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Specification of spatial models: A simulation study on weights matrices

Stanislav Stakhovych[1], **Tammo H.A. Bijmolt**[1]

[1] Marketing Department, Faculty of Economics and Business, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands (e-mail: s.stakhovych@rug.nl, t.h.a.bijmolt@rug.nl)

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Abstract. The correct specification of spatial models, and especially the choice of the spatial weights matrix, represents a crucial decision for researchers using georeferenced data. However, few guidelines exist on which weights matrix is most appropriate in certain cases. This paper therefore (1) studies the sensitivity of testing and estimating spatial models with different weights matrix specifications and (2) formulates recommendations for researchers applying spatial models regarding model selection and weights matrix specification. The research is based on Monte Carlo simulations with synthetic data.

JEL classification: C12, C13, C15, C52

Key words: Spatial econometrics, weights matrices, model selection, parameter estimates, Monte Carlo simulations

1 Introduction

Imagine a researcher who plans to analyse data that could be correlated over space. This researcher knows that ignoring spatial dependency could lead to inefficient and biased estimates, invalid inference procedures, and, as a result, wrong conclusions (Cliff and Ord 1981). In many disciplines, applying spatial econometric models provides the most common approach for avoiding such mistakes, as in regional sciences (Goodchild and Haining 2004), economics (Anselin 2002), and marketing (Bronnenberg 2005).

However, when applying spatial models, the researcher must determine several model specification issues, such as the structure of spatial dependence and the type of spatial weights. Some studies indicate that exogenous decisions, especially those related to the choice of the spatial weights matrices, are crucial and can affect the findings of the research project (for an overview, see Florax and de Graaff 2004). However, "[t]here is very little formal guidance in the choice of the 'correct' spatial weights in any given application" (Anselin 2002, p. 289). Therefore, we provide new insights into the consequences of alternative specifications of the spatial weights matrix by addressing the following primary questions: (1) How do specification testing decisions result from the type of weights matrix, and (2) how sensitive are the parameter estimates of a model to the type of weights matrix used? On the basis of the results of Monte Carlo simulations, we answer these questions and formulate practical recommendations.

The structure of the paper is as follows: section 2 briefly describes models, weights matrices and tests used in the experiment. In addition, we review the extant literature on weights matrices. Section 3 proceeds with the experimental design, and section 4 reports the findings of our Monte Carlo simulation experiment. Finally, in section 5, we provide conclusions, recommendations and limitations.

2 Spatial models, weights matrices and tests used in experiment

2.1 Types of spatial models

Taking spatial dependencies into account when dealing with spatial data is very important; neglecting them can cause many problems. For example, ignoring spatial lag structures causes ordinary least squares (OLS) estimators to become biased and inconsistent; ignoring spatial error structures makes OLS unbiased, but it becomes inefficient (Cliff and Ord 1981). Various models attempt to deal with spatial dependencies, the most commonly used of which are spatial lag and spatial error models (Equations 1 and 2 respectively).

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{1}$$

$$\begin{aligned} \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} &= \lambda\mathbf{W}\boldsymbol{\varepsilon} + \mathbf{u}, \end{aligned} \tag{2}$$

where $\mathbf{y}$ is an $n \times 1$ vector of observations on the dependent variable; $\mathbf{W}$ is an $n \times n$ spatial weights matrix; ρ and λ are spatial autoregressive parameters; $\mathbf{X}$ is an $n \times k$ matrix of observations on the exogenous variables, with associated $k \times 1$ regression coefficient vector $\boldsymbol{\beta}$; $\boldsymbol{\varepsilon}$ and $\mathbf{u}$ are vectors of error terms.

2.2 Spatial weights matrices

The spatial weights matrix $\mathbf{W}$ is an $n \times n$ nonnegative matrix that specifies the 'neighbourhood set' for each observation. A location (observation) appears as both row and column, and nonzero matrix elements w_{ij} indicate the relation between the observations (row) i and (column) j. By convention, self-neighbours are excluded, such that the diagonal elements $w_{ii} = 0$. Moreover, the weights matrix often appears in row-standardized form, with weights $w_{ij}^s = w_{ij} / \Sigma_j w_{ij}$. Row standardization facilitates an interpretation of the weights as constructing a weighted average of the neighbouring values through the so-called spatial lag operator, which can be applied to the dependent variable $\mathbf{W}\mathbf{y}$ (spatial lag model in Equation 1) or the error term $\mathbf{W}\boldsymbol{\varepsilon}$ (spatial error model in Equation 2) (Anselin 1988).

2.3 Literature review on spatial weights matrices

There are three important questions regarding weights matrices which arise for researchers working with spatial models: (1) How the weights matrix can be constructed? (2) Does specification of the weights matrix influence the tests? and, (3) Does it influence the parameter estimates?

First, the weights matrix is an important part of spatial modelling and is defined as the formal expression of spatial dependence between observations (Anselin 1988). The literature on specification of weights matrices is quite extensive and can be divided into three streams: (1) treating weights matrices as completely exogenous constructs; (2) letting the data determine them; and (3) estimating them. The first approach is often based on the geographical relations of observations or spatial units that contain these observations. Examples of weights matrices used in this approach are those determined by spatial contiguity, inverse distance, share of common border, centroids, *N* nearest neighbours, etc. (Cliff and Ord 1981; Anselin 1988; Anselin and Bera 1998). Next to geographical consideration, several specifications were suggested which arise from social networks and economic distance (i.e. Case et al. 1993; Conley and Ligon 2002; Leenders 2002). Another approach is to allow the existing data to determine the weights matrix. For example, Getis and Aldstadt (2004) propose a weights matrix that is based on the distance beyond which there is a specific change in the nature of spatial association. Aldstadt and Getis (2006) further develop an algorithm which constructs a spatial weights matrix using empirical data and simultaneously identifies the geometric form of spatial clusters. Finally, the third approach is to estimate the weights matrix. Due to the large number of weights matrix elements compared to the number of observations, certain constraints have to be imposed. For instance, Bhattacharjee and Jensen-Butler (2006) proposed a nonparametric approach for estimation of a spatial weights matrix based on consistent estimators for the spatial autocovariances. However, the weights matrix should be constrained to be symmetric, which in many cases does not represent the real-life situation. Going further, there are even attempts to get rid of weights matrix by using structural equations models with latent variables to model spatial dependence (Folmer and Oud 2008).

Second, there are several papers on the impact of weights matrices on testing procedures. It has been shown that tests on spatial dependence have greater power in less connected spatial configurations (Anselin and Rey 1991). Florax and Rey (1995) reveal that the power of a test increases when the connectivity function used in the test specification matches that of the data generating process. These results were explained by the recent research of Smith (2008) and Farber et al. (2009). The key to understand the power of tests according to these studies is the topology of the system as represented by the weights matrix (Farber et al. 2009), or the bias introduced in the estimation of spatial parameter by highly connected matrices (Smith 2008).

Finally, the third question regarding the influence of different weights matrix specifications on the estimation procedure gained also a lot of attention. Stetzer (1982) shows that the specification of weights is important for parameter estimation, especially when sample sizes are small and the data are autocorrelated. Griffith and Lagona (1998) prove that a misspecified weights matrix has serious impacts on finite-lattice properties of maximum likelihood (ML) estimators, one of which is a loss of efficiency. Also, in a recent paper Smith (2008) shows that strongly connected weights matrices introduce a downward bias for ML estimates of spatial dependence parameter.

Correct specification of spatial weights is an important and, unfortunately, not yet solved problem. Several guidelines and models for spatial weights were proposed (for example, Griffith 1996; Bavaud 1998), but still many unanswered questions remain. This paper aims to close one of the existing gaps in the literature by investigating the influence of weights matrices, which are constructed based on geographical relations between observations, on testing and estimating results.

2.4 Specification testing

The first logical step in analysing spatial data is finding the correct specification of the model. The two major specification search strategies are the Hendry approach and the classical approach (Florax et al. 2003). The Hendry approach starts with a very general, overparameterized model that gets progressively simplified through a sequence of specification tests. In contrast, the classical approach starts from the simplest specification and uses step-by-step modifications of the model to create a more complex one. Applied spatial econometric modelling uses the classical approach almost exclusively, and Florax et al. (2003) demonstrate that the Hendry approach is dominated by the classical approach with respect to detecting spatial dependence. Therefore, we use the classical approach in this paper.

