Safety in Sequential Decision-making

Mohammad Ghavamzadeh

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Outline

Safety: Problem Formulation

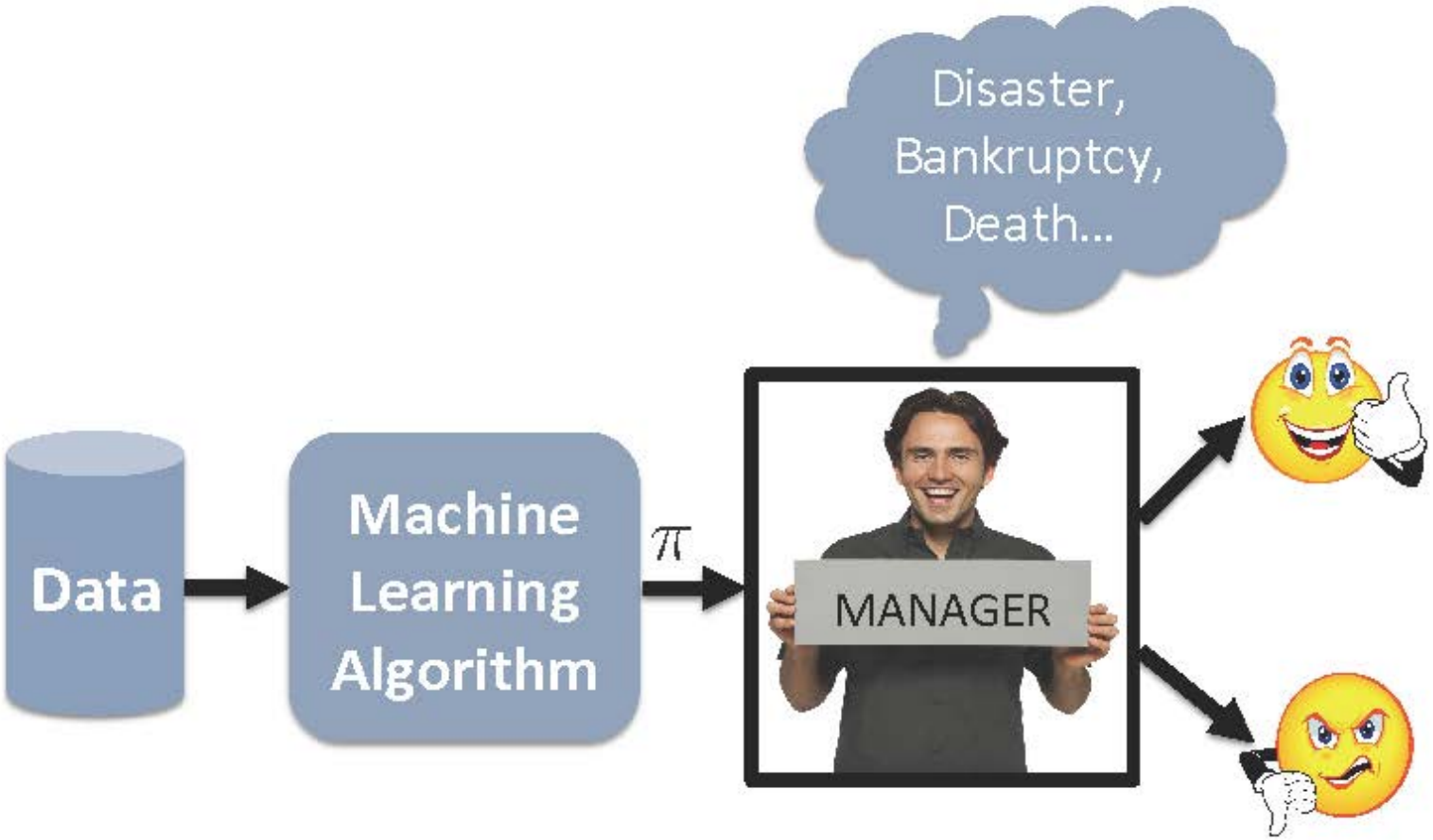
Different Approaches to Safety

Risk-sensitive Decision-making *(optional)*

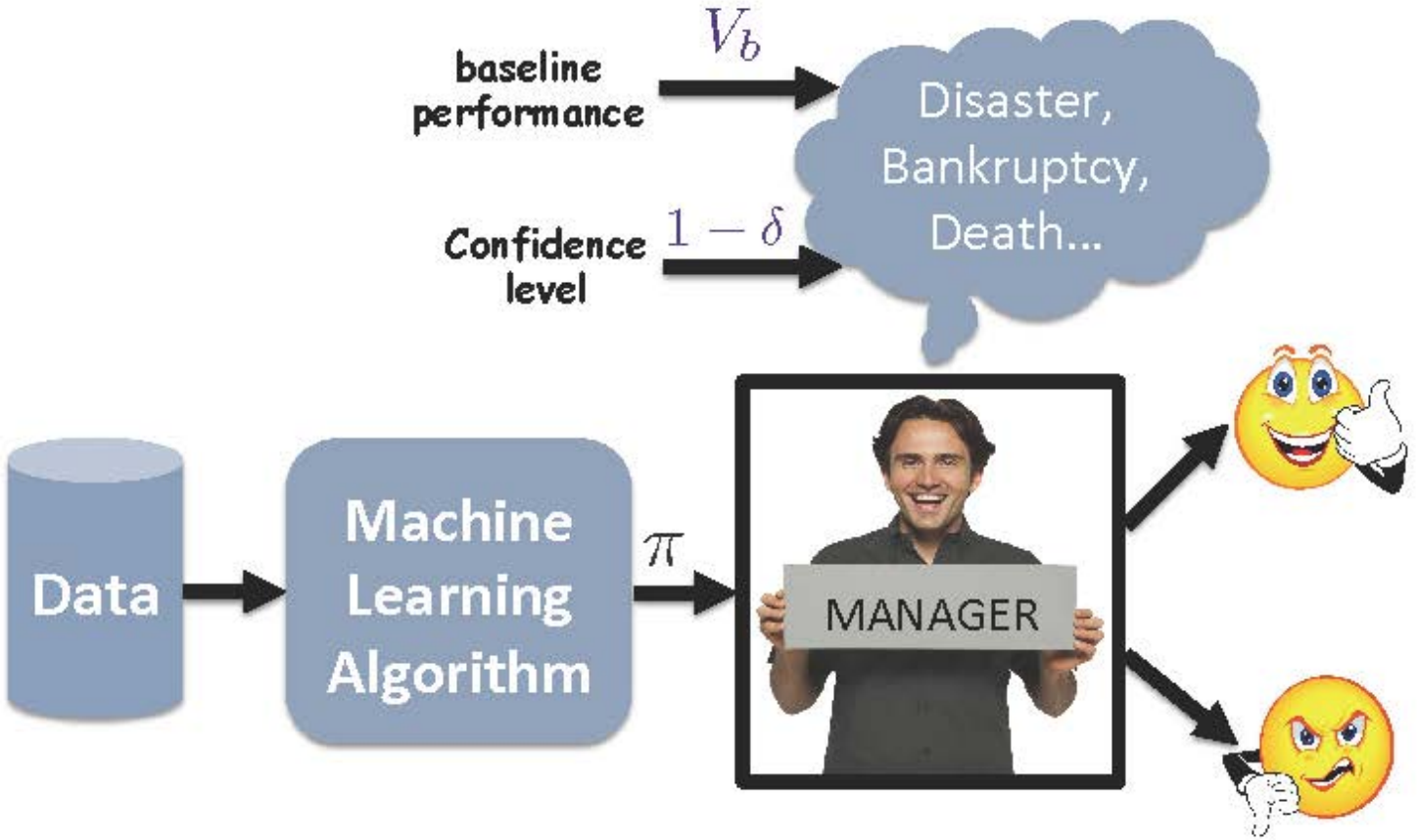
Safety w.r.t. Undesirable Situations *(optional)*

Outline

Safety: Problem Formulation



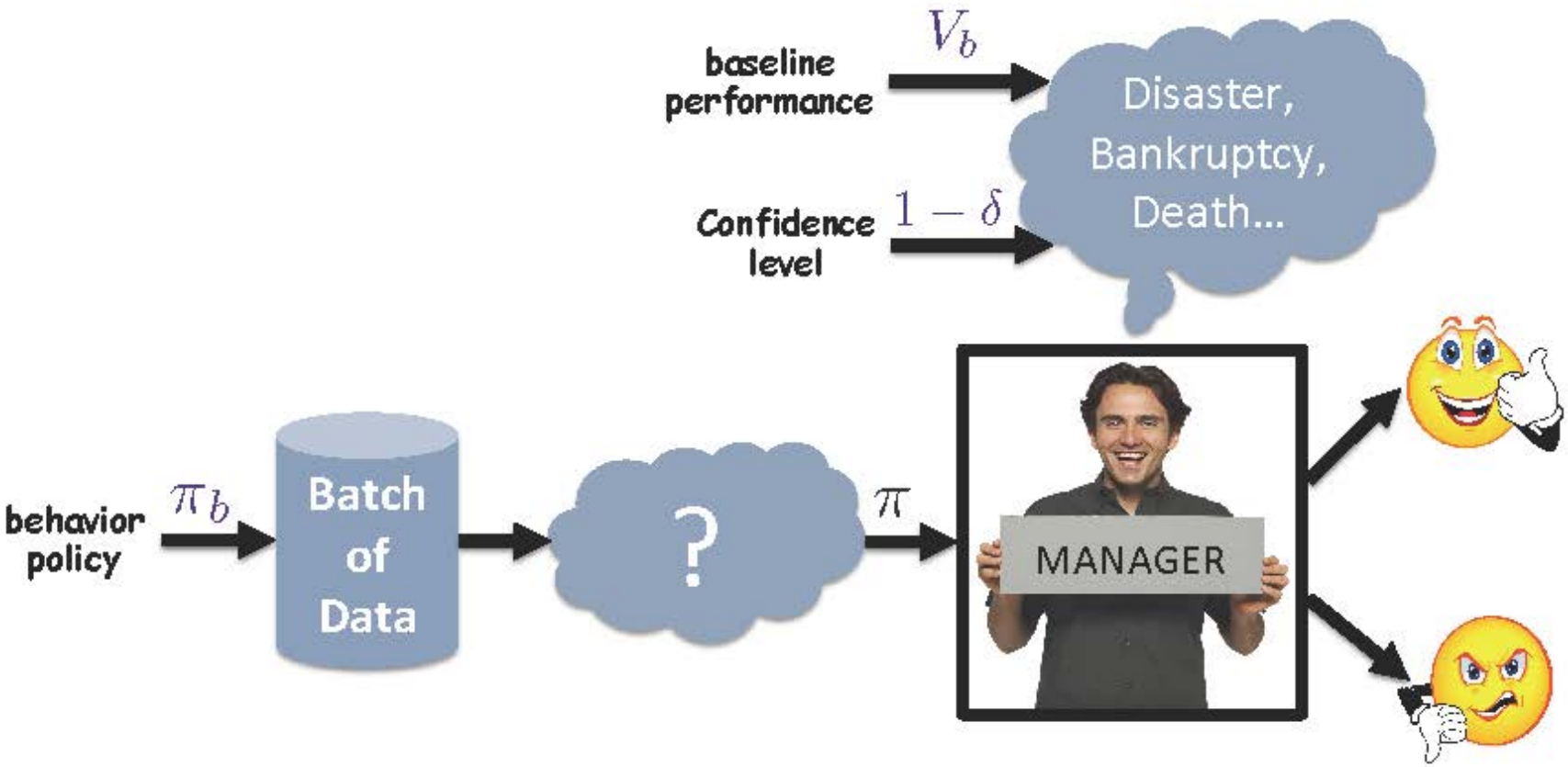
Safety: Problem Formulation



Outline

1. Model-free Approach

Offline Setting



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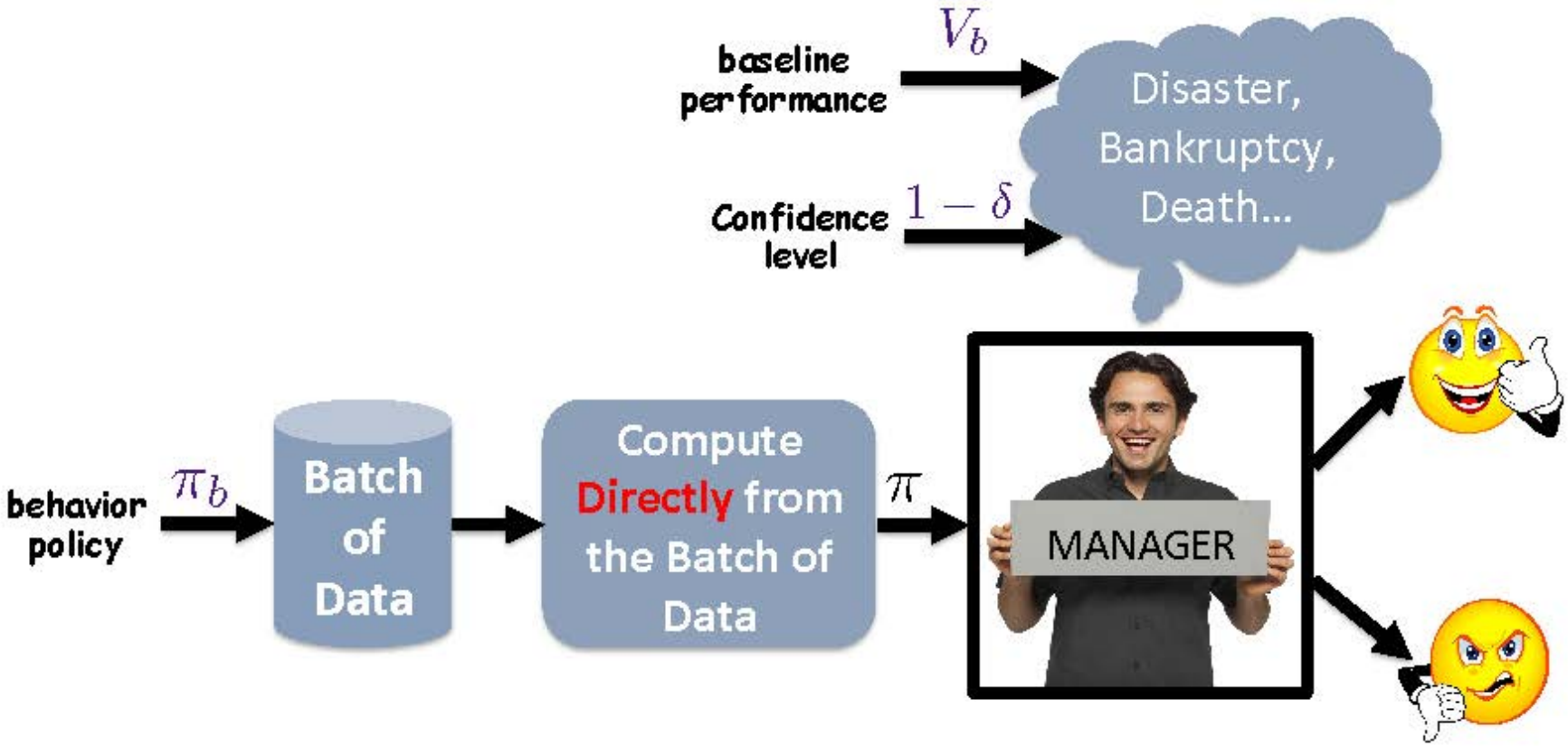

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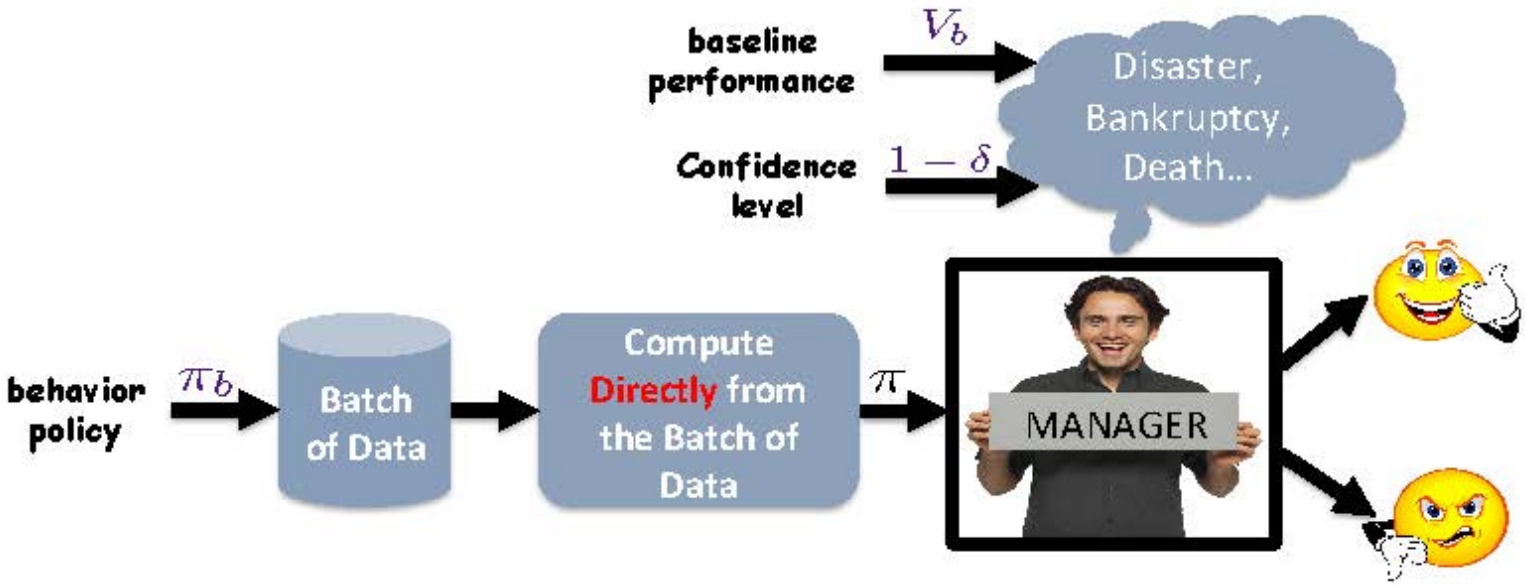

