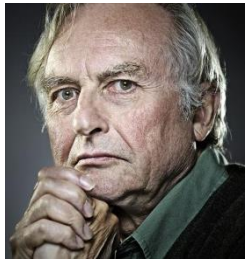
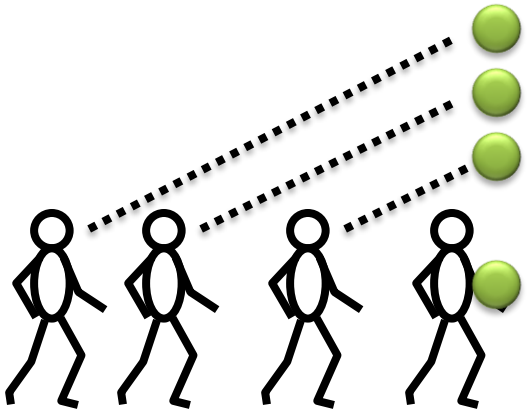
Policy Search

Sergey Levine

"When a man throws a ball high in the air and catches it again, he behaves as if he had solved a set of differential equations in predicting the trajectory of the ball … at some subconscious level, something functionally equivalent to the mathematical calculations is going on."

-- Richard Dawkins





McLeod & Dienes. Do fielders know where to go to catch the ball or only how to get there? Journal of Experimental Psychology 1996, Vol. 22, No. 3, 531-543

Overview

The problem setup

Model-free policy search via policy gradients

What if we know the model?

What if we don’t know the model, but are willing to learn?

Overview

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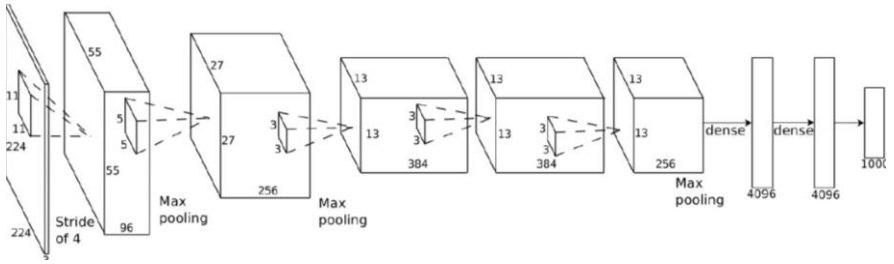
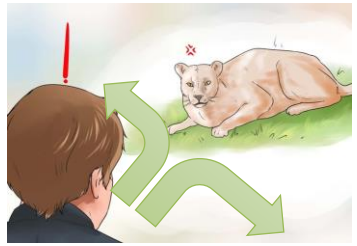
Terminology & notation

$\mathbf{o}_t$



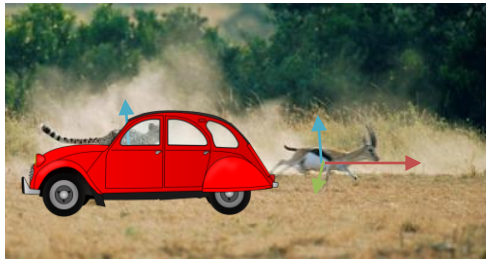
$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$



$\mathbf{a}_t$

$\mathbf{s}_t$ – state
$\mathbf{o}_t$ – observation
$\mathbf{a}_t$ – action

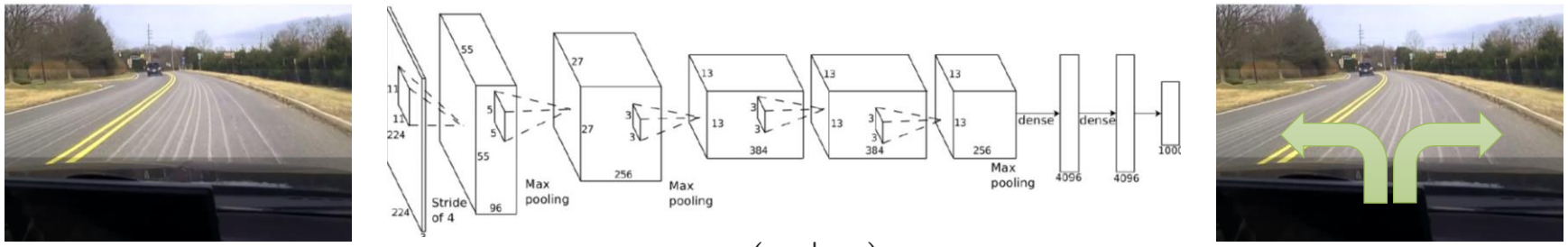
$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$ – policy
$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ – policy (fully observed)



$\mathbf{o}_t$ – observation

$\mathbf{s}_t$ – state

Terminology & notation



$\mathbf{o}_t$ $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$ $\mathbf{a}_t$

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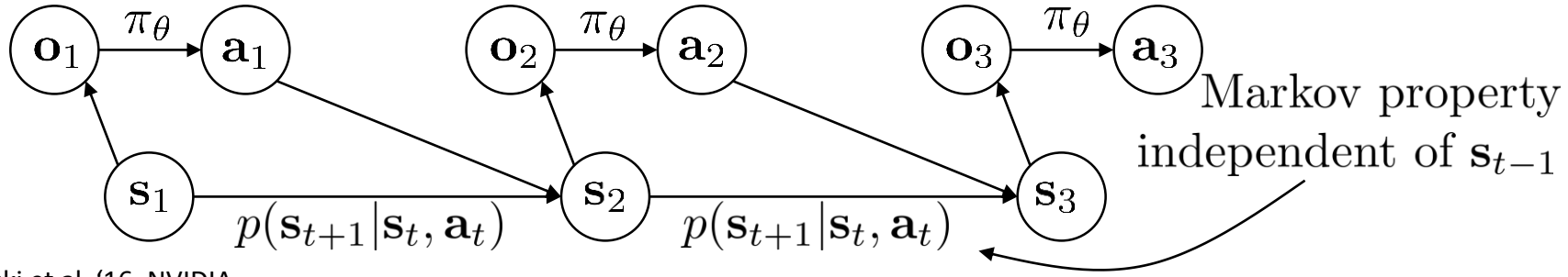
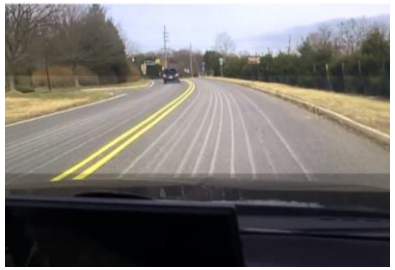
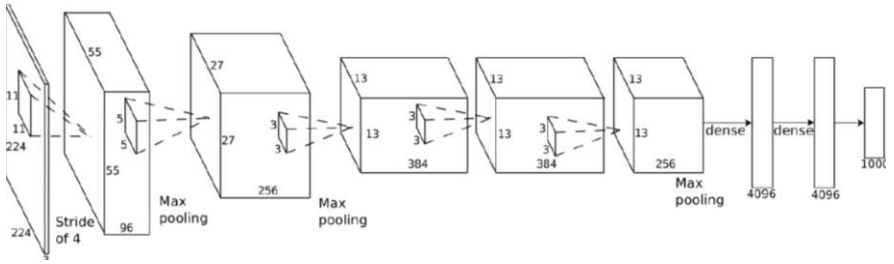


Images: Bojarski et al. ‘16, NVIDIA

Imitation Learning



$\mathbf{o}_t$

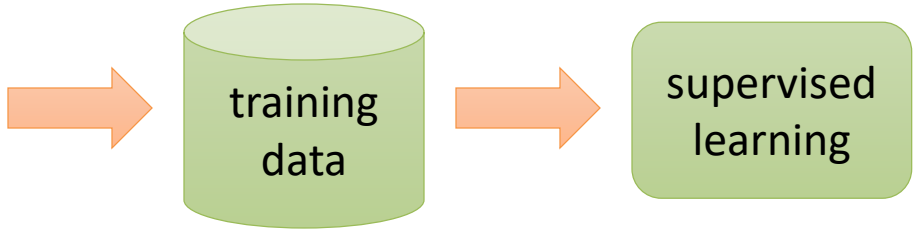


$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$

$\mathbf{a}_t$

$\mathbf{o}_t$

$\mathbf{a}_t$

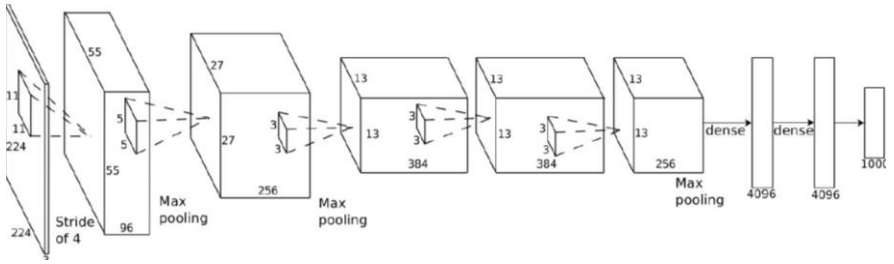


$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$

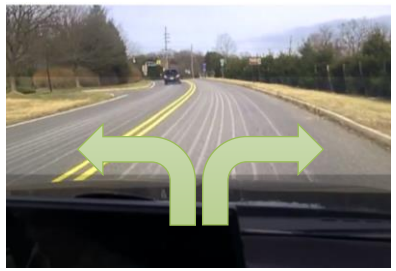
Images: Bojarski et al. ‘16, NVIDIA

Reward functions

$\mathbf{o}_t$



$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$



$\mathbf{a}_t$

which action is better or worse?

$r(\mathbf{s}, \mathbf{a})$: reward function

tells us which states and actions are better

$\mathbf{s}$, $\mathbf{a}$, $r(\mathbf{s}, \mathbf{a})$, and $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ define Markov decision process

high reward

low reward

Definitions

Andrey Markov

Markov chain

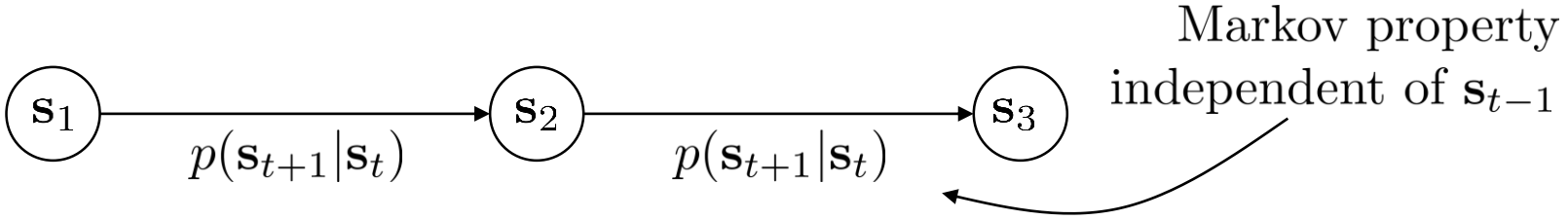
$\mathcal{M} = \{\mathcal{S}, \mathcal{T}\}$

$\mathcal{S}$ – state space — states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{T}$ – transition operator — $p(s_{t+1}|s_t)$

why "operator"? — let $\mu_{t,i} = p(s_t = i)$ — $\vec{\mu}_t$ is a vector of probabilities

let $\mathcal{T}_{i,j} = p(s_{t+1} = i|s_t = j)$ — then $\vec{\mu}_{t+1} = \mathcal{T}\vec{\mu}_t$



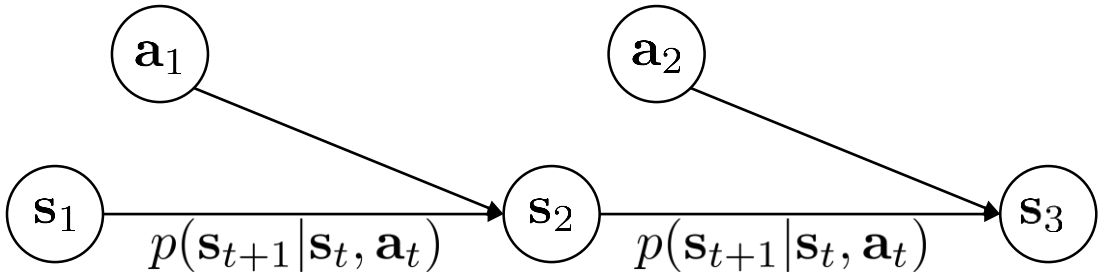
Definitions

Markov decision process $\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$

$\mathcal{S}$ – state space states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{A}$ – action space actions $a \in \mathcal{A}$ (discrete or continuous)

$\mathcal{T}$ – transition operator (now a tensor!)



Andrey Markov

Richard Bellman

Definitions

Markov decision process $\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$

$\mathcal{S}$ – state space states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{A}$ – action space actions $a \in \mathcal{A}$ (discrete or continuous)

$\mathcal{T}$ – transition operator (now a tensor!)

r – reward function $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$

$r(s_t, a_t)$ – reward

Andrey Markov

Richard Bellman

Definitions

partially observed Markov decision process $\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{O}, \mathcal{T}, \mathcal{E}, r\}$

$\mathcal{S}$ – state space states $s \in \mathcal{S}$ (discrete or continuous)

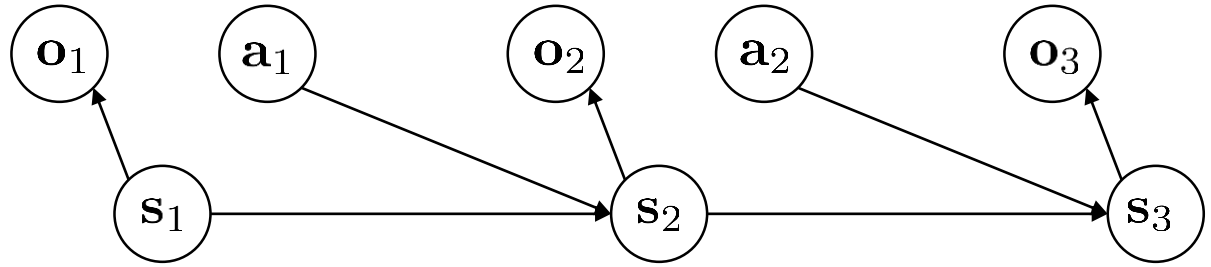
$\mathcal{A}$ – action space actions $a \in \mathcal{A}$ (discrete or continuous)

$\mathcal{O}$ – observation space observations $o \in \mathcal{O}$ (discrete or continuous)

