wrote:
April 8, 2014 at 22:59

Hi Sbegueria;

I'm new to this field, and hoping you can clarify some issues– although you don't explicitly indicate this, but should I assume that GSTAT and FIELDS use 'direct sequential simulation' (a new concept for me) versus synthesizing a covariance matrix COV (and then using Cholesky, etc.)? If so, is one limited to generating a grid only with GSTAT, whereas for a COV matrix, any point distribution can work, but at the computational cost of Cholesky?

thanks!

Rocio wrote:
August 19, 2014 at 02:55

Hi, the package produces simulation unconditional but i have the concern that algorithm used to simulation of those realizations.

Nish wrote:
January 27, 2015 at 04:07

Hi,

Thanks for this interesting post.Is that possible to generate spatially correlated multivariate random filed using gstat or any other package?

sbegueria wrote:
March 30, 2015 at 14:44

Hi Nish, although I've never tried it, in principle it should be possible via co-kriging. Please, let us know if you succeded.

Sebastian wrote:
June 10, 2015 at 08:14

Hi Santiago,
Excellent post.
I am using synthetic datasets in my doctoral research and I would appreciate if you could point me to papers where you (or others) have used the approach in this post to generate random fields.

Much appreciated
Sebastian

sbegueria wrote:
June 11, 2015 at 14:30

Sure, this is one example: Beguería S., Pueyo Y. (2009) A comparison of simultaneous autoregressive, generalized least squares models for dealing with spatial autocorrelation. Global Ecology and Biogeography 18, 273–279. – See more at: http://santiago.begueria.es/publications-

4/#sthash.TwPRoNaM.dpuf.
Also, but under review: Bias in the variance of gridded datasets leads to misleading conclusions about changes in climate variability.

Sebastian wrote:
June 17, 2015 at 09:07

Hi Santiago,
First of all, thank you so much for your reply. I don't want to abuse your kindness but in your 2009 paper (the one you cited above) I can't find any reference to generating synthetic spatially autocorrelated data with the method you show in this blog. Furthermore, the data mentioned is from Kissling & Carl (2008), which comes from the volcano dataset. There are also 2 datasets mentioned in which the spatial structure was degraded by sample reduction and coarsening.
Seems that I am missing something on how you used "gstat" or "fields" in generating synthetic data.
Please advice.
Thank you in advance.

sbegueria wrote:
June 17, 2015 at 14:10

Hi Sebastian, you're totally right. We used the dataset of Kissling and Carl on our 2009 paper. Well, I know I used this method for some application when I wrote the post, but I don't remember to have used it in any paper. There is one in press right now, but nothing you can refer to in your thesis, I'm sorry for that. Hopefully in some weeks there'll be one example, when this paper will appear online.

Sebastian wrote:
June 18, 2015 at 19:16

Thank you for your reply Santiago. Looking forward to your upcoming paper!

Andrew Hill wrote:
July 15, 2015 at 08:37

Would you know of code to do the reverse; if you have a general matrix, extract the spatial auto-correlation function?

sbegueria wrote:
April 4, 2016 at 20:54

In the case of spatial data, I'm most familiar with using the functions `variogram` and `fit.variogram` in the `gstat` package. You may want to have a look at the tutorial at https://cran.r-project.org/web/packages/gstat/vignettes/gstat.pdf.

parisa wrote:
September 22, 2015 at 20:52

Hi
I need to cross validate after simulation, how can I do that?
I used grf in package geor to simulate, I can cross validate its output but grf can't simulate the exact formula like z#1 or z#x and so on unlike your commands.
So how can I cross validate your command?

My English isn't good unfortunately.
Thanks in advance

Helen wrote:
September 27, 2015 at 18:45

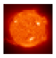
Thank you very much~It's great!

jn wrote:
October 20, 2015 at 13:55

Thank you for this post. It's very useful. I've still one question – how do you generate the first figure (with zero spatial correlation)?

sbegueria wrote:
April 4, 2016 at 20:59

Hi jn, as JoshO also noted there is an error in the first figure (it obviously has spatial correlation in it), I have corrected this in the post.



JoshO wrote:
November 19, 2015 at 21:20

Thanks for this nice post. There are a few +/- important errors that you might want to fix (if that's possible in a blog post): (1) Figure 0 does not, as claimed in the text "show[] an example of a random field with no spatial correlation."; (2) The code block following the phrase "For example, the following defines a model with a spatial trend in the y dimension (Figure 3)" does not do what it claims. It sets formula=z~1+x+y, when it should be formula=z~1+y.

sbegueria wrote:
April 4, 2016 at 21:35

Hi JoshO. You're right, I corrected both issues in the post, thanks a lot.