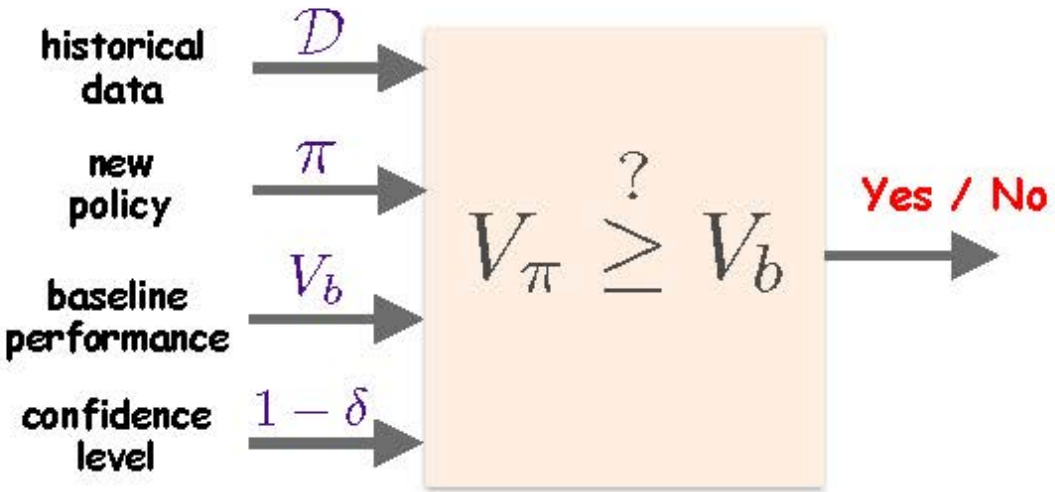

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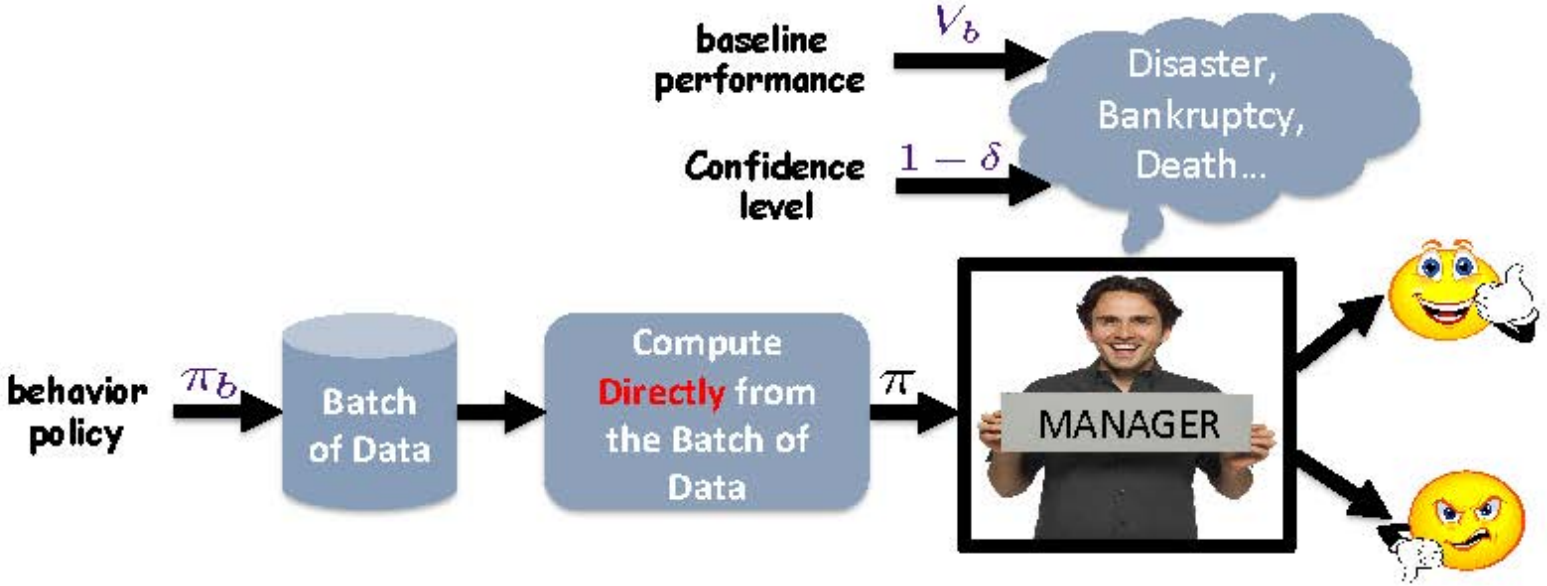

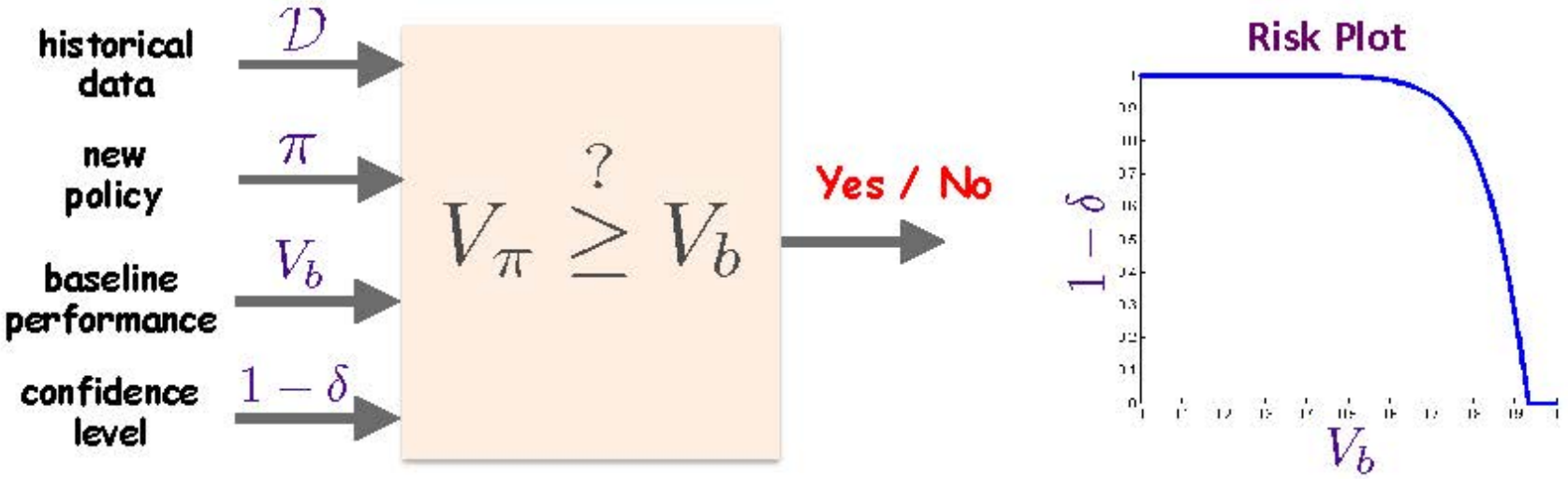

Publications

1. P. Thomas, G. Theocharous, and **MGH**. *"High Confidence Off-Policy Evaluation"*. ***AAAI-2015***.

2. P. Thomas, G. Theocharous, and **MGH**. *"High Confidence Policy Improvement"*. ***ICML-2015***.

3. G. Theocharous, P. Thomas, and **MGH**. *"Building Personal Ad Recommendation Systems for Life-Time Value Optimization with Guarantees"*. ***IJCAI-2015***.

Other Publications

- L. Bottou, J. Peters, J. Quinonero-Candela, D. Charles, D. Chickering, E. Portugaly, D. Ray, P. Simard, and E. Snelson. *"Counterfactual Reasoning and Learning Systems: The Example of Computational Advertising"*. **JMLR-2013**.

- A. Swaminathan and T. Joachims. *"Counterfactual Risk Minimization: Learning from Logged Bandit Feedback"*. **ICML-2015**.

- N. Jiang and L. Li. *"Doubly Robust Off-Policy Value Evaluation for Reinforcement Learning"*. **ICML-2016**.

- P. Thomas and E. Brunskill. *"Data-Efficient Off-Policy Policy Evaluation for Reinforcement Learning"*. **ICML-2016**.

- A. Swaminathan, A. Krishnamurthy, A. Agarwal, M. Dudik, J. Langford, D. Jose, and I. Zitouni. *"Off-policy Evaluation for Slate Recommendation"*. **NIPS-2017**.

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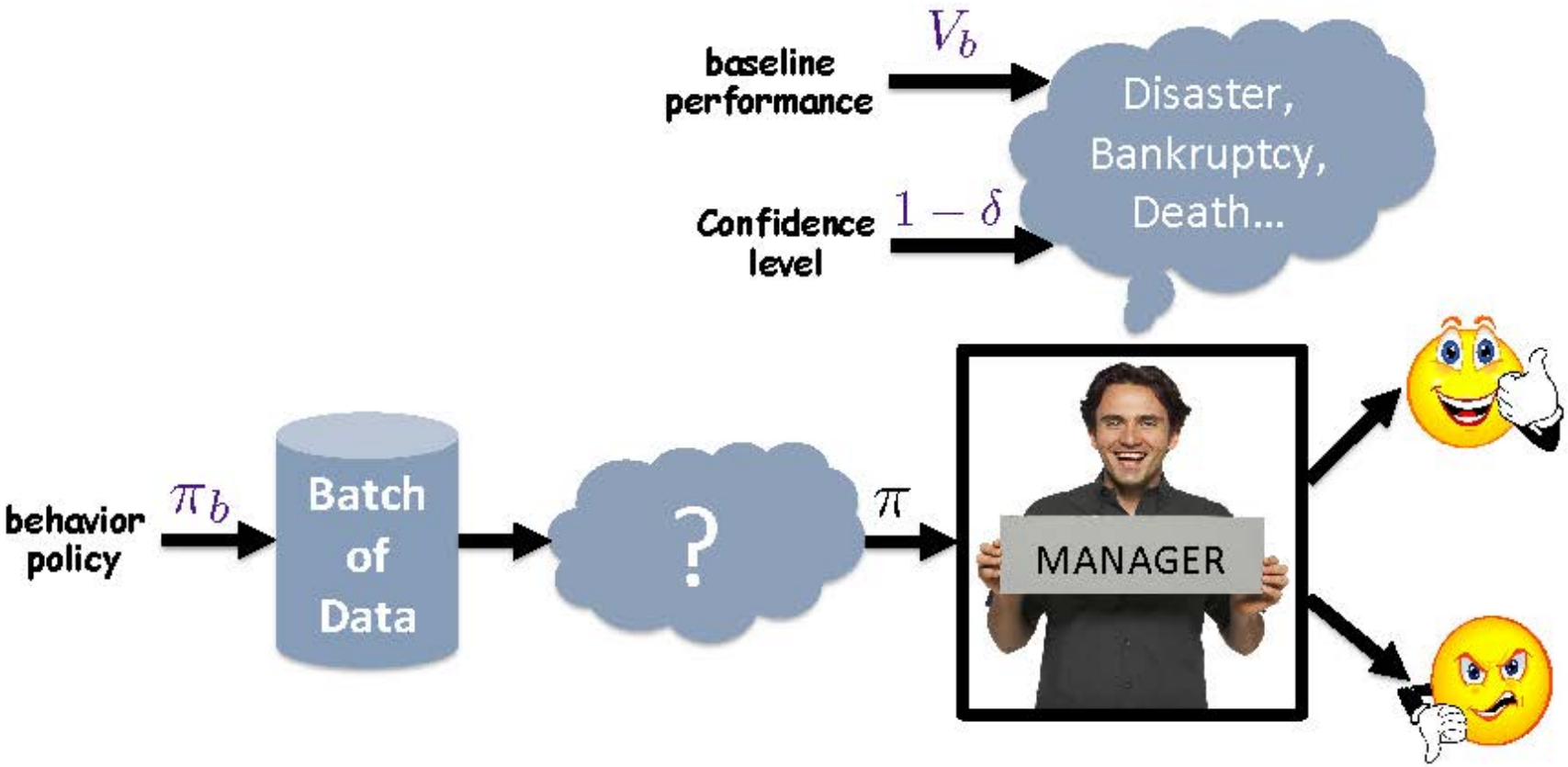
M. Farajtabar, Y. Chow, and **MGH**. *"More Robust Doubly Robust Off-policy Evaluation"*. **ICML-2018**.

2. Model-based Approach

Offline Setting



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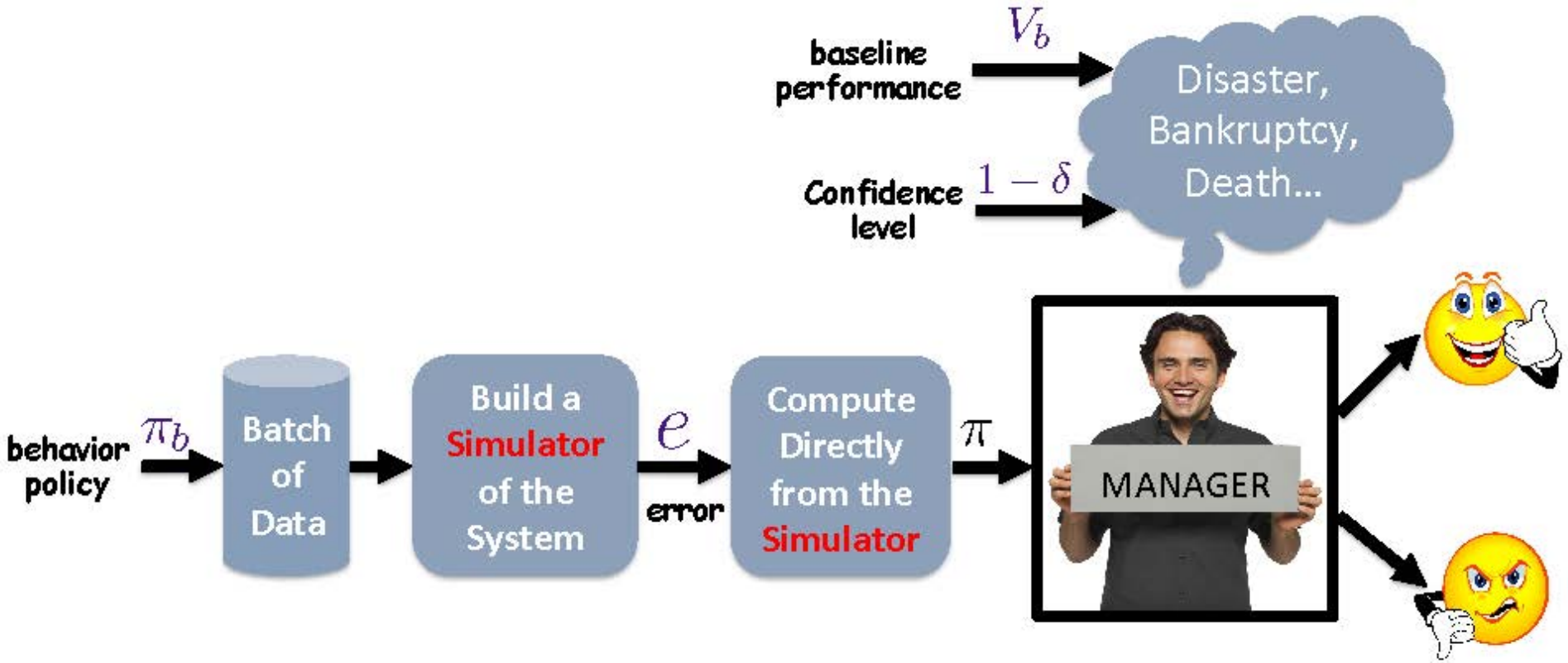