Various testing procedures capture spatial dependency in data, and the most common for spatial correlation are Moran's I statistics, the Kelejian-Robinson test, and Lagrange Multiplier (LM) tests. Each has certain specific advantages, but an important advantage of LM tests is their ability to test between spatial lag and spatial error specifications. Therefore, we apply LM tests in this study.

Two basic LM tests consider spatial dependency: LM_{lag} has power against the spatial lag model, whereas LM_{error} has power against the spatial error model. Both statistics are chi-square distributed with one degree of freedom. The LM_{error} test, as proposed by Burridge (1980), can be computed as follows:

$$\begin{aligned} \mathrm{LM}_{\mathrm{error}} &= \frac{\left(\hat{\mathbf{e}}'\mathbf{W}\hat{\mathbf{e}}/\hat{\sigma}^2\right)^2}{T} \\ T &= \mathrm{Tr}\left[(\mathbf{W}'+\mathbf{W})\mathbf{W}\right] \end{aligned} \tag{3}$$

where $\hat{\mathbf{e}}$ is a vector of OLS residuals, $\hat{\sigma}^2 = \hat{\mathbf{e}}'\hat{\mathbf{e}}/n$ with n as the number of observations, and Tr is a trace operator. The LM_{lag} test, as outlined by Anselin (1988), takes the following form:

$$\begin{aligned} \mathrm{LM}_{\mathrm{lag}} &= \frac{\left(\mathbf{e}'\mathbf{W}\mathbf{y}/\hat{\sigma}^2\right)^2}{nJ} \\ J &= \frac{1}{n\hat{\sigma}^2}\left[\left(\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}\right)'\mathbf{M}\left(\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}\right)+T\hat{\sigma}^2\right] \\ \mathbf{M} &= \mathbf{I}-\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' \end{aligned} \tag{4}$$

where $\mathbf{I}$ is an identity matrix of size n, and $\hat{\boldsymbol{\beta}}$ are OLS parameter estimates. Both LM tests consider the null hypothesis of no spatial dependency. To work under local misspecification, robust LM tests for spatial error autocorrelation in the potential presence of a spatially lagged dependent variable, as well as for endogenous spatial lag dependence in the potential presence of spatial error autocorrelation were designed (Anselin et al. 1996). However, a recent experiment shows that "the classical approach dominates the robust approach, in terms of power and accuracy" (Florax et al. 2003, p. 578). Therefore, we apply the classical LM diagnostics in our simulations. This approach suggests that if LM_{error} is significant and LM_{lag} is not, the spatial error model should be chosen, and vice versa. If both are significant, the specification associated with the more significant of the two tests is the correct choice. Finally, if both statistics are insignificant, researchers should use the spatial independent model.

3 Experimental design

To obtain insights into the effects of possible alternative specifications of the spatial weights, we conduct an experiment using Monte Carlo simulations. We generate synthetic data sets that systematically vary on several relevant factors (Table 1).

To generate and analyse the data sets, we proceed through the following steps.

Table 1. Design of the Monte Carlo simulations

Design factor	Levels	Number of levels
Type of spatial model	spatial lag model, spatial error model	2
Value of spatial parameter (ρ or λ)	[0.0,0.9] with 0.1 increment	10
Types of weights matrix for data generation	first order contiguity, N nearest neighbours, inverse distance	3
Types of weights matrix for testing and estimating the model	first order contiguity, N nearest neighbours, inverse distance	3
Relative variance of error terms (ξ)	0.2, 1.0, 5.0	3
Total number of unique simulations	$2 \times 10 \times 3 \times 3 \times 3 =$	540
Total number of replications		1,000
Total number of simulations	$540 \times 1000 =$	540,000

In the first step, we generate two-dimensional spatial patterns for the observations. At this point the question arises whether regular or irregular tessellations should be used for the experiment. Previous research, for instance, Stetzer (1982), Florax and Rey (1995), and Florax et al. (2003), made use of regular tessellations. However, Farber et al. (2009) recently presented results indicating that regular and irregular tessellations are not comparable in terms of their topologies and, more importantly, not comparable in the way they affect the performance of tests. Farber et al. (2009) argue in favour of using irregular tessellations for experimental spatial statistics, because they produce more realistic spatial patterns than regular tessellations. Therefore, we use Voronoi polygons instead of square lattices. In particular, we draw both co-ordinates of 100 points, which represent the same number of observations. Next, we construct the distinct geographical units by means of Voronoi tessellation. Such tessellation ensures that every spatial unit contains only one observation. To exclude the possible influence of certain spatial patterns on the results, we generate a new pattern for each replication. Further, on the basis of each generated spatial pattern, we compute the following weights matrices: (1) first-order contiguity (W_{cont}), the elements of which are $w_{ij} = 1$ when i and j are geographical neighbours, and 0 otherwise; (2) N nearest neighbours (W_{nnear}), the elements of which are $w_{ij} = 1$ when j is among the $N = 10$ nearest neighbours of i; and (3) inverse distance (W_{dist}) the elements of which are $w_{ij} = 1/d_{ij}$, where d_{ij} is the distance between units i and j. All weights matrices are row-standardized.

In the second step, we generate the explanatory variables $\mathbf{X}$ with the intercept and two variables drawn from the uniform distribution U(–5,5). The vector $\boldsymbol{\beta}$ consists of three coefficients: intercept $\alpha = 1$ and two slopes, $\beta_1 = 0$ and $\beta_2 = 1$. Further, we generate the error terms $\boldsymbol{\varepsilon}$ which are assumed to be normally i.i.d. distributed, with mean 0 and variance σ^2. At this point we introduce a new variable ξ, which is the fraction of the variance of error terms over the variance of $\mathbf{X}\boldsymbol{\beta}$, that is, $\xi = var(\boldsymbol{\varepsilon})/var(\mathbf{X}\boldsymbol{\beta})$ and set ξ equal to 0.2, 1.0, and 5.0. Variable ξ has the following interpretation. Under the condition of no spatial correlation, namely, $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, the relation between the coefficient of determination and ξ is $R^2 = 1/(1 + \xi)$. This equality holds because by definition $R^2 = var(\mathbf{X}\boldsymbol{\beta})/[var(\mathbf{X}\boldsymbol{\beta}) + var(\boldsymbol{\varepsilon})]$ and by construction $\xi = var(\boldsymbol{\varepsilon})/var(\mathbf{X}\boldsymbol{\beta})$, where $\mathbf{X}$ and $\boldsymbol{\beta}$ are set in the experiment. For comparison, the levels of ξ equal to 0.2, 1.0, and 5.0 are equivalent to R^2 equal to 0.83, 0.5, and 0.17 respectively under the condition of spatial independency. For instance, in their simulation paper Florax et al. (2003) use such a variance level that ensures that the coefficient of determination R^2 averages 0.55 for the model with no spatial correlation and they argue that this average provides a representative value for applied spatial research.

In the third step, we compute the dependent variable $\mathbf{y}$ for two models: the spatial lag model $\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ and the spatial error model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon} = \lambda\mathbf{W}\boldsymbol{\varepsilon} + \mathbf{u}$, using $\mathbf{W}$, $\mathbf{X}$, and the errors obtained in the second step. Specifically, we compute the dependent variable for all three weights matrices defined in the first step and for 10 different values of spatial parameters ρ and λ, taken from the interval [0.0, 0.9] at increments of 0.1. The broad range of spatial parameters ensures that all degrees of spatial dependency, from an absence (0.0) to very strong (0.9), come into play. We do not study negative spatial coefficients, which are very rare and have weak theoretical explanations.

In step four, we apply the LM tests described in section 2 to the generated data sets by specifying the weights matrix as part of the model input. The significance level of tests is set at 0.025, which is also used in Florax et al. (2003). We use all three possible **W** for the LM tests to assess how the application of different weights matrices influences the probability of identifying the true model specification. In other words, we cover nine combinations of the spatial weights matrices used to generate the data and test the model.

