Appendix

1 Artificial data

1.1 STARMA models

Considering N fixed locations in space, observations of a random variable are generated for T time periods. The model is specified by Eq. 1 [1],

$$\begin{aligned}\mathbf{z}(t) =& \sum_{k=1}^{p}\sum_{i=0}^{\lambda_k}\phi_{kl}\mathbf{W}^{(l)}\mathbf{z}(t-k) \\ &- \sum_{k=1}^{q}\sum_{i=0}^{m_k}\theta_{kl}\mathbf{W}^{(l)}\boldsymbol{\epsilon}(t-k)+\boldsymbol{\epsilon}(t)\end{aligned} \tag{1}$$

where $\boldsymbol{z}(t)$ is a $N \times 1$ vector of observations at time t, I is the identity matrix, $W^{(l)}$ is a $N \times N$ square matrix of weights where element (i, j) is only non-zero if locations i and j are neighbours of l^{th} order with rows summing to one, p is the autoregressive order, q is the moving average order, λ_l is the spatial order of the k^{th} autoregressive term, m_k is the spatial order of the k^{th} moving average term, ϕ_{kl} and θ_{kl} are parameters, and the $\epsilon_l(t)$ are random normal errors respecting Eqs. 2 and 3.

$$E[\epsilon_l(t)] = 0 \tag{2}$$

$$E[\epsilon_l(t)\epsilon_j(t+s)] = \begin{cases}\sigma^2 & l=k, s=0 \\ 0 & otherwise\end{cases} \tag{3}$$

Non-linear versions of STAR models (based on non-linear AR models in [2]) are generated by applying a non-linear function f (cf. Eq. 4) to each $\mathbf{z}_l(t-k)$, f being randomly selected between $\sin(x)$, $\cos(x)$, $\arctan(x)$, $\tanh(x)$ and $\exp(-\frac{x}{C})$, with $C = 1 \times 10^4$.

$$\mathbf{z}(t) = \sum_{k=1}^{p}\sum_{l=0}^{\lambda_k}\phi_{kl}\mathbf{W}^{(l)}f(\mathbf{z}(t-k)) \tag{4}$$

1.2 Stationarity conditions

Stationarity, meaning that the covariance structure of $\mathbf{z}(t)$ does not change with time, requires that every x_u that solves Eq. 5 lies inside the unit circle ($|x_u| < 1$).

$$det\left[x_u^q\mathbf{I} - \sum_{k=1}^{q}\sum_{i=0}^{m_k}\theta_{ki}\mathbf{W}^{(i)}x_u^{q-k}\right] = 0 \tag{5}$$

Low-order STARMA stationarity

A STARMA(2_{11}) is defined by the following equation:

$$
\begin{aligned}
\boldsymbol{z}(t) = (\phi_{10} I + \phi_{11} \boldsymbol{W^{(l)}}) \boldsymbol{z}(t-1) && (6)\\
+ (\phi_{10} I + \phi_{21} \boldsymbol{W^{l}}) \boldsymbol{z}(t-2) + \boldsymbol{\epsilon(t)} && (7)\\
+ (\theta_{10} I + \theta_{11} \boldsymbol{W^{(l)}}) \boldsymbol{\epsilon}(t-1) && (8)\\
+ (\theta_{10} I + \theta_{21} \boldsymbol{W^{(l)}}) \boldsymbol{\epsilon}(t-2) + \boldsymbol{\epsilon(t)} && (9)
\end{aligned}
$$

Stationarity restrictions for STARMA(2_{11}) models can be written as below for the AR component (ϕ_{kl} coefficients) [3].

$$
\begin{aligned}
-\phi_{20} + |\phi_{21}| &< 1 \\
|\phi_{10} + \phi_{11}| &< 1 - \phi_{20} - \phi_{21} \\
|\phi_{10} - \phi_{11}| &< 1 - \phi_{20} + \phi_{21}
\end{aligned}
$$

The same set of restrictions apply to the MA terms (θ_{kl}).

1.3 Random coefficient generation

Coefficients are generally randomly generated within intervals that present reasonable chance of respecting stationarity conditions. In the case of order 2_{11}, one of the coefficients is fixed at a random value first and the remaining three coefficients are generated within intervals informed by this first selection (cf. Tab. 1).

Table 1: Model coefficients, c_{XY} corresponding to ϕ_{XY} and/or θ_{XY}. Coefficients are fixed or generated within the presented intervals.

Model order	c_{10}	c_{11}	c_{20}	c_{21}
2_{10}	$[-2, 2]$	$[-2, 2]$	$[-1, 1]$	0
2_{01}	$[-2, 2]$	0	$[-1, 1]$	$[-2, 1]$
2_{11}	$[-1.227, 0.733]$	$[0.733, 1.277]$	$[-0.227, 1.773]$	-0.7333
2_{11}	$[-1.755, 0.245]$	$[-1.755, 1.755]$	$[-0.7555, 0.7555]$	0.245

Bibliography

[1] Phillip E Pfeifer and Stuart Jay Deutsch. A Three-Stage Iterative Procedure for Space-Time Modeling. *Technometrics*, 22(1):35—-47, 1980.

[2] Christoph Bergmeir and José M. Benítez. On the use of cross-validation for time series predictor evaluation. *Inf. Sci. (Ny).*, 191:192–213, 2012.

[3] P. E. Pfeifer and S. J. Deutsch. Stationarity and invertibility regions for low order starma models. *Commun. Stat. Comput.*, 9(5):551–562, 1980.