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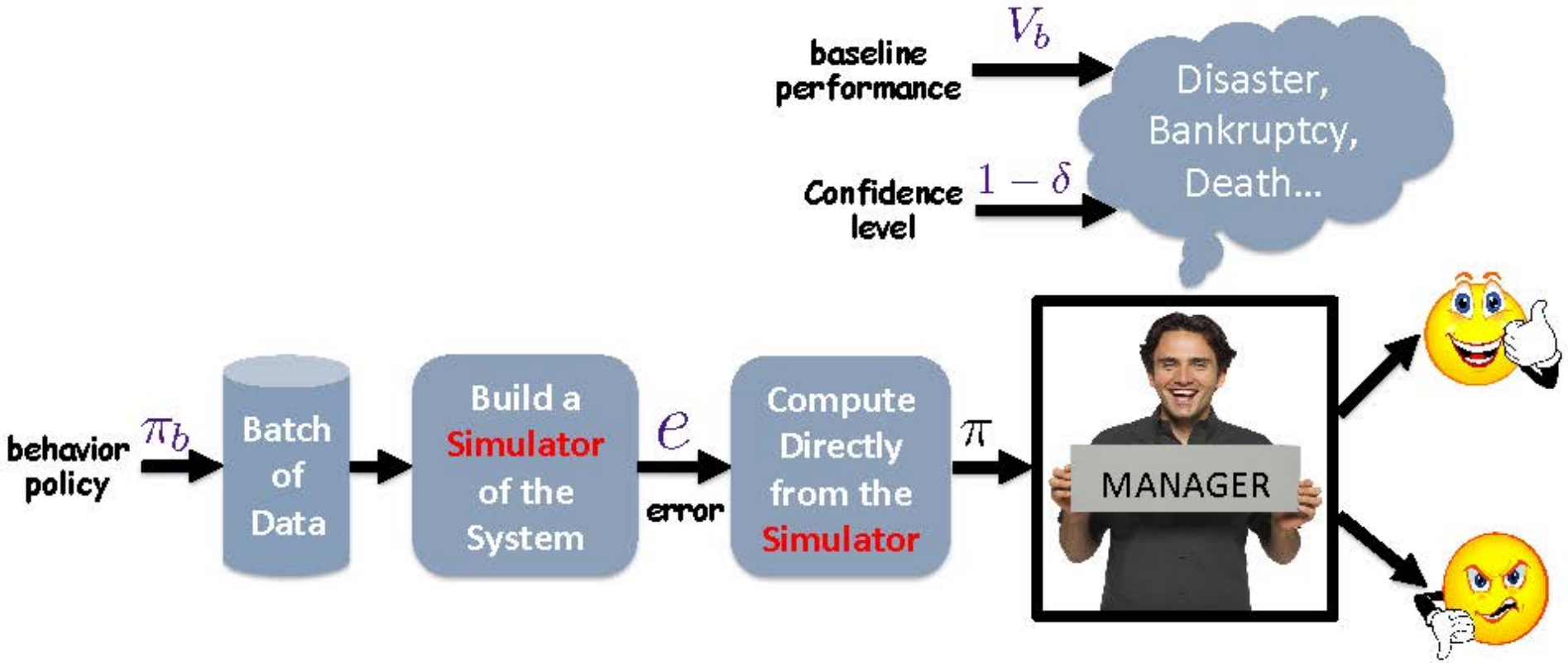


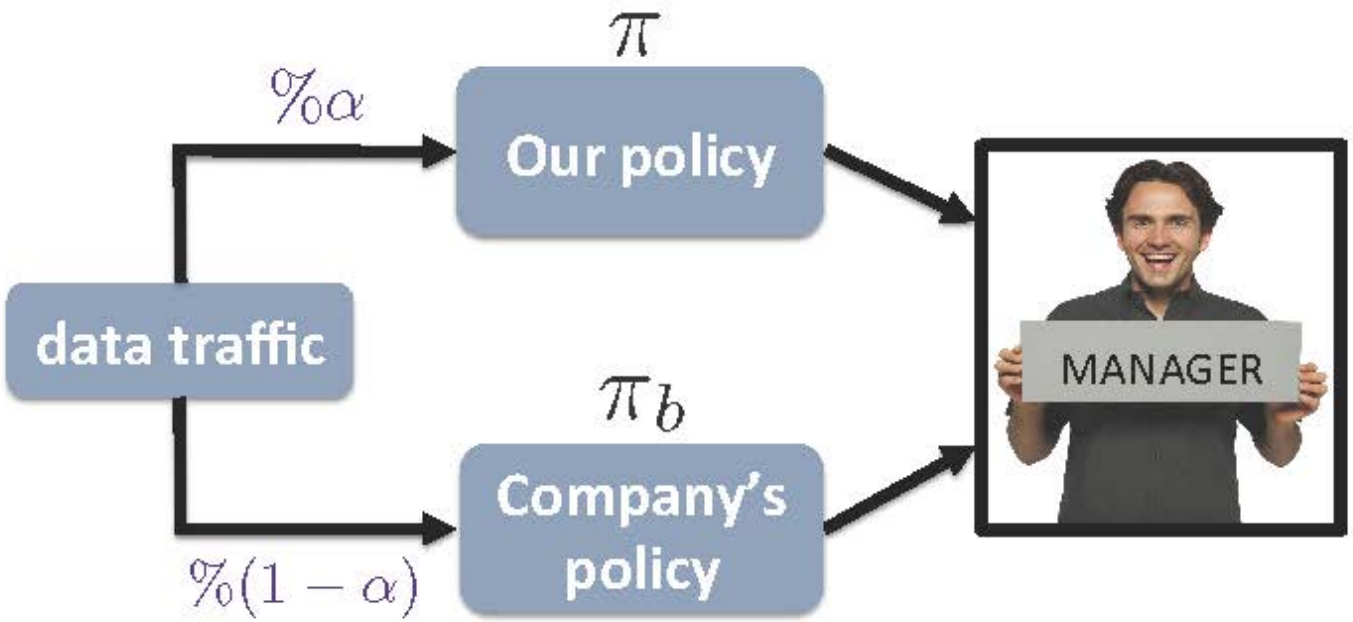
Main Question: Given the simulator and the error in building it, how to compute a policy that is guaranteed ***(with a given confidence level)*** to perform at least as well as a baseline???

Publications

1. M. Petrik, **MGH**, and Y. Chow. *"Safe Policy Improvement by Minimizing Robust Baseline Regret"*. ***NIPS-2016***.

3. Online Approach

Online Approach



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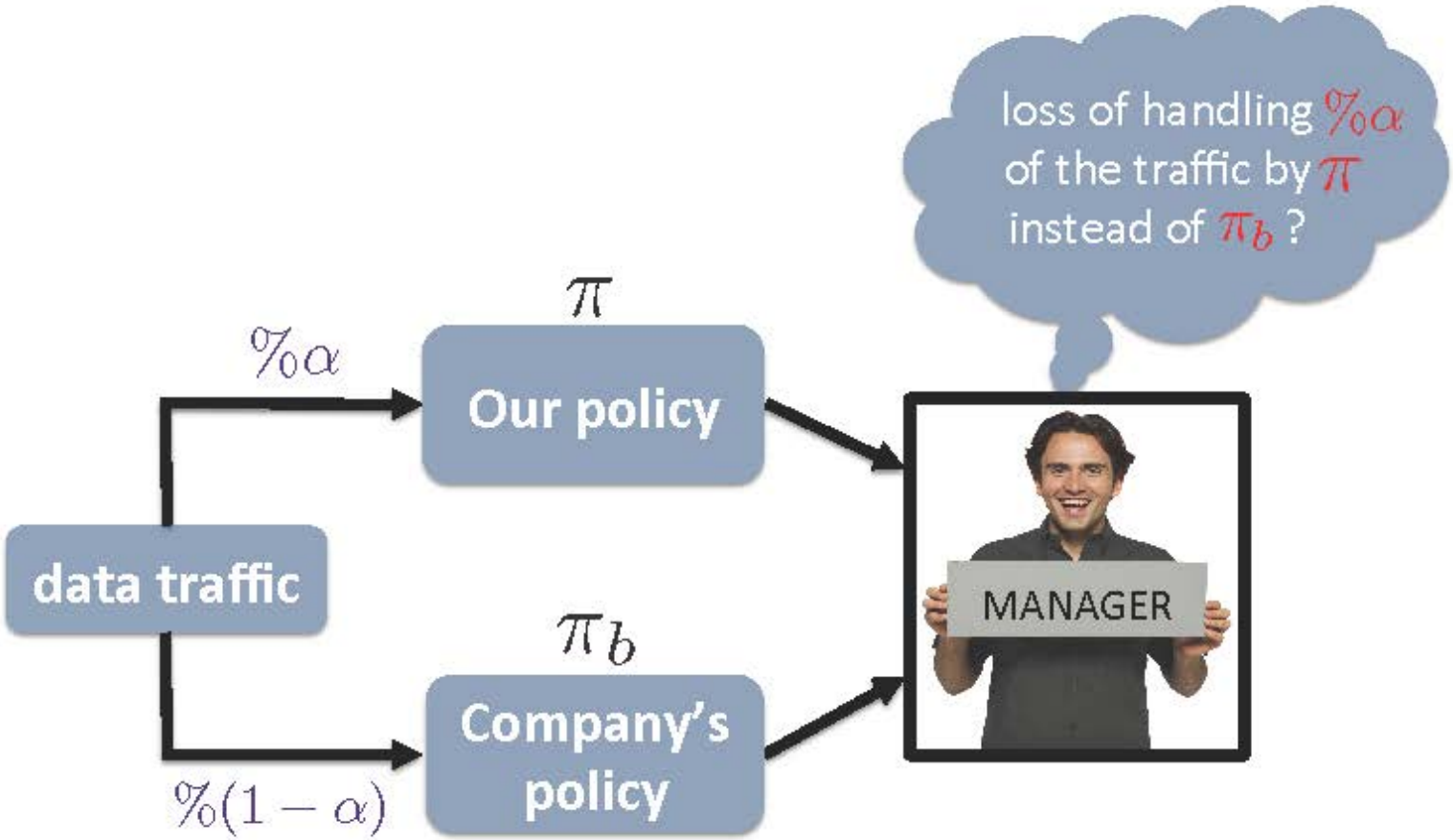

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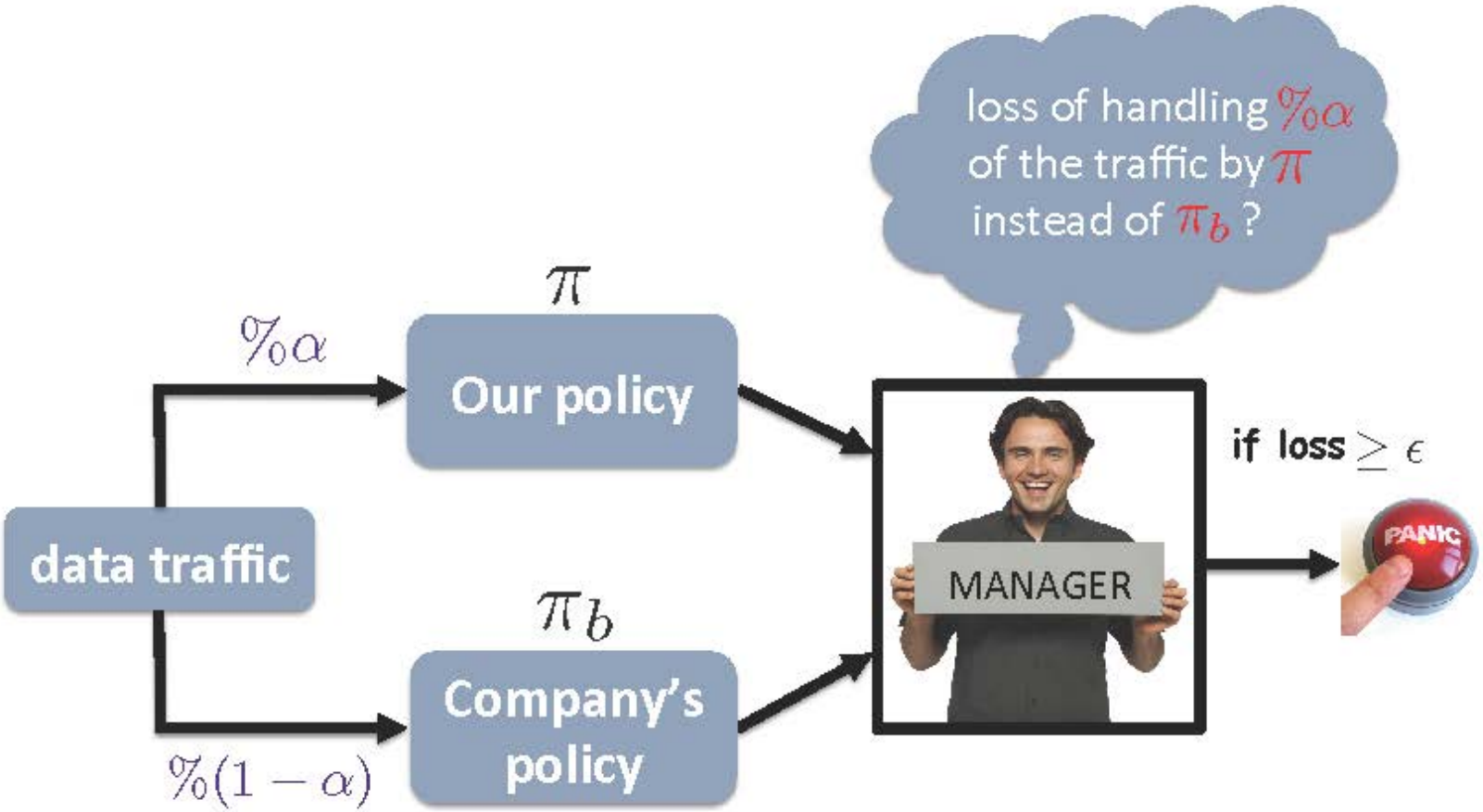

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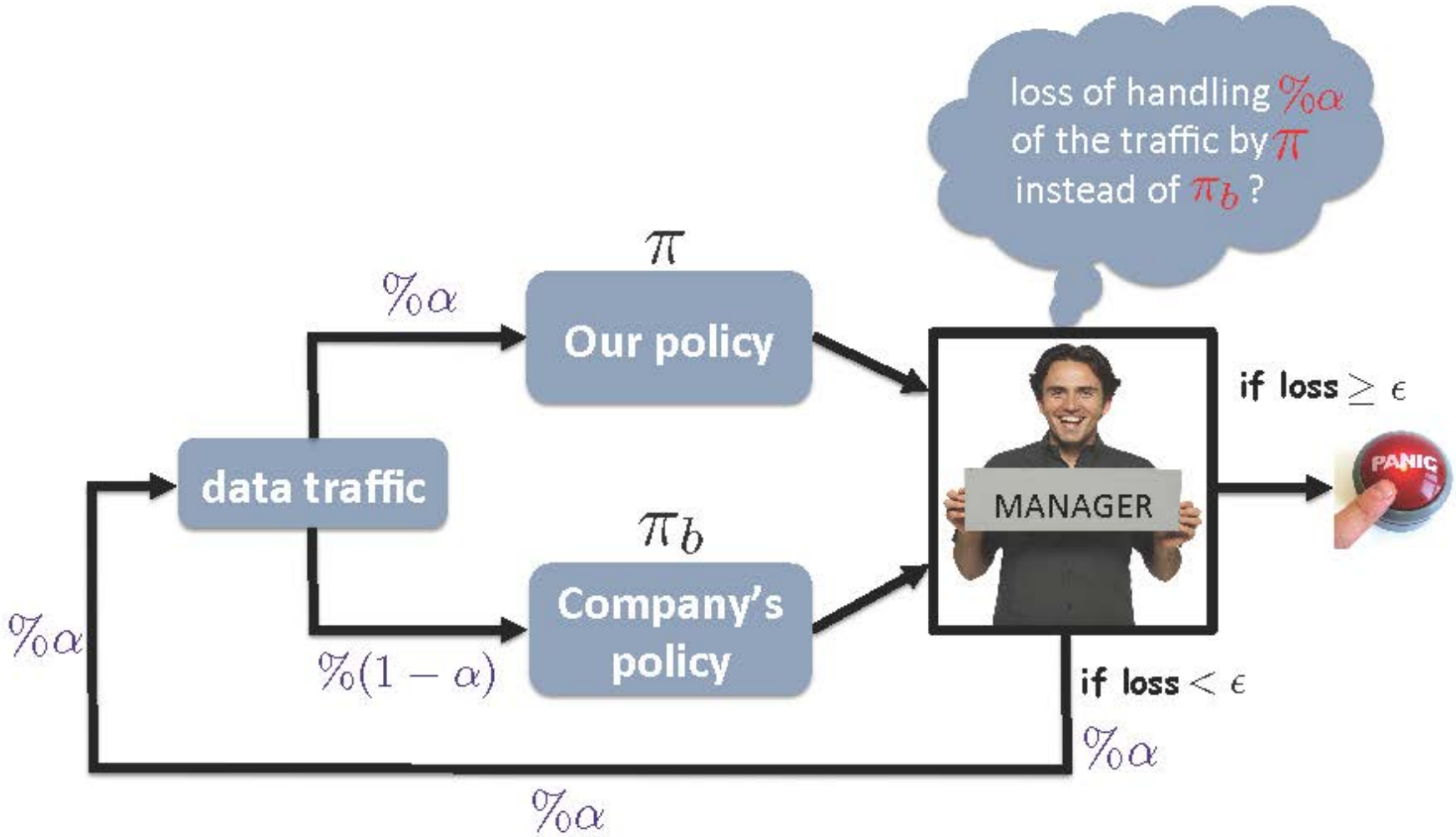

Publications

1. A. Kazerouni, **MGH**, Y. Abbasi-Yadkori, and B. Van Roy. *"Conservative Contextual Linear Bandits"*. ***NIPS-2017***.