Simulating spatial datasets with known spatial variability – APAD wrote:
November 8, 2016 at 22:09

[…] These steps are repeated until all observations are given a value (Bivand et al., 2013). Note that another interesting post tackles the simulation of these gaussian random fields in […]

Simulation de données spatiales avec une structure spatiale connue – APAD wrote:
November 15, 2016 at 18:25

[…] ce que toutes les observations reçoivent une valeur (Bivand et al., 2013). A noter cet autre post qui s'intéresse de près à la simulation de champs gaussians aléatoires sur R (et qui a bien […]

sbegueria wrote:
March 22, 2017 at 10:06

I'm not sure that I understood your question. You can incorporate the variogram parameters that match your data directly into the function.

raj wrote:
January 12, 2017 at 12:41

thank you for the post. but what if i have the correlation matrix already and i want to insert it as a given with the mean and cov?

NRS wrote:
February 14, 2017 at 21:28

I am trying to create a 10 x 10 matrix comprised only of whole numbers ranging from 1-10, with each integer represented 10 times (e.g., 10 1's and 10 2's), and a spatial correlation strength of 0.15. I've tried altering your code in both the 'gstat' and 'fields' packages, but cannot quite get the process to work. The closest that I have come is to constrain the matrix to numbers ranging from 1-10, but the output winds up including numbers with decimals. Do you have any suggestions? Thanks!

sbegueria wrote:
March 22, 2017 at 10:15

Hi Nathan. A fast solution could be to just round the values of your resulting field to have zero decimals with function `round` in R. Getting exactly 10 repetitions of each value between 1 and 10 is going to be much more difficult, and I'm not aware of any method to get that. Only approach that comes to mind is to repeat a number of simulated fields and reject all those that do not meet the criteria. There is a pretty efficient way to get a number of simulations with parameter `nsim` of gstat's `predict` method).

Stephanie wrote:
March 22, 2017 at 00:28

Hi, thanks for this post! I have one question as I am completely new to this. I am generating landscapes with various levels of spatial autocorrelation, however, the "landscape" is already composed of cells varying from 0-.50. Is there any way to do exactly as what you explained, but with a raster that has predefined values? In your figures above, I see that the cells have a range of values of about .4 to 1.6.

sbegueria wrote:
March 22, 2017 at 10:03

Hi Stephanie. The parameter `beta` specifies the mean of your Gaussian field. In my example I set it to 1, and that's why the data vary around that value. You can also control the standard deviation / variance of the field with the semivariogram parameters (`psill` and, if you want to use it, `nugget`).

Stephanie wrote:
March 22, 2017 at 15:34

Thanks a lot! This is exactly what I needed.

LEAVE A REPLY

Your email is *never* shared.Required fields are marked *

Comment

Name *

Email *

Website

Post Comment

← Generalized Least Squares regression:

SPEIbase, among the most downloaded items

LINKS

Flickr
GitHub
Global drought monitor
SPEI for R
SPEI site
Vimeo

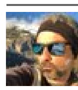
PHOTOS

Versión 2
Versión 2
Versión 2
Untitled

VIDEOS

EWS Aínsa Zona Zero 2108, Stage 5, Men
EWS Aínsa Zona Zero 2108, Stage 5, Women
Principio Almunias
Adiós 2016

TWEETS



Santiago Beguería
@sbegueria

Follow

@Trufforumofic @Europeanmico Roberto Serrano, del grupo EESA de la @EEAD_CSIC, presentando en el Congreso Internaci... twitter.com/i/web/status/1...
about 3 days ago via Twitter for iPhone

Reply Retweet Favorite

I just uninstalled the @Airbnb mobile app after discovering that it is sending tracking information (presumably inc... twitter.com/i/web/status/1...
about 2 months ago via Twitter Web App

Reply Retweet Favorite

Stoked to appear in the list of the 1% most cited scientists of 2018! twitter.com/AragonCsic/sta...
about 2 months ago via Twitter Web App

Reply Retweet Favorite

ABOUT

This is a personal website of Santiago Beguería, Physical Geographer and tenured scientist at the Spanish National Research Council (Consejo Superior de Investigaciones Científicas - CSIC).

SEARCH

Search for:

Search

META

Log in
Entries RSS
Comments RSS
WordPress.org