In the next step, we estimate the true spatial lag or error model with maximum likelihood method using the algorithms written by LeSage and available in the Spatial Econometrics Toolbox for MATLAB. Thus, we obtain model estimates for each of the three spatial weights matrices.

As part of the empirical research process in practice one must select a weights matrix from a set of alternatives for testing and estimating purposes. Therefore, in step six of the procedure, we use model fit criteria to determine which of the three weights matrices is most appropriate. For this purpose, we compare the information criteria $\mathrm{TC} = -2L\left(\hat{\rho}/\hat{\lambda}, \hat{\boldsymbol{\beta}}\right) + q(k)$ across the three models, as suggested by Leenders (2002), such that $L\left(\hat{\rho}/\hat{\lambda}, \hat{\boldsymbol{\beta}}\right)$ is the log-likelihood evaluated at the maximum likelihood estimates of the spatial parameter $\hat{\rho}$ (spatial lag model) or $\hat{\lambda}$ (spatial error model) and the regression parameters $\hat{\boldsymbol{\beta}}$, and $q(k)$ is a penalty function based on the number of model parameters k. The function $q(k)$ varies among the different information criteria; common specifications such as $q(k) = 2k$ or $q(k) = (\ln(n) + 1)k$ yield the Akaike (AIC; Akaike 1974) and consistent Akaike (CAIC; Bozdogan 1987) information criteria, respectively, where n is the number of observations. In our case, n and k are constant across all model specifications, so the decision heuristics of selecting the minimum AIC or CAIC reduces to selecting the model with the highest log-likelihood $L\left(\hat{\rho}/\hat{\lambda}, \hat{\boldsymbol{\beta}}\right)$.

Finally, in step seven, we compare the estimates of betas and spatial parameters with the true values. In particular, we analyse the mean squared error (MSE) of the parameter estimates with ANOVA.

The abovementioned steps of Monte Carlo simulation can be summarized as follows:

1. Generate a random spatial pattern and compute the corresponding weights matrices **W**.
2. Generate explanatory variables **X** and error terms $\boldsymbol{\varepsilon}$ with different variances.
3. Compute the dependent variable **y** for the spatial lag and error models with different types of weights matrices **W** and values of the spatial parameter.
4. Execute the specification testing procedure with the LM diagnostics using the three different weights matrices.
5. Estimate the true model (lag or error) with maximum likelihood method using the three different weights matrices.
6. Choose among the three alternative weights matrices **W**.
7. Compare the true parameter values with the ML estimates obtained using the true model specifications (spatial lag or spatial error model and correct weights matrix).

For each combination of design factors, we replicate the entire data generation and model estimation process 1,000 times (Table 1). Next, we examine the extent to which the LM tests correctly identify a spatial lag or error model (step four), how often the procedure identifies the correct weights matrix **W** (step six), and the extent to which the parameter estimates correspond to the true values (step seven). In particular, we assess the role of alternative spatial weights matrices with regard to these three issues. On every replication, the testing procedures suggest one of the weights matrices and one of the three model alternatives (i.e., spatial lag, spatial error, or spatial independent model). Averaging across 1,000 replications, we obtain the probability of finding the true model for different scenarios. In section 4.1, we analyse the model identification using ANOVA of the logit of such probabilities. In section 4.2, an ANOVA of the MSE analyses the estimates of spatial parameters and betas. As a measure of effect size, we also report partial eta-squared (η_p^2), which is computed as $\eta_p^2 = SS_{effect} / (SS_{effect} + SS_{error})$, where SS_{effect} is the variation attributable to a particular effect and SS_{error} is the error variation. It is important to note the difference between the classical eta-squared η^2 and partial eta-squared η_p^2. Classical eta-squared is defined as a proportion of the total variation that is attributed to an effect and computed as $\eta^2 = SS_{effect}/(SS_{total})$, where $SS_{total} = SS_{effect} + SS_{other\ effects} + SS_{error}$. Both measures lie in the interval from 0 to 1, but the crucial difference between them is that classical eta-squares η^2 sum to 1 and partial eta-squares η_p^2 do not.

4 Experimental simulation results

4.1 Model identification

Researchers likely are interested in the type of spatial model (lag or error model) to choose and the probability of applying the correct model. Therefore, we analyse the effects of design factors on the logit of the probability of finding the true model. We also consider which weights matrix is most appropriate in different cases.

4.1.1 Selecting between the spatial lag model and the spatial error model

As apparent from Equations 3 and 4, the LM tests depend on the spatial weights matrix **W**. Therefore, the probability of recovering the true spatial model may depend on the specification of the weights matrix. The main and interaction effects of the weights matrix and other design factors on the logit of the probability of recovering the true model appear in Table 2. We are especially interested in how the different combinations of weights matrices influence the probability of finding the true model and whether this probability depends on other factors, such as lag versus error models. We do not report the parameter estimates of the logit regression in the paper, because of the large number of coefficients. Instead, we discuss the direction and strength of the effects in the text and plot our main findings in Figures.

The ANOVA model fits the data very well, with an adjusted R^2 of 0.939. All effects are significant at the 0.01 level, except the interaction effect $Model \times W_{test}$ which is not significant. We plot the average probabilities of finding a true model in Figure 1(a) for the lag model and Figure 1(b) for the error model.

This initial analysis provides several important findings. First, the effect of the factor *Model* is significant and substantial, with $\eta_p^2 = 0.344$. The probability of detecting the true model is substantially higher for the spatial lag model than for the spatial error model. This finding confirms previous Monte Carlo studies, such as Anselin and Rey (1991), Anselin et al. (1996), and Florax et al. (2003).

Second, the size of the spatial parameter has a significant impact on the probability of recovering the true model, as indicated by the significance of the factor *Spat* and η_p^2 of 0.932. The

Table 2. Effects on the logit of the probability of recovering the true model

Source	Df	F	Sig.	η_p^2
Intercept	1	736.264	<0.001	0.596
Type of the model (*Model*)	1	262.585	<0.001	0.344
True value of spatial parameter (*Spat*)	9	763.657	<0.001	0.932
Weights matrix for generating data (W_{gen})	2	57.840	<0.001	0.188
Weights matrix for testing data (W_{test})	2	7.810	<0.001	0.030
Error terms' variance (ξ)	2	295.175	<0.001	0.541
$W_{gen} \times W_{test}$	4	95.448	<0.001	0.433
$Model \times W_{gen}$	2	7.252	0.001	0.028
$Model \times W_{test}$	2	0.981	0.376	0.004
$Model \times W_{gen} \times W_{test}$	4	6.861	<0.001	0.052
$Model \times Spat$	9	3.618	<0.001	0.061
$Model \times \xi$	2	31.832	<0.001	0.113

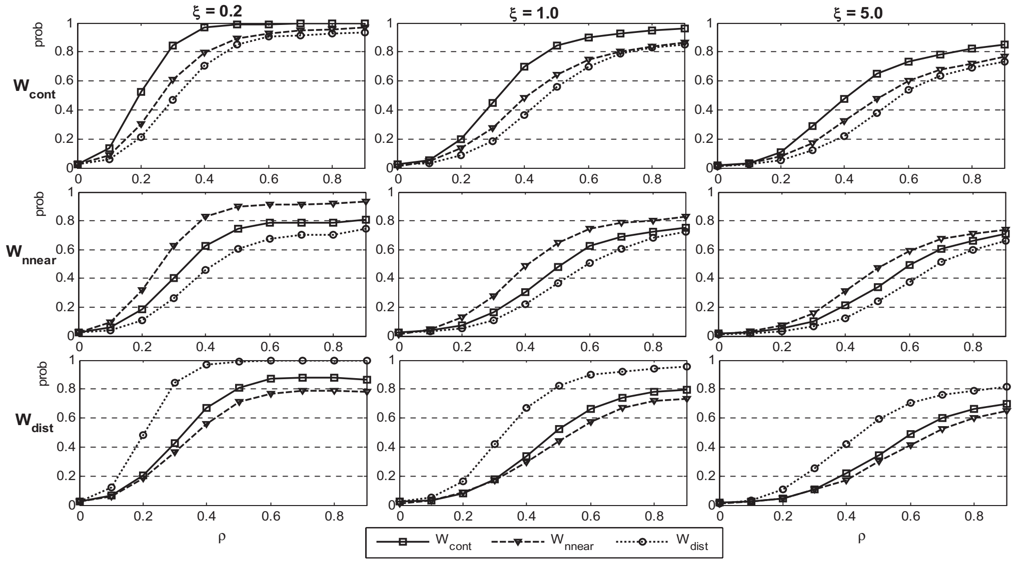
Notes: $R^2 = 0.944$ (Adjusted $R^2 = 0.939$).

higher the value of the spatial parameter, the greater the probability of finding the true model. Furthermore, the significant interaction $Model \times Spat$ ($\eta_p^2 = 0.061$) indicates that the true value of the spatial parameter differently influences lag and error models. Specifically, when we compare the corresponding plots in panels (a) and (b) of Figure 1, we find that the probability of correctly identifying the model increases faster at low to moderate values of the spatial parameter for the lag model, compared with the same probability for the error model.