2. Y. Wu, R. Shariff, T. Lattimore, and C. Szepesvári. *"Conservative bandits"*. ***ICML-2016***.

Other Publications

1. Y. Mansour, A. Slivkins, and V. Syrgkanis. *"Bayesian Incentive-Compatible Bandit Exploration"*. ***EC-2015***.

Outline

Model-free Approach

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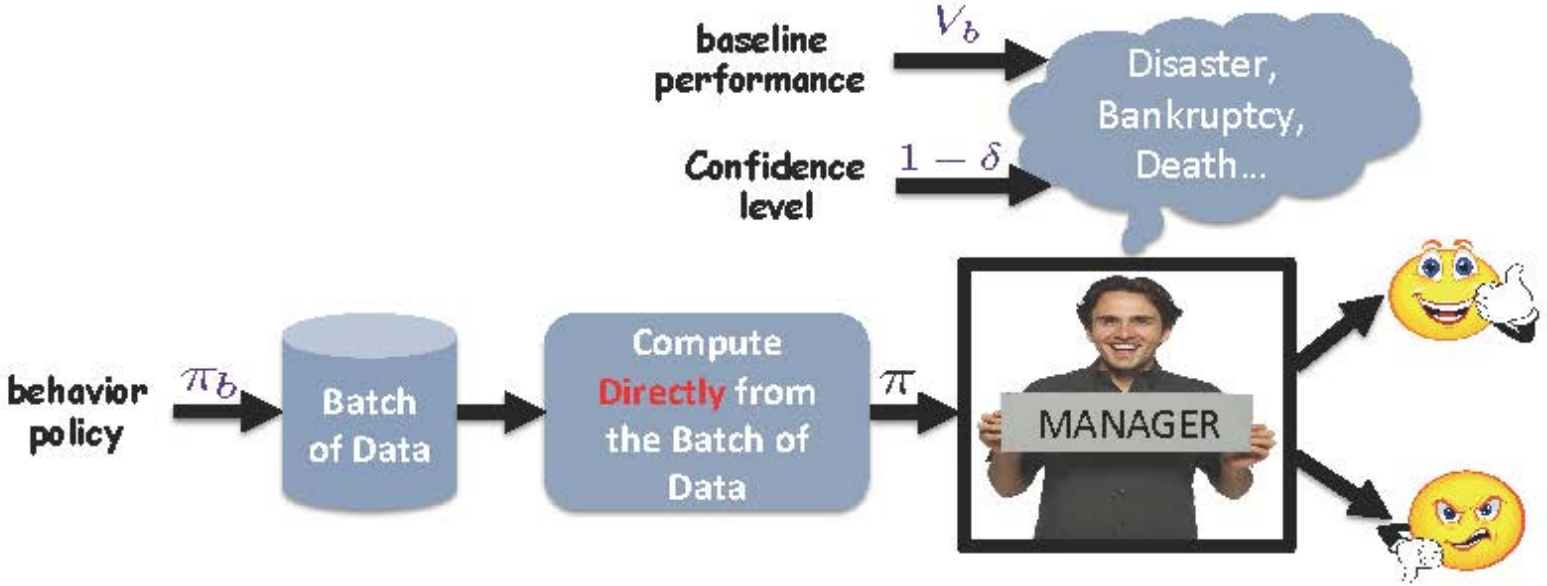

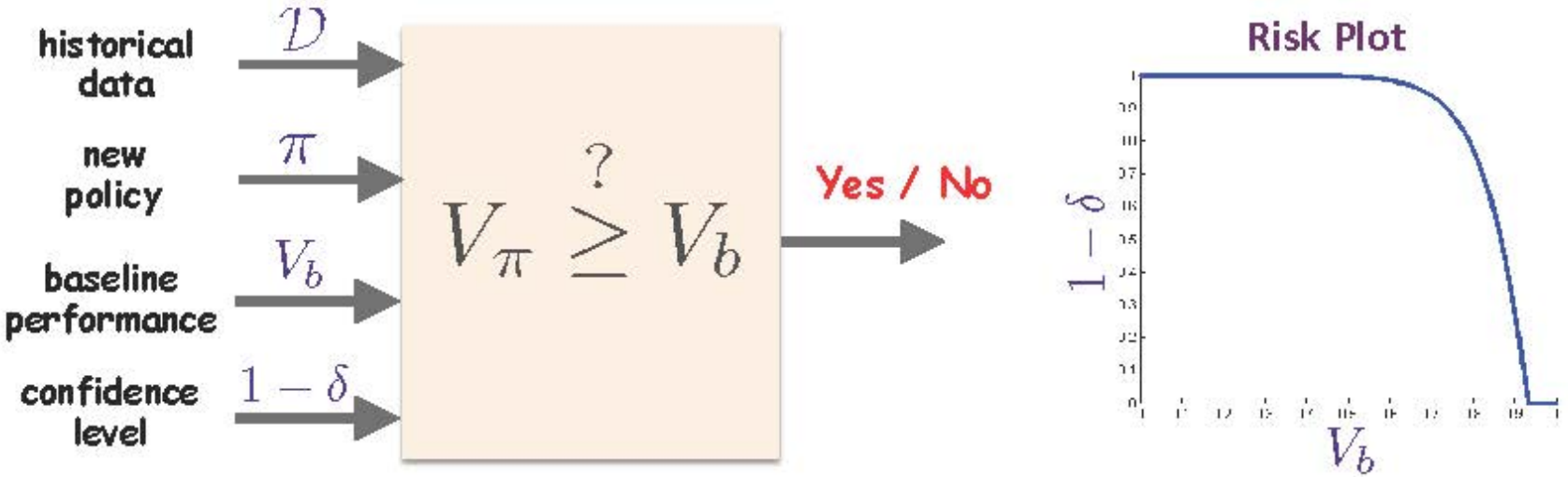

Problem Definition

new policy π has been computed using a finite number of samples

- Is it ***safe*** to deploy policy π?
- Is π guaranteed to perform at least as well as π_b?
- Is the performance of π guaranteed to be at least V_b?

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an important problem in many different fields including marketing, health, and finance

High-Confidence Off-Policy Evaluation

Problem Formulation

- **System Trajectory**

$$\tau = \{x_1, a_1, r_1, x_2, a_2, r_2, \ldots, s_T, a_T, r_T\} \qquad r_t = r(x_t, a_t)$$

- **Return of a Trajectory** *(assume rewards are in $[0,1]$)*

$$D(\tau) = \sum_{t=1}^{T} \gamma^{t-1} r_t \ \in \ [0, 1/1-\gamma]$$

- **Policy Performance**

$$V^{\pi} = \mathbb{E}\big[D(\tau)\big] \ \in \ [0, 1/1-\gamma] \qquad \text{if} \quad a_t \sim \pi(\cdot|x_t)$$

Problem Formulation

- **Historical Data**

$$\mathcal{D} = \left\{ (\tau_i, \pi_i) \right\}_{i=1}^{n}$$

τ_i ***has been generated by*** π_i

Problem Formulation

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- **Behavior Policies** $\pi_1, \ldots, \pi_n$ **Target Policy** π

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- **Baseline Performance** V_b

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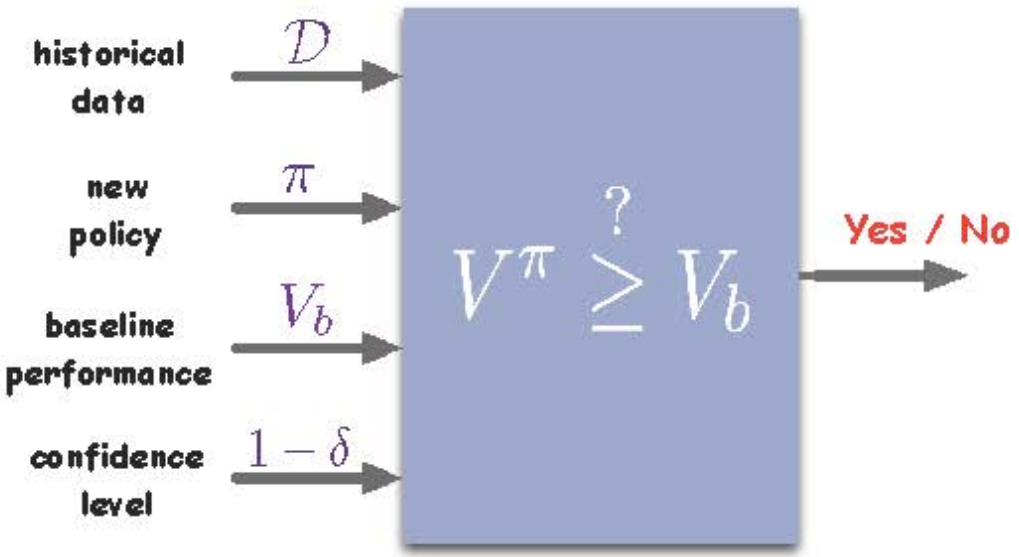

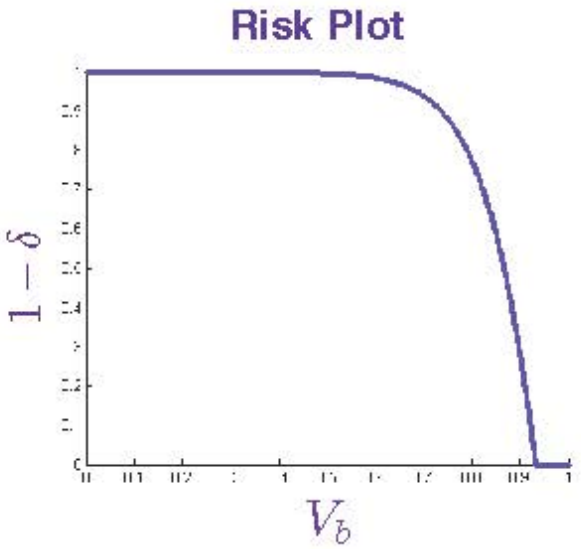

Weighted Importance Return

For any $(\tau_i, \pi_i) \in \mathcal{D}$

$$\widehat{D}(\tau_i, \pi, \pi_i) = D(\tau_i) \frac{\Pr(\tau_i|\pi)}{\Pr(\tau_i|\pi_i)} = \underbrace{D(\tau_i)}_{\text{return}} \underbrace{\prod_{t=1}^{T} \frac{\pi(a_t|x_t)}{\pi_i(a_t|x_t)}}_{\text{importance weight}}$$

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For each π_i, $\widehat{D}(\tau_i, \pi, \pi_i)$ is a random variable

(by generating a trajectory τ_i)

Weighted Importance Return

$\widehat{D}(\tau_i, \pi, \pi_i)$ is a random variable such that

- $\mathbb{E}\big[\widehat{D}(\tau_i, \pi, \pi_i)\big] \leq V^{\pi}$
  - if $\forall x, a \ni \pi_i(a|x) = 0 \Longrightarrow \pi(a|x) = 0$ then $\mathbb{E}\big[\widehat{D}(\tau_i, \pi, \pi_i)\big] = V^{\pi}$
- $\widehat{D}(\tau_i, \pi, \pi_i) \geq 0$ and $\mathbb{E}\big[\widehat{D}(\tau_i, \pi, \pi_i)\big] \in [0, 1/1-\gamma]$
- $\widehat{D}(\tau_i, \pi, \pi_i)$ may have a very large upper-bound ***(very long tail)***

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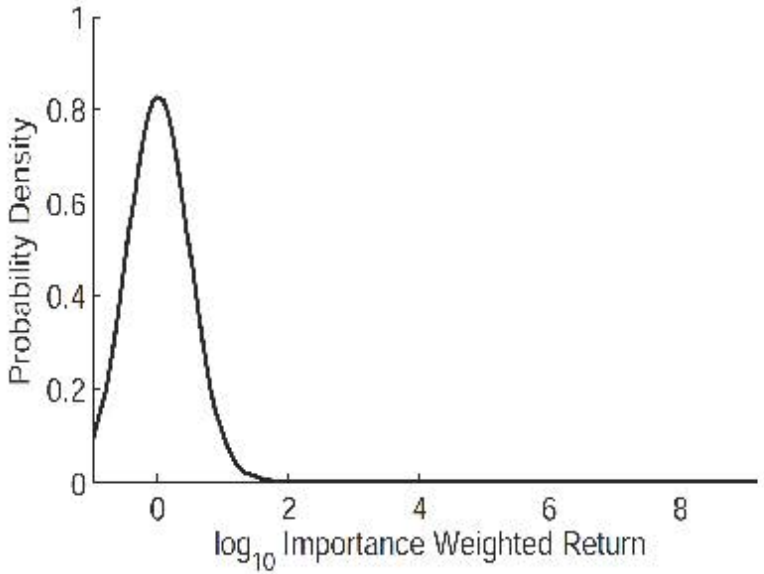

- two policies in the Mountain Car problem
- T = 20
- PDF is estimated from 100,000 trajectories
- sample mean = 0.191
- maximum observed WIR = 316
- upper-bound on WIR = $10^{9.4}$

Problem Formulation

Given the data set $\mathcal{D} = \left\{(\tau_i, \pi_i)\right\}_{i=1}^{n}$, we have

- n independent bounded random variables

$$X_1 = \widehat{D}(\tau_1, \pi, \pi_1) \quad X_2 = \widehat{D}(\tau_2, \pi, \pi_2) \quad \dots \quad X_n = \widehat{D}(\tau_n, \pi, \pi_n)$$

- for all $i \in \{1, \dots, n\}$,

$$\Pr\left(X_i \in [0, b_i]\right) = 1 \qquad \textbf{and} \qquad \mathbb{E}[X_i] = \mu \leq V^{\pi}$$

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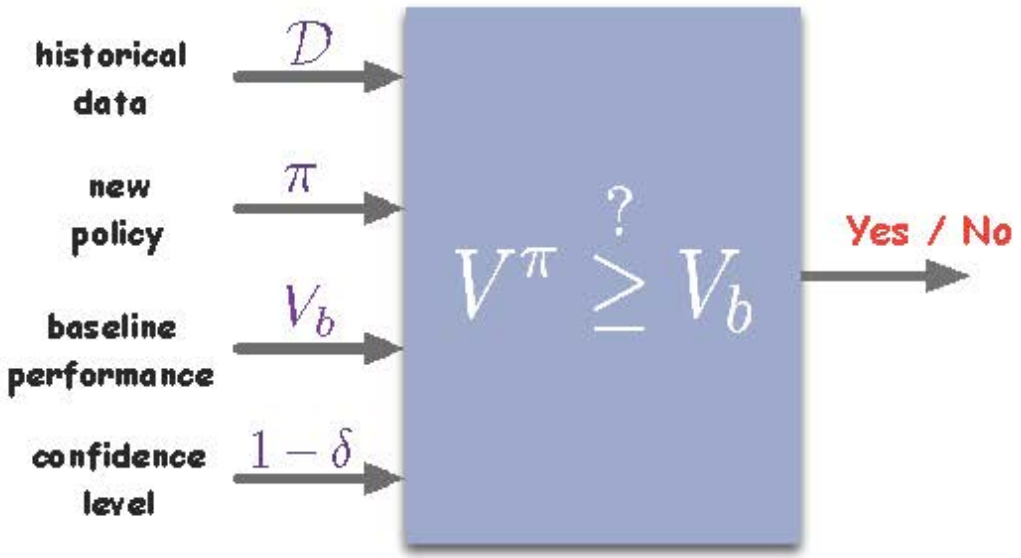
Main Objective: to derive a ***tight*** high probability (w.p. $\geq 1 - \delta$) lower-bound μ_{-} on μ

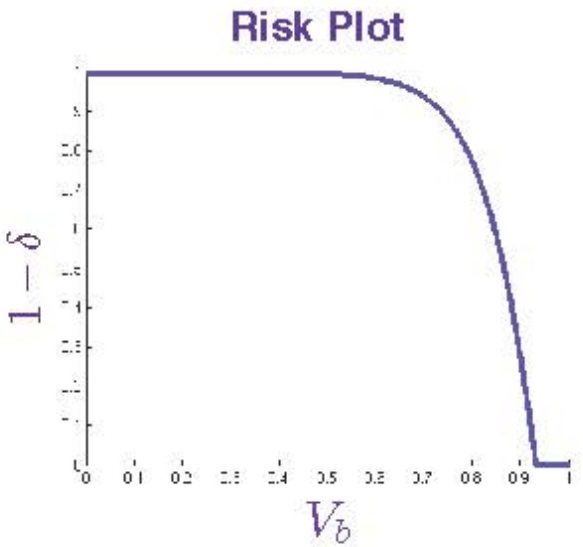
(note that b_i's can be very large)

