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A new test of conservation laws and Lorentz invariance in relativistic gravity

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Abstract. General relativity predicts that energy and momentum conservation laws hold and that preferred frames do not exist. The parametrized post-Newtonian formalism phenomenologically quantifies possible deviations from general relativity. The parametrized post-Newtonian parameter α_3 (which identically vanishes in general relativity) plays a dual role in that it is associated both with a violation of the momentum conservation law and with the existence of a preferred frame. By considering the effects of $\alpha_3 \neq 0$ in certain binary pulsar systems, it is shown that $|\alpha_3| < 2.2 \times 10^{-20}$ (90% CL). This limit improves on previous results by several orders of magnitude, and shows that pulsar tests of α_3 rank (together with Hughes–Drever-type tests) among the most precise null experiments of physics.

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1. Introduction

Conservation laws have long been a foundation of physics and careful tests of their validity have led to important discoveries, such as neutrinos. Most theories of gravity, starting with general relativity, are based on an invariant action principle and therefore predict that the energy and momentum of isolated gravitating systems are conserved. However, direct experimental constraints on those conservation laws are far less extensive than for the other fundamental forces of nature. Another cornerstone of physics is the absence of preferred frames in local experiments (local Lorentz invariance). This property may be violated in certain theories of gravity allowing for the existence of long-range vector fields.

The parametrized post-Newtonian formalism (PPN) phenomenologically quantifies possible deviations from general relativity. Such deviations are measured by a set of ten parameters: $\gamma - 1, \beta - 1, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$ [1]. To each of these parameters is associated a class of observable, non-Einsteinian effects. The parameter α_3 plays a special role in that it is associated both with a violation of the momentum conservation law, and with the existence of preferred frames ($\alpha_3 \equiv 0$ in general relativity).

The most striking observable effect induced by a putative $\alpha_3 \neq 0$ is the existence of a non-zero self-acceleration for a rotating body, perpendicular to its spin axis and absolute

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velocity [1–3]. As a consequence, the total acceleration of a body, which is a member of a gravitating system, can be decomposed as

$$\boldsymbol{a} = \boldsymbol{a}_{\mathrm{Newt}} + \boldsymbol{a}_{n\text{-body}} + \boldsymbol{a}_{\alpha_3} \tag{1}$$

where $\boldsymbol{a}_{\mathrm{Newt}}$ is a 2-body Newtonian-like $1/R^2$ force (modified by possible equivalence-principle-violation effects), $\boldsymbol{a}_{n\text{-body}}$ denotes relativistic n-body effects, and where $\boldsymbol{a}_{\alpha_3}$ is the self-acceleration resulting from the violation of momentum conservation associated with $\alpha_3 \neq 0$ [1–3]. For a nearly spherical body, rotating with angular velocity $\boldsymbol{\Omega}$,

$$\boldsymbol{a}_{A\alpha_3} = -\frac{1}{3}\alpha_3 \frac{E_A^{\mathrm{grav}}}{m_A c^2}(\boldsymbol{w} + \boldsymbol{v}) \times \boldsymbol{\Omega} \tag{2}$$

where E_A^{grav} is the gravitational self-energy of body A having mass m_A, moving with velocity $\boldsymbol{w} + \boldsymbol{v}$, with respect to the absolute rest frame. For later convenience, we have decomposed this velocity into the absolute velocity $\boldsymbol{w}$ of the centre of mass of the considered system, and the peculiar velocity $\boldsymbol{v}$ of body A with respect to the centre of mass frame. Instead of the gravitational self-energy E_A, one can introduce the (dimensionless) 'compactness' parameter $c_A = -2\partial \ln m_A / \partial \ln G$, which is approximately given by $c_A = -2E_A^{\mathrm{grav}}/m_A c^2$ [4]. We see from (2) that, given an absolute velocity $\boldsymbol{w}$, $\Omega c_A = 2\pi c_A/P$ defines a figure of merit for the selection of bodies testing α_3. For the Sun $c_A \sim 10^{-6}$, and for a white dwarf $c_A \sim 10^{-4}$. By contrast, neutron stars have compactness parameters of order unity. Considering a range of equations of state for neutron stars, in [4] it was found that c_A takes a median value of $0.21 m_A/M_\odot$. Therefore, rapidly rotating neutron stars are, by a very large factor, the best objects for constraining α_3. In the present paper, we shall show that existing data on certain long-period, quasi-circular binary pulsar systems allow one to constrain α_3 at the 10^{-20} level.