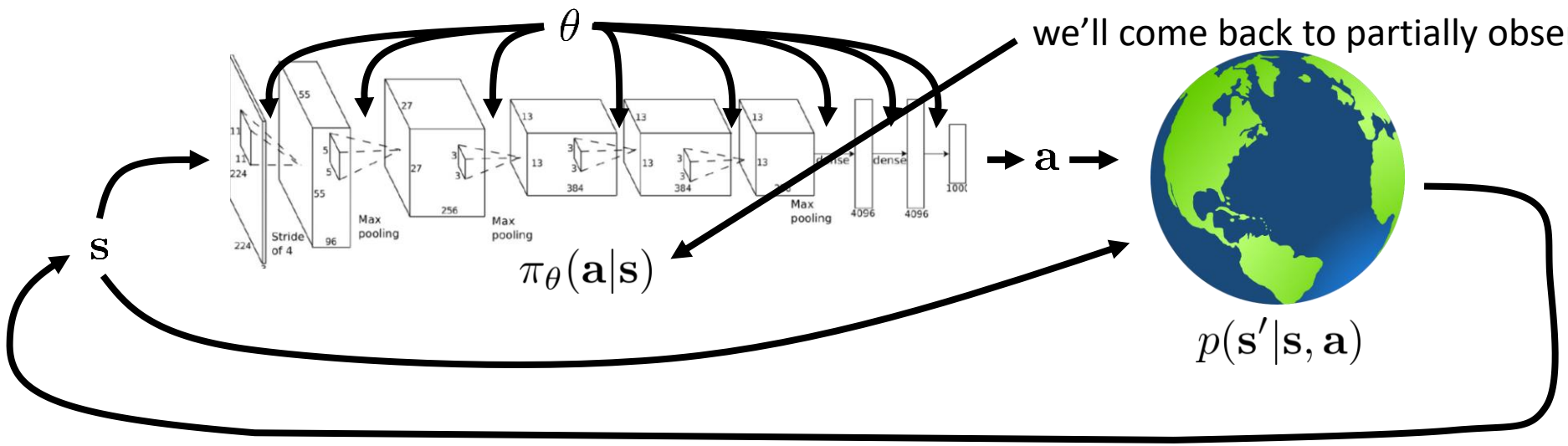
$\mathcal{T}$ – transition operator (like before)

$\mathcal{E}$ – emission probability $p(o_t|s_t)$

r – reward function $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$



The goal of reinforcement learning



$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)}_{p_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\theta^\star = \arg\max_\theta E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

The goal of reinforcement learning

$$\theta^\star = \arg\max_\theta E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$\theta^\star = \arg\max_\theta E_{(\mathbf{s},\mathbf{a}) \sim p_\theta(\mathbf{s},\mathbf{a})}[r(\mathbf{s}, \mathbf{a})]$$

infinite horizon case

$$\theta^\star = \arg\max_\theta \sum_{t=1}^{T} E_{(\mathbf{s}_t,\mathbf{a}_t) \sim p_\theta(\mathbf{s}_t,\mathbf{a}_t)}[r(\mathbf{s}_t, \mathbf{a}_t)]$$

finite horizon case

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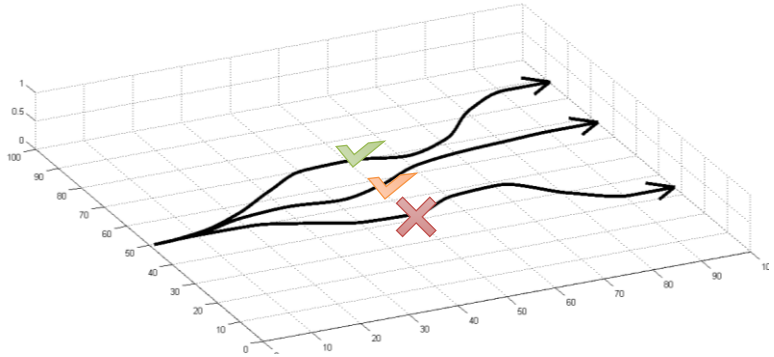
What if we know the model?

What if we don’t know the model, but are willing to learn?

Evaluating the objective

$$\theta^\star = \arg\max_\theta \underbrace{E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right]}_{J(\theta)}$$



$$J(\theta) = E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right] \approx \frac{1}{N}\sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

sum over samples from π_θ

Direct policy differentiation

$$\theta^\star = \arg\max_\theta \underbrace{E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right]}_{J(\theta)}$$

a convenient identity

$$\pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) = \pi_\theta(\tau)\frac{\nabla_\theta \pi_\theta(\tau)}{\pi_\theta(\tau)} = \nabla_\theta \pi_\theta(\tau)$$

$$J(\theta) = E_{\tau\sim p_\theta(\tau)}[\underbrace{r(\tau)}_{\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)}] = \int \pi_\theta(\tau) r(\tau) d\tau$$

$$\nabla_\theta J(\theta) = \int \nabla_\theta \pi_\theta(\tau) r(\tau) d\tau = \int \pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) r(\tau) d\tau = E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau) r(\tau)]$$

Direct policy differentiation

$$\theta^\star = \arg\max_\theta J(\theta)$$

$$J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[r(\tau)]$$

$$\nabla_\theta J(\theta) = E_{\tau\sim p_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau) r(\tau)]$$

$$\pi_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T) = p(\mathbf{s}_1)\prod_{t=1}^{T}\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$p_\theta(\tau)$

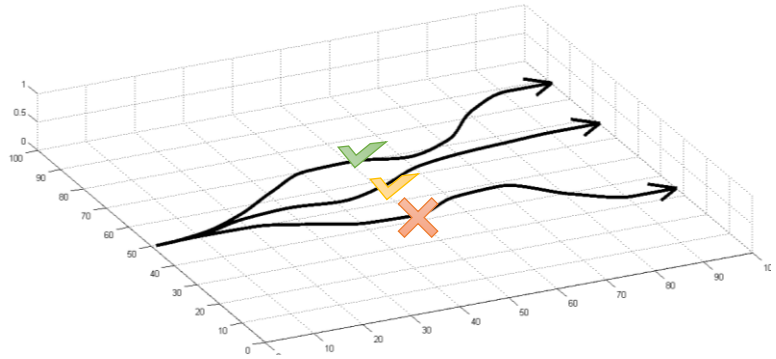
log of both sides

$$\log p_\theta(\tau) = \log p(\mathbf{s}_1) + \sum_{t=1}^{T}\log \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) + \log p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\nabla_\theta \left[\log p(\mathbf{s}_1) + \sum_{t=1}^{T}\log \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) + \log p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)\right]$$

$$\nabla_\theta J(\theta) = E_{\tau\sim p_\theta(\tau)}\left[\left(\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)\right)\left(\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)\right)\right]$$

Evaluating the policy gradient



recall: $J(\theta) = E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right] \approx \frac{1}{N} \sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$

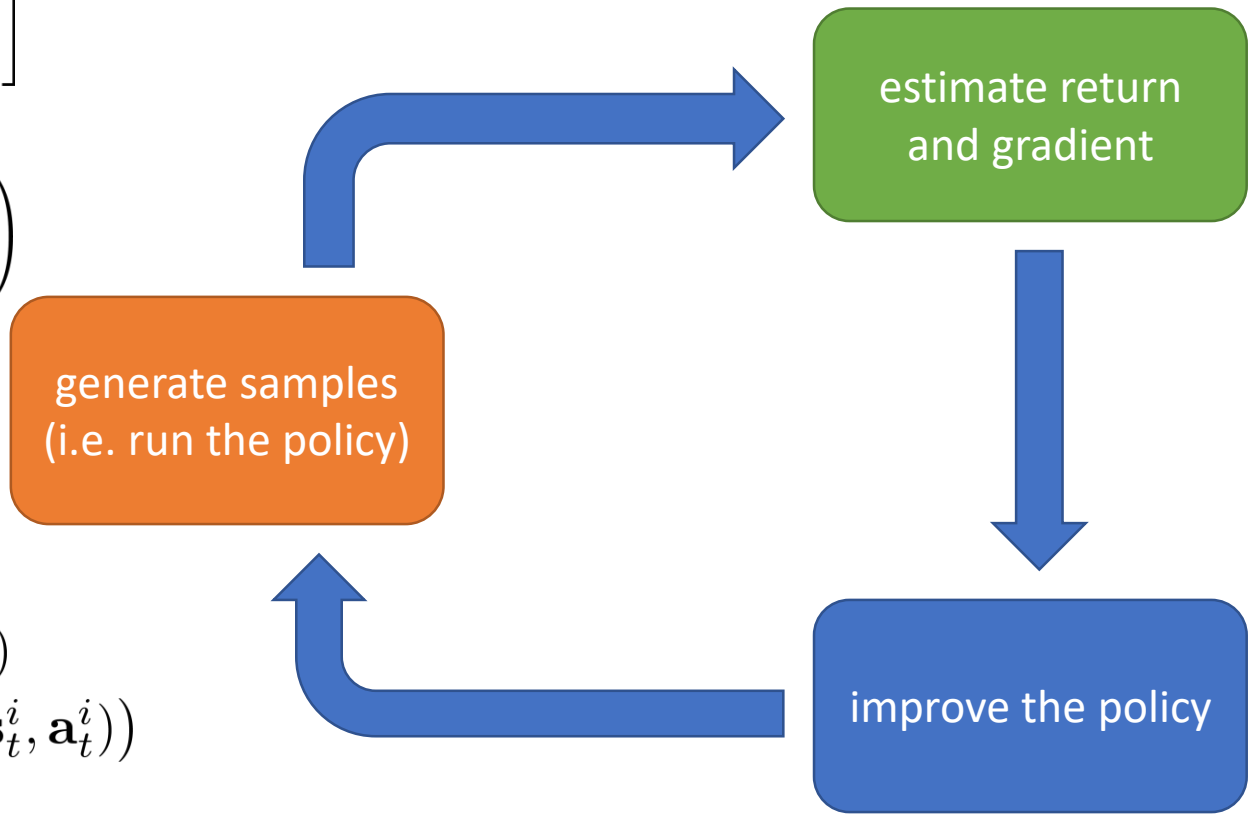
$$\nabla_\theta J(\theta) = E_{\tau \sim p_\theta(\tau)} \left[\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_t | \mathbf{s}_t) \right) \left(\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

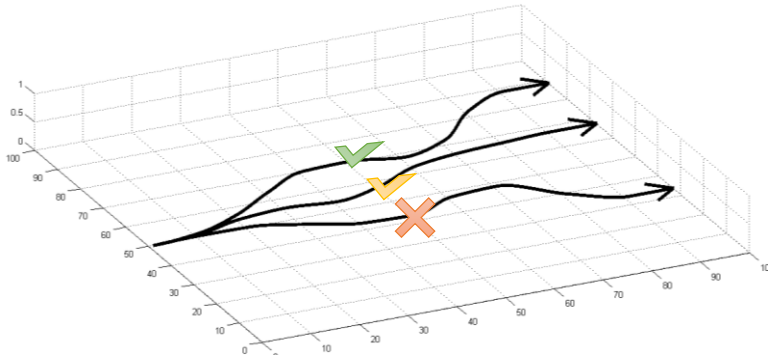
$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t | \mathbf{s}_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i | \mathbf{s}_t^i) \right) \left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i) \right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



Evaluating the policy gradient



$$\text{recall: } J(\theta) = E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right] \approx \frac{1}{N} \sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

$$\nabla_\theta J(\theta) = E_{\tau \sim p_\theta(\tau)} \left[\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_t | \mathbf{s}_t) \right) \left(\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

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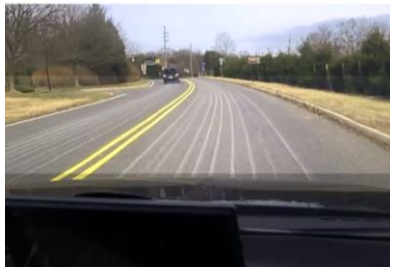
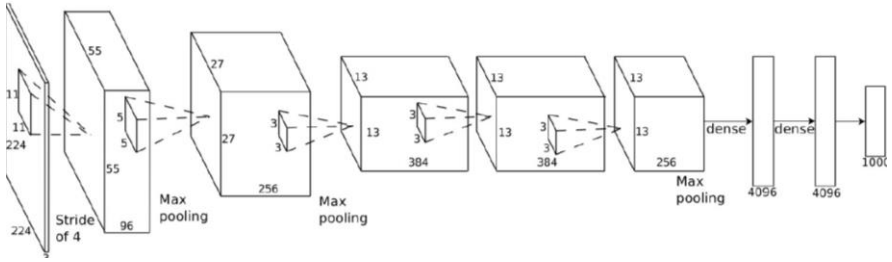
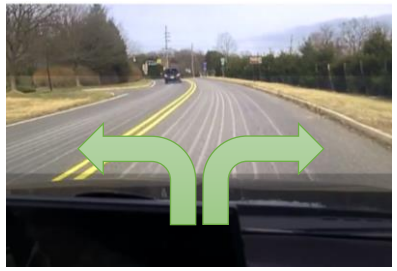
what is this?



$\mathbf{s}_t$



$\pi_\theta(\mathbf{a}_t | \mathbf{s}_t)$

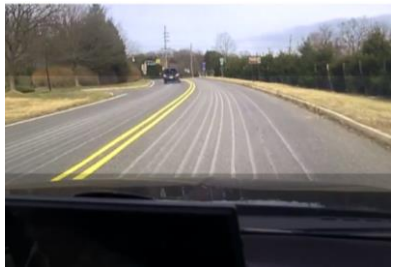


$\mathbf{a}_t$

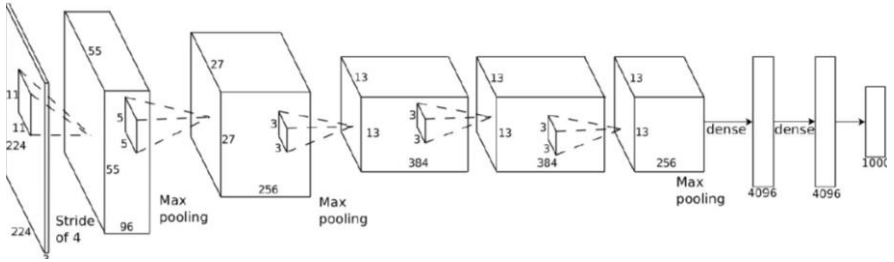
Comparison to maximum likelihood

policy gradient: $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$

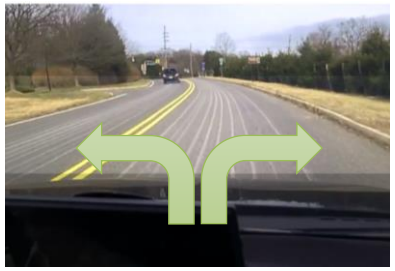
maximum likelihood: $\nabla_\theta J_{\mathrm{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \right)$



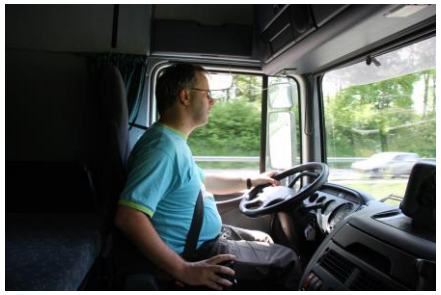
$\mathbf{s}_t$



$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$

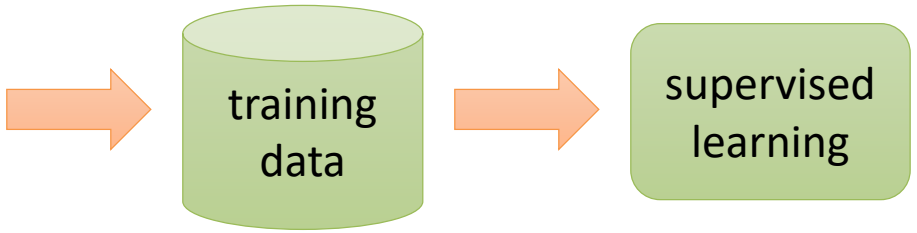


$\mathbf{a}_t$



$\mathbf{s}_t$

$\mathbf{a}_t$



$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$

What did we just do?