Problem Formulation

if $\mu_- \geq V_b$, ***then*** so does V^π ***(w.p.*** $\geq 1-\delta$***)*** ***Yes***

otherwise ***no answer***





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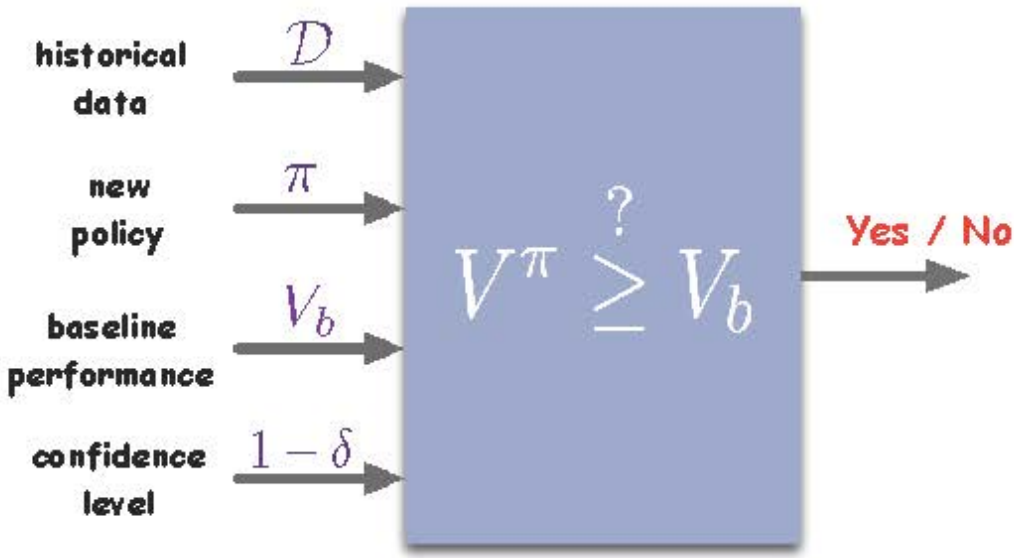

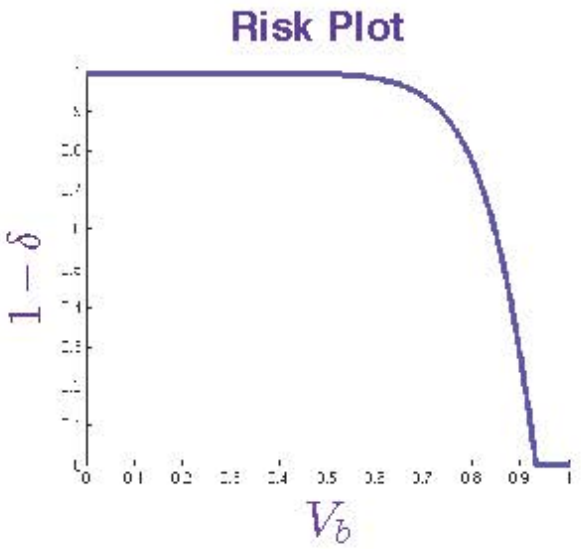

how to derive a tight lower-bound on μ***???***

(note that b_i***'s can be very large)***

Concentration Inequalities

Chernoff-Hoeffding (CH)

$$\mu \geq \frac{1}{n}\sum_{i=1}^{n} X_i - b\sqrt{\frac{\log(1/\delta)}{2n}} \qquad \text{w.p.} \geq 1-\delta$$

Maurer-Pontil empirical Bernstein (MPeB) *(Maurer & Pontil, 2009)*

$$\mu \geq \frac{1}{n}\sum_{i=1}^{n} X_i - \frac{7b\log(2/\delta)}{3(n-1)} - \frac{1}{n}\sqrt{\frac{\log(2/\delta)}{n-1}\sum_{i,j=1}^{n}(X_i - X_j)^2} \qquad \text{w.p.} \geq 1-\delta$$

Anderson-Massart (AM) *(Anderson, 1969; Massart, 1990)*

$$\mu \geq z_n - \sum_{i=0}^{n-1}(z_{i+1} - z_i)\ \min\left\{1, \frac{i}{n} + \sqrt{\frac{\log(2/\delta)}{2n}}\right\} \qquad \text{w.p.} \geq 1-\delta$$

$z_1 \leq z_2 \leq \ldots \leq z_n$ are sorted samples of the random variables $X_1, \ldots, X_n$

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New MPeB bound: by collapsing the tails of the distributions of the random variables and then bounding the means of the new distributions ***a from of Winsorization*** ***(Wilcox & Keselman, 2003)***

New MPeB Bound

Theorem

Let $X_1, \ldots, X_n$ be n independent bounded ***(in*** $[0, b_i]$***)*** random variables with the ***same*** mean $\mathbb{E}[X_i] = \mu$. Let $Y_1, \ldots, Y_n$ such that $Y_i = \min\{X_i, c_i\}$ be bounded versions of $X_1, \ldots, X_n$ with ***fixed*** thresholds $c_i > 0$. Then ***w.p.*** $\geq 1 - \delta$, we have

$$\mu \geq \underbrace{\left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \sum_{i=1}^{n} \frac{Y_i}{c_i}}_{\text{empirical mean}} - \underbrace{\left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \frac{7n \log(2/\delta)}{3(n-1)}}_{\text{term that goes to zero as } 1/n}$$

$$- \underbrace{\left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \sqrt{\frac{\log(2/\delta)}{n-1} \sum_{i,j=1}^{n} \left(\frac{Y_i}{c_i} - \frac{Y_j}{c_j}\right)^2}}_{\text{term that goes to zero as } 1/\sqrt{n}}.$$

New MPeB Bound

Theorem

Let $X_1, \ldots, X_n$ be n independent bounded ***(in*** $[0, b_i]$***)*** random variables with the ***same*** mean $\mathbb{E}[X_i] = \mu$. Let $Y_1, \ldots, Y_n$ such that $Y_i = \min\{X_i, c_i\}$ be bounded versions of $X_1, \ldots, X_n$ with ***fixed*** thresholds $c_i > 0$. Then ***w.p.*** $\geq 1 - \delta$, we have

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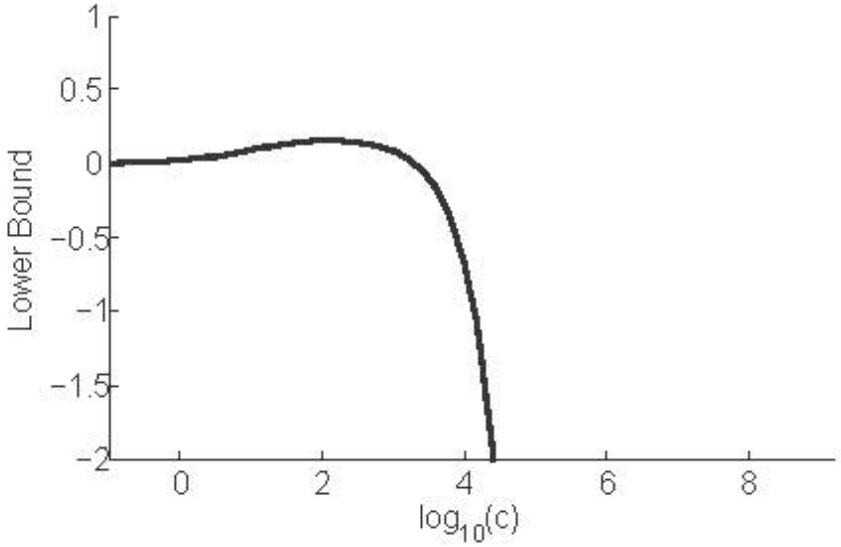
how to select c_i***'s independent of the realization of*** X_i***'s???***

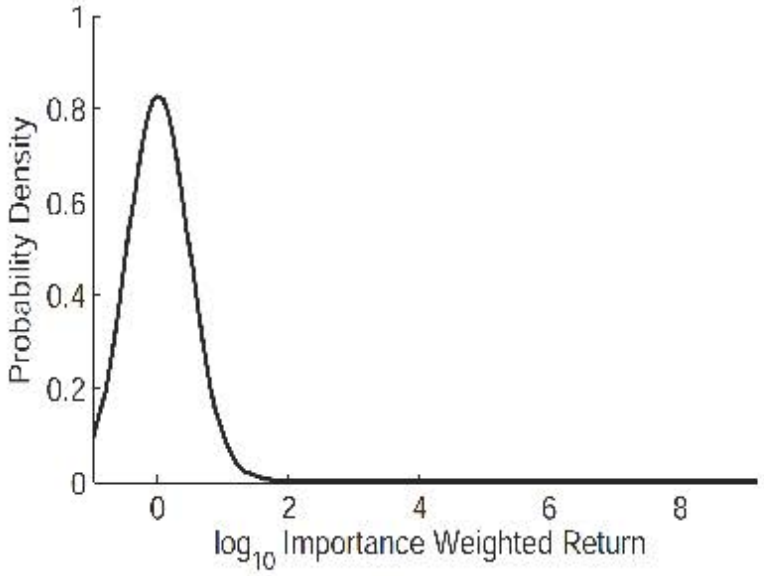
Selecting the Thresholds c_i's



- dividing the data $\mathcal{D}$ into two parts $\mathcal{D}_{\text{pre}}$ and $\mathcal{D}_{\text{post}}$
  ($1/20$ in $\mathcal{D}_{pre}$, $19/20$ in $\mathcal{D}_{post}$)
- calculating the optimal values of c_i by maximizing μ_- using $\mathcal{D}_{\text{pre}}$
  (computing c_i^*'s)
- calculating μ_- with c_i^*'s using $\mathcal{D}_{\text{post}}$
- ***if*** $\mu_- \geq V_b$, *return* ***Yes***, ***else*** *return* ***no answer***

Experimental Results

Results - Mountain Car Problem

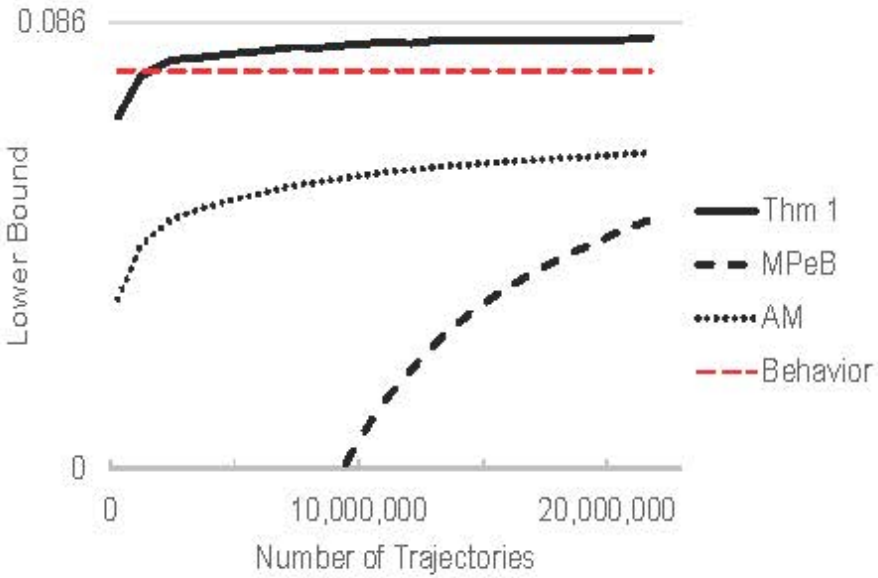


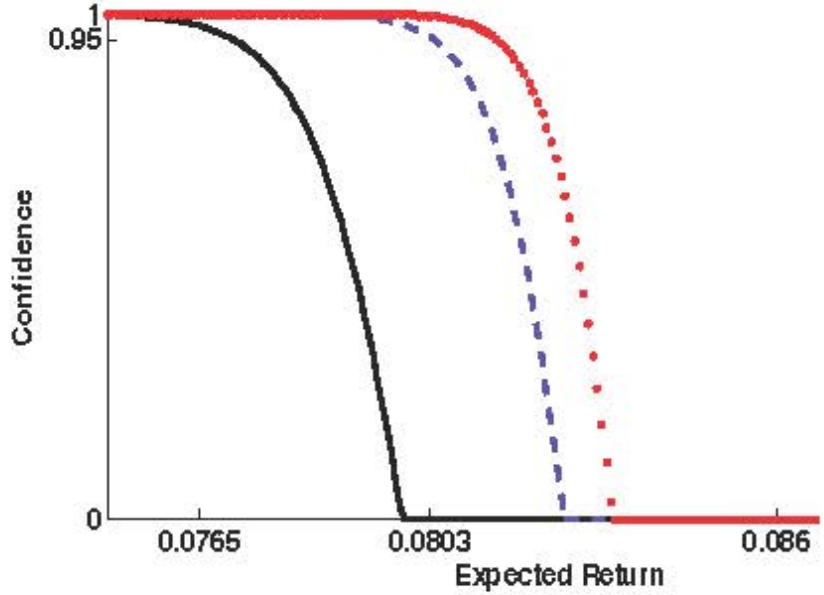
- two policies in the Mountain Car problem
- T = 20
- PDF is estimated from 100,000 trajectories
- sample mean = 0.191
- maximum observed WIR = 316
- upper-bound on WIR = $10^{9.4}$

95% *confidence lower-bounds*

	New MPeB	CH	MPeB	AM
μ_-	0.154	$-5{,}831{,}000$	$-129{,}703$	0.055

Results - Personalized Ad Recommendation *(simulated data)*





- target policy = a RL policy
- performance of behavior policy = 0.0765
- performance of target policy = 0.086
- trajectories of size T=20
- 95% confidence lower-bound

2M traj (behavior)
95% lower-bound = 0.077
5M traj (behavior)
95% lower-bound = 0.0803
5M traj (behavior) + 1M traj (target)
95% lower-bound = 0.081

Improving the Lower-Bound

MPeB lower-bound

general - no assumption on the data

One-sided Student's t-Test

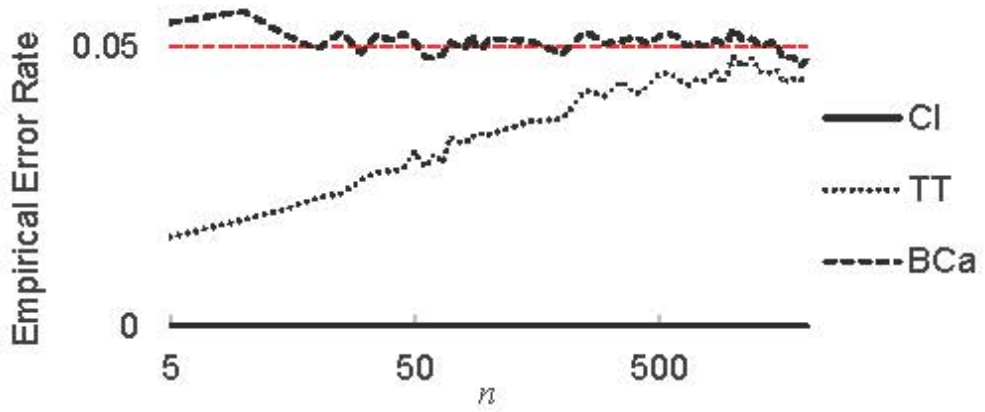
assumes $\widehat{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is normally distributed *(is true as $n \to \infty$ - CLT)*

Bias Corrected and accelerated (BCa) Bootstrap: *(Efron, 1987)*

use bootstrapping to estimate the true distribution of $\widehat{X}$ and then use it to produce a lower-bound

Improving the Lower-Bound

heavy upper-tail Gamma distribution

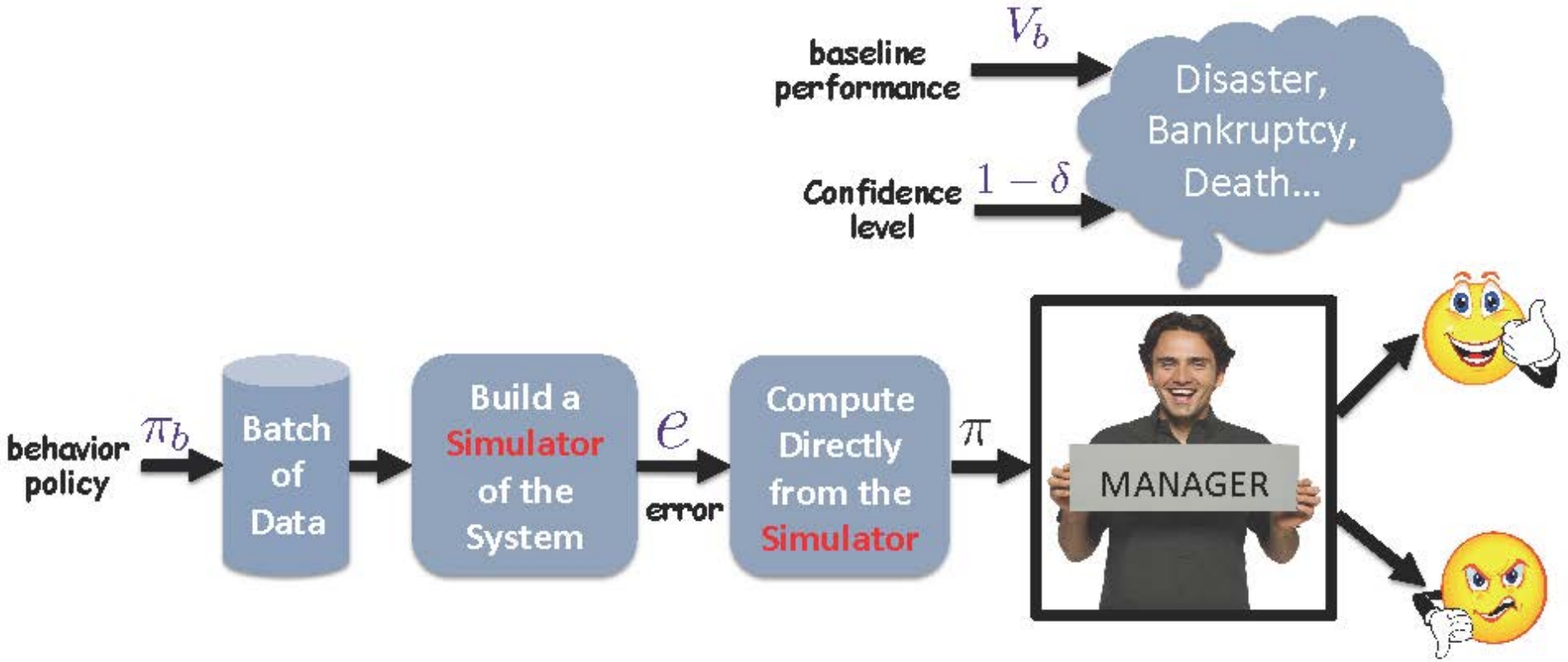


95% confidence lower-bound on the mean - 100,000 trials

Outline

Model-based Approach

Model-based Approach



Main Question: Given the simulator and the error in building it, how to compute a policy that is guaranteed ***(with a given confidence level)*** to perform at least as well as a baseline???