2. Previous limits

As shown by Nordtvedt and Will [1, 2], a non-zero α_3 induces a contribution to the perihelion precession of the planets in the solar system. The two planets with the best measurements of periastron advance are Earth and Mercury. By combining the observations for two planets it is possible to eliminate the terms involving the well known Eddington parameters γ and β, obtaining $|49\alpha_1 - \alpha_2 - 6.3 \times 10^5 \alpha_3 - 2.2\xi| < 0.1$ [1]. Using the limits on other PPN parameters, a limit of $|\alpha_3| < 2 \times 10^{-7}$ was thus obtained.

A tighter limit on α_3 has been obtained by considering the effect of the acceleration (2) on the observed pulse periods of isolated pulsars. The observed pulse period $P = P_0(1 + v_r/c)$ contains contributions from both the intrinsic pulse period P_0 and the Doppler effect resulting from the radial velocity v_r. Consequently any radial acceleration a_r contributes to the observed period derivative $\dot{P}$,

$$\frac{\dot{P}}{P} = \frac{\dot{P}_0}{P} + \frac{a_r}{c}. \tag{3}$$

Self-accelerations are directed perpendicular to both the spin axis and the absolute velocity of the spinning body. If self-accelerations were contributing strongly to the observed period derivatives of pulsars, roughly equal numbers of positive and negative observed period derivatives would be expected, since the spin axes and therefore the self-accelerations are randomly oriented. However the observed distribution (excluding those pulsars in globular clusters) contains only positive period derivatives, allowing a limit to be placed on α_3. This was done initially using normal pulsars [1] to obtain a limit of

$|\alpha_3| < 2 \times 10^{-10}$ and more recently using millisecond pulsars [5] to obtain a limit of $|\alpha_3| < 5 \times 10^{-16}$. The later case incorrectly included binary pulsars in the sample. (As will be shown in the following section the effects of self-accelerations in a binary system are more complicated.) Nevertheless, if one restricts oneself to a sample of isolated millisecond pulsars, a limit quantitatively similar to that above, though slightly less stringent, can be obtained.

3. Polarization of binary orbits

When considering a binary system, there is a perturbing self-acceleration felt by each body. These self-accelerations perturb both the centre of mass motion of the system, and the relative orbital dynamics. We focus on the perturbations of the relative motion which turn out to be a much more sensitive probe of α_3. The perturbation of the relative acceleration $\boldsymbol{a}_{\alpha_3} = \boldsymbol{a}_{A\alpha_3} - \boldsymbol{a}_{B\alpha_3}$, where A labels the pulsar and B its companion, can be written as

$$\boldsymbol{a}_{\alpha_3} = \boldsymbol{a}_{\alpha_3}^{\text{Stark}} + \boldsymbol{a}_{\alpha_3}^{\text{Zeeman}} \tag{4}$$

where

$$\boldsymbol{a}_{\alpha_3}^{\text{Stark}} = \frac{\alpha_3}{6} \boldsymbol{w} \times (c_A \boldsymbol{\Omega}_A - c_B \boldsymbol{\Omega}_B) \,, \tag{5}$$

$$\boldsymbol{a}_{\alpha_3}^{\text{Zeeman}} = \frac{\alpha_3}{6} \boldsymbol{v} \times (x_B c_A \boldsymbol{\Omega}_A + x_A c_B \boldsymbol{\Omega}_B). \tag{6}$$

Here Ω_A and $x_A = m_A/(m_A + m_B)$ are the angular velocity and mass fraction for the pulsar. The names given to the perturbing accelerations (5) and (6) have been chosen by analogy with the well known effects of constant external electric and magnetic fields on the classical dynamics of an atom. These two perturbations have very distinct effects on the relative orbital dynamics.