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

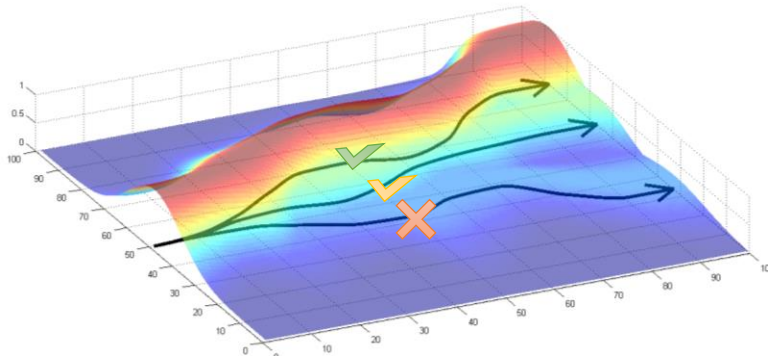
$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \underbrace{\nabla_\theta \log \pi_\theta(\tau_i)}_{\sum_{t=1}^{T} \nabla_\theta \log_\theta \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})} r(\tau_i)$$

maximum likelihood: $\nabla_\theta J_{\mathrm{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta \log \pi_\theta(\tau_i)$

good stuff is made more likely

bad stuff is made less likely

simply formalizes the notion of “trial and error”!

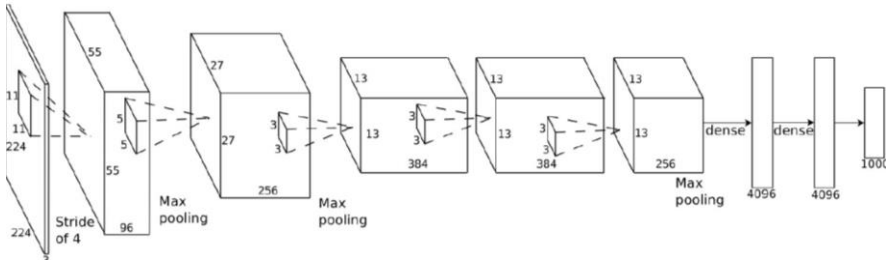


REINFORCE algorithm:

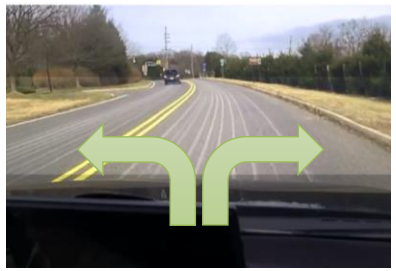
1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i) \right) \left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i) \right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Partial observability

$\mathbf{o}_t$



$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$



$\mathbf{a}_t$

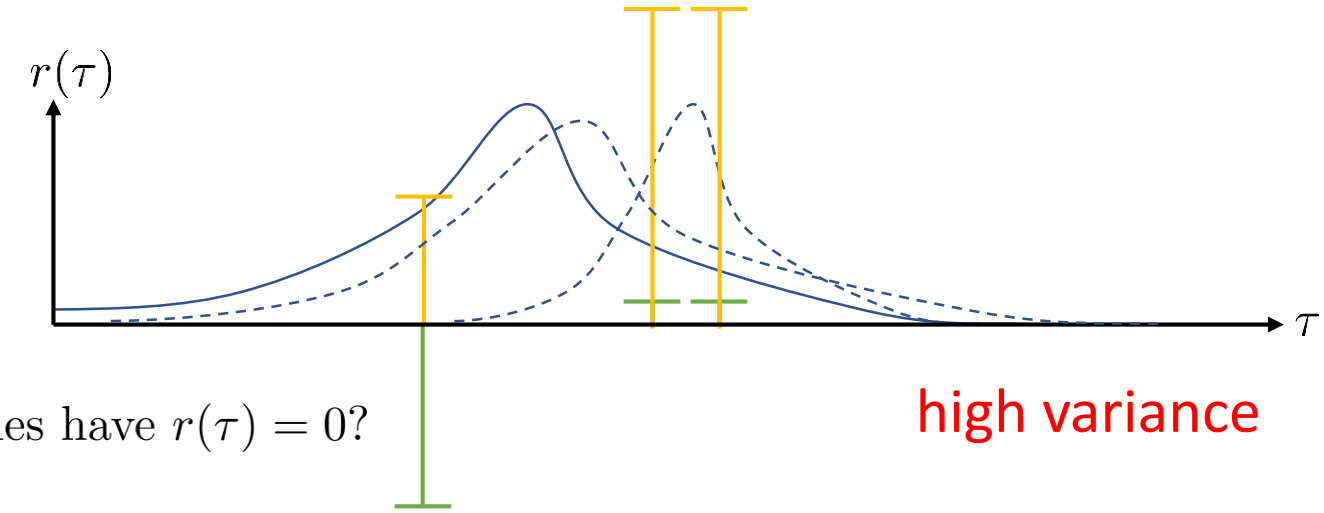
$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{o}_{i,t}) \right) \left(\sum_{t=1}^{T} r(\mathbf{o}_{i,t}, \mathbf{a}_{i,t}) \right)$$

Markov property is not actually used!

Can use policy gradient in partially observed MDPs without modification

What is wrong with the policy gradient?

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta \log \pi_\theta(\tau) r(\tau)$$



high variance

even worse: what if the two "good" samples have $r(\tau) = 0$?

Reducing variance

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

Causality: policy at time t' cannot affect reward at time t when $t < t'$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \hat{Q}_{i,t}$$

$$\hat{Q}_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$$

"reward to go"

$\hat{Q}_{i,t}$

Baselines

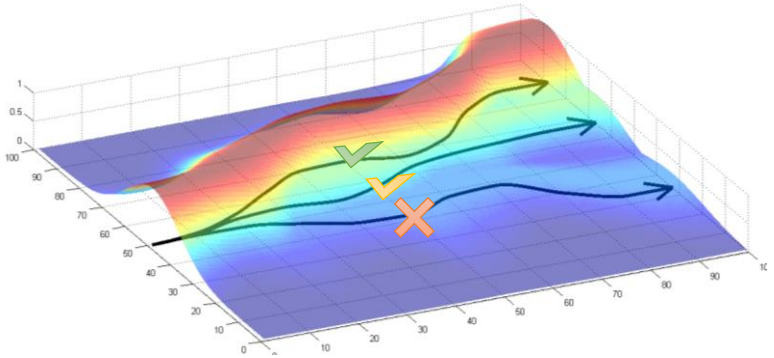
a convenient identity

$$\pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) = \nabla_\theta \pi_\theta(\tau)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log \pi_\theta(\tau)[r(\tau) - b]$$

$$b = \frac{1}{N}\sum_{i=1}^{N} r(\tau)$$

but… are we *allowed* to do that??



$$E[\nabla_\theta \log \pi_\theta(\tau) b] = \int \pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) b \, d\tau = \int \nabla_\theta \pi_\theta(\tau) b \, d\tau = b\nabla_\theta \int \pi_\theta(\tau) d\tau = b\nabla_\theta 1 = 0$$

subtracting a baseline is *unbiased* in expectation!

average reward is *not* the best baseline, but it's pretty good!

Policy gradient with automatic differentiation

Pseudocode example (with discrete actions):

Maximum likelihood:

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions, logits=logits)
loss = tf.reduce_mean(negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

Policy gradient with automatic differentiation

Pseudocode example (with discrete actions):

Policy gradient:

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# q_values – (N*T) x 1 tensor of estimated state-action values
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions, logits=logits)
weighted_negative_likelihoods = tf.multiply(negative_likelihoods, q_values)
loss = tf.reduce_mean(weighted_negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

$$\tilde{J}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \hat{Q}_{i,t}$$

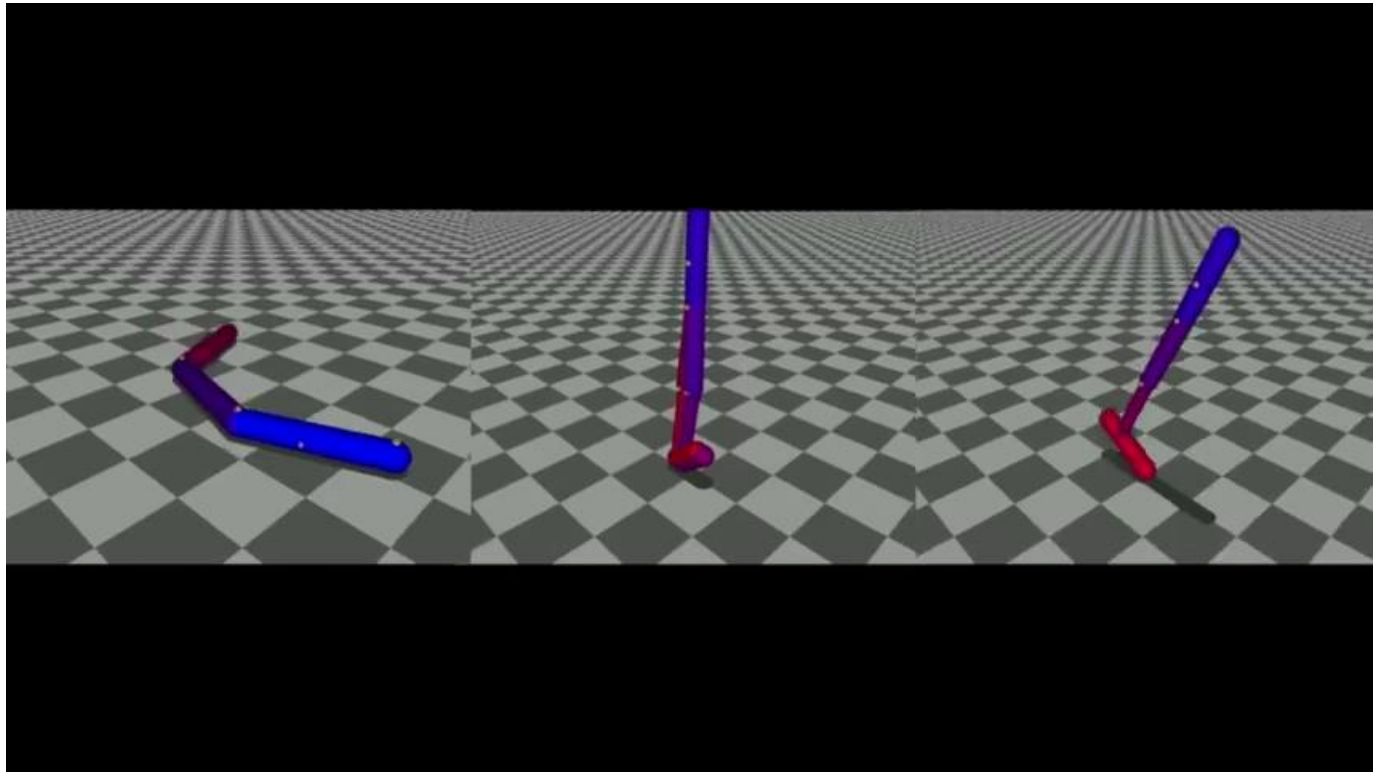
q_values

Policy gradient in practice

- Unfortunately only part of the story
  - Policy gradients have very high variance
  - Choosing step size is hard (much harder than regular SGD)
  - “Raw” (“vanilla”) policy gradients are hard to use
- What makes policy gradients easier to use?
  - Use natural gradient/trust region/etc.
    - Look up “natural policy gradient”
    - Look up “trust region policy optimization”
  - Use automated step size adjustment (e.g., ADAM)
  - Reduce your variance
    - Use a baseline
    - Use a huge batch size

Example: trust region policy optimization

- Natural gradient with automatic step adjustment
- Discrete and continuous actions
- Code available (see Duan et al. ‘16)



Schulman, Levine, Moritz, Jordan, Abbeel. ‘15

Policy gradients suggested readings

- Classic papers
  - Williams (1992). Simple statistical gradient-following algorithms for connectionist reinforcement learning: introduces REINFORCE algorithm
  - Sutton, McAllester, Singh, Mansour (1999). Policy gradient methods for reinforcement learning with function approximation, temporal decomposition
  - Baxter & Bartlett (2001). More discussion of temporal decomposition
  - Peters & Schaal (2008). Reinforcement learning of motor skills with policy gradients: very accessible overview of optimal baselines and natural gradient
- Deep reinforcement learning policy gradient papers
  - Schulman, L., Moritz, Jordan, Abbeel (2015). Trust region policy optimization: deep RL with natural policy gradient and adaptive step size
  - Schulman, Wolski, Dhariwal, Radford, Klimov (2017). Proximal policy optimization algorithms: deep RL with importance sampled policy gradient
- Practice on your own!
  - Homework 2 here: https://github.com/berkeleydeeprlcourse/homework

Overview

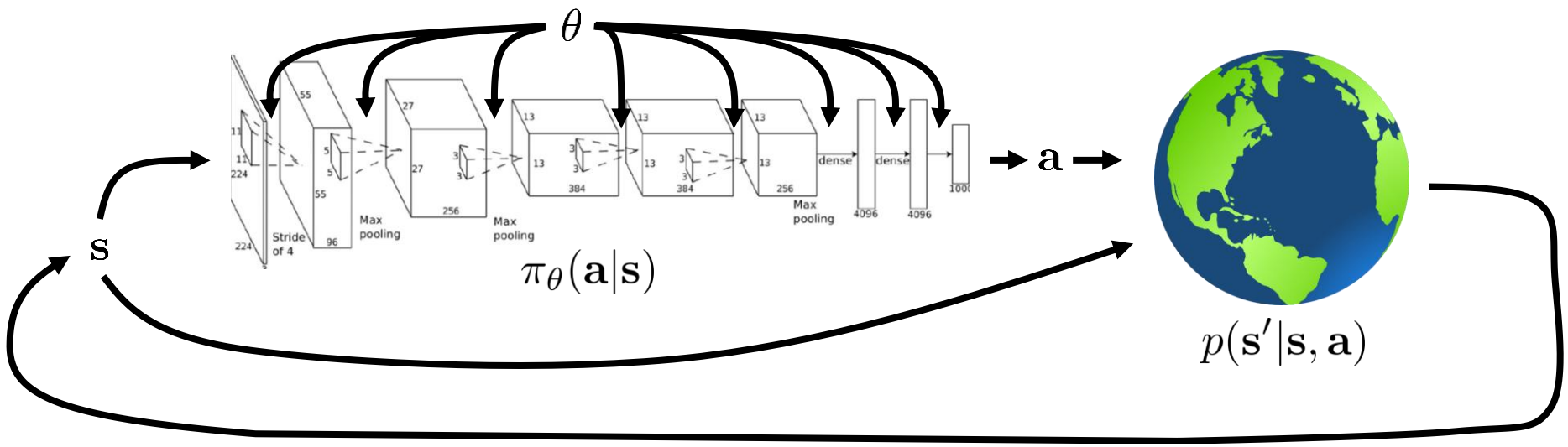
The problem setup

Model-free policy search via policy gradients

What if we know the model?

What if we don’t know the model, but are willing to learn?