Safe Policy Improvement by Minimizing Robust Baseline Regret

M. Petrik, **MGH**, and Y. Chow. *"Safe Policy Improvement by Minimizing Robust Baseline Regret"*. ***NIPS-2016***.

Problem Definition *(Model Uncertainty)*

- **True Dynamics** $P^\star$
- **Simulator** $\widehat{P}$
- **Error Function**

$$\forall (x,a) \in \mathcal{X} \times \mathcal{A} \quad : \quad ||P^\star(\cdot|x,a) - \widehat{P}(\cdot|x,a)||_1 \leq e(x,a)$$

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- **Uncertainty Set**

$$\Xi(\widehat{P}, e) = \Big\{ \xi : \mathcal{X} \times \mathcal{A} \to \Delta^{\mathcal{X}} \ : \\ ||\xi(\cdot|x,a) - \widehat{P}(\cdot|x,a)||_1 \leq e(x,a), \ \forall (x,a) \in \mathcal{X} \times \mathcal{A} \Big\}$$

Problem Definition *(Safety)*

- **Baseline Policy** *(deterministic)* π_b
- **Return of a Policy π in world ξ** $V(\pi, \xi)$
- **True Return of a Policy π** $V(\pi, P^\star)$
- **True Optimal Policy** $\pi^\star \in \arg\max_\pi V(\pi, P^\star)$
- **Policy π is safe if** $V(\pi, P^\star) \geq V(\pi_b, P^\star)$

Problem Formulation I

Solving the Simulator

$$\pi_{\text{sim}} \in \arg\max_{\pi} \; V(\pi, \widehat{P})$$

Problem Formulation I

Solving the Simulator

$$\pi_{\text{sim}} \in \arg\max_{\pi} \; V(\pi, \widehat{P})$$

- no guarantee that π_{sim} is ***safe***

Theorem: Bound on Performance Loss of π_{sim}

$$\Phi(\pi_{\text{sim}}) \triangleq V(\pi^\star, P^\star) - V(\pi_{\text{sim}}, P^\star) \leq \frac{2\gamma R_{\max}}{(1-\gamma)^2} ||e||_\infty$$

Problem Formulation II

Solving the Robust MDP

$$\pi_0 \in \arg\max_{\pi} \min_{\xi \in \Xi} V(\pi, \xi) \tag{1}$$

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- π_R is guaranteed to be ***safe***

Theorem: Bound on Performance Loss of π_R

$$\Phi(\pi_R) \leq \min \left\{ \frac{2\gamma R_{\max}}{(1-\gamma)^2} \left(||e_{\pi^*}||_{1,u^*_{\pi^*}} + ||e_{\pi_b}||_{1,u^*_{\pi_b}} \right), \Phi(\pi_b) \right\}$$

Our Proposed Formulation

Robust Policy Improvement

$$\pi_S \in \arg\max_{\pi} \min_{\xi \in \Xi} \Big(V(\pi, \xi) - V(\pi_b, \xi) \Big) \tag{2}$$

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- π_S is guaranteed to be ***safe***
- π_S can outperform π_R by an arbitrarily large margin

Comparison with the Robust Solution

Solving the Robust MDP

$$\pi_0 \in \arg\max_{\pi} \min_{\xi \in \Xi} V(\pi, \xi)$$

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$$\overbrace{\max_{\pi} \min_{\xi} \Big(V(\pi, \xi) - V(\pi_b, \xi) \Big)}^{\text{Robust Policy Improvement}} \geq \underbrace{\max_{\pi} \min_{\xi} V(\pi, \xi) - \max_{\xi} V(\pi_b, \xi)}_{\text{Robust MDP}}$$

Properties of Robust Policy Improvement Formulation

1. Policy Class

Theorem

Optimization problem (2) may not have solution in the space of deterministic policies and ignoring this may cause huge loss.

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3. Computational Complexity

Theorem

Optimization problem (2) is NP-hard.

Solutions to Robust Policy Improvement Problem

1. Exact Solution with Extra Information

Assumption: Markov chain induced by π_b is known

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Solutions to Robust Policy Improvement Problem

2. A Heuristic Solution

Assumption: Simulator is ***accurate*** for all the actions suggested by π_b

$$\forall x \in \mathcal{X}, \qquad \widehat{P}\big(\cdot\,|x, \pi_b(x)\big) \approx P^{\star}\big(\cdot\,|x, \pi_b(x)\big)$$

Solutions to Robust Policy Improvement Problem

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$$\forall x \in \mathcal{X}, \qquad \widehat{P}\big(\cdot\,|x, \pi_b(x)\big) \approx P^{\star}\big(\cdot\,|x, \pi_b(x)\big)$$

solve the robust MDP

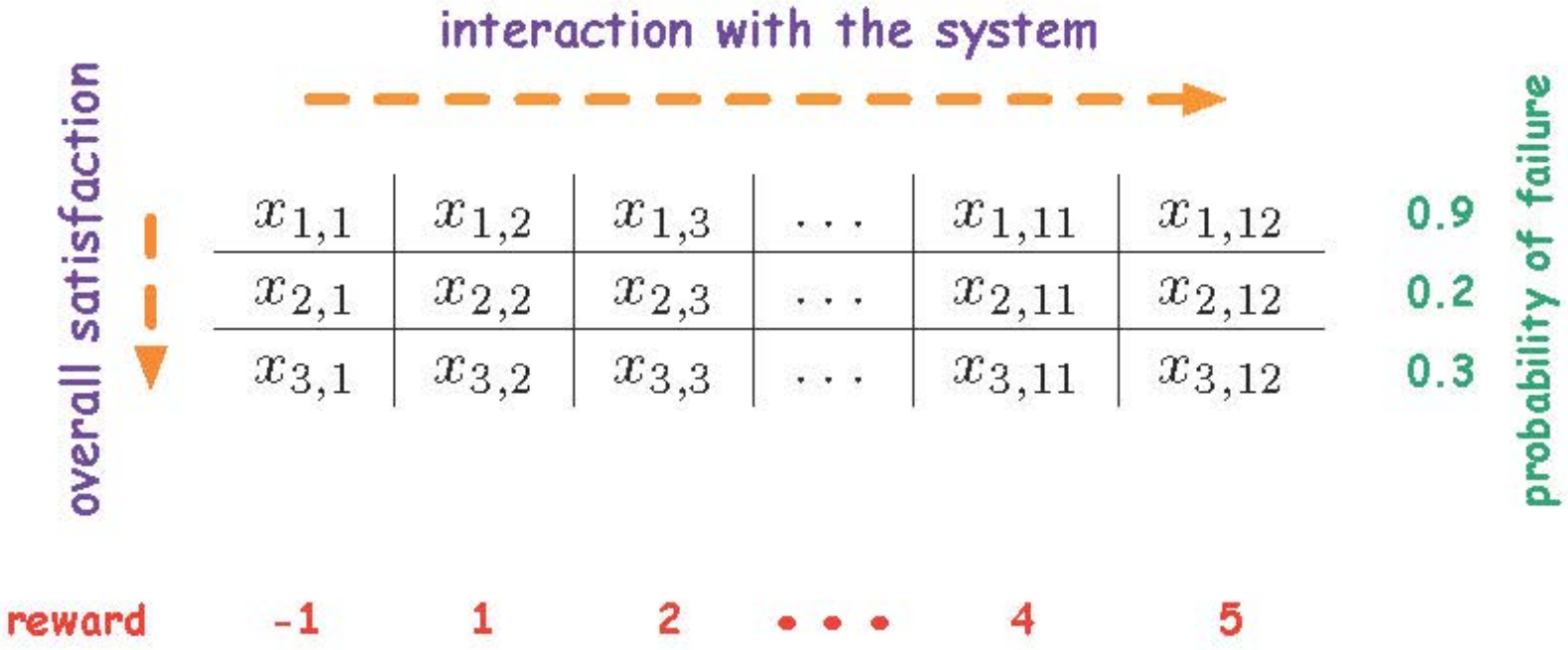
$$\underset{\pi \in \Pi_R}{\arg\max} \quad \underset{\xi \in \Xi(\widehat{P}, e)}{\min} \quad V(\pi, \xi)$$

where in the uncertainty set $\Xi(\widehat{P}, e)$, we force

$$\forall x \in \mathcal{X}, \qquad \widehat{P}\big(\cdot\,|x, \pi_b(x)\big) = P^{\star}\big(\cdot\,|x, \pi_b(x)\big)$$

Experimental Results

Grid Problem

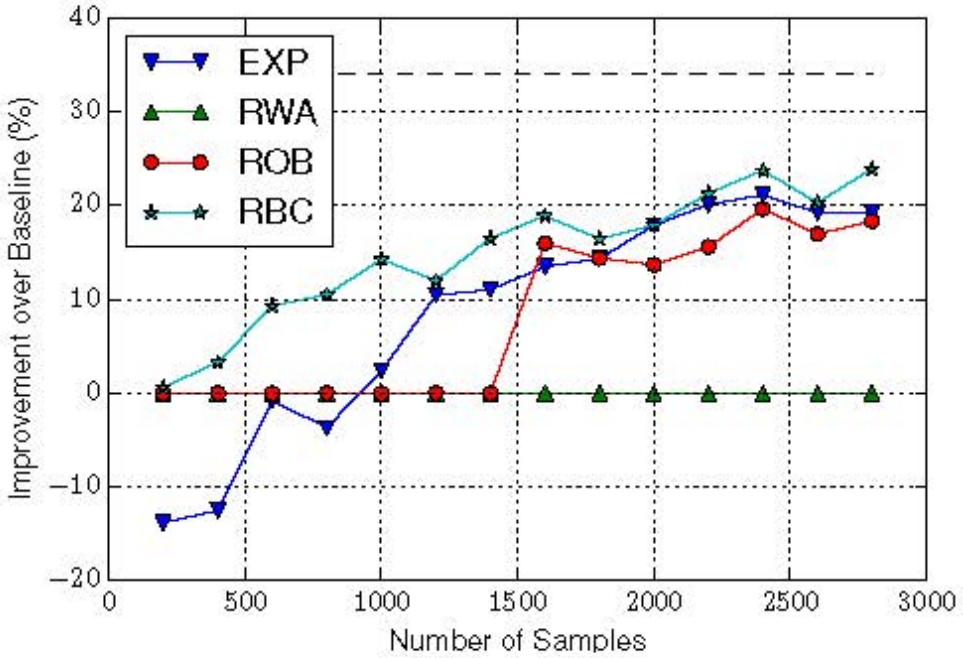


Actions: *left* *right* *up* *down*

Results - Grid Problem

baseline policy: optimal ignoring the row part of the state

simulator: built by samples from random policy



EXP: solving simulator

RWA: reward-adjusted MDP

ROB: robust MDP

RBC: robust policy improvement

Energy Arbitrage Problem

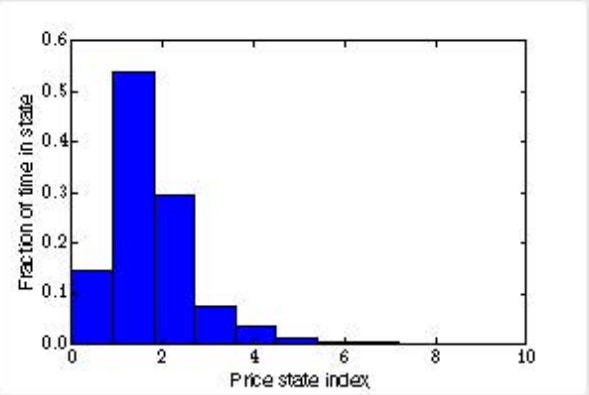
Problem Description

State: *charge level* *capacity* *price*

Action: amount of energy to buy or sell

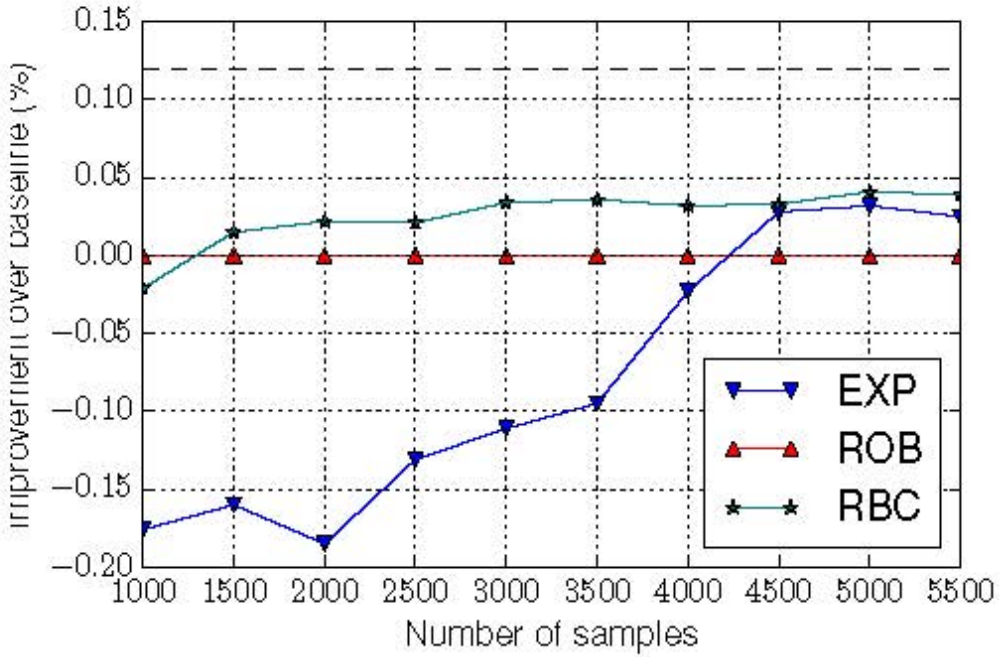
Dynamics:

- capacity degrades when energy is stored or retrieved
- model for charge level and capacity are ***known***
- model for price is ***unknown***
- price is discretized to 10 separate levels



Results - Energy Arbitrage Problem

baseline policy: optimal for the model in which price is discretized to 3 levels



EXP: solving simulator **ROB:** robust MDP

RBC: robust policy improvement

Summary

Safe Policy Improvement

Error Function $\forall(x,a) \in \mathcal{X} \times \mathcal{A} \quad : \quad ||P^*(\cdot|x,a) - \widehat{P}(\cdot|x,a)||_1 \leq e(x,a)$

Uncertainty Set

$$\Xi(\widehat{P}, e) = \left\{\xi : \mathcal{X} \times \mathcal{A} \to \Delta^{\mathcal{X}} \ : \ ||\xi(\cdot|x,a) - \widehat{P}(\cdot|x,a)||_1 \leq e(x,a),\ \forall(x,a) \in \mathcal{X} \times \mathcal{A}\right\}$$