By a generalization of Larmor's theorem, the 'Zeeman' contribution (6) is easily seen to cause a slow overall precession of the orbit with angular velocity

$$\boldsymbol{\Omega}_{\text{precession}} = -\frac{\alpha_3}{12} (x_B c_A \boldsymbol{\Omega}_A + x_A c_B \boldsymbol{\Omega}_B) \,. \tag{7}$$

The theory of the formation of binary pulsars leads us to expect that, to a first approximation, the spin vectors $\boldsymbol{\Omega}_A$, $\boldsymbol{\Omega}_B$ are parallel to the orbital angular momentum. This means that the effects of the perturbation (7) give rise to non-observable, very small additional contributions to the orbital period and the periastron precession.

By contrast, the 'Stark' contribution (5), which represents a constant perturbing acceleration, leads to a forced eccentricity, polarizing the orbit along a fixed direction. Such a 'Stark' polarization (which is the DC analogue of the Nordtvedt polarization of the lunar orbit [6]) has already been studied in certain binary pulsars when considering the effect of a violation of the strong equivalence principle [7]. The best systems for constraining the polarization effects caused by (5) are the long orbital period, quasi-circular binary millisecond pulsars with white dwarf companions. For a neutron star and white dwarf $c_B \ll c_A$ and $\Omega_B \ll \Omega_A$, so that (5) reduces to

$$\boldsymbol{a}_{\alpha_3}^{\text{Stark}} \simeq \frac{\alpha_3}{6} c_A \boldsymbol{w} \times \boldsymbol{\Omega}_A \,. \tag{8}$$

For this acceleration to be in a fixed direction, $\boldsymbol{w}$ and $\boldsymbol{\Omega}_A$ must remain fixed in space. Any precession of the pulsar spin axes will be very slow, if it occurs at all. The absolute velocity of pulsars in our galactic neighbourhood is taken to be $|\boldsymbol{w}| = 369\,\text{km}\,\text{s}^{-1}$, as determined from observation of the cosmic microwave background [8]. Some recent results [9] question the validity of the cosmic microwave background as an absolute reference frame. As a sample of pulsars are used, only the magnitude of $\boldsymbol{w}$ is important and this is similar for both [8, 9].

It is also required that the centre of mass of the binary system does not move appreciably in the galaxy during the build up of the polarization induced by (8), otherwise $\boldsymbol{w}$ will vary and the polarization force (8) must be decomposed as a sum of monochromatic terms. This is equivalent to a corresponding requirement for the test of the strong equivalence principle [7], i.e.

$$\frac{\omega_{\rm gal}}{\omega_{\rm R}} = \left(\frac{P_b}{1364(\text{days})}\right)^{5/3}\left(\frac{M}{1.7M_\odot}\right)^{-2/3} \ll 1 \tag{9}$$

where $\omega_{\rm gal}$ is the angular rotation rate of the galaxy near the sun, $\omega_{\rm R}$ is the relativistic periastron advance, P_b is the orbital period and $M = m_A + m_B$.