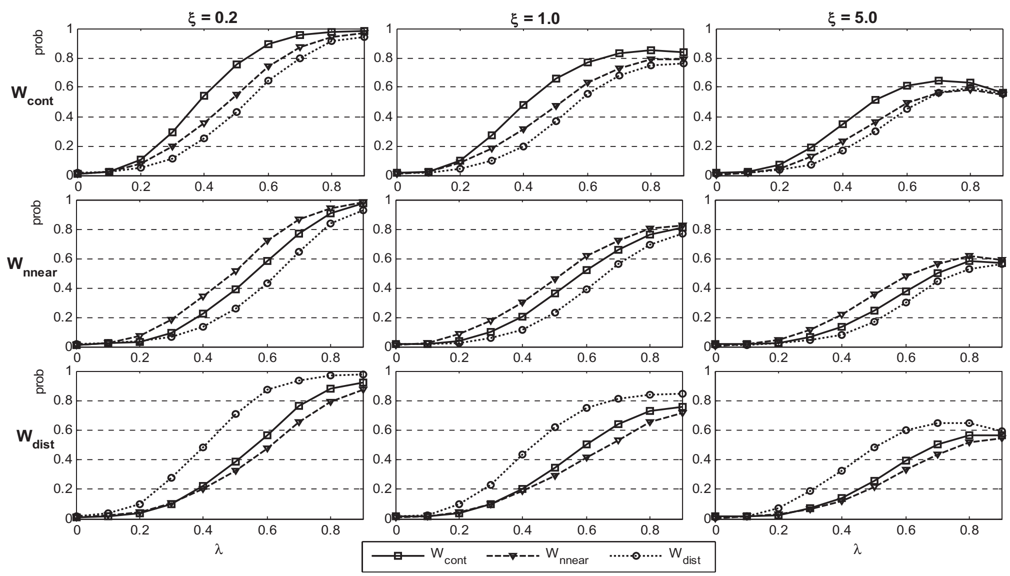
Third, the variance of the error terms significantly influences the probability of finding the true specification (significant ξ in Table 2, with $\eta_p^2 = 0.541$). This effect differs for lag and error models (significant $Model \times \xi$ with $\eta_p^2 = 0.113$); the parameter estimates (which are not explicitly reported here) and Figure 1 shows that the spatial error model suffers more from high error variance than does the spatial lag model.

With respect to the weights matrices, we conclude that their combinations, in terms of generating the data and testing the model, actually matter for the probability of finding the true specification ($W_{gen} \times W_{test}$ in Table 2 is significant, with $\eta_p^2 = 0.443$). Furthermore, these effects differ between lag and error models (interaction $Model \times W_{gen} \times W_{test}$ is significant, with $\eta_p^2 = 0.052$). As Figure 1(a) and Figure 1(b) show, if the weights matrix used to test the model is the same as the one used to generate the data, the probability of finding the true spatial model increases compared with other combinations of weights matrices. This result confirms findings by Stetzer (1982), Florax and Rey (1995) and Kelejian and Robinson (1998). Also, Figure 1 reveals that the first order contiguity weights matrix identifies the true model more frequently than other matrices. This is consistent with findings of Farber et al. (2009), namely that the degree of connectivity is negatively related to the power of a test. Specifically, they showed that less connected weights matrices (W_{cont} in our study) perform better in tests than the matrices with high connectivity (W_{dist} in our study).

Averaging the probability of finding a true model across all design factors except the weights matrices provides further interesting results (Table 3). When using the correct weights matrix, the $W_{cont} - W_{cont}$ combination performs best, with a 0.536 probability of recovering the true model; the second best is $W_{dist} - W_{dist}$ with a 0.522 probability, and the worst is $W_{nnear} - W_{nnear}$ with a 0.444 probability. When the right matrix is not chosen, the ranking is as follows: $W_{cont} - W_{nnear}$ achieves a probability of 0.451, followed by $W_{cont} - W_{dist}$ with 0.403 and the worst one is $W_{nnear} - W_{dist}$ with 0.311. This is consistent with Paez et al. (2008), namely that adding irrelevant connections to a matrix degrades the power of tests. Specifically, it explains why matrix W_{nnear} performs worse and W_{dist} worst than W_{cont} when the true matrix is first order contiguity (Table 3).



(a) Lag model



(b) Error model

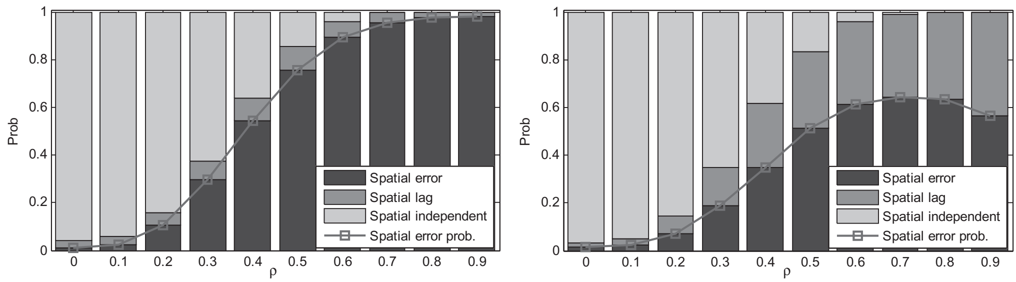
Fig. 1. Probability of finding the true model specification

Notes: Rows are weights matrices W used to generate the data; columns represent different values of ξ. The different curves correspond to alternative weights matrices W used to test the model. Abscissa on each plot is the true value of the spatial parameter (ρ or λ); ordinate is the probability of finding the true model specification.

Averaging the probabilities across the matrices used for data generation (averaging across columns in Table 3) produces the following results for model testing: The best performance comes from the first-order contiguity matrix, with an average probability of 0.430; second best is the inverse distance matrix with 0.412; and the third best is the N nearest neighbours weights matrix, whose probability of finding the correct specification is 0.411. However, the difference between the last two is negligible.

Table 3. Probability to find a true model depending on the types of weights matrices

		W for testing data			
		W_{cont}	W_{nnear}	W_{dist}	Total
W for generating data	W_{cont}	0.536	0.451	0.403	0.463
	W_{nnear}	0.374	0.444	0.311	0.376
	W_{dist}	0.381	0.338	0.522	0.413
	Total	0.430	0.411	0.412	0.418



(a) Error model, $\xi = 0.2$ (a) Error model, $\xi = 5.0$

Fig. 2. Distribution of specification testing decisions for spatial error as a true model, with $W_{gen} = W_{test} = W_{cont}$

In general, higher values of the spatial parameter lead to higher probabilities of finding the correct model specification. However, in the case of a spatial error model combined with a high error variance ($\xi = 5.0$), the probability of finding the true specification does not monotonically increase (Figure 1(b)) but rather increases up to $\lambda = 0.7$ and then decreases. To explore this finding in more details, we present, in Figure 2, the relative frequencies of identifying the model as a spatial lag, spatial error, or spatial independent model for two cases in which spatial error is the true model, and a first-order contiguity weights matrix is used for generating the data and further its testing. Hence, these investigations are extensions of the upper-left and upper-right plots of Figure 1(b). Nonmonotonicity occurs because for high error variances, the LM diagnostics identify the spatial error model as a spatial lag model. If the spatial error structure contains a large proportion of variance in the data and these errors induce a high correlation between neighbouring observations, the observed patterns in the data closely resemble those of a spatial lag structure.