Robust Policy Improvement

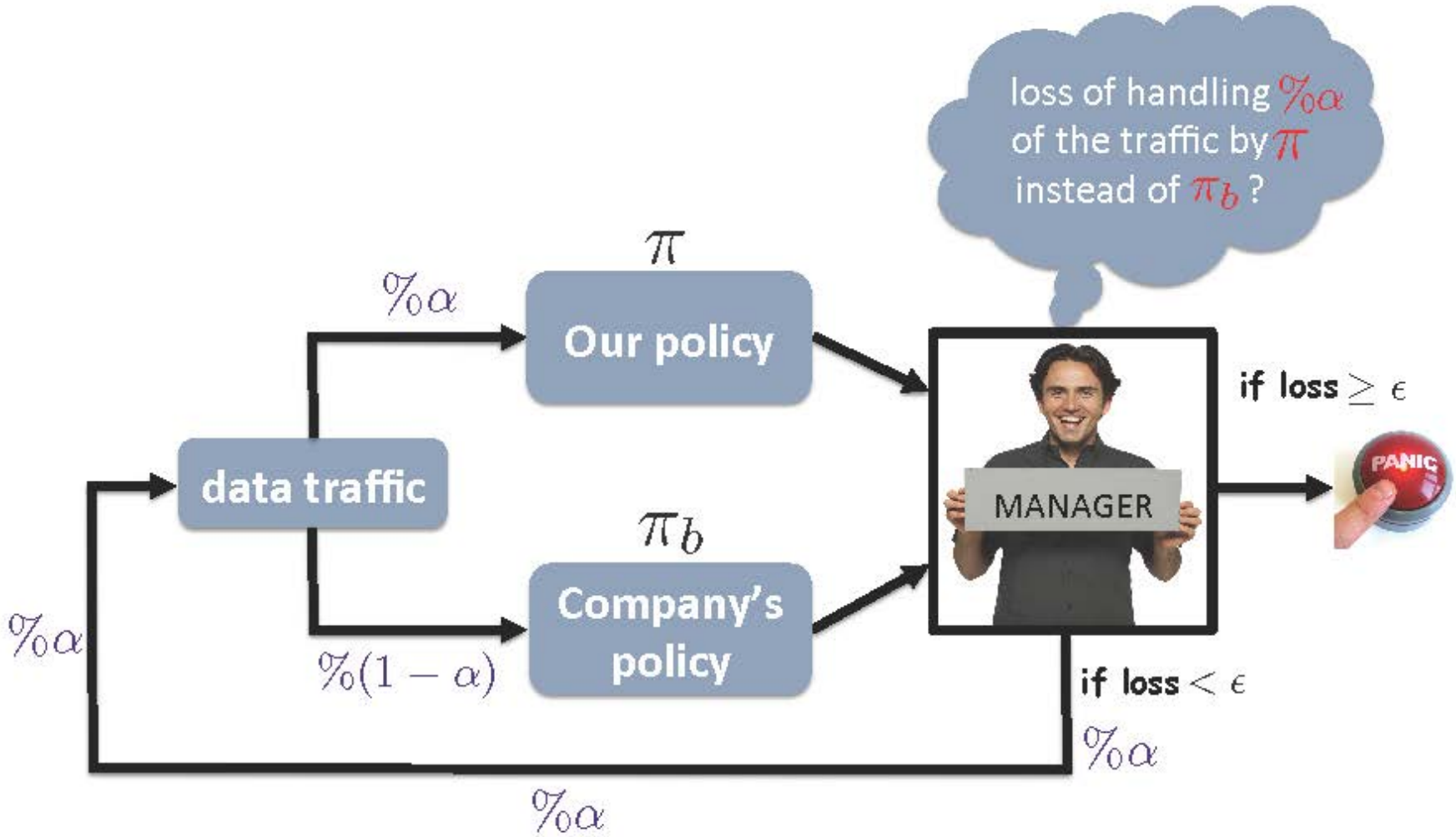
$$\pi_S \in \arg\max_{\pi} \min_{\xi \in \Xi} \Big(V(\pi, \xi) - V(\pi_b, \xi)\Big) \tag{3}$$

- π_S is guaranteed to be ***safe***
- π_S is less conservative than the other solutions
- (3) may not have solution in the space of deterministic policies
- Optimization problem (3) is NP-hard
- Under an assumption, (3) can be solved with a polynomial algorithm

Outline

Online Approach

Online Approach



Conservative Contextual Linear Bandits

A. Kazerouni, **MGH**, Y. Abbasi-Yadkori, and B. Van Roy. *"Conservative Contextual Linear Bandits"*. ***NIPS-2017***.

Contextual Linear Bandit

At each round t,

- Learner selects an action $a_t \in \mathcal{A}_t$ *($\mathcal{A}_t$ possibly infinite set)*
  - a_t is associated with the feature vector $\phi^t_{a_t} \in \mathbb{R}^d$

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- Learner observes a random reward

$$Y_t = \langle \theta^*, \phi^t_{a_t} \rangle + \eta_t$$

  - $\langle \theta^*, \phi^t_{a_t} \rangle = \mathbb{E}[Y_t] = r^t_{a_t}$ *(expected reward)*
  - η_t is conditionally σ-sub-Gaussian noise
  - $\theta^* \in \mathbb{R}^d$ is the ***unknown*** parameter

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Assumption: $||\theta^*||_2 \leq B$ and $||\phi^t_a||_2 \leq D$ and $r^t_a \in [0, 1]$

Contextual Linear Bandit *(Main Objective)*

Optimal Action

$$a_t^* = \arg\max_{a \in \mathcal{A}_t} \langle \theta^*, \phi_a^t \rangle$$

Minimizing (pseudo)-Regret

$$R_T = \sum_{t=1}^{T} \langle \theta^*, \phi_{a_t^*}^t \rangle - \sum_{t=1}^{T} \langle \theta^*, \phi_{a_t}^t \rangle$$

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same as contextual linear bandit ***except***

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same as contextual linear bandit ***except***

- there exists a ***baseline*** policy π_b that at each round t
  - selects action $b_t \in \mathcal{A}_t$
  - observes expected reward $r^t_{b_t} = \langle \theta^*, \phi^t_{b_t} \rangle$ ($r^t_{b_t}$ ***is known***)

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same as contextual linear bandit ***except***

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  - observes expected reward $r^t_{b_t} = \langle \theta^*, \phi^t_{b_t} \rangle$ ($r^t_{b_t}$ ***is known***)
- **Performance Constraint:** At each round t, $\alpha \in (0, 1)$

$$\underbrace{\sum_{i=1}^{t} r^i_{b_i} - \sum_{i=1}^{t} r^i_{a_i}}_{\textit{cumulative loss}} \leq \alpha \sum_{i=1}^{t} r^i_{b_i}$$

Conservative Contextual Linear Bandit

same as contextual linear bandit ***except***

- there exists a ***baseline*** policy π_b that at each round t
  - selects action $b_t \in \mathcal{A}_t$
  - observes expected reward $r^t_{b_t} = \langle \theta^*, \phi^t_{b_t} \rangle$ ($r^t_{b_t}$ ***is known***)
- **Performance Constraint:** At each round t, $\alpha \in (0, 1)$

$$\underbrace{\sum_{i=1}^{t} r^i_{b_i} - \sum_{i=1}^{t} r^i_{a_i}}_{\textit{cumulative loss}} \leq \alpha \sum_{i=1}^{t} r^i_{b_i} \quad , \quad \sum_{i=1}^{t} r^i_{a_i} \geq (1-\alpha) \sum_{i=1}^{t} r^i_{b_i}$$

small α more conservative, large α less conservative

A Conservative Contextual Linear Bandit Algorithm

Given α and $r^t_{b_t}$, it should

minimize (pseudo)-regret

$$R_T = \sum_{t=1}^{T} \langle \theta^*, \phi^t_{a^*_t} \rangle - \sum_{t=1}^{T} \langle \theta^*, \phi^t_{a_t} \rangle$$

satisfy performance constraint

$$\sum_{i=1}^{t} r^i_{a_i} \geq (1-\alpha) \sum_{i=1}^{t} r^i_{b_i}$$

A Conservative Contextual Linear Bandit Algorithm

At each round t, the CLUCB algorthm

A Conservative Contextual Linear Bandit Algorithm

At each round t, the CLUCB algorthm

- uses the previous observations and builds a ***confidence set*** $\mathcal{C}_t$ ***(w.h.p. contains*** θ^* ***)***

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A Conservative Contextual Linear Bandit Algorithm

At each round t, the CLUCB algorithm

- uses the previous observations and builds a ***confidence set*** $\mathcal{C}_t$ ***(w.h.p. contains*** θ^* ***)***
- computes the ***optimistic action***

$$a'_t = \arg\max_{a \in \mathcal{A}_t} \max_{\theta \in \mathcal{C}_t} \langle \theta, \phi^t_a \rangle$$

- plays the ***optimistic action*** $(a_t = a'_t)$, only if

$$\overbrace{\sum_{i \in S^o_{t-1}} r^i_{b_i}}^{\text{baseline rounds}} + \min_{\theta \in \mathcal{C}_t} \left(\underbrace{\langle \theta, \phi^t_{a'_t} \rangle}_{\text{round } t} + \overbrace{\sum_{i \in S_{t-1}} \langle \theta, \phi^i_{a'_i} \rangle}^{\text{optimistic rounds}} \right) \geq (1-\alpha) \sum_{i=1}^{t} r^i_{b_i}$$

plays the ***baseline action*** $(a_t = b_t)$, otherwise.

Construction of Confidence Sets *(Abbasi-Yadkori et al., 2011)*

At each round t,

given the observed data $\left\{(\phi^i_{a_i}, Y_i)\right\}_{i=1}^{|S_t|}$, CLUCB updates the ***confidence set*** as *(S_t = set of rounds we play optimistic)*

$$C_{t+1} = \left\{\theta \in \mathbb{R}^d \ : \ ||\theta - \widehat{\theta}||_{V_t} \leq \beta_{t+1}\right\}$$

$$\widehat{\theta}_t = (\Phi_t \Phi_t^\top + \lambda I)^{-1} \Phi_t Y_t \quad , \quad V_t = \lambda I + \Phi_t \Phi_t^\top$$

$$\beta_{t+1} = \sigma \sqrt{d \log\left(\frac{1 + (|S_t| + 1)D^2/\lambda}{\delta}\right)} + \sqrt{\lambda} B$$

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$$\beta_{t+1} = \sigma \sqrt{d \log\left(\frac{1 + (|S_t| + 1)D^2/\lambda}{\delta}\right)} + \sqrt{\lambda} B$$

Proposition: For any C_t and $\delta > 0$, we have $\mathbb{P}\left[\theta^* \in C_t, \ \forall t \in \mathbb{N}\right] \geq 1 - \delta$.

Regret Analysis

Assumption

There exists $0 \leq \Delta_l \leq \Delta_h$ and $0 < r_l$ such that at each round t,

$$\Delta_l \leq \Delta_{b_t}^t = r_{a_t^*}^t - r_{b_t}^t \leq \Delta_h \qquad \text{and} \qquad r_l \leq r_{b_t}^t.$$

Proposition

The regret of CLUCB can be decomposed into two terms as follows:

$$R_T(\text{CLUCB}) \leq R_{S_T}(\text{LUCB}) + |S_T^c|\Delta_h$$

Regret Analysis

Theorem

With probability at least $1-\delta$, CLUCB satisfies the performance constraint for all $t \in \mathbb{N}$ and has the regret bound

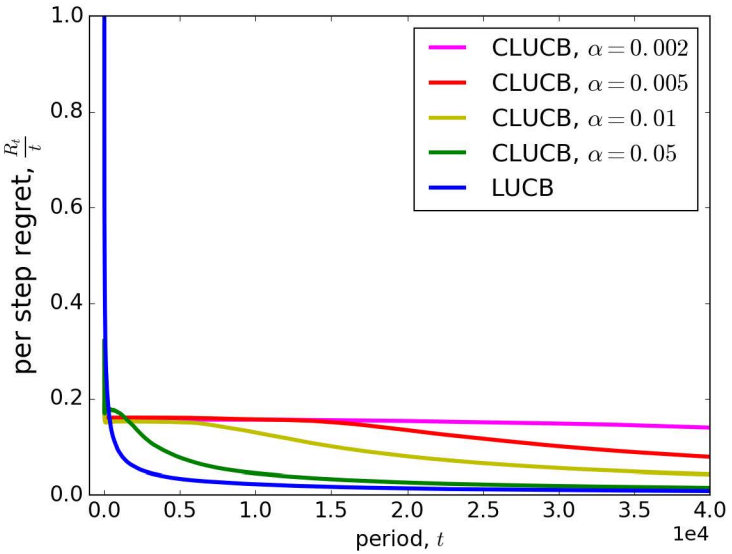
$$R_T(CLUCB) = O\left(\overbrace{\underbrace{d\sqrt{T}\log\left(\frac{DT}{\lambda\delta}\right)}_{R_{S_T}(\text{LUCB})}}^{O\left(\sqrt{T}\log(T)\right)} + \underbrace{\frac{K\Delta_h}{\alpha r_l}}_{\text{constant}} \right)$$

$$K = 1 + 114d^2 \frac{(B\sqrt{\lambda}+\sigma)^2}{\Delta_l + \alpha r_l} \left[\log\left(\frac{62d(B\sqrt{\lambda}+\sigma)}{\sqrt{\delta}(\Delta_l+\alpha r_l)}\right)\right]^2$$

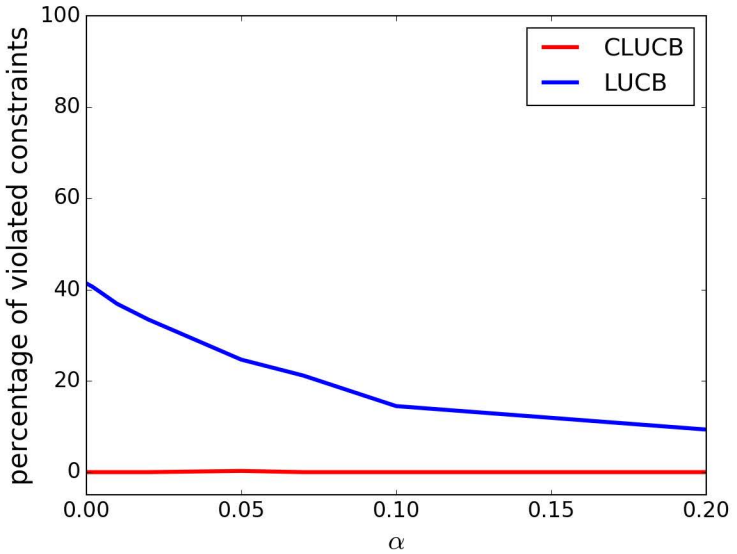
Experimental Results

$$d = 4, \quad \lambda = 1, \quad \delta = 0.001, \quad \theta^* \sim \mathcal{N}(\mathbf{0}, \mathbf{I_4}), \quad \eta_t \sim \mathcal{N}(0, 1)$$

$$\#\text{ of arms} = 100, \quad \text{baseline} = 10\text{'th best arm}, \quad \text{averaged over } 1,000 \text{ runs}$$



per-step regret



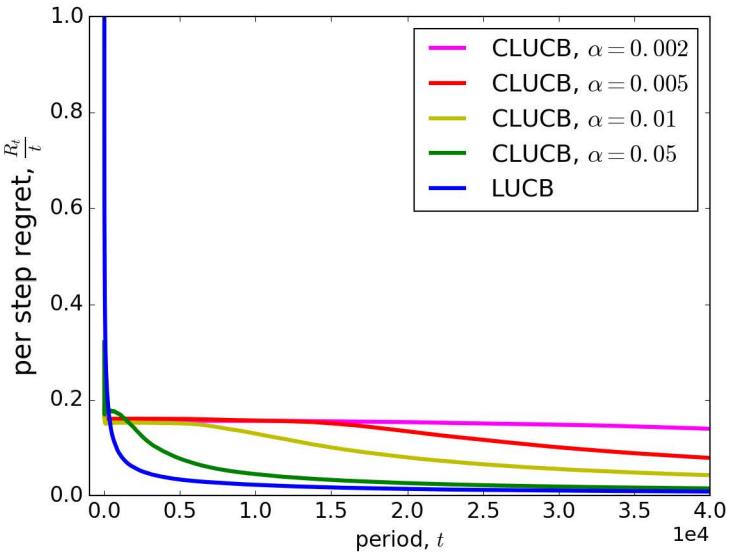
% constraint violation in first 1000 rounds

CLUCB starts to play optimistically more quickly for larger values of α

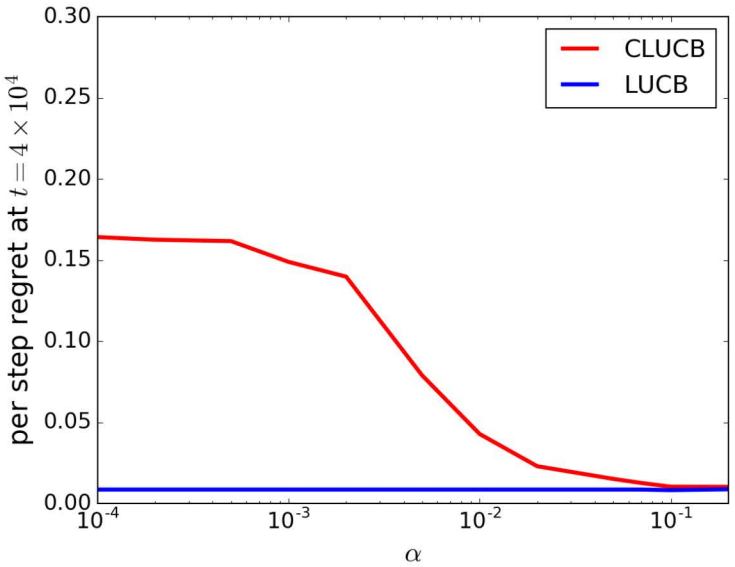
Experimental Results

$$d = 4, \quad \lambda = 1, \quad \delta = 0.001, \quad \theta^* \sim \mathcal{N}(\mathbf{0}, \mathbf{I_4}), \quad \eta_t \sim \mathcal{N}(0, 1)$$

$\#$ of arms $= 100$, baseline $= 10$'th best arm, averaged over $1,000$ runs



per-step regret



per-step regret at $t = 40,000$

performance of CLUCB converges to that of LUCB more quickly for larger values of α