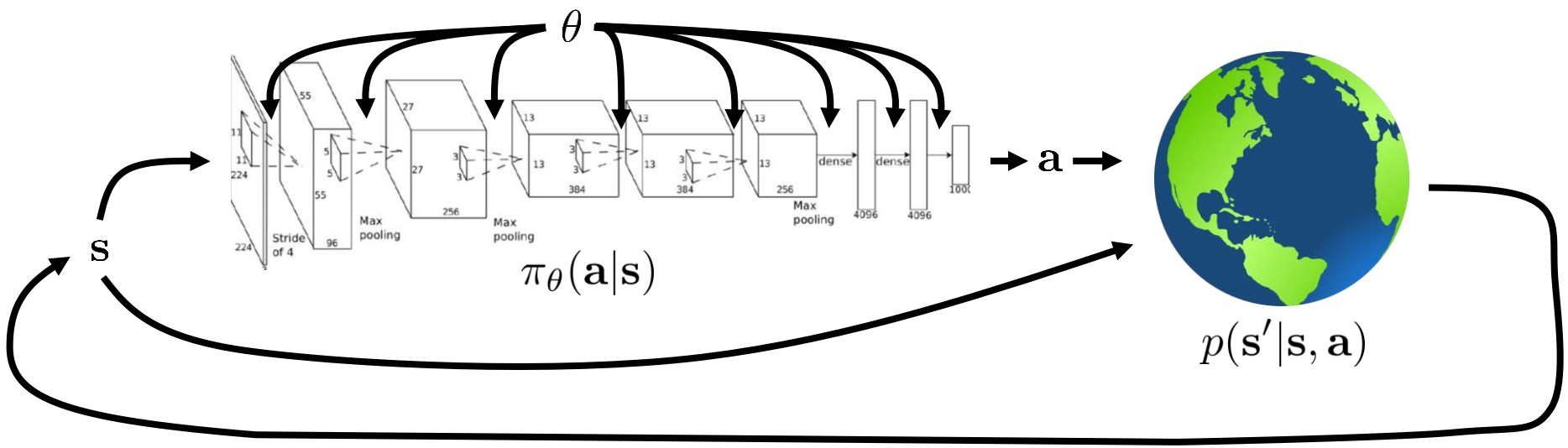
The reinforcement learning objective



$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)}_{p_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\theta^\star = \arg\max_\theta E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

Model-free reinforcement learning



$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)}_{p_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}$$

assume this is unknown

don't even attempt to learn it

$$\theta^\star = \arg\max_\theta E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

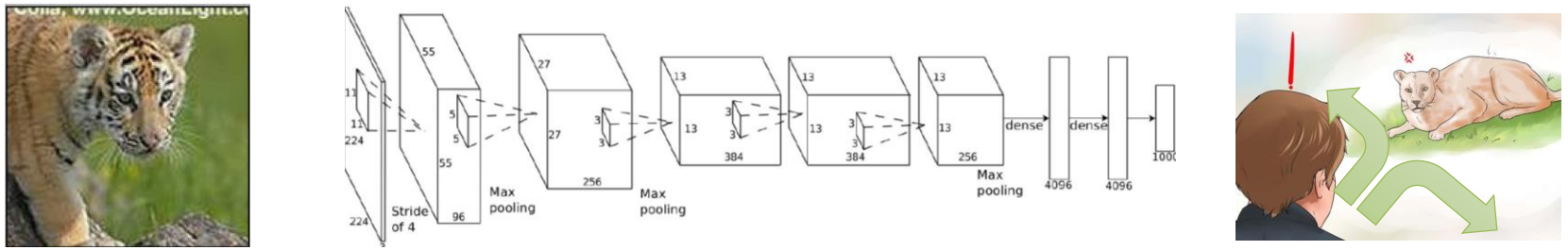
What if we knew the transition dynamics?

- Often we do know the dynamics
  1. Games (e.g., Go)
  2. Easily modeled systems (e.g., navigating a car)
  3. Simulated environments (e.g., simulated robots, video games)
- Often we can learn the dynamics
  1. System identification – fit unknown parameters of a known model
  2. Learning – fit a general-purpose model to observed transition data

Does knowing the dynamics make things easier?

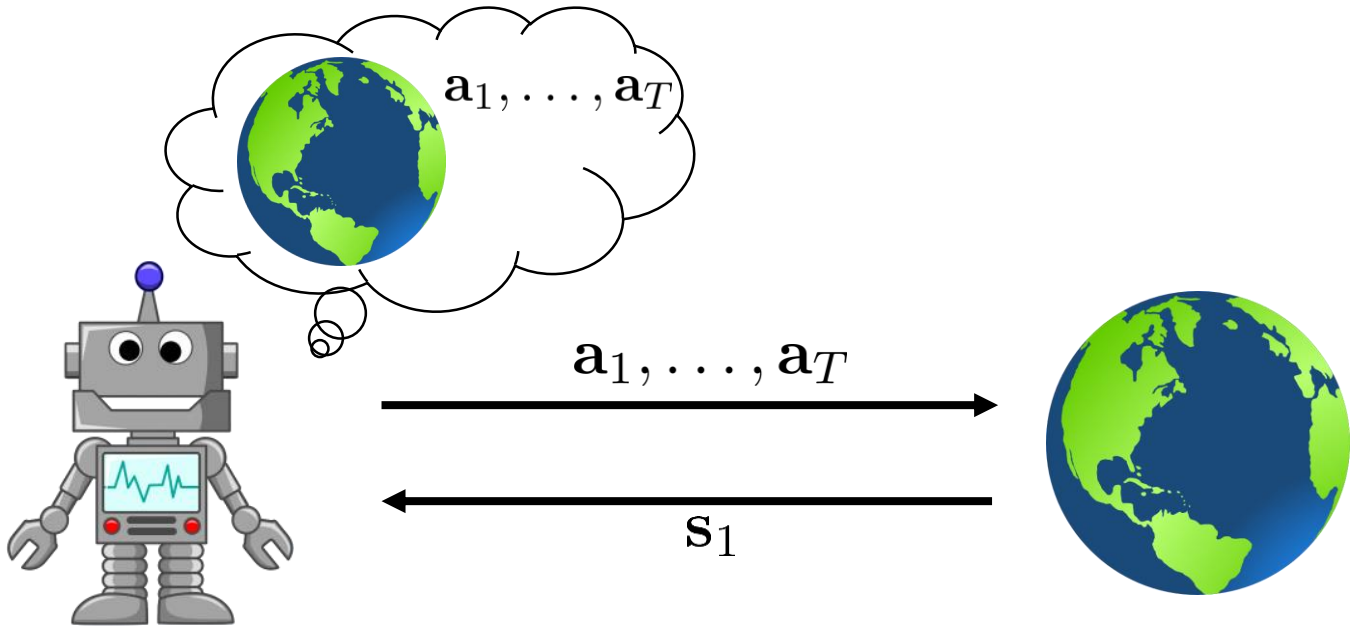
Often, yes!

The objective



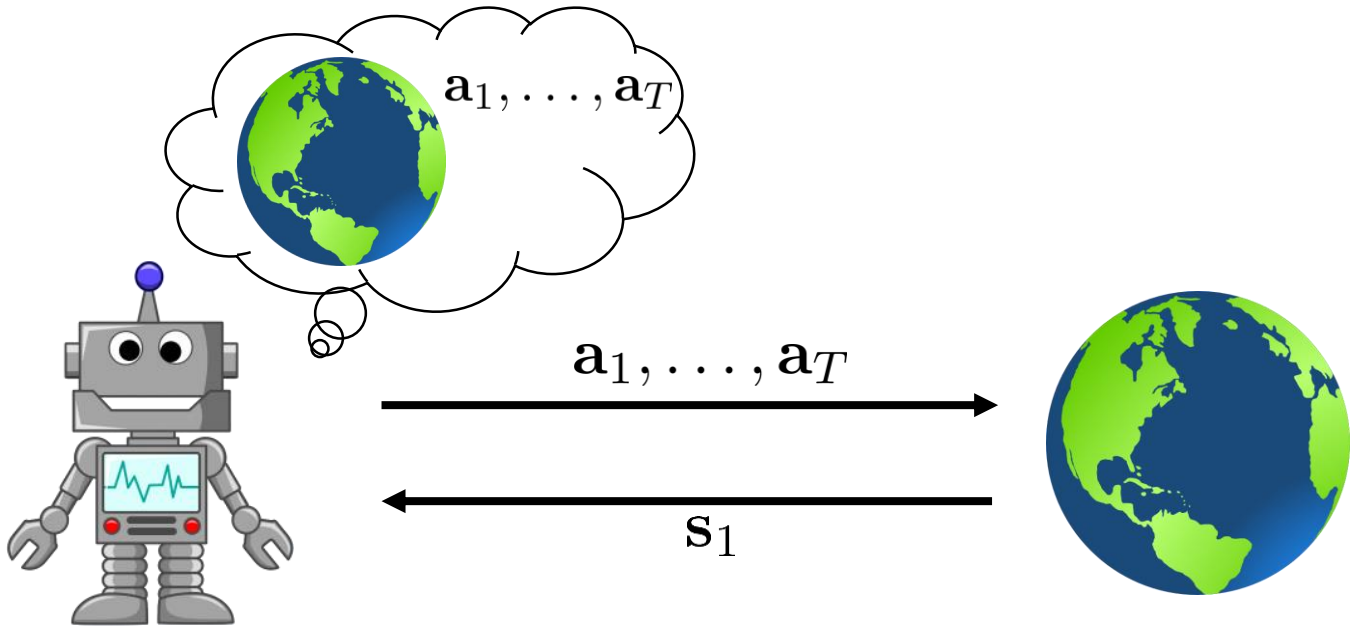
$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t) \text{ s.t. } \mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

The deterministic case



$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t) \text{ s.t. } \mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

The stochastic open-loop case



$$p_\theta(\mathbf{s}_1, \ldots, \mathbf{s}_T | \mathbf{a}_1, \ldots, \mathbf{a}_T) = p(\mathbf{s}_1) \prod_{t=1}^{T} p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

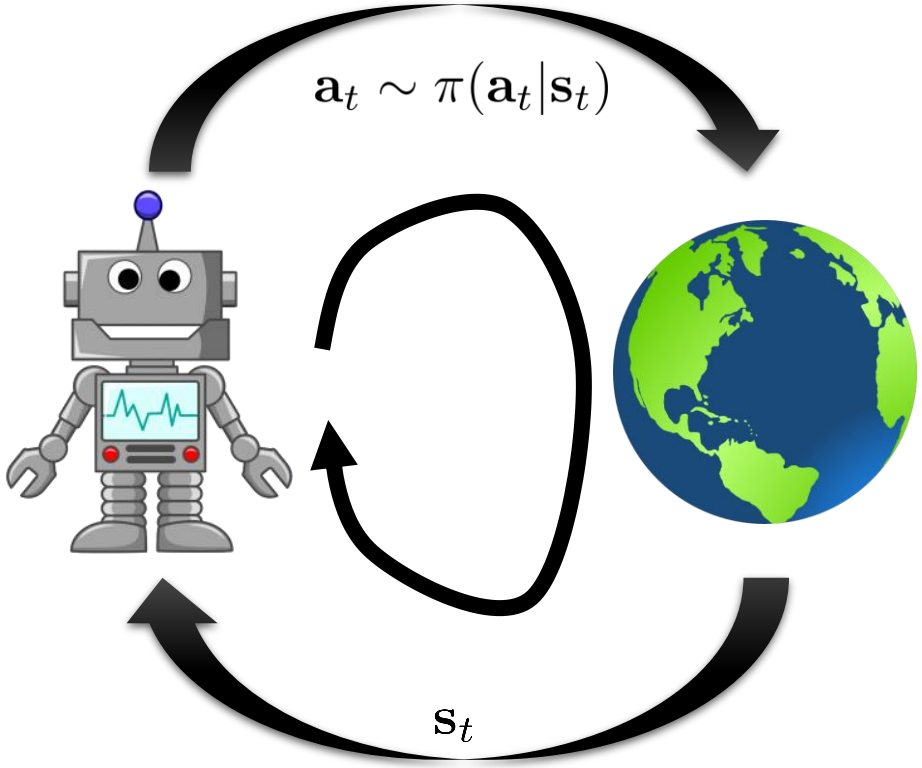
why is this suboptimal?

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} E \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) | \mathbf{a}_1, \ldots, \mathbf{a}_T \right]$$

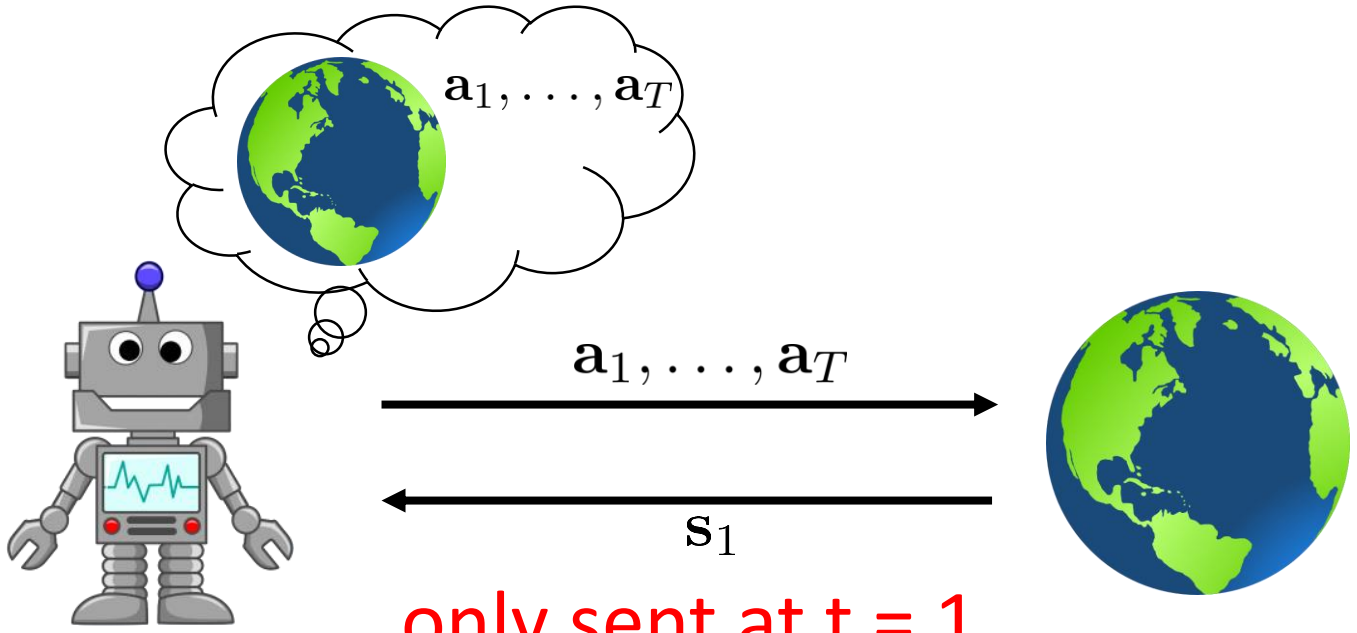
Aside: terminology

what is this “loop”?

closed-loop

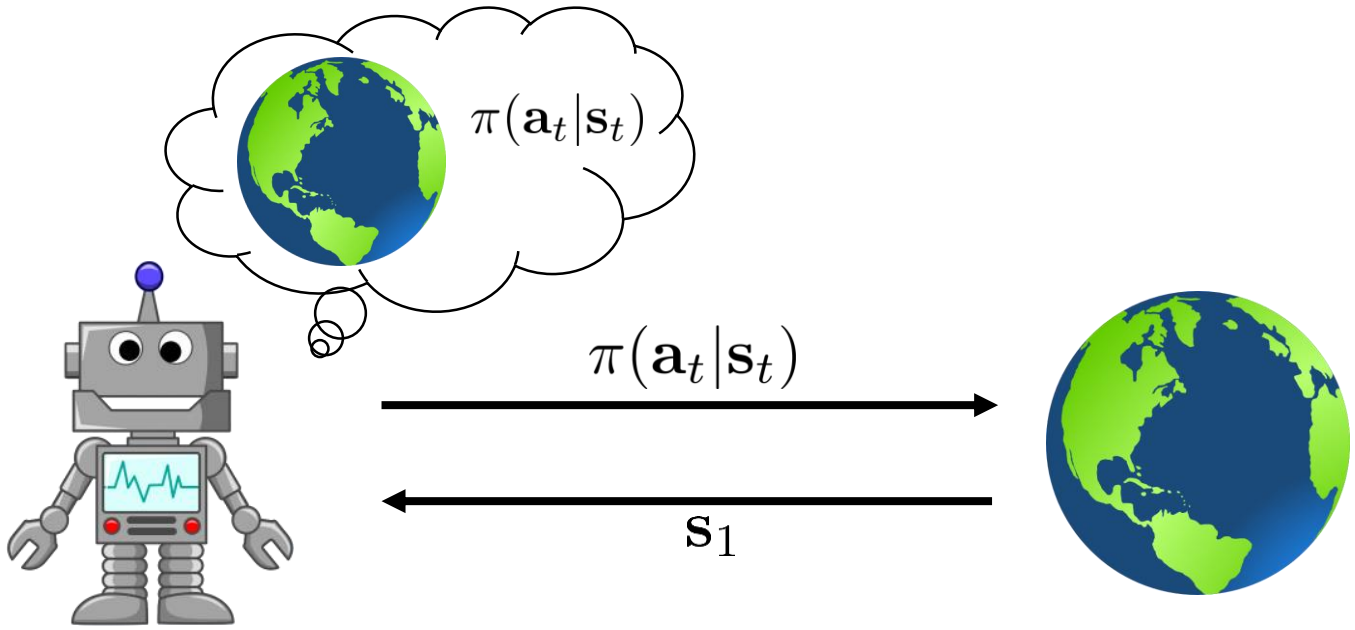


open-loop



only sent at t = 1,
then it’s one-way!