The equation of motion for a binary system with the perturbation (5) is very similar to the equation of motion for the strong equivalence principle test [7]

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} + \frac{GM\boldsymbol{r}}{r^3} = \boldsymbol{R} + \boldsymbol{a}_{\alpha_3}^{\rm Stark} \tag{10}$$

where $\boldsymbol{R}$ contains the relativistic forces responsible for periastron precession. We can use the results of [7]. Considering systems for which $e \ll 1$, while letting $\boldsymbol{w}\times\boldsymbol{\Omega}_A = w\Omega_A \sin\beta \boldsymbol{e}_u$ (where $\boldsymbol{e}_u$ is a unit vector) and $\Omega_A = 2\pi/P$, gives the solution

$$\boldsymbol{e}(t) = \boldsymbol{e}_{\rm R}(t) + \boldsymbol{e}_{\alpha_3} = \boldsymbol{e}_{\rm R}(t) + \alpha_3 \frac{c_A w}{24\pi}\frac{P_b^2}{P}\frac{c^2}{GM}\sin\beta\boldsymbol{e}_u\,. \tag{11}$$

Here $\boldsymbol{e}(t)$ is the total, observable eccentricity vector, $\boldsymbol{e}_{\rm R}(t)$ is the intrinsic eccentricity vector, which rotates with angular frequency $\omega_{\rm R}$ due to relativistic periastron precession and $\boldsymbol{e}_{\alpha_3}$ is the forced eccentricity vector caused by $\alpha_3 \neq 0$. It is assumed that since considerable mass transfer has taken place, the orbital and spin angular momenta are aligned, which results in the forcing term (5) due to α_3 being parallel to the orbital plane. Given an observed eccentricity $e_{\rm obs} = |\boldsymbol{e}(t)|$ measured at some time t, equation (11) may be used to place a limit on α_3. From equation (11) it can be seen that

$$P_b^2/eP \tag{12}$$

defines a figure of merit for selecting which binary pulsar systems will provide the best limit. This figure of merit differs from the one (P_b^2/e) selecting the best tests of the strong equivalence principle.

4. Confidence level of the limit

Unfortunately, in (11) there are two parameters which are not constrained by the observations, β and $\boldsymbol{e}_{\rm R}(t)$. Firstly, the angle β may be very small, causing α_3 effects to contribute weakly to the observed eccentricity. Secondly, there is a finite probability that $\boldsymbol{e}_{\rm R}(t)$ might have cancelled most of the term due to α_3, leaving only a small observed eccentricity. In order to impose a limit on α_3 at a particular confidence level, a quantitative assessment of the probability of these independent effects is needed. Given $\boldsymbol{w}$, the spin vector $\boldsymbol{\Omega}_A$ can point in an arbitrary direction on the unit sphere. Therefore the polar angle β is distributed with the probability law $\frac{1}{2}\sin\beta\,\mathrm{d}\beta$. In other words, the quantity $c_\beta = \cos\beta$ is distributed uniformly over $[-1, 1]$.

The problem of $\boldsymbol{e}_{\rm R}(t)$ potentially cancelling a second term was first considered by Damour and Schäfer [7], who decomposed the problem into three cases. When $\boldsymbol{e}_{\rm R}(t) \gg \boldsymbol{e}_{\alpha_3}$

or $\boldsymbol{e}_{\mathrm{R}}(t) \ll \boldsymbol{e}_{\alpha_3}$ it is sufficient to use the observed eccentricity e_{obs} for obtaining a limit. When $\boldsymbol{e}_{\mathrm{R}}(t) \simeq \boldsymbol{e}_{\alpha_3}$ and the angle between $\boldsymbol{e}_{\mathrm{R}}(t)$ and the opposite of $\boldsymbol{e}_{\alpha_3}$ is θ, then $e_{\mathrm{obs}} = e_{\alpha_3} 2|\sin(\theta/2)|$ was used. A slightly more precise approach was taken by Wex [10] who defined a function

$$S(\theta) = \begin{cases} |\sin\theta| & \text{if } |\theta| < \pi/2 \\ 1 & \text{if } |\theta| \geqslant \pi/2. \end{cases} \tag{13}$$