Summary

Conservative Contextual Linear Bandit

Given α and $r^t_{b_t}$,

minimize (pseudo)-regret

$$R_T = \sum_{t=1}^{T} \langle \theta^*, \phi^t_{a^*_t} \rangle - \sum_{t=1}^{T} \langle \theta^*, \phi^t_{a_t} \rangle$$

s.t. the performance constraint

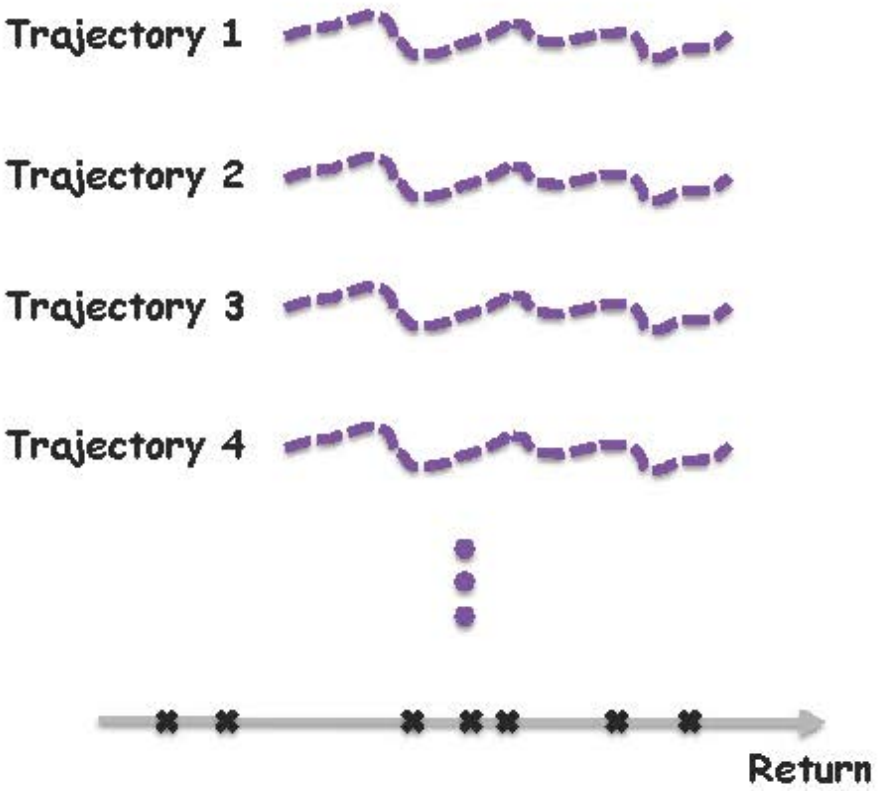
$$\forall t \in \{1, \dots, T\} \qquad \sum_{i=1}^{t} r^i_{a_i} - \sum_{i=1}^{t} r^i_{b_i} \leq \alpha \sum_{i=1}^{t} r^i_{b_i}$$

Outline

Risk-sensitive Decision-making
(brief overview)

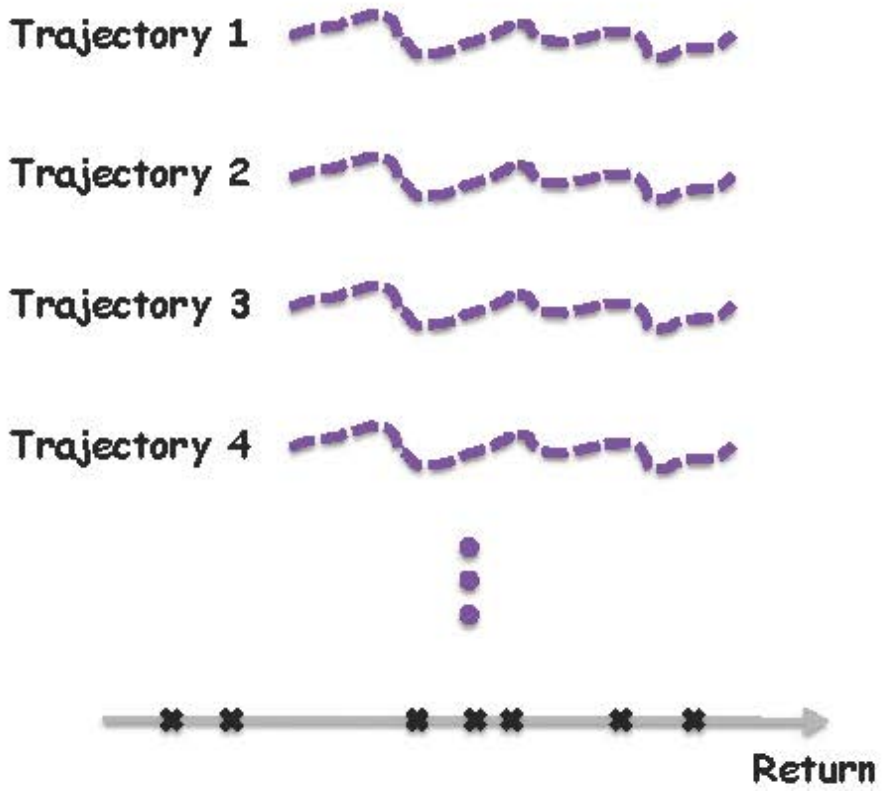
Risk-sensitive Decision-making

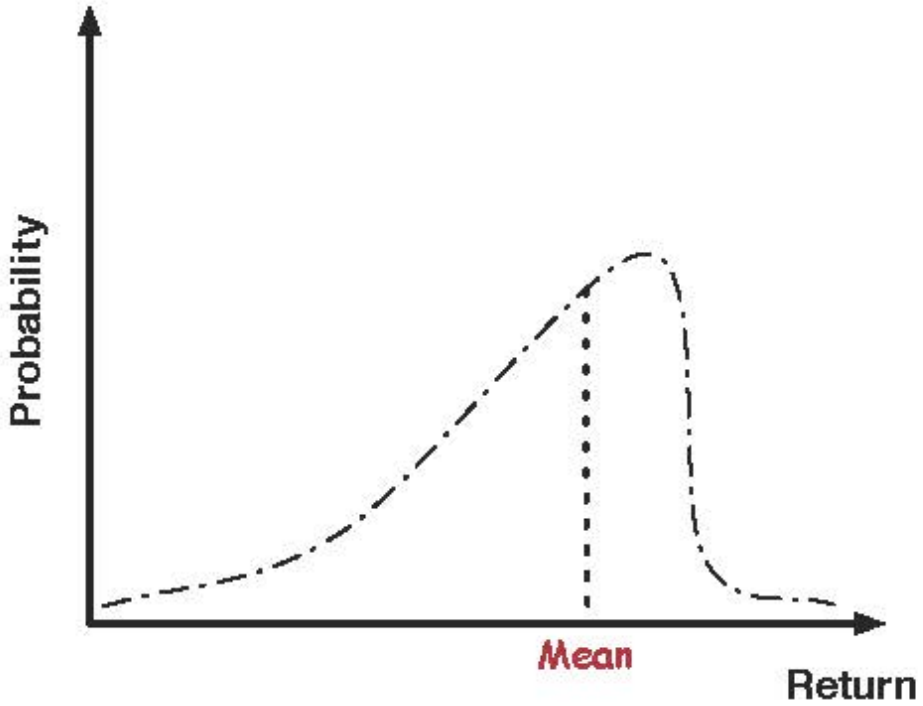
Policy π



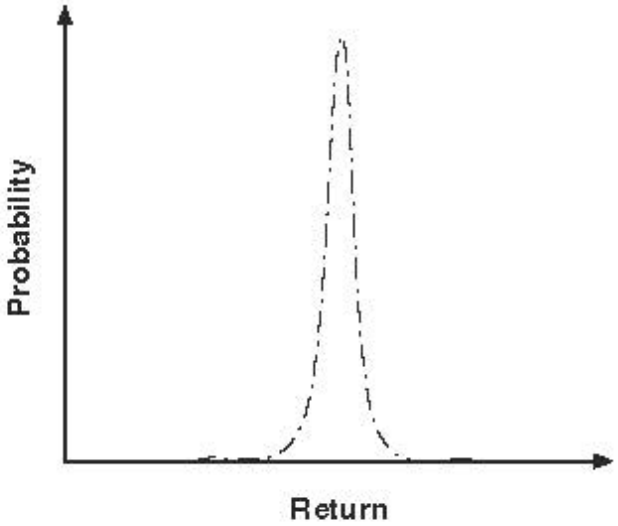
Risk-sensitive Decision-making

Policy π



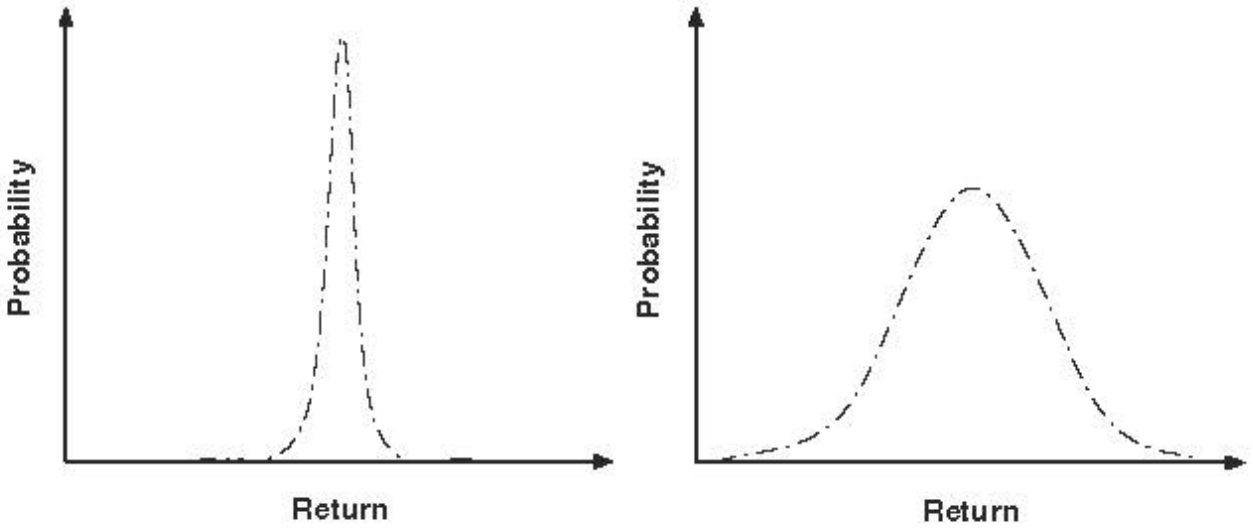


Risk-sensitive Decision-making



$$\max_{\pi} \ \text{Mean}(D^{\pi})$$

Risk-sensitive Decision-making



$$\max_{\pi} \quad \text{Mean}(D^{\pi})$$

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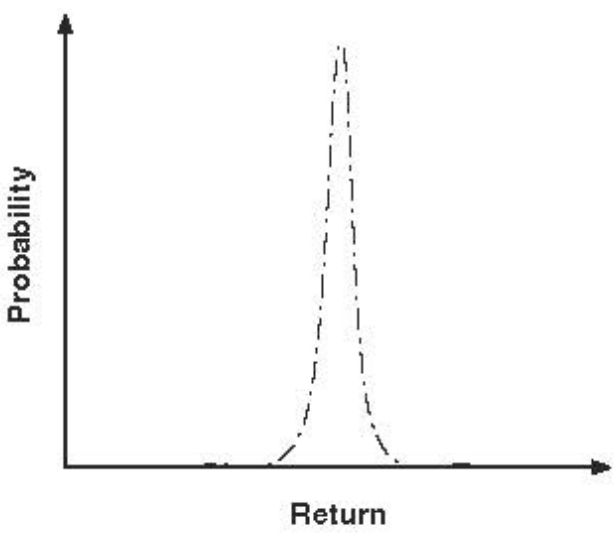
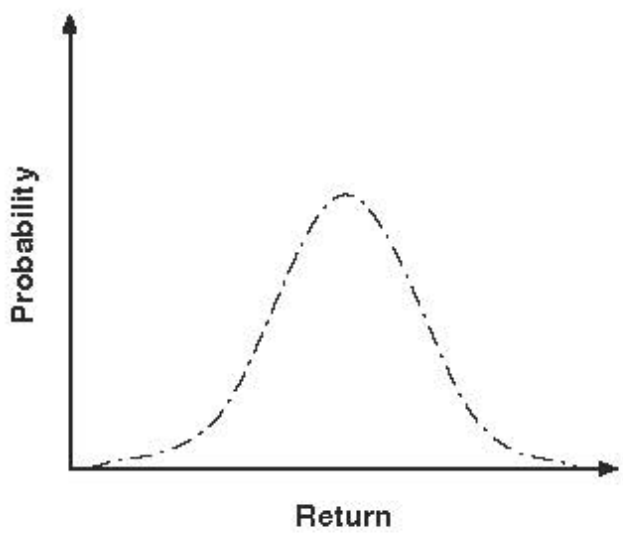
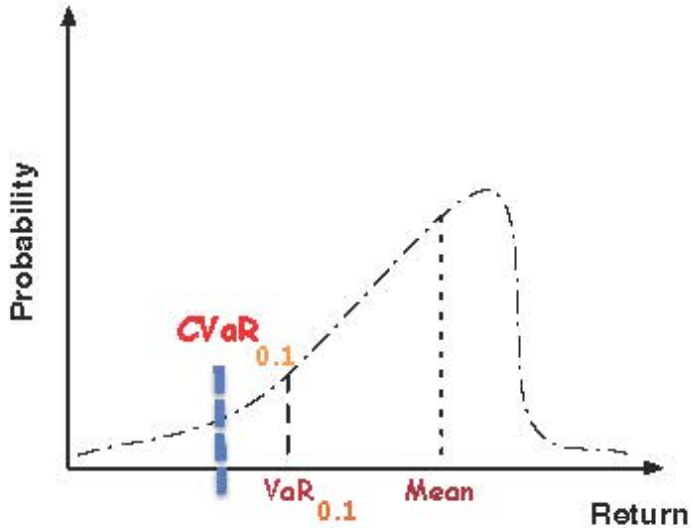
$$\text{s.t.} \quad \text{var}(D^{\pi}) \leq \beta$$

Risk-sensitive Decision-making







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$$\text{s.t.} \quad \text{var}(D^{\pi}) \leq \beta$$

$$\max_{\pi} \quad \text{Mean}(D^{\pi})$$
$$\text{s.t.} \quad \text{CVaR}_{\alpha}(D^{\pi}) \geq \beta$$

Publications

1. Prashanth L. A. and **MGH**. *"Actor-Critic Algorithms for Risk-Sensitive MDPs"*. ***NIPS-2013***.

2. Y. Chow and **MGH**. *"Algorithms for CVaR Optimization in MDPs"*. ***NIPS-2014***.

3. A. Tamar, Y. Chow, **MGH**, and S. Mannor. *"Policy Gradient for Coherent Risk Measures"*. ***NIPS-2015***.

4. Prashanth L. A. and **MGH**. *"Variance-constrained Actor-Critic Algorithms for Discounted and Average Reward MDPs"*. ***MLJ-2016***.

5. A. Tamar, Y. Chow, **MGH**, and S. Mannor. *"Optimization of Coherent Risk Measures"*. ***IEEE TAC 2017***.

6. Y. Chow, **MGH**, L. Janson, and M. Pavone. *"Risk-Constrained Reinforcement Learning with Percentile Risk Criteria"*. ***JMLR-2017***.

7. NEW! J. Lacotte, Y. Chow, **MGH**, and M. Pavone. *"Risk-sensitive Generative Adversarial Imitation Learning"*. ***submitted***.

8. NEW! B. Liu and **MGH**. *"A Block Coordinate Ascent Algorithm for Mean-Variance Optimization"*. ***submitted***.

Tutorial on

Risk-averse Decision-making & Control

Marek Petrik and Mohammad Ghavamzadeh

AAAI-2017

Outline

Safety w.r.t. Undesirable Situations
(brief overview)

Safe RL

CMDP Formulation

$$\min_{\pi} \mathbb{E}\left[\sum_{t=0}^{T-1} c(x_t, a_t) \mid x_0, \pi\right] \quad , \quad \text{s.t.} \quad \mathbb{E}\left[\sum_{t=0}^{T-1} d(x_t) \mid x_0, \pi\right] \leq d_0$$

Safe Policy Iteration (SPI)

1. finding the Lyapunov function

$$\max_{\epsilon: \mathcal{X} \to \mathbb{R}+} ||\epsilon||_1 \quad , \quad \text{s.t.} \quad \mathcal{T}_{d+\epsilon}^{\pi_k}[L_k](x) = L_k(x), \; \forall x \in \mathcal{X} \quad , \quad L_k(x_0) \leq d_0$$

$$L_k(x) = V_{d+\epsilon}^{\pi_k}(x), \; \forall x \in \mathcal{X}$$

2. policy evaluation $\quad V_k = V_c^{\pi_k}$
3. policy improvement $\quad \pi_{k+1} \in \arg\min_{\pi \in \mathcal{F}_{L_k}(x)} \mathcal{T}_c^{\pi}[V_k]$

$$\mathcal{F}_{L_k}(x) = \left\{\pi(\cdot|x) \mid \mathcal{T}_d^{\pi}[L_k](x) \leq L_k(x)\right\}$$

(a) all π_k's are safe, **(b)** π_{k+1} is no worse than π_k, **(c)** SPI converges

Safe RL

SPI / Large Penalty | **Optimal / Small Penalty**

Thank you!!

Mohammad Ghavamzadeh

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