The stochastic **closed**-loop case



$$p(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T) = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi(\mathbf{a}_t|\mathbf{s}_t) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\pi = \arg\max_{\pi} E_{\tau \sim p(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

form of π?

neural net global

time-varying linear $\mathbf{K}_t \mathbf{s}_t + \mathbf{k}_t$ local

(more on this later)

Open loop control with stochastic optimization

abstract away optimal control/planning:

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \underbrace{J(\mathbf{a}_1, \ldots, \mathbf{a}_T)}_{\text{don't care what this is}}$$

$$\mathbf{A} = \arg \max_{\mathbf{A}} J(\mathbf{A})$$

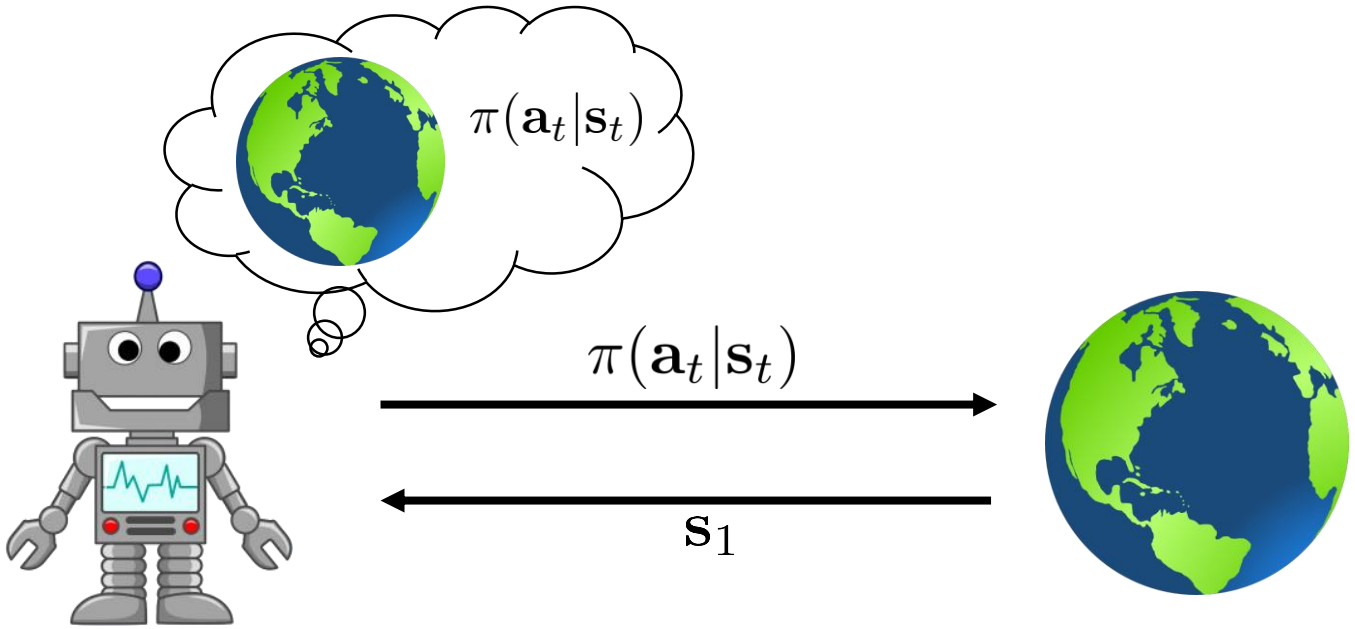
simplest method: guess & check "random shooting method"

1. pick $\mathbf{A}_1, \ldots, \mathbf{A}_N$ from some distribution (e.g., uniform)
2. choose $\mathbf{A}_i$ based on $\arg\max_i J(\mathbf{A}_i)$

What else can we do?

1. Smarter stochastic optimization: cross entropy method, CMA-ES
2. Continuous optimization: gradient ascent, LQR/iteration LQR
3. Discrete actions: Monte Carlo tree search
4. Many other solutions from planning and optimal control
   - Motion planning (quasistatic) problems: RRTs, PRMs
   - Collocation methods for trajectory optimization
   - Many many more

Closed loop control: policy search with a model

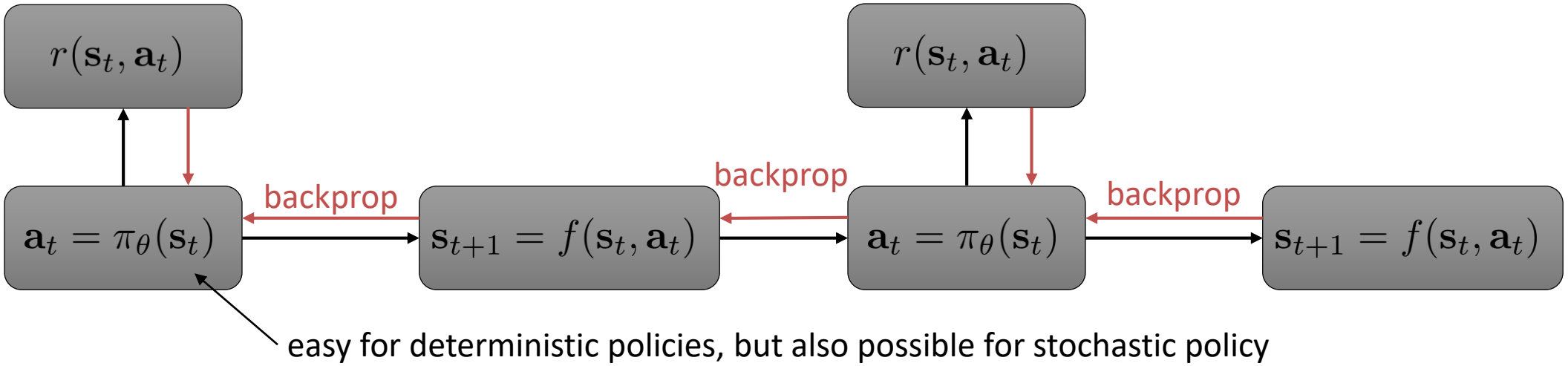


Can we use a **model** to learn a **policy?**

+ no planning at test-time

+ handles closed-loop control

Backpropagate directly into the policy?

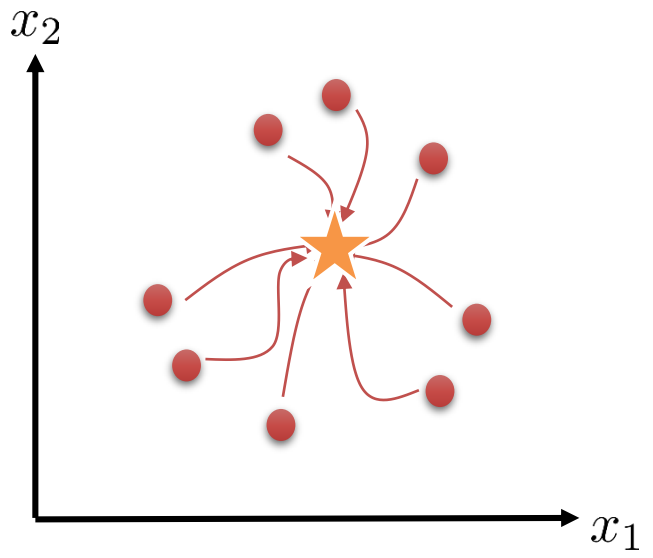


- this is possible
- it can be really hard (for the same reason that RNNs are hard)
- there are often better ways to do this

Model-free optimization with a model

- Just use policy gradient (or another model-free RL method) even though you have a model
- Sometimes better than using the gradients!
- See a recent analysis here:
  - Parmas et al. ‘18: PIPP: Flexible Model-Based Policy Search Robust to the Curse of Chaos

Learning policies without BPTT: Guided policy search



optimize trajectory $\mathbf{s}_0, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T$ from $\mathbf{s}_0 = \mathbf{s}_0^0$

optimize trajectory $\mathbf{s}_0, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T$ from $\mathbf{s}_0 = \mathbf{s}_0^1$

optimize trajectory $\mathbf{s}_0, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T$ from $\mathbf{s}_0 = \mathbf{s}_0^2$

model-based RL provides **data**

train $\pi_\theta(\mathbf{a}|\mathbf{s})$ using supervised learning

supervised learning provides **reward term**

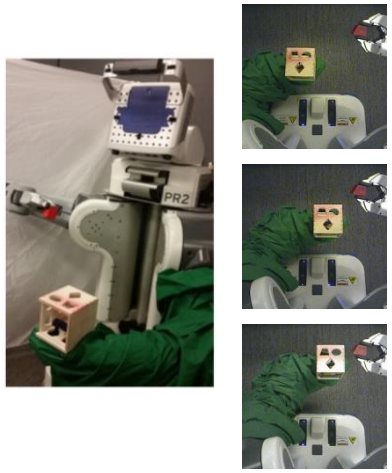
$$\tilde{r}(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \eta \log \pi_\theta(\mathbf{a}|\mathbf{s})$$

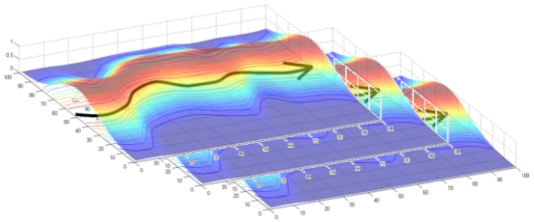
Levine*, Finn*, et al. End-to-end training of deep visuomotor policies. 2016.

Guided Policy Search

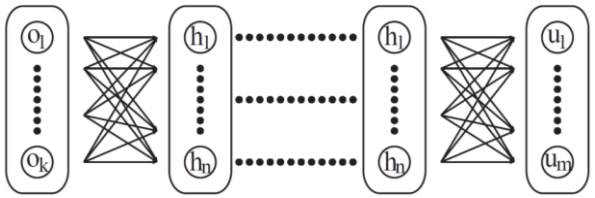
$\mathbf{s}_t \rightarrow \mathbf{a}_t$

$\mathbf{o}_t \rightarrow \mathbf{a}_t$

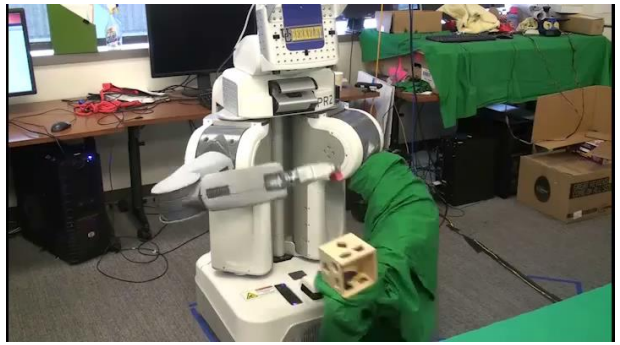




model-based RL



supervised learning



Overview

The problem setup

Model-free policy search via policy gradients

What if we know the model?

What if we don’t know the model, but are willing to learn?

What if we don't know the model?

If we knew $f(\mathbf{s}_t, \mathbf{a}_t) = \mathbf{s}_{t+1}$, no more learning, just planning!

(or $p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$ in the stochastic case)

So let's learn $f(\mathbf{s}_t, \mathbf{a}_t)$ from data, and *then* plan through it!

model-based reinforcement learning version 0.5:

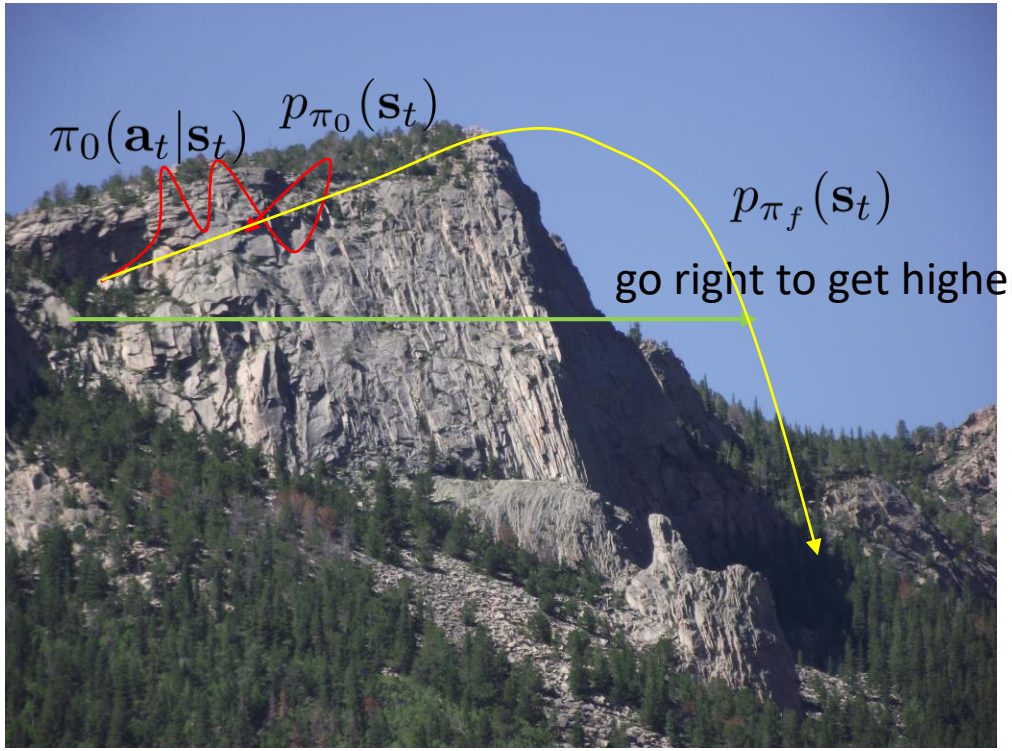
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

Does it work? Yes!

- Essentially how system identification works in classical robotics
- Some care should be taken to design a good base policy
- Particularly effective if we can hand-engineer a dynamics representation using our knowledge of physics, and fit just a few parameters

Does it work?

No!