Here the angle θ between $\boldsymbol{e}_{\mathrm{R}}(t)$ and $-\boldsymbol{e}_{\alpha_3}$ is taken to vary between $-\pi$ and π. Given the magnitude of the forced eccentricity $e_{\alpha_3} = |\boldsymbol{e}_{\alpha_3}|$ and the value of the angle θ, it is easily seen that when the magnitude of the intrinsic eccentricity $|\boldsymbol{e}_{\mathrm{R}}|$ is allowed to vary one can write the inequality

$$e_{\mathrm{obs}} \geqslant e_{\alpha_3}\ S(\theta)\,. \tag{14}$$

(Indeed, when $|\theta| < \pi/2$ the worst cancellation arises when the resulting eccentricity vector $\boldsymbol{e} = \boldsymbol{e}_{\mathrm{R}} + \boldsymbol{e}_{\alpha_3}$ is perpendicular to $\boldsymbol{e}_{\mathrm{R}}$, while when $|\theta| > \pi/2$ the worst case is $|\boldsymbol{e}_{\mathrm{R}}| = 0$.) Using the inequality (14) and rearranging (11) gives the upper bound

$$|\alpha_3| \leqslant \frac{24\pi}{c_A w} \frac{eP}{P_b^2} \frac{GM}{c^2} \frac{1}{S(\theta)\sin\beta} \equiv \frac{K_i}{S(\theta)\sin\beta}. \tag{15}$$

Given the quantity K_i determined from observed parameters, the probability that some number α is *smaller* than $|\alpha_3|$ is *smaller* than

$$F(K_i/\alpha) = \mathrm{Prob}[S(\theta)\sin\beta < K_i/\alpha]\,. \tag{16}$$

For sufficiently old systems, $\boldsymbol{e}_{\mathrm{R}}(t)$ has had the time to make many turns and we can consider that the orbital phase θ is randomly distributed over the interval $[-\pi, \pi]$. (The pulsars included in our sample below have old white dwarf companions, confirming that this oldness condition is satisfied.) Hence, since the distributions are symmetric about $\theta = 0$ and $c_\beta = 0$,

$$F\left(\frac{K_i}{\alpha}\right) = \mathrm{Prob}\left[S(\theta)\sqrt{1-c_\beta^2} < \frac{K_i}{\alpha}\right]_{|c_\beta|<1}^{|\theta|<\pi} = 4\,\mathrm{Prob}\left[S(\theta)\sqrt{1-c_\beta^2} < \frac{K_i}{\alpha}\right]_{0<c_\beta<1}^{0<\theta<\pi} \tag{17}$$

$$F\left(\frac{K_i}{\alpha}\right) = 1 - \sqrt{1-\left(\frac{K_i}{\alpha}\right)^2} + \frac{1}{\pi}\int_0^{\sqrt{1-(K_i/\alpha)^2}} \mathrm{d}c_\beta \sin^{-1}\left[\frac{K_i/\alpha}{\sqrt{1-c_\beta^2}}\right]. \tag{18}$$

Probabilities evaluated from this function using the Mathematica package to numerically integrate the second term are shown in table 1 for a range of values of K_i/α.

Since the observational results for pulsars are independent, the combined probability, given the existence of several pulsars with small values for K_i, satisfies the inequality

$$1 - (\mathrm{CL}/100) \equiv \mathrm{Prob}[|\alpha_3| > \alpha] < \prod_i F(K_i/\alpha)\,. \tag{19}$$

Here, we have introduced the confidence level (CL) with which one can reject the hypothesis that $|\alpha_3|$ is larger than some given α.

When the product on the right-hand side is equal to 0.1, $|\alpha_3| < \alpha$ at *better* than the 90% confidence level. Note that when $K_i \geqslant \alpha$, $F(K_i/\alpha) = 1$, so that including such a pulsar does not improve the limit.

Table 1. The probability that $S(\theta)\sin\beta < K_i/\alpha$.