Taking a close look at Figure 1(a) also reveals an interesting case. The upper lines on graphs (1,1) and (3,1) are similar in shape and the maximum achieved value of probability. However, the upper line on graph (2,1) differs by a downward shift. In other words, if the true data were generated with the *N* nearest neighbours weights matrix, the spatial dependency could be captured less precisely compared with other matrices, possibly because of the parameter *N* which defines how many neighbouring regions influence the target region. To clarify its influence on model identification, we run additional analyses on the *N* nearest neighbours weights matrix but consider both lag and error models. We take four different values of *N*, namely, 5, 10, 25, and 50, and present the probability lines for the lag and error models in Figure 3.

The average number of neighbours used to construct the first-order contiguity weights matrix is 4.5. Therefore, for *N* equals 5, the *N* nearest neighbours matrix performs similarly to the first-order contiguity one. Increasing *N* leads to a decrease in the probability of identifying

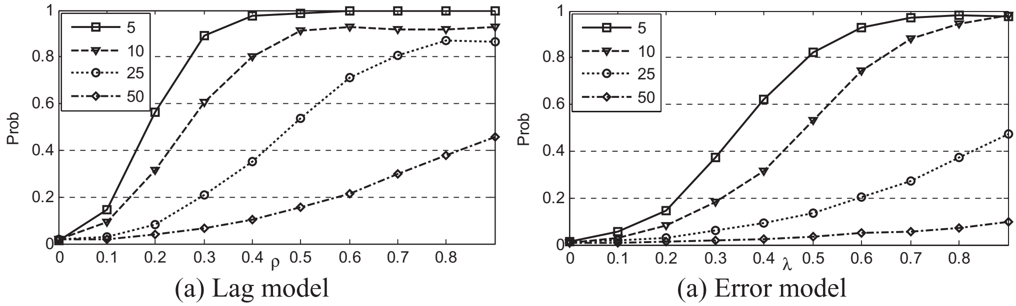


Fig. 3. Probability of finding the true specification for different values of N when $W_{gen} = W_{test} = W_{nnear}$ and $\xi = 0.2$

Table 4. Effects on the logit of the probability of recovering the true weights matrix

Source	Df	F	Sig.	η_p^2
Intercept	1	1,884.979	<0.001	0.925
Type of the model (*Model*)	1	108.919	<0.001	0.417
True value of spatial parameter (*Spat*)	9	244.865	<0.001	0.935
Weights matrix for generating data (W_{gen})	2	40.279	<0.001	0.346
Error terms' variance (ξ)	2	38.679	<0.001	0.337
$Model \times W_{gen}$	2	0.989	0.374	0.013
$Model \times Spat$	9	4.182	<0.001	0.198
$Model \times \xi$	2	33.627	<0.001	0.307

Notes: $R^2 = 0.944$ (Adjusted $R^2 = 0.934$).

the true model. Particularly poor performance starts after $N > 10$, especially for the spatial error model. Anselin and Rey (1991) reach a similar conclusion and assert that test strength erodes as the extent of connectivity increases.

4.1.2 Choosing the weights matrix

The main and interaction effects on the logit of the probability of recovering the true weights matrix appear in Table 4. The ANOVA model fits the data very well, with an adjusted R^2 of 0.934. All effects are significant at the 0.01 level, except the interaction effect $Model \times W_{gen}$, which is not significant. We plot the average probabilities of finding the true weights matrix in Figure 4.

An analysis of Figure 4 and Table 4 suggests the following findings. First, the factor *Model* is significant and substantial with $\eta_p^2 = 0.417$. The probability of detecting the true weights matrix is greater for the spatial lag model than for the spatial error model. To the best of our knowledge, this finding has not been reported previously.

Second, the size of the spatial parameter has a substantial impact on the probability of recovering the true weights matrix, as indicated by the significance of the factor *Spat* and η_p^2 of 0.935. The higher the value of the spatial parameter, the higher is the probability of finding the true spatial weights matrix. Furthermore, the significant interaction $Model \times Spat$ ($\eta_p^2 = 0.198$) indicates that the true value of the spatial parameter differently influences lag and error models. Specifically, the probability of correctly identifying weights matrices increases more quickly at

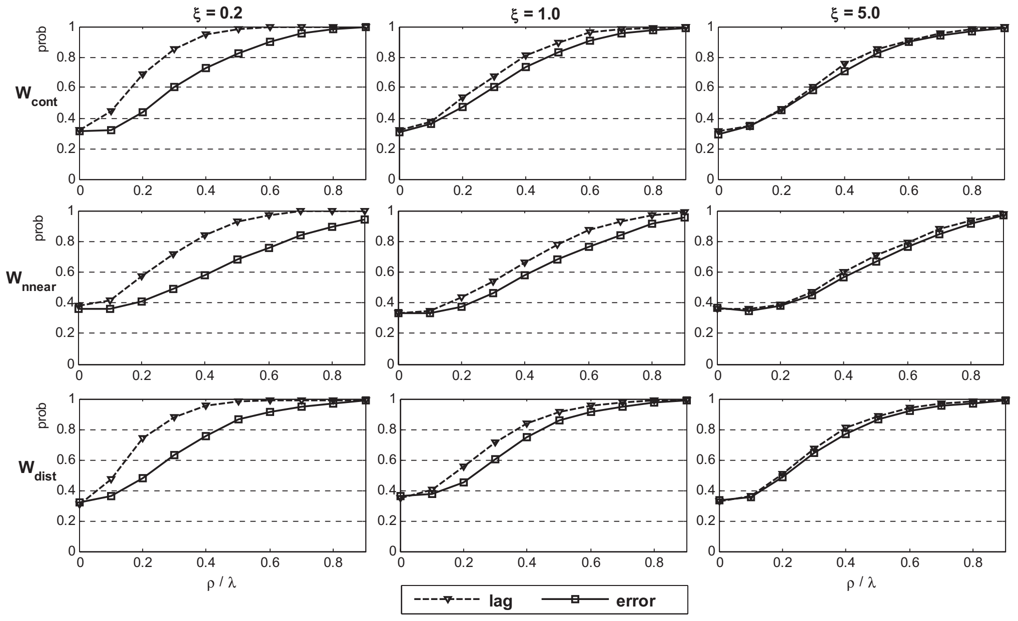


Fig. 4. Probability of identifying the correct weights matrix W by comparing information criteria of different specifications

low to moderate values of the spatial parameter for the lag model compared with for the error model (Figure 4).

Third, the variance of the error terms significantly influences the probability of finding the true weights matrix (significant ξ in Table 4 with $\eta_p^2 = 0.337$), but this effect differs for the lag and error models (significant $Model \times \xi$, with $\eta_p^2 = 0.307$). As the parameter estimates and Figure 4 show, the lag model is much more sensitive for high error variance than is the error model.

Figure 4 also reveals that if the spatial parameter equals 0, the probability of detecting the true weights matrix is approximately one-third, which is merely equal to a random guess probability. This finding is true for both lag and error models, as well as for different weights matrices that generate data and error terms' variances. Which matrix is chosen is of little relevance when there is no spatial correlation. However, the testing procedure occasionally produces false positives (in our study with a probability 0.029) indicating spatial dependency when it is absent, in which case selection of the weights matrix is relevant. Finally, and importantly, Figure 4 indicates that applying the information criteria selection procedure increases the probability of choosing the right weights matrix beyond one-third for all values of the spatial parameter greater than 0.

4.2 Parameter estimation

Assuming that the researcher successfully identifies the true data generating process (spatial lag or spatial error model), the following question arises: How sensitive are the parameter estimates to the type of weights matrix chosen for the estimation? We examine the estimated values obtained for the spatial parameters ρ and λ and the regression parameter $\beta_2 = 1$. The analysis and results for the parameter $\beta_1 = 0$ are largely the same as those for $\beta_2 = 1$, so we do not discuss them here. In addition, we do not discuss the effects on the intercept α because it has no meaningful interpretation in spatial models and usually tends to absorb variation produced by the measurement error (Paez et al. 2008).