1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

$$p_{\pi_f}(\mathbf{s}_t) \neq p_{\pi_0}(\mathbf{s}_t)$$

- Distribution mismatch problem becomes exacerbated as we use more expressive model classes

Can we do better?

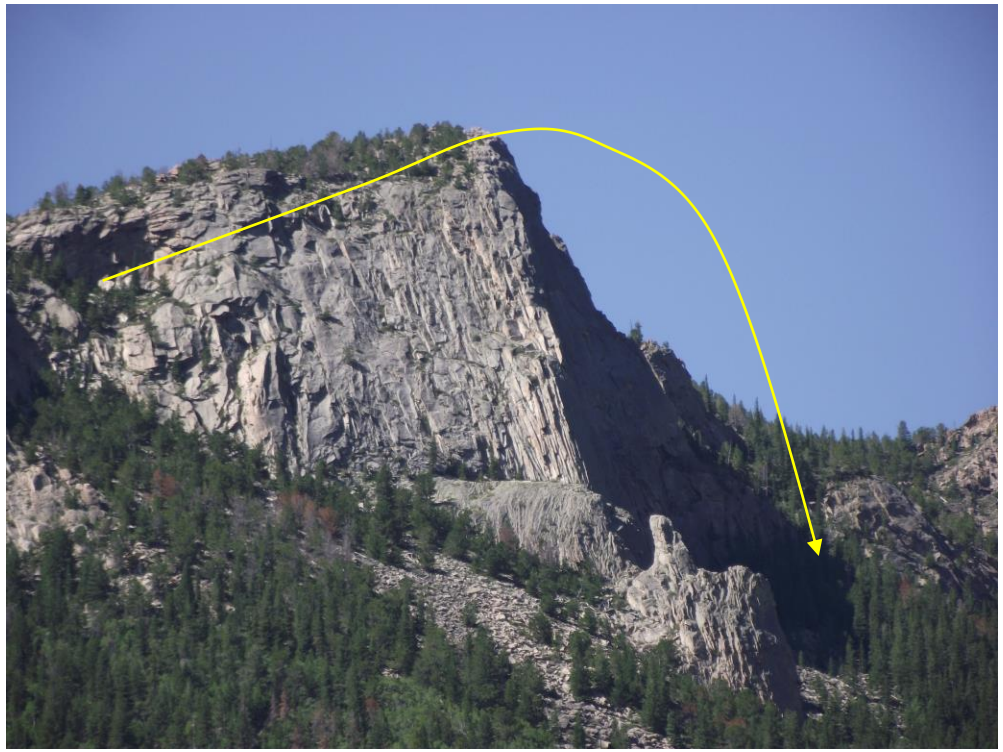
can we make $p_{\pi_0}(\mathbf{s}_t) = p_{\pi_f}(\mathbf{s}_t)$?

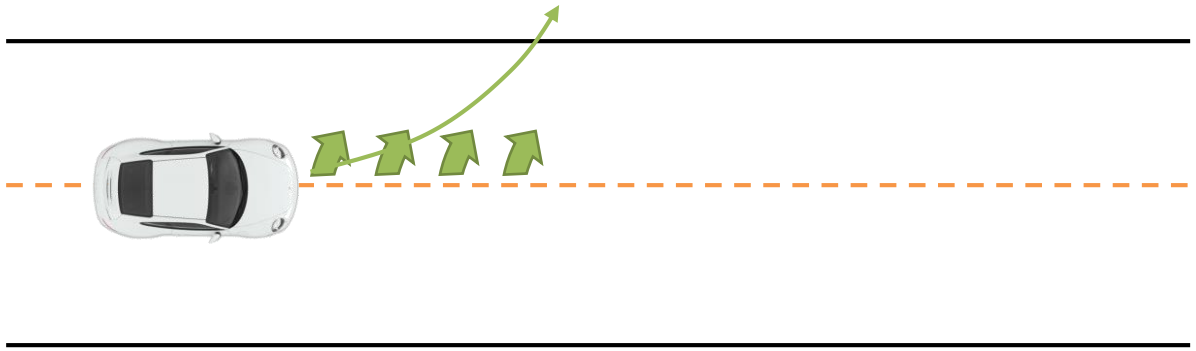
need to collect data from $p_{\pi_f}(\mathbf{s}_t)$

model-based reinforcement learning version 1.0:

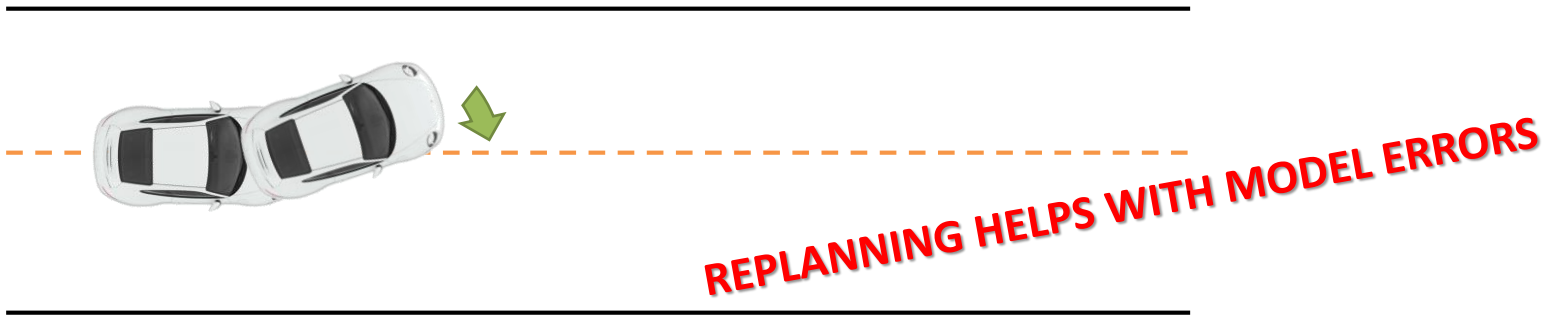
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute those actions and add the resulting data $\{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_j\}$ to $\mathcal{D}$

What if we make a mistake?





Can we do better?



model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps

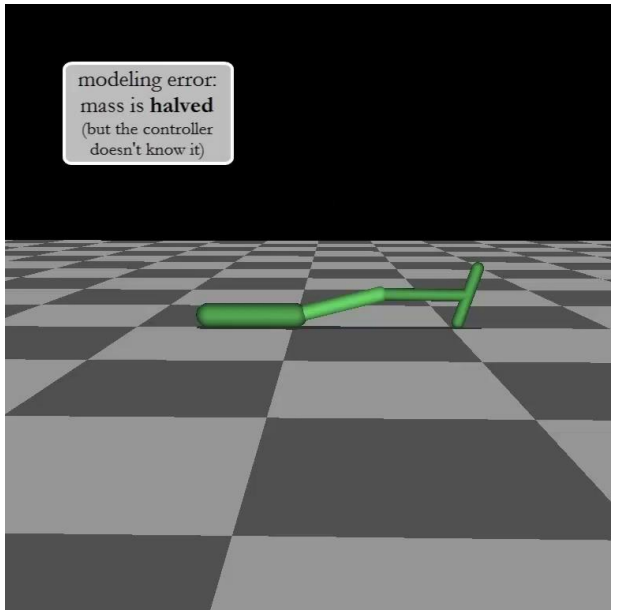
How to replan?

model-based reinforcement learning version 1.5:

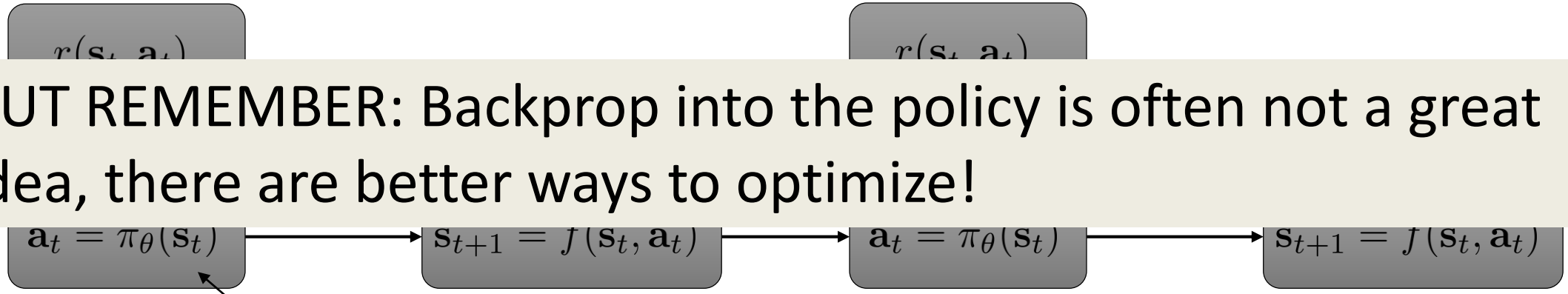
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps

- The more you replan, the less perfect each individual plan needs to be
- Can use shorter horizons
- Even random sampling can often work well here!



Backpropagate directly into the policy?



BUT REMEMBER: Backprop into the policy is often not a great idea, there are better ways to optimize!

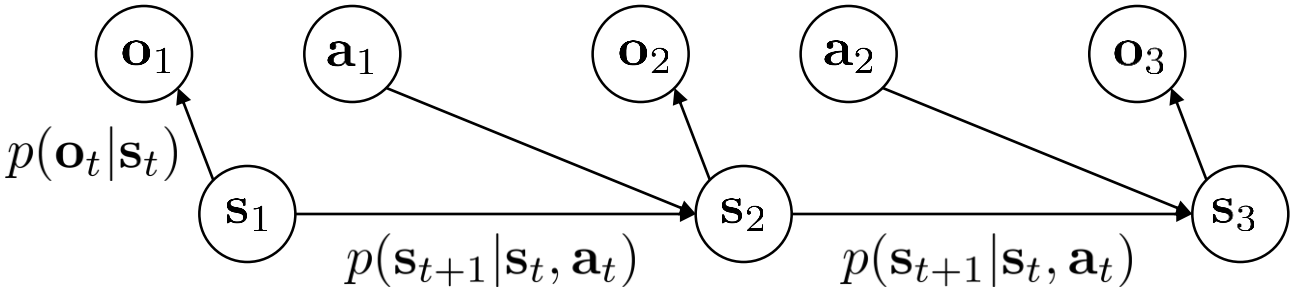
easy for deterministic policies, but also possible for stochastic policy (more on this later)

model-based reinforcement learning version 2.0:

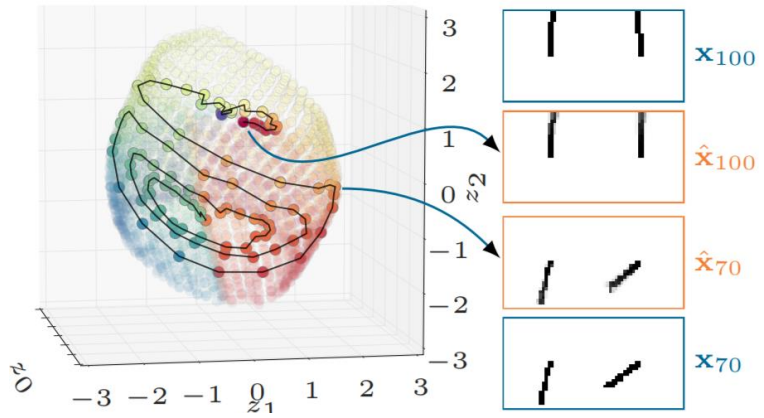
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. backpropagate through $f(\mathbf{s}, \mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
4. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to $\mathcal{D}$

What about observations?

Learn POMDP model



Example: Watter et al. ‘15: Embed to Control



Example: Finn et al. ‘16: Deep Spatial Autoencoder

Summary

- Version 0.5: collect random samples, train dynamics, plan
  - Pro: simple, no iterative procedure
  - Con: distribution mismatch problem
- Version 1.0: iteratively collect data, replan, collect data
  - Pro: simple, solves distribution mismatch
  - Con: open loop plan might perform poorly, esp. in stochastic domains
- Version 1.5: iteratively collect data using MPC (replan at each step)
  - Pro: robust to small model errors
  - Con: computationally expensive, but have a planning algorithm available
- Version 2.0: backpropagate directly into policy
  - Pro: computationally cheap at runtime
  - Con: direct backprop doesn’t usually work, but see previous section for alternatives

Model-based RL suggested readings

- Classic papers
  - Sutton. Integrated architectures for learning, planning, and reacting based on approximating dynamic programming. ICML 1990: Learn a model, then use this model with model-free RL to learn value functions and policies.
  - Deisenroth, Rasmussen. PILCO: A Model-Based and Data-Efficient Approach to Policy Search. ICML 2011: Direct backpropagation method based on Gaussian processes.
  - Ross, Bagnell. Agnostic System Identification for Model-Based Reinforcement Learning. ICML 2012: Nice discussion of distribution shift problem in model learning.
- More recent papers (some examples)
  - Watter et al. Embed to control. 2015: Model-based RL by learning latent dynamical systems and LQR.
  - Heess et al. Stochastic value gradients. 2015: Backprop through dynamics to aid in learning policy in actor-critic algorithm.
  - Levine*, Finn*, et al. End-to-end training of deep visuomotor policies. 2016: Detailed description of guided policy search and applications for robotic manipulation.
  - Finn et al. Deep Visual Foresight for Planning Robot Motion. 2017: Model-based RL with direct video prediction on raw pixels.
  - Nagabandi et al. Neural network dynamics models for model-based deep RL with model-free finetuning. 2018: Learns neural net dynamics & plans with random shooting.
- Practice on your own!
  - Homework 4 here: https://github.com/berkeleydeeprlcourse/homework

Case study: model-based policy search with GPs

Learning to Control a Low-Cost Manipulator using Data-Efficient Reinforcement Learning

Marc Peter Deisenroth
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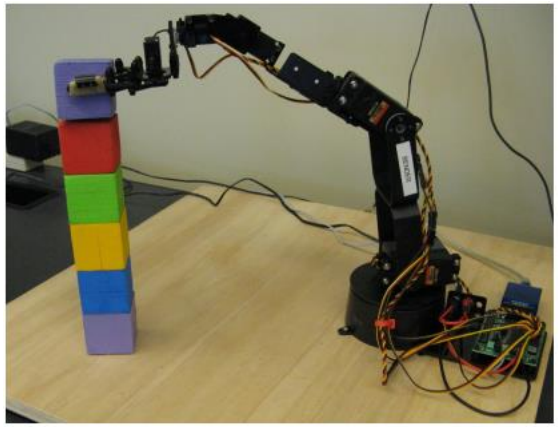
University of Washington
Seattle, WA, USA

Carl Edward Rasmussen
Dept. of Engineering
University of Cambridge
Cambridge, UK

Dieter Fox
Dept. of Computer Science & Engineering
University of Washington
Seattle, WA, USA



1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn GP dynamics model $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ to maximize $\sum_i \log p(\mathbf{s}'_i|\mathbf{s}_i, \mathbf{a}_i)$
3. backpropagate through $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
4. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to $\mathcal{D}$

Marc Peter Deisenroth, Carl Edward Rasmussen, Dieter Fox

Learning to Control a Low-Cost Manipulator using Data-efficient Reinforcement Learning