K_i/α	Probability	K_i/α	Probability
0.00	0.000	0.85	0.604
0.10	0.045	0.90	0.681
0.20	0.092	0.93	0.736
0.30	0.143	0.96	0.804
0.40	0.201	0.97	0.831
0.50	0.267	0.98	0.864
0.60	0.343	0.99	0.905
0.70	0.432	1.00	1.000
0.80	0.540		

5. The limit

In the most recent compilation of fast pulsars in binary systems [11], there are many systems which satisfy the constraints discussed in the previous two sections. Using the result of [4] for the median value of c_A quoted above, one finds $K_i = 1.93\times 10^{-13}(1+m_B/m_A)eP/P_b^2$, where the pulsar period P is expressed in milliseconds and the orbital period P_b in days. After evaluating K_i for each of those binary pulsars, the objects with the smallest K_i were chosen and are listed in table 2. The probability that $S(\theta)\sin\beta < K_i/\alpha$ was evaluated using (18) for several values of α which are shown in table 2. The combined probabilities and confidence levels from using several pulsars were then determined using (19) (see table 2).

Table 2. $F(K_i/\alpha)$ for the test systems, K_i and α are in units of 10^{-20}.

		α				
Pulsar	K_i	3.7	2.9	2.4	2.2	1.95
B1855+09	17.6	1.00	1.00	1.00	1.00	1.00
J2317+1439	12.6	1.00	1.00	1.00	1.00	1.00
J1455−3330	5.45	1.00	1.00	1.00	1.00	1.00
B1953+29	3.24	0.64	1.00	1.00	1.00	1.00
J1643−1224	2.30	0.36	0.54	0.80	1.00	1.00
J1640+2224	1.91	0.28	0.40	0.54	0.62	0.86
J2229+2643	1.83	0.27	0.37	0.50	0.58	0.76
J2019+2425	1.79	0.26	0.36	0.48	0.55	0.71
J1713+0747	1.76	0.25	0.35	0.47	0.54	0.68
$\prod_i F(K_i/\alpha)$		0.001	0.01	0.05	0.10	0.32
CL $>$		99.9%	99%	95%	90%	68%

Using, for instance, the 90% confidence level gives a limit $|\alpha_3| < 2.2\times 10^{-20}$. This is more than a factor of 10^4 better than the previous limit [5], and by far the tightest limit on any of the PPN parameters.

The only other ultra-high-precision null experiments giving limits of order 10^{-20} on a dimensionless theoretical parameter we are aware of are the recent Hughes–Drever-type tests [12–14]. Figure 14.2 of [1] shows the limits on the parameter $\delta = c_0^2/c_e^2 - 1$, where c_0 is the limiting speed of massive particles, and c_e the speed of light. It is remarkable that tests involving binary pulsars can rank, together with modern laser-cooled trapped atom experiments, among the most precise null experiments of physics.

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References

[1] Will C M 1993 *Theory and Experiment in Gravitational Physics* (Cambridge: Cambridge University Press)
[2] Nordtvedt K and Will C M 1972 *Astrophys. J.* **177** 775
[3] Nordtvedt K 1973 *Phys. Rev.* D **7** 2347
[4] Damour T and Esposito-Farèse G 1992 *Class. Quantum Grav.* **9** 2093
[5] Bell J F 1996 *Astrophys. J.* **462** 287
[6] Nordtvedt K 1968 *Phys. Rev.* **170** 1186
[7] Damour T and Schäfer G 1991 *Phys. Rev. Lett.* **66** 2549
[8] Fixsen D J *et al* 1994 *Astrophys. J.* **420** 445
[9] Lauer T R and Postman M 1994 *Astrophys. J.* **425** 418
[10] Wex N 1996 *Astron. Astrophys.* in press
[11] Camilo F 1996 *High Sensitivity Radio Astronomy* ed N Jackson (Cambridge: Cambridge University Press) in press
[12] Prestage J D *et al* 1985 *Phys. Rev. Lett.* **54** 2387
[13] Lamoreaux S K *et al* 1986 *Phys. Rev. Lett.* **57** 3125
[14] Chupp T E *et al* 1989 *Phys. Rev. Lett.* **63** 1541