4.2.1 Spatial parameters

The main and interaction effects on the MSE of the spatial parameters appear in Table 5. The ANOVA model fits the data moderately well, with an adjusted R^2 of 0.679. Most components are significant at the 0.01 level except the interactions $Model \times W_{gen}$ and $Model \times W_{gen} \times W_{test}$, which are not significant and $Model \times Spat$, which is significant at the 0.05 level. We present the average estimates and MSEs of the spatial parameters ρ and λ for both the lag and error models in Figure 5.

In analysing Figure 5 and Table 5, we derive several results. First, the spatial parameter for both lag and error models is slightly underestimated on average (Figure 5(a)). Second, the MSE of the spatial parameter is lower for the spatial lag model than that for the spatial error model (Figure 5(b) and significant *Model* in Table 5, with $\eta_p^2 = 0.178$).

Third, the MSE depends slightly on the true value of the spatial parameter (significant *Spat*, $\eta_p^2 = 0.171$), but this dependence does not differ between the lag and error models ($Model \times Spat$ interaction is not significant). Parameter estimates also show that MSE's dependence on the true spatial parameter is not linear and achieves its maximum in the interval [0.4,0.6].

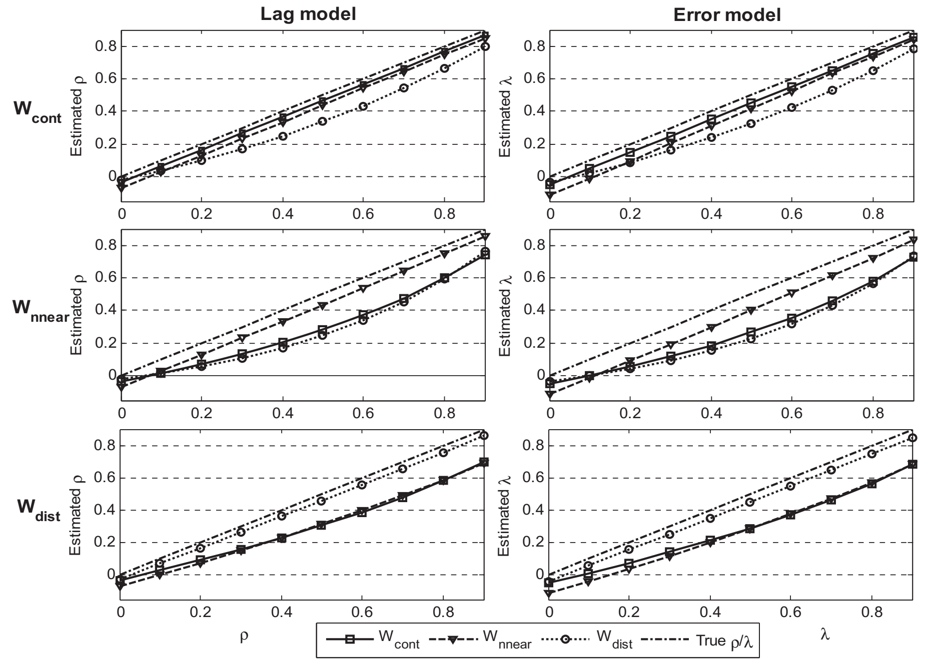
Fourth, the variance of the error terms increases the MSE of the spatial parameter (significant ξ in Table 5, $\eta_p^2 = 0.155$). However, this effect differs for lag and error models (significant $Model \times \xi$, $\eta_p^2 = 0.101$), such that the spatial error model suffers more from high error variance than does the spatial lag model.

As Table 5 shows, the use of the correct weights matrix for model estimation matters significantly for the precision of the spatial parameter estimation ($W_{gen} \times W_{test}$ is significant, $\eta_p^2 = 0.515$). If the weights matrix W used to test the data is the same as that used to generate the data, the MSE of the spatial parameter is lower than it would be otherwise. These effects do not differ significantly between the lag and error models ($Model \times W_{gen} \times W_{test}$ is not significant). Moreover, the MSE decreases monotonically with the increasing true value of the spatial parameter (if $W_{gen} \times W_{test}$).

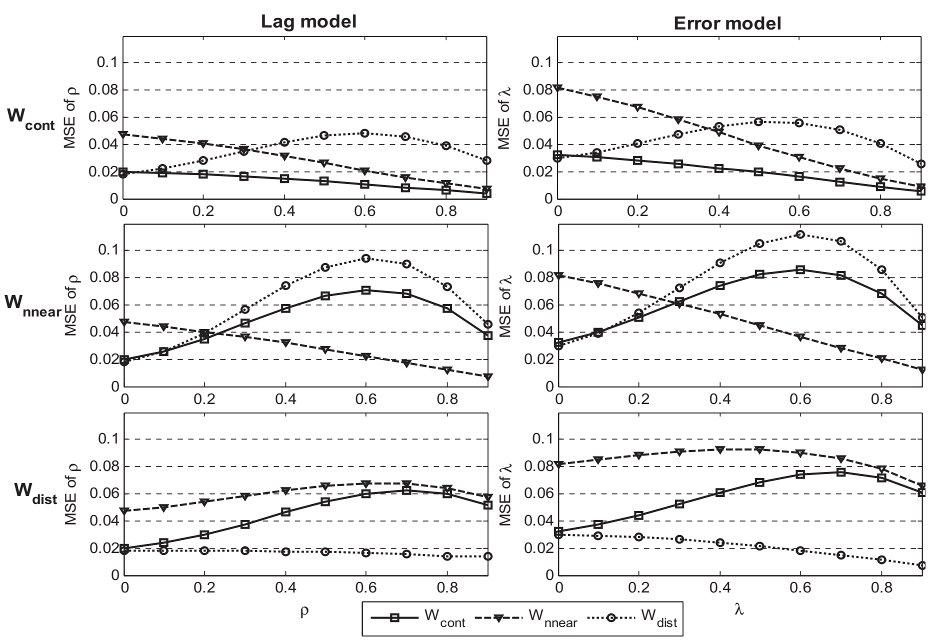
Table 5. Effects on the MSE of spatial parameters ρ/λ

Source	Df	F	Sig.	η_p^2
Intercept	1	4,470.868	<0.001	0.899
Type of the model (*Model*)	1	108.201	<0.001	0.178
True value of spatial parameter (*Spat*)	9	11.499	<0.001	0.171
Weights matrix for generating data (W_{gen})	2	109.271	<0.001	0.304
Weights matrix for testing data (W_{test})	2	14.868	<0.001	0.056
Error terms' variance (ξ)	2	45.878	<0.001	0.155
$W_{gen} \times W_{test}$	4	132.929	<0.001	0.515
$Model \times W_{gen}$	2	1.307	0.272	0.005
$Model \times W_{test}$	2	7.299	0.001	0.028
$Model \times W_{gen} \times W_{test}$	4	1.188	0.315	0.009
$Model \times Spat$	9	1.765	0.072	0.031
$Model \times \xi$	2	28.210	<0.001	0.101

Notes: $R^2 = 0.702$ (Adjusted $R^2 = 0.679$).



(a) Estimates of spatial parameters ρ and λ



(b) MSE of spatial parameters ρ and λ

Fig. 5. Estimates and mean squared error of the spatial parameters, averaged across ξ

Notes: Rows are weights matrices W used to generate data; columns are lag and error models. The different curves correspond to alternative weights matrices W used to estimate the model.