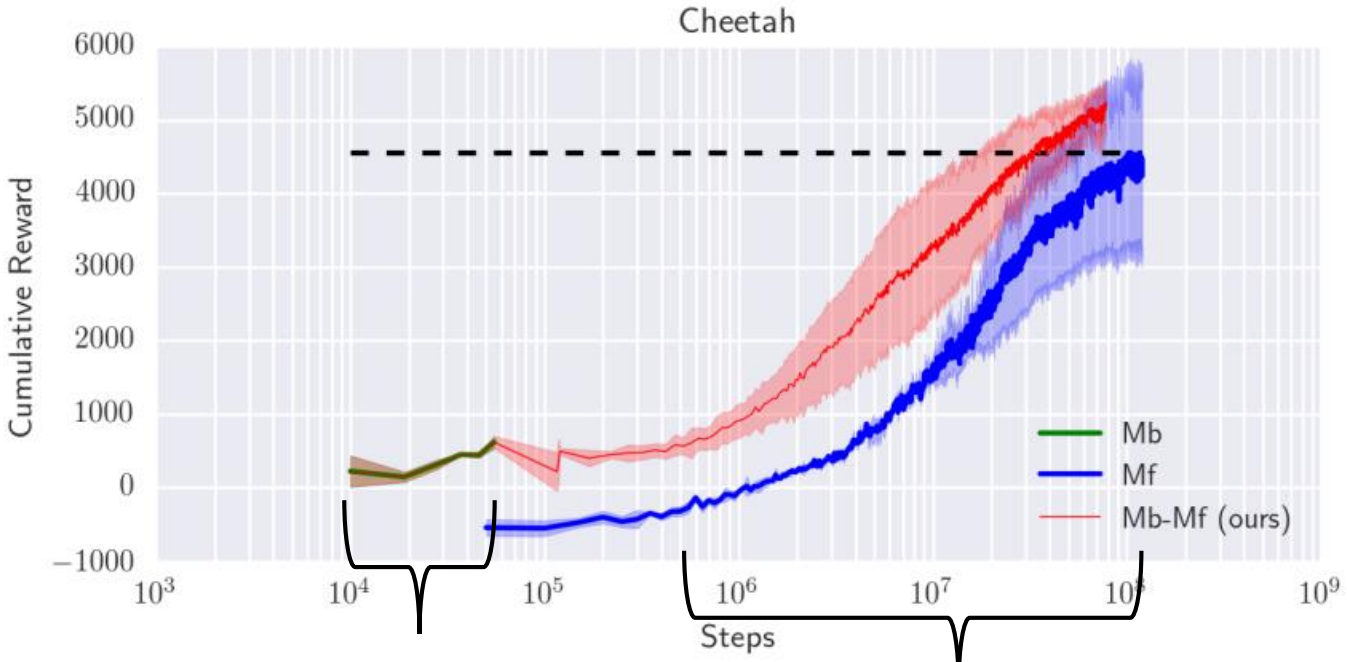
Neural Network Dynamics for Model-Based Deep Reinforcement Learning with Model-Free Fine-Tuning

Anusha Nagabandi, Gregory Kahn, Ronald S. Fearing, Sergey Levine
University of California, Berkeley

model-based reinforcement learning version 1.5:

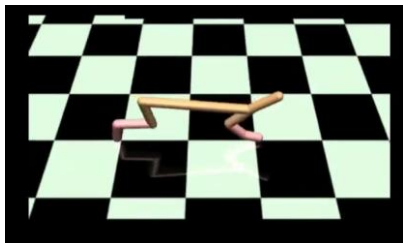
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions (random sampling)
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

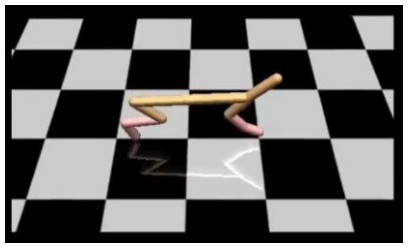
Bridging the Gap in Performance



pure model-based
(about 10 minutes real time)

model-free training
(about 10 days…)

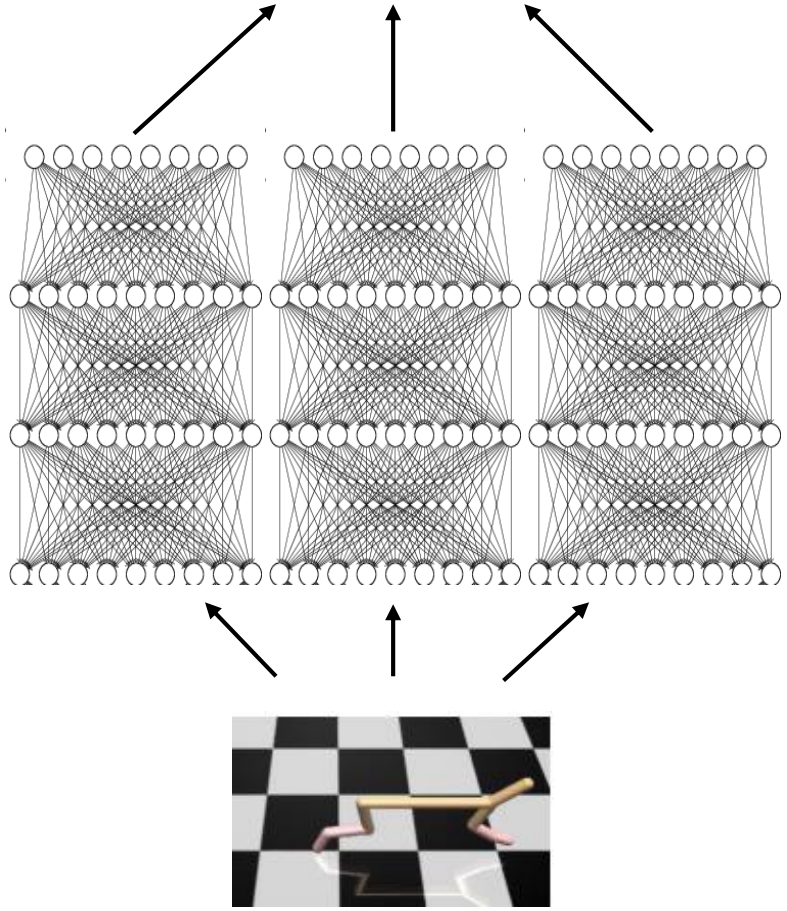




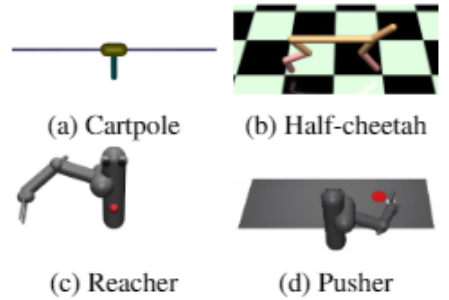
Nagabandi, Kahn, Fearing, L. ICRA 2018

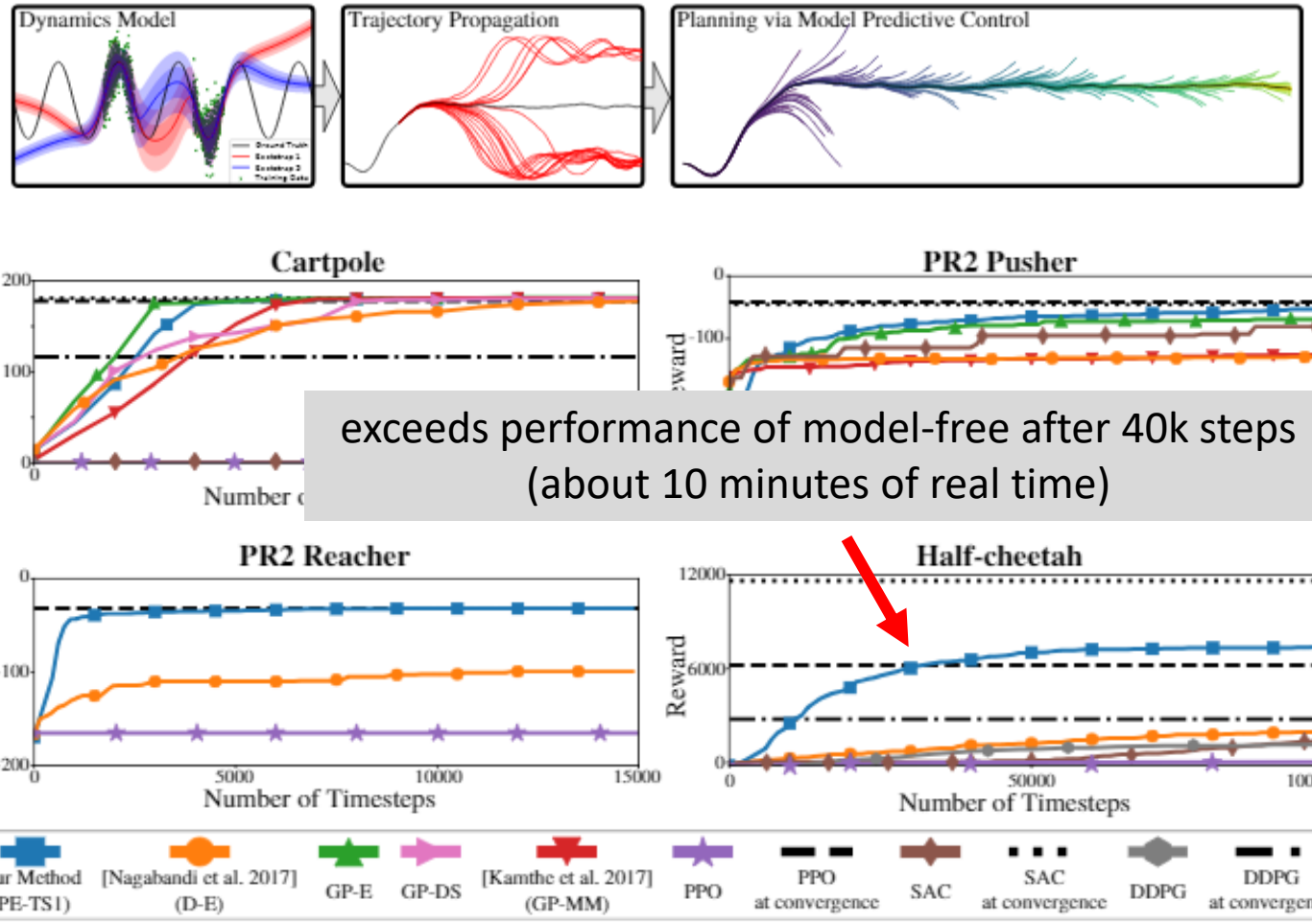
Incorporating Uncertainty

$$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$



PETS: Probabilistic Ensembles with Trajectory Sampling

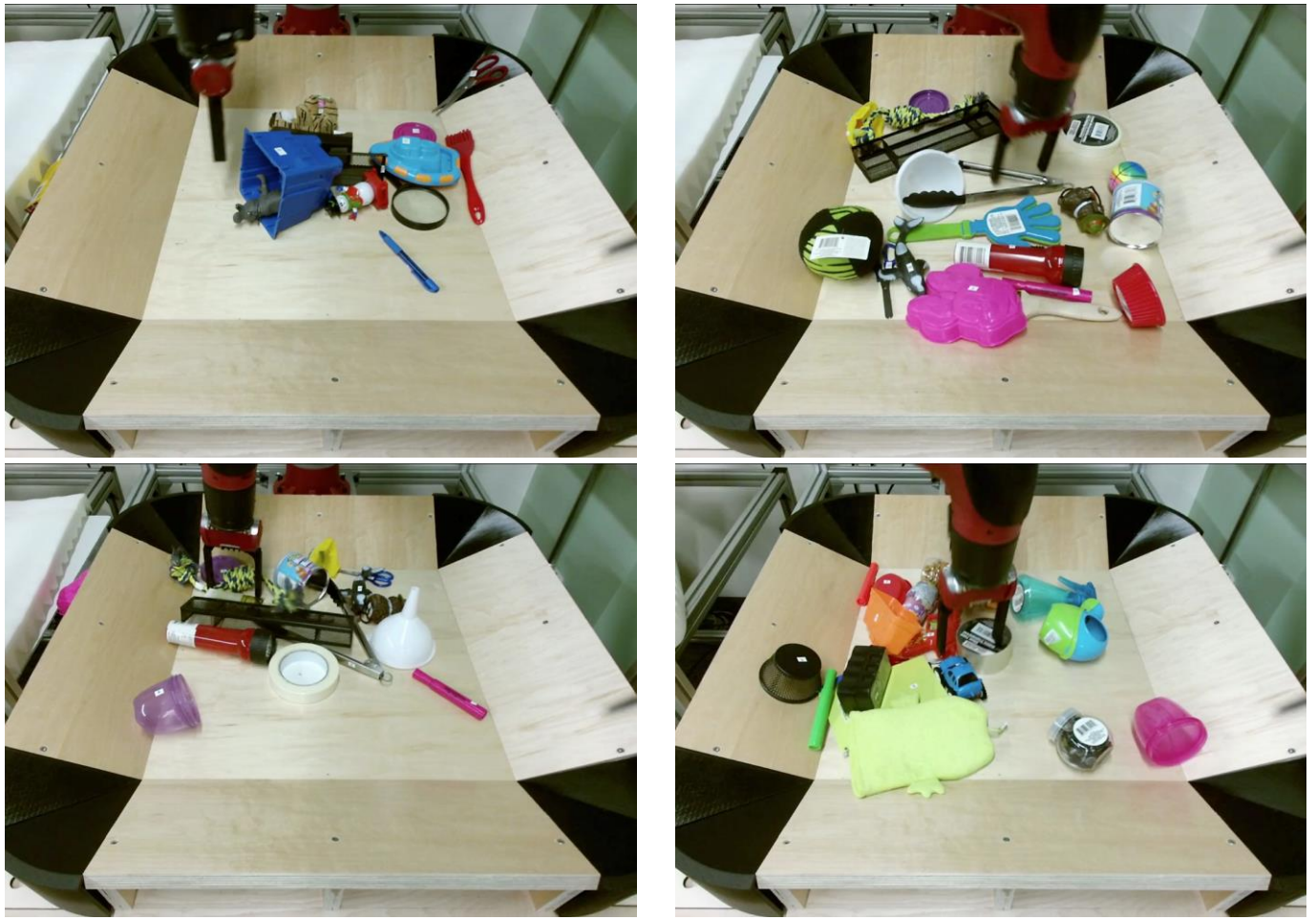




Deep Reinforcement Learning in a Handful of Trials using Probabilistic Dynamics Models

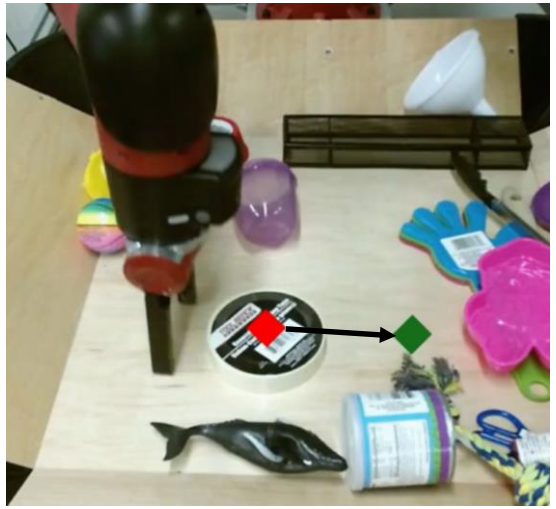
Chua, Calandra, McAllister, L.

Can we do this with pixels?



Ebert, Finn, Lee, Levine. 2017. Self-Supervised Visual Planning with Temporal Skip Connections. CoRL 2017.

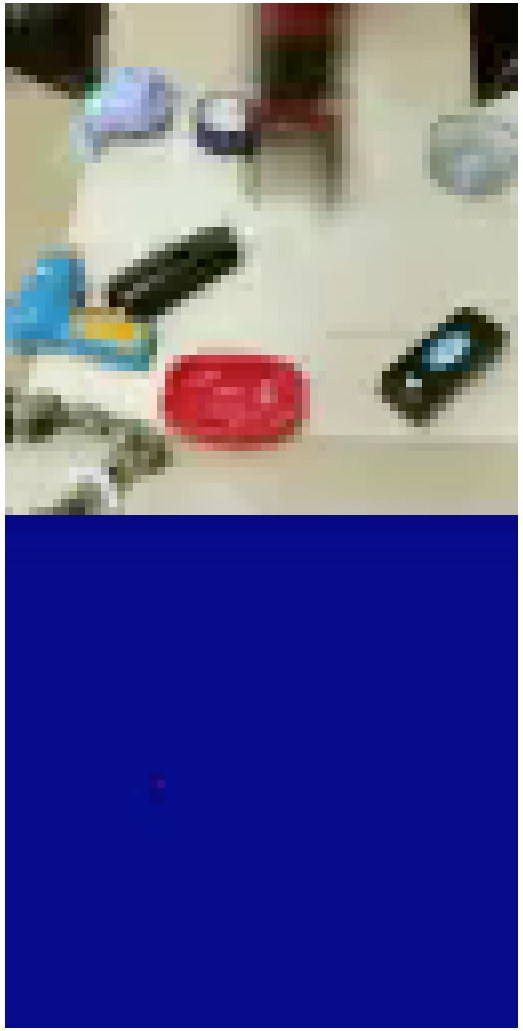
The model on pixels

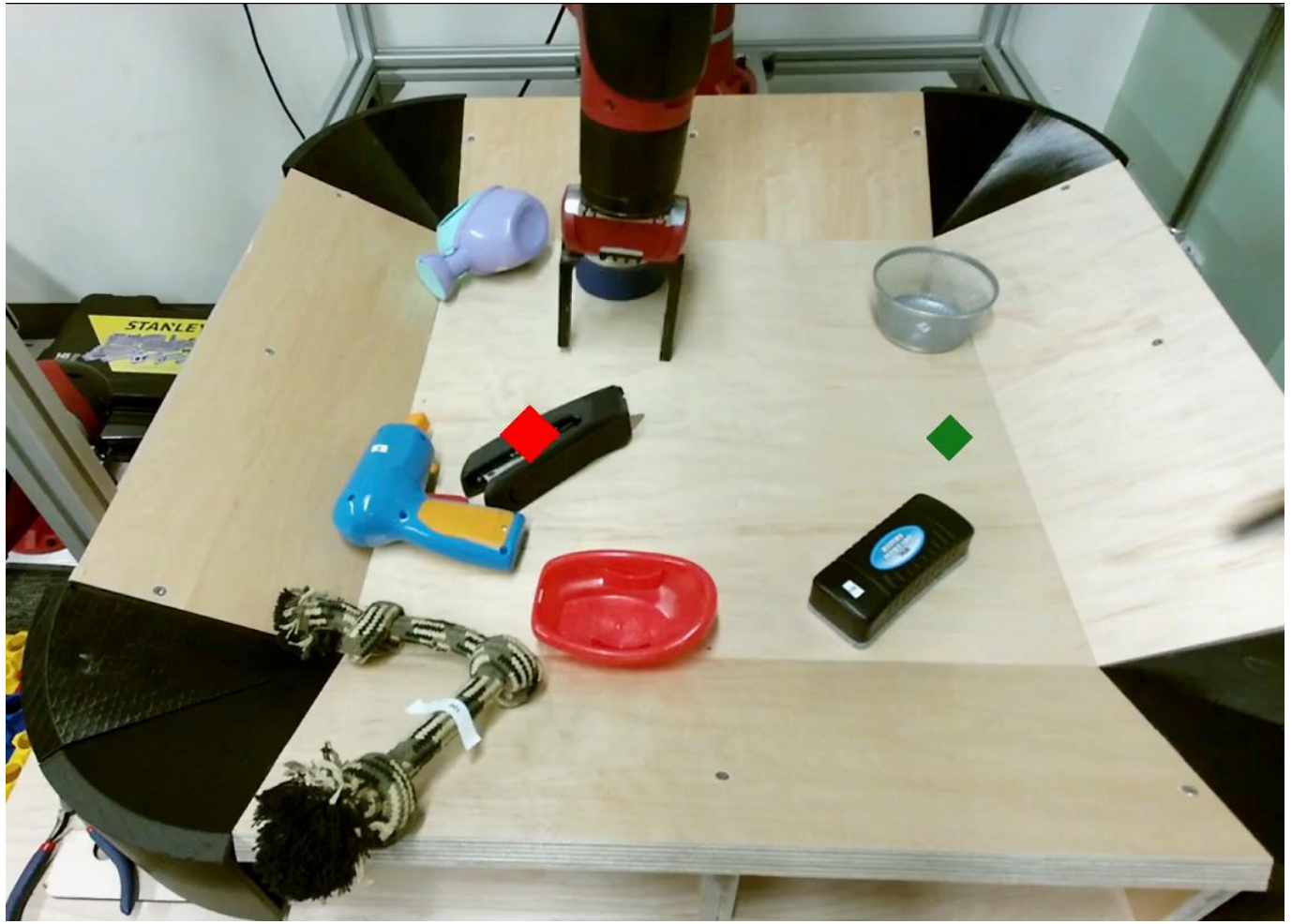


Designated Pixel

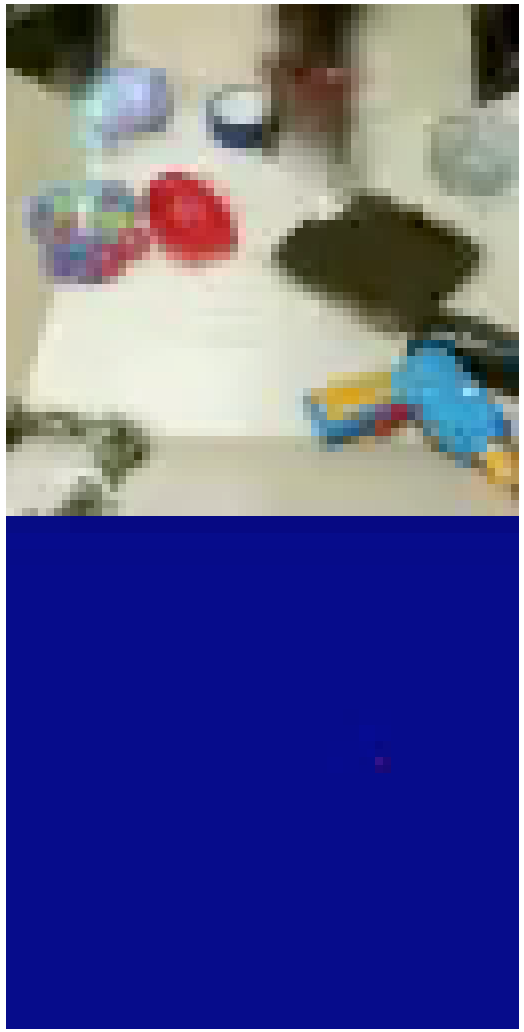
Goal Pixel

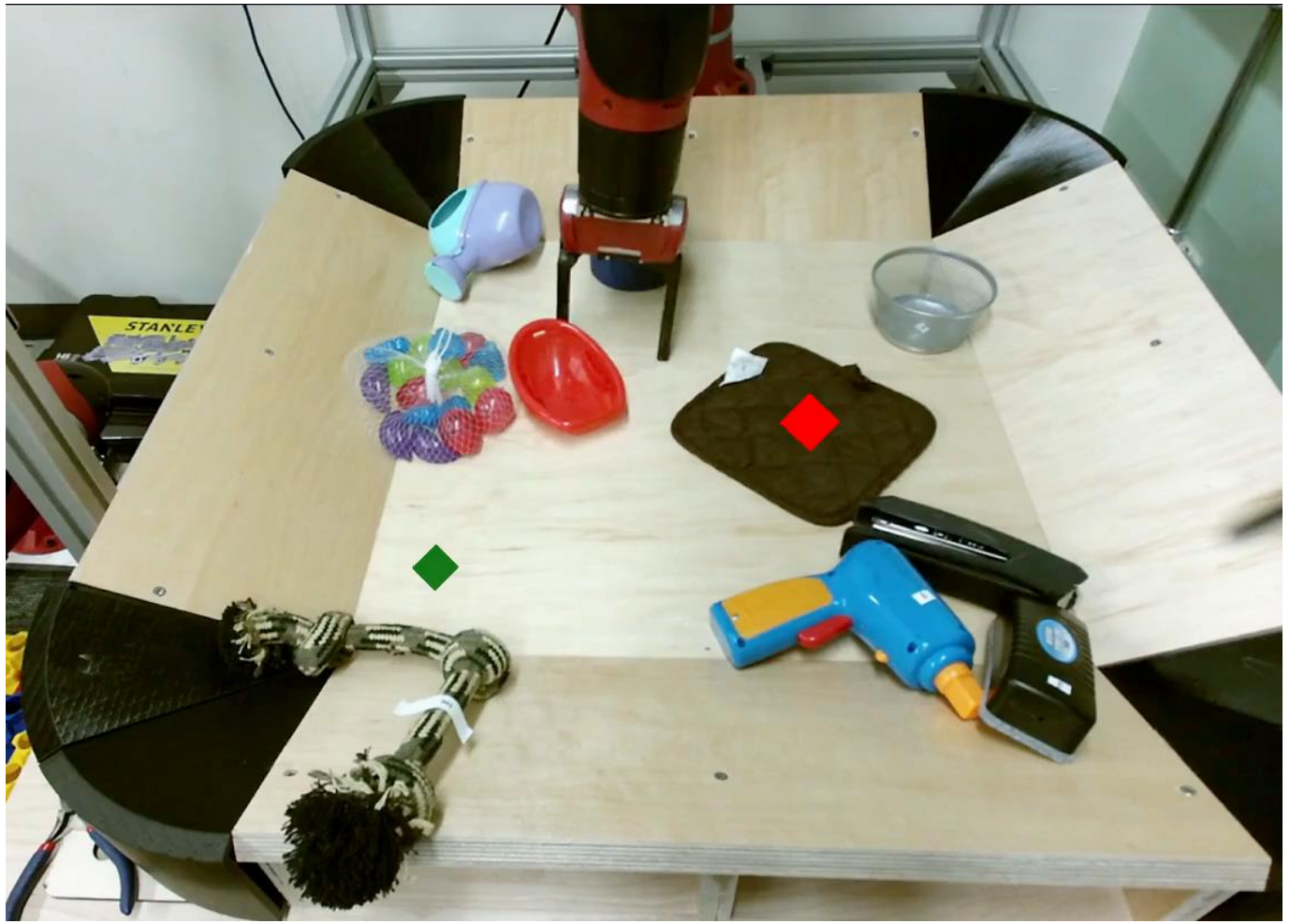
More examples





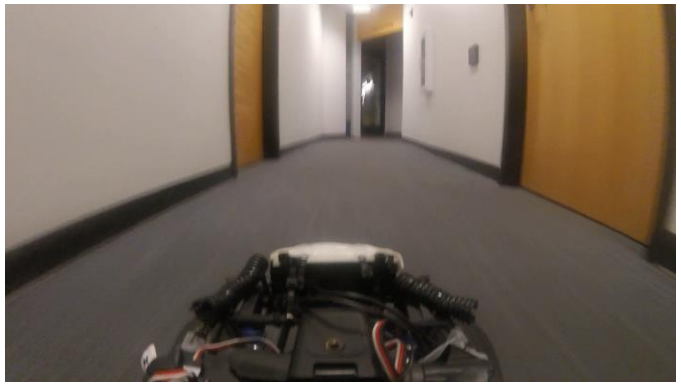
More examples



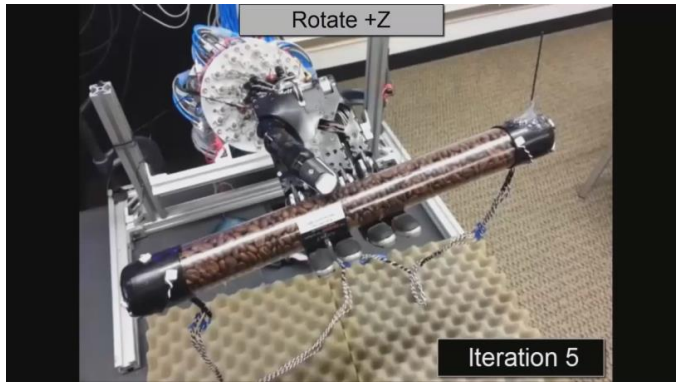


Reinforcement learning in robotics

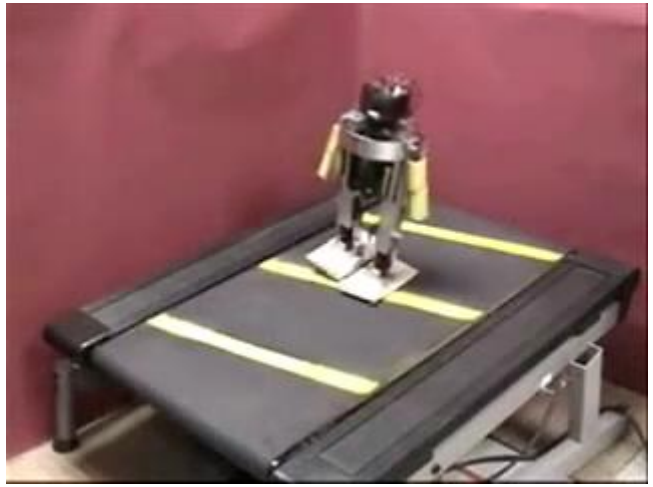
Kormushev et al. **Robot Motor Skill Coordination with EM-Based Reinforcement Learning**



Kahn et al. **Self-Supervised Reinforcement Learning with Generalized Computation Graphs**

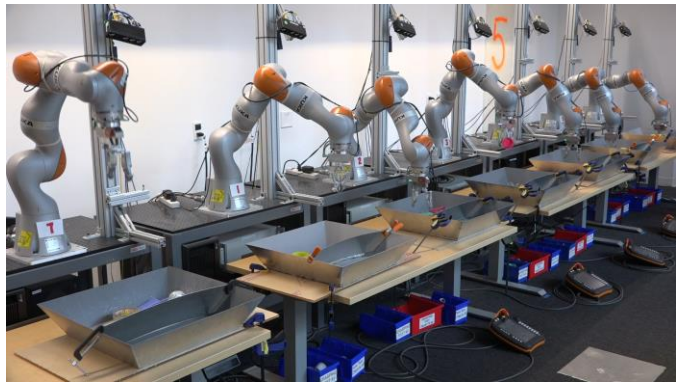


Kumar et al. **Optimal Control with Learned Local Models**



Tedrake et al. **Learning to Walk in 20 Minutes**

Nagabandi et al. **Neural Network Dynamics Models for Control of Underactuated Legged Millirobots**



Kalashnikov, Irpan, Pastor, Ibarz, Herzong, Jang, Quillen, Holly, Kalakrishnan, Vanhoucke, Levine. **QT-Opt: Scalable Deep Reinforcement Learning of Vision-Based Robotic Manipulation Skills**

Questions?

The problem setup

Model-free policy search via policy gradients

What if we know the model?

What if we don't know the model, but are willing to learn?