Table 6. MSE of spatial parameter depending on the types of weights matrices

		W for testing data			
		W_{cont}	W_{nnear}	W_{dist}	Total
W for generating data	W_{cont}	0.0158	0.0349	0.0414	0.0307
	W_{nnear}	0.0542	0.0374	0.0697	0.0538
	W_{dist}	0.0503	0.0698	0.0209	0.0470
	Total	0.0401	0.0474	0.0440	0.0438

Again, we examine the effects of the weights matrices' interactions by averaging the MSE of the spatial parameter across model types, true spatial parameters, and error variances. The data is presented in Table 6. The best performance comes from the $W_{cont} - W_{cont}$ combination, whose MSE equals 0.0158, followed by $W_{dist} - W_{dist}$ with 0.0209 and $W_{nnear} - W_{nnear}$ with 0.0374. Failing to choose the right matrix leads to the following ranking of combinations: $W_{cont} - W_{nnear}$ with MSE of 0.0349, $W_{cont} - W_{dist}$ with 0.0414, and worst of all performs the combination $W_{nnear} - W_{dist}$ with 0.0697. This is also consistent with Paez et al. (2008). Using the wrong matrix introduces irrelevant links in the case of W_{nnear} and W_{dist} when the true matrix is W_{cont} which increases the MSE of the spatial parameter. Moreover, the more connected matrices are ($W_{dist} > W_{nnear}$), the higher MSE ($0.0414 > 0.0349$ in Table 6). Figure 5 also shows that the spatial parameter is biased downwards. This confirms the findings of Smith (2008), where he proves that for models with strongly connected weights matrices, maximum likelihood estimates of the spatial dependence parameter are biased downwards.

Further, by averaging the MSEs across the matrices used for data generating (averaging across columns in Table 6) we achieve the following ratings: For estimation purposes the first-order contiguity matrix performs best of all, with an average MSE of spatial parameter 0.0401, closely followed by the inverse distance matrix with 0.0440, whereas the third best is the N nearest neighbours weights matrix with MSE of 0.0474.

4.2.2 Regression parameters

The effects on the MSE of the regression parameter $\beta_2 = 1$ appear in Table 7. The ANOVA fits the data very well, with an adjusted $R^2 = 0.932$. Most components are significant at the 0.01 level, except the interactions $Model \times W_{test}$ and $Model \times \xi$, which are not significant, and W_{test} with $Model \times W_{gen} \times W_{test}$, which are significant at the 0.05 level. We present the average estimates and MSEs of the $\beta_2 = 1$ parameter for both the lag and error models in Figure 6.

First, the regression parameter $\beta_2 = 1$ becomes on average more overestimated for the spatial lag model as the level of spatial dependency increases (first column of Figure 6(a)), whereas for the spatial error model it remains unbiased for the whole range of spatial dependency (second column of Figure 6(a)). Second, the MSE of β_2 is lower for the spatial error model than that for the spatial lag model (Figure 6(b) and significant *Model* in Table 7 with $\eta_p^2 = 0.054$). This is consistent with the aforementioned finding that the regression parameter β_2 is on average overestimated for the spatial lag model.

Third, the MSE depends on the true value of the spatial parameter (significant *Spat*, $\eta_p^2 = 0.306$), but this dependence differs between the lag and error models (significant interaction $Model \times Spat$, $\eta_p^2 = 0.145$). Specifically, according to the parameter estimates, for the same spatial parameter, the MSE of β_2 is higher in the lag model than in the error model.

Table 7. Effects on the MSE of regression parameter $\beta_2 = 1$

Source	Df	F	Sig.	η_p^2
Intercept	1	7,694.980	<0.001	0.939
Type of the model (*Model*)	1	28.780	<0.001	0.054
True value of spatial parameter (*Spat*)	9	24.462	<0.001	0.306
Weights matrix for generating data (W_{gen})	2	17.575	<0.001	0.066
Weights matrix for testing data (W_{test})	2	3.971	0.019	0.016
Error terms' variance (ξ)	2	3,490.057	<0.001	0.933
$W_{gen} \times W_{test}$	4	19.783	<0.001	0.137
$Model \times W_{gen}$	2	6.152	0.002	0.024
$Model \times W_{test}$	2	1.503	0.223	0.006
$Model \times W_{gen} \times W_{test}$	4	2.473	0.044	0.019
$Model \times Spat$	9	9.394	<0.001	0.145
$Model \times \xi$	2	0.009	0.991	0.000

Notes: $R^2 = 0.937$ (Adjusted $R^2 = 0.932$).

Table 8. MSE of regression parameter $\beta_2 = 1$ depending on the types of weights matrices

		W for testing data			
		W_{cont}	W_{nnear}	W_{dist}	Total
W for generating data	W_{cont}	0.0219	0.0257	0.0276	0.0251
	W_{nnear}	0.0232	0.0220	0.0246	0.0233
	W_{dist}	0.0286	0.0315	0.0226	0.0276
	Total	0.0246	0.0264	0.0250	0.0253

Fourth, the variance of the error terms has a very strong, increasing effect on the regression parameter's MSE (significant ξ, $\eta_p^2 = 0.934$), but this effect does not differ significantly between the lag and error models (insignificant $Model \times \xi$).

The precision of the regression parameter estimation depends on the combination used to generate and test the data ($W_{gen} \times W_{test}$ in Table 7 is significant, $\eta_p^2 = 0.137$). However, these effects hardly differ between the lag and error models (interaction $Model \times W_{test}$ is not significant, and $Model \times W_{gen} \times W_{test}$ is significant only at the 0.05 level).

Closer inspection of Figure 6 enables us to formulate the same conclusion we derived for the MSEs of the spatial parameters. If the weights matrix *W* used to test the data is the same as that used to generate the data, the MSE of the regression parameter is lower than would otherwise be the case.

Again, we examine the effects of the weights matrices' interactions by averaging the MSE of regression parameter β_2 across model types, true spatial parameters, and error terms' values (Table 8). The results show that the $W_{cont} - W_{cont}$ combination performs best, with an MSE equal to 0.0219 very close followed by $W_{nnear} - W_{nnear}$ with MSE of 0.0220; finally the last one is $W_{dist} - W_{dist}$, with MSE of 0.0226. Without the right matrix, the ranking of these combinations changes as follows: $W_{nnear} - W_{cont}$ with 0.0232, followed by $W_{nnear} - W_{dist}$ with 0.0246, and the worst performance comes from $W_{dist} - W_{nnear}$ with 0.0315. For the regression parameter, the ranking of weights matrices is not entirely the same as that for the spatial parameter. Averaging the MSEs across the data generating matrices (averaging across columns in Table 8) indicates again that the best performer is the first-order contiguity matrix with an average MSE of 0.0246, but the second best is the inverse distance matrix with 0.0250, and the third best is the *N* nearest neighbours weights matrix with an MSE of 0.0264.

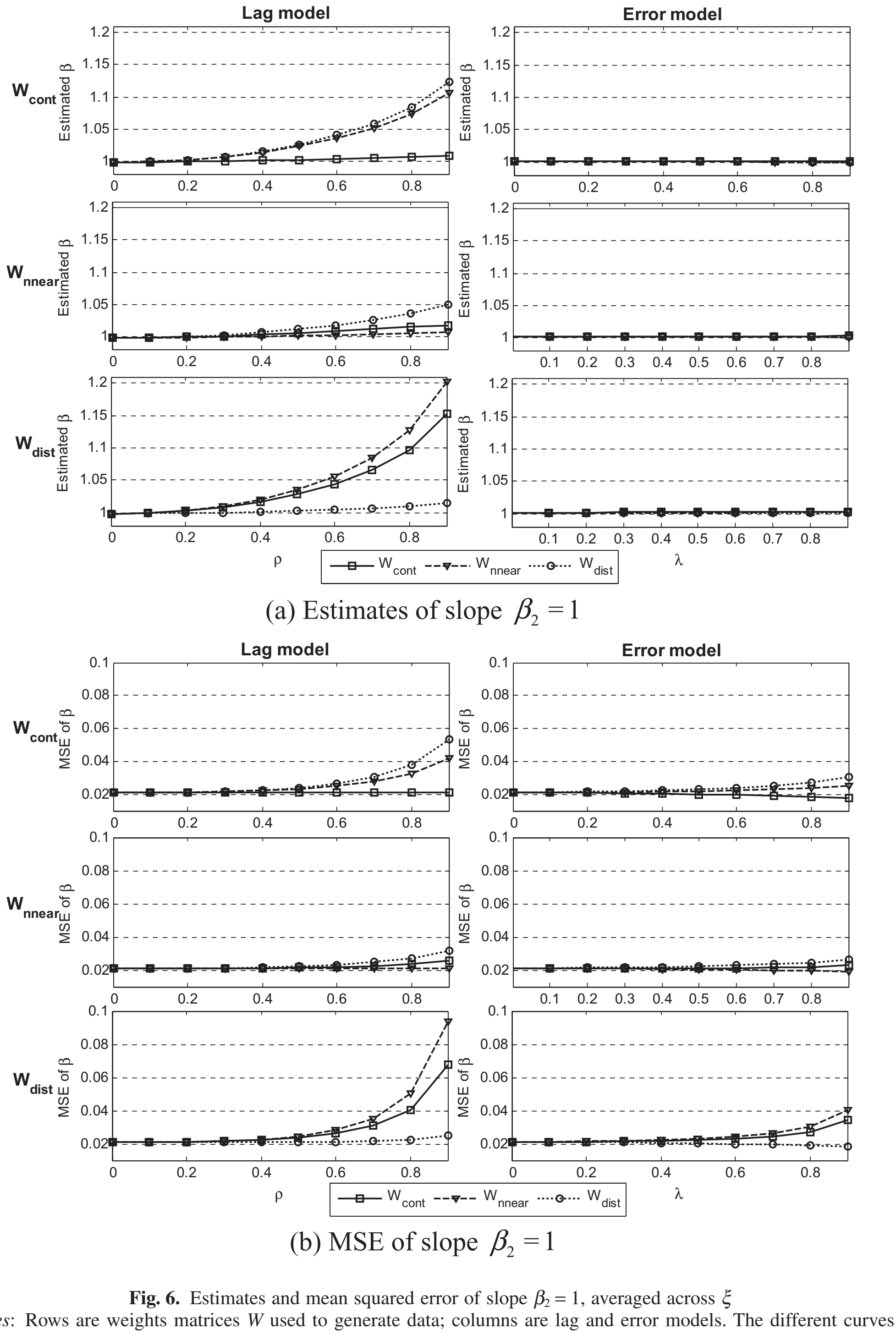


Fig. 6. Estimates and mean squared error of slope $\beta_2 = 1$, averaged across ξ

Notes: Rows are weights matrices W used to generate data; columns are lag and error models. The different curves correspond to alternative weights matrices W used to estimate the model.

Table 9. Effects of factors on the recovery of the true model and the true weights matrix, as well as on the accuracy of the parameter estimates

	Lag vs. error model	Strong spatial dependence	Low error terms' var.	$W_{test/estim} = W_{gen}$
High probability of detecting true model	+	+	+	+
High probability of choosing correct W	+	+	+	+
Low MSE of spatial parameter	+	+	+*	+
Low MSE of regression parameter	−	+	+*	+

Note: * Only for the spatial lag model.

5 Conclusions, recommendations, and further research

5.1 Conclusions

This research allows us to formulate conclusions about (1) choosing a spatial model specification, (2) choosing a spatial weights matrix, and (3) the precision of spatial and regression parameter estimates, conditional on the choice of the spatial model and spatial weights matrix. In Table 9, we present the effects of several prominent factors on the probabilities of detecting the true model and the true weights matrix, as well as on the accuracy of the parameters.

We also highlight several important conclusions. First, the probability of recovering the spatial lag as the true model is greater than that for the spatial error model, which confirms Anselin et al.'s (1996) findings. The probability to recover the true spatial error model is particularly low for small values of the spatial parameter. Second, the spatial lag model produces a lower MSE of the spatial parameter and higher MSE of regression parameters compared with the spatial error model, which is consistent with Florax et al.'s (2003) findings.

Third, consistent with Anselin and Rey (1991) and Farber et al. (2009), high connectivity of the weights matrix has a negative impact on the probability of detecting the true model specification. Moreover, strongly connected matrices introduce a downward bias for maximum likelihood estimates of spatial parameter as was shown in Smith (2008). Fourth, a high proportion of variance in the data, together with high correlation between neighbouring observations, leads to an incorrect identification of the spatial error structure as a spatial lag one. This finding has not been previously reported in the literature.

Fifth, spatial models estimated using the first-order contiguity weights matrix perform better on average than those using the *N* nearest neighbours and inverse distance weights matrices in terms of their higher probabilities of detecting the true model and the lower MSE of the parameters. Sixth and finally, a selection procedure for the weights matrix based on the log-likelihood or information criteria usually indicates the correct specification. In particular, the probability of detecting the correct weights matrix is moderate for low values of the true spatial parameter (less than 0.3) but approaches 0.8 or even unity as the spatial parameter increases.

The accuracy of the regression parameter is not very sensitive to the type of weights matrix for low spatial parameter values but is much more sensitive for higher values, though in this case, the probability of detecting the true weights matrix is also very high. In other words, for low values of the true spatial parameter, there is a higher probability of making a mistake in choosing the weights matrix. However, the consequences of this poor choice are not very dramatic, because the parameter estimates are quite close to the true ones. In contrast, for high values of the true spatial parameter, the wrong choice of a weights matrix can distort the parameter estimates severely, but the weights matrix is nearly always correctly identified by the information criteria–based procedure. To the best of our knowledge, this important pattern of findings has not been reported previously.

5.2 Recommendations

Combining these conclusions, we formulate several practical recommendations for researchers who deal with georeferenced data.

First, applying a weights matrix selection procedure that is based on information criteria increases the probability of identifying its true specification and benefits the researcher in terms of the precision of the parameter estimates compared with choosing the weights matrix arbitrarily. This finding is true for all degrees of spatial dependency.

Second, using the most simple spatial weights matrix, first-order contiguity (if theory does not suggest otherwise), offers the second-best option. It does not perform best in all cases, but in general, it increases the probability of detecting the true model and decreases the MSE of spatial and regression parameters.

5.3 Limitations and further research

We acknowledge several limitations of our study that also suggest directions for further research. First, we use a fixed sample size of 100 observations. Additional levels of sample size could be included in future simulation studies. It would be particularly relevant to examine cases with large sample sizes.

Further, more alternative models and weights matrices could be considered in additional research. We do not study other specifications of spatial models, for instance, the mixed model of spatial lag and spatial error, because these models are much less often used in practice. A valuable contribution would be also including spatial weights matrices that can be transformed to each other by manipulating their topology parameters.

Finally, it would be interesting to apply model and weights matrix selection procedures not sequentially but simultaneously. In other words, researchers might specify several alternatives of a model and several types of weights matrices. Furthermore, instead of fixing the weights matrix, researchers could estimate all combinations of models with weights matrices and choose the combination that produces information criteria with the lowest value.

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Especificación de modelos espaciales: Un estudio de simulación sobre matrices de pesos

Stanislav Stakhovych, Tammo H.A. Bijmolt

Resumen. La correcta especificación de modelos espaciales, y especialmente la elección de la matriz de pesos espaciales, representa una decisión crucial para investigadores que estén utilizando datos georreferenciados. Sin embargo, existen pocas recomendaciones acerca de que matriz de pesos es la más apropiada en ciertos casos. Por tanto este artículo (1) estudia la sensitividad de los ensayos y la estimación de modelos espaciales con especificaciones diferentes de la matriz de pesos y (2) formula recomendaciones para investigadores que estén aplicando modelos espaciales en relación con la elección del modelo y las especificaciones de la matriz de pesos. La investigación está basada en simulaciones de Monte Carlo con datos sintéticos.

JEL classification: C12, C13, C15, C52

Palabras clave: Econometría espacial, matrices de pesos, elección de un modelo, estimaciones de parámetros, simulaciones de Monte Carlo

要旨： 空間的モデルの的確な特定化、特に空間的重み行列の選択は、地理参照データを使う研究者にとり重大な決定を意味する。しかし、どの重み行列が特定のケースにおいて最適であるかについてのガイドラインはほとんど存在しない。そこで本論では、(1)異なる重み行列仕様を持つ空間的モデルの検証及び推計の感応度を分析し、(2)空間的モデルを利用する研究者に対し、モデル選択及び重み行列の特定化に関する推奨を策定する。本調査は合成データを用いたモンテカルロ・シミュレーションに基づいている。